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# Modelling the Depth of Jet Penetration in Abrasive Waterjet Contouring of Alumina Ceramics

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**Keywords:** Abrasive waterjet cutting; Depth of jet penetration; Contouring, Cutting performance model.

**Abstract.** Predictive mathematical models for the depth of jet penetration are presented for both straight-slit cutting and contouring by an abrasive waterjet (AWJ). The plausibility and predictive capability of the models are assessed and verified by an experimental investigation when cutting an 87% alumina ceramic. It shows that the predictions of the models are in good agreement with the experimental data.

## Introduction

Abrasive waterjet (AWJ) cutting is being increasingly accepted by industry as a most promising machining technology, because of its various distinct advantages over other cutting technologies. Over the last decades, considerable research and development effort has been made to understand this machining process and the associated sciences [1-3] and the various technological performance measures, such as the depth of jet penetration, with respect to the many process parameters [4-6]. These studies have resulted in some useful guides and solution strategies for the effective and efficient use of this technology in machining various materials. Furthermore, investigations have been made to develop new cutting techniques, such as forwards angling the jet, controlled nozzle oscillation and multipass cutting, to enhance the AWJ cutting performance and its application domain [1,4,7-9]. In order to more effectively control and optimize the AWJ machining process, it is essential that predictive mathematical models for the various cutting performance measures be developed and considerable research effort has been made towards this direction. Predictive models for some major cutting performance measures have been developed using empirical (i.e. regression analysis) and fundamental (or analytical) approaches. The most promising ones are those using the erosion theories, fractural mechanics and an energy conservation approach [1].

Despite these efforts, there is a general lack of predictive models for the various cutting performance measures. This lack is mainly because of the complex nature of the AWJ cutting process and the fact that many aspects of this process are still not fully understood, such as the particle interference and fragmentation for which there are no available models. As a result, most mathematical modeling approaches often result in predictive models with unknown factors, so that these models are of little practical use. Moreover, little work has been reported on AWJ contouring or profile cutting, although this is a more common cutting process than straight-slit cutting. As a special feature of jet cutting, striations or wavinesses are formed at the lower region of the machined surface. These striations have a backward drag angle whose magnitude increases as the jet cuts into the workpiece and the jet traverse speed increases. In profile cutting, this jet drag nature coupled with the varying jet traverse direction result in not only removing more material on the outer kerf wall, forming different kerf tapers on the two kerf walls [1], but also the reduction of jet energy in the direction of jet penetration, reducing the depth of jet penetration.

This paper presents a depth of jet penetration (or depth of cut) model for AWJ cutting of alumina ceramics. A dimensional analysis technique is used to formulate the jet penetration model for both straight and contour cutting. The plausibility and predictive capability of the model are assessed by analyzing the model predictions and by comparing with the experimental results.

**Depth of Jet Penetration Model**

AWJ cutting process involves a large number of parameters that affect the cutting performance, such as the depth of jet penetration. In addition, a number of phenomena, such as particle interference and fragmentation, exist in abrasive jet cutting. At this stage of development, there is no sufficient knowledge of these phenomena [1,6]. Therefore, to consider all these variables and phenomena is either impossible or results in many unknown parameters in the final equation, making the model unrealistic for practical use. For the same reasons, a number of mathematical models in the literature that were developed using fundamental approaches are very complicated and require a large number of parameters to be determined before the models are of any practical use. In the present study, a dimensional analysis technique will be used to develop the depth of jet penetration model considering the major process parameters, while experimental data will be used to allow for the unpredictable phenomena. The model for straight-slit cutting will be developed first.

The material removal rate, $V_t$, can be expressed as a function of the cross-sectional area of cutting front (depth of cut, $h$, multiplying kerf width, $w$) and jet traverse (feed) speed, $u$; namely,

$$V_t = hwu \tag{1}$$

By ignoring the variation of kerf width along the depth, and assuming that the kerf width is equal to the effective jet diameter (within which the particles have energy above the threshold value for removing target material), which is in turn equal to the nozzle diameter. Therefore, Eq. (1) becomes

$$V_t = hd_j u \tag{2}$$

Consequently, the depth of jet penetration is given by

$$h = \frac{V_t}{d_j u} \tag{3}$$

The material removed in AWJ cutting may be considered as an accumulation of material removed by numerous individual particles. If the abrasive mass flow rate is $m_a$ and the average mass of a particle is $m_p$, the material removal rate may be given by

$$V_t = C_0 \frac{m_a}{m_p} V_s \tag{4}$$

where $V_s$ is the material removed by a single particle, and $C_0$ is an efficiency factor to allow for the fact that not all particles are involved in the erosion process and some particles do not have sufficient energy to cut the material. If assuming that the shape of particles is spherical, the mass of a particle is given by

$$m_p = \frac{\pi}{6} d_p{}^3 \rho_p \tag{5}$$

where $d_p$ is particle diameter and $\rho_p$ is particle density.

It is now essential to develop the volume of material removed by individual particles. It has been established that the erosion process of brittle material, such as ceramics, is controlled by the formation and propagation of cracks [10,11]. Further, the material removed by a single particle can be estimated in terms of the target material properties (fracture toughness, hardness, flow strength etc.) and particle properties represented by the velocity, density, shape and size of a particle [12]. However, it is not realistic to include all material properties in modeling the AWJ cutting performance. In the present work, the material removed by a single particle is considered as a function of the particle mass $m_p$, particle velocity $v_p$, local particle attack angle $\alpha$, and the flow strength of the target material $\sigma$; i.e.

$$V_s = \phi(m_p, v_p, \alpha, \sigma) \tag{6}$$

A dimensional analysis technique [13] is employed to establish the relationship of material removal by a particle and the other variables in Eq. (6). With this technique, all variables appearing in a problem can be assembled into a smaller number of independent dimensionless products (or groups $\pi_i$) using the constraint that all products formed must have the same dimension. The relations connecting the individual variables can be determined by algebraic expressions relating each dimensionless product [13,14]. All the variables depend on three fundamental dimensions, i.e. length L, mass M and time T. Since $\alpha$ is already a dimensionless variable and can form a product on its own, one more independent dimensionless product can be formed and the two products are given by

$$\pi_1 = \frac{\sigma V_s}{m_p v_p^2}, \quad \pi_2 = \alpha \tag{7, 8}$$

Applying the functional relationship between these two products and the power law formulation, the dimensional equation is given by

$$\frac{\sigma V_s}{m_p v_p^2} = C_1 \alpha^a \tag{9}$$

or $$V_s = C_1 \frac{m_p v_p^2}{\sigma} \alpha^a \tag{10}$$

If assuming the particles are evenly distributed across the jet and their velocity is the same as that of their surrounding water, the particle velocity, $v_p$, can be obtained using the momentum transfer equation, i.e.

$$v_p = k_1 \left( \frac{m_w}{m_w + m_p} \right) v_j \tag{11}$$

where $v_j$ is the waterjet velocity before it is mixed with abrasives, $m_w$ is the water mass flow rate, and $k_1$ is a factor to consider the momentum transfer efficiency. The particle velocity, $v_p$, is assumed to be the velocity of the water-particle slurry jet. To work out the mass ratio term in Eq. (11) will make the model complicated. Therefore, the ratio term is approximated as a constant, $k_2$, to simplify the derivation. For the process conditions used in the experiments of this study, this approximation only results in less than 2.5% error for the mass ratio and even smaller error for the final depth of cut. Thus, Eq. (11) can be re-written as

$$v_p = k_1 k_2 v_j \tag{12}$$

If assuming that the energy loss in the system is negligible, the velocity of waterjet, $v_j$, can be found by using the Bernoulli's equation, viz,

$$v_j = \sqrt{2P / \rho_w} \tag{13}$$

where $P$ is water pressure and $\rho_w$ is water density. Substituting Eqs. (12) and (13) into Eq. (10) gives

$$V_s = 2C_1 \frac{(k_1 k_2)^2 P m_p}{\rho_w \sigma} \alpha^a \quad (14)$$

The magnitude of local particle attack angle, α, varies as the depth of cut increases. However, the exact nature of this variation is not clear and there is no model to describe this angle. Thus, the average particle attack angle is used in this study and its value is determined by a dimensional analysis technique. According to reported the investigations [15], jet attack angle depends on the profile of the cutting front which in turn depends on jet traverse speed $u$, abrasive mass flow rate $m_a$, material flow strength $\sigma$, and nozzle diameter $d_j$. For given abrasive particles, the abrasive mass flow rate, $m_a$, may be represented by the number of particles, $n$ for given particle density and size. Further, water pressure, $P$, is related to the water and particle velocity in a jet and affects material removal and particle flow direction. Therefore, water pressure should be included in the analysis. Thus, the average particle attack angle is given by

$$\alpha = f(P, u, n, d_j, \sigma) \quad (15)$$

Using dimensional analysis, three independent dimensionless groups can be formed; namely,

$$\pi_1 = \frac{\sigma}{P},\ \pi_2 = \frac{u}{n d_j},\ \pi_3 = \alpha \quad (16, 17, 18)$$

The three groups are related by the function

$$\pi_3 = \varphi(\pi_1, \pi_2) \quad (19)$$

Applying the power law formulation gives

$$\alpha = C_2 \left(\frac{\sigma}{P}\right)^b \left(\frac{u}{n d_j}\right)^c \quad (20)$$

Consequently, substituting Eqs. (5), (14) and (20) into Eq. (4), and replacing $n$ by $m_a/m_p$, give the general form of material removal rate as

$$V_t = C \frac{m_a P}{\rho_w \sigma} \left(\frac{\sigma}{P}\right)^{a_1} \left(\frac{\pi d_p^3 \rho_p u}{6 m_a d_j}\right)^{a_2} \quad (21)$$

where $C$, $a_1$ and $a_2$ are generalized constants. From Eq. (3), the depth of cut equation for straight-slit cutting is

$$h = C \frac{m_a P}{\rho_w \sigma u d_j} \left(\frac{\sigma}{P}\right)^{a_1} \left(\frac{\pi d_p^3 \rho_p u}{6 m_a d_j}\right)^{a_2} \quad (22)$$

In AWJ contouring, it has been reported that the depth of cut slightly increases as the curvature of the cut profile increases [16]. To consider this effect, a proportionality factor, $k$, is introduced which can be determined from experimental data. Thus the equation for contouring is

$$h_r = k h \quad (23)$$

Eqs. (22) and (23) are the general form of the depth of jet penetration models. For a given work material, the constants in the models can be determined from cutting tests.

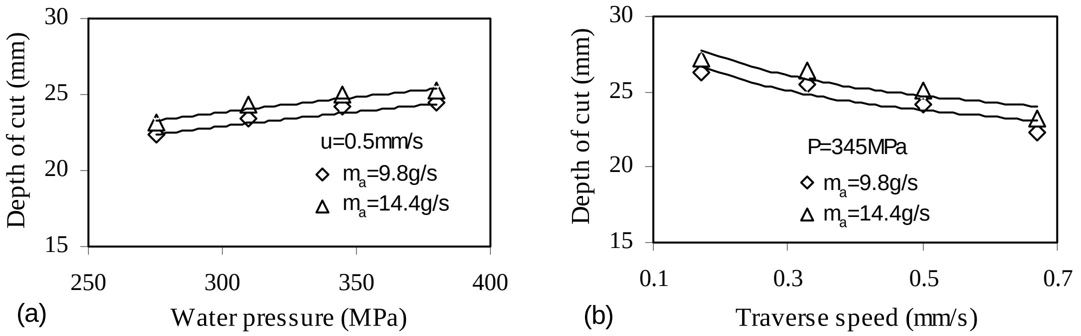


Fig. 1. Predicted and experimental depth of cut for straight cutting (standoff distance $d$=2mm).

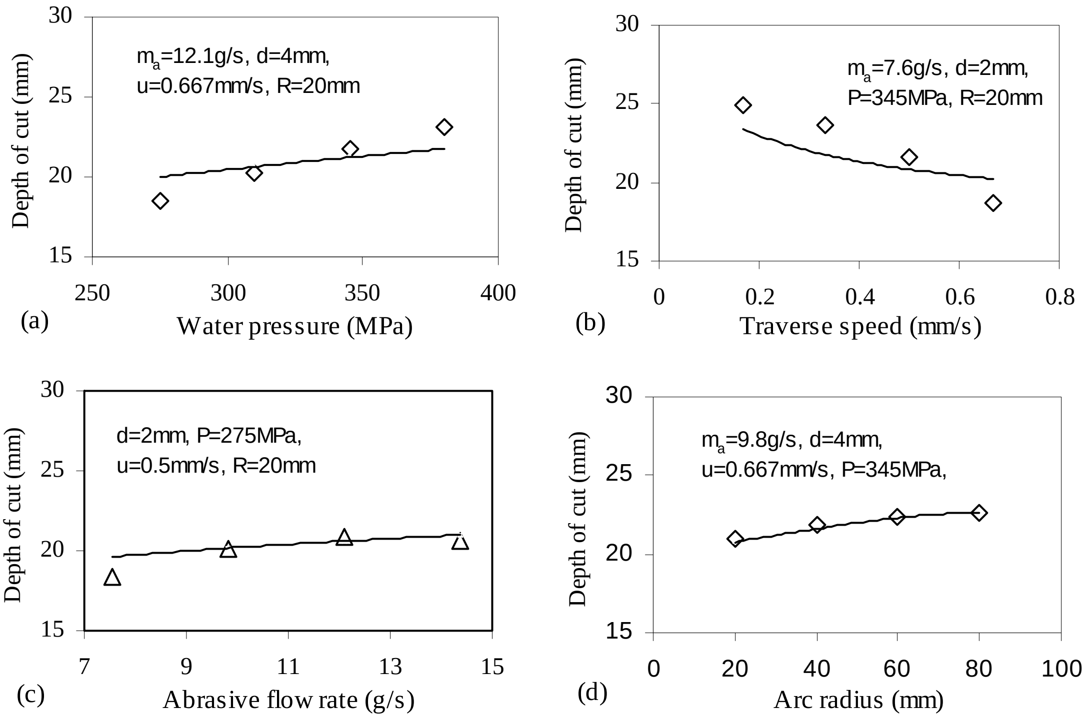


Fig. 2. Predicted and experimental depth of cut for contouring.

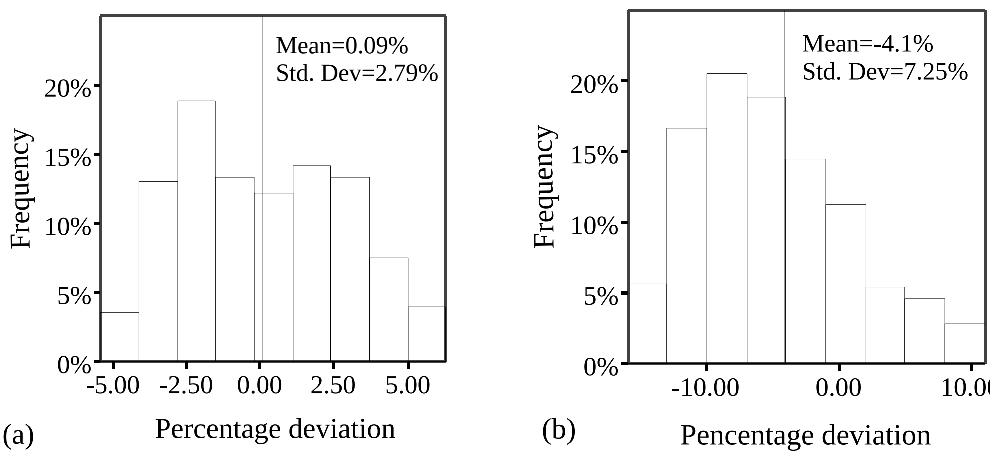


Fig. 3. Percentage deviations of the model predictions from the experimental data: (a) Straight-slit cutting, (b) Contouring.

## Experimental Work

The experiment was conducted on a Flow International waterjet cutter that was equipped with a model 20X dual intensifier high output pump (up to 380 MPa) and a five axis robot positioning system. The specimens were 87% alumina ceramic slabs of 25.4 mm thickness. The effect of contour curvature or arc radius was studied under different water pressures, nozzle traverse speeds, abrasive mass flow rates and standoff distances. Specifically, five levels of arc radius ($R$=0, 20, 40, 60 and 80mm), four levels of nozzle traverse speed ($u$=0.167, 0.333, 0.5 and 0.667mm/s), four levels of water pressure ($P$=275, 310, 345 and 380MPa), four levels of abrasive mass flow rate ($m_a$=7.6, 9.8, 12.1 and 14.4g/s) and four levels of standoff distance ($d$=2, 3, 4 and 5mm) were considered in the experimental design. The 'zero' radius represents a special case of contouring, i.e. sharp corner cutting, while its straight segments provided the opportunity to compare straight cutting with contouring. A single level of jet impact angle ($90^o$) was used. The abrasive was 80 mesh almandine garnet sand. Other parameters that were kept constant were: orifice diameter=0.33 mm, nozzle diameter=1.02 mm, and nozzle length=76.2 mm. Using a statistical experimental design (the orthogonal arrays), a total of 104 cuts of 30mm long were performed. A Sigma Scope 500 profile projector was used to measure the depth of cut for each test.

## Model Assessment

Based on the experimental data, a regression analysis has been carried out to determine the constants in Eqs. (22) and (23) at a 95% confidence interval. The final depth of cut equation for AWJ cutting of straight slits on an 87% alumina ceramic is given by

$$h = 5362.77 \frac{m_a\, P}{\rho_w\, \sigma\, u\, d_j} \left(\frac{\sigma}{P}\right)^{0.735} \left(\frac{\pi\, d_p{}^3\, \rho_p\, u}{6\, m_a\, d_j}\right)^{0.895} \qquad (24)$$

or

$$h = 3005.33 \frac{m_a{}^{0.105} P^{0.265}\, \rho_p{}^{0.895} d_p{}^{2.685}}{u^{0.105}\, \rho_w\, \sigma^{0.265}\, d_j{}^{1.895}} \qquad \mathbf{(25)}$$

and the corresponding equation for AWJ contouring is

$$h_r = \left(1 - \frac{1}{9.15 + 0.000164 R^{2.875}}\right) h \qquad (\text{for } R \neq 0) \qquad (26)$$

The above equations are valid for the conditions specified in the Experimental Work. In these equations, $h$, $d_j$ and $d_p$ are in mm, $m_a$ is in g/s, $u$ is in mm/s, $\rho_w$ and $\rho_p$ are in kg/m$^3$, and $P$ and $\sigma$ are in MPa.

Figs. 1 and 2 show the predicted trends (in lines) and experimental data (in symbols) for straight and contour cutting, respectively. It can be noticed from the predicted trend lines that the depth of cut increases with a decrease in jet traverse speed and an increase in water pressure, abrasive mass flow rate and contour curvature. These trends are consistent with the findings of earlier investigations [1,16]. An increase in water pressure increases the particle energy so that the overall material removal rate is increased. If the kerf width remains approximately unchanged with water pressure, this will increase the depth of cut from Eq. (1). By contrast, an increase in jet traverse speed reduces the number of particles impacting on a given length of kerf, reducing the depth of cut. Increasing abrasive flow rate increases the number of particles in the jet which in turn increases the material removal rate and depth of cut according to Eq. (4). As discussed earlier in this paper, the varying jet traverse direction reduces jet energy in the direction of jet penetration and hence reduces the depth of cut. Fig. 2(d) clearly shows this trend. It is encouraging that the predicted trends are in good agreement with the experimental data.

Quantitative comparisons were made based on the percentage deviation of the predicted results from experimental data, as shown in Fig. 3. It is again shown that the model predictions are in good agreement with the experiments. Thus, the developed models can be used for adequately predicting the depth of cut for straight and contour cutting of alumina ceramics by an AWJ.

## Conclusions

Predictive models for the depth of jet penetration in AWJ machining have been presented for both straight-slit cutting and contouring. The predictive capability of the models has been assessed both qualitatively and quantitatively. It has been shown that model predictions are in good agreement with the corresponding experimental results. The successful development of the models forms a step towards the optimum and effective use of the AWJ machining technology.


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