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GARETH L. FORBES and ROBERT B. RANDALL

ABSTRACT

Condition monitoring of blades within gas turbines has been and will continue to be of importance in all areas of their use. Non-intrusive measurement of blade condition is the ambition of most techniques, with many methods proposed, investigated and employed for such measurement, with the current dominant method using proximity probes to measure blade arrival time for subsequent monitoring. It is proposed however that the measurement of the casing vibration, due to the aerodynamic-structural interaction within a gas turbine, could provide a means of blade condition monitoring and modal parameter estimation. An analytical model has been developed of the response of a gas turbine casing to what is believed to be the dominant forces acting on it, viz: (i) the moving pressure waveform around each blade throughout its motion and (ii) the forces (and moments) applied through the stationary stator blades, to understand the form of the casing response. A simulated blade fault has then been added to the model to represent a damaged blade, of arbitrary form, by reducing the stiffness of one blade. The change in the contribution to the casing response measurements due to this simulated fault is explored.

INTRODUCTION

Gas turbines are driven by the heating and expansion of gases from combustion, causing fluid flow over the multiple blade rows inside the engine and providing useful work output. These extreme operating conditions of high temperatures and pressures within a turbulent flow environment mean that analysis of stresses and heat transfer of the engine working surfaces are of great importance in design and operation.

Measurement and modelling of blade vibrations, has developed for two main applications, determination of stress levels induced by the blades' dynamic motion and to quantify blade condition. A substantial amount of research has been conducted on forced blade motion to determine the stress levels within blades, with advancements taking into account the effects of wake passing, blade tip vortices as well as structural and aerodynamic loadings on blades [1]. Further extensions using computational methods to calculate blade mode shapes and pressure forces have been used in an effort to better estimate the structural and aerodynamic loadings [2].

Under operating conditions the introduction of a fault in the gas turbine blade will alter the blade vibration properties, and has led the development for prediction and measurement of blade vibration in order to make a determination of blade condition. The current dominant blade vibration measurement method, in the aero-industry [3, 4], for engine design, is blade tip timing (BTT), which is the measurement of blade arrival time at one or more locations around the casing periphery. BTT

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methods are currently able to satisfactorily measure asynchronous, non-integer multiples of shaft speed, blade vibrations such as rotating stall, flutter and compressor surge, but requires perforation of the casing. However, no one single method is able to fully characterise the vibration parameters of blades in service [5].

An alternative method has been proposed for non-intrusive measurements of blade vibrations by means of external accelerometer measurements on the casing of a gas turbine [6, 7]. The deterministic periodic blade forced vibrations were analytically and numerically modelled, with further extension including the addition of random fluctuations in the blade force, due to the turbulence within the flow. Incorporation of fluctuations due to turbulence is not customary in conventional blade forced vibration modelling and measurement; it has been shown however that the stochastic portion of the forces within a gas turbine can contain vital blade vibration information [7]. Within the following sections only the stochastic forced vibration of the casing will be modelled, to show the correlation between patterns in the casing vibration and blade vibration within the engine. Changes in the natural frequency of a single blade, and the effect this has on the casing vibration, is then investigated.

ROTOR BLADE FORCED VIBRATION

Flow over the rotor and stator blade aerofoils cause high and low pressure forces acting over the blade surface. Behind the trailing edge of leading blades, wakes are also created, as shown in Figure 1(b). As the rotating blade stages rotate around the engine the stationary pressure profiles around the stator blades, and the wakes behind up-steam blade rows, cause fluctuating forces on the rotor blades. In this study the stator blades will be assumed to remain stationary, this assumption can be partly justified due to the fact that few stator blade vibration problems have ever been reported in turbo-machinery [8]. Stator blade vibration could however be included in more detailed models. As the stator blades are assumed to be stationary the pressure profile and wakes from the trailing edge will remain stationary and the force on the rotor blades will be stationary and periodic with stator passing frequency, with a defined spatial shape. The force acting on rotor blades downstream of a stator blade row has been shown to have, in general, a damped impulse shape [9] (obviously this varies widely and is dependent on the specific turbine). Turbo-machinery flow conditions are however inherently turbulent, with background turbulence intensity levels often reported to be approx. 3-4%, and turbulence within the wake region is often five times greater than the background turbulence [10-12]. It is also noted that the frequency content of turbulence within a gas turbine will be somewhat band-limited; however in the context of this work the turbulence was assumed to be completely broadband.

The forces acting on the rotor blades are therefore modelled as a raised cosine with a period of half the stator blade pass frequency, and modulated by a random fluctuation due to turbulent flows. The Fourier series expansion of this force on the 'rth' blade can be expressed as:

$$f(t)_r = F_0 \left(b(t) + 1 \right) \left\{ \sum_{i=0}^{\infty} A_i \cos \left[i \left(\omega_{spf} t + \gamma_r \right) \right] \right\} \qquad (1)$$

$$\gamma_r = \frac{2\pi (r-1)}{b} - \mathrm{round} \left(\frac{s(r-1)}{b} \right) \frac{2\pi}{s} \qquad (2)$$

where A_i are the Fourier coefficients, ω_{spf} is the stator passing frequency, $b(t)$ is a uniformly distributed random variable with zero mean and a deviation of $\pm 7.5\%$.

The rotor blades are assumed to act as a simple oscillator, i.e. a single degree of freedom spring-mass-damper system, this has been stated to be an adequate model of a blade's response if only one natural frequency is near the excitation frequency [13]. The solution of the blade motion under the force given in equation (1), will only be solved, in this analysis, for the stochastic portion of the force, as the deterministic force component is not of interest in this present study (solution for the deterministic force component can however be found in [6]).

Solution of the rotor blade motion for the 'rth' blade can be simply seen to be given by:

$$X(f)_r = H(f)_r F(f)_r \tag{3}$$

where capitalization refers to the Fourier transform of the corresponding time signal and $H(f)_r$ is the transfer function of the 'rth' blade being:

$$H(f)_r = \frac{A_r}{\left(\omega_{nb}^2 - \omega^2\right) + 2\zeta_2 j\omega_{nb}\omega} \tag{4}$$

A_r being the gain factor for the 'rth' blade.

FORCES ON CASING AND STATOR BLADES

The casing of a gas turbine, under test conditions, can be excited by two groups of forces [14], viz: (a) forces from the engine and running gear through casing/bearing attachments, (b) forces from the aerodynamic/structural interaction within the engine. The second group of forces, (b), can then be further broken down into its believed constituents; (i) interaction with the rotating pressure profile around each rotor/stator blade, (ii) propagation of acoustic waves inside the casing, (iii) pressure fluctuations due to turbulent and impulsive flows.

The current model is of a 1.5 stage turbine, with a row of 5 inlet guide vanes, IGV's, followed by a row of 6 rotor blades, with a final row of 5 stator blades. The IGV's and stator blades are also not "clocked", i.e. they are aligned axially. A longitudinal section of the simulated blade rows is shown in Figure 1(a), a transverse sectional view is seen in Figure 2.
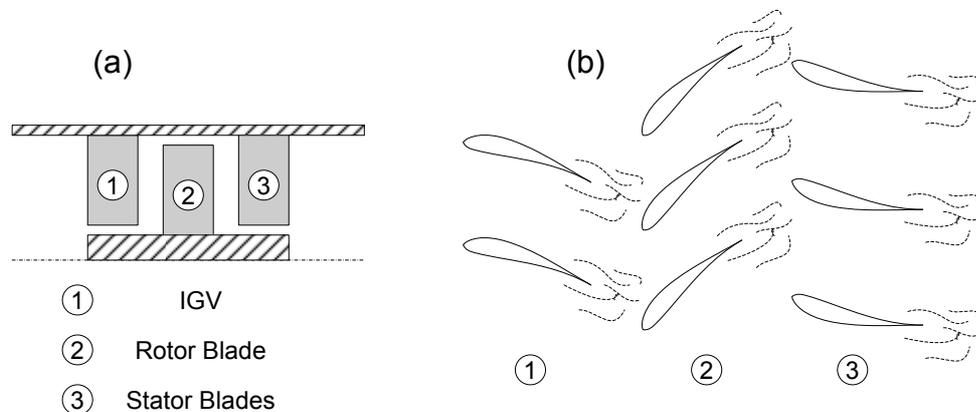

Figure 1 longitudinal section of 1.5 stage turbine (a), wake interaction between blade rows (b)
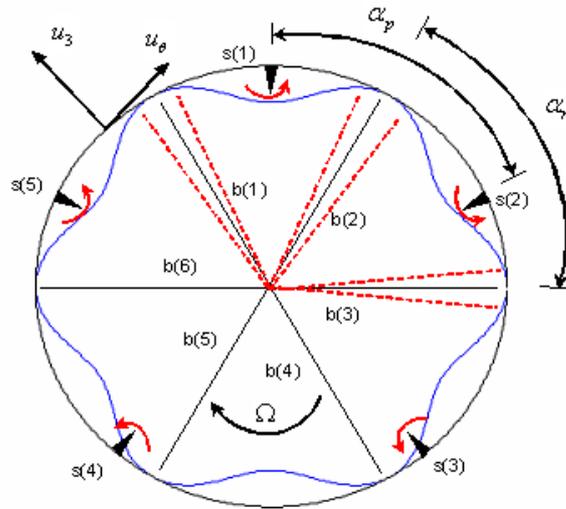
Figure 2 transverse sectional view of 1.5 stage turbine, s(1)-s(5) stator blade locations, b(1)-b(6) rotor blade locations

The pressure profile around each of the rotor blades which causes a fluctuating pressure on the inside surface of the casing as the blades rotate, is considered to follow the motion of the rotor blades as they also vibrate about their equilibrium position. The pressure from the 'r[th]' blade is once again given as a raised cosine, with a period of one rotor blade spacing with the Fourier expansion given as:

$$P_r = \sum_{i=0}^{\infty} A_i . P . e^{ji\left[\theta - \Omega t - x(t)_r - \alpha_r\right]} \tag{5}$$

The forces acting on the stator blades are modelled as a raised cosine of half the blade passing period as was the force on the rotor blades previously. The assumption of neglecting the stator blades' dynamic motion means that the force acting upon them will directly be applied to the casing surface, which in the limit could be expressed as the application of a local moment at the location of the stator blade. However as with the rotor blades, the phase of the force will be dependent on the rotor blade motion, such that the resulting moment for the 'p[th]' stator blade can be expressed as:

$$T_p = M_0 \left\{ \sum_{r=1}^{b} \sum_{i=0}^{\infty} A_i \cos\left[i\left(\Omega t - x(t)_r + \alpha_r - \alpha_p\right)\right] \right\} \delta\left(\theta - \alpha_p\right)\left(\frac{1}{R}\right) \tag{6}$$

ANALYTICAL CASING RESPONSE

The response of the casing under the rotating pressure force from the rotor blades and the excitation by the local moment due to the stator blade loadings, given by equations (5) and (6), can be undertaken in the same manner as the blade response, expressed in equation (3), for the stochastic portion of the blade motion. The casing was modelled analytically as an ideal circular ring, with the geometric and material properties given in Table 1. The solution method is similar for both the rotating pressure and the stator blade loadings, with solution of the generalised force vector then construction of the frequency response function and Fourier transform of the generalised force vector.

Table 1 Geometric and material properties

Density	$\rho = 7.85 \times 10^{-9}\,Ns^2/mm^4$
Young's Modulus	$E = 20.6 \times 10^4\,N/mm^2$
Mean Radius of ring	$R = 100\,mm$
Radial Thickness of ring	$h = 2\,mm$
Damping of ring	$\zeta_1 = 0.01$
Damping of blades	$\zeta_2 = 0.005$
Blade natural frequency	$\omega_{nb} = 500\,Hz$

Stator blade loadings

The generalised force vector for a moment load on an elastic ring is given by [15],

$$F_{nk} = \frac{1}{\rho h N_{nk}} \int_\theta U_3 \left[\frac{\partial (T_p R)}{\partial \theta} \right] d\theta \tag{7}$$

where the mode shapes are given as: (the solution is also only solved for k = 1, as the lowest natural frequency at k = 2, is an order of magnitude above the blade pass frequency)

$$U_3 = \cos n (\theta - \phi_p) \tag{8}$$

where

$$N_n = \pi \left(1 + C_n^2 \right) R$$
$$C_n \approx -1/n \tag{9}$$

The condition is also set such that

$$\phi_p = -\frac{\pi}{2n} + \alpha_p \tag{10}$$

The generalised force vector can be shown to be:

$$F_n = \frac{n M_0}{\rho h N_n} \sum_{r=1}^{b} \sum_{i=0}^{\infty} A_i \cos \left[i \left(\Omega t - x(t)_r + \alpha_r - \alpha_p \right) \right] \tag{11}$$

The frequency response function of the ring under the influence of the moment load T_p, given in equation (6), can now be expressed:

$$H_2(f) = \frac{n}{\rho h N_n} \frac{\cos n (\theta - \phi_p)}{\omega_n^2 \left[1 - (\omega/\omega_n)^2 + 2j\zeta_1 \omega/\omega_n \right]} \tag{12}$$

Such that the response of the ring under the load T_p, given in equation (6), is:

$$U_{31}(f) = \sum_{p=1}^{s} H_2(f) T_p(f) \tag{13}$$

Rotating pressure loading

A similar solution method is used for the rotating pressure load, firstly solving for the generalised force vector, given in [16]:

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$$F_n = \frac{1}{2\rho h N_n} \int_\theta P_r . conj(U_3) R d\theta \tag{14}$$

The mode shape is now given as:

$$U_3 = e^{jn(\theta - \alpha_r)} \tag{15}$$

Solution of the generalised force vector can be shown to now be:

$$F_n = \frac{A_n P R \pi}{\rho h N_n} e^{jn[-\Omega t - x(t)_r - \alpha_r]} \tag{16}$$

The frequency response function of the generalised force vector for the rotating pressure loading is now:

$$H_{3n}(f) = \frac{e^{jn(\theta - \alpha_r)}}{\omega_n^2 \left[1 - (\omega / \omega_n)^2 + 2j\zeta_1 \omega / \omega_n \right]} \tag{17}$$

Such that the response of the casing under the generalised force vector in equation (16), is given as:

$$U_{32}(f) = \sum_{n=2}^{\infty} H_{3n}(f) F_n(f) \tag{18}$$

The total response is then the sum of the contributions from the rotating pressure force and the stator blade loadings.

RESULTS

Results were obtained for the radial displacement of a circular ring under the loading conditions outlined earlier, with an input shaft speed, Ω, of 80 Hz. The power spectral density, PSD, of the output was calculated by averaging over an ensemble of 100 time records of 32 revolutions. The PSD of the same loading conditions applied to the transfer functions obtained from a Finite Element, FE, model of a thin cylindrical shell, of finite width of 10mm, is also presented. Any deterministic components of the casing radial displacement, were removed from the PSD's. The residual PSD, in Figure 3(a-b), can be seen to have narrow band peaks, which are the forcing function filtered by the blades' frequency response function, at multiples of shaft speed $\pm$ the blade natural frequency. The structural resonances can also be seen to be excited in the residual PSD's. Differences are apparent between the analytical and FE results due to the small differences in the structural resonance locations; however, the same harmonic series of narrowband peaks at multiples of shaft speed $\pm$ blade natural frequency can be seen.

Results were then obtained by lowering the natural frequency for one blade by 10%. This results in a new set of harmonic narrow band peaks at shaft $\pm$ the altered blade natural frequency, as seen in Figure 3(c).
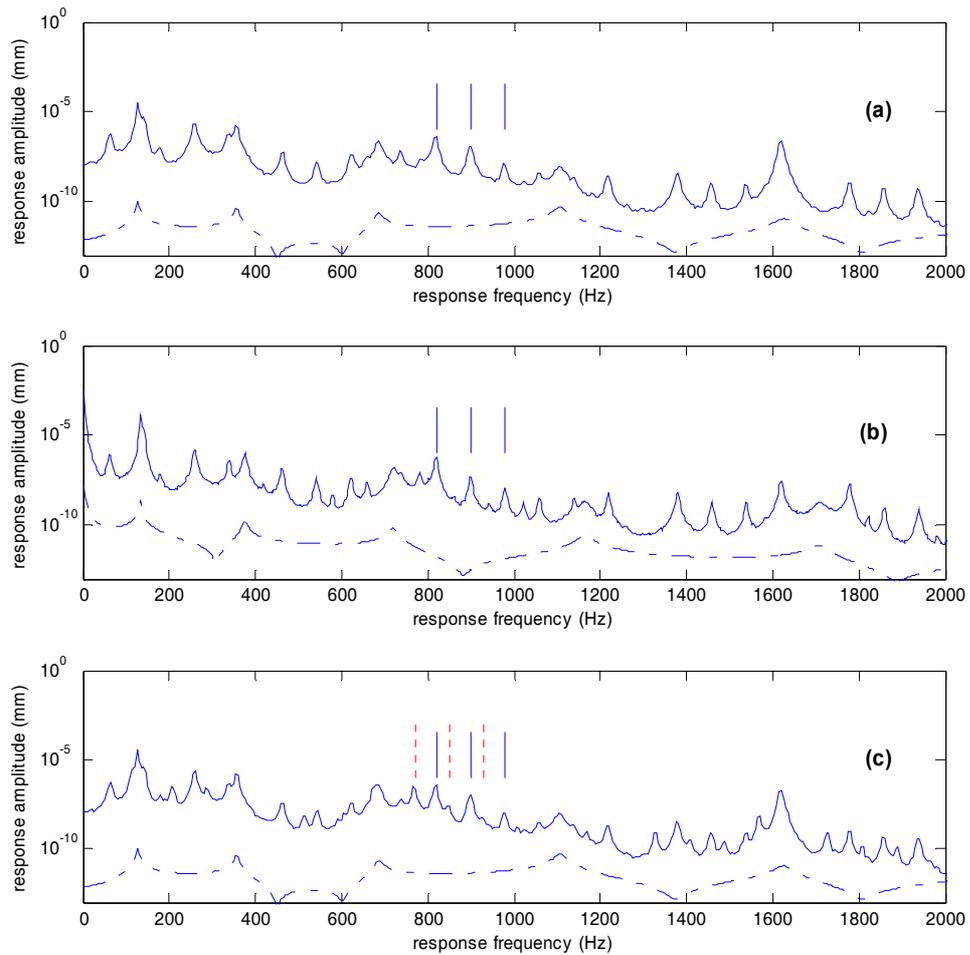
Figure 3 Residual PSD ——, structural transfer function - - - -, vertical dashed lines at multiples of shaft speed, vertical dotted lines at multiples of shaft speed ± blade natural frequency. (a) analytical solution no blade fault, (b) FEA solution no blade fault, (c) analytical solution with blade fault

CONCLUSIONS

Analytical and FE solutions to the forced casing vibration response under the simulated operating conditions have been presented. Cross validation of the analytical and FE model have shown that blade vibration information is present within the residual PSD of the radial casing response, with a harmonic series of narrowband peaks present at multiples of shaft speed ± the blade natural frequency. The reduction of natural frequency, by 10%, of one blade was then modelled and the changes in the residual PSD were then compared. An additional set of narrowband peaks was now seen to be present at multiples of shaft speed ± the altered blade natural frequency. Both these results highlight that information should be obtainable from casing vibration measurements, on the natural frequency of the blades within the engine.

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