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Publication details:
International Journal of Advanced Manufacturing Technology
v. 27
Chapter No. 7-8
pp. 693-702

Publication Date:
2006

Publisher DOI:
http://dx.doi.org/10.1007/s00170-004-2256-7

International Journal of Advanced Manufacturing Technology. Vol. 27/7-8 (2006), pp. 693-702.

A study of Abrasive Waterjet Cutting of Alumina Ceramics with Controlled Nozzle Oscillation

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Abstract An experimental investigation of abrasive waterjet (AWJ) cutting of alumina ceramics with controlled nozzle oscillation is presented and discussed. Particular attention is paid to the effect of small oscillation angles on the various cutting performance measures. It is found that nozzle oscillation at small angles can equally improve the major cutting performance measures if the cutting parameters are correctly selected. However, under high water pressures, high nozzle traverse speeds and large oscillation frequencies, nozzle oscillation may cause a decrease in some major cutting performance measures such as surface finish. Plausible trends of cutting performance with respect to the process parameters are amply discussed. Finally, a predictive mathematical model for the depth of cut is developed and verified.

Keywords: Abrasive waterjet cutting; Nozzle oscillation; Modelling; Cutting performance

1. Introduction

Abrasive waterjet (AWJ) machining is a powerful tool in cutting various materials, particularly difficult-to-cut materials. However, the cutting capacity of this technology in terms of the depth of cut and kerf quality is the major obstruction that limits its applications. In the last decade, a great deal of research effort has been made to develop new techniques to enhance the cutting performance and cutting capacity of the AWJ technology. Some newly developed techniques include cutting with forward angling the jet in the cutting plane, multipass cutting and controlled nozzle oscillation [1, 2, 3], among which controlled nozzle oscillation or cutting head oscillation has been found to be one of the most effective ways in improving the cutting performance without additional costs to the machining process. With this cutting technique, a pendulum-like nozzle forward and backward motion in the cutting plane at predetermined frequency and angular amplitude is superimposed to the normal nozzle traverse motion, as shown in Figure 1. The idea of controlled nozzle oscillation was introduced by Veltrup [4] and was then developed as a most effective cutting technique by many other researchers [5, 6]. When using the nozzle oscillation technique, oscillation angle and oscillation frequency are two of the major cutting parameters. It has been found that the nozzle oscillation cutting technique can significantly improve some major cutting performance measures such as the depth of cut and kerf surface roughness. It has been reported [1, 7] that the depth of the upper smooth zone in nozzle oscillation cutting can be increased by more than 30% compared with that without oscillation, while kerf surface finish as measured by the centre line average R_a can be improved by as much as 30% [6].

<Take in Figure 1>

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It appears that all the reported studies are primarily on the use of large nozzle oscillation angles (or angular amplitudes) of 10 degrees or more. Nozzle oscillation in the cutting plane (in the direction tangential to the curved profile in contouring) with such large oscillation angles results in theoretical geometrical errors on the component profile in contouring and is therefore not preferred in practice. As a result, it is necessary to investigate if nozzle oscillation at small angles can be employed to enhance the cutting performance. In addition, it has been noticed in our early experiments that if the oscillation parameters were not correctly selected for a given combination of the other process variables, nozzle oscillation could have an adverse effect on the cutting performance. However, there has been no adequate study to thoroughly understand this phenomenon, and examine how the cutting parameters affect cutting performance in nozzle oscillation cutting with small oscillation angles so as to suggest strategies for selecting the correct or optimum oscillation parameters. Furthermore, there has been no reported study on the development of predictive mathematical models for the cutting performance in AWJ machining with this novel cutting technique, although such models are essential for the optimum selection of operating parameters in process planning.

In this study, an experimental investigation is first undertaken to examine the effect of cutting parameters on the major cutting performance measures in AWJ cutting of an 87% alumina ceramic with nozzle oscillation at small angles. Particular attention is paid to how the oscillation parameters affect the major cutting performance measures and under what conditions nozzle oscillation can have adverse effect on the cutting performance. Regression analysis and analysis of variance (ANOVA) are performed to comparatively study the data acquired under the normal way of cutting (cutting without nozzle oscillation at a 90^o jet impact angle) and nozzle oscillation cutting, assess the importance of each variable to the cutting performance, and identify the best combinations of cutting conditions for the optimum cutting performance. Finally, a mathematical model for the depth of cut or depth of jet penetration is developed using a dimensional analysis technique and verified with the experimental data.

2. Experimental procedure

2.1 Work material and AWJ cutting machine

In this experiment, the specimens used were 87% alumina ceramics in the form of plate with the thickness of 12.7mm to represent brittle materials. The abrasive waterjet cutting system employed was the Flow International Waterjet Cutter driven by a “Model 20X” dual intensifier pumping system with the operating pressure of up to 380MPa. The motion of the nozzle is numerically controlled by a computer and a five-axis robot positioning system. The other basic components in this system consisted of an “M-263” abrasive delivery system, a “Paser II” abrasive jet cutting head, an “ASI CNC controller”, a water catcher tank and a remote terminal to program the machine.

2.2. Design of experiment

In AWJ cutting, a large number of variables have an impact on the cutting performance [1, 8]. To simplify the analysis, four major variables in normal AWJ cutting as identified in earlier studies [1, 3, 9] and two oscillation variables were chosen for investigation. These six variables include water pressure, nozzle traverse speed, abrasive mass flow rate, standoff distance between nozzle and workpiece surface, nozzle oscillation angle and oscillation frequency. Their levels and corresponding values are shown in Table 1. The other parameters that were kept constant during the tests included the nominal jet impact angle (90^o), orifice diameter (0.33mm), mixing tube or nozzle diameter (1.02mm) and abrasive material (80 mesh garnet sand).

<Take in Table 1>

The Taguchi experimental design array [10] was used to construct the cutting tests. Three groups of tests were adopted in the experimental design. The first group used the four-level, six-factor design scheme in Taguchi orthogonal arrays with all the six selected variables to study the influence of oscillation cutting on the cutting performance, which required 64 experimental runs (L_{64}). For comparatively studying the difference of cutting performance between oscillation cutting and normal cutting, another four-level, four-factor design scheme (L_{16}) was used, including four cutting variables; namely, water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. This resulted in 16 more runs. Furthermore, in order to facilitate the analysis using the as-measured experimental data, a third group was designed using some typical cutting conditions. This group of design included 30 tests. Thus a total of 110 runs were undertaken in this four-level, six-factor experiment.

2.3 Data acquisition

Measurements with the assistance of metrological instruments were conducted for the major cutting performance measures that included the depth of cut, top kerf width, bottom kerf width (for through cut only), minimum kerf width (at which kerf taper is evaluated), kerf taper, and surface roughness. Of theses quantities, kerf taper was calculated using the top kerf width, the minimum kerf width and the depth where the minimum kerf width was measured, while the other five quantities were directly measured from each cut. At least three measures for each quantity on each cut were made and the average was taken as the final reading.

The top kerf width, bottom kerf width, minimum kerf width and depth of cut were measured from the end view of the kerf profile by using a “SigmaScope 500” profile projector prior to separating the specimens. Kerf surface roughness was measured after the specimens were separated to expose the kerf walls. Surface roughness as assessed by the centre-line average R_a was taken using a “Surtronic 3+” stylus surface profile meter. A sample length of 12.5mm was chosen with a cut-off length of 2.5mm for all specimens. R_a values were measured at three different locations along the cut walls, i.e. 1mm, 3mm and 5mm, from the top kerf edge, although only the surface roughness at 1mm from the top kerf edge will be analysed here. The analysis for the major cutting performance measures is presented below.

3. Results and discussion

3.1 Surface roughness

It was noted from the measured surface roughness data that in the majority of the cases where nozzle oscillation was used, the surface is rougher than when cutting without nozzle oscillation under the corresponding cutting conditions. The average R_a value for the 94 cuts with nozzle oscillation is 6.43μm, while that for the 16 cuts under the normal way of cutting is 5.23μm. In general, it was found that when high water pressures and high nozzle traverse speeds were used, oscillation cutting increased surface roughness irrespective of the oscillation frequencies or angles used. The increased R_a values in oscillation cutting may be attributed to the jet instability and system vibration caused by cutting head oscillation.

It is interesting to note that under some conditions, nozzle oscillation can improve the surface finish compared to the normal way of cutting under the corresponding conditions. It appears that the proper combinations of cutting parameters are crucial to surface roughness. The study has found that when water pressure is high, low nozzle traverse speeds with small oscillation angle and medium to high oscillation frequency can improve surface finish. Further, when nozzle traverse speed is high while water pressure is selected at small to medium level together with small oscillation angle and high oscillation frequency, better surface finish can be achieved. Figure 2 shows two sample surfaces produced under normal and oscillation cutting. The improvement in surface finish by using nozzle oscillation is apparent if the cutting parameters are correctly selected.

When these good combinations are used, the improvement in surface finish by oscillation cutting with respect to the corresponding normal way of cutting is more pronounced at high nozzle traverse speed than at low speed. Similar to the normal way of AWJ cutting, oscillation cutting at low traverse speed was found to be able to reduce surface roughness.

<Take in Figure 2>

The results of ANOVA suggest that among the six independent variables tested, water pressure and oscillation frequency have the most influence on surface roughness while abrasive mass flow rate and oscillation angle have the least influence. The small effect of oscillation angle may be because of the small range and increment used in the study. The ANOVA of the oscillation cutting data has found that the optimum cutting conditions are: oscillation frequency at 2Hz, oscillation angle at 2°, nozzle traverse speed at 1mm s^{-1}, water pressure at 275MPa, standoff distance at 5mm and abrasive mass flow rate at 6.8g s^{-1}. Using this set of optimum cutting parameters can produce a surface roughness R_a of 2.05µm. A similar analysis has found that the optimum conditions for cutting without nozzle oscillation are nozzle traverse speed at 1mm s^{-1}, water pressure at 275MPa, standoff distance at 2mm and abrasive mass flow rate at 11.3g s^{-1}, and the resulting surface roughness R_a is 2.29µm. This analysis demonstrates that cutting with nozzle oscillation under optimized combination of cutting parameters can improve surface finish and use less abrasive particles than cutting without nozzle oscillation. However, if the combination of cutting conditions is incorrectly selected, cutting with nozzle oscillation can increase the surface roughness. This finding is somehow different from the previous investigations [5-7] in which the use of nozzle oscillation technique is shown to constantly improve the cutting performance.

The effect of oscillation frequency on surface roughness is shown in Figure 3. It can be seen from Figure 3(a) that surface roughness increases with oscillation frequency monotonically and linearly at a standoff distance of 2mm. However, standoff distance appears to affect the slope of the linear relationship between oscillation frequency and surface roughness, as shown in Figure 3(b). An increase in the standoff distance results in a decrease in the slope. The standoff distance at 5 mm is associated with a negative slope of the linear relationship, i.e. surface roughness decreases slightly with the oscillation frequency. This phenomenon is attributed to the fact that at high standoff distances, the waterjet scanning scope is widened under the same oscillation angle, which leads to an increased overlapping cutting action on the cutting front and hence reduces the surface roughness.

<Take in Figure 3>

<Take in Figure 4>

Figure 4 shows the effect of oscillation angle on surface roughness. It can be seen from the figure that initially surface roughness increases slightly with an increase in oscillation angle and reaches a maximum turning point. As the oscillation angle further increases, surface roughness starts to decrease. This may be a result of the scanning action of the jet on the cutting front and there appears to be an optimum scanning scope corresponding to a set of cutting parameters, similar to the above discussion about the effect of standoff distance. As the oscillation angle increases in the lower region, the jet scanning action cannot effectively cut off the “peaks” left on the cut surface, and causes jet turbulence or instability and system vibration that increase the surface roughness. Larger oscillation angles increases the overlap cutting action and the number of scanning actions on a given part of surface, so that the scanning action is dominant and reduces the surface roughness.

3.2 Depth of cut

The depth of cut for all the tests with nozzle oscillation shows an average increase of 27.7% with respect to the cutting without nozzle oscillation under the corresponding cutting conditions. In some extreme cases, nozzle oscillation cutting increased the depth of cut by as much as 82%. This advantage can be seen clearly from Figure 5. Nevertheless, there are still cases where the depth of cut in oscillation cutting is less than that of the corresponding normal cutting. This may be different from the usual intuition that oscillation technique can constantly increase this cutting quantity. The analysis shows that whether or not nozzle oscillation can increase the depth of cut is dependent on both oscillation angle and oscillation frequency. Dependent on the other operating parameters, there appears to be a critical or threshold value of the product of oscillation frequency and oscillation angle above which nozzle oscillation can increase the depth of cut comparing to the corresponding normal cutting. This critical value is related to the other process parameters and can be determined from the analysis of experimental data. For instance, when the water pressure is 380MPa, nozzle traverse speed is 1.33 mm s^{-1}, standoff distance is 3 mm and abrasive mass flow rate is 11.3 g s^{-1}, the critical value is found to be approximately 8.

<Take in Figure 5>

An analysis of the experimental data has found that the most dominant cutting parameters for depth of cut are oscillation frequency, nozzle traverse speed and water pressure, whereas standoff distance and oscillation angle have little effect. The result of ANOVA suggests that if 14Hz oscillation frequency, 6° oscillation angle, 380MPa water pressure, 0.67mm s^{-1} traverse speed, 3mm standoff distance, and 11.3g s^{-1} abrasive flow rate are used, oscillation cutting can produce the maximum depth of cut of 16.3mm. However, with the normal way of cutting the optimum combination of the cutting parameters were found to be water pressure at 345MPa, nozzle traverse speed at 0.67mm s^{-1}, standoff distance at 2mm, and abrasive mass flow rate at 11.3g s^{-1}, which yields the maximum depth of cut of 13.3mm. Thus, statistically nozzle oscillation cutting can increase the depth of cut by 23% with respect to the normal cutting technique under the respective optimum combinations of cutting parameters and under the same cutting rate and abrasive mass flow rate.

<Take in Figure 6>

The relationship between oscillation frequency and the depth of cut is plotted in Figure 6. Figure 6(a) shows that the depth of cut increases approximately linearly with the oscillation frequency. It is believed that high oscillation frequency increases the number of repeated scanning actions and reduces the particle interference, which in turn increases the overall abrasive cutting capacity and increases the depth of cut. As discussed earlier, under some conditions, an increase in oscillation frequency may result in a slight decrease in the depth of cut, as shown in Figure 6(b). This figure also shows that the slope of the linear relationship between oscillation frequency and the depth of cut is affected by the nozzle traverse speed. A higher nozzle traverse speed is associated with a more rapid increase of the depth of cut as the oscillation frequency increases. When a low traverse speed of 0.67mm s^{-1} is used, an increase in oscillation frequency in fact results in a decrease in the depth of cut. This may be explained that at low traverse speed, the jet scanning action cannot take effect and may cause increased particle interference and a reduction in the jet cutting capability.

<Take in Figure 7>

Figure 7 shows the effect of oscillation angle on the depth of cut from the experimental data. This effect is also dependent on the oscillation frequency. At relatively large oscillation frequencies (e.g. 10Hz and 14Hz), an increase in oscillation angle is associated with a steady decrease of the depth of cut while the decreasing rate slightly increases with the oscillation angle. This may be due to the fact that at high oscillation frequencies, an increase in oscillation angles increases the jet instability and particle interference which decreases the jet cutting capability and hence decrease this cutting performance measure.

By contrast, at low oscillation frequencies such as 2 Hz in Figure 7(a), an increase in oscillation angle results in a slight increase in the depth of cut, while the increasing rate decreases with the oscillation angle. As the oscillation angle further increase to beyond 6º, the depth of cut exhibits a decreasing trend. A maximum turning point for the depth of cut occurs at about 4º to 6º of oscillation angle when small oscillation frequencies are used.

For the other oscillation frequencies in the medium range of the tested conditions, the depth of cut appears to be somehow independent of the oscillation angles with only very slight decrease as this cutting variable increases, as shown in Figure 7(b) at an oscillation frequency of 6 Hz.

To this end, large oscillation frequencies with small oscillation angles are preferred to increase the depth of cut. Figure 7(b) also shows that standoff distance has an effect on the depth of cut. While an increase in the standoff distance may reduce the particle energy at the point of particles attacking the material, it may take an effect together with the oscillation angle and frequency. With a larger standoff distance, the jet scanning scope on the cutting front is increased, which may increase or decrease the depth of cut depending on the other parameters used, in a similar way to the oscillation angle.

3.3 Kerf taper

The value of kerf taper (T_R) for each cut was obtained from the other measured quantities, i.e. the top kerf width W_t, the minimum kerf width along the kerf profile W_m and the depth t at which the minimum kerf width was measured, based on the following equation

$$T_R = \frac{(W_t - W_m)}{2t} \tag{1}$$

The mean values of kerf taper for all the oscillation cutting and normal cutting tests were found to be 0.094 and 0.118, respectively. Based on these average kerf taper values, oscillation cutting reduced taper angle by 18.1% compared to normal cutting under the corresponding cutting conditions. ANOVA analysis of the experimental data suggested that oscillation frequency accounted for most of the kerf taper formation, while nozzle traverse speed, water pressure, and standoff distance assumed the second important role for kerf taper formation in oscillation cutting. By contrast, abrasive mass flow rate and oscillation angle had the least effect on kerf taper. The optimum combination for the smallest kerf taper has been found to be oscillation frequency of 14Hz, oscillation angle of 8º, nozzle traverse speed of 0.67mm s^{-1}, water pressure of 380MPa, standoff distance of 2mm, and abrasive mass flow rate of 11.3g s^{-1}. This optimum combination can produce a kerf taper of 0.028. Similarly, the analysis of the experimental data has found that statistically, the minimum kerf taper for the normal way of AWJ cutting is 0.061 under the optimum combination of cutting parameters, i.e. water pressure is at 380MPa, nozzle traverse speed at 0.67mm s^{-1}, standoff distance at 2mm, and abrasive mass flow rate at 11.3g s^{-1}. Thus, under the optimum parameter combinations, cutting with nozzle oscillation can reduce the kerf taper by 54% comparing with normal cutting under the same cutting rate and the same usage of abrasives.

It has been found that oscillation frequency and oscillation angle have a similar effect on kerf taper. Since oscillation frequency has little effect on kerf width, but has a significant effect on the jet penetration depth, an increase in oscillation frequency is associated with a reduced kerf taper from equation (1), as shown in Figure 8. This trend is consistent for cutting under various conditions. The effect of oscillation frequency on the kerf taper becomes interesting when traverse speed is varied, as shown in Figure 8(b). The slope of the kerf taper plots with respect to oscillation frequency becomes steeper as the traverse speed increases. A similar trend is observed in the effect of oscillation angle on the kerf taper.

<Take in Figure 8>

4. Mathematical model for depth of cut

4.1 Modelling conditions and assumptions

To model the depth of cut in AWJ cutting needs to consider a host of variables which make the modelling process complicated. In addition, a number of phenomena, such as the particle interference and fragmentation, exist in abrasive jet processing. At this stage of development, there is no sufficient knowledge of these phenomena [1, 11]. Therefore, to consider all these variables and phenomena is either impossible or results in many unknown parameters in the final equation, making the model unrealistic for practical use. In this study, a dimensional analysis technique is used, where necessary, to establish the mathematical relationship between this cutting performance measure and the process variables, while regression analysis of the experimental data is undertaken to determine the constants in the model.

With dimensional analysis, all variables appearing in a problem can be assembled into a smaller number of independent dimensionless products or groups (π_i) using the constraint that all terms of the model must have the same dimensions. The relations connecting the individual variables can be determined by algebraic expressions relating each π_i [12, 13]. To simplify the model development process, some assumptions need to be made, i.e.:

- Abrasive particles are distributed uniformly over the jet cross-sectional area.
- Abrasive particles have the same velocity with the surrounding water in the jet. The jet velocity variation along the jet stream is ignored.
- Jet side spreading is ignored. Thus, kerf width w is approximately equal to jet diameter d_j.

The steps involved in developing the model include:

1. Derive the expression for the overall material removal rate in terms of the material removed by individual abrasive particles and the number of particles in the jet for the time span.
2. Formulate the relation between the material removed by an individual particle and the other influencing variables.
3. Estimate the constants in the equation by the multiple regression analysis.

The underlying principle in constructing the predictive model for depth of cut is that the overall material removal rate is equal to the accumulated volume of material removed by individual abrasive particles in a given time span [14]. If assuming that the depth of cut is h, the overall material removal rate can be expressed as uhw, where u is nozzle traverse speed and w is the average kerf width. If the abrasive mass flow rate is $\dot{m}$ and the average mass of an individual particle is m, the number of abrasive particles in the jet per unit time is $\dot{m}/m$. The total accumulated volume of material removed by the abrasive particles can be represented as $R\,\dot{m}/m$, where R is the average material removed by an individual particle contributing to the material removal process. In abrasive waterjet cutting, not all particles in the jet will impinge the material or have sufficient energy to cut the target material. Some particles may collide with other particles and are not involved in the cutting action. To consider this phenomenon, an efficiency factor (K_e) may be introduced so that the following volumetric relation can be drawn to relate the overall material removal rate to the accumulated volume of material removed by individual particles

$$uhw = K_e \frac{\dot{m}}{m} R \tag{2}$$

By ignoring jet side spreading, it may be assumed that the average kerf width is equal to the jet diameter which in turn is approximated by the nozzle diameter, i.e. $w=d_j$. Hence,

$$uhd_j = K_e \frac{\dot{m}}{m} R \tag{3}$$

4.2 Material removal by individual particles

To determine the mathematical expression for the average volume of material removed by an individual particle, a dimensional analysis technique is used. It was noted that there are a number of variables that affect the material removed by a particle (R). The most dominant amongst these are the modulus of elasticity for the target material (E), particle velocity (v), particle attack angle (α) (the angle between the material surface and particle flow direction at the point of attack), and the average mass of an individual particle (m). Mathematically, the relationship between the material removed by a particle and the above variables can be written in the form of

$$R = f(E, v, m, \alpha) \tag{4}$$

This set of variables depends on three fundamental dimensions, i.e. length L, mass M, and time T. Since α is already a dimensionless variable, two independent dimensionless products can be formed from equation (4), i.e.

$$\pi_1 = \frac{RE}{mv^2} \tag{5}$$

$$\pi_2 = \alpha \tag{6}$$

It then follows from equations (5) and (6) that the functional relation between these two dimensionless products is

$$\pi_1 = f(\pi_2) \tag{7}$$

or

$$\frac{RE}{mv^2} = f(\alpha) \tag{8}$$

A non-dimensional quantity is proportional to the product of other dimensionless products raised to rational power [15]. Because of its simplicity and wide use, the power law formulation is applied to equation (8), and the complete dimensional equation is given as follows

$$\frac{RE}{mv^2} = A\alpha^{a_1} \tag{9}$$

Equation (9) can be rewritten as:

$$R = \frac{Amv^2}{E}\alpha^{a_1} \tag{10}$$

where A and a_1 are two constants introduced.

It has been reported that the particle attack angle (α) is a variable along the cutting front [1]. To determine the attack angle for each particle represents a difficult task. Therefore in this study, an average attack angle for all particles impinging the target material is used. To derive the relation between the average particle attack angle and its influencing factors, dimensional analysis is again used. According to the reported investigations for AWJ cutting without the use of nozzle oscillation [16, 17], the modulus of elasticity for target material (E), nozzle traverse speed (u), and average particle diameter (D) have a significant effect on the particle attack angle. When the nozzle oscillation cutting technique is used, the oscillation parameters have been found to be additional variables affecting the depth of cut, as in the foregoing analysis. In addition, the standoff distance between the nozzle and work surface has changed its role in affecting the cutting process in nozzle oscillation cutting. Specifically, the jet scanning or oscillating scope in the cutting front is not only related to the oscillation angle, but also to the standoff distance. A study has found the underlying theme why nozzle oscillation cutting can increase the depth of cut [1], which shows that the successive traces of particles on the cut surface with nozzle oscillation are steeper than those without nozzle oscillation. The changed particle traces in fact affect the particle attack angle. Consequently, additional three parameters, i.e. oscillation angle (θ), oscillation frequency (F) and standoff distance (H) are considered in determining the particle attack angle nozzle oscillation cutting.

Thus, the particle attack angle can be expressed as a function of the six variables, i.e.

$$\alpha = \Phi(\theta, F, H, u, D, E) \tag{11}$$

All these seven parameters in equation (11) are quantities with regards to three fundamental dimensions; namely length L, mass M, and time T. Since α and θ are already dimensionless variables, four independent dimensionless groups can be formed from the dimensional analysis, i.e.

$$\pi_1 = \alpha \tag{12}$$

$$\pi_2 = \theta \tag{13}$$

$$\pi_3 = \frac{FH}{u} \tag{14}$$

$$\pi_4 = \frac{D}{H} \tag{15}$$

Those groups are related by the function of

$$\pi_1 = \Phi(\pi_2, \pi_3, \pi_4) \tag{16}$$

Thus,

$$\alpha = \Phi\left(\theta, \frac{FH}{u}, \frac{D}{H}\right) \tag{17}$$

Again, based on the power law formulation, α can be expressed using those four groups as

$$\alpha = B\theta^{a_2}\left(\frac{FH}{u}\right)^{b_2}\left(\frac{D}{H}\right)^{c_2} \tag{18}$$

where B, a_2, b_2, and c_2 are all constants.

Substituting equation (18) into equation (10) and generalizing the constants and exponents by new constants C, a, b, and c give:

$$R = \frac{Cmv^2}{E}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{19}$$

4.3 Depth of cut model

Having established the equation for the average volume of material removed by an individual particle, the depth of cut can be achieved by substituting this equation (equation (19)) into the overall material removal rate equation (3). After making the necessary transformations, the depth of cut is given by

$$h = \frac{K_1\dot{m}v^2}{Ed_ju}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{20}$$

where $K_1 = CK_e$ and is a dimensionless constant.

If assuming that the water is incompressible and the friction loss in the supply system is negligible, the particle velocity can be calculated by using the Bernoulli's equation, i.e.

$$v = \sqrt{\frac{2P}{\rho_w}} \tag{21}$$

where P is water pressure and ρ_w is water density.

Thus, by introducing equation (21) into (20), the final form of the depth of cut equation can be given by

$$h = \frac{K\dot{m}P}{E\rho_w d_j u}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{22}$$

where $K=2K_1$, is an empirical constant to be determined by experiment.

4.4 Model verification

A mathematical model for the depth of cut has been established. This model is in its general form and the constants in the model need to be determined from the experimental data before it can be of any use. For this purpose, a regression analysis was performed on the experimental data obtained under the conditions given in section 2 of the paper. The constants K, a, b, and c in equation (22) have been determined at a 95% confidence interval. Substituting these constants into the depth of cut model results in

$$h = \frac{1.025\dot{m}P}{E\rho_w d_j u}\theta^{0.043}\left(\frac{FH}{u}\right)^{0.054}\left(\frac{D}{H}\right)^{0.044} \tag{23}$$

Re-arranging the equation gives:

$$h = 1.025\frac{\dot{m}P\theta^{0.043}F^{0.054}D^{0.044}H^{0.01}}{E\rho_w d_j u^{1.054}} \tag{24}$$

where the symbols and theirs units are specified in the Nomenclature.

Equation (24) is valid for nozzle oscillation cutting of an 87% alumina ceramic under the ranges of cutting parameters given in section 2 of the paper. An examination of the equation reveals that the form of the model is generally feasible and consistent with the experimental trends of the depth of cut with respect to the major process variables. Specifically, the model realistically represents the effects of water pressure, nozzle traverse speed, and abrasive mass flow rate as discussed earlier in this paper, i.e. an increase in the water pressure and abrasive mass flow rate and a decrease in nozzle traverse speed result in an increase in the depth of cut. It has been reported in early investigations [1] that larger particles carry more energy and have the potential to removal more material in a cutting action. This variable has a positive exponent in the numerator of the equation and is therefore considered to be correctly incorporated in the model. Similarly, the model correctly reflects the trends of the depth of cut with respect to the oscillation parameters discussed earlier in the paper. Overall, both the oscillation frequency and oscillation angle have a positive effect on the depth of cut. It is interesting to note that standoff distance has a positive exponent in the numerator. An analysis of the experimental data has found that in a considerable number of cases, an increase in standoff distance initially results in a slight increase in the depth of cut. As the standoff distance further increases, some tests show a slightly decreasing trend for the depth of cut. Thus it is not surprising that standoff distance has a positive effect on the depth of cut as represented by the model. Consequently, the basic form of the model is considered as correct.

<Take in Figure 9>
<Take in Figure 10>

In order to check the adequacy of the model, a qualitative assessment has been made by comparing the predicted depths of cut with the corresponding experimental data. Some typical and representative samples of the comparisons are given in Figure 9. It can be seen from the figures that the model's predictions are in good agreement with the experimental data. A quantitative comparison between the predicted and experimental depths of cut has also been carried out based on the percentage deviation of the model predicted value with respect to the corresponding experimental result. This is shown in the histogram in Figure 10. The comparison shows that the model's prediction yields an average percentage deviation of 3.3% with the standard deviation of 20.1%. The large standard deviation is attributed to the scatter of the experimental data which were used to determine the constants in the model. Thus, it can be deduced that the model developed can be used for adequate prediction of the depth of cut in process planning for the ranges of the parameters used in this study.

5. Conclusions

An experimental investigation has been carried out in AWJ cutting of an 87% alumina ceramic with controlled nozzle oscillation. The investigation focuses on some major cutting performance measures in terms of surface roughness, depth of cut, and kerf taper when nozzle oscillation cutting at small oscillation angles. It has been shown that similar to oscillation cutting at large oscillation angles, oscillation at small angles can equally have a significant impact to the cutting performance. It has been found that if the cutting parameters are not selected properly, nozzle oscillation cutting can reduce some major cutting performance measures. Plausible trends of the major cutting performance measures with respect to the various cutting variables have been analysed and the benefits of using the nozzle oscillation cutting technique amply demonstrated. Nozzle oscillation cutting at small oscillation angles can increase the depth of cut by as much as 82%. When the optimum cutting parameters are used for both nozzle oscillation and normal cutting, statistically the former can increase the depth of cut by 23% and reduce kerf taper by 54%. Similarly, under the optimum cutting conditions, nozzle oscillation can significantly reduce the surface roughness.

A predictive mathematical model for the depth of cut in AWJ cutting with controlled nozzle oscillation has been developed by using a dimensional analysis technique. A numerical analysis has verified the model and demonstrated the adequacy of the model's prediction. This model has provided an essential basis for the development of optimization strategies for the effective use of the AWJ cutting technology when the nozzle oscillation technique is used.

Acknowledgments

The work was supported by the Australian Research Council (ARC) through its Discovery Project scheme.

References

1. Wang J (2003) Abrasive Waterjet Machining of Engineering Materials, Trans Tech Publications, Switzerland.
2. Wang J (1999) Abrasive waterjet machining of polymer matrix composites: Cutting performance, erosive analysis and predictive models, Int. J. Adv. Manuf. Technol., 15:757-768.
3. Wang J, Kuriyagawa T, Huang C Z (2003) An experimental study to enhance the cutting performance in abrasive waterjet machining, Machining Science and Technology, 7/2:191-207.
4. Veltrup E M (1976) Application of oscillating nozzles for cutting and cleaning, Proc. Third Int. Symp. Jet Cutting Technology, Chicago, pp. C1-1/C1-13.
5. Siores E, Wong W C K, Chen L, Wager J G (1996) Enhancing abrasive waterjet cutting of ceramics by head oscillation techniques, Ann. CIRP, 45(1): 327-330.
6. Lemma E, Chen L., Siores E, Wang J (2002) Optimising the AWJ cutting process of ductile materials using nozzle oscillation technique, Int. J. Mach. Tools Manufact., 42(7):781-789.
7. Chen L, Siores E, Wang W C K (1998) Optimising abrasive waterjet cutting of ceramic materials, J. Mater. Proc. Technol., 74(1-3):251-254.

8. Hashish M (1984) A modelling study of metal cutting with abrasive waterjets, J. Eng. Mater. Technol., Trans. ASME, 106(1):88-100.
9. Wang J (1999) A study on abrasive waterjet cutting of metallic coated sheet steels, Int. J. Mach. Tools Manuf., 39:855-870.
10. Roy R K (1990) A primer on the Taguchi method, Society of Manufacturing Engineers, Dearborn, Michigan.
11. Wang J, Guo D M (2002) A predictive depth of penetration model for abrasive waterjet cutting of polymer matrix composites, J. Mate. Proc. Tech., 121/2-3:390-394.
12. Isaacson E, Isaacson M (1975) Dimensional methods in engineering and physics: reference sets and the possibilities of their extension, Edward Arnold, London.
13. Svobodny T (1998) Mathematical modelling for industry and engineering, Prentice Hall, Upper Saddle River, N.J..
14. Zeng J, Wu S, Kim T J (1994) Development of a parameter prediction model for abrasive waterjet turning, Proc. 12th Int. Conf. Jet Cutting Technology, Rouen, France, Mechanical Engineering Publication Limited, London, pp. 601-617.
15. Thurairajasingam E, Shayan E, Masood E (2002) Modelling of a continuous food pressing process by dimensional analysis, Computers and Industrial Engineering, 42(2-4):343-351.
16. Zeng J, Kim T J (1992) Development of an abrasive waterjet kerf model for brittle materials, Proc. 11th Int. Conf. Jet Cutting Technology, Bedford, UK, pp. 483-501.
17. Hashish M (1989) Model for abrasive-waterjet (AWJ) machining, J. Eng. Mater. Technol., Trans. ASME, 111(2):154-162.

Nomenclature

D	average particle diameter (mm)
d_j	nozzle diameter (mm)
E	modulus of elasticity for work material (GPa)
F	oscillation frequency (Hz)
h	depth of cut (mm)
H	standoff distance (mm)
K, a, b, c	constants
K_e	efficiency factor
m	average mass of a particle (g)
$\dot{m}$	abrasive mass flow rate (g s^{-1})
P	water pressure (MPa)
R	average volume of material removed by a particle (mm^3)
T_R	kerf taper
u	nozzle traverse speed (mm s^{-1})
v	particle velocity (mm s^{-1})
w	average kerf width (mm)
w_t	top kerf width (mm)
w_m	minimum kerf width (mm)
α	particle attack angle (degrees)
θ	oscillation angle (degrees)
ρ_w	water density (g mm^{-3})

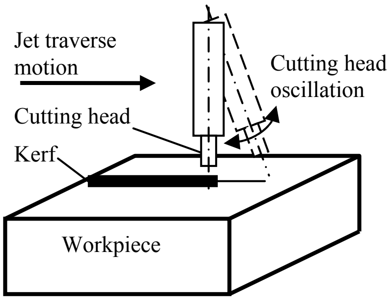


Figure 1. Schematic of controlled nozzle oscillation.

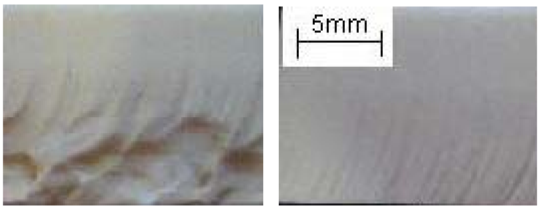


Figure 2. Samples of surfaces produced under normal cutting (left) and oscillation cutting (right): $\dot{m}=9.1\text{g s}^{-1}$, H=4mm, P=380MPa, u=0.67mm s^{-1}; For oscillation cutting: θ=4°, F=14Hz.

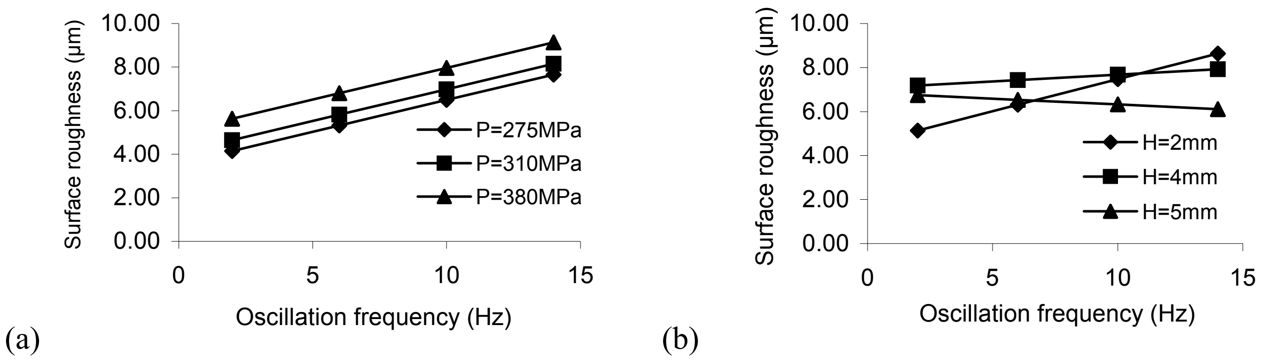


Figure 3. Effect of frequency on surface roughness: (a) $\theta=4^{\circ}$, u=1mm s^{-1}, H=2mm, $\dot{m}=9.1\text{g s}^{-1}$; (b) $\theta=4^{\circ}$, P=345MPa, u=1mm s^{-1}, $\dot{m}=9.1\text{g s}^{-1}$.

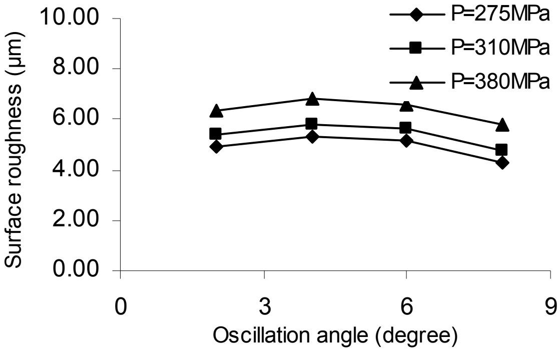


Figure 4. Effect of oscillation angle on surface roughness (F=6Hz, u=1mm s^{-1}, H=2mm, $\dot{m}$ =9.1g s^{-1}).

Figure 5. Samples of kerf profiles produced under normal cutting (left) and oscillation cutting (right): $\dot{m}$ =9.1g s^{-1}, H=2mm, P=310MPa, u=1.33mm s^{-1}; For oscillation cutting: θ=8°, F=10Hz.

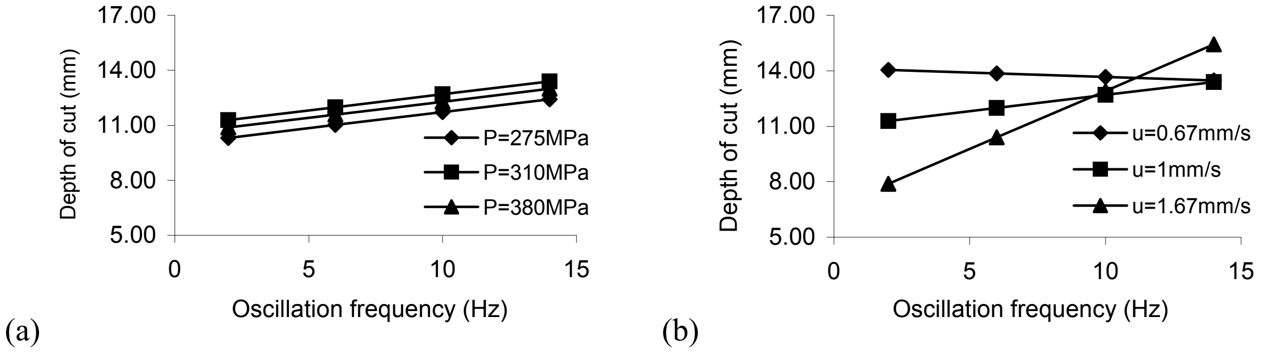
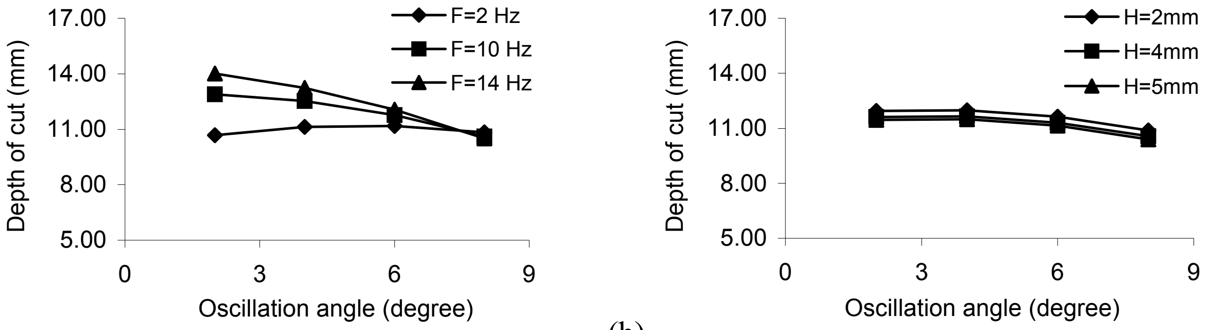
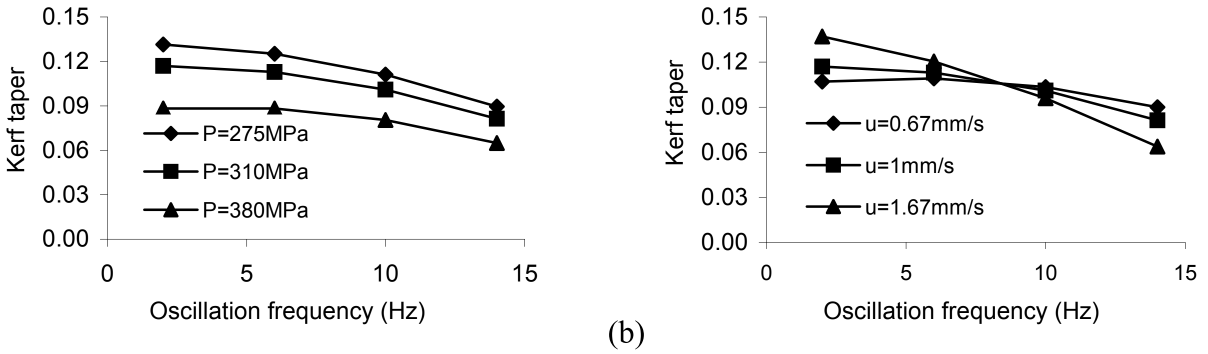


Figure 6. Effect of oscillation frequency on the depth of cut: (a) θ=4°, u=1mm s^{-1}, H=2mm, $\dot{m}$ =9.1g s^{-1}; (b) θ=4°, P=310MPa, H=2mm, $\dot{m}$ =9.1g s^{-1}.



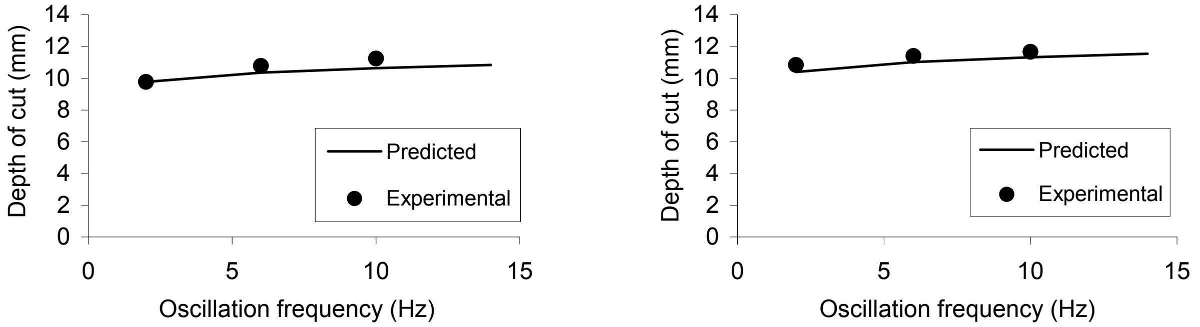
(a) (b)

Figure 7. Effect of oscillation angle on the depth of cut: (a) P=310MPa, u=1mm s^{-1}, H=3mm, $\dot{m}$ =9.1g s^{-1}; (b) F=6Hz, P=310MPa, u=1mm s^{-1}, $\dot{m}$ =9.1g s^{-1}.



(a) (b)

Figure 8. Effect of frequency on taper: (a) θ=4°, u=1mm s^{-1}, H=3mm, $\dot{m}$ =9.1g s^{-1}; (b) θ=4°, P=310MPa, H=2mm, $\dot{m}$ =9.1g s^{-1}.



(a) (b)

Figure 9. Comparisons between model predictions and experimental data: (a) θ=4°, P=310MPa, u=1.33mm s^{-1}, H=3mm, $\dot{m}$=11.3g s^{-1}. (b) θ=2°, P=275MPa, u=0.67mm s^{-1}, H=2mm, $\dot{m}$=6.8g s^{-1}.

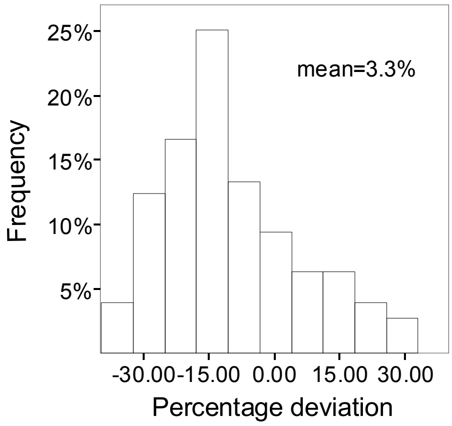


Figure 10. Histogram for percentage deviation between predicted and experimental depth of cut.

Table 1. Experimental design.

Process Variables	Level 1	Level 2	Level 3	Level 4
Abrasive mass flow rate $\dot{m}$ (g s^{-1})	6.8	9.1	11.3	13.6
Standoff distance H (mm)	2	3	4	5
Water pressure P (MPa)	275	310	345	380
Nozzle traverse speed u (mm s^{-1})	0.67	1.00	1.33	1.67
Oscillation angle θ (degrees)	2	4	6	8
Oscillation frequency F (Hz)	2	6	10	14