Effects of a porous jacket on sound radiated from a pipe

S. Kanapathipillai and K. P. Byrne
School of Mechanical and Manufacturing Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

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Conventional pipe laggings incorporate a porous jacket and an impervious outer cladding sheet. It has been observed during investigations of such pipe laggings that a simple porous jacket applied by itself to a pipe can actually increase the sound radiated. Reasons for this phenomenon are discussed. The effect of a rigid frame porous jacket around a pipe is examined theoretically for the breathing, bending, and ovaling modes of pipe vibration. The predicted insertion loss associated with the bending mode of pipe vibration is compared with the corresponding experimental result and some of the results of a parametric study are given. © 1996 Acoustical Society of America.

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INTRODUCTION

The usual way of attenuating the noise radiated by pipes is to lag them with constructions formed of porous layers or jackets such as glasswool blankets and impervious layers or jackets such as metal cladding sheets. Papers available in the readily accessible literature relating to the acoustic performance of pipe laggings generally have been concerned with presenting experimental results such as the lagging insertion losses. It appears that few attempts have been made to predict insertion losses by the use of models which are based on the fundamental properties of the layer elements such as the thickness and flow resistivity of the porous layer. This is not surprising in view of the difficulty which has been encountered in satisfactorily predicting the transmission of sound from the inside to the outside of a pipe. It has been found with regard to this latter problem that theoretical predictions of the sound transmission loss from the inside to the outside of a pipe can significantly overestimate (by 30 dB) the actual sound transmission loss. It has been suggested that this situation arises because unexpected vibrations of the pipe in bending modes can be induced by a variety of mechanisms.

Although practical pipe laggings usually incorporate a porous jacket surrounded by an impervious jacket which protects the porous jacket and enhances the acoustic performance of the construction, sometimes no impervious jacket is used. A surprising effect occurs with this simple porous jacket lagging. It has been observed that a simple porous blanket or jacket pipe lagging can, up to quite high frequencies, enhance the sound radiated from a pipe, that is, the porous jacket can produce a negative insertion loss.

I. POSSIBLE NEGATIVE INSERTION LOSS MECHANISMS

Examples of negative insertion losses produced by porous jackets around pipes have been reported in the literature. It is worthwhile reviewing some of these examples and considering possible mechanisms. Figure 1, which is taken from Ref. 2, gives the insertion loss produced by a 50-mm-thick fiberglass blanket applied to a 300-mm-diam pipe. It can be seen that a negative insertion loss of 9 dB was measured in the 315-Hz 1/3-oct band. Figure 2, which is taken from Ref. 5, shows the insertion loss curves for various thicknesses of preformed fiberglass pipe insulations applied to a 25-mm-diam copper pipe carrying a noisy water flow. It can be seen that large negative insertion losses extend up to very high frequencies. Further, the thicker the insulation the greater the low-frequency negative insertion loss. It is only at high frequencies where the expected behavior of increasing insertion loss with increasing insulation thickness occurs. There are several possible reasons for this low-frequency negative insertion loss effect and these are discussed in the following paragraphs.

The first possible reason is related to the forced vibration of the insulation. It appears that for many pipes most of the radiated sound is not associated with vibration of the pipe in the breathing modes but with vibration in the other modes and in particular, with bending modes. At low frequencies the sound power radiated by a rigid vibrating cylinder is proportional to the fourth power of the pipe diameter. Thus if the skeletal structure of the porous insulation is forced to vibrate as a result of its close mechanical contact with the pipe wall and as the outer diameter of the insulation is several times the diameter of the pipe it is insulating, as can easily happen with preformed thermal pipe insulations for small diameter pipes, the “fourth power of diameter” effect is likely to induce enhanced sound radiation. Preformed thermal pipe insulations often fit the associated pipes very neatly and so the skeletal structure of the insulation will be forced to vibrate.

The second possible reason why negative insertion losses are observed at low frequencies is more subtle and is the basis of the study described in this paper. A rigid porous jacket would inhibit the tangential acoustic particle motion of the fluid surrounding the pipe when it is vibrating in the bending, ovaling, and higher modes of pipe vibration. A pipe fitted with numerous radial fins which run parallel to the pipe axis also would be expected to inhibit tangential particle motions of the surrounding fluid and so such a finned pipe would appear to be larger and would be expected to function as a more efficient radiator in these modes.

Although low-frequency negative insertion losses are often observed with plane laggings formed of porous layers...
and impervious sheets applied to flat surfaces, such as the sides of rectangular ducts, these negative insertion losses are attributable to the resonant response of the impervious sheet on the air in the porous layer. A plane porous layer alone does not produce this low-frequency negative insertion loss effect. Similarly, a rigid frame porous jacket around a pipe would not be expected to produce low-frequency negative insertion losses in the pipe breathing mode as the particle motions would be purely radial. It is only in the higher modes which have tangential components of particle motion associated with them that a rigid frame porous jacket around the pipe could produce negative insertion losses.

II. PRINCIPLES OF THE ANALYSIS METHOD

The model of the pipe with a rigid frame porous jacket lagging is shown in Fig. 3. The rigid frame jacket is not in contact with the pipe surface and in fact there may be a substantial air gap between the pipe and the jacket. The pipe and the jacket are assumed to be infinitely long. The pipe is capable of supporting various forms of structural waves which travel along it in the positive z direction. The simplest form of structural wave is associated with the so-called breathing mode. The complex representation of the radial velocity, \( v_r \), of the pipe surface for this mode when the wave is harmonic is given by Eq. (1) with \( n = 0 \):

\[
v_r = V_r \cos n \phi \exp[j(\omega t - k_z z)].
\]  

Often the most important mode of pipe structural motion in terms of sound radiation is the bending mode for which \( n \) in Eq. (1) is 1. The ovaling mode has \( n \) equal to 2.

The purpose of the analysis is to find the frequency-dependent insertion losses produced by the porous jacket when the pipe is supporting each of the previously mentioned structural wave types.

The insertion loss which the rigid frame porous jacket produces for a particular mode at a particular frequency can be found from the ratio of the sound power radiated from a unit length of the pipe with and without the jacket present. It should be noted that sound energy can be radiated from the pipe only if \( k_z \), is less than the acoustic wave number \( k = \omega/c \). If \( k_z \) is less than \( k \) “conical” sound waves are radiated by the pipe and it is convenient to define the wave propagation direction by the angle \( \theta \) shown in Fig. 4. It can be seen that \( k_z = k \cos \theta \). The acoustic pressure and radial particle velocity just outside the outer surface of the jacket are assumed to vary with \( \phi \) according to \( \cos n \phi \) whether the jacket is present or not. Thus the radial intensities vary with \( \cos^2 n \phi \) with and without the jacket and so the ratio of the sound powers radiated from unit lengths of the bare and the lagged
pipe can be found from the ratio of the radial intensities at a convenient point just outside the outer surface of the porous jacket whether or not it is present. It is convenient to determine the radial intensities at the point \( r = b \), \( \phi = 0 \). The radial intensities can be obtained from the pressures and radial particle velocities. The radial particle velocities can be obtained from the pressures and radial impedances. Thus the basic strategy is to determine the radial impedance at the point \( r = b \), \( \phi = 0 \) and the pressures at this point with and without the porous jacket. The radial impedance at this point is of course unchanged by the presence or absence of the jacket.

Consider first the situation when the lagging is not present. The pressure and radial impedance at the point of interest can be found by expressing the harmonic form of the wave equation, Eq. (2), in cylindrical coordinates, assuming a separable solution, Eq. (3) and solving the resulting differential Eqs. (4) for \( R(r) \) and \( \Phi(\phi) \) to give the solutions defined by Eqs. (5):

\[
\nabla^2 p + k^2 p = 0. 
\]

\[
p = PR(r)\Phi(\phi)\exp\left[j(\omega t - k z)\right],
\]

\[
d^2 R \over dr^2 + {1 \over r} {dR \over dr} + \left(k^2 - k_s^2 - {n^2 \over \rho} \right) R = 0, \quad d^2 \Phi \over d\phi^2 + n^2 \Phi = 0.
\]

\[
R(r) = AJ_n(k r) + BN_n(k r), \quad \Phi(\phi) = C \cos n \phi.
\]

where \( A, B, C \) and \( N_n \) are constants and \( J_n \) and \( N_n \) are the \( n \)-th order Bessel and Neumann functions in which \( k_s^2 = k_0^2 - k_r^2 \). The radial particle velocity, \( v_r \), is found from the linearized Euler equation \( \partial p / \partial r = -\rho \partial v_r / \partial t \). This radial particle velocity at the pipe surface can be made equal to the radial velocity of the pipe surface. The radiation condition provides a further boundary condition and so the constants \( A \) and \( C \) can be determined. The pressure at the point \( r = b \), \( \phi = 0 \) then can be found as can the radial impedance. They are as follows:

\[
p = -jV_p \rho_c \left[ J_n(k b) - jN_n(k b) \right] \exp\left[j(\omega t - k z)\right],
\]

\[
z_r = -j \left[ J_n(k b) - jN_n(k b) \right].
\]

Equation (6a) can be written as \( p = pR(r)\Phi(\phi)\exp\left[j(\omega t - k z)\right] \). The radial intensity at the point \( r = b \), \( \phi = 0 \) is then given by \( 0.5 |p| |R|^2 \Re(z) \).

Consider now the situation in which the rigid frame porous jacket surrounds the pipe. There may be a significant air gap between the pipe surface and the inner surface of the jacket. The intention is to find the acoustic pressure at the reference point \( r = b \), \( \phi = 0 \) with this jacket in place so that the radial intensity at this reference point can be found as already described.

The simplest possible model which can be used to describe the propagation of acoustic waves in the gas which saturates the porous material considers only the viscous drag the porous material imposes on the motion of the gas particles. This drag can be quantified by the flow resistivity, \( R_1 \) of the gas in the porous material.

The propagation of acoustic waves in the gas of density, \( \rho \) which saturates the porous medium is governed by the equation

\[
c^2 \nabla^2 p = \frac{\partial^2 p}{\partial t^2} + \frac{R_1}{\rho} \frac{\partial p}{\partial t}.
\]

It can be readily shown from this equation that the wave number, \( k \) and the characteristic impedance, \( Z_0 \), relating to the propagation of plane harmonic waves within the gas contained in the porous material are given by

\[
k = \sqrt{k_w^2 - \frac{1}{\rho \omega}} \frac{\sqrt{R}}{\rho} \frac{1}{2}, \quad Z_0 = \rho c \left(1 - \frac{R_1}{\rho \omega} \right)^{1/2}.
\]

Frequently semiempirical expressions for \( k \) and \( Z_0 \) are used to partially overcome the inadequacies of the simple model which leads to the above equations. The expressions for \( k \) and \( Z_0 \) developed by Delany and Bazley\(^{6} \) and extended to low frequencies by Mechel\(^{7} \) are widely used.

The harmonic form of Eq. (7) then can be written as Eq. (9) and, as with Eq. (3), Eq. (10) is a useful form of solution for this equation:

\[
\nabla^2 p + k^2 p = 0.
\]

\[
p = PR(r)\Phi(\phi)\exp\left[j(\omega t - k z)\right].
\]

It can be seen from Eq. (8a) that if \( R_1 = 0 \), Eq. (9) reduces to Eq. (2), the simple acoustic wave equation which governs the propagation of acoustic waves in the airspace if it is present. Thus the airspace can be considered to be a porous jacket made of a zero flow resistivity porous material.

The harmonic acoustic waves which propagate in the porous jackets as a result of the harmonic structural wave which propagates along the pipe must have the same wave number in the axial direction, that is, the wave number of the structural waves, \( k_s \). The acoustic pressures and the radial particle velocities at the surfaces of the pipe and the jacket must be continuous. The preceding requirements suggest that the analysis to determine the acoustic pressure and hence radial intensity at the reference point could be undertaken by developing formulae which relate, for a given mode, frequency and axial wave number, the acoustic pressure and radial particle velocity at a particular position, say \( \phi = 0 \), on the inside and outside surfaces of a porous jacket. More specifically, one formula is needed to determine the radial impedance on the inside surface when that on the outside surface is known and another is needed to determine the acoustic pressure on the outside surface when that at the corresponding position on the inside surface is known. The development of these formulae is next considered.

After the assumed solution, Eq. (10) is substituted into Eq. (9) the differential Eq. (11), whose solutions are given by Eq. (12) are obtained:

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\[
\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left( k^2 - k^2_z \right) \frac{n^2}{r^2} R = 0, \quad \frac{d^2 \Phi}{dr^2} + n^2 \Phi = 0.
\]

(11)

\[
R(r) = A^* J_n(k, r) + B^* N_n(k, r), \quad \Phi(\phi) = C^* \cos n \phi.
\]

(12)

These equations are identical with Eqs. (4) and (5) except that the wave numbers \( k \) and \( k_z \) are now complex and so the Bessel and Neumann functions are functions of complex arguments. \( k_z \) is given by Eq. (13):

\[
k_z^2 = k^2 - k^2.
\]

(13)

The complex representation of the acoustic pressure, \( p \) at radius \( r \) is then

\[
p = p C^* \left[ A^* J_n(k, r) + B^* N_n(k, r) \right] \cos n \phi \times \exp[j(\omega t - k_z z)].
\]

(14)

The complex representation of the radial particle velocity, \( v_r \) can be found from the above equation and is given by

\[
v_r = -\frac{p C^* k_r}{j Z_0} \left[ A^* J_n(k, r) \right] \cos n \phi \exp[j(\omega t - k_z z)]
\]

(15)

These two equations can be used to develop the required expressions. The impedance equation which gives the radial impedance on the inner surface of a porous jacket at \( r = r_{in} \), that is \( Z_{in} \) when that on the outer surface at \( r = r_{out} \), that is, \( Z_{out} \) is shown is given by Eq. (16):

\[
Z_{in} = -\frac{k}{j} \frac{J_n(k, r_{in}) + \alpha N_n(k, r_{in})}{J_n(k, r_{in}) + \alpha N_n(k, r_{in})} \quad Z_{out} = -\frac{k}{j} \frac{J_n(k, r_{out}) + \alpha N_n(k, r_{out})}{J_n(k, r_{out}) + \alpha N_n(k, r_{out})}
\]

(16)

and the pressure equation, which gives the pressure at \( \phi = 0 \) on the outer surface, \( P_{out} \) when that at \( \phi = 0 \) on the inner surface, \( P_{in} \) is known is given by Eq. (17). The complex quantity \( \alpha \) which appears in both of these equations is given by Eq. (18):

\[
P_{out} = P_{in} \frac{J_n(k, r_{out}) + \alpha N_n(k, r_{out})}{J_n(k, r_{in}) + \alpha N_n(k, r_{in})},
\]

(17)

\[
\alpha = \frac{(Z_{out}/Z_0)(k/\alpha)(k/\alpha)N_n(k, r_{out}) + j J_n(k, r_{out})}{(Z_{out}/Z_0)(k/\alpha)(k/\alpha)N_n(k, r_{out}) + j N_n(k, r_{out})}.
\]

(18)

The insertion loss at a particular frequency for a particular mode is found by making \( V_r \) in Eq. (1) unity and then determining the radial intensities at the reference point \( r = b, \phi = 0 \) with and without the porous jacket. The determination of the radial intensity without the jacket is straightforward as already described. When the jacket is present the radial intensity is found in the following manner. The radial impedance at the reference point \( r = b, \phi = 0 \) can be found from Eq. (6b) and used in Eq. (16) to find the radial impedance at the inside of the porous jacket. If there is an air space, Eq. (16) can be used again to find the radial impedance at the pipe surface. The pressure at \( \phi = 0 \) on the pipe surface then can be found from this radial impedance and the unity pipe velocity. The pressure formula, Eq. (17), then can be used repeatedly to determine the pressures at \( \phi = 0 \) on the jacket surfaces.
IV. EXPERIMENTAL RESULTS

An attempt to verify the insertion losses predicted by the previously described procedure was made by measuring the insertion losses produced by fiberglass jackets fitted to a pipe, hung vertically, which was vibrating in the bending modes. The main features of the test rig are shown in Fig. 6.

A 6-m length of an ammonia pipe with an outside diameter of 48.3 mm and a wall thickness of 7.5 mm was hung from its upper end. An electrodynamic shaker was used to apply a fluctuating force to the upper end of the pipe so that the pipe was forced to vibrate and radiate sound. The pipe was selected so that over a wide frequency range the only modes which could be effectively excited and so radiate sound were the bending modes. This enabled the predicted insertion loss of the lagging associated with pipe vibration in the bending mode to be compared with the measured insertion loss for that mode. An analysis of the dispersion characteristics of the pipe shows that the breathing mode cutoff frequency is approximately at 50 kHz and the ovalling mode cutoff frequency is approximately at 21 kHz whereas the bending mode starts to propagate at zero frequency. The critical frequency for the bending mode, that is, the frequency above which sound will be radiated, is 234 Hz. The fiberglass jackets were located relative to the pipe by a cage formed of six highly tensioned wires which surrounded the pipe and were aligned with it such that there was an air gap of 6 mm between the surface of the pipe and the inner surface of the porous jacket.

The sound power radiated from a length of the bare and lagged pipe was determined by measuring the sound intensity at 60 points on a 1.5-m-long imaginary cylindrical surface surrounding the pipe. A Bruel & Kjaer (B & K) real time analyzer type 2133 and a Bruel & Kjaer sound intensity probe type 2519 were used to make the sound intensity measurements.

Figure 7 shows comparisons of the predicted and measured one third octave band insertion losses produced by two different porous jackets. Figure 7(a) contains the results for a 25-mm-thick jacket made of a low-density fiberglass, whose flow resistivity was 15 300 rayls/m. Figure 7(b) contains the results for a jacket which was also 25 mm thick but whose...
flow resistivity was 120,000 rayls/m. As previously noted the air gap was 6 mm. The predicted results given in these figures were obtained by using the expressions for $k$ and $Z_0$ given by Delaney and Bazley and Mechel. Although over a frequency range of 500 to 2500 Hz the agreement between the predicted and measured results is satisfactory the agreement is not as good outside this range. Difficulties encountered with the intensity measurements could account for the discrepancies, as could the semiempirical expressions for $k$ and $Z_0$.

V. SAMPLE RESULTS

The previously described procedure can be used to calculate the insertion losses produced by porous jackets applied to vibrating pipes. A large number of variables are needed to describe a particular configuration and so only a sample of the results produced can be given. Figure 8(a) and (b) show the predicted insertion losses associated with the bending mode for the ammonia pipe referred to previously. The curves in these figures give an indication of the effect of the porous jacket parameters such as jacket thickness, jacket material flow resistivity, and air gap. It can be seen that in the low-frequency region, which is of particular interest, the propensity for negative insertion losses to occur is reduced by increasing the air gap between the jacket and the pipe. The behavior of the system in the high-frequency region is essentially as expected in that the insertion losses increase with increasing jacket thickness and increasing flow resistivity of the jacket material.

VI. CONCLUSIONS

The results presented in this paper are derived from a simple model of a pipe surrounded by a porous jacket in which a key feature of the model is the matching of radial
velocities at interfaces between the pipe, the airspace if there is one, the porous jacket and the surrounding medium. The agreement between the predicted and measured results for several cases, although not perfect, is sufficiently good to indicate that the model is capable of providing useful predictions. The predicted results, obtained by use of this model, show that for the low-order pipe modes, other than the breathing mode, negative insertion losses at low frequencies generally will occur unless there is a significant air gap between the pipe and the porous jacket. This outcome indicates that a close fitting porous jacket inhibits the tangential particle motion close to the pipe surface in a similar manner as would longitudinal radial fins fitted to the pipe. Thus the pipe appears larger in diameter and so acts as a more effective radiator.