Investigation of the airflow instability in the print gap of inkjet printers

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**Publication Date:**
2023

**DOI:**
https://doi.org/10.26190/unswworks/25182

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Investigation of the airflow instability in the print gap of inkjet printers

Andre Fellipe Vilanova de Araujo Aquino

A thesis in fulfillment of the requirements for the degree of

Doctor of Philosophy

School of Mechanical & Manufacturing Engineering
Faculty of Engineering
The University of New South Wales

May 2023
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<tr>
<td>Journal or Book Name</td>
<td>Physical Review Fluids</td>
</tr>
<tr>
<td>Volume/Page Numbers</td>
<td>7/013804</td>
</tr>
<tr>
<td>Date Accepted/Published</td>
<td>28 January 2022</td>
</tr>
<tr>
<td>Status</td>
<td>published</td>
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<th>Full Title</th>
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### Publication's Details #3

<table>
<thead>
<tr>
<th><strong>Full Title:</strong></th>
<th>The evolution of print defects in inkjet printers at elevated print gap height</th>
</tr>
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<tbody>
<tr>
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<tr>
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</tr>
<tr>
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<table>
<thead>
<tr>
<th><strong>Full Title:</strong></th>
<th>Aerodynamic effects in a narrow-width inkjet printer</th>
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Abstract

There is a demand in the inkjet printing market for systems that can operate at elevated distances from the media. Large print gaps can accommodate media height variations and prevent the media from hitting the printhead, which can damage the nozzles and require expensive maintenance. At elevated print gap heights, however, the printing quality is likely to be compromised. This is because the ink droplets tend to be misplaced on the media, creating printing defects characterised by patterns that resemble tiger-stripes, wood-grain or sand-dunes. This effect has been linked to the airflow instability that misdirects the droplets from their original trajectory. The aim of this thesis is to advance our fundamental and practical understanding of these aerodynamic effects that compromise printing quality.

The study began with a numerical investigation of the two-dimensional onset of instability where the airflow became non-uniform in time. The simulations introduced the dispersed-phase continuum (DPC) approximation to model the interaction between the droplets and the surrounding airflow. For a combination of print gap heights and paper speed, the increase in droplet non-dimensionalised number density ($N$) induced an airflow transition from a steady to an unsteady regime, which is characteristic of the supercritical Hopf bifurcation. The airflow unsteadiness was related to the oscillation of the pair of vortices located near the injection-zone. The two-dimensional instability mode is believed to define the upper bounds of the printing envelope, where the airflow is expected to be uniform in time and spanwise direction.

Numerical analyses using both the DPC and particle-in-cell (P-in-C) models were performed in a three-dimensional domain to better reproduce the operation of inkjet printers. The results computed with both models were compared to assess the effectiveness of the DPC approximation and indicated that the DPC model better correlated to the P-in-C model when the droplet deceleration was taken into consideration. The DPC simulations also demonstrated that, with an increase in $N$, the airflow shifted from a uniform flow regime to a regime which is non-uniform in the spanwise direction, indicating a supercritical pitchfork bifurcation. The spanwise non-uniformity was characterised by the deformation of the main pair of vortices, forming a standing wave flow pattern. The P-in-C simulations captured similar flow features but with elevated computational cost, about a thousand times greater than the DPC model. Furthermore, due to the *pseudoturbulence* induced by the slip motion of droplets that perturbed the flow and excited the standing wave solution at low number densities, the P-in-C model predicted a smooth and continuous transition, which is referred to as an excited supercritical pitchfork bifurcation. Overall, the DPC model captured similar flow dynamics but with the main difference
between models being evident near the critical number density, $N_c$.

Experimental tests using a page-wide industrial inkjet printer rig further refined the understanding of the onset of instability and validated the numerical model based on the DPC approximation. With increases in number density, the prints developed a periodic variation in optical density across the page with darker vertical stripes creating a pattern that resembles corduroy fabric. With further increases in $N$, the stripes became oblique, creating the tiger-stripping pattern. This trend indicated that the transition to the corduroy printing defect defines the bounds of the printing envelope where the prints are uniform. It was then demonstrated that the corduroy printing defect is a result of the standing wave airflow regime captured in the simulation. Based on that, numerical analysis using the DPC model predicted the bounds of the printing envelope for a combination of print gap heights and paper speed.

Direct measurements of the airflow dynamics in a narrow-width inkjet printer were performed using the particle image velocimetry (PIV) technique. Narrow-width printing is of relevance to the packaging industry where labels with QR- and bar-codes are printed. The high-speed planar PIV measurements conducted in two plane orientations (spanwise and longitudinal directions) captured flow features similar to those observed in the simulations. The darker stripes seen in the non-uniform prints were caused by the spanwise disturbance of the airflow that concentrates droplets at specific regions on the paper. When the airflow became temporally non-uniform, the darker stripes became oblique. However, at the edges of the print-zone, tip vortices were observed due to the interaction of the injection-zone with the free-stream. When these vortices became stronger, they spread the droplets over a wider region on the paper, inducing a swelling printing defect.

In summary, the numerical and experimental methods developed in this thesis provided detailed information of the airflow dynamics. It has been demonstrated that non-uniform optical density printing defects such as the corduroy and tiger-stripe pattern are a result of airflow non-uniformity in the spanwise direction and in time. Furthermore, the novel DPC model introduced here has been demonstrated to be an effective alternative to model the interaction between droplets and a fluid carrier phase. Therefore, the results will be of interest to inkjet printer manufacturers as well as the research community focused on investigating particle driven flows.
Acknowledgement

First and foremost, I would like to thank my supervisors, Prof. Tracie Barber, Dr. Samuel Mallinson and Dr. Charitha de Silva.

Prof. Tracie Barber, thank you for introducing me to this fascinating project. Your excitement about science, nature and fluid dynamics is contagious and it makes the PhD journey so much lighter and enjoyable. Your technical expertise, ceaseless support and kind words in every feedback were fundamental to get me this far. I hope that we get to dive into this new world of surfboards hydrodynamics soon.

Dr. Samuel Mallinson, I am so grateful for the coffee meetup when we first met and discussed this project. Changing my career plans was a hard decision at the time, but, after that conversation, there was no doubt left. Furthermore, thank you for the continuous help, the knowledge you shared and for pushing me to go beyond my expectations. I believe that your guidance, support and mentorship were also far above my expectations.

Dr. Charitha de Silva, I am extremely grateful for your supervision throughout this project. When you joined this research, I felt that I had the ‘dream team’ of supervisors. Thank you for always making the time to help me with the experiments and your attention to detail. I am excited to continue our collaboration in the next endeavours.

The three of you have been role models and my source of inspiration. Thank you.

I am also very grateful for the support given by Dr. Sammy Diasinos, my master’s supervisor as you were of great importance to make this journey happen and I will always be thankful for it. The knowledge and experience developed from your supervision during my master’s program were crucial to achieve the goals of this project.

I also would like to express my gratitude to Memjet for funding this project and Memjet employees - Mike Hudson, Graeme Lowe and Glenn Horrocks - for assisting me in several moments of my PhD. A special acknowledgement to Geordie McBain, former Memjet employee, who worked as a mentor for more than two years and shared so much of his knowledge and expertise. You are one of the brightest people I have ever worked with.

Next, I would like to thank my colleagues at UNSW - Dr. Sanjiv, Dr. PJ, Dr. Bac, Dr. Olivia, Dr. Xinxing, Yani, Alex, Dr. Joseph and Dr. Jessie - who were always open to share their knowledge,
experience or just a good laugh. Thank you as well for taking my bowling game to another level.

Next, thanks to my great friends, Julius, Nicole, Matthew, Davidson and Felipe, for all the encouragement, the surfing sessions and the amazing Italian dinners. To my girlfriend Tine, the hard days were not so hard after talking to you. Thank you for being of so much support during the final phase of the PhD.

Finally, to my family who endured the distance and the time apart. I am in forever debt for your love and sacrifices. I could not have asked for more.

I am truly thankful for all of those who made a special contribution during this journey.
Publications and Presentations

List of Journal Publications

  
  – As the primary author, I conducted the numerical simulations and analysed the results by simultaneously incorporating feedback from the co-authors. The original draft was prepared by me and the ensuing reviewer comments were addressed by consulting with the co-authors. The content of this publication has been included in Chapter 3.

  
  – I was involved in developing the numerical model for this study. Additionally, I reviewed the manuscript in the submission process.

  
  – As the primary author, I conducted the experiments and analysed the results by simultaneously incorporating feedback from the co-authors. The original draft was prepared by me and the ensuing reviewer comments were addressed by consulting with the co-authors. The content of this publication has been included in Chapter 4.

  
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with the co-authors. The content of this publication has been included in Chapter 5.


  - As the primary author, I conducted the experiments and analysed the results by simultaneously incorporating feedback from the co-authors. The original draft was prepared by me and the ensuing reviewer comments were addressed by consulting with the co-authors. The content of this publication has been included in Chapter 6.

**List of conference articles**


  - As the primary author, I conducted the experiments and analysed the results by simultaneously incorporating feedback from the co-authors. The original draft was prepared by me and the ensuing reviewer comments were addressed by consulting with the co-authors. The results described in this publication verified the numerical model implemented in Chapter 3.


  - I was involved in developing the numerical model for this study. Additionally, I reviewed the manuscript in the submission process.


  - As the primary author, I conducted the experiments and analysed the results by simultaneously incorporating feedback from the co-authors. The original draft was prepared by me and the ensuing reviewer comments were addressed by consulting with the co-authors. Content in this publication has been reflected in Chapter 6.

**List of presentations**


• Australasian Conference on Computational Mechanics (ACCM 2021): The effectiveness of a novel method to model the entrainment effect of droplets on the airflow in the print gap of inkjet printers

• Australasian Fluid Mechanics Conference (AFMC 2022): Experimental and numerical investigation of the airflow in a narrow-width inkjet print-zone

• Asian Computational Fluid Dynamics (ACFD 2022): Numerical analyses to investigate the airflow dynamics in the print gap of a narrow-width printer
# Contents

- **Abstract** i
- **Acknowledgement** iii
- **Publications and Presentations** v
- **Contents** viii
- **List of Figures** xii
- **List of Tables** xxi

## 1 Introduction 1

- 1.1 Thesis outline 5

## 2 Literature Review 8

- 2.1 Fundamentals of inkjet printing 8
- 2.2 Inkjet printing application 13
- 2.3 Defining and assessing print quality 16
- 2.4 Aerodynamic effects that compromise print quality 20
- 2.5 Systems dynamics 26
- 2.6 Modelling of aerodynamic effects 30
- 2.7 Summary 37
3 Airflow instability in a two-dimensional print gap model

3.1 Introduction ................................................. 40

3.2 Methodology ................................................. 43

3.2.1 Numerical Model .................................... 43

3.2.2 Boundary Conditions ................................ 45

3.2.3 Geometry .............................................. 45

3.2.4 Spatial and temporal discretisation ................. 46

3.2.5 Non-dimensionalizing the print-zone ............... 47

3.3 Results ....................................................... 47

3.3.1 Characterizing the airflow instability ............... 47

3.3.2 Influence of paper speed and induced incoming cross-flow 54

3.3.3 Influence of print gap height ....................... 59

3.3.4 Dynamics of frequency .............................. 60

3.4 Conclusion .................................................. 60

4 Three-dimensional airflow instability and the assessment of the DPC model

4.1 Introduction .................................................. 63

4.2 Methodology ................................................ 68

4.2.1 Numerical model .................................... 68

4.2.2 Geometry and boundary condition ............... 71

4.2.3 Governing dimensionless parameters ............ 72

4.2.4 Flow metrics .......................................... 73

4.2.5 Transient statistics .................................. 73

4.2.6 Spatial discretisation ................................ 74

4.2.7 Temporal discretisation ............................. 75

4.3 Results ....................................................... 76
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3.1 Investigating the DPC assumptions</td>
<td>76</td>
</tr>
<tr>
<td>4.3.2 Bifurcation diagram</td>
<td>81</td>
</tr>
<tr>
<td>4.4 Conclusion</td>
<td>90</td>
</tr>
<tr>
<td>5 The evolution of printing defects</td>
<td>92</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>93</td>
</tr>
<tr>
<td>5.2 Methodology</td>
<td>96</td>
</tr>
<tr>
<td>5.2.1 Experiments</td>
<td>96</td>
</tr>
<tr>
<td>5.2.2 Numerical model</td>
<td>98</td>
</tr>
<tr>
<td>5.3 Results</td>
<td>100</td>
</tr>
<tr>
<td>5.3.1 Experimental tests</td>
<td>100</td>
</tr>
<tr>
<td>5.3.2 Numerical analyses</td>
<td>103</td>
</tr>
<tr>
<td>5.4 Conclusion</td>
<td>108</td>
</tr>
<tr>
<td>6 Aerodynamic effects in a narrow-width inkjet printer</td>
<td>110</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>111</td>
</tr>
<tr>
<td>6.2 Methodology</td>
<td>114</td>
</tr>
<tr>
<td>6.2.1 Printing rig</td>
<td>115</td>
</tr>
<tr>
<td>6.2.2 Laser and camera specifications</td>
<td>118</td>
</tr>
<tr>
<td>6.2.3 Working fluid</td>
<td>118</td>
</tr>
<tr>
<td>6.2.4 PIV process</td>
<td>119</td>
</tr>
<tr>
<td>6.2.5 Printed image post-procesing</td>
<td>120</td>
</tr>
<tr>
<td>6.3 Results</td>
<td>121</td>
</tr>
<tr>
<td>6.3.1 Influence of duty cycle</td>
<td>121</td>
</tr>
<tr>
<td>6.3.2 Influence of paper speed</td>
<td>129</td>
</tr>
<tr>
<td>6.3.3 Influence of print gap height</td>
<td>131</td>
</tr>
</tbody>
</table>
6.4 Conclusion

7 Conclusion and Future Work

7.1 Research Summary

7.2 Future work

7.3 Final remarks

Bibliography
# List of Figures

1.1 Comparative figure showing a good quality print without defects (a) and a bad quality print with features that resemble tiger-stripes (b). ........................................ 1

1.2 Schematic of the airflow physics in the print gap of inkjet printers. ........................... 2

1.3 Printed image showing the tiger-stripe defect where the magnified region demonstrates that the optical density variations are caused by misplaced satellites. Image reproduced from Mallinson et al. (2022). ................................. 3

1.4 Numerical analyses performed by Mallinson et al. (2016) (left) and Barnett and McDonald (2014) (right) showing the vortices created as result of the ejection of droplets in the print gap of inkjet printers. ........................................ 4

2.1 a) Continuous jet breakup in a CIJ printer and b) jet and droplet formation in a DoD printer. Image obtained from Hoath (2016). ........................................ 9

2.2 Schematic of the operation of Continuous Inkjet (CIJ), left, and Drop on Demand (DoD), right, inkjet printers. Figure obtained from Peng et al. (2017). .................. 10

2.3 Computational Fluid Dynamic (CFD) multiphase simulation of a DoD ejection chamber with thermal firing mechanism. Figure obtained from Hoath (2016). ........ 12

2.4 Operating regime of DoD printers with respect to Ohnesorge and Reynolds number. Figure obtained from Hoath (2016). ........................................ 14

2.5 Schematic of the crack propagation in Carbon Fibre Reinforce Polymer (CFRP) with a thermoplastic printed in hexagon pattern in the interlaminar region. Figure obtained from Zhang et al. (2015). ........................................ 15

2.6 Schematic showing the amplitude modulation screening (a), frequency modulation (b) and frequency modulation with randomness (c) screenings for a continuous tone. Figure adapted from Kipphan (2001). ........................................ 17
2.7 Letter X printed to exhibit the impact of non-overlapping dots on human perception. Figure obtained from Ritchie [2017].

2.8 Schematic diagram of the $L^*a^*b^*$ colour space where $L^*$ is the vertical axis and $a^*$ and $b^*$ define the axes of the horizontal plane. Figure obtained from P. and Sanchez [2020].

2.9 Schematics of the Couette-Poiseuille flow profile, which is a result of the interaction between the planar Couette flow induced by the relative motion between media and printhead and the planar Poiseuille flow created by the suction exerted by mist extraction systems.

2.10 Comparison of dot misplacement between the leading edge (left) and trailing edge (right) regions. Picture obtained from Wang [1999].

2.11 Vector plot coloured by velocity magnitude of a simulation of the airflow in the print gap of inkjet printers performed by Mallinson et al. [2016] showing two counter-rotating vortices. Printing from right to left.

2.12 Schematic of the airflow induced by the paper media approaching the print gap.

2.13 Printed pattern obtained from Rodriguez-Rivero et al. [2015] showing the wood-graining printing defect for a paper speed of 0.55 m/s, 13.1 pL droplets, 1000 nozzles fired and 5 mm print gap height. Bottom: the delimited region is displayed in more detail.

2.14 Numerical print pattern obtained from Mallinson et al. [2016] showing the optical density resultant of a simulation at elevated print gap height.

2.15 Schematic of the different jet regions. Figure obtained from Viskanta [1993].

2.16 Flow visualisation of a highly confined ($H/e=2$) impinging jet. Figure taken from Varieras et al. [2007].

2.17 Schematics showing the counter-rotating horse-shoe vortices created by the interaction between the cross-flow and the wall jet. Figure taken from Barata and Durao [2004].

2.18 Example of bifurcation diagrams: a) subcritical and b) supercritical bifurcations.

2.19 Flowchart diagram describing the framework implemented in the one-way (a) and two-way phase coupling algorithms.

2.20 Drag coefficient ($c_{drag}$) for different formulations compared to experimental data [Schlichting and Kestin [1961], p. 17] for droplet Reynolds number ($Re_d$) ranging from 0 to 100.
3.1 Schematic of the physics in the print gap, where $H$ is the print gap height, $W$ is the droplet velocity, $V_p$ is the paper speed, $b$ is the breadth of the print-zone and the origin of the coordinate system is marked by the white square. 

3.2 Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the white square represents the origin of the coordinate system and the white cross represents the probe location. 

3.3 Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$. $St = f H/W$, where $f$ is the frequency of oscillation. 

3.4 Discrete Fourier Transform of the case with $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$. 

3.5 Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.33 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the square represents the origin of the coordinate system and the white cross represents the probe location. 

3.6 Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.33 \cdot 10^{-3}$ and $V_p/W = 0.027$. 

3.7 Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 0.31 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the square represents the origin of the coordinate system and the white cross represents the probe location. 

3.8 Bifurcation diagram for $V_p/W = 0.027$ and $b/H = 1/3$. Top figure: non-dimensionalized amplitude of oscillation $(r^2 = r^2/W^2)$ versus dimensionless number density $(N)$. Bottom figure: initial dimensionless velocity ($u_0 = u_0/W$) versus number density $(N)$. 

3.9 Discrete Fourier Transform for cases $N = 1.3 \cdot 10^{-3}$, $N = 1.33 \cdot 10^{-3}$, $N = 1.41 \cdot 10^{-3}$ and $N = 1.49 \cdot 10^{-3}$. 

3.10 Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0$ where the square represents the origin of the coordinate system and the white cross represents the probe location. 

3.11 Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0$. 

3.12 System of vortex created by the entrained flow. (A) main vortices and (B) secondary recirculation regions.
3.13 Bifurcation diagram - non-dimensionalized amplitude of oscillation ($\tilde{r}^2 = \frac{r^2}{W^2}$) versus dimensionless number density ($N$) - for $V_p/W = 0$ and $b/H = 1/3$.  

3.14 Non-dimensionalised velocity magnitude ($\tilde{u} = \frac{u}{W}$) and planar vorticity ($\tilde{\omega}_x = \frac{\omega_x H}{W}$) fields - $b/H = 1/3$, $N = 1.57 \times 10^{-3}$ and $V_p/W = 0.041$ where the white square represents the origin of the coordinate system and the white cross represents the probe location.  

3.15 Bifurcation diagram - non-dimensionalized amplitude of oscillation ($\tilde{r}^2 = \frac{r^2}{W^2}$) versus dimensionless number density ($N$) - for $V_p/W = 0.041$ and $b/H = 1/3$.  

3.16 Bifurcation diagram - non-dimensionalized amplitude of oscillation ($\tilde{r}^2 = \frac{r^2}{W^2}$) versus dimensionless number density ($N$) for $V_p/W = 0.027$, $b/H = 1/3.3$ and $b/H = 1/3.6$.  

3.17 Strouhal number ($St = \frac{f H}{W}$) versus number density ($N$) for different printing conditions.  

4.1 Schematic of the physics in the numerical model where the origin of the system is located at the center of the injection face.  

4.2 Example print of a ten percent green noise gray image where the non-uniform air-flow in the print-zone gives rise to optical density variations. The magnified region shows the optical density variations are caused by misplaced satellites which are much smaller than the main droplets. Image reproduced from Mallinson et al. [2022].  

4.3 On the left: flow field snapshots adapted from Mallinson et al. [2022], on the right side: prints of flat gray on a A4 page provided by Memjet with print speed of 0.41 m/s, green noise of 10%, a.2) $b/H = 1/2.2$; b.2) $b/H = 1/3.2$; and c.2) $b/H = 1/4$.  

4.4 Flowchart exhibiting the solution algorithm for the P-in-C (a) and DPC (b) models.  

4.5 Comparison of velocity profiles captured by meshes with cell size of $0.0167H, 0.025H$ and $0.0375H$ in simulations using the DPC (a) and P-in-C (b) models.  

4.6 Comparison of velocity profiles in steady-state and transient simulations with time-step of $\Delta \tilde{t} = 0.25, 0.1$ and 0.05.  

4.7 Vector field overlayed on $Q$-criterion contour comparing the DPC and P-in-C models at a vertical plane located at $x/H = 0.9375$ for a case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$ and $\chi = 0$, a) DPC model that disregards the droplet deceleration, b) DPC that accounts for droplet deceleration, c) P-in-C snapshot at $\tilde{t} = 5 \times 10^4$ and d) P-in-C time averaged. A, B and C are, respectively, the upstream and downstream vortices and the vortex spottiness. Paper moving from left to right.
4.8 Point particles colored by their velocity magnitude (left) and horizontal velocity (right) overlaid on the flow field velocity magnitude contour indicating the droplets’ velocity variation as they transit across the print gap. Paper moving from left to right. 79

4.9 Velocity magnitude measured by a longitudinal transect located $x/H = 0.9375$ and $z/H = -0.5$ comparing the DPC with and without droplet velocity correction (dark red and blue lines respectively) and P-in-C (green dots) for a case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. 80

4.10 Top image: $x$-velocity signal measured by a probe located at $x/H = 0.9375$, $y/H = 0$ and $z/H = -0.5$. Bottom image: DFT of the velocity signal showing the frequency behaviour of the spottiness. Printing conditions: $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. Paper moving from top to bottom. 81

4.11 On the left side: top view of a $xy$ plane located at $z/H = -0.5$ showing isosurface of $Q$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\bar{u}_x$, comparing, while on the right side: $\bar{w}_x$ contour at a vertical plane located at $x/H = 0.9375$, both set of plots comparing a) Spanwise-uniform flow field regime with $N = 0.267 \times 10^{-3}$, b) Standing wave regime for a case with $N = 0.267 \times 10^{-3}$ and $\lambda/H = 1.667$ and c) Standing wave regime for a case with $N = 0.267 \times 10^{-3}$ and $\lambda/H = 2.667$. Printing conditions: $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. Paper moving from left to right. 82

4.12 Bifurcation diagram showing the transition from uniform flow to standing wave regime for DPC solutions with wavelength $\lambda/H = 2.143$ and 1.875 at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. 83

4.13 Isosurface of $Q$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\bar{u}_x$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C for a case with $b/H = 1/3$, $V_p/W = 0.027$, $V_p/W = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. a) DPC solution; b) P-in-C time average; and, snapshot at $\bar{t} = 10^{-2}$ c). Paper moving from left to right. 84

4.14 Lateral velocity profile taken at a spanwise transect located at $y/H = 0$ and $z/H = -0.5$ for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C models. 85

4.15 Bifurcation diagram at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C models. 86

4.16 Isosurface of $Q$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\bar{u}_x$ for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.267 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing: a) the DPC with $\lambda/H = 1.875$; b) P-in-C time averaged; and c) P-in-C snapshot at $\bar{t} = 5 \times 10^{-3}$. Paper moving from right to left. 87
5.1 Schematic of the physics in the numerical model where the origin of the system is located at the center of the injection face. .................................................. 94

5.2 On the left: flow field snapshots adapted from Mallinson et al. (2022), on the right side: prints of flat gray on an A4 page provided by Memjet with print speed of 0.41 m/s, green noise of 10%, a) $b/H = 4/7$; b) $b/H = 1/2$; and c) $b/H = 1/3$. ... 95

5.3 a) Schematic of the printing rig and b) Schematic of the printhead, with dimensions in millimetres (not to scale). ................................................................. 97

5.4 Example of the approach used to post-process the raw signal. .................................................. 98

5.5 Prints of flat cyan with different number densities on an A4 page converted to gray scale with contrast and brightness adjusted in ImageJ [Ferreira and Rasband, 2012] to facilitate the identification of printing defects: a) $N = 1.26 \times 10^{-4}$; b) $2.18 \times 10^{-4}$; c) $N = 2.89 \times 10^{-4}$ and d) $N = 5.43 \times 10^{-4}$ respectively showing uniform optical density, beginning of corduroy defect, transition to tiger stripes and fully developed tiger stripes. .................................................. 101

5.6 $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the darkness levels for uniform prints (a) and prints with the corduroy printing defect (b). .................................................. 102

5.7 DFT of the post-processed $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the wavelength spectrum for uniform prints (a) and prints with the corduroy defect (b). .................................................. 103

5.8 DFT of the post-processed $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the wavelength spectrum for uniform prints (a) and prints with the corduroy defect (b). .................................................. 104

5.9 Bifurcation diagram showing the transition from uniform flow to standing wave regime for DPC solutions with wavelength $\lambda/H = 2.08$ and $1.79$ at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ overlayed on the experimental bifurcation diagram (blue). .................................................. 106

5.10 Iso-surface of $Q$-criterion overlayed on a plane located at $z/H = -0.5$ coloured by $\tilde{u}_x$ with overlayed velocity vectors for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. .................................................. 107

5.11 Printing envelope diagram showing the critical number density ($N_c$) for print gap heights equal to $b/H = 1/3.6, 1/3$ and $b/H = 1/2.4$ and paper speed varying from $V_p/W = 0.0136$ to $0.0612$ .................................................. 108

6.1 Schematic of the physics in the numerical model where the origin of the system is located at the center of the injection face. .................................................. 112
6.2 Example of an image with ten percent of duty cycle, i.e., ratio of nozzles effectively active, where the non-uniform air flow in the print zone gives rise to optical density variations. The magnified region shows the optical density variations are caused by misplaced satellites which are much smaller than the main droplets. Image reproduced from Mallinson et al. (2022).

6.3 a) Image of the narrow-width inkjet printing device showing the flex connector, BETTI tile and chip with nozzles; b) Schematic view of the nozzles distribution on the chip.

6.4 a) Photograph showing the printing rig, high speed camera and computer that controls the printing process; b) Schematic view of the experiment when PIV measurements are taken for a plane in the longitudinal direction; c) Schematic view when measurements are taken for a plane in the spanwise direction.

6.5 Scan of the printed pattern (a) and (b) longitudinal PIV measurements taken at $x/H = 0$ for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the upstream and downstream vortex for different normalised times ($\tilde{t}$), where $\tilde{t} = t \times L/V_p$, after the start of the printing process ($\tilde{t} = 0$ s). Here the paper moves from left to right and the black dashed line represents the laser sheet.

6.6 Scan of the print (a), spanwise PIV measurements taken at $y/H = -2.3$ (b) and flow visualisation images (c) for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap and the droplet distribution for different normalised times ($\tilde{t}$), where $\tilde{t} = 0$ s is the start of the printing process. A indicates the vortex near the printhead, B and C highlight the reminiscent tip vortex, D shows the uniform droplet distribution at the centre of the print and E portrays the effect of the tip vortex on the droplet distribution near the paper. Here the paper moves out of the page.

6.7 Spanwise PIV measurements taken at $y/H = -1.7$ for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz. Note that this plane is located upstream of that shown in Figure 6.6. The velocity vectors overlayed on the velocity magnitude contour show the tip vortex (A) at the edge of the injection-zone.

6.8 Scan of prints (a) at $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz with number density equal to $N = 0.77 \times 10^{-4}$ (A), $N = 0.92 \times 10^{-4}$ (B), $N = 1.23 \times 10^{-4}$ (C) and $N = 1.54 \times 10^{-4}$ (D); enlarged view of the scanned prints (b) and $L^*$ profiles of the enlarged region (c).

6.9 Summary of printing tests conducted at $N = 1.54 \times 10^{-4}$ (f) to show the variability of the experimental print results among different tests.
6.10 Scan of the print (a) and longitudinal PIV measurements taken at $x/H = 0$ (b) for a case at $N = 1.54 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz. The velocity vectors overlayed on the velocity magnitude contour show the upstream and downstream vortex for different normalised times after the start of the printing process ($t = 0$ s). Here the paper moves from left to right and the black line represents the laser sheet.

6.11 Scan of the final print (a), spanwise PIV measurement taken at $y/H = -2.3.3$ (b) and flow visualisation images (c) for a case at $N = 1.54 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap and the droplet distribution for different normalised times ($t = t \times L/V_p$), where $t = 0$ s is the beginning of the printing process. A and B highlight new recirculation zones, C shows a region where the droplets are densely concentrated and D indicates the high optical density stripe created by region C. Here the paper moves out of the page.

6.12 Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.3.3$ (b) and flow visualisation images (c) for a case at $N = 2.31 \times 10^{-4}$ ($\xi = 1.97\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap, which includes a well defined tip vortex (A), and a uniform droplet distribution for different times. Here $t = 0$ s is the beginning of the printing process and the paper moves out of the page.

6.13 Print tests at $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz with number density equal to $N = 2.75 \times 10^{-4}$, $N = 3.14 \times 10^{-4}$ and $N = 3.92 \times 10^{-4}$; enlarged view of the scanned prints at locations A, B and C (b) and $L^*$ profiles of enlarged region (c).

6.14 Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.3.3$ (b) and flow visualisation images (c) for a case at $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show a number of new vortices across the print gap and a non-uniform droplet distribution.

6.15 Scan of the print (a), longitudinal PIV measurements taken at $x/H = 0$ (b) and flow visualisation images (c) for a case at $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz. The velocity vectors overlayed on the velocity magnitude show the upstream and downstream vortex for different normalised times after the start of the printing process ($t = 0$ s).

6.16 Scan of prints at $V_p/W = 0.013$, $b/H = 1/4.1$ and $f_n = 3.875$ kHz with number density equal to $N = 0.83 \times 10^{-4}$, $N = 1.04 \times 10^{-4}$ and $N = 2.08 \times 10^{-4}$; enlarged view of the scanned prints at locations A.1, A.2, B and C (b) and $L^*$ profiles of the enlarged region (c).
6.17 Scan of the print (a), spanwise PIV measurement taken at \( y/H = -2.33 \) (b) and flow visualisation images (c) for a case at \( N = 0.83 \times 10^{-3}, V_p/W = 0.013, b/H = 1/4.1 \) and \( f_n = 3.875 \text{kHz} \). The velocity vectors overlayed on the velocity magnitude contour show a wide tip vortex (A) and the droplets being spread by this vortex (B). Here the paper is moving out of the page.

6.18 Scan of the print (a), spanwise PIV measurement taken at \( y/H = -2.33 \) (b) and flow visualisation images (c) for a case at \( N = 2.08 \times 10^{-4}, V_p/W = 0.013, b/H = 1/4.1 \) and \( f_n = 3.875 \text{kHz} \). Here the paper is moving out of the page.

7.1 Printing envelope diagram showing the critical number density \((N_c)\) for print gap heights equal to \( b/H = 1/3.6, 1/3 \) and \( b/H = 1/2.4 \) and paper speed varying from \( V_p/W = 0.0136 \) to \( 0.0612 \).
List of Tables

2.1 Expressions for the drag correction factor as a function of Reynolds number ($\phi(Re_d)$) 36

6.1 Summary of the experimental conditions and equipment configuration. . . . . . 116

6.2 Experimental settings - laser frequency, double frame separation and firing frequency - for different paper speeds . . . . . . . . . . . . . . . . . . . . . . . . . . . . 119
List of Symbols

$\alpha$  
first Lyapunov coefficient

$\alpha_d$  
volumetric ratio between dispersed and fluid carrier phases

$\bar{V}_a$  
Poiseuille flow average velocity

$\Delta E$  
Euclidean difference in the $L^*a^*b^*$ colour scheme

$\Delta x$  
nozzle pitch

$\gamma$  
phase subscript

$\gamma_d$  
ink surface tension

$\lambda$  
standing wave wavelength

$\tilde{u}$  
non-dimensionalized airflow velocity

$F$  
body-force induced by the point particles in the Navier-Stokes equation

$F_k$  
body-force induced by the point particles at the $k$-cell

$f^m_k$  
force experienced by the $m$-th particle at the $k$-cell

$k$  
unit vector in $z$-direction

$U, u_d$  
dispersed phase velocity field

$u$  
airflow velocity field

$\mathcal{H}$  
volume-filtering kernel

$\mu$  
dynamic viscosity of air

$\mu_d$  
ink dynamic viscosity

$\omega$  
vorticity

$\rho_d$  
droplet density

$\rho_f$  
fluid carrier density
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>number of nozzles per unit area on the injection face</td>
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<tr>
<td>$a^*$</td>
<td>red-green value in the $L^*a^<em>b^</em>$ colour scheme</td>
</tr>
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<td>$b^*$</td>
<td>blue-yellow value in the $L^*a^<em>b^</em>$ colour scheme</td>
</tr>
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<td>$L^*$</td>
<td>lightness value in the $L^*a^<em>b^</em>$ colour scheme</td>
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<td>$\tilde{\chi}$</td>
<td>non-dimensionalized through-flow</td>
</tr>
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<td>$\tilde{\omega}$</td>
<td>vorticity</td>
</tr>
<tr>
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<td>non-dimensionalized pressure</td>
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<tr>
<td>$\tilde{Q}$</td>
<td>non-dimensionalized $Q$-criterion</td>
</tr>
<tr>
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</tr>
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<td>drag coefficient correction factor</td>
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<tr>
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<td>duty-cycle</td>
</tr>
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<td>frequency of airflow oscillation</td>
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<td>Couette flow rate</td>
</tr>
<tr>
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<td>transit Reynolds number</td>
</tr>
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<td>amplitude of airflow oscillation</td>
</tr>
<tr>
<td>$Re_d$</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$w$</td>
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</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates</td>
</tr>
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**Acronym**

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<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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</thead>
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</tr>
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</tr>
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<td>computational fluid dynamics</td>
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<td>carbon fibre reinforce polymers</td>
</tr>
<tr>
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</tr>
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<td>discrete fourier transform</td>
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<td>DoD</td>
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</tr>
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<td>particle resolved</td>
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<td>red, green and blue</td>
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Chapter 1

Introduction

Inkjet printers deposit ink droplets onto a substrate in predefined positions, creating a two-dimensional pattern. Due to the easy control of the printing pattern and the repeatability of the process, the use of inkjet printers is no longer limited to graphics applications and has been expanded to manufacturing electronics, solar panels, rapid prototypes, and reinforced composites (Arango et al., 2019; Hoath, 2016; Zapka, 2018). In these applications, fine features with resolutions of tenths of millimetres are reproduced, which demands highly accurate deposition of droplets on the substrate (Hoath, 2016).

The expansion of inkjet technology is limited by the restrictive print gap height \( H \) i.e., the distance from the media to the nozzles (Figure 1.2), at which inkjet printers currently operate.

Figure 1.1: Comparative figure showing a good quality print without defects (a) and a bad quality print with features that resemble tiger-stripes (b).
Small print gaps prevent the printer from accommodating media with variable or large thickness and increase the likelihood of the media striking the printhead, which can damage the nozzles, demanding expensive maintenance. At high print gap heights \((H > 1 \text{ mm})\), however, the print quality is likely to be compromised due to the misplacement of droplets. When a uniform coloured image (Figure 1.1(a)) is printed at elevated print gap heights, specific aerodynamic effects misplace the droplets, especially the small ones, on the paper, creating printing defects that resemble tiger-stripes, wood-grain or sand-dunes (Figure 1.1(b)) \cite{Hsiao2012,Hsiao2013,Mallinson2016,Rodriguez-Rivero2015}.

The droplets' response to the flow field can be defined according to their Stokes number (Equation 1.1), which is the ratio of the characteristic droplet time to the characteristic flow time. The satellite droplets result from the jet filament breakup, and, due to their low Stokes number \((S < 1)\), they are more likely to be carried by the airflow. However, the main droplets tend to have a ballistic trajectory as a result of their high Stokes number \((S > 10)\).

\[
S = \frac{\rho_d d^2 W}{18 \mu H}, \tag{1.1}
\]

Here, \(\rho_d\) is the droplet density, \(d\) is the diameter of the droplet, \(\mu\) is the dynamic viscosity of air and \(W\) is the droplet ejection velocity \cite{Mallinson2016}.

When large print gaps are employed \((H > 1 \text{ mm})\), the airflow can become unstable and this behaviour tends to misdirect the droplets. The airflow is considered unstable when it becomes
1. Introduction

Figure 1.3: Printed image showing the tiger-stripe defect where the magnified region demonstrates that the optical density variations are caused by misplaced satellites. Image reproduced from Mallinson et al. (2022).

non-uniform in time and/or in the spanwise direction. This is because, in a good quality page-width print, the droplets are homogeneously distributed in the longitudinal and spanwise directions, and, to achieve that, the base flow is expected to be nearly two-dimensional in the spanwise direction (x-direction) and stationary. At the onset of instability, the satellite droplets tend to land away from the main droplets. This process increases the number of dots in the region where the droplets have been misplaced, and, as a result, a high optical density region (dark feature) is perceived on the printed image. This is depicted in Figure 1.3 which shows the satellite droplets landing on the white space between the lines created by the main droplets. Thus, in those regions, the images appear darker. The different patterns in which the droplets are misplaced by the airflow induce specific macroscopic printing defects like those observed in Figure 1.1b (de A. Aquino et al., 2020; Mallinson et al., 2016, 2020).

The airflow in the print gap is characterised by two-counter rotating vortices that are created when the entrained flow resulting from the exchange of momentum between the droplets and the surrounding air, impinges on the paper (Figure 1.4). It has been reported that, at specific operating conditions, these vortices become unstable, misplacing the droplets and creating
1. Introduction

Figure 1.4: Numerical analyses performed by Mallinson et al. (2016) (left) and Barnett and McDonald (2014) (right) showing the vortices created as result of the ejection of droplets in the print gap of inkjet printers.

printing defects (Barnett and McDonald, 2014; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015). However, the conditions at which this instability occurs as well as the mechanism of instability are not yet fully understood. Previous studies have suggested that the instability is caused by the interaction with the incoming cross-flow induced by the relative motion of the media and the printhead (Barnett and McDonald, 2014; Hsiao et al., 2013; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015). This is unlikely to be the triggering mechanism since similar systems such as laminar impinging jets develop self-sustainable instabilities even without cross-flows (Varieras et al., 2007). This interaction, however, plays a key role on the overall airflow dynamics. The characteristics of the incoming cross-flow combined with the print gap height, nozzles spacing and firing frequency modulate the range of flow regimes and define the bounds of the printing envelope (Barnett and McDonald, 2014; Hsiao et al., 2013; Mallinson et al., 2022; Rodriguez-Rivero et al., 2015). The different flow regimes (stable, transitioning, unstable) are expected to have specific impacts on the droplet trajectory and, as a result, lead to different printing patterns.

The aim of this thesis is to refine the understanding of the flow dynamics that lead to printing defects like tiger-stripes. The investigation focuses on the range of flow regimes while relating the flow features to the occurrence and severity of printing defects. To achieve that, experimental and numerical analyses are employed. For the experimental analyses, printing tests and high speed planar particle image velocimetry (PIV) are performed to characterise the evolution of printing defects and characterise the airflow dynamics, respectively. For the numerical work, the novel dispersed-phase continuum (DPC) technique is employed. The DPC model assumes that the droplets can be modeled as a continuum source of momentum with a pre-
defined temporally averaged number density. This approximation is expected to be a more computationally efficient strategy than the well-established particle-in-cell (P-in-C), eulerian-eulerian (EE) and particle resolved (PR) models (Capecelatro and Desjardins, 2013a; Seydel, 2007). This thesis also assesses the DPC approximation by comparing it against the P-in-C model and experimental tests.

The combination of experiments and simulations allows a comprehensive understanding of the problem. This is because the simulations offer information about the three-dimensional flow features in the print gap, even in regions of difficult optical access. Furthermore, the numerical model can isolate the system from ambient disturbances, allowing a detailed investigation of the nature of the airflow instability, capture real-world conditions and complex physics that may not be fully captured by numerical models. However, experiments can be expensive, requiring the development of complex systems, and may have limited accessibility during situations like the COVID-19 pandemic. By integrating simulations and experiments, it was possible to validate and refine simulations, capture real-world complexities, and overcome the limitations associated with experiments alone.

1.1 Thesis outline

Chapter 1 summarises the rationale and the main physical concepts related to printing defects induced by aerodynamic effects, such as tiger-striping. The gaps in the literature are highlighted and the aim of the research is introduced. The main concepts are further detailed and evaluated throughout the following chapters.

Chapter 2 is a critical review of previous literature on the key aspects of the problem: fundamentals and applications of inkjet printers, printing quality, aerodynamic effects and droplet modelling. The critical discussion on the aerodynamic effects and various modelling techniques indicates the opportunity for development of an alternative approximation - the DPC strategy - to model the entrainment phenomenon. It is suggested to benchmark the DPC model against the well-established P-in-C technique.
Chapter 3 is a published manuscript entitled 'Investigation of the vortex instability in a two-dimensional inkjet injection zone (printing zone) using numerical analysis' in the Physical Review of Fluids journal. This study makes use of two-dimensional numerical simulations based on the DPC approximation to shed light into the dynamics of the airflow in the print-zone while characterising the upper bound at which the base flow meets the conditions for a uniform print: uniform in time and in the spanwise direction.

Chapter 4 is a submitted article (to be accepted pending minor revisions in the Physical Review of Fluids journal) entitled ‘The effectiveness of the dispersed-phase continuum model for investigating the airflow in the print gap of inkjet printers. This study assesses the effectiveness of the DPC model to model the time averaged momentum exchange between droplets and the airflow. Both the DPC and P-in-C are used to predict the dynamics of the system and characterise the onset of instability.

Chapter 5 is a submitted manuscript (under review in the International Journal of Heat and Fluid Flow) entitled ‘The evolution of printing defects in inkjet printers at elevated print gap height’. In this article, the optical density of images printed by a page-wide industrial inkjet printer is measured to characterise the onset of printed image non-uniformity and validate the DPC model. The numerical analyses are also used to estimate the operating envelope for a page-wide printing system.

Chapter 6 is a submitted manuscript (under review in the Experiments in Fluids journal) entitled ‘Aerodynamic effects in a narrow-width inkjet printer’. This characterises the airflow dynamics at the onset of instability of a narrow-width printer using particle image velocimetry (PIV). The key flow features are characterised and their impact on print quality are directly assessed. Tests at different paper speed and print gap height also demonstrate the effect of these parameters on the bounds of the print envelope. Narrow-width printing is of particular interest to the packaging industry where QR- and bar-codes are frequently printed on labels.

Chapter 7 is a final chapter where the key results and findings of the chapters are summarized and discussed and suggestions for the future directions are made. The significance of the investigations for the inkjet printing industry, particle-laden flow dynamics and airflow stability
is highlighted.
Chapter 2

Literature Review

Inkjet printing is a surprisingly complex technology that involves a wide number of sciences, including: MEMS, image processing, colour science, fluid dynamics and computational modelling. In this chapter, the fundamentals of inkjet printing are introduced and linked to the current state of inkjet printing applications, e.g., printed electronics, composites reinforcement and 3D-printing. Next, the key concepts that define print quality are presented and the different strategies for its assessment are explored. Focus is given here to critically review previous studies on aerodynamic effects that compromise print quality and discuss the different techniques used to model the interaction between droplets and a fluids carrier.

2.1 Fundamentals of inkjet printing

The inkjet printing process consists in building an image by firing a number of tiny ink drops onto the substrate. These droplets have a diameter ranging from 10 to 100 µm and are deposited in a predefined pattern. Digital image printing of graphics is the most common application of inkjet printing. In this application, droplets are deposited on the substrate in the x- and y-directions in such a way that the final dots on the printing media are perceived as a continuous figure (Hoath, 2016).
2. Literature Review

Figure 2.1: a) Continuous jet breakup in a CIJ printer and b) jet and droplet formation in a DoD printer. Image obtained from Hoath (2016).

The main components of an inkjet printing system are the printhead, substrate, ink and ink supply system. Ink travels through the ink supply system to the printhead where an actuator mechanism fires a jet of ink that breaks up into droplets. Those droplets are deposited on the substrate in a predefined pattern that reproduces the original image in a discretised manner. This process can be achieved with a single- or multi-pass printer. Single-pass inkjet printers, as the one investigated in this project, deposit a page-wide swath of droplets onto the moving media. This technology is considered to be more suitable for industrial applications due to its higher printing speed. The inkjet printing process differs from conventional printing technologies, such as lithography, flexography, gravure and screen, which use a physical form of template embodied on a roll, plate or screen to reproduce the image on the printing media (Pond, 2000).

In comparison to laser printing, inkjet printers have the advantage of providing elevated precision and printing quality due to their nature of creating homogeneously sized droplets at a determined speed and direction (Castrejon-Pita et al., 2013; Hoath et al., 2013; Pond, 2000). Furthermore, inkjet printers are linked to lower emissions of harmful volatile organic compounds (VOC’s) (Lee et al., 2001).

Drop on demand (DoD) and continuous inkjet (CIJ) are the predominant technologies employed in inkjet printers (Hoath, 2016; Pond, 2000; Zapka, 2018). The difference between these methods lies in the nature of the flow (Figure 2.1). Whilst CIJ technologies create a pressure perturbation that ejects a continuous jet of ink that breaks up into droplets (Figure 2.1a) (Castrejon-
2. Literature Review

Figure 2.2: Schematic of the operation of Continuous Inkjet (CIJ), left, and Drop on Demand (DoD), right, inkjet printers. Figure obtained from Peng et al. (2017).

Pita et al. (2013) [Ikegawa et al. 2014], DoD systems (Reddy et al. 2007) [Wijshoff 2015] [Yoo and Kim 2013] use a pressure pulse to create a single droplet ejection (Figure 2.1 b). At first, these two technologies were used to attend to different demands of the market since CIJ reached higher printing speed while DoD provided higher resolution and colour flexibility (Castrejón-Pita et al. 2021). However, improvements in DoD printing have increased the maximum speed at which these systems can operate, making them the main technology in inkjet printing (Hoath 2016).

In a CIJ printer, the pressure perturbation is finely tuned to break the continuous flow into a train of droplets (Castrejón-Pita et al. 2021) [Hoath et al. 2013]. This breakup process was first characterised by Plateau (1873) and Rayleigh (1878, 1892), who demonstrated that the distance between the resulting droplets is about 4.5 times the diameter of the jet. Due to the importance of these studies for the breakup of liquid jets and atomization of droplets (Lefebvre and McDonell 2017), the phenomenon is often referred to as the Rayleigh-Plateau instability.

Even though droplets are continuously formed, only a number of them are printed on the substrate to create the desired pattern (Hoath 2016) [Ikegawa et al. 2014]. This can be done with an airflow (Delametter et al. 2007) [Jeanmaire 2003] or charge deflection technique, which is the most common technology used in CIJ. It consists in electrically charging target droplets and us-
ing an electrostatic field to steer them into the desired location on the paper (Figure 2.2a). The remaining droplets that are unaffected by the electrostatic field land in a gutter, where they are recovered and recycled back to the ink supply system. Kodak, Domino, Scitex Iris, Stork, Imaje, VideoJet are the main manufactures of CIJ printers (Kumar and Baral, 2022a).

In the DoD technology, a pressure pulse is created to eject a single droplet. When the digital control system interprets that an ink drop needs to be deposited onto the paper to create the desired pattern (Hoath, 2016; Pond, 2000). A firing mechanism is used in the ejection chamber to create the pressure pulse, forcing an ink flow through the nozzle which leads to the jet formation. The pressure pulse can be produced either by a thermal, piezoelectric, electrostatic, acoustic or mechanical actuators (Hutchings and Martin, 2013; Wijshoff, 2010). Piezoelectric and thermal techniques are the most widely used in the industry (Choi et al., 2008; Kumar and Baral, 2022a). When a voltage pulse is sent to the piezoelectric actuator, it becomes deformed and bends, squeezes or pushes the ejection chamber (Wijshoff, 2010). This mechanical motion forces the ink inside the chamber to flow through the nozzle (Hoath, 2016; Hoath et al., 2013; Pond, 2000; Reddy et al., 2007; Yoo and Kim, 2013). For the thermal technique, the process consists in sending a voltage pulse to the heater to vaporise an amount of ink and create a vapour bubble. This pressurises the ink, forcing it through the orifice, which then forms a droplet (Figure 2.3). The thermal actuator can be sub-classified in either side- or roof-shooter depending on the location of the resistive heater. This technique allows the printers to be compact and have a high nozzle density (Yano, 1982). HP, Canon, Lexmark, Kodak, Memjet, Kyocera and Jetinks are some of the manufacturers that make use of the DoD technology (Kumar and Baral, 2022a).

During the ejection, the ink undergoes a fast stretching process that forms a main droplet attached to a long and thin filament (thread, tail or ligature). This process is characterised by extensional deformation rates of the order of 3000 s\(^{-1}\) and shear rates of magnitude higher than 10\(^5\) s\(^{-1}\) (Hoath et al., 2013). As the main droplet moves, the filament undergoes a severe constriction until it breaks up or pinch off at the nozzle exit. The filament may then contract to form an almost spherical droplet or break up into one or multiple smaller droplets, commonly referred to as satellite droplets (Figure 2.1). Studies have also shown the existence of a cascade
2. Literature Review

Figure 2.3: Computational Fluid Dynamic (CFD) multiphase simulation of a DoD ejection chamber with thermal firing mechanism. Figure obtained from Hoath (2016).

process, where the satellite droplets may also form secondary tails, which further breaks up into tertiary and quarternary tails (Fraters et al., 2020; Shi et al., 1994).

The stability of the filament is modulated by the balance between inertia, viscosity and surface tension forces acting on the liquid (Castrejón-Pita et al., 2021; Hoath, 2016; Zapka, 2018), which is defined by the Ohnesorge number ($Oh$) (Equation 2.1). In summary, these parameters dictate whether any disturbances due to vibration, aerodynamics, thermocapillarity and viscous effects will be dampened or grow, causing the filament breakup. We note here that for high aspect ratio filaments, the Rayleigh-instability plays a crucial role for the pinch-off effect in DoD printers (Anthony et al., 2019).

$$\text{Oh} = \frac{\mu_d}{\sqrt{\rho_d \gamma_d d}} \quad (2.1)$$

where $\mu_d$ is the ink viscosity, $\rho_d$ is the density, $\gamma_d$ is the surface tension, $d$ represents the droplet diameter and the subscript $d$ refers to the droplets.
When the viscous forces are significantly larger than the surface tension and inertial forces, i.e., $Oh$ is high, the droplet ejection is prevented. According to Castrejón-Pita et al. (2021), the maximum viscosity permitted for typical DoD inkjet inks is approximately 25 mPa·s and droplet ejection is impeded beyond that. However, if the liquid viscosity is too low, the filament becomes unstable and breaks up in several satellite droplets. There has been an extensive number of studies that described the range of $Oh$ for DoD inkjet printers: $0.1 < Oh < 1$ (Reis and Derby, 2000), $0.07 < Oh < 0.25$ (Jang et al., 2009). The difference in results may be due to the fact that the filament breakup depends on the pressure waveform and the presence of additives, such as polymers and surfactants. The presence of cells in bio-printing also tends to change the bounds of the printability envelope. For instance, Chen et al. (2023) indicated that stable droplet generation (satellite-free ejections) is achieved when $0.067 < Oh < 0.5$. According to Castrejón-Pita et al. (2021), modern jetting mechanisms have expanded the jettability printing envelope of DoD inkjet systems to Ohnesorge numbers larger than 2 and with inks of viscosity that exceed 25 mPa·s. The 3-cycle and the high laydown modes are examples of these modern systems that have pushed the jettability envelope. A more detailed description of the operation of these systems can be found in Jackson et al. (2019) and Castrejón-Pita et al. (2021).

The jettability or printability region is yet defined by additional constraints associated with the jet inertial forces, which is quantified by the jet Reynolds number $Re_{jet} = \frac{Wd}{\mu}$, where $W$ represents the droplet ejection velocity. This is because the jet must have enough energy to overcome the viscous and surface tension forces but without causing rebound or splash on the substrate. Figure 2.4 summarises the bounds of the suggested operating regime of DoD inkjet printers, with respect to Ohnesorge and Reynolds numbers (Hoath, 2016).

### 2.2 Inkjet printing application

The inkjet manufacturers have expanded the use of this technology to several other market segments, such as printed electronics, composites, spray coating, bioprinting and 3D manufacturing. This is because the process of depositing droplets on a substrate in a controllable, repeatable, and uniform manner allows a variety of structures to be built.
2. Literature Review

Figure 2.4: Operating regime of DoD printers with respect to Ohnesorge and Reynolds number. Figure obtained from Hoath (2016).

Digital image printing of graphics is still the most common application of inkjet printing. In this application, droplets are deposited on the substrate in the $y$- and $x$-directions.

The flexibility and low cost of the inkjet printing process have made it attractive to manufacture printed electronics (Zapka, 2018). The process involves the deposition of metallic or polymeric droplets on a substrate until they fuse to create electrodes. Metallic inks are either composed by a suspension of metallic nanoparticles or metal salt dissolved in a solvent (Arango et al., 2019; Hoath, 2016). Due to their higher conductivity and lower fusion temperature, metallic inks are usually preferred over polymeric inks (Hoath, 2016). Polymeric inks, however, are used to produce flexible electronics and photovoltaic devices. The inkjet process shows a great potential for the manufacturing of solar cells because no vacuum system is required, significantly reducing operational costs (Bidoki et al., 2007; Castrejon-Pita et al., 2013; El-Molla, 2017).

Inkjet printing of polymeric inks has also been employed to improve the mechanical properties of composites (Zhang et al., 2014a, 2014b, 2015). Laminated composites using Carbon fibre reinforced polymers (CFRP) show elevated stiffness-to-weight and strength-to-weight ratios, but they are susceptible to delamination, which is hard to detect and considered the most typical failure mode. Effort has been made to prevent this kind of mechanical failure by adding inter-
facial reinforcement and using thermoplastic toughening agents as a matrix additive. However, these strategies reduced the specific strength and elastic modulus of CFRP's due to the significant increase of the polymeric fraction. As inkjet printing is a non-contact, highly scalable, high resolution, and flexible printing technique, it has been used as an alternative solution. Droplets of thermoplastic polymer are printed in pre-defined patterns in the interlaminar region, increasing the bonding strength between layers without significant weight gain (Zhang et al., 2014a,b, 2015). The polymer particles acted as crack stoppers by increasing the energy necessary to propagate the crack (Figure 2.5).

The inkjet technology has been extended to many two-dimensional applications. It is also used in three-dimensional manufacturing (Zapka, 2018): the process consists of repeatedly building layers of two-dimensional cross-sections, which combine to create three-dimensional objects. Three-dimensional inkjet printing has been used to manufacture rapid prototypes, automotive components, turbine parts, prosthetic devices, biological tissues etc (Gibson et al., 2021). Evaporation, material phase change and powder bed printing are some of the techniques employed. A more detailed description of these techniques is given by Hoath (2016).

In all these applications, inkjet printing was selected due to its flexibility, cost, controllability, resolution and accuracy (Gibson et al., 2021; Hoath, 2016; Hutchings and Martin, 2013). How-
ever, these two last aspects still need further improvement. The high resolution comes at the cost of lower productivity. This has been counter-balanced by increasing firing frequency or the number of nozzles, which results in additional costs (Hoath, 2016; Zapka, 2018). To improve resolution, droplet size reduction is an alternative, but it is limited by the rheological ink properties and poor print quality due to droplet misplacement. Dot placement accuracy is associated with parameters such as jet characteristics, nozzle contamination, ink formulation, media absorption and aerodynamic effects. The latter is worse for longer flight paths and will be further discussed in the next sections.

2.3 Defining and assessing print quality

Print quality can be defined as “the overall merit or excellence of an image, as perceived by an observer neither associated with the act of photography nor closely involved with the subject matter depicted” (Keelan, 2002). Despite the subjective nature that is intrinsic to this topic, it is well documented that the perceived image quality depends on factors that include image resolution, number of grey levels, size and position of dots as well as ink and media properties and their interaction (Kipphan, 2001; Pond, 2000).

The resolution, i.e., the number of dots per inch (dpi) printed on the media, directly impacts the level of detail that an image can reproduce. This metric tends to be limited by the number of nozzles on the printhead and the frequency at which they are fired. In inkjet printers, DoD systems, especially those using thermal actuators, achieve considerably higher resolutions than CIJ systems (Kumar and Baral, 2022a).

The number of grey levels, also commonly referred to as bit-depth or colour-depth, defines the perceived transition between shades and colours (Kipphan, 2001). It has direct impact on the contrast and clarity of fine features and texts. For some systems, different grey levels are achieved by printing dots of different sizes, which is referred to as amplitude modulation (AM) screening. However, for binary systems, i.e., printheads with nozzles that can only fire a single droplet size, a frequency modulation (FM) technique is employed where the distance between
2. Literature Review

dots modulates the grey level. FM screening techniques are expected to better reproduce details (Kipphan, 2001). This FM process is achieved using a wide range of algorithms that can introduce randomness (Figure 2.6). In digital screening, a dithering algorithm arranges the dots within predefined cells or pixels in such a way that avoids the quantisation of grey levels and creates the illusion of colour depth. One of the first dithering algorithms was developed by Floyd-Steinberg (Floyd and Steinberg, 1976) and is still widely used in the industry. Other examples are the green- and blue-noise algorithms (Rodriguez et al., 2008).

The overall print quality is affected by the size of the droplets fired by the printhead. Smaller droplets produce smaller dots that result in better control over colour density and finer features (Kipphan, 2001; Pond, 2000; Ritchie, 2017). However, there is a limit to which droplet size can be reduced since a degree of dot overlap is desirable to induce the perception of a continuous feature (Kipphan, 2001; Pond, 2000; Ritchie, 2017) (Figure 2.7). Furthermore, smaller droplets tend to be more easily misdirected by aerodynamic effects, landing away from their predetermined final position. Droplet misplacement directly affects the print quality and can create defects like ragged edges, background hazes and/or mottle (Briggs et al., 1999b, 2000a).
2. Literature Review

Figure 2.7: Letter X printed to exhibit the impact of non-overlapping dots on human perception. Figure obtained from Ritchie (2017).

Choudhari et al. (2019). Inconsistent dot size also leads to printing defects as the dot size has direct impact on the perceived darkness level or optical density (Kipphan 2001; Pond 2000). This problem tends to be a result of inconsistent firing characteristics or ink properties across the printhead and over the life cycle.

For a good quality print, the ink properties must achieve specific printing requirements. In addition to the rheological characteristics previously mentioned, thermal and photo-sensitivity properties are other examples of significant relevance. For instance, the ink must be stable when exposed to light and humidity in order to avoid image colour degradation (Debeljak and Gregor-Svetec 2010). Furthermore, excessive evaporation is taken into consideration to avoid nozzles being clogged during and in between prints (Abbott 2018). The interaction of the ink with the media also adds complexity to the problem. Media characteristics such as bleeding, porosity, roughness and coating properties (Kumar and Baral 2022b) are fundamental to the perceived image quality.

A good quality print depends, thus, on numerous factors that have their individual and combined contribution. Understanding what defines a good quality print and its assessment is another complex topic of discussion. This is because the assessment of image or print quality attempts to signify the subjective and multi-dimensional nature of human perception (Le Pedersen, M.).

The assessment of image quality used to solely rely on subjective tests given that they are expected to provide the most accurate outcome (Ullah et al. 2023; Zhai and Min 2020). These
tests are conducted with human subjects that are asked to assess image quality according to different attributes (Lin et al., 2022). Due to the nature of the tests, however, they tend to be expensive and slow and, as a result, unfeasible for the industry. Objective measurements have become a faster and cheaper alternative to assess image quality (Pedersen et al., 2011). This strategy uses computational models to predict the perceived image quality by evaluating specific image attributes.

Various attributes have been used to assess image quality based on subjective and objective methods. Nussbaum (2010) investigated print quality evaluation based on features such as colour gamut, colour match, visual resolution and accuracy. Gast and Tse (2001) evaluated other attributes such as blur, noise and banding. Lindberg (2004) performed a more complete investigation with twelve different attributes: overall quality, tone quality, detail highlights, detail shadow, gamut, sharpness, contrast, gloss level, gloss variation, colour shift, patchiness, mottle, and ordered noise. In total, more than 25 different attributes have already been investigated, but there is still a certain level of debate regarding their individual or combined contribution to print quality (Pedersen et al., 2011). According to Pedersen et al. (2011), many of the attributes overlap and have a common denominator, and, as a result, they can be grouped into a more general set of attributes to reduce the dimensionality of the problem.

In an attempt to guide the assessment of print quality, the ISO 13660 and ISO 19751 standards have proposed a number of algorithms and attributes that include gloss uniformity, macro-uniformity, micro-uniformity, text and line quality and colour rendition (Briggs et al., 1999a; Chung and Rees, 2007). Macro-uniformity is of great relevance to graphics applications where large areas of uniform colour (lightness, hue, saturation, and combinations thereof) are printed. Defects like streaks, banding, mottle, moiré patterns and tiger-stripes are examples of non-conformity to this attribute (Rasmussen et al., 2006). The assessment of macro-uniformity usually consists in measuring the $L^*a^*b^*$ colour distribution of the image (Mizes et al., 2000). Similar to red-green-blue (RGB), CIELAB is a colour system used to define a colour based on the $L^*, a^*$ and $b^*$ triplet. The $L^*$ component represents the lightness values or luminosity, while $a^*$ and $b^*$ define the red/green and blue/yellow axes, respectively (Figure 2.8). The geometric distances (Euclidean difference, $\Delta E$), where
2. Literature Review

Figure 2.8: Schematic diagram of the $L^*a^*b^*$ colour space where $L^*$ is the vertical axis and $a^*$ and $b^*$ define the axes of the horizontal plane. Figure obtained from P. and Sanchez (2020).

$\Delta E = \sqrt{L^{*2} + a^{*2} + b^{*2}}$, in the $L^*a^*b^*$ colour space better represent real colour differences than in the RGB system (Kipphan, 2001).

In general, Euclidean differences smaller than $\Delta E < 1$ are considered imperceptible (Colombo et al., 2015). This range, however, tends to be affected by the spatial frequency sensitivity of the observer’s eyes (Goodman, 1997; Kelly, 1989). Mizes et al., 2000, for instance, indicated that periodic variations in optical density due to banding or streaks are more easily perceptible. For periodic defects with spatial frequency varying from 0.1 to 1.0 cycles/mm, the defects become perceptible for $\Delta L^*$ as low as 0.2. Thus, due to its periodic pattern, the tiger-stripe printing defect is more easily perceptible. It is important to note that, while banding and streaks are defects resultant of mechanical, electrical or chemical imperfections (Briggs et al., 2000b), tiger-stripes or wood/grain are created by specific aerodynamic effects (Hsiao et al., 2013).

2.4 Aerodynamic effects that compromise print quality

The characteristics of the airflow dynamics in the print gap of inkjet printers are of particular interest to the inkjet printing industry. This is because specific aerodynamic effects tend to
have a direct impact on the print quality.

The motion of droplets and particles in a fluid is highly dependent on their responsiveness to the state of the surrounding flow field. This responsiveness is characterised by the Stokes number (Equation 1.1), which is defined as the ratio between the characteristic particle time and the characteristic flow time (Crowe et al., 1995). This means that particles with a low Stokes number ($S < 1$) tend to follow the flow streamlines, while larger droplets with a high Stokes number ($S > 10$) tend to be unaffected by the airflow and present a ballistic trajectory (Mallinson et al., 2016; Wang, 1999). In inkjet printers, the satellite droplets, due to their lower Stokes number, tend to travel for longer and land away from the main droplets, creating printing defects (Wang, 1999).

The study of Wang (1999) indicated that the final position of the droplets could be severely impacted by the velocity profile of the incoming cross-flow. The incoming cross-flow was initially believed to have a linear Couette profile ($u_y(z) = \frac{V_p z}{H}$) due to the relative motion between the paper and the printhead. Here, $V_p$ is the media velocity and $u_y(z)$ is the y-direction velocity. This conclusion was misled by measurements conducted with inactive printers (Link et al., 2009) or a single-nozzle printhead (Hsiao et al., 2012). Mallinson et al. (2022) indicated, however, that in normal operating conditions, due to the interaction with the pressure gradient induced by the stream of droplets, the cross-flow profile differs from a linear Couette flow. Furthermore, inkjet printers tend to use a mist extraction system located downstream of the printhead to collect satellite droplets and prevent them from being inhaled (Balala and Baterna, 2019; Hoath, 2022; Ishikawa, 1986); such systems create a pressure gradient ($\Delta p$) that adds a Poiseuille component to the flow (Figure 2.9). The general formulation of a planar Poiseuille is given by Equation 2.2 where $l$ is the print gap length.

$$u_y(z) = \frac{\Delta p}{2\mu l} z(H - z)$$

(2.2)

The interaction of the stream of droplets and the incoming cross-flow takes time to develop and leads to different flow profiles at the beginning and at the end of the print may be observed.
2. Literature Review

Figure 2.9: Schematics of the Couette-Poiseuille flow profile, which is a result of the interaction between the planar Couette flow induced by the relative motion between media and printhead and the planar Poiseuille flow created by the suction exerted by mist extraction systems.

Figure 2.10: Comparison of dot misplacement between the leading edge (left) and trailing edge (right) regions. Picture obtained from Wang (1999).

This can cause aerodynamic effects referred to as leading and trailing edge effects (Figure 2.10) as Wang (1999) reported.

A series of studies have further characterised the flow features that evolve from the interaction between the incoming cross-flow and the stream of droplets (Barnett and McDonald, 2014; Hsiao et al., 2012, 2013; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015). From the exchange of momentum between the droplets and the surrounding airflow, referred to here as the entrainment effect, two counter-rotating vortices arise (Figure 2.11). These vortices play a significant role in the misplacement of droplets.

At specific operating conditions, especially at elevated print gap heights, the airflow in the print gap becomes oscillatory, leading to printing defects that resemble tiger-stripes or wood-grain (Barnett and McDonald, 2014; Barnett et al., 2016; Hsiao et al., 2013; Link et al., 2009; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015) as Figure 2.13 shows. Link et al. (2009) had hypothesised that this printing defect was a result of vortices created by the sudden contraction that the airflow experiences when approaching the forward-facing step created by the print-
2. Literature Review

Figure 2.11: Vector plot coloured by velocity magnitude of a simulation of the airflow in the print gap of inkjet printers performed by Mallinson et al. (2016) showing two counter-rotating vortices. Printing from right to left.

Figure 2.12: Schematic of the airflow induced by the paper media approaching the print gap.

Head geometry (Figure 2.12) This was based on the findings of Shockling et al. (2006) and Allen and Naitoh (2007) that related the vortices in a piston/cylinder geometry to a sudden geometrical disturbance on the stationary wall. However, experimental and numerical tests refuted this hypothesis as the sudden contraction did not produce airflow oscillations even when the paper speed was 20 m/s. This is in agreement with the studies of Lanzerstorfer and Kuhlmann (2012) on a forward-facing step that indicated that the airflow is asymptotically stable for Reynolds numbers below 680.

The airflow oscillation in the print gap of inkjet printers has been linked to the instability of the vortices induced by the entrainment effect Barnett and McDonald (2014), Mallinson et al. (2016), Rodriguez-Rivero et al. (2015). When these vortices become non-uniform in time and spanwise direction, the droplets are misplaced in specific patterns that lead to the features that characterise the tiger-striping printing defect.
Figure 2.13: Printed pattern obtained from Rodriguez-Rivero et al. (2015) showing the wood-graining printing defect for a paper speed of 0.55 m/s, 13.1 pL droplets, 1000 nozzles fired and 5 mm print gap height. Bottom: the delimited region is displayed in more detail.
Barnett and McDonald (2014) investigated the impact of a number of different printing parameters, including print gap height, firing frequency and nozzle spacing, on the severity of wood-graining. Based on print samples, the printing defect was more evident under conditions of high vortex unsteadiness, which tends to occur at reduced spacing between nozzles, low paper speed and/or high print gap heights. Particle tracking experiments and high-magnification images of the print samples indicated that the defect is a result of satellite misplacement, but under cases of high unsteadiness, main droplet misplacement becomes predominant. It was also demonstrated that the use of a low-density gas forced through the print gap tends to stabilise the vortices, and this was ascribed to the lower drag of the droplets and the reduced exchange of momentum between droplets and the surrounding fluid.

Rodriguez-Rivero et al. (2015) used time-resolved particle image velocimetry (PIV) and shadow-graphic images to observe that the vortices develop a shedding motion at elevated print gap heights. With the increase in firing frequency and droplet size and reduction in nozzle spacing, the printing defect became more evident, which is in agreement with the studies of Barnett and McDonald (2014). They suggested that this was a consequence of adjacent nozzles having a stronger interaction. Further, they indicated that smaller print gap heights and higher paper speeds tend to make the printing defect less evident.

Mallinson et al. (2016) made use of particle-in-cell (P-in-C) simulations to investigate the airflow dynamics. They observed that the vortex oscillation led to a periodic droplet misplacement, which, due to their lower Stokes number, was considerably larger for satellite droplets than for the main droplets. In this study, the authors developed an algorithm that is capable of producing numerical prints based on the final position of main and satellite droplets. The numerical prints showed that the divergence of the misplacement of droplets approximate the optical density variation that is seen in the tiger-stripe printing defect.

At the onset of instability, when the variations in airflow velocity are relatively small, the variation in optical density tends to be a result of the satellite droplets landing away from the main dots in regions of white space Barnett and McDonald (2014) Wang (1999). This increases the number of dots in those regions, and thus the perceived optical density (darker perception). At conditions of high airflow non-uniformity, however, the main droplet misplacement becomes
2. Literature Review

Figure 2.14: Numerical print pattern obtained from Mallinson et al. (2016) showing the optical density resultant of a simulation at elevated print gap height.

more significant (Rodriguez-Rivero et al., 2015). It is important to note that the misplacement of satellite and main droplets both contribute to the overall effect optical density, that depends on dot size and misplacement magnitude.

Mallinson et al. (2016) also demonstrated that increasing the flow rate across the print gap with a suction system or a roller tends to stabilise the airflow and mitigate the problem. This finding, however, challenges the suggestion made by previous studies that the interaction between the entrained airflow and the incoming cross-flow leads to printing defect (Barnett and McDonald, 2014; Hsiao et al., 2013; Rodriguez-Rivero et al., 2015). In agreement with Mallinson et al. (2016), tests with a guarded printhead to reduce the incoming cross-flow resulted in increasing the severity of the printing defect instead of reducing it (Hsiao et al., 2013).

2.5 Systems dynamics

It is noted here that the entrained airflow due to inkjet printing behaves like laminar impinging jets, which can develop a self-sustainable instability even in the absence of a cross-flow (Goldschmidt and Bradshaw, 1973; Ho and Nosseir, 1981; Varieras et al., 2007). This is because the jet of droplets is expected to create flow features similar to those seen in a laminar impinging jet.

The three main regions that characterise laminar impinging jets are the free jet, stagnation and wall jet (Figure 2.15, Viskanta, 1993). Once the jet exits the opening, internal viscous relaxation and external shear forces act on the flow stream, transforming the velocity profile. For free jets
or jets that are not highly confined, i.e. $H/e > 9$, where $H$ is the distance between the jet exit and the impinging wall and $e$ is the jet thickness (Figure 2.16), the flow in the free jet region develops a non-uniform radial velocity profile. The initial velocity profile depends on the geometry of the nozzles, and in the case of highly confined scenarios, the jet velocity profile may not be fully developed (Geers, 2004). At the stagnation point, axial momentum is converted into radial momentum, causing a strong lateral velocity component in the vicinity of the wall. When jets are in a highly confined environment, a pair of vortex dipoles - indicated by (2) in Figure 2.16 - is created. In the wall jet region, the flow develops a velocity field almost parallel to the wall, and a boundary layer starts to develop.

The vortices in laminar impinging jets induce a positive feedback response into the system,
where oscillations at the base of the jet are carried back into the jet by the eddies, creating a positive feedback loop, as noted by Varieras et al. (2007). Above specific values of Reynolds number and normalised gap height ($H/e$), this positive feedback induces a self-sustaining oscillation that makes the jet flap and the vortices oscillate. Given the similar underlying flow features between both systems, it is believed that the entrained airflow will present similar overall dynamics.

Laminar impinging jets in the presence of an incoming cross-flow develop additional flow features that are also expected to be observed in the print gap of inkjet printers. The interaction with the incoming cross flow creates a new system of vortices characterised by a horseshoe like structure. As the flow bypasses the jet, it carries some of the vortices further downstream, inducing tip vortices as those observed in Figure 2.17. Under strong cross-flows, the jet impingement can be delayed or even prevented (Barata and Durao, 2004; Geers, 2004). This indicates that strong cross-flows can induce large shifts in droplets’ trajectories and, as a result, create another type of printing defect. The tip vortices can also create edge effects, which will be further investigated in this thesis.

Systems like laminar impinging jets that start to develop a self-sustaining oscillation at a specific critical transition point are classified as a supercritical Hopf bifurcation. A system is classified as supercritical or subcritical depending on the dynamics of the transition (Drazin, 2002).
2. Literature Review

Figure 2.18: Example of bifurcation diagrams: a) subcritical and b) supercritical bifurcations.

Seydel [2009]. This is usually represented in bifurcation diagrams (Figure 5.9), which indicate the evolution of the system as a disturbance parameter is altered. It is said that a bifurcation is supercritical when there is a gradual variation between states with change in conditions. However, when the system is unstable in nature and changes in conditions lead to drastic variations in state, the system is classified as subcritical. Taylor-Couette flow [Caton et al., 2000 Pfister et al., 1988], Rayleigh-Benard [Bajaj et al., 1998 Bodenschatz et al., 2000] and the flow around a cylinder [Gallaire et al., 2016] are examples of supercritical bifurcations, while plane Couette [Daviaud et al., 1992, Tillmark and Alfredsson, 1992], Poiseuille [Bayly et al., 1988] and Couette-Poiseuille [Cowley and Smith, 1985] flows are examples of subcritical bifurcations.

The transition can be further categorised according to the dynamics of the different states. Hopf and pitchfork bifurcations are the most common categories. The Hopf bifurcation is characterised by a limited cycle oscillation, while the pitchfork is characterised by an exchange of stability where the system transitions between two stable regimes [Drazin 2002, Seydel 2009]. It is important to note that the existence of noise or imperfections in real systems can excite the transition between states at conditions lower than the critical transition point [Juel et al., 1997, Schumaker and Horsthemke, 1987]. This has been captured, for instance, in the study of a horizontal pendulum [Schmitt and Bayly, 1998]. This investigation demonstrated that the horizontal pendulum presented a supercritical pitchfork bifurcation when the system was pushed by a sinusoidal force but when the force was perturbed by a stochastic noise, the transition
became smooth and continuous in what is referred to as a saddle-node or excited transition (Figure).

Understanding the dynamics of the system is of importance to develop flow control strategies and define the system envelope. However, these analyses are yet to be done for the airflow in the print gap of inkjet printers. Furthermore, even though previous studies have reported that the airflow tends to be unstable at high print gap heights, low paper speed, high firing frequency and small nozzle spacing, the range of flow regimes are yet to be determined.

2.6 Modelling of aerodynamic effects

The mechanisms of the entrainment effect induced by the interaction between ink droplets and the surrounding airflow are part of the well-established field of dispersed multiphase flows (Brennen, 2005; Seydel, 2007). Hence, the underlying physics of the problem are similar to those seen in a wide range of industrial applications such as fluidized beds, fuel sprays in internal combustion engines and bubble columns (Almohammed, 2018; Baker et al., 2020; Keser et al., 2019; Seydel, 2007; Vié et al., 2016). Multiphase flows are classified as dispersed when discrete elements - solid particles, droplets or gas bubbles - that form the dispersed phase interact with a continuous phase. This classification differs from separate flows as those have two or more fluid phases separated by interfaces (Brennen, 2005; Seydel, 2007). It is worth noting that the term ‘particle’ and ‘dispersed phase’ are interchangeably used in this thesis to refer to the ink droplets.

Dispersed flows are further classified depending on the volumetric ratio of phases (Equation 2.3). This classification has a direct impact on the closure models used to approximate the interaction between particles and the fluid carrier. The works of Balachandar and Eaton (2010); Crowe et al. (2011); Elghobashi (1991, 1994); Elgobashi (2006) are of relevance in defining the range that classifies the interaction between phases. For \( \alpha_d < 10^{-6} \), the particles are carried by the flow with a negligible feedback response from the particles on the fluid phase. This means that a “one-way” coupling scheme (Fluid \( \rightarrow \) particles) can be employed. The feedback
2. Literature Review

Figure 2.19: Flowchart diagram describing the framework implemented in the one-way (a) and two-way phase coupling algorithms.

The response of the dispersed phase becomes relevant when \(10^{-6} < \alpha_d < 10^{-3}\) and “two-way” coupling schemes (Fluid ↔ particles) are necessary. These regimes \(\alpha_d < 10^{-3}\) are termed diluted systems, while the regime with \(\alpha_d > 10^{-3}\) is referred to as dense systems. In the case of dense flows, the particle-particle collisions become relevant to the physics of the system and have to be taken into account. It is worth mentioning that in some studies, mass loading is used instead of the particle volume fraction as a criterion to distinguish between the one-way and the two-way coupled regimes. The reason for this choice is that the momentum coupling terms of the fluid phase include the masses of the particles.

\[
\alpha_d = \frac{v_d}{v_f + v_d}
\]  

(2.3)

where \(\alpha_d\) is the volume fraction of the dispersed phase and \(v_d\) and \(v_f\) represent the volume of the dispersed and fluid phases, respectively.

In one-way coupling schemes, the fluid carrier phase is modelled as a single-phase flow, and
2. Literature Review

the particle motion is determined as a post-processing step. However, for systems where the coupling between phase is necessary, a vast number of strategies that vary in complexity, computational cost and accuracy are employed to formulate the complex physics. Figure 2.19 describes the main concept of the one- and two-way frameworks. In ascending order of cost and complexity, the two-way strategies are: eulerian-eulerian (EE), particle-in-cell (P-in-C) also referred to as eulerian-lagrangian (EL) or discrete phase model (DPM), and particle-resolved (PR) methods [Almohammed 2018; Seydel 2007].

In the EE two-fluid theory, the fluid and dispersed phases are modelled as two inter-penetrating continua using the Eulerian framework. This assumption results in separate conservation equations for each phase, but eliminates the need for modelling every single particle, which significantly reduces the computational cost of simulations. For a system where there is no exchange of mass between phases, the continuity (Equation 2.4) and momentum conservation (Equation 2.5) equations are written as:

$$\frac{\partial \alpha_\gamma \rho_\gamma}{\partial t} + \nabla \cdot (\alpha_\gamma \rho_\gamma \mathbf{u}_\gamma) = 0 \quad (2.4)$$

$$\frac{\partial \alpha_\gamma \rho_\gamma \mathbf{u}_\gamma}{\partial t} + \nabla \alpha_\gamma \rho_\gamma \mathbf{u}_\gamma \mathbf{u}_\gamma = -\alpha_\gamma \nabla p + \nabla \tilde{\tau}_\gamma + \alpha_\gamma \rho_\gamma \mathbf{g} + \mathbf{F} \quad (2.5)$$

where the subscript $\gamma$ identifies the phase, $\rho$ is the density, $t$ is the time, $p$ is the pressure field, $\mathbf{u}$ represents the velocity vector field, $\mathbf{g}$ is the gravity vector field, and $\tilde{\tau}$ is the stress tensor which includes the pressure and viscous stresses for each phase. For the fluid phase, the viscous stress is usually modelled using the gradient-viscosity model [Baker et al. 2020], whilst, for the dispersed phase, these stresses are calculated with closure models based on the kinetic theory of gases [Baker et al. 2020; Crowe et al. 2011]. $\mathbf{F}$ represents the interphase momentum exchange that arises from a number of fluid forces, including drag, lift, pressure gradient, added mass and buoyancy. It is important to note here that each phase has their respective velocity field, but they both share the same pressure field.

The EE model is believed to be more suitable for systems with densely packed and highly collisional flows such as fluidized beds [Du et al. 2006; Gryczka et al. 2009; Huilin et al. 2004].
The model, however, tends to fail to capture particle-phase velocity anisotropy \cite{Capecelatro_and_Desjardins_2013a,b}, particle trajectory crossing or, in general, scenarios where particles cannot be represented by an average fluid continuum. These drawbacks of the model tend to lead to erroneous results when modelling droplet jets impinging on surfaces \cite{Subramaniam_2013}.

In the P-in-C or Eulerian-Lagrangian approaches, the fluid carrier phase is treated as a continuum in an Eulerian framework and the individual particles are tracked using the equations of motion in a Lagrangian framework \cite{Capecelatro_and_Desjardins_2013a,b}. For systems like the one studied here where the particles are much smaller than the main flow features, the point particle approach is usually implemented. This means that the individual particles are treated as point sources of mass, momentum and energy, and the fluid volume fraction becomes unity \cite{Seydel_2007}. Assuming that there is no exchange of mass and energy between phases, the continuity (Equation 2.5) and momentum equations (Equation 2.7) for the fluid phase are defined as:

\[
\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{u}_f) = 0. \tag{2.6}
\]

\[
\frac{\partial \rho_f \mathbf{u}_f}{\partial t} + \nabla \rho_f \mathbf{u}_f \mathbf{u}_f = -\nabla p + \nabla \mathbf{\tau}_f + \rho_f \mathbf{g} + \mathbf{F}. \tag{2.7}
\]

According to \cite{Baker_et_al._2020}, \( \mathbf{F} \) is the volume-filtered summation of the interphase forces acting on each particle contained in a control volume (Equation 2.8).

\[
\mathbf{F} = \sum_{m=1}^{n=q} \mathcal{H} \mathbf{f}^m. \tag{2.8}
\]

Here, \( \mathcal{H} \) is the volume-filtering kernel for a defined control volume, \( \mathbf{f}^m \) is the force of the fluid acting on a single particle \( m \) and \( q \) is the total number of particles. The governing equations for the individual particles are then derived from Newton’s second law of motion:
2. Literature Review

\[ \frac{dx_d^n}{dt} = u_d^n. \] (2.9)

\[ v_d^n \rho_d \frac{du_d^n}{dt} = f^n + f_{\text{collisions}}^m + v_d^n \rho_d g. \] (2.10)

where \( f_{\text{collisions}} \) is the source term resulting from the particle-particle interactions and usually implemented in the four-way coupling strategies.

P-in-C methods have successfully modelled a wide variety of dispersed flows including fluidized beds, vertical channel risers, sprays, and impinging jets \cite{Capecelatro and Desjardins, 2013a}, which indicates that this technique has great potential to model the entrainment effect. One practical limitation, however, to the use of P-in-C models is the large computational cost involved in systems containing high concentrations of particles due to the sheer number of individual particles that must be tracked. Furthermore, the point-source formulation tends to produce inaccurate results in regions of high concentration of particles. This is because the point-source approach neglects the volume change of the continuous phase due to the presence of particles/droplets in a cell and, as a result, can lead to inaccuracies.

The PR approach is expected to provide the most accurate solution but is limited by the computational cost, becoming impractical to model flows involving a high number of particles like the cases investigated here. The PR model attempts to solve the continuity and momentum equations without any closure models and, as a result, resolving all flow length-scales.

In the end, the P-in-C, EE and the PR models \cite{Baker et al., 2020, M. Kuerten, 2016, Nasr et al., 2009, Vié et al., 2016} deal with the same challenge of being computationally efficient, while accurately predicting the dynamics of particle-driven flows \cite{Baker et al., 2020}. Part of the challenge is due to the modelling of forces acting on the dispersed phase. For the system investigated here, due to the high ejection velocity of the droplets, the drag is predominant in comparison to all the other forces. Due to the lack of particle rotation and the high particle-to-fluid density \( \rho_d / \rho_f >> 1 \), the Magnus, buoyancy, added mass and pressure gradient forces are
2. Literature Review

significantly small (Crowe et al., 2011; M. Kuerten, 2016). Furthermore, gravity is disregarded because of the negligible mass of the particles and the small print gap height. Therefore, we have that \( f^m = f^m_{\text{drag}} \).

Bond and Newton (1928) demonstrated that, due to the presence of surfactants, small droplets can be modelled as rigid spheres. Stokes (1851) was the first to define a theoretical expression that describes the drag experienced by spherules (Equation 2.11). However, this expression only holds for when the droplet Reynolds \( (Re_d = Wd/\mu) \) number value is nearly zero. The Stokes law states that the drag coefficient is inversely proportional to the Reynolds number. To extend this formulation for a wider range of Reynolds number values, Oseen (1913) proposed a novel formulation that accounts for inertial effects (Equation 2.12). This formulation, however, tends to overestimate the drag coefficient \( (c_{\text{drag}}) \) in comparison to experimental data (Schlichting and Kestin, 1961). A series of studies have then attempted to formulate expressions that better approximate the drag coefficient of spherules. These expressions add a correction factor \( (\varphi) \) to Stokes’ law (Equation 2.13).

\[
\begin{align*}
\dot{f}_{\text{drag}} &= 3\pi\mu dW \\
\dot{f}_{\text{drag}} &= 3\pi\mu dW(1 + \frac{3}{8}Re_d) \\
\dot{f}_{\text{drag}} &= 3\pi\mu dW(1 + \varphi(Re_d))8Re_d
\end{align*}
\] (2.11)  
(2.12)  
(2.13)

Figure 2.20 shows the drag coefficient for \( 0 < Re_d < 100 \) for different formulations compared against the experimental data captured by Liebster and Schiller-Schmiedel. For the range of Reynolds number values tested in this study \( (Re_d < 16) \), the expressions derived by White (1991, Eq. 3–225), Schiller and Naumann (1933) as well as Clift et al. (2005, p. 112, table 5.2) show good correlation in comparison to the experimental data. White’s formulation is chosen to be employed in this thesis due to its simplicity.
2. Literature Review

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\varphi(Re_d) =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oseen (1913)</td>
<td>$1 + \frac{3Re_d}{8}$</td>
</tr>
<tr>
<td>White (1991, Eq. (3–225))</td>
<td>$\frac{Re_d}{4(1+\sqrt{Re_d})} + \frac{Re_d}{60}$</td>
</tr>
<tr>
<td>Clift et al. (2005)</td>
<td>$0.1315Re_d^{0.82} - 0.05\log_{10}Re_d$</td>
</tr>
<tr>
<td>Proudman and Pearson (1957)</td>
<td>$\frac{3Re_d}{16} + \frac{9Re_d}{160}\log_{10}\frac{Re_d}{2} + O(Re_d^2)$</td>
</tr>
<tr>
<td>Goldstein (1929)</td>
<td>$1 + \frac{3}{16}Re_d - \frac{19Re_d}{1280} + \frac{71Re_d}{20480} + O(Re_d^3)$</td>
</tr>
<tr>
<td>Schiller and Naumann (1933)</td>
<td>$0.15Re_d^{0.687}$</td>
</tr>
<tr>
<td>Abraham (1970)</td>
<td>$(1 + Re_d^{1/2}/9.06)^2 - 1$</td>
</tr>
</tbody>
</table>

Table 2.1: Expressions for the drag correction factor as a function of Reynolds number ($\varphi(Re_d)$).

Figure 2.20: Drag coefficient ($c_{drag}$) for different formulations compared to experimental data [Schlichting and Kestin, 1961 p. 17] for droplet Reynolds number ($Re_d$) ranging from 0 to 100.
2.7 Summary

As reviewed in this Chapter, inkjet printing technology has been used for applications such as graphics, printed electronics, reinforced composites and rapid prototyping. This is because of the easy control of the printed pattern, high resolution and accuracy of droplet deposition. In graphics application, printing quality is defined according to a number of attributes, including, but not limited to colour gamut, colour match, visual resolution, blur, noise and banding. Print quality assessment can employ an objective method based on a mathematical model that evaluates particular attributes of a printed image and/or perform a subjective test, which consists in surveying a group of human subjects about the printed image quality. While subjective assessments are more accurate, objective algorithms are faster and cheaper.

It has been demonstrated that several factors can influence the quality of the printed image, including aerodynamic effects that can misdirect the droplets from their original trajectory. This is of particular interest at large print gap heights, where specific aerodynamic effects create printing defects that resemble tiger-stripes or wood-grain are observed.

Previous studies have indicated that the tiger-stripes or wood-grain printing defects are a result of the spatially and temporally non-uniform misplacement of droplets on the substrate. This misplacement is linked to the vortex instability in the near vicinity of the injection zone. Even though the impact of paper speed, firing frequency and $H$ on the severity and occurrence of tiger-stripes has been demonstrated, the range of the flow regimes (stable and unstable) is still unknown. This means that the transition or bifurcation point of the flow is undetermined, and, as a result, the operating envelope of inkjet printers is yet to be determined. The mechanism of instability proposed by previous studies, i.e., the interaction between incoming cross-flow and vortices, has yet to be confirmed as there is evidence to challenge it.

This project will target these knowledge gaps while investigating the occurrence and severity of tiger-stripes. Furthermore, an alternative approach - the dispersed-phase continuum (DPC) approximation - to model the entrainment effect is introduced and assessed (Chapters 3 and 4). This strategy treats the droplets as a continuum source of momentum that depends on a pre-
defined temporally averaged droplet number density which is calculated with the predefined trajectories and velocities of the droplets.
Chapter 3

Airflow instability in a two-dimensional print gap model

As highlighted in the previous chapters, at specific operating conditions, especially at elevated print gap heights, printing defects that resemble tiger-stripes, wood-grain or sand-dunes are likely to be observed, limiting the expansion of the inkjet printing technology. These printing defects are created when the airflow becomes unstable, misplacing the droplets in periodic patterns. In this chapter, a published journal entitled 'Airflow instability in a two-dimensional print gap model' is presented. Two-dimensional numerical analysis based on the novel DPC model are performed to shed light into the dynamics of the airflow instability and investigate whether the interaction between the incoming cross-flow and the entrained airflow triggers this instability. The two-dimensional model allows a detailed investigation of the base flow that is expected to create a uniform print. Furthermore, the transition from a stable (steady) to an unstable regime (unsteady) observed in this study defines the upper bounds of the printing envelope where the airflow no longer satisfies the conditions - uniformity in time and spanwise direction - for adequate printability.
Abstract

A numerical model was employed to investigate the vortex instability in a two-dimensional inkjet print-zone. The simulation models the entrainment effect of the droplets on the airflow via a novel dispersed-phase continuum method that, due to the separation of length scales, treats the force exerted by the main droplets as a continuum smooth field. The trajectory and speed of the main droplets are also assumed to be unaffected by the airflow. The results indicate the existence of a dimensionless droplet density threshold \((N_c)\) at which the vortex shifts from steady to oscillatory. This demonstrates that the two-dimensional instability has a supercritical Hopf type of bifurcation, i.e., the shift from stable to unstable is continuous but not smooth and the amplitude of oscillation follows the asymptotic square-root behavior characteristic of this type of bifurcation. Further, as shown by tests with stationary paper and no induced cross-flow, the mechanism of instability cannot be attributed to the interaction between the incoming cross-flow and the entrained airflow. Characterizing the two-dimensional airflow instabilities and their mechanism provides a better understanding of the airflow dynamics in the print gap of inkjet printers.

3.1 Introduction

Inkjet printers deposit ink droplets onto a substrate in predefined positions, creating a two-dimensional pattern. Due to the easy control of the printing pattern and the repeatability of the process, the use of inkjet printers is no longer limited to graphics applications and has been expanded to manufacturing electronics, solar panels, rapid prototypes, and reinforced composites (Arango et al., 2019; Hoath, 2016; Zapka, 2018). In these applications, fine features with resolution of tenths of millimeters are reproduced, which demands highly accurate deposition of droplets on the substrate (Hoath, 2016).

The expansion of inkjet technology to other applications, or even further within the graphics sector, remains limited by the restrictive print height \((H)\) with which inkjet printers can operate (Figure 3.1). Specifically, small print gaps prevent the printer from accommodating media
3. Airflow instability in a two-dimensional print gap model

Figure 3.1: Schematic of the physics in the print gap, where $H$ is the print gap height, $W$ is the droplet velocity, $V_p$ is the paper speed, $b$ is the breadth of the print-zone and the origin of the coordinate system is marked by the white square.

with variable or large thickness and increase the likelihood of the media striking the printhead, which can damage the nozzles, demanding expensive maintenance. At high print heights, however, the print quality is likely to be compromised due to the misplacement of droplets which tends to create printing defects (de A. Aquino et al. 2020; Hsiao et al. 2012, 2013; Mallinson et al. 2016, 2020; Rodriguez-Rivero et al., 2015; Wang, 1999).

These printing defects have been linked to specific aerodynamic effects that influence droplets trajectories (Hsiao et al., 2012; Link et al., 2009; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015). The airflow in the print gap is formed by an interaction between the cross-flow and the induced impinging air-jet (Figure 3.1), which is created by the droplets motion in an effect named here as airflow entrainment. Satellite droplets, due to their lower Stokes numbers ($S < 1$), are likely to be carried by the airflow, landing away from main droplets ($S > 10$), where the Stokes number is defined by Eq. 3.1

$$S = \frac{\rho_d d^2 | W - V_p |}{18 \mu H}. \quad (3.1)$$

where $\rho_d$ is the droplet density, $d$ is the diameter of the droplet, $\mu$ is the dynamic viscosity of air, $W$ is the droplet ejection velocity and $V_p$ is the paper speed. When the airflow oscillates at larger print gaps ($H < 1 \text{mm}$), the satellites are periodically misplaced on the paper (de A. Aquino et al. 2020; Mallinson et al. 2016, 2020). At scenarios of large unsteadiness, the
3. Airflow instability in a two-dimensional print gap model

main droplet can also be periodically misplaced (Hsiao et al., 2013), but this regime is already beyond the envelope that defines a functional printer.

Link et al. (2009) had initially proposed that droplet misplacement was created by vortices induced by the forward-facing step of the printhead geometry. However, numerical simulations and particle image velocimetry (PIV) experiments conducted in their study (Link et al., 2009) refuted this hypothesis as the vortices minimally affected the mean airflow in the print-gap. The study of Lanzerstorfer and Kuhlmann (2012) on forward-facing step also confirmed that the airflow is asymptotically stable for Reynolds numbers below 680. Other works using time-resolved PIV (t-PIV) (Rodriguez-Rivero et al., 2015) measurements and particle-in-cell tracking simulations (Barnett and McDonald, 2014; Mallinson et al., 2016) have highlighted that printing defects can be a result of the oscillation of the vortices created by the entrainment effect. They reported that, at specific operating conditions, the airflow becomes three-dimensional and the vortices become unstable, inducing a periodically misplacement of droplets on the paper.

While it has been demonstrated that the vortices tend to oscillate at low volumetric flow rate across the printzone (low paper speed and/or low downstream suction) and high droplet mass flux (Barnett and McDonald, 2014; de A. Aquino et al., 2020; Hsiao et al., 2012, 2013; Mallinson et al., 2016, 2020; Rodriguez-Rivero et al., 2015), little is known about the dynamics of the oscillation. Furthermore, the vortex instability has also been linked to the interaction between the incoming cross-flow and the entrained airflow, but there is evidence to challenge this assertion because laminar impinging jets of air can develop a self-sustained oscillation even when there is no cross-flow (Goldschmidt and Bradshaw, 1973; Ho and Nosseir, 1981; Varieras et al., 2007). Varieras et al. (Varieras et al., 2007) indicated that the self-sustained oscillation of a laminar impinging plane jet of air is characterized by a two-dimensional mode of oscillation that presents a super-critical Hopf bifurcation (Seydel, 2009). The natural frequency of oscillation of the impinging plane jet increases as the jet speed increases and the confinement height \(H\) decreases. Due to some physical similarities of the problems, it is expected that the entrained airflow in the print gap will present similar behavior.

We note, the base flow that creates uniform prints without tiger-stripe or wood-grain is expected to be homogeneous in time and the span direction. Based on that, two-dimensional nu-
numerical analyses are performed to shed light into the dynamics of the vortex in the print-zone and determine the range of flow regimes at different conditions. This will provide a refined understanding of the two-dimensional mode of oscillation while characterizing the upper bound at which the base flow meets the conditions for a uniform print. Tests are also conducted to determine whether the interaction between the incoming cross-flow and the entrained airflow triggers the instability.

3.2 Methodology

3.2.1 Numerical Model

The OpenFOAM solver \textit{pimpleFoam} is used to solve the governing equations for incompressible flows:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{F}(\mathbf{U} - \mathbf{u}) + \nu \nabla^2 \mathbf{u}. \tag{3.2}
\]

\[
\nabla \cdot \mathbf{u} = 0. \tag{3.3}
\]

where \( \mathbf{u} \) is the velocity field, \( t \) is time, \( p \) is pressure, \( \nu \) is the air kinematic viscosity, \( \mathbf{F} \) is the body-force induced by the point particles - main droplets - moving at velocity \( \mathbf{U} \).

The force exerted by the droplets is equal but of opposite sign to the drag the droplets experience, which is of the form of Stoke’s drag law with an empirical correction factor \((1 + \varphi) - \text{Eq. (3.5)}\). The empirical factor used here was calculated using Eq. (3.4) as [White (1991, Eq. (3-225)) defined and comes from a fit to experimental data for droplet Reynolds number lower than 200000. This covers the range of droplet Reynolds number \((\text{Re}_d)\) tested in this study, which is approximately \(4.5 < \text{Re}_d < 16\). Here, the Reynolds number is given by \( \text{Re}_d = \frac{|W_k - \mathbf{u}| d}{\nu} \), where \( k \) is the unit vector in z-direction and \( W_k = \mathbf{U} \). The Reynolds number for the flow across the print gap, on the other hand, is given by \( \text{Re}_H = \frac{V_p H}{\nu} \) and varies from 0 to 250.
3. Airflow instability in a two-dimensional print gap model

\[ \phi(Re_d) = \frac{Re_d}{4(1 + \sqrt{Re_d})} + \frac{Re_d}{60}. \] (3.4)

Previous studies (Barnett and McDonald, 2014; Mallinson et al., 2016) have modeled the droplets using the classic particle–in-cell (Lagrangian) method, which employs additional kinematic equations to track the droplets and, as a result, demands higher computational cost. Therefore, to prevent this additional computational cost, a novel dispersed-phase continuum (DPC) method is implemented in this study. This model assumes that, due to the separation of length scales (droplet diameter, droplet spacing, droplet wake length and airflow structure size), the dispersed-phase can be treated as a continuum. This results in \( F \) being a smooth continuous field given by Eq. (3.5). Here, \( n \) is the number density or number of droplets per unit volume in the predefined print-zone (Figure 3.1) and depends on the number of nozzles per unit area on the injection face (\( \sigma \)), the firing frequency (\( f_n \)) and the droplet speed (\( W \)) – Eq. (3.6). It is also assumed here that the main droplets have constant speed and, due to their high Stokes number, their trajectories are unaffected, as indicated in previous studies (Barnett and McDonald, 2014; Hsiao et al., 2012; Link et al., 2009; Mallinson et al., 2016). The DPC method takes approximately half of the time that the particle-in-cell method takes to calculate the same solution.

\[ F = 3\pi \nu n d(U - u)(1 + \phi). \] (3.5)

\[ n = \frac{\sigma f_n}{W}. \] (3.6)

To couple the pressure and momentum equations, the \textit{PIMPLE} scheme (Holzmann, 2016) is employed here due to its robustness. It is deemed that the outer loop of the \textit{PIMPLE} scheme has reached convergence when the pressure and momentum residuals fall to less than \( 10^{-5} \), as suggested by Holzmann (Holzmann, 2016). Pure Crank–Nicolson, a second-order implicit scheme, is used to discretise the time derivative, while the second-order discretisation of the gradient, divergence and Laplacian terms is achieved via the unbounded central differencing algorithm, named Gauss linear in the OpenFOAM idiom (Holzmann, 2016). The velocity terms are solved using the bi-conjugate gradient method with a diagonal incomplete LU preconditioner (Fletcher, 1976), whilst the geometric algebraic multi-grid method is used to solve
the pressure equation, with diagonal incomplete Cholesky Gauss–Seidel smoothing (Behrens [2008]).

The transient simulations are initialized using a steady-state solution of the entrained airflow calculated using the scikit-fem (Gustafsson and McBain [2020]) finite element method Python library. The code has been chosen as it provides greater numerical stability than the OpenFOAM steady-state solver at conditions where the flow is unstable. This higher stability is achieved by implementing the Newton iteration method (Gartling [1990]) and numerical continuation (Seydel [2009]). The flow field computed by scikit-fem is projected into the transient simulation, serving as the initial time step ($t = 0$ s).

### 3.2.2 Boundary Conditions

Both the inlet and outlet in scikit-fem are set as open boundaries, whilst the inlet in OpenFOAM is set with zero pressure gradient and a velocity profile that has been captured at the inlet of the scikit-fem simulations and the outlet has zero gauge pressure. The paper is modeled as a moving wall with no slip, traveling at a speed $V_p$, while the printhead is a stationary wall with no slip. The droplets have an ejection velocity of 14.95 m/s and are assumed to be solid spheres with diameter of 15.6 $\mu m$. The droplet diameter and velocity were estimated experimentally by measuring the average ejected mass of several thousand droplets using an accurate balance, and from high magnification stroboscopic imaging of the ejection process, respectively.

### 3.2.3 Geometry

A rectangular domain is created to model the print gap defined by the printhead and the media (Figure 3.1). The domain has the same length as a printhead system, with the inlet and outlet respectively located at 25 mm upstream and 45 mm downstream. These are, respectively, where the leading and trailing edge of the printhead are located. The model, however, simplifies the problem by using a flat roof.

A schematic of the problem is shown in Figure 3.1, where ink droplets are fired from the injec-
3. Airflow instability in a two-dimensional print gap model

transition face into the injection zone which has length \( b \) and extends from -0.5 mm to 0.5 mm. The coordinate system orientation arises from the perspective of the printed image on the paper: the media is located at \( z = 0 \) and the print nozzles are located at \( z = -H \), thus the nozzles fire in the positive \( z \)-direction. When the printed image is viewed from above, the paper moves downward, which is equivalent to negative \( y \)-direction in Fig 3.1.

3.2.4 Spatial and temporal discretisation

The domain is discretised using a structured mesh created in OpenFOAM. A uniform mesh with cell size of 0.022 mm is employed from \( y = -1.5 \) mm to \( y = 2.5 \) mm to ensure a high resolution in the region of interest near the print-zone. Upstream and downstream of this region, geometric growth rates of 1.05 and 1.03, respectively, are implemented in the \( y \)-direction to reduce the cell count and the computational cost. This results in a mesh with over 58,000 cells when \( H = 3 \) mm, which represents 1/3 of the cell count of a uniform mesh with cell size of 0.022 mm.

In an earlier grid independence study [de A. Aquino et al., 2020], the difference in the amplitude of oscillation of the velocity magnitude, when refining the mesh from a case with a uniform cell size of 0.022 mm to 0.0148 mm, is approximately 2.33% and the grid convergence index error between these two refinement levels is equal to 0.94%. The amplitude observed using the non-uniform mesh created here, in comparison to the uniform mesh, only differs by 1%, which is deemed as acceptable.

For the scikit-fem mesh, a similar strategy is implemented, with the difference that the cell size around the injection zone is equal to 0.033 mm. This is done because the velocity magnitude computed by scikit-fem only varies by less than 1% when the cell size is refined to 0.022 mm. Furthermore, due to the use of direct solvers in the scikit-fem code, further mesh refinements make the computational cost of the simulations unfeasible.

A time-step of \( 5 \times 10^{-6} \) s is set in the simulations, which results in a maximum Courant number \footnote{The Courant number is \( Co = u \times \Delta t / \Delta h \), where \( \Delta t \) and \( \Delta h \) are the time-step and the cell-size, respectively.} of approximately 1.3. The previous verification study [de A. Aquino et al., 2020] indicated...
3. Airflow instability in a two-dimensional print gap model

that the amplitude of velocity oscillation varies only by 2.4% when the time-step decreases to $1 \cdot 10^{-6}$ s while increasing the computational cost by over five times.

3.2.5 Non-dimensionalizing the print-zone

Equation 3.2 is non-dimensionalized using the ejection droplet speed $W$, and the print gap height $H$ as the characteristic scales. The corresponding derived scales for time is the transit time $H/W$, while for pressure it is $\rho W^2$, which gives the non-dimensionalized Navier–Stokes equation:

$$
\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \cdot \nabla \tilde{u} = -\frac{1}{\tilde{\rho}} \nabla \tilde{p} - 3\pi N(\tilde{u} + k)(1 + \phi) + R^{-1} \nabla^2 \tilde{u}.
$$

where $\tilde{u}$, $\tilde{p}$ and $\tilde{t}$ are the normalized $u$, $p$, $t$; $N$ is the dimensionless number density of drops given by $N = ndHv/W$ and $R$ represents the transit Reynolds number given by $R = HW/v$. The additional governing dimensionless parameters are the dimensionless print gap height or aspect ratio $(b/H)$, where $b$ is the breadth of the print-zone, and the dimensionless droplet diameter $(d/H)$.

3.3 Results

3.3.1 Characterizing the airflow instability

The flow regimes and the instability are characterized by measuring the airflow response for a matrix of cases defined by different number densities. The different number densities are set with variations in the number of nozzles firing droplets per unit area ($\sigma$) which affects the optical density of the colored figure printed on the paper. This section investigates a case with respectively.
3. Airflow instability in a two-dimensional print gap model

Figure 3.2: Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the white square represents the origin of the coordinate system and the white cross represents the probe location.

$V_p/W = 0.027$, $b/H = 1/3$ and $f_n = 15.5$ kHz in an attempt to reproduce the conditions that the market currently demands.

The velocity and vorticity fields for a case with $N = 1.57 \cdot 10^{-3}$ exhibit two main counter-rotating vortices near the injection-zone. These vortices are created by a similar mechanism as observed in impinging jets. The relative motion between droplets and surrounding air induces an impinging air-jet and this induced air-jet hits the media over an impact zone, where the axial momentum is converted through pressure into radial momentum. This leads to strong lateral velocities in the vicinity of the media. The low pressure region near the injection face sucks the surrounding air back into the impinging airflow, completing the recirculation loop and forming two vortices.

Due to the incoming cross-flow induced by the paper motion, the upstream and downstream vortices are distorted. The incoming cross-flow causes the upstream vortex to be compact and have a stronger core due to the higher rate at which axial momentum is converted into radial momentum; the downstream vortex is stretched, becoming weaker. At the bottom of the upstream vortex, a region of positive vorticity is created as the fluid layer adjacent to the paper moves in the negative $y$-direction while the bottom part of the upstream vortex adjacent to the paper moves in the positive $y$-direction, indicating the existence of a small clockwise vortex. At the bottom of the downstream vortex, a negative vorticity region is observed even though the adjacent fluid layers move in the same direction. This is because the bottom part of the downstream vortex has
3. Airflow instability in a two-dimensional print gap model

Figure 3.3: Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$.

Figure 3.4: Discrete Fourier Transform of the case with $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.027$. $St = f H/W$, where $f$ is the frequency of oscillation.
3. Airflow instability in a two-dimensional print gap model

Figure 3.5: Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.33 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the square represents the origin of the coordinate system and the white cross represents the probe location.

Figure 3.6: Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.33 \cdot 10^{-3}$ and $V_p/W = 0.027$. 
3. Airflow instability in a two-dimensional print gap model

a higher horizontal velocity than that of the paper.

The time snapshots show the non-steady behavior of the entrained airflow (Figure 3.2), where both upstream and downstream vortices present an unstable and periodic oscillation. This would likely cause periodic misplacement of droplets on the media in practical systems. Due to the airflow oscillation in the wake of the downstream vortex, it is believed that those droplets that stay in suspension (mist) far downstream of the injection zone can have their trajectories affected by the wake oscillation, leading to further misdeposition on the paper, as indicated in Rodriguez-Rivero et al (Rodriguez-Rivero et al., 2015).

A probe is positioned near the region of maximum variation in velocity in the upstream vortex, $z/H = -0.5$ mm and $y/H = 0$ to capture the velocity oscillation (Figure 3.3). The measured velocity indicates three phases: the first where the flow develops from a steady equilibrium to a transient state, the second where the oscillation exponentially grows and the last where the oscillation has fully developed at about 0.075 s, where the velocity magnitude reaches a maximum amplitude of $6.71 \cdot 10^{-3}$. The Discrete Fourier Transform (DFT) shows a spectrum composed of harmonics, see Figure 3.4. Here, the frequency of oscillation, $f$ is presented in terms of the Strouhal number, $St = f H/W$. The first peak indicates the fundamental frequency or first harmonic, with $St = 0.106$, while the other frequencies are simple harmonics defined by an integer multiple of the fundamental frequency.

The case of $N = 1.33 \cdot 10^{-3}$, Figure 3.5, shows that the two main recirculation zones are less evident than at higher number density values. This is a result of fewer droplets being fired into the injection zone, which leads to a lower transfer of momentum from the droplets to the surrounding air. This lower body-force induces a small oscillation in the flow field as indicated by the time snapshots (Figure 3.5) and further confirmed by the plots of velocity oscillation (Figure 3.6).

The amplitude of velocity oscillation only reaches a value of $3.75 \cdot 10^{-3}$, which represents a reduction of approximately 50% compared to the value computed with $N = 1.57 \cdot 10^{-3}$. The flow takes approximately 0.2 s to reach a converged amplitude of oscillation, which is 0.125 s longer than the case with higher number density. The airflow also oscillates with a dominant
3. Airflow instability in a two-dimensional print gap model

Figure 3.7: Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields – $b/H = 1/3$, $N = 0.31 \cdot 10^{-3}$ and $V_p/W = 0.027$ where the square represents the origin of the coordinate system and the white cross represents the probe location.

frequency $St = 0.104$ and the fundamental mode of oscillation has an amplitude one order of magnitude higher than the second harmonic. The predominant mode of oscillation has a practical importance as it directly determines the main length-scale of print defects, which can significantly affect the severity of the problem since the human perception of the printing defect depends on the eye sensitivity to different length-scales.

The case with $N = 0.31 \cdot 10^{-3}$ shows that the main vortices are significantly weaker and the rotational field of the downstream vortex is barely observable (Figure 3.7). Due to the reduction in the vorticity magnitude of the two main recirculation regions, a secondary flow feature is observed. Ridges in the vorticity field near the edges of the injection zone are seen and indicate the sharp variation in velocity that the flow experiences, caused by the motion of droplets.

The time snapshots indicate that the flow field is stable and, as a result, the droplet misplacement should not occur. This is further confirmed by the probe signal as no oscillation in velocity is measured over the 0.3 s of simulation. The steady behavior of this case indicates that there is a critical number density, higher than $N = 0.31 \cdot 10^{-3}$ and lower than $N = 1.33 \cdot 10^{-3}$, at which the flow transitions from stable to unstable.

The square of the amplitude of oscillation ($\tilde{r}^2$) versus the number density plot (Figure 3.8) shows the range of the flow regimes – stable and unstable – when the paper is moving at $V_p/W = 0.027$. Above a critical printing density ($N_c \approx 1.30 \cdot 10^{-3}$), the airflow induced by the droplets
3. Airflow instability in a two-dimensional print gap model

motion develops a self-sustainable oscillation with a saturated amplitude, characterizing the super-critical Hopf bifurcation type of instability \cite{Drazin:2002, Seydel:2009}. For any number density lower than $N_c$, the final solution is stable and steady and any small initial perturbation is dampened to reach a zero-amplitude equilibrium. Figure 3.8 indicates that for print densities larger than $N_c$, the square of the amplitude grows linearly as the number density is increased. This linear correlation can be approximated by the following equation: $r^2 = 0.529N - 0.0007$ where the root ($\hat{r}^2 = 0$) represents the estimated critical number density, the slope is equal to $(-\alpha)$ where $\alpha$ is the first Lyapunov coefficient. Therefore, the estimated critical number density is $1.32 \cdot 10^{-3}$ and $\alpha$ is equal to -1.89.

Figure 3.8 indicates that there is a small increase in frequency due to increments in number density, but all fundamental frequencies hover around 0.103-0.106 and the harmonics are multiple integers of the fundamental. Further, it is observed that the amplitude of the harmonics decrease as the number density reduces. The amplitude of the higher harmonics shows a more evident decay while the amplitude of the fundamentals experiences a more subtle reduction.
3. Airflow instability in a two-dimensional print gap model

The airflow instability has been characterized by fluctuations in the flow field linked to the oscillation of the main vortices near the print-zone. The oscillation shows an exponential growth in a first stage until it develops to a saturated amplitude. This oscillation is further characterized by a fundamental mode of oscillation and its harmonics. While the fundamental mode of oscillation is linked to the super-critical Hopf bifurcation, the higher harmonics arise from the non-linearity of the governing Navier–Stokes equation.

3.3.2 Influence of paper speed and induced incoming cross-flow

To investigate whether the onset of instability is triggered by the interaction between the induced cross-flow and the entrained airflow, as previous studies indicated in [Barnett and McDonald, 2014; Mallinson et al., 2016], a test with a stationary lower surface ($V_p/W = 0$) is performed, with $b/H = 1/3$ and $N = 1.57 \cdot 10^{-3}$. A following test with $V_p/W = 0.041$ is also conducted to investigate the impact of higher paper speeds on the stability of the airflow and on
3. Airflow instability in a two-dimensional print gap model

Figure 3.10: Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0$ where the square represents the origin of the coordinate system and the white cross represents the probe location.

Figure 3.11: Non-dimensionalised velocity ($\tilde{u} = u/W$) oscillation signal – $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0$. 
3. Airflow instability in a two-dimensional print gap model

Figure 3.12: System of vortex created by the entrained flow. (A) main vortices and (B) secondary recirculation regions.

The instability bifurcation.

The time snapshots indicate that even though there is no induced cross-flow, the entrained airflow becomes unstable (Figure 3.10). Due to the lack of cross-flow, two counter-rotating symmetrical vortices are created which oscillate at the same frequency but with a phase delay. In contrast to the cases with moving paper, the oscillation propagates from the injection zone to the inlet and the outlet. The velocity vectors overlaid on the contour of x-vorticity show that the impinging airflow creates two counter-rotating vortices (A), and also induces secondary recirculation cells (B) (Figure 3.12). These cells are a result of the main vortices forcing the adjacent layer of fluid to move along with them and the wall blockage (top and bottom surfaces) transforming linear momentum into angular momentum. These secondary zones propagate throughout the whole domain but become weaker as they move away from the injection zone as the velocity difference between adjacent layers decreases. Similar flow features can also be observed when the paper is moving, but they are less evident due to the incoming cross-flow.

The velocity signal captured by a probe located at \( y/H = -0.5/3 \) and \( z/H = -0.5 \) indicates that the instability takes approximately 0.075 s to develop a saturated amplitude of \( 9.69 \times 10^{-3} \). The DFT identified that the first three harmonics have Strouhal number equal to 0.062, 0.124 and 0.186 and the first harmonic being the predominant frequency of oscillation. The instability regime for a stationary paper is defined by \( r^2 = 0.2316N - 0.00027 \) (Figure 3.13). The estimated critical number density is equal to \( N_c = 1.16 \times 10^{-3} \) and \( \alpha \) is equal to -4.32. This indicates that the amplitude increases at a lower rate than when the paper is moving.

When the paper speed increases to \( V_p/W = 0.041 \) but \( H/b \) and \( N \) are kept constant, we can observe that the flow field becomes stable (Figure 3.14). The downstream vortex becomes more

\( 56 \)
3. Airflow instability in a two-dimensional print gap model

stretched and the upstream vortex becomes smaller in comparison to the cases with lower paper speed. The large variation in velocity over a smaller area results in higher vorticity for the upstream vortex. The velocity signal measured by a probe at $y/H = 0.5$ and $z/H = -0.5$ showed that the initial oscillation is completely dampened out to converge into a steady state flow within the 0.06 s of simulation.

The bifurcation diagram for $V_p/W = 0.041$ and $b/H = 1/3$ (Figure 3.15) shows that the transition occurs near a number density of $N = 1.69 \times 10^{-3}$. The instability regime near the onset of instability is defined by $r^2 = 1.71N - 0.0029$, which results in a critical number density equal to $N_c = 1.7 \times 10^{-3}$ and $\alpha$ equal to -0.585. In contrast to the stationary paper scenario that presented a higher $\alpha$, this lower $\alpha$ suggests that the shift in stability is more sensitive to variations in number density.

Previous studies had indicated that the paper speed modulates the flow stability (Arango et al., 2019; Mallinson et al., 2016, 2020; Rodriguez-Rivero et al., 2015). This is further confirmed by the results presented here, which also suggest that the critical number density tends to be
3. Airflow instability in a two-dimensional print gap model

Figure 3.14: Non-dimensionalised velocity magnitude ($\tilde{u} = u/W$) and planar vorticity ($\tilde{\omega}_x = \omega_x H/W$) fields - $b/H = 1/3$, $N = 1.57 \cdot 10^{-3}$ and $V_p/W = 0.041$ where the white square represents the origin of the coordinate system and the white cross represents the probe location.

Figure 3.15: Bifurcation diagram - non-dimensionalized amplitude of oscillation ($\tilde{r}^2 = r^2/W^2$) versus dimensionless number density ($N$) - for $V_p/W = 0.041$ and $b/H = 1/3$. 
3. Airflow instability in a two-dimensional print gap model

Figure 3.16: Bifurcation diagram - non-dimensionalized amplitude of oscillation ($r^2 = r^2/W^2$) versus dimensionless number density ($N$) - for $V_p/W = 0.027$, $b/H = 1/3.3$ and $b/H = 1/3.6$.

higher at higher paper speeds. This is believed to be related to the higher cross-flow that creates a stabilizing effect on the vortices as suggested by Mallinson et al. [Mallinson et al., 2016, 2020].

3.3.3 Influence of print gap height

Two sets of tests are conducted here to investigate the influence of print gap height on the stability of the airflow. The first set of tests has a print gap height of $b/H = 1/3.3$, while the second one has $b/H = 1/3.6$. Both tests are conducted with $V_p/W = 0.041$. The bifurcation diagram (Figure 3.16) shows that, when $b/H = 1/3.3$, the transition occurs near $N = 1.11 \cdot 10^{-3}$. The unstable linear regime has $N_c = 1.09 \cdot 10^{-3}$ and $\alpha = -0.81$. On the other hand, for $b/H = 1/3.6$, the transition occurs near $N = 0.96 \cdot 10^{-3}$, $N_c = 0.97 \cdot 10^{-3}$ and $\alpha = -0.61$. These results, therefore, demonstrate that increasing the print gap height tends to reduce the critical print density as well as increase the sensitivity of the airflow to disturbances.
3. Airflow instability in a two-dimensional print gap model

3.3.4 Dynamics of frequency

Further investigations on the dynamics of the frequency are performed using the auto-regression (AR) algorithm\(^2\). This algorithm has been chosen over the FFT because of the limited resolution of the latter to capture small variations in frequency\(^3\). Figure 3.17 displays the impact of number density on the Strouhal number for different printing conditions. For a given \(H/b\) and \(V_p/W\), the increase in \(N\) tends to slightly increase the fundamental frequency of airflow oscillation. The comparison between systems with same paper speed but different \(H/b\) indicates that higher aspect ratios only tend to lower the Strouhal number because it lowers the \(N\) at which the onset of instability occurs. The driving factor for the Strouhal number is \(N\) since, given the same \(N\), for instance \(N \approx 1.15 \cdot 10^{-3}\), it is observed that the airflow oscillates at the same fundamental frequency even though the systems have different aspect ratios. This demonstrates that, for a constant paper speed, \(N\) is the key parameter for the dynamics of the system, driving the fundamental frequency. This was already expected because \(N\) is the dimensionless number associated to the droplets body-force (Equation 3.7).

The increase in paper, however, leads to significant variations in Strouhal number. For each paper speed, there exists a specific curve that defines the effect of \(N\) on the frequency of oscillation of the system. The cross-alike markers group the curve for \(V_p/W = 0.027\), while the filled and hollow markers group \(V_p/W = 0.041\) and \(V_p/W = 0.0\), respectively. This variation also suggests that there is another dimensionless number that characterizes the impact of paper speed and/or volumetric cross-flow on the stability of the airflow.

3.4 Conclusion

Two-dimensional numerical simulations using a novel dispersed-phase continuum approach to model the entrainment effect of ink droplets were performed to investigate the type of airflow instability in the print gap of inkjet printers and investigate the instability mechanism. The results also indicated that the airflow instability is characterized by sinusoidal oscillations that

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\(^2\)Fu and He\(^4\) (2001) provide a detailed description of the auto-regression algorithm.
3. Airflow instability in a two-dimensional print gap model

Figure 3.17: Strouhal number ($St = f H/W$) versus number density ($N$) for different printing conditions.

grow until a saturated amplitude is reached. It was also demonstrated that as the number density increases, the flow shifts from stable to unstable at a specific transition point – the critical number density – and the amplitude of oscillation follows an asymptotic square-root behavior, characterizing a super-critical Hopf bifurcation type of instability. For a stationary paper, the transition occurs at a lower critical number density due to the lack of cross-flow which can have a stabilizing effect on the vortices near the injection zone. The stationary paper results, more importantly, indicated that even in the absence of induced cross-flow, the airflow can become unstable, contradicting the assertion of previous studies that attributed the mechanism of instability to the interaction between the induced cross-flow and the entrainment effect. The continuous variation in Strouhal number for systems with different print gap height demonstrated that the number density dictates the fundamental frequency of oscillation, while the paper speed variation translates the curve that defines the impact of $N$ on the frequency of oscillation of the system.
Chapter 4

Three-dimensional airflow instability
and the assessment of the DPC model

Chapter 3 presented two-dimensional numerical simulations of the airflow in the print gap of an inkjet printer using the DPC technique to model the interaction between the droplets and the surrounding airflow. In this chapter, an under review journal article entitled 'The effectiveness of the dispersed-phase continuum model for investigating the airflow in the print gap of inkjet printers' extends the analysis presented in Chapter 3. Three-dimensional numerical simulations using both DPC and P-in-C models are performed to investigate the three-dimensional onset of instability where the airflow becomes non-uniform in the spanwise direction. The results computed by both models are compared to assess and improve the effectiveness of the DPC model.

Abstract

To investigate the effectiveness of the dispersed-phase continuum (DPC) approximation to model the airflow in the print gap of inkjet printers, 3D simulations using the DPC model were
Three-dimensional airflow instability and the assessment of the DPC model compared against those using the classic particle-in-cell (P-in-C) approach. The DPC approximation, due to the separation of time scales, models the dispersed phase with a momentum source that depends on a predefined temporally averaged particle number density field. The results demonstrated that the steady DPC model correlated well to the time-averaged P-in-C solution when the former’s formulation accounted for the droplet deceleration. The steady DPC model requires less than 0.1% of the computational resources used by the transient P-in-C approach to compute the mean flow field. Further analyses indicated that the DPC model captured a supercritical pitchfork bifurcation, where the airflow shifted from a steady span-wise uniform regime to a standing wave regime at a critical number density. The P-in-C model computed a smooth continuous transition that characterizes an excited supercritical pitchfork bifurcation. An excellent correlation between the models was observed at print densities above the transition point, but at low number densities a certain level of discrepancy was observed. This was a result of the pseudo-turbulence or spottiness induced by the local and instantaneous motion of droplets that excited the standing wave solution even at low number densities. The results, thus, demonstrated that the DPC model is effective at estimating the mean flow field and approximating the bifurcation diagram, while being simultaneously more computationally efficient than the P-in-C model.

4.1 Introduction

The inkjet printing industry faces the challenge of developing systems that can operate at large print gap heights ($H > 1$ mm) to enable the expansion of inkjet technology to applications, such as rapid prototyping, manufacturing of electronics, solar panels and reinforced composites. Restrictive print gap heights in current designs prevent inkjet systems from accommodating media with variable or large thickness, and increase the likelihood of the media striking the printhead and damaging the nozzles. Further, at high print heights, aerodynamic effects tend to misplace the ink droplets on the paper, creating printing defects such as tiger-stripes and/or wood-grain. Effective solutions to these challenges involve the development of new inkjet printhead architectures.
4. Three-dimensional airflow instability and the assessment of the DPC model

These printing defects have been found to be associated with the airflow non-uniformity in time and span-direction across the print-zone [Barnett and McDonald (2014); Hsiao et al. (2012, 2013); Mallinson et al. (2016)]. The relative motion between the ink droplets and the surrounding air creates two counter-rotating vortices and, at specific operating conditions, these become unstable [de A. Aquino et al. (2020); Mallinson et al. (2020); Rodriguez-Rivero et al. (2015); Mallinson et al. (2022)] demonstrated, using numerical analyses based on the dispersed-phase continuum (DPC) model the existence of three different airflow regimes. At large $b/H$, where $b$ is the breadth of the print-zone (Figure 4.1), the airflow is stable and uniform in time and span-direction ($x$ direction), while at smaller $b/H$ the airflow is unsteady. At intermediate $b/H$, however, the airflow is uniform in time but non-uniform in the span-direction as it shows a spanwise periodicity. In this regime, the airflow’s $x$-velocity varies in a sinusoidal pattern and the main vortex becomes deformed with a standing wave pattern evident.

The airflow non-uniformity can then misplace the droplets on the paper, inducing the print pattern characteristic of tiger-stripes. We note here that one of the mechanisms that creates regions with higher optical density on the paper is a result of the misplacement of satellite droplets. The satellite droplets have lower Stokes number, $S < 1$, with

$$ S = \frac{\rho_d d^2 W}{18 \mu H} $$  \hspace{1cm} (4.1)

where $\rho_d$ is the droplet density, $d$ is the diameter of the droplet, $\mu$ is the dynamic viscosity of air and $W$ is the droplet ejection velocity [Mallinson et al. (2016)]. This means that the satellite
droplets are more likely to be carried by the airflow instability, landing away from the main drops and covering the white spaces between lines as seen in Figure 4.2 which has been reproduced from Mallinson et al. (2022). From Figure 4.2, it is also observed on the left and right sides of the bottom image that, depending on the operating conditions, the main droplets can be minimally affected by the airflow. This is supported by the fact that the line width and space between lines are consistent across the width of the page. At conditions of intense airflow non-uniformity, however, the main droplets are also expected to be misplaced (Rodriguez-Rivero et al., 2015). The main drop and satellite misplacement have their respective influence coefficients that depend on dot size, print density and misplacement magnitude that impact on the optical density variation. Mallinson et al. (2016) indicated that, in numerical analysis, the expected optical density variation can be predicted with the divergence of displacement of both main and satellite droplets. Since, in this study, focus is given to investigate the conditions at the onset of spanwise non-uniformity, it is expected that the main droplets are not sufficiently deflected to change their effect on the airflow.
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.3: On the left: flow field snapshots adapted from Mallinson et al. (2022), on the right side: prints of flat gray on a A4 page provided by Memjet with print speed of 0.41 m/s, green noise of 10%, a.2) $b/H = 1/2.2$; b.2) $b/H = 1/3.2$; and c.2) $b/H = 1/4$. 


Print samples provided by Memjet\textsuperscript{1} have indicated the existence of similar regimes as observed in the simulations. At low duty cycles (fraction of nozzles activated during printing) and $b/H$ ratios, the prints tend to be uniform (flat gray), but with higher duty cycles and/or aspect ratios, the prints develop a spanwise periodicity characterized by vertical bands with different aspect ratios. With further increments in duty cycles and/or aspect ratios, the prints appear more non-uniform in the longitudinal direction. It is believed that these printing patterns relate to the flow regimes identified in Mallinson et al. (2022). Whilst the temporally non-uniform (unsteady) airflow leads to longitudinally non-uniform prints, the spanwise flow field variation of the standing waves creates the reported vertical stripes. The different flow regimes and their respective impact on prints of uniform image density is summarized in Figure 4.3. This manuscript focuses on investigating the transition from the aforementioned uniform regime to the standing wave regime as this transition is expected to define the bounds of the printing envelope at which the prints are uniform.

The numerical analyses conducted by de A. Aquino et al. (2020, 2022b); Mallinson et al. (2022) introduced the DPC model to investigate the entrainment effect of the dispersed phase (ink droplets). This model was introduced as a less computationally expensive alternative to the classic particle-in-cell (P-in-C) or Lagrangian model that had already been implemented to investigate the entrainment effect Barnett and McDonald (2014); Fujii and Mori (2009); Mallinson et al. (2016). The P-in-C and the DPC approaches, along with the Direct Numerical Simulations M. Kuerten (2016); Nasr et al. (2009) and Eulerian-Eulerian models Baker et al. (2020); Vié et al. (2016), are part of a wide variety of models that deal with the same challenge of being computationally efficient, while accurately predicting the dynamics of particle-driven flows Baker et al. (2020).

The P-in-C model employs dynamic equations to track each individual particle of the dispersed phase while treating the fluid phase as a continuum Baker et al. (2020); Keser et al. (2019); Seydel (2007); Vié et al. (2016). The DPC approximation models the dispersed phase with a momentum source that depends on a predefined temporally averaged particle number density

\textsuperscript{1}Memjet is a multinational inkjet printer manufacturer (https://www.memjet.com/) and research partner in this project
4. Three-dimensional airflow instability and the assessment of the DPC model

This assumption eliminates the need of solving the dynamic equations to calculate the number density in each specific cell as seen in the P-in-C model and is expected to reduce the computational cost of the simulations. While a preliminary comparison of the flow features predicted by the DPC and P-in-C models and those evident in experimental, time-averaged, laser light-sheet visualization has already been presented Mallinson et al. (2022), further investigation needs to be undertaken to assess the effectiveness of the DPC model. In this manuscript, the effectiveness of the DPC model for predicting the change from a uniform regime to a standing wave regime is investigated and compared against the results using the P-in-C model.

4.2 Methodology

4.2.1 Numerical model

Both P-in-C and DPC models solve the continuity and momentum equations for incompressible laminar flows and assume the droplets as point sources:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{F} + \nu \nabla^2 \mathbf{u}, \tag{4.2}
\]

\[
\nabla \cdot \mathbf{u} = 0. \tag{4.3}
\]

where \( \mathbf{u} \) is the velocity field, \( t \) is time, \( p \) is pressure, \( \nu \) is the air kinematic viscosity, \( \mathbf{F} \) is the force per unit mass induced by the dispersed phase moving at velocity \( \mathbf{u}_d \), where the subscript \( d \) refers to the dispersed or particle phase.

The P-in-C model, implemented in the OpenFOAM solver reactingParcelFoam, computes the body-force in the \( k \)-th cell (\( \mathbf{F}_k \)) and at a given time-step as the summation of the forces exerted by \( q \) number of particles located in that \( k \)-th cell, where \( \mathbf{f}_k^m \) represents the force exerted by the \( m \)-th particle. We note here that, due to the separation between droplets and cell sizes tested in this study, \( q \) is either 0 or 1 and Equation 4.4 reduces to Equation 4.5. The P-in-C model
4. Three-dimensional airflow instability and the assessment of the DPC model

demands yet additional equations to track the motion of droplets over time. The position of
the \( m \)-th particle with volume \( \nu_d \) is given by the kinematic equation of motion (Equation 4.6) and Newton’s second law is used to compute the particle velocity (Equation 4.7). Figure 4.4.a shows the process used by the P-in-C model to compute the solution.

\[
F_k = \sum_{m=1}^{n_k} f_m^k. \tag{4.4}
\]

\[
F_k = \begin{cases} 
  f_k^1 & \text{if } q = 1 \\
  0 & \text{otherwise}
\end{cases} \tag{4.5}
\]

\[
\frac{dx_d^m}{dt} = u_d^m. \tag{4.6}
\]

\[
\nu_d \rho_d \frac{du_d^m}{dt} = f_m. \tag{4.7}
\]

In the dispersed-phase continuum (DPC) model, the dispersed properties are defined beforehand (Figure 4.4.b). Due to the separation of time scales (droplet relaxation time, flow characteristic time, droplet transit time and time difference between droplets visiting the \( k \)-th cell), it is assumed that the force term on the \( k \)-th cell with volume \( \nu_k \) is governed by the expected temporally averaged rate of visitation of the particles at the respective cell \( (n_k) \).

\[
F_k = \nu_k n_k f_m^k. \tag{4.8}
\]

In both models, \( f_m^m \) is equal, but of opposite sign, to the drag the droplets experience and is modeled with the Stoke’s law multiplied by an empirical correction factor (Eq. 4.9), \( 1 + \phi \) determined by White \cite{White1991} (Eq. 4.10). This method is based on a fit to experimental data for Reynolds number less than 200,000, which covers the range of droplet Reynolds number (\( \text{Re}_d \))
Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.4: Flowchart exhibiting the solution algorithm for the P-in-C (a) and DPC (b) models.

tested in this study, which is approximately $4.5 < Re_d < 16$ ($Re_d = \frac{|u_d - u_m|}{v}$). The Reynolds number for the flow across the print gap, on the other hand, is given by $Re_H = \frac{V_p H}{v}$ and is equal to 81.

$$f^m = 3\pi \mu(u_d^m - u)[1 + \varphi(Re_d^m)]. \quad (4.9)$$

$$\varphi(Re_d) = \frac{Re_d}{4(1 + \sqrt{Re_d})} + \frac{Re_d}{60}. \quad (4.10)$$

To couple the pressure and momentum equations in both models, the PIMPLE scheme [Holzmann 2016] is employed here due to its robustness. It is deemed that the outer loop of the PIMPLE scheme has reached convergence when the pressure and momentum residuals fall to less than $10^{-5}$, as suggested by Holzmann [Holzmann 2016]. Pure Crank–Nicolson, a second-order implicit scheme, is used to discretise the time derivative, while the second-order discretisation of the gradient, divergence and Laplacian terms is achieved via the unbounded central-
differencing algorithm, named Gauss linear in the OpenFOAM idiom [Holzmann] (2016). The velocity terms are solved using the bi-conjugate gradient method with a diagonal incomplete LU preconditioner [Fletcher] (1976), whilst the geometric algebraic multi-grid method is used to solve the pressure equation, with diagonal incomplete Cholesky Gauss–Seidel smoothing [Behrens] (2008).

4. Three-dimensional airflow instability and the assessment of the DPC model

4.2.2 Geometry and boundary condition

A rectangular cuboid domain was created to reproduce the print gap defined by the printhead and the media (see Figure 4.1). The media is a wall moving at \( V_p \) in the negative \( y \)-direction and located at \( z = 0 \), while the printhead surface is modeled as a stationary wall and is located at \( z = -H \). The lateral boundaries are located \( w \) apart to compute the wavelength with sufficient resolution and are set as periodic boundaries. The injection-zone, i.e., the region where the droplets are fired from the injection face, has length \( b = 1/3H \) extending throughout the span of the domain. For the DPC model, \( w \) is equal to \( 15H \) but, for the P-in-C model, due to the excessive computational cost of the simulations, \( w = 1.875H \) or \( 1/8 \) of \( 15H \) which is equal to the wavelength of one possible solution observed in the DPC simulations. This allows the P-in-C model to time resolve the standing wave without significantly affecting the flow field spatial resolution.

The inlet and outlet are respectively located \( 8.33H \) upstream and \( 15H \) downstream of the print-zone to allow the flow to fully develop. While the outlet has a specified constant pressure, the inlet is set with a Couette–Poiseuille velocity profile. The Couette component is induced by the paper motion while the Poiseuille component is a result of the pressure gradient created by aerosol suction systems and the drag created by the droplets. The velocity profile is, then, derived from the through-flow parameter, \( \chi \), which is the ratio of the added (Poiseuille) flow rate, \( q_a \), to the paper-induced (Couette) flow rate, \( q_p \), where \( \bar{V}_a \) is the average velocity of the Poiseuille part.

\[
\chi = \frac{q_a}{q_p} = 2 \frac{\bar{V}_a}{V_p}, \tag{4.11}
\]
4. Three-dimensional airflow instability and the assessment of the DPC model

In this study, the temporally averaged number density field \( n \) used in the DPC model is defined by Equation 4.12, where it depends on the number of firing locations (active nozzles) per unit area on the injection face \( \sigma \), the firing frequency \( f_n \) and \( u_d \). The number of active nozzles per unit area is given by \( \sigma = n_r \xi / \Delta xb \), where \( \xi \) is the duty-cycle which represents the fraction of nozzles that are activated with \( 0 \leq \xi \leq 1 \), \( n_r, b \) and \( w \) are the number of injection rows, injection-zone breadth and width (out of the page in Figure 4.1), respectively. To begin with, the droplet velocity was set as constant, disregarding the droplet deceleration. The impact of this assumption on the accuracy of the model is further described in the Results section.

\[
\begin{align*}
n & = \frac{\sigma f}{|u_d|}.
\end{align*}
\] (4.12)

4.2.3 Governing dimensionless parameters

Equation 4.2 is non-dimensionalized using the ejection droplet speed \( W \) and the print gap height \( H \) as the characteristic scales. Given that the problem is defined by eight parameters - \( W, d, H, b, V_p, n, v \) and \( \bar{V}_d \) - that have two dimensions, there must be six groups of dimensionless parameters governing this system, which are \( N, R, b/H, Re_d, V_p/W \) and \( \chi \). \( N \) is the dimensionless number density of drops given by \( N = n_o d H v / W \), where \( n_o \) is \( n \) at the injection face, and \( R \) represents the transit Reynolds number (\( R = H W / v = 2978 \)). The additional governing dimensionless parameters are the dimensionless print gap height or aspect ratio (\( b/H = 1/3 \)), droplet Reynolds number (\( Re_d = W d / v = 15.5 \)), dimensionless paper speed (\( V_p/W = 0.027 \)) and dimensionless through flow (\( \chi = 0 \)). Here, the continuum phase (air) has kinematic viscosity of \( v = 0.015 \text{mm}^2 \text{ms}^{-1} \). The droplets are ejected with velocity of \( W = 14.95 \text{m/s} \) at firing rate of \( f_n = 15.5 \text{kHz} \) and are assumed to be solid spheres with diameter of \( d = 15.6 \mu \text{m} \) [Mallinson et al. (2022)].
4. Three-dimensional airflow instability and the assessment of the DPC model

4.2.4 Flow metrics

The spanwise component of the airflow velocity \( u_x \) is the main metric used to correlate the misplacement of droplets on the paper, to the flow features observed in the simulations. This is because the optical density variation seen in Figure 4.3.b.2 primarily occurs in the spanwise direction, indicating that it is linked to variations in \( u_x \). To measure this variation in the flow field, a spanwise transect and a plane, both located at \( z/H = -0.5 \) were used in our analyses. The root mean square (RMS) of the \( u_x \) profile along the spanwise transect is then further calculated to quantify the strength of the standing wave.

To aid in the investigation of changes in the structure of the main vortex, isosurfaces and contours of \( Q \)-criterion are employed. The \( Q \)-criterion has been used in different applications to efficiently identify vortices. This relates to its formulation that subtracts the strain rate part of the vorticity magnitude (Equation 4.13). When \( Q > 0 \), the vorticity exceeds the strain rate, indicating the existence of a vortex. We note that the dimensionless \( Q \)-criterion is given by \( \tilde{Q} = Q(H/W)^2 \), while the dimensionless velocity is \( \tilde{u} = u/W \).

\[
Q = \frac{1}{2} (||\Omega||^2 - ||S||^2). \tag{4.13}
\]

4.2.5 Transient statistics

Unsteady flows, as measured in the P-in-C simulations, tend to present an initial transient phase (start up) where the flow develops from the initial conditions. To eliminate this start up effect and determine the interval that the signal is statistically stationary, the Transient Scanning Technique (TST) is employed. The TST calculates the uncertainty of the mean for a 95% confidence level and determines the interval at which the uncertainty of the mean exponentially decays with the realization time, characterizing a stationary signal [Brouwer et al. (2013)].

The TST applied to the RMS of the lateral airflow velocity measured by a transect indicates that the start up effect can take up to 200 000 time steps. The uncertainty of the mean for the
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.5: Comparison of velocity profiles captured by meshes with cell size of 0.0167H, 0.025H and 0.0375H in simulations using the DPC (a) and P-in-C (b) models.

stationary regime tends to be lower than 0.2%. This low uncertainty level is likely to be due to the long realisation times that reach up to 400,000 time steps. For the remainder of this manuscript, the mean values of the P-in-C simulations refer only to the stationary interval of the signal.

4.2.6 Spatial discretisation

The domain is discretised using a structured mesh created in OpenFOAM. A uniform mesh with cell-size of 0.025H is employed from \( \frac{y}{H} = -0.5 \) to \( \frac{y}{H} = 0.833 \) to ensure a high mesh resolution in the region of interest near the print-zone. Upstream and downstream of this region, geometric growth rates of 1.05 and 1.03, respectively, are implemented in the \( y \)-direction to reduce the cell count and the computational cost. In the \( x \)- and \( z \)-direction, all cells have length equal to 0.025H. This results in a mesh with over 5 million cells.

To assess the error associated with the spatial discretisation, tests with meshes with minimum
4. Three-dimensional airflow instability and the assessment of the DPC model
cell-size of 0.0167H, 0.025H and 0.0375H for a case with $N = 0.355 \times 10^{-3}$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ were compared. The velocity profile captured by a transect extending in the spanwise direction of the domain and located at $y/H = 0$ and $z/H = -0.5$ was used to undertake this comparison. For the DPC model, Figure 4.5a indicates that these three mesh sizes capture the profile of a standing wave but the coarsest mesh does not capture the same magnitude. Due to the agreement between the two refined meshes, a cell-size of 0.025H was used to reduce the computational cost of simulations in the present work. For the P-in-C model, however, the mean RMS of $\tilde{u}_x$ and the velocity profile are compared to deem convergence. The results indicate that the $\tilde{u}_x$-RMS when using the grid size equal 0.025H only differs by 0.5% in comparison to the most refined grid, while the coarsest mesh presents a difference of 12%. Figure Figure 4.5b also indicates that the grids with cell size of 0.025H and 0.0375H correlates well. It is believed, thus, any improvements provided by a more refined mesh do not seem to compensate the increase of over 2x in computational time and a cell-size of 0.025H was used to perform the analyses presented here.

4.2.7 Temporal discretisation

Time-steps of $\Delta \tilde{t} = 0.25, 0.1$ and 0.05, where $\tilde{t}$ is the droplet transit time defined by $H/W$, in a case with $N = 0.355 \times 10^{-3}$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ were tested. For the DPC model, since the airflow converges to a steady-state solution, a steady version of the model was also compared against the transient analyses. The velocity profile captured by the transect located at $y/H = 0$ and $z/H = -0.5$ (Figure 4.6a) indicates that the steady solver efficiently captures the flow profile without introducing any errors due to the lack of temporal resolution.

For the P-in-C model, however, due to its nature, the simulations must be transient. To assess the error associated with the temporal discretisation, the time average of the RMS of $\tilde{u}_x$ was computed for time-steps of $\Delta \tilde{t} = 0.1, 0.25$ and 0.5. It was observed that the mean RMS of $\tilde{u}_x$ captured when using a time step of $\Delta \tilde{t} = 0.25$ only differed by less than 1% in comparison to the simulations with $\Delta \tilde{t} = 0.1$, whereas the RMS of $\tilde{u}_x$ for $\Delta \tilde{t} = 0.5$ differs by 1.9%. Figure 4.6b also shows that the results with $\Delta \tilde{t} = 0.25$ correlates well to the simulation with $\Delta \tilde{t} = 0.1$ and, as
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.6: Comparison of velocity profiles in steady-state and transient simulations with time-step of $\Delta \tilde{t} = 0.25, 0.1$ and $0.05$.

a result, the following P-in-C numerical analyses are performed with $\Delta \tilde{t} = 0.25$.

4.3 Results

4.3.1 Investigating the DPC assumptions

A case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$ and $\chi = 0$ was initially used to assess the assumptions implemented in the DPC model. In previous studies de A. Aquino et al. (2020, 2022b); Mallinson et al. (2022), the DPC model approximated the droplet velocity to constant and equal to the ejection velocity. The results computed using the DPC model were compared against snapshots and the time average of the flow field computed by the P-in-C model. The time average of the statistically stationary interval creates a more homogeneous flow field and facilitates the comparison between the DPC and P-in-C models.
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.7: Vector field overlayed on $\tilde{Q}$-criterion contour comparing the DPC and P-in-C models at a vertical plane located at $x/H = 0.9375$ for a case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$ and $\chi = 0$, a) DPC model that disregards the droplet deceleration, b) DPC that accounts for droplet deceleration, c) P-in-C snapshot at $\tilde{t} = 5 \times 10^4$ and d) P-in-C time averaged. A, B and C are, respectively, the upstream and downstream vortices and the vortex spottiness. Paper moving from left to right.
The $\bar{Q}$-criterion at a vertical plane located at $x/H = 0.9375$ for a case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$ demonstrates that both models capture two counter-rotating vortices (A and B) with the upstream vortex (A) being more concentrated (Figure 4.7). These flow structures are similar to those seen in the 2D simulations performed by de A. Aquino et al. (2022b) with the DPC model. The P-in-C snapshot at $\bar{t} = 5 \times 10^4$ (Figure 4.7-c) tends to show a distorted and quite spotty $\bar{Q}$-criterion flow field (C) as a result of the instantaneous relative motion between particles, while the time-averaged data shows a more homogeneous field but with some residual spottiness (Figure 4.7-d). The DPC model, however, tends to overpredict the strength and size of the counter-rotating vortices (Figure 4.7-a). This is a result of the constant dispersed phase velocity field assumption initially used in the DPC model. This is further confirmed in Figure 4.8 that shows the droplets decelerating to 0.51 of their initial velocity, resulting in significant loss in momentum.

To account for the droplet deceleration in the DPC model, a one-way formulation that first estimates the final velocity of a single particle when moving in still air for a distance $H$. This was achieved by integrating the equations of motion (Equations 4.6 and 4.7) over time until the particle had traveled the specified distance. It was assumed that the only force acting on the particle is the drag which is modeled with the Stokes law multiplied by White’s empirical correction factor. This formulation gave an error of approximately 1% in comparison to the final velocity computed by the P-in-C model. Once this process was performed, the droplet deceleration was implemented in the DPC model by assuming that $u_d$ in Equation 4.8 varies linearly from the ejection velocity to the final velocity. The use of a linear interpolation is deemed as acceptable since the droplet velocity variation during flight differs only marginally from a straight line, which is confirmed by the studies of Barnett and McDonald (2014).

When the DPC model accounts for the droplet deceleration, the flow structures better correlate to those seen in the P-in-C model. The strength and size of the vortices are equivalent in both models (Figure 4.7-b). A transect taken in the longitudinal direction of the domain at $x/H = 0.9375$ and $z/H = -0.5$ quantitatively characterizes the correlation between the models. Figure 4.9 clearly indicates that the DPC with droplet deceleration shows a strong correlation to the P-in-C model. The DPC captures well both the maximum velocity and the gradient of veloc-
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.8: Point particles colored by their velocity magnitude (left) and horizontal velocity (right) overlayed on the flow field velocity magnitude contour indicating the droplets’ velocity variation as they transit across the print gap. Paper moving from left to right.

Further investigations of the main droplet velocity indicates that, for the paper speed and through flow tested here, the droplets longitudinal velocity reaches up to 0.14% of the ejection velocity. Given the transit time of the droplets, it is estimated that their longitudinal displacement does not exceed $6.67 \times 10^{-4}H$. This demonstrates that the incoming cross-flow minimally displaces the droplets during flight and, as a result, it is expected that there is no substantial need to account for the longitudinal displacement of droplets nor their longitudinal velocity in the DPC model.

The agreement between the DPC and P-in-C model is a strong indicator that treating the discrete phase as a continuum smooth field is a valid strategy to compute the mean flow field. The main difference between the two models comes from the spottiness predicted using the P-in-C model that is a result of the local and instantaneous slip velocity of the particles that perturbs the base flow (Figure 4.10). The FFT of the stationary interval of the velocity signal measured by a probe located at $x/H = 0.9375$, $y/H = 0$ and $z/H = -0.5$ is used to characterize the airflow spottiness. It is seen that the spottiness manifests as a dense spectrum with no peak in Strouhal number ($St = f_n H/W$) and where the power density decays with $St$. The spottiness, although existent in real printing systems, is not expected to directly induce the tiger stripes printing defects. This is because this printing defect is characterized by specific time and length scales while the spottiness behaves as noise. The specificity of the time and length scales are the factors
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.9: Velocity magnitude measured by a longitudinal transect located $x/H = 0.9375$ and $z/H = -0.5$ comparing the DPC with and without droplet velocity correction (dark red and blue lines respectively) and P-in-C (green dots) for a case with $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. 
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.10: Top image: $x$-velocity signal measured by a probe located at $x/H = 0.9375$, $y/H = 0$ and $z/H = -0.5$. Bottom image: DFT of the velocity signal showing the frequency behaviour of the spottiness. Printing conditions: $N = 0.267 \times 10^{-3}$, $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$, $R = 2978$ and $Re_d = 16$

that contribute to the objectionability of the printing defects as the human eye is very perceptive of patterns while tolerant to blur or noise [Sharma and Bala (2017)].

4.3.2 Bifurcation diagram

The effectiveness of the DPC model for predicting the range of the flow regimes is investigated by characterizing the airflow response to an array of cases defined by different number densities ($N$). We note, this study extends the work done in 2D [de A. Aquino et al. (2020, 2022b) to a 3D domain, while also assessing the accuracy of the DPC model. Cases with $V_p/W = 0.027,$
Three-dimensional airflow instability and the assessment of the DPC model

$b/H = 1/3$ and number density ranging from $N = 0.267 \times 10^{-3}$ to $N = 0.355 \times 10^{-3}$ are performed in an attempt to reproduce some of the conditions that the market currently demands.

For low number densities, the flow field captured by the DPC model is uniform in time and spanwise direction. The base flow field is characterized by a pair of spanwise-uniform counter-rotating vortices as indicated in Figure 4.7 and 4.11-b.1 and described by de A. Aquino et al. (2020). Due to the interaction with the incoming cross-flow, the upstream vortex, referred to as the main vortex, is more concentrated than the downstream vortex. The main vortex extends across the spanwise direction of the domain forming a nearly cylindrical shape.

For high number densities, the vortices become stronger due to the higher entrainment effect and their cores become deformed, and a standing wave pattern is evident (Figure 4.11). This standing wave is characterized by a specific wavenumber. Two possible different solutions with 7 and 8 wavenumber have been captured by the model with the imposed span of $15H$, which translates to wavelengths ($\lambda$) of $\lambda/H = 2.143$ and 1.875, respectively. The wavelength measured in the simulations correlates well to that of the prints performed at Memjet, which range from $\lambda/H = 1.667$ to 2.667.

To characterize the range of flow regimes (spanwise-uniform flow field and standing wave), the root mean square (RMS) of $\tilde{u}_x$ measured by a spanwise transect located at $y/H = 0$ and $z/H = -0.5$ is employed. It is seen in Figure 4.12 that, for each solution ($\lambda/H = 2.143$ and 1.875), there is a bifurcation point or critical print density ($N_c$) at which the RMS shifts from zero to a finite value, indicating that the airflow transitions from the uniform regime to the standing wave regime. At the critical print density, the disturbance induced by the entrainment effect promotes an exchange of stability that characterizes a supercritical pitchfork bifurcation Schmitt and Bayly (1998). The two solutions have similar bifurcation diagrams that define the exchange of stability. While the solution with $\lambda/H = 2.143$ transitions at $N_c = 0.306 \times 10^{-3}$, the solution with $\lambda/H = 1.875$ bifurcates at $N_c = 0.305 \times 10^{-3}$. It is also observed that, for $\lambda/H = 2.143$, the RMS of the standing wave reaches higher magnitudes. We note here that to compute the two possible solutions, the cases were initialized with either a Couette–Poiseuille flow field or through numerical continuation Seydel (2009).
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.11: On the left side: top view of a $xy$ plane located at $z/H = -0.5$ showing isosurface of $\tilde{Q}$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\tilde{u}_x$ comparing, while on the right side: $\tilde{\omega}_x$ contour at a vertical plane located at $x/H = 0.9375$, both set of plots comparing a) Spanwise-uniform flow field regime with $N = 0.267 \times 10^{-3}$, b) Standing wave regime for a case with $N = 0.267 \times 10^{-3}$ and $\lambda/H = 1.667$ and c) Standing wave regime for a case with $N = 0.267 \times 10^{-3}$ and $\lambda/H = 2.667$. Printing conditions: $V_p/W = 0.027$, $b/H = 1/3$, $\chi = 0$ $R = 2978$ and $Re_d = 16$. Paper moving from top to bottom.
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.12: Bifurcation diagram showing the transition from uniform flow to standing wave regime for DPC solutions with wavelength $\lambda/H = 2.143$ and 1.875 at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$.

The results show that the transition to the standing wave regime in a 3D domain occurs at lower number densities than the supercritical Hopf bifurcation captured in a 2D domain, which occurred at $N_c = 1.30 \times 10^{-3}$. This suggests that the airflow becomes non-uniform in the spanwise direction before it becomes non-uniform in time.

For the comparison between the DPC and the P-in-C, a domain with $w = 1.875H$ width is used. This is because of the larger computational cost of the P-in-C and the long realization time to reach statistical stationarity. The comparison between the DPC and the P-in-C models for a case with $N = 0.355 \times 10^{-3}$ shows excellent agreement. Figure 4.13 shows that the time-averaged P-in-C captures the same flow field pattern and the same vortex deformation as that observed in the DPC. The P-in-C snapshot at $\tilde{t} = 10^5$ (Figure 4.13-c) shows that the spottiness tends to create a corrugated deformed vortex without significantly altering the downstream $\bar{u}_x$ flow field magnitude. A longitudinal transect at $y/H = 0$ and $z/H = -0.5$ (Figure 4.14) con-
Three-dimensional airflow instability and the assessment of the DPC model

It is observed that the time-averaged P-in-C shows a similar standing wave profile as that of the DPC but the profile tends to have a 3% lower amplitude.

The bifurcation diagram when using the P-in-C differs from the DPC diagram as it shows a smooth continuous curve rather than a transition point. The DPC and P-in-C agree well for number densities above the transition point. However, at low number densities, below and near the transition point, the P-in-C shows a nonzero RMS, which is a result of the spottiness that perturbs the flow field. This is because the standing wave mode is repeatedly excited by the stochastic forcing spottiness and the balance between the forcing and the rate of decay leads to a nonzero mean at statistical stationarity. Systems subjected to a stochastic excitation have already been observed to have a similar response \cite{Juel1997, Schumaker1987}. For instance, in the study of a horizontal pendulum \cite{Schmitt1998}, it was demonstrated that the system presented a supercritical pitchfork bifurcation when the pendulum was pushed by a sinusoidal force but when the force was perturbed by a stochastic noise, the bifurcation diagram became a smooth continuous curve in what has been referred to as a perturbed supercritical pitchfork bifurcation.

To investigate the discrepancies between the DPC and the P-in-C at low number densities, the isosurface of $\tilde{Q}$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\tilde{u}_x$ for a case with $N = 0.267 \times 10^{-3}$ is exhibited in Figure 4.16. The isosurface computed by the P-in-C shows that the standing wave vortex core deformation is minimal, nearly unnoticeable in comparison to the lumps created by the spottiness. The contour of $\tilde{u}_x$ shows that the magnitude of $\tilde{u}_x$ tends to be close to zero. However, the flow field starts to demonstrate the patterns observed in the case with $N = 0.355 \times 10^{-3}$ (Figure 4.13) where the lateral velocity shows a sinusoidal variation from positive to negative.

The DPC model serves, thus, as means of predicting the time averaged flow features and approximating the evolution of the standing wave regime while also being much more computationally efficient than the P-in-C model. The DPC model takes no more than 1.5 hours when run on 96 processors to reach a steady converged solution for a $15H$ wide domain and approximately 0.2 hours for a domain with width of $w = 1.875H$. On the other hand, the P-in-C takes
Figure 4.13: Isosurface of $\tilde{Q}$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\tilde{u}_x$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$. a) DPC solution; b) P-in-C time average; and, snapshot at $\tilde{t} = 10^5$ c). Paper moving from left to right.
Figure 4.14: Lateral velocity profile taken at a spanwise transect located at $y/H = 0$ and $z/H = -0.5$ for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C models.
4. Three-dimensional airflow instability and the assessment of the DPC model

Figure 4.15: Bifurcation diagram at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing the DPC ($\lambda/H = 1.875$) and P-in-C models.
Figure 4.16: Isosurface of $\tilde{Q}$-criterion overlayed on a plane located at $z/H = -0.5$ and colored by $\tilde{u}_x$ for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.267 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ comparing: a) the DPC with $\lambda/H = 1.875$; b) P-in-C time averaged; and c) P-in-C snapshot at $\tilde{t} = 5 \times 10^4$. Paper moving from right to left.
over 240 hours to run $\tilde{t} = 10^4$ (40,000 time steps) of simulation when run on the same 96 processors for a full width domain. This simulation time is, however, insufficient to reach statistically meaningful results and increasing the simulation time is unfeasible for the computational resources available. The P-in-C simulations with a domain width of $w = 1.875H$ takes approximately 280 hours to reach $\tilde{t} = 10^5$ (400,000 time steps) of flow realization. In some cases, the start up effect can take up to $\tilde{t} = 5 \times 10^4$ of realization time, which significantly contributes to the inefficiency of the P-in-C model. The efficiency of the DPC model is due to its formulation that does not track the Lagrangian particle and allows running a steady solver to compute the mean flow field. Such a reduction in computational cost allows a refined search to identify the critical transition point.

4.4 Conclusion

Three dimensional numerical simulations were performed to investigate the effectiveness of the dispersed-phase continuum approximation to model the entrainment effect of the droplets in the print gap of inkjet printers. The results computed by the DPC model were compared against the classic transient particle-in-cell (P-in-C) model. The DPC assumes that, due to the separation in time scales, the dispersed-phase can be modeled with a momentum source that depends on a predefined temporally averaged number density field.

Initial tests were performed to assess the assumptions made within the DPC model. It was observed that the droplet deceleration must be accounted for to improve the accuracy of the DPC model. The horizontal motion of the particles, on the other hand, tends to minimally affect the flow field and, as a result, it can be disregarded for the conditions tested in this study. The main difference between between the models is that the P-in-C captures the pseudo-turbulence or spottiness created by the instantaneous slip velocity of the particles. The DPC computes a steady flow field that correlates well to the time-averaged solution of the P-in-C model. The main advantage of the DPC model is its efficiency as the computational time to predict the mean flow field is about one fourteen-hundredth of that required for the P-in-C calculation. This is due to the DPC formulation that does not track the particles and the base flow being
4. Three-dimensional airflow instability and the assessment of the DPC model

steady, allowing to run the DPC model with a steady solver.

Further analyses were performed to characterize the transition from a uniform in time and spanwise direction flow field to a standing wave regime since this sheds light into the printing conditions at which prints are expected to be uniform. While the DPC model captured a critical number density where the flow field shifted from uniform to non-uniform in spanwise direction, the P-in-C model predicted a smooth continuous transition. At number densities above the critical print density, very good correlation was observed between models, but below the critical number density, a certain level of disagreement was exhibited as the P-in-C captured a small and finite standing wave amplitude. This was a result of the *spottiness* that excited the standing wave solution even at low number densities. Thus, the DPC model is effective at estimating the time averaged flow features and approximating the evolution of the standing wave regime, while also being dramatically more computationally efficient than the P-in-C model. The reduction in computational cost allows the characterization of the flow field at a wide range of different printing conditions.

In comparison to the 2D DPC results presented in [de A. Aquino et al. (2022b)], the 3D DPC and P-in-C simulations captured similar flow features, i.e., two main counter-rotating vortices located upstream and downstream of the injection-zone. While the upstream vortex was stronger and smaller due to the interaction with the incoming cross-flow, the downstream vortex was stretched. It is also demonstrated that the pitchfork bifurcation captured in a 3D domain occurred at a lower number density than the super-critical Hopf bifurcation predicted by the 2D model. This suggests that the exchange between a uniform regime to a non-uniform regime in the spanwise direction tends to define the bounds of the printing envelope.
Chapter 5

The evolution of printing defects

In chapter 4, the transition from a uniform in time and spanwise direction flow field to a standing wave regime was characterized and it has been demonstrated that the DPC model is an effective strategy to estimate the range of the airflow regimes. In this chapter, a submitted journal article entitled ‘The evolution of printing defects in inkjet printers at elevated print gap height’ refines the understanding about the onset of three-dimensional instability and its impact on the print quality. Printing tests are performed with a page-wide industrial inkjet printer to investigate the onset of printed image non-uniformity at a specific combination of paper speed and print gap height and to validate the DPC model. The optical density uniformity is assessed by analysing the spatial frequency spectrum of $L^*$ transects taken across the page of the scanned printed images. Due to the efficiency of the DPC method and the limited access to the experimental rig during the COVID-19 pandemic, numerical simulations are further performed to define the bifurcation point for a combination of different printing conditions. The combination of experiments and numerical analysis offers a synergistic approach for a comprehensive understanding of the problem.
Abstract

Image uniformity is a key requirement for inkjet printers. When the gap between the print-face and media is large, one of the main causes of non-uniformity is the misplacement of droplets due to air-flow instabilities. Here, we characterize the onset of printed image non-uniformity by measuring the optical density variation of images printed with varying numbers of active nozzles. The optical density data are obtained by scanning printed images, which are corrected to remove any scanner-related effects. Several transects across the image are extracted, the peak spatial frequency is computed using Fourier transform analysis, and the results from these different transects are averaged. The amplitude of the main frequency peak in image density variation exhibits the same onset behaviour as seen in numerical simulations of the fluid flow in the printhead-media gap. Using numerical simulation, we extend the range of the study to estimate the operating envelope for the printing system.

5.1 Introduction

Due to easy control of the printing pattern and the accuracy of the printing process, the use of inkjet printers has been expanded to manufacturing electronics, solar panels, rapid prototypes, and reinforced composites (Arango et al., 2019; Hoath, 2016; Zapka, 2018). This expansion, however, has been limited by the small print gap heights \( H \) at which current systems operate (Figure 5.1). At large print gap heights printing defects, e.g., tiger-stripes and/or wood-grain, are observed due to aerodynamic effects that misplace the droplets (de A. Aquino et al., 2020; Hsiao et al., 2012, 2013; Mallinson et al., 2016, 2020; Rodriguez-Rivero et al., 2015; Wang, 1999). Small print gaps, however, prevent the printers from accommodating media with variable thickness and increase the likelihood of the media striking the printhead and damaging the nozzles.

These printing defects have been found to be associated with the airflow non-uniformity in time and spanwise direction across the print-zone (Barnett and McDonald, 2014; Hsiao et al., 2012, 2013; Mallinson et al., 2016). The relative motion between the ink droplets and the sur-
The evolution of printing defects

rounding air creates two counter-rotating vortices and, at specific operating conditions, these vortices become unstable \cite{deA.Aquino2020,Mallinson2020,Rodriguez-Rivero2015}. At large $b/H$, where $b$ is the breadth of the print-zone and $H$ is the print gap height, the vortices are uniform in time and across the domain ($x$-direction), whilst, at smaller $b/H$, they become unsteady. At intermediate $b/H$, however, the vortices become deformed, creating a standing wave \cite{Mallinson2022}.

The airflow non-uniformity tends to misdirect the droplets, misplacing them in patterns that lead to the printing features that characterise the tiger-stripe printing defect. The different microscopic patterns in which the droplets are misplaced induce specific macroscopic printing defects. At small $b/H$, a uniform printed image shows oblique stripes, whereas, at large $b/H$, the print is uniform and no printing defects are observed \cite{Barnett2014,deA.Aquino2022,Mallinson2022,Rodriguez-Rivero2015}. While a uniform flow field is linked to a uniform print, the unsteady airflow regime is indicated as the reason for the oblique stripes. \cite{Rodriguez-Rivero2015} observed that the upstream vortex tends to entrain both main and satellite droplets and when the vortices become unstable, usually at small $b/H$, high particle density and/or low paper speeds, they periodically misplace both main and satellite droplets. We note here that, at the onset of instability, the printing defects are led by the misplacement of satellite droplets that, due to their lower Stokes numbers ($S < 1$), are easily carried by the airflow. The Stokes number is defined as $S = \frac{\rho_d d^2 W}{18 \mu H}$, where $\rho_d$ is the droplet density, $d$ is the diameter of the droplet, $\mu$ is the dynamic viscosity of air and $W$ is the
5. The evolution of printing defects

droplet ejection velocity (Mallinson et al., 2016). As the satellite droplets land away from the main droplets, it artificially increases the density of dots on the paper, increasing the optical density over that region.

Figure 5.2: On the left: flow field snapshots adapted from Mallinson et al. (2022), on the right side: prints of flat gray on an A4 page provided by Memjet with print speed of 0.41 m/s, green noise of 10%, a) \( b/H = 4/7 \); b) \( b/H = 1/2 \); and c) \( b/H = 1/3 \).

It has been demonstrated by de A. Aquino et al. (N.Db) that, at intermediate \( b/H \), vertical stripes of low and high optical density alternate across the page in the spanwise direction. The authors related this printing pattern to the airflow standing regime observed by Mallinson et al. (2022) and indicated that the spanwise non-uniform regime defines the lower bounds of the print envelope since it occurs before the airflow becomes unsteady. They demonstrated, using the dispersed-phase continuum (DPC) approximation, that the airflow shifts from a steady spanwise uniform regime to a standing wave regime at a critical droplet number density, which characterises a supercritical pitchfork bifurcation. Comparisons between the DPC and the particle in cell model (P-in-C) indicated that the DPC model effectively computed the mean flow field using 0.1% of the computational resources required by the P-in-C model. However, the P-in-C analyses demonstrated that the standing wave regime can be excited by the pseudo-
5. The evolution of printing defects

_turbulence or spottiness_ induced by the slip motion of the droplets even at low duty cycles (that is, the fraction of nozzles activated during printing). This discrepancy was more evident near the critical number density but minimal for other conditions and it was suggested that the DPC model is an effective approximation to model the overall flow dynamics.

The work presented here aims to further refine the understanding about the airflow standing wave instability and its impact on the print quality. Experimental tests using an industrial inkjet printer are initially performed at a specific combination of paper speed and print gap height to investigate the evolution of the printing defects and validate the DPC model. The flow patterns predicted by the simulations are identified and correlated to the printing defects. Furthermore, additional numerical analyses are performed to define the bounds of the printing envelope at a combination of different printing conditions.

5.2 Methodology

5.2.1 Experiments

Experimental tests were conducted with a printing rig that reproduces the operation of an industrial inkjet printer when printing on a continuous moving substrate. A Duralink printhead, which is a commercial drop-on-demand inkjet printer developed by Memjet[^1] was used to perform the printing tests. The printhead has 10 rows of nozzles \( n_r \) with nozzle pitch \( \Delta x \) of 31.75 µm and resolution of up to 1600×1585 dots per inch (dpi) with drop size of 2.1 pL. A sequence of different images of various print densities are performed in a single run. The image pattern is designed with the blue noise dither (Rodriguez et al., 2008) that has 8 levels of grey scale and is composed of approximately 38 000 lines fired at the ejection frequency \( f_n \), resulting in a print length \( L \) of 0.6 m. The chosen image length allows the airflow and printing defects to become fully developed for most of the prints. We also note here that the prints are set 0.5 m apart to reset the airflow to a Couette profile before a new print starts.

[^1]: Memjet is an international inkjet printer manufacturer (https://www.memjet.com/) and research partner in this project
5. The evolution of printing defects

Figure 5.3: a) Schematic of the printing rig and b) Schematic of the printhead, with dimensions in millimetres (not to scale).

Here, the printhead is stationary and has width and length respectively equal to 220 mm and 67 mm. The injection zone is located 25 mm away from the leading edge of the printhead and the nozzles are positioned at $z = -H$ above the paper. The substrate, located at $z = 0$, moves at $V_p$ in the negative $y$-direction of the coordinate system, where the motion is induced by a custom-made system of rollers that are powered by a motor, driving the paper continuously during the set of experiments.

The main droplet ejection velocity is approximately equal to $W = 15 \text{ m/s}$, while the main droplet size is approximately equal $d = 15.6 \mu\text{m}$ (Mallinson et al., 2022). Due to the Plateau-Rayleigh instability (Plateau, 1873; Rayleigh, 1878, 1892), the high aspect ratio filament that follows the main droplet tends to break up (Anthony et al., 2019) into two or three satellite droplets of size approximately ten times smaller than the main droplet. Glossy paper with weight 80 g/m$^2$ was used as a the substrate in all tests conducted. The air dynamic viscosity and density are assumed to be, respectively, equal to 18.6 $\mu\text{Pa.s}$ and 1.1839 kg/m$^3$.

For the purpose of post-processing, the final prints are digitised with an Epson Expression 1200XL scanner of resolution of 300 dpi that uses a moving charged-coupled device (CCD) sensor to detect the light reflected by the image. The scanner features a 48-bit colour depth, allowing the system to capture sharp contrast levels and a wide gamut of colours. The CCD sensor uses arrays of blue, green and red filters to transform the print into a RGB dataset. The data is

97
normalised based on the calibration colour scheme of the scanner and converted to the $L^*a^*b^*$ colour space. The $L^*$ component represents the lightness values, whilst $a^*$ and $b^*$ indicate the four colours of human vision: red, green, blue and yellow (Kipphan, 2001). Profiles of $L^*$ across the $x$-direction are taken to measure the darkness variation that characterises the printing defects investigated in this study. To minimise the noise due, for instance, to the image dither and the scanning process, a window average that extends for 10 mm in the $y$-direction is initially employed (Figure 5.4a) and the result is subsequently exponentially averaged in the spanwise direction (Figure 5.4b). To reduce the low frequency effect resulting from the ambient light that affects the scanning process, a sixth order polynomial curve fit is subtracted from the filtered signal (Figure 5.4c). A discrete Fourier transform (DFT) is then taken of this processed signal, allowing clear identification of the dominant wavelengths.

![Figure 5.4: Example of the approach used to post-process the raw signal.](image)

### 5.2.2 Numerical model

The OpenFOAM solver *pimpleFoam* is used to solve the governing equations for incompressible laminar flow where the droplets are treated as point sources of momentum:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{F} + \nu \nabla^2 \mathbf{u}. \quad (5.1)$$
∇ · \mathbf{u} = 0. \quad (5.2)

where \(\mathbf{u}\) is the air velocity field, \(\rho\) is the air density, \(t\) is time, \(p\) is pressure, \(\nu\) is the air kinematic viscosity, \(\mathbf{F}\) is the force per unit mass induced by the dispersed phase moving at velocity \(\mathbf{u}_d\), where the subscript \(d\) refers to the dispersed or particle phase.

The DPC approximation is chosen here to model the force exerted by the dispersed phase on the surrounding air due to the low computational cost (de A. Aquino et al., N.Db). This model assumes that, due to the separation of time scales (droplet relaxation time, flow characteristic time, droplet transit time and time difference between droplets visiting the \(k\)-th cell), the force term on the \(k\)-th cell with volume \(v_k\) is governed by the expected temporally averaged rate of visitation of the particles (\(n_k\)). Here, the temporally averaged number density field (\(n\)) is defined by Equation (5.3) where \(\sigma\) is the number of firing locations (active nozzles) per unit area on the injection face, \(\mathbf{u}_d\) is the droplet velocity field and \(f_n\) is the firing frequency. To improve the model accuracy, it is assumed that, due to the drag force, the droplets decelerate as they travel from the printhead to the media (de A. Aquino et al., N.Db). The number of active nozzles per unit area is given by \(\sigma = n_f \xi / \Delta x b\), where \(\xi\) is the duty-cycle with \(0 \leq \xi \leq 1\) and \(b\) and \(w\) are the injection-zone breadth and width (out of the page in Figure 5.1), respectively.

\[
\mathbf{n} = \frac{\sigma f_n}{|\mathbf{u}_d|}. \quad (5.3)
\]

The force per unit mass induced by the dispersed phase at the \(k\)-th cell is given by:

\[
\mathbf{F}_k = v_k n_k \mathbf{f}_m. \quad (5.4)
\]

where the force exerted by the particles (\(\mathbf{f}_m\)) is equal, but of opposite sign, to the drag the droplets experience. Here, the force is modelled using the Stoke’s law multiplied by an empirical correction factor (Eq. 5.5), \(1 + \varphi\) determined by White (White, 1991) (Eq. 5.6). This factor is based on a fit to experimental data for the Reynolds number values less than 200 000, which covers the range tested in this study \((4.5 < Re_d < 16)\), where \(Re_d = \frac{|\mathbf{u}_d - \mathbf{u}_d|}{\nu}\).
5. The evolution of printing defects

\[ f^n = 3\pi \mu (u_d^m - u)[1 + \varphi(Re_d^m)]. \quad (5.5) \]

\[ \varphi(Re_d) = \frac{Re_d}{4(1 + \sqrt{Re_d})} + \frac{Re_d}{60}. \quad (5.6) \]

A rectangular cuboid domain similar to that used in de A. Aquino et al. (N.Db) was created to reproduce the print gap defined by the printhead and the media in the experiments (see Figure [5.1]). The inlet and outlet are respectively located 25 mm upstream and 45 mm downstream to match the dimensions of the printhead geometry. While the outlet has a specified constant pressure, the inlet is set with a linear Couette velocity profile that varies from 0 m/s at \( z = 0 \) to \( -V_p \) at \( z = -H \) [Link et al., 2009; Mallinson et al., 2022]. Given the large ratio between the printhead width and the print gap height and the spanwise periodicity of the flow, the numerical model has a width of \( w = 45 \) mm and the lateral faces are set as periodic boundaries. The droplet ejection velocity \( (W) \), the ejection rate \( (f_n) \) and duty cycle \( (\xi) \) are set to match the conditions used in the experimental tests. The mesh as well as the discretisation methods are equivalent to those described in de A. Aquino et al. (N.Db).

5.3 Results

5.3.1 Experimental tests

Experimental tests for an array of number densities ranging from \( N = 0.353 \times 10^{-4} \) to \( 6.71 \times 10^{-4} \) at increments of \( N = 1.823 \times 10^{-5} \) are conducted for a case with \( V_p/W = 0.02 \) and \( b/H = 1/3.6 \) to investigate the evolution of printing patterns. Here \( N \) represents the dimensionless number density and is given by \( N = ndHv/W \). The different number densities are set with variations in the duty cycle \( (\xi) \) which directly affects the optical density of the coloured figure printed on the paper.
Figure 5.5 shows that from $N = 0.353 \times 10^{-4}$ to $2.18 \times 10^{-4}$ the print is uniform across the page. With the increase in number density, the prints start to exhibit a spanwise periodicity characterised by vertical bands that alternate between lower and higher optical density, resembling a corduroy pattern. With further increments in the number density, from $N = 2.89 \times 10^{-4}$, the corduroy stripes become oblique at the edges of the print and, at $N = 5.43 \times 10^{-4}$, the instability has fully propagated towards the centre of the page with prints exhibiting the fully developed pattern that characterises the tiger-stripe printing defect.

Focus is given to investigate the corduroy printing defect given that it is the first printing defect to develop and, as a result, it defines the lower bounds of the printing envelope. Figure 5.6 exhibits the $L^*$ profiles taken for $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$. The profiles show small $L^*$ variation across the page for low number densities, where most of the $L^*$ fluctuation is yet due to noise which can be caused by the ambient light, dither, scanning process and paper wrinkles. We note here that the case with $N = 1.82 \times 10^{-4}$ starts to reveal an oscillatory behaviour, indicating that the flow is reaching the onset of instability. For prints that exhibit the corduroy printing defect, the darkness levels across the page vary in a periodic pattern characterised by a specific wavelength. The troughs and peaks respectively represent the darker and lighter bands and the amplitude of the sinusoidal wave tends to increase as the number density increases and
the printing defect develops.

Figure 5.6: $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the darkness levels for uniform prints (a) and prints with the corduroy printing defect (b).

To characterise the spectrum that composes the $L^*$ signal, the DFT algorithm is employed. Here, the wavelength is given in dimensionless terms, where $\tilde{\lambda} = \lambda / H$. For very low number densities ($N < 1.63 \times 10^{-4}$), the spectrum is composed of broadband noise and a residual long wavelength peak adjacent to the zero frequency bin. A peak near $\tilde{\lambda} = 1.88$ develops as the number density increases, indicating the onset of instability. At larger number densities, the peak amplitude increases and becomes the predominant wavelength of the signal, indicating the existence of the corduroy printing defect.

Figure 5.8 shows the evolution of the fundamental wavelength, given by the DFT coefficient, for the tests conducted in this study. It is seen that, at low number densities, the fundamental wavelength has a finite amplitude due to noise and, with the increase in number density, the amplitude of the fundamental wavelength coefficient grows continuously. This is the char-
The characteristic response of an excited supercritical pitchfork bifurcation. Instead of presenting a shift between states or solutions at a critical number density as a (non-excited) supercritical pitchfork bifurcation, it has a continuous and smooth transition. A similar response was observed when the entrained airflow was modelled using the Particle-in-Cell (P-in-C) approach (de A. Aquino et al. [N.Db]). We note here that, even though low number density cases present a sinusoidal component in the signal, the amplitude of this component is minimal and, as a result, the observers do not perceive any printing defects but rather a uniform optical density.

Figure 5.7: DFT of the post-processed $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the wavelength spectrum for uniform prints (a) and prints with the corduroy defect (b).

5.3.2 Numerical analyses

To validate the numerical model and identify the flow patterns that are expected to lead to printing defects, numerical simulations using the DPC model are performed reproducing the
Figure 5.8: DFT of the post-processed $L^*$ profiles taken for number densities equal to $N = 1.26, 1.81, 2.18$ and $2.89 \times 10^{-4}$ showing the wavelength spectrum for uniform prints (a) and prints with the corduroy defect (b).
same conditions as used in the experimental tests. Thus, the simulations were conducted at $V_p/W = 0.02$ and $b/H = 1/3.6$ with number density varying from $N = 1.885 \times 10^{-5}$ to $2.45 \times 10^{-5}$.

Two possible solutions with different wavelengths are captured by the simulations. The calculation tends to converge to either solution depending on the initial conditions (de A. Aquino et al. N.Db). To compute the two possible solutions, the cases were initialised with either a Couette–Poiseuille flow field or through numerical continuation (Seydel 2009). Both solutions, however, present a supercritical pitchfork bifurcation in which the airflow transitions from a uniform flow field to a standing wave regime at a specific critical number density, which differs from the transition observed with the printing samples and the P-in-C simulation. This is because of the temporally averaged rate of visitation of the particles that eliminates the noise induced by their slip motion. The transition point captured with the simulations, however, correlates well to the critical point at which the corduroy printing defect becomes visible. While the numerical simulations predict the airflow transition at $N_c = 1.94 \times 10^{-5}$, the experimental tests showed that the printing defects start at $N_c = 2.186 \times 10^{-5}$. We note here that the airflow tends to transition at a lower number density than the printing defect because there is a minimum droplet misplacement at which the printing defect becomes visually perceptible.

Good correlation is also observed in terms of the wavelength predictions. The DFT of the standing wave profile, measured from a transect located at $y/H = 0$ and $z/H = 0.5$, shows dominant wavelengths equal to $\tilde{\lambda} = 2.08$ and 1.79 for each specific solution, which represents a difference between 7 to 10% (0.5 mm to 0.7 mm) in comparison to the wavelength measured in the print samples. These findings suggest that specific flow patterns that arise after the airflow transition are associated with the droplet misplacement.

Figure 5.10 shows a plane located at $z/H = 0.5$ coloured by lateral velocity ($\tilde{u}_x$) with velocity vectors overlayed on it and a isosurface of $\tilde{Q}$-criterion for a case with $N = 2.186 \times 10^{-5}$. Contour of $Q$-criterion show the location of vortex cores (Mallinson et al. 2022). Two main mechanisms are expected to be linked to the droplet misplacement on the paper. Bands with low and high lateral velocity alternate across the width of the domain. It is believed that, in those bands with high lateral velocity, the satellite droplets are pushed away from the main droplet trajectory, landing in between main dots. This effect increases the number of dots on those regions of the
5. The evolution of printing defects

Figure 5.9: Bifurcation diagram showing the transition from uniform flow to standing wave regime for DPC solutions with wavelength $\lambda/H = 2.08$ and $1.79$ at $b/H = 1/3$, $V_p/W = 0.027$, $\chi = 0$, $R = 2978$ and $Re_d = 16$ overlayed on the experimental bifurcation diagram (blue).
paper and leads to stripes with higher optical density. Furthermore, it has been observed that the main vortex entrains some of the satellite droplets and misdirect them. Thus, the nodes of the deformed vortex may further misdirect the droplets, adding a $y$-direction component of misplacement that contributes to the higher optical density seen in those bands.

Figure 5.10: Iso-surface of $\bar{Q}$-criterion overlayed on a plane located at $z/H = -0.5$ coloured by $\bar{u}_x$ with overlayed velocity vectors for a case with $b/H = 1/3$, $V_p/W = 0.027$, $N = 0.355 \times 10^{-3}$, $\chi = 0$, $R = 2978$ and $Re_H = 16$.

Given the good correlation between the DPC simulations and the experiments, further numerical analyses are performed to estimate the bounds of the print envelope based on the critical number density at a combination of print gap heights and paper speeds. Tests were conducted at print gap heights equal to $b/H = 1/3.6$, $1/3$ and $b/H = 1/2.4$ with paper speed varying from $V_p/W = 0.0136$ to $0.0612$.

The results indicate that, due to the increase in paper speed, the critical number density increases in a quadratic behaviour that can be approximated by a second order polynomial fit. At smaller $b/H$, the airflow transition occurs at a lower $N_c$. The lower gradient of the transition curve at smaller $b/H$ also suggests that the system tends to be more prone to the exchange of stability under these operating conditions than at higher $b/H$. The filled area in grey represents the transition region for $3.6 < b/H < 2.4$. Any tests that fall above that region are expected to lead to a standing wave regime, whereas tests below that will lead to a uniform flow field. Therefore, to avoid non-uniform prints, printing tests should be performed under a combination of
5. The evolution of printing defects

\(N, b/H\) and \(V_p/W\) that result in a uniform flow field.

![Figure 5.11: Printing envelope diagram showing the critical number density \((N_c)\) for print gap heights equal to \(b/H = 1/3.6, 1/3\) and \(b/H = 1/2.4\) and paper speed varying from \(V_p/W = 0.0136\) to 0.0612](image)

5.4 Conclusion

Printing tests using an industrial inkjet printer rig were performed to investigate the evolution of printing defects as number density increases at a specific combination of print gap height and paper speed. The optical density variation of these prints was measured to characterise printing defects. The tests indicated that, with an increase in number density, the prints tend to transition from a uniform optical density to a regime with vertical periodic stripes across the page in a pattern that is referred to as the corduroy printing defect. With further increments in number density, the stripes at the edge of the prints become oblique and the instability propagates towards the centre of the page until all stripes have become oblique. The DFT analysis of
the optical density signal indicated that the corduroy printing defect is characterised by a fundamental wavelength whilst the uniform prints are mainly composed of broadband noise. The evolution of the fundamental wavelength for different number densities suggests that the system has an excited supercritical-pitchfork bifurcation where the amplitude of the fundamental wavelength continuously increases as the number density increases.

Numerical simulations of the entrained airflow in the print gap were performed using the DPC method to shed light into the flow features that induce the corduroy printing defect. The results showed that the flow shifts from a uniform regime to a standing wave regime at a critical number density as reported by de A. Aquino et al. [N Db]. This critical number density relates to the point at which the corduroy stripes start to be observed. The flow features predicted by the simulations are also correlated to the print patterns captured in the experiments. The bands of the flow with high lateral velocity and vortex deformation predicted by the simulations are expected to misplace the droplets and induce the stripes with higher optical density, resulting in the corduroy defect. While the high lateral velocity create a lateral misplacement component, the vortex deformation entrains the satellite droplets and displace them in the longitudinal direction. The simulations were extended to a range of different paper speeds and print gap heights to estimate the bounds of the printing envelope. The critical number density tends to increase with an increase in paper speed, whereas the increase in print gap heights tends to reduce the critical number density.
Chapter 6

Aerodynamic effects in a narrow-width inkjet printer

In previous chapters, numerical and experimental analyses were performed to characterise the onset of instability in a two-dimensional domain (chapter 3), an infinite-span model (chapter 4) and a page-wide printer (chapter 5), respectively. In chapter 5, a surrogate analysis was conducted to correlate the flow features observed in the simulations to the printing features. In this chapter, a submitted journal article entitled ‘Aerodynamic effects in a narrow-width inkjet printer’ presents direct measurements of the airflow in the print gap of narrow-width printers. Narrow-width printing is of relevance to the packaging industry where labels with QR- and barcodes are printed. High-speed planar PIV measurements are performed in two different plane orientations to characterise the three-dimensional flow features and investigate the onset of instability. The impact of the airflow dynamics on the printed images is then further characterised with transects of $L^*$ taken across the printed image.
6. Aerodynamic effects in a narrow-width inkjet printer

Abstract

During inkjet printing at elevated print gap heights image non-uniformities become evident due to droplet misplacement and this compromises the print quality. The airflow dynamics that cause such printing defects in the print-zone of a narrow-width inkjet printer are characterised using the high-speed particle image velocimetry (PIV) technique. Narrow-width printing is of relevance to the packaging industry where bar- and QR-codes are printed. Tests are performed with varying numbers of active nozzles at combinations of paper speed and print gap height. The results indicate that printing defects which resemble corduroy-stripes are observed with an increase of the number of active nozzles. The higher optical density stripes seen in the non-uniform prints are caused by the spanwise disturbance of the airflow that concentrate droplets at specific regions of the print gap. The concentration of droplets in those regions lead to an increased number of dots on the paper, and thus a higher optical density stripe. When the airflow becomes temporally non-uniform, the darker stripes move across the print, creating an oblique pattern. At the edges of the print-zone, tip vortices are created by the interaction of the injection-zone with the free-stream. When these vortices become stronger, they spread the droplets over a wider region on the paper, creating a swelling printing defect.

6.1 Introduction

Inkjet printers have become a key technology not only for digital imaging, but also for manufacturing electronics, solar panels, rapid prototypes, and reinforced composites (Arango et al., 2019; Hoath, 2016; Zapka, 2018). In all these applications, there is a demand for less restrictive systems that operate at elevated print gap heights ($H$) (Figure 6.1) to accommodate media with variable/high thicknesses, e.g., cardboard. At large print gap heights, however, printing defects, resembling tiger-stripes and/or wood-grain, are observed due to the flow field in the print gap that induces the misplacement of droplets (Hsiao et al., 2013; Rodriguez-Rivero et al., 2015; Wang, 1999) and especially satellite droplets (de A. Aquino et al., 2020; Mallinson et al., 2016, 2020).
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.1: Schematic of the physics in the numerical model where the origin of the system is located at the centre of the injection face.

The droplets produced during inkjet printing can be classified according to their Stokes number: $S = \frac{\rho_d d^2 |W - V_p|}{18 \mu H}$, where $\rho_d$ is the droplet density, $d$ is the diameter of the droplet, $\mu$ is the dynamic viscosity of air, $W$ is the droplet ejection velocity and $V_p$ is the paper speed. The satellite droplets, due to their lower Stokes numbers ($S < 1$), are more likely to be carried by the airflow and misplaced on the paper. At the onset of instability, the satellite droplets are expected to land away from the main droplets ($S > 10$), which increases the density of dots on the paper, and thus the optical density (perceived darkness level) over that region. This is noted in Figure 6.2, which shows the satellite droplets landing on the white space between the lines created by the main droplets. We note here that the misplacement of the main droplets may also affect the optical density variation observed in tiger-stripes, but this tends to occur at operating conditions far from the onset of instability (Mallinson et al., 2016, 2022). For instance, at low print densities, the tiger-stripes are predominantly caused by satellite misplacement, while main droplet misplacement was observed to be more significant at higher print densities ( Barnett and McDonald, 2014).

The tiger-stripe printing defect has been linked to specific flow dynamics that misdirect the droplets (Hsiao et al., 2012; Link et al., 2009; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015). Specifically, the exchange of momentum between droplets and the surrounding airflow creates a flow system similar to those seen in laminar impinging jets where the dominant structures are two counter-rotating vortices located upstream and downstream of the injection zone. At specific operating conditions, these vortices can become non-uniform in time and in the spanwise direction, which then affects the droplets’ trajectories.
At large $b/H$, where $b$ is the breadth of the print-zone, the vortices are stable and uniform in time and spanwise direction ($x$-direction); at intermediate $b/H$, they become deformed and non-uniform in the spanwise direction, creating a standing wave flow field. At smaller $b/H$, however, the vortices become unsteady. Print samples performed by Memjet and presented in [de A. Aquino et al. (N.Da)] indicate that: at large $b/H$, the print is uniform, i.e., no printing defects are present; at intermediate $b/H$, the prints show vertical bands of different optical density with a spanwise periodicity, and this is referred to as the corduroy printing defect; at small $b/H$, oblique stripes that characterise the tiger-stripe defect are captured. These findings suggest that the different airflow regimes lead to specific printing patterns.

Experimental tests [de A. Aquino et al. (N.Da)] characterised the corduroy printing defect and demonstrated that it can be triggered by an increase in duty cycle ($\xi$), i.e., the fraction of nozzles activated during printing. At low $\xi$, the prints are uniform but, at moderate $\xi$, the prints develop the corduroy printing defect. At higher $\xi$, the vertical strikes at the edge of the print become oblique and, with further increments, the instability propagates towards the centre of the page until all stripes have become oblique and the tiger-stripe defect has fully developed. Numerical simulations performed in that same study also suggested that the corduroy defect is preceded

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1Memjet is an international inkjet printer manufacturer (https://www.memjet.com/) and research partner in this project
6. Aerodynamic effects in a narrow-width inkjet printer

by the standing wave airflow regime. The authors indicated that the flow regions with high lateral velocity and the vortex deformation induce droplet misplacement, resulting in stripes with higher optical density.

These studies have indirectly explained the mechanisms of droplet misplacement due to aerodynamic effects. Efforts must be placed, however, to directly characterise the airflow dynamics and their impact on print quality and only a restricted number of studies have taken this approach. For instance, Mallinson et al. (2016) created numerical prints based on the divergence of the misplacement of the droplets and observed that the airflow oscillation caused optical density variations that resemble tiger-stripes. Barnett and McDonald (2014) employed particle tracking to investigate the droplet’s trajectories and observed that those droplets entrained in the upstream vortex were misplaced with the vortex oscillation. Rodriguez-Rivero et al. (2015) used time-resolved particle image velocimetry (PIV) to measure the flow field and indicated that the wood-grain printing defect is caused by what they referred to as vortex shedding near the print-zone.

The work presented here aims to extend these previous direct analyses by investigating the flow field spanwise variation and defining the range of flow regimes under different printing conditions for a narrow-width printer. Focus is given to conditions of narrow-width printing due to its relevance to the packaging industry where QR- and bar-codes are frequently printed on labels. In the present work, we apply high speed planar particle image velocimetry (PIV) measurements at different orientations to obtain a more detailed picture of the flow features. These are then related to the optical density variation on the printed image. Tests are conducted at different printing conditions, that is, a combination of duty cycle, paper speed and gap height, to investigate the influence of printing parameters on the bounds of the printing envelope.

### 6.2 Methodology

To recreate the flow field in the print gap of industrial inkjet printers when printing narrow images such as QR- and bar-codes, a narrow-width inkjet printing rig was developed and man-
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.3: a) Image of the narrow-width inkjet printing device showing the flex connector, BETTI tile and chip with nozzles; b) Schematic view of the nozzles distribution on the chip.

The rig is composed of three main systems: a printer, a moving platform upon which the paper is placed and the high-speed PIV system. Tests have been conducted in the Thermofluids Laser Lab in the School of Mechanical and Manufacturing Engineering at the University of New South Wales (Sydney, Australia).

6.2.1 Printing rig

The inkjet printer has the same design features as the Duralink printhead produced by Memjet. The device is composed of a single chip mounted on a Back Etch Tile Test Interface (BETTI) tile with a flex connector (Figure 6.3a). The chip has 10 rows of nozzles ($n_r$) or 5 colour planes with nozzle pitch ($\Delta x$) of 31.75 $\mu$m placed evenly across a rectangular area of dimensions $b \times w$ approximately equal to $1 \times 20$ mm (Figure 6.3b) with 6400 nozzles per chip and a resolution of up to 1600×1585 dots per inch (dpi) with drop size of 2.1 pL. The BETTI device can be operated in a flexible manner compared to industrial inkjet printers, allowing full control over several ejection parameters, such as voltage pulse, frequency, temperature and dither. For the tests presented here, the ejection parameters are summarised in Table 6.1. The BETTI tile is mounted on a platform that secures its position while keeping it attached to the electronic board that drives the printing process.
6. Aerodynamic effects in a narrow-width inkjet printer

<table>
<thead>
<tr>
<th><strong>Printhead parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-heat temperature</td>
<td>40 °C</td>
</tr>
<tr>
<td>Firing Frequency ($f_n$)</td>
<td>3.875, 7.750 and 11.625 kHz</td>
</tr>
<tr>
<td>Voltage pulse width</td>
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<tr>
<td>Dither</td>
<td>Floyd Steinberg</td>
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<td>Resolution</td>
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<table>
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<tr>
<td>Ink density</td>
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<tr>
<td>Ink viscosity</td>
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</tr>
<tr>
<td>Surface tension</td>
<td>37.2 mPa.m @ 25 °C</td>
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<tr>
<td>Main droplets diameter</td>
<td>15.6 µm</td>
</tr>
<tr>
<td>Satellite droplets diameter</td>
<td>1 - 2 µm</td>
</tr>
<tr>
<td>Smoke particles diameter</td>
<td>0.2 - 1 µm</td>
</tr>
<tr>
<td>Droplet ejection velocity</td>
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</tr>
<tr>
<td>Air density</td>
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</tr>
<tr>
<td>Air viscosity</td>
<td>18.6 µPa.s @ 25 °C</td>
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</table>

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</tr>
<tr>
<td>Frequency</td>
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<tr>
<td>Pulse duration</td>
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<tr>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Double frame separation</td>
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<tr>
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<tr>
<td>Frame rate</td>
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<tr>
<td>Spanwise plane - pixel resolution</td>
<td>71 pixels/mm</td>
</tr>
<tr>
<td>Lens</td>
<td>k2 DistaMax</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the experimental conditions and equipment configuration.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.4: a) Photograph showing the printing rig, high speed camera and computer that controls the printing process; b) Schematic view of the experiment when PIV measurements are taken for a plane in the longitudinal direction; c) Schematic view when measurements are taken for a plane in the spanwise direction.

The printed image is designed based on the Floyd-Steinberg ([Floyd and Steinberg, 1976](#)) dithering algorithm. Given that the BETTI tile works with 8-bit images, there are 256 levels of grayscale available for printing. The image is composed of 16000 lines of dots fired at the ejection frequency \( f_n \) which results in a uniform print with a rectangular shape \( L \times s \) of 260×20 mm, where the width is defined by the width of the chip \( w \). At the leading edge of the image, a “spit” section of 10 mm with maximum darkness level (255) or \( \xi = 100\% \) is printed. This ensures that all nozzles are unclogged and humidified prior to the tests. Succeeding the spit section, we have the region of interest where the print quality is investigated and the PIV measurements are undertaken. The grayscale used in this section is based on the duty cycle implemented for each specific test.

A linear motor and encoder system (Akibris DGL200-AJM80-B2) were used to create the motion of the platform that carries the paper. The platform has a width and length of 150×485 mm, respectively, while the paper extends for 335 mm. The paper is positioned 150 mm away from the platform leading edge to allow the flow to develop before the ejection of droplets and the substrate movement was programmed with sufficient lead-in and lead-out distances to ensure a constant speed during printing. We note here that to keep the distance between drops on the paper constant, the ejection frequency and paper speed \( V_p \) are scaled at the same rate as.
6.2.2 Laser and camera specifications

A double pulse Nd:YAG laser with a wavelength of 527 nm that has an output of 10 mJ/pulse at 1kHz with a pulse duration of approximately 100 ns was used to illuminate the seeding particles. A LaVision laser arm was coupled to a 125 mm plano-convex cylindrical lens to diverge the laser beam into a sheet. The plano-convex cylindrical lens was adjusted to minimise the thickness of the laser sheet to approximately 1mm. The dual pulse laser was synchronised to a FASTCAM SAX2 high-speed camera and data was captured at a resolution of 1024x1024 pixels. To reach the desired magnification of the print gap, the camera was coupled to a k2 DistaMax long-distance microscopic lens and two 2× magnification teleconverters, which resulted in a field of view approximately equal to 12.7×12.7 mm.

Two sets of experiments are performed to capture and characterise the longitudinal (yz) and spanwise (xz) planes. This is achieved by interchanging the position of the camera and laser as indicated in Figure 6.4. For the longitudinal plane, double-frame images \( (A_t + B_t) \) were taken at frequencies that varied from 0.5 to 1.5 kHz with straddling time ranging from 125-375 µs. For the spanwise plane, single-frames images were taken at the same frequency rate. This approach has been selected because \( x \) and \( z \)-velocity components have lower magnitudes and using double-frames was not necessary to maintain a movement of 5 pixels between frames. The longitudinal plane is located at the centre of the printhead \( (x = 0) \), whilst the spanwise plane is positioned at \( y = -7 \) mm downstream of the nozzles. We note here that the frequency and separation time between frames varies according to the paper speed and ejection frequency as shown in Table 6.2.

6.2.3 Working fluid

The ambient temperature is approximately 25 °C in the temperature-controlled lab and as a result the air dynamic viscosity and density are assumed equal to 18.6 µPa.s and 1.1839 kg/m³,
6. Aerodynamic effects in a narrow-width inkjet printer

Table 6.2: Experimental settings - laser frequency, double frame separation and firing frequency - for different paper speeds

<table>
<thead>
<tr>
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<th>Low paper speed</th>
<th>High paper speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper speed (m/s)</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Laser and camera frequency (kHz)</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Double frame separation (ns)</td>
<td>375</td>
<td>125</td>
</tr>
<tr>
<td>Firing Frequency (kHz)</td>
<td>3.875</td>
<td>11.625</td>
</tr>
</tbody>
</table>

respectively (White, 1991). The droplet ejection velocity, for the ejection parameters used here, is approximately equal to \( W = 15 \text{ m/s} \), while the main droplet size is about \( d = 15.6 \mu\text{m} \). The droplet velocity was measured in a previous study using high magnification stroboscopic imaging while the droplet mass was estimated experimentally by weighing the average ejected mass of several thousand droplets (Mallinson et al., 2022). The ink used in the experiments has a density of approximately \( 1045 \text{ kg/m}^3 \), surface tension of \( 37.2 \text{ mPa.m} \) at \( 25^\circ \text{C} \) and viscosity of \( 2.05 \text{ mPa.s} \) at \( 35^\circ \text{C} \).

The filament that follows the main droplet during ejection tends to break up into two or three satellite droplets of size approximately one-tenth of the main droplet, due to the Plateau-Rayleigh instability (Plateau, 1873; Rayleigh, 1878, 1892). A LaVision aerosol generator was used to seed the airflow in the print gap with a polydisperse cloud of particles of diameter ranging from 0.2 to \( 1\mu\text{m} \). For the flow conditions in these tests, this translates to maximum Stokes number values of approximately 5, 0.04, and 0.02 for the main, satellite and smoke particles, respectively.

6.2.4 PIV process

The images were captured and processed using the commercial PIV processing software DaVis 8.4.0 (LaVision GmbH). To correct any distortion due to the lenses imperfection and camera-laser non perpendicularity, images of a dotted bi-planar calibration target (Type 058-5 from LaVision) were used in the calibration process. The pin-hole method was employed to locate the dotted points and map the pixel space to real-world coordinates. This process resulted in a pixel error lower than 0.6 pixels. To improve the cross-correlation process, the raw images
were pre-processed using a geometric mask to delete the regions of elevated brightness due to surface reflection. A threshold elimination process that nulls low-intensity particles was also implemented to reduce the influence of ghost particles (Scarano, 2012).

For the longitudinal plane measurements, the dual frames \((A_t + B_t)\) were cross-correlated using a multi-step procedure starting with an interrogation window size of 48x48 that reduces to 24x24 pixels with an overlap of 50%. For the spanwise plane, a similar multi-step window size was implemented but, due to the low magnitude of the \(x\) and \(z\)-velocity components, a time series cross-correlation was used instead, where image \((A_t)\) is cross-correlated with the third subsequent image of the temporal series \((A_t+3\times\Delta t)\). This approach results in adequate pixel movement to provide sufficient dynamic velocity range between frames. To post-process the final vectors, the universal outlier detection method coupled to a median filter (Westerweel and Scarano, 2005) with a window size of 5 pixels was initially employed. A smoothing method based on polynomial fitting of first order was further employed to reduce the spatial noise of the final results.

We note here that, because of their low Stokes number, both satellite and smoke particles were tracked for the high-speed PIV analysis. The results were slightly biased towards the satellite droplets, as they were relatively larger and moved at higher velocities. This also created some noise that was filtered during the post-processing step. In future analysis, pre-processing techniques are suggested to be implemented to remove the influence of satellite droplets.

### 6.2.5 Printed image post-processing

The printed images are digitised with an Epson Expression 1200XL scanner with resolution of 600 dpi and colour depth of 48-bits. The scanner uses a moving charged-coupled device (CCD) sensor to detect the light that is decomposed into red-green-blue (RGB) dataset. The data is normalised based on the calibration colour scheme of the scanner and converted to the \(L^*a^*b^*\) colour space. The \(L^*\) component represents the lightness values, whilst \(a^*\) and \(b^*\) indicate the four colours of human vision: red, green, blue and yellow. Profiles of \(L^*\) across the \(x\)-direction are taken to measure the optical density variation that characterises the print de-
fects investigated in this study. To minimise the noise due, for instance, to the image dither and the scanning process, a window average that extends for 10 mm in the $y$-direction is initially employed and the result is subsequently exponentially averaged in the span-wise direction in a process similar to that taken in [de A. Aquino et al.](#).

### 6.3 Results

#### 6.3.1 Influence of duty cycle

The flow regimes and the flow features are characterised by determining the system response for an array of cases defined by different number densities (Equation 6.1). The different number density values, ranging from $N = 0.77 \times 10^{-4}$ to $N = 1.48 \times 10^{-4}$, are set with variations in the duty cycle ($1.96 < \xi < 3.9\%$) which directly affects the optical density of the coloured figure printed on the paper. This section investigates a case with $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875 \text{kHz}$.

\[
n = \frac{f_n n \xi}{\Delta x b d H W^2}.
\]  

The longitudinal plane taken at the midspan of the printhead for a case with $N = 0.77 \times 10^{-4}$ ($\xi = 1.96\%$) shows a system of vortices created by the entrainment effect (Figure 6.5). The incoming cross-flow induced by the paper motion increases the strength of the upstream vortex, in comparison to the downstream vortex. It is also observed that, at the beginning of the print ($\tilde{t} = 0.15$), where $\tilde{t} = t \times L/V_p$, the vortices are stronger due to the elevated number density during the initial declogging "spit" interval. Over time, the vortices reduce in size until the airflow becomes developed. The temporal response of the airflow indicates that the system is stable and only minimal oscillations are observed. These are believed to be a result of the noise of the measurements and the *pseudo-turbulence* or *spottiness* induced by the slip motion of the particles.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.5: Scan of the printed pattern (a) and (b) longitudinal PIV measurements taken at $x/H = 0$ for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the upstream and downstream vortex for different normalised times ($\tilde{t}$), where $\tilde{t} = t \times L/V_p$, after the start of the printing process ($\tilde{t} = 0$ s). Here the paper moves from left to right and the black dashed line represents the laser sheet.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.6: Scan of the print (a), spanwise PIV measurements taken at $y/H = -2.3$ (b) and flow visualisation images (c) for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap and the droplet distribution for different normalised times ($\tilde{t}$), where $\tilde{t} = 0$ s is the start of the printing process. A indicates the vortex near the printhead, B and C highlight the reminiscent tip vortex, D shows the uniform droplet distribution at the centre of the print and E portrays the effect of the tip vortex on the droplet distribution near the paper. Here the paper moves out of the page.

Figure 6.7: Spanwise PIV measurements taken at $y/H = -1.7$ for a case at $N = 0.77 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875$ kHz. Note that this plane is located upstream of that shown in Figure 6.6. The velocity vectors overlayed on the velocity magnitude contour show the tip vortex (A) at the edge of the injection-zone.
The $xz$-plane at $y/H = -2.3$, i.e., downstream of the injection zone, shows the spanwise flow features from $x/H = -2/3$ to $x/H = 4$. Before the printing starts (Figure 6.6.b.1), there is a lateral flow towards the centre of the printhead, indicating that the flow differs from a two-dimensional linear Couette flow. This is because, as the airflow bypasses the blockage induced by the printhead, it develops a small lateral component of velocity and becomes three-dimensional. When the print starts, the lateral flow becomes stronger due to the low-pressure region in the wake of the injection zone and an additional vortex structure (A in Figure 6.6.b2) is observed at $y/H = 0.2$ and $x/H = 2.8$. Near the centre of the print gap ($x/H = 0$), the flow approaching from both sides combines, creating an up- and downwash motion. Furthermore, at $y/H = 0.5$ and $x/H = 3$, there is a high-velocity region with a downwash component (B in Figure 6.6.b3) that is reminiscent of a wing tip vortex (C in Figure 6.6.c3). Due to the distance of the spanwise plane from the injection-zone, the core of the tip vortex is no longer visible. For the $xz$-plane located at $y/H = -1.7$, however, the vortex core is clearly captured (Figure 6.7). This tip vortex is a result of the discontinuity of the ejection of droplets at the edge of the injection zone, where the impinging airflow induced by the motion of the main droplets hits the paper and leaks towards the free-stream region, creating a coherent structure that is carried downstream by the incoming cross-flow. This mechanism is similar to that observed in laminar impinging jets interacting with an incoming cross-flow [Barata and Durão, 2004].

The flow dynamics in the print gap have a direct impact on the droplet deposition across the print gap. Given the lack of temporal and spatial spanwise disturbances in this system, the droplets (brighter particles) near the paper are evenly spread across most parts of the print (D in Figure 6.6.c4), except near the edges of the injection-zone, where a number of droplets have been swept by the tip vortex (E in Figure 6.6.c). The overall dynamics of the system translate, thus, to a small variation in optical density across the print.

Printing defects, however, are perceived with an increase in the number density. At $N = 0.77 \times 10^{-4}$, variations in optical density are small as indicated in the $L^*$ profiles (Figure 6.8.c) and, as a result, at lower number densities, the prints are expected to be uniform. At $N = 0.92 \times 10^{-4}$, however, nearly straight stripes that characterise the corduroy printing defect [de A. Aquino et al., N.Da] are observed (Figure 6.8) and with further increase in number density, the stripes
become more evident. This trend is evident in the $L^*$ plots where larger variations in $L^*$ are observed with the increase in number density.

We noted that prints produced by different tests using the same printing conditions can be slightly different from case to case as observed in Figure 6.9. This is because we are exploring the regime in close proximity to the onset of instabilities. Further, variations in ambient temperature, the uncertainty of the print gap height, the flatness of the paper, the temperature of the BETTI chip, and the chip itself have an impact on the system response. The difference in ejection properties between chips can reach up to 5%, which is expected to affect the final print uniformity at conditions near the onset of instability. These differences, however, are not expected to affect the understanding of the flow dynamics that lead to printing defects.

For a test case with $N = 1.54 \times 10^{-4}$ ($\xi = 3.9\%$), the longitudinal PIV measurement (Figure 6.10) shows that, with the increase in number density, the upstream and downstream vortices become stronger and the airflow reaches higher velocities. This is due to the greater number of droplets that results in a higher exchange of momentum with the surrounding air. The airflow
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.9: Summary of printing tests conducted at $N = 1.54 \times 10^{-4}$ (f) to show the variability of the experimental print results among different tests.

shows a small level of oscillation with the size of the vortex core varying during printing. These oscillations indicate a temporal flow non-uniformity that is accompanied by the stripes slightly moving across the print for this specific test (Figure 6.10.a).

The $xz$-planar PIV test with $N = 1.54 \times 10^{-4}$ (Figure 6.11) shows that in comparison to the case with $N = 0.77 \times 10^{-4}$, the vortex near the printhead becomes larger and moves closer to the centre of the printhead. The main difference, however, is that the spanwise airflow reaches higher velocities and becomes disorganised, with a large velocity variation across the print gap, and the presence of smaller recirculation regions near the centre of the printhead. Figure 6.11.c also shows that the droplets are non-uniformly distributed across the print gap with a region where the droplets are densely concentrated (C in Figure 6.11.c). It appears that in this region a greater number of droplets are deposited on the paper, creating the high optical density stripe observed in the prints. Even though the flow velocity varies over time, the main flow features are temporally stable with little movement across the domain, which reflects the fact that region C is located between $0 < x/H < 1$ over the whole printing process and the stripes show little to no obliqueness.
Figure 6.10: Scan of the print (a) and longitudinal PIV measurements taken at \( x/H = 0 \) (b) for a case at \( N = 1.54 \times 10^{-4} \) (\( \zeta = 3.9\% \)), \( V_p/W = 0.013 \), \( b/H = 1/3 \) and \( f_n = 3.875 \) kHz. The velocity vectors overlayed on the velocity magnitude contour show the upstream and downstream vortex for different normalised times after the start of the printing process (\( \tilde{t} = 0 \) s). Here the paper moves from left to right and the black line represents the laser sheet.
Figure 6.11: Scan of the final print (a), spanwise PIV measurement taken at $y/H = -2.33$ (b) and flow visualisation images (c) for a case at $N = 1.54 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.013$, $b/H = 1/3$ and $f_n = 3.875\text{kHz}$ showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap and the droplet distribution for different normalised times ($\tilde{t} = t \times L/V_p$), where $\tilde{t} = 0\text{s}$ is the beginning of the printing process. A and B highlight new recirculation zones, C shows a region where the droplets are densely concentrated and D indicates the high optical density stripe created by region C. Here the paper moves out of the page.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.12: Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.33$ (b) and flow visualisation images (c) for a case at $N = 2.31 \times 10^{-4}$ ($\xi = 1.97\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show the flow features across the print gap, which includes a well defined tip vortex (A), and a uniform droplet distribution for different times. Here $\tilde{t} = 0$ s is the beginning of the printing process and the paper moves out of the page.

6.3.2 Influence of paper speed

To investigate the influence of paper speed on the flow dynamics, tests were conducted with $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz and $\xi$ ranging from 1.97% to 3.9% to match the lower paper speed experiments. This array of duty cycles leads to $2.31 \times 10^{-4} < N < 4.62 \times 10^{-4}$ due to the firing frequency being scaled with the paper speed.

The spanwise flow field for a case with $N = 2.31 \times 10^{-4}$ ($\xi = 1.97\%$) and $V_p/W = 0.04$ (Figure 6.12) indicates that in comparison to a case with $V_p/W = 0.013$ and same duty cycle the higher paper speed and number density lead to elevated flow velocities across the print gap. Even before printing, a higher lateral velocity is observed in comparison to the lower paper speed tests. At $z/H = 0.6$ and $y/H = 3.3$, a clear tip vortex is captured on this plane (A), suggesting
that these printing conditions create a more energised vortex that travels further downstream of the print zone. The flow field in this case is quite organised with lower lateral velocity regions at the centre of the print. This leads to a uniform droplet distribution across most of the print zone apart from near the edge where they tend to be spread by the tip vortex (Figure 6.12c). The uniform droplet distribution further results in a uniform optical density on the print.

The experimental print tests (Figure 6.13) show that, at $N = 2.75 \times 10^{-4}$, faded stripes appear indicating that this case is near the onset of instability, whereas at $N = 3.14 \times 10^{-4}$, the stripes are easily perceived. At $N < 2.31 \times 10^{-4}$, no stripes are evident and the optical density is approximately uniform across the page. These results demonstrate that at higher paper speed, the transition from uniform to non-uniform prints tends to occur at a higher number density and higher duty cycle. This is because the higher momentum in the incoming cross-flow tends to stabilise the flow field. While for $V_p/W = 0.0133$ the transition occurs at $0.77 \times 10^{-4} < N < 0.92 \times 10^{-4}$ ($1.96 < \xi < 2.35$), for $V_p/W = 0.04$, it happens at $2.75 \times 10^{-4} < N < 3.14 \times 10^{-4}$ ($1.96 < \xi < 2.35$). It is also observed that, for the number densities and print length tested here, the stripes are oblique rather than vertical as is observed in the lower paper speed tests (Figure 6.8).

The spanwise flow field of a case with $V_p/W = 0.04$ and $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$) is presented in Figure 6.14. Under these conditions, a number of additional vortices are observed across the print gap and the flow field becomes spatially and temporally non-uniform. Based on Figure 6.14c, the droplets are carried by the high-velocity fluid layers and tend to be trapped in regions where flows in opposite directions converge and stagnate. Since the vortices and the stagnation regions move across the print gap over time, the stripes become oblique as portrayed in Figure 6.14a.

The longitudinal PIV measurements shed further light on the dynamics of the flow system with $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$). Figure 6.15 shows that the airflow reaches higher velocities and the downstream vortex becomes stretched due to the higher momentum in the incoming cross-flow. Both upstream and downstream vortices, however, tend to oscillate over time, changing their structures and alternating between periods of large and small cores. By correlating the flow visualisation images (Figure 6.15c) to the actual print (Figure 6.15a), it is observed that the
6.3.3 Influence of print gap height

Additional experiments were conducted with \( V_p/W = 0.013, b/H = 1/4.1, f_n = 3.875 \text{kHz} \) and \( \xi \) ranging from 1.57\% to 3.9\% which translates to \( 0.83 \times 10^{-4} < N < 2.08 \times 10^{-4} \). This allowed an investigation of the influence of higher print gap heights on the flow dynamics and, therefore, on print quality.

Figure 6.16 shows that the high optical density stripes appear at \( N > 0.83 \times 10^{-4} (\xi > 1.57\%) \), indicating that larger print gaps induce an earlier transition to non-uniform prints. Even though the test at \( N = 0.83 \times 10^{-4} \) does not exhibit any dark stripes, edge printing defects are observed, due to the large lateral misplacement of droplets. This print width increase is referred to as a
Figure 6.14: Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.3.3$ (b) and flow visualisation images (c) for a case at $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625$ kHz showing velocity vectors overlayed on velocity magnitude contour. The results show a number of new vortices across the print gap and a non-uniform droplet distribution.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.15: Scan of the print (a), longitudinal PIV measurements taken at $x/H = 0$ (b) and flow visualisation images (c) for a case at $N = 4.62 \times 10^{-4}$ ($\xi = 3.9\%$), $V_p/W = 0.04$, $b/H = 1/3$ and $f_n = 11.625\,\text{kHz}$. The velocity vectors overlayed on the velocity magnitude show the upstream and downstream vortex for different normalised times after the start of the printing process ($\bar{t} = 0\,\text{s}$).
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.16: Scan of prints at \( V_p/W = 0.013 \), \( b/H = 1/4.1 \) and \( f_n = 3.875 \text{ kHz} \) with number density equal to \( N = 0.83 \times 10^{-4} \), \( N = 1.04 \times 10^{-4} \) and \( N = 2.08 \times 10^{-4} \); enlarged view of the scanned prints at locations A.1, A.2, B and C (b) and \( L^* \) profiles of the enlarged region (c).

swelling effect.

The PIV measurements in the \( xz \)-plane for a case with \( N = 0.83 \times 10^{-4} \) (Figure 6.17) shows that the larger print gap height allows the flow to reach higher lateral velocities. Despite that, the flow field is still stable in time and uniform in the span direction which leads to a uniform distribution of the droplets near the paper (Figure 6.17c). At 0.3 s after starting the printing process (Figure 6.17b.2 and c.2), the tip vortex begins to form, sweeping the droplets away from the print-zone and creating the swelling printing defect. This defect is more evident at larger print gaps because the tip vortex grows in size and moves further away from the centre of the print-zone, which results in a larger lateral misplacement of the droplets.

For \( N = 2.08 \times 10^{-4} (\xi = 3.9%) \), the flow field becomes disturbed and spatially non-uniform as vortices similar to those observed in the higher paper speed tests (Figure 6.14) are captured here. However, in contrast to the higher print gap tests, the vortices and stagnation regions appear to be fixed in the domain and, as a result, the stripes present little to no obliqueness.
Figure 6.17: Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.3.3$ (b) and flow visualisation images (c) for a case at $N = 0.83 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/4.1$ and $f_n = 3.875$ kHz. The velocity vectors overlayed on the velocity magnitude contour show a wide tip vortex (A) and the droplets being spread by this vortex (B). Here the paper is moving out of the page.
6. Aerodynamic effects in a narrow-width inkjet printer

Figure 6.18: Scan of the print (a), spanwise PIV measurement taken at $y/H = -2.33$ (b) and flow visualisation images (c) for a case at $N = 2.08 \times 10^{-4}$, $V_p/W = 0.013$, $b/H = 1/4.1$ and $f_n = 3.875$ kHz. Here the paper is moving out of the page.
This relates well to the dynamics observed in the lower print gap height tests and indicates that the stripe obliqueness is led by the lateral motion of the main flow features.

6.4 Conclusion

High-speed planar PIV experiments were performed in a narrow-width inkjet printer to characterise the airflow dynamics that create non-uniform prints. Tests were conducted for an array of different number densities, which was determined by duty cycles ranging from $1.57 < \xi < 3.9\%$, at combinations of paper speeds and print gap heights. Longitudinal and spanwise planes were used to characterise the three-dimensional flow features. The PIV results provided detailed information about the flow field dynamics whilst the particle illuminated images highlighted the droplet distribution in the print gap.

The results indicated that there is a transition point (critical number density) where the printed image non-uniformity becomes perceptible. At low number densities, the airflow is stationary, i.e., small airflow oscillation during the printing, and uniform across most of the spanwise direction of the injection-zone. At the edges of the injection-zone, tip vortices are observed due to the impinging air jet induced by the motion of droplets leaking into free-stream. At higher duty cycles, however, the airflow becomes disturbed with additional recirculation zones spread across the spanwise direction of the injection-zone. These patterns tend to concentrate the droplets at specific regions of the print-gap, leading to a higher number of dots on the paper and thus higher optical density stripes. Given that the airflow was temporally uniform, the dark stripes on the print were stationary, resulting in the corduroy printing defect.

The tests conducted at a higher paper speed showed that, due to the higher momentum in the incoming cross-flow, the transition point occurred at a higher number density. The stronger cross-flow also contracted the upstream vortex and stretched the downstream one. For this set of experiments, above a critical number density, the flow became temporally non-uniform which led to oblique high optical density stripes. This was a result of the spanwise disturbance moving across the injection-zone and the upstream and downstream vortex cores varying in
size during the printing process.

With the increase in print gap height, the printing defect became evident at a lower number density. The airflow spanwise disturbance was also more evident than at a lower print gap height. Due to stronger tip vortices, the droplets at the edges of the print were laterally pushed away from the injection-zone, which created an additional printing defect referred to as a swelling defect.
Chapter 7

Conclusion and Future Work

This final chapter reviews the advances made on the aim of characterising the airflow instability in the print gap of inkjet printers that lead to printing defects. To do so, this chapter discusses the novel numerical and experimental work that was performed, the main results and insights gained, the avenues for potential future work, and the overall significance and implications of this study.

7.1 Research Summary

Inkjet printing technology is continuously evolving to meet the demands of different sectors of the market. In all the different applications, the accurate deposition of droplets on the media is a shared requirement. However, under specific operating conditions, especially at elevated print gap heights, aerodynamic effects tend to misplace the ink droplets on the media, compromising the printing quality. To characterise these aerodynamic effects that create printing defects like tiger-stripes and/or wood-grain, numerical and experimental investigations were performed in this thesis. Focus was given to investigating the airflow dynamics in graphics applications with emphasis on wide prints, but tests were also conducted for narrow-width printing, which is of relevance to the packaging industry.
Chapter 3 introduced two-dimensional numerical analyses using the dispersed-phase continuum approximation to model the ink droplets. These were performed to shed light on the dynamics of the airflow instability and the range of flow regimes. As observed in previous studies (Barnett and McDonald, 2014; Mallinson et al., 2016; Rodriguez-Rivero et al., 2015), two vortices were captured in the vicinity of the injection zone as a result of the impingement of the air-jet induced by the motion of the droplets. These vortices, located upstream and downstream of the injection zone, became oscillatory at elevated print gap heights. For a combination of print gap height and paper speed, the airflow transitioned from a steady state to an oscillatory regime with a limited oscillation amplitude at a critical non-dimensionalized number density, $N_c$, which is characteristic of a supercritical Hopf bifurcation. This transition defined the upper bounds of the printing envelope, where the system was expected to be uniform in time and spanwise direction. The transition was affected by the paper speed and the print gap height. While higher print gap heights made the system more prone to instability as the transition occurred at a lower number density, higher paper speeds, due to the higher momentum in the incoming cross-flow, stabilised the airflow and pushed the transition to a higher value of $N_c$. Furthermore, the frequency of the oscillation was also affected by the paper speed, number density and print gap height. This has practical implications since the objectionability of the defect is a function of its spatial frequency.

To expand the study to a more practical configuration, Chapter 4 presented three-dimensional numerical analyses using the DPC model in an infinite-span domain and compared them to the classic P-in-C formulation to assess the effectiveness of the former. An infinite-span model was employed since it assumed that, in a page-wide print where the ratio $H/w$ is quite small, the base flow at the centre of the printhead is minimally affected by edge effects. The DPC simulations showed that the airflow shifted from a uniform in time and spanwise regime to a non-uniform in the spanwise direction regime. This is typical of a supercritical pitchfork bifurcation. The spanwise non-uniformity is characterised by the upstream and downstream vortices becoming deformed, resulting in a standing wave regime where the airflow spanwise component of velocity varied in a sinusoidal manner. This airflow regime is expected to be linked to specific printing defects, referred to here as the corduroy printing defect, where the optical density varies in a periodic manner across the page, forming stripes or bands of higher
optical density. The comparison between the DPC and the P-in-C models showed that droplet deceleration must be accounted for in the DPC model to improve its accuracy. Furthermore, due to the pseudo-turbulence or spottiness induced by the slip motion of the droplets that perturbed the airflow and excited the standing wave regime, even at low number densities, the P-in-C model predicted an excited super-critical pitchfork bifurcation. The time-averaged flow field captured by both models showed a certain level of disagreement near the critical number density but correlated well with other conditions away from the critical point. The overall results indicate that the DPC approximation is an effective model for estimating the range of flow regimes and, therefore, the bounds of the printing envelope: the DPC model took approximately 0.01% of the computational time used in the P-in-C approximation to reach a converged time-averaged solution.

In Chapter 5, experimental tests using a page-wide industrial inkjet printer rig were presented to investigate the onset of print non-uniformity and the evolution of printing defects. For a combination of print gap height and paper speed, an increase in number density induced a non-uniform optical density across the page, characteristic of the corduroy printing defect. With further increases in number density, the vertical stripes of the corduroy defect became oblique, becoming the tiger-stripe defect. This phenomenon first occurred at the edges of the print, but, with further increases in number density, the instability propagated towards the centre of the page until it settled at elevated number densities. The stripes’ obliqueness indicates that the airflow has also become temporally non-uniform. Transects of $L^*$ taken across the page indicated that the optical density in a uniform print is composed of broadband noise while the corduroy printing defect is composed by a specific wavelength added to the broadband noise. This fundamental wavelength tends to evolve with an increase in number density with a behaviour similar to the excited super-critical pitchfork bifurcation. This is because, at low number densities, the broadband noise of a uniform print contains a small amplitude of the fundamental wavelength. Additional simulations performed using the DPC simulation showed that there is a correlation between the corduroy printing defect and the airflow transition to a standing wave regime. The defect only became perceptible at number densities above the critical number density of the airflow. Furthermore, the standing wave regime is also defined by a fundamental wavelength that varies from $\lambda / H = 2.143$ (6.5 mm) to 1.875 (5.625 mm), which
correlates well to the fundamental wavelength measured in the experiments. The regions of
the airflow where the vortex is deformed, and the spanwise component of velocity is higher,
tend to misplace the droplets, especially the satellite droplets that are pushed away from the
main droplet, increasing the optical density on the paper.

Direct measurements using the PIV technique in a narrow-width inkjet printer rig – designed
and manufactured for this project – are presented in Chapter 6. Narrow-width inkjet printing is
of relevance to the packaging industry, which requires labels with QR- and bar-codes. Despite
certain particularities of a narrow-width printer, several physical features here can be corre-
lated to page-wide or infinite-span systems. The results indicated that similar to the tests con-
ducted in Chapter 5, the prints transitioned from a uniform to a non-uniform optical density
across the page with an increase in number density. This is a result of the airflow becoming dis-
turbed and non-uniform across the print gap and presenting a number of recirculation zones
spreading across the spanwise direction of the print gap. These recirculation zones concen-
trate the droplets at specific regions of the print-gap, leading to a higher number of dots on
the paper and, thus, high optical density stripes. The tests conducted at a higher paper speed
showed that the increase in momentum in the incoming cross-flow increases the critical num-
ber density. For the higher paper speed tests, the airflow became non-uniform in time and in
spanwise direction, which led to oblique high optical density stripes. The transient behaviour
of the flow was characterised by the recirculation zones moving across the injection zone and
the upstream and downstream vortices varying in size during the printing process. With an
increase in print gap height, the printing defect became evident at a lower value of number
density. The airflow spanwise disturbance was also more evident at a higher print gap height.
The main difference observed in this particular study is the existence of tip vortices, which are
a result of the impinging air-jet that is induced by the motion of the droplets leaking towards
the free-stream. These vortices are expected to be observed in a page-wide printer but nonex-
istent in an infinite-span model. At higher print gap heights, the tip vortices became stronger
and pushed the droplets laterally away from the injection zone, which created an additional
printing defect referred to as the swelling defect. These edge effects are expected to be more
relevant for narrow-width printing due to the small aspect ratio of the system.
7.2 Future work

As summarised above, progress has been made towards improving the accuracy of the DPC model by taking into account the droplet deceleration in the momentum formulation. In the future, further improvements can be made to account for the spanwise and longitudinal displacement of the droplets. It is expected that under high paper speeds and/or downstream suction, the longitudinal displacement will be considerably larger than in the tests conducted here and, as a result, it may affect the dynamics of the airflow instability. The DPC model can be coupled to a one-way particle tracking model to predict the trajectory and misplacement of satellite droplets. Based on the final position of the droplets, numerical prints as presented by Mallinson et al. (2016) can be created, and the mechanism of droplet misplacement can be further investigated. Future work may also extend the DPC simulations of a narrow-width inkjet printing condition presented in de A. Aquino et al. (2022a), so that the results can be further compared against the experimental tests (Chapter 6). The numerical simulations can complement the experimental data by providing a more detailed picture of the underlying flow features and investigating the influence of printhead width on the airflow instability. Additional can also be performed to investigate parameters not yet tested, such as the width of the print-zone, nozzle-spacing, moderate to high paper speeds and downstream suction.

With respect to the experimental tests, the PIV analysis conducted in this work tracked smoke particles and droplets. With a tailored filter based on particle brightness and velocity, the main droplets can possibly be distinguished from the smoke particles and the satellite droplets and, thus, filtered from the cross-correlation process. This strategy is believed to reduce the temporal noise and improve the accuracy of the experiments, especially in regions near the injection zone where the main droplets tend to be concentrated. Furthermore, to provide a more comprehensive understanding of the flow features, 3D PIV measurements show great potential (Gunasekera et al., 2020). The PIV measurements taken in this study were limited to planar measurements, and to characterise the three-dimensional flow features, tests were performed with different plane orientations. However, due to the variability of the experiments, a certain level of discrepancy was observed between tests. A 3D PIV measurement can characterise the
7. Conclusion and Future Work

$u, v, w$ components of velocity in a single test. The three-dimensional data set is also expected to provide a more detailed picture of the droplets' trajectory during the printing process.

7.3 Final remarks

Taken together, the new techniques, findings, and fundamental insights of this study have significantly progressed our capability and understanding of the airflow instabilities that lead to non-uniform prints in inkjet printers. It has been shown that the airflow non-uniformity in time and space leads to printing defects, such as corduroy- and tiger-stripes. The occurrence and severity of these defects depend on the combination of printing parameters, such as print gap height, number density and paper speed. The curves in Figure 7.1 estimate the bounds of the printing envelope where the airflow is uniform and the prints are expected to be uniform. The advances in computational techniques to model droplets are not only of interest to inkjet printer manufacturers but also to the community investigating particle driven-flows since the DPC approximation has been demonstrated to be a computationally efficient alternative to model the exchange of momentum between the dispersed phase and the fluid carrier.

Based on the findings presented here, several potential mitigation strategies can be explored to minimize the tiger-striping problem. Figure 7.1 indicates that the airflow tends to become unstable at high print gap heights and low paper speeds. At higher paper speeds, the airflow becomes stable due to the stronger incoming cross-flow. This suggests that mitigation strategies can be designed to increase the volumetric flow rate across the print gap. One approach to achieve this is by implementing a suction system downstream or a blowing system upstream. The former is a common solution as it not only increases the volumetric flow rate but also aids in collecting mist particles (Balala and Baterna 2019; Ishikawa 1986), minimizing the risk of droplet inhalation. However, there are alternative solutions, such as the roller system presented and investigated by Mallinson et al. (2016). This roller system was positioned downstream of the injection zone to create a pressure gradient and enhance the volumetric flow rate across the print gap. Nevertheless, it is crucial to carefully tune the volumetric flow rate to avoid an excessive longitudinal displacement of the droplets, which could induce secondary printing defects.
7. Conclusion and Future Work

like ghosting.

It has also been demonstrated here that the onset of instability is triggered by the disturbance resulting from the motion of the droplets. This disturbance depends on drag the droplets experience and the droplet number density ($N$), which is defined by the density of nozzles on the printhead, firing frequency, droplet velocity, print gap height, and droplet size. While most of these parameters are limited by the printhead construction, considering the tiger-striping problem during the early stage of printhead design can allow adjustments to be made to minimize airflow instability. Another method to minimize the airflow disturbance has been presented by [Barnett and McDonald, 2014], where a low density fluid (Helium) was forced into the print gap. This process effectively reduced the overall flow viscosity, resulting in a lower droplet number density and reduced drag force. Additionally, the lower flow viscosity increased the droplets’ Stokes number, reducing the impact of the flow field on the droplets’ trajectories.

Overall, the findings presented in this study are expected to strengthen the next generation of inkjet printers and be of use to develop mitigation strategies to prevent the occurrence of printing defects.
Figure 7.1: Printing envelope diagram showing the critical number density ($N_c$) for print gap heights equal to $b/H = 1/3.6$, $1/3$ and $b/H = 1/2.4$ and paper speed varying from $V_p/W = 0.0136$ to $0.0612$. 
Bibliography


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