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Publication details:
Key Engineering Materials
v. 291-292
pp. 459-464
1013-9826 (ISSN)

Publication Date:
2005

Key Engineering Materials, Vol. 291-292 (2005), pp. 459-464.

Mathematical Models for the Hydrodynamic Characteristics of Abrasive Waterjets

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Keywords: Abrasive waterjet, CFD simulation, Particle velocity models, AWJ cutting.

Abstract. Predictive models for the particle velocity in an abrasive waterjet (AWJ) are developed following a CFD (computational fluid dynamics) study. A numerical study is then carried out to assess the models. It is shown that the predictive models can adequately predict this particle characteristic in an AWJ.

Introduction

Abrasive waterjet (AWJ) cutting is one of the most recently developed non-traditional manufacturing technologies. It is being increasingly used to machine a wide range of metals and non-metals, particularly 'difficult-to-cut' materials such as ceramics and marbles [1,2] and layered composites [3,4]. However, this technology is still under development and its many aspects are yet to be fully understood. At a fundamental level, an understanding of the dynamic characteristics of an AWJ, such as particle velocity distribution, is required for optimizing the cutting head design and jetting parameters for eventually optimising the jet performance, as well as for understanding some kerf geometrical features formed by AWJ in order to improve the kerf quality. This jet characteristic information is also essential for the development of cutting performance model where the jet or particle velocity and trajectory are the key factors for material erosion by particles. However, very little reported research has been directed towards achieving a fundamental understanding of the dynamic behaviour of AWJs [1,5].

In this paper, the dynamic characteristics of particles in an AWJ, typically the particle velocity distribution, are investigated and mathematically modelled based on the findings of a CFD (computational fluid dynamics) simulation study. The models are then verified both qualitatively and quantitatively by assessing the model predictions with the corresponding CFD simulated results. For this purpose, the particle dynamic characteristics from a CFD study are reviewed first.

Characteristics of Particle Velocity

In the CFD simulation of particle dynamic characteristics for AWJs [5,6], a pure waterjet with the peak inlet velocities at the nozzle exit ranging from 600 to 900m/s and nozzle diameters of 0.8, 1.0 and 1.2mm was first considered using a Fluent6 flow solver. The CFD simulation considered the jet flow domain from the nozzle exit to 50mm downstream. In the absence of knowledge and experimental data, the flow at the nozzle exit was studied first and it was found that the 1/7th power law distribution [7] gave reasonable results. Thus, the 1/7th law velocity distribution was used as the inlet condition for water velocities. After a converged solution of water flow field was obtained, abrasive particles were added into the jet flow across the inlet at various radial positions. It was assumed that the effect of particles on the continuum in the waterjet was negligible, as were the particle-particle interactions. The initial particle velocity at the inlet was set to the velocity of its surrounding water, i.e., particle velocity distribution at the nozzle exit also followed the 1/7th power law distribution. The particle velocities were calculated using the Discrete Phase Model in Fluent.

In addition, the particle shape was assumed to be spherical and the CFD study used garnet particles of four different diameters, i.e. d_p=0.08, 0.12, 0.16 and 0.20mm. The density of the garnet was 4100kg/m^3. This CFD model has been verified experimentally and used to evaluate the jet characteristics which are discussed below.

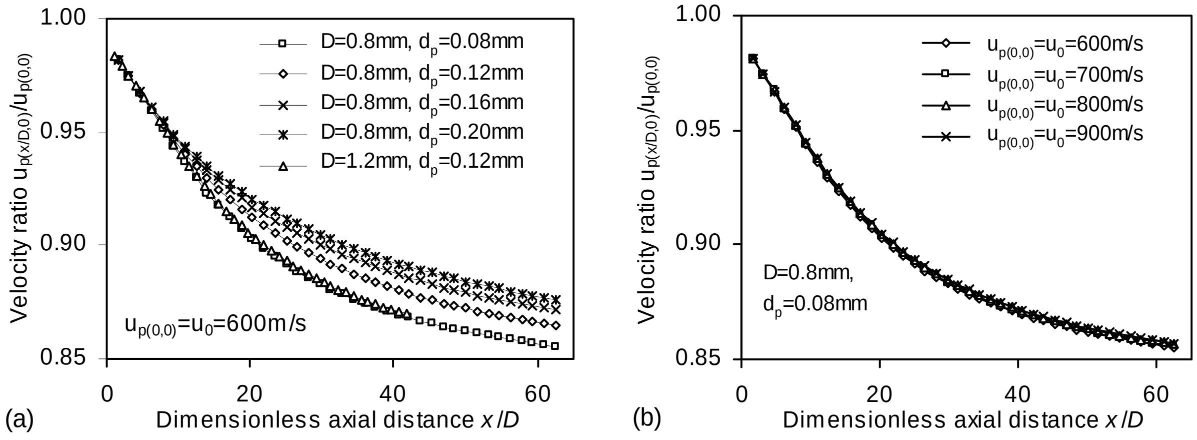

Fig. 1. Particle velocity along the jet centreline with respect to *x*/*D*.

Fig. 1 shows that particle velocity (or the ratio of downstream particle velocity to that at the nozzle exit on the jet centreline) decreases with the dimensionless axial distance *x*/*D* for the particles released at the jet centreline, where *D* is nozzle diameter and *x* is axial distance from the nozzle exit. The decrease in the velocity of downstream particles gives less energy for the AWJ to cut the lower part of a workpiece. Fig. 1(a) indicates that for the particles moving in the same sized nozzle (such as D=0.8mm), the effect of particle diameters on the particle velocity along the jet centreline is initially minimal. However, as the jet travels further downstream, the effect of particle diameter becomes obvious. Larger particles are associated with smaller velocity decay rates than smaller particles because of their larger mass and momentum. This characteristic indicates that the use of larger particles can increase the cutting capability of AWJ, e.g. increase the material removal rate or depth of cut.

It is also noticed from Fig. 1(a) that the velocity variation for a particle of 0.08mm in diameter travelling in a 0.8mm nozzle is almost the same as that for a particle of 0.12mm travelling in a 1.2mm nozzle. It can be further noticed that the ratio of particle diameter to nozzle diameter for the two situations is the same (d_p/D=0.1). A detailed study has found that the particle diameter combines with the nozzle diameter in affecting the particle velocity. The effect of initial peak velocity on the particle motion is shown in Fig. 1(b), from which it can be seen that the particle velocity ratio decays along the centreline and is independent of the value of the initial peak velocity.

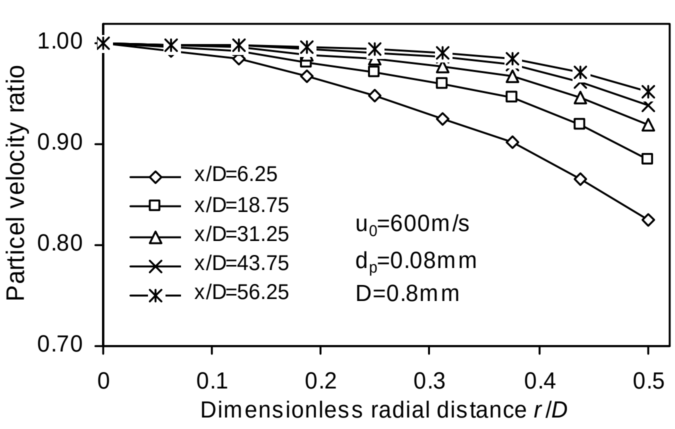

Fig. 2. Particle velocity ratio $u_{p(x/D,\ r/D)}/u_{p(x/D,\ 0)}$ along the radial direction.

Some typical results of the particle velocity distribution along the jet radial direction at different downstream locations, x/D=6.25, 18.75, 31.25, 43.75 and 56.25, are shown in Fig. 2, where r is the distance along the jet radial direction from the jet centre. It can be noticed that the particle velocity at a given jet cross-section decreases as the radial distance from the jet centre increases. Furthermore, the ratio of particle velocity to that at the corresponding jet centre increases as the jet travels downstream, i.e. the velocity decay in the jet inner region is faster than in the jet outer region, so that the particle velocity evolves from an initial 1/7th power law distribution to a more top flat-hat profile as the jet travels downstream.

Development of Mathematical Models for Particle Velocity

To develop predictive models for the various cutting performance measures in AWJ cutting, such as the material removal rate and depth of jet penetration, requires the knowledge of the jet dynamic characteristics. For instance, to use the erosion theories [8,9], it is essential to know the particle velocity at the point of particle attack as well as the particle moving direction. For this reason, mathematical models for particle velocity are developed based on the foregoing CFD study. The velocity models for a particular (or reference) sized particle is developed along the jet centreline and radial direction successively. The effect of particle size on particle velocity is then considered to arrive at the general particle velocity models.

Velocity Variation Along Jet Centreline. As shown in Fig. 1, the ratio of particle velocity at a downstream location on the jet centerline to the initial jet velocity at the jet centre decreases as the jet travels away from the nozzle and is independent of the initial peak velocity. Consequently, the velocity decay for a particular sized particle along the jet centreline can be expressed as

$$\frac{u_{p1(x/D,\,0)}}{u_{p(0,\,0)}} = f\left(\frac{x}{D}\right) \tag{1}$$

where $u_{p1(x/D,\,0)}$ is the velocity for a certain sized particle at the point (x/D, 0) on the jet centreline, $u_{p(0,\,0)}$ is the initial particle velocity at location (0, 0), and it is equal to the initial peak water velocity. It should be noted that the x–r coordinates used in this study originate at the jet centre and the nozzle exit, where x is along the jet flow direction and r is in the jet radial direction.

By analysing the relationship between the dimensionless velocity and dimensionless axial distance (x/D), the variation of particle velocity along the jet centreline can be expressed by

$$\frac{u_{p1(x/D,\,0)}}{u_{p(0,\,0)}} = A_1\left(\frac{x}{D}+A_2\right)^{-A_3} \tag{2}$$

The coefficients, A_1, A_2 and A_3, have then been statistically determined at a 95% confidence level using the CFD simulated results. When the reference particle size is selected such that the dimensionless particle diameter (d_p/D) is 0.1, the velocity equation for garnet particles in the jet centreline can be given by

$$\frac{u_{p1(x/D,0)}}{u_{p(0,0)}} = 1.118\left(\frac{x}{D}+5.567\right)^{-0.065} \tag{3}$$

A statistical analysis has found that the coefficient of determination (R^2) in obtaining the coefficients for the above equation is 0.998, which indicates the adequacy of the equation within the axial distance domain considered in this study (i.e. 50mm from the nozzle exit).

Velocity Variation in Jet Cross-Section. As discussed before, particle velocity approaches to a more flat profile from an initial 1/7th power law distribution. Thus, the particle velocity profile at a given jet downstream cross-section may be expressed mathematically as

$$\frac{u_{p1(x/D,\ r/D)}}{u_{p1(x/D,\ 0)}} = \left(1 - 2\,\frac{r}{D}\right)^{B_1} + B_2\left(2\,\frac{r}{D}\right) \qquad (4)$$

where $u_{p1(x/D,\ r/D)}$ is the velocity of a certain sized particle at the point (x/D, r/D), and $u_{p1(x/D,\ 0)}$ is the particle velocity at the point (x/D, 0), B_1 and B_2 are coefficients and are the function of dimensionless axial distance x/D.

The determination of B_1 and B_2 is based on the following analysis. Firstly, it is assumed that the initial particle velocity distribution follows the one-seventh law; therefore, when $x/D=0$, B_2 should be equal to zero and B_1 is equal to 1/7 such that Eq. (4) becomes the one-seventh law equation. Secondly, both B_1 and B_2 increase as the jet travels downstream so that the particle velocity profile becomes a top flat-hat. When x/D is infinity, the two coefficients approach to 1. Therefore, B_1 and B_2 can be expressed by

$$B_1 = 1 - \frac{1}{\frac{7}{6} + b_1\,(x/D)^{b_2}} \qquad (5)$$

$$B_2 = 1 - \frac{1}{1 + b_3\left(x/D\right)^{b4}} \qquad (6)$$

The coefficients, b_1, b_2, b_3 and b_4, have then been determined statistically using the CFD simulation data at a 95% confidence level based on the reference dimensionless particle diameter (d_p/D) of 0.1. For garnet particles, Eqs. (5) and (6) become

$$B_1 = 1 - \frac{1}{\frac{7}{6} + 0.9371(x/D)^{0.4447}} \qquad (7)$$

$$B_2 = 1 - \frac{1}{1 + 0.4906\,\left(x/D\right)^{0.6421}} \qquad (8)$$

A statistical analysis of the particle velocity profile data from the CFD simulation gave a 0.998 coefficient of determination (R^2), which indicates the adequacy of Eq. (4) in predicting the particle velocity profiles at different jet downstream cross-sections.

Overall Particle Velocity Model. By substituting $u_{p1(x/D,\ 0)}$ from Eq. (3) into Eq. (4), the overall particle velocity model for garnet particles with a dimensionless particle diameter of 0.1 can be given by

$$u_{p1(x/D,\ r/D)} = u_{p(0,\ 0)}\left(1.118\left(\frac{x}{D} + 5.567\right)^{-0.065}\right)\left(\left(1 - 2\frac{r}{D}\right)^{B_1} + 2B_2\,\frac{r}{D}\right) \qquad (9)$$

where $B_1$and B_2 are given in Eqs. (7) and (8).

As mentioned earlier, particle mass affects the change rate of particle velocity owing to its momentum effect. For a given particle density, this effect can be considered as a result of the particle diameter or size. The above modelling work has considered only a reference particle size (i.e. $d_p/D=0.1$). It has been discussed that particle size combines with nozzle diameter in affecting particle velocity variation, so that a correction factor in relation to the dimensionless particle diameter, d_p/D, can be introduced to consider the effects of particle size. Mathematically, the particle velocity model that takes into account the effects of particle size can be given by

$$u_{p(x/D,r/D)} = \phi\left(\frac{d_p}{D}\right) u_{p1(x/D,\ r/D)} \tag{10}$$

In Eq. (10), if d_p/D is 0.1, the function $\phi(d_p/D)$ is equal to unity. In addition, the particle velocity increases with dimensionless particle diameter as it travels in the region close to the jet centreline as shown in Fig. 3(a), while Fig. 3(b) shows an inverse trend for the particle travelling in the region far away from the jet centre. Considering these effects arrives at the following equation

$$u_{p(x/D,\ r/D)} = \left(\frac{d_p}{0.1D}\right)^{C_1} u_{p1(x/D,\ r/D)} \tag{11}$$

where C_1 has been statistically determined from the CFD simulation data with the coefficient of determination (R^2) of 0.974, and is given by

$$C_1 = (0.3128 - r/D)^3 \tag{12}$$

By substituting Eq. (9) into Eq. (11), the velocity of a garnet particle at any location within an AWJ can be found from

$$u_{p(x,r)} = u_{p(0,0)}\left(\frac{d_p}{0.1D}\right)^{C_1}\left(1.118\left(\frac{x}{D}+5.567\right)^{-0.065}\right)\left(\left(1-2\frac{r}{D}\right)^{B_1}+2B_2\left(\frac{r}{D}\right)\right) \tag{13}$$

where B_1, B_2 and C_1 are given in Eqs. (7), (8) and (12), respectively.

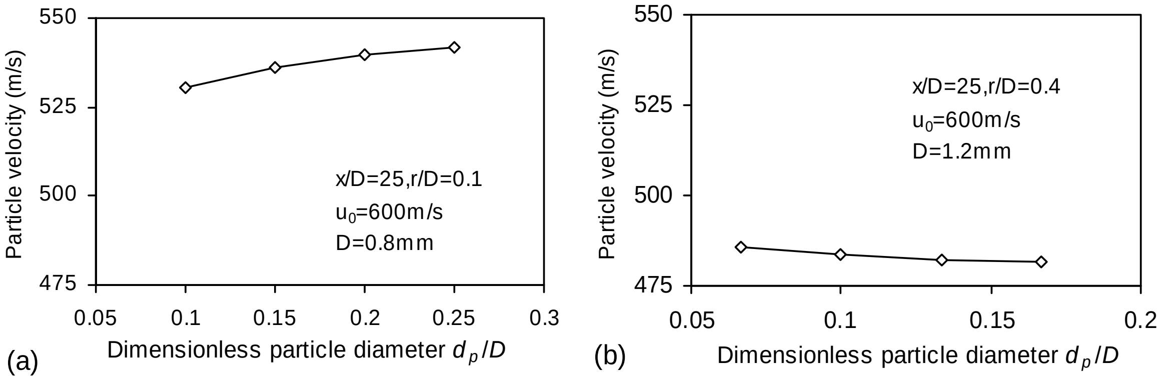

Fig. 3. The effect of dimensionless particle diameter on the particle velocity.

Model Assessment

The plausibility of the model is assessed by comparing the predict trends (in lines) with the corresponding CFD result (in symbols), as shown in Fig. 4, where Fig. 4(a) shows the predicted trend of particle velocity along the jet centreline, and Fig. 4(b) shows the predicted particle velocity profiles at five different jet downstream locations. The figures indicate that Eq. (13) has been formulated in the correct form and can give adequate predictions of the particle velocity for the range of input variables considered in this study.

To conduct a quantitative assessment of the predictive model, the comparisons of 2160 pairs of model predicted and CFD simulated data have been carried out. The percentage deviation of the predicted particle velocities from the corresponding CFD results is –0.5% on average with a standard deviation of 1.56%. Consequently, the developed model can be used to adequately predict the velocity of particles at any locations within an AWJ for the jet travel domain and the range of parameters considered in this study.

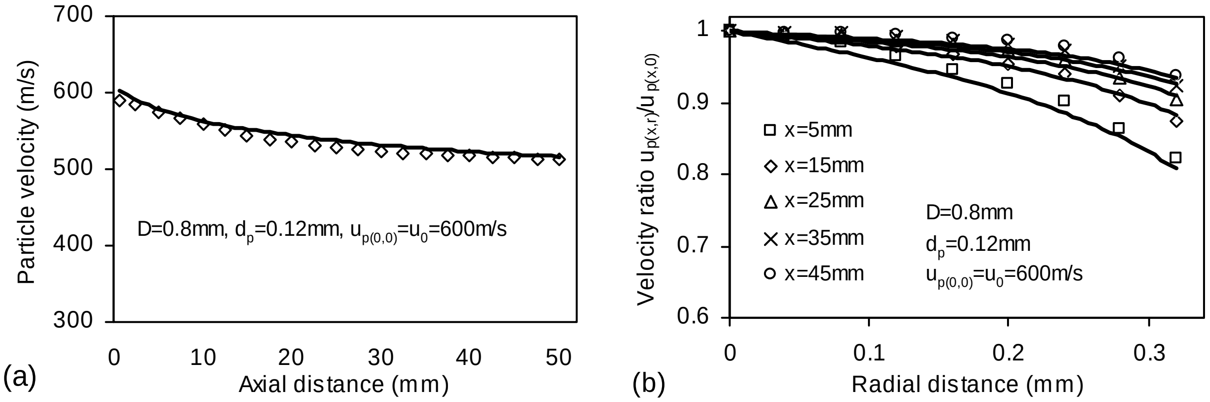

Fig. 4. Comparisons of predicted and CFD simulated particle velocities.

Conclusions

The knowledge of the dynamic characteristics of abrasive particles is essential for the understanding of the cutting process and cutting mechanisms in AWJ machining, as well as for modeling the various cutting performance measures. From a CFD simulation of AWJs, it has been found that the particle velocity decays as the jet travels away from the nozzle, and smaller particles decelerate more rapidly than large particles. Along the jet radial direction, the particle velocity distribution evolves towards a more top-flat-hat profile as the jet flows downstream. Based on the understanding of particle characteristics from this CFD study, mathematical models for particle velocity within an AWJ have been developed and numerically assessed. It has been shown that the model predictions are in excellent agreement with the corresponding simulated data from a verified CFD model. These particle velocity models form the essential basis for modeling the cutting performance in AWJ machining.

Acknowledgements

The authors wish to thank the Australian Research Council (ARC) for financial support to this project under the Discovery-Project scheme.

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