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Processing techniques for compensating for multiple scattering in TDM and other spectrally shadowed multiplexing systems

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Abstract

A theoretical analysis of multiple scattering is undertaken. The effect it has upon systems that do not take multiple scattering events or spectral shadowing into account through their signal processing routines is examined. An exact first order compensation scheme and higher order non-exact compensation schemes are developed. A comparison is made between the system performance that is obtained from the uncorrected, first-order corrected and higher-order corrected systems. The comparison makes use of a worst-case time-independent scenario.

Keywords: Multiple scattering, TDM, Spectral shadowing.

1. Introduction

The effects of multiple reflections and spectral shadowing place the greatest restrictions upon the system design of Time Division Multiplexed (TDM), and other related, optical fibre sensor networks. Spectral shadowing occurs by upstream sensors depleting some of the spectrum (thus "shadowing") sent to the downstream sensors. Multiple reflections occur due to part of the signal reflecting off several sensors before it is finally detected. These effects arise as, whereas as the signals from sensors in a parallel architecture and combine additively, the signals from a series architecture and combine multiplicatively. For the time independent case, the multiplication is that of the transfer matrices of each sensor along the array. For the time dependent case, the situation is much more complicated due to the presence of a time reverse propagator operator in each transfer matrix [1]. In either case the resultant signal will not just be a sum of the signals from each sensor, but rather will include nonlinear terms from pairs and groups of sensors.

Signal processing techniques are typically designed for linear systems where a signal can be easily decomposed into its constituent components. The primary tool of signal processing, the Fourier transform, operates on linear vector spaces and thus only gives an accurate representation of the frequency spectrum of a signal when the signal is a linear functional of its constituent components. Any nonlinearities produce mixed frequency components which can be misread as being due either to a frequency component that does not exist amongst the constituent signals or as the behaviour of an existing frequency component that isn't representative of its true behaviour.

2. Characterising Multiple Scattering

In a TDM system a multiply reflected signal having the same path length as that of a deeper penetrating singly reflected signal will arrive at the detector at the same time (see figure 1). Unless the effect of multiple reflections can be removed by a proper processing of the signal, then the sensor assigned to that time channel will experience crosstalk from the effects of the sensors that produce the multiply reflected signal. The worst case is when all sensors have their resonant reflection at the same wavelength and the furthest downstream sensor is interrogated. Morey has produced an analysis of the worst case scenario [2] when a set of FBGs are all equally spaced and have the same reflectivity and Bragg wavelengths, but has only taken into account the case of first order multiple reflections. Similar analyses of crosstalk in TDM systems have been presented [3-5], but these either consider only the first order multiple reflections (as in the case of [3] and [4]) or ignore multiple reflections altogether (as in the case of [5]). For a large number of sensors, it will be shown that the higher orders will be a significant contribution to the detected signal.

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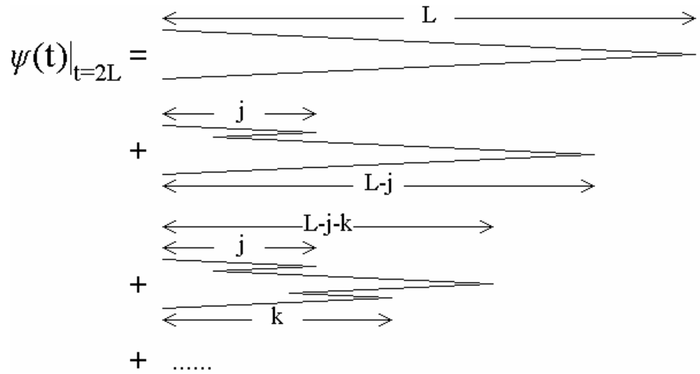

Figure 1. Signals from various paths that reach the detector at the same normalised time (2L) in a TDM sensor system.

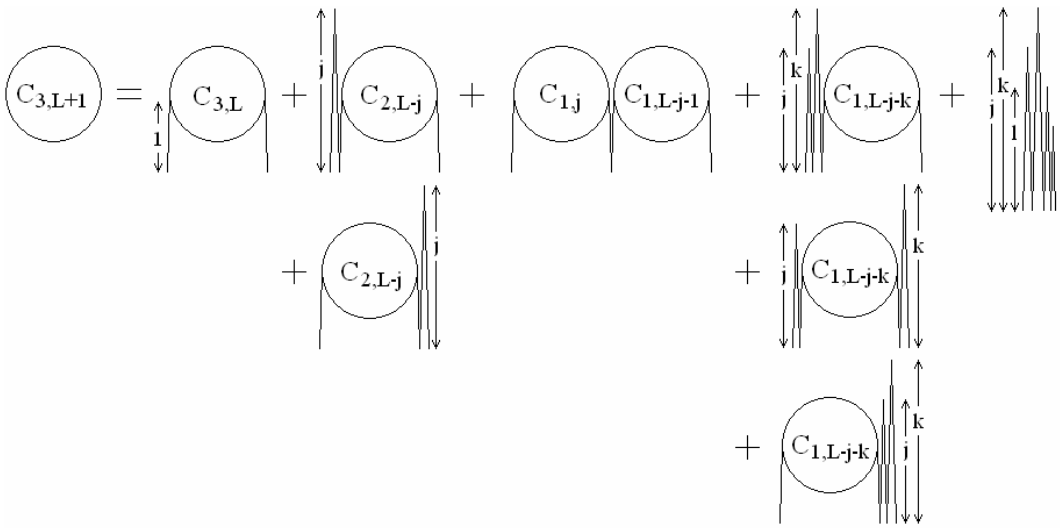

Figure 2. Scattering processes contributing to 3rd order multiple reflections in a series array of identical sensors.

Examining the multiple reflection problem, when the reflectivity and resonant wavelengths are all the same, is a simple combinatorial problem of summing up the number of different paths of length $2L$ (measured from the first sensor) that light can take while making $2j+1$ reflections. Let this number be $c_{j,L}$. The total signal for the Lth time channel is given by:

$$S_L = \sum_j c_{j,L} R^{2j+1} (1-R)^{2(L-j)}, \tag{1}$$

where the form of $c_{j,L}$ is as follows:

$$\begin{aligned} c_{0,L} &= 1 \\ c_{1,L} &= \frac{L(L-1)}{2} \\ c_{2,L} &= \frac{L(L-1)^2(L-2)}{12} \\ c_{3,L} &= \frac{L(L-1)^2(L-2)^2(L-3)}{144} \end{aligned} \quad . \tag{2}$$

These equations are found by recursively over L and summing over the cases of number of reflections with the sensor at the zero position. For example:

$$c_{3,L+1} = c_{3,L} + \left[2\sum_j c_{2,L-j} + \sum_j c_{1,j} c_{1,L-j-1} \right] + 3\sum_{j,k} c_{1,L-j-k} + \sum_{j,k,l} 1 , \tag{3}$$

is the recursive relation that gives the above equation for $c_{3,L}$. Diagrammatically this is the sum of the scattering diagrams shown in figure 2 with each term of the recursion corresponding to a diagram or vertical pair or triplets of diagrams when the term is multiplied by 2 or 3 respectively.

As can be seen to be the case, at least as far as j=2, the contribution to the signal S_L of each successively higher order multiple reflection scales as:

$$\frac{S_{j+1,L}}{S_{j,L}} = \frac{R^2 (L-j)(L-j-1)}{(1-R)^2 (j+1)(j+2)}, \tag{4}$$

where $S_{j,L}$ is the summand of equation (1). Thus it can be seen that a truncation at first order is clearly not valid for large TDM systems having L = 49 or 99 and the crosstalk will be underestimated.

By careful system design the problem of multiple scattering can be mitigated for the case of time dependant systems that make use of single pulses of light to interrogate the sensors (such as traditional TDM but not CDMA or FDM). By arranging the relative positions of the sensors along the array in such a manner that the multiply reflected signals occur at a time that is either slotted in between the intervals of time where the singly reflected signals are detected or sometime after all the singly reflected signals are detected. The repetition rate of the pulsing of the light source needs to be slow enough to allow the sensor array to reach a quiescent state before another pulse of light is injected into the system.

Due to the way in which the problem scales with sensor capacity, for large TDM arrays this will involve impractically long lengths of fibre used as delay lines and as such there will need to be a compromise made between restricting the degree to which the system scales (both in terms of the length of fibre and the time required for the system to approach a quiescent state) and the permitted degree of overlap in the time domain of the singly reflected signals and the multiply reflected signals. However, given a low degree of overlap between the singly and multiply reflected signals, multiple reflections can be ignored completely from the analysis of the system state and it will only be first order spectral shadowing that will need to be considered as will be discussed in the next section.

3. First Order Compensation

A sensor array is indexed by integers k,j such that sensors are pairwise given the same value of k, but different values of j, when multiple reflections may occur between them (e.g. those multiplexed using TDM or CDMA etc.) and pairwise given different values of k when multiple reflections may not occur between them (e.g. those multiplexed using WDM). For the continuous-wave, time-independent case, the combined reflection spectrum can be broken down accordingly as:

$$R(\lambda) = A(\lambda) \sum_{\kappa} R_{\kappa}(\lambda) . \tag{5}$$

where $A(\lambda)$ is the source power spectrum and $R_\kappa(\lambda)$ is the combined reflection spectrum of the sensor sub-array with only sensors having index k=κ and the same ordering of these sensors as was the case for the original array. In order to breakdown the more complicated $R_\kappa(\lambda)$, an inductive approach is required. Dropping the variable λ from use, as it is implicit to all terms here, let $\tilde{R}_{\kappa,j}$ represent the combined reflection of sub-array κ from the sensors up to but excluding sensor j for a normalised source of unity power. The combined reflection of sub-array κ from the sensors up to and including sensor j, $r\left(\tilde{R}_{\kappa,j}, R_{\kappa,j}\right)$, is found by adding up all the possible scattering events between the block of reflectivity $\tilde{R}_{\kappa,j}$ and the sensor of reflectivity $R_{\kappa,j}$ as:

$$\begin{aligned} r\left(\tilde{R}_{\kappa,j}, R_{\kappa,j}\right) &= 1 - \left(1-\tilde{R}_{\kappa,j}\right)\left(1-R_{\kappa,j}\right)\left(1 + \left(\tilde{R}_{\kappa,j} R_{\kappa,j}\right) + \left(\tilde{R}_{\kappa,j} R_{\kappa,j}\right)^2 + \ldots\right) \\ &= 1 - \frac{\left(1-\tilde{R}_{\kappa,j}\right)\left(1-R_{\kappa,j}\right)}{\left(1-\tilde{R}_{\kappa,j} R_{\kappa,j}\right)} \end{aligned} . \tag{6}$$

Representing the operator $r\left(\bullet, R_{\kappa,j}\right)$ by $\tilde{\mathbf{r}}_{\kappa,j}$, the combined reflection spectrum is found as:

$$R_{\kappa} = A \underset{j}{\circ} \tilde{\mathbf{r}}_{\kappa,j}\,, \tag{7}$$

where $\underset{k,j}{\circ}$ is taken as the composition over all possible k,j in the order in which they occur in the sensor array. The multiplicative nature of this operator can be taken care of using an isomorphism to convert the multiplicative operators to additive operators. The natural mapping to choose would be the logarithmic mapping $\log_2 x$ as $\log_2 ab = \log_2 a + \log_2 b$. Here the base 2 mapping is chosen because powers of 2 are faster to perform on a computer than powers of any other exponent.

Separating multiplicative terms using this logarithmic mapping gives:

$$\log_2\left(1-\tilde{\mathbf{r}}_{\kappa,j}\left(\tilde{R}_{\kappa,j}\right)\right) = \log_2\left(1-\tilde{R}_{\kappa,j}\right) + \log_2\left(1-R_{\kappa,j}\right) - \log_2\left(1-\tilde{R}_{\kappa,j}R_{\kappa,j}\right). \tag{8}$$

Iterating recursively over k,j gives:

$$\log_2\left(1-\frac{R}{A}\right) = \sum_{k,j}\log_2\left(1-R_{k,j}\right) - \log_2\left(1-O\left(R^2\right)\right). \tag{9}$$

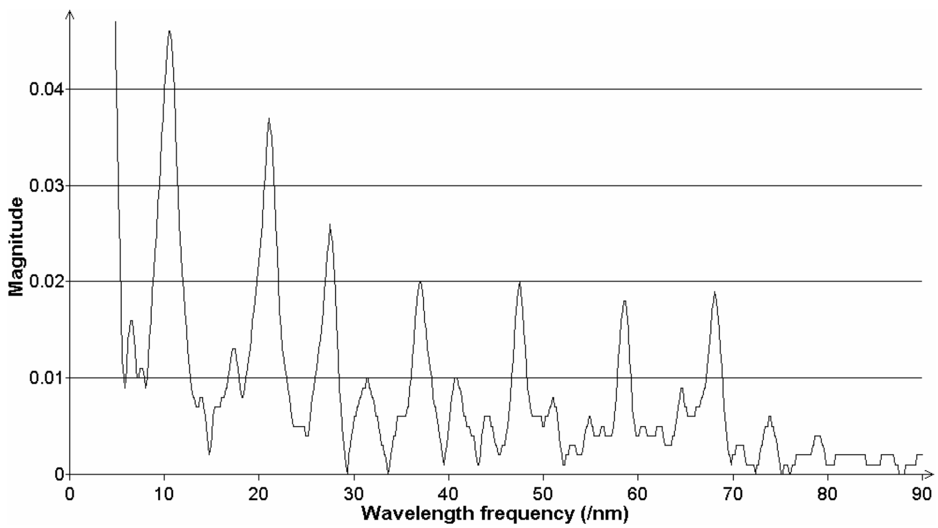

Figure 3. Fourier spectrum of 8 spectrally overlapping carrier modulated FBG sensors with no compensation for spectral shadowing or multiple scattering.

For first order compensation, the final term will be taken as an error term as it contains the higher order terms in the expansion. This term is due to the higher order spectral shadowing terms associated with multiple reflections. Without the error term this equation matches that used by the approach in [6] where first order spectral shadowing is taken into account but multiple scattering and higher order spectral shadowing are not. However as it is a linear functional of the individual sensor reflectivities, it can be processed using Fourier analysis and other signal processing techniques.

For the case of TDM, which is time dependant, the reflected signal is:

$$\frac{R\left(\lambda, t+2\Delta_{k,j}\right)}{A(\lambda,t)} = \sum_{k} R_{k,j}\left(\lambda, t+\Delta_{k,j}\right)\prod_{m=1}^{j-1}\left(1-R_{k,m}\left(\lambda, t+\Delta_{k,m}\right)\right)\left(1-R_{k,m}\left(\lambda, t+2\Delta_{k,j}-\Delta_{k,m}\right)\right), \tag{10}$$

where the $\Delta_{k,j}$ are the time delays for light to reach sensor k,j. Assuming that the system state changes slowly compared to the timescales $\Delta_{k,j}$, each sensor reflectivity, $R_{k,j}(\lambda,t)$, can be found recursively as:

$$R_{k,j}\left(\lambda,t+\Delta_{k,j}\right)=\frac{R\left(\lambda,t+2\Delta_{k,j}\right)\chi_{I(k)}(\lambda)}{A(\lambda,t)\prod_{m=1}^{j-1}\left(1-R_{k,m}\left(\lambda,t+\Delta_{k,m}\right)\right)^2}, \quad (11)$$

where $\chi_{I(k)}(\lambda)$ is the characteristic function on the interval $I(k)$ which selects the appropriate WDM window for sensors with index k; being equal to zero for all wavelengths not within this window.

Results given in [7] apply the routine of equation (9) to a set of eight strongly overlapping fibre Bragg gratings; each having a different peak in their Fourier spectrum; the index j linearly corresponding to the wavelength frequencies of each sensor. The tests show a decay in the higher wavelength frequency components of the spectrum; however, the improvement over an uncorrected scheme can quite clearly be seen as is evidenced by comparing figure 3 and figure 4 where the Fourier spectra are shown for the uncorrected and first order corrected methods respectively. The first order corrected scheme has much better isolation between the wavelength frequency peaks, whereas the uncorrected scheme overlaps more leading to crosstalk of the phase information between the sensors in this region. The magnitude of the peaks for the first order corrected case is almost twice as large as that for the uncorrected case giving an improvement in the signal to noise ratio and thus the accuracy of determining the state of each sensor.

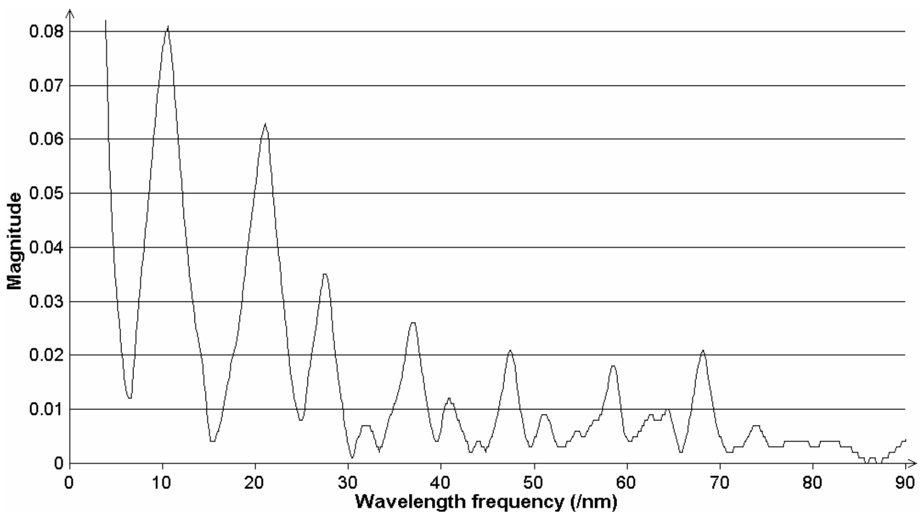

Figure 4. Fourier spectrum of 8 spectrally overlapping carrier modulated FBG sensors with first order compensation for spectral shadowing.

4. Higher order Compensations

The error term given in equation (9) is a complicated transcendental function of the entire unknown system state. That is in order to exactly determine the state of the sensor system, it will be required to first calculate the error term, which in turn requires an entire knowledge of the system state to perform. Thus any attempt to compensate for the error term will be best done using an adaptive algorithm where two options are available. One is that an exact zero/first order compensation scheme can be used to approximately determine the system state and thus the error term which is then corrected for and reprocessed to give a more accurate value for the system state. The other is to use the measured system state of the previous iteration to approximate the error term for the current iteration. In order to do this latter technique, it will be necessary to assume that the system state moves slowly throughout its configuration space as will be the case for most quasi-static sensor systems. The latter technique has the advantage over the former in that the system state only has to be processed once, whereas for the former it has to be processed twice.

From a knowledge of the system state, a set of individual sensor spectra $\{R_{k,j}(\lambda)\}$ is constructed and the $\widetilde{R}_{k,j}$ terms are found iteratively using equation (6). They are thence substituted in to calculate the error term as:

$$-\log_2\left(1-O\left(R^2\right)\right)=-\sum_k\sum_j\log_2\left(1-\widetilde{R}_{k,j}R_{k,j}\right) \quad (12)$$

Which in turn is then used to convert the nonlinear LHS of equation (9) into the linearised first term of the RHS of equation (9).

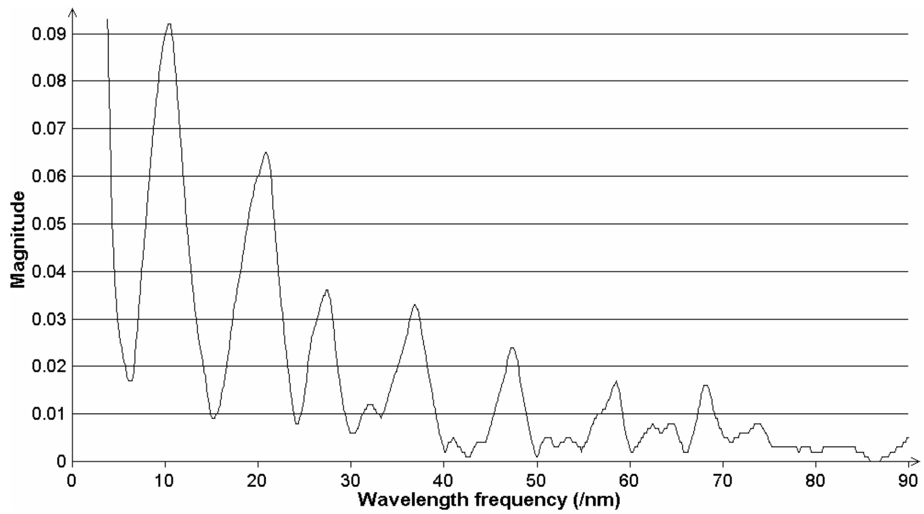

Figure 5. Fourier spectrum of 8 spectrally overlapping carrier modulated FBG sensors with full compensation for spectral shadowing and multiple reflections.

The results from performing the first order compensation given in figure 4 are reprocessed using this methodology and shown in figure 5. A visual improvement on the first order scheme, however, is only marginal; seen more so in the lower wavelength frequencies along with a slight increase in the magnitude of the signal.

One drawback to the higher order scheme is the strict requirement on calibrating the individual sensor spectra $\{R_{k,j}(\lambda)\}$ accurately in order to ensure that the algorithm converges stably. This is particularly important for higher wavelength frequency components where miscalibration causes beat frequencies in $\log_2(1\text{-}RR)$ to alter the lower wavelength frequency components and will thus, with further iterations, spread throughout the entire system causing errors.

5. Conclusion

A study of multiple scattering in sensor networks was carried out. Three approaches for dealing with multiple scattering were presented, an uncorrected, a first order corrected and a higher order corrected scheme. Compared with the uncorrected case, a vast improvement was obtained by applying a first order compensation scheme demonstrating less crosstalk and a higher signal to noise ratio. The higher order corrected scheme gave a marginal improvement again, however, due to the difficulties involved in calibration, this method might not provide a sufficient level of stability for some sensor networks.

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