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USE OF THE COLEBROOK-WHITE EQUATION IN PIPE NETWORK ANALYSIS PROGRAMS

by

T.R.Fietz

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Abstract

Computer execution times for subroutines using the Colebrook-White equation for friction factor (and its explicit approximations) are compared with times for subroutines using the Hazen-Williams formula. The effect of using these subroutines on the total computer execution time of nodal and loop method programs is discussed.

Notation

a, b, c	coefficients in Wood's approximation for λ
C_1	constant
C_{hw}	Hazen-Williams coefficient
d	pipe diameter
f	function of
F	function of
g	acceleration due to gravity
G	function of
h	head loss due to friction in a prismatic pipe
H	node (or junction) head
k	equivalent sand grain roughness size
K	resistance coefficient = $\frac{8\,\ell}{\pi^2\, g d^5}$
ℓ	pipe length
p	function of
Q	flowrate (or discharge) = $\frac{\pi d^2 V}{4}$
r	function of
R	Reynolds Number = $\frac{Vd}{\nu}$
V	mean velocity
ν	kinematic viscosity
λ	Darcy-Weisbach friction factor = $\frac{2hgd}{\ell V^2}$
ϵ	convergence limit

1. Introduction

For analysis of steady flow in pipe networks by nodal methods, a relationship Q(h) between Q and h is required, as well as the derivative Q'(h). For loop methods h(Q) and h'(Q) are required.

The empirical Hazen-Williams and Manning formulae are frequently used to relate Q and h, and the derivatives are simple expressions.

Use of the Darcy-Weisbach equation, in conjunction with the Colebrook-White equation for λ, is preferable. Not only are the results more accurate (Refs. 19, 16, 20), but the computer program is then applicable to flows other than water. Application of the Colebrook-White equation in pipe network analysis programs has often been avoided in the past, particularly for loop methods when iterative solutions of the equation have resulted in significant increases in computer execution times.

This note compares the computer times required to use subroutines employing the Hazen-Williams formula and the Colebrook-White equation (and some explicit approximations to it) for both nodal and loop methods of analysis. For modern computer programs, the increase in computer execution time required to use the Colebrook-White equation instead of the Hazen-Williams formula is shown to be slight.

2. Basic Equations for Head Loss and Friction Factor

The Hazen-Williams formula, the Darcy-Weisbach equation, and several forms of the Colebrook-White equation are shown in Table 1. Also shown are three explicit approximations to the Colebrook-White equation.

The accuracy of the Moody and Wood approximations in the transition zone is shown in Table 2. The Barr approximation is not shown in Table 2 as it gives λ values within $\pm$ 2% of the true value over the entire range of $\frac{k}{d}$ and R values shown in Table 2.

3. Equations for Nodal Methods

Equations for nodal methods are shown in Table 3. These apply to both the simple node (Refs. 15, 2) and the simultaneous node (Refs. 14, 8, 5) methods of analysis. Taking H_i as the head at the node in question, and H_j as the head at the far end of the pipe, then the head loss is given by equation (11). Equations (12) to (15) apply for the convention that flows towards node i are positive. Note that $Q'(H_i)$ is always negative with this convention.

Table 1: Basic Equations

Eqn. No.	Ref. No.	Name	Equation
1	12	Hazen-Williams, SI units	$h = \frac{4.73\, Q^{1.852}\, \ell}{d^{4.87}\, C_{hw}^{1.852}}$
2		Darcy-Weisbach, Velocity form	$h = \lambda \frac{\ell}{d} \frac{V^2}{2g}$
3		Darcy-Weisbach, Discharge form	$h = \frac{8\,\ell}{\pi^2 g d^5} \lambda\, Q^2 = K \lambda\, Q^2$
4	6	Colebrook-White, $\frac{1}{\sqrt{\lambda}}$ form	$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda}}\right)$
5		Colebrook-White, $\sqrt{\lambda}$ form	$\sqrt{\lambda} = -\left[2 \log_{10} e \log_e\left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda}}\right)\right]^{-1}$
6		Rough pipe, $\sqrt{\lambda}$ form	$\sqrt{\lambda} = -\left[2 \log_{10} e \log_e\left(\frac{k}{3.7d}\right)\right]^{-1}$
7		Colebrook-White, λ form	$\lambda = \left[2 \log_{10} e \log_e\left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda}}\right)\right]^{-2}$
8	17	Moody approx. for λ	$\lambda = .0055\left[1 + \left(2\times10^4 \frac{k}{d} + \frac{10^6}{R}\right)^{1/3}\right]$ $\left(5\times10^{-6} < \frac{k}{d} < 10^{-2},\ 4\times10^3 < R < 10^7\right)$
9	22	Wood approx. for λ	$\lambda = a + bR^{-c}$ $a = .094\left(\frac{k}{d}\right)^{.225} + .53\left(\frac{k}{d}\right)$ $b = 88\left(\frac{k}{d}\right)^{.44},\ c = 1.62\left(\frac{k}{d}\right)^{.134}$ $\left(10^{-5} < \frac{k}{d} < 4\times10^{-2},\ 4\times10^3 < R < 10^8\right)$
10	3	Barr approx. for λ	$\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{k}{3.7d} + \frac{5.13}{R^{.89}}\right)$ $\left(10^{-6} < \frac{k}{d} < 10^{-2},\ 3\times10^3 < R < 10^8\right)$

Table 2: Accuracy of Explicit Approximations for λ

k/d	% Error Exceeding ± 2%					
	$R=10^4$	10^5	10^6	10^7	10^8	
10^{-6}	-	- -3.5	- -5.4	-	- -	Wood, eqn. (9) Moody, eqn. (8)
10^{-5}	-3.2	-3.8 -3.4	-4.4	+2.1	-	
5×10^{-5}	+2.4	 -2.9	+3.8	+3.7 -2.9	-	
10^{-4}	+2.3	 -2.3	+5.3	+4 -3.1	-	
4×10^{-4}		+2.7	-6 -2.7	+4.5 +3.7	+3.5 -	
10^{-3}		+3.7	-5.2 +3.7	+4.4 +4	-	
4×10^{-3}	-2.4	+2.8 +2.2	+3.2 +2.7	+3 -2.8	+3 -	
10^{-2}					-	

Table 3: Equations for Nodal Methods

Eqn. No.	Name	Equation
11	Head loss	$h = H_j - H_i$
12	Hazen-Williams discharge	$Q(H_i) = \text{Sign}(h) \left(\frac{\lvert h \rvert}{l}\right)^{.54} \frac{d^{2.63}\, Chw}{2.314}$
13	Hazen-Williams derivative	$Q'(H_i) = -\ .54 \left(\left\lvert \frac{Q(H_i)}{h} \right\rvert\right)$
14	Darcy and Colebrook-White discharge	$Q(H_i) = -\text{Sign}(h) \log_{10} e \frac{\pi d^2}{2} \sqrt{\frac{2gd \lvert h \rvert}{l}}$ $\log_e \left[\frac{k}{3.7d} + \frac{2.51\nu}{d} \left(\sqrt{\frac{2gd \lvert h \rvert}{l}} \right)^{-1} \right]$
15	Darcy and Colebrook-White derivative	$Q'(H_i) = \frac{-1}{2\lvert h \rvert} \left[\lvert Q(H_i) \rvert + \log_{10} e \frac{\pi d^2}{2} \frac{2.51\nu}{d} \left\{ \frac{k}{3.7d} + \frac{2.51\nu}{d} \left(\sqrt{\frac{2gd \lvert h \rvert}{l}} \right)^{-1} \right\}^{-1} \right]$

4. Equations for Loop Methods

Equations for loop methods are shown in Table 4, which apply to both the simple loop (Refs. 8, 2, 11) and simultaneous loop (Refs. 10, 21) methods. The term normally included in the h(Q) equations to indicate the direction of the loop through the pipe has been omitted. Note that h'(Q) is always positive.

Table 4: Equations for Loop Methods

Eqn. No.	Name	Equation
16	Hazen-Williams head loss	$h(Q) = \text{Sign}(Q)\ 4.73 \left(\frac{\lvert Q\rvert}{C_{hw}}\right)^{1.852} \frac{\ell}{d^{4.87}}$
17	Hazen-Williams derivative	$h'(Q) = 1.852 \left(\left\lvert\frac{h(Q)}{Q}\right\rvert\right)$
18	Darcy head loss	$h(Q) = \text{Sign}(Q)\ K \lambda Q^2$
19	Darcy derivative	$h'(Q) = K \left(2 \lambda \lvert Q\rvert + \frac{\partial \lambda}{\partial Q} Q^2\right)$
20	Darcy and Colebrook-White derivative	$h'(Q) = 2K\lambda Q \left[1 - 2 \log_{10} \text{ex} 2.51 \left\{\left(\frac{2.51}{R\sqrt{\lambda}} + \frac{k}{3.7d}\right) R + 2 \log_{10} \text{ex} 2.51\right\}^{-1}\right]$
21	Darcy and Moody approx. derivative	$h'(Q) = K \left[2\lambda Q - .0055 \times 10^6 \times \frac{Q}{3R} \left(2 \times 10^4 \frac{k}{d} + \frac{10^6}{R}\right)^{-2/3}\right]$
22	Darcy and Wood approx. derivative	$h'(Q) = KQ \left[2\lambda - c(\lambda - a)\right]$
23	Darcy and Barr approx. derivative	$h'(Q) = 2K\lambda Q \left[1 - 2 \log_{10} \text{ex} 5.13 \times .89\sqrt{\lambda} R^{-.89} \left(\frac{k}{3.7d} + \frac{5.13}{R^{.89}}\right)^{-1}\right]$

Equations (16) and (17) use the Hazen-Williams formula.

Equation (18) gives the head loss from the Darcy equation. The friction factor λ for use in equation (18) may be found by iterative solution of one of the forms of the Colebrook-White equation (equation (4), (5) or (7)) or from one of the explicit approximations (equation (8), (9) or (10). h'(Q) is given by equation (19) (Ref. (13)). When λ is found from the Colebrook-White equation h'(Q) is found from equation (20). When one of the approximations is used, then the corresponding h'(Q) expression is given by equation (21), (22) or (23).

5. Iterative Solution of the Colebrook-White Equation

Equations for this section are shown in Table 5. The successive substitution (Refs. 23, 7, 13) and Newton's method (Ref. 21) have been suggested for finding the root λ of the implicit Colebrook-White equation.

Table 5: Equations for Iterative Solutions

Eqn. No.	Name	Equation
24	Successive substitution	$\sqrt{\lambda_{j+1}} = -\left[2 \log_{10} e \log_e \left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)\right]^{-1} = f\left(\sqrt{\lambda_j}\right)$
25	Successive substitution	$\sqrt{\lambda_{j+1}} = (1-C_1)\sqrt{\lambda_j} - C_1 \left[2 \log_{10} e \log_e \left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)\right]^{-1}$
26	Newton's method, $\frac{1}{\sqrt{\lambda}}$ form	$\lambda_{j+1} = \lambda_j - \frac{F(\lambda_j)}{F'(\lambda_j)} = p(\lambda_j)$
27		$F(\lambda_j) = \frac{1}{\sqrt{\lambda_j}} + 2 \log_{10} e \log_e \left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)$
28		$F'(\lambda_j) = \frac{-1}{2\lambda_j^{1.5}}\left[1 + 2 \log_{10} e \times \frac{2.51}{R}\left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)^{-1}\right]$
29	Newton's method, λ form	$\lambda_{j+1} = \lambda_j - \frac{G(\lambda_j)}{G'(\lambda_j)} = r(\lambda_j)$
30		$G(\lambda_j) = \lambda_j - \left[2 \log_{10} e \log_e \left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)\right]^{-2}$
31		$G'(\lambda_j) = 1 - 2 \log_{10} e \times \frac{2.51}{R}\left(\frac{k}{3.7d} + \frac{2.51}{R\sqrt{\lambda_j}}\right)^{-1}$
32	Convergence criterion	$\left\lvert\frac{\lambda_{j+1} - \lambda_j}{\lambda_j}\right\rvert < \epsilon$

Equation (5) is already in a form suitable for applying successive substitution. Equation (24) results, where λ_j is the current estimate of the root and λ_{j+1} is a better estimate by iteration. Iteration is terminated when the relative convergence criterion, equation (32), is satisfied. Taking the convergence limit $\epsilon = .001$ suffices for pipe network programs.

Convergence of equation (24) is assured as $|f'(\sqrt{\lambda_n})| < 1.0$, where λ_n is the true solution (Refs. 7,4). Convergence is slowest for small values of k/d, in particular at low R values. Convergence of the successive substitution method can sometimes be improved by redefining $f(\lambda_j)$ in equation (24) to yield equation (25) (Ref. 4). The optimum value of the constant C_1 lies between 0.8 and 0.9 and a mean value of 0.85 has been used for C_1.

The starting value λ_1 to commence iteration, can be either λ from the rough pipe formula (equation (6)) (Ref. 23), or an arbitary value for all turbulent flows, ranging from .024 to 1.0 (Refs. 24,7,13). Equations (26) to (31) are for application of Newton's method, using either equation (4) or equation (7) as the basic equation. At the root λ_n $p'(\lambda_n)$ (equation (26) or $r'(\lambda_n)$ (equation (29)) tends to zero. For convergence the starting value for friction factor, λ_1, should be "near" the true value λ_n (Ref. 4), and $p'(\lambda_j)$ or $r'(\lambda_j)$ should decrease from one iteration to the next. Possible values of λ_1 are the rough pipe λ (equation (6)); an arbitrary value for all turbulent flows; or λ from one of the approximate formulae, equation (8), (9), or (10).

6. Comparative Computer Times

The methods described in Sections 3,4 and 5 above, have been translated into computer program subroutines. The execution times requires to call these subroutines many times have been found for the transition zone between smooth and rough wall turbulent flow in the range $10^{-6} < \frac{k}{d} < 10^{-2}$ and $10^4 < R < 10^8$. In the case of the iterative subroutines mean execution times in the transition zone have been found. Execution times have been divided by those for the Hazen-Williams nodal or loop subroutines to find comparative times. Comparative times may vary slightly with the computer and compiler used, so values to the nearest 0.1 have been quoted.

Comparative times for nodal method subroutines are shown in Table 6.

Comparative times for loop method subroutines are shown in Table 7.

7. Discussion of Results

The results in Table 6 show that the time for $Q(H_i)$ and $Q'(H_i)$ computations in nodal methods is increased by 30% if the Colebrook-White equation (in conjunction with the Darcy equation) is used instead of the Hazen-Williams formula.

Table 6: Nodal Method Subroutines, Comparative Computer Times

Subroutine	Description	Eqn. for $Q(H_i)$	Eqn. for $Q'(H_i)$	Comparative Time
HWN	Hazen-Williams	12	13	1.0
CWN	Darcy and Colebrook-White	14	15	1.3

Table 7: Loop Method Subroutines, Comparative Computer Times

Subroutine	Description	Eqn. for $h(Q)$	Eqn. for $h'(Q)$	Eqn. for iteration	Starting	Comparative Time
HWL	Hazen-Williams	16	17			1.0
WL	Wood approx.	18	22			1.0
ML	Moody approx.	18	21			1.5
BL	Barr approx.	18	23			1.5
S1	Successive substn.	18	20	24	.02	3.1
S2		18	20	24	Eqn. (6)	3.2
S3		18	20	25, C_1 = .85	Eqn. (6)	3.5
S4		18	20	24	1.0	3.6
N1	Newton's Method	18	20	26	Eqn. (10)	3.9
N2		18	20	26	Eqn. (9)	5.7
N3		18	20	26	Eqn. (6)	5.8
N4		18	20	29	Eqn. (9)	6.5
N5		18	20	29	Eqn. (6)	6.9
N6		18	20	26	.02	Diverges $10^{-6} < \frac{k}{d} < 10^{-5}$ $10^{7} < R < 10^{8}$

Notes (1) Wood's coefficients a, b and c available to subroutine WL

(2) Convergence ϵ =.001 in Eqn.(32) for iterative subroutines.

For loop methods, the results in Table 7 show that the time required for h(Q) and h'(Q) determinations will increase by factors ranging from 1.0 to 6.9 (depending on the subroutine used) if Colebrook-White based equations are used instead of the Hazen-Williams formula.

Wood's approximation for λ (subroutine WL) may be used instead of the Hazen-Williams formula in loop methods without increasing the execution time. The error in λ introduced by using Wood's approximation ranges from about -4% to 6% (see Table 2) but this is still preferable to using the Hazen-Williams formula.

Barr's approximation for λ (subroutine BL) involves an increase in execution time for h(Q) and h'(Q) calculations of 50% compared to using the Hazen-Williams formula. Barr's approximation gives λ values within $\pm$ 2% of the true value over the whole range of practical pipe flows, and this accuracy is more than adequate for network analysis (Ref. 1).

Moody's approximation for λ (subroutine ML) takes the same time as Barr's, and, as it is less accurate, its use is not justified.

Of the iterative subroutines, S1 to S4 and N1 to N5, the subroutine S1 is the least time consuming, taking about twice as long as subroutine BL (using Barr's approximation), and three times as long as HWL (using Hazen-Williams formula). Use of the iterative subroutine S1 does not seem justified, however, in view of slight increase in accuracy over subroutine BL. An ill-chosen subroutine, for example N5, can be quite time consuming when compared to subroutine HWL. This may account for some previous avoidance of use of the Colebrook-White equation in loop method programs.

8. Effect of the Head-Discharge Relation on Total Execution Time

The effect of the head-discharge relation used depends on the proportion of the total time spent in using the relation, which in turn depends on the method of analysis and the size of the network.

For the simple nodal and loop methods, where node head adjustments and loop flow corrections, respectively, are made one at a time, the head-discharge relation is used continuously. For the simultaneous nodal and loop methods, the head-discharge relations are used only at the beginning of each iteration step involving in the solution of a set of linear simultaneous equations. The solution of the simultaneous equations accounts for most of the computer time.

Execution times for the various sections of a program are not often given in the literature. The effect of network size on total execution time is usually given. Some approximate estimates of the effect of changing from the Hazen-Williams formula for pipe flow to the Colebrook-White equation for several methods of analysis are given in Table 8. These probably apply to medium sized networks up to about 300 pipes. Lack of data prevents estimation of the effect of network size on the proportion of the total time spent in using head-discharge relations.

Table 8: Effect of h - Q Relation on Total Execution Time, Networks of about 300 Pipes

Method of Analysis	Ref. No.	Estimate of: Time using h - Q relation / Total execution time x 100%	Subroutine A, Hazen-Williams	Subroutine B, Colebrook-White	Approx. % increase in total execution time using subr. B in place of A, to nearest 5%.
Simple Node	15	90	HWN	CWN	30
Simultaneous Node	9	10	HWN	CWN	5
Simultaneous Node, sparsely oriented	5	5	HWN	CWN	0
Simple Loop, initial pipe Q and loops supplied as data	11	90	HWL	WL	0
				BL	45
Simple Loop, initial pipe Q and loops found by program	18	60	HWL	WL	0
				BL	30
Simultaneous Loop	10, 21	10	HWL	WL	0
				BL	5
				N5	60

9. Conclusions and Recommendations

1. For simple nodal method programs using the Colebrook-White equation instead of the Hazen-Williams formula will increase execution time by about 30%.

2. For modern simultaneous node method programs, the head-loss relation used for pipes has negligible effect on execution time.

3. For loop method programs, Wood's approximation may be used instead of the Hazen-Williams formula without affecting execution time. Wood's approximation introduces errors in λ ranging from -4 to + 6% but this is still preferable to using the Hazen-Williams formula.

4. Barr's approximation is recommended for use in loop methods programs, being of adequate accuracy ($\pm$ 2%) and less time consuming than iterative solutions of the Colebrook-White equation.

5. For simple loop method programs using Barr's approximation in place of the Hazen-Williams formula increases execution time from 30% to 45%, depending on the method used for determining initial pipe discharges and loop layout.

6. For modern simultaneous loop method programs a negligible increase in execution time will result when Barr's approximation is used instead of the Hazen-Williams formula.

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