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THE FLOW OF A TWO LAYER FLUID OVER A BROAD CRESTED WEIR

by

I. R. Wood and K. K. Lai

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Preface

The work reported herein represents part of the programme of basic research within the Water Research Laboratory sponsored by a grant of funds from the Australian Research Grants Committee. The theoretical work was carried out by Associate Professor I. R. Wood and the experimental program was carried out by Mr. K. K. Lai under Associate Professor Wood's direction.

R. T. Hattersley,
Assoc. Professor of Civil Engineering,
Officer-in-Charge.

Summary.

In this paper the method used by Wood (1968, 1970) is extended to cover the case of the flow of a stably layered fluid from a reservoir through a contraction with a round crested weir at its minimum width. The conditions under which a single layer may be separated from a two layer system by having this lighter layer alone flowing over a weir are first examined. The conditions under which two layers continuously decrease in depth from the reservoir to and downstream of the weir are determined. It is shown that in this case the theory involves computations not only at the section of minimum width but also at a section upstream of this point (the virtual point of control). For a weir shape, chosen so as to simplify the algebra, complete solutions are obtained.

For the case of the flow of single layer, the depth of flow over the weir depends only on the depth of the upstream layer, and is two thirds of that depth. For the two layer system it is shown that the depth of the layers over the weir depend not only on the depth upstream but also on the width of the crest and indirectly on the geometry of the crest and the contraction.

Some simple experiments were carried out to verify the major conclusions of this theory. The method presented should have applications in predicting flow in numerous engineering fields where more than one layer is flowing and where viscous effects are likely to be small.

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1. Introduction

The problem of the selective withdrawal of a fluid from a stably stratified reservoir is one of obvious practical importance (Brooks and Koh 1969). A particular case of this general problem involves the flow of two layers over a weir and for this case the problem is to determine for various upstream conditions the relative discharge in each layer and hence the properties of the total discharge. Where both layers in a two layered system were flowing under air and over a sharp crested weir the problem was examined experimentally by Schlag (1959). In this case the curvature of the streamlines over the weir was important and no simple theory could be developed. When, however, the weir is round crested and the flow is gradually varied the theory used by Wood (1968, 1970) may be used to obtain a complete solution. In obtaining this solution it has been assumed that the fluid flows from a reservoir through a smoothly contracting channel in which there is a definite minimum width and that the crest of the round crested weir is at this minimum width. Under these conditions it is reasonable to make the hydrostatic approximation and to use arguments that are extensions of the one dimensional ones used in open channel hydraulics.

The fluid is considered as inviscid and only steady flows are considered. This latter restriction is of minor importance provided that the reservoir is sufficiently large so that the time for a particle to travel through the contraction and over the weir is short compared to the time for the streamline patterns to change due to the withdrawal of fluid from the reservoir.

2. Theory

Consider a channel leading from an infinite reservoir as in Figure 1a. Let there be a broad crested weir in the channel as in Figure 1b. Let the datum be arbitary and let the depths and densities of the layers in the infinite reservoir be Y_o, Y_1 and Y_2 and ρ_o, $\rho_o + \Delta\rho_1$, and $\rho_o + \Delta\rho_1 + \Delta\rho_2$ respectively. Further let the height of the weir at any section x be h, and the depth of the layers at this section be y_o, y_1 and y_2. Let the width of the contraction be b (x), the discharge in each of the flowing layers be Q_1 and Q_2 and the maximum height of the weir at the section x = 0 be H.

It will be assumed that the fluid is inviscid and that the vertical curvature of all of the streamlines is sufficiently small such that the flow is gradually varied and the pressures may be taken as hydrostatic.

Then Bernoulli's equation for each of the flowing layers may be written as

$$\frac{1}{2}\frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{by_1}\right)^2 + y_1 + y_2 + h = Y_1 + Y_2 \tag{1}$$

and

$$\frac{1}{2}\frac{\rho_2}{\Delta\rho_2 g}\left(\frac{Q_2}{by_2}\right)^2 + \alpha_{12}\, y_1 + (1+\alpha_{12})(y_2+h) = \alpha_{12} Y_1 + (1+\alpha_{12}) Y_2 \tag{2}$$

where

$$\rho_1 = \rho_o + \Delta\rho_1$$

$$\rho_2 = \rho_o + \Delta\rho_1 + \Delta\rho_2$$

$$\alpha_{12} = \Delta\rho_1 / \Delta\rho_2$$

As in the previous two papers (Wood 1968 and 1970) these two equations together with the condition that $\frac{dy_1}{dx}$ and $\frac{dy_2}{dx}$ remain finite determine the final steady state flow over the weir.

(a) The Flow of the Single Layer over the Weir

Consider firstly the case where there is no flow in layer 2, (Fig. 1c). In this case we will consider that the shape of the bump (h(x)) and the total depth behind the bump ($Y_1 + Y_2$) are known. However, the individual values of Y_1 and Y_2 are not known. It is then required to determine the minimum value of Y_1 such that there is no flow in the lower layer.

Simple broad crested weir theory yields for the weir discharge (Henderson 1966)

$$Q^2 = \frac{2}{3}\frac{\Delta\rho_1}{\rho_1} g\, Y_T\, b_m^2 \left(\frac{2}{3} Y_T\right)^2$$

where $Y_T = Y_1 + Y_2 - H$

b_m = width at the top of the crest

H = the maximum height of the weir

Equation (1) then becomes

$$\frac{A}{y_1^2\, b^2} + y_1 + y_2 + h = Y_1 + Y_2 \qquad (3)$$

where $A = \frac{4}{27} Y_T^3\, b_m^2$

Downstream of the point of contact of the interface with the solid boundary of the weir (point A, Fig. 1c) we have $y_2 = 0$ and equation (3) becomes

$$\frac{A}{y_1^2 \; b^2} + y_1 + h = Y_1 + Y_2 \qquad (4)$$

Thus if we assume the point of contact of the interface with the weir occurs at a particular value of (x) then the value of y_1 (= y_c) may be computed from the known values of b, h, $Y_1 + Y_2$ and A. This value will be in the region of subcritical flow and thus only the larger of the two real solutions will be physically realistic (Henderson 1966). The smaller of the two solutions represents a supercritical flow and occurs downstream of the crest.

Upstream of the point of contact there are values of y_2 and for the lower layer we have from equation (2) with $Q_2 = 0$

$$\alpha_{12} y_1 + (1 + \alpha_{12}) (y_2 + h) = \alpha_{12} Y_1 + (1 + \alpha_{12}) Y_2 \qquad (5)$$

Solving for $(y_2 + h)$ from the above and substituting into equation (3) we get

$$(1 + \alpha_{12}) \frac{A}{y_1^2 \; b_1^2} + y_1 = Y_1 \qquad (6)$$

This equation must also hold at the point of contact and thus having computed y_c from equation (4) the appropriate value of Y_1 may be obtained by substituting $y_1 = y_c$ into equation (6) and solving. By varying the position of the point of contact of the interface with the solid surface (A of Fig. 1c) the curve of Y_1 versus x(c) can be obtained and the minimum value of Y_1 selected.

It is now proposed to show that search for the minimum value of Y_1 should only commence when $(1 + \alpha_{12}) \mathbb{F}_1^2 < 1$ where $\mathbb{F}_1^2 = \frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{by_1}\right)^2 \frac{1}{y_1}$

Differentiating equation (6) with respect to x we get

$$\frac{dy_1}{dx} = \frac{(1+\alpha_{12})\mathbb{F}_1^2 \frac{y_1}{b}\frac{db}{dx}}{1-(1+\alpha_{12})\mathbb{F}_1^2}$$

and thus from (5) $$\frac{d(y_2 + h)}{dx} = \frac{-\alpha_{12}\mathbb{F}_1^2 \frac{y_1}{b}\frac{db}{dx}}{(1-(1+\alpha_{12})\mathbb{F}_1^2)} \qquad (7)$$

Since it is required that $\frac{d(y_2 + h)}{dx}$ be of the opposite sign to $\frac{db}{dx}$ it is apparent that at the point of contact

$$(1 + \alpha_{12}) \mathbb{F}_1^2 < 1 \qquad (8)$$

and the search for the minimum value of Y_1 should start only after this requirement is satisfied. A computer program was written to obtain the minimum of Y_1 for a range of values of Y_T for weirs of variable geometry and the result is plotted in Fig. (2).

(b) The Case of Both Layers flowing over the Crest

For this case the method used by Wood (1968, 1970) is followed and the conditions that $\frac{dy_1}{dx}$ and $\frac{dy_2}{dx}$ remain finite for all x are determined.

Defining $y_1' = \frac{y_1}{Y_1}$, $y_2' = \frac{y_2}{Y_1}$, $Y_2' = \frac{Y_2}{Y_1}$, $h' = \frac{h}{Y_1}$, and $H' = \frac{H}{Y_1}$

and substituting into equations (1) and (2) we get

$$\frac{1}{2}\frac{\rho_1}{\Delta\rho_1 g}\left(\frac{Q_1}{by_1}\right)^2 \frac{1}{Y_1} + y_1' + y_2' + h' = 1 + Y_2' \qquad (9)$$

and

$$\frac{1}{2}\frac{\rho_2}{\Delta\rho_2 g}\left(\frac{Q_2}{b y_2}\right)^2 \frac{1}{Y_1} + \alpha_{12}\, y_1' + (1+\alpha_{12})(y_2' + h') = \alpha_{12} + (1+\alpha_{12})\, Y_2' \qquad (10)$$

If Q_1 and Q_2 and α_{12} and the conditions at infinity Y_2' are known these two equations determine y_1' and y_2' in terms of b at every x. Two further equations are required to solve for Q_1 and Q_2. These equations come from the condition that as b varies from its large value at $x = -\infty$ through its minimum at $x = 0$ to a large value at $x = +\infty$ it is required that the depths of both layers continuously decrease from their values in the reservoir (Y_1 and Y_2) at $x = -\infty$ to very small values at $x = +\infty$. These conditions determine a possible flow. To obtain this flow it is necessary to obtain the conditions for which $\frac{dy_1'}{dx}$ and $\frac{dy_2'}{dx}$ are always finite.

Differentiating (9) and (10) with respect to x and solving for $\frac{dy_1'}{dx}$ and $\frac{dy_2'}{dx}$ we get

$$\frac{dy_1'}{dx} = \frac{1}{b}\frac{db}{dx}\frac{D_2}{D_1} \qquad (11)$$

$$\frac{dy_2'}{dx} = \frac{1}{b}\frac{db}{dx}\frac{D_3}{D_1} \qquad (12)$$

where $D_1 = (1 - \mathbb{F}_1^2)(1 + \alpha_{12} - \mathbb{F}_2^2) - \alpha_{12}$

$$D_2 = \left[-\frac{dh'}{dx}\Big/\left(\frac{1}{b}\frac{db}{dx}\right) + \mathbb{F}_1^2 y_1'\right](1 + \alpha_{12} - \mathbb{F}_2^2)$$
$$+ (1+\alpha_{12})\frac{dh'}{dx}\Big/\left(\frac{1}{b}\frac{db}{dx}\right) - \mathbb{F}_2^2 y_2'$$

$$D_3 = (1 - F_1^2)\left[F_2^2 y_2' - (1 + \alpha_{12}) \frac{dh'}{dx} \Big/ \left(\frac{1}{b}\frac{db}{dx}\right)\right]$$

$$+ \alpha_{12} \frac{dh'}{dx} \Big/ \left(\frac{1}{b}\frac{db}{dx}\right) - \alpha_{12} F_1^2 y_1'$$

and $$F_1^2 = \frac{\rho_1}{\Delta\rho_1 g} \frac{Q_1^2}{b^2 y_1^3}$$

and $$F_2^2 = \frac{\rho_2}{\Delta\rho_2 g} \frac{Q_2^2}{b^2 y_2^3}$$

Now at the crest of the weir $\frac{db}{dx}$ and $\frac{dh'}{dx}$ equal zero and hence from equations (11) and (12) we have $\frac{dy_1'}{dx}$ and $\frac{dy_2'}{dx}$ equal zero or $D_1 = 0$. We require the interfaces to be continuously sloping and can therefore exclude the case where the slopes are zero. Further, if at any other point $D_1 = 0$ then to obtain finite values of $\frac{dy_1'}{dx}$ and $\frac{dy_2'}{dx}$ then D_2 and D_3 must equal zero.

It can be shown that the condition D_2 and D_3 equal zero together implies that D_1 also equals zero. It is these two conditions which ultimately enable the relationships between Q_1, Q_2, y_1' and y_2' to be obtained.

The condition that $D_1 = 0$ at the minimum width (where $\frac{db}{dx} = 0$ and $\frac{dh'}{dx} = 0$) gives us three equations at this point [(9), (10) and $D_1 = 0$]. However, a fourth equation is required and it is therefore necessary to examine the variation of D_1 with x.

In the reservoir where $x = -\infty$, $y_2' = Y_2'$ and $\mathbb{F}_1^2$ and $\mathbb{F}_2^2$ tend to zero. Hence D_1 $[(1-\mathbb{F}_1^2)(1+\alpha_{12}-\mathbb{F}_2^2)-\alpha_{12}]$ tends to one. Similarly when $x = +\infty$, $\mathbb{F}_1^2$ and $\mathbb{F}_2^2$ tend to large values and D_1 tends to a large positive value. It can also be shown that D_1 has only one turning point and thus the graph of D_1 versus x is as in Figure (3) and the equation

$$D_1 = 0 \tag{13}$$

holds at the position of minimum width and at some other point upstream of the position of minimum width. This second point is called the point of virtual control (Wood 1968). In order that $\frac{dy_1'}{dx}$ and $\frac{dy_2'}{dx}$ remain finite at this point, then from equations (11) and (12)

$$D_2 = 0 \tag{14}$$

and

$$D_3 = 0 \tag{15}$$

Thus at the section of the virtual control we have four equations (9, 10, 14 and 15) and if the height h' at the position of virtual control is known these equations may be solved for the four unknowns $v_1 = \frac{Q_1}{by_1}$, $v_2 = \frac{Q_2}{by_2}$, y_1' and y_2'. Up to this stage the position of the datum has not been defined. It is convenient to define the datum as a horizontal line through the point of virtual control and to measure all the depths in the reservoir from this line. It also simplifies the expressions for D_2 and D_3 if the bump shape is defined by

$$\frac{dh}{dx} = -\gamma bm \frac{1}{b}\frac{db}{dx} \tag{16}$$

where γ is a constant which defines the shape of the bump as

$$\frac{H-h}{bm} = \gamma \log_e \frac{b}{bm} \tag{17}$$

It also greatly simplifies the algebra if the following variables are used:-

$$C^2 = \frac{\rho_2}{\rho_1}\frac{V_2^2}{V_1^2}, \quad \phi = \frac{\rho_1}{2\Delta\rho_1 g}\frac{V_1^2}{Y_1}, \quad y_1' \text{ and } y_2'$$

Using these variables the Bernoulli equations (1) and (2) become

$$\phi + y_1' + y_2' + h' = 1 + Y_2' \tag{18}$$

and

$$\alpha_{12}\phi C^2 + \alpha_{12} y_1' + (1+\alpha_{12})(y_2' + h') = \alpha_{12} + (1+\alpha_{12})Y_2' \tag{19}$$

These equations are solved for y_1' and $y_2' + h'$. Then noting that $h'=0$ at the virtual control we obtain from $D_2 = 0$ (Equation 14).

$$\phi = \frac{-[Y_2'(1+\alpha_{12}-\alpha_{12}C^2) - \alpha_{12}\beta C^2]}{\alpha_{12}[\alpha_{12}C^4 + C^2(-3-2\alpha_{12}) + (1+\alpha_{12})]} \tag{20}$$

where $\beta = \dfrac{\gamma bm}{Y_1}$

and from $D_3 = 0$ (equation 15)

$$\phi = \frac{-[(\alpha_{12}C^2 - \alpha_{12}) - 0.5\beta(3+3\alpha_{12}-\alpha_{12}C^2)]}{\alpha_{12}[\alpha_{12}C^4 + C^2(-3-2\alpha_{12}) + (1+\alpha_{12})]} - \frac{\beta}{2\alpha_{12}\phi[\alpha_{12}C^4 + C^2(-3-2\alpha_{12}) + (1+\alpha_{12})]} \tag{21}$$

When equations (20) and (21) are satisfied the equation (13) ($D_1 = 0$) is also satisfied. Equations (20) and (21) lead to the following quadratic for C^2

$$A c^4 + B c^2 + D = 0 \qquad (22)$$

where
$$A = (n - \alpha_{12})(2n - \alpha_{12}\beta)$$
$$B = 2m(n - \alpha_{12}) + 2n(m + \alpha_{12}) + \beta\{3p(n - \alpha_{12}) + \alpha_{12}(\alpha_{12} + m) - 2\alpha_{12} m\}$$
$$D = (m + \alpha_{12})(2m + \beta p) + 2\beta mp$$

and
$$m = (1 + \alpha_{12})Y_2'$$
$$n = -\alpha_{12}(\beta + Y_2')$$
$$p = 1 + \alpha_{12}$$

This equation is solved for c^2 and it is apparent that two values of c^2 will satisfy all the conditions at the virtual control and it remains to determine which of the pair of values satisfies the particular flow situation.

Consider the plot of $D_1 = (1 - \mathbb{F}_1^2)(1 + \alpha_{12} - \mathbb{F}_2^2) - \alpha_{12}$ for the case of $\alpha_{12} = 1$ (Figure 4). The curves FG and HI represents $D_1 = 0$ and between the curves D_1 is negative. The point A $(1 + \alpha_{12}, 1)$ represents the position in the reservoir and the point E where $(1 - \mathbb{F}_1^2)$ and $(1 + \alpha_{12} - \mathbb{F}_2^2)$ are both large and negative represents a point in the channel downstream of the reservoir. Now as the surfaces from the upstream reservoir to the downstream reservoir are smooth and continuous then as we move from the upstream reservoir to the downstream channel $(1 - \mathbb{F}_1^2)$ and $(1 + \alpha_{12} - \mathbb{F}_2^2)$ vary continuously. Hence in the reservoir where the first value at which $D_1 = 0$ occurs (at B) $(1 - \mathbb{F}_1^2)$ and $(1 + \alpha_{12} - \mathbb{F}_2^2)$ are both positive. This condition enables us to select the correct value of c^2.

The remaining properties (y_1', y_2' and ϕ) at the virtual control are then calculated from equations (20), (18) and (19). Now all the properties at the virtual control are known and the ratio of the discharges in the layers can be computed from

$$\left(\frac{Q_2}{Q_1}\right)^2 = \frac{\rho_1}{\rho_2}\, c^2 \left(\frac{y_2'}{y_1'}\right)^2 \qquad (23)$$

and for each flow situation a new constant

$$\frac{\rho_2}{\rho_1}\left(\frac{Q_2}{Q_1}\right)^2 = c^2\left(\frac{y_2'}{y_1'}\right)^2 = Q_{21}^2 \qquad (24)$$

may be defined. Since the flow is steady the value of Q_{21} must be independent of x. Now y_2' and y_1' obtained from equations (18) and (19) are substituted into equation (22) and we obtain

$$\phi = \frac{Q_{21} - c\,(Y_2' - h')}{Q_{21} + \alpha_{12}\,(1 - c^2)(c + Q_{21})} \qquad (25)$$

This equation must hold at the position of minimum width (i.e. at the crest of the weir) and thus at this point equation (25), the two Bernoulli equations (18) and (19) and $D_1 = 0$ (equation (13)) hold. There are, however, five unknowns ϕ, c^2, y_1', y_2' and the difference in elevation between the crest of the weir and the position of the virtual control (h_m). The additional condition comes from having to simultaneously satisfy the equation of continuity for each layer and the relationship between the elevation of the crest of the weir above the virtual control and the width of the contraction.

The method of solution was to assume a value of c^2 at the minimum width (c_m^2) and calculate the other values of ϕ_m, y_{1m}', y_{2m}' and h_m' $\left(\frac{h_m}{Y_1}\right)$ at the minimum width to satisfy equations (9), (10), (13) and (24). The equation of continuity for layer (2) between the virtual control where the variables are y'_{2v}, ϕ_v, b_v, and c_v and the minimum width is

$$y'^2_{2v}\ \phi_v\ b_v^2\ c_v^2 = y'^2_{2m}\ \phi_m\ b_m^2\ c_m^2 \tag{26}$$

and was then used to calculate the ratio of $\frac{b_m}{b_v}$. This was then substituted into equation (17) and a new value of h_m and hence $h_m' = h_m/Y_1$ was obtained. The process was continued until both values of h_m' agreed and the solution was then complete.

The results were then converted into values where the depth was measured from the crest of the weir as in Figure 5 and all depths were written in terms of b_m the width of the contraction at the crest of the weir.

Discussion of the Results

The results for a single layer system show the effects of (1) changes in the density difference between the flowing and stationary layer and (2) the geometry of weir and bump on the depth of withdrawal. As was expected, the smaller the density difference between the flowing and stationary layer (the greater the value of α_{12}) the greater the depth from the crest to the stationary layer. The effect of the geometry of the weir crest and of the contraction was, however, surprising. Steepening the weir crest (i.e. increasing 'e' in the equation for the weir crest $\frac{H-h}{b_m} = e\left(\frac{x}{b_m}\right)^2$) and increasing the radius of curvature of the contraction (i.e. increasing 'a' in the equation defining the contraction $\left[\frac{b}{2b_m} = 1 + a\left(\frac{x}{b_m}\right)^2\right]$)

has the effect of increasing the depth from the crest to the stationary layer.

This effect of the geometry is also important for the case where both layers are flowing. For the case where $\gamma = 0$ (i.e. no bump) it was shown by Wood (1968) and Yih (1969) that y_1 and y_2 at the minimum width were both two thirds of Y_1 and Y_2 respectively. No such beautifully simple result was obtained in this case. Indeed y_1' and y_2' depend on the density differences, Y_2', and the geometry of the weir and the contraction in a most complicated manner. For large values of α_{12} the value $\frac{y_1 + y_2}{Y_1 + Y_2}$ is always close to the value of two thirds expected for a single layer. Indeed for $\gamma = 1.6$ and $\alpha_{12} = 100$ the maximum departure from the value of two thirds was only 0.007. For this case, however, the values of y_2/Y_2 depart markedly from two thirds. The results for any particular value of Y_1, Y_2 and b_m can be obtained from Figure 6 where $\frac{y_2}{Y_1 + Y_2}$ is plotted against $\frac{Y_1}{Y_1 + Y_2}$ for various values of $\frac{Y_1}{b_m}$. If $Y_1 = Y_2$ then for $\frac{Y_1}{b_m}$ of 27, $\frac{y_2}{Y_2} = 0.76$ and for $\frac{Y_1}{b_m} = 0.64$ then $\frac{y_2}{Y_2} = 1.14$. This shows that decreasing the width of weir not only decreases the discharge through contraction but also changes the ratio of the flows in the two layers.

For the case of $\gamma = 1.6$ and $\alpha_{12} = 1$ both of the values of $\frac{y_1 + y_2}{Y_1 + Y_2}$ and $\frac{y_2}{Y_1 + Y_2}$ depart markedly from the value of two thirds (Figures 7 and 8). In this case the value of $\frac{y_1 + y_2}{Y_1 + Y_2}$ tends to two thirds when $Y_1/(Y_1 + Y_2)$ tends to a small value. The result depends on $\frac{Y_1}{b_m}$ but the effect is small with the smaller values of $Y_1/(Y_1+Y_2)$ The values of $y_2/(Y_1 + Y_2)$ are less dependent on the values of Y_1/b_m than was the case for the larger values of α_{12} (compare Figures 6 and 8).

Experiments

Two sets of experiments were carried out to confirm the major features of this analysis. In both series of experiments reservoirs were formed by placing a contraction of shape $\frac{b/2}{b_m} = 1 + 0.0277 \left(\frac{x}{b_m}\right)^2$ in a flume that was eight feet long, two feet wide and one foot deep. For work with case (a) (the condition for the minimum value of the upper layer such that there is no flow in the lower layer) a weir of shape $\frac{H - h}{bm} = 0.020 \left(\frac{x}{bm}\right)^2$ was used, and for case (b) (two flowing layers) a weir of shape $(H - h)/bm = 1.60 \log_e (b/bm)$ was placed in the centre of the contraction and the two reservoirs were separated by a sliding gate in the contraction. In both cases b_m was 0.052 ft.

For the experiments the flume was partially filled with fresh water and the two coloured layers of salt water of different densities (layers 1 and 2) were slowly pumped in beneath the top layer of fresh water. (An exception to this method was case (a) where $\alpha_{12} = 200$. In this case only fresh water and salt water were used as the upper interface was between air and fresh water). In all cases circulation velocities in the fresh water (ρ_o) above the flowing layers were kept very small by having large flow areas above the level of the flowing layers.

Case (a): Determination of the minimum Y_1 for no flow in the lower layer.

After the layers had been set up in the flume the gate was removed and the flow commenced. A constant discharge was then maintained in layer (1) for some hours and the flow was observed until the flow in layer (2) ceased. The conditions at which this occurred were then recorded and it can be seen in Figure 2a that the agreement between the experimental and theoretical predictions for α_{12} ranging from one to 200 was satisfactory. As was to be expected, for a given $(Y_1 + Y_2 - H)$, the viscous effects made the depth below the weir at which the flow ceased slightly greater than the inviscid theory would predict.

Further experiments to verify the effect of geometry changes are planned once larger scale experimental equipment becomes available.

Case (b): Flow in both Layers

In this case it was proposed to check the theory by making measurements of the depths of the layers over the weir crest. In order to check the performance of the equipment a number of experiments were carried out with a single layer flowing over the weir. In this case water was pumped into the reservoir upstream of the contraction and allowed to flow over the weir in the centre of the contraction. The velocity head in the reservoir was small enough to be neglected and under these conditions the one dimensional inviscid theory predicts that the depth over the weir would be 2/3 of that far upstream. In the experiments depths were measured over the weir and upstream in the reservoir and Figure 9 shows a comparison between the experimental points and theoretical line. It is to be noted that for large depths in the reservoir the experimental points were above those predicted. Experiments carried out previously with the flow of a single layer through a contraction (Wood 1970) also gave depths through the contraction that were always greater than the theoretical value of 2/3. In this case it was shown that the measured depth would be the critical depth plus one third of the displacement thickness. It is believed that it is this same effect that is causing the discrepancy in this case. From these preliminary experiments it was concluded that errors at least of the order of +5 pc. could be expected in the two layer experiments.

The two layer experiments were commenced by removing the gate and a short time after this the flow settled down and the reservoir level changes became slow. A typical experiment is illustrated in Figure 10.

Once the level changes in the reservoirs became slow the depths in the reservoirs and at the contraction were continuously measured. Values $\frac{y_1 + y_2}{Y_1 + Y_2}$ and $\frac{y_2}{Y_1 + Y_2}$ were then plotted against the values of $\frac{Y_1}{Y_1 + Y_2}$. Theoretical values of $\frac{y_1 + y_2}{Y_1 + Y_2}$ and $\frac{y_2}{Y_1 + Y_2}$ were then obtained using the measured values of α_{12}, $\frac{Y_1}{Y_1 + Y_2}$ and $\frac{Y_1}{b_m}$ and Figures 7 and 8. Straight lines were then drawn through these deduced points. Typical results are shown in Figures 11 and 12.

It can be seen that the agreement between the experimental and theoretical values was reasonable. Indeed the trend in the curves of $\frac{y_1 + y_2}{Y_1 + Y_2}$ is the same as obtained for a single layer system. That is the experimental depths are greater than those deduced. As in the single layer experiments this was particularly noticeable when the depths over the weir were large.

It therefore appears that the theory is satisfactory for deducing the depths of flow of a two layer system over a weir provided the flow is gradually varied. Values of the discharge in each layer can be computed from the deduced values of $\frac{y_1 + y_2}{Y_1 + Y_2}$ and $\frac{y_1}{Y_1 + Y_2}$ (from Figures 7 and 8) and from the two Bernoulli equations (9) and (10).

Acknowledgements:

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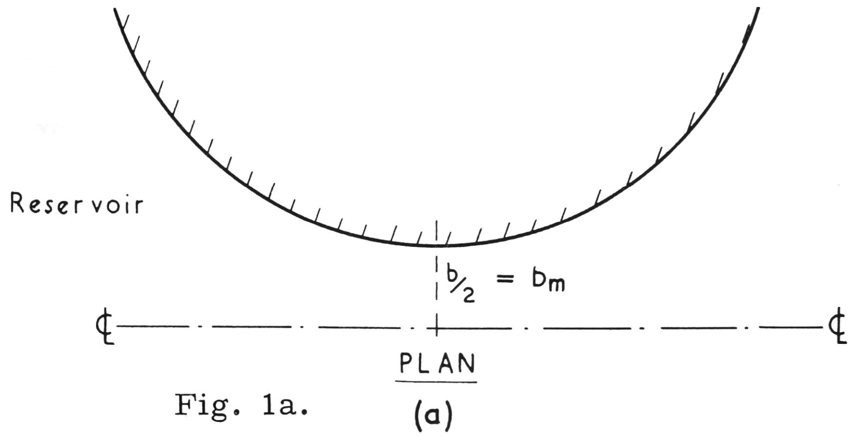


Fig. 1a.

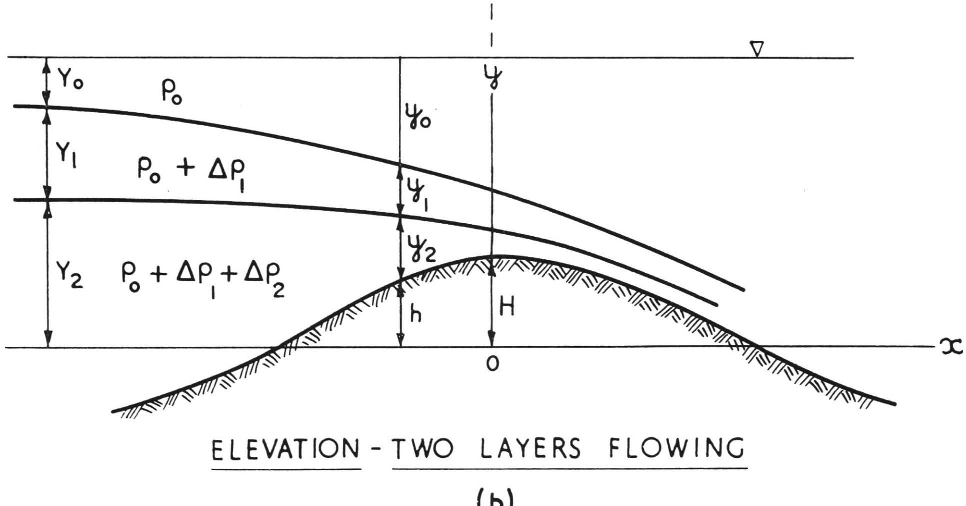


Fig. 1b.

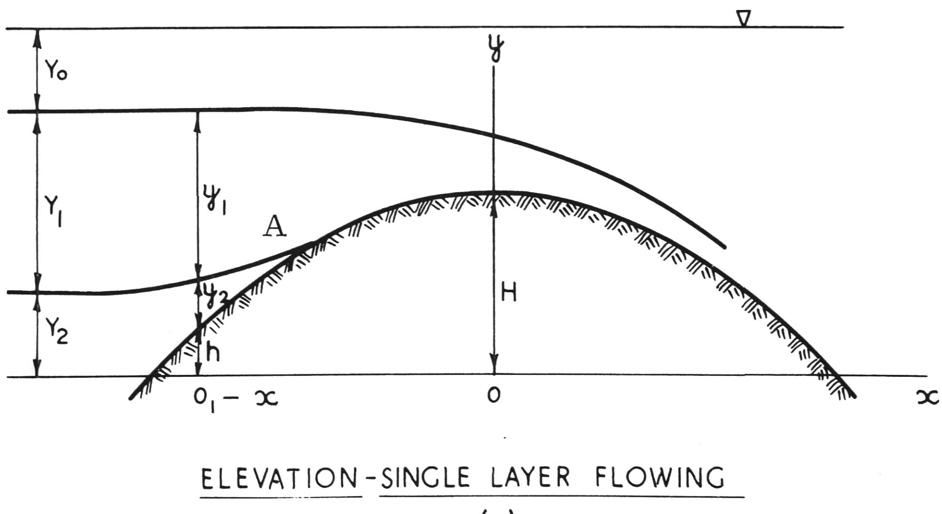
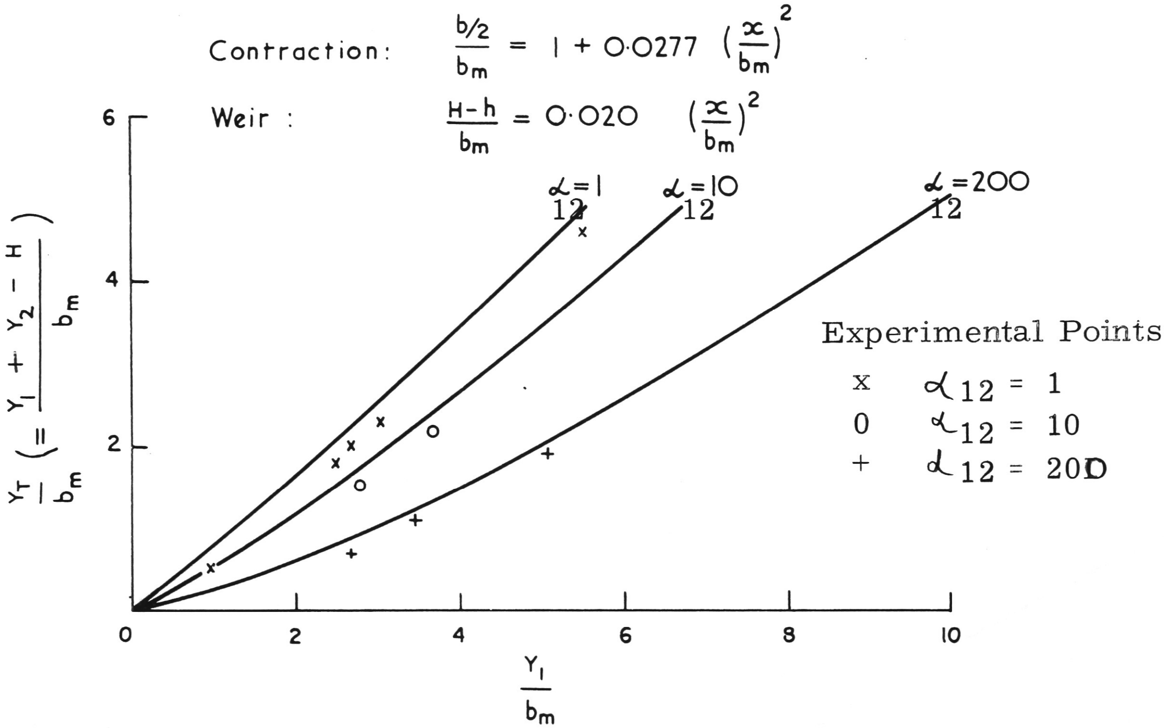
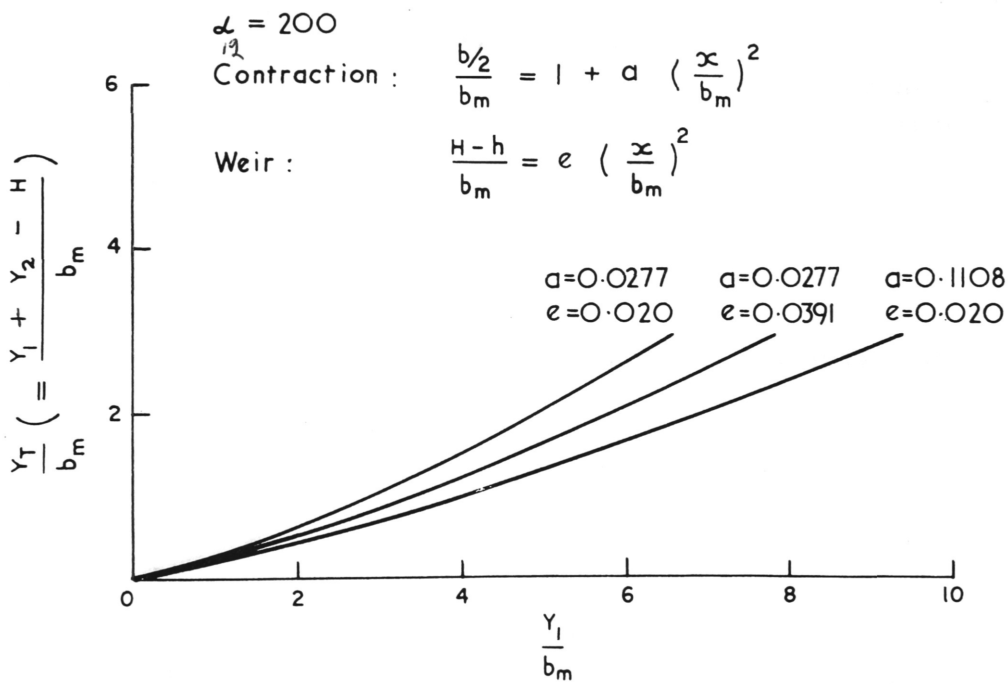


Fig. 1c.

Fig. 1: The plan and elevation of layered flows over a weir at the exit to a reservoir.



(a.) EFFECT OF α



(b.) EFFECT OF GEOMETRY

Fig. 2: The flow of a single layer over a weir.
(See list of figures for details).

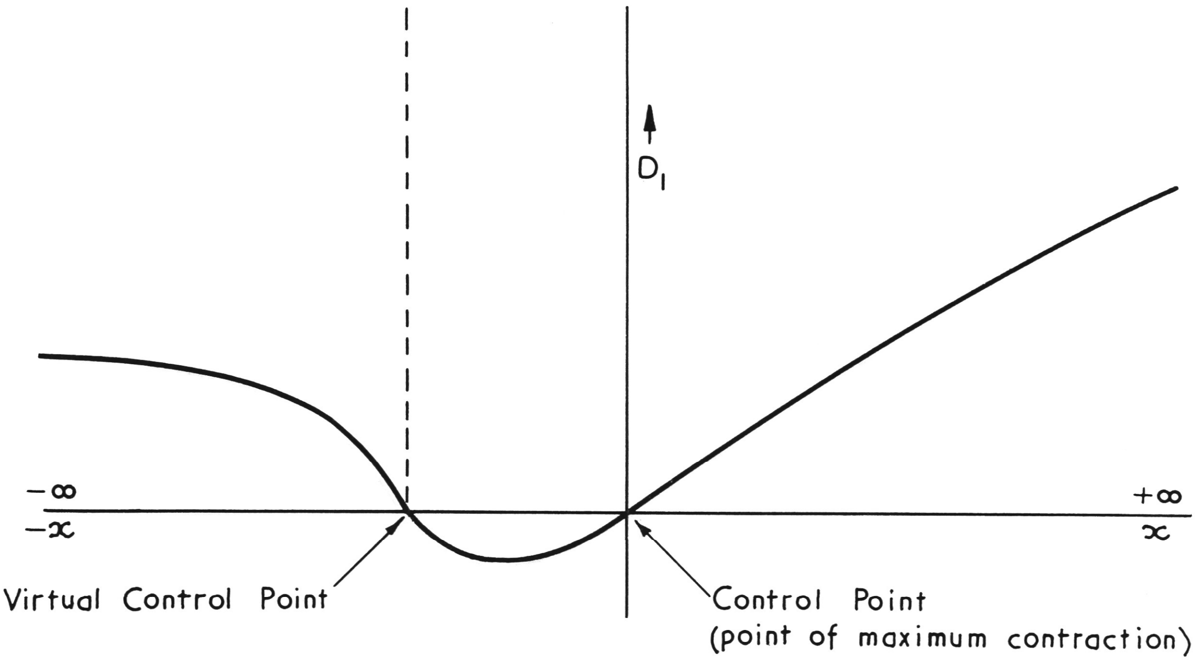


Fig. 3: The points of control in a two layer flow.

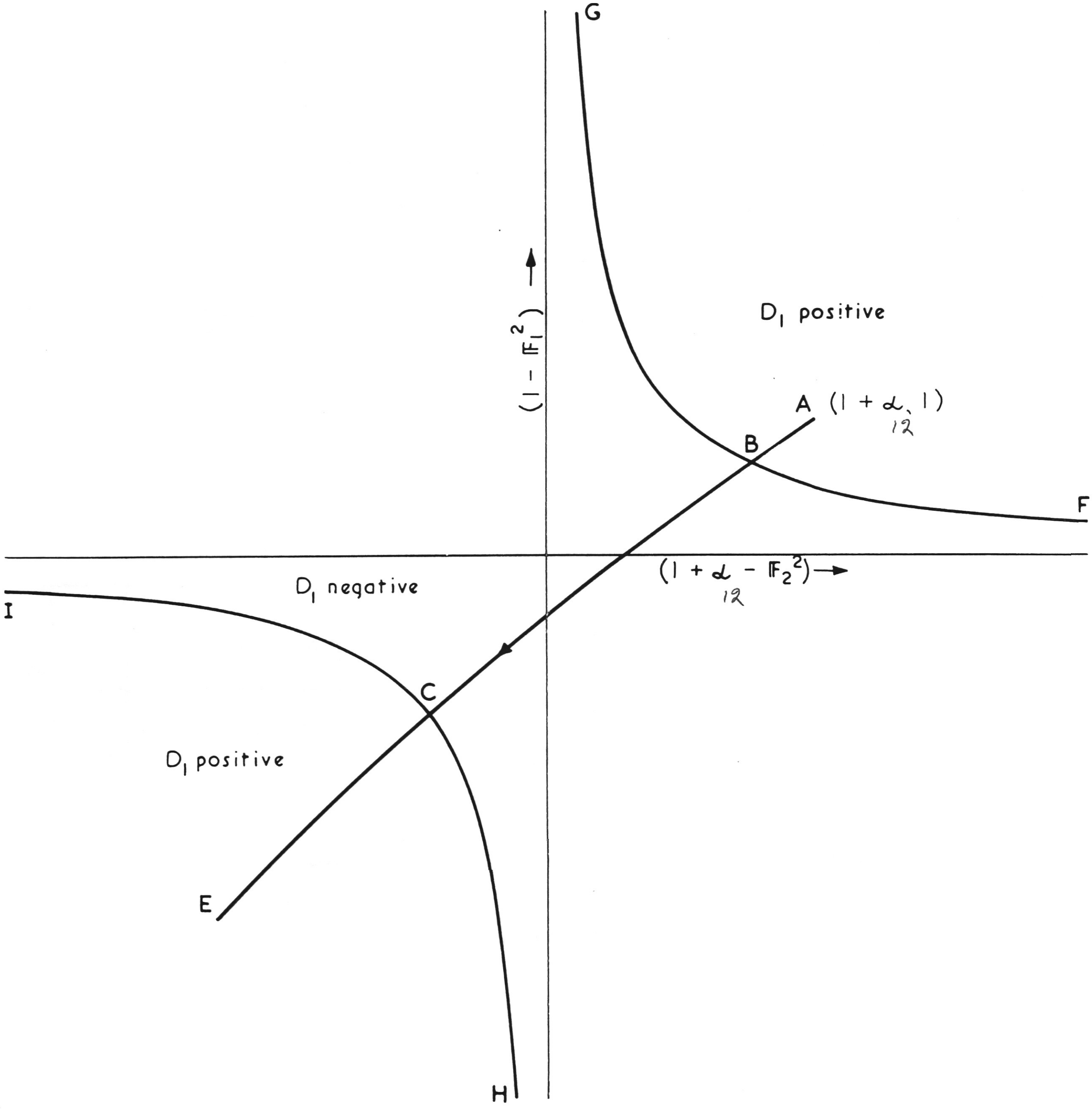


Fig.4: A plot of $(1 - \mathbb{F}_1^{\,2})(1 + \alpha_{12} - \mathbb{F}_2^{\,2}) - \alpha_{12} = D_1$

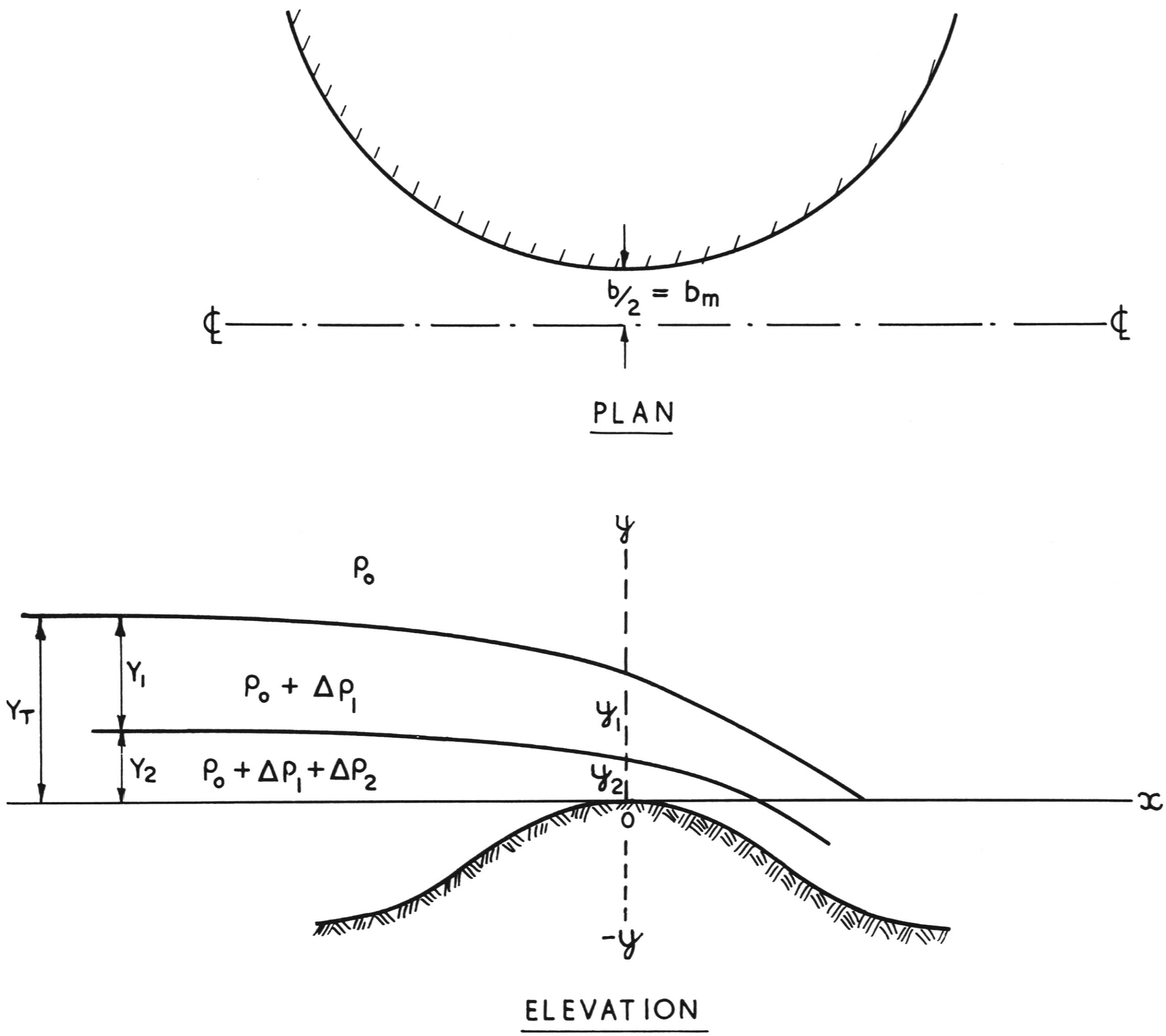


Fig. 5: The nomenclature used in plotting the results for the two layered flow.

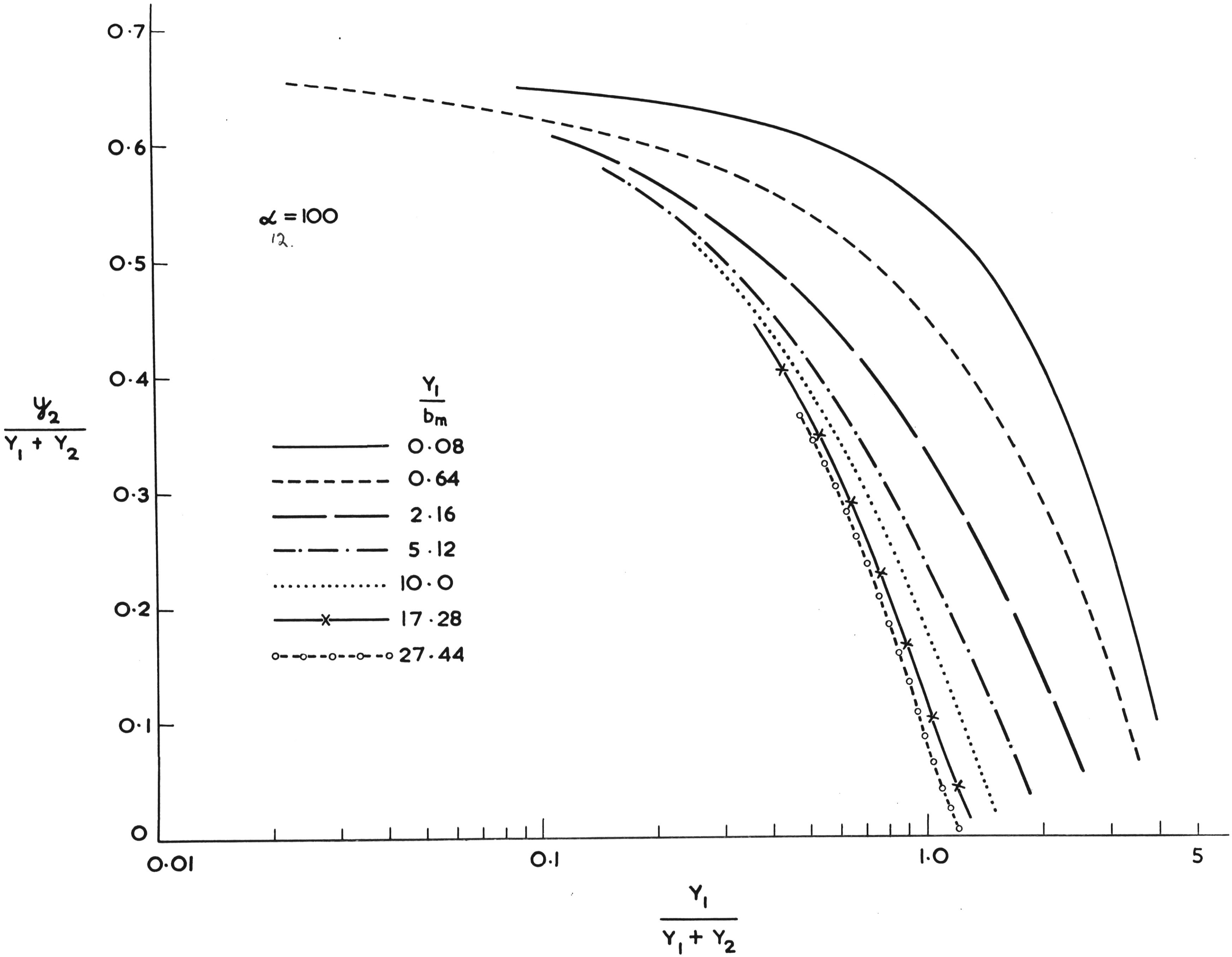


Fig. 6: The two layered flow for $\alpha_{12} = 100$. A plot of $y_2/(Y_1 + Y_2)$ at the minimum width against $Y_1/(Y_1 + Y_2)$ for a range of Y_1/b_m.

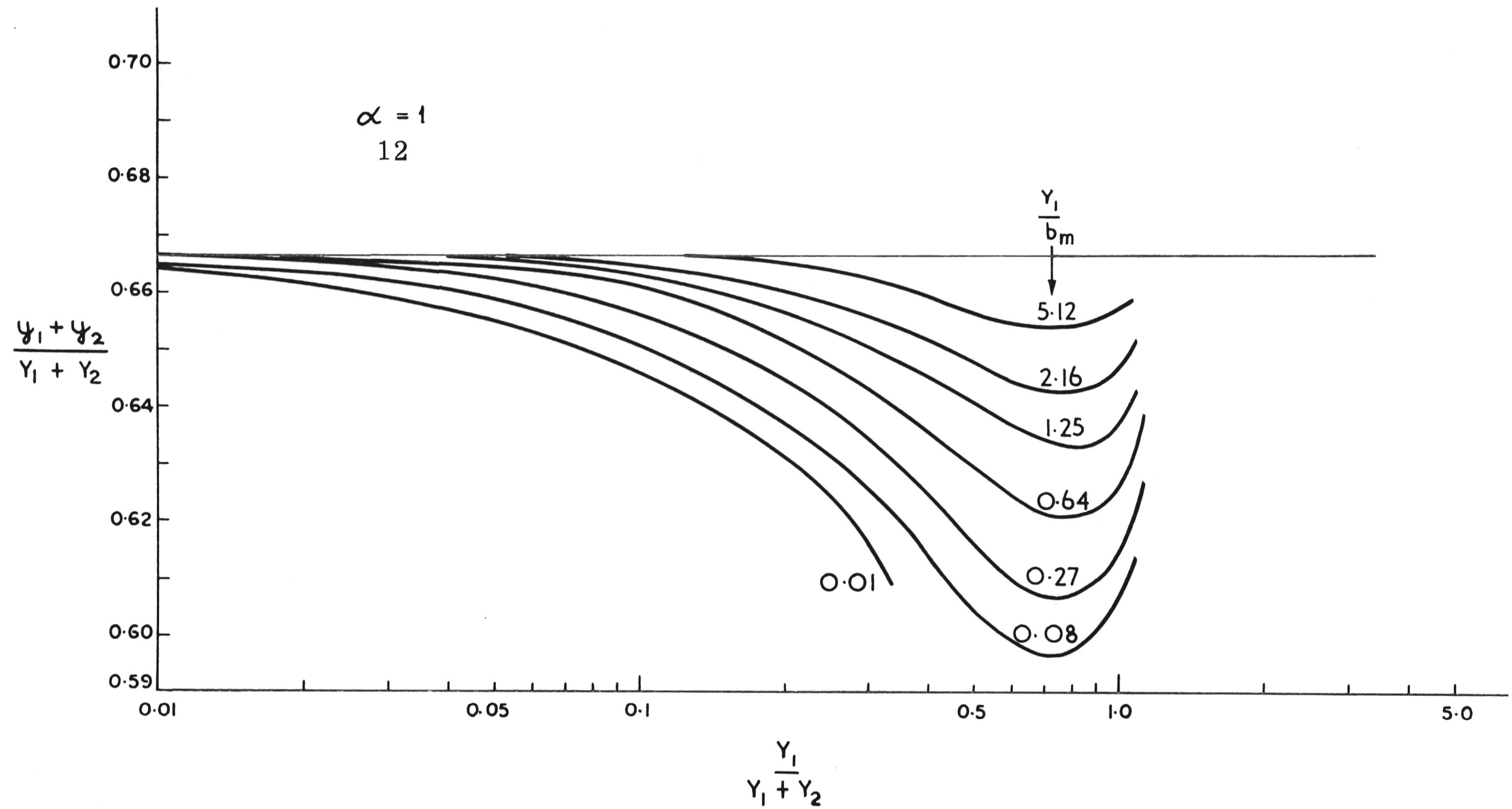


Fig. 7: The two layered flow for $\alpha_{12} = 1$. A plot of $(y_1 + y_2)/(Y_1 + Y_2)$ at the minimum width against $Y_1/(Y_1 + Y_2)$ for a range of Y_1/b_m.

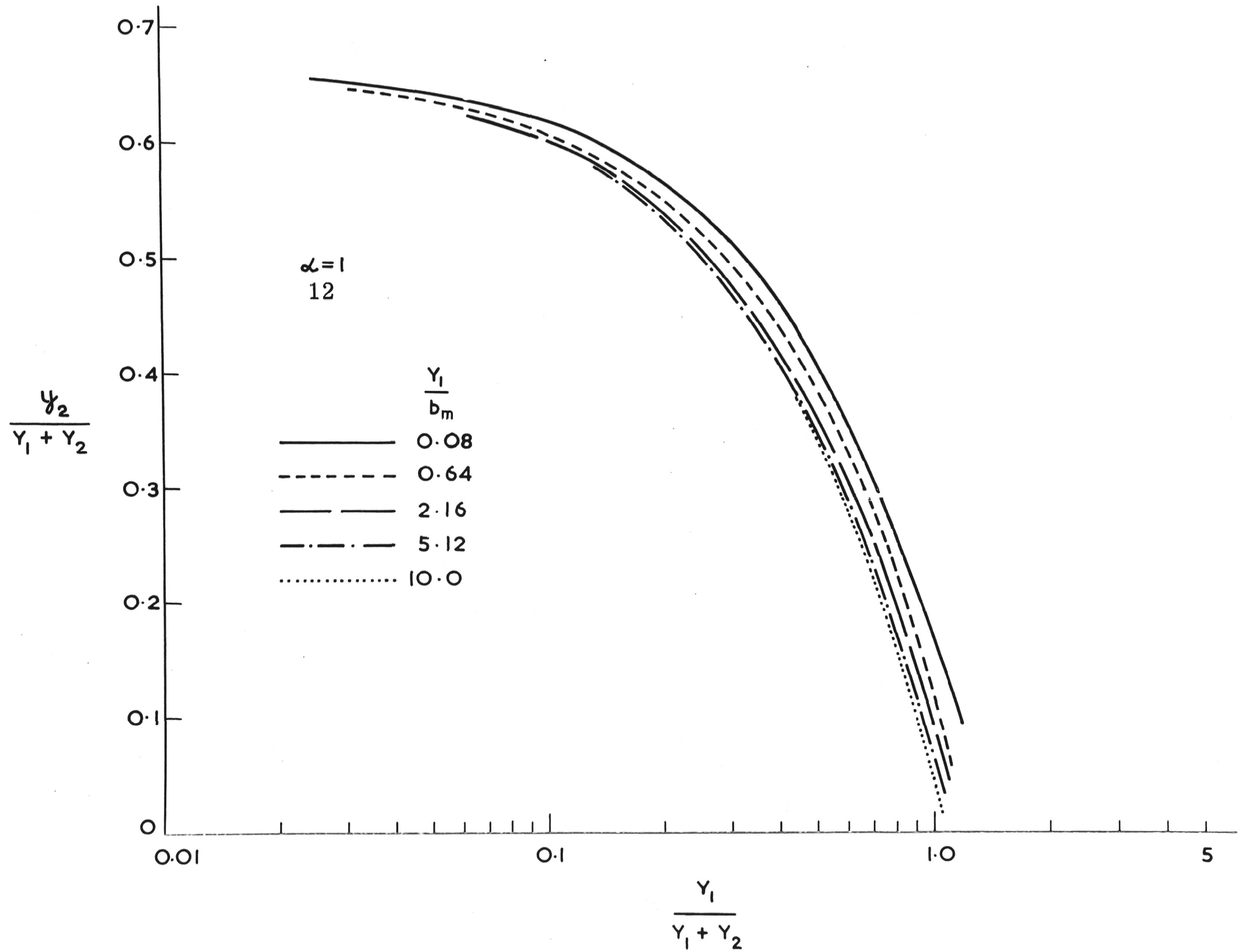


Fig. 8: The two layered flow for $\alpha_{12} = 1$. A plot of $y_1/(Y_1 + Y_2)$ at the minimum width against $Y_1/(Y_1 + Y_2)$ for a range of Y_1/b_m.

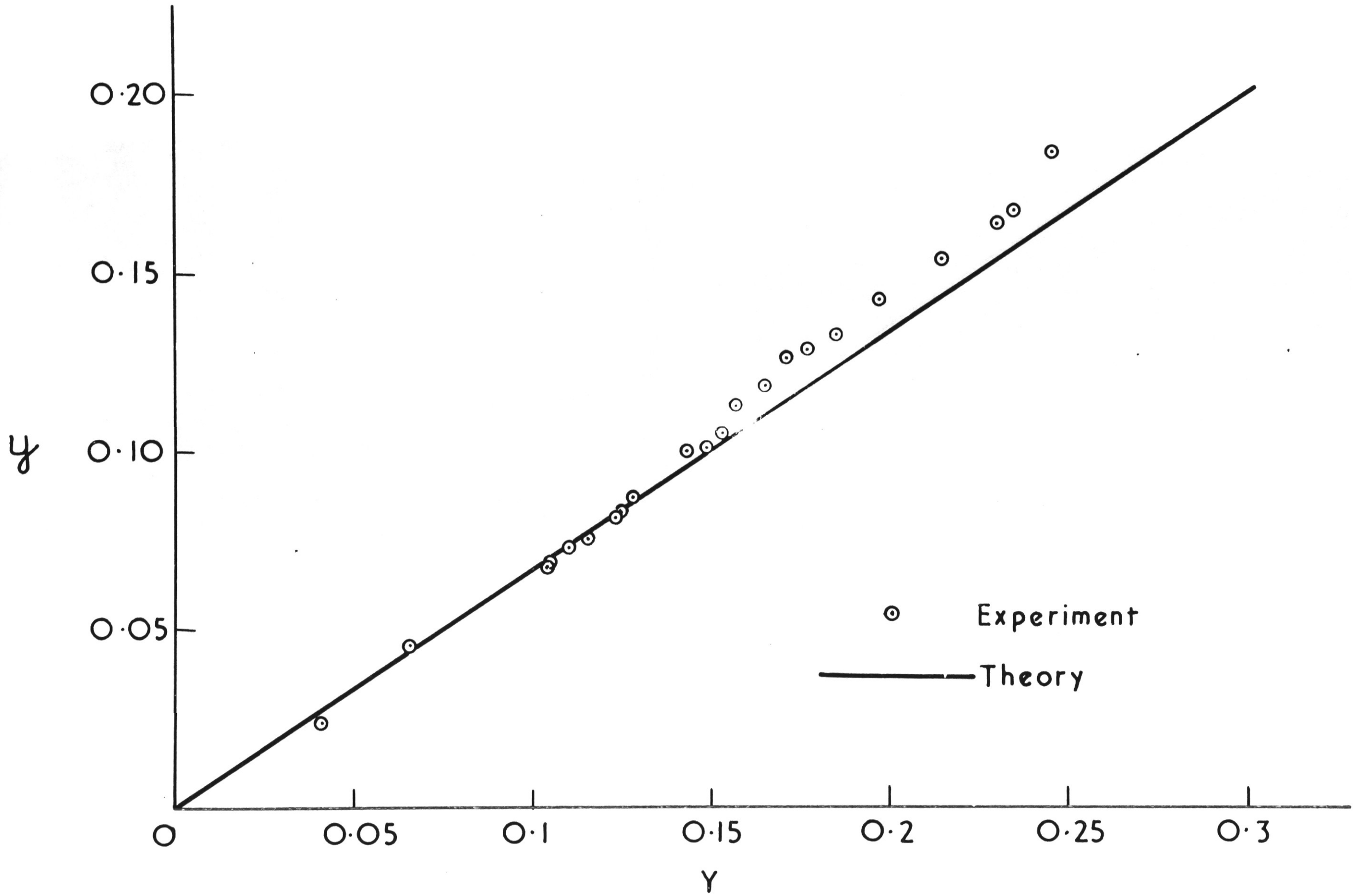


Fig. 9: The results for a single layer flowing over a weir.

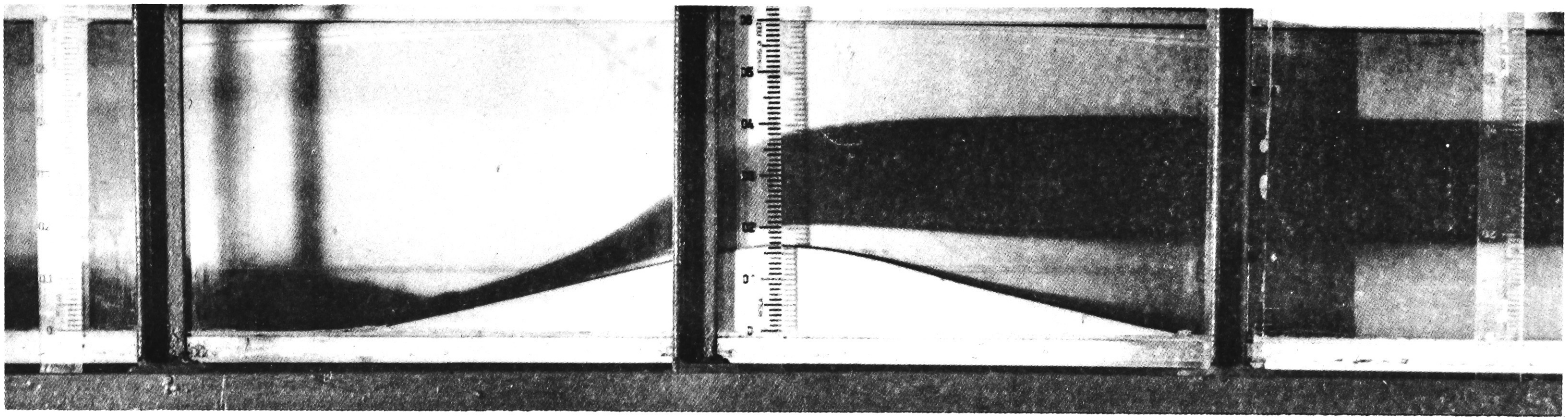


Fig. 10: A typical two layered experiment.

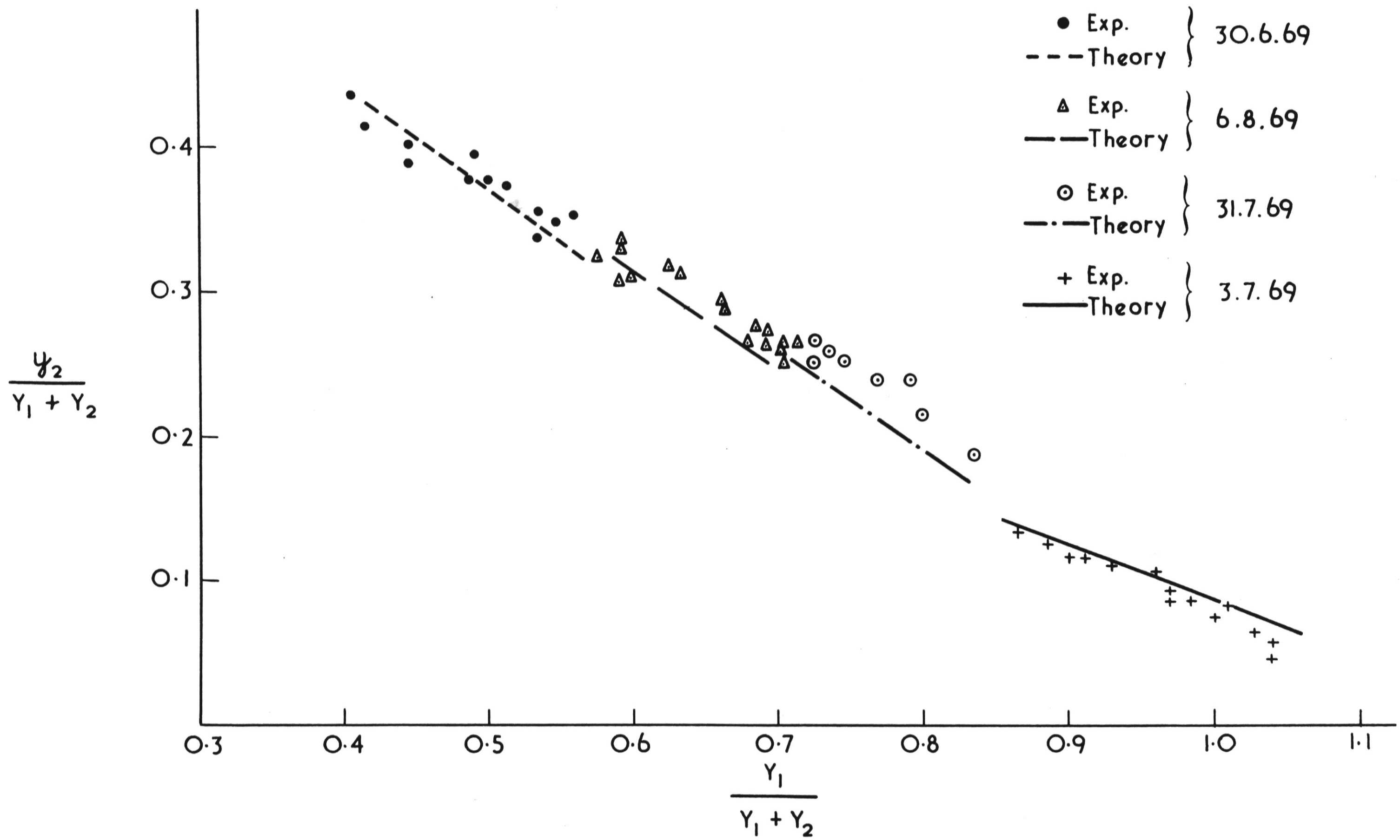


Fig. 11: The two layered flow. Typical experimental results. A comparison of the predicted results for $y_2/(Y_1 + Y_2)$ and the experimental results.

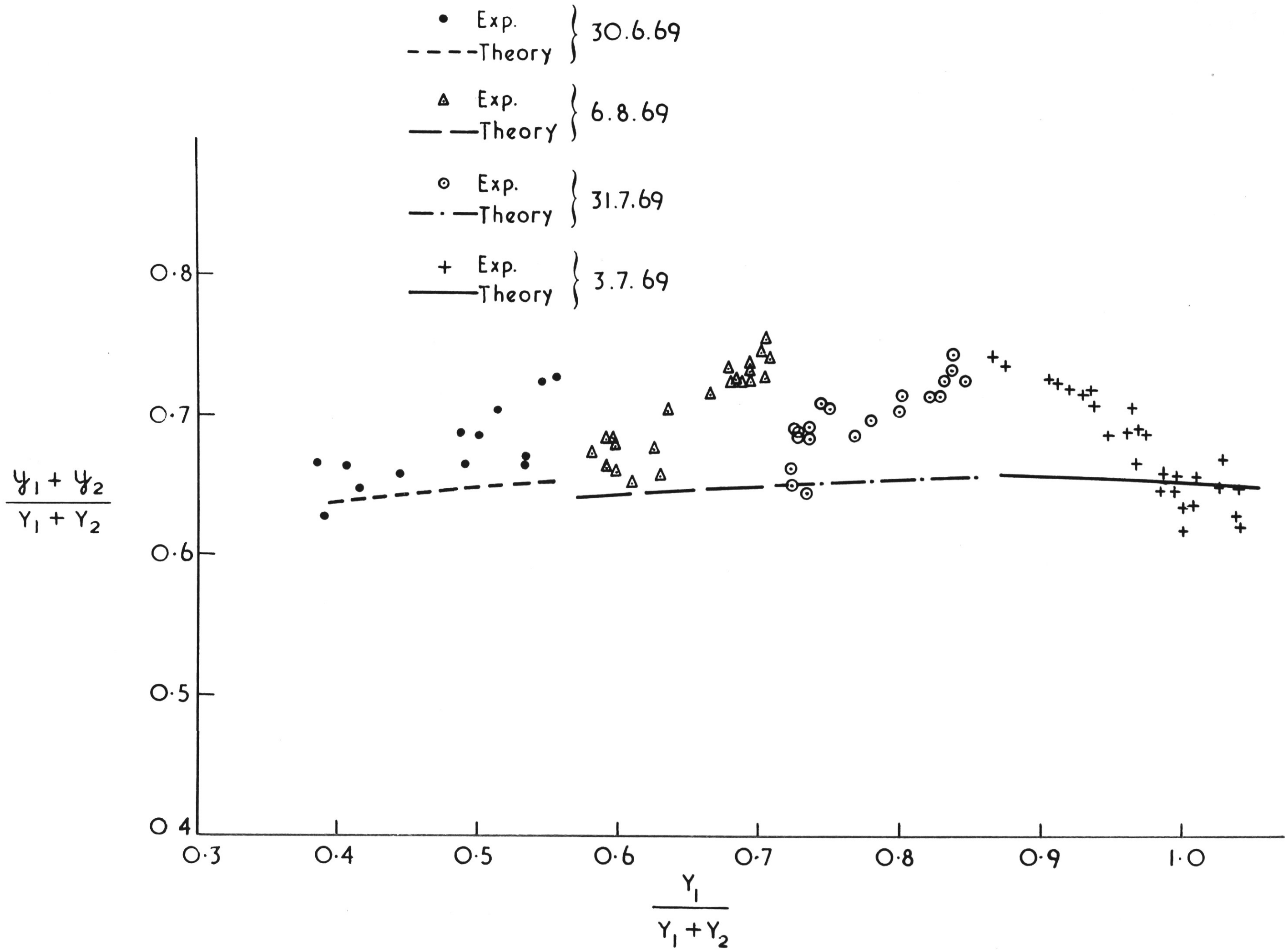


Fig. 12: The two layered flow. Typical experimental results. A comparison of the predicted results for $(y_1 + y_2)/(Y_1 + Y_2)$ and the experimental results.