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Author:
Wang, Jun

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Predictive depth of jet penetration models for abrasive waterjet cutting of alumina ceramics

J. Wang*
School of Mechanical and Manufacturing Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

Abstract
A study of the depth of jet penetration (or depth of cut) in abrasive waterjet (AWJ) cutting of alumina ceramics with controlled nozzle oscillation is presented and discussed. An experimental investigation is carried out first to study the effects of nozzle oscillation at small angles on the depth of cut under different combinations of process parameters. Based on the test conditions, it is found that nozzle oscillation at small angles can improve the depth of cut by as much as 82% if the cutting parameters are correctly selected. Depending on the other cutting parameters in this study, it is found that a high oscillation frequency (10-14 Hz) with a low oscillation angle (4-6^{o}) can maximize the depth of cut. Using a dimensional analysis technique, predictive models for jet penetration when cutting alumina ceramics with and without nozzle oscillation are finally developed and verified. It is found that the model predictions are in good agreement with the experimental results with the average percentage errors of less than 2.5%.

Keywords: abrasive waterjet cutting, nozzle oscillation, depth of jet penetration, machining

Nomenclature

A, B's, C's, j's, K_1, K_2, a, b, c, d, e, x, y, z	constants
D	average particle diameter (mm)
d_j	nozzle diameter (mm)
E	modulus of elasticity (MPa)
F	oscillation frequency (Hz)
H	depth of jet penetration (mm)
H_d	material dynamic hardness (MPa)
h_1	depth of jet penetration in normal cutting (mm)
h_2	depth of jet penetration in oscillation cutting (mm)
H	standoff distance (mm)
k_e	particle impingement efficiency
k_m	momentum transfer efficiency
MRR	material removal rate (mm^3/s)
m	average mass of a particle (g)
m_a	abrasive mass flow rate (g/s)
m_w	water mass flow rate (g/s)
P	water pressure (MPa)
R	average volume of material removed by a particle (mm^3)
u	nozzle traverse speed (mm/s)
v	particle velocity (m/s)
v_j	waterjet velocity (m/s)
w	average kerf width (mm)

* Tel: +61-2-9385 5784; Fax: +61-2-9663 1222.
Email: jun.wang@unsw.edu.au (J. Wang).

α	particle attack angle (degrees)
α_1	average particle attack angle in normal cutting (degrees)
α_2	average particle attack angle in oscillation cutting (degrees)
σ_f	material flow stress (MPa)
θ	oscillation angle (degrees)
ρ_w	water density (g/mm^3)

1. Introduction

As an advanced manufacturing technology, abrasive waterjet (AWJ) cutting is being increasingly used in various industries. In the last decades, a large amount of research effort has been made to understand the process and improve its cutting performance such as the depth of cut [1,2,3]. However, the cutting capacity of this technology in terms of depth of cut (or depth of jet penetration) and kerf quality is still the major obstruction that limits its applications. Considerable research and development effort has been made in recent years to develop new techniques to enhance the cutting performance of this technology such as the depth of cut and surface finish. Some newly developed techniques include cutting with forward angling the jet in the cutting plane, multipass cutting and controlled nozzle oscillation [4,5,6]. Among these new techniques, controlled nozzle or cutting head oscillation has been found to be one of the most effective ways in improving the cutting performance without additional costs to the process. With this cutting technique, a pendulum-like nozzle forward and backward motion in the cutting plane at predetermined frequency and angular amplitude is superimposed to the normal nozzle traverse motion, as shown in Fig. 1. It has been found that the nozzle oscillation cutting technique can significantly improve some major cutting performance measures such as the depth of cut and surface roughness. It has been reported [4,**7**,8] that the depth of the upper smooth zone in nozzle oscillation cutting can be increased by more than 30% as compared with that without oscillation, while kerf surface finish as measured by the centre line average R_a can be improved by as much as 30%.

It appears that the reported studies in controlled nozzle oscillation cutting are primarily about the use of large oscillation angles (or angular amplitudes) of 10 degrees or more. Nozzle oscillation in the cutting plane (in the direction tangential to the curved profile in contouring) with such large oscillation angles results in theoretical geometrical errors on the component profile in contouring and is therefore not preferred in practice. As a result, it is necessary to investigate if nozzle oscillation at small angles can be employed to enhance the cutting performance. Furthermore, it has been noticed in early experiments in the authors' laboratory that if the oscillation parameters were not correctly selected, nozzle oscillation could have an adverse effect on the cutting performance. Therefore, it is necessary to understand this phenomenon and develop predictive mathematical models for the major cutting performance measures, such as the depth of cut, in AWJ cutting with this novel cutting technique. Such mathematical models are essential for the development of strategies for selecting the optimum operating parameters in process planning.

<Take in Fig. 1>

This paper presents a study on the depth of cut and the associated predictive models in AWJ cutting with controlled nozzle oscillation. The analysis is based on an experimental investigation to cut an 87% alumina ceramic with nozzle oscillation at small angles. Predictive mathematical models for the depth of cut in AWJ cutting with and without nozzle oscillation are then developed using a dimensional analysis technique. The models are finally verified by comparing the model predictions with the corresponding experimental data.

2. Experimental study of the depth of jet penetration in nozzle oscillation cutting

2.1 Experimental work

In the experiment, 87% alumina ceramic plates of 12.7mm thickness were cut by a Flow International waterjet cutter driven by a "Model 20X" dual intensifier pumping system with the operating pressure of up to 380MPa. The main properties of the specimens are given in Table 1. The motion of the nozzle is numerically controlled by a computer and a five-axis robot positioning system. Four major variables in normal AWJ cutting (i.e. cutting without nozzle oscillation and at a 90^{o} jet impact angle) as identified in earlier studies [4] and two oscillation variables were chosen for investigation. These six variables include water pressure, nozzle traverse speed, abrasive mass flow rate, standoff distance between nozzle and workpiece surface, nozzle oscillation angle, and oscillation frequency. Their levels and corresponding values are shown in Table 2. The selection of these process parameters was made based on their ranges of practical applications and the machine system limitations. Small oscillation angles of less than 10^{o} were selected in order to assess the improvement in cutting performance so that the oscillation does not result in significant kerf geometrical errors. According to earlier studies and findings [4,**7**], the oscillation cutting used a 90^{o} jet impact angle as the neutral or original position while the jet oscillated in the nozzle traverse direction, as shown in Fig. 1.

The other parameters that were kept constant during the tests included the nominal jet impact angle (90^{o}), orifice diameter (0.33mm), mixing tube or nozzle diameter (1.02mm) and abrasive material (80 mesh garnet).

<Take in Table 1>
<Take in Table 2>

The Taguchi experimental design array [9] was used to construct the cutting tests. Three groups of tests were considered in the experimental design. The first group used the four-level, six-factor design scheme in Taguchi orthogonal arrays with all the six selected variables in order to study the influence of oscillation cutting on the cutting performance. This design scheme required 64 experimental runs. For comparatively studying the difference of cutting performance between oscillation cutting and normal cutting, another four-level, four-factor design scheme was used; the four cutting variables were water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. This resulted in 16 more runs. Furthermore, in order to facilitate the analysis using the as-measured experimental data, a third group was designed using some typical cutting conditions from Table 2. This group of design included 30 tests. Thus a total of 110 runs were undertaken in this experimental investigation.

All the major cutting performance measures, such as the depth of cut, surface roughness, kerf width and kerf taper, were acquired from the specimens with the assistance of metrological instruments, i.e. a SigmaScope 500 profile projector and a Surtronic 3+ stylus surface profilometer. Of theses quantities, kerf taper was calculated using the top kerf width, the minimum kerf width and the depth where the minimum kerf width was measured, while the other quantities were directly measured from each cut. At least three measures for each quantity on each cut were made and the average was taken as the final reading. A detailed analysis of the effects of the process parameters on these cutting performance measures have been reported in reference [8]. This paper pays attention to the depth of cut only.

2.2 Effects of nozzle oscillation parameters

The experimental results showed that the effects of water pressure, nozzle traverse speed, abrasive mass flow rate and stand distance on the depth of cut are the same as previously reported [4], i.e. an increase in the water pressure or abrasive mass flow rate is associated with an increase in the depth of cut, while the reverse trend applies when the nozzle traverse speed or standoff distance is increased. Since the effects of these variables on the depth of cut have been qualitatively and quantitatively studied previously [4,8], the current work focuses on the effects of nozzle oscillation and the two oscillation parameters, i.e. oscillation frequency and oscillation angle.

Nozzle oscillation has been found to affect the depth of cut significantly. The depth of cut for all the tests with nozzle oscillation shows an average increase of 27.7% with respect to the cutting without nozzle oscillation under the corresponding cutting conditions. In some extreme cases, nozzle oscillation cutting increased the depth of cut by as much as 82%. Nevertheless, there are still cases where the depth of cut in oscillation cutting is less than that of the corresponding normal cutting (without nozzle oscillation). The analysis shows that whether or not nozzle oscillation can increase the depth of cut is dependent on both oscillation angle and oscillation frequency. In general, it appears that if a small oscillation angle (e.g. 2^{o}) is used together with a small oscillation frequency (e.g. 2Hz), the depth of cut may be reduced by the nozzle oscillation process. The results also showed that a small oscillation angle (e.g. 2^{o}) with a large oscillation frequency (e.g. 10Hz) within the tested ranges can increase the depth of cut.

The result of ANOVA suggests that if a 14Hz oscillation frequency and a 6º oscillation angle are used under 380MPa water pressure, 0.67mm/s traverse speed, 3mm standoff distance, and 11.3g/s abrasive flow rate, oscillation cutting can produce the maximum depth of cut of 16.3mm within the ranges of the conditions tested. However, with the normal cutting the optimum combination of the cutting parameters within the tested ranges were found to be water pressure at 345MPa, nozzle traverse speed at 0.67mm/s, standoff distance at 2mm, and abrasive mass flow rate at 11.3g/s, which yields the maximum depth of cut of 13.3mm. The reason for 345MPa, rather than 380MPa, being the optimum pressure for normal cutting may be because of the increased particle interference and fragmentation at high water pressure under normal cutting, reducing the overall cutting efficiency. Thus, statistically nozzle oscillation cutting can increase the depth of cut by 23% with respect to the normal cutting technique under the respective optimum combinations of cutting parameters tested (and under the same jet traverse speed and abrasive mass flow rate).

The relationship between oscillation frequency and the depth of cut is plotted in Fig. 2. Fig. 2(a) shows that the depth of cut increases approximately linearly with the oscillation frequency. It is believed that a higher oscillation frequency increases the number of repeated scanning actions and reduces the particle interference, which in turn increases the overall abrasive cutting capacity and the depth of cut. Under some conditions, an increase in oscillation frequency may result in a slight decrease in the depth of cut, as shown in Fig. 2(b) when u=0.67mm/s. This figure also shows that the slope of the linear relationship between oscillation frequency and depth of cut is affected by nozzle traverse speed. A higher nozzle traverse speed is associated with a more rapid increase of the depth of cut as the oscillation frequency increases. When a low traverse speed of 0.67mm/s is used, an increase in oscillation frequency in fact results in a decrease in the depth of cut. This may be explained that at low traverse speed, the jet scanning action cannot take effect and may cause increased particle interference and a reduction in the jet cutting capability.

<Take in Fig. 2>

Fig. 3 shows the effect of oscillation angle on the depth of cut from the experimental data. This effect is also dependent on the oscillation frequency. At relatively large oscillation frequencies (e.g. 10Hz and 14Hz), an increase in oscillation angle is associated with a steady decrease of the depth of cut while the decreasing rate slightly increases with the oscillation angle. This may be due to the fact that at high oscillation frequencies, an increase in oscillation angle increases the jet instability which decreases the jet cutting capability and hence decrease this cutting performance measure.

By contrast, at low oscillation frequencies such as 2 Hz as shown in Fig. 3(a), an increase in oscillation angle results in a slight increase in the depth of cut, while the increasing rate decreases with the oscillation angle. As the oscillation angle further increase to beyond 6º, the depth of cut exhibits a decreasing trend. A maximum turning point for the depth of cut occurs at about 4º to 6º of oscillation angle when small oscillation frequencies are used.

<Take in Fig. 3>

For the other oscillation frequencies in the medium range of the tested conditions, the depth of cut appears to be somehow independent of the oscillation angles with only very slight decrease as this cutting variable increases, as shown in Fig. 3(b) at an oscillation frequency of 6 Hz.

To this end, large oscillation frequencies (10-14Hz) with small oscillation angles (4-6^{o}) are preferred to increase the depth of cut based on this experimental study. Fig. 3(b) also shows that standoff distance has an effect on the depth of cut. While an increase in the standoff distance may reduce the particle energy at the point of particles attacking the material, it also affects the scope of oscillations. With a larger standoff distance, the jet scanning scope on the cutting front is increased, which may increase or decrease the depth of cut depending on the other parameters used, in a similar way to oscillation angle.

2.3 Concluding remarks

The experimental investigation on AWJ cutting of an 87% alumina ceramic has shown that nozzle oscillation at small angles can also enhance the depth of cut as compared to oscillation at large angles of more than 10^{o}. It has been found that if the cutting parameters are not selected properly, nozzle oscillation cutting can reduce this major cutting performance measure. If the oscillation parameters are correctly selected, nozzle oscillation cutting at small oscillation angles can increase the depth of cut by as much as 82%. When the optimum cutting parameters are used for both nozzle oscillation and normal cutting, statistically the former can increase the depth of cut by 23%. Depending on the other cutting parameters, the optimum depth of cut can be achieved at a high oscillation frequency (10-14Hz) together with a small oscillation angle (4-6^{o}) based on the test conditions. In order to quantitatively predict the depth of cut for process planning and optimization, predictive mathematical models are required and those are developed below for cutting 87% alumina ceramics with nozzle oscillation and without nozzle oscillation.

3. Predictive depth of jet penetration models

The development of the depth of cut (or jet penetration) models for AWJ cutting with and without nozzle oscillation involves the consideration of a host of operating variables. This makes the modelling process extremely difficult. Furthermore, there are a number of phenomena associated with AWJ cutting, such as particle interference and fragmentation, that have not yet been well understood and there are no mathematical models to represent these phenomena [4,10]. As a result, to theoretically model the depth of jet penetration is either not possible at this stage of development or results in very complicated equations with many unknown parameters, making the model unrealistic for practical use [4]. By contrast, dimensional analysis [11,12] is a powerful analytical technique in describing the relationship between physical engineering quantities (such as the depth of jet penetration) and independent variables. It will be used in this study to develop mathematical equations for the depth of cut in terms of process variables, while the constants in the equations will be determined from experiments.

With dimensional analysis [11,12], all variables appearing in a problem can be assembled into a smaller number of independent dimensionless products or *Pi* (π_i) groups. The dimensional homogeneity requires that all terms in a mathematical relationship must have the same dimensions regardless of the choice of units for each variable. For this purpose, the Pi theorem can be used to find the proper dimensionless products. The relationships connecting individual variables can be determined by algebraic expressions relating each π_i group, thus reducing the total number of variables. Several assumptions have to be made in order to develop the models. These include:

(1) Abrasive particles are distributed uniformly over any jet cross-sectional area.
(2) The velocity of an abrasive particle is the same as that of its surrounding water in the jet, and the jet velocity variation along the jet flow direction is ignored, and
(3) Kerf width is considered to be approximately equal to the jet diameter.

The underlying premise in the construction of the depth of cut models is that the overall material removal rate is equal to the accumulated volume of material removed by individual abrasive particles in the given time span [8,13]. If assuming that the depth of cut is h, the nozzle traverse speed is u and the average kerf width is w, the overall material removal rate (MRR) can be given by

$$MRR = u\,h\,w \tag{1}$$

If the average contribution of a particle to the material removal is R, the overall material removal rate can be re-expressed as

$$MRR = u\,h\,w = \frac{m_a}{m} R \tag{2}$$

where m_a is the abrasive mass flow rate, m is the average mass of an abrasive particle. In AWJ cutting, not all particles in the jet will impinge the material or have sufficient energy to cut the target material. Some particles may collide with other particles and are not involved in the cutting action. To consider this phenomenon, an efficiency factor, K_e, may be introduced. Thus, if assuming that the average kerf width, w, is approximately equal to the jet diameter which is in turn equal to the nozzle diameter, d_j, Eq. (2) becomes

$$MRR = u\,h\,d_j = K_e \frac{m_a}{m} R \tag{3}$$

In Eq. (3), k_e will need to be determined from experimental data along with other constants, and R is a parameter yet to be determined and related to its influencing variables by using a dimensional analysis technique as follows.

3.1 Material removal by individual particles

Although a large number of variables affect the material removal process in AWJ cutting, based on the material erosion models by particles [14-17], four parameters are dominant in controlling the removal process. These dominant parameters are related to the properties of work material, the mass of an individual abrasive particle, m, the impact velocity of the particle, v, and the particle attack angle (i.e. the angle between the material surface and particle moving direction at the point of impact), α. For brittle materials, Zeng and Kim [18] incorporated a host of work material properties in their modelling work, including the ratio of dynamic hardness, H_d, to the modulus of elasticity, E. The authors also modelled the material removal by a single impact in another work [19] using this ratio (i.e. H_d/E) and the material flow stress, σ_f, to allow for different material erosion modes. In the present study, the material flow stress and the ratio of dynamic hardness to the modulus of elasticity are used to account for the effect of work material properties. Thus, the material removal by a particle can be expressed by

$$R = f(\sigma_f, \frac{H_d}{E}, v, m, \alpha) \tag{4}$$

where the mass of an individual particle is taken from the average value based on the average particle size (assuming in spherical shape) and particle material density.

The set of variables in Eq. (4) is expressed in terms of three fundamental dimensions, i.e. length L, mass M, and time T, which means that three repeating variables must be chosen to construct this model. Based on the common rules in the selection of repeating variable [11], σ_f, v and m can represent all the three fundamental dimensions and thus are selected as the repeating variables. According to the *Pi* theorem of dimensional analysis [11], the number of independent dimensionless products is equal to the number of variables in the equation minus the number of repeating variables. Therefore, three independent dimensionless products can be formed by the six governing variables in Eq. (4). Noting that α and H_d/E are already dimensionless variables, the three independent dimensionless products can be formed as

$$\pi_1 = \frac{R\sigma_f}{mv^2} \tag{5}$$

$$\pi_2 = \alpha \tag{6}$$

$$\pi_3 = \frac{H_d}{E} \tag{7}$$

Based on the dimensional analysis technique, the functional relation between these three dimensionless products in Eqs. (5) to (7) is

$$\pi_1 = f(\pi_2, \pi_3) \tag{8}$$

or

$$\frac{R\sigma_f}{mv^2} = f(\alpha, \frac{H_d}{E}) \tag{9}$$

The function in the abovementioned equation is yet to be mathematically determined. According to reference [8,20], a non-dimensional quantity is proportional to the product of other dimensionless groups raised to a rational power. Because of the simplicity and wide use of this power law formulation, it is used in this study, so that the complete dimensional equation is given by

$$\frac{R\sigma_f}{mv^2} = A\alpha^{j_1}\left(\frac{H_d}{E}\right)^{j_2} \tag{10}$$

where A, j_1 and j_2 are constant. Eq. (10) can be rewritten as

$$R = \frac{Amv^2}{\sigma_f}\alpha^{j_1}\left(\frac{H_d}{E}\right)^{j_2} \tag{11}$$

In determining the particle attack angle α in the above equation, the cutting processes with and without nozzle oscillation are considered separately.

3.2 Cutting without nozzle oscillation

The particle attack angle in AWJ cutting depends on the curvature or orientation of surface being impacted and the moving direction of the particle at the impact site. From the study of kerf formation process (or macro mechanism of AWJ cutting) [4,19,21,22], an AWJ forms a complete kerf in a step formation process and the surface curvature of the cutting front changes as the jet cuts into the workpiece. Likewise, the particle moving direction changes as the particle flows away from the nozzle exit, mainly as a result of jet tailback or drag effect [4]. Therefore, it is extremely difficult to model the attack angle of each individual particle. To simplify this analysis, the average particle attack angle from the kerf top to bottom is used and mathematically modelled by using a dimensional analysis technique.

After analysing the effects of various major variables, the attack angle can be expressed as a function of six parameters that include nozzle traverse speed u, water pressure P, standoff distance H, average particle diameter D and the material properties, H_d/E and σ_f, i.e.

$$\alpha_1 = \phi\left(u, P, H, D, \sigma_f, \frac{H_d}{E}\right) \tag{12}$$

where α_1 is the average particle attack angle in AWJ cutting without nozzle oscillation. Similar to the foregoing dimensional analysis, nozzle travel speed u, standoff distance H, and material flow stress σ_f are selected as repeating variables, so that four *Pi* groups can be formed from the seven variables in Eq. (12); namely

$$\pi_1 = \alpha_1 \tag{13}$$

$$\pi_2 = \frac{D}{H} \tag{14}$$

$$\pi_3 = \frac{P}{\sigma_f} \tag{15}$$

$$\pi_4 = \frac{H_d}{E} \tag{16}$$

Those groups are related by the function of

$$\alpha_1 = \phi\left(\frac{D}{H}, \frac{P}{\sigma_f}, \frac{H_d}{E}\right) \tag{17}$$

By applying the power law formation method [20], the following functional relation is obtained

$$\alpha_1 = B_1\left(\frac{D}{H}\right)^{x_1}\left(\frac{P}{\sigma_f}\right)^{y_1}\left(\frac{H_d}{E}\right)^{z_1} \tag{18}$$

where B_1, x_1, y_1 and z_1 are dimensionless constants. By replacing α in Eq. (11) with α_1 from Eq. (18), the material removal by a particle is given by

$$R = \frac{B_2 m v^2}{\sigma_f}\left(\frac{D}{H}\right)^{x}\left(\frac{P}{\sigma_f}\right)^{y}\left(\frac{H_d}{E}\right)^{z} \tag{19}$$

where B_2, x, y and z are used to generalise the constants, such that $B_2 = AB_1^{j_1}$, $x=x_1 j_1$, and $y = y_1 j_1$ and $z=z_1 j_1 j_2$..

Substituting Eq. (19) into Eq. (3) and replacing the depth of cut h with h_1 to denote cutting without nozzle oscillation give

$$h_1 = \frac{B_3 m_a v^2}{\sigma_f u d_j}\left(\frac{D}{H}\right)^{x}\left(\frac{P}{\sigma_f}\right)^{y}\left(\frac{H_d}{E}\right)^{z} \tag{20}$$

where constant B_3 generalises the two constants, B_2 and K_e.

It is now necessary to determine the particle velocity, v, in the above equation. If assuming that the energy loss in the system is negligible and that the water is incompressible, the water velocity in a jet, v_j, before mixing with abrasive particles can be found by using the Bernoulli's equation, i.e.

$$v_j = \sqrt{\frac{2P}{\rho_w}} \tag{21}$$

where P is water pressure, and ρ_w is water density. If assuming that the particle is entrained by the water to increase its velocity and at the point of particle attacking the material surface, the particle has gained the same velocity as its surrounding water, the particle velocity can be obtained using the momentum transfer equation, i.e.

$$v = k_m\left(\frac{m_w}{m_w + m_a}\right)v_j \tag{22}$$

where m_w is the water mass flow rate, m_a is abrasive mass flow rate, and k_m is a factor to consider the momentum transfer efficiency.

The water mass flow rate may be obtained by considering the water pressure (and hence the water flow speed from the orifice) and the cross-sectional area of the orifice; however, it is a variable because of the change in water pressure and orifice diameter. Likewise, the abrasive mass flow rate is an input variable that may change from a process to another process. Thus, to work out the mass ratio term in Eq. (22) will make the model complicated. Therefore, to simplify the derivation, the mass ratio term is approximated as a constant, B_4. For the process conditions used in the experiments of this study, this approximation only results in a less than 2.5% error for the mass ratio and even smaller error for the final depth of cut. Thus, Eq. (22) can be re-written as

$$v = k_m B_4 v_j \tag{23}$$

where B_4 is constant. Substituting Eqs. (23) and (21) into Eq. (20) yields the final equation for the depth of cut for AWJ cutting without nozzle oscillation, i.e.

$$h_1 = \frac{K_1 m_a P}{\sigma_f u \rho_w d_j}\left(\frac{D}{H}\right)^x \left(\frac{P}{\sigma_f}\right)^y \left(\frac{H_d}{E}\right)^z \tag{24}$$

where K_1 is a constant generalizing all the constants ($=2B_3 B_4^2 k_m^2$) and can be determined by experiments.

3.3 Cutting with nozzle oscillation

It has been found in previous studies that nozzle oscillation affects the particle attack angle as evidenced by the particle-work interface traces [4,7]. Thus, in deriving the mathematical expression for particle attack angles, the nozzle oscillation parameters, oscillation angle θ and frequency F, must be taken into account in addition to the six parameters considered in cutting without nozzle oscillation. Furthermore, it should be noted that standoff distance in AWJ cutting with nozzle oscillation affects the jet scanning or oscillating scope in the cutting front which may further affect the particle attack angle. For the same reasons as stated in Section 3.2, the average particle attack angle is used for this study. Consequently, the average particle attack angle for cutting with nozzle oscillation can be expressed as a function of the seven variables, i.e.

$$\alpha_2 = \psi\left(\theta, F, u, P, H, D, \sigma_f, \tfrac{H_d}{E}\right) \tag{25}$$

where α_2 is the average particle attack angle in oscillation cutting. Using the same technique in modelling the attack angle for cutting without oscillation and by identifying u, H and σ_f as repeating variables, six independent dimensionless groups can be formed from the nine variables in Eq. (25), i.e.

$$\pi_1 = \alpha_2 \tag{26}$$

$$\pi_2 = \theta \tag{27}$$

$$\pi_3 = \frac{FH}{u} \tag{28}$$

$$\pi_4 = \frac{D}{H} \tag{29}$$

$$\pi_5 = \frac{P}{\sigma_f} \tag{30}$$

$$\pi_6 = \frac{H_d}{E} \tag{31}$$

Thus, Eq. (25) becomes

$$\alpha_2 = \psi\left(\theta, \frac{FH}{u}, \frac{D}{H}, \frac{P}{\sigma_f}, \frac{H_d}{E}\right) \tag{32}$$

By using the power law formulation, α_2 can be given by

$$\alpha_2 = C_0\,\theta^{a_1}\left(\frac{FH}{u}\right)^{b_1}\left(\frac{D}{H}\right)^{c_1}\left(\frac{P}{\sigma_f}\right)^{d_1}\left(\frac{H_d}{E}\right)^{e_1} \tag{33}$$

where C_o, a_1, b_1, c_1, d_1 and e_1 are constants. Substituting Eq. (33) into Eq. (11) to replace α with α_2 gives

$$R = \frac{C_1 m v^2}{\sigma_f}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c\left(\frac{P}{\sigma_f}\right)^d\left(\frac{H_d}{E}\right)^e \tag{34}$$

where C_1, a, b, c, d and e are all constants such that $C_1 = AC_o^{j_1}$, $a=a_1j_1$, $b=b_1j_1$, $c=c_1j_1$, $d=d_1j_1$ and $e=e_1j_1j_2$. Consequently, by substituting Eq. (34) into Eq. (3) and making the necessary transformations give

$$h_2 = \frac{C_2 m_a v^2}{\sigma_f\, d_j\, u}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c\left(\frac{P}{\sigma_f}\right)^d\left(\frac{H_d}{E}\right)^e \tag{35}$$

where h_2 denotes the depth of cut in oscillation cutting, while $C_2=C_1K_e$ and is a dimensionless constant.

By substituting Eqs. (21) and (23) into Eq. (35), the final form of depth of cut equation for AWJ oscillation cutting is given by

$$h_2 = \frac{K_2 m_a P}{\sigma_f \rho_w d_j u}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c\left(\frac{P}{\sigma_f}\right)^d\left(\frac{H_d}{E}\right)^e \tag{36}$$

where K_2 is a constant generalizing all the constants ($=2C_2\,B_4^2\,k_m^2$) and can be determined by experiments along with the other constants in the equation, a, b, c, d and e.

4. Model assessment

The mathematical models in Eqs (24) and (36) developed earlier are in their general form for brittle materials. However, before the models can be of any use, the constants in the models need to be determined first. For this purpose, a regression analysis of the experimental data obtained in Section 2.1 of the paper has been performed. The constants in Eqs. (24) and (36) have been determined at a 95% confidence interval with the material dynamic hardness H_d=10400MPa, the modulus of elasticity E=276000MPa and the flow stress σ_f=20800MPa . Substituting these constants into the equations gives

$$h_1 = (1.974\times10^{-6})\frac{m_a\,P^{1.186}D^{0.156}}{\rho_w\,d_j\,u\,H^{0.156}} \tag{37}$$

and

$$h_2 = (0.145 \times 10^{-6}) \frac{m_a \, \theta^{0.229} \, P^{1.604} \, F^{0.169} \, D^{0.289}}{\rho_w \, d_j \, u^{1.169} \, H^{0.12}} \tag{38}$$

where the units for the parameters are given in the Nomenclature. Eqs. (37) and (38) are valid for cutting the 87% alumina ceramics under the cutting conditions given in Section 2.1 of the paper. An assessment of the models has revealed that the forms of the models are generally feasible and consistent with the experimental trends of the depth of cut in cutting both with and without nozzle oscillation. As shown in Fig. 4 when cutting without nozzle oscillation, the model realistically presents the effects of cutting parameters, i.e. an increase in the water pressure and abrasive flow rate is associated with an increase in the depth of cut, while a decreasing trend is shown for the depth of cut as the traverse speed or standoff distance increases. When cutting with nozzle oscillation, the model in Eq. (38) again correctly predicted the trends of the depth of cut with respect to the process parameters, as shown in Fig. 5. The figure shows that the predicted trends in terms of the water pressure, nozzle traverse speed, standoff distance and abrasive flow rate are consistent with the findings of earlier investigations [4] and the experimental results shown in symbols. Fig. 5 also shows that in general, an increase in the oscillation frequency and oscillation angle results in an increase in the depth of cut, although the increase rate is decreasing as these two parameters increase so that compromised or optimum oscillation parameters may exist. Consequently, it is considered from this qualitative assessment that the models for cutting with and without nozzle oscillation correctly predicted the trends of the depth of cut with respect to the process variables.

<Take in Fig. 4>
<Take in Fig. 5>

It is interesting to note that the corresponding exponents in the models for cutting with and without nozzle oscillation may bear different values. It is understood [4] that nozzle oscillation not only clears the way for particles to effectively cut the material and reduces interferences between particles through a scanning cutting action, but also changes the particle-work interactions and the erosive process (which may involve different erosive mechanisms). Thus, it is not surprising that different exponential values for the same variable were determined in the two models.

<Take in Fig. 6>

A quantitative assessment of the models has been carried out based on the percentage deviation of the model predictions with respect to the experimental results under the corresponding cutting conditions. This is shown in the histograms in Figs. 6(a) and (b) for cutting without and with nozzle oscillation respectively. It can be seen that the average deviation for the depth of cut in the cases where normal cutting (without nozzle oscillation) was used is 2.4% (or -2.4%) with a standard deviation of 19.99%. When cutting with nozzle oscillation, the average deviation for the depth of cut is 2.3% (or -2.3%) and the standard deviation is 17.20%. The large standard deviation is attributed to the scatter of the experimental data that were used to determine the constants in the models. Consequently, it may be stated that the depth of jet penetration models developed can give reasonably good predictions both qualitatively and quantitatively, and can be used for adequate prediction of this cutting performance measure in process planning.

5. Conclusions

An experimental investigation of the depth of jet penetration in AWJ cutting of alumina ceramics with controlled nozzle oscillation has been carried out and reported. It has been shown that nozzle oscillation cutting can cause negative effect on the depth of cut if the cutting parameters are not correctly selected. Such a negative effect generally occurred when a small oscillation angle (e.g. 2^o) was used with a small oscillation frequency (e.g. 2Hz). It has also been shown that if the cutting parameters are properly selected, nozzle oscillation can increase the depth of cut by as much as 82%. Based on the test conditions in this study, the combinations of high oscillation frequencies (10-14Hz) and small oscillation angles (4-6^o) are recommended for maximizing the depth of cut in nozzle oscillation cutting.

In order to mathematically predict the depth of cut for process planning and optimization, predictive models for this cutting performance measure in AWJ cutting without and with nozzle oscillation have been developed. The general forms of the models are applicable for brittle materials, such as ceramics and marbles, while the final models with the constants determined from experimental data have been developed specifically for cutting an 87% alumina ceramic. The models have been assessed both qualitatively and quantitatively and shown to be able to give adequate predictions for the depth of cut with an average percentage error of less than 2.5% within the conditions tested in this study.

A challenge derived from this study is to apply the nozzle oscillation cutting technique to AWJ contouring by developing a motion control mechanism that can oscillate the nozzle or cutting head in the direction tangential to the cutting profile. This work is being undertaken and it is hoped to report on this study shortly.

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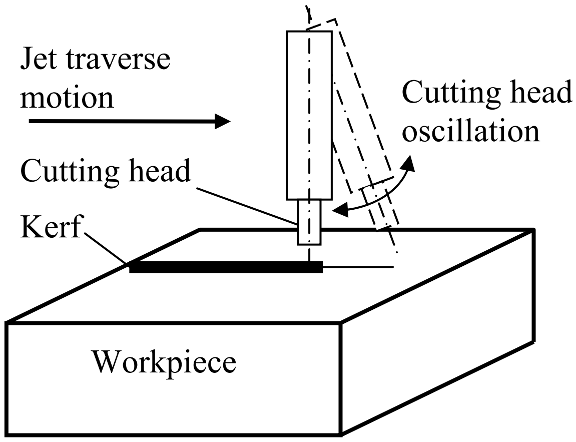


Fig. 1. Schematic of cutting head oscillation.

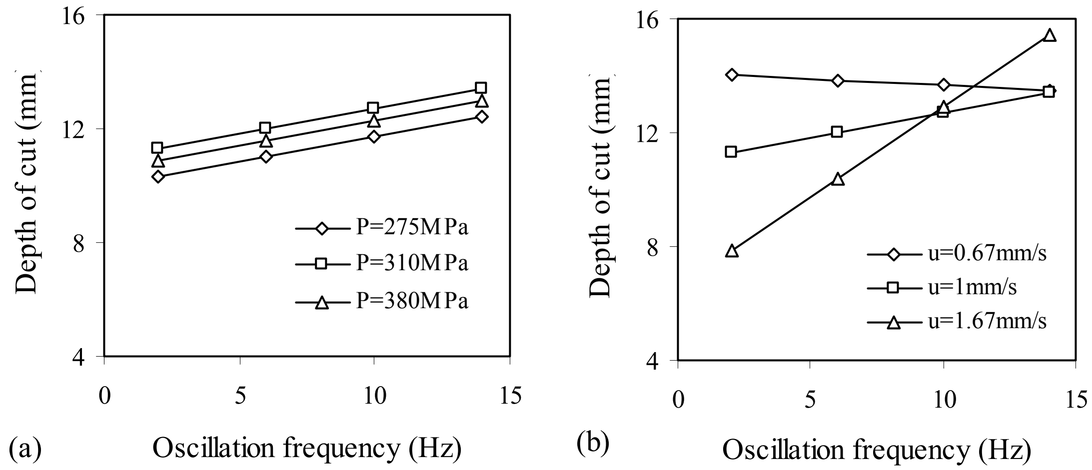


Fig. 2. Effect of oscillation frequency on the depth of cut: (a) θ=4°, u=1mm/s, H=2mm, m_a=9.1g/s; (b) θ=4°, P=310MPa, H=2mm, m_a=9.1g/s.

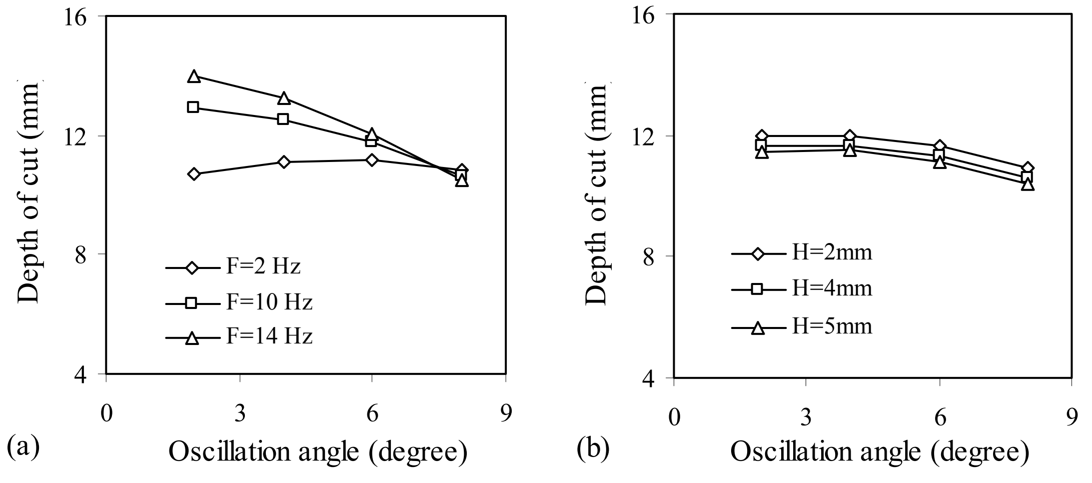


Fig. 3. Effect of oscillation angle on the depth of cut: (a) P=310MPa, u=1mm/s, H=3mm, m_a=9.1g/s; (b) F=6Hz, P=310MPa, u=1mm/s, m_a=9.1g/s.

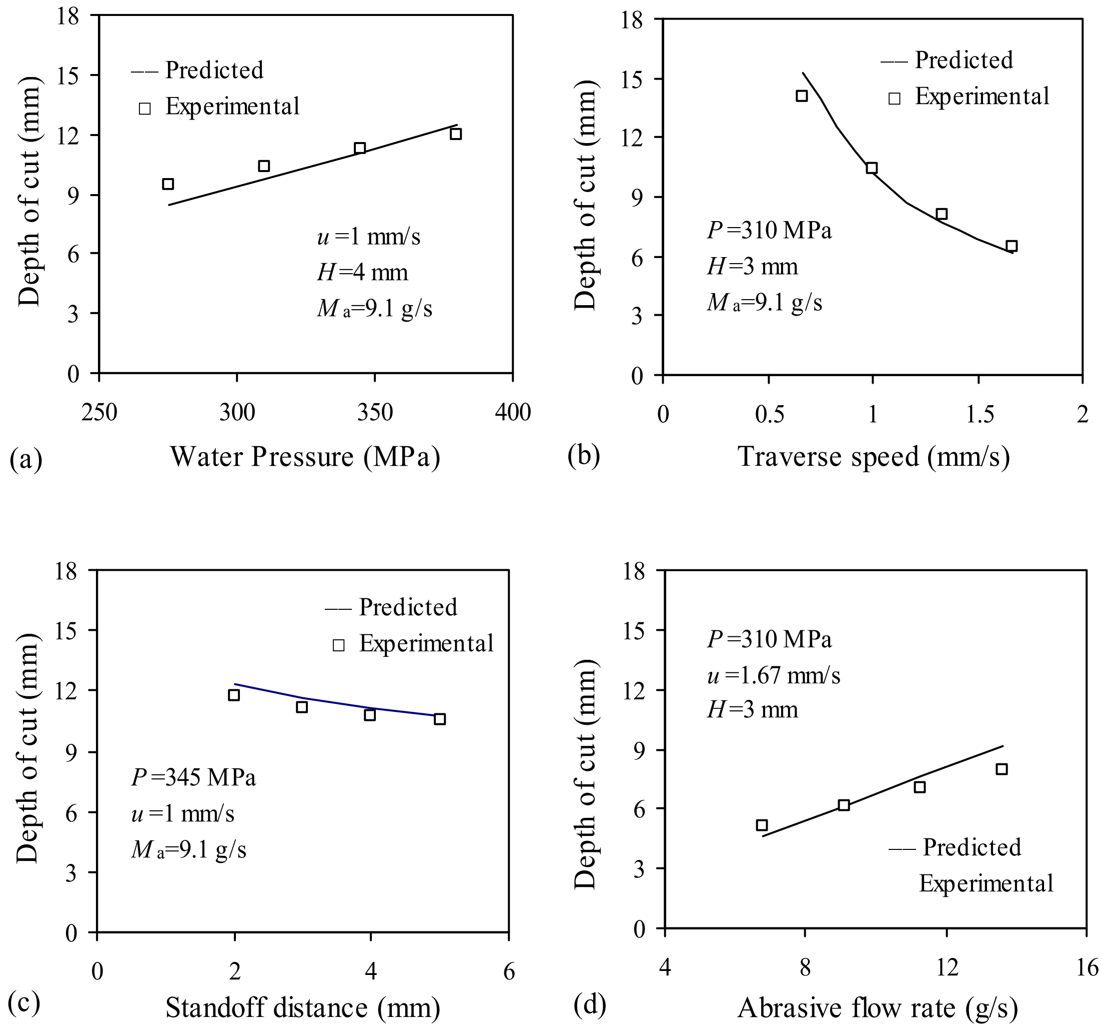


Fig. 4. Comparisons between model predictions and experimental data for cutting without nozzle oscillation.

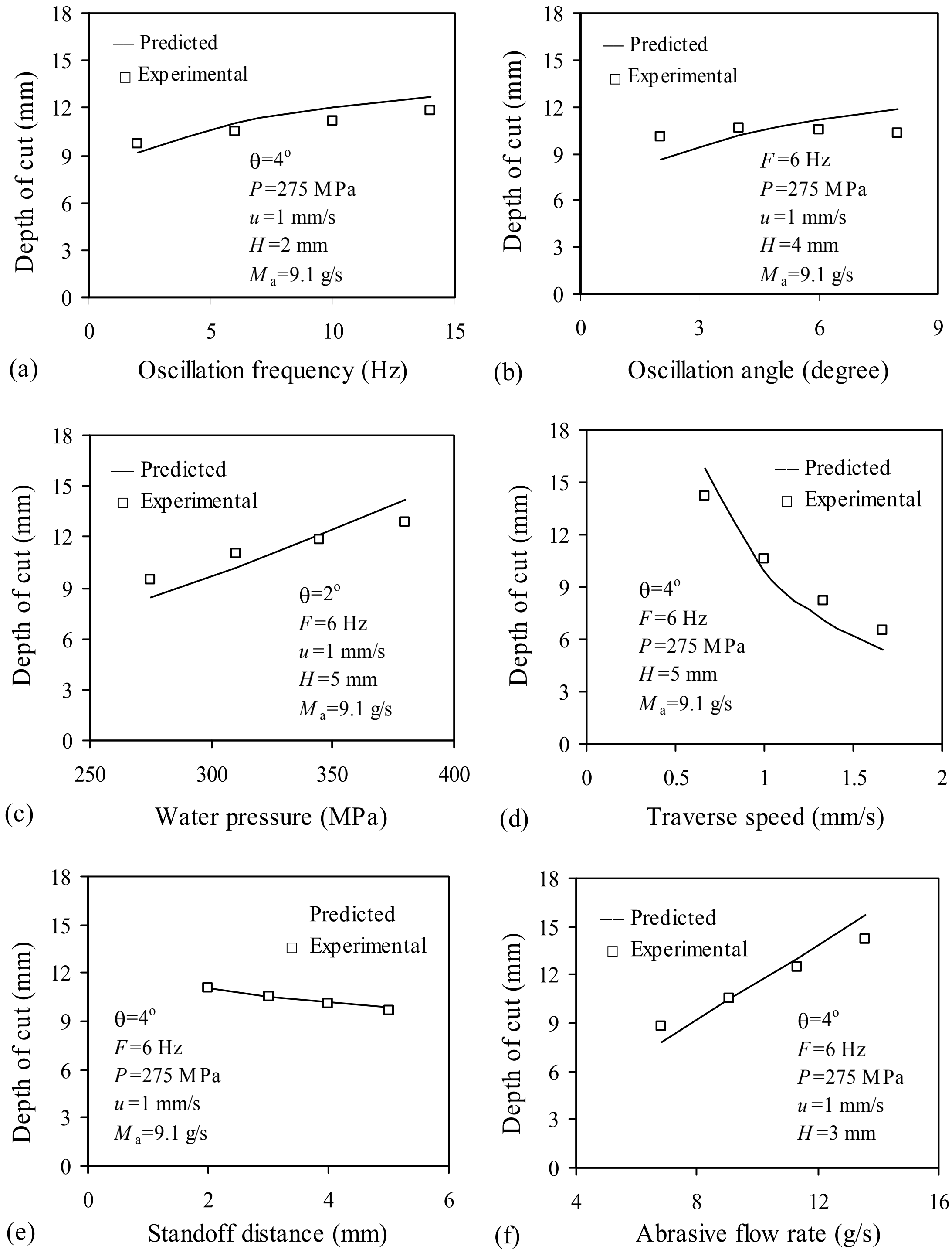
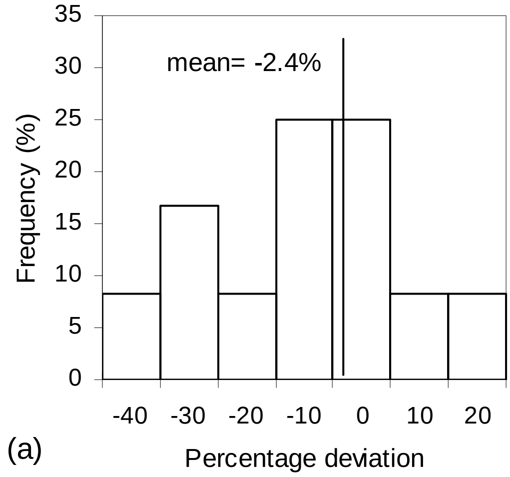


Fig. 5. Comparisons between model predictions and experimental data for nozzle oscillation cutting.



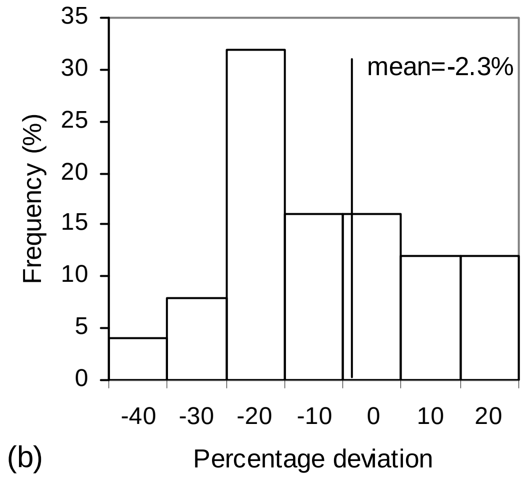


Fig. 6. Percentage deviations of predicted from experimental depth of cut. (a) Without nozzle oscillation; (b) With nozzle oscillation.

Table 1. Main physical and mechanical properties of the specimens.

Hardness, Knoop 1000g (MPa)	10400	Modulus of elasticity (MPa)	276000
Compressive strength (MPa)	2480	Flow stress (MPa)	20800
Flexural strength (MPa)	336	Average crystal size (μm)	1.6
Tensile strength (MPa)	221		

Table 2. Experimental design.

Process Variables	Level 1	Level 2	Level 3	Level 4
Abrasive mass flow rate m_a (g/s)	6.8	9.1	11.3	13.6
Standoff distance H (mm)	2	3	4	5
Water pressure P (MPa)	275	310	345	380
Nozzle traverse speed u (mm/s)	0.67	1.00	1.33	1.67
Oscillation angle θ (degrees)	2	4	6	8
Oscillation frequency F (Hz)	2	6	10	14