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Computer-aided economic optimization of end-milling operations

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Abstract

Optimization analysis, strategy and CAM software for single pass end-milling on CNC machine tools are outlined and discussed based on criteria typified by the maximum production rate and allowing for a range of machine tool and component surface roughness constraints. It is shown that the deterministic optimization approach involving mathematical analyses of constrained economic trends and their graphical representation on the feed-speed domain provides a deeper understanding of the influences of constraints and a clearly defined strategy which guarantees the global optimum solutions. Numerical simulation studies have amply demonstrated the economic benefits of using this strategy over handbook recommendations as well as in assessing, selecting and improving machine tool design specifications.

Keywords
Optimization strategy, end-milling, machining, CAM software, process planning

1. Introduction

The selection of cutting conditions is an important step in process planning of machining operations. While the notion of an optimum speed for maximum material removal rate in single pass turning was recognized by Taylor [1] early this century, the progress in developing realistic optimization strategies for selecting the economic cutting conditions in process planning has been very slow [2]. As a result, the selection of cutting conditions in machining operations has often relied on experience, 'rules of thumb' and handbook recommendations. A recent survey [3] has indicated that the cutting conditions set on CNC machine tools by such approximate practices are far from optimal. Furthermore, about 75% of total available production time forecast to be spent on machining components in computer based automation and flexible manufacturing compared to about 5% in conventional 'manual' machine tools [4] has highlighted the considerable scope for cost savings through efficient and effective machining. The use of economic cutting conditions may indeed be essential to offset the high capital and operating costs in modern manufacturing involving high technology equipment and systems and to increase the competitiveness of manufacturing firms.

Traditionally, economic optimization of machining involves the selection of feed and cutting speed according to a variety of economic criteria such as the minimum cost per component, maximum production rate or maximum profit rate [2, 5, 6]. A realistic optimization study should also allow for the many technological and practical constraints which limit the feasible domain for the selection of optimum cutting conditions. However, this work has proved to be surprisingly difficult requiring intricate mathematical analyses and computer assistance, depending on quantitatively reliable mathematical functions of machining performance measures and detailed specifications of the machine tools, cutting tools and components which act as constraints on the permissible feeds and speeds [7]. While promising progress has been made in the optimization of turning operations since Taylor's work [1] whereby various practical and technological constraints have been considered [8-13] and well defined optimization strategies and computer software have been developed [8, 9], the optimization of milling operations has received little attention and often relied on the use of available computer-aided mathematical programming and numerical search techniques in attempts to provide the optimum feed and speed [14-18]. These 'computer packaged strategies' neither guarantee global optimum solutions nor provide clearly defined economic characteristics and solution strategies which allow for the ready identification of trends in the way in which the optimum solution can change with alternative constraints. Therefore, it is necessary to develop a computer executable strategy which can show the effects of the different constraints and guarantee the global optimum solution.

In this paper, optimization analysis, strategy and CAM software module for the selection of economic cutting conditions in single pass end-milling on computer numerically controlled (CNC) machine tools are outlined. The analysis is based on the popular economic criterion of minimum production time (or maximum production rate) while the strategy developed applies for the minimum cost per component criterion due to the proven mathematical similarity of the two objective functions. The constraints considered are the machine tool feed and speed boundaries, machine tool maximum feed force, spindle torque and power constraints as well as the component surface roughness constraint when finish milling is considered. Numerical simulation studies have verified the optimization strategy and the software module and shown the economic benefits of using the optimization strategy over handbook recommendation in process planning. The use and importance of this strategy in the selection and design of milling machines for economic machining are also considered in the simulation studies.

2. Objective function and constraints

Based on the popular minimum production time and cost per component criteria, the objective functions for single pass end-milling can be given by the well-known equations [2, 7]:

$$T_T = T_L + T_c + T_R \frac{T_{ac}}{T} \tag{1}$$

and

$$C_T = x\left(T_L + T_c + T_R^{'} \frac{T_{ac}}{T} \right) \tag{2}$$

where $T_R^{'} = (T_R + y/x)$ and T, T_c, T_{ac}, T_L, T_R, x and y are as defined in the nomenclature.

Despite the dearth of tool-life data in end-milling noted in the research literature, two comprehensive machining data handbooks [19, 20] have been found to provide the tool-life function as well as the values of the empirical constants for a range of tool-workpiece material combinations suitable for economic machining studies. The tool-life was given in the typical extended Taylor-Type equation

$$T = \frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{1/n_1} a_r^{1/n_2} a_a^{1/n_3} z^{1/n_4}} \tag{3}$$

Some of the process variables for single pass end-milling are illustrated in Figure 1. The cutting time T_c for end milling of a cut length ℓ can be approximately expressed as:

$$T_c \approx T_{ac} = \frac{\ell D}{\mu V f_z z} \tag{4}$$

It can be noted that if the labour and overhead cost rate, x, and the cutter cost per failure, y, are minimized and constant through good management and purchasing policy, equations (1) and (2) are mathematically similar. Hence, the characteristics and strategies for minimizing T_T and C_T are similar although the optimum feed and speed for the two criteria are not necessarily the same under the same constraint conditions. Thus, only the analyses for the minimum time per component T_T equation will be explicitly presented in this paper.

Substituting equations (3) and (4) into equation (1) gives

$$T_T = T_L + \frac{\ell D}{\mu V f_z z} + \frac{\ell T_R}{\mu K} V^{1/n-1} f_z^{1/n_1 -1} a_r^{1/n_2} a_a^{1/n_3} z^{1/n_4} D^{1-1/n_5} \tag{5}$$

This is the fundamental form of the objective function which has to be optimized. As is usual in single pass optimization studies, only the cutting speed V and feed per tooth f_z have to be optimized, since it is expected that the loading/unloading time T_L and cutter replacement time T_R have been minimized using work study techniques and well-designed handling devices while the milling cutter (D, z) has been adequately selected *a priori* for the given workpiece dimensions (ℓ, a_r and a_a) and tool-work material combination.

In practice, the cutting speed V and feed per tooth f_z must be selected to minimize T_T in equation (5) without violating a number of constraints, such as those imposed by the machine tool, which in fact limit the feasible domain of speed V and feed per tooth f_z and result in a constrained optimum T_T or C_T. For the present study of single pass end-milling operations, the machine tool limiting feed force, F_{fmax}, spindle torque, T_{qmax}, maximum power, P_{max}, as well as the feed and spindle speed boundaries (v_{fmin}, v_{fmax}, N_{min}, N_{max}) are considered. The component surface roughness requirement will also be included in the study for finish peripheral end-milling operations. These constraints can be expressed mathematically as follows [7]:

(i) ***Feed force constraint***

$$f_z \le f_{zFf} = \left(\frac{F_{f\,\max}}{A\, a_a^{\beta}\, a_r^{\varepsilon}\, z\, D^{-\delta}} \right)^{1/\alpha} \tag{6}$$

(ii) ***Spindle torque constraint***

$$f_z \le f_{zTq} = \left(\frac{2\, T_{q\,\max}}{G_1\, a_a^{\beta}\, a_r^{\varepsilon}\, z\, D^{1-\delta}} \right)^{1/\alpha} \tag{7}$$

(iii) ***Machine tool maximum power constraint***

$$P = G V\, f_z^{\alpha}\, a_a^{\beta}\, a_r^{\varepsilon}\, z\, D^{-\delta} \le P_{\max} \tag{8}$$

(iv) ***Machine tool speed and feed boundary constraints***

$$\pi D N_{\min} = V_{\min} \le V \le V_{\max} = \pi D N_{\max} \tag{9}$$

$$\frac{\pi D v_{f\min}}{zV} = f_{z\min} \le f_z \le f_{z\max} = \frac{\pi D v_{f\max}}{zV} \tag{10}$$

(v) ***Component surface roughness constraint***

It has been found by an extensive survey [7] that although some theoretical surface roughness equations have been reported, there is a general lack of published data to support these equations so that this constraint cannot be accurately allowed for in the machining optimization until reliable surface roughness equations and associated data become available. For the purpose of the present study, the 'theoretical' or 'ideal' peak-to-valley height equation in reference [2] has been employed when peripheral end-milling is considered and the resulting constraint expression is given by

$$f_z \le f_{zRt} = 2\left(D R_{t\max}\right)^{1/2} \tag{11}$$

In the above constraint equations, A, G, G_1, α, β, ε and δ are empirical constants dependent on the tool-work material combination, cutter geometry and the units used and can be found in the comprehensive Chinese handbooks [19, 20], F_{fmax}, T_{qmax}, P_{max}, N_{min}, N_{max}, v_{fmin} and v_{fmax} are constraints given in the machine tool specifications, and R_{tmax} is the maximum surface roughness (peak-to-valley) limit. It is evident that the magnitudes of these constraints limit the permissible domain for the optimization of cutting speed V and feed per tooth f_z in equation (5). Furthermore, the maximum F_{fmax}, T_{qmax} and R_{tmax} constraints, which only limit the permissible feed per tooth f_z and are mutually exclusive, can be generalised by a feed per tooth limit f_{zFTR}, i.e.

$$f_z \le f_{zFTR} = \min\left\{f_{zFf}, f_{zTq}, f_{zRt}\right\} \tag{12}$$

For rough milling, equation (12) can be simplified as

$$f_z \le f_{zFT} = \min\left\{f_{zFf}, f_{zTq}\right\} \tag{13}$$

From a detailed study of the machining performance data in the Chinese handbooks [19, 20] it has been found that the exponents in the tool-life and power equations have the following relationships: $1/n > 1/n_1 > 0$, $1/n > 1$ and $1 > \alpha > n/n_1$ while $1/n_1$ can be greater than, equal to or less than 1. Based on these common ranges of the machining performance exponents, the optimization strategy will be developed below.

3. Development of optimization strategy and software

A global minimum time per component (maximum production rate) requires that the partial derivatives of the objective function in equation (5) with respect to the cutting speed and feed per tooth are zero, i.e.

$$\frac{\partial T_T}{\partial V} = \frac{\ell D}{\mu V^2 \ f_z \ z}\left[\frac{T_R}{T}\left(\frac{1}{n}-1\right)-1\right]=0 \tag{14}$$

$$\frac{\partial T_T}{\partial f_z} = \frac{\ell D}{\mu V \ f_z^2 \ z}\left[\frac{T_R}{T}\left(\frac{1}{n_1}-1\right)-1\right]=0 \tag{15}$$

from which the economic tool-life equations with respect to the cutting speed and feed per tooth can be found and are respectively given by

$$\frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} = T_R\left(\frac{1}{n}-1\right) = T_V \tag{16}$$

$$\frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} = T_R\left(\frac{1}{n_1}-1\right) = T_F \tag{17}$$

Since $n \neq n_1$ for the common tool-work material combinations, as mentioned earlier, equations (16) and (17) (or equations (14) and (15)) cannot be simultaneously satisfied for a minimized T_R and hence a unique pair of V and f_z does not exist for a global minimum T_T. Therefore it is necessary to study the T_T characteristics in order to establish a strategy for selecting the V and f_z such that the production time per component is minimized.

Figure 2(a) illustrates the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci on an f_z-V graph. It has been proved [7] that for the usual values of the empirical tool-life equation exponents $1/n>1/n_1>1$, the $\partial T_T/\partial f_z=0$ curve is above and to the right of the $\partial T_T/\partial V=0$ curve on the f_z-V diagram. Further, a local optimum T_T with respect to V always exists for a given f_z, since $1/n>1$, which can be found on the curve described by equation (16), i.e. the $\partial T_T/\partial V=0$ curve. Similarly, the f_z for a local optimum T_T can be obtained from equation (17) (on the $\partial T_T/\partial f_z=0$ curve) for a given cutting speed V when $1/n_1>1$. The time per component T_T characteristics along the $\partial T_T/\partial V=0$ locus can be found by substituting equation (16) into equation (5) from which

$$T_T = T_L + \frac{\ell}{\mu}\left[\frac{T_R}{n\,K}\right]^n (1-n)^{n-1}\, f_z^{n/n_1-1} a_r^{n/n_2} a_a^{n/n_3} z^{n/n_4-1} D^{1-n/n_5} \tag{18}$$

Thus, since $n/n_1<1$, T_T will decrease along the $\partial T_T/\partial V=0$ curve as f_z increases or V decreases, as indicated by the arrowheads in Fig. 2(a). It can also be proved that T_T along the $\partial T_T/\partial f_z=0$ locus (when $1/n_1>1$) possesses similar characteristics to those of the $\partial T_T/\partial V=0$ curve, as shown in Fig. 2(a).

However, when $1/n>1$ but $1/n_1 \leq 1$, as is possible for some tool-work material combinations noted in the handbooks [19, 20], $\partial T_T/\partial f_z$ in equation (15) is negative and equation (17) does not apply. Thus the necessary condition for a local minimum with respect to f_z (i.e. $\partial T_T/\partial f_z=0$) can never be satisfied and the minimum T_T for a given V occurs when f_z is as high as possible [7], as shown in Fig. 2(b). By contrast, equations (14) and (16) still apply from which the optimum cutting speed can be found for a given f_z. It can be shown again that the T_T value decreases along the $\partial T_T/\partial V=0$ locus as f_z increases or V decreases.

The above T_T characteristics lead to the popular strategy of selecting V and f_z in the 'high feed-low speed' region in the vicinity of the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ (when $1/n_1>1$) loci. However, this strategy is not always valid in selecting the optimum cutting speed and feed per tooth since in practice a number of technological and practical constraints as noted earlier have to be satisfied. The effects of groups of related constraints on the selection of optimum cutting conditions are considered separately below before developing the overall strategy allowing for the combined effect of all the constraints.

3.1. Effects of machine tool feed and speed boundary constraints

For CNC machine tools, any feed speed and spindle speed within the specified boundaries are considered to be available for the selection of optimum cutting conditions. Thus when these boundary limits are considered, it is found that they define an available f_z-V domain with the upper curve occurring at v_{fmax} and the lower boundary at v_{fmin}, as shown in Figure 3. The relationship between the feed speed v_f, feed per tooth f_z and cutting speed V can be expressed as

$$f_z = \frac{v_f}{z\,N} = \frac{D\,v_f}{\mu\,z\,V} \tag{19}$$

The maximum and minimum spindle speeds establish the cutting speed boundaries, for a given cutter diameter, as vertical lines on the f_z-V diagram. By comparing the slopes of the $\partial T_T / \partial V = 0$ and $\partial T_T/\partial f_z=0$ loci with that of v_f curve at their intersections, it has been proved [7] that these curves cross in the way shown in Fig. 3 for the common range of machining performance exponents noted above.

The T_T trends along a constant v_f curve can be found by substituting equation (19) into equation (5), from which it is found that for $1/n>1/n_1$, T_T decreases as V decreases, as indicated by the arrowheads on the v_f as well as v_{fmin} and v_{fmax} curves in Fig. 3. The T_T characteristics on the constant V lines can be established readily by superimposing the $\partial T_T / \partial V = 0$ and $\partial T_T/\partial f_z=0$ curves on an f_z-V diagram. The T_T reducing direction is again shown by the arrowheads on the cutting speed boundaries in Fig. 3. It is apparent that the time per component T_T characteristics along the V limiting lines depend on the relative positions of the $\partial T_T/\partial f_z=0$ (when $1/n_1>1$) locus with respect to the V_{min} and V_{max} boundaries, so does the optimum (f_z, V) solution.

From a detailed study of the T_T characteristics when only feed and cutting speed boundary constraints are considered, three possible optimum solutions all on the V_{min} limit have been identified and the corresponding limiting conditions established, as shown in Fig. 3 where the feed per tooth f_{zFVn} at the intersection of the $\partial T_T/\partial f_z=0$ curve and V_{min} limiting boundary can be found from equation (17) with $V=V_{min}$ and is given by

$$f_{zFVn} = \left(\frac{K\,D^{1/n_5}}{T_R(1/n_1 - 1)V_{\min}^{1/n}\,a_r^{1/n_2}\,a_a^{1/n_3}\,z^{1/n_4}} \right)^{n_1} \tag{20}$$

It is interesting to note that the location of the $\partial T_T/\partial V=0$ curve on the f_z-V diagram does not affect the optimum solutions. When $1/n_1 \leq 1$, the $\partial T_T/\partial f_z=0$ locus does not exist and T_T will monotonically decease along the constant V_{min} and V_{max} boundaries as f_z increases (Fig. 3(d)) according to the characteristics noted earlier. In this case the optimum solution is always at (f_{zmax}, V_{min}) which is coincident with the solution shown in Fig. 3(a).

3.2. Effects of machine tool feed force, torque and power and component surface roughness constraints

The machine tool feed force, spindle torque and the component surface roughness constraints have been generalised and represented by an upper feed per tooth limit f_{zFTR}. In the 'high' cutting speed region of the machine tool operating range, the machine tool maximum power constraint will come into play and limit both the feed speed and cutting speed from which a constrained optimum can be selected.

In order to establish an optimization strategy, it is again necessary to study the T_T trends on the f_z-V diagram while considering the effects of these constraints. As shown in Figure 4, the maximum power P_{max} and the $\partial T_T/\partial V=0$ curves intersect such that for the usual set of exponent values, $1>\alpha>n/n_1>0$, the slope of the P_{max} curve is less than that of the $\partial T_T/\partial V=0$ locus at the point of intersection and so the curves cross in the way shown in the figure. It can also be shown [7] that the maximum power constraint curve intersects the $\partial T_T/\partial f_z=0$ curve in the same manner as the $\partial T_T/\partial V=0$ curve on the f_z-V graph.

The T_T trend along the P_{max} locus may be found by substituting V from equation (8) (with $P=P_{max}$) into equation (5). This shows that for the common conditions of $n/n_1<\alpha<1$ and $1/n>1/n_1>0$, T_T decreases along the P_{max} locus as f_z increases, as shown by the arrowheads in Fig. 4. Thus if f_z below the intersection of P_{max} and $\partial T_T/\partial V=0$ curves, the optimum cutting conditions will be on the $\partial T_T/\partial V=0$ locus; otherwise, the optimal solution will lie on the P_{max} curve as the cutting conditions on the $\partial T_T/\partial V=0$ locus are not feasible. The portions of the P_{max} and $\partial T_T / \partial V = 0$ curves on which the optimum point is likely to lie are shown by solid lines in Fig. 4.

When both the machine tool power P_{max} and the generalised feed per tooth f_{zFTR} constraints are considered jointly, the T_T characteristics can be found by superimposing the loci of these constraints on the f_z-V diagram as shown in Fig. 4. The resulting optimum solution will be on the generalised f_{zFTR} constraint, at its intersection with either the P_{max} curve or the $\partial T_T / \partial V = 0$ curve, depending on which intersection is at a lower speed. The two cutting speeds where f_{zFTR} intersects with the P_{max} and $\partial T_T / \partial V = 0$ curves can be found by substituting f_{zFTR} into equations (8) and (16), respectively, and given by

$$V_{FP} = \frac{P_{\max} D^{\delta}}{G\, f_{zFTR}^{\alpha}\, a_r^{\varepsilon}\, a_a^{\beta}\, z} \tag{21}$$

$$V_{TV} = \frac{K D^{1/n_5}}{T_R (1/n-1)\, f_{zFTR}^{1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} \tag{22}$$

These two constrained optimum (f_z, V) solutions are illustrated in Fig. 4 and are possible depending on the relative positions of the $\partial T_T/\partial V=0$ curve and the constraint loci on the f_z-V diagram.

3.3. Optimization strategy and software

The above study has separately considered small groups of constraints. In practical situations, the economic trends and combined effects of all the constraints have to be considered when end-milling on CNC machine tools. This results in more complex strategy which benefits greatly from computer assistance for its implementation after the various possible constrained optimum solutions and the corresponding limits identifying these solutions are established.

By applying the above analyses, the economic trends for the combined effects of machine tool and component surface roughness constraints can be found by superimposing the upper feed per tooth limit, f_{zFTR}, and the P_{max} limiting speed-feed curve together with the feed speed and cutting speed boundaries as well as the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci on the f_z-V diagram. Since the relative positions of these curves on the f_z-V diagram can vary depending on the magnitudes of the constraints, the machining performance data and the cut and cutter geometry, the active constraints on which the optimum conditions may lie can also vary. To establish the optimization strategy, it is necessary to identify the various constrained optimum solutions on the 'active' constraints, and the associated limiting conditions for each solution.

A detailed mathematical study of the T_T trends on the f_z-V domain has resulted in 11 distinctly different solutions being identified for a total of 22 different limiting conditions or cases representing all possible relative positions of the $\partial T_T/\partial V=0$, the $\partial T_T/\partial f_z=0$ and the various constraints' loci [7]. These are shown in Figure 5 where the arrowheads indicate the T_T decreasing direction and the 'dot' highlights the optimum feed and speed. The limiting conditions for identifying each constrained optimum f_z and V solution are given in the captions of each diagram in Fig. 5.

It is noteworthy that among the 11 possible solutions, one represents the situation where single pass milling is not feasible since at least one of the practical constraints will be violated for the 'input' conditions so that multipass milling operations or an alternative machine tool must be considered. It is also interesting to note that in only four of the 22 different cases do the solutions match with the popularly acclaimed strategy of selecting the largest possible feed with the speed being found from $\partial T_T/\partial V=0$ [2].

Thus, despite the complexity of the constrained optimization analyses and strategy requiring computer assistance, the approach assisted by the f_z-V graphical presentation has provided a means of guaranteeing and identifying the various possible global optimal solutions. In addition, the active constraints associated with each global solution can be identified. A computer-aided strategy and software for arriving at the required constrained global optimum f_z and V solution as well as the optimum T_T (or C_T) when end-milling on CNC machines have been developed. This strategy and the associated software involves only a simple sequential testing of the limiting conditions for identifying the appropriate global solution and thus greatly reducing the computer processing time. The inputs required are machine tool constraints (v_{vfmin}, v_{fmax}, N_{min}, N_{max}, F_{fmax}, T_{qmax}, P_{max}), component surface requirement (R_{tmax}), cutter and cut geometry (D, z, a_r, a_a, ℓ), time and cost (if C_T criterion is considered) factors (T_L, T_R, x, y), and machining performance information in the force, torque, power and tool-life equations. For a given set of inputs, the program will produce a unique optimum feed and speed solution and the corresponding minimum time (or cost) per component. If required, the output will also include the active constraints for the specific input conditions.

4. Numerical simulation studies

4.1 Benefits of using optimization strategy over handbook recommendations

Many machining data handbooks (e.g. [21]) only provide initial recommendations for some of the cutting conditions, such as the feed per tooth and cutting speed in milling, leaving most of the problem to the process planner to solve. However, the comprehensive cutting conditions handbooks [19, 20] provide not only complete tool-life, force, torque and power equations (and the associated data or constants) for a wide range of tool-work material combinations, but also detailed information and specifications for a number of milling machines. Furthermore, these handbooks provide a methodology for selecting a standard cutting tool as well as the feed and speed for single pass rough end-milling such that the machine tool constraints are not violated. A study [7] has clearly indicated that the handbook solutions are feasible, but not necessarily optimal. Nevertheless, these handbooks provide a unique opportunity to assess the developed optimization strategy and software as the optimized times (or costs) should always be superior to those from the handbooks. Further, the benefits of using optimization strategy over handbook recommendations can be evaluated.

In order to assess the penalty of using the handbook recommendation rather than the optimization strategy for end-milling, a simulation study based on the maximum production rate has been carried out. The handbook and optimized results have been compared on the basis of the percentage increase in the time per component T_{Tr} from the handbook over the optimized T_{To}, i.e.

$$\frac{\Delta T_T}{T_{To}}100 = \left[\frac{T_{Tr} - T_{To}}{T_{To}}\right]100 = \left[\frac{T'_{Tr} - T'_{To}}{T'_{To}}\right]\left[1 - \frac{T_L}{T_{To}}\right]100 \qquad (23)$$

where the T'_T is the time per component when T_L is zero.

It is apparent from the linearity of equation (23) that the maximum penalties of using handbook recommendations (or maximum benefits of using the optimum conditions) will occur when the loading/unloading and idle time T_L is as small as possible (ideally zero). Also these penalties reduce linearly to zero as T_L/T_{To} increases to 1. Thus this equation highlights the need to continually reduce the non-productive times in machining operations. It also follows that in modern flexible manufacturing, where the non-productive times are minimized and are small proportions of the total production times, the use of optimization strategy becomes more important than in the past.

For the purposes of this numerical study, rough end-milling was simulated on a Cincinnatti CNC machining centre SABRE-750 to cut a plain carbon steel (≤0.6% C) of ultimate tensile strength σ_b=657 MPa with high speed steel (HSS) cutters (30° helix angle and 15° normal rake angle). The machine tool specifications or constraints are given in Table 1. The cut geometry were selected to span the recommended range in the handbook [20] and have at least three levels. Thus, six levels of radial depths of cut, a_r, and length of cut, ℓ, and three levels of axial depth of cut, a_a, were tested at one level of non-productive time T_L and one level of cutter replacement time T_R. The 108 end-milling cases are given in Table 2, where the cutter diameter and number of teeth were selected according to the handbook [20] recommendations.

Examining the optimum solutions has revealed that for all the cases tested the optimum feed and speed are considered to be practically feasible based on the ranges of these parameters recommended in the handbook [20]. Table 3 gives the optimum results for the cases when ℓ =600 mm and D=16 mm. In addition, all the optimized times per component are less than those from the handbook cutting conditions. Figure 6 shows the overall penalties of using handbook recommendation over the optimization strategy found in this simulation study for alternate loading/unloading times, where the range and average penalties are shown to decrease linearly as the proportions of non-productive time T_L/T_{To} increases according to equation (23). Based on the ideal loading/unloading time of zero, the overall results show that the use of handbook recommendations will lead to an average increase of 222% in T_T over the optimized times with a range of 89% to 407%. It is apparent that these overall maximum penalties are significant and emphasize the need to use optimization strategy for efficient and economic production.

The histogram in Figure 7 shows the T_T penalties of using handbook rather than optimized cutting conditions for the 108 conditions at T_L=1 min. considered in this study. It is noted that the average percentage time penalty is about 154% with a range of 64% to 332%. It is again apparent that even for $T_L \neq 0$ the penalties of using handbook cutting conditions are significant, so that in general substantial benefits would be gained if these cutting conditions were optimized using the above strategy.

4.2 Machine tool selection and design considerations

In process planning, the above optimization strategy and software may be used on a case by case basis to select the most appropriate available machine tool in a production facility. A broader approach may also be adopted, whereby the optimization software is used to evaluate and select appropriate machine tools for the expected population of components to be machined.

Taking the latter approach, three modern CNC machining centers have been evaluated for end-milling of a plain carbon steel of ultimate tensile strength of 657 MPa, as noted earlier, with HSS cutters. The machine tool specifications are given in Table 1, where differences in power, torque, feed force as well as feed speed and spindle speed ranges are evident. In particular, the torque constraint in the SABRE-750 is considerably larger than those for the standard ANCA-MMC800 and the modified ANCA-MMC800-1 machines, while the feed force constraint for the SABRE-750 is considerably smaller than that for the two ANCA machines.

Based on the 108 end-milling cases described in Table 2 and the maximum production rate criterion, the results of applying the optimization strategy for each machine tool are summarized in Table 4. It is interesting to note that the average optimum time per component T_{To} for the ANCA-MMC800 machine is 5.68 min. which is very similar to that for the SABRE-750 of 5.73 min., while a higher average T_{To} of 8.52 min. applies for the modified ANCA-MMC800-1 machine. A further study of these results has shown that although the standard ANCA-MMC800 and SABRE-750 machines gave comparable average T_{To}, the 'active' constraints in the optimization were different with the former machine being limited by its lower torque of 48 NM and the latter machine being constrained by the lower feed force of 3100 N. The differences in performance of these two machines are noted in the range of T_{To} values in Table 4. The even lower torque of 24 NM for the 'high-speed' modified ANCA-MMC800-1 machine was found to be the 'active' constraint again. This explains why the average T_{To} increased to 8.52 min. for this machine. It is also interesting to note that none of the machines utilized the available maximum power for the range of conditions tested.

In general, it appears that the SABRE-750 could be improved by increasing the feed force limit, while the two ANCA machines require considerably more torque. The latter is particularly so if face milling with generally larger cutter diameters (resulting in larger machining torque) than those of end-mills is conducted on the two machines, whereby the torque limits will severely restrict the selection of cutting conditions while the force constraint may never come into play. Thus, the use of the optimization strategy for assessing and improving the design specification and capabilities of machine tools has been amply demonstrated in this investigation.

5. Conclusions

Realistic and clearly defined optimization strategy for single pass end-milling on CNC machine tools, allowing for the many practical constraints, has been presented based on the criteria typified by the minimum production time per component. Despite the complexity, the detailed optimization analysis assisted by the feed-speed diagrams has provided a deeper understanding of the economic characteristics and the influence of the constraints and machining performance data, and a means of guaranteeing the global optimum solution.

The numerical studies have shown the substantial penalties in production time per component incurred when using handbook recommended rather than optimized cutting conditions in end-milling operations, and highlighted the increased benefits of using optimization strategies in process planning in modern computer controlled and automated manufacture, where the proportions of non-productive times are low and continually being improved. These studies have also demonstrated the importance and use of the optimization strategy in assessing and selecting machine tools in production as well as in improving the machine tool capabilities and specifications at the design stage for economic and efficient production.

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Nomenclature

a_a, a_r	axial and radial depths of cut, respectively
A, G, G_1, α, β, ε, δ	constants in cutting force, torque and power equations
C_T	average cost (excluding material) per component
C_{To}	optimum C_T
D	cutter diameter
f_z, f_{zo}	feed per tooth, optimum feed per tooth
f_{zmin}, f_{zmax}	min. and max. permissible f_z due to machine tool feed and speed constraints
f_{zFf}	feed per tooth limit due to feed force constraint
f_{zFTR}	feed per tooth limit due to combined feed force, torque and surface roughness constraints
f_{zRt}	feed per tooth limit due to surface roughness constraint
f_{zTq}	feed per tooth limit due to torque constraint
F_{fmax}	machine tool maximum feed force constraint
K, n, n_1, n_2, n_3, n_4, n_5	tool-life equation constants
ℓ	length of cut
N_{min}, N_{max}	machine tool min. and max. spindle speed constraints

P	machining power
P_{max}	machine tool maximum power constraint
R_{tmax}	max. surface roughness (peak-to-valley) constraint
T	tool-life in time units
T_{ac}	actual cutting time
T_c	cutting (feed engagement) time
T_L	loading/unloading and idle time per component
T_{qmax}	machine tool maximum torque constraint
T_R	cutter replacement time per failure
T_T	average total production time per component
T_{To}	optimum T_T
T_{Tr}	T_T from handbook recommended V and f_z.
v_f	feed speed
v_{fmax}, v_{fmin}	machine tool max. and min. feed speed constraints
V, V_o	cutting speed (=πDN), optimum cutting speed
V_{max}, V_{min}	machine tool max. and min. cutting speed limits for a given D
x	labour and overhead cost rate
y	cutter cost per failure
z	number of teeth on milling cutter
μ	constant of proportionality (= $1/\pi$)

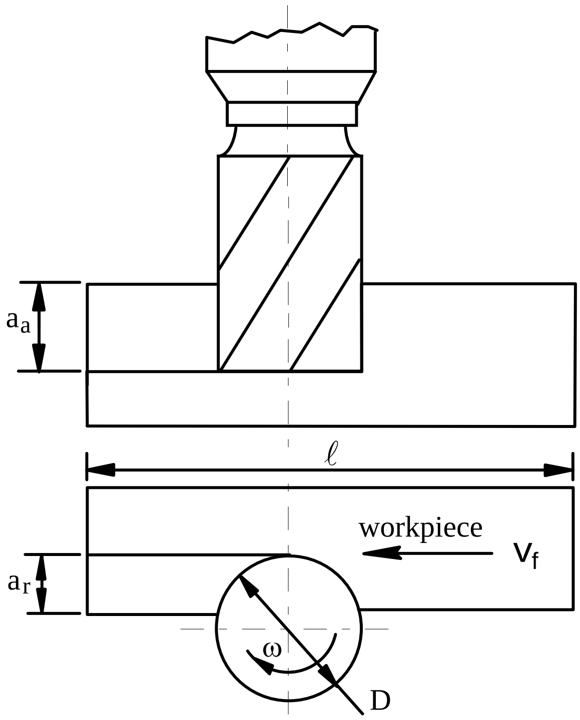
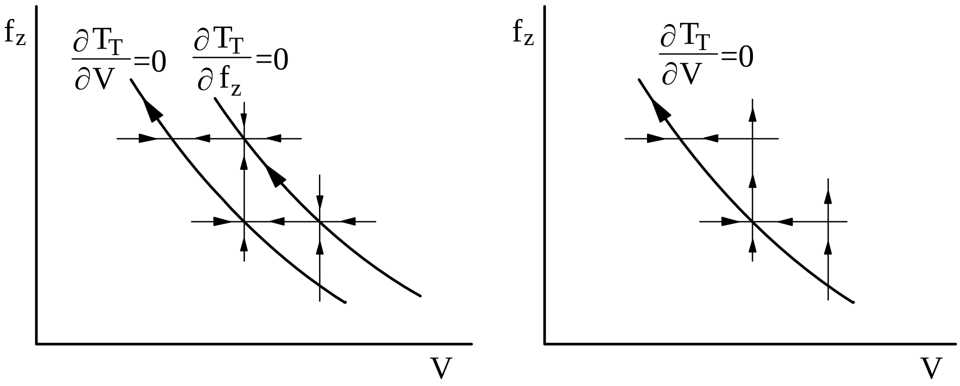
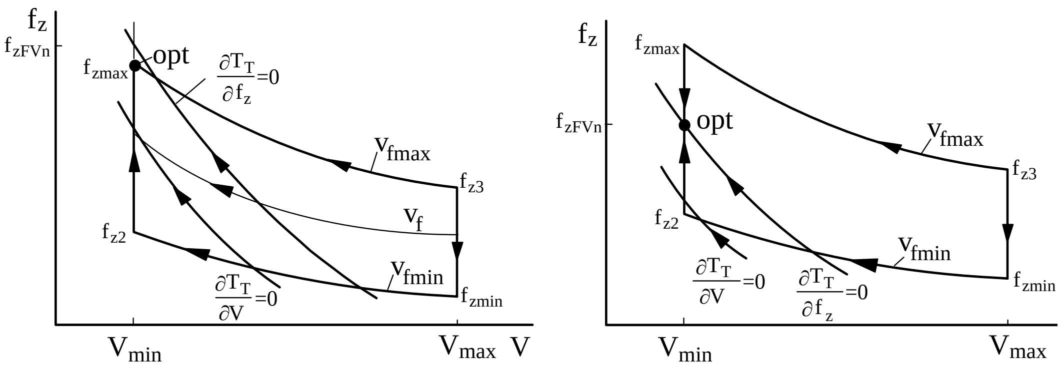


Fig. 1. Some process variables in single pass end-milling.

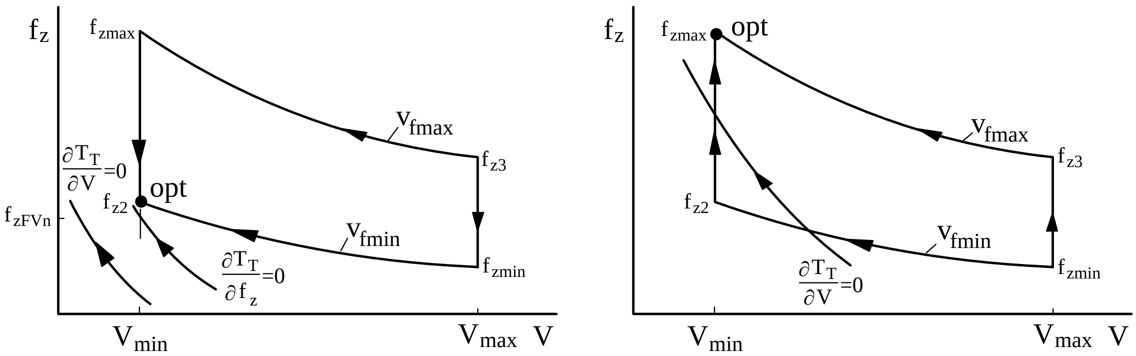


(a) $1/n > 1/n_1 > 1$ (b) $1/n > 1 \geq 1/n_1 > 0$

Fig. 2. Diagrammatic presentation of time per component characteristics.



(a) $1/n > 1/n_1 > 1$ and $f_{zFVn} \geq f_{zmax}$ (b) $1/n > 1/n_1 > 1$ and $f_{zmax} > f_{zFVn} > f_{z2}$



(c) $1/n > 1/n_1 > 1$ and $f_{zFVn} \leq f_{z2}$ (d) $1/n > 1 \geq 1/n_1 > 0$

Fig. 3. Time per component characteristics and the various constrained optimum solutions under feed and speed boundary constraints.

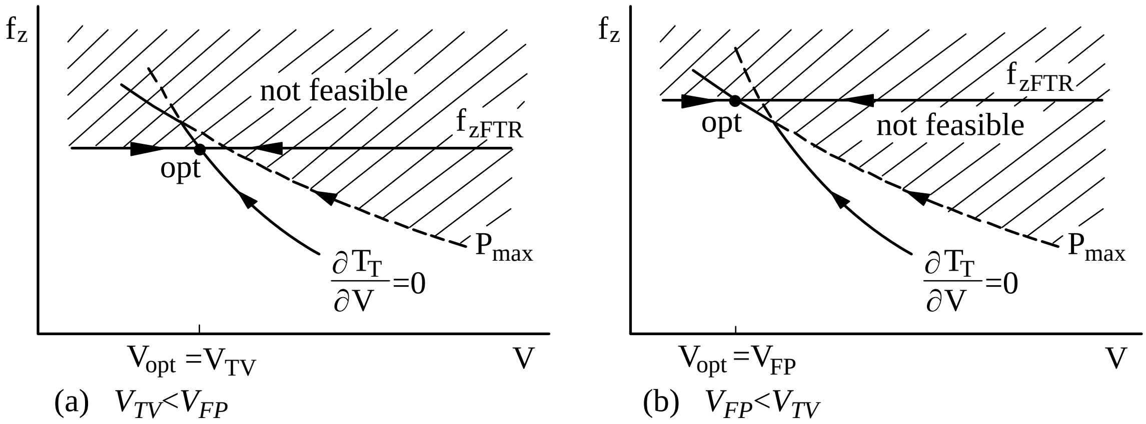


Fig. 4. Time per component characteristics and the various constrained optimum solutions under feed force, torque, power and surface finish constraints.

Fig 5 is not available on e-file.

Fig. 5. Various possible constrained optimum solutions for single pass end-milling on CNC machine tools.

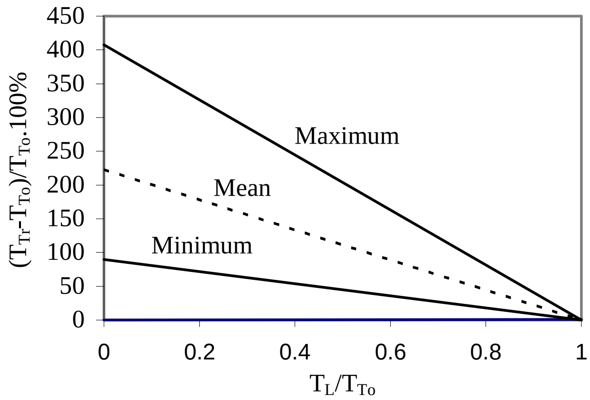


Fig. 6. Production time penalties for alternate non-productive loading/unloading times.

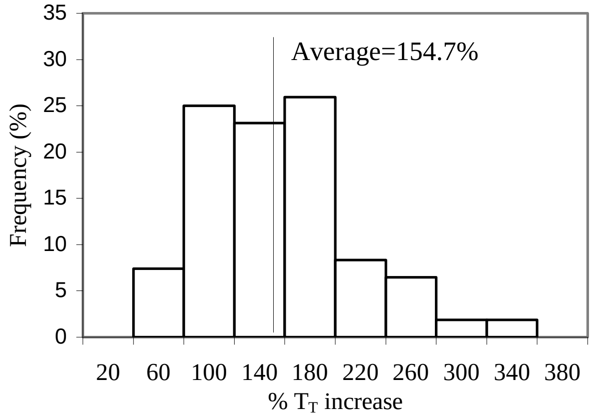


Fig. 7. Production time penalties of using handbook recommendation over optimization.

Table 1. Specifications for the three CNC machining centers.

	SABRE-750	ANCA-MMC800	ANCA-MMC800-1
Feed speed (mm/min)	3 - 1,200	0 - 5,000	0 - 5,000
Spindle speed (rpm)	60 - 8,000	0 - 5,000	0 - 10,000
Max. power (KW)	11.2	7.5	7.5
Max. torque (NM)	143	48	24
Feed force (N)	3,100	8,000	8,000

Table 2. Cutting conditions for simulation study.

Radial depth of cut a_r (mm)	3, 5, 8	12	16, 20
Cutter diameter D (mm)	16	25	40
Number of teeth z	3	3	4
Axial depth of cut a_a (mm)	20	30	40
Length of cut ℓ (mm)	150, 300, 450, 600, 750, 900		
Non-productive time T_L=1 min. Cutter change time T_R=0.2 min			

Table 3. Sample results of optimum feed and speed solutions (ℓ=600 mm, D=16 mm, z=3).

a_r (mm)	a_a (mm)	f_{zo} (mm)	V_o (m/min)
3	20	0.202	64.9
3	30	0.127	82.9
3	40	0.085	98.7
5	20	0.121	68.5
5	30	0.069	87.5
5	40	0.046	104.1
8	20	0.069	72.0
8	30	0.039	92.0
8	40	0.026	109.5

Table 4. Summary of optimum T_T (in minutes) for CNC machining centre simulation.

	SABRE-750	ANCA-MMC800	ANCA-MMC800-1
Average T_{To}	5.73	5.68	8.52
Maximum T_{To}	20.18	23.84	37.69
Minimum T_{To}	1.26	1.17	1.27
T_{To} range	18.92	22.67	36.42