The use of manipulative materials in early place value instruction: a cognitive load perspective

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The use of manipulative materials in early place value instruction: a cognitive load perspective

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Teaching mathematical concepts is often accompanied by the use of worked examples, and the use of manipulative materials. Worked examples have been shown to be an effective method of instruction with novice learners, as shown by higher test performance and shorter acquisition times. Often worked examples are accompanied by illustrations of manipulative materials, and the physical use of such materials. One such example is the use of Multi-base Arithmetic Blocks (MAB) to teach place value. Whilst the use of worked examples alongside illustrations of MAB in instructional material is common, their efficacy has not been investigated in young students. Using the concepts of cognitive load theory, which investigates how the learner's limited working memory and vast long-term memory can be used to efficiently design educational material, this research examines the effects of using worked examples alongside MAB to teach place value to young students. Experiment 1 examined whether it was possible to facilitate a more efficient transition from the manipulative material to the abstract concept of place value. Using the abacus and MAB, and two methods of instruction, the results of Experiment 1 showed no significant differences in student performance between the methods of instruction, or the manipulative materials. Using worked examples in Experiment 2, no significant differences in the performance of students using either the abacus or MAB, were found. Experiment 3 reduced the level of the reading comprehension in the instructional material. The control group, with no access to MAB, performed better in the post-test than the group using MAB. Experiment 4 examined whether the use of MAB produced a redundancy effect, by providing identical information in three different formats. No significant differences were found due to the complexity of the instructional material. In Experiment 5, the expertise of students with respect to the experimental materials was decreased. The non-MAB group performed better in the post-test and the delayed test, than the group using MAB. Thus, despite the widespread use of MAB in primary school to teach place value, this research suggests that the blocks may produce a redundancy effect, leading to an increased extraneous load, and negatively affecting learning.

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Abstract

Teaching mathematical concepts is often accompanied by the use of worked examples, and the use of manipulative materials. Worked examples have been shown to be an effective method of instruction with novice learners, as shown by higher test performance and shorter acquisition times. Often worked examples are accompanied by illustrations of manipulative materials, and the physical use of such materials. One such example is the use of Multi-base Arithmetic Blocks (MAB) to teach place value. Whilst the use of worked examples alongside illustrations of MAB in instructional material is common, their efficacy has not been investigated in young students. Using the concepts of cognitive load theory, which investigates how the learner's limited working memory and vast long-term memory can be used to efficiently design educational material, this research examines the effects of using worked examples alongside MAB to teach place value to young students. Experiment 1 examined whether it was possible to facilitate a more efficient transition from the manipulative material to the abstract concept of place value. Using the abacus and MAB, and two methods of instruction, the results of Experiment 1 showed no significant differences in student performance between the methods of instruction, or the manipulative materials. Using worked examples in Experiment 2, no significant differences in the performance of students using either the abacus or MAB, were found. Experiment 3 reduced the level of the reading comprehension in the instructional material. The control group, with no access to MAB, performed better in the post-test than the group using MAB. Experiment 4 examined whether the use of MAB produced a redundancy effect, by providing identical information in three different formats. No significant differences were found due to the complexity of the instructional material. In Experiment 5, the expertise of students with respect to the experimental materials was decreased. The non-MAB group performed better

in the post-test and the delayed test, than the group using MAB. Thus, despite the widespread use of MAB in primary school to teach place value, this research suggests that the blocks may produce a redundancy effect, leading to an increased extraneous load, and negatively affecting learning.

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1 Introduction

Place value, defined as the numerical value that a digit has by virtue of its position in a number (A. Stevenson, 2010), is at the core of most basic arithmetic manipulations and procedures. Understanding place value is, therefore, a fundamental building block for the future mathematical understanding and performance of students (Kamii, 1986; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Nataraj & Thomas, 2007; Schmittau & Vagliardo, 2006).

Currently, the teaching of place value concepts is often accompanied by the use of manipulative materials, such as Multi-base Arithmetic Blocks (MAB blocks), which are designed to provide a concrete representation for the number bases. However, it remains unclear whether the use of such materials increases the extraneous load on the student by requiring the student not only to understand the blocks and how to manipulate them, but also to comprehend the number concepts they are trying to represent. Herein lies a problem of dual representation, with students being required to translate between the concrete blocks and the meaning of number bases, using their limited cognitive resources. Thus, it remains unclear whether the use of MAB or other manipulative materials in early mathematics education is an efficient way to assist students in understanding place value concepts, due to the implications of cognitive load theory (Sweller, 2011). Children's understanding of the base-10 number system emerges only with formal schooling and as a direct result of teaching. However, children do possess intuitive skills that involve the ability to bundle objects into sets.

The aim of this research is to draw on notions of evolutionary educational psychology (Geary, 2008a) and on cognitive load theory and the associated effects (Sweller, van Merriënboer, & Paas, 1998), in order to examine whether certain manipulative materials might present a more intuitive representation of number concepts for children or whether the use of such materials may negatively affect learning due to the high level of processing required by the student. The fundamental argument of this thesis is that using manipulative materials as part of instructional material may lead to the redundancy effect, which occurs when identical information is presented to the learner simultaneously, but in different formats. This could result in a negative impact on learning. In exploring this argument, worked examples were used as an instructional method. These have been shown to improve performance and learning outcomes when used with novice learners. Worked examples are also commonly used in mathematics education, and, therefore, provide a familiar format to students. Additionally, using worked examples in the instructional material ensured a consistent method of intervention across the different student groups.

The second chapter presents a discussion of human cognitive architecture, with consideration given to working and long-term memory concepts. Cognitive number processing is also discussed, as pertinent to the subject matter of this thesis. The chapter concludes with a general overview of evolutionary biological primary and secondary skills, as proposed by Geary (2008a) in his contribution to the body of knowledge of cognitive load theory.

Chapter 3 provides a general overview of cognitive load theory, including a discussion of the three types of cognitive load and of the potential measurement mechanisms for each. The chapter concludes by discussing the cognitive load effects relevant to this thesis.

Chapter 4 discusses the acquisition of the Hindu-Arabic number system, by explaining the basis of the system and the instructional materials currently used in place value mathematics education.

The following chapter, Chapter 5, provides a brief overview of the literature relevant to this thesis. It presents a general overview of the series of experiments conducted as part of this thesis, which investigated the efficiency of teaching place value concepts with manipulative materials, from the perspective of cognitive load theory.

The experimental data and the results of these experiments are presented in Chapters 6-10. The results partially support the hypothesis and provide some evidence to question the use of MAB blocks as an aid in teaching place value concepts.

Chapter 11 summarises the major findings of the study, taking into consideration the concepts and effects of cognitive load theory. The chapter also considers the limitations of this study and proposes future research directions. Both theoretical and practical implications for teaching place value, with the assistance of manipulative materials, are also presented in this chapter.

2 Human Cognitive Architecture

2.1 Chapter Overview

The system of cognitive architecture is defined by Langley, Laird and Rogers (2009, p. 141), as something that specifies the "…underlying infrastructure for an intelligent system…". Similarly, human cognitive architecture refers to the structure and organisation of our cognitive resources that allows humans to integrate, process and use information (Sweller, 2011; Sweller et al., 1998). Human cognitive architecture is composed of memory structures that are fundamental to the human ability to think, learn and solve problems. Current research proposes two vital memory structures, working memory and long-term memory, as the basis of human cognitive architecture (Baddeley, 1998, 1999, 2000; Baddeley & Hitch, 1974). Understanding how these memory structures function can assist with designing the most effective instructional material.

Working memory is essential for conscious thought, but is only able to process a small number of elements at the one time, with the number often placed at 7 ± 2 (Miller, 1956). The limited working memory capacity can be improved through a process known as chunking, in which related elements are grouped together in long-term memory to form meaningful structures. Long-term memory can hold an unlimited number of elements on a permanent basis (Sweller, 2011). Storage in long-term memory occurs through the use of cognitive constructs, known as schemas, where related information is stored together for easy retrieval when required.

Cognitive load theory refers to the total amount of cognitive load required in working memory to complete a specific task. The processing capacity of each individual differs, based on the extent of their knowledge held in long-term memory in regards to a specific task. Learners who have more in-depth knowledge and understanding of a particular task are considered to be experts. Conversely, novices do not possess in-depth knowledge and understanding of the same task. This means that the cognitive load of an expert is often much lower than that of a novice when completing the same task. Catering for both expert and novice learners is an important consideration in the design of effective instructional material.

The theory has recently been expanded to take into consideration an evolutionary perspective of the development of human cognitive architecture (Geary, 2002, 2005, 2008a; Geary & Bjorklund, 2000; Muller, 2010; Paas & Sweller, 2011; Sweller, 2004, 2008). One of the primary goals of evolutionary educational psychology is to uncover inherent biases to learning, and to use knowledge of those biases for instructional design and reform. In his evolutionary framework, Geary (2008a) draws a clear line between two types of skills and knowledge, primary and secondary. Knowledge that is acquired effortlessly, and that humans have evolved to acquire naturally, is referred to as primary knowledge. This is in contrast to secondary knowledge, which requires specific instruction. Geary (1996) suggests six areas of primary mathematical knowledge: numerosity, ordinality, counting, estimation, basic arithmetic and geometry. He believes that building on an evolutionary knowledge can be more effective than using non-evolutionary instructional theories (Geary, 2007; Muller, 2010).

Currently, early primary school education is not making full use of children's primary spontaneous skills to facilitate the acquisition of secondary knowledge (Geary, 2008a; Muller, 2010). Educators should be more mindful of primary intuitive knowledge and how this knowledge may be exploited to help acquire secondary knowledge more effectively (Keil, 2008). The use of physical materials in the classroom could present one way of exploiting this primary knowledge. Physical materials, commonly referred to as manipulative materials, are widely used in the classroom to help students understand abstract mathematical concepts. Using such materials is inherently a primary skill. Children may use the materials intuitively in their play in order to learn. Mix, Huttenlocher and Levine (2002, p. 130) state that

"…working with concrete materials serves to help children bridge the gap between conventional symbols and their pre-existing concepts. This suggests that educators should place greater emphasis on the connections between symbols and experiences rather than simply providing the experiences themselves.". Whilst some manipulative materials may be a very useful tool in the classroom, they must be used correctly. For this to occur, children must have a basic understanding of the concept being taught and not simply play with the manipulative material (Smith, 2008). For this reason, research is still equivocal as to whether there is a clear and consistent benefit in using manipulative materials in the classroom rather than more traditional methods of teaching (Bartolini Bussi, 2011; Kaminski, Sloutsky, & Heckler, 2008; Sowell, 1989; Uttal, Scudder, & DeLoache, 1997; Wright, 2013).

2.2 Memory and modeling memory

An understanding of human cognitive architecture, composed of memory structures such as sensory memory, working memory and long-term memory, is essential in establishing the fundamentals of learning, problem solving and instructional design. A number of different memory models have been put forward. One of the initial propositions defining a memory model made up of multiple stores, was developed by Atkinson and Shiffrin (1968). It suggests a flow of information through the system in the following sequence (Figure 1). Incoming information enters the sensory register (sensory memory) and resides there for a brief period of time. From sensory memory, information flows onto the working memory store, where it can also only be kept for a short period of time. Through a process called rehearsal, information can then be moved into long-term memory. The long-term store is a permanent repository for information and cannot be easily accessed, although it allows for the retrieval of information from the long-term memory to the short-term memory, as required. Information that does not undergo the process of rehearsal is lost over a short period of time.

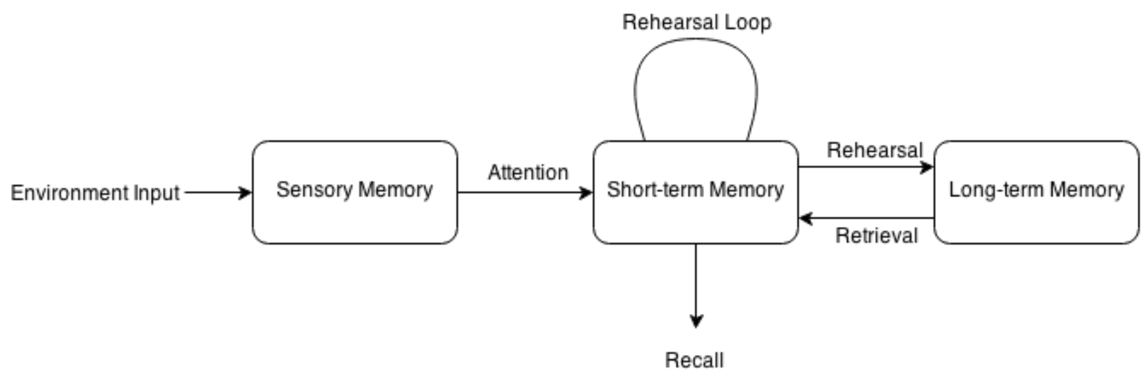


Figure 1: Memory Model composed of three different stores first proposed by Atkinson and Shiffrin (1968)

The Atkinson-Shiffrin model (R. C. Atkinson & Shiffrin, 1968) was one of the most influential memory models of its time, in part because it laid out a comprehensive view of information processing in the memory system. However, this is a fairly simplistic model of human cognitive architecture, and many improvements have been made to this model over the years. In particular, there has been a change in thinking about short-term memory function, and this shift is reflected in the use of the term working memory, instead of short-term memory, to describe what was once considered to be an extremely brief, unitary store.

2.3 Working Memory

Building on the initial research by Atkinson and Shiffner (1968), a working model of memory was proposed by Baddeley and Hitch (1974). It assumed that short-term memory is not only a unitary store, but it can also provide a workspace for complex cognitive activity. Baddeley and Hitch (1974) showed that working memory was composed of three basic parts: the central executive, the visuo-spatial sketch pad, and the phonological loop. The central executive is the driver of the overall system, assigning data to either of the two other components. It is also responsible for mental arithmetic and problem solving. The visuo-spatial sketchpad is where information presented in a visual or spatial format is stored and processed. Finally, the phonological loop deals with spoken and written material, consisting of two further components: the phonological store, dealing with speech, and the articulatory control linked to speech production (Figure 2).

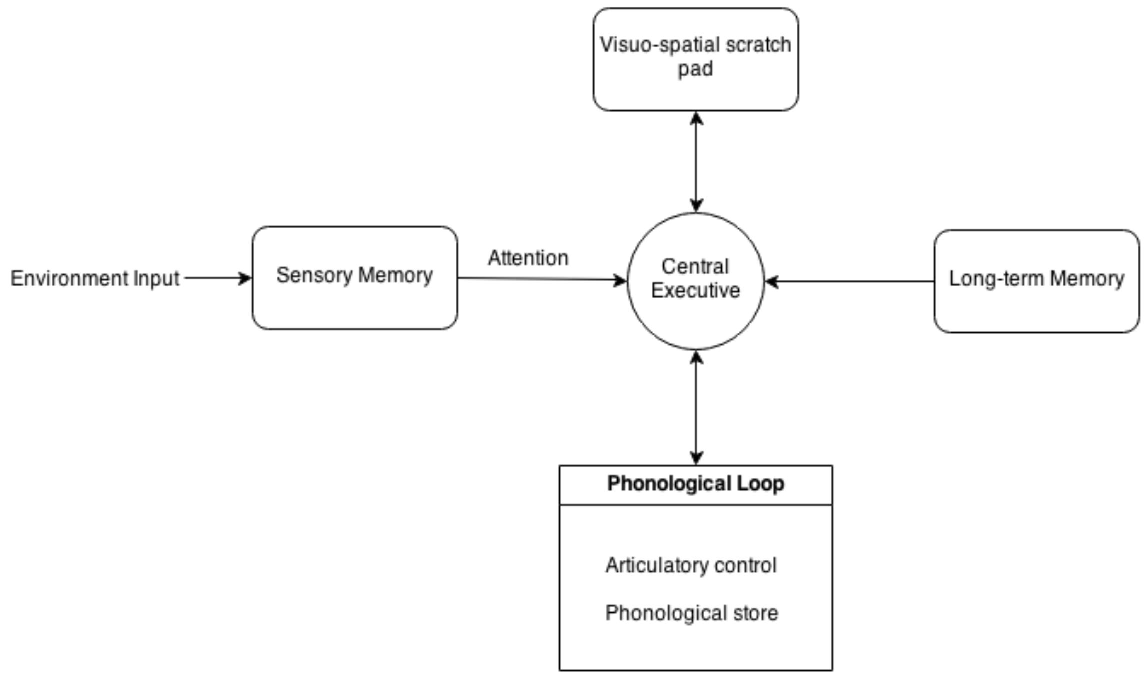


Figure 2: The working memory model as proposed by Baddeley and Hitch (1974)

Further research resulted in the addition of the fourth component to the working memory structure, the multimodal episodic buffer, which comprises a limited and temporary storage of information held in a multimodal code (Baddeley, 2000) (Figure 3). The episodic buffer is also controlled by the central executive, which is responsible for amalgamating information from multiple sources into a unitary episode. The buffer serves as a modeling space, separate from the long-term memory, but an important stage in long-term episodic learning (Baddeley, 2000).

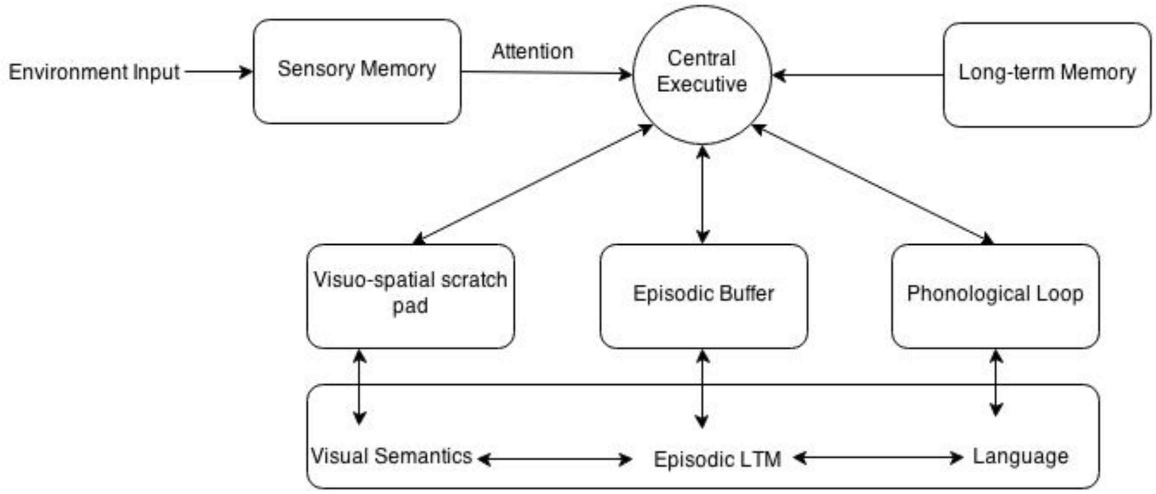


Figure 3: Updated Model of Working Memory (Baddeley, 2000)

The central aspect of the working memory model developed by Baddeley and Hitch (1974) is the ability of working memory to act as more than just a temporary storage space. According to the model, working memory is a dynamic construct, a system consisting of two temporary storage facilities, the first to process visual information, and the second to process auditory information, and, in addition, a system to control these stores. The central executive unit is the system that controls both visual and auditory processing. This is the component that most strongly differentiates current thinking about working memory from historical views of short-term memory. The central executive is responsible for deciding what information is stored and where the information should be stored. Additionally, it is responsible for any integration, coordination and cognitive manipulation of the information held in the separate stores. This component determines how to maximize the cognitive resources available, and how to suppress irrelevant information that would overuse the resources (Baddeley, 1998). The model further differentiates between the processing of phonological and visual information. Audio-based information directly enters the phonological store, a passive storage facility within the phonological loop, and is held there until it decays a couple of seconds later. Written material must first be converted to articulatory code before it can enter the phonological store, and can also be stored there for up to two seconds. The second part of the phonological loop, the articulatory control, rehearses the information from the phonological store and circulates the information until it is remembered. The articulatory control is also responsible for converting written material into audio code and transferring it into the phonological store (Baddeley, 1999) (Figure 4).

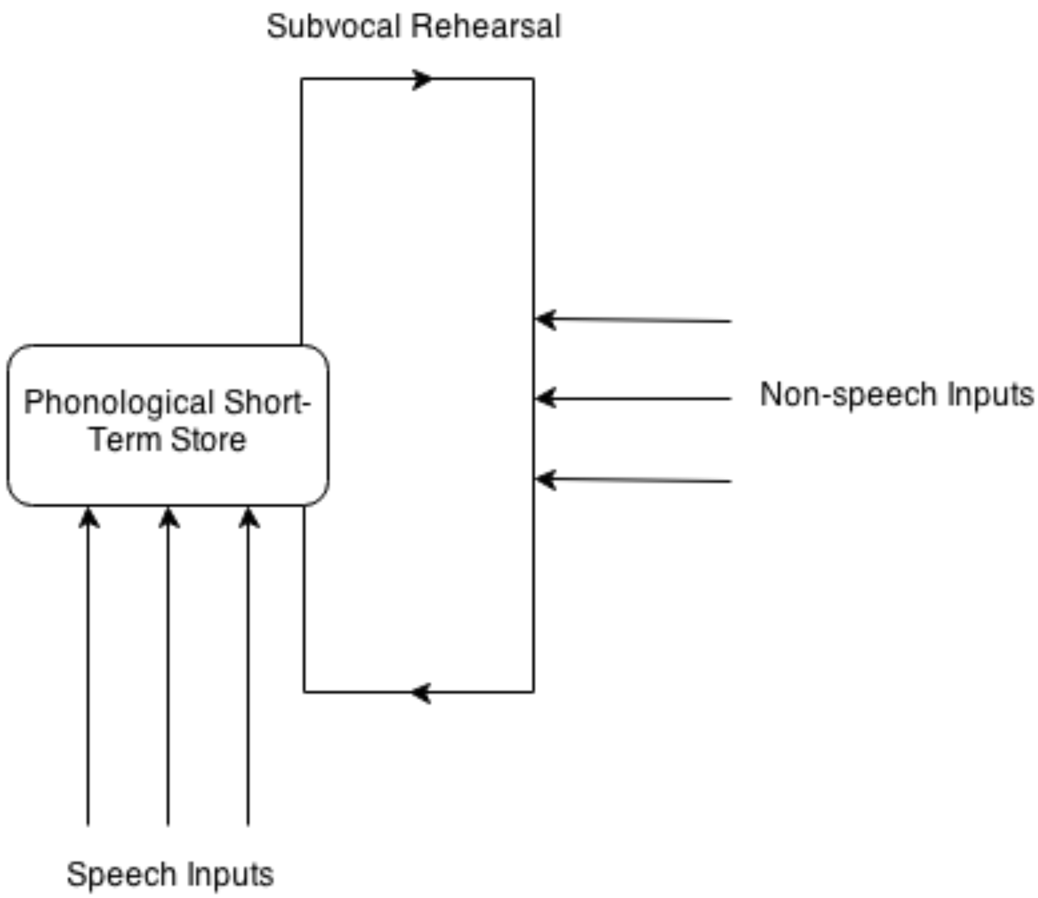


Figure 4: Phonological Loop (Baddeley, 1998)

The visuo-spatial sketchpad, a second temporary memory store, allows visual and spatial information to be held so that it can be processed. Research has shown that the visuo-spatial sketchpad can operate simultaneously with the phonological loop, allowing both visual and auditory information to be processed without one affecting the efficacy of the other (Cornoldi & Vecchi, 2004).

Sweller (2004) postulates against the existence of the central executive unit, and the lack of empirical evidence provides a strong indication against its existence. Sweller (Sweller, 2004) postulates that the central executive is not a general biological structure, but rather a specific learned structure, not available when processing novel information. He proposes that schemas, in long term memory, act as a central executive funtion for the working memory. If such a schema is not availble, as is the case with novel information, learners attempt a series of random combinations followed by tests to establish the effectiveness of those combinations, which can act as a substitute for the central executive function (Sweller, 2003, 2004).

However, the model of working memory, consisting of two different storage buffers for visual and audio information, is strongly supported by dual-task studies, which require the concurrent performance of two tasks that could otherwise be performed separately (Baddeley & Hitch, 1974). In specific examples, verbal concurrent tasks affect the storage of verbal information, but not visuo-spatial information, whereas visuo-spatial tasks affect the storage of visuo-spatial information but not auditory information (Cocchini, Logie, Sala, MacPherson, & Baddeley, 2002; Olive, 2003).

Novel information enters working memory via sensory memory and is then processed in the working memory structure. However, the ability to process complex information is hindered by the limited capacity of working memory. Miller (1956) suggests that most adults can only store in their short-term memory between five and nine (seven plus or minus two) items that have not been previously learnt, and then only for a brief duration. This limit also applies to working memory, if information storage or maintenance only is involved; if processing is also involved then this limit could be further reduced. If information needs to be processed or manipulated in any manner, the number of elements that can be kept in working memory could be as few as two or three (Cowan, 2001). If not rehearsed, novel information can only be held in working memory for about 15-30 seconds (Driscoll, 2005). These limitations have important implications for instructional design and the way new information is taught to students. It is important that new information is taught in ways that compensate for the limited capacity and duration of working memory. Instructional design that requires students to process many elements of complex novel information at the same time will lead to failure in the learning of that material (Sweller, 2002).

Information that has already been learnt and is stored in long-term memory is available for processing only after it is first retrieved and transferred temporarily into working memory. Unlike information entering from sensory memory, information retrieved from long-term memory is not hindered by the limited processing capacity of working memory. Since information in the long-term memory is stored in an organised form, there may be no limits to the amount of such information (Sweller, 2002). Consequently, there are further implications for instructional design based on whether students have prior knowledge of the material being taught. Students who have prior knowledge of new material that has already been organised in their long-term memory will be able to process new information very quickly, as opposed to those students who are hearing or seeing this information for the very first time.

2.4 Chunking Theory

When discussing storage limitations of short-term memory, Miller (1956) suggested seven plus or minus two as the maximum number of chunks, or groups, that a person can hold. Chunking, initially proposed by Miller (1956) and de Groot (1965), and theorized by Chase and Simon (1973), is an important mechanism in circumventing working memory limitations and increasing the ability to hold more information than a maximum of seven singular elements. Chunking involves combining together related pieces of information according to familiar patterns, in order to form a single chunk, which is then treated in working memory as one element. One of the most commonly used examples of chunking is the way phone numbers are grouped. A number such as 3412975837 might be hard to remember when each of the ten numbers has to be remembered separately, but when chunked into three smaller singular units, such as 3412 975 837, it becomes possible and easier to remember, especially if some of the units contain familiar combinations of numbers (e.g., associated with some well-remembered dates, codes, et.).

More recent research has focussed on the mechanisms of chunking. This research has cast some doubt over whether chunks are stored in working memory or in our long-term memory (Gobet, 2000; Guida, Gobet, Tardieu, & Nicolas, 2012). Gobet (2000) proposed that some of the information encoded as chunks is stored in long-term memory. A study by Ericsson, Chase and Faloon (1980) found that an average undergraduate college student, who had been engaged in performing memory span tasks for more than 230 hours, was able to increase his memory span from 7 digits to 79 digits. The student utilised a mnemonic technique to chunk the numbers into familiar structures. The use of mnemonics allowed the student to relieve the burden placed on his working memory through an association with the already existing knowledge of sports records in his long-term memory. For example, 3492 was recorded by the student as "3 minutes and 49 point 2 seconds, near world-record mile time" (Ericsson et al., 1980). The study led Ericsson and his colleagues (1980) to develop their theory of skilled memory and to conclude that, whilst it is not possible to increase the capacity of working memory with extended practice, it is possible to increase the memory span with the use of chunking, based on the related associations in long-term memory.

To attain exceptional memory performance, prior knowledge must be used to encode and store items in long-term memory for later retrieval (Ericsson, Delaney, Weaver, & Mahadevan, 2004). With practice and rehearsal, the encoding and retrieval processes can be greatly sped up, causing dramatic improvements in memory performance. This is consistent with skilled memory theory and its generalization to long-term working memory (Ericsson et al., 2004). Recent studies have further confirmed that at least some of the information contained in chunks is stored in long-term memory, due to the absence of the effect of an interfering task

(Ericsson & Kintsch, 1995; Gobet & Simon, 1996; Kintsch, 1998). The study by Ericsson et al. (2004) proposed that some chunking information was stored in the long-term memory. Their experiment looked at a person with superior memory abilities, and found that the subject's encoding techniques were a result of memorizing the first 10,000 digits of π as numerous 10-digit groups, consistent with the chunking information residing in long-term memory (Ericsson et al., 2004). In the case of a domain expert, a process meant to be occurring solely in working memory can also involve storage and retrieval of chunks from long-term memory (Guida et al., 2012). Such studies further demonstrate that some chunking information can be stored in long-term memory, thereby greatly improving memory skills by overcoming the limited capacity of working memory.

2.5 Working Memory and Mathematics

Considerable research conducted in the last decade suggests that working memory plays a significant role in mathematical cognition (Alloway, Gathercole, Willis, & Adams, 2004; Ashcraft & Krause, 2007; Berg, 2008; Bull & Scerif, 2001; Geary, 1990; Holmes & Adams, 2006; Passolunghi, Vercelloni, & Schadee, 2007; Siegel & Ryan, 1989; St Clair Thompson, Stevens, Hunt, & Bolder, 2010; Swanson & Sachse-Lee, 2001; van der Sluis, van der Leij, & de Jong, 2005; Zheng, Swanson, & Marcoulides, 2011). Raghubar, Barnes, and Hecht (2010) noted that "…the very nature of many mathematical tasks would seem to require or at least be supported by working memory…". Indeed, research has shown that children with a higher working memory capacity tend to learn early mathematics more efficiently (Hoard, Geary, Byrd-Craven, & Nugent, 2008; Passolunghi, Mammarella, & Altoè, 2008; Passolunghi et al., 2007; Passolunghi & Siegel, 2001, 2004).

Hecht (2002) proposed that children use different strategies to solve maths problems, some of the strategies drawing heavily on working memory resources. Research by Lee, Ng and Ng (2009) used a multitask approach to examine whether working memory is associated with generating a model of the problem and then solving the problem. They tested 255 students, all of them 11-years-old, using algebraic problem solving tasks, and found that differences in working memory capacity accounted for about a quarter of the variance in building a model of the problem and then solving that problem. Literacy, however, accounted for another 10% of variability in algebraic problem solving skills (K. Lee et al., 2009). This is consistent with other research by Zheng, Swanson and Marcoulides (2011) who proposed that whilst all working memory components play significant roles in children's ability to solve mathematics problems, reading skills can compensate for some of the gaps in working memory measures. Conversely, Swanson and Beebe-Frankenberger (2004), using a large sample of first, second and third graders, found that in the area of word problems, working memory predicted solution accuracy independent of reading ability. A review of literature by LeFevre et al. (2005) supports the notion that working memory is heavily involved in problem solving, when the problems reach sufficiently high levels of complexity.

There is, however, some debate as to the importance of the different components of working memory to the various mathematical skills (Ashcraft, 1992; Fürst & Hitch, 2000; Hecht, 2002; Raghubar et al., 2010). Meyer et al. (2010) suggested that the central executive and the phonological loop are vital in facilitating mathematical learning in the early stages (e.g., at Grade 2 level), whereas the visuo-spatial sketchpad plays an increasingly vital role in later stages of learning mathematical skills. Another study that is consistent with these results (Passolunghi et al., 2007) tested 170 children at the beginning and at the end of Year 1, approximately aged 6-7 years. From the results, the research similarly proposed that, in the early stages of mathematical learning at school, working memory and, specifically, the central executive unit, is a distinct and significant predictor of success. The study found no link to phonological ability as a predictor of mathematical success (Passolunghi et al., 2007).

In contrast to the above findings, other studies have found that school-aged children rely heavily on the phonological loop for subvocal rehearsal, ensuring accuracy during mental calculations involved in arithmetic problem solving (Logie, Gilhooly, & Wynn, 1994). Using a large sample of 8- to 11-year-olds, Adams and Hitch and Donlan (1998) found that the phonological loop is essential for the ability to solve arithmetic problems. The role of the visuo-spatial sketchpad is not as clear, although in more recent work, Holmes and Adams (2006) found that both the visuo-spatial sketchpad and the central executive, but not the phonological loop, were excellent predictors of curriculum-based mathematical success in early numerical development.

2.6 Long-Term Memory

Long-term memory is an organised store of information, altered through the acquisition of knowledge and it is widely accepted that an unlimited number of elements related in meaning can be stored together in long-term memory on a permanent basis (Baddeley, 1999; Ericsson & Kintsch, 1995; Sweller, 2004).

A series of experiments by De Groot (1965) examined the differences between chess grand masters and novice chess players. The chess grand masters were able to win almost all games, not because of their superior reasoning skills, but due to the fact that they were familiar with, and held in their long-term memory, the largest number of various chess board compositions from real game situations and their corresponding best moves. In this way, they were able to easily predict, with their prior experience, the best move from a large number of various board setups. A computer model built to replicate this process estimated that a chess grand master holds between 10,000 and 100,000 chess board configurations in his or her long-term memory (H. A. Simon & Gilmartin, 1973).

Later studies by Simon and Chase (1973) extended the experiment to include both real game positions and random positions. Their findings indicated that in random positions, chess grand masters performed on par with novice players, whilst in real game positions, chess grand masters performed significantly better in choosing their next move. This further reinforced De Groot's findings, linking long-term memory to higher-level cognitive activities such as problem solving and thinking (Chi, Glaser, & Rees, 1982; Sweller, Ayres, & Kalyuga, 2011). These findings have been replicated in areas other than chess, such as algebra (Sweller & Cooper, 1985), physics (Chi, Feltovich, & Glaser, 1981), and mechanics (Reif & Heller, 1982). Thus, it is widely accepted that expertise is heavily dependent on knowledge held in long-term memory (Sweller, 2002).

Tulving (1972) made one of the earliest distinctions between episodic, semantic and procedural long-term memories. Episodic memory is that part of long-term memory responsible for storing episodes in our lives associated with a certain time and place (Tulving, 1972). An example of episodic memory is recalling a city through memories of the last holiday taken there. Semantic memory is that part of long-term memory responsible for storing general information about the world, for example, basic knowledge of countries, continents and oceans, or knowledge required for the use of language. The above two types of knowledge are involved in conscious thought, if activated and retrieved into working memory. On the other hand, procedural memory does not require conscious thought and is that part of long-term memory responsible for motor skill memory, for example, the ability to ride a bicycle. The different types of memory differ greatly in the kind of prior experience involved: episodic and procedural memories require prior experience, whilst semantic memory does not (Greenberg & Verfaellie, 2010). Cohen (1980) was able to demonstrate a distinction between declarative and procedural knowledge in experiments with patients suffering from amnesia. Declarative knowledge is concerned with facts, and the recollection of this type of knowledge requires some degree of consciousness. Semantic and episodic memory structures are responsible for storing declarative knowledge. Conversely, procedural knowledge is based on motor skills and is automated; it does not require conscious thought for recollection.

Whilst amnesiac patients have trouble in acquiring new episodic or semantic information after the onset of amnesia, their knowledge of procedural information remains largely unaffected, and they are able to acquire new procedural skills (N. J. Cohen, 1980).

2.7 Schemas

Information in long-term memory is organised and stored in domain-specific structures, known as schemas. In their research, Rumerlhart and Ortony (1977) identified four essential characteristics of schemas, which work together to establish the efficient building blocks of memory. The first characteristic is that schemas have variations in their values, based on the context and environment of a specific situation. To demonstrate this point, Rumelhard and Ortony (1977) used the example of GIVE as a schema with three variables: a giver, a gift and a recipient. On different occasions, different variables will take on different values, whilst the relationships internal to the GIVE schema will remain constant. Another example is COMPUTER as a schema, which can include the motherboard, the shell, the graphics card, memory and RAM. Each of these variables can vary largely in their dimensions or other characteristics, but the schema itself remains consistent in that these variables are present.

Secondly, schemas can embed one within the other (Rumelhart & Ortony, 1977). To use the previous example of a COMPUTER schema, one of the elements, such as the shell, can form a sub-schema consisting of plastic, screws, clips, etc. This sub-schema is embedded within the dominating schema of the COMPUTER. There are certain advantages to embedding schemas, primarily the ability to look at the ‘big picture’ without the need to delve deeper into the internal structures. A second advantage is that a deeper understanding can be achieved if reference is made to the internal structure of the lower-level elements. This means that the COMPUTER schema can remain a simple schema with just the main elements, and without the need to examine every screw and clip holding elements in place, that is, until these lower-level structures need to be accessed.

The third characteristic is the ability of schemas to represent all levels of abstraction. This refers to the fact that schemas relate not only to concrete objects, such as the computer example used previously, but also to feelings and intangible concepts (Rumelhart & Ortony, 1977).

Finally, the fourth characteristic of schemas is their ability to represent actual knowledge rather than simply dictionary-based definitions and to "…represent knowledge in the kind of flexible way which reflects human tolerance for vagueness, imprecision, and quasi-inconsistencies…" (Rumelhart & Ortony, 1977, p. 111).

Schemas are engaged at all stages of cognitive processing. They not only allow the categorization of information based on similar elements, but also determine how that information will be used, thereby facilitating retrieval at a later stage when the information is required (Sweller et al., 1998). Basically, schemas are responsible for facilitating the flow of information through the cognitive processing system (Rumelhart, 1980) and are referred to as the "…building blocks of cognition…" (Rumelhart, 1980). Schemas help to reduce cognitive load by allowing a person to ignore any irrelevant information that would otherwise unnecessarily burden their working memory (Sweller & Chandler, 1994). For example, when one sees a dog, a schema allows the person to recognise the animal simply as a 'dog', despite there being many breeds of dogs. Because this information is stored in a schema in long-term memory, working memory does not need to be burdened with the different breed-specific elements presented in a single dog.

Although schemas are stored in the long-term memory, their construction occurs in working memory, where new material is first manipulated. Retrieval of relevant schemas from long-term memory depends on three factors, firstly, the strength of the information stored within the schema, secondly, the extent to which the incoming information matches with the information currently stored within the schema, and, finally, how recently and frequently the necessary schema has been used (Thorndyke & Hayes-Roth, 1977).

Schemas play an important role in increasing the capacity of working memory. Complex schemas, which store many elements of related information, can be treated in working memory as a single entity. This results, firstly, in an increased memory capacity and, secondly, in the ability to overcome the limitations of working memory that apply to new material with no associated schemas (Kirschner, Sweller, & Clark, 2006). Experts possess more complex and developed schemas in their long-term memory (van Merriënboer & Ayres, 2005) than non-experts, which is one of the factors differentiating a novice from an expert learner. This could be due to the fact that domain expertise has to be acquired slowly over years of deliberate practice (Ericsson, Krampe, & Tesch-Römer, 1993). As an example of this, a chess expert requires at least ten years of consistent and continuous practice to be able to store tens of thousands of board configurations in their long-term memory (H. A. Simon & Gilmartin, 1973). Another example is an algebraic schema, which allows a domain expert to easily transform $\frac{a}{b} = c$ into $a = bc$. Despite the fact that there are three elements and a set of relationships between them, this equation can be easily transformed by anyone with basic algebraic knowledge, due to the fact that the elements and relations are grouped into, and stored in, a single schema in long-term memory. Schema acquisition is directly related to the capacity of working memory and also to whether the cognitive load resulting from the material being learnt is in line with the cognitive capacity of the learner (Verhoeven, Schnotz, & Paas, 2009). Once acquired, schemas make processing in working memory a relatively simple task, because a schema acts as only one element, in contrast to the several elements present in the initial algebraic equation (Pollock, Chandler, & Sweller, 2002). The construction and automation of schemas are essential to the process of gradually transforming novices into experts (Verhoeven et al., 2009).

2.8 Schema Automation

Once a schema has been constructed, it needs to be practiced in order to promote automation (Kalyuga, Ayres, Chandler, & Sweller, 2003; van Merriënboer & Sweller, 2005). Schema automation occurs when information stored in schemas can be processed automatically with no conscious effort, bypassing the need to be processed within the working memory structure. To successfully achieve schema automation, a task requires much practice and repetition, and an automated schema can only develop if the task is consistent across a range of different problem situations (Schneider & Shiffrin, 1977; van Merriënboer & Sweller, 2005). An example of task consistency is the use of software applications, such as word processing tools, where the user makes use of the same functionality, located in the same area of the application, and produces the same result each time, allowing for maximized efficiency in task performance due to schema automation. In an early experiment looking at the effects of automation, Schneider and Shiffrin (1977) showed that the ability to differentiate letters from numbers is an automated skill in an adult. Approximately 2100 trials were required for participants to be able to differentiate between letters and letters as fast as they could differentiate between letters and numbers.

Automation allows familiar tasks to be performed quickly and efficiently without mistakes, and is an important element of instructional design. Efficient instructional design should, therefore, encourage both schema construction and schema automation (van Merriënboer & Sweller, 2005), as both these processes are essential for successful learning, "…Cognitive mechanisms of schema acquisition and transfer from consciously controlled to automatic processing are the major learning mechanisms and foundation of our intellectual ability and skilled performance…" (Kalyuga, 2010, p. 61).

2.9 Cognitive Architecture of Numerical Processing

Number competence in humans begins in early infancy and develops over time to encompass more complex number processing and manipulation skills (Geary, 1995). Various models of numerical cognition have been proposed, and have caused much debate (Jamie I. D. Campbell & Epp, 2004).

2.9.1 Modeling number cognition

McCloskey, Caramazza and Basili (1985) proposed a basic model of the cognitive architecture for numerical processing. The model examined three cognitive mechanisms involved in number processing: number comprehension, number production, and the execution of simple calculations. The model considered the functions of numerical comprehension and numerical production to be independent of each other (Figure 5). Basically, the model proposes a modular presentation of number cognition: comprehend numbers first, then perform number manipulations , and finally produce the result. An expanded model with all the components included is shown in Figure 6.

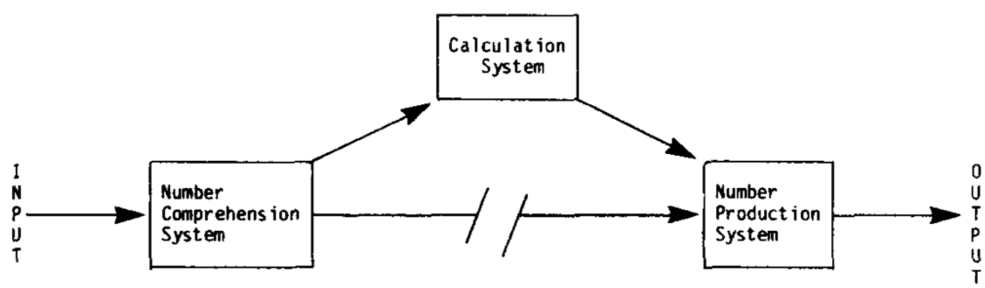


Figure 5: The functions of numerical comprehension and numerical production are independent to each other (McCloskey et al., 1985)

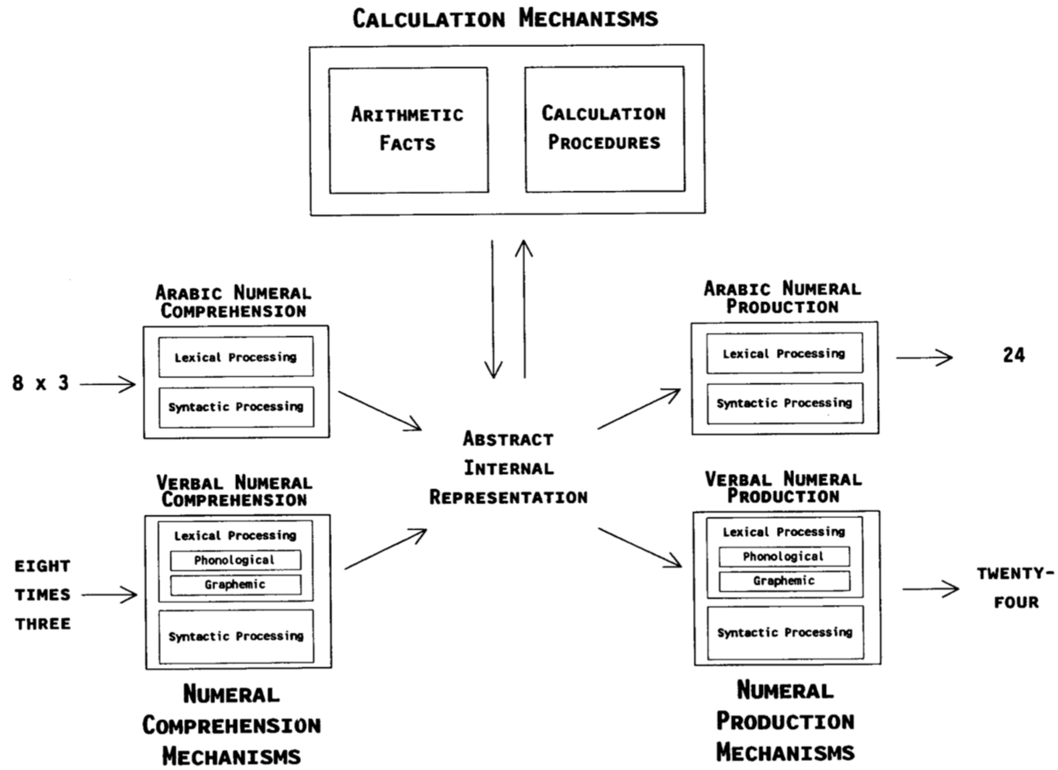


Figure 6: Cognitive Architecture for numerical processing (McCloskey et al., 1985)

Numeric comprehension and numeric production elements also distinguish between numbers spoken or written in alphabetical form, e.g. *one hundred and thirty-two* and numbers written in digital form, e.g. *132*. Within both forms of comprehension and production there is a distinction between lexical and syntactic processing components. Lexical processing involves the comprehension or production of individual elements in a number, such as the digit 3 or the word three (McCloskey et al., 1985). Syntactic processing involves the processing of the relationships amongst the different elements in order to produce the number as a whole. For example, using the number 132 again, lexical processing requires the understanding of the separate digits 1, 3, and 2 in the number, whereas syntactic processing ensures that the relationship between the positions of the digits determines the complete numeral. The model assumes a single 'abstract' semantic quantity code that is common to all the components (Jamie I. D. Campbell & Epp, 2004). Therefore, as an example, when a person is presented with a mathematical problem, the input (digital or verbal) needs to be translated into an abstract code, then the problem needs to be solved, and finally, the abstract code translated back into an output code (digital or verbal) (García-Orza, León-Carrión, & Vega, 2003). Accordingly, in mathematics, difficulties with reading numbers aloud could be independent from actual errors in calculations. Thus, this model assumes that mathematical processes, such as number production, number estimation and memory for number facts, operate independently of the way in which numerals are represented (written, verbal or Arabic). However, Campbell & Epp (2004), in their study examining the encoding approach in Chinese-English bilinguals, have argued against this model and have proposed that there are indeed mechanisms for format specific number judgements and calculations (Jamie I. D. Campbell & Epp, 2004; Dehaene, 2004).

In contrast to the abstract code model (McCloskey et al., 1985), an encoding-complex hypothesis has been proposed (Jamie I. D. Campbell, 1994). The hypothesis is based on empirical evidence of the existence of format-specific retrieval in number processing (Bernardo, 2001; Blankenberger & Vorberg, 1997; Jamie I. D. Campbell, 1994; Jamie I. D. Campbell & Epp, 2004; Sciama, Semenza, & Butterworth, 1999). According to this model, numerical skills are based on two factors: the modality of the representation, as opposed to abstract code, and the number processing tasks that are required and which involve common cognitive mechanisms. This means that number skills would be based on multiple forms of internal representation of number and realized in a variety of different ways (Thevenot & Barrouillet, 2006). Based on this hypothesis, another model, proposing a different view of number cognition, has been presented by Dehaene (1992), and is referred to as the triple-code model (Figure 7). The triple-code model proposes that number processing is based on three types of code: visual Arabic form, an auditory-verbal code, and an analog-magnitude representation (Dehaene, 2004). The triple-code model is, therefore, modular in respect to the representational code, in contrast to the abstract mode model, which is modular in respect to numerical function. The auditory-verbal code facilitates verbal number input and output, some counting processes, and the retrieval process for simple addition and multiplication facts. García et al. (2003) considers the multiplication tables to be the best examples of using verbal code for calculation, because the tables would be retrieved as automatic verbal associations, due to the rote learning of tables. The visual digital form mediates input and output, parity judgements and certain multi-digit operations, whereas the analog-magnitude form provides the basis for number comparisons and approximate calculation and estimation, and is also thought to contribute to subitising capabilities (Jamie I. D. Campbell, 1994; Dehaene, 1992). This model, unlike the abstract code model, assumes that the form in which the numbers are presented could affect numerical manipulation and calculation (Jamie I. D. Campbell & Epp, 2004). For example, extensive work with Arabic-based digits would develop an Arabic-specific inhibitory process, which would reduce reading-based interference in arithmetic (Jamie I. D. Campbell & Epp, 2004).

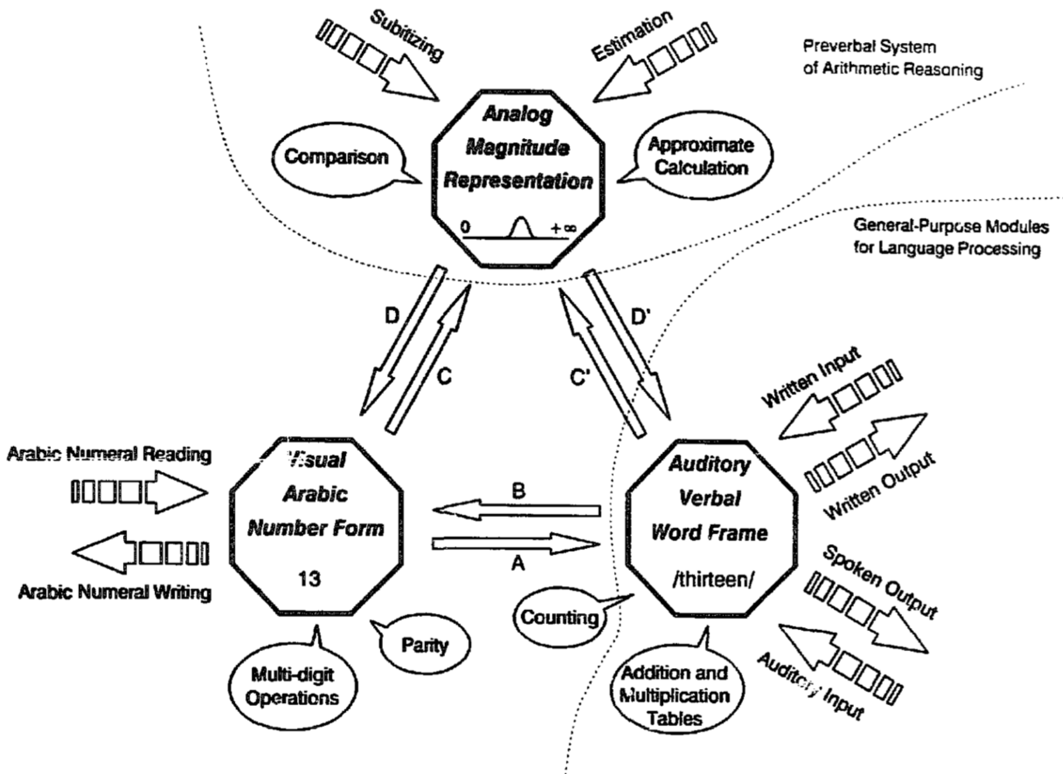


Figure 7: The triple-code model (Dehaene, 1992)

This research maintains consistent representations of number in its instructional material and uses only familiar systems of Arabic numerals and their associated place value. This was done to reduce any negatory preffects on numerical manipulation and calculation required within the proposed tasks.

2.10 Biologically Primary and Secondary Knowledge

Early numerical competence in humans has only recently become the subject of research in developmental psychology. Previously, this field was dominated by the theories of Piaget, who argued that young children do not possess any conceptual understanding of number and largely undertake activities such as counting by rote (Geary, 1994c). The notion of arithmetic in the first year of human life was unthinkable (Dehaene, 1997). Recently, David Geary (Geary, 1994a, 1994d, 1996, 2008a) proposed an evolutionary-based framework that accounted for early numerical skills. This framework can assist in the understanding of biological and cultural influences on children's cognitive and academic development. Geary's theory proposes the presence in humans of an innate understanding, on a basic level, of psychology, biology and physics (folk domains), which ensured evolutionary human survival. Human folk domains comprise a set of innate motivations for learning, which can provide a framework for formal education to build upon. The ability to suppress the intuitive knowledge that comes with the human folk domain ensures that humans are able to create and learn evolutionary novel information (Geary, 2008a). This leads to the presence of two forms of knowledge: biologically primary knowledge, such as defined by the human folk domains, and biologically secondary knowledge, that is, evolutionary novel knowledge that is acquired beyond the intuitive innate motivations.

Biologically primary knowledge is the knowledge that we are born with, or that is easily acquired and which has undergone selection pressure. Biologically primary abilities are evolved abilities that allowed our ancestors to survive. It is this intuitive knowledge that allows humans to effortlessly learn how to speak, despite the high motor complexity of the task. This kind of knowledge requires minimal instruction and is thought to be acquired with very little effort.

In contrast, biologically secondary knowledge is acquired through explicit instruction and effort. This knowledge is harder to acquire and relies on the culture and environment of the child, and is introduced to deal with novel ecological problems that our ancestors never faced. It involves such skills as reading, writing, and higher mathematics. The cognitive processes associated with the acquisition of secondary knowledge rely heavily on conscious processing of information in working memory. Cognitive load theory, therefore, primarily deals with instructional aspects of the acquisition of secondary knowledge (Sweller, 2011).

This distinction between primary and secondary knowledge has direct consequences for instruction both in the classroom and outside. First, due to their evolved mechanisms, biologically primary abilities are universal and are typically the same in all normally developed humans across cultures and nationalities (Geary, 1995). The acquisition of biologically secondary abilities depends on the culture and motivation of the child. Whilst biologically primary skills are endogenous and do not require much external motivation, biologically secondary skills require external reinforcement. Children must often be coerced into practicing biologically secondary skills, such as mathematics (Geary, 1994d). Learning about children's extrinsic motivations can help to develop better strategies to teach evolutionary novel or biologically secondary knowledge.

The ability to rely on more primary forms of learning, intuitive to children, with subtle guidance from teachers, could prove to be effective in teaching novel concepts. It is, therefore, vital that we examine what these primary forms of learning are and how these can be used in the classroom. In mathematics education, of particular interest is the relationship between primary systems and the learning of the base-10 number system used globally, which Geary (2007) believes is essential for modern mathematics. Resnick (1984, p. 126) has stated that "...the initial introduction of the decimal system and positional notation system based on it is, by common agreement of educators, the most difficult and important instructional task in mathematics in the early school years...". One such difficulty in teaching place value occurs due to the conceptual nature of the number structure and the difficulty in knowing what students are thinking with regard to the numbers.

Biologically primary knowledge is composed of many different skills, in the fields of both numeracy and literacy. This research focusses only on numeracy skills. Geary (Geary, 1994a) suggested six potential categories for biologically primary mathematical abilities, clustered together in a coherent numerical domain. Each of these categories emerges early in childhood, and evidence shows that these biologically primary abilities are gender neutral (Geary, 1996; E. S. Spelke, 2005). These are:

i. Numerosity – the ability to determine small sets of items without actually counting them;

ii. Ordinality – basic understanding of ordinal relationships in terms of more than or less than;
iii. Counting – there appears to be a preverbal counting system for very small subsets of numbers, up to perhaps three or four items;
iv. Simple Arithmetic – a sensitivity to increases or decreases in number groups, again this sensitivity is limited to small subsets
v. Estimation - inexact estimation of magnitude or size
vi. Geometry - implicit understanding of shapes and spatial relationships

The following sections will examine each of the mathematical categories for biologically primary skills in more detail.

2.10.1 Numerosity

Recent studies have provided sufficient evidence to show that children are indeed born with core cognitive skills in mathematics that provide some basic numerosity understanding (Dehaene, 1997; Geary, 2008a). Numerosity, or subitising, refers to the ability to recognise small quantities without the need to count. Subitising allows for the exact recognition of up to four items by infants and plays an important role in numerical development (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & de Rijt, 2009; Peucker & Weißhaupt, 2013). In fact, Baroody (1986) views subitising as a fundamental skill in children's further development of number concepts. Adults and some animals are able to discriminate and estimate numerosities larger than three or four (Dehaene, 1997; Xu & Arriaga, 2010). However, the accuracy of estimation is dependent on the geometrical pattern of the items to be subitised. For instance, the number of objects tends to be overestimated when they are consistently spread out, and underestimated when they are inconsistently distributed, a function of our visual system parsing the objects into small groups (Dehaene, 1997; Frith & Frit, 1972; Ginsburg, 1976). The ability to quickly and accurately determine small numerosities is evident in infants in their first week of life (Geary, 1996). By six months of age, and from at least four months of age, infants are able to perceive and represent small numbers of objects (S. E. Antell & Keating, 1983; Mandler & Shebo, 1982; Starkey & Cooper, 1980; Strauss & Curtis, 1981; Van Loosbroek & Smitsman, 1990).

In a number of experiments by van Loosbroek and Smitsman (1990), infants were visually habituated to a static display of a small number of items, from one to four. After habituation, infants were presented with a different display each varying in the number of items. Typically, collections would differ in their colour, density and configuration. In their study, van Loosbroek and Smitsman (1990) showed that infants would focus longer on displays containing different numbers of items, rather than on the new displays of the habituated number. These results indicate infants' sensitivity to changes in numerosity and an early perception of subitising small numerosities.

2.10.2 Ordinality

Ordinality describes the basic understanding of ordinal relationships, that is, discriminating between number relationships that are 'greater than' or 'less than'. Research has shown that basic ordinality is evident in 18-month old infants (Strauss & Curtis, 1981). In one of the first experiments testing only for ordinal relationships, Cooper (1984) habituated infants to successively presented pairs of displays. Each pair maintained a constant ordinal relationship, with varying absolute values between trials, the values ranging from 1 to 4. On habituation, infants were always shown a small number followed by a larger number, or vice versa. Then, during the experiment, infants were shown a series of relationships that were presented in one of three ways: the same way as the habituation, opposite to the habituation or with equal numerical values. The trend showed that 10- to 12-month-old infants looked longer at equal numerical relationships, but failed to look longer at the reversed relationship to the habituation. Conversely, 14- to 16-month-old infants looked longer at both the equal numerical relationships and at the reversed relationships to the habituation (R. G. Cooper, 1984). The results of this study could indicate a basic understanding of ordinality in infants, in particular infants older than 12 months of age. However, the results were not controlled for surface area of displays and elements shown on displays (Brannon, 2002).

Two more recent studies have found evidence of basic ordinality at ten months of age (Feigenson, Carey, & Hauser, 2002) and at eleven months of age (Brannon, 2002). In their experiment, Feigenson et al. (2002) had 10- to 12-month-old infants choose between two quantities of crackers. The infants watched an experimenter sequentially hide two different quantities of crackers in either of two baskets, first using one cracker in one basket and two in the other, and then two crackers in one basket and three crackers in another basket. With these choices, infants always chose the larger quantity of crackers. However, when the numbers were increased in a 3 v 4 ratio, 2 v 4 ratio, and 3 v 6 ratios, the infants chose randomly among the quantities (Feigenson et al., 2002). This study has been reproduced with similar results (Siegler & Opfer, 2003; Xu & Arriaga, 2010), showing that infants possess an innate rudimentary understanding of ordinality, both in phylogeny and ontogeny (Brannon, 2002).

2.10.3 Counting

There appears to be an initial set of principles that allows children to count some small subsets of numbers, up to three or four, before knowledge of number words can develop. One of the first experiments that established that babies could recognise small numbers was conducted by Starkey and Cooper (1980). In these series of experiments, a number of slides were presented to a total of 72 babies, aged between 16 and 30 weeks. The slides contained either two or three dots, arranged in different patterns and with a variety of spreads (Figure 8).

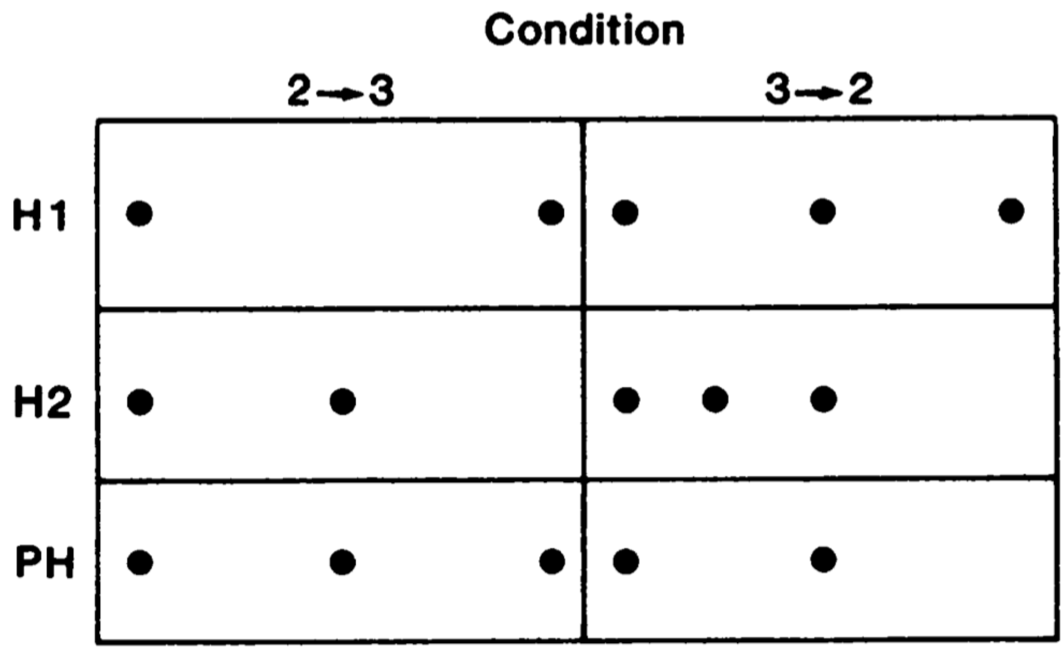


Figure 8: The series of slides presented to babies in a series of experiments determining the basic ability to count (Starkey & Cooper, 1980). H1 and H2 represent the habituation slides, whereas PH is the post-habituation slide.

The babies were habituated with a slide containing the same number of dots until such time as each of the babies started losing interest in this repetitive stimulus. However, once a different slide with three dots were presented to the babies, the fixation time, 1.9 seconds prior to the switch, changed to 2.5 seconds on the first different slide, showing that the baby is able to detect a change in number, and therefore have a very basic principle of counting (Starkey & Cooper, 1980). A few years later, Antell and Keating (1983) demonstrated a similar result, but with newborns. Their experiments demonstrated that babies are able to discriminate between small numerosities even a few days after birth. Another series of experiments conducted by Strauss and Curtis (1981) used arrays of commonly observed objects as opposed to just dots to demonstrate that the ability to count to small numbers does not rely only on object recognition, but also on the numerosity of the image itself. The objects pictured in the photographs were all different, photographed at different angles and from various distances. None of this variation in object representation modified the behavior of babies, who were able to notice the change in number. Using moving objects, van Loosbroek and Smitsman (1990) demonstrated that infants in the first few months of life were able to count objects even in a moving environment.

All of the above experiments relied heavily on the visual perception of babies. To investigate the ability of babies to discriminate between numerosities associated not just with changes in visual stimuli, a follow-up experiment by Starkey, Spelke and Gelman (1983) was designed to take into account audio sequences. In this experiment, 6-, 7- and 8-month old babies were provided with two visual stimuli from two separate projectors. One of the slides would show two common objects, whereas the second slide would show three objects randomly arranged. Simultaneously, the baby was presented with an audio sound of a drumbeat. The results of this study demonstrated that after habituation, the babies were more attentive to those slides that matched the numerosity of the drumbeats. This experiment implied that numerical representation in infants is not a result of visual or auditory perception (Dehaene, 1997). Two recent studies (Izard, Sann, Spelke, & Streri, 2009; Izard & Spelke, 2009), further confirmed the ability of infants to discriminate between small numerosities. Their experiment first introduced auditory sequences with a fixed number of syllables, followed by images of the same and different numbers of items. The results of the study showed that newborns focused their gaze consistently longer on displays that were in ratio 3:1 with the auditory representation during the habituation stage of the experiment (Izard et al., 2009). As in the previous study, babies were able to differentiate between varying numerosities, providing further evidence for abstract numerical representation early in life (Izard et al., 2009).

2.10.4 Simple Arithmetic

Some research suggests that 5-month old infants are aware of basic addition and subtraction, in particular of the effect of adding or subtracting one item from a small set of items. Wynn (1992) used puppets to demonstrate basic addition and subtraction. An infant was shown one puppet on a stage, a screen was then put up and a second puppet placed behind the screen, thereby producing a simple '*1 + 1 = 2'* equation. In one of the displayed outcomes ("impossible outcome"), infants were never shown the two puppets together, and when the screen was lowered, only one puppet could be seen on stage (Figure 9 demonstrates the setup used in the experiments).

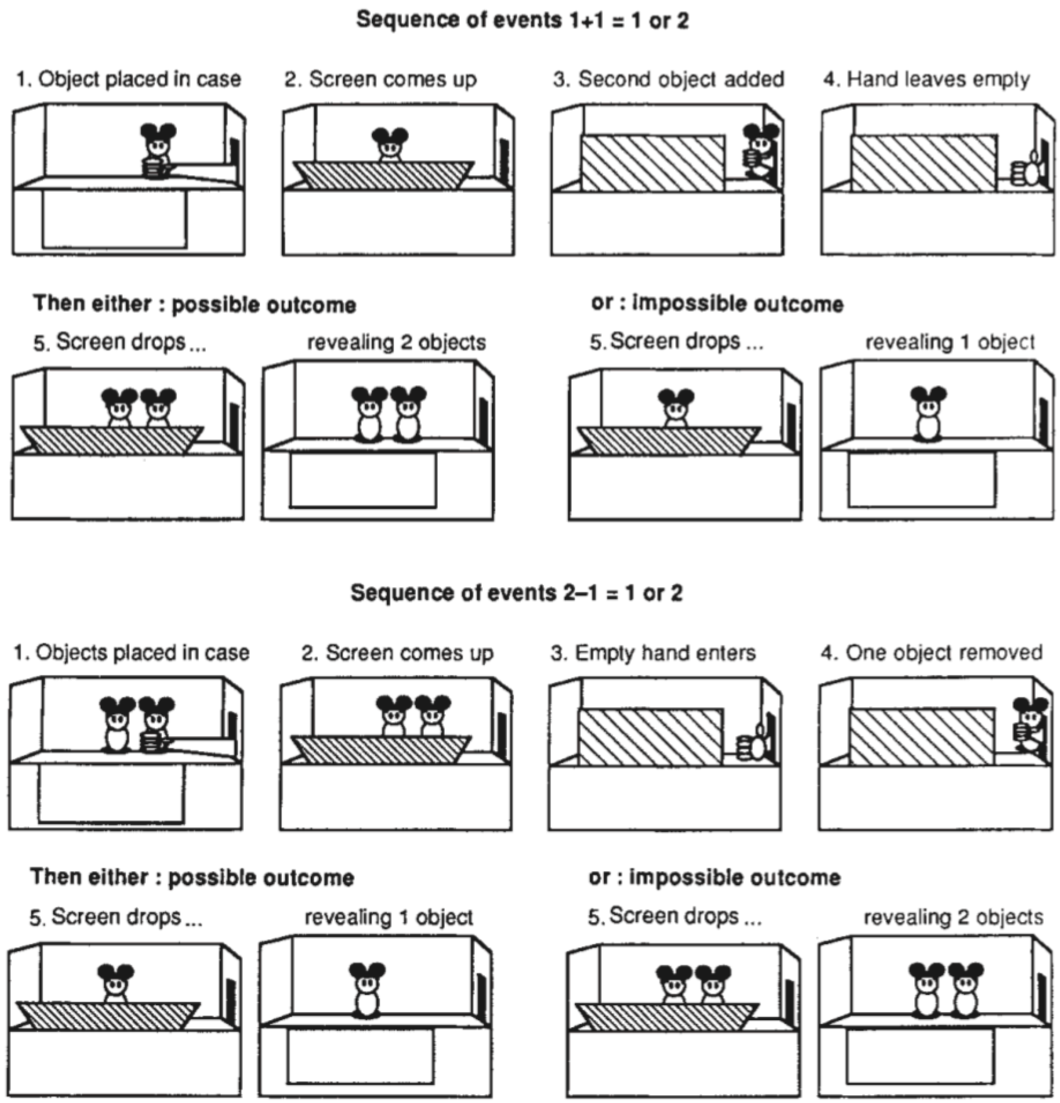


Figure 9: The setup with puppets to demonstrate the innate ability to do basic addition and subtraction in infants (Wynn, 1992)

The time spent focusing on the stage was measured and compared for the possible outcome, i.e. 1 + 1 = 2 and for the impossible outcome, i.e. 1 + 1 = 1. The results indicated that infants spent on average a second longer looking at the impossible outcome (1 + 1 = 1) in comparison to the possible outcome (1 + 1 = 2). The experiment then looked at the subtraction component of simple arithmetic, using the same setup as for addition. It was found that infants spent as much as three seconds longer looking at an impossible outcome (2 - 1 = 2), than the possible outcome (2 - 1 = 1). The study has been replicated with 2- and 3-year-olds (Houdé, 1997), and further evidence exists from other studies, demonstrating that infants are concerned not by the variability in the object being added or subtracted, but rather by the impossibility of some arithmetic operations (Kobayashi, Hiraki, Mugitani, & Hasegawa, 2004; Koechlin, 1997; McCrink & Wynn, 2004; T. J. Simon, Hespos, & Rochat, 1995). A study by Kobayashi (2004) used both auditory and visual modalities, and the results suggested that infants formed expectations about numerosity behind a screen based on both the visual stimuli presented and the auditory tones that they heard, thereby again replicating and further extending on Wynn's (1992) initial findings regarding innate arithmetical abilities.

2.10.5 Estimation

The hypothesized primary mathematical ability of estimation refers to the ability to approximate quantities, magnitudes or sizes. Estimation is a process that many children find difficult within the confines of the school curriculum, but this process is essential for future mathematical success (Siegler & Opfer, 2003). Current research suggests that the number representations found in human infants depend on a mechanism for estimating, but not determining, exact numerosities (Xu & Spelke, 2000). Prior to the development of language and number words, a language-independent representation of number magnitude is used for quantity manipulation and estimation (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; McCrink & Wynn, 2007). In fact, research using brain imaging evidence (Dehaene et al., 1999) suggested that estimation relies on non-verbal visuo-spatial cerebral networks, in line with infants' developing language skills.

Xu and Spelke (2000) attempted to directly test whether infants are capable of representing approximate numerosities, a direct function of estimation. Their study found that, provided the difference between two numerosities is sufficiently large, infants are able to distinguish between an 8- and a 16-element display, when all other variables of the display are controlled. However, when the difference between numerosities was reduced, infants could no longer discriminate between the two. This result is in line with prior research by Starkey and Cooper (1980), which also found that infants were unable to discriminate four from six elements. These findings also complement an experiment conducted by Wynn and Bloom (1999) studying infant ability to enumerate a collection of elements. This research was replicated using auditory stimuli as well, with similar results (Lipton & Spelke, 2004).

Further expanding on this work, a more recent study by McCrink and Wynn (2004) examined ratio abstraction in 6-month-old infants. Infants were presented with multiple examples of the same ratio during the habituation phase, using images of Pac-man and pellets. They were then presented either with new ratios or with the habituated ratio, and were successfully able to discriminate two ratios that differed by at least a factor of 2. The results of this study support the concept of an approximate magnitude-estimation system in infants (McCrink & Wynn, 2004). Sections *2.10.1* Numerosity and *2.10.3* Counting looked in more detail at an infant's ability to infer small numerical subsets and estimate quantities and magnitudes. The next section describes the ability to estimate sizes of objects.

2.10.6 Geometry

Core geometry knowledge refers to the implicit understanding of shapes and spatial relationships, and is one of the hypothesized primary mathematical abilities. Indeed, sensitivity to geometric information is essential for the development of mathematical knowledge, including identifying objects and establishing locations (Dehaene, Izard, Pica, & Spelke, 2006). Over the last few decades, research has found that human infants are sensitive to length, angle and direction in visual format (S. E. G. Antell & Caron, 1985; Bomba & Siqueland, 1983; Izard & Spelke, 2009; McGurk, 1972; Schwartz, Day, & Cohen, 1979; Slater, Mattock, Brown, & Bremner, 1991; E. Spelke, Lee, & Izard, 2010). Slater et al. (1991) investigated infant sensitivity to angles only a few hours after birth, and their results suggested that infants were indeed sensitive to angle variations. A more recent study showed that 5½-month-old infants were capable of using geometric cues to differentiate different corners of an isosceles triangle, implying a basic understanding of angles (Lourenco & Huttenlocher, 2008). Similar experiments also demonstrated that infants were sensitive to variations in length (Newcombe, Huttenlocher, & Learmonth, 1999) and, specifically, could differentiate lines of different lengths (Slater, Mattock, & Brown, 1990). Sensitivity to length in infancy was observed even after objects had changed orientation or position (Izard & Spelke, 2009).

A number of experiments have also revealed that infants are capable of extracting object shape even from partly occluded objects (Kellman & Spelke, 1983; Slater, Morison, et al., 1990). Schwartz and Day (1979) demonstrated that infants could recognise differences between squares and rectangles, even when the orientation or the angular relationship of the shape had changed. Expanding on this work, Bomba and Siqueland (1983) used a series of dot patterns placed together to form shapes, to demonstrate that infants are also capable of distinguishing a triangle shape and treating it as a different object from a rectangle.

Baillargeon and Graber (1987) demonstrated that infants are able to distinguish between the sizes of different objects. In this experiment, infants were presented with short and tall objects, which moved behind a screen with a window in it, allowing infants to see the taller objects moving, but not the shorter objects. Results of the study showed that 5½-month-old infants looked significantly longer if the window in the screen showed a shorter object moving through, but not a taller object, indicating sensitivity to the size of the objects. Expanding on these experiments further, Baillargeon (1991; 1987) used a block hidden behind a screen, which was rotated in an arc formation in relation to the block. In a possible case, the screen was rotated only until it reached the top of the block, while in an impossible case, the screen was rotated unhindered by the block, either in a full 180° arc or partway through the top of the block. The results showed that, whereas 6½-month-old infants looked longer at both impossible events, 4½-month-old infants only looked longer at one of the impossible events, the 180° arc rotation. This indicated a difference in the way height is distinguished by infants of different ages (Lourenco & Huttenlocher, 2008), consistent with earlier findings by Huttenlocher, Newcombe and Sandberg (1994) showing infant sensitivity to height.

2.10.7 Biologically primary skills and the base-10 system

A better way to design instructional material that involves secondary skills is to base such material on the foundation of children's intuitive primary skills. However, whether base-10 instructional material can be designed giving consideration to such intuitive skills has not yet been explored in depth. Geary (2007) believes that in the case of place value, or the base-10 system, the secondary mathematics is too far removed from the primary skills to be easily and intuitively learnt. Explicit instruction is required if these concepts are to be attained. Similarly, Sweller (Sweller, 2012), mentions that it is an error to assume that the difficulty in acquiring secondary knowledge can be alleviated or eliminated by using techniques that are appropriate for primary knowledge. It might therefore not be possible to eliminate the dificulties that children often face when learning the base-10 number system, but this issue does require further research. However, materials that are designed on the foundations of primary skills could provide a more efficient transition by learners to secondary skills.

There are some primary abilities that can potentially underlie the attainment of the base-10 system. For example, infants and young children implicitly organise collections of objects into sets; however, the sets are not clustered around the 10 set. One of the prime examples of this, often used by children as a basic learning strategy, is counting with fingers, grouped into two groups of 5. The primary understanding of the number line, and the decomposition of the number line into sets of ten, with the sets organised into clusters, may therefore facilitate the learning of the base-10 system.

Additionally, the ability to physically interact with the environment at large is a natural primary ability, and could thus be helpful in acquiring secondary skills. Children are inherently motivated to engage in play activities, facilitating the development of primary skills (R J Sternberg & Ben-Zeev, 2012). However, it is not clear whether such activities would be sufficient for the acquisiton of secondary skills. In the arena of the base-10 system, manipulative materials are often used as physical objects that may allow for an easier translation to abstract mathematical concepts. Tools such as blocks and other counting objects are frequently used in both free play and organised play activities to establish deeper understanding of place value concepts. However, Geary (2008b) mentions that whilst free play is an important component of education, and is one of the most effective ways for building on pre-exisiting evolutionary based cognitive structures to enhance primary skills, it does have instructional limitations. Formal direct instruction is still the most effective approach in teaching complex secondary knowledge (Geary, 2008b; Sweller, 2012). The base-10 system is considered to be biologically secondary knowledge. Hence, it is a skill that requires specific instruction. However, the level of understanding of the concept of place value can vary greatly. One factor that can affect understanding and subsequent learning of place value is the first language of the student. Research shows that children from Asian language backgrounds find it easier to comprehend number concepts, compared to children from European-derived languages (Miura, Okamoto, Kim, Steere, & Fayol, 1993). This is largerly due to the direct correspondence between the number word for values greater than 10, and the place value of that number (Fuson, 1990; Miura & Okamoto, 1989; Miura et al., 1993).

Predominantly, however, children's knowledge of the base-10 system, globally, emerges in a formal school setting and only as a result of teaching practices that are designed to impart this knowledge to the child.

2.11 Summary of Chapter

This chapter provided a summary of human cognitive architecture, taking into account the evolutionary framework proposed by Geary (Geary, 1996, 2000, 2008b). Human cognitive architecture is composed of two distinct memory constructs: working memory and long-term memory. Working memory is where conscious and effortful processing occurs. This structure is limited in its capacity and can only hold a small number of elements simultaneously (Miller, 1956). This is in contrast to long-term memory, which appears to be limitless in its capacity and is the basis of the human ability to solve complex problems. The limitless capacity of long-term memory is due to the acquisition and subsequent storage of cognitive constructs, known as schemas, in long-term memory. Schema acquisition and schema automation are critical for efficient and fluent learning. Their importance to learning should be considered in instructional design. The automation of schemas allows their efficient and effortless retrieval from long-term memory, without putting further burden on the limited capacity of working memory.

An evolutionary-based framework for learning was proposed by Geary (Geary, 1996, 2000, 2008b), drawing a distinction between primary skills, which are effortless to learn and can be acquired spontaneously, and secondary skills, which require effort and direct instruction for acquisition. A series of mathematical biologically primary skills has been hypothesized by Geary (1994a), and includes:

i. Numerosity as the ability to determine small sets of items without actually counting them;
ii. Ordinality, the basic understanding of ordinal relationships;
iii. Counting, a pre-verbal counting system for small subsets of numbers;
iv. Simple arithmetic, a sensitivity to basis increases or decreases in small numbers;
v. Estimation, inexact estimation of magnitude or size; and
vi. Geometry, a basic sensitivity to different shapes, angles and spatial relationships.

These primary skills are spontaneously learnt and often appear in infanthood. During formal schooling, primary skills are built upon to acquire secondary skills. It is vital that children’s spontaneous primary abilities are considered when designing instructional material, as this can lead to more efficient and fluent learning models.

Chapter 3 will discuss cognitive load theory and how it aims to improve instructional design, in order to encourage a more efficient facilitation of schema acquisition and automation (Sweller et al., 1998).

3 Cognitive Load Theory

3.1 Introduction

Cognitive Load Theory (CLT) is based on the assumption that human cognitive architecture consists of a limited working memory interacting with an unlimited long-term memory. The theory proposes that instructional design be developed on this basis, with consideration given to the implications of limited working memory (P. Chandler & Sweller, 1991; Sweller, 1989, 2002, 2004; Sweller et al., 2011).

One of the most important elements in the learning process is schema construction and automation, a process that occurs in the working memory. Accordingly, one of the main goals of instruction is to aid in the successful construction and automation of schemas in working memory. Work within the cognitive load framework has largely concentrated on the design of instructional methods that make efficient use of working memory (Paas, Renkl, & Sweller, 2003). Instructional methods developed on the basis of CLT have proven successful in a variety of different fields (Paas, Tuovinen, Tabbers, & Van Gerven, 2003; Sweller, 1999, 2004; van Merriënboer & Ayres, 2005). In contrast, instructional materials that do not take CLT into account have proven to be less efficient, requiring more time and mental effort for the learning process to take place.

3.2 Cognitive Load Types

The ease with which information can be processed in working memory depends on the cognitive load imposed by the task. Cognitive Load Theory proposes three different types of cognitive load: intrinsic, extraneous and germane cognitive load. Working memory may, therefore, be affected by intrinsic cognitive load, based on the nature of the material itself, extraneous cognitive load, based on the way in which the material is presented, and germane load, which is the load imposed by the actual learning process. Intrinsic and extraneous cognitive loads are considered to be additives. This means that if there is low intrinsic load, then a high extraneous load might not make a difference to the learning process, due to the overall cognitive load being within the boundaries of working memory capacity. When added together, they provide the overall cognitive load on the working memory during a task. In order for material to be efficiently learned, the amount of cognitive load cannot exceed the working memory capacity available for learning (Figure 10). This theory drives the development of alternative instructional formats and methods.

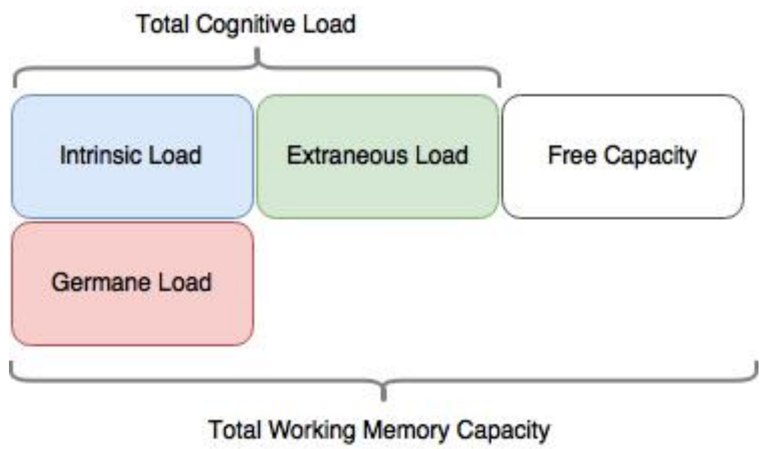


Figure 10: Cognitive Load Theory suggests three types of cognitive load, of which intrinsic and extraneous are additive. Germane load is representative of the working memory resources allocated to dealing with intrinsic load.

Research suggests that learner understanding and performance improve if extraneous load is reduced, in particular when intrinsic load is high (P. Chandler & Sweller, 1991; Kirschner et al., 2006; Richard E Mayer & Moreno, 2003; Sweller et al., 1998; van Merriënboer & Ayres, 2005; van Merriënboer & Sweller, 2005, 2010; Verhoeven et al., 2009).

The next section will examine the three types of cognitive load in more detail.

3.2.1 Intrinsic Cognitive Load

Intrinsic cognitive load is measured by the number of elements that must be processed simultaneously and their interconnectedness to each other, commonly referred to as element interactivity, within the context of the instructional material (Sweller et al., 1998; van Merriënboer & Sweller, 2005). Element interactivity depends on the difficulty of the material that needs to be learnt and the expertise of the learner (Gerjets & Scheiter, 2003; Kalyuga, 2011; Sweller et al., 1998). The inherent complexity of the material that needs to be learnt is, therefore, directly related to the amount of intrinsic cognitive load placed on the working memory.

Tasks that are low in element interactivity, that is, which do not contain many elements that need to be amalgamated, but rather elements that can be learnt in isolation to each other, impose a relatively low load on the working memory (Ayres, 2006). This is in contrast to high element interactivity material, requiring users to deal simultaneously with several complex elements that cannot be learnt in isolation. High element interactivity causes high cognitive load and can reduce the capacity of working memory available for optimal schema construction (van Merriënboer & Sweller, 2005, 2010).

Intrinsic cognitive load is hard to control. It cannot be altered by instructional intervention, without actually modifying the material that needs to be learnt, which is not a feasible solution in the majority of cases (Ayres, 2006; van Merriënboer & Sweller, 2010). However, intrinsic load is essential in learning and understanding instructional material, and, therefore, it is vital to accommodate this load without exceeding working memory capacity (Kalyuga, 2011).

Traditionally, intrinsic cognitive load has been regarded as independent of the specific instructional design methods (Kalyuga, 2011), making it difficult to manage. One strategy for managing intrinsic cognitive load is to manipulate learning material based on the individual learner's level of expertise. As expertise develops in a particular domain, it is possible to decrease the intrinsic cognitive load, due to the fact that the task interactions become incorporated into schemas and become automated (Ayres, 2006). Gerjets, Scheiter and Catrambone (2006) argue that there are instructional-based manipulations that can be carried out to achieve a reduction in intrinsic cognitive load and suggest using a modular-example approach, focussing on the role of smaller individual meaningful units of the overall problem. Additionally, Kalyuga (2011) provides the example of achieving a reduction in intrinsic cognitive load by either substituting the original material with a simpler task, or by removing complex elements from the original material during the initial stages of learning.

Another strategy to reduce the amount of intrinsic cognitive load was first explored by Pollock et al. (2002), and involved teaching isolated elements to begin with, before combining the elements together to teach the complete concept. Clarke, Ayres and Sweller (2005) similarly demonstrated better information transfer in the learning of spreadsheet skills, by firstly teaching novice spreadsheet users how to use spreadsheets, thereby isolating those elements, and only then attempting mathematical tasks. Consistent with these results, Ayres (2006) examined how intrinsic cognitive load can be reduced in algebra. His results found that learners showed improvements in algebra performance when elements of the material were isolated and then learnt prior to the material being presented as a whole (Ayres, 2006). Furthermore, Ayres (2006) demonstrated that low-expertise learners benefit from an initial reduction in interactivity when learning mathematical tasks. Conversely, high-expertise learners benefit from an initial high-level interactivity (Ayres, 2006), a factor attributed to the decrease in germane cognitive load (See *3.2.3* Germane Cognitive Load for more information). Both these studies relied on improved schema acquisition through prior learning. Previous studies also demonstrated the efficiency of prior training (a pre-training method) in reducing intrinsic cognitive load, with learners being provided with specific prior knowledge before the intended materials were learnt (Clarke et al., 2005; Richard E Mayer, Mathias, & Wetzell, 2002; Richard E Mayer & Moreno, 2003).

In their research, Mayer and Moreno (2003) discussed a better information transfer when students first had to study the components making up a system before being introduced to the system as a whole and its operation. Clarke et al. (2005) similarly found that the learning process was enhanced when students were provided with prior training in relevant elements before being introduced to the full element interactivity. These strategies do not directly affect the intrinsic cognitive load of a particular task, but they do manipulate the task to reduce its complexity in the initial learning stages.

3.2.2 Extraneous Cognitive Load

Cognitive load theory and many of the cognitive load effects were initially developed in an attempt to reduce or eliminate the amount of extraneous cognitive load in tasks with a high level of intrinsic cognitive load (Kalyuga, 2011). In contrast to intrinsic cognitive load, which is generated by the instructional material, extraneous cognitive load is generated by the manner in which the instructional material is presented to the learner. Extraneous cognitive load is detrimental to learning. It is caused by inappropriately designed instructional material that fails to take into consideration human cognitive architecture and the limited capacity of working memory, and is not essential in achieving instructional goals (Sweller & Chandler, 1994). For example, the presentation to learners of redundant information, the processing of which requires extra working memory resources, can impact the overall capacity available for processing the required instructional information.

Most cognitive load effects (see *3.3* Cognitive Load Effects for more information) arise from the need to reduce extraneous cognitive load, and have been demonstrated to exist only where high intrinsic cognitive load is present (R. Carlson, Chandler, & Sweller, 2003; Sweller, 2002). Instruction designed to reduce extraneous load has no effect on learning simple material, or material with low intrinsic load. However, for teaching complex material, it is essential that extraneous cognitive load is kept to a minimum, freeing up valuable memory resources to deal with intrinsic cognitive load, thereby inducing germane load to achieve the goals (van Merriënboer & Sweller, 2010) (Figure 11)

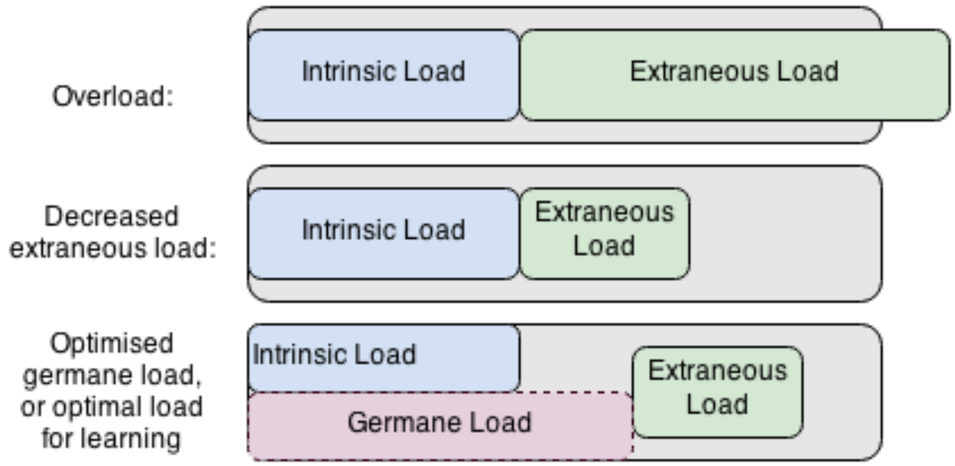


Figure 11: The additive nature of cognitive load, demonstrating an overload on the working memory (top), a decrease in extraneous load freeing up more resources for intrinsic load processing (middle), and optimised germane load, allowing for optimal learning with the presence of high intrinsic load to occur (bottom) (van Merriënboer & Sweller, 2010)

3.2.3 Germane Cognitive Load

The concept of germane cognitive load was added into cognitive load framework at a later time by Sweller et al. (1998). Germane cognitive load is considered to be an effective or positive load, as working memory resources are engaged in processing intrinsic cognitive load, leading directly to learning through schema acquisition and automation. Van Merriënboer and Sweller (2005, p. 152) suggest that "…Effective instructional methods encourage learners to invest free processing resources to schema construction and automation, evoking germane cognitive load…" Interestingly, it has been found that an increase in effort or motivation can also lead to an increase in cognitive resources allocated to a particular task, and this can directly translate to germane cognitive load, if the resources are being used in schema acquisition and automation (Paas, Renkl, et al., 2003; van Merriënboer & Sweller, 2005).

Cognitive load theory aims to optimise germane load in order to enhance the process of learning (Figure 11, bottom). A number of design guidelines for instructional material have been proposed, aiming to optimise germane load in learning. Germane cognitive load was first proposed to help account for the effects in variability of instructional materials (Sweller et al., 1998). Variability in problem content and context, as applied to problem conditions, encourages learners to acquire and construct schema (van Merriënboer & Sweller, 2005). Problem variability ensures that when learners are confronted with similar problems, they are able to identify similar features and distinguish relevant information required to solve the problem. Initially, high variability increases cognitive load during practice, but thereafter it leads to schema construction and transfer of learning (van Merriënboer & Sweller, 2005). Another kind of possible variability known as contextual interference, refers to the interference resulting from practising various problem situations within the same context of practice. For example, if a series of problems performed one after another rely on exactly the same set of skills, then the contextual interference is low. Conversely, if a series of problems performed one after another rely on different skill sets, i.e. different problems presented to the learner in a random practice schedule, then interference is high. Van Merriënboer et al. (2002) have reported a number of studies that examine the role of context interference. Research suggests that a high context interference stimulates the construction of schema (van Merriënboer et al., 2002). However, for high context interference to be effective, learners are required to mentally integrate a large quantity of newly acquired knowledge during practice (van Merriënboer et al., 2002).

The use of self-explanations is another design guideline that aims to optimise germane load. This guideline states that worked examples should be enriched with prompts for self-explanations from learners. In their study of probability based problems, Renkl and Atkinson (2003) used worked examples that guided learners by asking them which probability rule was applied during each step within the solution. They found a strong effect on transfer test performance for learners who received self-explanation prompts in comparison to the learners who did not (Renkl & Atkinson, 2003), suggesting the efficiency of the self-explanation principle in inducing germane load.

The introduction of germane cognitive load into the cognitive load framework (Sweller et al., 1998) was not based on empirical evidence. It was used as a way to account for the intentional cognitive effort required for learning to take place, and the associated demands on working memory. More recently, it has been postulated that germane cognitive load is not an independent source of cognitive load, but rather pertains to the working memory resources associated with intrinsic cognitive load (Kalyuga, 2011; Sweller, 2010). This new conceptulisation aims to remove the germane cognitive load from the framework and instead refers to the use of ‘germane resources’, such as variability in practice or self-explanation, associated with allocating working memory resources to learning (Kalyuga, 2011; Leppink, Paas, Gog, Vleuten, & Merriënboer, 2014; Sweller, 2010).

3.3 Cognitive Load Effects

3.3.1 Worked Example Effect

The worked example effect was initially described in relation to cognitive load theory by Sweller and Cooper (1985), and it is probably one of the most recognised of the cognitive load theory effects (Sweller, 2011). The effect describes the ability of learners to learn more by studying a problem and its solution, as opposed to just being presented with the problem alone (Figure 12).

$$\frac{5x}{6} + 15 = 30$$

$$\frac{5x}{6} = 30 - 15$$

$$\frac{5x}{6} = 15$$

$$5x = 15 \times 6$$

$$5x = 90$$

$$x = \frac{90}{5}$$

$$x = 18$$

Figure 12: An algebra example of a worked-example effect, where the problem and solution are presented together to the learner for optimal schema formation

In their research, Sweller and Cooper (1985) compared two groups of students with a limited knowledge of algebraic transformations, i.e. problems that require students to change the subject of a formula. One group was involved with conventional problem solving techniques, whereas the second group was given a series of problems, with the first problem in each series having a worked example. The results of the study (Sweller & Cooper, 1985) suggested that, in a situation where the students were required to complete four pairs of similar problems, the worked example group required significantly less time than the conventional problem solving group to complete the acquisition phase. Additionally, the worked examples groups performed better and made significantly fewer errors in the test phase, when both groups were subjected to a number of problems similar to those encountered in the acquisition phase (Sweller & Cooper, 1985). This initial research seemed to indicate the ability of worked examples to facilitate the acquisition of schemas (Sweller & Cooper, 1985).

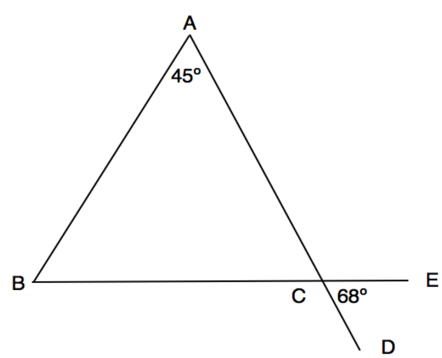
Further research has shown that for novices, worked examples are more effective for learning and transfer, with less mental effort expended in the process of learning (R. K. Atkinson, Derry, Renkl, & Wortham, 2000; Rourke & Sweller, 2009; van Gog, Kester, & Paas, 2011; van Gog & Rummel, 2010). Conversely, learners with a higher level of expertise no longer required worked examples, and research has found that, for such learners, worked examples could be ineffective and even have a negative effect on learning when compared to problem solving (Kalyuga et al., 2003; Kalyuga, Chandler, Tuovinen, & Sweller, 2001). Conventional instruction that consists only of presenting the learner with the problem forces that learner to resort to problem solving strategies, such as means-ends analysis, where the learner is constantly searching for an operator to reduce the difference between the problems state and the goal state (Sweller, 1988). Despite being able to arrive at the correct solution to a problem using means-ends analysis, it remains a weak strategy, due to the high extraneous load imposed on working memory. The high extraneous load does not contribute to schema acquisition, and therefore reduces learning efficiency. The worked examples strategy allows the learners to focus their limited working memory on studying each step of the solution and thereby developing a schema to assist in solving similar problems in the future (Sweller & Cooper, 1985).

Another study showing the benefits of worked examples was conducted by Zhu and Simon (1987). In this study, the researchers found that a three-year mathematics curriculum could be completed in two years if a worked examples strategy was utilised, as opposed to conventional problem solving. Further to this, the study found that students in the worked examples groups performed slightly better in subsequent tests than the conventional problem solving group (Zhu & Simon, 1987).

Schema acquisition, using the worked-examples strategy, can go beyond the ability to solve the specific type of problem; it has been shown that general rules can be abstracted from examples, and parts of the solutions can be adapted to other problems (J. R. Anderson & Fincham, 1994; J. R. Anderson, Fincham, & Douglass, 1997; G. A. Cooper & Sweller, 1987; Rourke & Sweller, 2009). The worked examples effect can therefore be a powerful tool in improving the efficiency of instructional design, leading to improved learning.

3.3.2 Split-Attention Effect

Section 3.3.1 Worked Example Effect reviewed the benefits that worked examples can provide to the design of instructional material and to learning. However, many worked examples consist of two or more sources of mutually referring information (P. Chandler & Sweller, 1992). For a worked example to be effective, it needs to be presented in such a way as to take into consideration the limited capacity of working memory. One cognitive load phenomenon that needs to be of concern when presenting two or more sources of mutually referring information is the split-attention effect. Split-attention effect refers to the need to mentally integrate information, which is physically presented apart. A common example of the effect can be seen often in the area of geometry, where a diagram and a set of statements are presented to the learner (Figure 13).



Find $\angle ABC$

① $\angle ECD = \angle ACB = 68°$ (vertically opposite angles)

② $\angle BAC + \angle BCA + \angle CBA = 180°$ (angle sum of a triangle)

$45° + 68° + \angle CBA = 180°$

$113° + \angle CBA = 180°$

$\angle CBA = 180° - 113°$

$\angle CBA = 67°$

Figure 13: An example of a conventional geometry worked example, where the diagram and the accompanying set of statements are physically separated (Tarmizi & Sweller, 1988)

Mental integration of information presented in such a way requires the learner to search and match each statement with the diagram. In accordance with cognitive load theory, this process of searching and matching text to diagram has the same cognitive repercussions as searching for operators to solve a problem during means-ends analysis (P. Chandler & Sweller, 1992). Both cases require a process that is not essential to learning, such as searching and matching, which uses up the limited cognitive capacity available for learning. For example, in a study of the effectiveness of worked examples, Tarmizi and Sweller (1988) investigated the use of worked examples in the area of circle geometry. The nature of content, in particular geometry or science-based content, often requires both a diagram and a set of descriptive statements, with both elements being essential for learning. Results showed that there was no significant difference in the time taken to solve the presented problems between the worked examples group and the control group (Tarmizi & Sweller, 1988). In fact, in the acquisition phase, students in the worked examples group spent the same amount of time on the question solutions as the conventional problem-solving group, and in the test phase the worked-examples group spent longer on the questions than the control group with equivalent scores on all questions rewritten (Tarmizi & Sweller, 1988). Other research has confirmed the hypothesis that worked examples requiring mental integration of physically separate information are no more effective than conventional problem-solving strategies (Berends & van Lieshout, 2009; Kalyuga, Chandler, & Sweller, 1999; Ward & Sweller, 1990; Yeung, Jin, & Sweller, 1997). Therefore, for optimal learning, the worked example used in Figure 13, should be presented without the need to mentally integrate physically separated information (Figure 14).

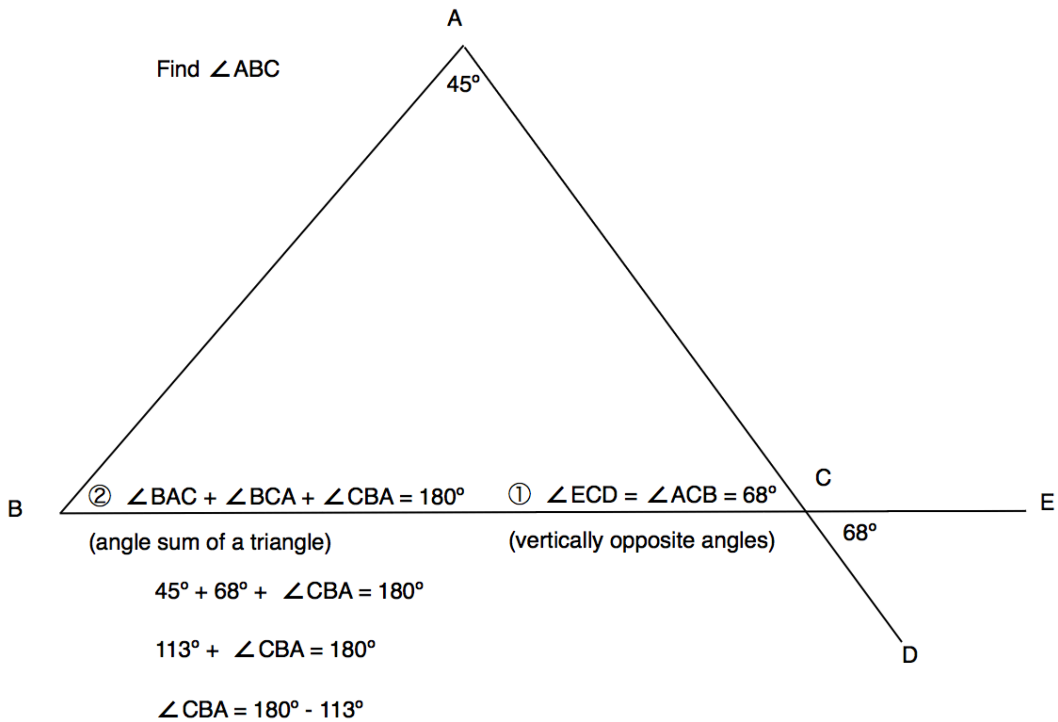


Figure 14: Worked example that does not require mental integration of spatially separated material (Tarmizi & Sweller, 1988)

However, to transition from a spatially separated format to a format that integrates both sources of information requires more than just a change in spatial proximity (P. Chandler & Sweller, 1992). Indeed, in order to integrate a set of statements into a diagram, the text needs to be segmented and the relevant segments positioned in close proximity to the related element in the diagram, thereby labeling elements of that diagram. In previous work, it has been demonstrated that segmentation of learning material facilitates learning (Clark & Mayer, 2011; Richard E Mayer, 2005). It would appear that the structure of the text is able to guide the learner's attention during reading. Segmentation of text is able to provide learners with information about which text elements hold meaning, thereby facilitating a more efficient learning process (Florax & Ploetzner, 2010). Florax and Ploetzner (2010) examined whether the split-attention effect in instructional material is the result of physical integration of text and image, or whether it is due to the labelling of the image and segmentation of text to accomplish this physical integration. The results of the study replicated the effects of split-attention in retention of material, with the only significant effect on learning outcomes due to text segmentation (Florax & Ploetzner, 2010). This provides evidence for the positive effect of segmented material to learning.

Split-attention effect occurs when a combination of images and text are spatially separated. The effect has been replicated in several other situations, including computer manuals, where a user is required to read a paper manual to learn the function of particular menu options before looking at the screen to see the necessary menu options on the device itself (Sweller & Chandler, 1994). The users must split their attention between the device and/or program itself, and the manual and then mentally integrate these two sources, in order to learn how to use the new device or programme. Chandler and Sweller (1996) found that for instructional formats involving high element interactivity, a self-contained manual that physically integrated the necessary pieces of information was superior to other instructional formats involving continual interaction with the computer program.

Huff, Bauhoff and Schwan (2012), examined the split-attention effect in learning tasks requiring comparison and mental integration of two pictures. Building on previous work (Huff & Schwan, 2010), Huff et al. (2012) examined troubleshooting tasks, where learners are required to find an error in a mechanical device and compare it to a working reference device or a picture in a manual of a working reference device. The research used vexing images for the first group, presenting two pictures in such a way that learners were able to switch between the images just by moving their heads. This meant that learners in the 'vex mode' group were able to still see the second representation, just by moving their head but keeping their eyes fixed on the same point. The second group was given the images on two separate screens, requiring them to split their attention between the two screens, positioned directly next to each other. The results indicated that it was not the separation of the two images that was responsible for the split-attention effect, but the re-orientation process after the visual focus was switched. This meant that in the troubleshooting tasks, the performance of the learners using the split screens was inferior to that of the learners using the 'vex mode' images.

The prevention of the split-attention effect is regarded as a basic design guideline in developing material with high element interactivity. The elimination of this effect results in more cognitive resources being available for schema acquisition and automation, thereby supporting the process of learning.

3.3.3 Redundancy Effect

Another basic design guideline in creating instructional material is the prevention of the redundancy effect. The redundancy effect occurs when the same information is presented more than once, or in different formats that are not required for learning. The effect has been demonstrated in a number of studies (Bobis, Sweller, & Cooper, 1993; P. Chandler & Sweller, 1991, 1996; Kalyuga et al., 1999; Sweller & Chandler, 1994). Chandler and Sweller (1991) found that learning can be improved by removing textual statements describing the contents of a diagram. The study involved thirty Year 9 students who had no previous knowledge of blood circulation around the heart, lungs and body. The students were split into a diagram-only group (Figure 15), a conventional group (Figure 16) or a modified group, where the diagram and textual statements were integrated (Figure 17).

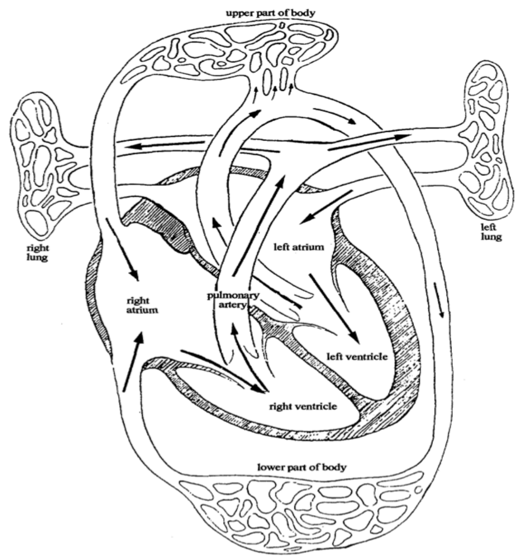


Figure 15: Diagram only group (P. Chandler & Sweller, 1991)

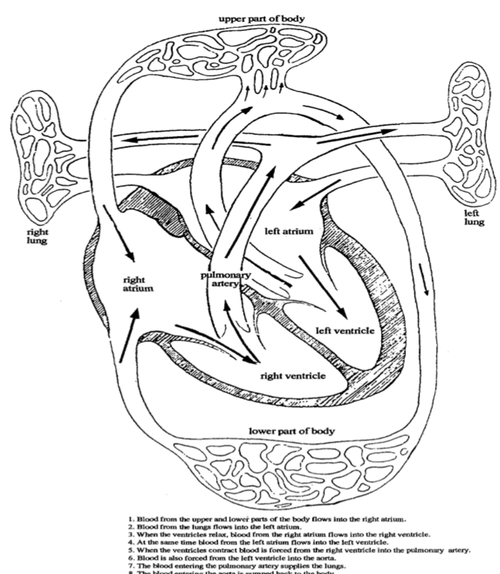


Figure 16: Conventional group: diagram and accompanying text spatially split (P. Chandler & Sweller, 1991)

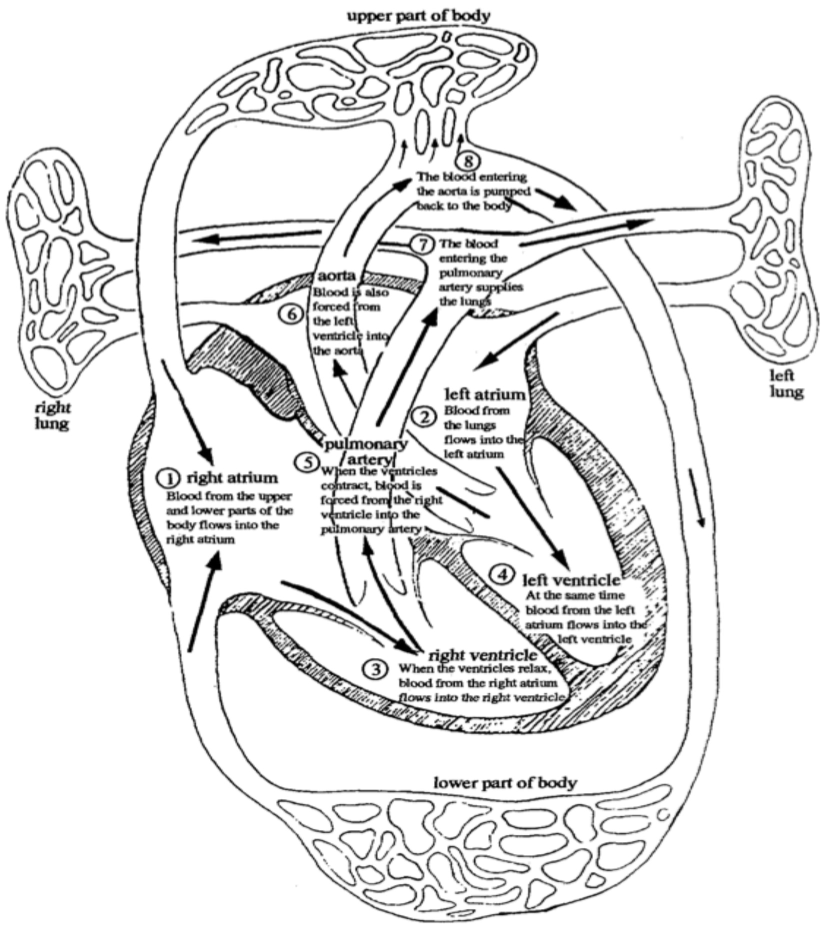


Figure 17: Modified - diagram and text integrated (P. Chandler & Sweller, 1991)

The diagram contained labels showing the different parts of the heart, and the arrows within the diagram indicated the direction of blood flow. The textual statements in the modified diagram of the heart reproduced the information that was already evident from the arrows and the labelling of the parts of the heart, and was, therefore, redundant. The study found that students presented with the diagram only learnt more than those students presented either with the conventional diagram and accompanying textual statements (spatially split), or the modified diagram and accompanying textual statements (spatially located together). In the conventional and modified diagrams, processing the redundant textual statements required extra cognitive resources to carry out the task, thereby leaving fewer cognitive resources available for the essential learning process of schema acquisition and automation. Indeed, whether the text and diagram were integrated or separated had no positive influence on learning, since the textual elements in each experimental condition were redundant and the diagram could be used in isolation for learning.

Kalyuga et al. (1999) also demonstrated the redundancy effect using multimedia computer-based training material. In their research, thirty-four trade apprentices and trainees were separated into three groups for the purpose of learning how to use fusion diagram and solder. The first of the groups was provided with a visual explanatory text only, the second group with auditory instructions only and the third group was provided with both the visual and the auditory instructions, with the auditory instructions being a direct replica of the visual explanatory text. The auditory instructions only group outperformed the auditory and visual instruction group, due to the replicated material inhibiting learning through the increased cognitive load. The dual-mode presentation of the material resulted in the worst performance out of all three cases, evidence that the redundancy effect overrode all benefits of the dual-mode presentation (Kalyuga et al., 1999).

Berends and van Lieshout (2009) examined the effects of different types of illustrations in arithmetic word problems. Four different types of illustrations were used, the “Bare” contained no accompanying illustrations for the word problem, the “Useless” contained a graphic that did not in any way contribute to problem solving, the “Helpful” contained a pictorial representation of the problem and the “Essential” contained information necessary to solve the problem in conjunction with the word problem itself (Figure 18).

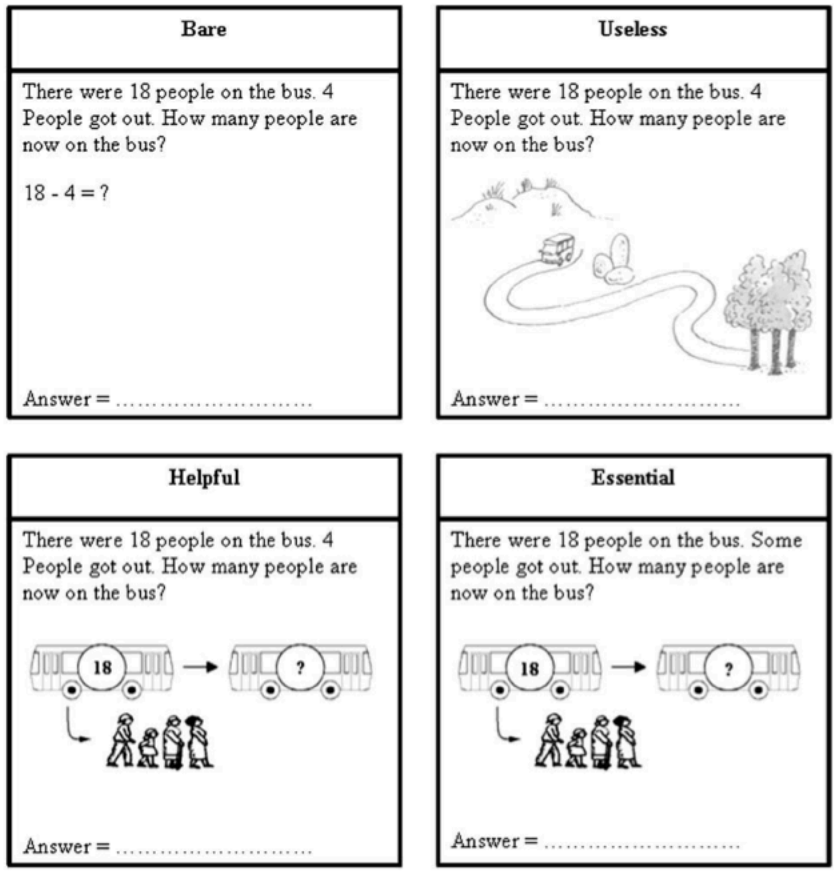


Figure 18: Four types of illustrations used in the experiment (Berends & van Lieshout, 2009)

The results indicated that it took longer to reach the correct solution to a word problem if the problem was accompanied by the 'Useless' illustration, than it did if the problem was produced without an illustration, or 'Bare'. However, in both cases the time take to reach a correct solution were not affected by the excessive demands on the working memory, due to the simple nature of the problems (Berends & van Lieshout, 2009). The 'Helpful' illustration contained redundant information, with the diagram being accompanied by a word problem repeating the already relevant information, yet the redundant information did not have a significant effect, with response times and accuracy remaining the same as those for the 'Useless' group. This contradicted the hypothesis of the study, which was that 'Bare' type of illustration would provide the best influence on the speed and accuracy of performance, followed by the ‘Useless’, ‘Helpful’, and ‘Essential’ types in that order. Such a hypothesis was based on the influence of the redundancy effect, where information that is duplicated leads to a higher extraneous load and therefore more cognitive resources being used on processing. The contradiction of such a hypothesis does not necessarily mean that illustrations are detrimental to learning, but rather, demonstrate that illustrations can slow down overall processing (Berends & van Lieshout, 2009).

It has been established that the level of expertise of the learner in a particular domain impacts the effect of redundancy on learning (Kalyuga, Chandler, & Sweller, 1998). Kalyuga et al. (1998) found that novice learners benefitted the most from an instructional design that included both a diagram and physically integrated text. However, as the level of expertise increased, the best results were then obtained from instructional material that included diagrams only, with all text having been eliminated (Kalyuga et al., 1998). Similar findings were obtained in the area of language comprehension and vocabulary learning (Yeung et al., 1997). Yeung et al. (1997) found that as language comprehension increased, the learner’s ability to comprehend a passage decreased, when the passage included vocabulary definitions physically integrated into the text. This further validated the findings that with increasing expertise, some information becomes redundant and has a negative impact on learning. These findings are indicative of the process of schema acquisition – as schemas form, the inclusion of some explanatory text becomes redundant, and the replicated/redundant information uses up valuable cognitive resources on processing, thereby hindering the process of learning.

3.3.4 Modality Effect

Modality effect refers to the presentation of two pieces of information that cannot be learnt in isolation, in two different modalities, such as visually and orally, thereby facilitating mental integration of material. The effect can only be obtained if the two sources of information cannot be processed and learnt in isolation. The split-attention effect (*3.3.2* Split-Attention Effect) is evident when two sources of visually presented information are spatially separated and require mental integration. Both sources of information rely on the visuo-spatial sketchpad to process the information, thereby causing overload of scarce cognitive resources. However, there is evidence that presenting information in dual modalities can increase the effective capacity of working memory (Brünken, Plass, & Leutner, 2004; for full meta-analysis, see Ginns, 2005; Jeung, Chandler, & Sweller, 1997; Richard E Mayer, Bove, Bryman, Mars, & Tapangco, 1996; Richard E Mayer & Moreno, 1998; Mousavi, Low, & Sweller, 1995).

For instance, Mayer and Moreno (1998) demonstrated that students receiving auditory explanations concurrently with a computer animation outperformed students who received the same computer animation accompanied by textual statements. In fact, their research found that multimedia learners can integrate diagrammatical and textual information more efficiently if the text is delivered via audio (Richard E Mayer & Moreno, 1998).

Mousavi et al. (1995) examined the effects of dual instructional mode in the context of geometry education. In their research, Mousavi et al. (1995) used two groups, both groups working through geometry worked examples. One of the groups received a diagram with an auditory explanation, while the second group received a diagram accompanied by written text. The first experiment did not control for learning time, and allowed all the students to study the worked examples at their own pace, with the diagram/auditory explanation group taking a longer time to learn the geometry problem (due to listening to the narration from the beginning to the end). In the testing phase, the students in the diagram/auditory group consistently solved problems faster than the diagram/text group (Mousavi et al., 1995). In the second set of experiments, the learning time variable was controlled, but the modality effect remained (Mousavi et al., 1995), suggesting that dual modality instruction can lead to more effective learning.

In a series of experiments, Tindall-Ford, Chandler and Sweller (1997) also investigated the modality effect with worked examples within an adult learning setting, using elementary electrical engineering instructions. The results reinforced previous findings that, for learning, an audio narration with an illustration was superior to a purely visual illustration.

Harskamp, Mayer and Suhre (2007) examined the modality effect in the context of science education. The research used twenty-seven secondary school students, who received a web-based multimedia lesson in biology. Some of the students received a science lesson composed of a series of illustrations accompanied by written text, whilst the others received a series of illustrations accompanied by concurrent narration. Students who received the illustrations accompanied by concurrent narrations outperformed the students receiving the illustrations with concurrent text on subsequent transfer tasks (Harskamp et al., 2007). Additionally, the study found an interaction effect in the post-test scores between the learning times spent on each module and the modality of presentation. The illustration and concurrent narration group outperformed the illustration and concurrent text group once more, but only for students who required less time to learn (Harskamp et al., 2007). Mayer (2005) found that the modality effect is strongest in a situation where the images and words are presented quickly with no opportunity to replay the presentation. For those learners who move at a slower pace, replaying of the material is available, thereby allowing more time to rehearse the text and the images. This can lead to a higher expertise level in the domain, which would help to compensate for visually presented material.

Kalyuga, Chandler and Sweller (2000) demonstrated that the modality effect is only applicable for novice learners; as expertise starts to build, the advantages given by the modality effect virtually disappear and can even lead to detrimental effects to learning. In a set of experiments using instructions for industrial manufacturing machinery, novice learners benefitted the most from receiving instructions in visual format with concurrent auditory explanations. However, once expertise started to build, the modality effect reversed and the group with the visual only presentations started learning more efficiently. The auditory explanations became redundant information (Kalyuga et al., 2000). This is referred to as the expertise reversal effect (Kalyuga et al., 2003). The reverse modality effect was demonstrated in a series of experiments by Leahy and Sweller (2011). Two experiments were conducted as part of the research. Both experiments tested the ability of primary school students to read graphs showing temperature variations throughout the day, using a visual only presentation and a visual presentation accompanied concurrently by audio explanations. In the first experiment, complex statements were used to explain the visual illustration, whilst in the second experiment, exactly the same information was presented as segmented text in a simpler format. Results of the first experiment indicated a reverse modality effect, whereas the second set of experiments indicated a conventional modality effect. It was hypothesized that the reverse modality effect was due to the use of complex instructions. The transitory nature and the length of the auditory explanations meant they could not be easily and effectively processed in the working memory. This contrasted with the written information presented, which was permanent and therefore could be more effectively processed in working memory.

3.3.5 The Expertise Reversal Effect

When designing instructional material, the expertise of the learner must be given careful consideration, as it may influence the effectiveness of this material (Kalyuga et al., 2003, 1998). The expertise referred to is the learner's prior experience and knowledge of the domain being taught. Thus, a novice learner is someone with very little or no prior knowledge of a specific domain. A novice learner lacks the schemas necessary to process complex information using limited working memory capacity and requires instructional guidance to help bridge the gap of the missing schemas and aid in schema construction. This is in contrast to an expert learner, who is able to activate relevant schemas, due to their prior knowledge and experience of a specific domain. Expert learners do not benefit from additional instructional guidance, and, in fact, such guidance can lead to "…an overlap between schema-based and the redundant instruction-based components of guidance… " (Kalyuga et al., 2003, p. 24). As a direct result of such redundancy caused by the superfluous instructional guidance, the expert learner may be subjected to cognitive overload whilst attempting to deal with information that they already have schemas for. More experienced learners may benefit from the elimination of instruction-based guidance, thereby reducing cognitive load and increasing learning efficiency. This is known as the expertise-reversal effect, whereby instructional techniques that are highly effective for novice learners, can lose their effectiveness, or even have a negative effect on learning, when used with experts (Kalyuga et al., 2003). Numerous studies have provided evidence that the expertise-reversal effect can be extended to a variety of instructional methods and a wide array of learners (Bokosmaty, Sweller, & Kalyuga, 2014; Kalyuga et al., 2003; Leslie, Low, Jin, & Sweller, 2011). Accordingly, instructional material must be tailored to suit the learner's level of expertise (Kalyuga & Sweller, 2004).

3.4 Summary of Chapter

The goal of cognitive load theory is to use the knowledge of human cognitive architecture to develop effective instructional design. The theory utilises the knowledge of limited working memory, the limitless capacity of long-term memory and the knowledge of how schema acquisition and automation occur to achieve its goal. The theory was first proposed with regards to designing instructional material in the 1980s (Sweller, 1988), and since then has used aspects of human cognition to generate instructional effects that can assist in more efficient instructional design.

Cognitive load theory has identified three different types of load: intrinsic, extraneous and germane. Intrinsic load refers to that type of load found in the material itself, and is dependent on the complexity of the material that needs to be learnt. Intrinsic load is very hard to control without changing the very information that needs to be learnt. Extraneous load is the load that the majority of cognitive load theory effects are concerned with. It is the load that is extraneous to the learning process and is encountered as a result of badly presented or designed instructional material. Extraneous load interferes with the process of schema acquisition and automation, thereby reducing the efficiency of learning. Germane load is the load produced by the learning process itself, complementing intrinsic and extraneous loads. It describes the working memory resources allocated to dealing with intrinsic load, and is considered to have a positive influence on learning. Accoding to the dual model of working memory that considers the additive property of intrinsic and extraneous loads, germane load is "…generated by intentional additional activities designed to further enhance schema acquisition beyond that associated with intrinsic load…" (Kalyuga, 2011, p. 9)

Some of the effects that have been established as a result of cognitive load theory include: the redundancy effect, the split-attention effect, the modality effect and the worked examples effect. Each of these effects should be considered in order to create the most efficient forms of instructional material. The worked-examples effect occurs when the learner is presented with a worked example of a problem and its solution, thereby freeing up limited working memory resources for learning. This is in contrast to using the more conventional means-ends analysis to solve a problem, where the user is presented with a problem and must successfully reach a solution, thereby using all available cognitive resources on processing potential steps for a solution. The worked examples effect is the most efficient when presented to novice learners, but has been found to hinder learning when expertise is built up in a particular domain.

Split-attention effect occurs when two independent but related sources of information are presented to the learner spatially separated, but still requiring mental integration. The learner must search and match statements, the processing required being similar to that needed for conventional means-ends analysis. The need to process redundant information uses up cognitive resources that would otherwise be used for learning. This has the effect of hindering schema acquisition and automation, thereby preventing effective learning.

Using two different modalities to present information to a learner utilises the working memory more efficiently, because different modalities are linked to different areas of working memory. The modality effect is experienced by novice learners, studying two sources of material that cannot be understood in isolation to each other.

The expertise-reversal effect occurs both when instructional methods that are more effective for novice learners are not effective for expert learners, and also, when instructional methods effective for expert learners are found to be ineffective for novices.

The next chapter discusses the concept of number sense in young primary-aged children, and the design of instructional material that is used to teach number sense, specifically place value characteristics of the base-10 number system; this is then related to cognitive load theory and the cognitive load effects discussed thus far.

4 Acquisition of the Hindu-Arabic number system

4.1 Introduction

The Hindu-Arabic number system, commonly referred to as the base-10 number system, is the most universally used number system in the world. Indeed, its simplicity has been one of the main reasons why it is considered "…one of the greatest inventions of the human mind…" (Zhang & Norman, 1995, p. 272) There are three main features that help to clearly define the base-10 number system:

1) The system is composed of *digits 0-9*, that can be used in combination to represent all possible numbers);
2) The system groups its numbers by *sets of ten* (Figure 19);

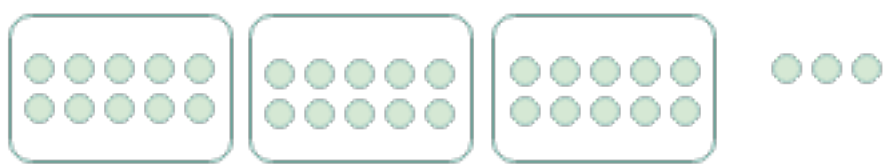


Figure 19: The system groups its numbers by sets of ten

3) The system uses *place value* (based on powers of 10) to assign a value to a digit based on that digit's position within a number (Figure 20).

Another defining characteristic of the Hindu-Arabic number system is the use of the zero, which is vital to being able to represent a placeholder in the place value notation. For example, in the number 105, the 0 in the tens column indicates that there are no tens in this number, however, the zero still holds a place between the 1 and the 5 to indicate the total value of the number is in the hundreds.

A decimal point in a number signifies that the digits to the left are whole numbers, whereas the numbers to the right of the decimal point are fractional portions of a whole number. From any place in the system, the next position to the left is 10 times greater and the position to the right is one tenth greater than the previous place value.

These features make it possible to write all known numbers in an *expanded form*, as the sum of the digits multiplied by the place value of each of the digits. For example the number 6424 is represented in Figure 20.

Place Value	Thousands	Hundreds	Tens	Ones
Digit	6	4	2	4
Expanded Form	6×1000	4×100	2×10	4×1

Figure 20: Representing the number 6424 using the base-10 number system

Children's understanding of number grows over time. Very early number sense is developed with language, through the use of rhymes, songs and everyday language in a social environment. Children learn the Hindu-Arabic number system early by learning how to count objects and record the numerals. When children first construct number, they construct a system of ones, where 16 would be a collection of 16 ones (Chandler & Kamii, 2009). Early number counting is based on rote learning, the simple memorization of all the number words without an association with the meaning of each of the numbers (Kennedy, Tipps, & Johnson, 2007). Children at this stage are not yet able to think about tens and ones simultaneously (Chandler & Kamii, 2009).

Rational counting occurs later and involves children connecting actual quantities with the words. Counting is a complex cognitive task, requiring five key principles to come together:

1) The one-to-one principle – objects can only be counted once for each item;
2) The stable order principle – numbers that are counted are arranged in a sequence that does not change with any number of counts;
3) The cardinal principle – the cardinal number is represented by the last number in a sequence and represents the number of objects in a set;

4) The order-irrelevance principle – the order in which objects are counted is irrelevant;
5) The abstraction principle – any collection of real or imagined objects can be counted.

Throughout the primary years of education, children start learning about place value notation, and how to manipulate large numbers with the help of that notation.

4.2 Place Value

Place value refers to the positional value of a digit within a set of digits and is one of the defining characteristics of the base-10 number system. The value of a digit as written depends entirely on the position of that digit within a string of digits. For example, whilst numbers 72 and 27 utilise the same digits, the position of the digits determines the actual value of the final number. The concept of positional relevance in building numbers is powerful, as it can be expanded or contracted to represent very large or very small numbers without difficulty. Before formal education commences, children think of numbers larger than 10 as a collection of ones, not as a collection of tens and ones, or of hundreds, tens, and ones, etc. (Mix, Prather, Smith, & Stockton, 2014). Formal schooling is needed in order to develop the concept of tens and ones and of place value in general (Fuson, 1990; Fuson et al., 1997; Rouder & Geary, 2014), which in turn provides children with the ability for more efficient number processing and manipulation in the future.

A lot of emphasis is currently being placed on teaching place value in Australian primary schools. The Australian Curriculum Board (National Curriculum Board, 2009, p. 8) describes the role of place value as important in order to "…develop deep understanding of whole numbers to build reasoning in fractions and decimals and develop their conceptual understanding of place value. With these understandings, students are able to develop proportional reasoning and flexibility with number through mental computation skills. These understandings extend students' number sense and statistical fluency…”. The Australian Mathematics Syllabus expects students to develop an understanding of place value in Stage 1 (Years 1 and 2), “...applying an understanding of place value and the role of zero to read, write and order two-digit numbers; state the place value of digits in two-digit numbers...”. At this stage, students are also expected to be able to count by tens on and off the decade, this ability made much simpler with a more concrete understanding of place value.

Using the latest available data from the Growing Up in Australia: The Longitudinal Study of Australian Children (LSAC), Daragonova and Ainley (2012) examined the teacher ratings of student performance in the area of place value understanding. An examination of the teacher ratings used in the LSAC study showed that students displayed difficulty grasping place value concepts. Results in early primary years (6-7 year olds, with an average age of 6 years and 10 months) showed only 31% of children demonstrated place value skills competently and consistently, while 29% displayed average competence. The remaining 40% of children were either not competent in the skill (7%), just beginning to demonstrate place value understanding (12%), or demonstrated place value skills below average competence (21%) (Daraganova & Ainley, 2012). Similarly, these statistics did not differ much in the middle primary school years (8-9 year olds, with an average age of 8 years and 10 months), where 37% of children showed an above average understanding of place value and 31% demonstrated average competence with place value. The remaining 32% displayed place value skills that were below average (Daraganova & Ainley, 2012).

The mimimum standards for numeracy in Year 3, as defined by the National Assessment Program (NAPLAN, 2016), state that students should be able to compare and order whole two-digit numbers. In addition, children should be able to use place value knowledge up to the hundreds to interpret representations of whole numbers. This demonstrates a very basic understading of place value. Such a level of knowledge does not cover three-digit and larger numbers, even though at that level students are expected to know how to manipulate numbers well into the thousands. Studies show that children who use decomposition in multi-digit computation tasks, such as breaking up a number based on its positional properties, have overall improved mathematical performance (Geary, Hoard, Byrd-Craven, & Catherine DeSoto, 2004; Geary, Hoard, Nugent, & Bailey, 2013; Laski, Ermakova, & Vasilyeva, 2014). This type of number processing and manipulation requires students to have a much more in-depth understanding of place value. The results of the numeracy component of the National Assessment Program – Literacy and Numeracy (NAPLAN) indicate that 95.5% of students in Australia scored at or below the national minimum standard, a level that only requires children to order and compare two-digit numbers. This demonstrates a very basic understanding of place value. Ordering and comparing three-digit numbers is a Band 3 requirement in the NAPLAN, and only 62.5% of students in Australia were able to meet this level of achievement ("NAP - Results," n.d.).

Understanding place value concepts involves building connections between key ideas, including quantifying sets of objects by grouping them into 10s and treating those groups as units (Fuson, 1990; Steffe, Cobb, & von Glasersfeld, 1988), and using the structure of the written notation to capture this information about groupings. The introduction of multi-digit numbers in school could be difficult to understand even for relatively advanced students (Mix et al., 2014). However, understanding the positional properties or the place value feature of the Hindu-Arabic number system can greatly enhance learning subsequent secondary mathematics material and is essential for understanding and applying multi-digit numbers in further arithmetic and everyday life situations (Bailey, 2015; Chan & Ho, 2010; Chan, Au, & Tang, 2014; Dietrich, Huber, Dackermann, Moeller, & Fischer, 2016; Fuson, 1990; Ho & Cheng, 1997; Kamii, 1986; Mann, Moeller, Pixner, Kaufmann, & Nuerk, 2012; Moeller et al., 2011; Nataraj & Thomas, 2007; Ross, 1989).

Additionally, a significant correlation has been shown between children's place value skills in early primary school and subsequent problem solving ability (Mix et al., 2014). Schmittau and Vagliardo (2006, p. 590) have emphasised the importance of understanding the system of place value, a concept which "...not only [does it] connect[s] to many important concepts... [but] it is also a prerequisite for any real understanding of the base-10 system...". Mix et al. (2014, p. 1306) refer to place value concepts as "…the gateway for conceptualizing large quantities and more complicated mathematical operations…". For example, when multi-digit addition or subtraction is started at school, prior to trading being introduced in the more complex equations, children are often taught to align their numbers to the right for both numerals that they are adding or subtracting. However, they are often not taught why there is a need to align those digits. It would be better for children to understand that each place in the base-10 system needs to be aligned underneath each other perfectly in order to complete the sum easily. This ensures that not only trading becomes easier once it is taught, but also that there is good understanding when decimal place value is introduced. The same rule will still apply to the decimal place value system: line up each of the ones, tens, hundreds etc columns to get the correct answer. Lining up numbers based on the place of each digit in the number would show a good conceptual understanding that the number 20, for example, does not simply refer to a set which is composed of 20 objects, but rather that it also has 2 tens and 0 ones. The understanding that multi-digit numbers represent place value groups of units, tens, hundreds, etc can further influence the sophistication of the problem-solving strategies that the child can use to solve complex arithmetic problems (Geary, 1994d).

Fuson (1990) proposed that the difficulty young children find with place value notation could be related to the vast differences in the spoken and written number systems. These differences make it difficult to see the relationship between written and spoken numbers, without explicit instruction. In fact, research has shown that the base-10 system is more obvious to Asian students, who develop a much better conceptual understanding of place value early in school (Bjorklund & Pellegrini, 2002; Fuson, 1990; Geary, 1994a, 1996; Moeller et al., 2011; Ngan Ng & Rao, 2010). This is due to language differences in the way numbers are spoken out loud. In particular, counting in an Asian language promotes an understanding of numbers that is directly related to the presentation of numbers in the traditional base-10 system (Miura et al., 1993). For example, in Japanese the teen numbers, such as 11, 12, 13 etc. are spoken as “ten-one”, “ten-two”, “ten-three” and so on. The number 30 is spoken as “three-ten”. This means that Asian children have a better understanding of number concepts and have been shown to use more sophisticated techniques to solve complex mathematical problems earlier in their education (Geary, 1996; Ngan Ng & Rao, 2010). English and most European and Slavic languages experience the problem of the words not truthfully representing the positional properties of the base-10 system, making it very hard for speakers of those languages to understand the concept of place value and leading to further problems in mathematics. The earlier and more complete conceptual understanding of place value by Asian students is one of a number of factors including educational and social influences, often leading to higher mathematical achievement in these students, observed as early as the middle of the first year of schooling (Miura et al., 1993; H. W. Stevenson, Chen, & Lee, 1993).

Other studies have proposed that children simply lack the logical capacity at a young age to comprehend the complexity of the place value notation (Chandler & Kamii, 2009). However, research has indicated that children do not enter the early primary years without some understanding of numbers, or sensitivity to statistical patterns in multi-digit numerals and corresponding verbal names. This sensitivity and prior knowledge comes from the child’s everyday environment, and includes such things as phone numbers, page numbers in books, and street addresses (Mix et al., 2014). Consistent with this idea of prior knowledge is a study by Barrouillet, Thevenot and Fayol (2010), which found that, prior to entering school, children may be able to differentiate between legal and illegal strings of lexical primitives, for example when comparing three hundred and forty nine to three nine forty hundred. A recent study conducted by Byrge, Smith and Mix (2013) reinforced these findings by demonstrating that preschool children have the ability to write multi-digit numbers with some success, for the most part only making ‘intelligent’ errors. For example, the number 113 would be written as 10013, with the children writing one hundred and then followed by the number 13. Whilst not correct, it directly relates the spoken number to its written abstraction. Whilst Moeller et al. (2011) refer to such errors as precursors for later difficulties in mathematics, Byrge et al. (2013) suggests that these types of errors are only a precursor to later difficulties in mathematics if the expanded writing persists in the face of formal place value instruction. In fact, Byrge et al. (2013) have shown that expanded number writing, or ‘intelligent’ errors, may provide another important milestone in the development of a child’s understanding of place value, as they are based on the children’s initial and early understanding of multi-digit numbers. This type of understanding could then present a potential foundation upon which formal instruction of place value could begin.

Whilst the reasons for the difficulties experienced in learning place value concepts by children are many and varied, it is evident that the concept of place value itself requires instruction for mastery and is not intuitive knowledge. It could be highly beneficial to base any such formal instruction upon a child’s intuitive knowledge and understanding of the place value notation (Byrge et al., 2013).

Various researchers have attempted to describe the progressive stages that apply to children’s understanding of two-digit place value (Chan et al., 2014; Cobb, 1995; Fuson, 1990; Fuson et al., 1997; Fuson & Briars, 1990; Miura & Okamoto, 1989; Miura et al., 1993; Resnick, 1984; Steffe et al., 1988; Young-Loveridge, 2002). The authors have not always been in complete agreement on the stages involved in place value understanding. Ross (1989) and Fuson et al. (1997) agree on five stages of conceptual understanding of place value, with a similar approach to each of the stages: unitary multi-digit, decade and ones, sequence tens and ones, separate tens and ones, and finally an integrated sequence and separate tens. This thesis will use the five-stage conceptual understanding framework described by Ross (1989) to assess place value understanding in children, according to the following stages (Table 1).

Table 1: Five stages of place value understanding, as described by Ross (1989)

Stage	Description
Stage 1: Whole numerals	No meaning assigned to individual digits in a number; number is seen as a whole.
Stage 2: Positional properties	Meaning is assigned to individual digits in a number; however, despite knowing that the digit on the right is ones and the digit of the left is tens, the meaning of these does not yet encompass quantities.
Stage 3: Face value	Interpret each digit as representing a number indicated by its face value. Students do not yet recognise that the number represented by the tens column is a multiple of ten.
Stage 4: Construction zone	Tentative knowledge that the number in the tens column is not just a number but is a set of tens.
Stage 5: Understanding	Students understand that the individual digits in multi-digit numbers represent a part of the whole quantity split into ones, tens, hundreds and so on.

4.3 Teaching Place Value

4.3.1 Use of manipulative materials

Manipulative materials in mathematics have long been used to help teach conceptual information that requires physical demonstration. Hynes (1986, p. 11) defines manipulative materials, as "…concrete models that incorporate mathematical concepts, appeal to several senses, and can be touched and moved around by students…”. However, a more recently proposed definition of manipulative materials also includes the concept of engagement with the materials being used, “…A manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered…” (Swan & Marshall, 2010, p. 14). Manipulative materials can serve as a useful tool that can enhance learning and understanding. Early research suggests that the act of manipulation itself can aid the students in experiencing the patterns and relationships that are the focus of mathematics (Adler, 1966). For example, the NSW Mathematics K-10 Syllabus (Board of Studies Teaching and Educational Standards - NSW, n.d.) recommends using models, such as base 10 material, interlocking cubes and bundles of sticks to help teach place value (Australian Curriculum/Maths/Number and Algebra/028 (ACMNA028) Requirement: Group, partition and rearrange collections of up to 1000 in hundreds, tens and ones to facilitate more efficient counting). Using physical objects and physical actions to represent the world is intuitive, and there are many instances where this is demonstrated in the context of real-life (Marley & Carbonneau, 2014). For example, building a model of a house, using paddle-pop sticks and other material, is often the preferred method of explaining and demonstrating certain structures in architecture class.

Many studies have advocated the use of manipulative materials, such as base-10 blocks or bundling sticks, to teach place value concepts (Baroody, 1990; Fuson, 1990; Hiebert & Wearne, 1992; Nataraj & Thomas, 2007; Ross, 1989). However, there has also been other research that has suggested that manipulative materials could only be used at the children’s own level of abstraction. That is, a child must first understand the concept before manipulative materials could be used to represent that concept (Fosnot & Dolk, 2001). In the case of place value, a child must first understand the basic principles of tens and ones to further benefit from the use of manipulative materials (Chandler & Kamii, 2009). Some of the research that opposes the use of manipulative materials in teaching the base-10 system includes the work of Kamii (2000; Kamii & Joseph, 2004) and Fosnot and Dolk (2001). Fosnot and Dolk (2001) consider the use of manipulative materials with the base-10 structure already built-in to be non-beneficial to the learning process. "…Building structure into manipulatives is not always beneficial by itself…abacus, Cuisenaire rods … Base-10 blocks all have a base-10 structure built in. The problem with these materials is that while the structures in them are apparent to adults, they are not always apparent to children… If a child has not constructed the idea [of tens and ones], she does not see the rod as one ten; she sees it simply as a unit…" (Fosnot & Dolk, 2001, pp. 103–104).

Despite opposing views as to whether or not manipulative materials can aid in learning the place value notation of the base-10 system, one study showed that 81.9% of 820 teachers in over 250 schools in Western Australia used base-10 blocks as a teaching aid in the classroom, and considered them to be one of the three most important tools (Swan & Marshall, 2010). However, individual teacher interviews conducted in the same study found that whilst teachers believed that manipulative materials could assist in learning, they could not identify the exact elements of the materials that were responsible for the learning process (Swan & Marshall, 2010). This could potentially indicate a lack of training for the educators using the materials, or a gap in understanding how to teach children to translate effectively between the representational manipulative material and the abstract mathematical concepts.

To be effective, manipulative materials cannot be used in an erratic manner. The use of the materials must be accompanied by clear and relevant explanations and instruction from teachers, ensuring that the link between the manipulative material used and the mathematical concept being taught is explicit (Stein & Bovalino, 2001; Swan & Marshall, 2010).

This study will utilise two different manipulative materials, the abacus and base-10 blocks, as aids in the understanding of place value concepts.

4.3.1.1 Base-10 blocks

The pioneering work of Dienes (1961) related to the structured materials developed to support children as they learnt about place value, referred to as Multibase Arithmetic Blocks (MAB). The MAB are composed of units, small cubes of wood; longs, made up of as many units long as the base being used; flats, representing the square of the base being used; and blocks, representing the cube of the base being used (Figure 21).

Figure 21: MAB blocks

These blocks remain one of the most popular manipulative materials in the classroom (English & Halford, 1995; Marshall & Swan, 2013; Swan & Marshall, 2010). Base-10 blocks come in four standard sizes, depicting the ones, tens, hundreds and thousands columns. Arguably, base-10 blocks are good at relating the value of the number to the size of the block, making it apparent that as size increases so does the magnitude of the number. Base-10 blocks are good at showing children the magnitude of numbers, although it is hard to relate this back to the positional property of the base-10 system. The concept of place value is based on the position of the digit within a number. It is the position of that digit within the number that affects the digit's magnitude and not the other way around, as the base-10 blocks may suggest. Base-10 blocks are arguably effective at demonstrating an initial understanding of trading, where ten single blocks can be swapped for a block of ten, and this can be useful in children's learning of place value (English & Halford, 1995). However, despite the common use of this manipulative, there have been contradictory findings in the literature about the effectiveness of base-10 blocks in teaching place value (Fuson, 1990; Resnick, 1984). A more recent study by Mix et al. (2014) explored the use of base-10 blocks to teach place value to 24 kindergarten students, with an average age of 4 years and 9 months. The children were divided into two equal groups, one of the groups using base-10 blocks to teach place value and the second group using symbols-only training for teaching place value. The study found that base-10 blocks were not particularly transparent to children. In fact, the results indicated that the potential correspondence between the differently sized blocks and the place value of the numbers that the blocks were meant to represent was not clear, with performance in block-based tasks consistently poor (Mix et al., 2014). Whilst training with the symbols-only method led to consistent improvements on certain tasks, base-10 block training did not lead to any improvements at all. The results go so far as to imply that perhaps the use of base-10 blocks requires prior knowledge of the written place value system, in order to be successful as a manipulative material intended for improving understanding of place value mix (Mix et al., 2014).

One of the most common ways to test understanding of place value in schools is to show children different combinations of base-10 blocks, arranged in order of magnitude, or canonical order, and asking them to represent the numbers that these blocks symbolize. The problem with this approach is that children commonly rote learn how to perform this type of task. They simply count up the different blocks and then put the two numbers together, whilst perhaps still not having a good concept of actual place value notation. Performing such a task correctly would appear to indicate that a child understands the concept of place value and the positional notation represented by digits in a number, although this might not actually be the case; the child may simply have learnt by rote how to put together base-10 blocks, without any real understanding of the magnitude of the digits within a number (Ross, 1989). Chan et al. (2014) proposed modifying this type of testing and using a method referred to as strategic counting. Using the strategic counting testing method, students would be given base-10 block representations arranged in non-canonical order, thereby requiring an understanding of base-10 properties in order to rearrange and group sets together to produce meaningful answers.

The contradictions in the efficiency of base-10 blocks as a training tool to teach place value could be attributed to a number of factors, incuding the child's first language, the quality of the teaching material accompanying the use of the manipulatives, teachers' understanding of place value, and the length of time spent on the topic.

4.3.1.2 The Slavonic abacus

Another manipulative, widely used in Asian countries and Eastern Europe, is the abacus. Throughout history, the abacus has played a part in showing the positional representation of numbers (Barbarin & Wasik, 2009). The simplified Slavonic abacus is composed of a hundred beads and allows counting up to one hundred (
Figure 22).

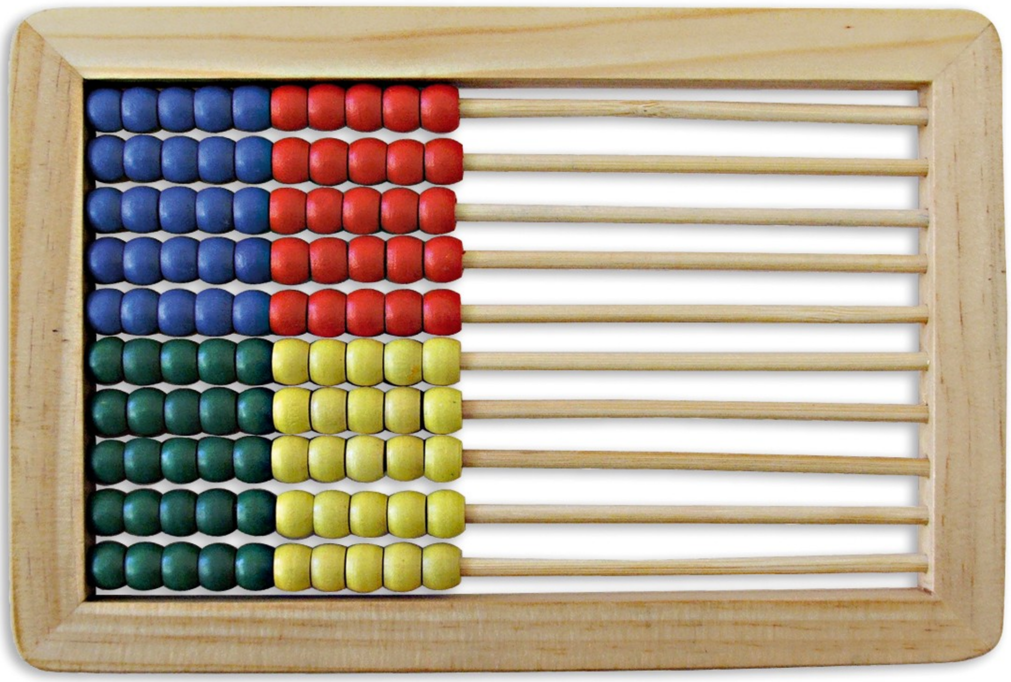


Figure 22: Slavonic abacus

When the beads are positioned on the right, they are 'off' and are therefore not forming a number. Pushing beads to the left switches them 'on' and this is how numbers are represented on the abacus. There are ten rows altogether, with each row grouping beads into tens. This grouping in a row is then halved again with each half of the beads in different colours. This correlates to the number of fingers on each hand, and may therefore aid in subitising. Additionally, the first five rows will differ in colour to the last five rows, so as to allow easy subitising for each group of fifty. For example, the number 37 can be displayed on an abacus using the movement of beads in Figure 23.

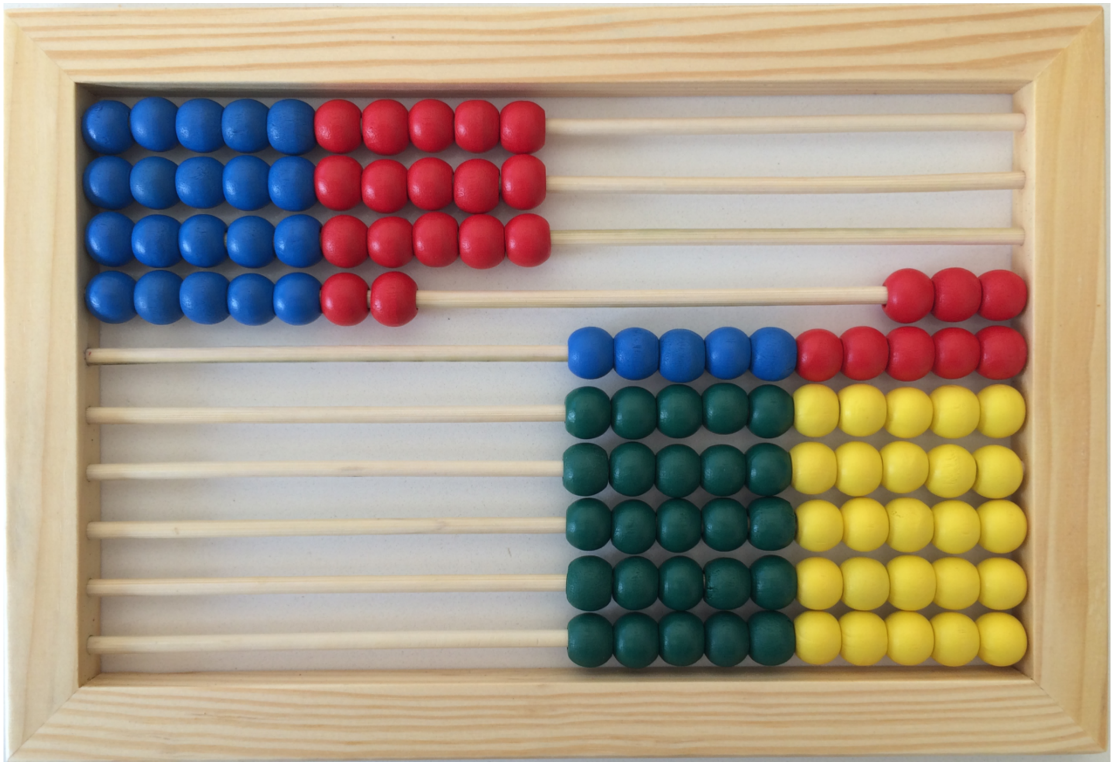


Figure 23: Abacus showing the number 37 (with its complement to a 100 of 63 clearly and easily subitised)

The main advantage of using a Slavonic abacus is that it clearly encourages the grouping of numbers in sets of ten, one of the properties of the Hindu-Arabic number system. It also does not require a prior understanding of multi-digit numbers; it is enough for students to know the verbal sequence of numbers to be able to count the beads on the Slavonic abacus.

Additionally, each row of the Slavonic abacus is made up to two sets of five beads, each set in different colours. The use of the small number of beads in different colours and its direct correlation to the number of fingers on each hand can support the child's ability to subitise. Subitising, or the ability to automatically recognise small numerosities without counting is an innate ability, and is therefore considered to be a biologically primary skill (Geary, 1994e). Furthermore, our biological ability to make sets of numbers is also supported, as there are two sets of five on each row, and a set of ten on each wire. This also reflects clearly the base-10 property of the Hindu-

Arabic number system. The Slavonic abacus physically lends itself to counting in the base-10 setting, where each full line of an abacus is called a ’ten line’ and a maximum of nine beads are allowed to be placed on a line. This setup helps to facilitate mental visualisation of a group of ‘ten’, as each row reresents a group of ten and allows to the child to count in a base-ten setting. The abacus can then facilitate the understanding of place value, by physically representing, for example, that the number seventy-six is made up of 7 tens, or seventy, and 6 ones. Such a visualisaition can potentially improve schema formation for the base-10 property of number.

A recent longitudinal study examined whether using a mental abacus, a technique used to perform mathematical calculations using a mental image of an abacus, can improve students’ mathematical abilities and lead to improvements in basic cognitive capacities, such as working memory (Barner, Alvarez, Sullivan, & Frank, 2016). Using a mental abacus relies primarily on the visuospatial working memory, as well as some motor procedures that are learnt during training with a physical abacus. The technique is based around the structure of the abacus, which is consistent with known working memory limits. The results of the study suggested that mental abacus training did not lead to consistent gains in the cognitive abilities of the students (Barner et al., 2016). However, mental abacus training did lead to measurable gains in ability to perform accurate mathematical computations. These gains emerged after a single year of training with a physical abacus, prior to the students learning the mental abacus technique. Towards the conclusion of this study, Barner et al. (2016) found that physical abacus expertise was significantly correlated with higher mathematical performance.

4.4 Summary of Chapter

Evolutionary educational psychology focusses on the ideas that relate evolutionary principles to educational foundations (Muller, 2010). The goals of evolutionary educational psychology are to determine what motivates learning and what motivational biases influence a child's ability to acquire evolutionary novel knowledge, as demanded by our system of schooling. Achieving these goals would result in more effective teaching methods and the ability to teach children in a way that is more naturally aligned to their intuitive skills and knowledge. Evolutionary educational psychologists claim that currently there is a great disparity between the way that we teach children and how children actually prefer to be taught (Muller, 2010).

Mathematics is an important area where educators need to be able to understand how children prefer to be taught, but, to date, we do not fully understand the mechanisms that influence children's mathematical learning (Geary, 1994b). Of particular interest are the motivational biases that can influence the way children come to an understanding of the abstract concept of number and its relationship to its symbolic representation, a concept which appears to be very difficult to grasp for children (Baroody, 1990; Fuson, 1992; Hiebert & Wearne, 1992). In particular, the positional property of the Hindu-Arabic number system, or the base-10 system, causes the most difficulty. Understanding the positional notation of our number system is essential to the development of number sense, and also forms the basis for four fundamental number operations (Jordan, Kaplan, Ramineni, & Locuniak, 2009). Chan et al. (2014, p. 78) refer to place value as "…crucial in learning arithmetic…" Understanding the base-10 system in early primary school years is essential, as research has shown that flawed understanding of the place-value positions of tens and units in first grade is related to difficulties in later arithmetic performance (Chan & Ho, 2010; Chan et al., 2014; Dietrich et al., 2016; Fuson, 1990; Ho & Cheng, 1997; Jordan et al., 2009; Moeller et al., 2011).

Place value errors are often syntactic, with children often using expanded notation to write down numbers, for example, the number 326 written as 300206. These mistakes then follow through to multi-digit addition and subtraction (Fuson, 1990). There are a number of different ideas relating to the conceptual structures in place value understanding. This thesis will be using the conceptual structures proposed by Ross (1989) and will examine place value understanding in five different stages. The first stage has the children assign no meaning to the individual digits in a number, whilst the second stage sees meaning assigned to individual digits, although the number 23 is still seen as 23 ones. The third stage sees an understanding of the face value of digits, and the fourth stage sees the tentative knowledge about tens and ones. The final stage sees the understanding of a multi-digit number as individual digits split into ones, tens, hundreds and so on.

The use of manipulative materials is common in teaching concepts in mathematics, although there is some debate as to the efficiency of manipulative materials in teaching place value notation in particular. A very common manipulative used to teach place value concepts are base-10 blocks. Base-10 blocks are good at showing children the magnitude of numbers; however, it is often difficult to relate this back to the positional property of the base-10 system, as the blocks differ in size only. Another manipulative widely used in Asian and Eastern European countries is the abacus. The main advantage of the Slavonic abacus is that it clearly encourages grouping of numbers in sets of ten, one of the key properties of the Hindu-Arabic number system. It also does not require a prior understanding of multi-digit numbers; it is enough for students to know the verbal sequence of numbers to be able to count the beads on the Slavonic abacus.

This thesis will utilise base-10 blocks and a Slavonic abacus as manipulative materials to aid in teaching the concepts of place value. The next chapter looks at the literature pertinent to this thesis and provides a broad overview of the series of experiments conducted as part of this study.

5 Research in the field

5.1 Research pertinent to this thesis

Multi-digit numbers present a complex cognitive challenge to children, and the understanding of their functionality is only just starting to emerge (Nuerk, Moeller, Klein, Willmes, & Fischer, 2011). Interest in children's learning and processing of multi-digit numbers has increased significantly over the last few years (Mann, Moeller, Pixner, Kaufmann, & Nuerk, 2011). Not only is place value and its relationship to multi-digit numbers a difficult concept for young children to grasp, but the abstract mathematical idea of place value is also a challenging topic to teach (Bailey, 2015). Von Aster and Shalev (2007), in their research, present the idea that children are first introduced to multi-digit numbers as an expansion of single digit numbers, at a time when they start to acquire the compound number words of two-digit numbers. It is, therefore, vital that place value instruction starts in early education. However, it still leaves the question of how to best teach such concepts at a young age. Development of multi-digit number understanding and processing in children is vital, not simply to their ability to count, but also as an important stepping stone to other basic numerical tasks including transcoding, estimation, magnitude comparison and generalization to basic arithmetic operations (Nuerk et al., 2011).

Manipulative materials are concrete objects that are often used around the world to assist with learning and understanding new mathematics concepts. Such materials offer students the opportunity to explore concepts at both the visual and tactile level, through hands-on experience (McNeil & Jarvin, 2007). Research into the use of manipulatives in the classroom to teach mathematics concepts has shown mixed results. Some manipulatives have been found to help students under some conditions (Aburime, 2007; Mix et al., 2016), whilst other studies have shown there to be no benefit from using manipulative materials. Still others have shown that the use of such manipulative materials can actually hinder learning under particular conditions (McNeil & Jarvin, 2007). Carbonneau, Marley and Selig (2013) provided a good meta-analysis of the current state of research into the use of manipulatives in the classroom in primary and high schools. The meta-analysis covered some very early studies by Fennema (1972) and similarly by Friedman (1978), who showed that manipulatives can lose their value after the first year of schooling. However, despite the mixed results of recent studies, the popularity of manipulative materials in the classroom remains strong. The use of manipulatives can pose a cognitive challenge to students' limited cognitive resources, in the form of dual representation, where a given manipulative not only needs to be represented as an object on its own, but also as a transparent symbol of whatever mathematical concept it is intended to explain (Uttal et al., 1997). Some early work by Boulton-Lewis (1998) found that when children interacted with manipulatives, their limited cognitive resources were all focused on representing and manipulating the objects, leaving no resources to establish actual understanding of concepts. It is, therefore, vital to continue research into how the use of manipulatives could affect students' limited cognitive resources and how this may impact learning and understanding in the mathematics domain.

Another common way in which novel information is presented in mathematics workbooks is with the help of worked examples. Worked examples present the learner with a problem and also the worked-out solution showing the steps that need to be taken to reach a goal state. This method of presentation means that a learner can dedicate all their cognitive resources to developing a schema for solving similar problems. Worked examples are particularly beneficial with novice learners in comparison to other standard problem solving techniques, provided the examples are well designed (van Loon-Hillen, van Gog, & Brand-Gruwel, 2012). For example, an early study by Tarmizi and Sweller (1988) found that if the worked example presented information in a split-attention format, the worked example effect was not observed, and was only observed again when the same information sources were presented in an integrated manner.

Worked examples in mathematics have been widely researched (Chen, Kalyuga, & Sweller, 2015; G. A. Cooper & Sweller, 1987; Spanjers, Gog, & van Merriënboer, 2011; Sweller & Cooper, 1985; Takir, 2012; Zhu & Simon, 1987); however, these have been, for the most part, conducted with high school or early university students. Only a handful of known studies have focused on the efficiency of worked examples when used with primary school children, (Hu, Ginns, & Bobis, 2015; Mwangi & Sweller, 1998; van Loon-Hillen et al., 2012). Van Loon-Hillen et al. (2012) found that the only difference between groups that had access to worked examples and those groups that did not was in the acquisition time, the time taken to study the presented material. However, in the study conducted by van Loon-Hillen et al. (2012), students were subjected to a realistic maths teaching environment within their classroom curriculum, with access to worked examples, which they could chose to use or not. This made it very difficult to measure accurately the benefits to learning of the worked example, which was a limitation of that study (van Loon-Hillen et al., 2012). Previous research by Mwangi & Sweller (1998), and more recent work by Hu (2014; Hu et al., 2015), had also shown positive learning outcomes when the worked example was utilised in primary school mathematics instruction. The efficacy of using such worked examples was dependent on the structure and content of instructional material.

5.2 Rationale for study

NAPLAN results indicate that Australian children have great difficulty learning the characteristics of the base-10 number system (Daraganova & Ainley, 2012), in particular the concept of place value, which is essential for understanding expanded notation, multi-digit addition and subtraction, and other basic concepts. Children in early primary years often do not understand that the number is made up of a series of quantifiable ones, tens and hundreds (and so on).

The goal of this research project is to investigate whether there is a more efficient way to teach the properties of the base-ten system to early primary school-aged children, based on contemporary understanding of the characteristics and origins of human cognitive architecture. The research project looks at the role that manipulative materials, such as MAB blocks, and other base-10 materials, play in teaching place value concepts, from the viewpoint of cognitive load theory. This is done with the help of worked examples, which are used throughout the study to present novel concepts to students. It is envisaged that the use of the worked example strategy will aid in reducing the imposed extraneous load, and lead to a more consistent instructional design across the many groups of students tested. The main research question of this study therefore is:

Do base-10 manipulative materials represent a biologically primary skill that can be used to efficiently transition to secondary knowledge of the base-10 system, thereby improving children's understanding of number and place value?

The general hypothesis of this thesis is that the use of base-10 manipulative materials for teaching place value may create the redundancy effect, thereby minimising any positive effect that the use of such materials might have on learning. Whilst the use of base-10 materials may have its foundations in children's biologically primary skills, the fact that the use of such materials must be accompanied by explicit guidance, in order to provide any benefit to learning, may mean that such tools are not an efficient way to transfer between primary and secondary knowledge.

5.3 An introduction to the experiments in the thesis

The previous chapters have discussed the difficulty with teaching and learning the concepts of number and place value. The current study was designed with the intention of making a further contribution to this discussion, by investigating the ways in which place value is taught and the effect this has on student cognitive resources.

This thesis used five experiments to test the hypothesis that using base-10 manipulative material, and in particular MAB blocks, within the construct of a worked example, can lead to the redundancy effect and, therefore, to a lack of improvement in learning. This could be due to the fact that MAB blocks, one of the most commonly used manipulative materials for teaching place value in the Australian classroom, do not correspond to our primary biological knowledge. Such tools require children to learn the abstract connection between the meaning of the blocks and the actual number system. Learning both the concepts of the blocks and the number system represent biologically secondary knowledge and require effort and skill to learn. This is unlike the common and intuitive use of blocks to build or play with, which represent biologically primary knowledge. Thus, the use of the manipulative material requires additional learning, not only of the function and meaning of the manipulative material but also the relationship that the material has to the number system, which places an even higher load on children's limited cognitive resources.

Experiment 1 investigated whether a positive impact on learning the concept of place value could be achieved using different manipulative materials, and, specifically, whether a particular manipulative material could more neatly fit with our biologically primary knowledge, thereby providing a more intuitive way of grasping the concept of place value. Students were randomly assigned to one of four groups: groups 1 and 2 were given MAB blocks to assist with place value learning, with group 1 learning the five different stages of place value understanding gradually and group 2 learning only the fifth stage of place value understanding, without focusing on the preceding stages. Groups 3 and 4 were given Slavonic abaci to assist in learning, with group 3 learning the five stages of place value understanding gradually, and group 4 learning only the fifth stage without focusing on the preceding stages. Students spent the day undertaking place value related activities using the relevant manipulative material to assist them in learning. One of the hypotheses proposed that using the abacus, instead of the MAB blocks would lead to better performance in learning place value concepts, due to the more intuitive structure of the abacus.

Experiment 2 was designed to remove the real-life teaching bias found in Experiment 1, by utilizing the concept of worked example based learning, with the assistance of the same two manipulative materials used in Experiment 1, and the addition of a control group which was not provided with any manipulative materials to assist them in completing the set tasks. The aim of this experiment was to establish whether worked examples with certain manipulative materials were more beneficial than standard worked examples that did not use any manipulative materials.

Experiment 3 reduced the complexity of the worked examples by reducing the amount of literacy required in question comprehension and once again used worked examples with MAB blocks, Abacus or control groups (no manipulative materials) to see whether worked examples with particular manipulative materials are more beneficial to learning. It was argued that worked examples that do not utilise the images or the physical element of the manipulative materials would result in better performance in comparison to manipulative materials being used.

Experiment 4 once again utilised the concept of teaching/learning with worked examples, but reduced the sample size to only using one manipulative material, as no differences were found in Experiments 1, 2 or 3 for the different manipulative materials in learning the concepts of place value. This experiment was designed to test the hypothesis that the use of MAB blocks is in fact detrimental to learning the concepts of place value as it causes a Redundancy Effect both when presented on paper for the student to study in their own time and as a physical manipulative material.

Experiment 5 used more advanced learners with more complex learning material to again test whether MAB blocks, often used to assist students in gaining the early concepts of place value, may in fact hinder learning, as they pose a Redundancy Effect both when presented in their physical form and also as images within the worked examples.

6 Experiment 1 - Place Value

6.1 Introduction

The series of experiments making up this thesis aimed to investigate whether the current methods of teaching place value with the aid of manipulative materials could be improved, based on cognitive load theory. Experiment 1 was designed to explore whether teaching place value could be improved with a more gradual approach to number understanding established first, and also whether teaching with MAB blocks as is the current standard, is superior to teaching with any other manipulative material, such as a Slavonic abacus.

It was predicted that the gradual transition using the abacus, where each stage of place value understanding was covered and practised individually before the following stage was introduced, would produce better results in genuine understanding of the place value topic than using the MAB blocks in an identical transition. Similarly, using an abacus to teach place value in an integrated fashion from the beginning, without first focussing on the separate stages, was predicted to produce better outcomes than using MAB blocks in an identical transition. This would reflect the gradual building of schemas during the intervention/acquisition phase, and thus lower levels of the inherent intrinsic cognitive load. It was hypothesized that the use of the Slavonic abacus would be more intuitive to the students' basic understanding of number construction due to its one-to-one correspondence with counting, i.e. one bead counts as one object. This is in contrast to MAB blocks, that do not represent number correspondence intuitively but attempt to do it visually through object size.

6.1.1 Participants

Experiment 1 was conducted with sixty Year 2 and remedial Year 3 students, aged 7-8, picked at random from two Sydney Eastern Suburbs tutoring centres. The children were a mix of both genders, and a range of private and public schools. The centre specialises in both remedial education and gifted students, so there was a range of students with different academic abilities. Students who chose to participate were randomly assigned to one of four groups:

1) Gradual transition group with the abacus as a tool (n = 15)
2) Gradual transition group with base-10 blocks as a tool (n = 15)
3) All-at-once transition with the abacus as a tool (n = 15)
4) All-at-once transition with base-10 blocks as a tool (n = 15)

6.1.2 Materials and Procedure

A 2 (manipulative materials: abacus vs. base-10 blocks) × 2 (method of instruction: gradual transition vs. all-at-once transition) mixed factorial design was used with 15 participants in each of the groups (Table 2).

Table 2: A 2×2 mixed factorial design in Experiment 1

	Gradual Transition	*All-at-once Transition*
Abacus	15	15
Base-10 blocks	15	15

Children were invited to participate in the study throughout the Term 1 period, and were then tested in Term 2 of the school year, after they had covered basic number concepts at school but before they have had an opportunity to study place value concepts in depth. The study took place over four consecutive weekends, with each session lasting the whole day. All the materials used as part of this experimental setup can be found in Appendix A: Experimental materials used in Experiment 1. Prior to commencing teaching, children were given a pre-test, examining their prior knowledge and current understanding of place value concepts (up to 45 minutes in length). This was followed by one of four intervention strategies, with regular breaks given to the children. The day was structured to include five sessions of half an hour in length each with a 10-minute break between sessions and a longer lunch break. Each session involved one teacher, who was conducting the lesson, and a second teacher, to assist with class control and any administrative matters. The same two teachers were used for all sessions.

There were two types of manipulative materials used: the Slavonic abacus (one-hundred frame) and MAB blocks. The Slavonic abacus is made up of ten rows of ten beads each; with each group of five beads on any single row of the abacus in a different colour to ensure that efficient subitising is possible. The MAB blocks were familiar to the children, as they used them during their normal school time activities in developing their understanding of place value. Each set used in the experiment consisted of shorts (units), longs (tens) and flats (hundreds). When making numbers in the thousands, children were asked to put together the necessary number of flats to show understanding of number magnitude.

Children in two of the groups received a gradual instruction of the base-10 system, providing a segmented approach to learning based on each stage of learning place value, as described by Ross (1989). Accordingly, each of the five half-hour sessions covered each individual stage of understanding of place value (Table 3).

Table 3: Stages of place value understanding in the pre-test and post-test

Stage	Description	Task to test understanding	Example of question
Stage 1: Whole numerals	No meaning assigned to individual digits in a numeral, number is seen as a whole.	Transcoding – Question 1 (6 items)	Numbers are read out loud to students who are required to write them down (up to

			and including hundreds)
Stage 2: Positional properties	Meaning is assigned to individual digits in a numeral, however, despite knowing that digit on the right is ones and digit of the left is tens, the meaning of these does not yet encompass quantities.	Number Patterns/Counting - Question 2 (4 items)	12, 22, 32, ?
Stage 3: Face value	Interpret each digit as representing a number indicated by its face value. Students do not yet recognise that the number represented by the tens column is a multiple of ten.	Basic Maths operations – Question 3 (10 items)	12 + 52 = ? 74 – 31 = ?
Stage 4: Construction zone	Tentative knowledge that the number in the tens column is more than just a number but is a set of tens.	Representation of numbers - Question 4 (7 items) and Question 5 (8 items)	Write the number 32 in words. Write the number ‘one hundred and three’ in numbers.

Stage 5: Understanding	Students understand that the individual digits in multi-digit numbers represent a part of the whole quantity split into ones, tens, hundreds and so on.	Expanded Notation - Question 6 (4 items) Face Value – Question 7 (4 items) Place Value - Question 8 (3 items)	Write the following in expanded notation: 341 =? + ? + ? What is the face value of the tens in the number 652? What is the total value of the tens in the number 349?

The first session focussed on transcoding numbers, starting from smaller and building up to greater numbers. The students practised saying numbers out loud, covering numbers from one-digit to three- or four- digits. The use of zero as a placeholder was also discussed and used. The relevant manipulative materials, either the Slavonic abacus or the MAB blocks were also used as part of this activity to aid in understanding. The second session discussed the different place value positions in a number, starting on the right and from the lowest possible unit and building up to larger three digit numbers. The next session focussed on using numbers up to three digits and on being able to specify face value for a required digit within a number. In the fourth session, the children began to construct numbers, and discussed what makes up a two-, three- or greater digit number. Starting from a one-digit number and building up to a three-digit number, the students participated in activities that involved deconstructing the numbers into ones, tens and hundreds. Finally, the last session brought all these concepts together, with the children being able to deconstruct numbers and easily understand the difference between face value of numbers and total value of numbers.

Children in the remaining two groups received an all-at-once lesson, which covered the last stage of understanding without focussing separately on the preceding stages. Their material was delivered also in five half-an-hour sessions; however, the lesson was continuous in determining place value in specific numbers. A discussion of numbers starting with one-digit numbers and continuing up to three-digit numbers was undertaken, using place value to write and record numbers, but without specifically discussing the total values of digits within a two- or more digit number. For example, the children in the gradual groups worked on transcoding numbers in the first session, discussing place value positions in the second session, specifying face value in the third session, constructing numbers in the fourth session and finally working with deconstructing and constructing numbers in the last session by specifying what place value each digit represented. This is in contrast to the children in the all-at-once group, who worked on construction and deconstruction of numbers with one digit in the first session, two-digit in the second session and three-digit numbers in the third session. The fourth and fifth sessions were used to work through different number deconstruction, with children specifying what place value each digit in a number represented.

After the lesson, children were tested again to establish any benefits they might have received from either of the interventions in understanding the base-10 system. The post-test was composed of ten questions, and used similar questions to the ones used in each of the interventions, and in the pre-test. The questions in the test were divided according to particular areas of understanding they measured, based on the five stages of place value understanding (Ross, 1989). Each test item was marked as either correct or incorrect, with no partial marks being given. The stage of place value understanding was deduced from the results of each test.

It was predicted that the gradual transition, where each stage of place value understanding was covered and practiced individually before the following stage was introduced would produce better results in genuine understanding of the place value topic than if the place value was simply taught in an integrated way from the beginning without focussing on the separate stages first.

6.2 Data Analysis

To check the approximate normality of distribution, values of skewness and kurtosis were examined. These values were converted to z-scores by dividing them by their standard error, if the resulting score was greater than 1.96, a potential violation of the independent groups t-test assumption of normality was evident (Field, 2007). The Shapiro-Wilks test of normality was also used to test for parametric distribution assumptions. Where normality assumption was not violated, the independent groups t-test assumption of equality of variances was examined using Levene's test. Non-significant results of this test meant that analysis with a standard independent groups t-test was undertaken. However, a significant Levene's test result led to the use of the t-test suitable for heterogenous variances within the SPSS software package. The experimental data reported as part of this thesis combined tests of significance, controlling the Type 1 error rate at 0.05, with estimates of the standardised mean difference effect size (d). This thesis uses the benchmarks for effect size magnitude as defined by Cohen (1988), "small, d = .2,", "medium, d = .5," and "large, d = .8 and above".

6.3 Results

The scores received by each of the students in the pre-test were measured and analysed to understand base-line knowledge in all the groups, and to ensure that there were no significant differences in the cohorts of students between the Abacus

(n = 30, M = 79.97, SD = 12.11) and MAB blocks (n = 30, M = 76.93, SD = 15.62), F (1, 59) = .714, p = .402.

The independent variables in this experiment were the types of materials and the method of instruction, while the dependent variables under analysis were total post-test scores. Table 4 shows the means and standard deviations of each of the variables.

Table 4: Descriptive statistics for place value understanding

		Pre-test %	Post-test %
Abacus			
(N = 15)	Gradual	77.87 (12.94)	81.13 (14.03)
(N = 15)	All-at-once	82.07 (11.27)	82.67 (14.13)
(N = 30)	Total	79.97 (12.11)	81.9 (13.85)
MAB			
(N = 15)	Gradual	73.40 (20.61)	78.87 (18.46)
(N = 15)	All-at-once	80.47 (7.33)	84.0 (6.89)
(N = 30)	Total	76.93 (15.62)	81.43 (13.94)
Total			
(N = 30)	Gradual	75.63 (17.06)	80.0 (16.15)
(N = 30)	All-at-once	81.27 (9.38)	83.3 (10.94)
(N = 60)	Total	78.45 (13.94)	81.67 (13.78)

Using a two-way ANCOVA with the pre-test total scores used as a covariate, to take into account variations in children's understanding of concepts prior to the instructional session, it was found that there were no statistically significant differences in the post-test mean marks between the abacus or the MAB blocks groups, F(1, 59) = 1.146, p = .289, ηp^2 = .020 (small effect size) – i.e. no significant main effect of the type of materials. Similarly, there was no statistically significant difference in post-test total mean marks between the gradual transition and the all-at-once transition instructional methods, $F(1, 59) = .527$, $p = .471$, $\eta p^2 = .009$ (small effect size) – i.e., no significant main effect of the instructional method.

Additionally, there was no significant interaction between the manipulative material (abacus v. MAB blocks) and the method of instruction (gradual v. all-at-once), $F(1, 59) = .088$, $p = .768$, $\eta p^2 = .002$ (small effect size).

A brief examination of any potential gender effects, using the independent samples t-test, indicated that there were no statistical differences in the performance of males and females in either the pre-test scores, $t(58) = 1.000$, $p = .322$, or the post-test scores, $t(58) = 0.647$, $p = .520$ in the overall marks across all four strategies of teaching.

No other statistically significant differences were detected between the different materials or instructional methods used, or any interactions between the materials and methods.

6.4 Discussion

Experiment 1 was designed to test the hypothesis that teaching place value with the aid of MAB blocks would be inferior to teaching place value with a Slavonic abacus. It was hypothesized that because blocks do not directly translate to the numbers that they represent, it would be more beneficial to use an abacus. When using MAB blocks, the student is expected to decide which block is bigger based on the size of the block. No direct counting is involved, and choosing the correct block size can be easily learnt by rote. In contrast, each bead of the abacus is the same size and is counted as one, with beads arranged in groups of ten. This could reflect a more dirrect correlation between the physical object and counting. Additionally, the experiment attempted to establish whether a gradual teaching method (Ross 1989), would be more beneficial to students than an all-at-once transition.

The results of this experiment did not support the hypothesis. There were no significant differences in the use of MAB blocks or the abacus to teach the concepts of place value and there were no significant differences in the methods of teaching. Additionally, no interactions between the method of teaching and the materials used were found. This result could be due to the children's familiarity with MAB blocks, a material that they often use within the setting of their normal classroom. Observationally, children were able to use the MAB blocks in such a way that showed that they had simply "memorised the answer", that is, they knew how to use the blocks to construct the necessary numbers, without actually making the connection between the blocks and numbers, and showing no inherent understanding of the concept of place value. Some issues were also observed with children using the Slavonic abacus. The abacus was used to help the children count out numbers; however, all counting was mostly done in ones and the logical tens mapping of each line was not used by the students for the most part. This lack of connection between the meaning of each number and the deconstruction of each number into its relevant base-10 values was evident in the common mistakes made by the children in their post-tests. A small number of children had trouble writing numbers such as "one hundred and two" and would write these numbers as "12". This was repeated in the representation task in both the pre- and post-tests, where children did not include the zero placeholders when converting words to numerals (Figure 24). This demonstrated a very early stage of place value understanding, where no meaning was assigned to the actual values that each digit within a particular number represented.

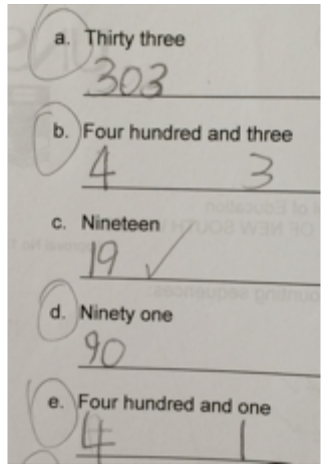


Figure 24: Example of mistakes made in the representation task, converting words to numerals

An inability to bridge the 100s when counting off the decade was another common mistake made by more than half (58.3%) of all the children tested (Figure 25) in their post-test results. This mistake indicated stages 1 or 2 in the level of understanding; however, this was specifically observed with three-digit numbers only.

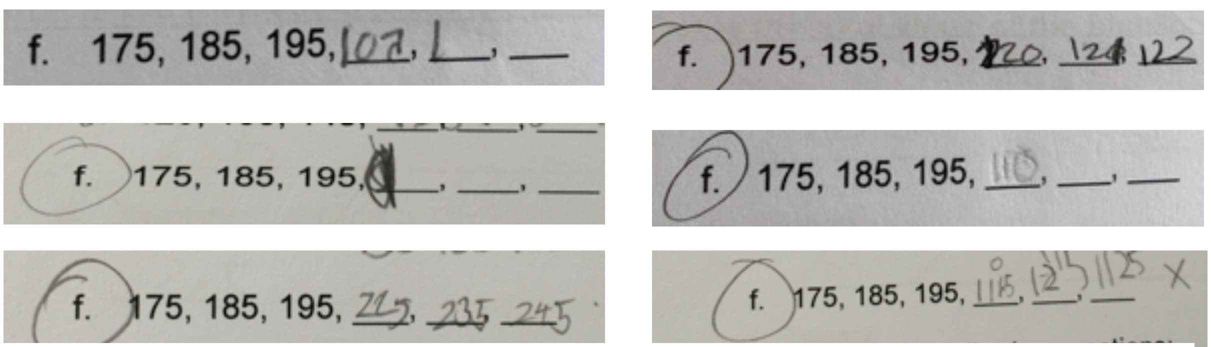


Figure 25: Mistakes showing an inability to bridge the 100's when counting off the decade in the post-test

One of the most common mistakes made was in specifying expanded notation of a number. A large majority of children in their post-test, specifically 78.3% of all children tested, were not able to complete this question, or could not complete it correctly. The mistakes made while attempting this question showed a lack of understanding of how numbers are built, and that each digit within a number represented a given quantity based on its location (Figure 26).

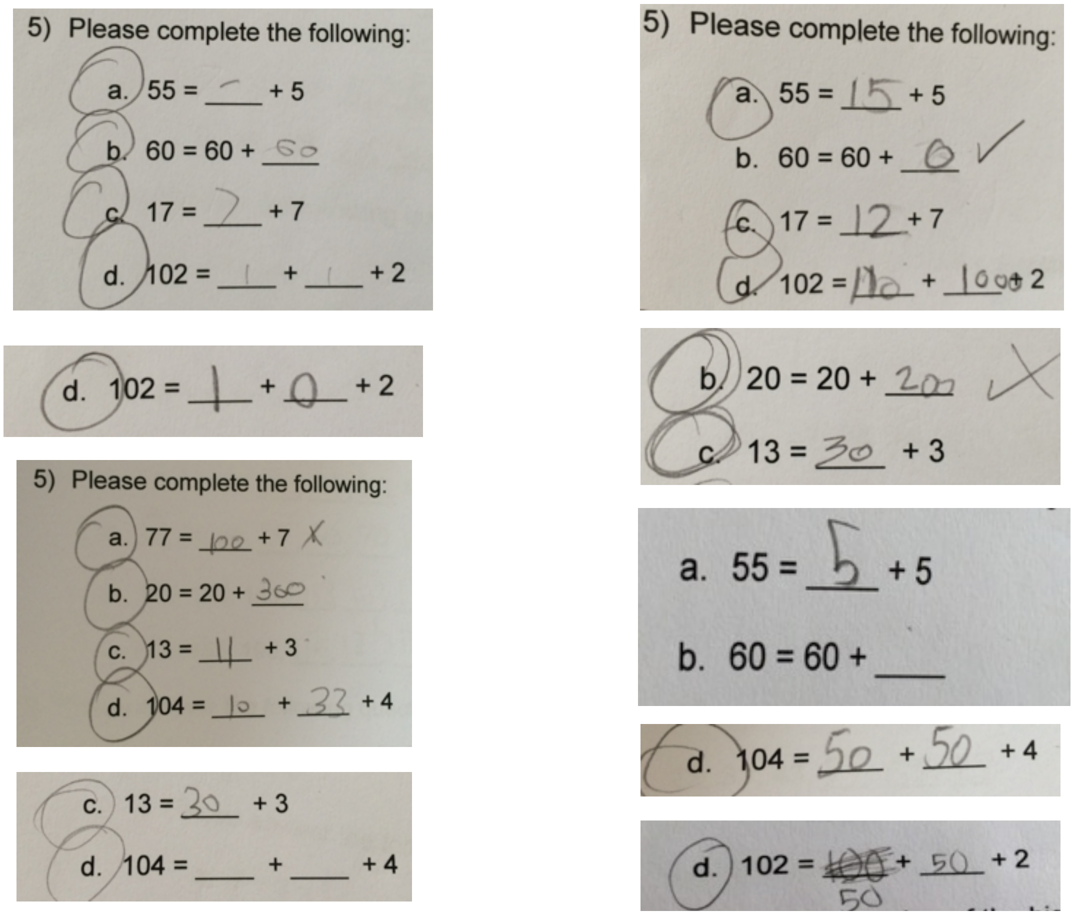
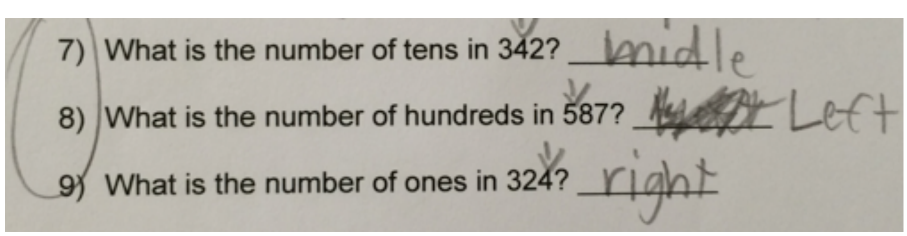


Figure 26: Common mistakes made in expanding a number

Stage 5 of understanding, where children are easily able to determine what place value is, expand a particular number based on place values for both single and multi-digit numbers, was only attained by 8% of the children tested in the post-test. The remainder of the children mostly left this section blank or attempted to complete this section with mistakes, which showed a lack of understanding of the concept (Figure 27).



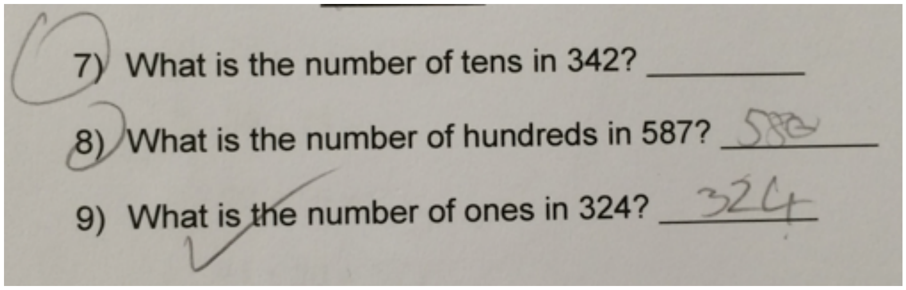


Figure 27: Mistakes made by children in specifying place value in numbers

All these mistakes were made despite the fact that, throughout the intervention, children were able to successfully use MAB blocks and the Slavonic abacus to construct numbers. Thus, observationally, whilst children understood what the different MAB blocks represented, they were not actually associating the blocks with the numerical values. Similarly, the children were not exploting the base-10 characteristics of the Slavonic abacus. The children had simply learnt how to build the numbers correctly using the blocks or to count out the numbers using individual beads, without any actual understanding of the quantities that the numbers and blocks or the lines of beads represented. It appeared that for those children who could easily construct numbers, and who had a good understanding of place value, the representative MAB block sizes actually correlated easily to the numerical value desired. Similarly, children with a good understanding of place value were able to count in tens on the Slavonic abacus by using a whole line of beads at once. However, for those children who had not yet made number associations, MAB blocks or the Slavonic abacus, did not appear to help bridge the gap between the physical objects and their meaning. Children were able to construct numbers more quickly using the Slavonic abacus, as each bead represented a single count, an association that was easier to make for the children and requiring fewer of the limited cognitive resources. This was in contrast to the MAB blocks, where patterns in the structure of the blocks and the association between the blocks and the numbers, were difficult to construct using three different sized blocks.

7 Experiment 2 – The Worked Example and Place Value

7.1 Introduction

The results of the first experiment showed no significant differences in student performance between the gradual or all-at-once transition or the two different manipulative materials used (abacus vs. MAB blocks). The lack of any differences could be attributed to the real-time method of instruction in the classroom. Whilst great care was taken to ensure consistency across all the groups with the same teachers and the same material presented to the students during instruction, there were still differences that could not be controlled, due to the real-time nature of classroom instruction. These included differences in questions posed by children, and the differing personalities of the children themselves, with some children more willing to ask questions to improve their understanding as necessary. In order to achieve a more consistent approach in the acquisition stage of the experimental setup, Experiment 2 based its instructional materials on the worked example effect, which states that providing worked examples is a more efficient way to teach novice students, in comparison to straight problem solving (G. A. Cooper & Sweller, 1987; Sweller, 1988).

The children in Experiment 1 demonstrated a lack of place value knowledge, in particular, when attempting number decomposition. The students scored an average mark of 60.9% in post-test scores and an average mark of 55.6% in pre-test scores in this area. Experiment 2 thus focussed on expanded notation (number decomposition), an area that children had little or no prior knowledge in, and was therefore the most likely area to produce the worked example effect.

The aim of Experiment 2 was to test the hypothesis that a worked example using the students' primary skills of movement and grouping knowledge and representation of number, when using an abacus, would be superior to that of using the MAB blocks as the manipulative material, in learning the secondary skills of place value and number knowledge, particularly when focussing on problems involving expanded notation and face value.

7.2 Method

7.2.1 Participants

Experiment 2 was conducted with forty-eight Year 2 students, from a selection of three private schools in the Eastern Suburbs, and Northern Suburbs of Sydney. My School website (www.myschool.edu.au) was used to better understand the backgrounds of each of the schools. School 1, located in metropolitan Sydney, had a student population of 2044, with 17% of students having a language background other than English. My School classified 83% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school. School 2, also categorised as 'metropolitan', had a student population of 753, with 83% of students having a language background other than English. My School classified 36% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a less advantaged school. The last school, School 3, also a metropolitan school, had a student population of 861, with 2% of students having a language background other than English. My School does not report the data for the Index of Community Socio-Educational Advantaged for this school. Students were all aged approximately 7-8 years, and were randomly assigned to three groups of sixteen students each: Abacus worked example, MAB blocks worked example, No visuals worked example (control).

7.2.2 Materials and Procedure

Students were tested in groups of three, with one student using an Abacus worked example and having access to an abacus, the second student using an MAB blocks worked example and having access to MAB blocks to aid them, and the third student in each test group using the No visuals worked example with no access to any of the manipulative materials. All the materials used as part of this experimental setup can be found in Appendix B: Experimental materials used in Experiment 2. Prior to commencing the testing, students were given a pre-test to establish their current understanding of place value. The pre-test was composed of eight questions, with questions focussed on the areas of expanded notation, face value and total value of place value. Following this, students were tested in two phases: the acquisition phase and the test phase. In the acquisition phase, students were presented with four sets of place value questions, each set consisting of a worked example followed by a similar question for the students to solve. Each student was given four minutes for each set of questions: two minutes to study the worked example and a further two minutes to solve the similar question. Therefore, for completing all four sets of problems, each student needed 16 minutes in total. Students in all of the three groups were given the same amount of time in the acquisition phase of the experiment. Students were asked to follow the worked example solution when solving the similar question, with the worked example available to students at all times in the acquisition phase. Students were allowed to have as many attempts as needed at a particular question, as long as their total time per question did not exceed two minutes.

The acquisition phase was followed by the test phase, in which a common test was used to examine the effect, if any, of the acquisition phase on learning the concept of place value. The test phase was composed of seven questions, without any worked examples but similar to the problems used in the acquisition phase. Each student was given a maximum of one minute to solve each of the 7 questions, for a total of 7 minutes. If a question remained unsolved after 1 minute, the child was guided to the next question. The time taken by each student to solve the questions was measured. Each correct answer was worth 1 mark, resulting in a total possible score of 7 marks. Children did not have access to the worked examples during the testing phase.

7.3 Results

The time taken to complete the pre-test and the score received by each of the students in the pre-test were measured and analysed to understand base-line knowledge in all the groups, and to ensure that there were no significant differences in the cohorts of students between the Abacus (n = 16, M = 33.04, SD = 20.68), MAB blocks (n = 16, M = 34.82, SD = 22.72), and Control (n = 16, M = 33.93, SD = 26.53) groups, F (2, 45) = 0.023, p = .977.

In addition, in the acquisition phase, the variables under analysis were the learning times, and also the learning scores. In the test phase, the variables under analysis were the testing time and the total test scores. The total learning time was another dependent variable used in analysis, and measured the amount of time each student spent studying the worked examples in the acquisition task. Table 5 shows the means and the standard deviations of each of the variables.

Table 5: Descriptive Statistics for the variables

	Abacus (N=16)	MAB blocks (N=16)	Control (N=16)	Total (N=48)
Pre-Test Time (mins)	3.64 (1.21)	3.37 (0.92)	3.22 (1.10)	3.41 (1.07)
Pre-Test Score %	33.04 (20.68)	34.82 (22.72)	33.93 (26.53)	33.93 (22.95)
Learning Time (mins)	12.29 (1.83)	12.63 (1.30)	8.80 (0.72)	11.24 (2.20)
Acquisition Phase Score %	68.75 (1.06)	60.94 (1.26)	64.06 (0.96)	64.58 (1.09)
Testing Phase Time (mins)	2.7 (1.38)	2.57 (1.42)	2.55 (1.36)	2.61 (1.36)
Testing Phase Score %	50.0 (31.73)	47.32 (32.04)	58.93 (22.05)	52.08 (28.80)

A one-way between groups analysis of variance was conducted to explore the impact of the manipulative material used on place value learning, as measured by the testing phase scores, testing times, learning times and acquisition phase scores. Using the pre-test mark as a covariate, there were no significant differences in the mean testing phase scores between the abacus (n = 16, M = 50.0, SD = 31.73), MAB blocks (n = 16, M = 47.32, SD = 32.04), and Control (n = 16, M = 58.93, SD = 22.05) groups, F (2,44) = 1.135, p = .331, ηp^2 = .049.

There were also no significant differences in the mean acquisition phase scores between the Abacus (n = 16, M = 68.75, SD = 1.06), MAB blocks (n = 16, M = 60.94, SD = 1.26), and Control (n = 16, M = 64.06, SD = 0.96) groups, F (2,44) = 0.046, p = .955, ηp^2 = .002.

No significant differences were also found in the mean testing phase time across the three different groups, Abacus (n = 16, M = 2.7, SD = 1.38), MAB blocks (n = 16, M = 2.57, SD = 1.42), and Control (n = 16, M = 2.55, SD = 1.36) groups, F (2,44) = 0.364, p = .697, ηp^2 = .016.

However, there was a statistically significant difference in the mean learning times, that is, the time taken during acquisition to study the worked examples, between the three groups of Abacus (n = 16, M = 12.29, SD = 1. 83), MAB blocks (n = 16, M = 12.63, SD = 1.30), and Control (n = 16, M = 8.80, SD = 0.72) groups, F (2,44) = 40.124, $p < 0.0005$, $\eta p^2 = .646$. A Tukey post-hoc test showed that the increase in learning times from 8.8 ± 0.72 in the control group to 12.63 ± 1.3 in the MAB blocks group was statistically significant ($p < 0.0005$); d = 8.45. Additionally, the increase in learning times from the control group (8.8 ± 0.72) to 12.29 ± 1.83 in the Abacus group was also statistically significant ($p < 0.0005$); d = 6.12. This meant that the students in the control group took less time to study and learn the worked examples, than the students in those groups using manipulative materials.

7.4 Discussion

Informed by the lack of significant findings in Experiment 1, Experiment 2 modified the mode of instructional guidance by using worked examples. Additionally, based on the results in Experiment 1 pre- and post-test scores, the test materials in Experiment 2 were modified to include only face value and expanded notation questions, which were the areas that the learners had the most difficulting in grasping. The procedure of Experiment 2 was subsequently also changed to accommodate the inability of such young students to focus for a prolonged period of time on the same task. It was hypothesized that these changes would result in a significant difference in the use of materials when teaching place value, and, in particular, that using the abacus would be more intuitive to students than the MAB blocks in place value problem solving.

The results of the experiment did not support the hypothesis. There were no significant differences found in the use of either MAB blocks or abacus to support place value instructional material, as indicated by the post-test scores in the testing phase. Throughout the experiment, it was observed that learners were having great difficulty comprehending the highly text-based instructions given to them for each of the conditions. Some of the students even required explanations of the English terms used in the questions. The difficulty reading the textual information prior to even attempting the mathematical question, possibly resulted in an increase in extraneous load and fewer cognitive resources remaining to cope with the presented mathematical problem. Whilst there were no significant differences between the two manipulative materials, the finding that students took less time to study and learn the worked examples in the control group, in comparison to both the MAB blocks and the Abacus group could be easily explained. The students in the control group were only required to contend with the text-based information, as opposed to the combination of text, pictures and physical material. Therefore the worked examples within the instructional material presented to the learner, even without visuals or manipulative materials, presented such a high cognitive load, that the worked examples were not able to act as substitute schemas to aid in construction of students' own schemas (Kalyuga et al., 2001), and therefore were not effective in producing any improvements in learning.

8 Experiment 3 – The Worked Example and Place Value (Simplified comprehension)

8.1 Introduction

The results of Experiment 2 indicated that there were no significant differences in the performance of students using different manipulative materials. This result can be attributed to the high level of reading comprehension required to understand the mathematical problems presented as worked examples. An assumption based on the results and observations made in Experiment 2 is that the worked examples provided to the students were too complex and were causing a cognitive overload prior to the children attempting to understand the concept of place value.

Therefore, Experiment 3 aimed to reduce the reading comprehension required for the children by presenting the place value problems in a more readable manner, with simplified instructions and explanations. This was expected to help reduce the extraneous load on the students, thereby allowing their limited cognitive resources to be used in understanding and familiarising themselves with the place value worked example.

Additionally, students were tested in groups of three with all three students using the same manipulative material at the same time. This was done to ensure limited distractions for the students, and to avoid any disappointment when no manipulative materials were provided.

The hypothesis of Experiment 3 remained the same as that of Experiment 2. However, the method differed in the instructional material and the way groups were formed. It was hypothesized that a worked example using the child's primary skills of movement and grouping, with the abacus as the manipulative material, would be superior to that of using the MAB blocks as the manipulative material, in learning the secondary skills of place value and number knowledge, particularly when focusing on problems involving expanded notation and face value.

8.2 Method

8.2.1 Participants

Experiment 3 was conducted with forty-five Year 2 students, selected from three private schools in the Eastern and Western suburbs of Sydney. School 1 had a student population of 1206, with 46% of students having a language background other than English. My School (www.myschool.edu.au) classified 81% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school. School 2 had a student population of 2036, with 21% of students having a language background other than English. My School classified 50% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore also considered to be a fairly advantaged school. The third school had a student population of 1012, with 68% of students having a language background other than English. My School classified 79% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school. Students were all aged approximately 7-8 years, and were randomly assigned to three groups: Abacus worked example (15 students), MAB blocks worked example (15 students), No visuals worked example (15 students).

8.2.2 Materials and Procedure

Students were tested in groups of three, with each group of three students having access to either an abacus (Abacus worked example group), MAB blocks (MAB blocks worked example group), or no manipulative materials at all (No visuals worked example). In addition to having access to the physical materials, the students' worked examples contained illustrations of the relevant manipulative materials for each of the examples. All the materials used as part of this experimental setup can be found in Appendix C: Experimental materials used in Experiment 3.

Prior to commencing the testing, students were given a pre-test, composed of 7 place value questions, to establish their current understanding of place value. Following this test, students were tested in two phases: the acquisition phase and the test phase. In the acquisition phase, students were presented with four sets of place value questions, each set consisting of a worked example followed by a similar question for the students to solve. Each student was given four minutes for each set of questions: two minutes to study the worked example and a further two minutes to solve the similar question. Therefore, for completing all four sets of problems, each student needed 16 minutes in total. Students in all of the three groups were given the same amount of time in the acquisition phase of the experiment. Students were asked to follow the worked example solution when solving the similar question, with the worked example available to students at all times in the acquisition phase. Students were allowed to have as many attempts as needed at a particular question, as long as their total time per question did not exceed two minutes.

The acquisition phase was followed by the test phase, in which a common test was used to examine the effect, if any, of the acquisition phase on learning the concept of place value. The test phase was composed of 7 questions, without any worked examples but similar to the problems used in the acquisition phase. Each student was given a maximum of one minute to solve each of the 7 questions, for a total of 7 minutes. If a question remained unsolved after 1 minute, the child was guided to the next question. The time taken by each student to solve the questions was measured. Each correct answer was worth 1 mark, resulting in a total possible score of 7 marks. Children did not have access to the worked examples during the testing phase.

8.3 Results

The score received by each of the students in the pre-test was measured and analysed to understand base-line knowledge in all the groups, and to ensure that there were no significant differences in the cohorts of students between the Abacus (n = 15, M = 51.67, SD = 14.07), MAB blocks (n = 15, M = 44.17, SD = 28.69), and Control (n = 15, M = 52.5, SD = 32.46) groups, F (2, 44) = 0.457, p = .636. Additionally, the interaction of the treatment and three different schools used was also found to be non-significant, F (2,44) = 1.127, p = .359).

In the acquisition phase, the variables under analysis were the learning times, the amount of time each student spent studying the worked examples in the acquisition phase, the number of attempts made per question by each of the three groups, and the learning scores. In the test phase, the variables under analysis were the time taken to complete the test and the total test scores. Table 6 shows the means and standard deviations of each of the variables.

Table 6: Descriptive Statistics for the variables

	Abacus (N=15)	MAB blocks (N=15)	Control (N=15)	Total (N=45)
Pre-test Score %	51.67 (14.07)	44.17 (28.69)	52.50 (32.46)	49.44 (25.97)
Learning Time (mins)	12.44 (1.39)	11.58 (1.30)	9.55 (0.71)	11.25 (1.69)
Acquisition Phase Score %	91.67 (20.41)	91.67 (15.43)	100 (0.00)	94.44 (14.97)
Testing Phase Time (mins)	2.33 (1.21)	1.63 (0.81)	2.22 (1.64)	2.06 (1.27)
Testing Phase Score %	72.5 (20.70)	65.0 (31.05)	86.31 (17.92)	74.60 (25.00)
Question 1 Attempts	1.2 (0.56)	1.27 (0.59)	1.07 (0.26)	1.18 (0.49)
Question 2 Attempts	1.87 (0.74)	1.40 (0.63)	1.27 (0.59)	1.51 (0.69)
Question 3 Attempts	1.07 (0.26)	1.27 (0.46)	1.27 (0.46)	1.20 (0.40)
Question 4 Attempts	1.27 (0.46)	1.33 (0.82)	1.33 (0.49)	1.31 (0.60)

A one-way between groups analysis of variance was conducted to explore the impact of the use of manipulative materials on place value learning, as measured by the testing phase scores, testing times, and learning times and acquisition phase scores. Using the pre-test mark as a covariate, there were significant differences in the mean testing phase scores between the Abacus (n = 15, M = 72.5, SD = 20.7), MAB blocks (n = 15, M = 65.00, SD = 31.05), and Control (n = 15, M = 86.31, SD = 17.92) groups, F (2, 44) = 3.180, p = .05, $\eta_p{}^2$ = .134. A follow up Tukey post-hoc test was run to find that the significant difference was between the MAB and Control groups, with p = .049. Additionally, using the learning time as another covariate, there were significant differences in the mean test phase scores between the Abacus (n = 15, M = 72.5, SD = 20.7), MAB blocks (n = 12, M = 68.75, SD = 33.5), and Control (n = 13, M = 84.2, SD = 18.4) groups, F (2, 39) = 3.315, $p < .05$, $\eta_p{}^2$ = .159. A follow up post-hoc test using the LSD adjustment was run to discover the significant difference between the Abacus and Control group with p = .024, and MAB and Control group with p = .022.

There were no significant differences in the mean acquisition phase scores between the Abacus (n = 15, M = 91.67, SD = 20.41), MAB blocks (n = 15, M = 91.67, SD = 15.43), and Control (n = 15, M = 100, SD = 0) groups, F (2,44) = 1.424, p = .252, $\eta_p{}^2$ = .065.

Additionally, no significant differences were found in the mean testing phase time across the three different groups, Abacus (n = 15, M = 2.33, SD = 1.21), MAB blocks (n = 15, M = 1.63, SD = 0.81), and Control (n = 15, M = 2.22, SD = 1.64) groups, F (2,44) = 0.67, p = .201, $\eta_p{}^2$ = .075.

However, there was a statistically significant difference in the mean learning times, that is the time taken during acquisition phase to study the worked examples, between the three groups of Abacus (n = 15, M = 12.44, SD = 1. 39), MAB blocks (n = 15, M = 11.58, SD = 1.30), and Control (n = 15, M = 9.55, SD = 0.71), F (2,44) = 21.207, $p < .01$, $\eta_p{}^2$ = .541. A Tukey post-hoc test showed that the increase in learning times from 9.55 ± 0.71 in the Control group to, 11.58 ± 1.30 in the MAB blocks group was statistically significant ($p < .01$). Additionally, the increase in learning times from the Control Group (9.55 ± 0.71) to 12.44 ± 1.39 in the Abacus group was also statistically significant ($p < .01$). This could indicate that it took less time to study the worked examples presented in the control group, than those groups that used manipulative materials.

8.4 Discussion

Experiment 3 was designed to replicate Experiment 2, with modifications to the learning materials given to the students in the acquisition phase. Students were required to solve place value problems using either an abacus, MAB blocks, or no manipulative materials for the control group. All students were first presented with worked examples of place value problems, that were also illustrated by using an abacus, MAB blocks or no manipulative materials, depending on the group they were assigned to. It was hypothesized that, with the reduced reading comprehension required, students would be able to use their limited cognitive resources to process the instructional place value material. This increase of working memory resources available to students was expected to be reflected in better post-test performance. Additionally, it was anticipated that due to the greater one-to-one correspondence of the abacus beads in relation to countable objects, and the intuitive grouping of the beads in tens, the abacus would make a superior reprepresenation of place value in comparison to MAB blocks.

The results of Experiment 3 only partially supported this hypothesis. Firstly, the reduced time taken by the control group, in comparison to both MAB block and abacus groups, to study the worked examples, can be explained. Because the instructional material for the control group was not accompanied by the use of manipulative materials, or relevant illustrations, the group was required to process less information. Since children were only subjected to numerical information, they were able to process the information faster due to an increase in their available working memory resources.

Secondly, whilst no differences were found between the use of the MAB blocks and an abacus in number decomposition tasks, a Tukey post-hoc test showed a significant difference in the post-test results between the group using MAB blocks and the control group. The students in the control group, who were given no access to physical manipulative material or their visual representation, performed better in the post-test than those students that had access to, and were given visual representations of, MAB blocks to represent place value problems. Such a difference could be due to the potential redundancy effect the MAB blocks may have caused to the student. The numerical information and the visual representation of the MAB blocks were each sufficient on their own to allow the student to understand the concept, yet they are often presented in conjunction with each other in the classroom and in instructional material. This leads to the basis of Experiment 4, which aims to explore whether the use of MAB blocks, both visually and physically, may lead to the redundancy effect, which in turn may have a negative impact on learning.

9 Experiment 4 – The Worked Example, Place Value and Manipulative Material Redundancy

9.1 Introduction

The results of Experiment 3 indicated that worked examples without MAB manipulatives may be beneficial to young primary-aged children establishing their early understanding of place value concepts. However, these worked examples needed to be sufficiently simple in terms of the language and reading comprehension skills required not to overwhelm the student's limited cognitive resources, and yet appropriately difficult to activate the learning pathways.

Students in the MAB block worked example group were provided with illustrations of blocks to demonstrate number deconstruction and were also allowed to use physical MAB blocks to help construct and manipulate given numbers. Similarly, children in the Abacus worked example group were provided with illustrations of an abacus to demonstrate number deconstruction and were also allowed to use the physical abacus to help construct and manipulate given numbers. The third group of children, the Control worked example group were provided with no illustrations or manipulative materials to assist with number construction and manipulation. The children in the Control worked example group performed better in the post-test, in comparison to the students in the MAB worked example group. There were no significant differences between the Abacus worked example group and the MAB block group, or the Abacus worked example group and the Control worked example group.

Since no significant differences in performance were found between the two types of manipulative materials used, it was decided that, in Experiment 4, only MAB blocks will be utilised. The MAB blocks were chosen due to their popularity and widespread use in Australian classrooms. It was also observed in Experiment 3, that the large quantity of blocks required to construct three- and four-digit numbers would often interfere with students being able to keep track of such numbers and their associated blocks. Students using blocks often made counting and handling errors. These errors were likely related to the amount of information that was required to be processed by their working memory at any one time. Keeping track of both the blocks necessary for the numerical deconstruction and also the concept of place value that the blocks represent, may have resulted in the complete exhaustion of their limited cognitive resources. The instructional material, in the form of worked examples, was therefore modified in Experiment 4, to include simpler numbers that did not require a large number of blocks in construction.

Experiment 4 aimed to explore the impact on children's learning and acquisition of the concept of place value, specifically number deconstruction, when using MAB blocks in worked examples, both illustratively and physically. It was hypothesized that children who did not have access to any manipulative materials would perform better in the testing phase, as measured by the post-test, post-test transfer and delayed test, than those children given access to MAB blocks.

9.2 Method

9.2.1 Participants

Experiment 4 was conducted with fifty-two Year 2 students, selected from two private schools in the Eastern suburbs of Sydney. According to the My School website (www.myschool.edu.au), the first school had a student population of 1029, with 14% of students having a language background other than English. My School classified 76% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school. School 2 had a student population of 777, with 7% of students having a language background other than English. My School classified 77% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school, similarly to the first school. Students were all aged approximately 7-8 years, and were randomly assigned to two groups: MAB blocks worked example (24 students), No visuals worked example (28 students).

9.2.2 Materials and Procedure

Students were tested in groups of three, with each group of three students having access to either the MAB blocks (MAB blocks worked example group), or no manipulative materials at all (No visuals worked example). Students in the MAB blocks group, had access to the physical materials as well as illustrations of MAB blocks in the paper-based worked examples. The second group, No visuals worked example group, were provided with the same worked examples, except no access was given to physical blocks, and no illustrations were included with their examples. All the materials used as part of this experimental setup can be found in Appendix D: Experimental materials used in Experiment 4.

Prior to commencing the testing, students were given a pre-test, composed of 7 place value questions, to establish their current understanding of place value. Following this test, students participated in two phases: the acquisition phase and the test phase. In the acquisition phase, students were presented with 9 place value statements, each of which detailed how to deconstruct a number up to a hundred, increasing by tens, such as 19, 29, 39, and so on. There were no paired questions associated with these worked examples during the acquisition phase. Each student was given two minutes per statement to study the worked example, and to construct the number using MAB blocks, if they were in the MAB block group. Therefore, for completing all nine problems, each student needed 18 minutes in total. Students in all of the three groups were given the same amount of time in the acquisition phase of the experiment. Students were allowed to have as many attempts as needed to construct a particular numerical value within a question, so long as their total time per question did not exceed two minutes. At the conclusion of their allocated question time, if the student had not constructed the correct numerical value, a solution was demonstrated to them.

The acquisition phase was followed by the test phase, in which a common test was used to examine the effect, if any, of the acquisition phase on learning the concept of place value. The test phase was composed of 10 post-test questions, without any worked examples, but similar to the problems used in the acquisition phase using different numerical values. Each student was given a maximum of 1 minute to solve each of the 10 questions, for a total of 10 minutes. If a question remained unsolved after 1 minute, the child was guided to the next question. Each question was worth 1 mark, resulting in a total possible score of 10 marks. Children did not have access to the worked examples during the testing phase. The post-test was then followed up by a post-transfer test, where 6 questions needed to be answered. The questions were different in structure to the worked examples, but in answering them, a student may be able to use information that could have been learnt from the worked examples. Each student was again given 1 minute for each question, and were able to make as many attempts as needed, as long as the overall time did not exceed 1 minute.

One week later, a delayed test was given to the participating students to ascertain whether any delayed place value knowledge existed. The delayed test was composed of sixteen questions. The first 10 questions directly related to the worked examples that were given to the students a week earlier. The subsequent 6 questions did not directly relate to the worked examples, but required the knowledge from the worked examples to be extended. Each student was given 1 minute per question, for a total of sixteen minutes to be spent on the delayed test. The students were allowed to make as many attempts as they needed to respond to the questions, as long as their overall time did not exceed 1 minute per question, with sixteen minutes in total to complete the test.

9.3 Results

The score received by each of the students in the pre-test was measured and analysed to understand base-line knowledge in all the groups, and to ensure that there are no significant differences in the cohorts of students between the MAB blocks (n = 24, M = 69.79, SD = 27.783), and Control (n = 28, M = 79.76, SD = 16.582) groups, $F(1, 51) = 2.551$, $p = .117$, $\eta_p^2 = .049$.

The variables under analysis in the test phase were the scores for the post-test, the post-test transfer, and the delayed test scores. Table 7 shows the means and standard deviations of each of these variables.

Table 7: Descriptive Statistics for the variables

	MAB blocks (N=24)	Control (N=28)	Total (N=52)
Pre-test Mark %	69.79 (27.78)	79.76 (16.58)	75.16 (22.78)
Post-test Mark %	86.39 (30.07)	87.8 (20.18)	87.15 (24.98)
Post-test Transfer Mark %	73.04 (30.46)	77.19 (24.26)	75.27 (27.1)
Delayed Test Mark %	n = 19	n = 25	n = 44
	79.79 (29.98)	81.44 (18.33)	80.73 (23.75)

A one-way between groups analysis of variance was conducted to explore the impact of the redundancy effect on place value learning, as measured by the testing phase scores, including the post-test scores, post-test transfer scores and the delayed test scores. Using the pre-test mark as a covariate, there were no significant differences in the mean testing phase scores in the post-test between the MAB blocks (n = 24, M = 86.39, SD = 30.07), and Control (n = 28, M = 87.8, SD = 20.18) groups, $F(1, 51) = 0.688$, $p = .411$, $\eta_p^2 = .014$. Additionally, there were no significant differences in the post-test transfer results between the MAB blocks (n = 24, M = 73.04, SD = 30.46), and Control (n = 28, M = 77.19, SD = 24.26) groups, $F(1,51)$ =

0.420, $p = .520$, $\eta_p^2 = .008$. The delayed test was completed the week after the post-test and the post-test transfer. Due to absences of some children on that day, the delayed test could only be given to 44 children, 19 of those in the MAB worked example group and 25 of those in the Control group. Similarly to the post-test and the post-test transfer results, no significant differences were found in the delayed test results between the MAB block group ($n = 19$, $M = 79.79$, $SD = 29.98$) and the Control group ($n = 25$, $M = 81.44$, $SD = 18.32$), $F(1, 43) = 2.139$, $p = .151$, $\eta_p^2 = .050$.

9.4 Discussion

Experiment 4 was designed to test the hypothesis that the utilisation of MAB blocks in worked examples in Year 2 place value material may induce a redundancy effect and could therefore lead to difficulties in learning the concept of place value effectively. Students were asked to study worked examples that demonstrated numbers up to the hundreds jumping in groups of tens. All students were instructed to say these number skips out loud and, if in the MAB blocks group, were also required to construct the numbers using the MAB blocks. It was expected that students not using the MAB blocks would perform better than those students that had access to MAB blocks, as measured by the post-test, the post-test transfer and the delayed test scores in the testing phase.

The results of Experiment 4 did not support this hypothesis. The high scores of the students in the pre-test, post-test, post-test transfer and delayed test in the testing phase of the experiment indicated a high level of prior knowledge. This demonstrates the presence of the ceiling effect, with students unable to acquire any further knowledge. The high level of student performance in the test phase meant that a better alignment between the difficulty of the material and the expertise of the students was necessary. The New South Wales Mathematics K-10 Syllabus (Board of Studies Teaching and Educational Standards - NSW, n.d.), defines the ability of students to count collections to a hundred by partitioning the numbers using place value, as an outcome required from Stage 1.2 (Year 2). However, since the experimental data was collected in the later stages of Year 2 (at the conclusion of Term 3 and Term 4), students might have already built up enough prior knowledge to learn the concepts easily. This prior knowledge would have meant that they were able to complete the questions in the pre-test and the test phases without a a great deal of learning taking place in the acquisition phase. This was one possible reason for misjudging the prior knowledge base of the students.

It was expected that repeating this experiment with a different cohort of learners and using a more complex set of questions would yield significant differences between the MAB worked example group and the control group. By using learners who are at Stage 2.1 of schooling (end of Year 3), and increasing the complexity of the testing material to the expected learning outcome of Stage 2.2 (Year 4) to *‘use place value to partition numbers of up to five digits and recognise this as 'expanded notation''* (Board of Studies Teaching and Educational Standards - NSW, n.d.; NESA, 2015)*,* could circumvent the ceiling effect observed in Experiment 4. This hypothesis was tested in Experiment 5.

10 Experiment 5 - The Worked Example, Place Value and Manipulative Redundancy (increased expertise and complexity of instructional material)

10.1 Introduction

Experiment 4 demonstrated no statistical significance between the group using MAB blocks, illustratively and physically, and the control group. One possible explanation for this lack of significance could be a higher level of prior topic knowledge than was initially observed with children at this stage of schooling. This indicated that either the level of expertise of the students needed to be changed to include more novice learners, or the complexity of the material needs to be increased, so that students were provided with new material, of which they had little or no prior experience.

In Experiment 5, the complexity of the material was increased to include number decomposition into the tens of thousands. This level of number comprehension was associated with Stage 2.2 (end of Year 4) of learning, as prescribed by the New South Wales Mathematics K-10 Syllabus (NESA, 2015). Due to the extreme complexity of material, students were selected at the conclusion of Year 3 to participate in the experiments in the first two weeks of Term 1 of Year 4. This was done to ensure that numbers of such magnitude had not yet been covered in the classroom and that students had no prior knowledge of how to decompose and bridge such numbers into the thousands and the tens of thousands. This was to be established by the pre-test. The learning materials of Experiment 5 were modified not only to include the more complex number deconstructions, but also to add a paired problem to each of the worked examples to help with concept acquisition. It was hypothesized in Experiment 5 that the students who were presented with illustrations of MAB blocks and required to use physical MAB blcoks, to assist with number construction and decomposition, would be hindered in their learning by the redundancy effect caused by the blocks.

10.2 Method

10.2.1 Participants

Experiment 5 was conducted with thirty-eight Year 4 students, selected from two private schools in the Eastern suburbs of Sydney. According to the My School website (www.myschool.edu.au), the first school had a student population of 1029, with 14% of students having a language background other than English. My School classified 76% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school. School 2 had a student population of 777, with 7% of students having a language background other than English. My School classified 77% of the students as being in the top quartile of the Index of Community Socio-Educational Advantaged. It is therefore considered to be a very advantaged school, similarly to the first school. Students were all aged approximately 8-9 years, and were randomly assigned to two groups: MAB blocks worked example (20 students), No visuals worked example (18 students).

10.2.2 Materials and Procedure

Students were tested in groups of three, with each group having access to either MAB blocks (MAB blocks worked example group), or no manipulative materials (No visuals worked example). The MAB blocks groups was given both illustrations and physical MAB blocks to manipulate. The Control group did not have any illustrations included with their acquisition text, but in all other respects the questions and text were identical to the MAB blocks group. All the materials used as part of this experimental setup can be found in Appendix E: Experimental materials used in Experiment 5.

Prior to commencing the testing, students were given a pre-test, comprised of 7 place value questions, in order to establish their current understanding of place value. Following this test, students participated in two phases: the acquisition phase and the test phase. In the acquisition phase, students were presented with 10 questions based on number expansion in place value and the solutions thereto. Each question and solution detailed how to deconstruct a number up to tens of thousands. Each worked example was followed by a paired question, using different numerical values but otherwise the same as the worked example. Students were allowed to refer back to the worked example when solving the paired problem. Each student in the MAB blocks group was given 2 minutes per question to study the worked example and then to construct the illustrated number using MAB blocks. Students in the Control group were given 2 minutes to study each worked example. A further 1 minute was provided to students in both groups to solve the paired problem. Therefore, for completing all 10 paired sets of problems, each student needed thirty minutes in total. Students in the two groups were given the same amount of time in the acquisition phase of the experiment. Students were allowed to have as many attempts as needed at constructing a particular numerical value within a question, as long as their total time per question did not exceed two minutes. At the conclusion of their allocated question time, if the student had not constructed the correct numerical value, a solution was demonstrated to them.

A test phase followed the acquisition phase, in which a common test was used to examine the effect, if any, of the acquisition phase. The test phase was comprised of a post-test, with 10 questions, similar to the problems used in the acquisition phase, but using different numerical values and without any worked examples. Each student was given a maximum of 1 minute to solve each of the 10 questions, for a total test time of 10 minutes. If a question remained unsolved after 1 minute, the child was guided to the next question. Each question was worth one mark, resulting in a total possible score of 10 marks. Children did not have access to the worked examples during the testing phase. The post-test was then followed up by a post-transfer test, where 6 questions needed to be answered. The questions were different in structure to the worked examples, but in answering them, a student may be able to use information that could have been learnt from the worked examples. Each student was again given 1 minute for each question, and were able to make as many attempts as needed, as long as the overall time did not exceed 1 minute.

One week later, a delayed test was given to the participating students to ascertain whether any delayed place value knowledge existed. The delayed test was composed of sixteen questions. The first 10 questions directly related to the worked examples that were given to the students a week earlier. The subsequent 6 questions did not directly relate to the worked examples, but required the knowledge from the worked examples to be extended. Each student was given 1 minute per question, for a total of sixteen minutes to be spent on the delayed test. The students were allowed to make as many attempts as they needed to respond to the questions, as long as their overall time did not exceed 1 minute per question, with sixteen minutes in total to complete the test.

10.3 Results

The variables under analysis in the test phase were the scores for the post-test, the post-test transfer, and the delayed test scores. Table 8 shows the means and standard deviations of each of these variables.

Table 8: Descriptive Statistics for the variables

	MAB blocks (N=20)	Control (N=18)	Total (N=38)
Post-test Mark %	74 (0.883)	91.1 (0.758)	82.1 (1.189)
Post-test Transfer Mark %	62.5 (1.803)	62.2 (2.045)	62.4 (1.895)
Delayed Test Mark %	79.0 (1.252)	95.6 (0.784)	8.68 (1.338)

A one-way between groups analysis of variance was conducted to explore the impact of the redundancy effect on the use of manipulative materials in place value learning, as measured by the testing phase scores, including the post-test scores, post-test transfer scores and the delayed test scores. Using the pre-test mark as a covariate, there were statistically significant differences in the mean testing phase scores in the post-test between the MAB blocks ($n = 20$, $M = 74.0$, SD = 0.883), and Control ($n = 18$, $M = 91.1$, SD = 0.758) groups, F (1, 37) = 52.883, $p < 0.01$. There were no significant differences in the post-test transfer results between the MAB block group ($n = 20$, $M = 62.5$, $SD = 1.803$), and Control group ($n = 18$, $M = 62.2$, $SD = 2.045$), F *(1, 37)* = 0.005, $p = .946$. The delayed test was completed a week following the post-test transfer. There were no children absent in this instance and all children who participated the previous week were able to complete the delayed test. Statistically significant differences were found in the delayed test performance marks between the MAB block group ($n = 20$, $M = 79.0$, $SD = 1.252$) and the Control group ($n = 18$, $M = 95.6$, $SD = 0.784$), F *(1, 37)* = 25.872, $p < 0.01$.

10.4 Discussion

The purpose of Experiment 5 was to test whether the use of MAB blocks, both visually within worked examples, and physically when students are using these materials hands-on, could lead to a redundancy effect that could have a negative impact on learning outcomes. The method used in Experiment 5 repeated that of Experiment 4, but used testing material that was higher in complexity. This was done to ensure that the worked example effect could be obtained, which could only occur if the presented material was adequately complex for the learner.

The results of Experiment 5 showed improved performance of the control group in the testing phase, as measured by the mean post-test and the delayed test marks. Within the cognitive load theory framework, this finding could be explained by the redundancy effect (Sweller, 2012). The use of manipulative materials, in the form of MAB blocks, to explain place value, could lead to an increased extraneous load and therefore could be detrimental to learning. According to the redundancy effect, if learners are presented with a diagram and text simultaneously that provide exactly the same information, the redundancy effect may be triggered, causing a lack of working memory resources to process the necessary information (P. Chandler & Sweller, 1991). Since Experiment 5 used illustrations and textual information, in addition to physical use of blocks, this could have triggered the redundancy effect and led to a high extraneous load on the student, based on the instructional material. Learners were not able to ignore the partly split information, potentially neutralizing the redundancy effect caused by the MAB block illustrations, because they were also required to construct the number decompositions with the physical blocks, using both the illustrative and written numerical information. Students were thus subjected to the potential redundancy of the blocks with each of the questions in the acquisition phase. The sources of information presented to the learner were self-contained, that is they could each be understood on their own, which would imply the presence of the redundancy effect. However, these types of questions are often presented to the student in this way in their normal learning environment, in an Australian classroom. In this instance, the use of MAB blocks in place value instructional material may have presented a redundancy to student learning. The objects increased the extraneous load by repeatedly drawing children's attention away from the task, thereby consuming limited working memory resources.

11 General Discussion

11.1 Summary of key findings

Mathematics curricula are generally designed on the assumption that instruction based on manipulative materials should precede instruction using mathematical symbols. Accordingly, formal manipulative materials are frequently employed in the classroom from the very early stages of education. Instructional techniques using manipulative materials give students the opportunity to interact with physical objects to learn more about abstract mathematical concepts. However, literature remains ambivalent on the advantages of such materials. Gürbüz (2010) and Sherman & Bisanz (2009) carried out studies which found the use of these materials to be an effective approach to improving student's achievement in mathematics. Other studies, however, have found that the use of manipulative materials either made no difference to students' performance or actually inhibited their performance (Canobi et al., 2003; Carbonneau, Marley & Selig 2013a; Uttal et al., 1997; Sowell 1989; McNeil et al., 2009; Amaya et al., 2008). These contradictions in research results may arise due to the variability in the types of manipulative materials used, the method of instructional guidance and in potential characteristics of the learning environment and the learner (Carbonneau, Marley & Selig 2013b; Carbonneau & Marley 2015).

One of the possible problems posed by the use of manipulative materials is their dual representation (Uttal et al., 1997). The materials not only have to be seen as objects in their own right, but also have to symbolise the concept or procedure they are attempting to demonstrate. Translating between these physical objects and the abstract concepts they demonstrate can be a difficult process, and one that requires intensive use of the limited cognitive resources that children possess. When children interact with manipulative materials, they are largely focused on the objects in their hands, thereby committing their cognitive resources to manipulating them, and understanding the abstract concept represented by the manipulative materials. This process may not leave sufficient cognitive resources for the processes required to understand the actual mathematical concepts (Boulton-Lewis 1998).

MAB blocks are one of the most common manipulative materials used in the classroom. These blocks are designed to assist in the teaching of place value, with blocks grouped into ones, tens, hundreds, thousands and so on. These groups are based on the size of the block, and are often referred to as a 'unit' for a single block, a 'long' for a group of ten blocks, and a 'flat' for a group of a hundred blocks. Students are not always aware that there are ten blocks stuck together in a long, and a hundred blocks stuck together in a flat or a thousand small cubes stuck together to make a thousand block. Using language such as ones, longs, flats is also not helpful in developing a link between the relevant block and the number that it is meant to represent. This can lead to a further disconnection between the manipulative material and the mathematical concept that it aims to represent.

Another example of a base-10 manipulative material is a Slavonic abacus. The Slavonic abacus is made up of ten rows of beads, with ten beads on each of the rows. The beads are coloured in such a way, that each row and each column is made up of five beads of one colour and five beads of another colour. Each row of beads corresponds to the fingers on the children's hands, that is, five beads of one colour can count as the fingers on one hand, and the second colour beads on the same row represent the fingers on the second hand. This ensures that children are easily able to subitise both in the ones and in the tens, up to one hundred. The Slavonic abacus can offer children a way to envision a number, as a whole, with each beads correlating to one single count. This may provide a more intuitive way to count and to represent numbers up to one hundred.

This thesis proposes that MAB blocks are not intuitive to students' basic number schemas and can, therefore, lead to the redundancy effect. Students often learn to copy a particular procedure, such as number expansion, to reach a specific goal using MAB blocks, and do not relate these objects to the numbers themselves. With the aim to contribute further to the body of research on the use of manipulative materials in learning mathematical concepts, this study specifically focused on investigating the use of manipulative materials in learning the concept of place value, taking into consideration cognitive load theory and its associated effects (Sweller et al., 1998).

Five experiments were carried out with students in Years 2 to 4 as participants. Experiment 1 investigated the impact a gradual (one stage at a time) or an all-at-once (all stages at the same time) transition had with two types of manipulative materials, an abacus or MAB blocks, on learning the concept of place value. Students were randomly assigned to one of four groups, in a 2 (gradual vs. all-at-once transition) $\times$ 2 (abacus vs. MAB blocks) mixed factorial design. It was proposed that using an abacus as a manipulative material, and using a gradual transition across the five stages of place value understanding, would be a more effective way to learn the concept. The proposition was based on the argument that the abacus would be a more intuitive material that could aid in schema construction, due to its easy representation of the base-10 system. In addition, it was proposed that the gradual transition would ensure no cognitive overload in learning. However, these arguments were not supported by the findings, showing that there were no significant differences in the results regardless of the variables under test.

Experiment 1 showed no significant differences between the gradual intervention and the all-at-once teaching intervention. Experiment 2, therefore, built on this finding by instead using the worked example effect to provide a consistent intervention across the groups of students. Three groups of students were used in this experiment. The first group was assigned the abacus, the second group used MAB blocks and the control group was not given any manipulative materials. The test material focused on only the last stage of place value understanding, number expansion and decomposition, which was the area where students had the most difficulty in Experiment 1. Experiment 2 was designed to investigate whether using the abacus in learning place value would be a more effective method for schema construction due to its one-to-one count correspondence and easy division into groups of tens beads. The results of this experiment did not support the hypothesis, as no significant differences were found amongst the three groups.

Experiment 3 was based on the observation that the worked examples presented to the students in Experiment 2 were too cognitively demanding, in terms of the level of English comprehension required, thereby causing cognitive overload before any learning could take place. Accordingly, Experiment 3 repeated Experiment 2, but with worked examples that did not require a high level of English comprehension. As in Experiments 1 and 2, Experiment 3 was designed to compare the efficiency of using an abacus versus MAB blocks to learn the concept of place value. The results of the experiment showed that the control group (no manipulative materials) performed better than the group using MAB blocks. There were no significant differences found between the group using an abacus and the group using MAB blocks. Additionally, no significant differences were observed between the group using an abacus and the control group. This result thus led to the proposition that the use of MAB blocks could potentially lead to the redundancy effect, thereby causing ineffective learning.

In order to further investigate the difference found in Experiment 3 between the group using MAB blocks and the control group, Experiment 4 was designed to include a delayed learning test in addition to an immediate post-test, given to the students a week after the intervention. Instructional material was modified to include more questions that focused on expanded number notation, thereby forcing the students to interact with the MAB blocks. The rationale of this experiment was based on the assumption that the use of MAB blocks led to the redundancy effect and therefore hindered effective learning of the concept of place value. Based on this, it was predicted that the students in the control group would perform better than those using the MAB blocks, leading to better outcomes in the post-test and the delayed learning test. The results did not support this argument, showing no significant differences between the group using MAB blocks and the control group. However, this result could be due to the ceiling effect observed throughout this experiment. Children scored very highly on both the pre-test and the post-test, thereby indicating that no actual learning had taken place. Due to the timing of this experiment at the conclusion of the school year, students had already become familiar with the concepts used in the acquisition phase, and so the worked examples did not display novel concepts for these students. Since the worked examples did not present novel information, no learning could take place.

Experiment 5 then built on this information by increasing the expertise of students and increasing the complexity of the instructional material. This was done by using more difficult worked examples, with numbers into the thousands and tens of thousands, and additionally breaking them up in unconventional ways to test for a deeper understanding of the concept of place value. Students were tested at the beginning of term 1 of the following school year and, therefore, had not yet learnt the material included in the worked examples. Two groups were used as part of this experiment, one group using MAB blocks, and the second group using no manipulative materials. The argument proposed by this experiment, similar to that of Experiment 4, was that MAB blocks, used both in worked examples and physically, could lead to the redundancy effect in the instructional material, resulting in ineffective learning. The findings of this final experiment showed significant differences in the performance of those students using MAB blocks and those not using any manipulative materials. Students performed better in the post-test and the delayed test when no manipulative materials were used to support the worked examples. This could indicate that the use of MAB blocks, physically and as part of worked examples, might have led to the redundancy effect and to less effective instructional material. In addition, it was observed that MAB blocks were often used in a task-irrelevant way, distracting the student. This could also result in the redundancy effect being observed. This is the most significant finding to emerge from this study showing that learning to handle manipulative materials, specifically MAB blocks, when learning the concept of place value, may not lead to a deeper understanding of this concept and improvements in student performance. The finding is grounded in cognitive load theory, as the use of MAB blocks may result in the redundancy effect and cause difficulty for students translating between the blocks and the concept of place value they are designed to demonstrate, due to the limited cognitive resources available to the student.

11.2 Theoretical Implications

Recent work by Mix et al. (2016) found mixed results when using MAB blocks to help teach children the concept of place value. The study found that, for some children, there was a clear advantage using the blocks, whilst for others, the blocks were only beneficial once they had already learnt the concept of place. The study found that one factor influencing the benefits of using the blocks was the number of years that children had been trained in their use. The more training the chidren had had, the more beneficial it was to use the blocks. Another study (Mix et al., 2013), found the meaning of MAB blocks was not always obvious to children, requiring some prior knowledge of number to be explicit in the student’s mind, before the blocks could be understood. This defeats the purpose of using blocks to teach the concept of place value, if children might need to have a basic understanding of place value in order to learn how to use the blocks.

The aim of this study was to consider the manipulative materials, specifically MAB blocks, from the viewpoint of cognitive load theory. Based on the findings of this study, there are two implications arising from the use of MAB blocks in teaching mathematical concepts. Firstly, research has shown that children may find it very difficult to transfer knowledge from the manipulative material to its symbolic written form they are meant to represent. This is in line with the current research showing that knowledge gained in one context may not transfer to another context, because of the mismatch between the format in which the material was first encoded and the way in which it is needed to be recalled (Uttal et al., 2013). The difficulty of such a general transfer in memory is also linked to the well-documented difficulty of maintaining the dual representation of a concrete object and the abstract concept it is demonstrating (Uttal et al., 1997; DeLoache 2000; Uttal et al., 2013). It was observed throughout the experimentation phase that children would often not use the blocks for their intended purpose, to help them solve a particular question. In fact, the children were often observed noting an answer down and then, as an afterthought, because the question required them to do so, constructing the number expansion using blocks. Often, the block response would be correct, but would not match the abstract symbolic representation written on paper by the student. This indicated, firstly, that blocks and symbols were seen as concepts in their own right and not linked, and, secondly, that difficulties were experienced translating between blocks and their symbolic place value representation.

The results of this research also extend the findings of some recent studies on the efficacy of manipulative materials in teaching mathematical concepts at an early stage of education (Uttal et al., 2013; Kamina & Iyer, 2009; Carbonneau & Marley, 2015; McNeil & Jarvin 2007; Moyer & Jones, 2004; Uttal et al., 1997) by examining them in the context of cognitive load theory. Recent research (K J Carbonneau & Marley, 2015) indicates that the use of manipulative materials can also be ineffective due to problems with dual representation. In addition, the current study has shown that the use of MAB blocks to teach the concept of place value may also contribute to the redundancy effect, which may reduce their efficacy in teaching this concept. In particular, Experiment 5 indicated that the students in the control group (without any manipulative materials) performed better solving place value problems of number expansion and decomposition, due to the potential redundancy effect created by the MAB blocks in the second group. Therefore, cognitive load factors should be considered when selecting optimal instructional strategies in this area.

11.3 Educational Implications

The results of this study may challenge the well-established practice of using MAB blocks to demonstrate the concept of place value to children to help them understand the base-10 number system. Firstly, there was an issue with dual representation. Students perceived the blocks as objects in their own right, without having any association with the number concepts that the blocks were meant to demonstrate. Hence, a problem with translating from a concrete object to the abstract concept of number was observed. Construction with blocks including bundling and grouping is a primary skill, and the process of using the blocks in this way comes intuitively. However, in this study the blocks did not directly translate to the secondary skill of mastering the base-10 number system. Such a system needs to be explicitly taught as a concept in its own right.

Alternatively, teaching such a concept requires instructional material comprising three components: learning to play with the blocks, learning how the number system is structured and, thirdly, translating between these two representations. Indeed, when using either the blocks or the abacus, it was often observed that children engaged in task-irrelevant activities, such as using blocks for construction, and making music with an abacus. It could be beneficial to abandon the use of these manipulative materials and persevere with explicit instruction in how the number system works. From observations of students in Year 2, it would appear that they might not yet have the general schemas required to build any deep understanding of the base-10 number system. It could be appropriate for such concepts to be taught at a later stage and, in the meantime, there could be a greater focus on creating better number schemas in memory, that could aid in the construction, manipulation and processing of numbers at later stages.

It was observed in the experiments of this study that children could successfully carry out routine tasks, such as pointing to a number in the tens or hundreds column, without actually understanding the concept of number. It was obvious that children had simply learnt by rote the processes required to complete such tasks, without showing any deep understanding of the concept of place value. For example, the link between MAB blocks and their direct numerical representation should not be assumed knowledge in young children. Learners did not understand the concept that the blocks were designed to illustrate, instead treating the blocks independently of the numbers. Students manipulated the blocks, based purely on their knowledge of blocks, as opposed to any relationship that these blocks might have had to the numbers.

The results of this study might also have more general educational implications, which leads to the second, and perhaps most important, point. This study has indicated that using MAB blocks with worked examples may lead to the redundancy effect, and, therefore, their use may be detrimental to learning. Worked examples are often presented in this way in instructional material for students. Although the redundancy effect and its repercussions are well researched in literature, there is not much data dealing with this effect in younger students. This study has indicated that using simple worked examples, requiring simple reading comprehension skills, and with no redundant illustrations, could be the most beneficial means for a young student learning the concept of place value. The need for simple instructions that require only basic comprehension skills can be attributed to young learners still developing their secondary skill of reading. Complicated instructions that require complex comprehension skills use up limited cognitive capacity that would otherwise be used in learning mathematical concepts. The efficacy of not using redundant illustrations or tools to teach place value concepts was demonstrated in Experiment 5. Students performed better in their post- and delayed- tests, in the group that did not use MAB blocks. The improved performance in the delayed test could also be an indication of better schema construction, encouraged by the use of simple worked examples without the distraction of the MAB blocks. In this situation, students were able to focus all their cognitive resources on the concept of place value.

11.4 Limitations of Study and Future Research Directions

Some of the limitations of this research may be utilised to inform future research directions exploring the use of manipulative materials in teaching place value, from the perspective of cognitive load theory. Firstly, only two manipulative materials (the abacus and MAB blocks) were used in the experiments. Other materials, such as cuisenaire rods, place value mats, grouped paddle-pop sticks and others can also be used in order to manipulate the concepts of place value in a concrete way. However, these other materials were not used as part of this study. The results would have been more generalized if a greater variety of manipulative materials could have been used to teach the concept of place value.

Secondly, due to the time constraints imposed by school timetables, only a limited number of worked examples was presented to the students in each of the experimental situations. Therefore, advantage could not be taken of the fact that more worked examples could have been used for students to acquire more general and robust schemas.

Thirdly, it is important to consider that there are variations in school curriculum milestones. Therefore, when students from different schools were used as part of the same experimental group, it could have been possible that some of their prior learning outcomes had affected their ability to solve questions and understand the concepts presented in this study.

Fourthly, this study did not measure the cognitive load placed on each student, as it is very difficult to measure cognitive load in children under 15 years of age (H. Lee, 2014; H. Lee, Plass, & Homer, 2006). Because the students in this experimental study were all so young, with the oldest aged just 8 years, no attempt were made to measure the levels of cognitive load using, for example, subjective rating scales (Paas, Tuovinen, et al., 2003). Some alternative measures could possibly be used with this category of students. The ability to provide a valid and reliable estimation of the amount of load experienced whilst completing a task is a complex undertaking, even more so when it concerns such a young cohort of students (H. Lee, 2014). A small number of studies have used a modified subjective scale with cartoon faces to help young students rate their cognitive load whilst solving questions (Hu et al., 2015; van Loon-Hillen et al., 2012). However, no known studies have used such scales with children as young as 6-7, as used in this series of experiments. From converstations with students after the experiments have taken place, it appeared that the young students were keen to label any task undertaken as 'easy'. This was observed even when it was clear that the students found the questions and the content to be difficult. Thus, it could be possible that measuring cognitive load, with the use of a subjective scale, in such a young cohort of students could be not possible. This could present an interesting area for further research.

Finally, only private school children were tested in Experiments 2, 3, 4 and 5. It is not possible to say whether these results can be generalized across the category of schools: private, public, catholic and independent schools.

Despite these limitations, the results do appear to have a solid theoretical grounding in already well-established cognitive load theory and, therefore, may have significant implications for the current widespread use of MAB blocks. Further work in the area could be undertaken both to reinforce, and to expand on, these findings.

Some research has already shown that objects high in perceptual detail are not effective in establishing deep mathematical understanding of concepts (Goldstone & Sakamoto 2003; Sloutsky et al., 2005). Children often use all of their limited cognitive resources to focus on the details of the manipulative material, with no resources left to establish the connection between the manipulative material and the concept it is attempting to represent. However, MAB blocks do not present a rich perceptual experience, as the blocks are simple cubes devoid of shape or interesting structure. McNeil (2007) proposed that this aspect of MAB blocks could make it easier for students to view them as mathematical tools, and to focus on the concepts that the blocks are trying to teach. However, the findings of this research do not indicate this to be the case. It was often observed that children would use these blocks in task irrelevant ways, using them to build and play with, instead of understanding that they represented mathematical concepts. A possible further area of research could examine whether the different types of blocks could make a difference. For example, there are coloured blocks available, or blocks that are able to divide into units and click together to make longs, flats, and so on, thereby potentially encouraging a more fluent translation between the dual representations. Additionally, it would be interesting to investigate whether using the physical blocks only or being given visual representations of the blocks, or a combination of both, would lead to a change in student performance and understanding. This could further lead into examination of whether using blocks as part of animated simulation is beneficial to student learning. Because simulations are able to break up the cubes and, therefore, show that a long is made of 10 small unit blocks, there is potential that it would aid in a better dual representation of the concept and not cause a cognitive overload.

This study examined the use of blocks and an abacus as manipulative materials with students in their second and fourth year of schooling. It would be beneficial to investigate whether the expertise of the student impacts the way MAB blocks and other manipulative materials are used and whether they provide more value at the beginning of schooling or at later stages. For example, the blocks could be used at later stages of schooling to expand upon a concept once the concept has already been learnt. This is in contrast to using the blocks at the beginning of schooling to teach the concept initially.

Lastly, a longitudinal study would be of benefit to examine whether long-term cognitive benefits can be obtained by the use of manipulative materials, or whether the concept of dual representation and its effect on basic and initial schema construction would hinder the learning process. It would be interesting to start at the beginning of a child's formal education, and then track the frequency of the use of such material and its effects. A control class would run in parrallel and be given no access to manipulative materials. Such a study would have to be carried out in a real-life context, with no formal interventions. Each class would be provided with worked examples of place value questions at varying stages of the year.

11.5 Conclusion

The present thesis argues that the use of manipulative materials (such as MAB blocks or an abacus) in the early primary school curriculum does not support the creation of number composition schemas in the young learners, and may, indeed, hinder learning. This has implications for the design of instructional materials that incorporate both illustrations and the physical use of MAB blocks and other base-10 material, when teaching individual components of number deconstruction.

The results suggest that the use of MAB blocks in worked examples, both illustratively and physically, may cause an increased cognitive load on the student, and generate a redundancy effect. The findings of the present research may thus lead instructional designers and primary school teachers to create learning activities that do not incorporate the use of MAB blocks, whether as illustrations or physical, into teaching the concept of place value.

The study demonstrated that cognitive load factors are essential to be considered while designing or selecting instructional materials for this area of education. Established cognitive load theory techniques could be used to improve learning outcomes.

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Appendices

Appendix A: Experimental materials used in Experiment 1

Pre-test

1) Please complete the following counting sequences:

   1, 2, 3, 4, 5, ___, ___, ___

   7, 8, 9, ___, ___, ___

   27, 28, 29, ___, ___, ___

   10, 20, 30, 40, ___, ___, ___

   23, 33, 43, ___, ___, ___

   145, 155, 165, ___, ___, ___

2) Please answer the following questions:

   7 + 3 =

   15 - 3 =

   8 + 6 =

   12 + 10 =

   22 + 8 =

   10 + 4 =

   35 - 31 =

   35 - 15 =

   10 + 25 =

   31 + 40 =

3) Please write these numbers as numerals:

Twenty three

____

Two hundred and three

____

Ninety nine

____

Seventeen

Seventy one

Seven hundred and twenty two

Ten

Fifteen

4) Please write these numerals in words:

9

19

99

91

138

831

402

5) Please complete the following:

55 = _ + 5

60 = 60 + _

17 = _ + 7

102 = _ + _ + 2

230 = 200 + _ + _

6) What is the total value of the highlighted number?

93 = __________

5**9** = __________

1**0**2 = __________

370 = __________

7) What is the number of tens in 342? __________

8) What is the number of hundreds in 587? __________

9) What is the number of ones in 324? __________

Post-test

1) The following numbers are to be written down to dictation: 6, 8, 3, 9, 7, 13, 28, 32, 57, 12, 21, 65, 42, 88, 24, 301, 147, 741, 205, 502

2) Please complete the following counting sequences:

3, 4, 5, _, _, _

17, 18, 19, _, _, _

37, 38, 39, _, _, _

20, 30, 40, _, _, _

123, 133, 143, _, _, _

175, 185, 195, _, _, _

3) Please answer the following questions:

6 + 4 =

17 - 3 =

5 + 6 =

22 + 10 =

52 + 8 =

10 + 9 =

37 - 32 =

38 - 18 =

10 + 15 =

28 - 17 =

41 + 20 =

4) Please write these numbers as numerals:

Thirty three ____________

Four hundred and three ____________

One hundred and ninety nine ____________

Nineteen ____________

Ninety one ____________

Four hundred and one ____________

Seven hundred and fifty five ____________

Two hundred and twenty two ____________

Twenty ____________

Sixteen ____________

5) Please write these numerals in words:

9 ____

19 ____

99 ____

91 ____

15 ____

51 ____

127 ____

721 ____

502 ____

808 ____

6) Please complete the following:

77 = _ + 7

20 = 20 + _

13 = _ + 3

104 = _ + _ + 4

100 + _ + _

7)What is the total value of the highlighted number?

73 = _

56 = _

306 = _

52 = _

220 = _

8) What is the number of tens in 152?

9) What is the number of hundreds in 227?

10) What is the number of ones in 362?

Appendix B: Experimental materials used in Experiment 2

Pre-test

1) Write the following using expanded notation:

25 = □ + □

2) Write the following using expanded notation:

905 = □ + □ + □

3) What is the face value of the digit 5 in the number 2**5**?

____

4) Circle the numbers that have a face value of 6 in the tens.

106 6 63 66 616 67

5) What is the total value of the digit 9 in the number 39?

____

6) What is the total value of the digit 5 in the number 507?

____

7) What number is made up of 300, and 20 and 2?

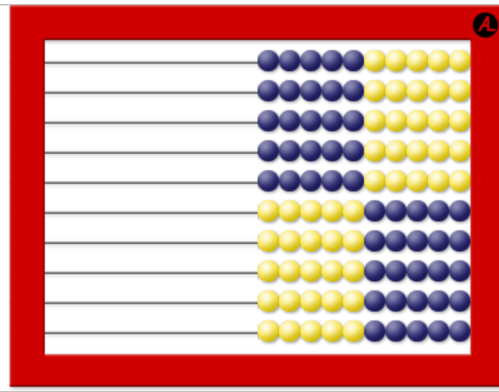
____

8) I have 9 hundreds, and 9 ones, what number am I?

____

Acquisition phase – abacus

Background:

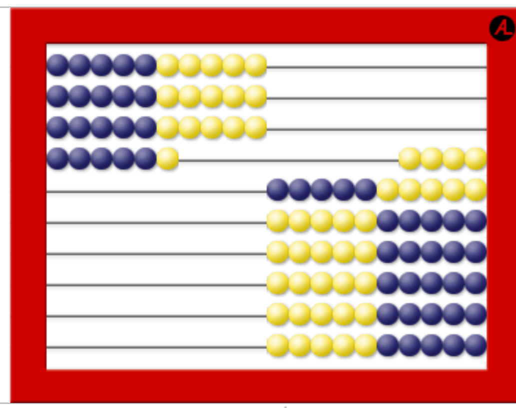
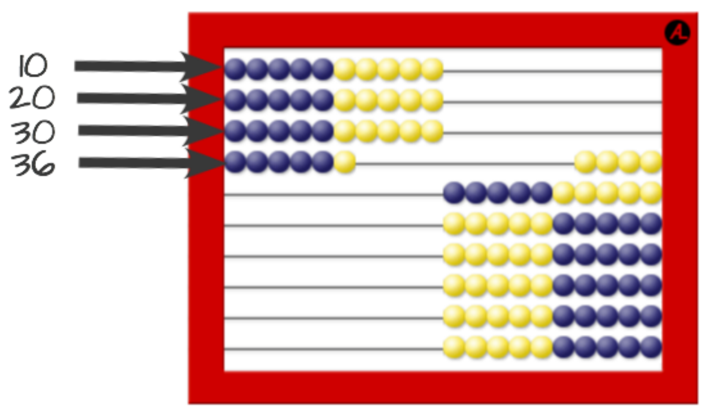
Use a one hundred-frame with ten lines and ten beads on each of the lines. The abacus has all its beads in the 'off position on the right. To switch the beads on we will move them from the right to the left. Notice that one line has 10 beads on it.	

Question:

1) Please write the following using expanded notation:

$36 = \square + \square$

Solution:

Use place value to construct numbers. The number 36 is made up of 3 tens which is 30 ones. In addition there are 6 ones.	
So to expand 36, we will write: $36 = 30 + 6$	

Question:

2) Now, use the worked example to solve this question, by writing the following using expanded notation:

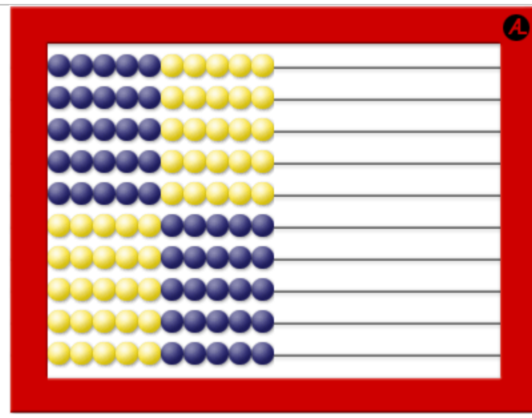
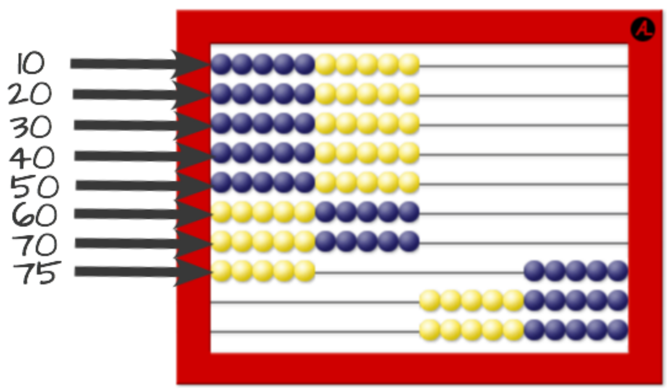
$47 = \square + \square$

Question:

3) Create a three-digit number that has a hundreds face value of 1, tens face value of 7, and a ones face value of 5

Solution:

<table>
<tr><td>Use place value to construct numbers.

We need a number that has a hundreds face value of 1, which is 100 ones. In addition we need a tens face value of 7, which is 70 ones, and a ones face value of 5.</td><td>

</td></tr>
<tr><td>So a number that has a hundreds face value of 1, a tens face value of 7, and a ones face value of 5, is a number that is made of 100, 70 and 5, equal to 175.</td><td></td></tr>
</table>

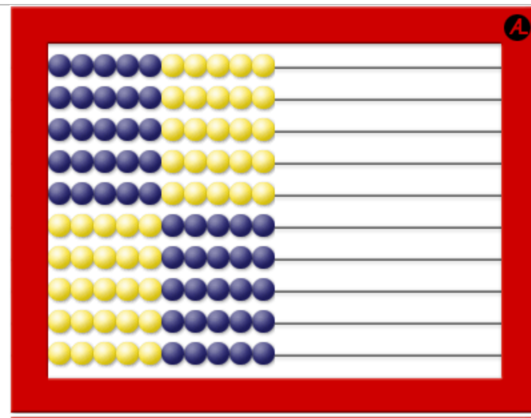
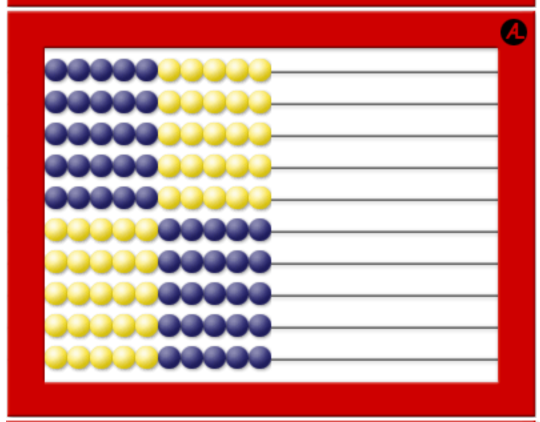
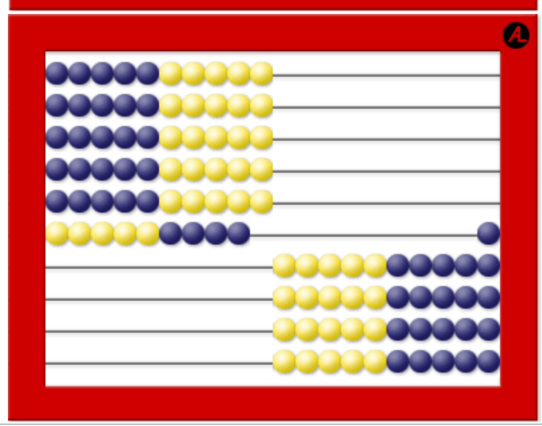
Question:

4) Now, use the worked example to solve this question. Create a three-digit number that has a ones face value of 9, a tens face value of 4, and a hundreds face value of 2.

Question:

5) What is the total value of the digit 5 in the number **259**?

Solution:

Use place value to construct numbers. The number 259 is made up of 2 hundreds, which is equal to 200 ones; 5 tens equal to 50 ones, and 9 ones.	  
So the total value of 5 in the number 259, is 5 tens, which is equal to 50.	

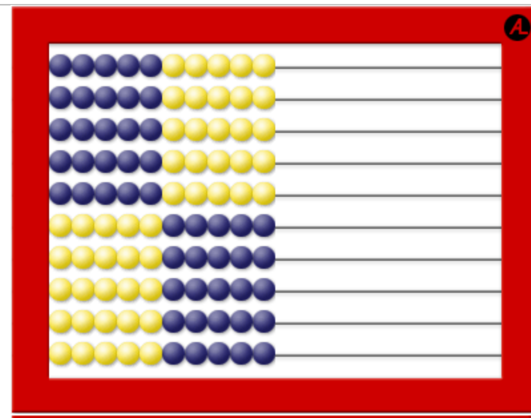
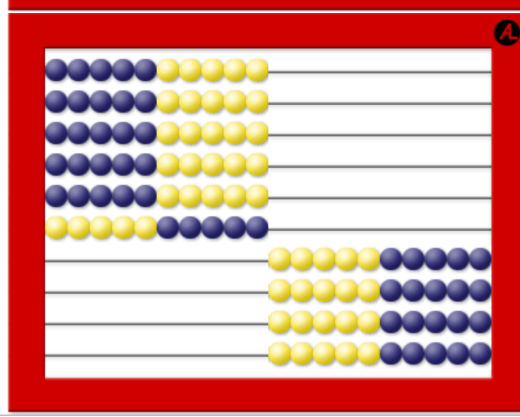
Question:

6) Now, use the worked example to solve this question. What is the total value of the digit 4 in the number 843?

Question:

7) I have 1 hundreds and 6 tens, what number am I?

Solution:

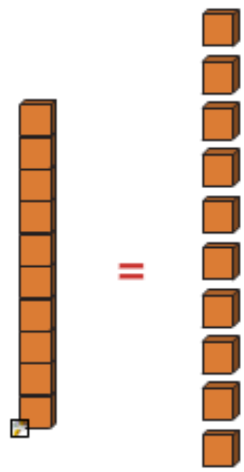
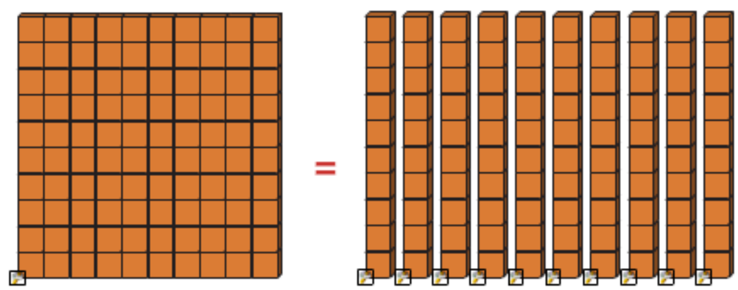
Use place value to construct numbers. 1 hundreds means that there are 1 lots of 100, which is equal to 100 ones. In addition 6 tens means that there are 60 ones.	
So a number that has 1 hundreds and 6 tens, is made up 100 + 60, which is 160	

Question:

8) Now, use the worked example to solve this question. I have 8 tens and 2 ones, what number am I?

Acquisition phase – MAB

Background:

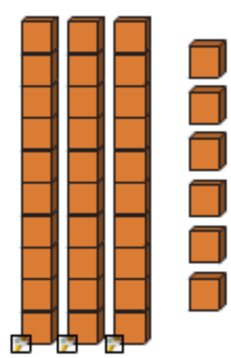
We use the bigger blocks for the digits on the left and the smaller blocks for the digits on the right. Notice that a long block is made up of ten small blocks joined together.	 

Question:

1) Please write the following using expanded notation:

$36 = \square + \square$

Solution:

Use place value to construct numbers. The number 36 is made up of 3 tens which is 30 ones. In addition there are 6 ones.	
So to expand 36, we will write: 36 = 30 + 6	

Question:

2) Now, use the worked example to solve this question, by writing the following using expanded notation:

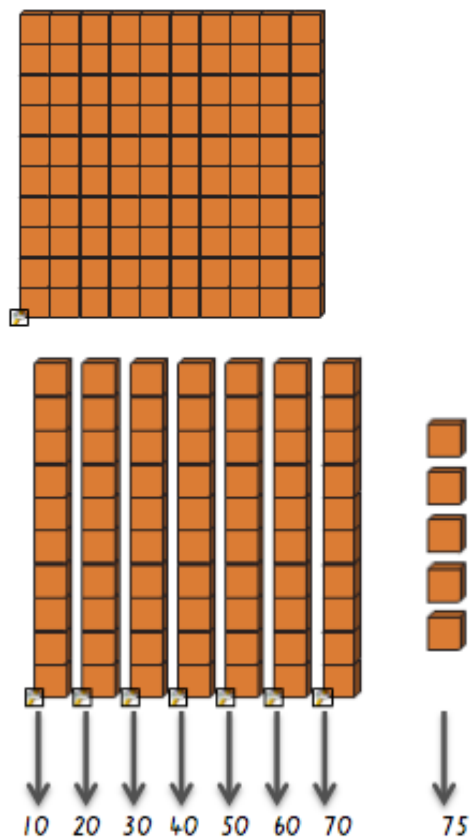
47 = ☐ + ☐

Question:

3) Create a three-digit number that has a hundreds face value of 1, tens face value of 7, and a ones face value of 5.

Solution:

<table>
<tr>
<td>Use place value to construct numbers.

We need a number that has a hundreds face value of 1, which is 100 ones. In addition we need a tens face value of 7, which is 70 ones, and a ones face value of 5.</td>
<td>
</td>
</tr>
<tr>
<td>So a number that has a hundreds face value of 1, a tens face value of 7, and a ones face value of 5, is a number that is made of 100, 70 and 5, equal to 175.</td>
<td></td>
</tr>
</table>

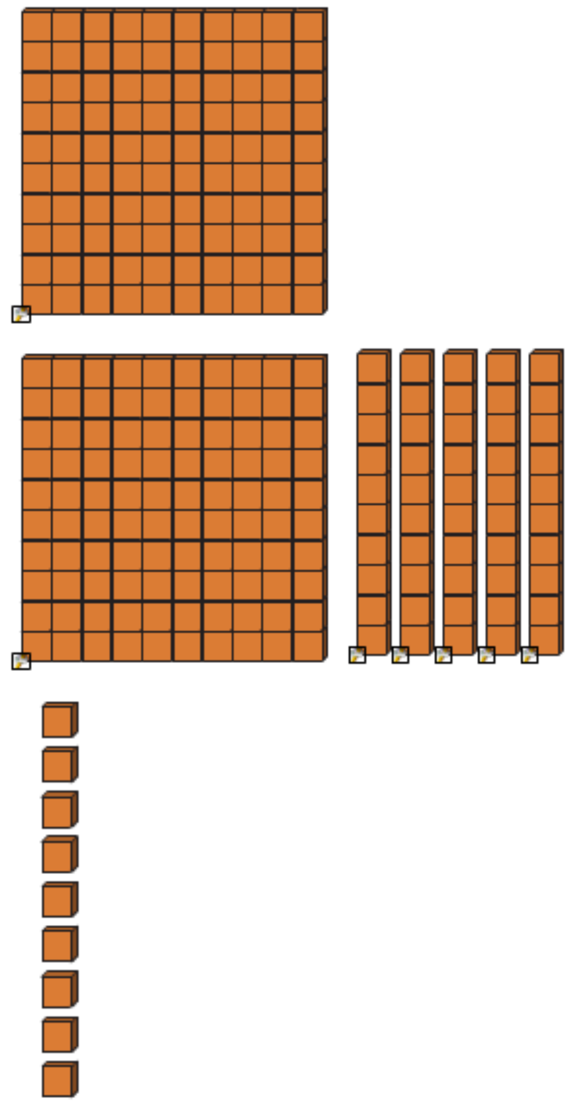
Question:

4) Now, use the worked example to solve this question. Create a three-digit number that has a ones face value of 9, a tens face value of 4, and a hundreds face value of 2.

Question:

5)   What is the total value of the digit 5 in the number **259**?

Solution:

Use place value to construct numbers. The number 259 is made up of 2 hundreds, which is equal to 200 ones; 5 tens equal to 50 ones, and 9 ones.	
So the total value of 5 in the number 259, is 5 tens, which is equal to 50.	

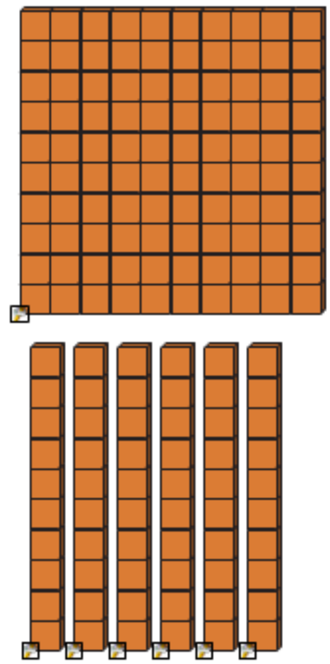
Question:

6)   Now, use the worked example to solve this question. What is the total value of the digit 4 in the number 843?

Question:

7) I have 1 hundreds and 6 tens, what number am I?

Solution:

Use place value to construct numbers. 1 hundreds means that there are 1 lots of 100, which is equal to 100 ones. In addition 6 tens means that there are 60 ones.	
So a number that has 1 hundreds and 6 tens, is made up 100 + 60, which is 160	

Question:

8) Now, use the worked example to solve this question. I have 8 tens and 2 ones, what number am I?

Acquisition phase – no visuals

Question:

1) Please write the following using expanded notation:

36 = □ + □

Solution:

Use place value to construct numbers. The number 36 is made up of 3 tens which is 30 ones. In addition there are 6 ones.	
So to expand 36, we will write: 36 = 30 + 6	

Question:

2) Now, use the worked example to solve this question, by writing the following using expanded notation:

$47 = \square + \square$

Question:

3) Create a three-digit number that has a hundreds face value of 1, tens face value of 7, and a ones face value of 5.

Solution:

Use place value to construct numbers. We need a number that has a hundreds face value of 1, which is 100 ones. In addition we need a tens face value of 7, which is 70 ones, and a ones face value of 5.	
So a number that has a hundreds face value of 1, a tens face value of 7, and a ones face value of 5, is a number that is made of 100, 70 and 5, equal to 175.	

Question:

4) Now, use the worked example to solve this question. Create a three-digit number that has a ones face value of 9, a tens face value of 4, and a hundreds face value of 2.

Question:

5) What is the total value of the digit 5 in the number **259**?

Solution:

Use place value to construct numbers. The number 259 is made up of 2 hundreds, which is equal to 200 ones; 5 tens equal to 50 ones, and 9 ones.	

So the total value of 5 in the number 259, is 5 tens, which is equal to 50.	

Question:

6) Now, use the worked example to solve this question. What is the total value of the digit 4 in the number 843?

Question:

7) I have 1 hundreds and 6 tens, what number am I?

Solution:

Use place value to construct numbers. 1 hundreds means that there are 1 lots of 100, which is equal to 100 ones. In addition 6 tens means that there are 60 ones.	
So a number that has 1 hundreds and 6 tens, is made up 100 + 60, which is 160	

Question:

8) Now, use the worked example to solve this question. I have 8 tens and 2 ones, what number am I?

Post-test

1) Write the following using expanded notation:
   25 = ☐ + ☐

2) Write the following using expanded notation:
   905 = ☐ + ☐ + ☐

3) What is the face value of the digit 5 in the number **25**?
   ____

4) Circle the numbers that have a face value of 6 in the tens.
   106 6 63 66 616 67

5) What is the total value of the digit 9 in the number **39**?

6) What is the total value of the digit 5 in the number **507**?

____

7) What number is made up of 300, and 20 and 2?

____

8) I have 9 hundreds, and 9 ones, what number am I?

____

Appendix C: Experimental materials used in Experiment 3

Pre-test

1) Expand:

25 = ____ + ____

905 = ____ + ____ + ____

2) State the value of the bold number in 2**5**

____

3) Circle the numbers that have 6 in the tens (there can be more than one number!).

106   6   63   66   616   67

4) State the value of the bold number in 9**3**9

____

5) State the value of the bold number in **5**07

____

6) What number is made up of 300, and 20 and 2?

____

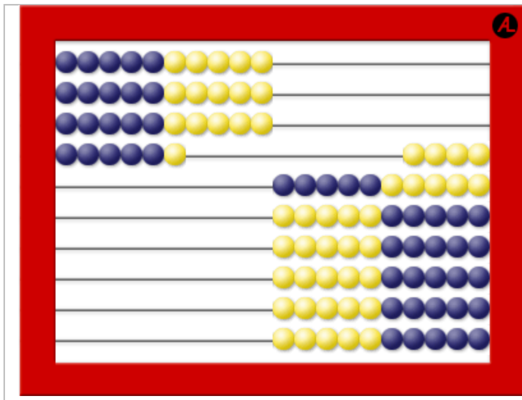
7) I have 9 hundreds, and 9 ones, what number am I?

____

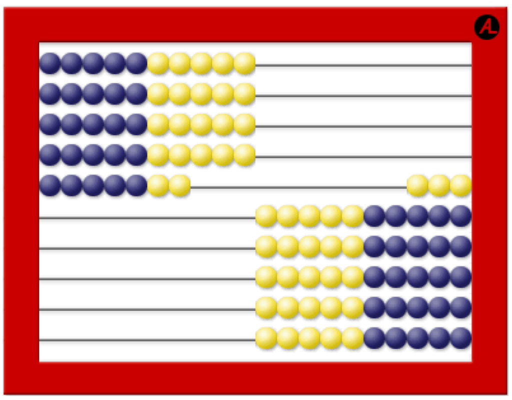
Acquisition phase – abacus

1) Expand the numbers. The first one has been done for you.

36 = 30 + 6



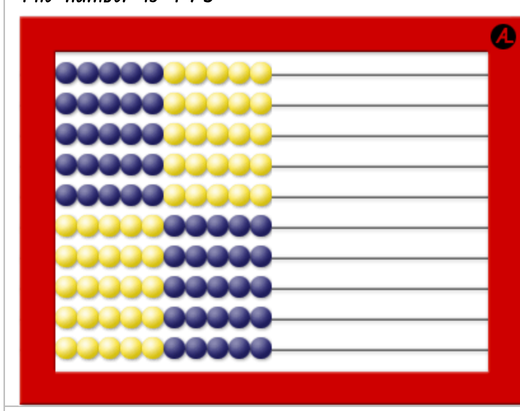
47 = ___ + ___

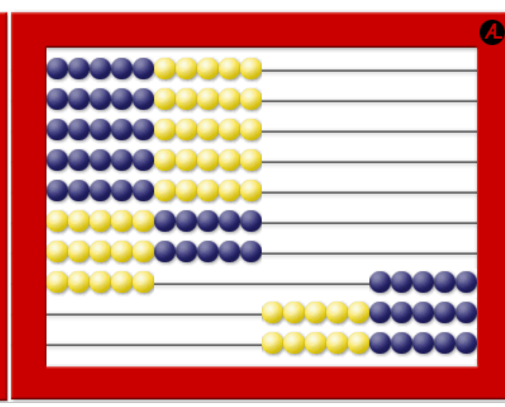


2) Guess what the numbers are. The first one has been done for you.

What is a three-digit number that has a **Hundreds** face value of 1; **Tens** face value of 7; **Ones** face value of 5

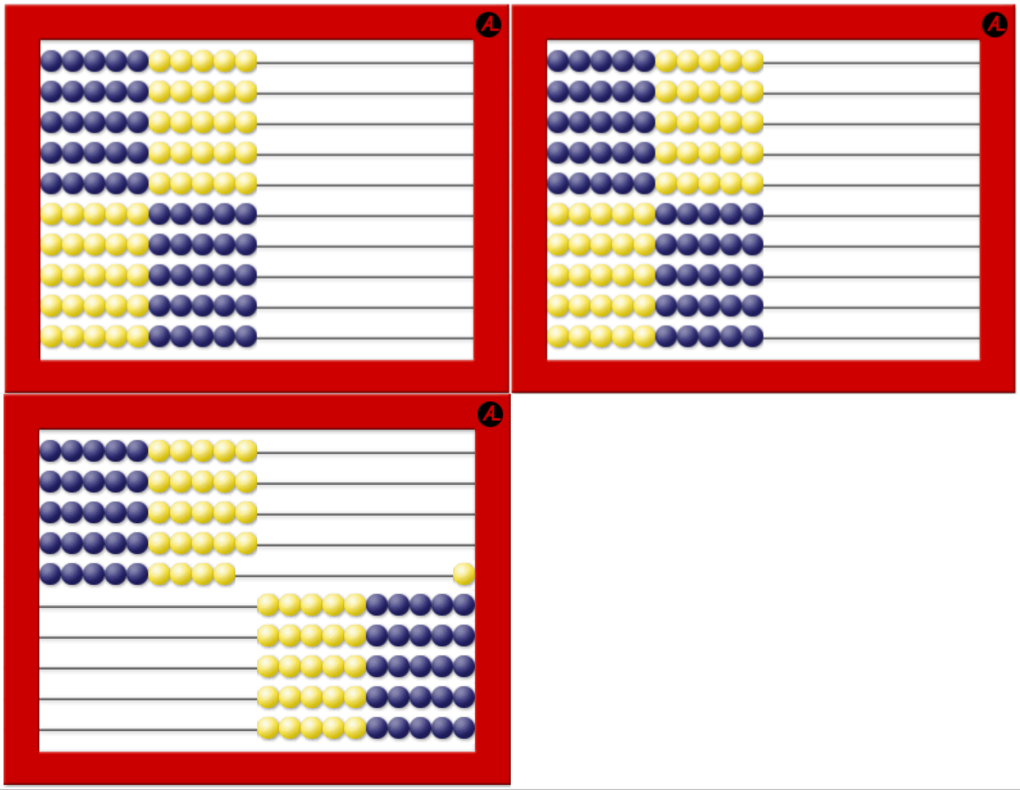
The number is 175





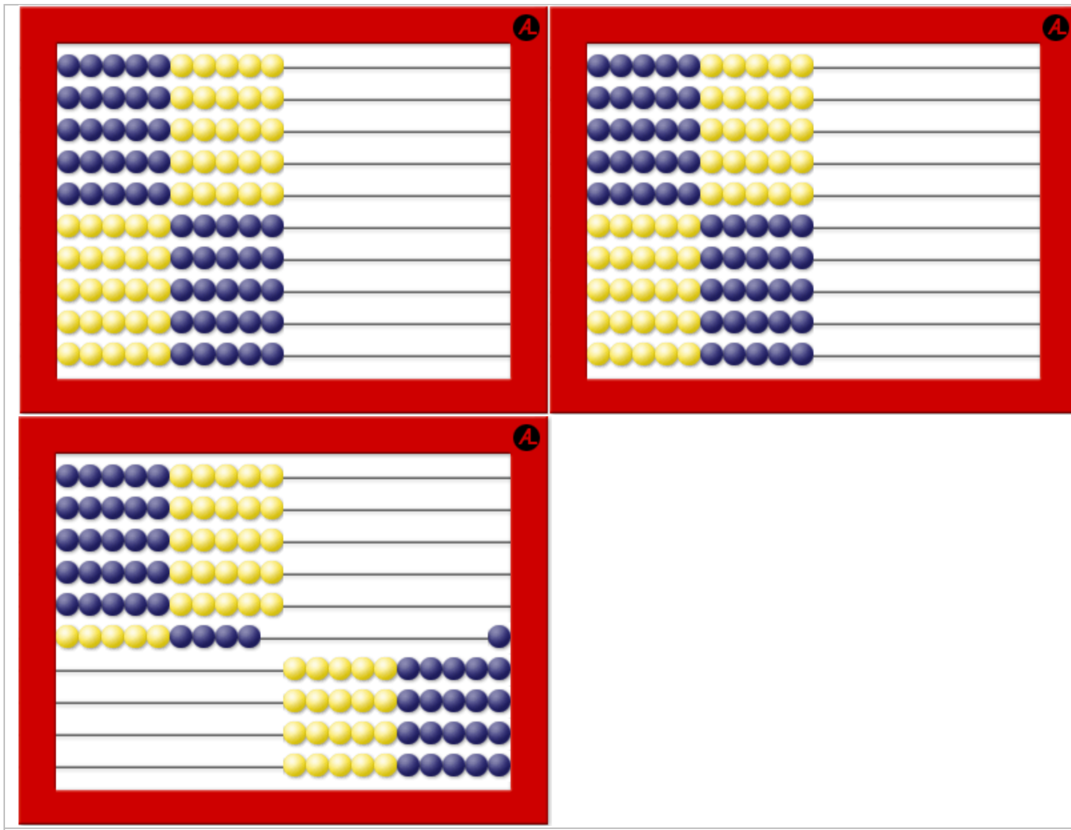
What is a three-digit number that has a **Ones** face value of 9; **Tens** face value of 4; **Hundreds** face value of 2

The number is ______________



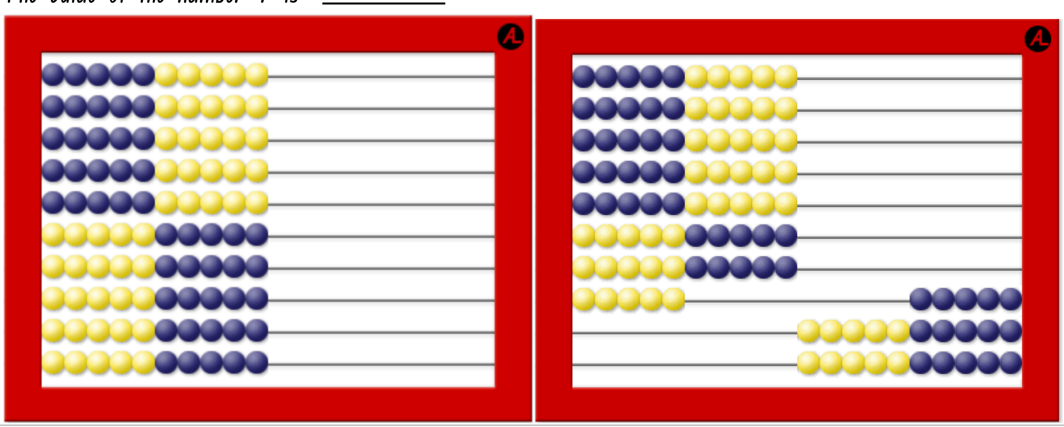
3) State the value of the bold number. The first one has been done for you

259 The value of the number 5 is <u>50</u>



175

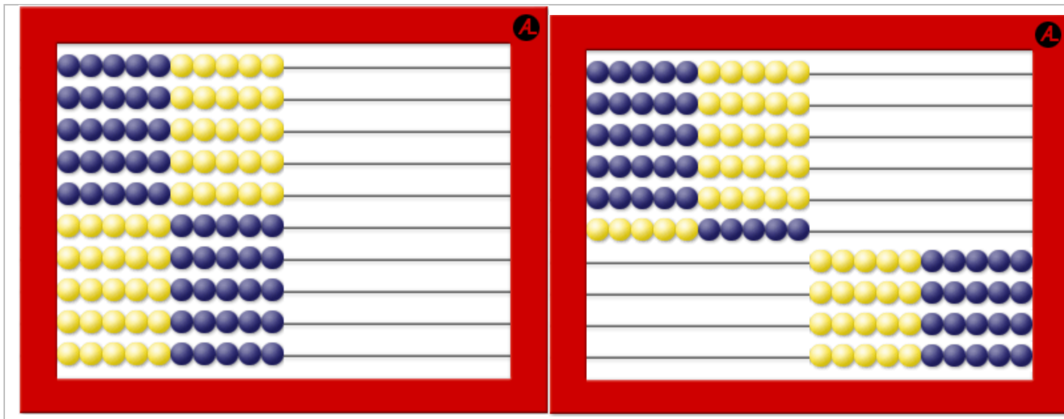
The value of the number 7 is ________



4) Guess what the numbers are. The first one has been done for you.

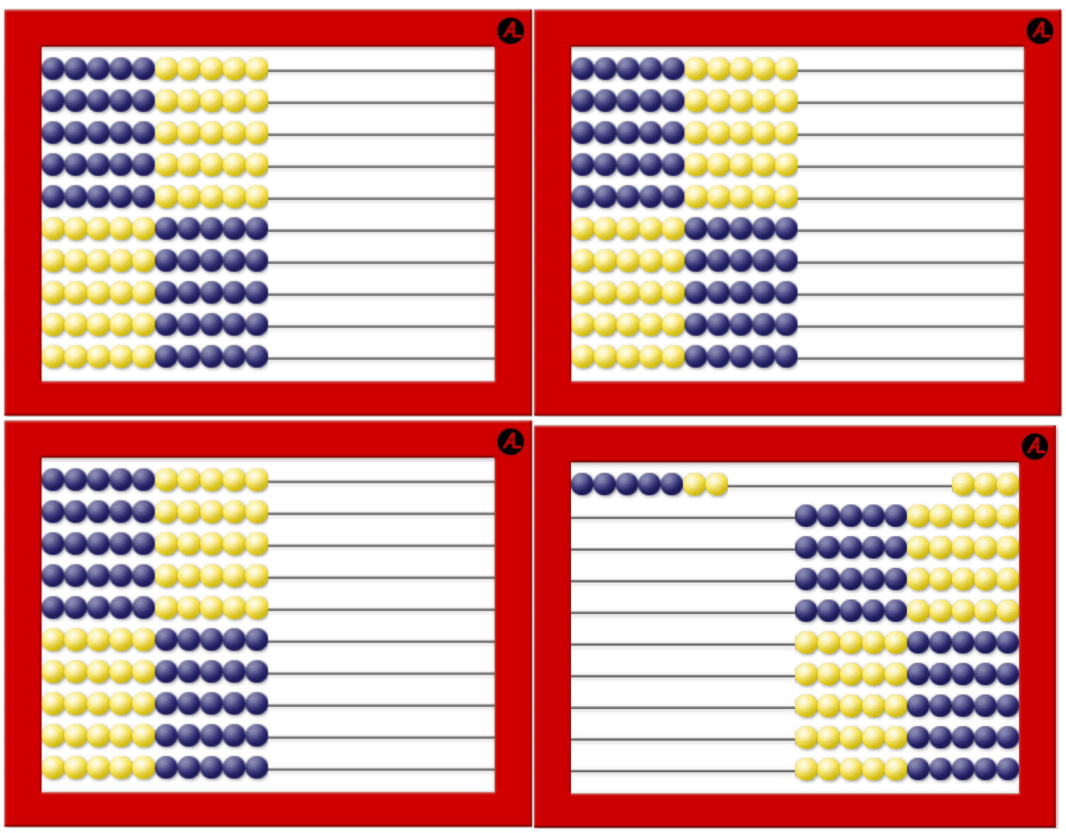
I have 1 hundreds and 6 tens, what number am I?

I am <u>160</u>



I have 3 hundreds and 7 ones, what number am I?

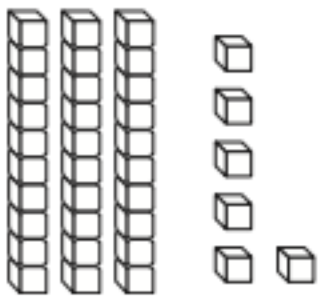
I am ______________________



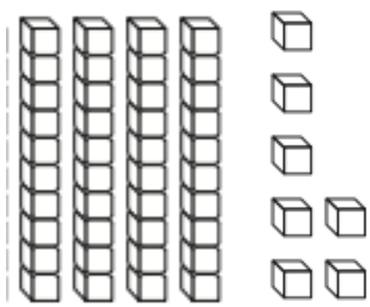
Acquisition phase – MAB

1) Expand the numbers. The first one has been done for you.

36 = 30 + 6



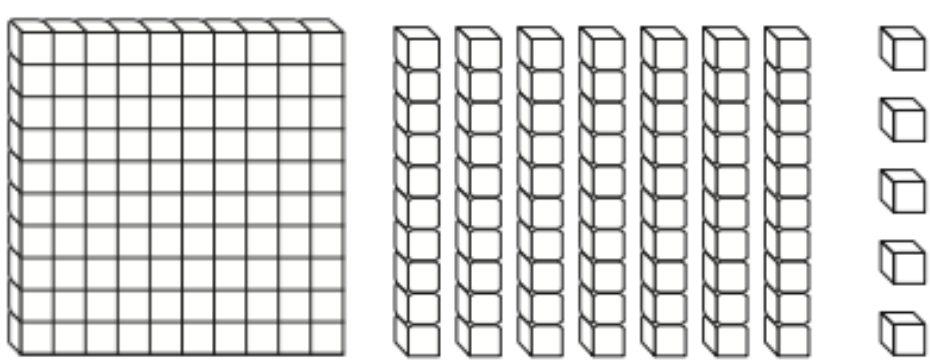
47 = ___ + ___



2) Guess what the numbers are. The first one has been done for you.

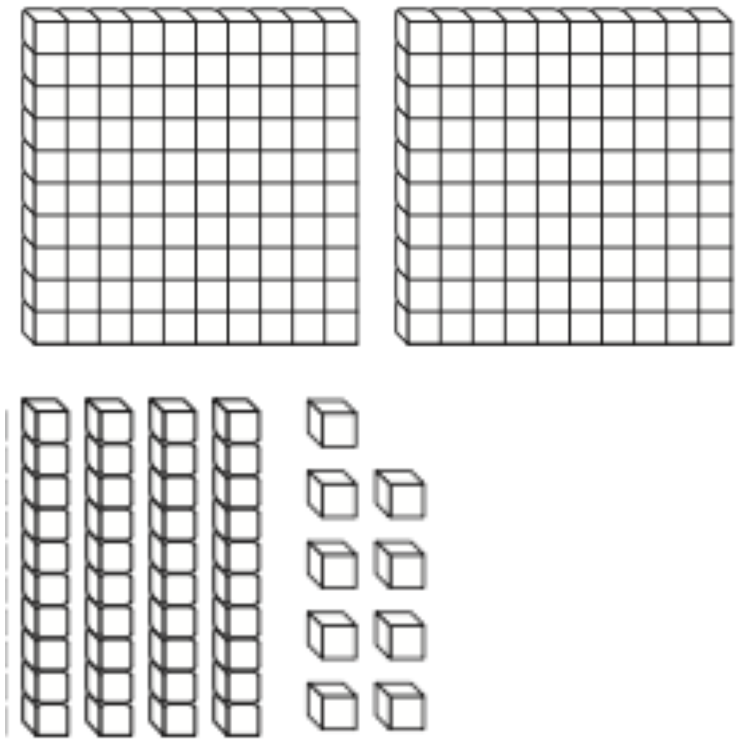
What is a three-digit number that has a **Hundreds** face value of 1; **Tens** face value of 7; **Ones** face value of 5

The number is 175



What is a three-digit number that has a **Ones** face value of 9; **Tens** face value of 4; **Hundreds** face value of 2

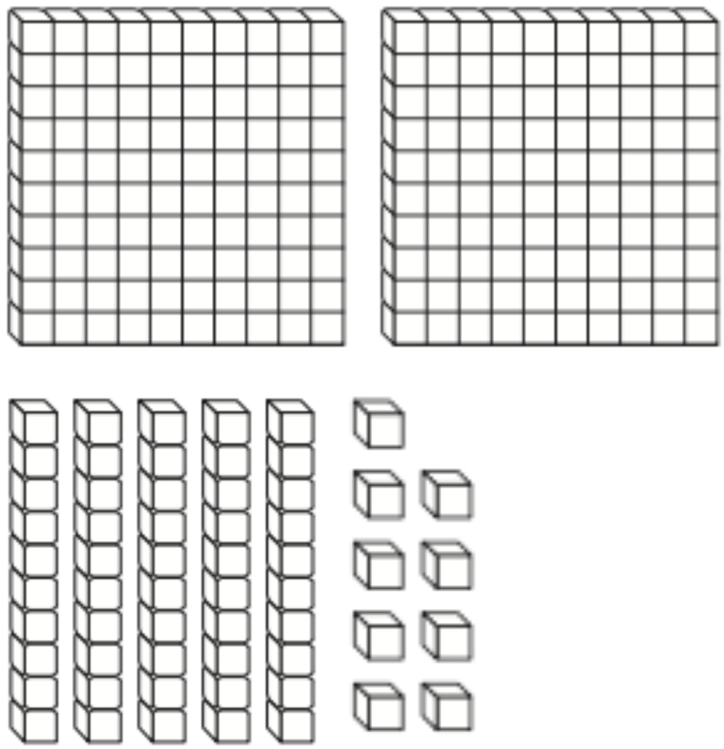
The number is ______________



3) State the value of the bold number. The first one has been done for you

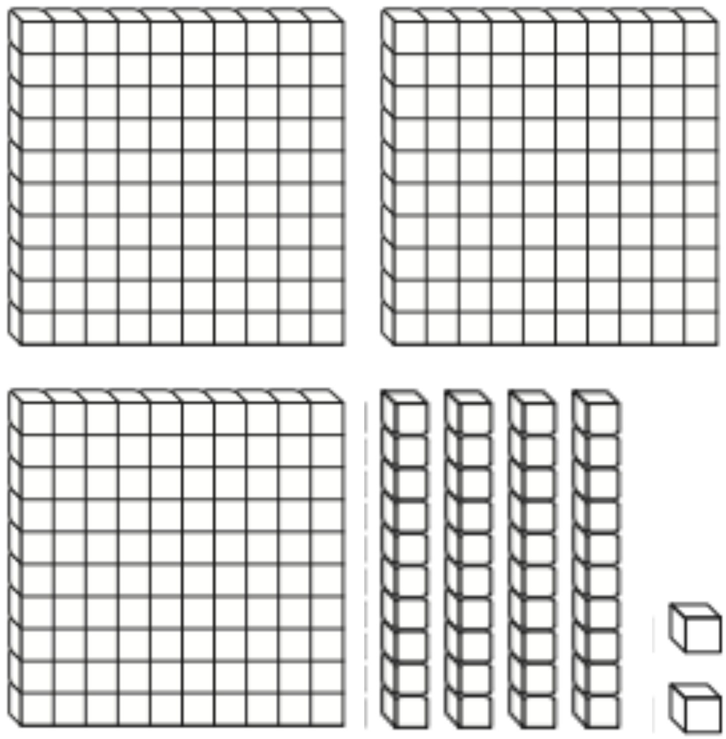
259

The value of the number 5 is <u>50</u>



342

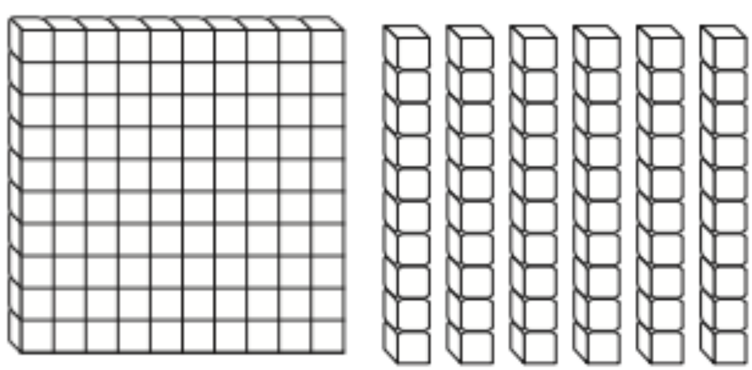
The value of the number 4 is ________



4) Guess what the numbers are. The first one has been done for you.

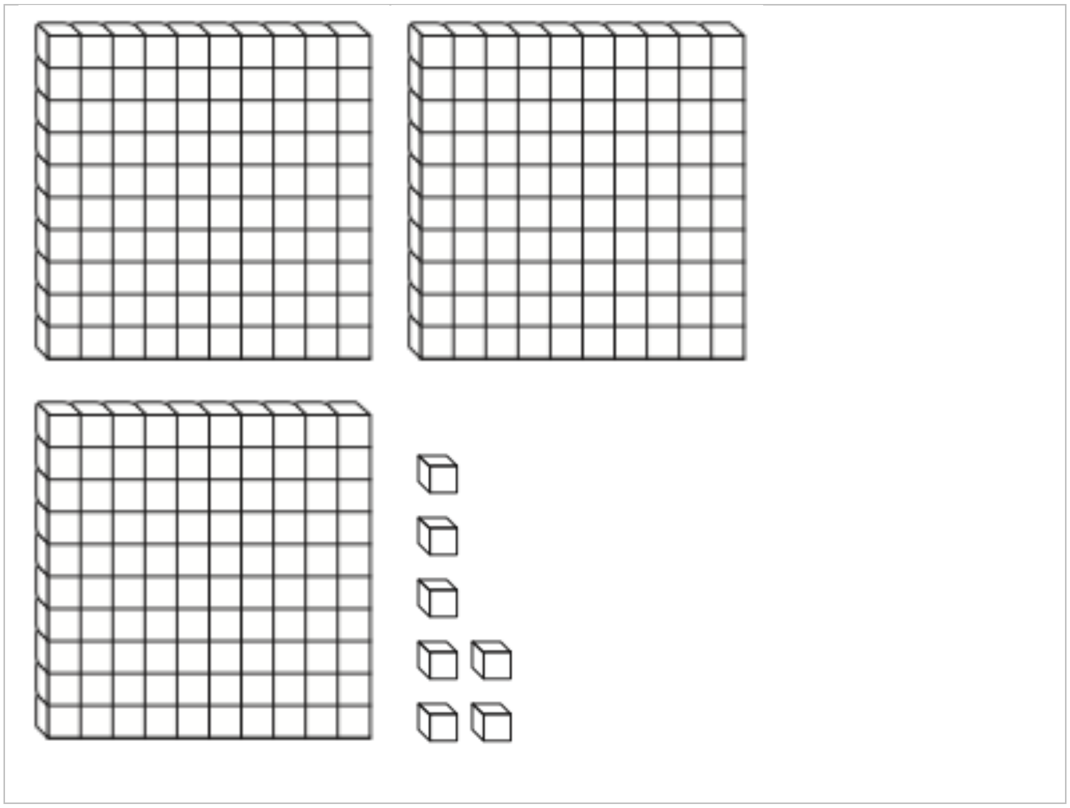
I have 1 hundreds and 6 tens, what number am I?

I am 160



I have 3 hundreds and 7 ones, what number am I?

I am ____________________



Acquisition phase – no visuals

1) Expand the numbers. The first one has been done for you.

36 = 30 + 6
47 = ___ + ___

2) Guess what the numbers are. The first one has been done for you.

What is a three-digit number that has a **Hundreds** face value of 1; **Tens** face value of 7; **Ones** face value of 5 **The number is 175**

What is a three–digit number that has a **Ones** face value of 9; **Tens** face value of 4; **Hundreds** face value of 2

The number is ________________

3) State the value of the bold number. The first one has been done for you

2**5**9

The value of the number 5 is 50

3**4**2

The value of the number 4 is __________

4) Guess what the numbers are. The first one has been done for you.

I have 1 hundreds and 6 tens, what number am I?

I am 160

I have 3 hundreds and 7 ones, what number am I?

I am ______________________

Post-test

1) Expand:

72 = _____ + _____

380 = _____ + _____ + _____

2) State the value of the bold number in 3**7**

____

3) Circle the numbers that have 3 in the ones (there can be more than one number!).

136 36 63 273 532 13

4) State the value of the bold number in 2**4**9

____

5) State the value of the bold number in **9**32

____

6) What number is made up of 500, and 60 and 7?

____

7) I have 3 hundreds, and 4 ones, what number am I?

____

Appendix D: Experimental materials used in Experiment 4

Pre-test

9) Circle the numbers that have 6 in the tens (there can be more than one number!).

106 6 63 66 616 67

10) Circle the numbers that have 1 in the hundreds (there can be more than one number!).

106 1016 16 2166 616 61

11) What number is made up of 300, and 20 and 2?

____

12) I have 9 hundreds, and 9 ones, what number am I?

____

13) Expand:

25 = _____ tens + _____ ones

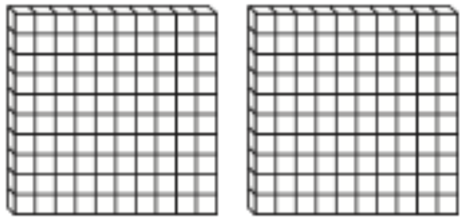
14) Expand:

931 = _____ + _____ + _____

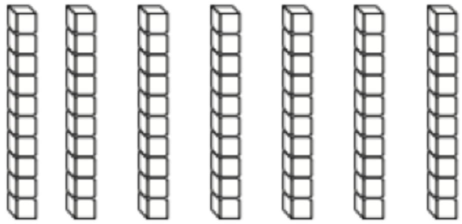
Acquisition phase – MAB

1) What number am I? I have a 5 in the ones column, the number of tens is 7 and there are 2 in the hundreds column.

I have 2 in the hundreds,



7 in the tens,

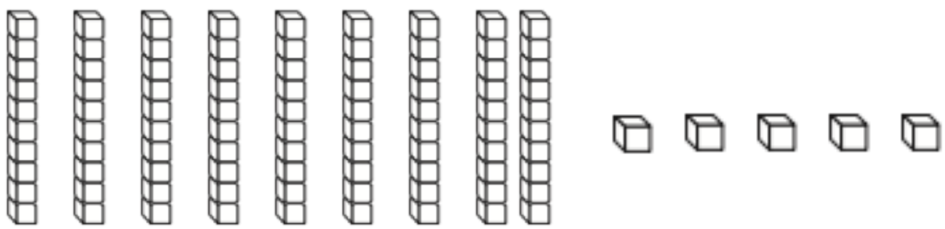


5 in the ones

So I am 275

2) Expand the number 95:

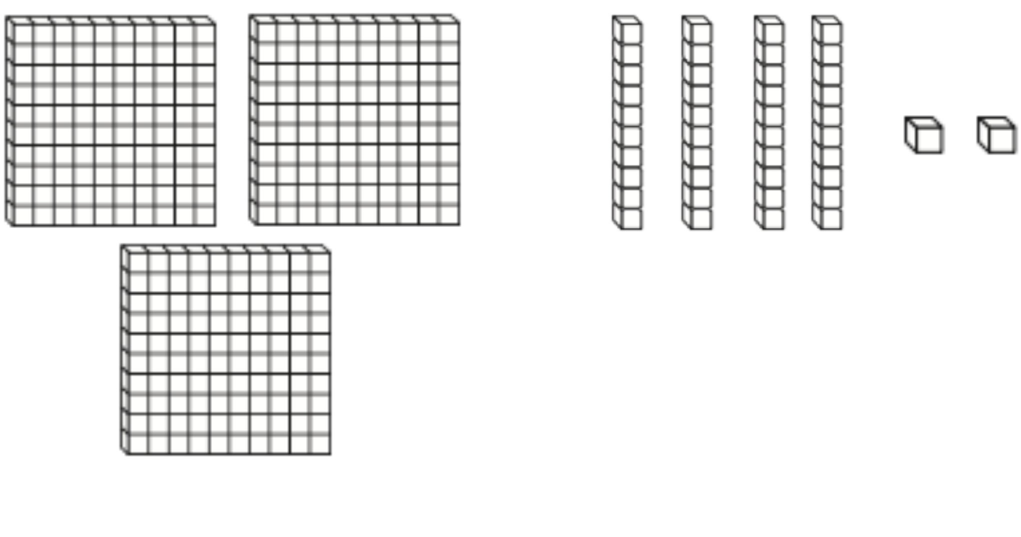
95 = 9 tens + 5 ones



95 = 90 + 5

3) Expand the number 342:

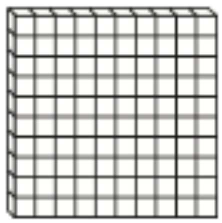
342 = 3 hundreds + 4 tens + 2 ones



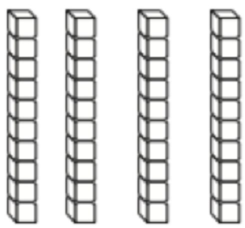
$342 = 300 + 40 + 2$

4) What number am I? I have a 3 in the ones column. The number in the tens column is 4. I have 1 hundreds.

I have 1 in the hundreds,



4 in the tens

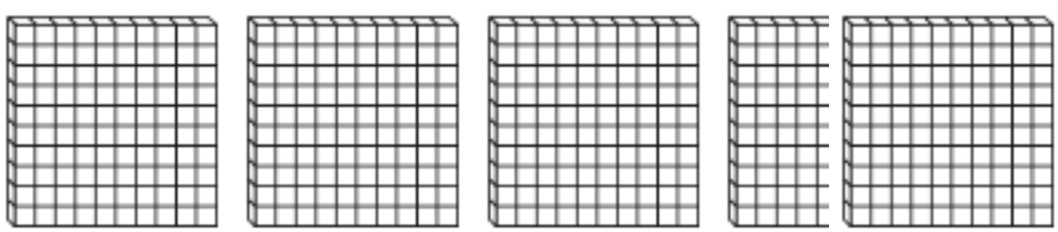


3 in the ones

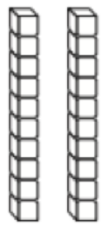
So I am 143

5) What number am I? I have a 2 in the tens column, 7 in the ones column and I have 5 in my hundreds column.

I have 5 in the hundreds,



2 in the tens

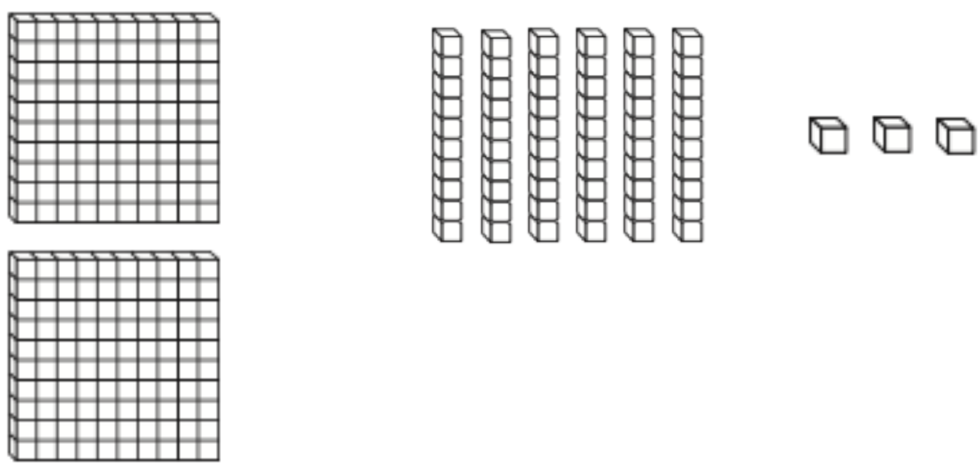


7 in the ones

So I am 527

6) Expand the number 263:

263 = 2 hundreds + 6 tens + 3 ones



263 = 200 + 60 + 3

Acquisition phase – no visuals

1) What number am I? I have a 5 in the ones column, the number of tens is 7 and there are 2 in the hundreds column.

I have 2 in the hundreds,
7 in the tens
5 in the ones
So I am 275

2) Expand the number 95:

95 has 9 tens and 5 ones, so

95 = 9 tens + 5 ones

3) Expand the number 342:

342 has 3 hundreds, 4 tens and 2 ones, so

342 = 3 hundreds + 4 tens + 2 ones

4) What number am I? I have a 3 in the ones column. The number in the tens column is 4. I have 1 hundreds.

I have 1 in the hundreds
4 in the tens
3 in the ones
So I am 143

5) What number am I? I have a 2 in the tens column, 7 in the ones column and I have 5 in my hundreds column.

I have 5 in the hundreds,
2 in the tens
7 in the ones
So I am 527

6) Expand the number 263:

263 has 2 hundreds, or 200, 6 tens or 60 and 3 ones or 3, so

263 = 200 + 60 + 3

Post-test

1) What number am I? I have a 9 in the ones column, the number of tens is 2 and there are 8 in the hundreds column.

I am ____________________

2) What number am I? I have a 7 in the ones column. The number in the tens column is 2. I have 2 hundreds.

I am ____________________

3) What number am I? I have a 1 in the ones column. My hundreds column has 6 and I have 3 in my tens column.

I am ____________________

4) What number am I? I have a 4 in my hundreds column, an 8 in my tens column and my ones column has 2.

I am ____________________

5) Expand:

34 = ________ + ________

6) Expand:

416 = ________ + ________ + ________

7) Expand:

999 = ________ + ________ + ________

Post-test transfer

1) Expand:

17 = ________ tens + ________ ones

2) Expand:

819 = ________ hundreds + ________ tens + ________ ones

3) Expand:

1252 = ________ + ________ + ________ + ________

4) Expand:

3124 = ________ + ________ + ________ + ________

5) Circle the numbers that have 2 in the hundreds (there can be more than one number!).

1226 142 23 219 2052 202

6) What number am I? I have a 7 in my hundreds column. My tens column has 3 less than the hundreds. And my ones column has 2 less than the hundreds column.

I am ____________________

Delayed test

1) What number am I? I have a 4 in my hundreds column, an 8 in my tens column and my ones column has 2.

I am ____________________

2) Expand:

417 = ________ hundreds + ________ tens + ________ ones

3) Expand:

904 = ________ hundreds + ________ tens + ________ ones

4) Expand:

3419 = ________ + ________ + ________ + ________

5) Expand:

9999 = ________ + ________ + ________ + ________

6) Circle the numbers that have 6 in the hundreds (there can be more than one number!).

6266 602 1623 6006 4056 6606

7) What number am I? I have an 8 in my hundreds column. My tens column has 2 less than the hundreds. And my ones column has 4 less than the hundreds column.

I am ____________________

Appendix E: Experimental materials used in Experiment 5

Pre-test

1) Circle the numbers that have 6 in the tens (there can be more than one number!).

106	1 663	616	67

2) Circle the numbers that have 1 in the hundreds (there can be more than one number!).

1 016	2 166	611	161

3) Circle the numbers that have 7 in the thousands (there can be more than one number!).

7 316	71 777	1 732	17 356

4) What number is made up of 3 000, and 10 and 2?

____

5) I have 9 thousands, and 9 ones, what number am I?

____

6) Arrange the following numbers in order from largest to smallest:
1 346, 942, 1 326, 924, 10 002, 9 009

____

____

7) Arrange the following numbers in order from smallest to largest:
10 987, 10 978, 1 724, 724, 6 002, 6 020

____

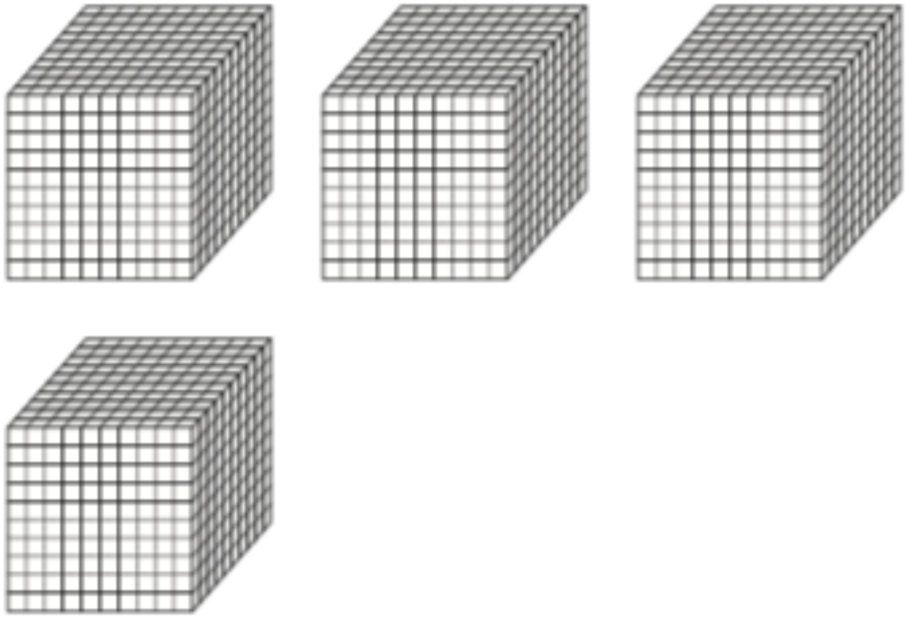
____

Acquisition phase – MAB

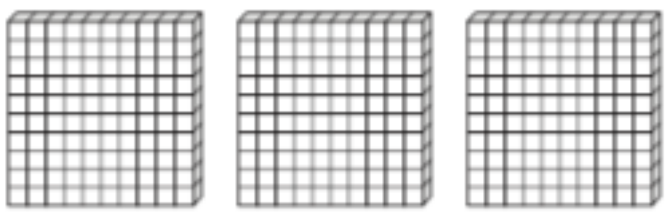
1) Expand the number 4 307:

4 307 has

4 thousands (4 000)



3 hundreds (300)



0 tens (0)

and 7 ones (7)

This means: 4 307 = 4 000 + 300 + 0 + 7

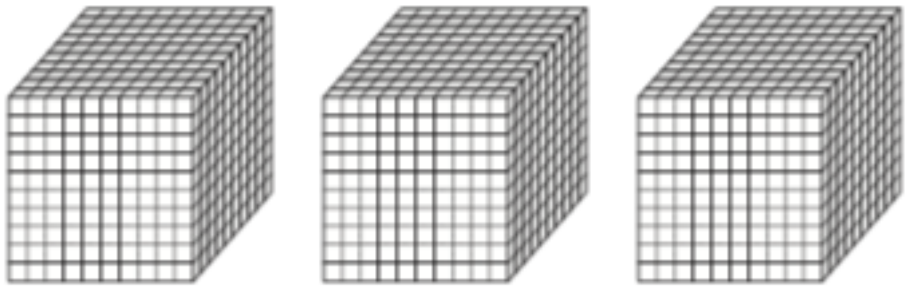
2) Now solve a similar problem

Expand the number 8 031:

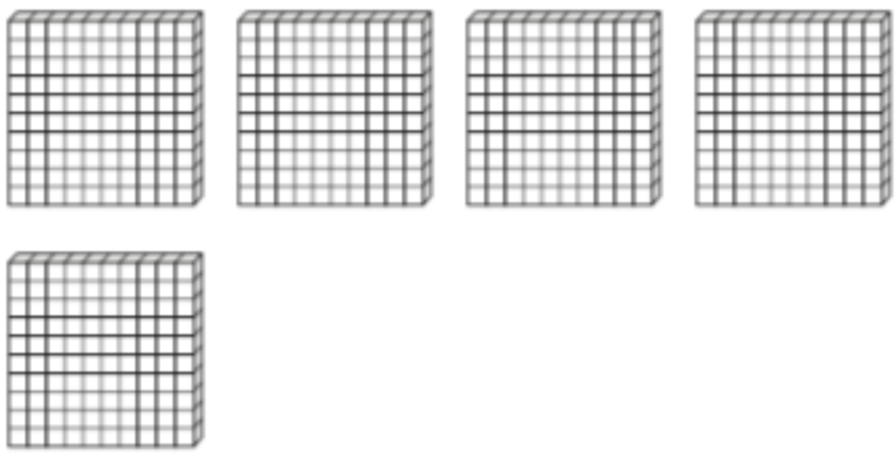
3) Expand the number 3 584

3 584 has

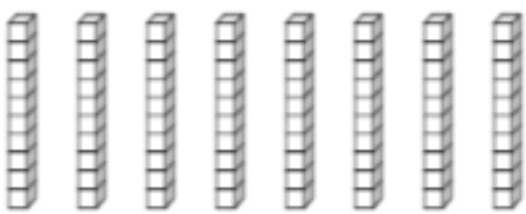
3 thousands (3 000)



5 hundreds (500)



8 tens (80)



and 4 ones (4)

This means: 3 584 = 3 000 + 500 + 80 + 4

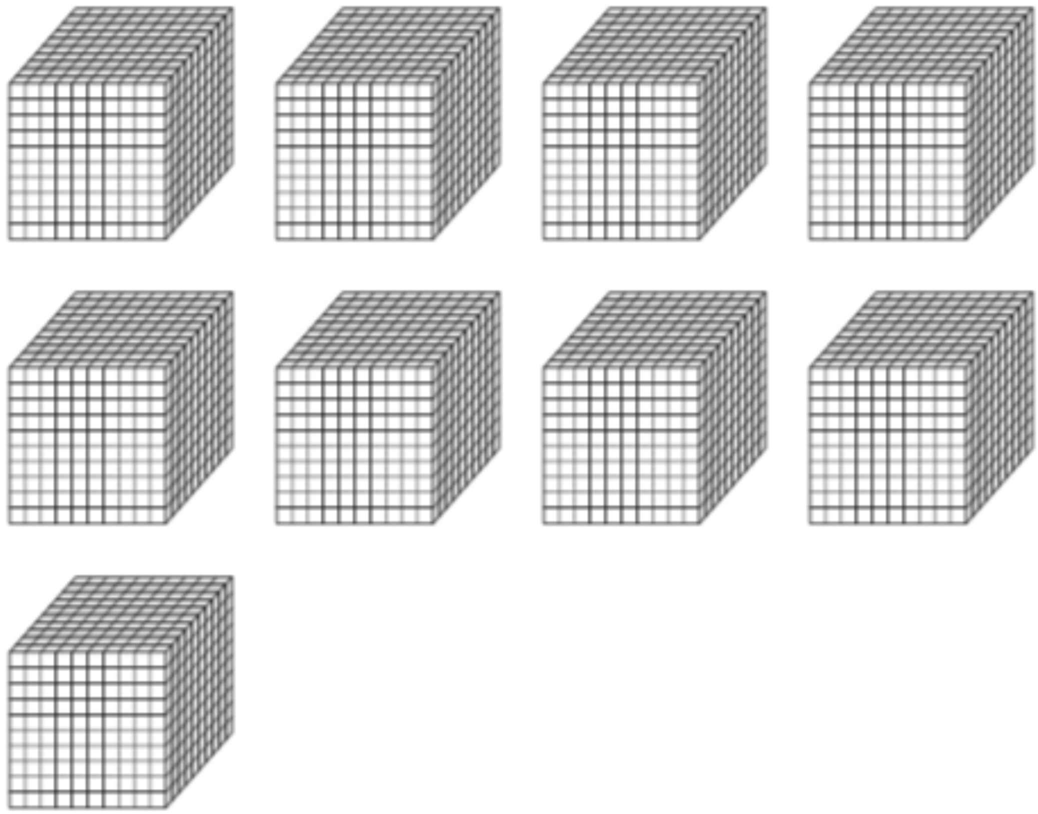
4) Now solve a similar problem

Expand the number 9 929:

5) Expand the number 9 075

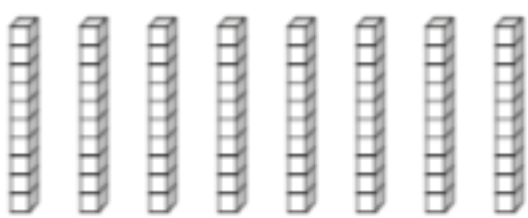
9 075 has

9 thousands (9 000)



0 hundreds (0)

7 tens (70)



and 5 ones (5)

This means: 9 075 = 9 000 + 0 + 70 + 5

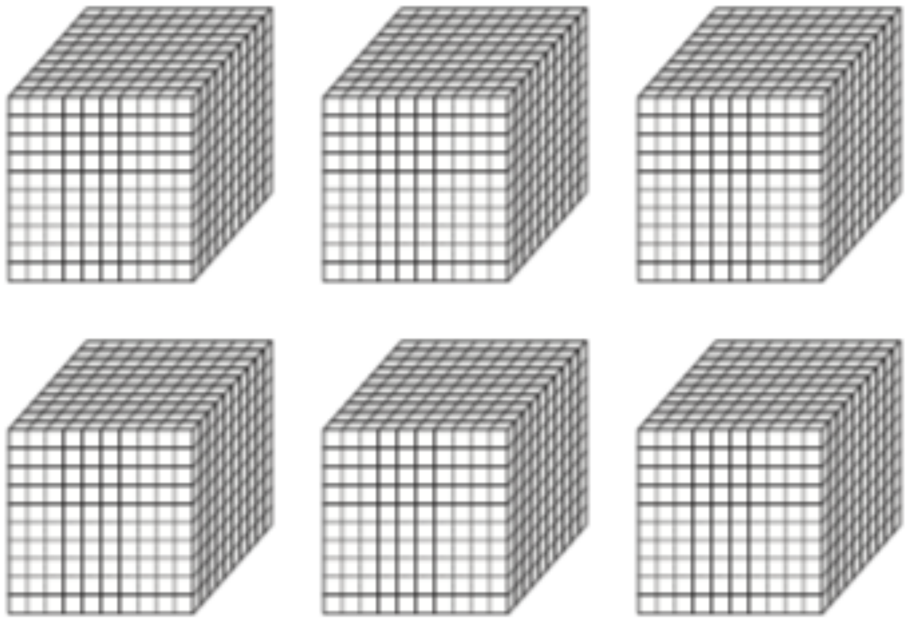
6) Now solve a similar problem

Expand the number 7 380:

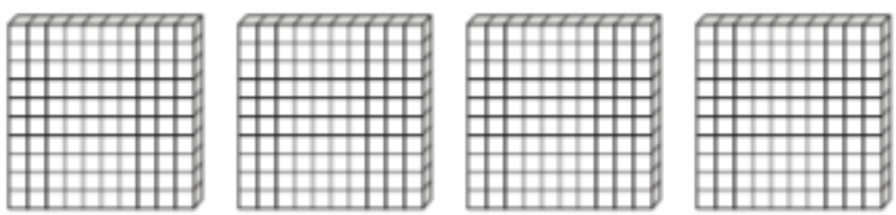
7) Expand the number 6 444

6 444 has

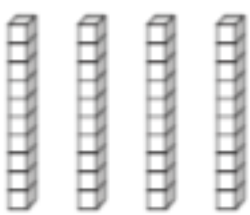
6 thousands (6 000)



4 hundreds (400)



4 tens (40)



and 4 ones (4)

This means: 6 444 = 6 000 + 400 + 40 + 4

8) Now solve a similar problem

Expand the number 3 888:

Acquisition phase – no visuals

1) Expand the number 4 307:

4 307 has

4 thousands (4 000)

3 hundreds (300)

0 tens (0)

and 7 ones (7)

This means: 4 307 = 4 000 + 300 + 0 + 7

2) Now solve a similar problem
Expand the number 8 031:

3) Expand the number 3 584

3 584 has

3 thousands (3 000)

5 hundreds (500)

8 tens (80)

and 4 ones (4)

This means: 3 584 = 3 000 + 500 + 80 + 4

4) Now solve a similar problem
Expand the number 9 929:

5) Expand the number 9 075

9 075 has

9 thousands (9 000)

0 hundreds (0)

7 tens (70)

and 5 ones (5)

This means: 9 075 = 9 000 + 0 + 70 + 5

6) *Now solve a similar problem*
Expand the number 7 380:

7) *Expand the number 6 444*

6 444 has

6 thousands (6 000)

4 hundreds (400)

4 tens (40)

and 4 ones (4)

This means: 6 444 = 6 000 + 400 + 40 + 4

8) *Now solve a similar problem*
Expand the number 3 888:

Post-test

1) *Expand:*

934 = ________ + ________ + ________

2) *Expand:*

1 416 = ________ + ________ + ________ + ________

3) *Expand:*

1 999 = ________ + ________ + ________ + ________

4) *Expand:*

9 499 = ________ + ________ + ________ + ________

5) Expand:

7 351 = ________ + ________ + ________ + ________

6) Expand:

4 099 = ________ + ________ + ________ + ________

7) Expand:

5 930 = ________ + ________ + ________ + ________

8) Expand:

2 024 = ________ + ________ + ________ + ________

9) Expand:

6 004 = ________ + ________ + ________ + ________

10) Expand:

9 999 = ________ + ________ + ________ + ________

Post-test transfer

1) Expand:

5 819 = ________ thousands + ________ hundreds + ________ tens + ________ ones

2) Expand:

13 124 = ________ + ________ + ________ + ________ + ________

3) 9 000 + 600 + 0 + 4 = ____________

4) 4 + 500 + 80 + 3 000 = ____________

5) 700 + 0 + 9 + 7 000 = ____________

6) A number has 13 tens and 6 ones. What is the number?

____

7) 5 thousands + 32 tens + 8 ones = ________________

8) 11 thousands + 8 hundreds + 37 ones = ______________

9) Fill in the missing number:

2088 + ______ = 2188

10) Fill in the missing number:

6 109 + ______ = 6 149

Delayed test

1) Expand:

692 = _______ + _______ + _______

2) Expand:

4 111 = _______ + _______ + _______ + _______

3) Expand:

7 916 = _______ + _______ + _______ + _______

4) Expand:

2 008 = _______ + _______ + _______ + _______

5) 6 000 + 600 + 60 + 6 = __________

6) 9 + 600 + 70 + 2 000 = __________

7) A number has 18 tens and 7 ones. What is the number? ____________________

8) 32 hundreds + 9 tens + 6 ones = __________

9) Fill in the missing number: 4 508 + __________ = 4 538

10) 3 thousands + 21 tens + 4 ones = __________