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ANALYSIS OF PIPE TESTS

by

T.R. Fietz, and K.B. Higgs

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ANALYSIS OF PIPE TESTS

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Abstract

Analysis of pipe tests to determine friction coefficients and their uncertainty is discussed. Optimisation methods are applied to finding the pipe diameter, friction coefficient, and fitting loss coefficient from a pipe test. A FORTRAN program implementing these methods is included.

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ABSTRACT

Analysis of pipe tests to determine friction coefficients and their uncertainty is discussed. Optimisation methods are applied to finding the pipe diameter, friction coefficient, and fitting loss coefficient from a pipe test. A FORTRAN program implementing these methods is included.

NOTATION

a	Coefficient for least squares regression line
b	Slope for least squares regression line
C_o to C_5	Coefficients
d	Diameter
d_g	Global d
d_h	Upper limit of d range for search
d_{high}	Upper d value
d_l	Lower limit of d range for search
d_{low}	Lower d value
e_d	Uncertainty in d
e_k	Uncertainty in k
e_l	Uncertainty in l
e_Q	Uncertainty in Q
e_S	Uncertainty in S
e_t	Uncertainty in t
e_μ	Uncertainty in μ
e_ν	Uncertainty in ν
e_ρ	Uncertainty in ρ
f	Darcy friction factor
f(x)	Function of x
$f(x_*)$	Minimum of f(x)
$f^1(x)$	First derivative of f(x)
g	Gravitational acceleration
h	Piezometric head
h_f	Drop in HGL due to friction (or surface resistance) head loss
$\bar{h}$	Mean of h observations
h_o	h on best fit HGL
H_f	Friction (or surface resistance) head loss
H_L	Total head loss
H_o	Observed total head
H_v	Fitting (or minor) head loss
HGL	Hydraulic grade line (piezometric head line)
I_d	Number of iterations for d search
I_k	Number of iterations for k search
I_{K_L}	Number of iterations for K_L search
k	Equivalent sand grain roughness
k_g	Global k

k_h	Upper limit of k range for search
k_l	Lower limit of k range for search
k_{max}	k at upper limit of uncertainty
k_{mean}	k from mean properties
k_{min}	k at lower limit of uncertainty
K_L	Fitting loss coefficient
K_{L_g}	Global K_L
K_{L_h}	Upper limit of K_L range for search
K_{L_l}	Lower limit of K_L range for search
l	Distance along pipe
$\bar{l}$	Mean of l observations
L	Pipe length
n	Number of sets of pipe test observations
N	Number of iterations
p	Static pressure
Q	Discharge
Q_o	Observed discharge
Q_{high}	Upper Q value
Q_{low}	Lower Q value
r	Golden section ratio
R	Residual head loss or discharge
$\mathbb{R}$	Reynolds number
S	Energy gradient (downward +ve)
S_{high}	Upper S value
S_{low}	Lower S value
S_{mean}	Mean S
S_*	Published S value
t	Temperature
t_p	Student's t coefficient
V	Pipe mean velocity
x	Abscissa, independent variable
x_*	x at function minimum
x_1	Lower limit of x range containing minimum
x_2	Upper limit of x range containing minimum
x_3, x_4	Intermediate x values between x_1 and x_2
y	Ordinate, dependent variable
z	Elevation of pipe above datum

γ	Unit weight of water
δx	x interval
Δd	d tolerance
Δh_{max}	Maximum perturbation about best fit HGL
Δk	Small k increment
Δl	Spacing of head tapping points
ΔS	Small S increment
Δt	Temperature increment
Δx	Range of uncertainty in x_*
θ_d	Sensitivity coefficient for d
θ_Q	Sensitivity coefficient for Q
θ_S	Sensitivity coefficient for S
θ_ν	Sensitivity coefficient for ν
μ	Dynamic viscosity of water
ν	Kinematic viscosity of water
ν_o	Observed kinematic viscosity
ν_{high}	Upper ν value
ν_{low}	Lower ν value
ρ	Density of water

INTRODUCTION

Computation of total head loss due to friction (or surface resistance) and minor (or fitting) loss is necessary for the hydraulic design of water supply pipelines and networks.

The reliability of the head loss estimates is affected by the uncertainty in the design values selected, in particular the value characterising the pipe material for the calculation of friction loss.

Design values for computing friction loss and minor loss are derived from pipe tests by fitting assumed mathematical relations to the test data. In this report the Darcy-Weisbach equation is taken as the relation for friction loss, in conjunction with the Colebrook-White equation for finding the equivalent sand grain roughness, which is the design value specifying the pipe wall material. The assumed mathematical relation for minor loss uses a constant minor loss coefficient applied to the pipe velocity head.

This report aims to:

(i) Develop statistically proper, programmable methods for determining the equivalent sand grain roughness and its uncertainty from a pipe test where the pipe diameter is known.

(ii) To show that the least squares minimisation method may be used for analysis of pipe test data where various combinations of pipe diameter, equivalent sand grain roughness and minor loss coefficient are required.

A FORTRAN computer program implementing the proposed methods is included, together with the results of applying the program to both synthetically generated and published pipe test data.

BASIC EQUATIONS

Figure 1 shows the head loss components for steady flow of water under pressure in a circular pipe. The total head loss H_L may be expressed as:

$$H_L = H_f + H_v \tag{1}$$

where H_f is the friction loss and H_v is the fitting loss, assumed concentrated at the end of the pipe length l being considered.

The fitting loss is given by:

$$H_v = K_L \frac{V^2}{2g} \tag{2}$$

where K_L is the fitting "loss coefficient" and V the pipe mean velocity. K_L depends mainly on fitting shape, and to a lesser extent on pipe Reynolds Number, pipe roughness, proximity to other fittings, etc. K_L may be determined accurately as a function of Reynolds Number and other factors [31], but for water supply calculations K_L is usually taken as a constant value for a particular fitting shape [2].

For a prismatic pipe the friction loss H_f is equal to the drop in the Hydraulic Grade Line h_f, given by the Darcy-Weisbach equation:

$$h_f = f \frac{L}{d} \frac{V^2}{2g} \tag{3}$$

The Darcy friction factor f depends on the Reynolds Number $\mathbf{R}$ and the pipe relative roughness $\frac{k}{d}$, where k is the "equivalent sand grain roughness", a linear measure of the roughness of the pipe wall. The most popular expression relating f to $\mathbf{R}$ and $\frac{k}{d}$ is the Colebrook-White equation [14]:

$$\frac{1.0}{\sqrt{f}} = -2 \log_{10} \left[\frac{k}{3.7\,d} + \frac{2.51}{\mathbf{R}\sqrt{f}} \right] \tag{4}$$

The Colebrook-White equation works well for small (less than 300 mm dia.) water supply pipes, both new and aged. For larger conduits deviations from the Colebrook-White function are frequently reported, for example from tests on concrete pipes [38], concrete tunnels [18], coated steel pipes [6], [9], [30], steel lined tunnels [18], and unlined rock tunnels at low flow rates [34]. Barr [3], [4], has derived formulae to cover these deviations but the expressions require six coefficients while Equation (4) requires only the equivalent sand grain roughness k.

For discussion of pipe test results the Moody Chart [32] provides the most convenient graphical representation of Equation (4).

Equation (4) is implicit in the friction factor f, but may be readily solved iteratively by "successive substitution" [43]:

$$\frac{1.0}{\sqrt{f_{i+1}}} = -2 \log_{10} \left[\frac{k}{3.7\,d} + \frac{2.51}{\mathbf{R}\sqrt{f_i}} \right] \tag{5}$$

where f_i, f_{i+1} are f values for iterations i, i+1 respectively. To commence iterations f_1 may be taken as 1.0 or as f for a rough pipe, obtained by eliminating the $\frac{2.51}{\mathbf{R}\sqrt{f_i}}$ term in Equation (5).

Putting $V = \frac{4Q}{\pi d^2}$ in Equations (2) and (3), Equation (1) becomes:

$$H_L = \left[f \frac{L}{d} + K_L \right] \frac{8\,Q^2}{\pi^2\, g d^4} \tag{6}$$

When the pipe flow is wholly rough wall turbulent (on the far right of the Moody chart, Figure 4) the friction factor f takes on a constant value. The minor loss coefficient K_L is also constant so some ambiguity in f and K_L values might be expected when Equation (6) is used in procedures to analyse test results from a pipe operating as wholly rough wall turbulent flow and with minor losses.

Putting $V = \frac{4Q}{\pi d^2}$ in the Reynolds Number $\mathbf{R} = \frac{Vd}{\nu}$ makes the friction factor f in Equations (4) and (5) a function of (Q, k, d, ν), where ν is the kinematic viscosity of water.

Equation (6) may be re-arranged to make Q the dependent variable:

$$Q = \left\{ \frac{H_L \pi^2 g d^4}{8 (f \frac{L}{d} + K_L)} \right\}^{0.5} \tag{7}$$

When the pipe flow is smooth wall turbulent, or transitional from smooth wall turbulent to wholly rough wall turbulent, the friction factor f in Equation (7) is a function of Q, so Equation (7) is implicit in Q and requires an iterative solution.

Tests on a pipe of known length L yield n sets of observations of discharge Q_o, head loss H_o, and water temperature, from which the kinematic viscosity ν_o may be found from tables. Equations (6) and (7) represent the assumed mathematical relations to be fitted to the test observations. A least squares minimisation procedure may be used to find the required pipe properties [10].

If Equation (6) is used the predicted head loss H_L (Q_o) is found by using the observed discharge Q_o in Equation (6). The residual head loss R is the difference between the predicted head loss and the observed head loss H_o. An adequate estimate of the pipe properties is assumed to occur when the sum of the squares of the residuals ΣR^2 is a minimum for all n sets of test observations.

For minimisation in terms of head loss the function to be minimised is therefore:

$$F(k, d, K_L) = \sum_{i=1}^{n} \left\{ H_{o_i} - H_L(Q_{o_i}) \right\}^2 = \sum_{i=1}^{n} R_i^2 \tag{8}$$

In Equation (8) F (k, d, K_L) indicates that one or more of k, d and K_L are unknown values. The term $H_L(Q_{o_i})$ is the head loss predicted by using Q_{o_i} in Equation (6), and R_i is the residual head loss.

Minimisation in terms of head loss residuals is preferable to minimisation in terms of discharge residuals [Equation (9) below] for two reasons:

(i) The least squares fitting procedure assumes that the dependent variable varies randomly about the fitted curve while the independent variable is known with negligible error [10]. As will be shown subsequently, the uncertainty in the head loss observation H_o is greater than the uncertainty in the discharge measurement Q_o, so H_o should be taken as the dependent variable.

(ii) The iterative solution required to find the predicted discharge $Q(H_o)$ from Equation (7), for use in Equation (9), is avoided.

Alternatively Equation (7) may be used for minimisation in terms of discharge where the function to be minimised is:

$$F(k, d, K_L) = \sum_{i=1}^{n} \left\{ Q_{o_i} - Q(H_{o_i}) \right\}^2 = \sum_{i=1}^{n} R_i^2 \tag{9}$$

Again $F(k, d, K_L)$ indicates that one or more of k, d, and K_L are unknown. The term $Q(H_{o_i})$ is the discharge predicted by using H_{o_i} in Equation (7) and R_i is the residual discharge.

One method of minimisation requires the partial derivatives $\frac{\partial F}{\partial k}$ and $\frac{\partial F}{\partial d}$ for Equation (9) with the loss coefficient $K_L = 0$. The derivatives are:

$$\frac{\partial F}{\partial k} = \sum_{i=1}^{n} 2 \left\{ Q_{o_i} + C_4 C_5 \right\} \frac{C_4}{3.7d} \tag{10}$$

$$\frac{\partial F}{\partial d} = \sum_{i=1}^{n} 2 \left\{ Q_{o_i} + C_4 C_5 \right\} \left\{ \frac{-C_1}{C_3} \left[\frac{kd}{3.7}^{0.5} + 1.5\, C_2 \right] + 2.5\, C_1 C_5 d^{1.5} \right\} \tag{11}$$

$$\text{where } C_o = \left[2g \frac{H_{o_i}}{L} \right]^{0.5}, \quad C_1 = (\log_{10} e) \frac{\pi}{2} C_o, \quad C_2 = \frac{2.51\nu}{C_o}$$

$$\text{and } C_3 = \frac{k}{3.7d} + \frac{C_1}{d^{1.5}}, \quad C_4 = C_1 d^{2.5}, \quad C_5 = \log_e(C_3)$$

DETERMINATION OF DIAMETER d, EQUIVALENT SAND GRAIN ROUGHNESS k, AND LOSS COEFFICIENT K_L FROM PIPE TESTS

Case 1: Given d, to Find k

This is the most common case for analysis of both laboratory and field pipe tests. Any minor losses due to joints, or slight changes in alignment in installed pipes, are absorbed in selecting the k value so K_L is taken as zero.

Minimisation in terms of head loss residuals has been adopted for the reasons discussed previously. In Equation (8) k is the unknown and the required value k_g (global k) may be found by searching for the minimum of $\sum R_i^2$ between stated limits k_l and k_h, using one of the minimisation procedures described in Appendix A. The search procedure is shown schematically in Figure 2 (A).

Note that only one set of observations of Q_o, H_o, and v_o is required to calculate f and $\mathbb{R}$ and to then select $\frac{k}{d}$ from the Moody Chart. Similarly only one set of observations is required for the least squares method outlined above, although as many sets as possible should be used to improve the reliability of the k estimate.

Case 2: Given d and k, to Find K_L

This case is applicable to a pipeline of known d and k which has appreciable minor losses generated by non-prismatic components such as bends, partially closed valves, etc. The procedure finds a single K_L value applicable over the range of test discharges

A similar least squares method to that used for Case 1 is employed. The unknown variable in Equation (8) is now K_L.

The search procedure is shown schematically in Figure 2(B) where the search limits are K_{L_l} to K_{L_h} and the global minimum is K_{L_g}.

Only one set of observations of Q_o, H_o and ν_o is required to find K_{L_g} but as many sets as possible should be used to assess the suitability of Equation (6) as a model for the $H_L - Q$ relation for the pipe.

Case 3: Given d, to Find k and K_L

This case might be applied to test results from a pipeline of known d but where k has changed with use and appreciable minor losses are suspected.

Here the unknown variables in Equation (8) are k and K_L so the minimum of a two-variable function is required. As shown in Figure 3, $\sum R_i^2$ as a function of k and K_L may be visualised as a three dimensional surface. The global minimum with values k_g and K_{L_g} occurs at the lowest point of the valley line.

Numerous techniques for optimising (i.e. finding the global minimum) of a multivariate function are known [25]. For the present report it has been decided to use a crude but readily visualised and easily programmed parametric search technique. The technique involves recursive application of a procedure for minimisation of a one-variable function and was first applied to analysing pipe test results by Stuckey [39].

The search procedure is shown schematically in Figure 2(C). The search is made between outer limits k_l and k_h. For each k value an inner search is made in the normal direction between limits K_{L_l} and K_{L_h} to locate the local minimum K_L. At the global minimum the values k_g and K_{L_g} locate the global minimum value of $\sum R_i^2$.

If I_k is the number of iterations required to search in the k direction, and I_{K_L} to search in the K_L direction, then the total number of iterations is $I_k \times I_{K_L}$.

The minimum number of sets of test observations required to find k and K_L is n = 2.

Case 4 : Find d and k

This case might be applied to test results from an old inaccessible pipeline to obtain an "effective diameter d" and "an effective sand grain roughness k" which best fits the test results. The unknown variables in Equation (8) are d and k.

The search procedure is shown schematically in Figure 2 (D). The outer search is between limits k_l and k_h and the inner search between d_l and d_h.

Stuckey [39] solved this case using minimisation of the discharge residuals defined by Equation (9).

Case 5 : Find d, k and K_L

This case may be applied to test results from a pipeline with little information as to size, condition or presence of fittings. Case 5 may be applied as an alternative to Case 4. The sum of the squares of the residual head losses $\sum R_i^2$ indicates which solution is a better fit to the test data.

This case requires searching at three levels, as shown schematically in Figure 2 (E).

The procedure may be visualised as examining numerous three dimensional surfaces, each labelled with a K_L = constant value, to find the surface which has the smallest local minimum value of $\sum R_i^2$. The smallest local minimum becomes the global minimum with the required variable values K_{L_g}, d_g and k_g.

If the number of iterations required to search for K_L, d and k is I_{K_L}, I_d and I_k respectively, then the total number of iterations to find the global minimum is $I_{K_L} \times I_d \times I_k$. The computer time required for a three-variable function makes this the practical limit of application of parametric searching methods for use on micro and mini computers.

The minimum number of sets of test observations required to find d, k and K_L is n = 3.

SELECTION OF A MINIMISATION PROCEDURE

Two procedures for minimisation of a one-variable function are described in Appendix A. These have been applied to the synthetically generated pipe tests described below with the following results :

Minimisation by Bisection of the First Derivative Function

This was tried for Case 4 problems, that is to find d and k, using Equation (9) as the function to be minimised, with $K_L = 0$ for this case.

The outer search is between limits k_l and k_h with the object of making $\frac{\partial F}{\partial k}$ from Equation (10) equal to zero by the method of bisection. For each k value during the outer search an inner search is made between d_l and d_h to make $\frac{\partial F}{\partial d}$ equal to zero by bisection.

The method performed satisfactorily in that the global minimum values k_g and d_g were correctly predicted. From Equations (A1) and (A2) in Appendix A, the number of iterations for convergence by bisection is about 0.7 times the number for a golden section search. The additional computer time required to evaluate the partial derivatives $\frac{\partial F}{\partial k}$ and $\frac{\partial F}{\partial d}$ appears to outweigh the apparent saving in the lower number of iterations so minimisation by bisection was not pursued further.

Minimisation by Golden Section Search

This method was found to be reliable and to converge for Cases 1 to 5 of pipe test analysis, and has been adopted as the standard procedure.

For Cases 3, 4 and 5, requiring two or more recursive applications of the golden section search algorithm, individually labelled subroutines have been used in the computer program as recursion is not implemented in some versions of FORTRAN.

COMPUTER PROGRAM

A computer program for solving pipe test Cases 1 to 5 is shown as Appendix C. The program is written in FORTRAN 77 and was used on a Digital MicroVAX II computer.

The program is developmental in that it was assembled using components from other programs to demonstrate the techniques described above. The program is not intended as an example of good or efficient programming.

PROGRAM TESTING USING SYNTHETIC PIPE TEST RESULTS

The program was tested by application to synthetic pipe test results generated by using Equation (6) to find head losses for at least ten equally spaced discharges. No attempt has been made to apply a random component to the head loss H_o values generated, although this could be done using the method described for generating random piezometric head values in the "Uncertainty in the Energy Gradient S" section below. The pipes selected cover the range of operation for typical water supply pipes [21]. Some of the data is plotted on a Moody Chart, shown as Figure 4. Some very rough pipes, which would plot above the maximum f value on Figure 4, were also tested.

The program predicted the unknowns for Cases 1 to 4 without fail. The only tests where any discrepancy occurred was in application of Case 5 to very rough pipes with minor losses approximately equal to friction losses. In these cases the predictions of d, k and K_L values were improved by using discharge increments proportional to $\log_{10}(Q)$. The test discharges were calculated by dividing $\left\{\log_{10}(\max. Q) - \log_{10}(\min. Q)\right\}$ into the required number of increments and then taking the antilogs to find the discharges. This compresses the flowrates towards the lower end of the range. Some typical results are shown in Table 1 below:

TABLE 1: Case 5, Find d, k and K_L, Very Rough Pipes

Pipe: Cast Iron, Tuberculated, High Minor Losses	d (mm)	k (mm)	K_L
Values used to generate data	245.9	10	40
Predicted values, linear Q increments	245.04	10.221	31.81
Predicted values, $\log_{10}$ (Q) increments	246.6	9.826	46.59
Pipe: Cast Iron, Tuberculated, High Minor Losses	**d (mm)**	**k (mm)**	**K_L**
Values used to generate data	245.9	15	100
Predicted values, linear Q increments	243.48	16.058	67.9
Predicted values, $\log_{10}$ (Q) increments	244.95	15.402	87.54

For all Case 5 applications shown in Table 1 the searches converged to give $\sum R_i^2$ of zero, i.e. a head loss calculated using the predicted values of d, k and K_L for a given discharge was equal to the test head loss for that discharge. The solution is therefore dependent on the discharge selection for the pipe tests. Minimisation using discharge as the working variable may improve the solution but this has not been tried.

APPLICATION OF THE PROGRAM TO PUBLISHED PIPE TEST RESULTS

The criteria for selecting pipe test results from the literature were, firstly, an adequate number of sets of Q_o, H_o and v_o observations, and secondly, preferably more than two head measuring tapping points along the line. Several tapping points are required to estimate the uncertainty in the slope of the hydraulic grade line, as discussed later. In selecting the sets of test data from a particular source some obvious "outliers" [31] were arbitrarily rejected.

Some of the pipe test data selected are shown on the Moody Chart, Figure 5 and the program test results are shown in Table 2. The tests labelled "mean properties", "properties for min. k", and "properties for max. k" are explained in the section on estimation of the uncertainty in k below.

Published k values were available for tests 5, 8 and 9, thus allowing comparison with k values from the Case 1 analyses. The published k values for tests 5 and 8 were obtained by the standard method of plotting (f,$\mathbb{R}$) points on the Moody Chart and selecting by eye a $\frac{k}{d}$ = constant line of best fit. The published k value for test 9 was obtained by comparing a logarithmic plot of observed (Q_o, H_o) values with that for new pipe. The known k value for the new pipe was increased to allow for the increased head loss in the aged pipe being tested. The Case 1 analyses of tests 5, 8 and 9 gave k values ranging from 30% greater to 40% smaller than the published k values. The range of uncertainty in k for tests 5 and 9 was estimated from Case 1 analyses of tests 6 and 7, and tests 10 and 11, respectively. For test 5 the k value was 0.055 mm and the range of uncertainty was from 0.038 mm to 0.077 mm, to 95% confidence level. For test 9 the k value was 0.0269 mm and the range of uncertainty from 0.0 mm to 0.060 mm.

The differences between published k values and those from the present method, and the wide range of uncertainty in the k values, suggests that a more critical view should be taken of published k values.

The Case 3 analyses are based on Equation (6), which allows for both friction loss and fitting loss whereas the Case 1 analyses are based on Equation (3), which allows for friction loss only. Comparing the residual errors $\sum R_i^2$ shows which model is the better fit to the pipe test data. An obvious reduction in $\sum R_i^2$ occurs for tests 1, 2 and 3, showing that the Case 3 method gives a better fit. Slight reductions in $\sum R_i^2$ occur for tests 8 and 9 but these could be truncation errors so no firm conclusion can be drawn. Introduction of fitting loss had no effect on tests 4 and 5.

The Case 4 and Case 5 analyses provide the best test of the mathematical model, in particular in predicting the pipe diameter. A Case 4 analysis is based on Equation (3), for friction loss only, and estimates d and k for a set of pipe test observations Q_o, H_o and v_o. For the present analysis d is known for all of the prismatic pipes tested. The Case 5 results gave d values within 5% of the known values. The residual errors $\sum R_i^2$ are smaller than those from Case 1, indicating that the d and k values from Case 4 provide a better fit to the pipe test observations than the known d and the k given by the Case 1 analysis. Some improvement in the mathematical model appears to be possible, particularly in Equation (5), used to estimate the friction factor f. For an "unseen" pipe the values for d and k predicted by a Case 4 analysis are "effective d" and "effective k".

The Case 5 analyses are based on Equation (6), which includes friction and fitting losses, and estimates d, k and K_L. The estimated d values are within 6% of the known values, except for test 1 within 8%, and test 4, where d is overestimated by 32%. The effective d, k and K_L values found by a Case 5 analysis gave a lower residual error $\sum R_i^2$ than that found using a Case 3 analysis for most of the tests examined.

TABLE 2: RESULTS OF ANALYSIS OF PUBLISHED PIPE TESTS

Test No.	Source	Ref. No.	Pipe Details	Site	No. Data Sets	No. Head Taps	Published Values		Case 1		Case 3			Case 4			Case 5			
							d(mm)	k(mm)	k(mm)	$\sum R_i^2$	k(mm)	K_L	$\sum R_i^2$	d(mm)	k(mm)	$\sum R_i^2$	d(mm)	k(mm)	K_L	$\sum R_i^2$
1	Heywood, 1925	26	4 in. Galv. iron All 30 data sets	Lab.	30	2	103.48		0.131	0.02289	0.016	0.57	0.01150	98.21	0.031	0.01184	111.77	0.0	1.53	0.01103
2	Heywood, 1925	26	4 in. Galv. iron 14 sets known temp.	Lab.	14	2	103.48		0.131	0.02059	0.012	0.61	0.01072	97.62	0.025	0.01088	110.40	0.0	1.39	0.01054
3	Heywood, 1925	26	2 in. Galv. iron 16 sets known temp.	Lab.	16	2	51.15		0.262	0.003075	0.10	1.3	0.00227	49.30	0.1262	0.002267	49.30	0.1262	0.0	0.002267
4	Burke, 1955	9	Steel penstock, hot enamel coat	Field	21	2	1289.0		0.060	0.159372	0.060	0.0	0.159373	1304.54	0.0892	0.158247	1698.0	0.0	3.07	0.15389
5	Levin, 1972	30	Machined casting, cold bitumen spray, mean properties.	Lab.	13	20	209.5	0.042	0.055	1.120746	0.055	0.0	1.120755	221.68	0.229	0.73428	221.68	0.229	0.0	0.73428
6	Levin, 1972	30	As above, min. k properties	Lab.	13	20	209.08		0.038	1.58576										
7	Levin, 1972	30	As above, max. k properties	Lab.	13	20	209.92		0.077	0.901908										
8	Dudgeon, 1983	17	Steel, cement mortar lined	Lab.	18	2	287	0.01	0.008	0.00661	0.0068	0.011	0.006603	286.86	0.007	0.0066033	289.66	0.004	0.223	0.006602
9	Foster, 1968	23	Asbestos cement, mean properties	Field	9	4	363.22	0.046	0.0269	0.0001916	0.0248	0.046	0.0001914	362.79	0.0236	0.0001909	362.79	0.0236	0.0	0.0001909
10	Foster, 1968	23	As above, min. k properties	Field	9	4	362.48		0	0.00136										
11	Foster, 1968	23	As above, max. k properties	Field	9	4	363.96		0.060	0.00127										

ESTIMATION OF THE UNCERTAINTY IN THE EQUIVALENT SAND GRAIN ROUGHNESS k FROM CASE 1 ANALYSES

Levin [30] and Dudgeon [17] have combined the uncertainties of the other variables in Equation (3) to estimate the uncertainty in the friction factor f. This uncertainty in f only applies to the particular pipe tested whereas an uncertainty in k can be applied to any diameter pipe made of the same material.

Replacing $\log_{10}$ by $\log_e$ and using $\mathbf{R} = \dfrac{4Q}{\pi d \nu}$ in Equation (4) gives:

$$\frac{1.0}{-2C_1\sqrt{f}} = \log_e\left[\frac{k}{3.7d} + \frac{2.51\,\pi d \nu}{4Q\sqrt{f}}\right] \tag{12}$$

where $C_1 = \log_{10}(e)$.

Taking antilogs and re-arranging gives an expression for k:

$$k = 3.7d\left[e^{\frac{1.0}{-2C_1\sqrt{f}}} - \frac{2.51\,\pi d \nu}{4Q\sqrt{f}}\right] \tag{13}$$

Using the energy gradient $S = \dfrac{h_f}{L}$ and $V = \dfrac{4Q}{\pi d^2}$ in Equation (3) gives f for use in Equation (13):

$$f = \frac{g\pi^2 d^5 S}{8Q^2} \tag{14}$$

The problem is to incorporate the uncertainties in the independent variables, S, D, Q and ν to find the uncertainty in the dependent variable k defined by Equation (13). Two methods have been tried, direct combination and statistical combination.

Direct Combination of Uncertainties

If the mean value of S is S_{mean} then S may vary from $S_{low} = S_{mean} - e_s$ to $S_{high} = S_{mean} + e_s$, where e_s is the uncertainty in S to some confidence level, say 95% [28]. Similarly d may vary from d_{low} to d_{high}, and so on. Possible combinations of S_{low}, S_{high}, d_{low}, d_{high} etc have been tried for data from several of the synthetic pipe tests described previously (with estimated uncertainties to find S_{low}, S_{high} etc) and for data from Levin [30] (with published uncertainties). The combinations found to give the k extremes were:

For minimum k: S_{low}, d_{low}, Q_{high}, ν_{high}

For maximum k: S_{high}, d_{high}, Q_{low}, ν_{low}

When converted to equivalent f and $\mathbf{R}$ values the data sets for minimum and maximum k plot as lines "parallel" to the mean properties data set. This is shown for the test data from Levin [30] on Figure 5.

Using S_{low}, d_{low} etc, a Case 1 analysis is applied to find the estimate of the minimum k. Similarly for the maximum k estimate. This method has been used for test numbers 6, 7, 10, and 11 in Table 2.

Statistical Combination of Uncertainties

Using the Simplified Method of combination of uncertainties [28], [35], the uncertainty in k due to uncertainties in the independent variables is:

$$e_k = \left\{ \left[\theta_s e_s\right]^2 + \left[\theta_d e_d\right]^2 + \left[\theta_Q e_Q\right]^2 + \left[\theta_v e_v\right]^2 \right\}^{0.5} \tag{15}$$

where θ_s is the sensitivity coefficient for S, equal to $\frac{\partial k}{\partial S}$ from Equations (13) and (14), and so on for the other variables. Finding the partial derivatives is tedious, so the partial derivatives have been approximated by finite increments [28], that is $\theta_s \approx \frac{\Delta k}{\Delta S}$ where Δk is the change in k due to a small change ΔS in S.

This method has been applied to results from a careful series of laboratory pipe tests by Levin [30]. A 1% change in variables was used to calculate approximate sensitivity coefficients θ and values of uncertainties e were as published. Results for four sets of data are shown in Table 3 below:

TABLE 3. Results of Statistical Combination of Uncertainties

Test data from Levin [30] for laboratory tests on cold bitumen sprayed pipe

$\frac{Q}{Q_{max}}$	$(\theta_s e_s)^2$	$(\theta_d e_d)^2$	$(\theta_Q e_Q)^2$	$(\theta_v e_v)^2$	e_k (mm) from Eqn. (15)	$\theta_s e_s$ (mm)
1.0	3.11 E-10	1.09 E-11	1.45 E-11	4.26 E-16	1.834 E-2	1.76 E-2
0.7	2.59 E-10	1.17 E-11	1.76 E-11	1.024 E-15	1.698 E-2	1.61 E-2
0.392	1.87 E-10	1.21 E-11	2.75 E-11	3.06 E-15	1.504 E-2	1.37 E-2
0.073	1.39 E-10	6.6 E-11	1.88 E-10	6.94 E-13	1.98 E-2	1.17 E-2

The results in Table 3 indicate that the energy gradient term makes the greatest contribution to e_k, except at very low discharges where the discharge term is of equal or greater importance. The pipe diameter term is insignificant, except at very low discharges, and the viscosity term can be ignored.

For the upper two-thirds of discharges, at least, taking the S term only in Equation (15) gives an approximate $e_k = \theta_s e_s$, which is slightly less than e_k found by using all terms.

Application of the statistical combination method at the design stage of a pipe test program should guide selection of measurement techniques and indicate areas where repetition of observations can reduce the uncertainty in k estimates.

The statistical combination method can only be applied to one set of test data at a time so some means of combining the n estimates of the uncertainty e_k values from n sets of data is required. On the other hand, the direct combination method above yields a "best fit" estimate of minimum k from the Case 1 analysis.

Comparison of Uncertainties in k from the Direct Combination and the Statistical Combination Methods

The mean uncertainty in k from the direct combination method has been taken as:

$$e_k = 0.5 \left\{ \left[k_{max} - k_{min.} \right] \right\} \tag{16}$$

where k_{max} and k_{min} are the results of Case 1 analysis for data sets for minimum k and maximum k respectively.

The mean uncertainty for n data sets using the statistical combination method has been taken as the mean value $\frac{1}{n}\sum_{i=1}^{n} e_{k_i}$.

As expected [35], using the test data of Levin [30], the direct combination method gave a mean e_k about 10% greater than the mean e_k using statistical combination for four of the data sets.

The direct combination method is therefore slightly conservative in estimating the uncertainty in k but is recommended for its simplicity and ability to give "best fit" estimates of k_{min} and k_{max}.

ESTIMATION OF UNCERTAINTIES IN THE INDEPENDENT VARIABLES

S, d, Q and ν

Measurement Methods for Pipe Tests

A review of methods of measurement of pipe, flow and water properties is given in Appendix B. The uncertainties associated with measurement of these properties influence the uncertainty estimates for S, d, Q and ν.

Uncertainty in the Energy Gradient S

As shown in the section above, the uncertainty e_s in S probably has the predominant effect when finding the uncertainty e_k in k. Estimating S and its uncertainty e_s is usually the most difficult task in analysing pipe test results, particularly for large diameter smooth pipes, or smaller pipes at low discharges, where the head loss is small and S is very small.

For steady flow of an incompressible fluid in a prismatic pipe the energy line (and the parallel hydraulic grade line HGL) is theoretically linear, but even the most careful pipe tests produce a scatter of piezometric head values about a straight line. When tests are repeated at the same value of discharge the head observations at a particular cross-section show some random variation. Examples of piezometric head measurements for some laboratory and field tests are shown in Figures 6 and 7 respectively.

The deviations of the observed piezometric head measurements from a straight line of best fit are often too great to be explained by combining the uncertainties in the measurements used to determine the piezometric head h, particularly with laboratory tests, for example Figure 6(A). It is suggested that S, and its uncertainty e_s, be found by:

(i) Adjusting the observations of the component variables used to find h to correct for any systematic errors, e.g. by calibration of pressure gauges.

(ii) Then treating the HGL observations as a statistical problem of fitting a straight line where "x is known without error and y can vary". In the present case x is the distance l along the pipe and y is the piezometric head h. The uncertainty e_l in l is usually negligible compared to the uncertainty e_h in h.

A least squares linear regression provides a simple means of fitting a straight line to the n sets of (l, h) data [1], [10], [33], [35]. This method assumes that the h values are normally distributed about the regression line with constant variance.

The regression line is given by:

$$h = a + bl \tag{17}$$

$$\text{where } b = \frac{\sum_1^n (l-\bar{l})\,(h-\bar{h})}{\sum_1^n (l-\bar{l})^2} \tag{18}$$

$$\text{and } a = \bar{h} - b\bar{l} \tag{19}$$

In Equations (18) and (19) $\bar{h}$ and $\bar{l}$ are mean values, given by $\bar{h} = \frac{1}{n}\sum_1^n h$ and $\bar{l} = \frac{1}{n}\sum_1^n l$, respectively. The energy gradient S is then equal to $-b$ and the line of best fit passes through $(\bar{l}, \bar{h})$. S values found using this method are shown on some of the HGL lines in Figures 6 and

7, together with the published value S_*.

Assuming the estimates of b from repeated measurements follow Student's t distribution for a small number of observations n [10] [33], the uncertainty in b, to some prescribed confidence level, gives the uncertainty e_s in the energy gradient:

$$e_s = t_p \left\{ \frac{\sum_1^n (h-h_o)^2}{\sum_1^n (l-\bar{l})^2 \, (n-2)} \right\}^{0.5} \tag{20}$$

where t_p is a coefficient from Student's t distribution, n is the number of sets of (l,h) observations, and h_o is the h value computed from the line of best fit for a given l value.

The coefficients t_p may be obtained from tables [35] or from a graph [10]. t_p values for confidence levels appropriate for pipe test analysis are shown graphically on Figure 8. A confidence level of 95% indicates that S is likely to be in the range $(S-e_s)$ to $(S+e_s)$ for 95% of the time.

In Figure 8 the abscissa is the number of degrees of freedom, equal to $(n-2)$. At least three head tapping points are needed if an estimate of e_s is required. Figure 8 (B) (the linear plot) provides guidance in selecting the minimum number of head tapping points required to keep t_p (and hence e_s) "reasonably low". For example, for a 95% confidence level six degrees of freedom, requiring eight tapping points, appears to be desirable.

Equation (20) shows that e_s is reduced by increasing the number n of head tapping points, which increases the value of the $(n-2)$ term in denominator.

Equation (20) also indicates that e_s is reduced by increasing the spacing of the head tapping points. This effect has been investigated by applying Equation (20) to synthetic pipe test data. For given d, k, ν and tapping spacing Δl, n sets of (l, h_o) points on the HGL are calculated. The h_o values are then perturbed by multiplying a perturbation Δh_{max} by a random number between -1.0 and $+1.0$. Results for two sets of data are shown in Figure 9. Increasing the tapping spacing Δl makes the energy gradient S approach the known value for the unperturbed HGL and decreases the uncertainty e_s.

Equation (20) suggests a possible measurement strategy might be to reduce e_s by increasing the $(l-\bar{l})$ values, that is to concentrate the head tapping points at the upstream and at the downstream ends of the test section. There are two objections to this strategy:

(i) There would be no check on the assumed linearity of the HGL over the intermediate section of pipe. An obstruction causing localised minor loss could go undetected [23].

(ii) A statistical test would be required to show that the observations at each end were samples from the same population.

Investigation of any non-linearity of the HGL, and a check on the assumption of constant variance of the h values, would require repetition of the h observations for the same discharge, preferably taking observations over several cycles with Q increasing and Q decreasing [1]. This is usually impractical for field tests, which have to fit in with operating schedules, but could be performed for laboratory tests with little additional cost.

Uncertainty in Pipe Diameter d

Measurement of d volumetrically by filling a length of pipe with water should give $\frac{e_d}{d} < 0.5\%$, e.g. Levin [30] obtained $\frac{e_d}{d} = 0.2\%$ for a 200 mm diameter pipe.

Measurements by precision mechanical instruments such as micrometers should give $\frac{e_d}{d} < 0.5\%$. Sufficient measurements should be taken to allow an elementary statistical analysis (see Appendix B).

When a d value is taken from manufacturers' catalogues the allowable tolerance in manufacture should be recognised. The allowable tolerance $\frac{\Delta d}{d}$ may range from 4.5% for small (75 mm diameter) pipes to about 1.0% for larger (600 mm diameter) pipes [20].

Uncertainty in Discharge Q

Orifice plates, venturis and flow nozzles manufactured, installed and operated according to standard codes should have $\frac{e_Q}{Q} \leq 1.25\%$ [22], [27], [28]. A pitot tube traverse, with appropriate corrections, should give $\frac{e_Q}{Q} \leq 1\%$ [7]. Electromagnetic flow meters are claimed to give $\frac{e_Q}{Q} \leq 1\%$ [30]. Laboratory volumetric tanks give $\frac{e_Q}{Q}$ from 0.25% [38] to 0.5% [17], while volumetric gauging in natural reservoirs gives $\frac{e_Q}{Q} \leq 2\%$ [9]. Q measurement by stream gauging gives $\frac{e_Q}{Q}$ about 6% [28].

Uncertainty in Water Properties

Providing the water temperature can be found within ±0.5°C, the uncertainty in density due to temperature variation can be neglected. The relative uncertainty in density $\frac{e_\rho}{\rho}$ is about 0.15%, due to systematic uncertainty in published ρ values [22].

Dynamic viscosity μ is much more sensitive to temperature change than density. A change in temperature from 20 to 21°C will decrease μ by about 2.5%. The uncertainty e_μ for a temperature uncertainty $e_t = \Delta t$ may be taken as the mean of the μ deviation:

$$e_\mu = 0.5 \left| \mu_{t-\Delta t} - \mu_{t+\Delta t} \right| \tag{21}$$

The percentage uncertainty $\frac{e_\nu}{\nu}$ in the kinematic viscosity $\nu = \frac{\mu}{\rho}$ may be taken as equal to $\frac{e_\mu}{\mu}$ as $\frac{e_\rho}{\rho} < 5 \frac{e_\mu}{\mu}$, the criterion for ignoring the smaller value [28].

CONCLUSIONS

1. It has been shown that least squares minimisation in terms of head loss residuals is a viable method for estimating the equivalent sand grain roughness k from a pipe test. The proposed method has a statistical basis and is preferable to the common method of drawing a line of best fit through plotted points on a Moody chart.

2. The uncertainty in the k value may be derived by direct combination of the estimated uncertainties in the pertinent variables. The uncertainty in the energy gradient S appears to be the major source of uncertainty in k. The uncertainty in S can be reduced by increasing the number of head tapping points and by increasing the spacing between the points.

3. Use of a total head loss relation (friction loss plus fitting loss) gave a better fit than friction loss only for some of the published pipe tests. This suggests that for some pipelines a K_L value, as well as a k value, should be specified.

4. The proposed least squares minimisation techniques have been shown to permit adequate estimation of "effective" diameter, k and K_L design values when various combinations of these values are unknown.

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APPENDIX A: MINIMISATION OF A ONE-VARIABLE FUNCTION

Referring to Figure 10(A), the value of x_* is sought to define the single local minimum $f(x_*)$ known to occur in the range x_1 to x_2. The function f(x) may be "round bottomed" or "sharp bottomed" as shown.

(i) **Minimum by Bisection of First Derivative Function** [25]

If the first derivative function $f^1(x)$ can be found then it crosses the x axis at x_*, as shown in Figure 10(B). The method of bisection (or interval halving) [10] may be used to obtain the root x_* of $f^1(x)$. The interval between x_1 and x_2 is bisected at x_3 and the algebraic signs of $f^1(x_1)$ and $f^1(x_3)$, and $f^1(x_2)$ and $f^1(x_3)$, compared. The curve crosses the x axis when the signs are opposite, i.e. between x_1 and x_3 in Figure 10(B). The interval between x_1 and x_3 is then bisected and the procedure repeated until the required interval of uncertainty Δx is reached. Each iteration (after the first) requires only one evaluation of $f^1(x)$ and the approximate number of iterations N required is:

$$N = \frac{\log_e \left[(x_1 - x_2)/\Delta x \right]}{\log_e 2} \tag{A1}$$

(ii) **Minimum by Golden Section Search** [13], [25]

This method is similar to the method of bisection in that the x interval containing the minimum is reduced by iteration. Each iteration reduces the interval from δx to $r\delta x$, where $r=0.5(\sqrt{5}-1)$, the golden section ratio, which satisfies the equation $r^2 = 1-r$. Referring to Figure 11(B), if x_* is known to be between x_1 and x_2, then for the first iteration f(x) is evaluated at x_3 and x_4. For the second and subsequent iterations only one determination of f(x) is required. The number of iterations N required to contain x_* within an interval of uncertainty Δx is:

$$N = \frac{-\log_e \left[(x_2 - x_1)/\Delta x \right]}{\log_e r} \tag{A2}$$

APPENDIX B : REVIEW OF MEASUREMENT METHODS FOR PIPE TESTS

Measurement of Pipe Diameter d

The diameter of pipes small enough to be easily handled may be measured volumetrically by filling a length of pipe with water [26], [30], [37]. With very small pipes care should be taken to remove air bubbles adhering to the walls by wiping [37]. For a small rough pipe the diameter obtained by water filling may be less than that measured by sharp ended calipers or greater than that measured with flat ended calipers [37].

For small pipes sample rings may be cut from the pipe for measurement by calipers [37]. For large pipes permitting access, a beam micrometer may be used [9].

In all cases sufficient measurements of d should be taken to permit calculation of the standard deviation and thence an estimate of the uncertainty in diameter e_d to the required confidence level [28].

Measurement of Pipe Length L

Distance between pressure tappings may be determined accurately by steel tape for laboratory tests or by survey traverse for field tests [23]. Uncertainty in pipe length is usually insignificant compared to uncertainty in the other variables, except for buried pipes where construction records have been lost.

Measurement of Pipe Elevation

The elevation z of tapping points is required when the pressure head $\frac{p}{\gamma}$ is being measured to locate the HGL (see Figure 1). For laboratory tests care is taken to lay the pipe straight and level using precise survey levelling, except where cases of joint displacement or deflection are being investigated [38].

For field tests survey levelling from adjacent bench marks is used [23]. Taking pressure readings at zero discharge to deduct the pressure head $\frac{p}{\gamma}$ from a known static HGL has also been used [12], [23]. For very large pipes the surges produced by shutting a downstream control valve to give a static HGL may persist for several hours [12].

When the HGL is detected by piezometer tubes, or differential manometer, the indicated head loss is independent of the pipe slope so no elevation data is required.

Measurement of Pressure

The most common way of accessing the flow to measure pressure at a cross-section is to drill one or more holes through the wall of the pipe. The centre-lines of these wall tapping holes are normal to the pipe centre-line. For field tests on manufactured pipes a tapping band is required so usually only one hole per cross-section is used [9], [16], [23], [40]. For large tunnels and penstocks pressure tappings may be installed during construction specifically for pipe tests after commissioning [18], [42]. Vertical shafts used during construction have been used subsequently as piezometers for head loss tests [8], [34]. For laboratory tests one [24], [26], [29], [41], two [30], or four [15], [17] wall tapping holes per cross-section have been used.

When two or more tapping holes are used at a cross-section they are usually connected by a ring manifold to average the pressure, with a single offtake to the pressure measuring device [5], [6], [17]. To avoid air accumulation at an obvert tapping, or blocking by debris of an invert tapping, tapping holes should preferably be located in sectors of the cross-section between $+\frac{\pi}{4}$ and $-\frac{\pi}{4}$ rad. to the horizontal [5]. The angle between adjacent tapping holes should be constant.

Wall tappings overestimate the static pressure slightly, and the error increases as the diameter of the tapping hole increases [19]. The detailed specifications for wall tappings for constriction meters should also be applicable to pipe tests [27].

For manufactured pipes connected by flanged, socketed or sleeved joints the wall tappings are usually made a short distance upstream of the joint [17], [23], presumably to minimise flow interference from the joint upstream.

The number of tapping cross-sections which have been used to define the HGL for friction tests varies from two to twenty. Some laboratory tests have used a sufficient number to make an adequate statistical estimate of the energy gradient and its uncertainty [30], [38]. Many investigators, however, while careful in measuring pressure, discharge, etc, have used only two or three taps [15], [17], [26], [29], [36]. For field tests two or three tapping cross-sections are common, occasionally more are used [23], [34], [40].

Alternative means of accessing the flow for pressure measurement include annular slots [37] and static pressure probes [36], [38]. Static pressure probes may have an advantage over wall tappings for very rough pipes, where presence of a roughness element close to the hole may affect the pressure, or for brittle pipes where fracture of the wall on drill breakthrough produces a conical hole.

Devices used for measuring pressure include simple piezometric tubes [8], [17], [18], [30], [34], manometers and differential manometers [9], [18], [26], [29], [37], [38], Bourdon pressure gauges [16], [23], dead weight testers [6], [16], and pressure transducers [6], [41]. Distortion of pressure signals in plastic connecting tubes has been investigated by Carolus et al. [11].

Measurement of Discharge

Laboratory tests have used volumetric tanks [17], [26], [38], constriction meters [15], pitot traverse [36], and electromagnetic flow meters [30]. Field tests have used orifice plates [23], venturi meters [9], [34], [41], volumetric tanks [16], stream gauging [8], salt-velocity and colour-prism methods [9], and correlation of discharge with turbine power output [6], [18].

When standard designs for constriction meters are used the calibration coefficients are given by the relative standard code.

Measurement of Fluid Properties

The fluid properties required for water flow are density ρ and dynamic viscosity μ, both functions of water temperature. Unit weight $\gamma = \rho g$ is required for finding pressure head $\frac{p}{\gamma}$. The kinematic viscosity $\nu = \frac{\mu}{\rho}$ is required for use in friction factor equations.

Temperature read with a cheap mercury-in-glass thermometer has an uncertainty e_t of about 0.5°C. Temperature read by an industrial quality electronic thermometer has an e_t of about 0.1°C [22].

APPENDIX C: COMPUTER PROGRAM

```
c         PROGRAM PIPE.TEST
c         FIND DIAMETER, ROUGHNESS K & MINOR LOSS COEFFICIENT FROM PIPE TESTS
c         BY LOCATING MINIMUM LOG10 (SUM SQUARES RESIDUAL HEAD)
c                 WITH GOLDEN SECTION SEARCH
c
c         AUTHORS
c           T.R. FIETZ AND K.B. HIGGS
c           UNIVERSITY OF NEW SOUTH WALES, WATER RESEARCH LABORATORY
c           AUGUST, 1989
c
c         VARIABLES
c
c         *=COMMON VARIABLE
c          *ANU(NP)=KINEMATIC VISCOSITY (M^2/S) (FROM PIPE TESTS)
c           ICASE=CASE NUMBER
c                 =1,casen(1)='GIVEN DIA., FIND SAND GRAIN ROUGHNESS K'
c                 =2,casen(2)='GIVEN DIA.& K, FIND MINOR LOSS COEFF'
c                 =3,casen(3)='GIVEN DIA., FIND K & MINOR LOSS COEFF'
c                 =4,casen(4)='FIND DIA. & K'
c                 =5,casen(5)='FIND DIA., K & MINOR LOSS COEFF'
c           CASEN(5)=CASE DESCRIPTIONS
c           D=PIPE DIA. (MM FOR I/O, M FOR CALCNS.)
c          *DC=CURRENT DIA.(M)
c           DG=DIA. AT GLOBAL MIN. (MM FOR I/O, M FOR CALCNS.)
c          *DHIG=UPPER LIMIT OF DIA. FOR SEARCH (MM FOR I/O, M FOR CALCNS.)
c          *DINC=DIA. INCREMENT CONTAINING GLOBAL MIN. (MM FOR I/O, M FOR CALCNS.)
c          *DLOW=LOWER LIMIT OF DIA. FOR SEARCH (MM FOR I/O, M FOR CALCNS.)
c           FC=CURRENT DARCY F
c           FNAME=SEQUENTIAL FILENAME FOR TEST DATA
c              FILE INCLUDES JOB,L,NP,Q(NP),H(NP),ANU(NP)
c          *G=GRAVITY
c          *H(NP)=MEASURED HEAD LOSS (FRICTION + MINOR LOSS)(M) (FROM PIPE TESTS)
c          *HN(NP)=ESTIMATED HEAD LOSS(M) (FROM GLOBAL SEARCH)
c           HC=CURRENT CALCULATED HEAD LOSS(M)
c          *JOB=PROJECT NAME
c           K=SAND GRAIN ROUGHNESS (MM FOR I/O, M FOR CALCNS.)
c          *KC=CURRENT K
c           KG=K AT GLOBAL MIN. (MM FOR I/O, M FOR CALCNS.)
c          *KHIG=UPPER LIMIT OF K FOR SEARCH (MM FOR I/O, M FOR CALCNS.)
c          *KINC=K INCREMENT CONTAINING GLOBAL MIN. (MM FOR I/O, M FOR CALCNS.)
c          *KLOW=LOWER LIMIT OF K FOR SEARCH (MM FOR I/O, M FOR CALCNS.)
c          *LEN=PIPE LENGTH(M)
c          *MC=CURRENT MLC
c           MG=MLC AT GLOBAL MIN.
c          *MHIG=UPPER LIMIT OF MLC FOR SEARCH
c          *MINC=MLC INCREMENT CONTAINING LOCAL MIN.
c          *MLOW=LOWER LIMIT OF MLC FOR SEARCH
c          *NP=NO. OBSERVATIONS FROM PIPE TESTS
c          *NU=CURRENT KINEMATIC VISCOSITY (M^2/S)
c          *PI=pi
c          *Q(NP)=FLOWRATE(M^3/S) (FROM PIPE TESTS)
c           QC=CURRENT Q(M^3/S)
c           SUMSQS=LOG10 SUM SQUARES AT GLOBAL MIN.
c
c         SUBROUTINE CALLS
c
c         CASE 1 - GIVEN DIA., FIND SAND GRAIN ROUGHNESS K
c                 KIP
c                 AGOLD <- KSUMEH <- HLOSS <- YAOF
c                 HLOSS <- YAOF
c         CASE 2 - GIVEN DIA. & K, FIND MINOR LOSS COEFF
c                 MIP
c                 AGOLD <- MSUMEH <- HLOSS <- YAOF
c                 HLOSS <- YAOF
c         CASE 3 - GIVEN DIA., FIND K & MINOR LOSS COEFF
c                 KIP
c                 MIP
c                 AGOLD <- MMID <- CGOLD <- MSUMEH <- HLOSS <- YAOF
c                 HLOSS <- YAOF
c         CASE 4 - FIND DIA. & K
```

Appendix C: Computer Program *(Contd)*

```
c               DIP
c               KIP
c               AGOLD <- DMID <- CGOLD <- DSUMEH <- HLOSS <- YAOF
c               HLOSS <- YAOF
c       CASE 5 - FIND DIA., K & MINOR LOSS COEFF
c               DIP
c               KIP
c               MIP
c               AGOLD <- BIG <- BGOLD <- DMID <- CGOLD <- DSUMEH <- HLOSS <- YAOF
c               HLOSS <- YAOF
c
        implicit real*8 (a-h,o-z)
        common /set1/ anu(40),q(40),h(40),hn(40),np
        common /set2/ g,len,dc,kc,mc,nu,pi
        real*8 len,k,kc,mc,nu
        dimension casen(5)
        character job*200,casen*40
        common /set3/ dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
        real*8 dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
        real*8 kg,mg
        character fname*15
        external ksumeh,msumeh,mmid,dmid,big
        g = 9.8d0
        pi = 3.141593d0
        casen(1)='GIVEN DIA., FIND SAND GRAIN ROUGHNESS K '
        casen(2)='GIVEN DIA. & K, FIND MINOR LOSS COEFF   '
        casen(3)='GIVEN DIA., FIND K & MINOR LOSS COEFF   '
        casen(4)='FIND DIA. & K                           '
        casen(5)='FIND DIA., K & MINOR LOSS COEFF         '
        print *,'PIPE FLOW ANALYSIS PROGRAM'
c       ****************** GET TEST DATA FROM DISK ********************
c       if getarg is not available, delete next line and remove c from next read
        call getarg(1,fname)
        print *
        print *, 'INPUT FILE NAME FOR TEST DATA:- ',fname
c       read 1,fname
        open (1,file=fname)
        rewind 1
        read(1,1) job
1       format(a)
        read(1,*) len
        read(1,*) np
        if(np.gt.40) then
          print *,'WARNING - Program Dimensions Exceeded'
          print *,'Number of Data Values=',np,', Dimensions Set to 40'
          stop
        endif
        read(1,*) ( q(i),h(i),anu(i), i=1,np )
        close(1)
c       ****************** INPUT CASE DETAILS **************************
        print *
        print *,job
        print *
100     print *,' CASE         CASE'
        print *,'NUMBER       DETAILS'
        write(6,2) (i,casen(i),i=1,5)
2       format(2x,i3,4x,a40)
        print *
        print *, 'INPUT CASE NUMBER '
        read *,icase
c       ****************** INPUT PIPE DETAILS **************************
        if(icase.lt.1.or.icase.gt.5) then
          print *,'Try again'
          goto 100
        endif
        if(icase.eq.1) then
          print *, 'INPUT PIPE DIA. IN MM. '
          read *,D
          D = D / 1000d0
          call kip (klow,khig,kinc)
```

Appendix C: Computer Program *(Contd)*

```
        elseif(icase.eq.2) then
          print *, 'INPUT PIPE DIA. IN MM. '
          read *,D
          D = D / 1000d0
          print *, 'INPUT SAND GRAIN ROUGHNESS, IN MM. '
          read *,K
          K = K / 1000d0
          call mip (mlow,mhig,minc)
        elseif(icase.eq.3) then
          print *, 'INPUT PIPE DIA. IN MM. '
          read *,D
          D = D / 1000d0
          call kip (klow,khig,kinc)
          call mip (mlow,mhig,minc)
        elseif(icase.eq.4) then
          call dip (dlow,dhig,dinc)
          call kip (klow,khig,kinc)
        else
          call dip (dlow,dhig,dinc)
          call kip (klow,khig,kinc)
          call mip (mlow,mhig,minc)
        endif
c       ********************* FIND GLOBAL MINIMUM ********************
        if(icase.eq.1) then
          dc = d
          mc = 0d0
          call agold(klow,khig,kinc,ksumeh, kg,sumsqs)
          dg = d
          mg = 0d0
        elseif(icase.eq.2) then
          dc = d
          kc = k
          call agold(mlow,mhig,minc,msumeh, mg,sumsqs)
          dg = d
          kg = k
        elseif(icase.eq.3) then
          dc = d
          call agold(klow,khig,kinc,mmid, kg,sumsqs)
          dg = d
          mg = mc
        elseif(icase.eq.4) then
          mc = 0d0
          call agold(klow,khig,kinc,dmid, kg,sumsqs)
          dg = dc
          mg = 0d0
c       CASE 5
        else
          call agold(mlow,mhig,minc,big, mg,sumsqs)
          kg = kc
          dg = dc
        endif
c       *********** FIND % ERROR IN ESTIMATED HYD. GRAD. ***************
        do 200 i = 1,np
          qc = q(i)
          nu = anu(i)
          call hloss(qc,dg,kg,mg, hc)
          hn(i) = hc
200     continue
c       ********************* PRINT RESULTS *************************
        print *
        print *
        print *,'               PROGRAM HED1'
        print *,'               ******* ****'
        print *
        print *,'JOB:-',job
        print *
        print *,'CASE TYPE:- ',icase,'  ',casen(icase)
        print *

        print *
```

Appendix C: Computer Program *(Contd)*

```
      print *,'              PIPE TEST RESULTS'
      print *,'              **** **** *******'
      if(icase.lt.4) write(6,3) d*1000d0
3     format('INPUT PIPE DIAMETER             =',f8.2,' mm')
      if(icase.eq.2) write(6,4) k*1000d0
4     format('INPUT SAND GRAIN ROUGHNESS      =',f8.4,' mm')
      print *
      print *,'    Q        HEAD      NU*1E6'
      print *,'  (L/S)       (M)     (M^2/S)'
      write(6,5) ( q(i)*1000d0, h(i), anu(i)*1e6 ,i=1,np)
5     format(f9.2,2x,f8.4,2x,f8.2)
      print *

      print *
      print *,'              GLOBAL SEARCH DETAILS'
      print *,'              ****** ****** *******'
      if(icase.eq.4.or.icase.eq.5)
     + write(6,6) dlow*1000d0, dhig*1000d0, dinc*1000d0
6     format('DIA. FROM                ',f9.1,' MM, TO ',f9.1,
     + ' MM, IN INTERVAL  ',f9.6,' MM')
      if(icase.ne.2) write(6,7) klow*1000d0, khig*1000d0, kinc*1000d0
7     format('K  FROM                  ',f9.3,' MM, TO ',f9.3,
     + ' MM, IN INTERVAL  ',f9.6,' MM')
      if(icase.eq.2.or.icase.eq.3.or.icase.eq.5)
     + write(6,8) mlow, mhig, minc
8     format('MINOR LOSS COEFF. FROM',f9.3,'     TO ',f9.3,
     + '    , IN INTERVAL  ',f9.6)
      print *

      print *
      print *,'              GLOBAL MINIMUM'
      print *,'              ****** *******'
      if(icase.eq.4.or.icase.eq.5) write(6,9) dg*1000d0
9     format('ESTIMATED PIPE DIAMETER       =',f10.2,' mm')
      if(icase.ne.2) write(6,10) kg*1000d0
10    format('ESTIMATED SAND GRAIN ROUGHNESS=',f10.4,' mm')
      if(icase.eq.2.or.icase.eq.3.or.icase.eq.5) write(6,11) mg
11    format('ESTIMATED MINOR LOSS COEFF.   =',f10.3)
      write(6,12) 10.**sumsqs
12    format('SUM SQUARES RESIDUAL H        =',F14.10)
      print *

      print *
      print *,'              ESTIMATED HEAD LOSS'
      print *,'              ********* *********'
      print *
      print *,'   Q(L/S)  HEAD(M)      HEAD(M)     PERCENT.'
      print *,'   FROM     FROM      FROM GLOBAL   ERROR IN'
      print *,'   TESTS    TESTS       SEARCH      HYD.GRAD.'
      do 300 i = 1,np
        pe = (hn(i) - h(i)) / h(i) * 100d0
        write(6,13) q(i)*1000, h(i), hn(i), pe
13      format(f9.2,2x,f9.4,3x,f9.4,3x,f8.3)
300   continue
      print *

      print *
      print *,'              END OF PROGRAM'
      end

c     ****************************************************************
c     SUBROUTINE DIP, INPUT DIA. RANGE & INCREMENT FOR SEARCH
c     ****************************************************************
      subroutine dip (dlow,dhig,dinc)
      real*8 dlow,dhig,dinc
      print *
      print *, 'INPUT LOWER LIMIT OF DIA. FOR SEARCH, IN MM. '
      read *,dlow
      dlow = dlow / 1000d0
      print *, 'INPUT UPPER LIMIT OF DIA. FOR SEARCH, IN MM. '
```

Appendix C: Computer Program *(Contd)*

```
      read *,dhig
      dhig = dhig / 1000d0
      print *,'INPUT DIA. INCREMENT TO CONTAIN GLOBAL MIN., IN MM.,
     + SUGGEST 1E-5'
      read *,dinc
      dinc = dinc / 1000d0
      end

c     ***********************************************************
c     SUBROUTINE KIP, INPUT K RANGE & INCREMENT FOR SEARCH
c     ***********************************************************
      subroutine kip (klow,khig,kinc)
      real*8 klow,khig,kinc
      print *
      print *, 'INPUT LOWER LIMIT OF K VALUE FOR GLOBAL SEARCH, IN MM.'
      read *,klow
      klow = klow / 1000d0
      print *, 'INPUT UPPER LIMIT OF K VALUE FOR GLOBAL SEARCH, IN MM.'
      read *,khig
      khig = khig / 1000d0
      print *, 'INPUT K VALUE INCREMENT TO CONTAIN GLOBAL MINIMUM, IN MM.,
     + SUGGEST 1E-6'
      read *,kinc
      kinc = kinc / 1000d0
      end

c     ***********************************************************
c     SUBROUTINE MIP, INPUT MLC RANGE & INCREMENT FOR SEARCH
c     ***********************************************************
      subroutine mip (mlow,mhig,minc)
      real*8 mlow,mhig,minc
      print *
      print *, 'INPUT LOWER LIMIT OF MINOR LOSS COEFF. FOR SEARCH '
      read *,mlow
      print *, 'INPUT UPPER LIMIT OF MINOR LOSS COEFF. FOR SEARCH '
      read *,mhig
      print *, 'INPUT MINOR LOSS COEFF. INCREMENT TO CONTAIN GLOBAL MIN.,
     + SUGGEST 1E-6'
      read *,minc
      end

c     ***********************************************************
c     SUBROUTINE KSUMEH, FIND LOG10(SUM SQUARES RESIDUAL H)
c     K INPUT, DIA. & MLC HELD CONSTANT
c     ***********************************************************
      subroutine ksumeh (k9, sum)
c      K9=SAND GRAIN ROUGHNESS(M), INPUT
c      SUM=LOG10(SUM SQUARES), RETURNED
c      DC,MC,NP,NU,Q(*),H(*),ANU(*) COMMON
      implicit real*8 (a-h,o-z)
      common /set1/ anu(40),q(40),h(40),hn(40),np
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,len,kc,mc,nu,m9
      sum = 0d0
      do 10 i = 1,np
        qc = q(i)
        hm = h(i)
        nu = anu(i)
        d9 = dc
        m9 = mc
        call hloss(qc,d9,k9,m9, hc)
        sum = sum + (hm - hc) ** 2
10    continue
      sum = log10 (sum)
      end

c     ***********************************************************
c     SUBROUTINE MSUMEH, FIND LOG10(SUM SQUARES RESIDUAL H)
c     MLC INPUT, DIA. & K HELD CONSTANT
c     ***********************************************************
c
```

Appendix C: Computer Program *(Contd)*

```
      subroutine msumeh (m9, sum)
c      M9=MINOR LOSS COEFF., INPUT
c      SUM=LOG10(SUM SQUARES), RETURNED
c      DC,KC,NP,NU,Q(*),H(*),ANU(*) COMMON
      implicit real*8 (a-h,o-z)
      common /set1/ anu(40),q(40),h(40),hn(40),np
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,len,kc,mc,nu,m9
      sum = 0d0
      do 10 i = 1,np
        qc = q(i)
        hm = h(i)
        nu = anu(i)
        k9 = kc
        d9 = dc
        call hloss(qc,d9,k9,m9, hc)
        sum = sum + (hm - hc) ** 2
10    continue
      sum = log10 (sum)
      end

c     ****************************************************************
c     SUBROUTINE DSUMEH, FIND LOG10(SUM SQUARES RESIDUAL H)
c     DIA. INPUT, K & MLC HELD CONSTANT
c     ****************************************************************
      subroutine dsumeh (d9, sum)
c      D9=DIA.(M), INPUT
c      SUM=LOG10(SUM SQUARES), RETURNED
c      HM=MEASURED HEAD LOSS (FROM PIPE TESTS)
c      HC=CALCULATED HEAD LOSS (FROM COLEBROOK WHITE)
c      KC,MC,NP,NU,Q(*),H(*),ANU(*) COMMON
      implicit real*8 (a-h,o-z)
      common /set1/ anu(40),q(40),h(40),hn(40),np
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,len,kc,mc,nu,m9
      sum = 0d0
      do 10 i = 1,np
        qc = q(i)
        hm = h(i)
        nu = anu(i)
        k9 = kc
        m9 = mc
        call hloss(qc,d9,k9,m9, hc)
        sum = sum + (hm - hc) ** 2
10    continue
      sum = log10 (sum)
      end

c     ****************************************************************
c     SUBROUTINE HLOSS, FIND HEAD LOSS FOR GIVEN Q, DIA., K, NU & MLC
c     ****************************************************************
      subroutine hloss (qc,d9,k9,m9, hc)
c      QC=FLOWRATE(M^3/S), INPUT
c      D9=DIA.(M), INPUT
c      K9=SAND GRAIN ROUGHNESS(M), INPUT
c      M9=MINOR LOSS COEFF., INPUT
c      HC=CALCULATED HEAD LOSS(M), RETURNED
c      VC=VELOCITY(M/S)
c      REYN=REYNOLDS NO.
c      FC=CURRENT DARCY F
c      G, PI, NU, LEN COMMON
      implicit real*8 (a-h,o-z)
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,m9,len,kc,mc,nu,kd
      kd = k9 / d9
      reyn = 4d0 * qc / (pi * d9 * nu)
      if(reyn.lt.2100) then
        fc = 64d0 / reyn
      else
        call yaof(kd,reyn, fc)
```

Appendix C: Computer Program *(Contd)*

```
      endif
      vc = 4d0 * qc / (pi * d9 * d9)
      hc = (fc * len / d9 + m9) * vc * vc / (2d0 * g)
      end

c     ***************************************************************
c     SUBROUTINE DMID, FIND LOG10(SUM SQUARES RESIDUAL H)
c     K INPUT, MLC HELD CONSTANT
c     ***************************************************************
      subroutine dmid (k9, f9)
c     FIND LOCAL MIN.DIA FOR K9 INPUT, RETURNS LOG10(SUM SQUARES RESIDUAL H)
c      K9=SAND GRAIN ROUGHNESS(M), INPUT
c      F9=LOG10(SUM SQUARES), RETURNED
c      DC,KC,MC,DLOW,DHIG,DINC COMMON
      implicit real*8 (a-h,o-z)
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,len,kc,mc,nu
      common /set3/ dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      real*8 dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      external dsumeh
      kc = k9
      call cgold(dlow,dhig,dinc,dsumeh, dc,f9)
      end

c     ***************************************************************
c     SUBROUTINE BIG, FIND LOG10(SUM SQUARES RESIDUAL H), MLC INPUT
c     ***************************************************************
      subroutine big (m9, g9)
c     FIND LOCAL MIN. K, THEN LOCAL MIN. DIA. FOR M9 INPUT
c      M9=MINOR LOSS COEFF., INPUT
c      G9=LOG10(SUM SQUARES RESIDUAL H), RETURNED
c      DC,KC,MC,KLOW,KHIG,KINC COMMON
      implicit real*8 (a-h,o-z)
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 m9,len,kc,mc,nu
      common /set3/ dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      real*8 dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      external dmid
      mc = m9
      call bgold(klow,khig,kinc,dmid, kc,g9)
      end

c     ***************************************************************
c     SUBROUTINE MMID, FIND LOG10(SUM SQUARES RESIDUAL H)
c     K INPUT, DIA. HELD CONSTANT
c     ***************************************************************
      subroutine mmid (k9, f9)
c     FIND LOCAL MIN. MLC FOR K9 INPUT
c      K9=SAND GRAIN ROUGHNESS(M), INPUT
c      F9=LOG10(SUM SQUARES RESIDUAL H), RETURNED
c      DC,KC,MC,MLOW,MHIG,MINC COMMON
      implicit real*8 (a-h,o-z)
      common /set2/ g,len,dc,kc,mc,nu,pi
      real*8 k9,len,kc,mc,nu
      common /set3/ dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      real*8 dlow,dhig,dinc,klow,khig,kinc,mlow,mhig,minc
      external msumeh
      kc = k9
      call cgold(mlow,mhig,minc,msumeh, mc,f9)
      end

c     ***************************************************************
c     SUBROUTINE YAOF, FIND DARCY F, TURBULENT PIPE FLOW
c     ***************************************************************
      subroutine yaof (kd,reyn, fc)
c     SOLUTION OF COLEBROOK-WHITE EQUATION BY YAO'S METHOD
c     REF. APPLE MANNZONE1, 5.10.85
c       KD=PIPE RELATIVE ROUGHNESS, INPUT
c       REYN=REYNOLDS NO., INPUT
c       FC=DARCY F, RETURNED
```

Appendix C: Computer Program *(Contd)*

```
      implicit real*8 (a-h,o-z)
      real*8 kd
      z8 = kd / 3.7d0
      if(z8.le.0d0) then
        x7 = sqrt(0.02d0)
      else
        x7 = - 1d0 / (2d0*log10 (z8))
      endif
      y7 = x7 * x7
100   if(z8.lt.0d0) z8 = 0d0
      x8 = - 1d0 / (2d0*log10 (z8 + 2.51d0 / (reyn * x7)))
      fc = x8 * x8
      if( abs ((fc - y7) / y7).lt..0001d0) goto 200
      y7 = fc
      x7 = x8
      goto 100
200   return
      end

c     ****************************************************************
c     SUBROUTINE AGOLD, MIN. OF UNIMODAL FUNCTION BY GOLDEN SECTION SEARCH
c     ****************************************************************
      subroutine agold (xlo,xhi,dx,funx, xmin,fxmin)
c      REF. CHENEY & KINCAID, P.462
c      XMIN AT FXMIN, MIN. PT. OF F(X), RETURNED
c      XMIN IN RANGE XLO TO XHI
c      DX=X INTERVAL CONTAINING XMIN
c      XMIN TOLERANCE = + OR - DX/2
c      FUNX=ENTRY NAME FOR SUBROUTINE GIVING F(X)
      implicit real*8 (a-h,o-z)
      external funx
      gsr = ( sqrt (5.d0) - 1d0) * 0.5d0
      x0 = xlo
      x3 = xhi
      itnum = 1 + log (abs(dx / (x3 - x0))) / log (gsr)
      x2 = x0 + gsr * (x3 - x0)
      call funx(x2, y2)
      x1 = x0 + gsr * gsr * (x3 - x0)
      call funx(x1, y1)
      do 100 i = 1,itnum
        if(y2.gt.y1) then
        x3 = x2
        x2 = x1
        y2 = y1
        x1 = x0 + gsr * gsr * (x3 - x0)
        call funx(x1, y1)
        else
        x0 = x1
        x1 = x2
        y1 = y2
        x2 = x0 + gsr * (x3 - x0)
        call funx(x2, y2)
        endif
100   continue
      xmin = (x1 + x2) * 0.5d0
      call funx(xmin, fxmin)
      end

c     ****************************************************************
c     SUBROUTINE BGOLD, MIN. OF UNIMODAL FUNCTION BY GOLDEN SECTION SEARCH
c     IDENTICAL WITH AGOLD
c     ****************************************************************
      subroutine bgold (xlo,xhi,dx,funx, xmin,fxmin)
      implicit real*8 (a-h,o-z)
      external funx
      gsr = ( sqrt (5.d0) - 1d0) * 0.5d0
      x0 = xlo
      x3 = xhi
      itnum = 1 + log (abs(dx / (x3 - x0))) / log (gsr)
      x2 = x0 + gsr * (x3 - x0)
```

Appendix C: Computer Program *(Contd)*

```
        call funx(x2, y2)
        x1 = x0 + gsr * gsr * (x3 - x0)
        call funx(x1, y1)
        do 100 i = 1,itnum
          if(y2.gt.y1) then
          x3 = x2
          x2 = x1
          y2 = y1
          x1 = x0 + gsr * gsr * (x3 - x0)
          call funx(x1, y1)
          else
          x0 = x1
          x1 = x2
          y1 = y2
          x2 = x0 + gsr * (x3 - x0)
          call funx(x2, y2)
          endif
100     continue
        xmin = (x1 + x2) * 0.5d0
        call funx(xmin, fxmin)
        end

c       ****************************************************************
c       SUBROUTINE CGOLD, MIN. OF UNIMODAL FUNCTION BY GOLDEN SECTION SEARCH
c       IDENTICAL WITH AGOLD
c       ****************************************************************
        subroutine cgold (xlo,xhi,dx,funx, xmin,fxmin)
        implicit real*8 (a-h,o-z)
        external funx
        gsr = ( sqrt (5.d0) - 1d0) * 0.5d0
        x0 = xlo
        x3 = xhi
        itnum = 1 + log (abs(dx / (x3 - x0))) / log (gsr)
        x2 = x0 + gsr * (x3 - x0)
        call funx(x2, y2)
        x1 = x0 + gsr * gsr * (x3 - x0)
        call funx(x1, y1)
        do 100 i = 1,itnum
          if(y2.gt.y1) then
          x3 = x2
          x2 = x1
          y2 = y1
          x1 = x0 + gsr * gsr * (x3 - x0)
          call funx(x1, y1)
          else
          x0 = x1
          x1 = x2
          y1 = y2
          x2 = x0 + gsr * (x3 - x0)
          call funx(x2, y2)
          endif
100     continue
        xmin = (x1 + x2) * 0.5d0
        call funx(xmin, fxmin)
        end
```

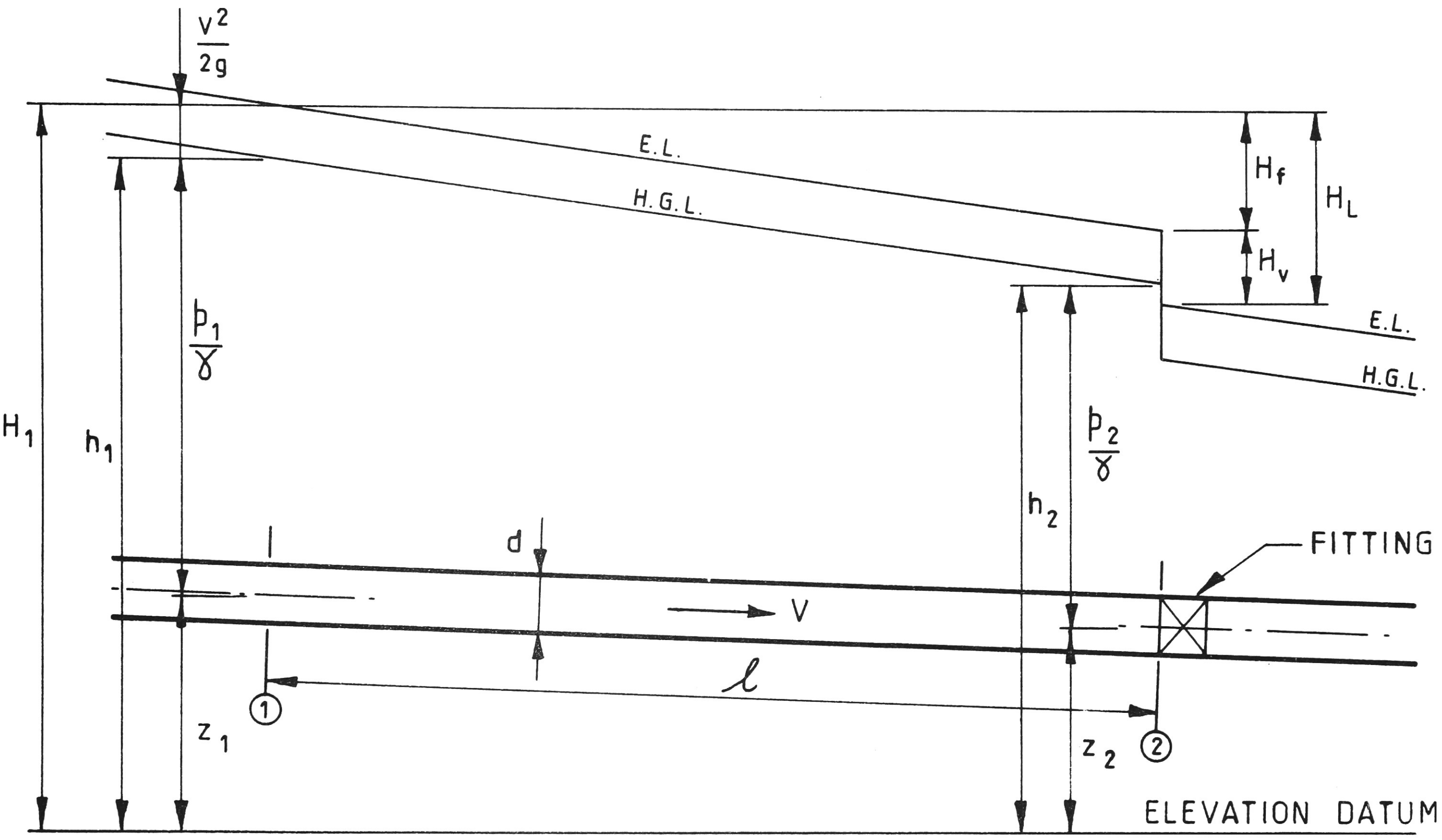
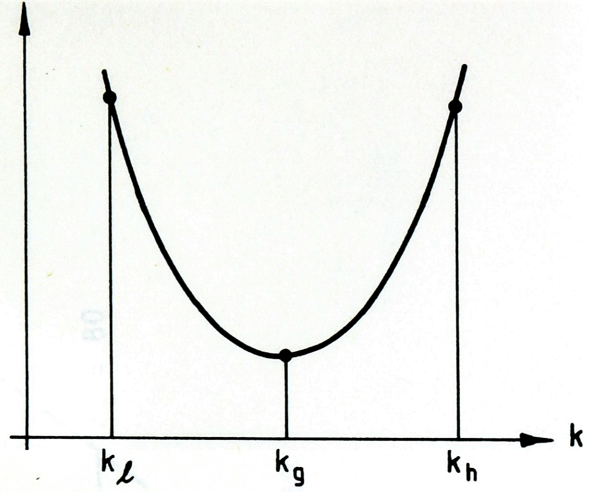
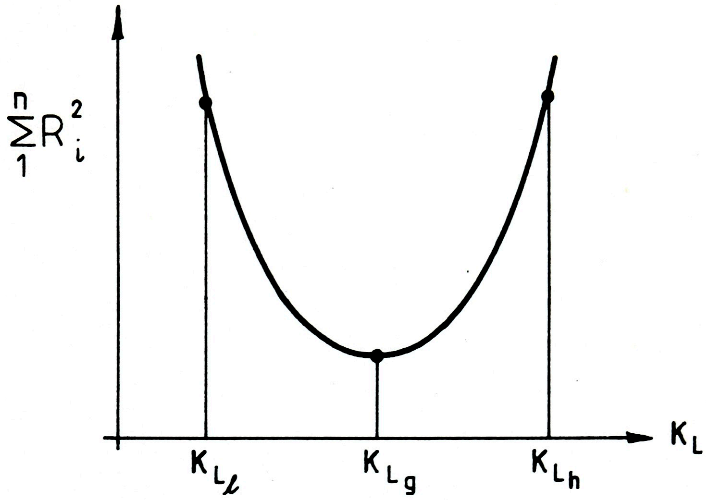


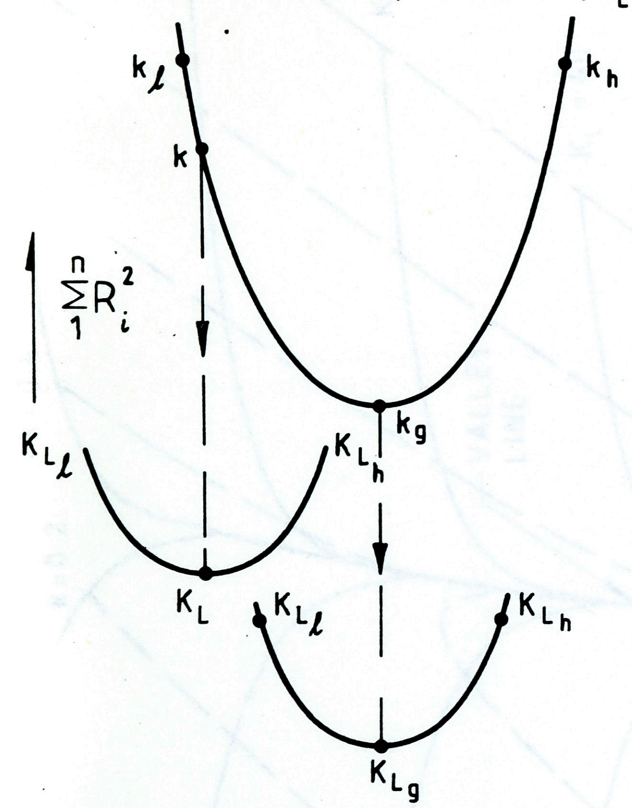
FIGURE 1 HEAD LOSS NOTATION



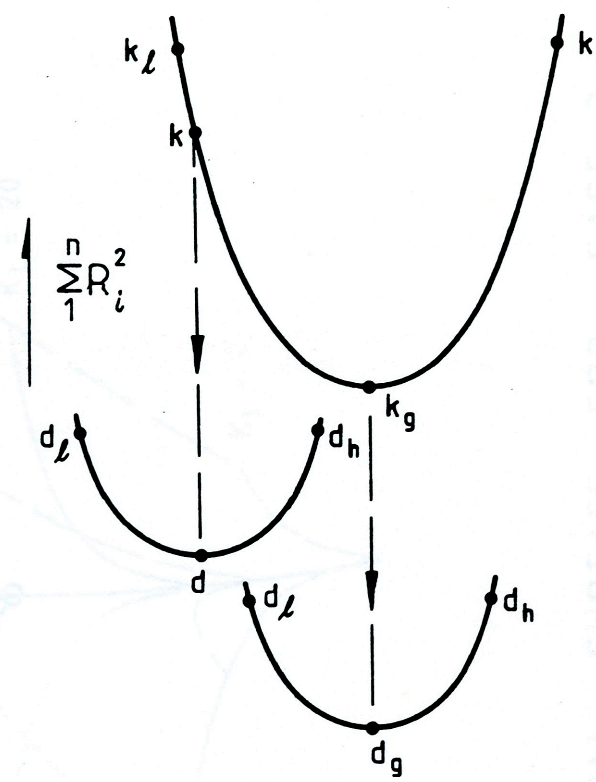
(A) CASE 1 : GIVEN d, FIND k ($K_L = 0$)



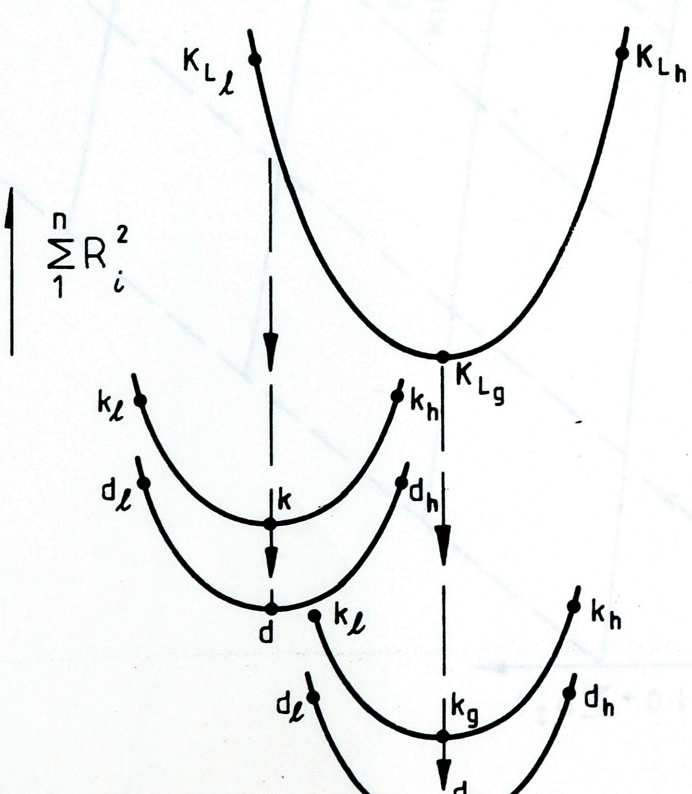
(B) CASE 2 : GIVEN d AND k, FIND K_L



(C) CASE 3 : GIVEN d, FIND k AND K_L



(D) CASE 4 : FIND d AND k ($K_L = 0$)



(E) CASE 5 : FIND d, k AND K_L

FIGURE 2 FIVE CASES OF PIPE TEST ANALYSIS

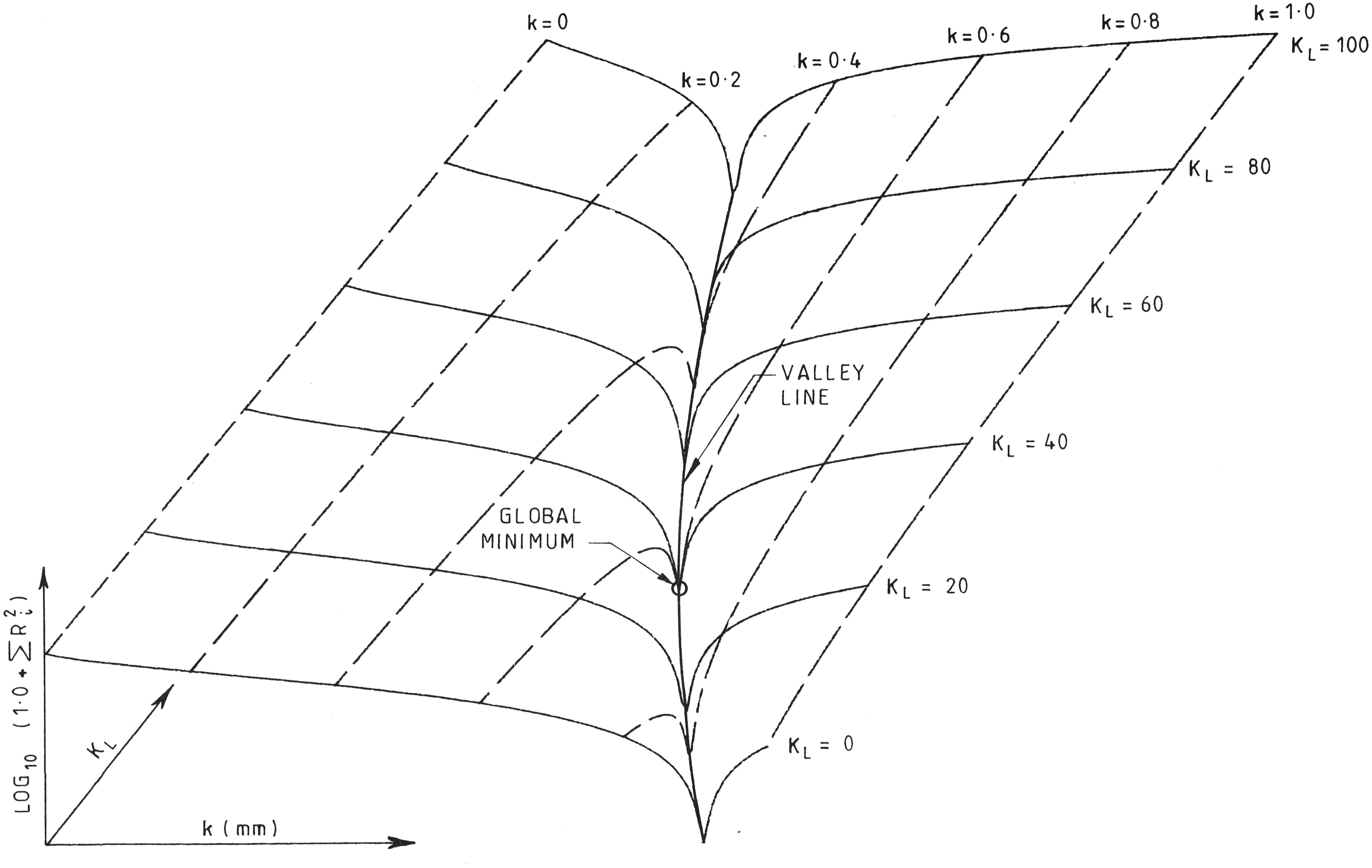
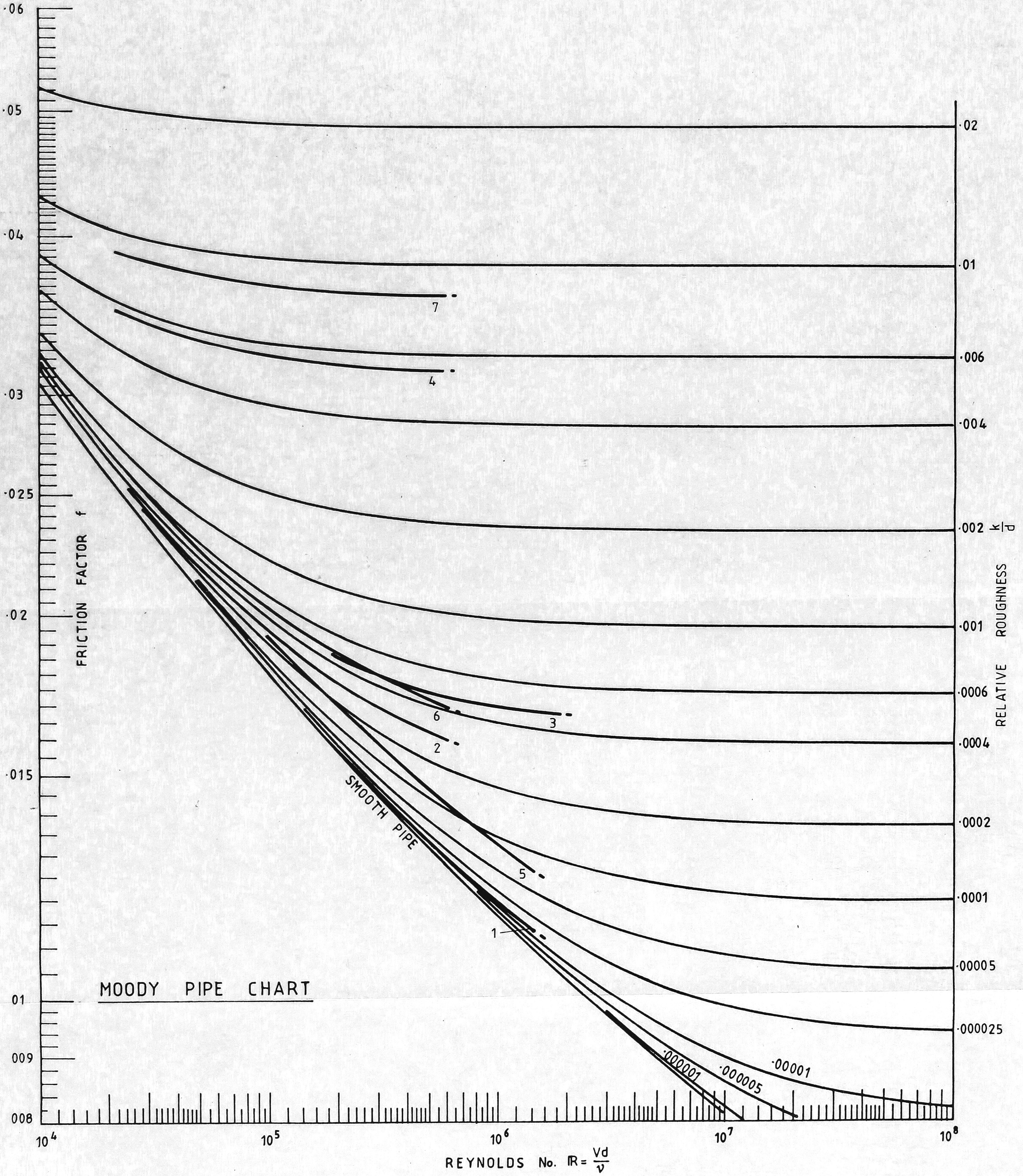
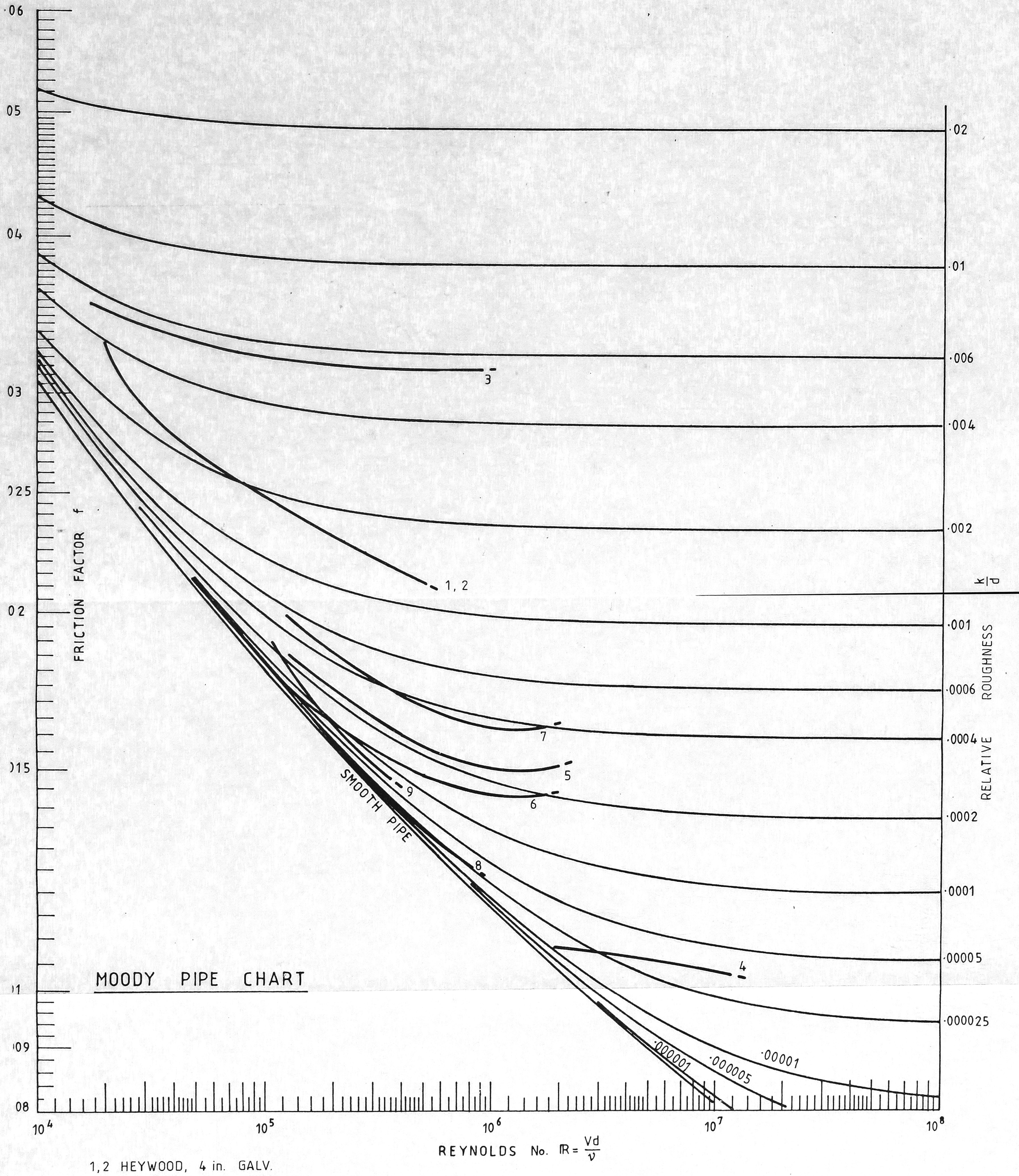


FIGURE 3 THREE DIMENSIONAL SURFACE FOR CASE 3, GIVEN d, FIND k AND K_L



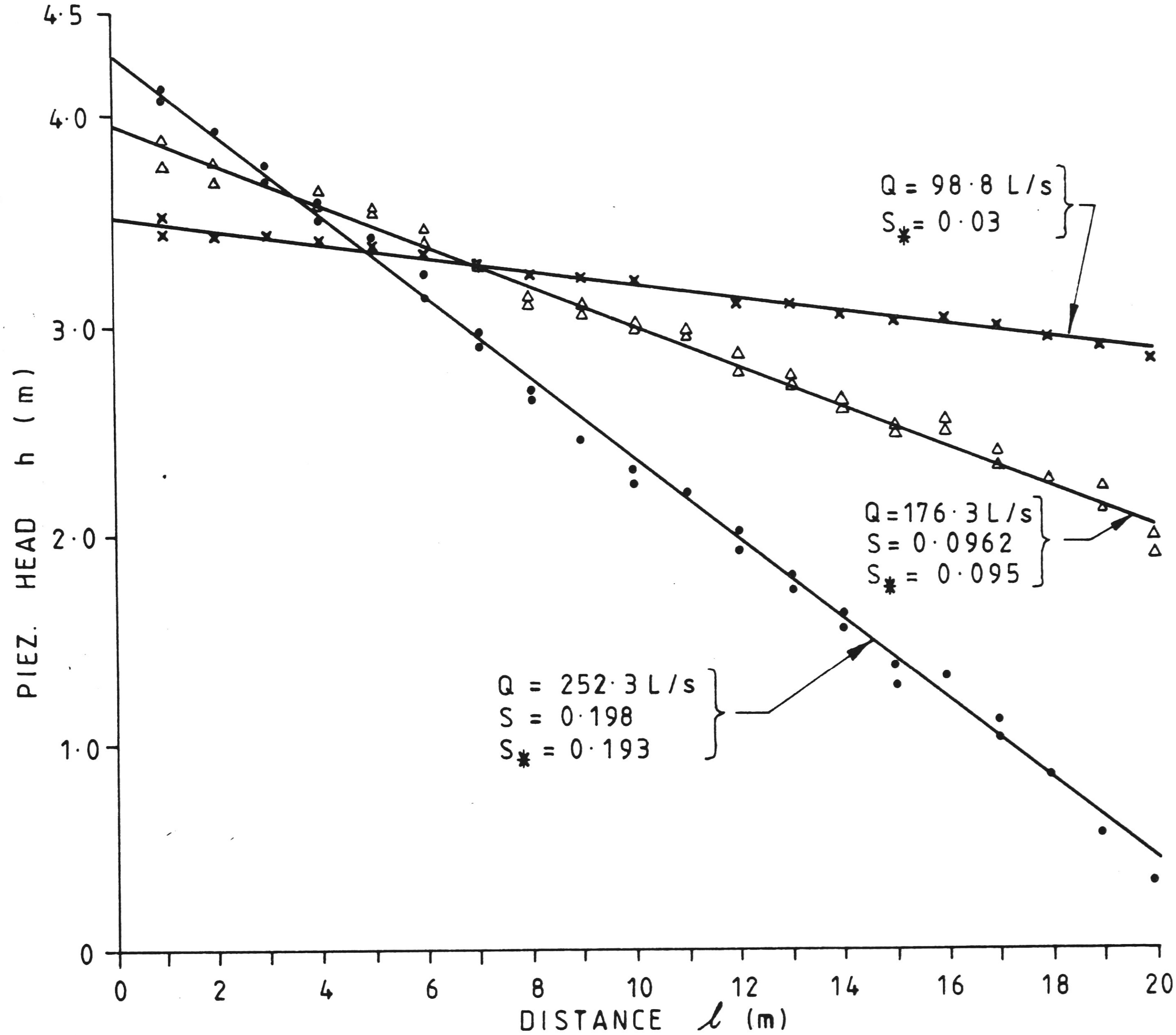
1 POLYETHYLENE, k = 0·003 mm $K_L = 0$
2 CONCRETE, GOOD, k = 0·03 mm $K_L = 0$
3 CI UNCOATED, GOOD, k = 0·15 mm $K_L = 0$
4 CI UNCOATED, POOR, k = 0·6 mm $K_L = 0$
5 POLYETHYLENE, k = 0·003 mm $K_L = 5$
6 CONCRETE, GOOD, k = 0·03 mm $K_L = 10$
7 CI UNCOATED, POOR, k = 0·6 mm $K_L = 40$

FIGURE 4 SYNTHETIC PIPE TESTS

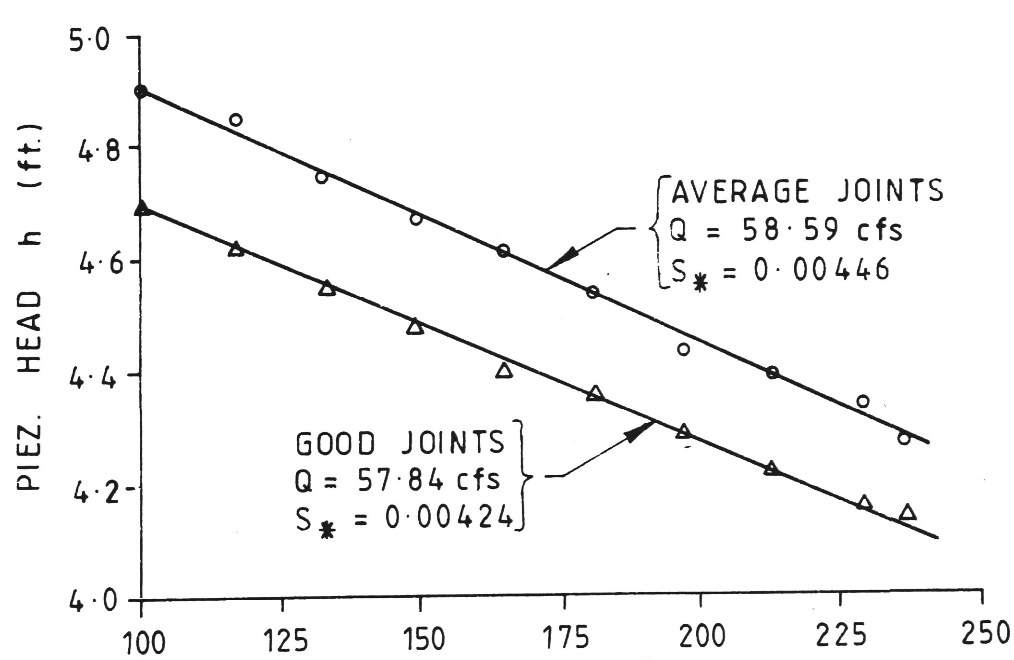


1, 2 HEYWOOD, 4 in. GALV.
3 HEYWOOD, 2 in. GALV.
4 BURKE, 4·2 ft. ENAMELLED STEEL PENSTOCK
5 LEVIN, 210 mm COLD BITUMEN SPRAYED, MEAN PROPERTIES
6 AS 5, MIN. k PROPERTIES
7 AS 5, MAX. k PROPERTIES
8 DUDGEON, 287 mm CEMENT LINED STEEL
9 FOSTER, 15 in. ASBESTOS CEMENT

FIGURE 5 PUBLISHED PIPE TESTS



(A) 209·5 mm DIA. COLD SPRAYED BITUMEN PIPE [30]

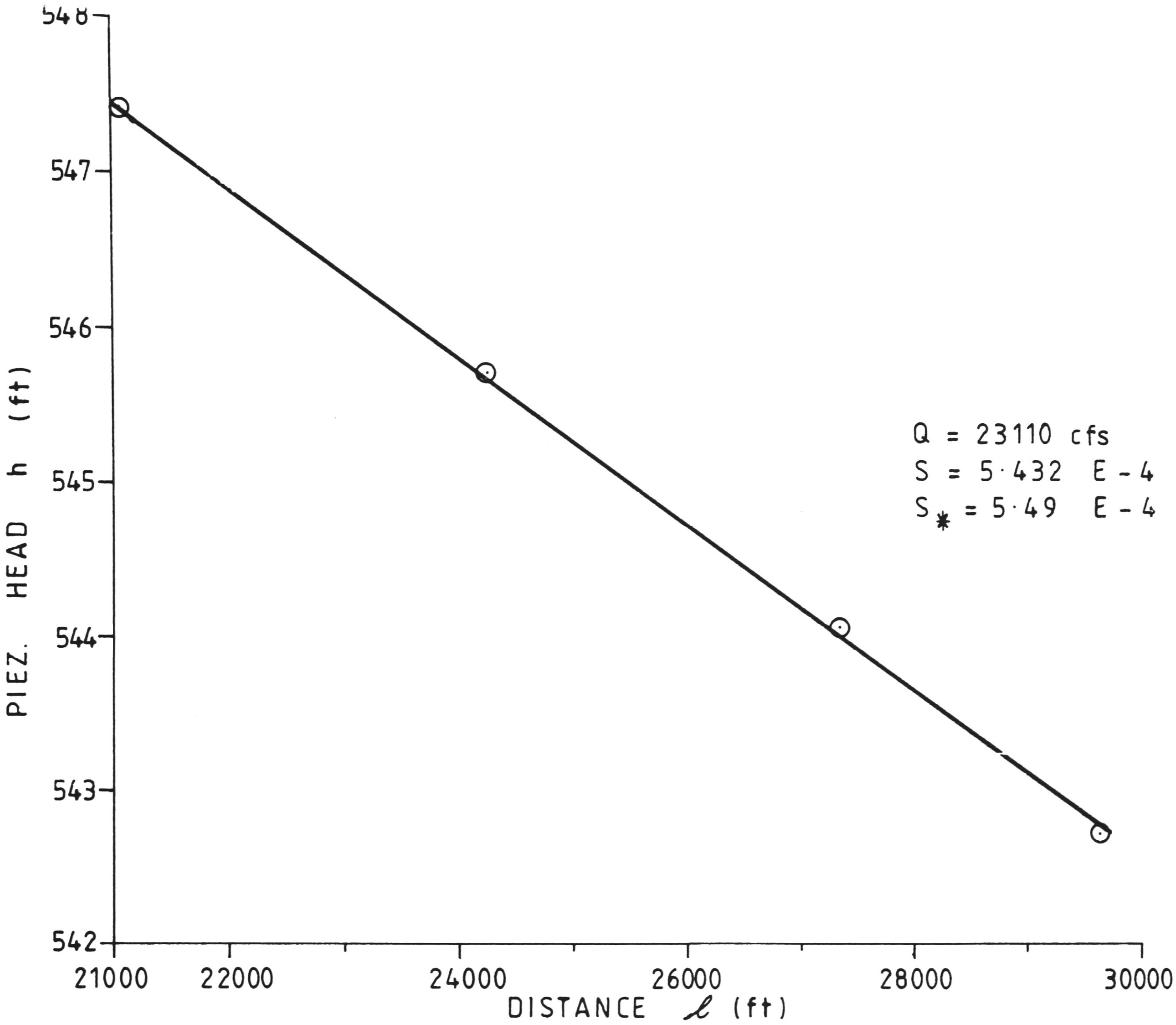


(B) 36 IN. DIA. CAST CONCRETE PIPE [38]

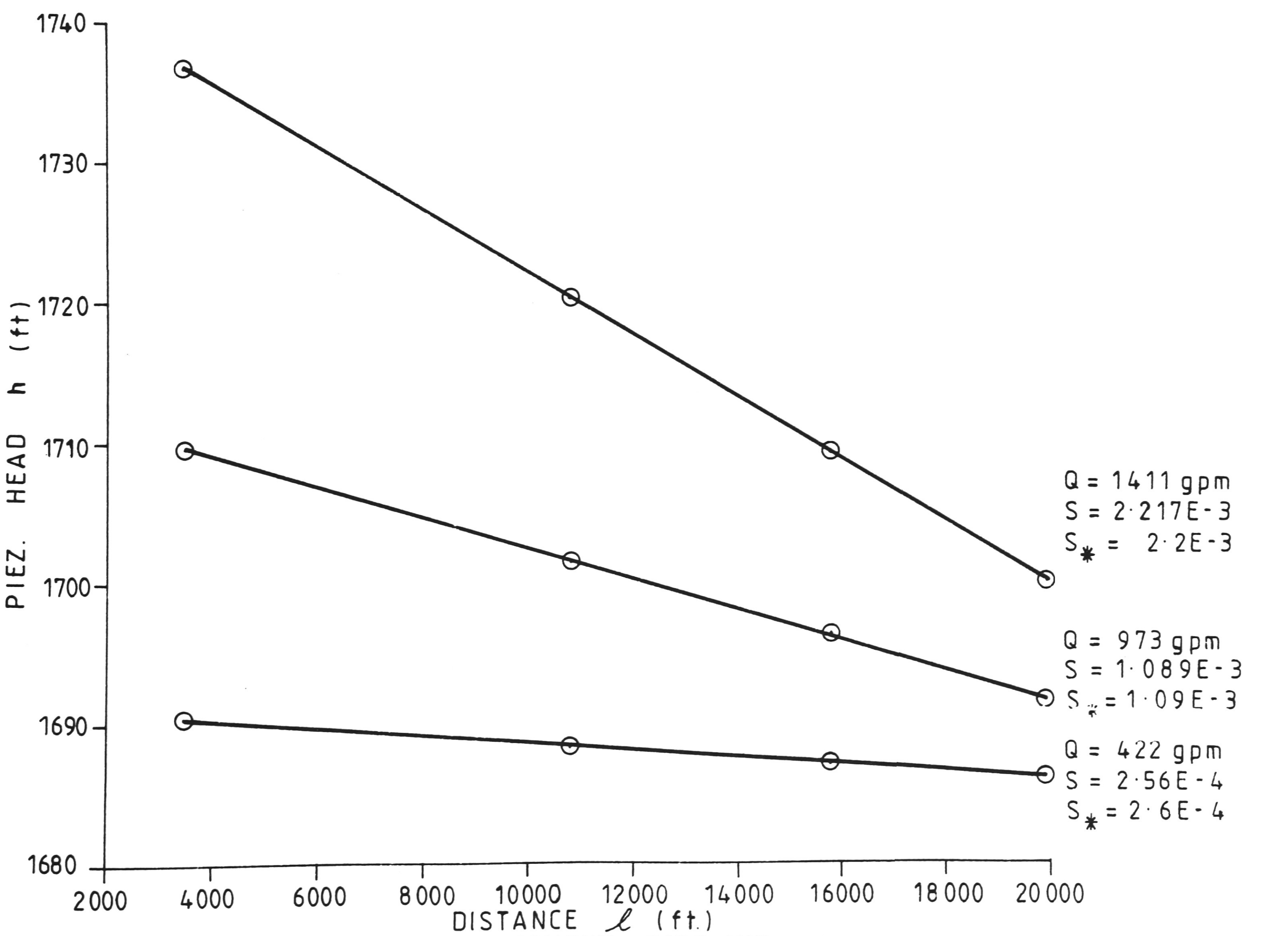
FIGURE 6 TYPICAL HGL, LABORATORY TESTS,

S = LEAST SQUARES ENERGY GRADIENTS

S_* = PUBLISHED ENERGY GRADIENT



(A) 45 FT. DIA. CONCRETE TUNNEL [8]



(B) 15 IN. DIA. ASBESTOS CEMENT PIPELINE [23]

FIGURE 7 TYPICAL HGL FIELD TESTS

S = LEAST SQUARES ENERGY GRADIENT, S* = PUBLISHED ENERGY GRADIENT

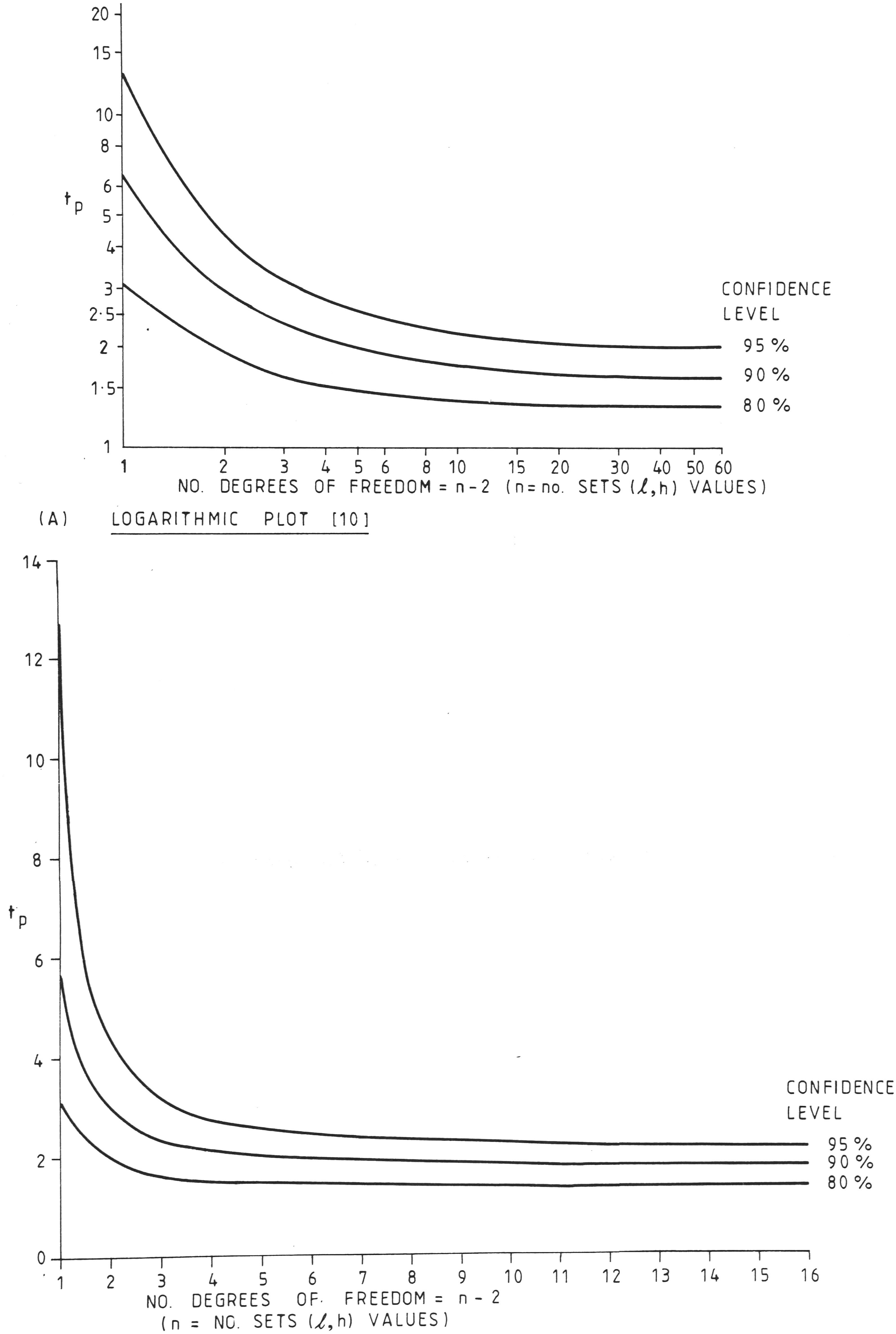


FIGURE 8 COEFFICIENTS t_p FROM STUDENT'S t DISTRIBUTION

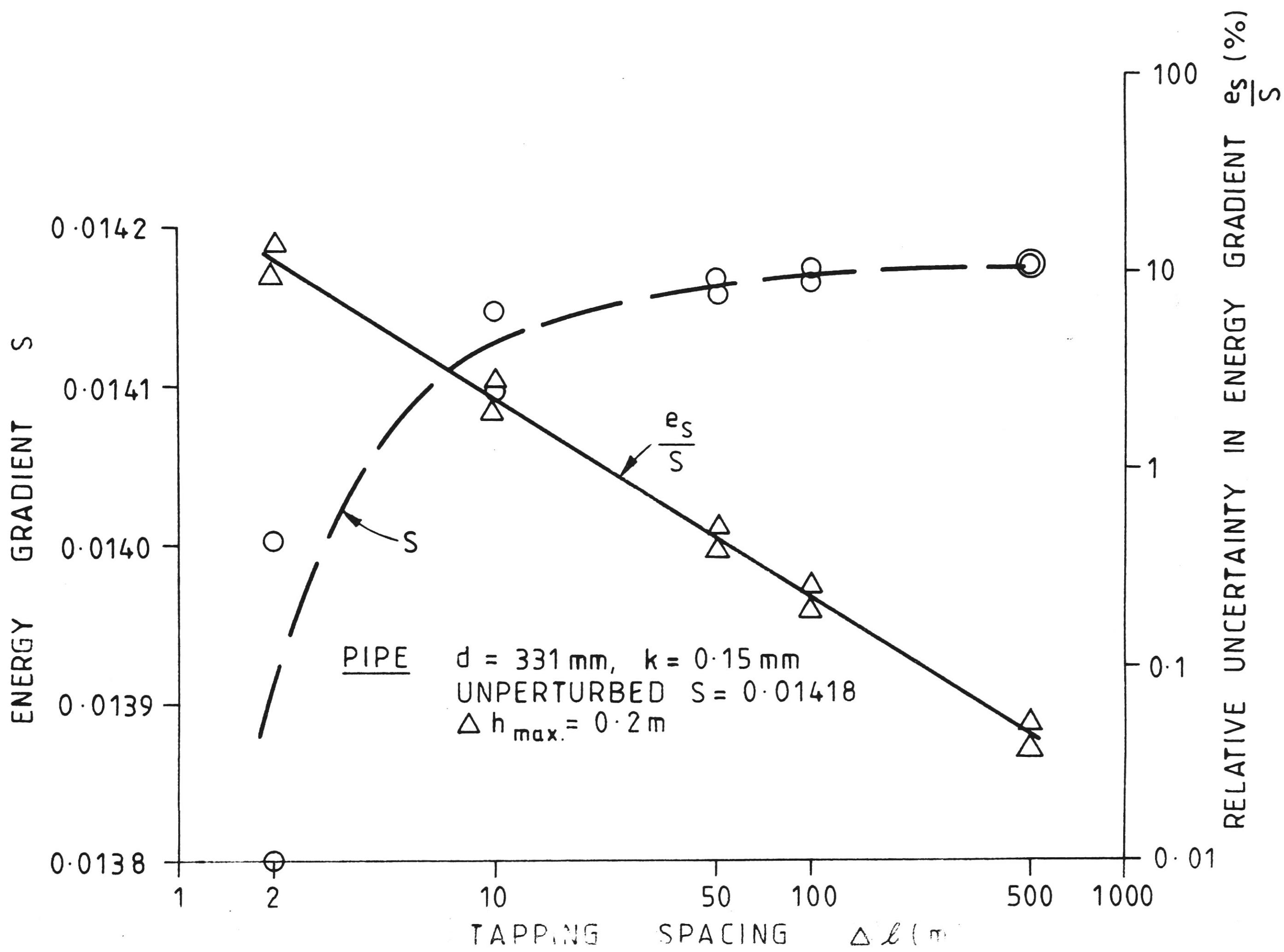
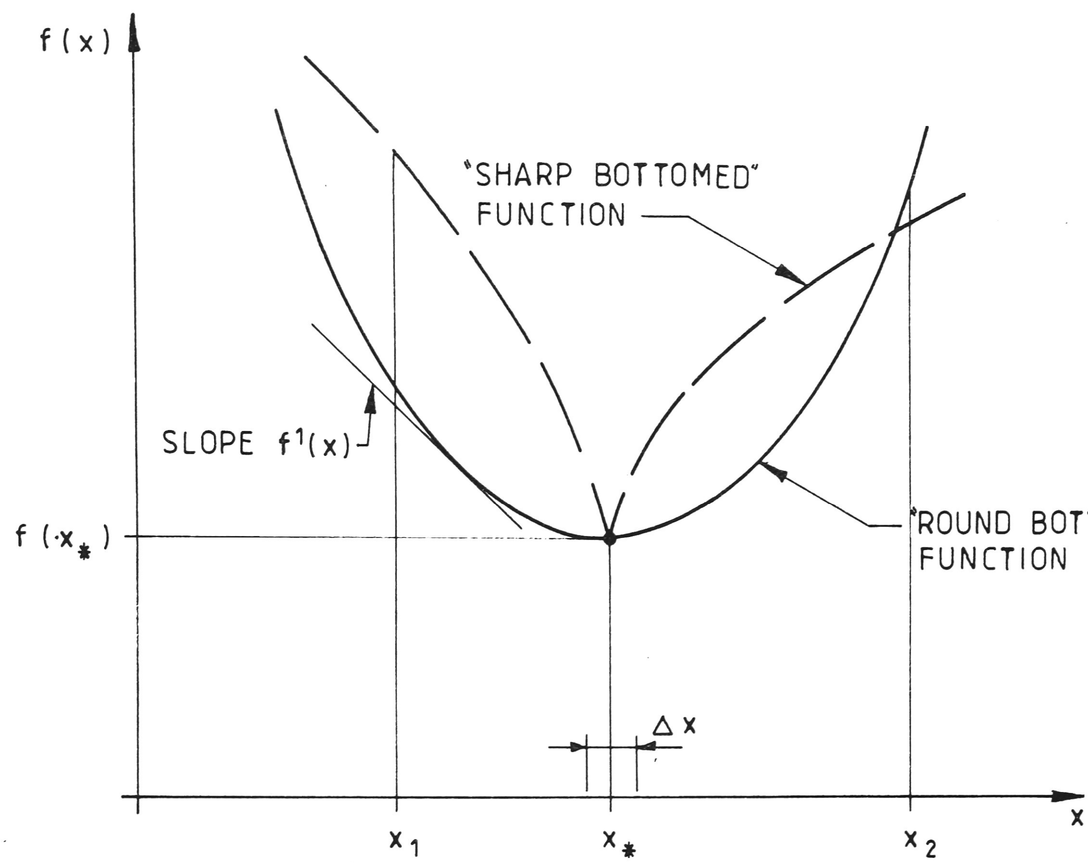
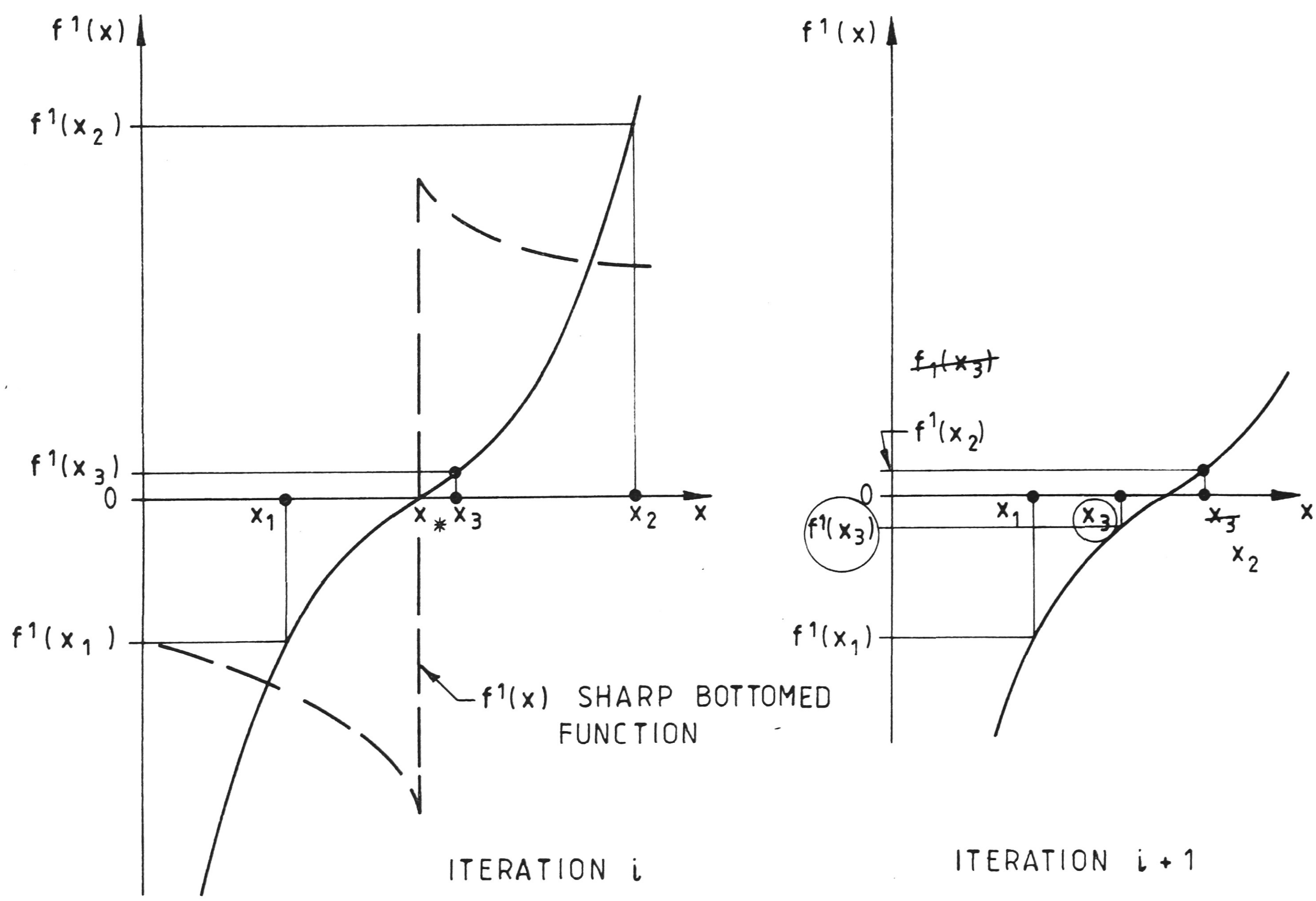


FIGURE 9 EFFECT OF TAPPING SPACING

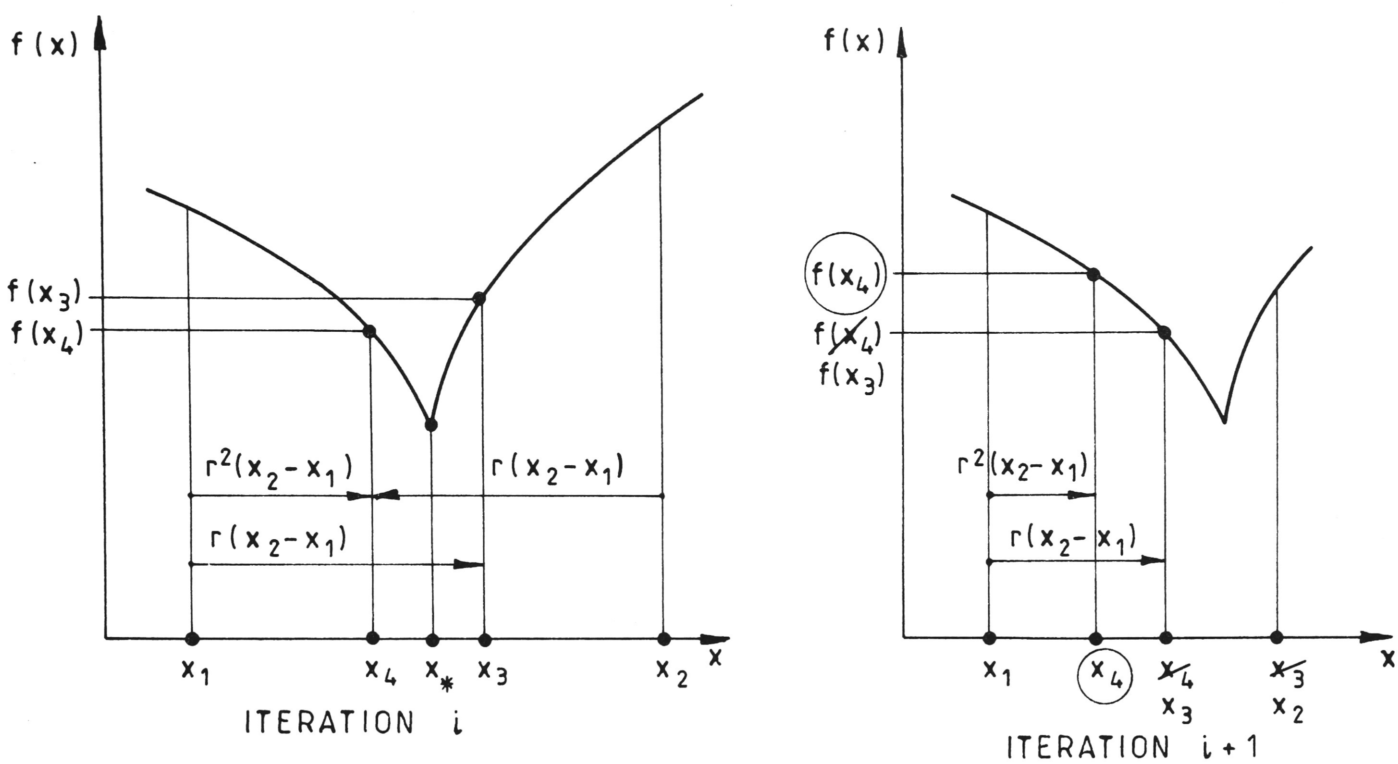


(A) MINIMUM OF ONE-VARIABLE FUNCTION

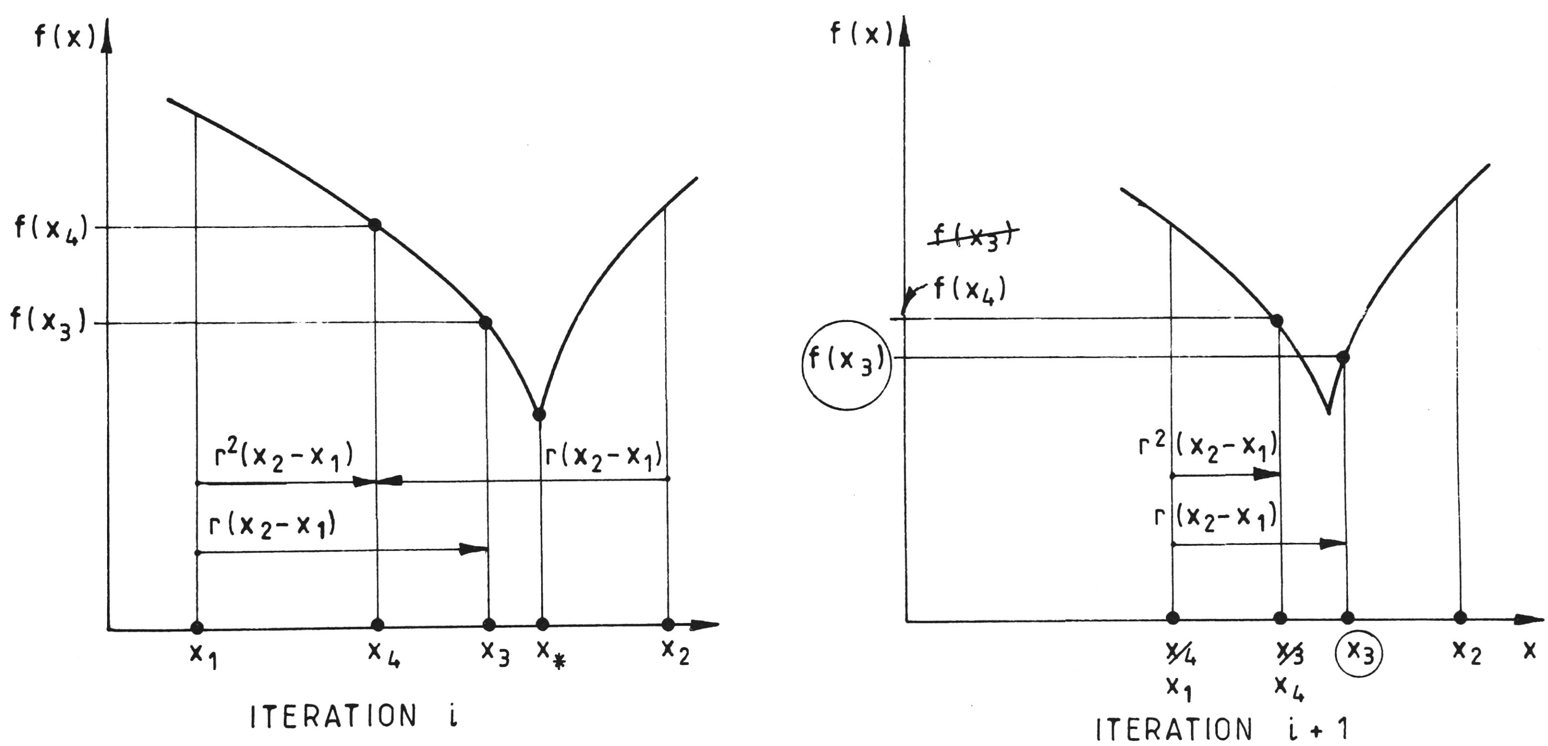


(B) MINIMUM BY BISECTION OF FIRST DERIVATIVE FUNCTION

FIGURE 10 MINIMUM OF A ONE-VARIABLE FUNCTION



(A) CASE OF $f(x_3) > f(x_4)$



(B) CASE OF $f(x_3) \leq f(x_4)$

FIGURE 11 MINIMISATION OF A ONE-VARIABLE FUNCTION BY GOLDEN SECTION SEARCH