Measurement of load coefficient of D.C. resistors by A.C. means.

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Measurement of Load Coefficient
of D.C. Resistors by A.C. Means.

a thesis submitted to the
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Submitted by
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DECLARATION.

This thesis has not been submitted to any other University or institution for the award of a higher degree.

(Signed) G.E. Beard B.Sc.(Tech.)

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SUMMARY

Comparative load coefficient measurements were made on several direct current resistors using two types of loading current of equal effective value. The first current consisted entirely of direct current which was supplied to the measuring bridge through a reversing switch so as to facilitate the direct current reversals used in obtaining bridge balance. The second consisted of alternating current with a small superimposed direct current which was supplied through the reversing switch and used to indicate bridge balance as above. The advantages and disadvantages of the A.C. loading technique are discussed together with the circuit modifications required when using this method in association with the Kelvin and Wheatstone Bridges. Effects are discussed which are considered most likely to produce significant differences between the direct current resistance changes obtained when the above loading conditions are used. The requirements of the alternating loading current supply circuit are discussed and consideration is made of the possibility of using an electrolytic D.C. blocking capacitor. A comparison is made of heavy direct and alternating current supplies with reference to cost, capacity, adjustability etc.

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INTRODUCTION

In direct current resistance intercomparisons made with high measurement accuracy on precision standard resistors, it is essential to know the load coefficients of all resistors in the measurement circuit. The load coefficient of a standard resistor is defined most usefully as its change in resistance in proportional parts from its negligible load value at constant temperature, under the given conditions of loading. Negligible load condition is dependent on both the accuracy with which the resistance measurements are to be made and the magnitude of the load coefficient of the resistor under test; the lower its value of load coefficient, the greater may be its level of negligible load power dissipation for a given resistance measurement accuracy.

The accuracy with which the load coefficient of D.C. resistors can be determined by traditional means is dependent on many factors: for example

(1) The absolute short term stability[1] with time at constant temperature and power dissipation level. Typical rates of short term drift in presently available precision resistance standards may be obtained from results presented by Miller.[1] The rate of drift of the balance point of a multiple bridge[2] containing two 10 ohm and two 100 ohm standard resistors is given as varying between 1 part in 10^9 and 1 part in 10^8 per hour. Obviously it is useless to try to measure the change in resistance between two conditions of loading effected over a certain time interval to an accuracy which is better than the order of the inherent instability of the resistor under test over the same time interval. Associated with this is;

(11) The stability of all other resistance elements in the resistance measuring circuit which have an effect on the measured value of resistance change obtained for the Unknown;

(111) The uncertainty with which the temperature of the immediate environment and the accuracy with which the temperature coefficient of all temperature sensitive elements in the

measuring system are known. The presence of fluctuating temperature gradients between the temperature measuring devices and the immediate environment of the elements of the measuring system, or the presence of temperature gradients across the elements themselves due to insufficient stabilisation time being allowed after a change in circuit conditions, will increase the intercomparison uncertainty greatly;

(1V) The sensitivity of the bridge null detector used to indicate bridge balance. Sufficient detector sensitivity to produce the desired accuracy of measurement is required, although there exists a maximum useable detector sensitivity owing to the presence of Thermal Noise and Brownian Motion in the bridge circuit and galvanometer;

(V) The accuracy with which changes in the resistance value of the resistors of the measuring circuit are predictable with variations in the loading condition of the resistor under test. Normally, the whole or a significant portion of the loading current passing through the resistor under test passes also through at least one other resistor in the measuring circuit, and hence an accurate knowledge of this resistor's load coefficient is required[3,4];

(V1) The accuracy with which the loading current is known and its stability with time once set. The accuracy with which the loading current can be measured or set places an ultimate limit on the accuracy with which a particular load coefficient can be measured, and the required accuracy and stability is directly dependent on the magnitude of the load coefficients associated with the overall measuring system;

(V11) The presence of spurious emfs of thermal or electro-chemical origin. The effects of most of these can be removed if they are constant with time, with the employment of supply or galvanometer reversals while obtaining balance[4]. Those whose effects are unavoidable must be removed;

(V111) Changes in the conditions of measurement, such as the rate of heat transfer from the resistance element may cause large variations in the resulting load coefficients obtained;

(1X) The accuracy of adjustment and the calibration of variable dials used in the measurement. Components presently available are not likely to cause trouble in this regard;

(X) The presence or not of leakage paths shunting various parts of the measuring circuit. These effects cause systematic errors, but they can normally be removed with the incorporation of guarding systems around the most sensitive bridge points. The presence of constant leakage resistances are not extremely serious when measuring only resistance changes but if they are voltage sensitive or otherwise variable, a guarding system must be used.

Several of these factors place fundamental limitations on the attainable measurement accuracy, by appearing as instabilities or errors in the load coefficient of the resistor under test. Other factors are limitations caused by the inadequacy of the components used in the measurement, while others are due to the fundamental requirements of the system of measurement used. Limitations of the latter type may be removed with improvements in the technology of the measuring system.

Traditional means of measuring load coefficients such as the Kelvin Double Bridge or the direct current Potentiometer, require that the full direct loading current should pass through one other resistor of the measuring circuit. Usually this resistor is at least a factor of ten, lower in resistance than the Unknown[3,4,5], so as to reduce the power dissipated in the Standard by the corresponding factor. Hence, if the load coefficient of the Standard is of the same order as that of the Unknown, the accuracy with which the load coefficient of the Standard is required to be known is of the order of one tenth of the accuracy required for the load coefficient of the Unknown. But the problem increases with the investigation of extremely low values of resistance of the order of 0.001 ohm and lower. To measure a resistor of this order at full load, another resistor is needed to use as Standard, whose load coefficient is known or calculable[5] at the appropriate current level.

There is the problem associated with heavy current low resistance standards of the supply of the necessary levels of loading current. The large battery banks required in order to obtain currents of the order of 10 000 amps, are extremely expensive to purchase and to maintain. Reversal of D.C. supply current if performed at this level, can be extremely hazardous, not to mention the problem of keeping the current steady due to changing resistance in the battery supply circuit around the reversing switch. There is also the problem of metering heavy currents, requiring possibly a third heavy current standard to use as a meter shunt.

All these problems are removed by a measuring system which uses Alternating Current to load the resistor under test, while maintaining a small no-load Direct Current to use for measurement. The method is general and may be used at any value of resistance. But the greatest advantages occur at low resistances and high loads, where the associated direct current loading system becomes extremely cumbersome.

There are four advantages obtained with the A.C. loading technique.

(1) The load coefficient of the Standard does not need to be accurately known, provided it is known that the D.C. measuring current used will cause negligible change in it. The current passing through the Standard will be made up of the D.C. measuring current and an extremely small portion of the A.C. loading current, so that the effective current in the Standard will be little more than the D.C. measuring current alone. Hence the Standard used may be of a relatively low current rating. Its resistance value may be equal to that of the resistance under test if required, so that increased detection sensitivity and hence measurement accuracy are attainable.

(11) The amount of direct current required to be reversed by the

reversing switch is much lower, being only the measuring current. Hence resistance variations around the reversing switch will not be as critical as previously experienced in maintaining the loading current constant, since the D.C. supply circuit resistance will be higher for the same D.C. supply voltage. Also the proportion of the effective loading current supplied by the D.C. circuit will be much lower and so the effects of fluctuations here reduced correspondingly. Practically, it is an advantage to have as high a resistance as possible in the D.C. supply circuit, since this will limit the level of alternating current which must be considered and allowed for in the D.C. supply circuit.

(111) With D.C. loading the actual loading of the resistor under test is reduced to zero during the period of supply reversal as used normally on the Kelvin Bridge. This interruption causes loading conditions to change slightly and hence its effect could be to give a value of load coefficient which is slightly in error. This problem is normally surmounted by employing an automatic reversing system which reverses regularly every minute or so, the time interval being dictated by the period required for thermo-electric effects or dielectric absorption to die away. The results obtained using this system give another variable to the method; that of the period between reversals. The method of A.C. loading however, does not suffer from this limitation since the loading current is supplied continuously to the resistor under test, and to a first approximation, remains constant during D.C. reversal.

(1V) The ease of metering the loading current is increased greatly when using alternating loading currents since a current transformer and ammeter may be used instead of an ammeter in association with a large meter shunt whose load coefficient may not be negligible at the level of loading required.

There are of course several features which must be considered in relation to the practical aspects of this method, but no insurmountable problems were encountered. This thesis deals primarily with the establishment of the method of A.C. loading as a feasable technique for the determination of the load coefficients of D.C. resistors. The possible differences considered to be most important between a D.C. loading condition and an apparently equivalent A.C. loading condition are discussed together with the results of measurements made on several commercial Standard resistors. The requirements of the alternating loading current supply circuit are considered and special adaption of the Kelvin and Wheatstone Bridges to suit the method is discussed.

CHAPTER 1

Error Effects

It is initially desirable to consider the mechanisms by which the self-heating produced by a combination of alternating and direct loading currents may differ from that produced by a direct current alone. All relevant mechanisms have been considered by Miller[1] and Gibbings[6]. Their interest centred on the possible differences existing between the alternating and direct current resistance values of a given resistor, while this chapter is concerned with the investigation of effects which cause the temperature or current distribution along a standard resistor under alternating current loading to differ from that experienced under direct current loading, or effects which produce differences in the measured value obtained for the direct current resistance of a standard at a particular power dissipation level when it is loaded with different relative alternating and direct current levels of the same effective value.

Most of the effects considered cannot be calculated exactly, owing to the awkward geometry and convection cooling mechanisms used in typical standard resistors. But with several of the effects, an estimate of a maximum value was obtained by assuming limiting cases. It will be appreciated that with all error effects, their relative importance will depend firstly upon the magnitude of the load coefficient of the standard resistor being investigated and secondly upon the accuracy with which its load coefficient is required to be known.

(1.1)

The Contribution of Peltier and Thomson Effects to the Measured Value Obtained for the Load Coefficient of a Direct Current Standard Resistor.

The temperature distribution and heat flow in a resistor carrying direct current will in general differ from the temperature distribution and heat flow in the same resistor carrying an equal r.m.s. alternating current, due to Peltier and Thomson Effects, provided the frequency of the alternating current is above that where the temperature alternations cease to follow the current alternations[6]. This frequency will depend upon the thermal capacity and cooling conditions of the particular standard resistor under consideration, but Miller's[1] curves published for 1 ohm Evanohm open wire resistors, indicate that the threshold frequency above which some difference may be expected in lower values of resistance, as suggested by Gibbings[6], is probably below 20 radians per second.

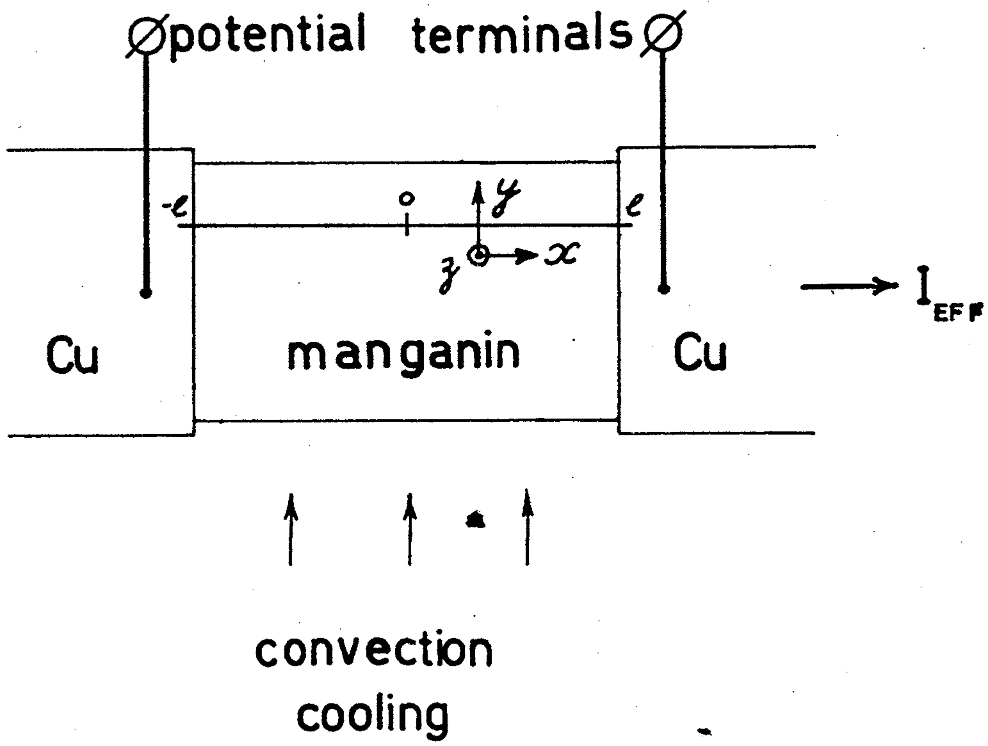
The presence of Peltier and Thomson Effects in load coefficient measurements made by loading with alternating current and measuring resistance changes with a small direct current, is not as serious as is their presence in resistance measurements performed by Miller's method[1], where any changes in thermo-electric effects from the direct current case is exhibited as a direct error in the measured resistance value, since the practical direct current resistance value includes any thermo-electric effects. In load coefficient measurements, the direct measuring current will cause assymetry in the temperature distribution at either end of the resistor due to Peltier and Thomson Effects at the junctions of the resistance material with the copper current terminals. It can be shown[7] that the Peltier Coefficient and the difference between the Thomson Coefficients of two dissimilar metals a and b making up a typical thermocouple are given by

$$\pi(T) = T\frac{dF}{dT} \tag{1.1}$$

and

$$\propto_{T_a} - \propto_{T_b} = -\frac{d^2F}{dT^2} \tag{1.2}$$

where $\pi(T)$ is the Peltier Coefficient. The quantity of heat that is reversably taken in or given out at a junction when q coulombs of electric charge pass is $\pi(T)$ q joules.



One
Type of Heavy Current Resistance Standard
with potential terminals taken from the copper current leads
Fig I

α_{T_a} and α_{T_b} are the Thomson Coefficients of the two metals a and b respectively at the temperature T,

T is the Absolute Temperature in degrees Kelvin and

F is the measured Seebeck Emf.

For small temperature differences between hot and cold junctions, the relationship between the Seebeck Emf F and the temperature difference $(T - T_F)$ between the hot and cold junction may be represented as

$$F = A(T - T_F) + 0.5B(T - T_F)^2 \qquad (1.3)$$

The importance of thermo-electric effects on the direct current resistance value obtained for a typical standard resistance under load will depend upon the temperature distribution in the resistance at that load. Temperature distributions are extremely difficult to calculate due to the difficulty of representing convection cooling in a practical resistor under load, but if the convection cooling is efficient, temperature gradients within the resistor will be small. Thermo-electric effects will be of greatest significance in low valued standard resistors where any dissimilar junctions between the resistance material and the current terminal connections are located between the potential leads. Such a resistor is shown in Fig.1. If as a first approximation it may be assumed that the resistance element temperature at any point (x, y, z) in Fig. 1 is a function of x only, then the one dimensional heat flow conditions in an elemental length δX situated at a point x along the manganin resistance element under steady-state conditions are given by[8,9]

$$\underbrace{ak_m \frac{d^2\theta}{dx^2}}_{\text{Conduction}} - \underbrace{\alpha_{T_m} I_{DC} \frac{d\theta}{dx}}_{\text{Thomson Effect}} - \underbrace{H_P(\theta - \theta_F)}_{\text{Convection}} + \underbrace{\frac{I^2_{EFF}\rho_m}{a}}_{I^2R} - \underbrace{p\sigma\varepsilon_m(T^4 - T_F^4)}_{\text{Radiation}} = 0 \qquad (1.4)$$

where

a is the cross-sectional area of the elemental length δX,

p is the perimeter of the element of length δX,

α_{Tm} is the Thomson Coefficient (assumed constant for a small range of T),

σ is the Steffan-Boltzmann Constant,

$\mathcal{E}_m$ is the emmissivity,

k_m is the thermal conductivity of the manganin resistance element,

ρ_m is the electrical resistivity of the manganin resistance element,

(θ, T) is the temperature of the element (θ is in ^{o}C, T is in ^{o}K),

(θ_F, T_F) is the temperature of the cooling oil in the neighbourhood of the resistor (assumed constant under steady state conditions),

I_{DC} represents the direct current flowing through the elemental length δX and

I_{EFF} represents the effective value of the loading current $\sqrt{I_{DC}^2 + I_{AC}^2}$

H is the outer conductivity of the manganin resistance element.

In practice the value of H will depend upon factors such as the viscosity, velocity and turbulance of the cooling oil. It will also be a function of y in Fig. 1 since heat transfer from the resistance element of Fig. 1 becomes related to the problem of the convection cooling of a flat plate in a laminar or turbulent oil flow. However the initial assumption made with regard to the temperature distribution in the resistance element is associated with the assumption that H is a constant for the system.

If the cooling oil is assumed to flow past the resistance element at sufficient speed to keep the temperature differences $(\theta - \theta_F)$ and $(T - T_F)$ small, then the radiation term in equation (1.4) may be neglected. Equation (1.4) is then soluble provided the coefficients listed above are constant with x and steady state conditions apply.

The general solution of equation (1.4) is given by

$$\theta(x) = \theta_F + \frac{I_{EFF}^2 \rho_m}{Hpa} + C_1 e^{mx} + C_2 e^{nx} \qquad (1.5)$$

where

$$\left.\begin{aligned} m &= \frac{\alpha_{T_m} I_{DC}}{2\ ak_m} + \sqrt{\left[\frac{\alpha_{T_m} I_{DC}}{2\ ak_m}\right]^2 + \frac{Hp}{ak_m}} \\ n &= \frac{\alpha_{T_m} I_{DC}}{2\ ak_m} - \sqrt{\left[\frac{\alpha_{T_m} I_{DC}}{2\ ak_m}\right]^2 + \frac{Hp}{ak_m}} \end{aligned}\right\} \qquad (1.6)$$

m and n will in general be real, m being positive and n being negative. If, as is usual in a resistor in which convection cooling is the major means of heat transfer,

$$\frac{Hp}{ak_m} \gg \left[\frac{\alpha_T I_{DC}}{2\ ak_m}\right]^2 \qquad (1.7)$$

then

$$m \simeq -n \simeq \sqrt{\frac{Hp}{ak_m}} \qquad (1.8)$$

There are two regions of interest in the resistance element in this case, situated at $x = \pm \ell$.

At $x = \ell$

$$\underset{\text{Conduction}}{q_{m,c}(\theta_\ell)} + \underset{\text{Conduction}}{ak_m \frac{d\theta}{dx}} + \underset{\text{Peltier Effect}}{\pi_{m,c}(T_\ell)\, I_{DC_{m,c}}} = 0 \qquad (1.9)$$

where

$\pi_{m,c}(T_{\ell})$ is the Peltier coefficient of the junction of the manganin resistance element with the copper current terminals at absolute temperature T_{ℓ},

$q_{m,c}(\theta_{\ell})$ is the rate at which heat energy is removed from the copper-manganin junction by conduction in the current terminal at temperature θ_{ℓ} with direct current flowing from manganin to copper, and

$I_{DC_{m,c}}$ is the direct current flowing from manganin to copper in magnitude and direction.

The Peltier coefficient $\pi_{m,c}(T)$ is called positive if heat energy is absorbed as positive charge flows from manganin to copper.

Similarly at $x = -\ell$

$$q_{c,m}(\theta_{-\ell}) - ak_m \frac{d\theta}{dx} - \pi_{m,c}(T_{-\ell})\, I_{DC_{c,m}} = 0 \qquad (1.10)$$

Conduction Conduction Peltier Effect

where

$q_{c,m}(\theta_{-\ell})$ is the rate at which heat energy is removed from the copper-manganin junction by conduction in the current terminal at temperature $\theta_{-\ell}$ with direct current flowing from copper to manganin,

$I_{DC_{c,m}}$ is the direct current flowing from copper to manganin in magnitude and direction.

Substituting boundary conditions (1.9) and (1.10) into equation (1.5) while using equation (1.8), the value of $\theta(x)$ is given by

$$\theta(x) = \theta_F + \frac{I_{EFF}\rho_m}{Hpa} - \frac{1}{\sinh 2\ell\sqrt{\frac{HP}{ak_m}}} \cdot \left[\frac{q_{m,c}(\theta_\ell) + \pi_{m,c}(T_\ell) I_{DC_{m,c}}}{\sqrt{Hpak_m}} \cosh\left[\sqrt{\frac{HP}{ak_m}}(x+\ell)\right] + \frac{q_{c,m}(\theta_{-\ell}) - \pi_{m,c}(T_{-\ell}) I_{DC_{c,m}}}{\sqrt{Hpak_m}} \cosh\left[\sqrt{\frac{HP}{ak_m}}(x-\ell)\right]\right] \qquad (1.11)$$

If $e^{\ell\sqrt{\frac{Hp}{ak}}}$ may be assumed to be very large then

$$\theta(-\ell) \simeq \theta_F + \frac{I^2_{EFF}\rho_m}{Hpa} - \frac{q_{c,m}(\theta_{-\ell}) - \Pi_{m,c}(T_{-\ell})\, I_{DC_{c,m}}}{\sqrt{Hpak_m}} \qquad (1.12)$$

and

$$\theta(\ell) \simeq \theta_F + \frac{I^2_{EFF}\rho_m}{Hpa} - \frac{q_{m,c}(\theta_{\ell}) + \Pi_{m,c}(T_{\ell})\, I_{DC_{m,c}}}{\sqrt{Hpak_m}} \qquad (1.13)$$

The temperature difference between the points $x = \pm\ell$ is given by

$$\theta_{-\ell} - \theta_{\ell} = \frac{q_{m,c}(\theta_{\ell}) - q_{c,m}(\theta_{-\ell}) + I_{DC}\left[\Pi_{m,c}(T_{-\ell}) + \Pi_{m,c}(T_{\ell})\right]}{\sqrt{Hpak_m}} \qquad (1.14)$$

The Seebeck e.m.f. versus temperature difference as measured by the Author for a pair of manganin-copper junctions was found to possess a slope $\frac{dF}{dT}$ of approximately $0.5\,\mu V/°C$ at 20 °C. Equation (1.1) shows that the Peltier Coefficient at $\pm\ell$ will be of the order of 150uV at 300 °K and thus the Peltier cooling will be small.

Substitution of $(\theta_{-\ell} - \theta_{\ell})$ from equation (1.14) for $(T - T_F)$ in equation (1.3) gives the relationship between the Seebeck e.m.f. appearing between the potential terminal connections of the resistor shown in Fig. 1. If the quadratic coefficient B is small and the temperature differential $(\theta_{-\ell} - \theta_{\ell})$ is small this relationship becomes

$$F = A\,\frac{q_{m,c}(\theta_{\ell}) - q_{c,m}(\theta_{-\ell}) + I_{DC}\left[\Pi_{m,c}(T_{-\ell}) + \Pi_{m,c}(T_{\ell})\right]}{\sqrt{Hpak_m}} \qquad (1.15)$$

The presence of this e.m.f. may be represented by the D.C. voltage drop across a small resistor connected in series with the manganin resistance material. The thermal resistance R_{Th} is obtained from equation (1.15)

$$R_{Th} = \frac{A}{\sqrt{Hpak_m}} \left[\frac{q_{m,c}(\theta_\ell) - q_{c,m}(\theta_{-\ell})}{I_{DC}} + \Pi_{m,c}(T_{-\ell}) + \Pi_{m,c}(T_\ell) \right] \tag{1.16}$$

Since the Peltier Coefficient $\Pi(T)$ is a function of the Absolute Temperature, as shown in equation (1.1), and the temperature difference $(\theta_{-\ell} - \theta_\ell)$ is small, the sum $\Pi_{m,c}(T_{-\ell}) + \Pi_{m,c}(T_\ell)$ appearing in equation (1.16) above will be approximately independent of changes in $(\theta_{-\ell} - \theta_\ell)$. The 'thermal resistance' R_{Th} therefore, will in general consist of the sum of two terms, one which varies with the relative level of the direct measuring current and one which is independent of it.

The presence of Peltier cooling at each junction of the resistance element with the copper current terminals will cause the temperature distribution at each end of the resistance element to be dependent upon the magnitude of the direct current which passes. However, if the resistor is constructed symmetrically about the centre of the resistance element, and if bridge balance is performed with the use of direct current supply reversals, then the resistance value obtained will, to a first order, be independent of the changing temperature distributions and the magnitude of the direct measuring current used. (This point is discussed further in Chapter 9).

It will be appreciated that the above representation of the conditions of temperature distribution in a heavy current standard resistor under load are at best, rather poor approximations to the true situation, since it was assumed that convection cooling of the resistance element was constant as was the temperature over any cross-section. In large flat-plate-construction resistors, heat is exchanged between the resistor and the cooling oil by a process of natural convection aided by forced convection, provided usually by a motor driven stirring system which forces the cooling oil more quickly past the resistance element. This type of cooling results in temperature gradients across the width and length of the resistor which are usually impossible to calculate exactly. The presence of a non-symmetrical temperature distribution along the resistance element and the current terminals will mean that a nett quantity of heat will be liberated or absorbed in the resistance element and current terminals due to Thomson cooling and the presence of a direct current flowing in the direction of the nett temperature gradient. The quantity of heat transferred will depend upon the direction and magnitude of the direct current flow through the temperature gradient, and the resulting temperature change over any elemental length, will cause the direct current resistivity of this elemental length to change, due to its temperature coefficient. The magnitude of this effect will be small however in resistors constructed from manganin, since both Peltier and Thomson coefficients are small. Also, the use of forced convection cooling tends to eliminate large temperature gradients by keeping element temperatures at relatively low magnitudes. It is usual for precision standard resistors to be constructed with reasonable symmetry so that large assymetrical temperature gradients do not occur.

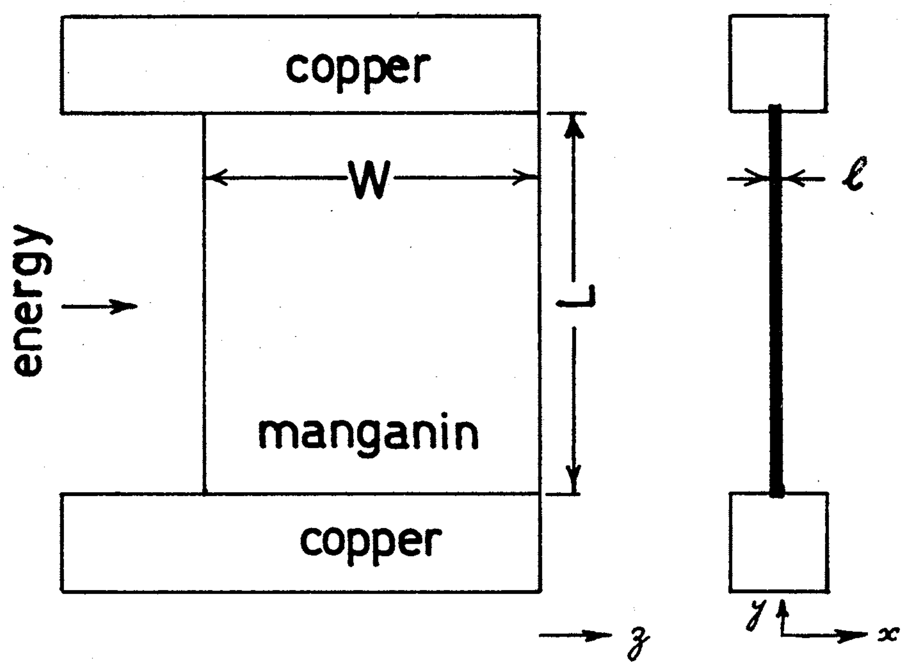
(1.2)

The Contributions made by 'Skin Effect' to the Possible Errors Caused by a Departure of the Loading Conditions from those Experienced at D.C.

(1.2.1)

Large Flat-Plate-Construction Resistor.

It is a well known phenomenon that as the frequency of an alternating current passing through a solid conductor is increased from zero (D.C.), the current density in the direction of flow is changed from a uniform value over the whole cross-section, to a condition where it is greatest at the outside



Heavy Current

Standard Resistor

Fig II

surfaces of the conductor. This effect is small at low frequencies, but becomes of greater significance when conductors of large cross-sectional dimensions (as in low valued Standard resistors) are considered[1]. In general, standard resistors of low value are constructed from sheets of resistance material rather than from conductors of circular cross-section and so the former condition will be the most important for consideration.

A typical construction for a low valued standard resistor appears in Fig. 11 where a resistance element is shown whose length L is greater than its width W, which is much greater than its thickness ℓ. If a sinusoidally varying electromagnetic wave is considered to be moving along the resistors current terminals from left to right, it will move with a velocity c essentially equal to the velocity of light in free space. If it is assumed also that the resistance of the current terminals is negligible, then the effective value of the electric field at any point on a cross-section of the resistance element taken parallel to the L and W dimensions will be constant. If this cross-section is considered to be the two outside surfaces of the resistance element whose dimensions are L and W, and if areas around the junction of the resistance element with the current terminals are neglected, then the electromagnetic waves from both surfaces will diffuse into the resistance element with a velocity given by c/n, found from a solution of Maxwell's equations in a conducting medium where

$$n = \sqrt{\frac{\sigma\mu}{2\omega\varepsilon_0}} \quad ; \qquad (1.17)$$

σ is the conductivity of the resistance material
μ is its relative magnetic permeability
ω is the frequency of the sinusoidal electromagnetic wave
ε_0 is permittivity of free space

providing that the frequency of the wave is such that

$$n^2 \gg \mu\varepsilon \qquad \text{for the medium} \tag{1.18}$$

where ε is the relative permittivity of the resistance material.

Substituting values for Manganin[10] while considering

$\sigma = 10^6$ mho. m^{-1}, angular frequency $\omega = 314$ and

$\mu = 1$, since Manganin is classed as non-magnetic[10],

(1.19)

$$n \simeq 1.5 \times 10^9 \tag{1.20}$$

The velocity of propagation of an electromagnetic wave along the current terminals is much greater than the velocity with which the wave diffuses into the resistance material. Thus all electromagnetic waves on a cross-section of the resistance element parallel L and W, may be considered to have the same phase. The problem then, of finding the current distribution in the resistance element, becomes one in which there are two in-phase signals impinging on either side of a large flat conductor of thickness ℓ and which interact as they propagate into the conductor. It is assumed that $E_x = E_z = 0$ for all x, y, z and t. This will generally be true, except at the extreme edge of the resistance element and in the immediate vicinity of the junction of the resistance element with the current terminals.

Solution of Maxwell's equations for the above conditions, gives the result for the electric field E_y at any point x where $0 \leqslant x \leqslant \ell$ in Fig. 11, such that

$$E_y = \exp(j\omega t)\left[A \exp(1 + j1)\left(-\frac{x}{\delta}\right) + B \exp(1 + j1)\left(\frac{x - \ell}{\delta}\right)\right] \tag{1.21}$$

where δ is the 'skin depth'[11] and is given by

$$\delta = (\tfrac{1}{2}\sigma\mu\mu_0\omega)^{-\frac{1}{2}} \tag{1.22}$$

Applying the Boundary Conditions

$$E_y(0,t) = E_y(\ell,t) = E_0 \exp(j\omega t) \tag{1.23}$$

$$E_y = \frac{E_0}{1+\exp(1+j1)(-\frac{\ell}{\delta})} \exp(j\omega t)\left[\exp(1+j1)(-\tfrac{x}{\delta}) + \exp(1+j1)\left(\tfrac{x-\ell}{\delta}\right)\right] \tag{1.24}$$

is obtained, which reduces to

$$E_y = E_0\left\{1-\tfrac{1}{6}\left(\tfrac{x}{\delta}\right)\left(\tfrac{\ell}{\delta}\right)^3+\tfrac{1}{2}\left(\tfrac{x}{\delta}\right)^2\left(\tfrac{\ell}{\delta}\right)^2-\tfrac{2}{3}\left(\tfrac{x}{\delta}\right)^3\left(\tfrac{\ell}{\delta}\right)+\tfrac{1}{3}\left(\tfrac{x}{\delta}\right)^4\right\} \exp j\left(\omega t+\left(\tfrac{x}{\delta}\right)\left(\tfrac{x-\ell}{\delta}\right)\right) \tag{1.25}$$

using the series expansions for $\exp(\theta)$, $\cos(\theta)$, and $\sin(\theta)$ for small θ. So that

$$J_y = \sigma E_0\left\{1-\tfrac{1}{6}\left(\tfrac{x}{\delta}\right)\left(\tfrac{\ell}{\delta}\right)^3+\tfrac{1}{2}\left(\tfrac{x}{\delta}\right)^2\left(\tfrac{\ell}{\delta}\right)^2-\tfrac{2}{3}\left(\tfrac{x}{\delta}\right)^3\left(\tfrac{\ell}{\delta}\right)+\tfrac{1}{3}\left(\tfrac{x}{\delta}\right)^4\right\} \exp j\left(\omega t+\left(\tfrac{x}{\delta}\right)\left(\tfrac{x-\ell}{\delta}\right)\right) \tag{1.26}$$

To determine the resistance variation with frequency due to 'skin effect', the total average power dissipation per unit length of the resistance element is equated to $\frac{1}{2}\left|I^2\right| R_{ac}$[11] to obtain

$$\frac{R_{ac}}{R_{dc}} = 1 + \frac{1}{720}\left(\frac{\ell}{\delta}\right)^4 \tag{1.27}$$

for small $(\frac{\ell}{\delta})$.

There are two ways in which the skin effect may cause errors in the measured value of load coefficient.

(a) The first is due to the increased value of resistance with frequency, meaning that the power dissipated is higher for the same r.m.s. loading current and the second

(b) is due to the fact that the redistribution of the current density will cause a change in the heat dissipated as a function of x as x moves between 0 and ℓ. Hence a change in the temperature distribution obtained with D.C. loading will occur causing an error in the measured value obtained for the resistance, because of the resistor's temperature coefficient.

Consider the first possibility (a).

If the 'skin depth' δ is evaluated for Manganin at 50 c/s using (1.19) it will be found to be about 7 cm. Under normal conditions of construction of heavy current resistance standards, ℓ will be around 1 mm. or less, so that (ℓ/δ) will be equal to about 0.01.

Inspection of equation (1.27) shows that for a sheet Manganin construction resistor where ℓ = 1mm., the error in the measured value of load coefficient obtained would be about 1.5 parts in 10^{11} of the measured load coefficient, which is negligible, especially if the load coefficient is small initially.

Consider the second possibility (b).

Variations in temperature over the thickness of the resistance element caused by the redistribution of the current density are considered here to be of minor importance in relation to the errors caused by the increase in resistance with frequency. This conclusion was reached in the light of the small dimensions of the element over which the temperature gradient is expected, the high thermal conductivity of metals and alloys in general and the small variation in J(x,t) with distance x as noted by equation (1.26) where fourth order changes in (x/δ) only occur.

(1.2.2)

Isolated Straight Wires of Circular Cross-Section.

For isolated straight wires of circular cross-section, it can be shown[12] that

$$\frac{R_{ac}}{R_{dc}} = \mathrm{Re}\left[\frac{\gamma a}{2} \cdot \frac{I_0(\gamma a)}{I_1(\gamma a)}\right] \qquad (1.28)$$

where

a is the radius of the wire

γ is the propagation constant $= (1 + j1)(1/\delta) =$

$= \sqrt{2}/\delta \;\; e^{j\pi/4}$

$I_0(\gamma a)$ is a Bessel function of the first kind and zero order defined by the series

$$I_0(\gamma a) \equiv 1 + \frac{(\frac{1}{2}\gamma a)^2}{(1!)^2} + \frac{(\frac{1}{2}\gamma a)^4}{(2!)^2} + \frac{(\frac{1}{2}\gamma a)^6}{(3!)^2} + \ldots \qquad (1.29)$$

$I_1(\gamma a)$ is a Bessel function of the first kind and first order defined by the series

$$I_1(\gamma a) \equiv (\tfrac{1}{2}\gamma a)\left[1 + \frac{(\frac{1}{2}\gamma a)^2}{1!\ 2!} + \frac{(\frac{1}{2}\gamma a)^4}{2!\ 3!} + \frac{(\frac{1}{2}\gamma a)^6}{3!\ 4!} + \ldots\right] \qquad (1.30)$$

The thickest wires used in the construction of standard resistance coils would be around 2 mm in diameter, so that the maximum value expected for the ratio of a/δ would be about 0.01.

Substituting this value for a/δ in the equation (1.28) gives the value of the ratio

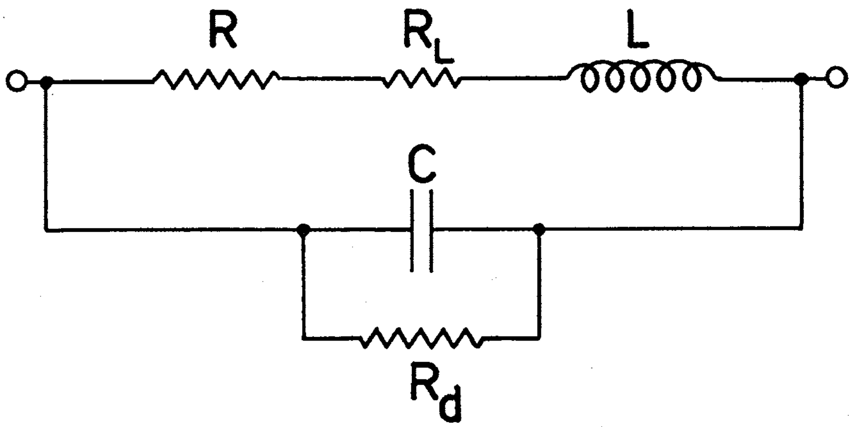
$$\frac{R_{ac}}{R_{dc}} \simeq 1 + 2.10^{-10} \qquad (1.31)$$

which is a negligible increase.

(1.2.3)

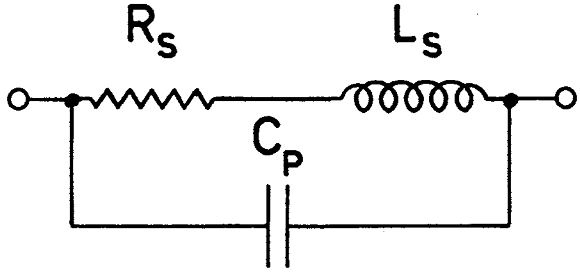
Discussion for Practical Resistors.

Calculations have demonstrated that the presence of 'skin effect' in resistance elements which are of similar construction to the examples considered will be negligible. The results obtained may reasonably be expected to apply to heavy current standards of cylindrical construction and to precision standards whose resistance elements are wound in helical form, provided



MILLER'S[1] EQUIVALENT CIRCUIT
OF A TYPICAL LABORATORY STANDARD RESISTOR

Fig III a



SIMPLE EQUIVALENT CIRCUIT
OF A RESISTOR

Fig III b

the diameters of typical cylinders and helices are relatively large in relation to the thickness of the resistance element.

(1.3)

Series Inductance, Shunt Capacitance and Conductance and Eddy Current Losses in the Alternating Current Loading Conditions.

(1.3.1)

Miller[1] suggests that the main capacitances in most direct current standard resistors occur between the metal former, which is used to improve the thermal properties of the resistor, or the shield can and the resistance element, and between the adjacent turns of the resistance element in higher valued resistance coils. He adds that the thin layer of dielectric used to insulate the winding may have an appreciable loss factor, and that the result of all these distributed effects may be represented by the parallel combination of a capacitance C and loss resistance R_d connected in parallel with the main resistance element.

Considering the equivalent circuit of a standard resistor to be as shown in Fig. 111 (a)[1], it can be shown that the equivalent series resistance R_x with variation in frequency is given by

$$R_x \simeq (R+R_L)\left(1 - \frac{R+R_L}{R_d} + \frac{\omega^2 L^2}{(R+R_L)R_d} + 2\left(\frac{\omega}{\omega_o}\right)^2 - \omega^2 (R+R_L)^2 C^2\right) \quad (1.32)$$

provided that $\left[R + R_L\right] \ll R_d$ and $\omega/\omega_o \ll 1$

where $\omega_o{}^2 = \frac{1}{LC}$,

R_L is the loss resistance due to eddy current losses and skin effect in the resistance element and any nearby conducting materials and

R_d is the loss resistance of the dielectric forming the capacitance C.

The ratio R/R_d increases linearly with frequency since $\tan\delta = R_d\omega C$ was found to be virtually constant with frequency[1]. Therefore

$$\frac{R}{R_d} = R\omega C/\tan\delta \simeq \frac{R + R_L}{R_d} \qquad (1.33)$$

The ratio R/R_d may be quite significant in high valued resistance standards, even at low value of ω[1].

Small changes in the value of R_x do not affect the measurement of load coefficient nearly as much as they do the measurement of resistance using Miller's method[1], since they only have an effect on the power dissipated. R_L was found to increase approximately with ω^2 and even though its presence may be significant when using Miller's method of resistance measurement, the value of R_L is generally so small in practice at low frequencies, that it may be neglected as far as its affect on the level of the power dissipated in the resistor under load is concerned.

To obtain an idea of just how large the value of the parasitic elements in the equivalent circuit must be in order to cause concern with load coefficient measurements in a practical case, consider the 1 ohm standard resistor L-259235 to be discussed in Chapter 4, whose load coefficient will be shown to be approximately + 250 parts in 10^7 per watt. Under normal circumstances the uncertainty with which the loading current may be set places a practical limit on the attainable accuracy of measurement for load coefficient. If the loading current may be set sufficiently accurately and if the smallest change detectable in the bridge balance is equivalent to about $\pm$ 1 part in 10^7 of the Unknown, then the maximum error permissable in the power supplied would correspond to an increase or decrease in load coefficient of about 1 part in 10^7 at 5 watts. Hence the maximum increase or decrease in R_x permissable with frequency would be 1/1250 proportional parts or 8 parts in ten thousand.

If this change in R_x is due solely to one parasitic element, the corresponding values of R_d, L and C which would cause it, are shown in Table (1.1) below, for a loading frequency of 50 c/s.

TABLE (1.1)

Parasitic Element of Fig. 111 (a)	Value Required to Cause 0.08% Change in R_x of equation (1.32) R_x = 1 ohm, ω = 314 rads/sec.
R_d	1250 ohms
R_L	800 microhms
L	second order effect, depends on R_d and C
C	120 uF

If a second standard resistor with a resistance value of 0.001 ohm and a similar level of load coefficient is considered, the corresponding values of the circuit elements will be as shown in Table (1.2).

TABLE (1.2)

Parasitic Element of Fig. 111 (a)	Value Required to Cause 0.08% Change in R_x of equation (1.32) R_x = 0.001 ohms, ω = 314 rads/sec
R_d	1.25 ohms
R_L	0.8 microhms
L	second order effect, depends on R_d and C
C	0.12 F

Hence for reasonable values of parasitic elements R_d, R_L, L and C in the equivalent circuit of Fig. 111 (a) at normal power frequencies, the effect of these elements on the level of the power dissipated will be negligible.

(1.3.2)

It has been calculated in section (1.2) that the contribution of skin effect to the resistance R_L for both wire and sheet construction standard resistors will be negligible. The presence of adjacent turns of the resistance element will cause an increase in the eddy current loss due to the proximity effect, but it will remain of similar order[1]. The presence of the metal former normally used to support the windings of low power standard resistors will contribute significantly to the resistance value of low valued standards[1], but in heavy current, low resistance standards, where no metal former is used, eddy current losses will be small. The eddy current contribution of a particular standard resistor to the resistance R_L of its equivalent circuit of Fig. 111 (a) are impossible to calculate without exact knowledge of the specific coil dimensions, but some limiting cases may be considered.

If it is assumed that the resistance coil is in the form of a long single-layer solenoid closely wound onto a non-magnetic metal former of equal length and that there exists complete magnetic coupling between the resistance coil and the former, then the self-inductances of the resistance coil and the coil former (single turn), are given by

$$L_c = \frac{\mu_o N_c^2 A}{\ell}$$

$$L_f = \frac{\mu_o A}{\ell} \qquad (1.34a)$$

respectively

where

A is the area of the cross-section of the solenoid formed by the resistance coil,

N_c is the number of turns in the resistance coil and

ℓ is the length of the former.

Similarly if there is complete magnetic coupling between the resistance coil and its former, the value of the mutual inductance will be given by

$$M_{cf} = \sqrt{L_f\ L_c}$$

$$= \frac{\mu_o\ N_c\ A}{\ell} \qquad (1.34b)$$

If the loss resistance value of the former is equal to R_f, it can be shown that this effective loss resistance referred to the primary side of the closely coupled air-cored transformer consisting of the resistance coil and its short circuited former, is given by

$$R_p = \frac{R_f\ \omega^2\ L_c\ L_f}{R_f^2 + \omega^2 L_f^2} \qquad (1.35)$$

Substituting for L_c, L_f and A gives

$$R_p = \frac{R_f (2\pi f)^2 \mu_o^2\ N_c^2\ (\pi r^2)^2}{\ell^2 \left[R_f^2 + \frac{(2\pi f)^2 \mu_o^2\ (\pi r^2)^2}{\ell^2} \right]}$$

The value of R_f would be expected to be of the order of milliohms, hence if $f = 50$ c/s, $r \simeq 0.02$ meter, and $\ell \simeq 0.1$ meter, the value of R_p reduces to

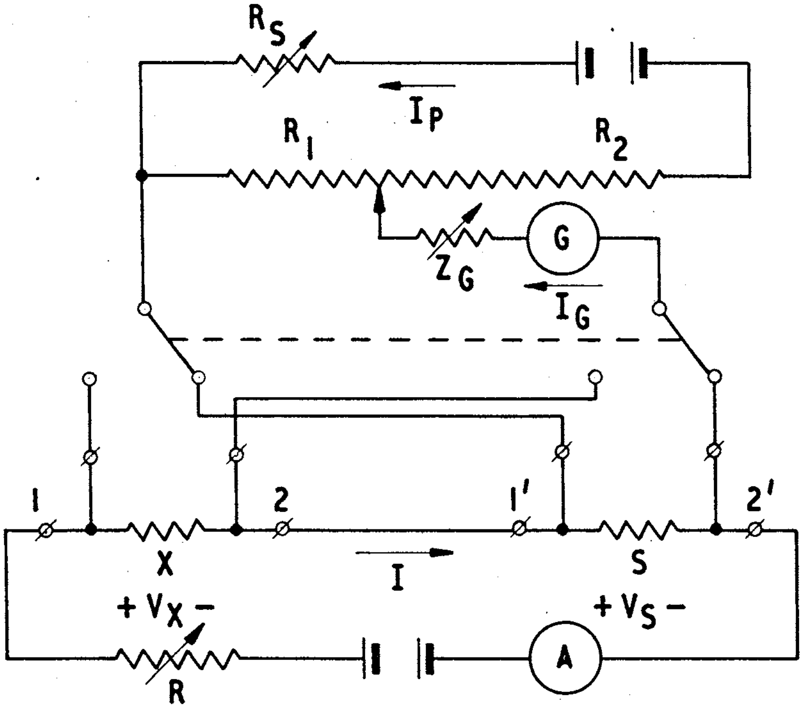
$$R_p = \frac{(2\pi f)^2 \mu_o^2\ N_c^2\ (\pi r^2)^2}{R_f\ \ell^2} \qquad (1.36)$$

If $f = 50$ c/s, $r = 0.02$m, $N_c = 10$ turns, $\ell = 0.1$m, $R_f = 10^{-3}$ ohm and $\mu_0 = 4\pi \cdot 10^{-7}$ Weber/ampere-meter, the value of R_p reduces to approximately 2.6×10^{-6} ohm.

Upon increasing the value of N_c to 100 turns, the value of R_p will increase to approximately 2.6×10^{-4} ohms, or by a factor of 100.

Resistance values of this order added to the resistance R_L as the contribution of eddy current losses in the equivalent circuit of Fig. 111 (a) only become of importance in standard resistors of low resistance value of the order of 0.1 and 0.01 ohms, where the added resistance value due to R_L becomes a significant proportion of the total. If the resistance coil is wound in bifilar fashion, the effective value of N_c will be reduced substantially with a corresponding reduction in the effective value of R_p.

It can be seen with reference to the calculations of section (1.2) that the contributions to R_L from eddy current effects in the winding former will be the major proportion of the total. Also it can be seen from the calculations above that the effective value of N_c is an extremely important quantity in determining the effective value of the resistance R_L. The contribution of R_L to the load coefficient of the resistor under consideration will depend on the magnitude of its load coefficient, but the effective value of R_L as calculated above will be a maximum for a given combination of parameters, since the coupling coefficient between the resistance coil and its winding former will generally be less than unity and the inductance values as calculated in equation (1.34a) will be too large. The effect of eddy current losses in other sections of the standard resistor structure will generally be substantially lower than that calculated for the winding former, since the mechanical separation between resistance coil and any other section will generally be very much greater than that of the resistance coil and its former.



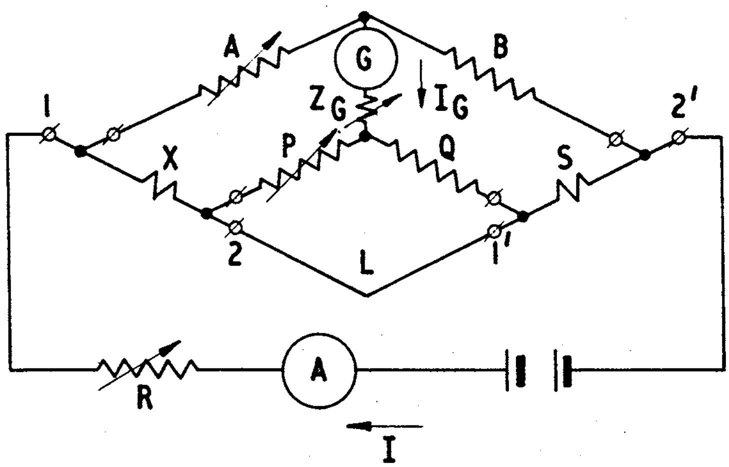
$I_G = 0$ at balance

$$\frac{V_X}{V_S} = \frac{I_X}{I_S} = \frac{X}{S}$$

for constant I & I_P

CONSTANT CURRENT POTENTIOMETER

Fig. IV(a)



$I_G = 0$ at balance

and

$$X = \frac{A}{B} \cdot S + \frac{QL}{P+Q+L}\left[\frac{A}{B} - \frac{P}{Q}\right]$$

KELVIN DOUBLE BRIDGE

Fig. IV(b)

CHAPTER 2

Direct Current Measurement Considerations.

(2.1)

Choice of Measuring System.

It is generally considered that the methods best suited for inter-comparison of 4-terminal direct current standard resistors of low resistance value are the direct current comparator,[13, 14] the potentiometric method[15] and the method based upon the Kelvin Double Bridge. Since a direct current comparator was unavailable, the choice of measuring system for measurements on low values of resistance was limited to either the D.C. potentiometer or the Kelvin Double Bridge. For resistance values above about 10 ohms, where the effect of connecting leads may be accurately allowed for, the Wheatstone Bridge method or one of its modifications[2] may be used.

(2.1.1)

Potentiometric Methods versus Kelvin Double Bridge.

Fig. 1V (a) and (b) show the classical schematic diagrams of a constant current Potentiometric resistance measuring circuit and a Kelvin Double Bridge respectively. There are three advantages which the Kelvin Bridge has over the Potentiometer.

The first is that the direct measuring current passing through the Unknown and Standard (marked X and S respectively), may be permitted to be less stable in measurements made with the Kelvin Bridge than in measurements made with the potentiometer. Moderate random fluctuations in the supply current of the Kelvin Bridge add only an annoyance factor and at worst introduce errors only if fluctuations are large enough to change the effective loading current.

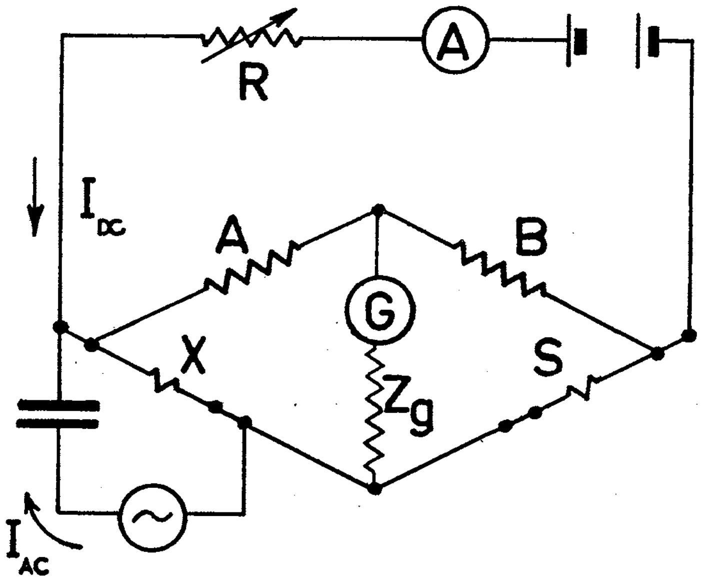
Measuring current drift during resistance measurements made potentiometrically can be accommodated by taking a series of timed readings, but the rate of drift must be accurately known.

This adds a large uncertainty factor if drifts are large.

There are commercially available, potentiometers in which the ratio of the direct voltage drops present across the Unknown and Standard may be measured with an extremely short period elapsing between voltage balances, hence current stability requirements are relaxed considerably. These units contain two sets of wiper arms which are connected to the same set of potentiometer resistance coils. The voltage drop across the Standard resistor is balanced with one set of dials and the voltage drop across the Unknown is balanced with the other. The voltage drops across the Unknown and Standard may be connected to their respective potentiometer dials in turn, simply by moving a switch. Hence balances may be checked with time separations of fractions of a second.[16]

The second advantage is in relation to thermal e.m.fs. Any thermal e.m.fs which appear in the galvanometer circuit of Fig. 1V (a) add directly as an error in voltage measurement. If a resistance ratio, Unknown to Standard of say 10 to 1 is used, the error obtained can be quite large. Galvanometer reversals may be used but the result of reversal may simply be to add worse thermals than before. The Kelvin Bridge however, uses supply reversals to eliminate thermal effects.[4,17] This method provides an electrical galvanometer zero which is displaced in general from its mechanical zero and provided the thermals vary sufficiently slowly with time, the result is satisfactory in practice. Supply reversals are also possible with the potentiometer, but this means that both potentiometer and measuring direct current supplies have to be interrupted simultaneously or else the galvanometer circuit opened.

Precision potentiometers presently available make voltage measurements below about 2.2 volts and may have subdivisions down to 0.1 uV. Hence to obtain measured differences in resistance of parts in 10^7, measurements should be made on voltages of the order of 1 volt. In tests performed on a 1 ohm resistor, this would constitute a power dissipation level in it of 1 watt,



Wheatstone Bridge

FEATURING PARALLEL- FED A.C. LOADING.

Fig V

which would in general be too high to be considered as negligible load.

Bridges in general, do not suffer from this limitation and provided sufficient galvanometer sensitivity is available, the measurement accuracy is limited only by the environment or thermal noise and Brownian Motion in the detector.

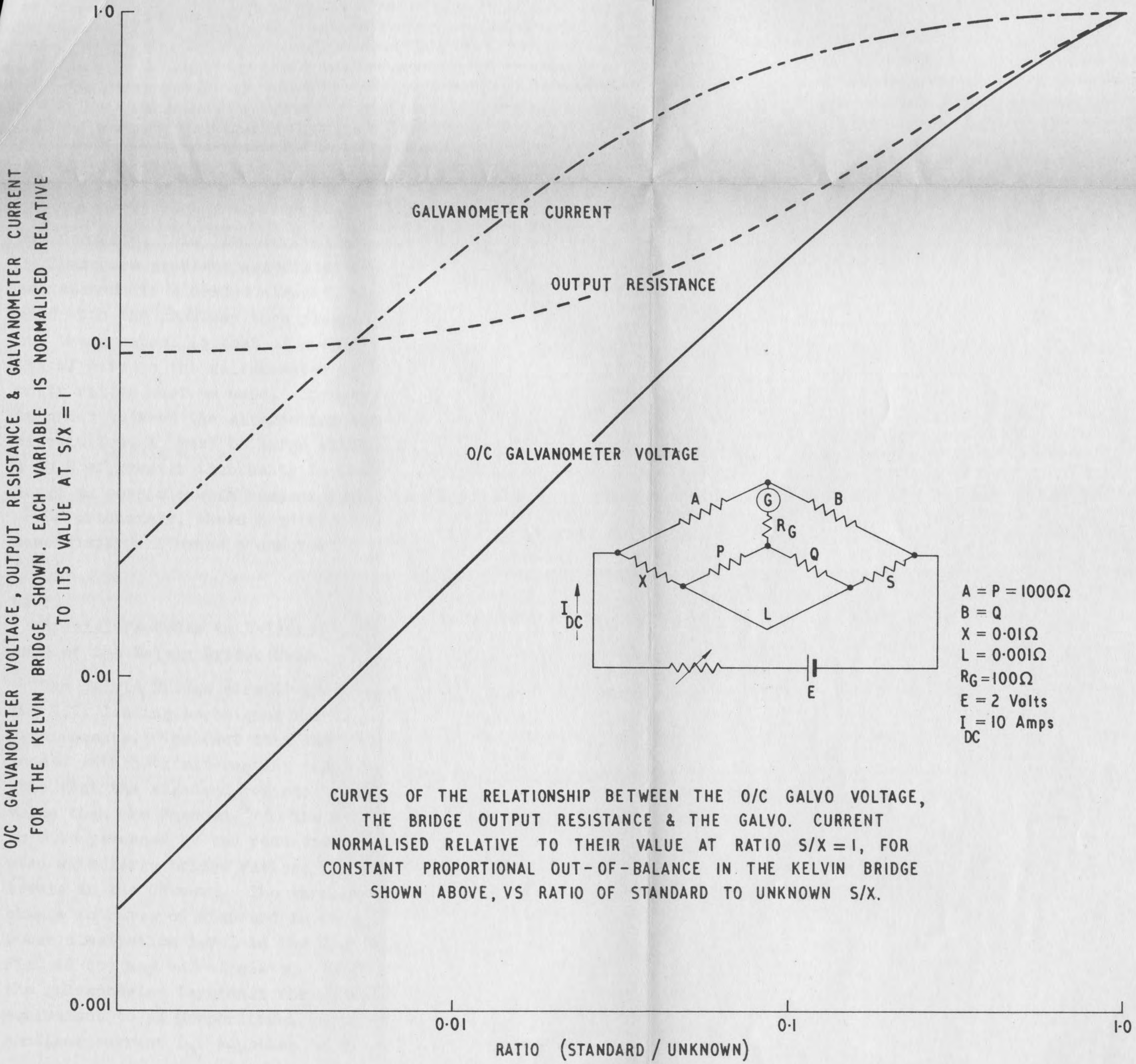
Apart from these limitations there are points peculiar to the method of Alternating Current loading.

The greatest problems encountered with this method are in keeping alternating currents in parts of the system which should not be loaded, to levels at which their presence is undetectable. This is especially important in the galvanometer circuit. The presence of the potential leads to the Unknown and Standard in the A, B, P and Q arms of a Kelvin Bridge means that the resistance values used in these Arms must be of sufficient value to make the potential lead resistances negligible. High bridge arm impedances are indeed desirable if the loading effects of alternating currents in the bridge arms are to be kept low. The disadvantages of having to make the A and B arm resistances large in the Kelvin Bridge, are off-set by the improved discrimination available with a bridge generally, and the elimination of any problems associated with measuring current stability or galvanometer thermals as in a potentiometric method.

(2.1.2)

Use of the Wheatstone Bridge at Higher Values of Resistance.

The value of resistance at which 4-terminal and 2-terminal measurements produce identical resistance values will vary with the construction of the resistor under test and the measurement accuracy required. Above this value the Wheatstone Bridge becomes more attractive than the Kelvin Bridge because it does not have the increased output impedance contributed by the parallel combination of the P and Q arms of the Kelvin Bridge. For most types of construction of the higher values of resistance and for normal laboratory resistance inter-comparison



CURVES OF THE RELATIONSHIP BETWEEN THE O/C GALVO VOLTAGE, THE BRIDGE OUTPUT RESISTANCE & THE GALVO. CURRENT NORMALISED RELATIVE TO THEIR VALUE AT RATIO S/X = 1, FOR CONSTANT PROPORTIONAL OUT-OF-BALANCE IN THE KELVIN BRIDGE SHOWN ABOVE, VS RATIO OF STANDARD TO UNKNOWN S/X.

Fig. VI

measurements, this limiting value is about 10 ohms.

There are problems associated with the A.C. loading of high resistances in a bridge circuit, and some of these are associated with the problems that plague high resistances generally, but the greatest is that of keeping the alternating current and its effects in the galvanometer at a minimum, when unity or near unity ratios must be used. Inspection of Fig. V will show that in order to keep the alternating current in the galvanometer circuit low, Z_g must be large with respect to X, and a ratio of A to B of greater than unity is desirable while A should be as large as possible with respect to X.

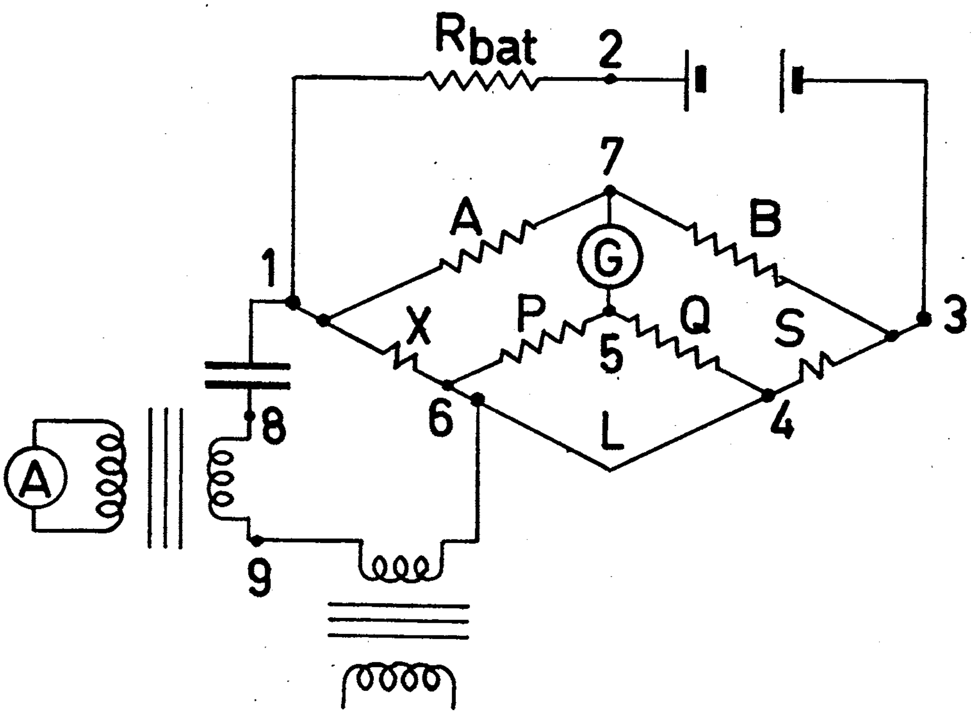
Unfortunately, these conditions reduce the bridge's D.C. sensitivity and hence a compromise solution must generally be found.

(2.2)

Special Features in Relation to the Resistance Values of the Arms of the Kelvin Bridge Used.

The Kelvin Bridge circuit used for the investigation of A.C. and D.C. loading techniques had to satisfy both A.C. and D.C. requirements. The fact that the resistor under test had to be loaded with both alternating and direct currents, made it desirable that the standard resistor used as Standard was lower in value than the Unknown.[4,5] The optimum value of the ratio of A to B is governed by the reduction in bridge sensitivity associated with large bridge ratios, for constant power dissipation levels in the Unknown. The variation of bridge sensitivity with change in ratio of Standard to Unknown, for a constant D.C. power dissipation level in the Unknown in the Kelvin Bridge of Fig. 1V (b) may be calculated. The open circuit voltage across the galvanometer terminals for a bridge unbalance condition equivalent to δX proportional parts in the Unknown and a constant current I_{DC} supplied to the bridge is given by

$$E_{o/c} = \frac{I_{DC}\, BX\delta X}{A + B + X + S + \frac{(P + Q)L}{P + Q + L}} \qquad (2.1)$$



KELVIN BRIDGE

Showing Possible Bridge Earthing

Points.

Fig VII

The output impedance at the galvanometer terminals of a balanced Kelvin Bridge is given by

$$R_{out} = \frac{(A + X + P)\ (B + S + Q)}{A + B + X + S + P + Q} \tag{2.2}$$

The curves shown in Fig. V1 are for $E_{o/c}$, galvanometer current and R_{out}, normalised to their values at S/X = 1, against Bridge Ratio S/X for the typical case of

$$\begin{aligned} A &= P = 1\ 000 \text{ ohms} \\ X &= 0.01 \text{ ohms} \\ E &= 2 \text{ Volts} \\ I_{DC} &= 10 \text{ Amps} \\ R_g &= 1\ 00 \text{ ohms} \end{aligned} \tag{2.3}$$

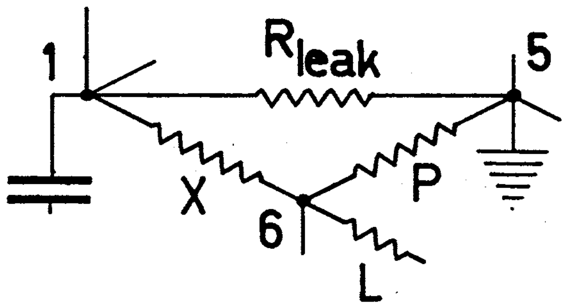
(2.3)

The Kelvin Bridge Earth Point and the Associated Guarding Circuit Found Necessary.

When loading the Unknown with alternating current it is desirable to earth the bridge circuit or the alternating current supply at some convenient point so as to keep possibly dangerous alternating potential differences at reasonable levels. The possibility of using an earth balance is reasonable, but it has the disadvantage that the earth balancing resistors used should be of low resistance value. Earthing the bridge circuit poses problems with D.C. leakage currents, especially around the galvanometer, and so some care has to be taken when setting out the bridge so as to minimise them.

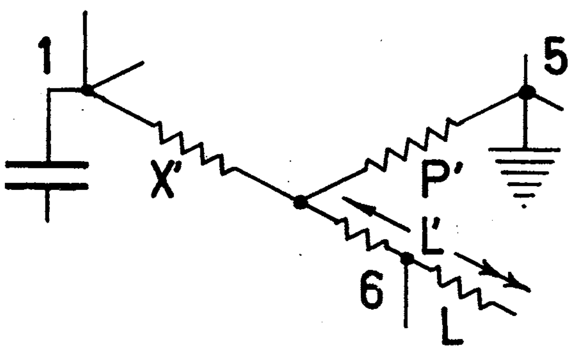
Referring to the Kelvin Bridge of Fig. V11, it may be seen that the leakage across the A, B, X and S arms will be the most troublesome, since the resistance of these bridge arms affect the bridge balance condition given by equation (2.4) below to a first order of magnitude.

$$X = \underbrace{\frac{A}{B} \cdot S}_{\text{Wheatstone Balance}} + \underbrace{\frac{L\ Q}{P + Q + L}\left[\frac{A}{B} - \frac{P}{Q}\right]}_{\text{Error Term}} \tag{2.4}$$



Leakage Between Bridge Points 1 & 5

Fig VIII a



Equivalent Circuit of Fig VIIIa

Fig VIII b

Fig VIII

In a Kelvin Bridge used to measure load coefficient the A, B, P and Q arm resistances are invariably much larger than the resistances X and S and hence the most sensitive arms relatively to leakage paths are the A and B arms. Thus the bridge points 1, 3 or 7 should not be earthed if other alternatives exist. Point 2 should be discarded also since leakage to earth from the bridge points 4, 5, 6 and 7 would affect one side of the bridge more than the other. Provided good quality capacitors are used, points 8 and 9 will be at the same D.C. potential as point 6, but the alternating voltages to earth at any of the bridge points may become dangerous at high A.C. loads if either of points 8 or 9 are earthed.

Leakage to earth from point 7 would shunt only the galvanometer if point 5 were earthed and so cause no error in bridge balance. But because of the relatively high impedance of the P and Q arms, if a leakage to earth is present from either of the bridge points 1 or 3, the effect on the X or S arms could be significant. A leakage resistance to earth from bridge point 1 equal to R_{leak} may be represented as shown in Fig. V111 (a), which by use of the delta-star transformation, may be represented by Fig. V111 (b), where

$$X' = \frac{X\, R_{leak}}{P + X + R_{leak}} \tag{2.5}$$

$$P' = \frac{P\, R_{leak}}{P + X + R_{leak}} \tag{2.6}$$

$$L' = L + \frac{X\, P}{P + X + R_{leak}} \tag{2.7}$$

Inspection of the expression for X' in equation (2.5) above shows that

$$X' \simeq X\left(1 - \frac{P}{R_{leak}}\right) \tag{2.8}$$

since $X \ll P$ and $P \ll R_{leak}$

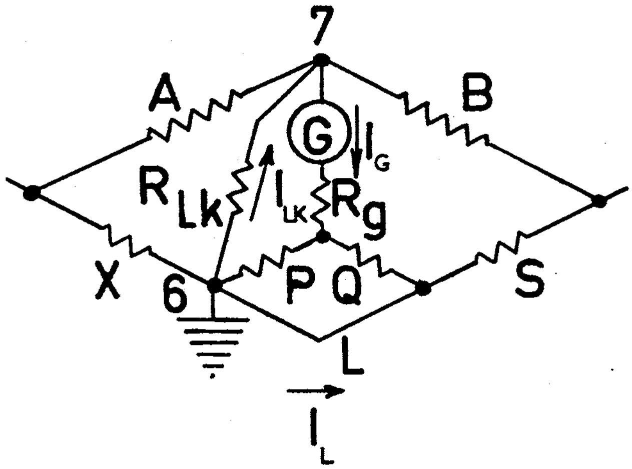
Hence an error of P/R_{leak} proportional parts in the resistance value of the X arm may result from the presence of R_{leak} at the bridge point 1.

Any electrostatic charge in motion in the neighbourhood of the bridge points 1 and 3 will force large transient currents through the galvanometer due to the relatively large impedance of the P and Q arms. Sensitivity of the galvanometer to movement by the operator near the bridge points 1 and 3 has been experienced by the Author when setting up the first Kelvin Bridge used. A large earthed electrostatic screen was placed in front of the whole bridge set-up to try to reduce this effect. The difficulty was that the bridge points 1 and 3 were connected to the D.C. reversing switch so that when reversal took place, the proximity of the operator's hand caused large galvanometer deflections. These were found to be reduced somewhat if the operator earthed himself before approaching the reversing switch, but even then the galvanometer deflections were annoying.

It was found in practice that the sensitivity to operator motion near the reversing switch was reduced drastically with point 6 earthed. The area around bridge point 7 was still quite sensitive to operator motion, but this was reduced with the aid of the large electrostatic screen mentioned above.

It can be shown that the resistance R_{leak} to earth from the bridge point 1 will produce a value of X' which is in error by $- X/R_{leak}$ proportional parts. This leakage effect is much smaller than that given by equation (2.8). The advantage of earthing the bridge point 5 with regard to leakage currents to earth from the bridge point 7 no longer applies, since the presence of a leakage resistance from the bridge point 7 will cause an error deflection in the galvanometer which will not cancel upon supply reversal. These points may be seen by consideration of Fig. 1X and the following discussion.

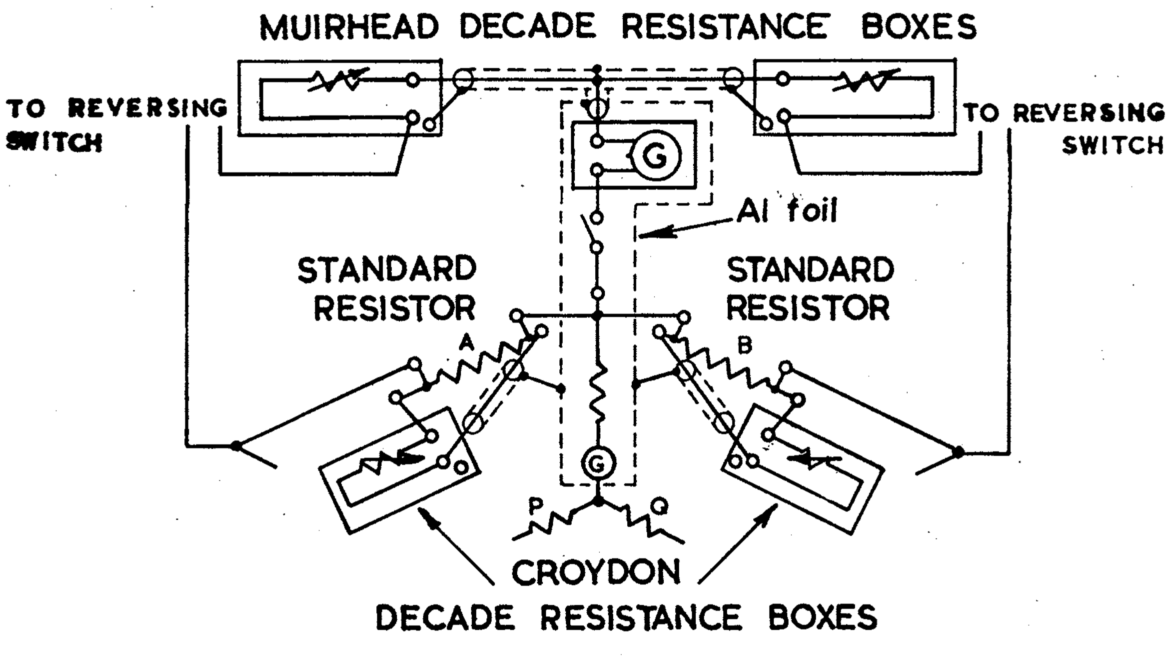
Let the bridge be balanced before R_{leak} is connected and let the current in the link be I_L so that the current through the P



KELVIN BRIDGE

with leakage from bridge point 7
to earth

Fig IX



SCHEMATIC DIAGRAM

of guard balancing circuit
for low valued resistors.

Fig X

and Q arms will be equal to $I_L L/(P + Q)$. This current causes a voltage drop of $I_L LP/(P + Q)$ across the P arm.

After connecting R_{leak}, where $R_{leak} \gg P, R_g$, a current I_{leak} will be obtained in the galvanometer which is approximately equal to $I_L LP/R_{leak}(P + Q)$. Upon reversal of the current I_L the same current I_{leak} will flow in the opposite direction, causing the deflection obtained previously to reverse. Thus, in order to reduce the galvanometer deflection to zero, the bridge must be unbalanced and a systematic error produced.

Measurements made with a Megohm Meter on the Croydon resistance boxes used as shunts for the A and B arms of several of the Kelvin Bridges used in practice showed leakage resistances from terminals to case to be about 5 by 10^9 ohms. With a measured galvanometer sensitivity of 80 000 mm/uA and typical values of circuit elements, it was found that this order of leakage was sufficient to cause several parts per million error in the Unknown in measurements made using an unguarded bridge.

A guard was placed around the bridge point 7 by constructing all its connecting leads from coaxial cable. The inner was used as the connecting lead and the outer used as a guard. All guards were connected together and in turn, connected to the centre point of two resistance decade boxes, which were connected between bridge points 1 and 3 (see Fig. X). A galvanometer was connected between bridge points 7 and the guard through a tapping key, for use in obtaining guard balance. 'Guard balance' was obtained at the start of each day's measurements, and spot checks taken at various times throughout the day.

(2.4)

Other Guarding Systems Considered Necessary.

In the Wheatstone Bridge set up for the performance of load coefficient measurements on 1 000 ohm standard resistors (Chapter 6), it was necessary to make the battery circuit impedance high in relation to the resistances being tested, so as to reduce the alternating current level flowing in the battery

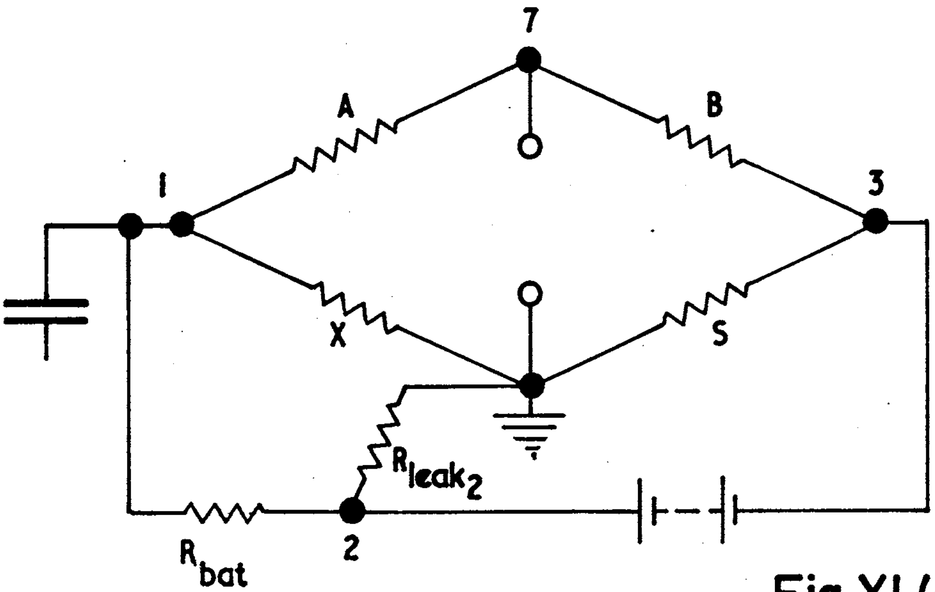
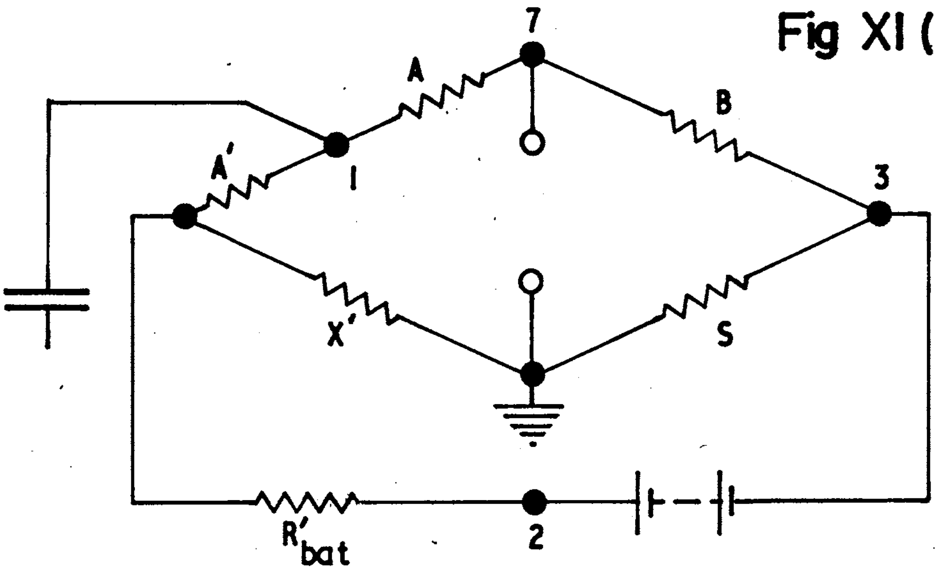


Fig XI (a)

Fig XI (b)



WHEATSTONE BRIDGE WITH LEAKAGE TO EARTH FROM BRIDGE POINT 2.

Fig XI.

circuit. This alternating current must be allowed for when calculating the actual loading current, and hence it is desirable to keep it as small as possible so that the relative accuracy with which it is required to be known, may be reduced.

Since the junction of the A and B arms with the galvanometer in the Wheatstone Bridge was brought to earth potential to direct current at balance, the guarding system of section (2.3) above was removed.

The battery supply voltage used was 90 volts and the battery circuit series resistance R_{bat} was 150 000 ohms approximately. Because of this large value of resistance required in the battery circuit, and because of the relatively large values of resistance in the X and S arms, it was decided that leakage resistances to earth from other bridge points would have to be investigated more closely. It can be shown that any leakage to earth across the X or S arms will appear as a direct error of $-\frac{X}{R_{leak_X}}$ and $-\frac{S}{R_{leak_S}}$ proportional parts respectively, so that R_{leak_X} and R_{leak_S} must be made as large as possible.

If a leakage resistance R_{leak_2} to earth exists from bridge point 2 in Fig. X1(a), a delta-star transformation may be applied to the three resistors R_{bat}, R_{leak_2} and X, and so produce the equivalent circuit of Fig. X1(b) where

$$X' = X\left(1 - \frac{R_{bat}}{R_{leak}}\right) \tag{2.9}$$

$$A' = X \cdot \frac{R_{bat}}{R_{leak}} \tag{2.10}$$

$$R_{bat}' = R_{bat}\left(1 - \frac{R_{bat}}{R_{leak}}\right) \tag{2.11}$$

provided $R_{bat} \gg X$.

It may be seen by inspection of the circuit of Fig. X1(b) that for D.C. balance conditions, a resistance A' is added in

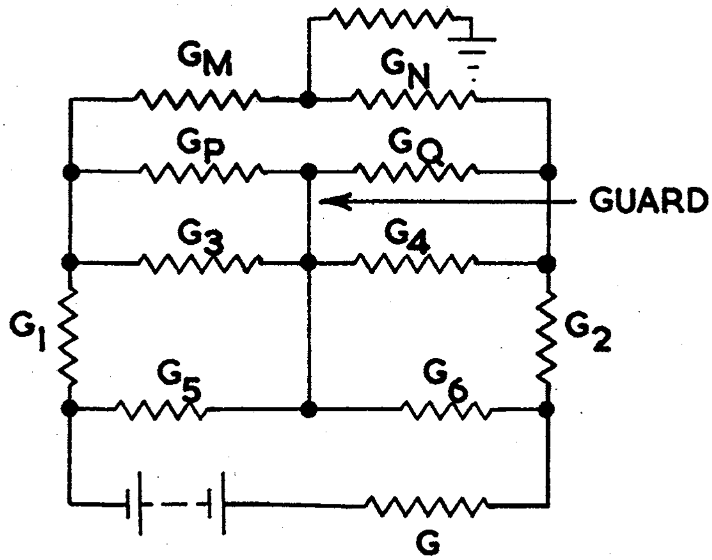
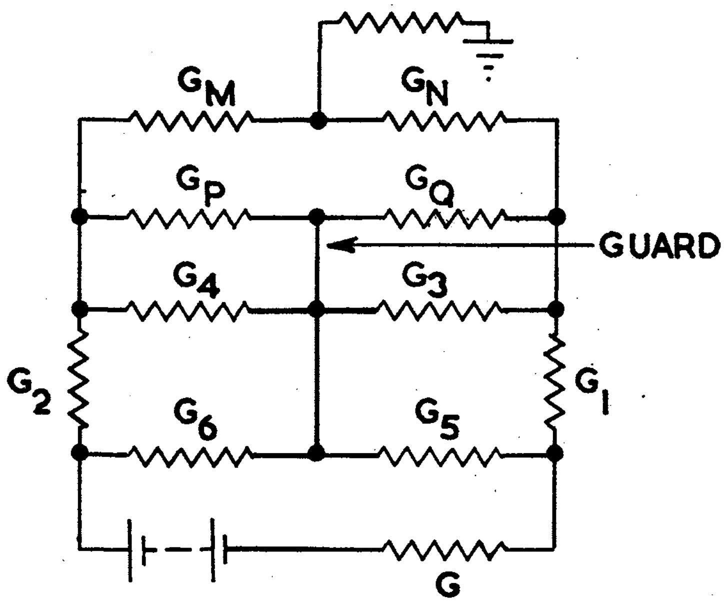


Fig XII (a)

Fig XII (b)



BALANCING OF GUARD

WITH VARYING LEAKAGES TO GUARD

Fig XII

series with the A arm and a resistance A' subtracted from the X arm.

The error caused by the presence of R_{leak_2} may be reduced by dividing the resistance R_{bat} into two equal resistances R_{bat_1} and R_{bat_2}, and placing them one on either side of the battery. If approximately equal leakage resistances exist to earth from either side of the battery, then their effects on bridge balance condition would cancel.

A guard system was placed around the battery supply circuit and all leads connected to the bridge points 1 and 3 of Fig. X1 (a). The guard was brought to earth potential with the adjustment of decade resistances placed between bridge points 1 and 3. It was found that in practice, in order to make the 'guard balance' condition independent of the direction of bridge measuring current flow, the resistances R_{bat_1} and R_{bat_2} had to be adjusted away from approximate equality. The values were varied so that the sum $R_{bat_1} + R_{bat_2}$ was a constant value depending on the bridge measuring current required.

Consider the two circuits as shown in Fig. X11 (a) and (b). Assume that the main resistance bridge is balanced so that the bridge points 7 and 5 of Fig. X1 (a) may be assumed to be isolated, thus forming the conductances G_M and G_N in Fig. X11 (a) and (b) from the main bridge arms after delta-star transformation. The conductances G_P and G_Q are the 'guard balance' conductances, while G_1 and G_2 are the conductances of R_{bat_1} and R_{bat_2} respectively.

There will in any practical circuit be always some leakage resistances to guard from various points in the battery circuit. These leakages will normally be distributed along connecting leads etc. but their presence may be represented by lumped resistances from convenient guard or circuit points without significant error. The circuit of Fig. X11 (b) represents the circuit of Fig. X11 (a) with the battery supply circuit reversed.

Analysis of these circuits will show that the two relationships which must be satisfied such that the voltage between the guard circuit and earth is zero are ...

$$G_1G_6(G_2 + G_4 + G_M + G_Q) - G_2G_5(G_1 + G_3 + G_M + G_P)$$
$$= G_1G_2 \quad (G_3 - G_4) + (G_P - G_Q) \qquad (2.12)$$

$$G_1G_6(G_2 + G_4 + G_M + G_P) - G_2G_5(G_1 + G_3 + G_M + G_Q)$$
$$= G_1G_2 \quad (G_3 - G_4) - (G_P - G_Q) \qquad (2.13)$$

for the condition that $G_M = G_N$.

For both relationships to apply simultaneously

$$G_P = G_Q \qquad (2.14)a$$
$$G_3 = G_4 \qquad (2.14)b$$
$$G_1 = G_2 \cdot \frac{G_5}{G_6 + G_2\frac{(G_6 - G_5)}{G_3 + G_M + G_P}} \qquad (2.14)c$$

It was found in practice that the 'guard balance' condition could be obtained with the adjustment of R_{bat_1} and R_{bat_2} only after balance of the guard resistors. Hence it must be assumed that the condition (2.14)b applied approximately. There were extra circuit elements connected to bridge points 1 and 3 which will be discussed in Chapter 3, but their leakages to guard will be accounted for in practice, in the values of G_P and G_Q of Fig. X11 (a) and (b).

It was noticed also, that when the 'guard balance' was adjusted correctly, the electrical zero of the main bridge galvanometer coincided with the zero obtained when the battery circuit was open circuited. With the 'guard balance' circuit correctly adjusted, the guard is always at earth potential, assuming that the open circuited conductance of the reversing switch between battery circuit and bridge is very much less than the conductances in the 'guard balance' circuit, and hence any leakage path from the guard to bridge point 7, which is also at earth potential when the main bridge is balanced, will carry no current.

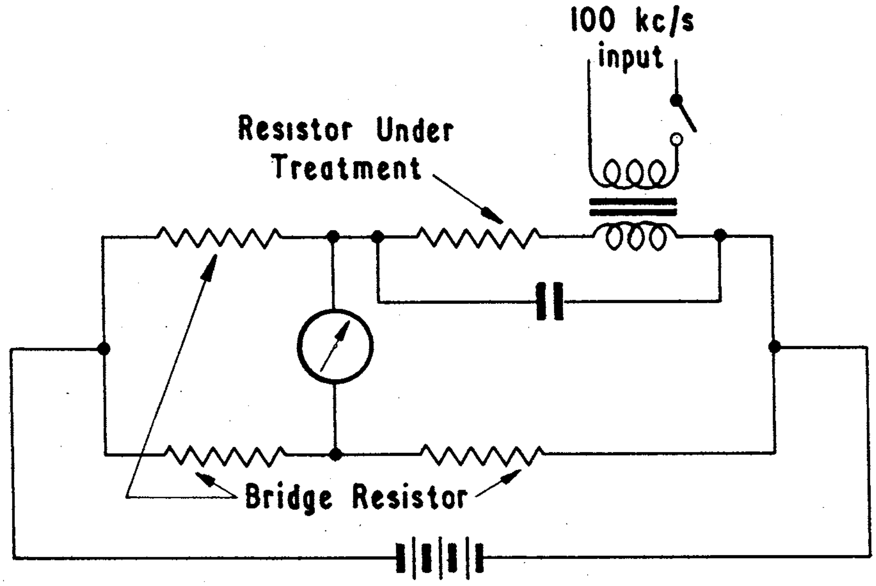


Fig. XIII

SERIES FEEDING OF ALTERNATING LOADING CURRENT AS USED BY GIBBINGS[6]

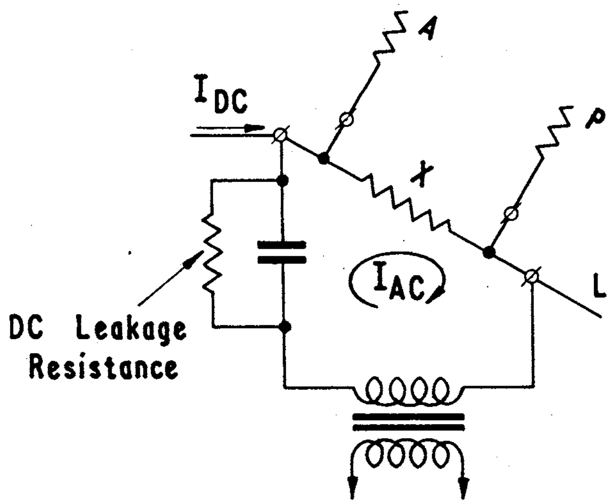


Fig. XIV

SHUNT FEEDING OF ALTERNATING LOADING CURRENT AS USED IN THIS THESIS.

CHAPTER 3

Problems Associated With A.C. Loading.

(3.1)

Choice of Loading Connection.

The A.C. loading of standard resistors can be performed by either series or shunt feeding. Another method uses a bridge type of connection[18] but it is not suitable for loading standard resistors of usual construction.

Series feeding has been used by Gibbings[6] to detect the presence of zero load coefficients in calculable A.C. to D.C. transfer standards. The circuit used is reproduced in Fig. X111 and it may be seen there that the secondary of the loading transformer is in series with the resistance being adjusted. This method of loading is suitable when measurements are to be made on relatively high values of resistance as was the case here (100 and 1000 ohms), for then the resistance of the supply transformer's copper secondary winding, with its high temperature coefficient of resistance, is small compared to resistances being measured and resistance changes due to its temperature rise, probably negligible.

If shunt feeding of the Unknown is used however, the resistance error in the Unknown caused by the presence of the loading circuit may be made small. In Fig. X1V, the resistance shunting the Unknown is made up of the D.C. series resistance of the secondary of the loading transformer in series with the leakage resistance of the D.C. blocking capacitor C, plus any circuit leakage from connecting leads etc. With good quality capacitors and reasonable insulation in all connecting leads, the leakage resistance will be of sufficient magnitude that its effect may be neglected. Shunt feeding was used on all resistors tested in this thesis.

The bridge connection method was used by Morgan[18] while testing steel cored aluminium conductors. The resistance under

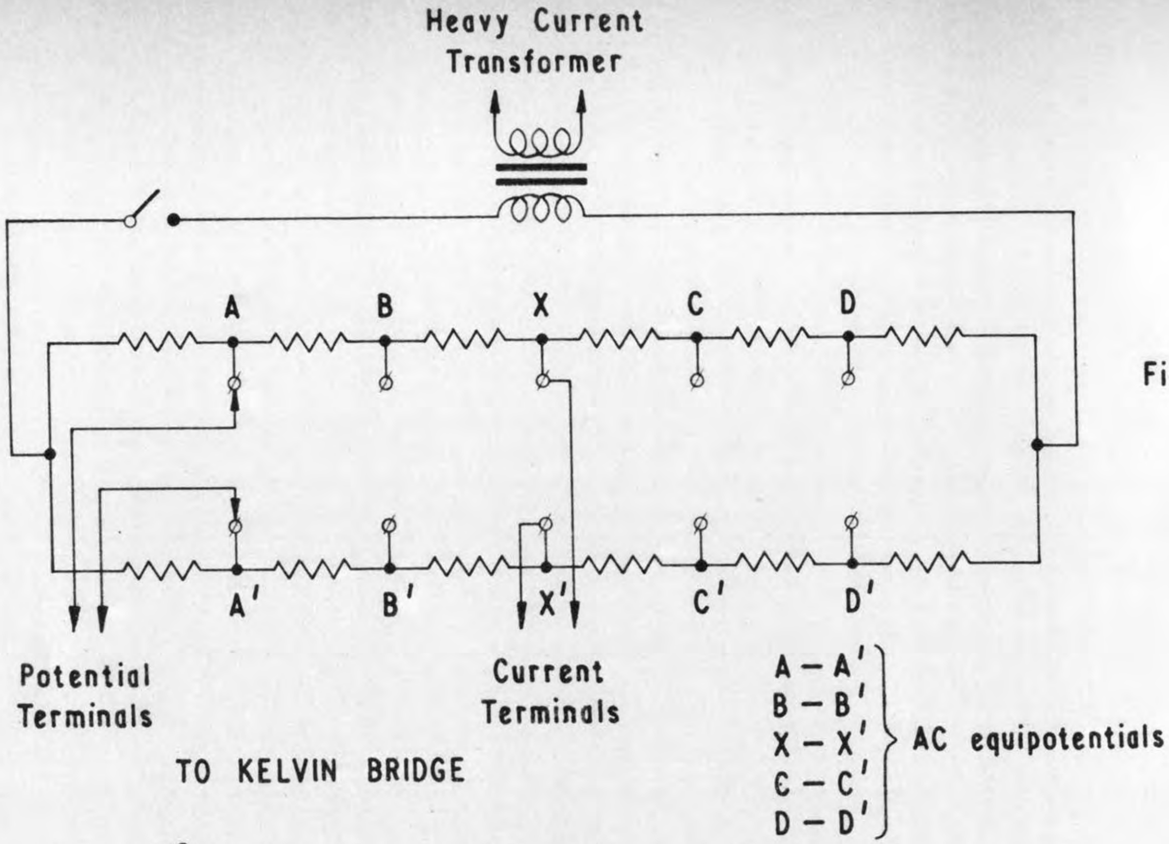


Fig. XV

MORGAN'S[18] METHOD OF MEASURING THE DC RESISTANCE OF CABLE WHILE UNDER AC LOAD.

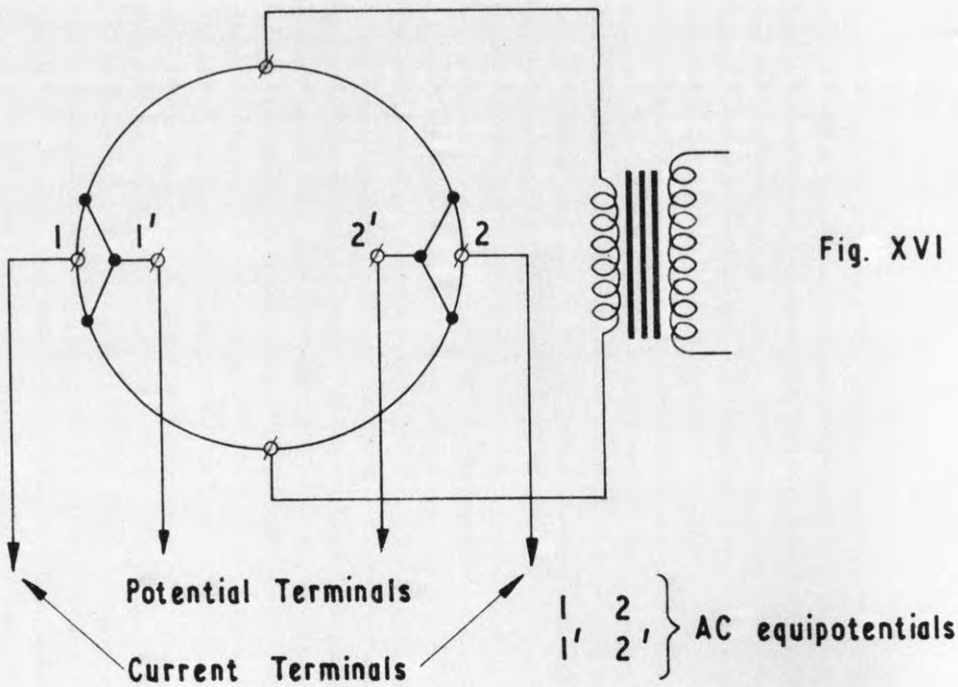


Fig. XVI

POSSIBLE METHOD OF CONSTRUCTION FOR A HEAVY CURRENT RESISTOR TO ENABLE THE USE OF MORGAN'S LOADING METHOD.

test is constructed in four sections, see Fig. XV, which form a balanced Wheatstone Bridge for both D.C. and A.C. connections. The alternating loading current is applied between two D.C. equipotentials so that the effect of the resistance of the transformer secondary winding on the measured D.C. resistances is eliminated. Similarly the D.C. connections to the resistor are at A.C. equipotentials, provided shunt reactances are balanced, and hence there is no alternating current passed to other parts of the measurement circuit (notably the galvanometer). It is possible to envisage the type of construction of a heavy current standard resistor which would utilise this method, one is shown in Fig. XV1, but the loading of existing standards with this method of connection would be impossible.

(3.2)

Reduction of the Effects and Level of Alternating Currents in the Galvanometer Circuit.

(3.2.1)

At the frequency of the alternating loading current the galvanometer system response is usually several tens of decibels down from that existing at D.C., but the fundamental component of the alternating current in the galvanometer circuit has to be reduced to a level where its effect on the galvanometer deflection is not noticeable.

If it may be assumed that

$$X \text{ and } S \ll A, B, R \text{ and } Z_g \text{ and that } Z_g \cdot R \gg B.S \qquad (3.1)$$

then the galvanometer alternating current in the Kelvin Bridge of Fig. 1V (b) is given by

$$I_{g_{AC}} = \frac{I_{AC}X}{(Z_g + \frac{AB}{A+B} + \frac{PQ}{P+Q})(1 + \frac{A}{B})} \qquad (3.2)$$

The system used for the reduction of alternating currents in the galvanometer throughout this thesis was to insert a coil of high inductance and low D.C. resistance tuned to the supply frequency, in series with the galvanometer, so as to make Z_g large to currents of the supply frequency, and low to direct currents.

The possibility of incorporating added impedances in the bridge arms in order to reduce the alternating currents in the galvanometer circuit was rejected because of their effect on the bridge balance condition. An impedance placed in series with the galvanometer has no effect on the bridge balance condition.

It was found during A.C. loading tests, that the amplitude stability of the alternating current supply was very important. Variations in amplitude produced an increase in the random noise level of the galvanometer which meant that with large variations, the balance condition was found with less accuracy, due to the difficulty of determining exactly where the galvanometer zero lay upon D.C. reversal. This problem became worse with the increase in the value of the resistance being tested, when an increased alternating current level passed through the galvanometer with the relative increase in X, for constant power dissipation level in X.

The alternating currents which are most troublesome when obtaining bridge balance are at frequencies very near D.C., where the galvanometer sensitivity is approaching its D.C. value and Z_g is greatly reduced.

(3.2.2)

Alternating Supply Current Increases in Amplitude (Galvo Zero Fluctuations).

It can be shown that if the alternating voltage applied at time $t = 0^+$ increases its amplitude in a step at time $t = a$, so as to be represented by the function

$$e(t) = A_1 H(t) \sin\omega t + A_2 H(t - a) \sin\omega t \qquad (3.3)$$

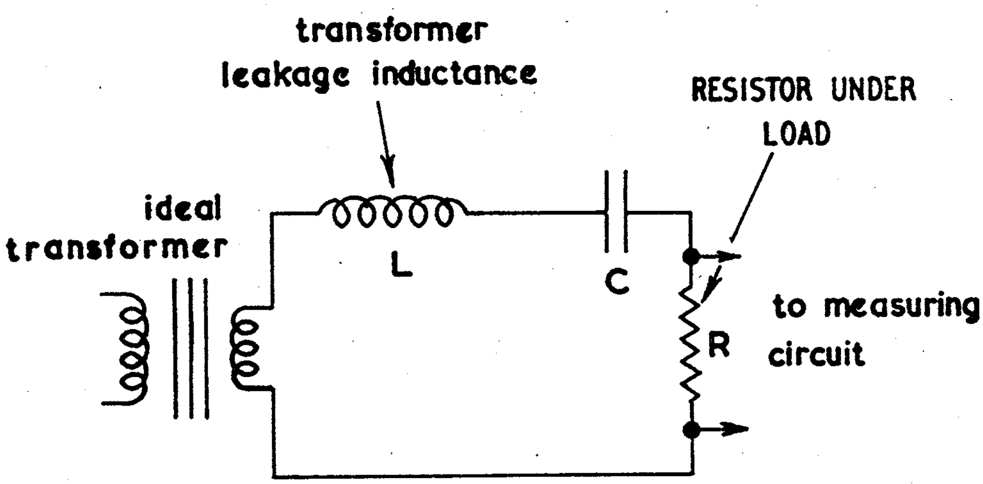


Fig XVII

General alternating current

loading circuit.

where

$H(t - t_1)$ is the Heaviside unit shifting function defined by

$$H(t - t_1) = 0 \qquad -\infty \leqslant t \leqslant t_1^-$$

$$= 1 \qquad t_1^+ \leqslant t \leqslant \infty$$

then the corresponding functional representation of the current through the resistor under test, assuming the shunting effects of all other impedances in the bridge circuit on the resistor under test to be negligible, is dependent on two factors;

(a) The relationship between the L, C and R of the equivalent series RLC circuit representing the alternating loading current supply circuit as shown in Fig. XV11, and

(b) The point of the voltage cycle at which the step occurs.

For simplicity in calculation, it is assumed that the transient conditions which occur after the switching on of the current at time $t = 0^+$ have been allowed to decay to zero before time $t = a$. This means that the current at time t where $t < a^-$ will be given by

$$i(t) = \frac{A_1}{R\sqrt{\left\{1 + \left(\frac{1 - \omega^2 LC}{R\omega C}\right)^2\right\}}} \cos(\omega t - \phi) \qquad (3.4)$$

where $$\phi = \tan^{-1} \frac{\omega CR}{1 - \omega^2 LC}$$

In respect to factor (b) above, if the current is calculated for the conditions where the supply voltage is passing through (1) a maximum and (11) a minimum, the resulting expressions will give an indication of what to expect if time $t = a$ occurs at any other point in the cycle. Similarly, for factor (a), there are considered to be three conditions likely to be encountered in a practical case.

(1) Where the capacitive reactance of the circuit is very much

greater than the inductive reactance at the supply frequency, which in turn is very much greater than the total series resistance of the circuit. This condition might be expected to be fulfilled while loading a very low valued resistance at high currents.

(11) Where the resistance of the circuit is very much greater than the capacitive reactance, which in turn is very much greater than the inductive reactance at the supply frequency. This condition might be expected to be fulfilled when loading a relatively high valued resistor, especially when supplying current from an electronic power amplifier as was the case in several of the tests performed in this thesis.

(111) Where the capacitive and inductive reactances are equal at the supply frequency and very much greater than the total series resistance of the circuit. This condition might be expected to be fulfilled as a special case of condition (1).

It can be shown that the resulting currents for each of the six possible combinations of factors (a) and (b) are given by

Condition (a) (1), (b) (1).

$$i(t) \simeq A_1 \omega C \cos(\omega t - \phi) + A_2 \omega_o C\, H(t - a)\left\{\frac{\omega}{\omega_o}\cos(\omega t - \phi) + e^{-\frac{R(t-a)}{2L}} \sin\omega_o(t - a)\right\} \tag{3.5}$$

where $\phi \simeq \tan^{-1}\omega CR$ and $\omega_o^2 = \frac{1}{LC}$

Condition (a) (1), (b) (11).

$$i(t) \simeq A_1 \omega C \cos(\omega t - \phi) + A_2 \omega C\, H(t - a)\left\{\cos(\omega t - \phi) - e^{-\frac{R(t-a)}{2L}} \cos(\omega(t - a) - \theta)\right\} \tag{3.6}$$

where $\phi \simeq \tan^{-1}\omega CR$ and $Q_o = \frac{\omega_o L}{R}$

where $\omega_o^2 = \frac{1}{LC}$ and $\theta = \tan^{-1} \frac{1}{2Q_o^2}$

Condition (a) (11), (b) (1).

$$i(t) \simeq \frac{A_1}{R} \cos(\omega t - \phi) +$$

$$\frac{A_2}{R} H(t-a)\left\{ \cos(\omega t - \phi) + Q_o^2 e^{-\frac{(t-a)}{RC}} - e^{-\frac{R(t-a)}{L}} \right\} \tag{3.7}$$

where $\phi \simeq \tan^{-1} \omega CR$ and $Q_o = \frac{\omega_o L}{R}$

where $\omega_o^2 = \frac{1}{LC}$

Condition (a) (11), (b) (11).

$$i(t) \simeq \frac{A_1}{R} \cos(\omega t - \phi) +$$

$$\frac{A_2}{R} H(t-a)\left\{ \cos(\omega t - \phi) - 2Q\, e^{-\frac{R(t-a)}{2L}} \sinh \frac{R(t-a)}{2L} \right\} \tag{3.8}$$

where $\phi \simeq \tan^{-1} \omega CR$ and $Q = \frac{\omega L}{R}$

Condition (a) (111), (b) (1).

$$i(t) = \frac{A_1}{R} \sin \omega_o t +$$

$$\frac{A_2}{R} H(t-a)\left\{ \sin \omega_o t + e^{-\frac{R(t-a)}{2L}} \sqrt{1 + \frac{1}{4Q_o^2}}\ \sin(\omega_o(t-a) - \theta) \right\} \tag{3.9}$$

where $\theta = \tan^{-1}(2Q_o)$, $Q_o = \frac{\omega_o L}{R}$ and $\omega_o^2 = \frac{1}{LC}$

(ω_o is the supply radian frequency)

Condition (a) (111), (b) (11).

$$i(t) = \frac{A_1}{R} \sin \omega_o t +$$

$$\frac{A_2}{R} H(t - a)\left\{ \sin \omega_o t - e^{-\frac{R(t-a)}{2L}} \sin(\omega_o(t - a))\right\} \quad (3.10)$$

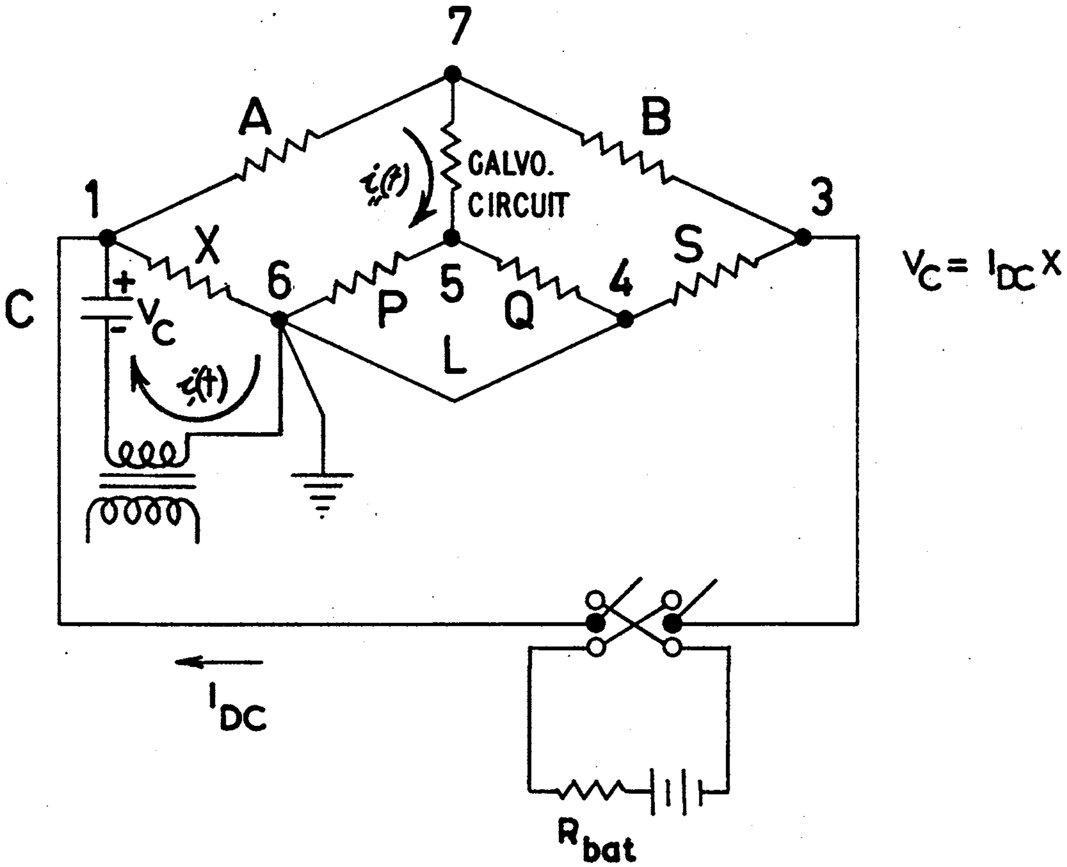
Consideration of the above six equations in the light of the frequency response of the galvanometer system indicates that the largest galvanometer response would be obtained under condition (a) (11), since here a decaying exponential of current is present which, depending on the ratio of R to L and the value of RC, may be considered to be a slowly varying direct current. Conditions (a) (1) and (a) (111) on the other hand, consist of the sum of two relatively high frequencies, whose effect would be attenuated by the frequency response of the galvanometer system. It is recollected in retrospect, that the worst cases encountered in practice during measurements were those in which condition (a) (11) might have been considered to apply.

(3.3)

Capacitive Discharge Obtained Upon Reversal of the Direct Measuring Current.

When performing load coefficient measurements on the Tinsley 5 milliohm air cooled shunt and the Cambridge Heavy Current Standard 10 milliohm resistor discussed in Chapters 7 and 8, it was noticed that at D.C. supply reversal, the galvanometer spot gave a kick of several centimeters in amplitude. This was thought at the time to be due to mutual inductive pick-up between the mu-metal cored galvanometer series inductor and the D.C. supply leads feeding the bridge, situated in reasonably close proximity, since the effect was found to be dependent on the magnitude of the direct current used.

Upon connecting the measuring circuit for load coefficient measurements on the 1 ohm Cambridge standard L-259235 discussed



Discharge circuit for the D.C. blocking capacitor used in the alternating loading current supply circuit, at time t after direct current interruption.

Fig XVIII

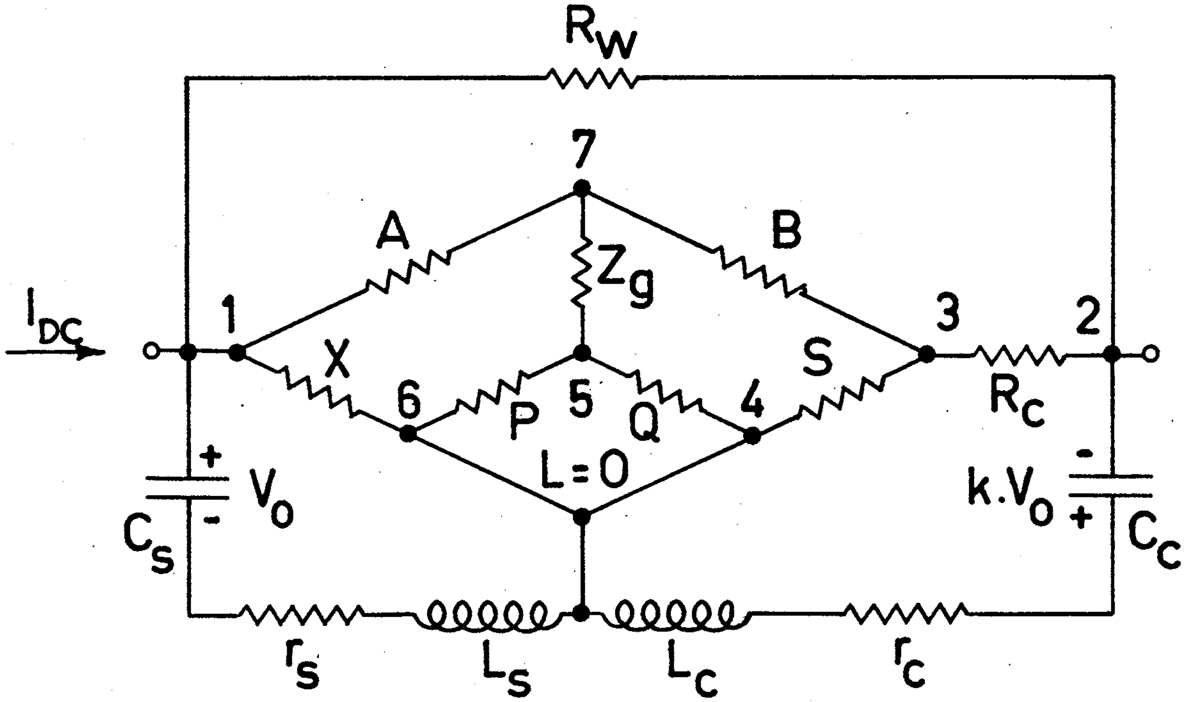
in Chapter 4, it was found that upon reversing the direct measuring current, the galvanometer deflection obtained was so large, that the galvanometer spot disappeared off-scale for several seconds. It was realised that the original supposition as to its cause was in error, since in this case, the D.C. supply current level was much lower, while the mutual inductive coupling mentioned above would have been of similar order.

After further consideration it was realised that the origin of the kick was capacitive rather than inductive. Consider the balanced Kelvin Bridge shown in Fig. XV111. If the D.C. supply current I_{DC} has been flowing for some time, all transient effects will have died out. All voltages around the bridge circuit will be balanced, and in particular, the voltage drop across X will be balanced by a voltage drop across S. The voltage drop which appears across X will also appear across the capacitor C, since the secondary of the supply transformer provides continuity.

Upon opening the reversing switch, the current I_{DC} falls instantaneously to zero leaving a charge on the capacitor C. C will discharge from bridge point 1 to earth through all elements of the bridge circuit, including the galvanometer. The initial current amplitude and the time constant of the discharge will depend upon the magnitude of the bridge resistances, the current I_{DC} and the size of the capacitance C. But as the resistance of X (and S) increase relative to the other bridge resistances, the amplitude of the initial current through the galvanometer becomes larger with an associated increase in amplitude of capacitive discharge kick experienced.

The method proposed for the elimination of the galvanometer kick is to balance the instantaneous voltage across X at all times with another voltage across S which is in the ratio of S/X to the instantaneous voltage across X. Desirable features for any correction circuit are

(a) It can be applied at all bridge ratios of A to B and A to X.
(b) It is possible to reduce the value of the correction



Modified Kelvin Bridge used to eliminate capacitive discharge current in the galvanometer upon direct current reversal.

Fig XIX

capacitance required to a value substantially lower than that of the supply capacitance for reasons of cost. (Further discussion of this point is carried out in Chapter 14).

(c) Its balance condition can be obtained with the minimum of trouble.

(d) It does not substantially reduce bridge detector sensitivity.

(3.3.1)

Link Resistance = Zero.

Consider the Kelvin Bridge circuit as shown in Fig. X1X, assuming for simplicity that the link resistance L = O. It is basically a Kelvin Bridge circuit but with several important changes. There is an extra resistance R_c in the battery circuit and two series circuits consisting of C_s, r_s and L_s and C_c, r_c and L_c respectively. There is also a resistance R_w connected from the supply circuit connection of A, X and r_s to the supply circuit connection of R_c and r_c. This resistance represents the presence of a driven guarding system as discussed previously in section (2.3).

If a direct current I_{DC} flows from 1 to 2, the voltages that will develop across C_s and C_c will approach V_o and $k.V_o$ respectively as shown. The problem is to obtain the relationship between the impedances of Fig. X1X such that at the instant that the direct current is reduced to zero and forever after, the instantaneous galvanometer current remains at zero. It is assumed that the effects of stray capacitance etc. can be neglected.

The initial conditions of the system are then

	I_{DC}	$= 0$	at time	$t = 0$
Voltage across	C_s	$= V_o$	at time	$t = 0^+$
Voltage across	C_c	$= k.V_o$	at time	$t = 0^+$
All currents		$= 0$	at time	$t = 0^+$ (3.11)

The Laplace Transform of the galvanometer current of Fig.X1X may be shown to be equal to zero for all p provided

$$0 = \frac{kV_o}{p}\Big[AR_wXS + r_s(AR_wS + XS(A + B + R_c + R_w) + BXR_c) + (pL_s + \frac{1}{pC_s})(AR_wS + XS(A + B + R_c + R_w) + BXR_c)\Big]$$
$$- \frac{V_o}{p}\Big[BR_wXS + R_cR_w(B + S)X + r_c(BR_wX + XS(A + B + R_c + R_w) + BXR_c) + (pL_c + \frac{1}{pC_c})(BR_wX + XS(A + B + R_c + R_w) + BXR_c)\Big] \qquad (3.12)$$

At balance $A.S. = B.X$ (3.13)

Based upon the initial conditions, if a direct current is passing through X, the voltage across S + the voltage across R_c must be equal to the factor k times the voltage across X at bridge balance.

Therefore

$$k.X = S + (1 + \frac{X}{A}).R_c \quad \text{at balance} \qquad (3.14)$$

and

$$R_c = \frac{BS(kM - 1)}{B + S}$$

$$\text{where } M = \frac{A}{B} = \frac{X}{S} \qquad \text{at balance} \qquad (3.15)$$

By substituting equations (3.15) and (3.13) in equation (3.12) the relationship

$$0 = \frac{V_o}{p}\Big[(kr_s - r_c)(BXR_w + XS(A + B + R_c + R_w) + BXR_c) + \left(k\left(pL_s + \frac{1}{pC_s}\right) - \left(pL_c + \frac{1}{pC_c}\right)\right).(BXR_w + XS(A + B + R_c + R_w) + BXR_c)\Big]$$

is obtained (3.16)

The conditions necessary in order to make equation (3.16) apply at all p are that

$$r_c = k.r_s \tag{3.17}$$

$$L_c = k.L_s \tag{3.18}$$

and $$C_c = C_s / k \tag{3.19}$$

Referring back again to Fig. X1X, the impedance Z_s may be taken as a reasonable representation of the secondary of the A.C. supply transformer. In practice, r_s may also include any resistance considered necessary for power factor correction of the supply circuit when connected to a high power electronic amplifier, such as the Savage 1 kW Amplifier used in load coefficient tests on several of the resistors discussed in later chapters. For the elimination of the capacitive kick, it is found that the most important adjustments to be made are those of R_c, r_c and C_c. Matching L_s and L_c seems to be relatively uncritical. This may be expected however, since the voltage $L_s.\frac{di(t)}{dt}$ would be quite low for usual values of decay time constants. If the values of C_s and r_s are too large, the time taken for the decay of all transient currents to zero may be too long. Hence, for any slight error in the desired value of the correction elements, the resulting galvanometer deflection may take too long to disappear.

The procedure used in practice for setting up the correction circuit was

(1) Choose a suitable value of k.

(11) Calculate the value of the B arm resistance from the existing circuit conditions

(111) Determine M.

(1V) Calculate the desired value of R_c using equation (3.15). R_c can be made up of a standard resistor, shunted by a decade box of higher value. It can be shown that the value of the shunt setting required for R_c is given by

$$R_{shunt} = \frac{R_c \cdot R_{std}}{R_{std} - R_c} \tag{3.20}$$

(v) Calculate the values of C_c, L_c and r_c using equations (3.17), (3.18) and (3.19).

The only practical point regarding the value of k chosen, is that it must be such that the D.C. measuring current is within the rating of the resistors used to make up R_c. The higher the value of k chosen, the higher is the resulting value of R_c required. (equation (3.15)).

It is instructive to the understanding of the balance conditions for capacitive kick elimination, if a star-delta transformation is applied to the three star connected resistors B, S and R_c. It can be shown using equations (3.13) and (3.15) that the resulting delta resistor

$$B + (1 + \frac{B}{S})\, R_c = k.A \tag{3.21}$$

and

$$S + (1 + \frac{S}{B})\, R_c = k.X \tag{3.22}$$

(3.3.2)

Link Resistance Not Negligible in Respect to Unknown and Standard.

If the Link resistance is not negligible in respect to X and S it can be shown that zero galvanometer current is passed at all p only if the bridge resistances are chosen such that

$$\frac{L_x}{L_s} = \frac{P}{Q} = \frac{A}{B} = \frac{X}{S} = M \tag{3.23}$$

$$\text{and} \quad kM = 1 \tag{3.24}$$

(3.4)

Limitations on the Accuracy Available for Load Coefficient Measurement Due to the Limited Accuracy of the Alternating Current Measuring Device Used.

In loading the resistor under test with alternating current and measuring resistance changes with direct current, there is energy supplied to it from two sources and usually through three separate metering circuits. It is obvious that the greatest accuracy of calibration will be required of the metering device which measures the greater proportion of the loading power, and under normal conditions of loading, this will be the alternating current metering device.

In all measurements made in this thesis on load coefficients, the D.C. power supplied to the Kelvin Bridge circuit used, remained constant at its no-load value, when loading with alternating current. The alternating current level flowing in the battery circuit will depend upon the level of power dissipation required in the Unknown. This current level will normally be found by a separate measuring system.

The total power supplied to the Unknown X will be given by P_x where

$$P_x \simeq P_{AC} - P_{bat\ AC} + P_{xDC} \qquad (3.25)$$

provided the alternating current levels passing through all other bridge circuit resistances are small. If the permissible errors in P_x, $P_{bat\ AC}$ and P_{xDC} are $\pm \delta P_x$, $\pm \delta P_{bat\ AC}$ and $\pm \delta P_{xDC}$ proportional parts respectively, then

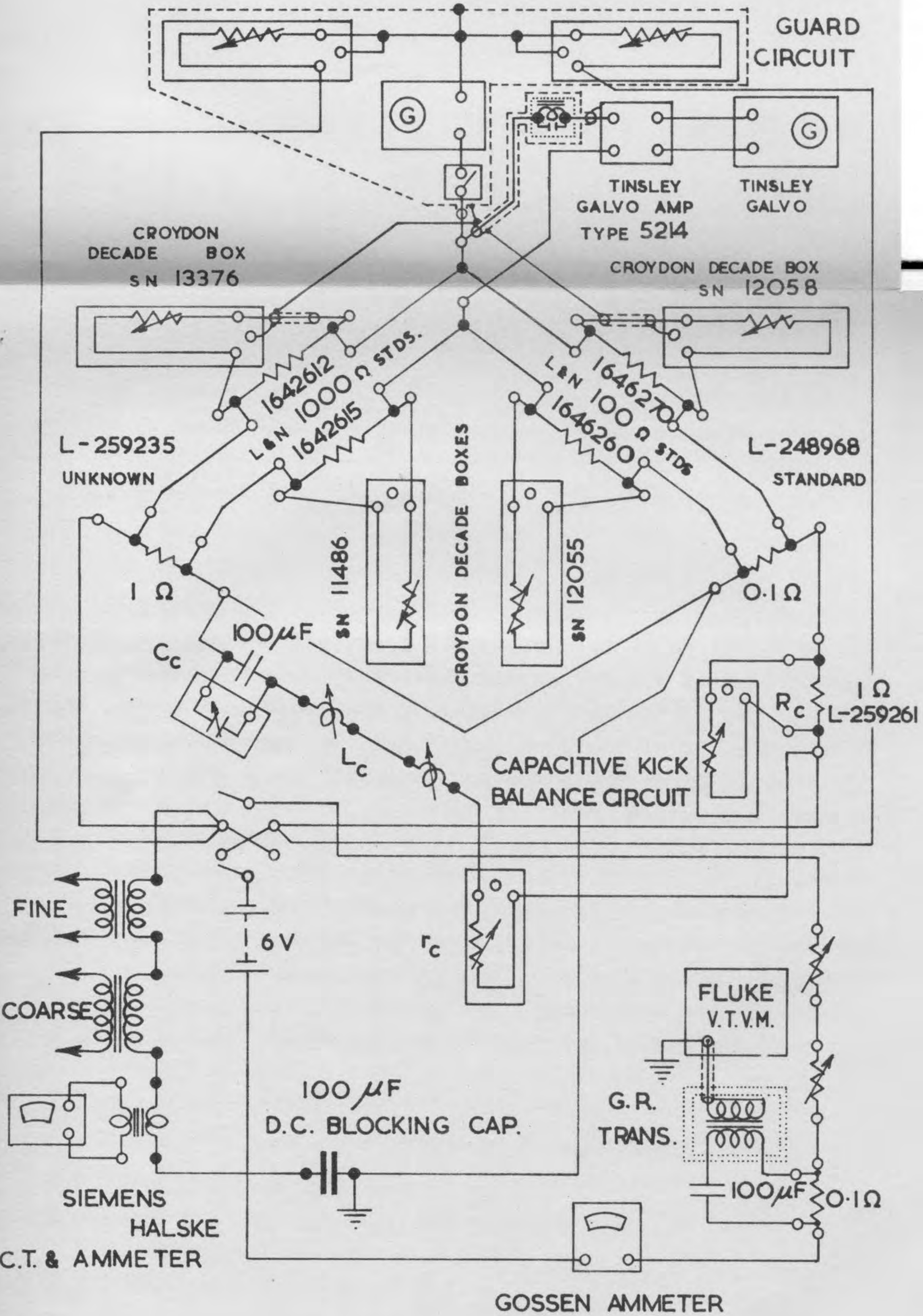
$$\delta P_x = \frac{P_{AC}}{P_x} \cdot \delta P_{AC} + \frac{P_{XDC}}{P_x} \cdot \delta P_{XDC} + \frac{P_{bat\ AC}}{P_x} \cdot \delta P_{bat\ AC}$$

(3.26)

If the errors δP_{AC}, δP_{xDC} and $\delta P_{bat\ AC}$ are produced by uncertainties in the respective current levels measured then

$$\delta P_x = 2 . \frac{P_{AC}}{P_x} . \delta I_{AC} + 2 . \frac{P_{xDC}}{P_x} . \delta I_{xAC} + 2 . \frac{P_{batAC}}{P_x} . \delta I_{batAC} \qquad (3.27)$$

The value of δP_x used above is dependent directly on the load coefficient of the resistance under test and the unbalance sensitivity of the bridge used. In most practical cases, the limiting value of the load coefficient measurement uncertainty is governed directly by the accuracy with which the loading current can be set.



SCHEMATIC DIAGRAM
of the Kelvin Bridge used in
measurements made on the Cambridge 1Ω resistor L-259235

Fig XX.

CHAPTER 4

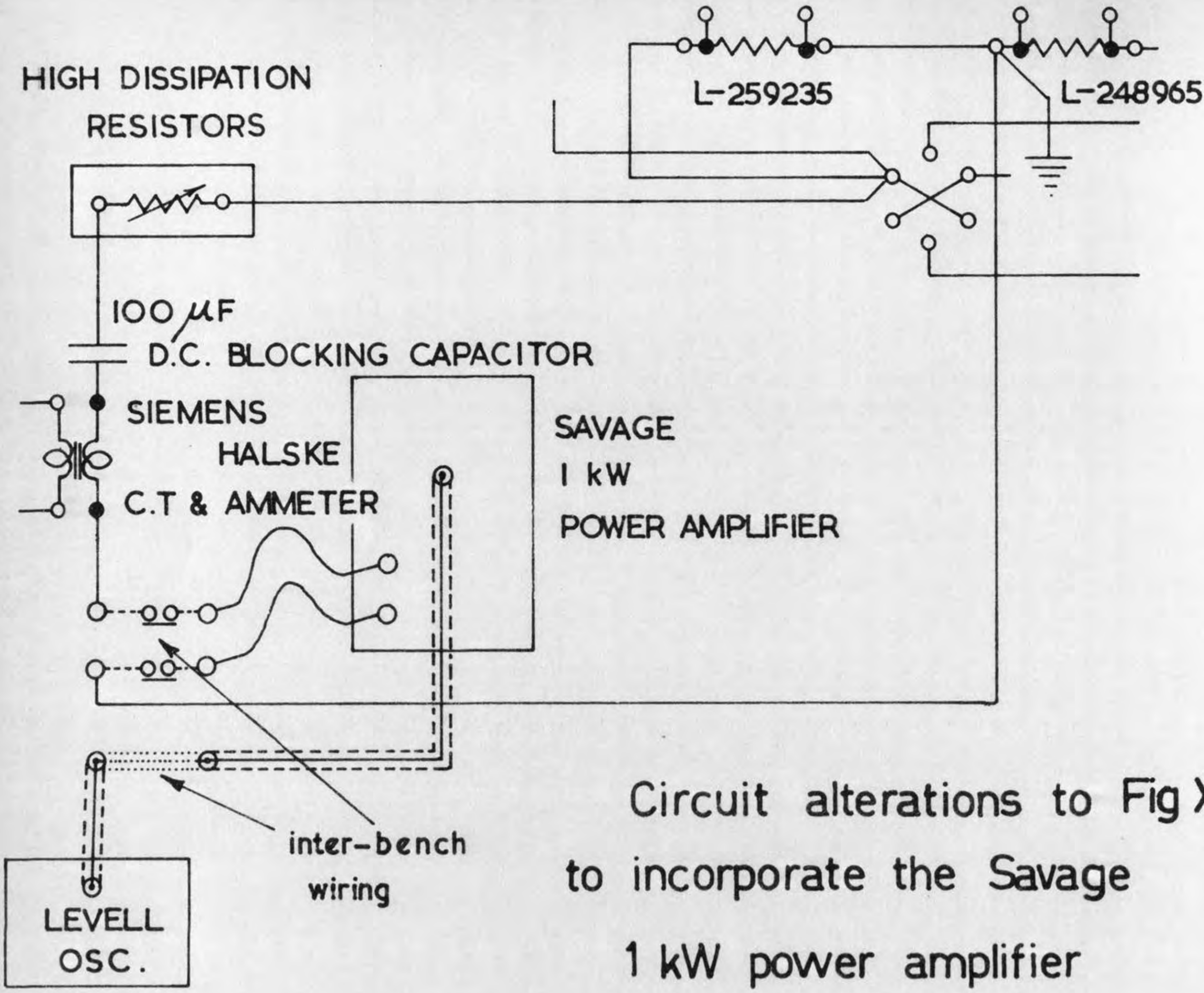
Load Coefficient Measurements on One Ohm Cambridge Resistor S/N L-259235 Using Both A.C. and D.C. Loading Techniques.

(4.1)

In order to establish the feasability of the A.C. loading of D.C. resistors as an alternative to the total D.C. loading when making load coefficient determinations, measurements of load coefficients on several resistance values and types of construction were performed.

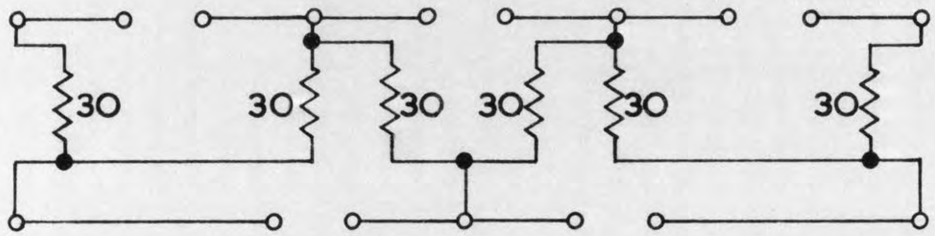
A Kelvin Bridge circuit was wired up as shown in Fig. XX. The resistor used as the Standard was a Cambridge 0.1 ohm N.P.L. construction manganin resistor S/N 248968 of similar construction to the Unknown. The decade resistance shunts in the A, B, P and Q arms were set to resistance values nominally 100 times that of the respective standard shunted. Both the A and P arm shunts were adjusted in unison when balancing the bridge, in an attempt to keep the correction term in the balance condition of the Kelvin Bridge due to the presence of the P and Q arms at a constant value.[3]

Galvanometer fluctuations were troublesome in the preliminary load coefficient measurements made with the 50 c/s A.C. mains providing the loading current. When it was realised that the fluctuations observed in the galvanometer could be correlated with fluctuations noticed in the deflection of the Siemens Halske ammeter monitoring the alternating loading current supply, it was decided to try loading the resistor with a current derived from a Brian Savage 1 kilowatt Power Amplifier, driven from a Levell battery powered oscillator. The combination of the Savage amplifier and the Levell oscillator was considered to have good amplitude stability. This modification required a modification of the circuit diagram shown in Fig. XX, with the supply transformers and auto-transformers being replaced with



Circuit alterations to Fig XX to incorporate the Savage 1 kW power amplifier

Fig XXI.



WIRING DIAGRAM

for the high dissipation resistance tank.

Fig XXII.

the Savage supply system as shown in Fig. XX1, which incorporated a high dissipation oil filled resistor to act as a power factor correction element, such that the Savage power amplifier worked into a predominantly resistive load.

The wiring diagram of this resistor is shown in Fig. XX11. Values of series resistance were possible ranging from 5 to 180 ohms, with the appropriate series and parallel connections of the resistance sections.

To eliminate the beating effects noticed in the galvanometer deflection when using the Levell-Savage power amplifier system near mains frequency, the loading current supply frequency was reduced to 40 c/s. This meant re-tuning the mu-metal cored series inductor to parallel resonance at 40 c/s also.

The Cambridge resistor was placed in an oil bath separate from that containing the remaining standards forming the other arms of the bridge. Each bath was fitted with its own stirrer and 0.01 ^{o}C thermometer so as to ensure an even mixing of the oil and an accurate indication of the temperature of the environment of each standard resistor. Potential lead connections between the resistor under test and the rest of the bridge were made with two thick copper braided leads. All connecting leads between the standard resistors forming the other bridge arms were of 10 S.W.G. copper wire, kept as short as possible, and submerged below the surface of the oil so as to eliminate the effects of air currents on them and their connections to the plated terminals of the standards. The link connection was made with a thick stranded conductor, whose resistance value was found to be 0.62 $\pm$ 0.02 milliohm. It was calculated that the braided leads, while being mostly in air, were of low enough resistance for typical wind currents and temperature gradients to have negligible effect on the bridge balance condition, assuming a bridge balance sensitivity equivalent to $\pm$ 1 part in 10^7 of the Unknown.

Oil bath temperatures were decreased where necessary with the aid of refrigerated water which passed through a metal heat exchanger placed under the bath concerned. Temperatures were raised with the aid of a small imersion heater placed in the bath concerned. The laboratory in which the measurements were made was air conditioned and temperature controlled nominally to a temperature of 20 ^{o}C. It was found that if the baths were left idle for a period of a day or so, each bath temperature would settle at about 19.3 ^{o}C, but if the stirrers were allowed to run continuously, the baths would stabilise at a temperature about 2 ^{o}C higher, the bath containing the Unknown being at the higher temperature, and the baths reaching their stable temperatures in about 4 to 5 hours from the time the stirrers were set in motion. Rises in temperature made in one bath caused the temperature of the other bath to increase slightly, due to incomplete thermal isolation between the two baths.

(4.2)

Measurement Procedure.

Before load coefficient measurements were made on the 1 ohm Cambridge resistor L-259235, its temperature coefficient at no-load (0.09 watt) and that of its associated bridge circuit were determined.

About 4 or 5 balances were taken, in general, under each condition of temperature setting. 20 minutes were allowed for temperature stabilisation after changing oil bath temperatures before further measurements were performed.

The process was repeated at sufficient temperatures to enable a reasonable accuracy for the determination of temperature coefficients to be obtained.

Results were then processed by digital computer, using the program TEMPCURV, as shown in Chapter 12 in order to calculate the values of α_{20} and β for the 1 ohm resistor under test and the bridge circuit. These computed values of α_{20} and β were later used in the processing of the results of the load coefficient measurements.

For load coefficient measurements, a similar measurement process was followed as for tests on the Unknown for temperature coefficient, except that for each set of Unknown bath temperatures, both A.C. and D.C. loading conditions were applied, taking 4 or 5 readings with A.C. loading and then 4 or 5 readings with D.C. loading, before changing the temperature of the bath. Load coefficient measurements were made at $\frac{1}{2}$, 1, 2 and 5 watts and the results processed by the digital computer using the program LOADCURV, shown also in Chapter 12.

Measurements were made at the 1 watt power dissipation level to determine the variations occurring in the load coefficient of the Unknown as the frequency of the alternating loading current was increased. For these measurements a thermocouple was used to measure the supply of approximately 1 amp A.C. to the Unknown at values of supply frequency up to 3 000 c/s.

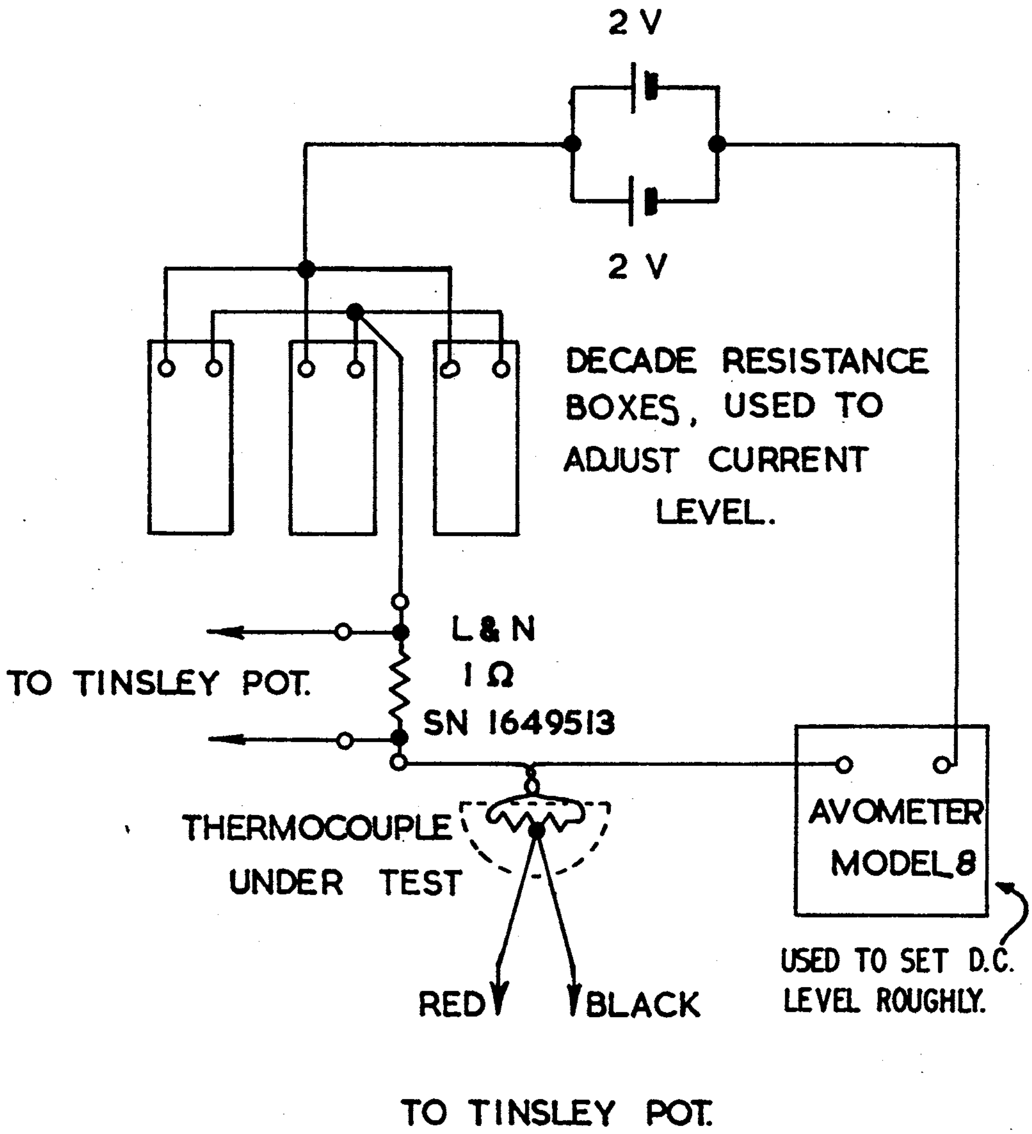
(4.3)

Measurement of Currents Used.

(4.3.1)

Direct Current and Low Frequency Alternating Current Measurements.

Direct currents measurements were made using a Gossen multi-range moving iron ammeter Plant No. 78414, fitted with a mirror scale and possessing an estimated current measurement accuracy of about $\pm$ 0.3 per cent of full scale deflection. Alternating current measurements contained problems to be considered in association with shunt paths and also the wide range of frequency used in measurement. Current measurements in the A.C. loading circuit at 40 c/s were made with a 5 amp Siemens mirror scaled, sub-standard moving iron ammeter in association with a matching Siemens Halske current transformer of multiple ratio, ranging from 0.1/5 to 60/5. The current measuring accuracy at 40 c/s was estimated to be about $\pm$ 0.2 per cent of full scale deflection, since the meter accuracy at 50 c/s is given as better than 0.2 per cent of full scale deflection, and the transformer is assumed to contribute negligible ratio error when working with



Calibration circuit for Thermocouples.

Fig XXIII.

its matching meter. (see Chapter 7).

(4.3.2)

High Frequency Current Measurements.

Currents of higher frequencies were measured with the aid of a sealed vacuum thermocouple,Best type S.23, which was immersed in oil to reduce the effect of wind currents past it. Rated heater current was 1 000 mA with a nominal couple output of 7 millivolts at rated current. Calibration of this thermocouple was carried out using the system shown in the circuit diagram of Fig. XX111 at D.C. The heater current level was set to a specified value with the AVOmeter model 8 and then by measuring the D.C. voltage drop across the 1 ohm standard resistor S/N 1649513 with the Tinsley Portable Potentiometer type 3184 for more precise adjustment. Currents over a small range of values only were of interest, since the frequency coefficient of load coefficient determination was confined to a single power dissipation of 1 watt. A relationship between the couple voltage and the heater current of the form

$$E_{couple} = k\, I^2_{heater} \qquad (4.1)$$

was assumed to exist.

The results of the calibration are shown in Table (4.1).

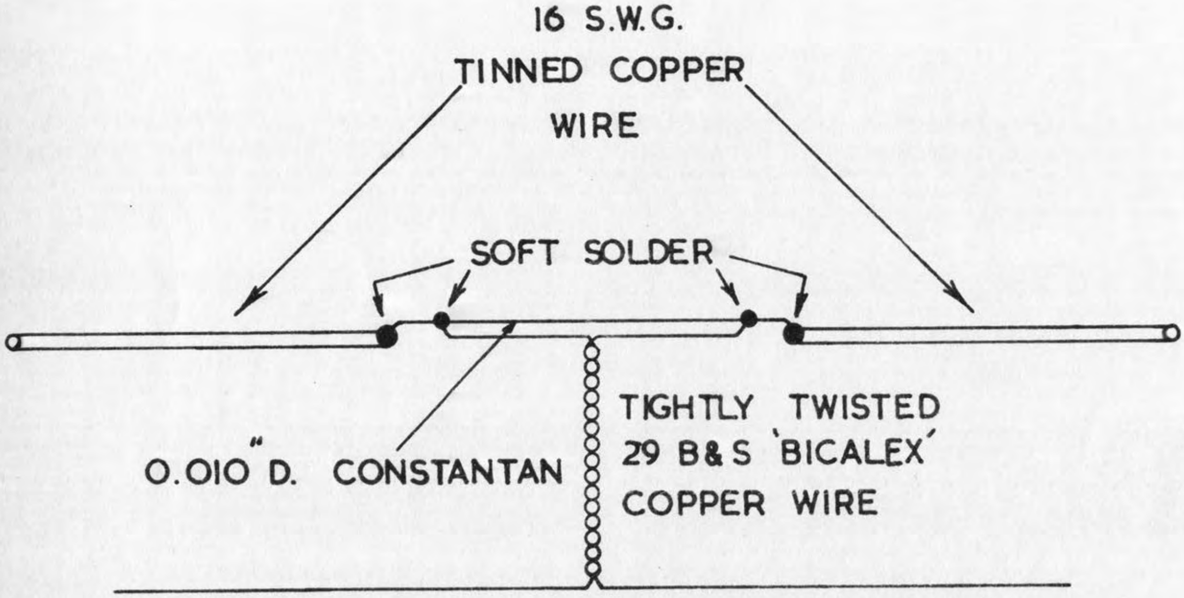
Investigations of the day to day drift in the value of k for S.23 and of its change with temperature suggested that it was almost unaffected by changes in temperature to the accuracy with which measurements were made, but that a day to day drift of about 0.1 per cent in its value might be expected. With this information, it was decided to make a check reading on the value of k each morning prior to commencing measurements. Resistance changes of less than 0.02 per cent from its no-load value are to be expected at a current of 1 amp for the 1 ohm resistor used in the calibration of S.23. The resistance value at no-load and 25 °C was equal to 1.0000_2 ohm, found from the maker's calibration certificate.

TABLE (4.1)

Calibration of Best Type S.23 Thermocouple

S.23 Heater Current (mA)	Tinsley Potentiometer Type 3184		k $\left[\frac{mV}{A^2}\right]$
	Range	Dial Reading	
1 000	0.01	0.747_6	7.47_6
1 000		0.747_6	7.47_6
1 000		0.747_6	7.47_6
850		0.540_0	7.47_4
870		0.566_1	7.47_9
890		0.592_2	7.47_6
910		0.619_4	7.48_0
930		0.646_8	7.47_8
950		0.674_8	7.47_7
960		0.689_0	7.47_6
970		0.702_9	7.47_0
980		0.717_7	7.47_3
990		0.732_5	7.47_4
1 000		0.747_8	7.47_8
1 010		0.762_6	7.47_6
1 020		0.777_5	7.47_3
850		0.540_2	7.47_7
1 000		0.747_8	7.47_8
1 000		0.747_8	7.47_8
1 000		0.747_7	7.47_7
1 000		0.747_6	7.47_6

mean value of k = $7.47_6 \ \frac{mV}{A^2}$, range of deviation $\begin{matrix} + 0.00_4 \\ - 0.00_6 \end{matrix} \ \frac{mV}{A^2}$



CALCULABLE 0·1 Ω RESISTOR

Fig XXIV.

(4.3.2)

Battery Circuit Alternating Current Measurements.

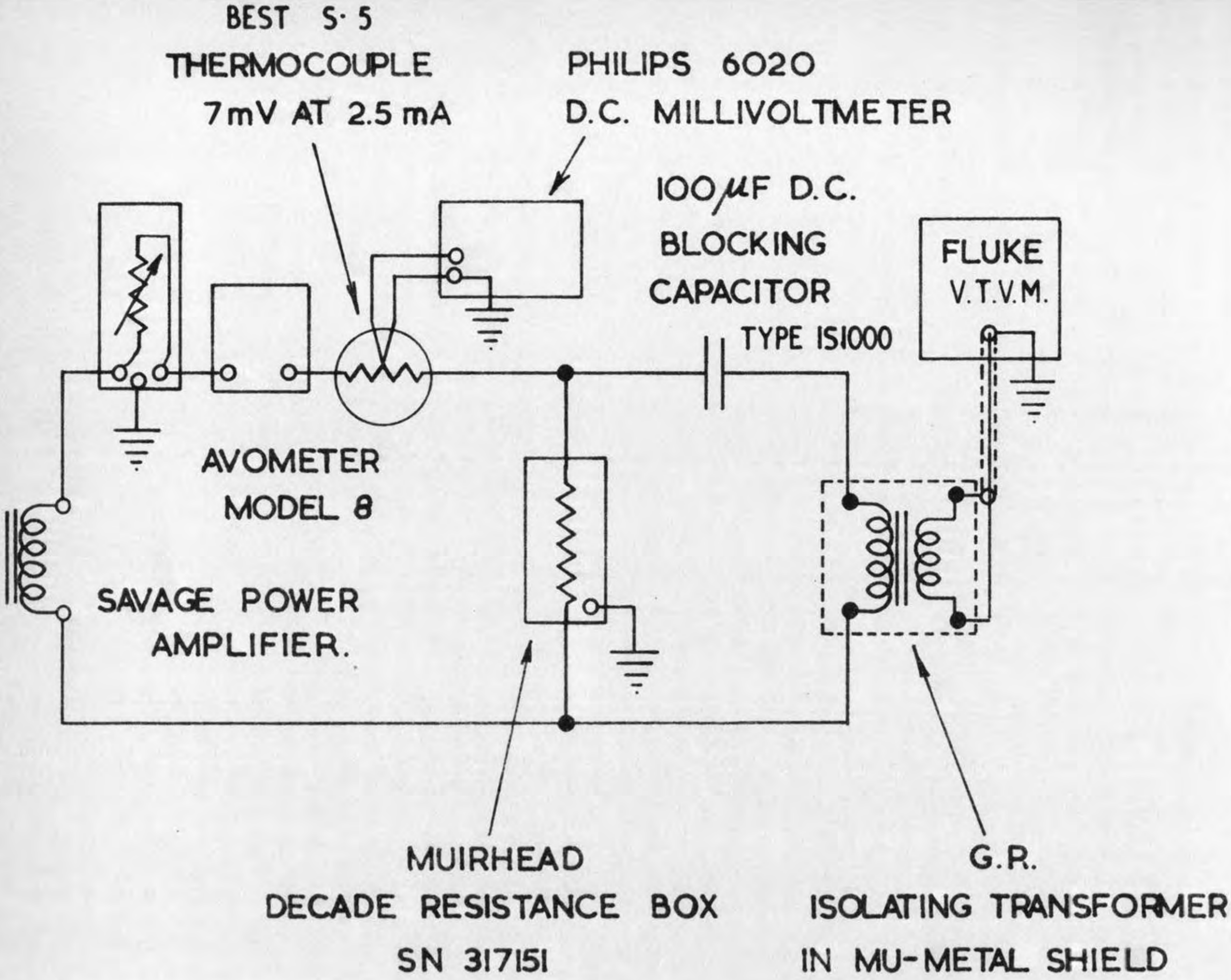
Measurement of the alternating current passing around the battery circuit was performed with a John Fluke A.C. voltmeter S/N 571, to measure the alternating voltage drop across a non-inductive 4-terminal 0.1 ohm resistor (Fig. XX), specially constructed by the Author from 14 S.W.G. Manganin wire. Isolation of the battery circuit from the Fluke voltmeter earth was obtained with a General Radio isolating transformer type 578-B Plant No. 10405 having a turns ratio of 4/1, situated in a mu-metal shield and having a nominal frequency range of 20 c/s to 5 kc/s. A series capacitance of 100 uF was found to be necessary in order to get well away from series resonance effects around the supply frequency. It was found that series resonance occurred at 50 c/s with a series capacitance of approximately 0.3 uF, indicating a reflected input inductance for the G.R. transformer with the input circuit of the Fluke attached, of about 34 henrys.

The non-inductive resistor, mentioned above, was measured 4-terminally in the Cintel Self and Mutual Inductance Bridge type CT 1 and S/N 119, at a frequency of 1592 C/S. The bridge indication was 'zeroed' with two straight pieces of 16 S.W.G. copper wire connected between corresponding current and potential terminals. A calculable resistor[19] constructed from 0.010" diameter constantan was produced to give a nominal resistance of 0.1 ohms and calculable inductance, see Fig. XX1V. It was used to determine whether the bridge was reading correctly.

Relating the measured value of the series inductance of the calculable resistor to the value calculated using the formula[19]

$$L = 0.005\ell \left[2.3 \log_{10} \left(4\frac{\ell}{d} - 0.75 \right) \right] \qquad (4.2)$$

where L is the calculated inductance in microhenrys,
ℓ is the length of the wire in inches and



Calibration of the Battery Circuit
Alternating Current Measuring Circuit.

Fig XXV.

d is its diameter in inches, which is applicable provided ℓ/d is large, the following values were obtained

Calculated value of $L = 1.16 \cdot 10^{-8}$ henrys
Measured value of $L = 1.2 \pm 0.1 \cdot 10^{-8}$ henrys.

The results of the measurements made on the special non-inductive resistor were

	Resistance	Inductance
	1.00 ± 0.005 ohm	1.50 ± 0.05 microhenrys
scale factor	0.1	0.1

The D.C. resistance of the special non-inductive 0.1 ohm resistor was obtained by measuring it against a Leeds and Northrup 1 ohm standard resistor S/N 1649513. The resistance value obtained was 0.1023 ± 0.0001 ohm.

Calibration of the battery circuit alternating current measuring circuit was performed using the circuit of Fig. XXV at several supply current frequencies. The non-inductive resistor for this calibration was obtained from a non-inductive Muirhead decade resistance box S/N 317151. Check measurements using the Cintel Self and Mutual Inductance Bridge to measure the series inductance of the two decade settings used, viz 1.6 and 3.8 ohms, gave values of 0.1 and 1.5 ± 0.1 microhenrys respectively. The current passing through the 'Best' thermocouple type S.5 as indicated by the reading on the Philips 6020 D.C. millivoltmeter was kept constant with change in supply frequency, with the adjustment of the series resistance shown, so that the voltage drop across the dropping resistor R was also constant with frequency. In this way, the relative response of the measuring system with changing frequency could be determined.

As an indication of the use of the measuring circuit to find the value of the alternating current flowing around the battery circuit at any particular loading current frequency, the following example is attached.

Consider one case which occurred during the frequency coefficient of load coefficient investigation, carried out at a loading frequency of 1 000 c/s, a power dissipation of 1 watt in the Unknown and with a D.C. measuring current of 0.3 amp. The alternating current required to be passed, because of the presence of the 0.3 amp measuring current, was

$$I_{AC} = \sqrt{1. - 0.09}\ \text{A} = 954\ \text{mA} \qquad (4.3)$$

From the value of k obtained from the measurements on the Best type S.23 thermocouple prior to commencing that day (see above), the reading of the Tinsley Portable Potentiometer type 3184, used to measure the couple voltage of the Best type S.23, would be

$$0.746_9 \cdot 0.954^2 = 0.679_7 \qquad (4.4)$$

Setting the loading current to give this reading on the Tinsley Potentiometer caused a deflection in the Fluke voltmeter on its 1 millivolt range of 0.68_0 mV. From the frequency response curve for the measuring system at 1 000 c/s with the Fluke on its 1 millivolt range, together with the value of the calculated current used in obtaining the results

$$0.91_0\ x = 3.66 \qquad (4.5)$$

therefore $$x = \frac{3.66}{0.91} = 4.02$$

where x is a multiplying factor corresponding to an effective turns ratio for the system at that frequency.

Therefore 0.68_0 mV corresponds to $$\frac{0.68 \cdot 4.02}{0.1023} = 26._7\ \text{mA} \qquad (4.6)$$

passing through the non-inductive dropping resistor, since its resistance was measured to be 0.1023 ohms.

Thus the total input loading current should be raised to the value 981 mA to allow for the $26._7$ mA flowing around the battery circuit. Increases in the supply current level made to allow for the corresponding battery circuit current level were repeated until the small increases required in the supply current level produced no noticeable increase in the battery circuit current level.

The method used in calibrating the battery circuit alternating current measuring system required that the Fluke voltmeter deflection should be linearly related to the input current to the system, at constant frequency, otherwise the use of equation (4.5) above would have caused an error in the value of the supply current calculated. To investigate the linearity of the system roughly, several readings were taken using the calibration circuit of Fig. XXV and reducing the reading of the Fluke voltmeter to half its value, reading the corresponding deflection of the Philips 6020 D.C. millivoltmeter monitoring the couple voltage of the Best thermocouple type S.5, and using the value of k found for the thermocouple to calculate the new value of input current, testing to see if a linear relationship held. The results obtained indicated a negligible departure from linearity.

(4.3.3)
A.C. - D.C. Load Coefficient Differential and Loading Current Measurement Errors.

It was found, as will be shown later, that the load coefficient of the resistor under test was approximately 25 p.p.m. per watt. Calculation showed that the maximum available bridge sensitivity, corresponding to a change of 1 part in 10^7 in the Unknown, could only be used up to 1/10 of rated power dissipation. Using the $\pm$ 0.2 and $\pm$ 0.3 per cent meters available to measure the alternating loading supply current and the D.C. loading

Fig. XXVI.

CURVES OF CHANGE IN CORRECTION TO THE CAMBRIDGE 1 Ω MANGANIN RESISTOR S/N L-259235 FROM ITS NO-LOAD 20°C VALUE VS OIL BATH TEMPERATURE AS A FUNCTION OF BOTH A C & D C LOADING.

supply current respectively, meant that at rated power dissipation in the Unknown, the accuracy of the load coefficient obtained was $\pm$ 6 parts in 10^7, so that as an extreme, the load coefficients obtained at rated power under both D.C. and A.C. loading conditions, may differ by up to 12 parts in 10^7 without indicating a fundamental difference between the two loading methods.

(4.4)

Presentation of Results.

(4.4.1)

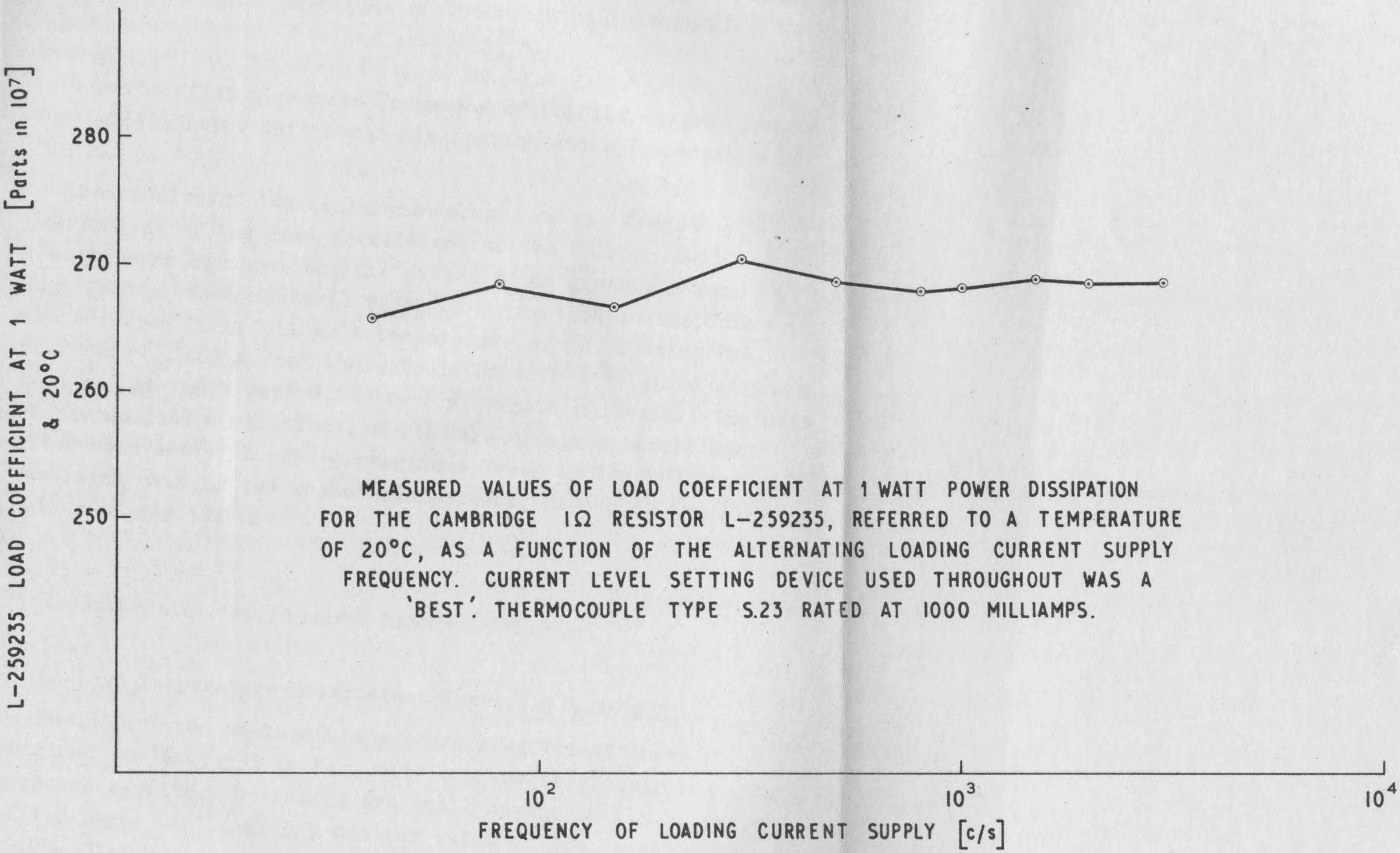
Temperature Coefficient of Unknown and Bridge Circuit at No-Load.

The no-load temperature coefficient curve of the Unknown was plotted, placing in selected measured points in order to demonstrate the magnitude of the residuals obtained. This curve is presented as the no-load curve in the load coefficient curves shown in Fig. XXV1.

(4.4.2)

Load Coefficient Measurements of the Unknown at $\frac{1}{2}$, 1, 2 and 5 Watts Using D.C. and 40 c/s Alternating Loading Currents.

The temperature coefficient curves obtained for the N.P.L. construction Cambridge 1 ohm Manganin standard resistor S/N L-259235 at loading values of $\frac{1}{2}$, 1, 2 and 5 watts are presented in Fig. XXV1 for both D.C. and 40 c/s A.C. loading conditions. The results are presented such that the resistance value of this 1 ohm resistor is referred to the no-load 20 $^{\circ}$C condition. The temperature coefficient curves under the various conditions of power dissipation are plotted from the parabola of best fit to the results of the D.C. load coefficient determination and the A.C. load coefficient curves of best fit added in broken lines, where their calculated values differed sufficiently from the calculated D.C. load coefficient values to be able to be plotted without undue crowding. Measured values of resistance change



MEASURED VALUES OF LOAD COEFFICIENT AT 1 WATT POWER DISSIPATION FOR THE CAMBRIDGE 1Ω RESISTOR L-259235, REFERRED TO A TEMPERATURE OF 20°C, AS A FUNCTION OF THE ALTERNATING LOADING CURRENT SUPPLY FREQUENCY. CURRENT LEVEL SETTING DEVICE USED THROUGHOUT WAS A 'BEST.' THERMOCOUPLE TYPE S.23 RATED AT 1000 MILLIAMPS.

Fig. XXVII

from the 20 $^{\circ}$C no-load condition for both A.C. and D.C. loading were then added to the calculated curves of best fit in order to demonstrate the magnitude of the residuals obtained.

(4.4.3)

Load Coefficient versus Frequency of Loading Current Supply for L-259235 at 1 Watt Power Dissipation and a Temperature of 20 $^{\circ}$C.

The results of the measurements made on the frequency coefficient of the load coefficient of the Unknown L-259235 at 1 watt power dissipation, are presented in graphical form in Fig. XXV11. Each measured value of correction to the Unknown was referred to an oil bath temperature of 20 $^{\circ}$C using the value of α_{20} and β from the calculated curves obtained at no-load and at the 1 watt D.C. power dissipation levels. The mean of the no-load corrections, which were obtained before and after each set of loaded corrections, were subtracted from the resulting mean of the loaded corrections, producing the plotted points of Fig. XXV11.

(4.5)

Assessment of the Measured Results Obtained.

(4.5.1)

No-Load Temperature Coefficient Curve for L-259235.

The calculated no-load temperature coefficient curve obtained for L-259235 in Fig. XXV1 represents closely the measured results. Residuals are small having an R.M.S. value of 1.8 parts in 10^7 with a maximum value of 3.4 parts in 10^7 at 20.00_5 $^{\circ}$C.

(4.5.2)

Load Coefficient Values Obtained Under A.C. and D.C. Loading Conditions.

The calculated curves of best fit to the measured values of resistance change from the 20 °C no-load value while under alternating and direct current loading may be seen by inspection of Fig. XXV1 to be in close agreement. The greatest differences occur at the 5 watt loading condition where approximately 10 parts in 10^7 may be seen at about 20 °C. The differences obtained were well within the range of up to 12 parts in 10^7 at 5 watts which may be expected due to uncertainties in the setting of both the alternating and the direct loading current levels (Section (4.3.3)). Temperature measurement uncertainties, resistor stability etc. will represent portion of the differences obtained, but the technique of loading alternately with alternating and direct currents with small time separation between, undoubtedly reduced the effect of environmental changes. The residuals obtained between the individual measured values of best fit and the calculated curve while under load may be seen from Fig. XXV1 to be larger than those obtained at no-load. This was partly due to the increased low frequency noise noticeable in the galvanometer spot while under A.C. loading conditions and the presence of an increased rate of drift in the temperature of the Unknown oil bath when under load. The stability of the alternating and direct loading current levels with time was undoubtedly a contributing factor.

(4.5.3)

Frequency Coefficient of Load Coefficient at 1 Watt Power Dissipation Level Referred to an Oil Temperature of 20 °C.

The change in load coefficient of L-259235 at 1 watt referred to an oil temperature of 20 °C as a function of the frequency of the loading current may be seen to be negligible by inspection of Fig. XXV11. The range of variation obtained may be seen to be about 5 parts in 10^7. This is not excessive when it is realised that the series of measurements required in order to obtain the results of Fig. XXV11 were performed with an Unknown oil bath temperature range of approximately 0.6 °C at 22 °C over a time interval of several days.

(4.5.4)

Agreement Between the Mean 1 Watt Load Coefficient Value From Fig. XXV11 and the 1 Watt Load Coefficient Value at 20 ^{o}C from Fig. XXV1.

Inspection of Figs. XXV1 and XXV11 will show that the mean 1 watt load coefficient values obtained at 20 ^{o}C differ by approximately 8 parts in 10^7. This infers that a difference in 1 watt load coefficient of 8 parts in 10^7 was obtained at an Unknown oil bath temperature of 22 ^{o}C. The difference obtained represents a difference of approximately 3 per cent in the 1 watt load coefficient values obtained. This value is too large to be due solely to the uncertainties with which the loading current levels were known in each case. The presence of a differing oil temperature distribution and cooling conditions around the L-259235 probably was the major cause, since differences of approximately 0.02 to 0.03 ^{o}C in temperature between the resistance element of the Unknown and the temperature as measured by the Unknown oil bath thermometer would account for it completely. Inspection of the order of the residuals obtained in the 1 watt curves of Fig. XXV1 and Fig. XXV11 indicates that the 8 parts in 10^7 difference although large is not excessive.

CHAPTER 5

Load Coefficient Measurements on the 1 ohm High Dissipation Evanohm Resistor R5 Using Direct and Alternating Loading Current Techniques

(5.1)

The resistor referred to as the 1 ohm high dissipation resistor R5 was one of five Evanohm resistors constructed at the National Standards Laboratory to a design by Briggs.[20]

The A.C. - D.C. load coefficient investigation being performed by the Author coincided quite closely in time with the D.C. load coefficient work being performed on these resistors by Mr. F. C. Brown of N.S.L. and it was decided to use a load coefficient measurement on one of these resistors up to a power dissipation level of 36 watts as a means of checking both the load coefficient measurements made already, by using a different and independent technique, and of checking the A.C. - D.C. technique itself. Load coefficient determinations were required under both A.C. and D.C. loading conditions, since the D.C. load coefficient values obtained originally by Mr. Brown were not made with sufficient accuracy to satisfy the measurement accuracy being attempted by the Author.

As a guide to assessing the results of measurements made on R5 it is worthwhile to note that the total resistance of all the copper present in R5 was estimated by Mr. F.C. Brown to be approximately 0.22 milliohms. The copper was situated such that its temperature would be significantly lower than that of the evanohm resistance element, under all conditions of loading.

(5.2)

Load Coefficient Measurements.

(5.2.1)

Alternating Current Load Coefficient Measurement Procedure.

The experimental method and calculation procedures used to obtain the A.C. load coefficient values of R5 were similar to those used when investigating the load coefficients of L-259235 under alternating current conditions as discussed in Chapter 4. L-259235 in its oil bath was simply replaced by R5 in its oil bath in the Kelvin Bridge circuit shown in Fig. XX1. The oil bath stirrer used in the measurements on L-259235 was not required, since R5 was constructed with its own stirrer, fitted with a stroboscope pattern used for the measurement of the stirrer speed in conjunction with the 100 c/s flickering of the laboratory lighting. The stirrer speed used for load coefficient measurements on R5 was 400 r.p.m., adjustable to within a few revolutions per minute. 400 r.p.m. value was treated as a reference value so that the environmental conditions of R5 could be reproduced reasonably accurately. Measurements were made at a stirrer speed of 300 r.p.m. and a power dissipation level in R5 of 16 watts so as to enable the effect of stirrer speed fluctuations on the measured value obtained for the load coefficient to be assessed.

Load coefficient measurements were made at power dissipation levels of 4, 16 and 36 watts, corresponding to effective loading current levels of 2, 4 and 6 amps. It was calculated that there existed a significant power dissipation level in the A and B arms at the higher values of A.C. power dissipation in R5, owing to the corresponding increase in the amplitude of the alternating current passing through the A and B arm of the Kelvin Bridge. The method used for this determination, and the associated calculations of loading current in 1642612 under different loading conditions of L-259235 and R5 are discussed in Chapter 6.

No-load temperature coefficient measurements were made on R5 also by Mr. R. E. Holmes of the National Standards Laboratory using an equal arm Warshawsky Multiple Bridge[2] made up of 1 ohm standard resistors and using a bridge supply current of 200 mA in order to pass 100 mA through each bridge arm. Bridge balance was obtainable to within $\pm$ 1 in 10^8 of the Unknown.

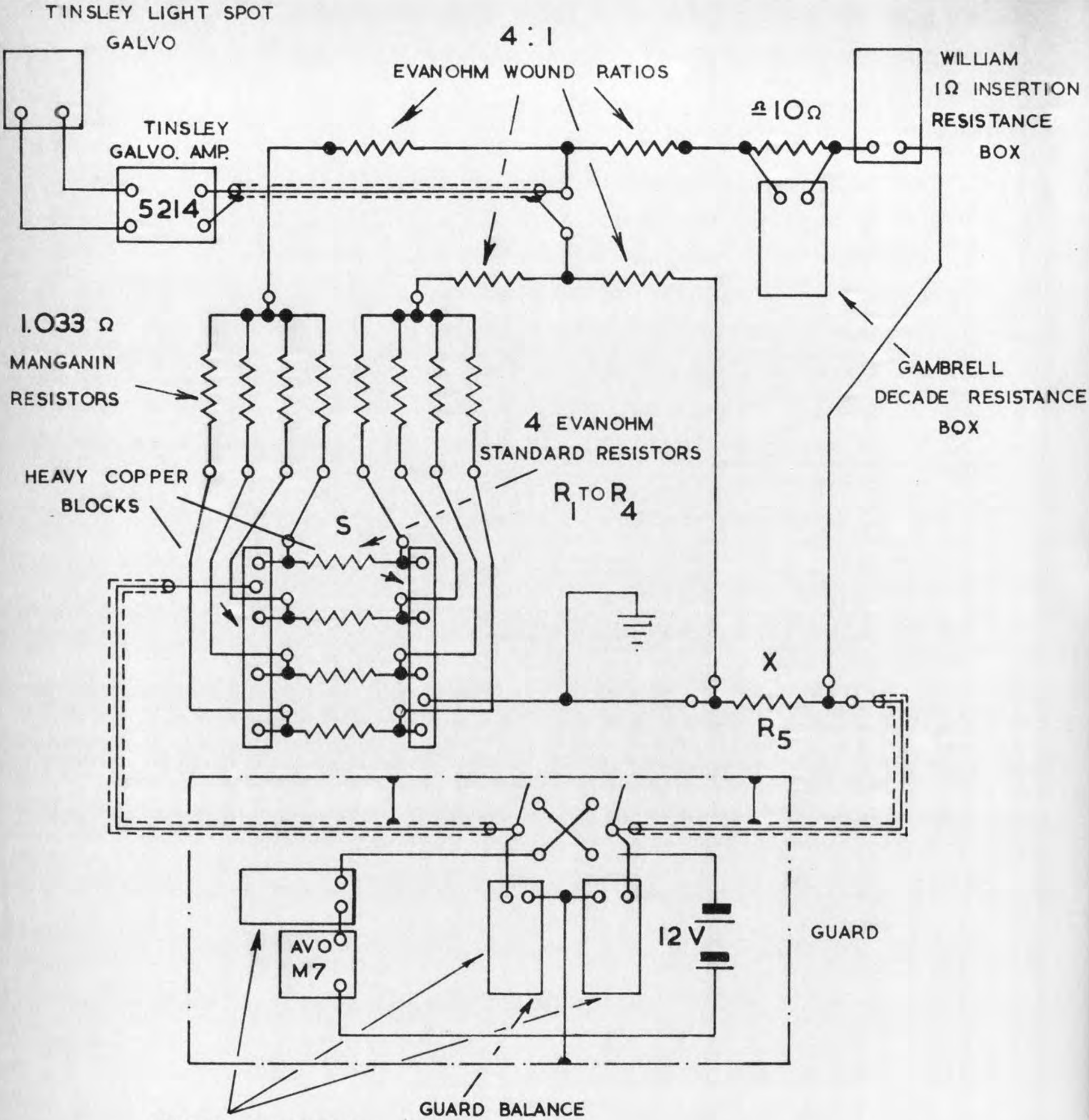


Fig XXVIII

Schematic Diagram
of the Kelvin Bridge used in obtaining
D.C. load coefficients of R_5.

(5.2.2)

Direct Current Load Coefficient Measurement Procedure.

When performing direct current load coefficient determinations on R5 at or near its 6 amp full-load current level using a Kelvin Bridge, the major problem encountered was that the resistor used as the Standard had to be capable of passing the 6 amps of direct current without greatly altering its resistance value and be of sufficiently high resistance value to provide a resistance ratio which would produce a reasonable bridge unbalance sensitivity. This problem was overcome in the measurements made by the Author on R5 with the use of a bridge ratio of 4 : 1 using a Standard made up of the four other Evanohm high dissipation resistors R1, R2, R3 and R4 connected in parallel with the use of 1.033 ohm manganin paralleling resistors.[21] The arrangement of the Kelvin Bridge measuring circuit used in measuring the load coefficient of R5 is shown in the circuit diagram of Fig. XXV111.

In order to obtain the necessary 4 : 1 ratio in the bridge arms A and B and P and Q, two specially constructed resistance ratios were wound from 30 S.W.G. polyeurathane coated Evanohm wire onto a 4 inch diameter tubular copper former. These resistance ratios were bifilar wound such that the main bridge ratio arms, A and B, consisted of approximately equal resistance sections of approximately 250 ohms each. The Evanohm winding wire used was selected to possess an extremely small temperature coefficient of resistance and a large resistance area was used for each of the five sections mentioned above in order to increase their power dissipation capability and hence reduce its load coefficient of resistance ratio.

The A arm of the wound ratio was adjusted to a resistance value of approximately 999 ohms. The remaining 1 ohm was made up by the insertion resistance of a 4-dial William shunted-dial resistance box, which was to provide the adjustments necessary to obtain bridge balance. The balance condition of the Kelvin Bridge was adjusted to give a convenient value on the 4-dial shunted-dial resistance box with the adjustment of the other resistance box, which is shown connected as a shunt across a 10 ohm section of the A arm in Fig. XXV111.

Parallel connecting the four 1 ohm high dissipation Evanohm standard resistors in order to provide a Standard meant that, with 36 watts power dissipation in the Unknown, there was only 2.25 watts power dissipation in each of them. The load coefficients of R1, R2, R3 and R4 determined by Mr. F. C. Brown at 36 watts previously, were considered to be of sufficient accuracy to act as a basis in the assessment of the load correction to be applied to the Standard at this level of power dissipation, on the assumption that the load coefficients at constant temperature were directly proportional to the power dissipated.[3,4]

The guard circuit shown surrounding the direct current supply circuit and its associated leads was found to be necessary in order to eliminate the effect of leakage currents from the battery circuit to earth on the resistance values of the Standard and Unknown. Prior to the commencement of direct current load coefficient measurements on R5, temperature coefficient measurements had to be made on the bridge circuit. The Bridge Circuit consisted of the wound ratios and the Standard, both of which were completely immersed in the one stirred oil bath.

Load coefficients of R5 were measured at direct current values of 2, 4 and 6 amps, so as to correspond with the readings taken under A.C. loading conditions. In processing the results obtained, use was made of the temperature coefficient of the Bridge circuit and of the Unknown itself obtained previously. Allowances had to be made for the power dissipated in the Standard and the wound ratio. The correction applied to the bridge balance conditions due to the power dissipated in the Standard was calculated from the load coefficient of each high dissipation Evanohm standard resistor obtained by Mr. F.C. Brown at 36 watts. (See section 5.2.3)

With a galvanometer sensitivity of approximately 80 000 mm/uA, 1 part in 10^7 change in the Unknown could be detected. The link resistance used was of the order of 10^{-3} ohm, so that the error term in the Kelvin Bridge balance condition of equation (2.4), due to the inequality of the major and minor bridge ratios, was negligible when resistance changes were being measured. Similarly, the fact that one side of the link was earthed produced a negligible effect on the bridge balance conditions, due to leakage to earth from the high impedance galvanometer point at the junction of the A and B arms, provided a leakage resistance above about 10 Megohms was obtained.

(5.2.3)

Calculation of the Load Corrections at Constant Temperature for 4 Parallel Connected Nominally Equal Resistors.

For four nominally equal resistors R1, R2, R3 and R4 with small corrections in proportional parts equal to δ_1, δ_2, δ_3 and δ_4 respectively, connected in parallel so as to produce a resistor of nominal resistance value equal to $\frac{R}{4}$, then it can be shown that δ_p, the correction in proportional parts to the parallel combination of the four nominally equal resistors R is given by

$$\delta_p = \frac{\delta_1 + \delta_2 + \delta_3 + \delta_4}{4} \qquad (5.1)$$

If δ_r is a function of T_r and W_r only, where T_r is the temperature of the environment of the rth resistor and W_r is the power dissipated in that resistor, then if

$$\left.\frac{\partial \delta_r}{\partial W_r}\right|_{T_r} = k_r \qquad r = 1,2,3,4 \qquad (5.2)$$

it can be shown that

$$4d\delta_p = \left(\frac{\partial\delta_1}{\partial W_1}\right)_{T_1} dW_1 + \ldots\ldots + \left(\frac{\partial\delta_4}{\partial W_4}\right)_{T_4} dW_4$$

$$+ \left(\frac{\partial\delta_1}{\partial T_1}\right)_{W_1} dT_1 + \ldots\ldots + \left(\frac{\partial\delta_4}{\partial W_4}\right)_{W_4} dT_4 \qquad (5.3)$$

If $\quad W_1 = W_2 = W_3 = W_4 = \frac{1}{4} W_T$

and $\quad T_1 = T_2 = T_3 = T_4 = T$

then $\quad dW_1 = dW_2 = dW_3 = dW_4 = \frac{dW_T}{4} \qquad (5.4a)$

and $\quad dT_1 = dT_2 = dT_3 = dT_4 = dT \qquad (5.4b)$

so that

$$d\delta_p = \frac{1}{16}\left\{ \left[\left(\frac{\partial\delta_1}{\partial W_1}\right)_{T_1} + \ldots + \left(\frac{\partial\delta_4}{\partial W_4}\right)_{T_4}\right] dW_T + 4\left[\left(\frac{\partial\delta_1}{\partial T}\right)_{W_1} + \ldots + \left(\frac{\partial\delta_4}{\partial T}\right)_{W_4}\right] dT \right\} \qquad (5.5)$$

Under most practical conditions it is found that

$$T_r = \phi(W_r, t) \qquad (5.6)$$

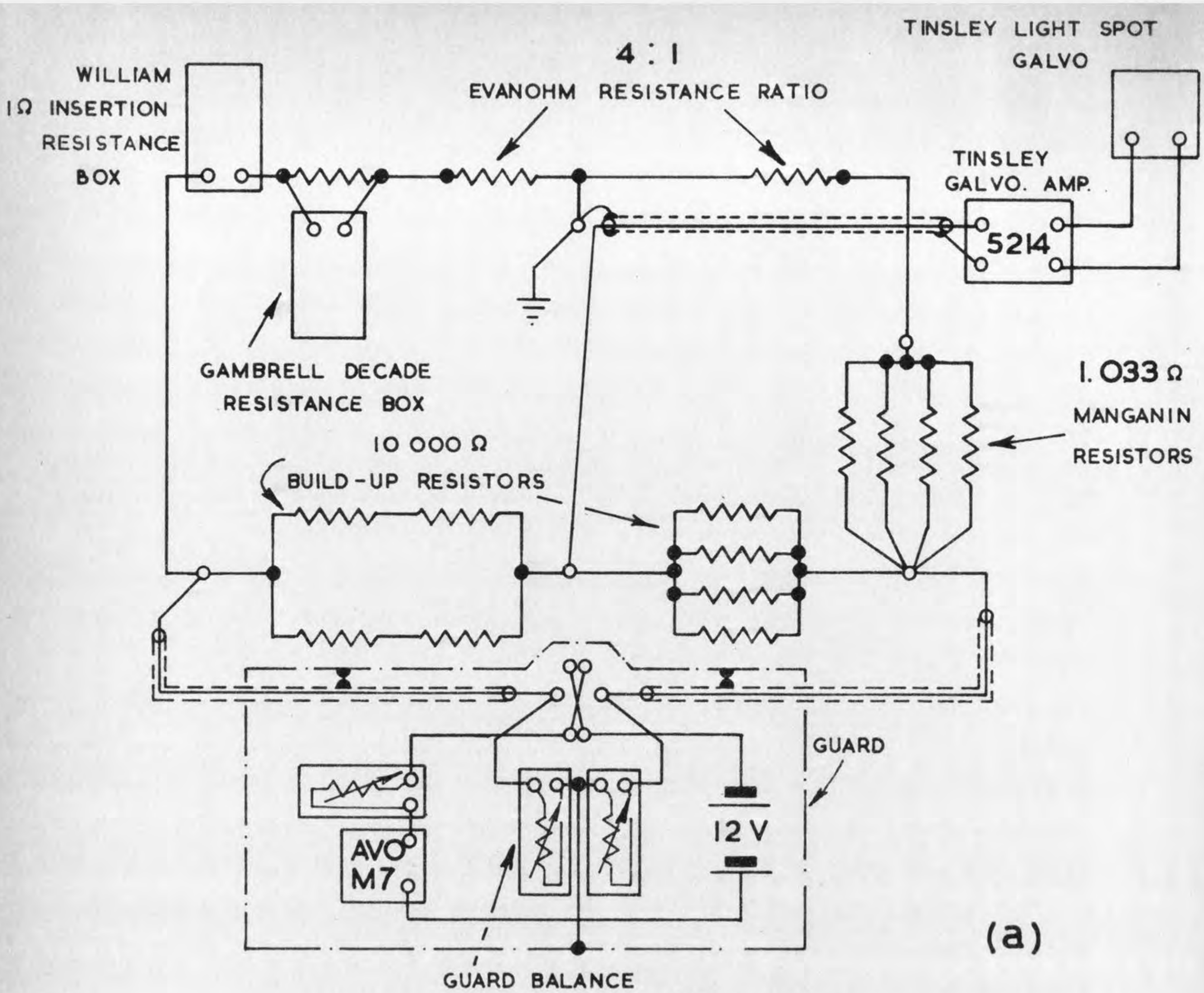
where t is the time.

For a reasonably controlled oil bath, or for one which has a large thermal mass, it is reasonable to expect that

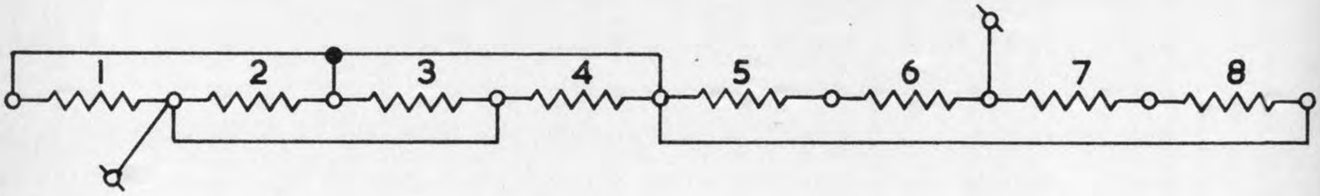
$$\frac{dT_r}{dW_r} \simeq 0 \qquad (5.7)$$

Therefore it may be shown from (5.5) that

$$(\delta_{pFL} - \delta_{pNL})_T = \frac{1}{16} \int_{WT_{NL}}^{W_{T_{FL}}} \left\{ \left(\frac{\partial\delta_1}{\partial W_1}\right)_T + \left(\frac{\partial\delta_2}{\partial W_2}\right)_T + \ldots + \left(\frac{\partial\delta_4}{\partial W_4}\right)_T \right\} dW_T \qquad (5.8)$$



Schematic Diagram
of the Wheatstone Bridge used in finding the Load Coefficient of the Evanohm Wound Ratio.



1-100,000 Ω Build-up Box connections (b)
used in obtaining a 4 : 1 ratio.

Fig XXIX.

Substituting (5.2)

$$(\delta_{pFL} - \delta_{pNL})_T = \frac{1}{16} (k_1 + k_2 + k_3 + k_4)(W_{TFL} - W_{TNL}) \quad (5.9)$$

(5.2.4)

Load Coefficient of the Evanohm Wound Ratio Used in the Kelvin Bridge when making Full D.C. Load Coefficient Measurements on R5.

The load coefficient of ratio for the Evanohm Wound Ratio used in the full D.C. load coefficient measurements made on R5, was carried out in the bridge oil bath used in section (5.2.2) with the aid of a 10 000 ohm manganin Build-up Box,[22] which had been removed from its container and immersed completely in the oil, to form the Wheatstone Bridge shown in Fig. XX1X (a). Because the centre point of the wound ratio was earthed, in order to reduce the sensitivity of the galvanometer system to operator motion in the vicinity of the bridge circuit, the guard circuit shown was considered necessary so that leakage currents from the battery circuit to earth were kept negligible.

The inter-connections used on the build-up box in order to obtain the required resistance values were made with small lengths of 16 S.W.G. bare copper wire of negligible resistance. The actual build-up box resistance coils used in obtaining the desired ratio may be seen by inspection of Fig. XX1X (b).

Stirring of the oil bath was accomplished with the aid of a motor driven stirrer and oil bath temperatures were measured with the aid of a 0.01 ^{o}C mercury-in-glass thermometer. A knowledge of the exact temperature coefficient for any bridge section was considered to be unnecessary, as it was intended to perform the load coefficient measurements on the Evanohm wound ratio early one morning when the room temperature was relatively constant and hence the oil temperature changes with time found to be negligible. The galvanometer system used in conjunction with these measurements had a current sensitivity of about

80 000 mm/uA as before, and in order to detect a change of 1 part in 10^7 in the X arm of the Wheatstone Bridge, it was found necessary to use a no-load bridge current of 1.2 mA D.C. because of the increase in bridge output resistance encountered in this case. The level of power dissipation in the wound ratio was higher than the 0.11 mW obtained in the Kelvin Bridge of section (5.2.2) under no-load conditions, but the increase was found to cause negligible error, as will be shown by the results of the load coefficient determination presented in section (5.4). Three series of four Bridge balances were performed using D.C. bridge currents of 1.2 mA (no-load) and 12 mA (full load) and then 1.2 mA again, so as to measure the load coefficient of ratio of the Evanohm wound ratio with a change in power dissipation level of 100 times. Oil bath temperatures were noted in each instance and the readings found to remain constant at 22.10_5 °C, with a laboratory air temperature of 20.8 °C throughout. It was considered to be a reasonable assumption that 2.5 mW in the 10 000 ohm coils of the A arm would not change the resistance value of any of these coils by an amount greater than several parts in 10^8 from its negligible load value, since they were all in close contact with stirred oil and their temperature coefficients were all less than 10 parts per million per °C at 20 °C.

The full load bridge current of 12 mA used meant that any change in the resistance value of the wound ratio at 36 mW in 1 000 ohms would be about 1/3 that experienced at the 100 mW in 1 000 ohm level used for the load coefficient determination.

(5.3)

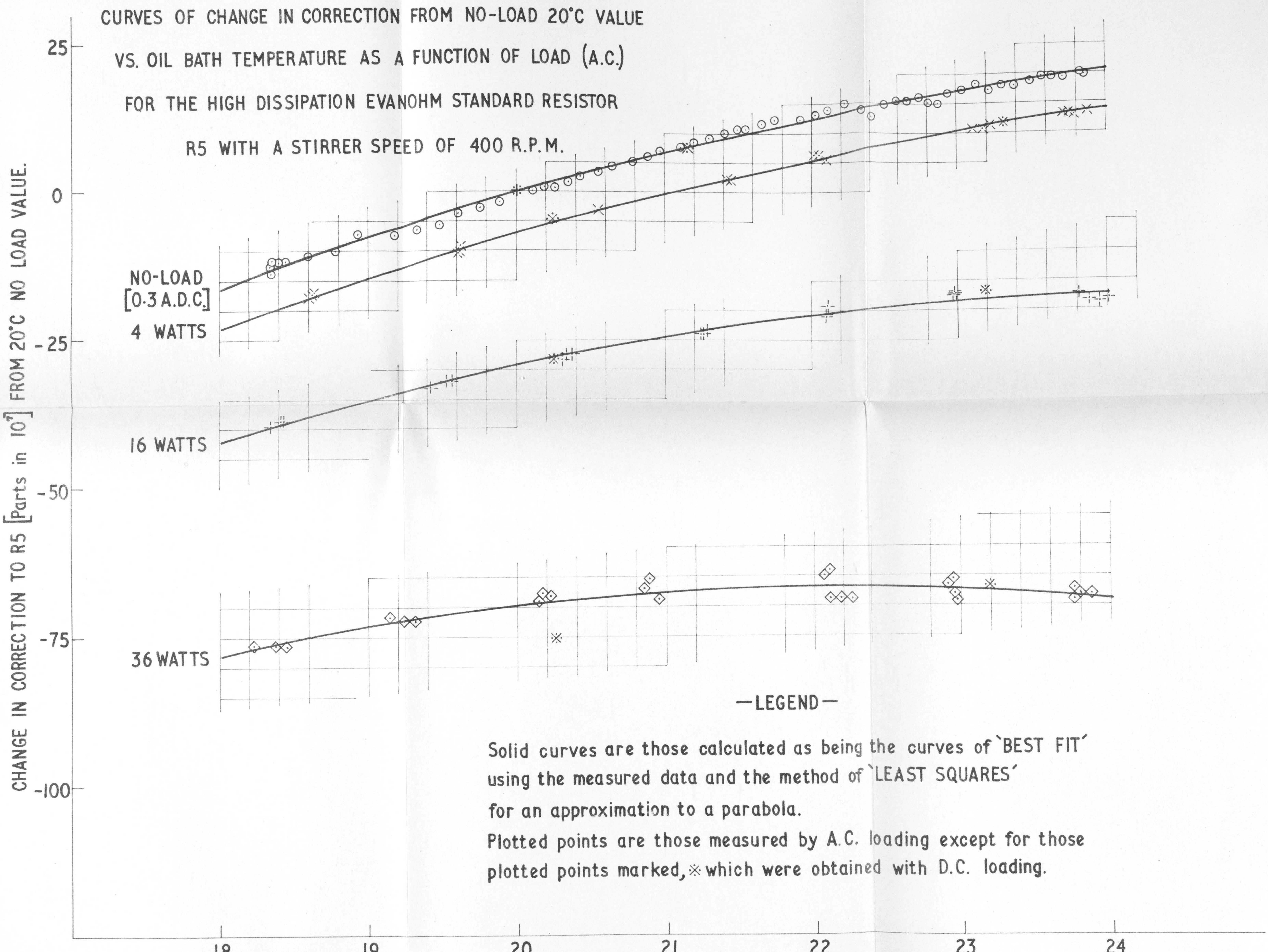
Measurement of Currents Used.

Current level setting and measurement techniques similar to those used in measurements on L-259235 were employed in the load and temperature coefficient measurements performed by the Author on R5.

The meter used during the D.C. load coefficient measurements to set the D.C. loading current was an AVOmeter model 7. It was a lower precision instrument than the Siemens Halske moving iron meter used by the Author in setting the alternating loading currents, but the current measurement accuracy required was such that the difference had little effect on the accuracy of the results obtained for values of load coefficient, except at the highest value of power dissipation in R5, 36 watts. To off-set this reduction in accuracy, the AVOmeter was spot checked at current levels of 4 and 6 amps. The results obtained showed that the AVOmeter read low by about 0.4 per cent of F.S.D. at 4 amps but that its correction was negligible at 6 amps. Reproducability of setting was found to be good, so that a 0.5 per cent of F.S.D. current measurement accuracy claimed for the meter was considered to be reasonable.

The value of the load coefficient of R5 was found to be approximately - 75 parts in 10^7 at 20 °C and 36 watts power dissipation. If the uncertainty with which the level of the loading current was set was to produce an error of less than $\pm$ 1 part in 10^7 in the load coefficient of R5 at 36 watts power dissipation, then the maximum error permissable when setting the effective value of the loading current was approximately $\pm$ 0.7 per cent. The calibration accuracy claimed for the Siemens Halske meter ($\pm$ 0.2 per cent of F.S.D.) and for the AVOmeter model 7 (0.5 per cent of F.S.D.), meant that the loading current measurement uncertainty obtained caused errors of less than $\pm$ 0.5 parts in 10^7 and $\pm$ 1 part in 10^7 in the load coefficient of R5 obtained at 20 °C with a power dissipation in R5 of 36 watts under alternating and direct current loading conditions respectively.

The direct current passing through the wound ratios and the guard balancing circuit of Fig. XXV111 will be 0.1 per cent and 0.01 per cent respectively of the loading current passing through R5 and the Standard. These relatively small amounts of direct current, which are by-passed from the current through R5, but which are measured by the AVOmeter, will cause negligible


CURVES OF CHANGE IN CORRECTION FROM NO-LOAD 20°C VALUE
VS. OIL BATH TEMPERATURE AS A FUNCTION OF LOAD (A.C.)
FOR THE HIGH DISSIPATION EVANOHM STANDARD RESISTOR
R5 WITH A STIRRER SPEED OF 400 R.P.M.
25
0
-25
-50
-75
-100
NO-LOAD
[0·3 A.D.C.]
4 WATTS
16 WATTS
36 WATTS
CHANGE IN CORRECTION TO R5 [Parts in 10⁷] FROM 20°C NO LOAD VALUE.
18
19
20
21
22
23
24
OIL BATH TEMPERATURE [°C]
—LEGEND—
Solid curves are those calculated as being the curves of 'BEST FIT'
using the measured data and the method of 'LEAST SQUARES'
for an approximation to a parabola.
Plotted points are those measured by A.C. loading except for those
plotted points marked, ※ which were obtained with D.C. loading.
Fig. XXX.

errors in the calculated value of the effective loading current in R5 and the Standard.

(5.4)

Presentation of Results.

(5.4.1)

No-Load Temperature Coefficient of the Unknown and the Bridge Circuits Used for both A.C. and D.C. Loading of R5.

The results of the no-load temperature coefficient measurements on R5 and its associated Bridge circuit used to measure A.C. load coefficients were processed using the computer program TEMPCURV, as shown in Chapter 12. The Bridge circuit used when making the D.C. load coefficient measurements on R5 was found to have a value for α_{20} of approximately + 3 parts in $10^7/\ ^{\circ}C$, with small β.

(5.4.2)

Load Coefficient Measurements of R5 at 4, 16 and 36 Watts Using Both D.C. and 40 c/s Alternating Loading Currents.

The temperature coefficient curves obtained for the high dissipation Evanohm resistor R5 at power dissipation levels of 4, 16 and 36 watts are presented in Fig. XXX for 40 c/s A.C. loading conditions, with the results presented such that its resistance value is referred to its no-load 20 $^{\circ}C$ resistance value. The D.C. load coefficient values obtained for R5 are presented at two oil bath temperatures. Measured values of resistance change from the 20 $^{\circ}C$ no-load value were added to the calculated curves of best fit in order to demonstrate the magnitude of the residuals obtained.

The load coefficients obtained by Mr. F. C. Brown for the high dissipation Evanohm resistors R1, R2, R3 and R4 gave the correction to be applied to the Standard under various conditions of D.C. loading using equation (5.9)

With I_{DC} = 6 amps

$$\delta_{p9W} - \delta_{pNL} \simeq -4.3 \text{ parts in } 10^7 \qquad (5.10)$$

With I_{DC} = 4 amps

$$\delta_{p4W} - \delta_{pNL} \simeq -1.9 \text{ parts in } 10^7 \qquad (5.11)$$

With I_{DC} = 2 amps

$$\delta_{p1W} - \delta_{pNL} \simeq -0.5 \text{ parts in } 10^7 \qquad (5.12)$$

If the load coefficient of R1, R2, R3 or R4 used in calculating the correction to the Standard was in error by a certain amount, then the corresponding error in the correction to be added to the resistance value of R5 at 36 watts because of this, would be approximately 1/64 of the error in the original load coefficient. Estimates made suggest that the load coefficients obtained by Mr. Brown on R1, R2, R3 and R4 would be accurate to within $\pm$ 5 parts in 10^7.

The effect on the load coefficient of R5 at 16 watts power dissipation corresponding to a change in stirrer speed from 400 to 300 RPM was to increase its negative value by approximately 25 per cent. It is estimated that the error in the set value of stirrer speed of R5 would not have been greater than an amount corresponding to 1 revolution of the stroboscope pattern every 5 seconds. This difference corresponds to a maximum error in the speed of the stirrer of $\pm$ 3 per cent.

(5.4.3)

Measurements of Load Coefficient of Ratio for the Evanohm Wound Ratio Used in the D.C. Load Coefficient Determination on R5.

The load coefficient of the Evanohm wound ratio was found to be less than 1 part in 10^7, for a change in the bridge current between 1.2 mA and 12 mA. It was reasoned then, that the change experienced at a bridge current of 7.2 mA and below would be negligible.

(5.5)

Assessment of the Results Obtained on R5.

(5.5.1)

The overall results of the measurements made on the load coefficients of R5 were pleasing and several points of interest occurred. The agreement between the A.C. and D.C. load coefficient values obtained at the two oil bath temperatures was within 2 parts in 10^7 under all conditions except those existing at the 36 watts power dissipation level, where the D.C. load coefficient value differed from that of the A.C. load coefficient value by - 5 parts in 10^7 at the lower oil bath temperature. The exact cause of this is unknown, but in the light of the agreement existing between the A.C. and D.C. load coefficient values obtained at the 36 watts power dissipation level at the higher oil bath temperature, it is assumed reasonable that the 5 parts in 10^7 disagreement is due to the difficulty in obtaining exactly reproducable environmental conditions from time to time.

Reference to section (5.3) will show that at 36 watts the probable errors in setting the A.C. and D.C. loading currents could cause differences between the A.C. and D.C. load coefficients obtained of up to 2 parts in 10^7.

The effect of stirrer speed fluctuations on the value of the load coefficient obtained for R5 is difficult to assess exactly. If it may be assumed that small fluctuations in stirrer speed will affect only the temperature of that portion of the Evanohm resistance element which is situated in an area of free oil flow, then from the result of the discussion presented in section (5.5.3) below, small stirrer speed fluctuation will affect only about 50 per cent of the load coefficient existing at a particular power dissipation level. It is shown by Briggs[20] that the resistance increases under load occurring in areas where forced convection only is applicable will be inversely proportional to the bulk oil velocity. If the bulk oil velocity at any point may be assumed to be directly proportional to the stirrer speed[23], then the resistance increase under load in a forced convection-cooling area will be inversely proportional to the stirrer speed.

$$\text{Therefore} \quad \frac{LC_1}{LC_2} = K\,\frac{SS_1}{SS_2} \qquad (5.13)$$

where K is a constant of proportionality and equal to unity, LC_1 and LC_2 are the load coefficient values in cases 1 and 2 respectively and
SS_1 and SS_2 are their respective stirrer speeds.

For stirrer speed fluctuations of the order of $\pm$ 3 per cent, changes of approximately $\pm$ 1.3 parts in 10^7 in the value of the load coefficient obtained for R5 at 36 watts may be expected. Hence the difference between the A.C. and D.C. load coefficient value obtained due to stirrer speed uncertainties at 36 watts may be up to approximately 2.5 parts in 10^7. The fact that load coefficient measurements made on R5 at 16 watts at stirrer speeds of 400 and 300 RPM showed the overall negative load coefficient to increase by 25 per cent, indicates that stirrer speed change of the order of 100 RPM in 400 are too large to be classed as small.

Difference in oil flow pattern through R5 in the cases of A.C. and D.C. load coefficient determinations, due to the placement of R5 relative to other objects in its oil bath and differing depths of immersion, will cause an unknown effect on the load coefficient values obtained, although it is felt that the stirrer speeds were so large that effects experienced from these sources were small. Temperature effects in the oil baths used are considered to be unlikely sources of error, in the light of the results obtained in section (10.1). Resistance instabilities in the Unknown and its associated Standard and Kelvin Bridge arms are also reckoned to contribute less than $\pm$ 1 or 2 parts in 10^7 to any difference in resistance change occurring. The bridge unbalance sensitivity available during measurements at all A.C. loads may have contributed up to $\pm$ 1 part in 10^7 to any difference occurring between the A.C. and D.C. load coefficient values obtained at these loads.

(5.5.2)

No-Load Temperature Curve for R5.

Close inspection of the no-load temperature coefficient curve of R5 presented in Fig. XXX will suggest that there appears to be a discontinuity of slope in the form of a 'hinge point' occurring at an oil bath temperature of approximately 21.8 $^{\circ}$C. Normally, because of its rather small effect, this condition would have been considered to have been due to the effect of experimental error and bridge balance uncertainties. But results of measurements made by Mr. R. E. Holmes of N.S.L. with the increased sensitivity and bridge resistor stability available, produced a similar effect.

(5.5.3)

Negative Load Coefficient - Positive Slope with Temperature at 20 $^{\circ}$C, Under All Levels of Loading Used.

It has been stated by Miller[3] and Wenner[4] that the steady state load coefficient of a standard resistor is proportional to its change in resistance with temperature and inversely proportional to its heat dissipating capacity. The abnormal behaviour of R5, with changing oil temperature and load, can be satisfactorily explained by postulating a non-uniform distribution of heat dissipation along its length while under load, and by examining the effect produced by a small amount of copper (0.22 ohm x 10^{-3}) situated at the junction of the Evanohm resistance element and the current posts.

Consider first the effect of the copper on the measured values of α_{20} obtained under all conditions of load.

The copper is situated such that when the temperature of the oil bath containing R5 is raised, the temperature of both copper and Evanohm will rise correspondingly. When significant power is dissipated in the Evanohm resistance element, the temperature of the Evanohm resistance element will rise relative to that of the oil, but the temperature of the copper will remain relatively close to that of the oil. Hence, any effect that the changing temperature of the copper, with its temperature coefficient of the order of 0.4 per cent/ ^{o}C rise, had on the overall resistance change in R5 when making temperature coefficient measurements, will be removed when R5 is loaded at constant temperature. But if a temperature coefficient measurement is to be made under load by altering the oil temperature, the effect of the copper will be similar to that obtained at no-load.

It was realised by Mr. R.C. Richardson of N.S.L. that the effect of the 0.22 milliohms of copper present in R5 was enough to cause no-load temperature coefficients for the Evanohm alone to change from the + 6.7 parts in 10^7 per degree C at 20 ^{o}C, as obtained from the composite no-load curve of Fig. XXX, to a slightly negative value. Hence if the temperature rise in the copper above that of the oil was negligible or very small under all conditions of power dissipation in R5, then the measured values obtained for the load coefficients of R5 at an oil temperature of 20 ^{o}C should have been negative.

The element temperature rise required to account for the - 80 parts in 10^7 load coefficient obtained for R5, while assuming a uniform temperature distribution along the element and using the values of α_{20} and β as calculated by Richardson, was 11 ^{o}C. This was considered by Briggs[23] to be excessive. His estimated maximum was 5.6 ^{o}C rise at the 36 watts power dissipation level in the unimpeded oil flow at 400 RPM stirrer speed.

(5.5.4)

Effect of Non-Uniform Element Temperature Distribution.

If it is assumed for simplicity that the resistance element consists of two sections 1 and 2 only, each with different capabilities for heat dissipation under load, then the operating temperature of each of the two portions of the resistance element will be different from that of the cooling oil and different from each other. The difference in temperature between that of the resistance element and its surrounding oil under load may be denoted by T_{D_1} and T_{D_2} for the elements R_1 and R_2 respectively and since the relationship between the resistance $R(T, T_{D_1}, T_{D_2})$ of the element and the temperature T of the oil at no-load ($T_{D_1} = T_{D_2} = 0$) is given by

$$R(T,0,0) = (R_{120} + R_{220})\left[1 + \alpha_{20}(T-20) + 0.5\beta(T-20)^2 \right] \quad (5.14)$$

where R_{120} and R_{220} are the resistance values of R_1 and R_2 at a temperature of 20 °C respectively, then the resistance of the element under load and at an oil temperature T is given by

$$R(T, T_{D_1}, T_{D_2}) = (R_{120} + R_{220}) \cdot$$

$$\left[1 + \alpha_{20} \frac{R_{120}(T + T_{D_1} - 20) + R_{220}(T + T_{D_2} - 20)}{R_{120} + R_{220}} + 0.5\beta \frac{R_{120}(T + T_{D_1} - 20)^2 + R_{220}(T + T_{D_2} - 20)^2}{R_{120} + R_{220}} \right] \quad (5.15)$$

If

$$T_{D_2} > T_{D_1} > 0, \quad \text{then} \quad (5.16)$$

$$R(T, T_{D_1}, T_{D_2}) - R(T, T_{D_1}, T_{D_1})$$

$$= R_{220}(T_{D_2} - T_{D_1})\left[\alpha_{20} + 0.5\beta(T_{D_1} + T_{D_2} + 2(T-20))\right] \quad (5.17)$$

$R(T, T_{D_1}, T_{D_1}) - R(T, 0, 0)$

$$= (R_{120} + R_{220})\, T_{D_1}\left[\alpha_{20} + 0.5\beta(T_{D_1} + 2(T-20))\right] \qquad (5.18)$$

Since α_{20} and β of R5 are negative, the values of the left hand sides of the equations (5.17) and (5.18) will be negative. Hence a non-uniform temperature distribution in R5, where element temperatures greater than those existing in the free-oil flow, T_{D_1} are to be found, will produce a greater negative load coefficient at constant oil temperature than would be expected from an example where a uniform element temperature distribution, equal to the temperature existing in the free-oil flow T_{D_1} was obtained.

Differentiation of equation (5.15) with respect to temperature T shows that the oil temperature at which the value of $R(T, T_{D_1}, T_{D_2})$ is maximum is given by

$$T_{max} = 20 - \left[\frac{R_{120}\, T_{D_1} + R_{220}\, T_{D_2}}{R_{120} + R_{220}} + \frac{\alpha_{20}}{\beta}\right] \qquad (5.19)$$

Similarly substitution of (5.19) into (5.15) shows that the maximum resistance value $R(T_{max}, T_{D_1}, T_{D_2})$ is given by

$$R(T_{max}, T_{D_1}, T_{D_2}) = (R_{120} + R_{220})$$
$$\left[1 - \frac{\alpha_{20}^{\,2}}{\beta} + 0.5\beta\frac{R_{120}\, R_{220}}{(R_{120} + R_{220})^2}(T_{D_2} - T_{D_1})^2\right] \qquad (5.20)$$

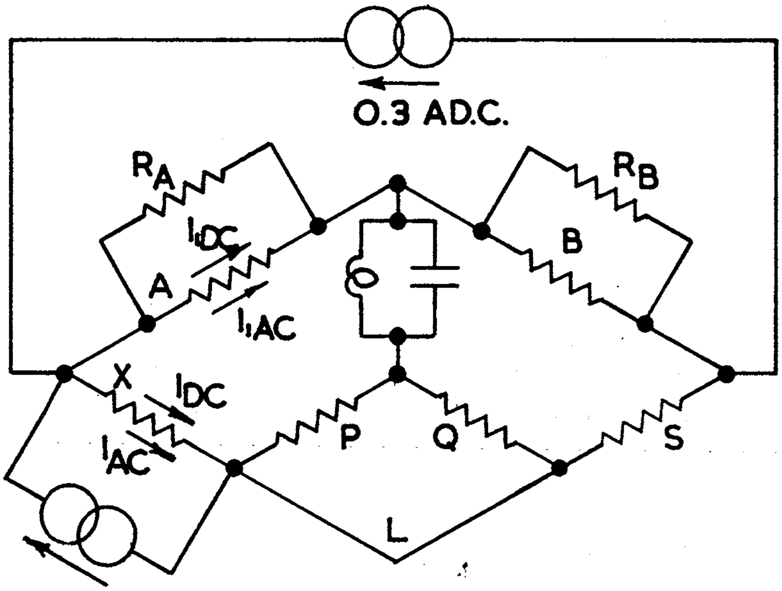
For all normally used standard resistance alloys, the value of β is negative whereas α_{20} may be positive or negative. From equation (5.19) it may be seen that if $T_{D_2} > T_{D_1}$, then T_{max} will occur at a lower temperature than if $T_{D_2} = T_{D_1}$ or if $T_{D_1} = T_{D_2} = 0$. Similarly equation (5.20) shows that the maximum value of $R(T, T_{D_1}, T_{D_2})$ will be lower than that experienced at no-load ($T_{D_1} = T_{D_2} = 0$), if the element temperature distribution is non-uniform under load. If the temperature distribution along a resistance element under load is uniform then the maximum value of $R(T, T_{D_1}, T_{D_1})$ will be equal to the maximum resistance value obtained at no-load, even though this maximum value will occur at a lower oil temperature.

Inspection by the Author of the method of construction used in the resistor R5 showed that its element was supported in four slotted P.T.F.E. spacers which shielded section of the resistance element from the free-oil flow.

The total shielded length was only about 10 per cent of the overall length of the resistance element, but it was considered likely that portion of the effect obtained was due to this shielding by the P.T.F.E. supports. Investigation of this theory was made by Mr. F. C. Brown. Two of the four slotted P.T.F.E. spacers were removed and load coefficient measurements were made on R5 at the same levels of power dissipation used by the Author. The results showed a reduction of about 20 parts in 10^7 in the negative load coefficient obtained at 36 watts at 20.4 °C. Extrapolation to the condition where all four spacers were removed indicated that approximately half the negative load coefficient was accounted for.

Further inspection of the mechanical construction of R5 showed that one of the two brackets used as bearing mounts for the stirrer of R5, was situated between the stirrer propellor and the resistance element in such a position that a significant interruption of the free oil flow past the neighbouring portion of the resistance element would be experienced.

The structure of R5 was such, that in order to alter the placement or to remove the bracket in question, a major mechanical operation would have been required, with the possibility of damage occurring to the Evanohm resistance element meanwhile. However, the presence of the support bracket in the oil flow path may account for the remaining negative load coefficient.



Typical Kelvin Bridge
featuring parallel-fed A.C. loading

Fig XXXI.

CHAPTER 6

Load Coefficient Measurements on 1 000 Ohm Leeds and Northrup Resistor S/N 1642612

(6.1)

For tests on the 1 ohm high dissipation Evanohm resistor R5 mentioned in Chapter 5, it was calculated that, during measurements at the higher values of power dissipation in R5, there was sufficient alternating current passed through the A and B arms of the Kelvin Bridge used to cause what might be a finite change in their resistance value due to their load coefficients.

Consider the circuit as shown in Fig. XXX1 neglecting any stray circuit impedances, the value of the alternating current I_{1AC} through the 1 000 ohm L and N resistor S/N 1642612 situated in the A arm, may be calculated as

$$I_{1ac} \simeq \frac{1}{1100.1} \cdot I_{ac} \qquad (6.1)$$

after substituting typical circuit values, with an error of less than 1 per cent.

Table (6.1) may be produced from the values of I_{ac} used in Chapters 4 and 5, and equation (6.1).

TABLE (6.1)

I_{dc} (A)	I_{ac} (A)	I_{1dc} (mA)	I_{1ac} (mA)	Power in 1 000 ohms (mW)
0.3	0	0.3	0	0.09 (DC)
	0.632		0.57_4	0.41_9
	0.95_4		0.86_7	0.84_2
	1.38_2		1.25_6	1.67
	1.97_7		1.80	3.32
	2.22		2.01	4.15
	3.99		3.62	13.2_3
	6.00		5.45	29.8

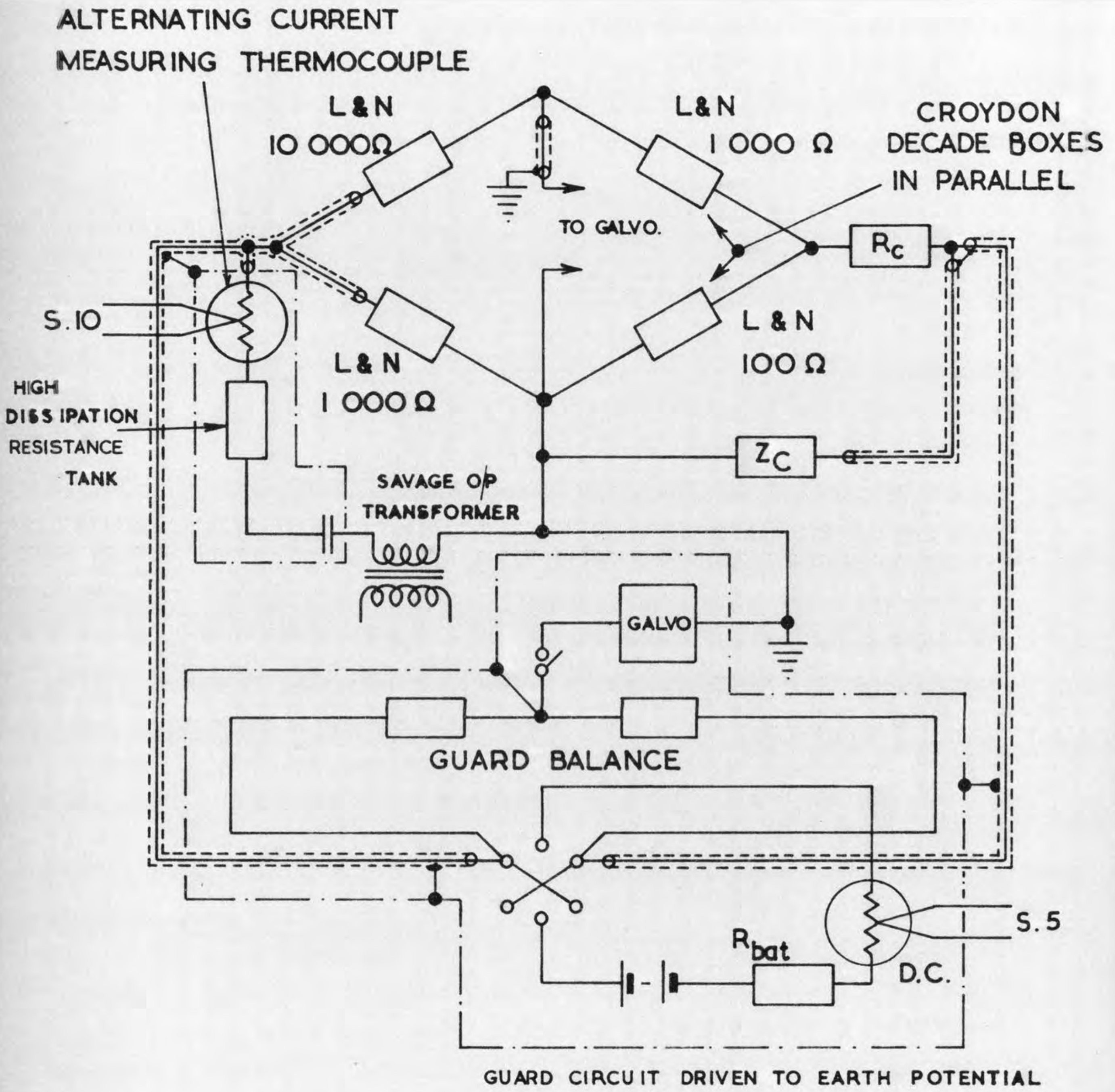
The power dissipation level in the L and N 100 ohm resistor 1646270 of the B arm was of the order of a factor of ten less than that in the 1 000 ohm A arm. An estimation of its change under load in the conditions encountered in practice may be made using the discussion of section (6.3).

For small loads, the relationship between the load coefficient and the power dissipated may be considered to be linear.[3,4] With this in mind it was decided to measure the load coefficient of 1642612 at power dissipation levels of 29.8, 13.2_3 and 0.84_2 mW, interpolating where necessary, for values of loading in between.

(6.2)

Choice of Measurement Technique.

It was decided that the advantages of the Kelvin Bridge for low values of resistance used in measurements made previously, did not apply for values of resistance around 1 000 ohms and it was decided to employ a Wheatstone Bridge technique. A.C. loading techniques were decided upon owing to the loading in 1642612



10 to 1 Wheatstone Bridge
using L&N 'Rosa' construction standard resistors.

Fig XXXII.

and 1646270 during tests on R5 at 16 and 36 watts being 40 c/s alternating current.

Initial considerations suggested that, in order to limit the alternating loading currents passing through the A, B and S arms of the bridge to values where their resulting A.C. power dissipation levels were negligible while loading the Unknown, the bridge ratios used might be

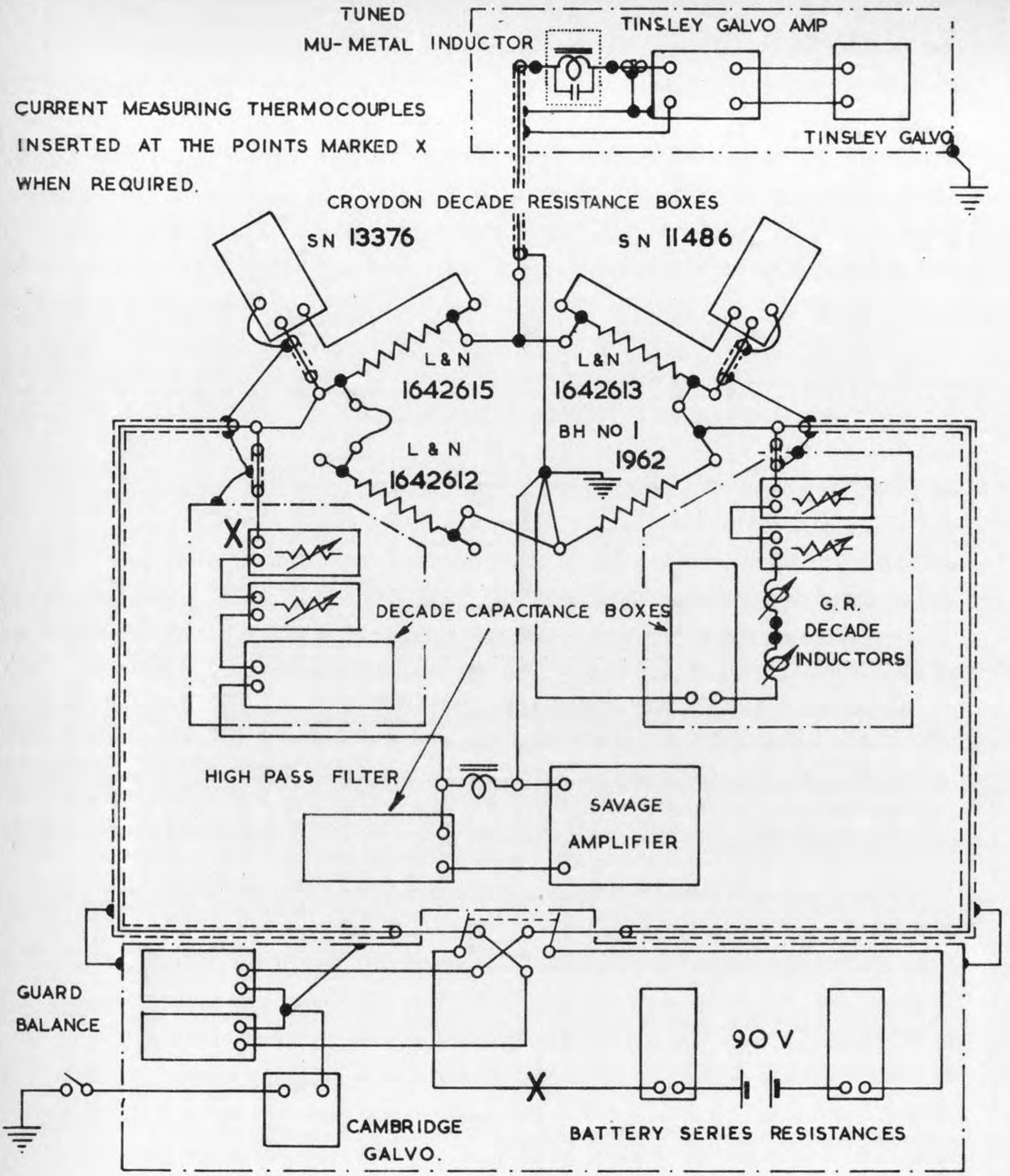
$$A/B = A/X = 10\,000/1\,000 \qquad (6.2)$$

The bridge unbalance sensitivity available when using large bridge ratios will be lower than the unbalance sensitivity available while using an equal arm Wheatstone Bridge, for the same D.C. power dissipation level in the Unknown, hence a compromise situation must be obtained.

The bridge circuit was wired as shown in Fig. XXX11 incorporating the capacitive kick correction circuit and the guard circuit as discussed in section (2.4).

After approximately balancing the bridge, guard and capacitive kick correction circuits, it was found that a slow drift occurred in the galvanometer spot of several centimetres over a period of about 30 seconds after supply reversal, which made balance to 1 part in 10^7, or even 1 part per million of the Unknown impossible. Calculations were made to check the time constants of the loading and capacitive kick correction circuits, but they were found to be small. Several other explanations for this behaviour were considered, such as dielectric absorption, electro-chemical emfs present in the 10 000 ohm resistor and, of course, the usual inductive kick, but all effects had to be considered in the light of the long time constant experienced.

Section of an article[24] dealing with the transient response to be expected of a typical Rosa type construction resistance standard, upon supply application (or reversal) was brought to the attention of the Author. This article is particularly pertinent to the transient condition experienced here, since the



Schematic Diagram of the Wheatstone Bridge used when investigating 1 000 ohm resistors.

Fig XXXIII.

Leeds and Northrup 1 000, 10 000 and 100 ohm resistance standards used were of Rosa construction. The conclusions drawn were that[24] "its principal cause is the formation of a small cell, or battery, on the surface of the header in the older type of standard resistor. Chemical analysis of scrapings from phenolic headers which have been in use for several years has shown that the scrapings contain relatively large amounts of calcium, silicon, magnesium, lead and iron. The electrochemical cell formed on the header by such substances can cause almost unpredictable transient effects."

Further measurements indicated that the 10 000 ohm resistance standards were unsuitable and that even the 1 000 ohm coils exhibited a small electro-chemical effect. It was decided to try an equal arm bridge, thus attempting to eliminate the effects of the electro-chemical emfs across the headers by balancing them out.

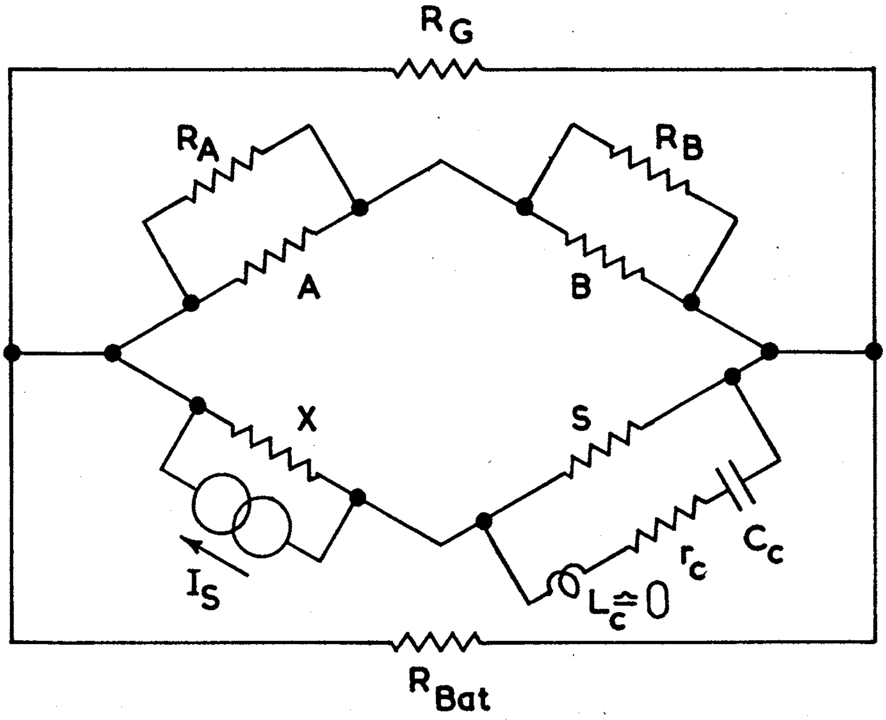
The circuit obtained is shown in Fig. XXX111 and investigation showed that, upon reversal, the drifts experienced in the galvanometer spot were negligible.

(6.3)

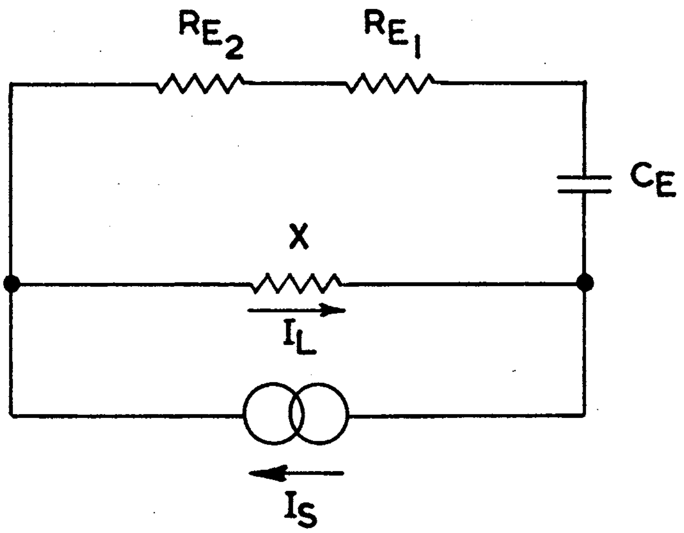
Estimation of the Order of Resistance Change at Small Power Dissipation Levels.

In an equal arm Wheatstone Bridge approximately 1/4 of the alternating current supplied to the bridge passes through the circuit consisting of the three resistors forming the A, B and S arms of the bridge, loading them with approximately 1/9 of the level of the power dissipated in the Unknown. This level is of similar order to the relative power dissipation level experienced in the L and N 100 ohm resistance standard 1646270 used in the corresponding B arm of several of the Kelvin Bridges of Chapters 4 and 5.

The presence of small amounts of copper in the resistance elements of 100 and 1 000 ohm resistors, as was found in the resistor R5, may be assumed to have a negligible influence on



(a)



(b)

Equivalent Circuits of Fig XXXIII.

Fig XXXIV.

their temperature or load coefficients. Hence the load coefficients of all similarly constructed and situated high valued resistance standards might be expected to be approximately in the ratio of their temperature coefficients α_{20}, for small power dissipation levels in them, irrespective of their resistance value.

As a means of testing this assumption in relation to the three L and N 1 000 ohm Rosa construction standard resistors 1642612, 1642613 and 1642615, an investigation was made of their load coefficients. The 1 000 ohm standard B.H. No. 1 1962 was measured also for completeness.

(6.4)

Calculation of the Relationship Between the Loading Current Passing Through the A, B and S Arms of the Wheatstone Bridge and the Loading Current in the Unknown.

The equal arm Wheatstone Bridge used in the measurement of the load coefficients of the 1 000 ohm resistors is as shown in Fig. XXX111. Considering the alternating currents flowing in the various branches of the bridge, the circuit of Fig. XXX111 may be modified to obtain Fig. XXX1V (a), which can be simplified with the combination of parallel resistors and impedances to the circuit as shown in Fig. XXX1V (b) where

$$R_{E_1} = S\left(\frac{1 + \omega^2 r_c (S + r_c) C_c^2}{1 + \omega^2 (S + r_c)^2 C_c^2}\right) \qquad (6.3)$$

$$R_{E_2} = \frac{R_{BAT} R_G \left[\dfrac{AR_A}{A + R_A} + \dfrac{BR_B}{B + R_B}\right]}{R_{BAT} + R_G \left[\dfrac{AR_A}{A + R_A} + \dfrac{BR_B}{B + R_B}\right] + R_{BAT}\, R_G} \qquad (6.4)$$

$$C_E = C_c\left[\frac{1 + \omega^2 (S + r_c)^2 C_c^2}{\omega^2 S^2 C^2}\right] \qquad (6.5)$$

$$\text{and} \quad I_S = I_L \cdot \frac{1 + j\omega C_E\,(X + R_{E_1} + R_{E_2})}{1 + j\omega C_E\,(R_{E_1} + R_{E_2})} \tag{6.6}$$

Substitution of circuit values gives

$$R_{E_1} \simeq 952 \text{ ohms} \tag{6.7}$$

$$R_{E_2} \simeq 1780 \text{ ohms} \tag{6.8}$$

$$C_E \simeq 542 \text{ uF} \tag{6.9}$$

$$\text{and} \quad I_S \simeq 1.36_5\, I_L \tag{6.10}$$

The current through the A and B arm resistances will be given by

$$I_A = I_B = \frac{1}{1 + \frac{(A + B)(R_G + R_{BAT})}{R_G\, R_{BAT}}} \cdot 0.36_5\, I_L \tag{6.11}$$

$$\simeq 0.32_8\, I_L \tag{6.12}$$

Similarly calculating the current passing through the S arm of the bridge gives

$$I_S = \frac{1 + j\omega r_c\, C_c}{1 + j\omega(r_c + S)C_c} \cdot 0.36_5\, I_L \tag{6.13}$$

$$\text{or} \quad I_S \simeq 0.34_9\, I_L \tag{6.14}$$

upon substitution of circuit values.

For a load of 1/20 watt in 1 000 ohms an effective current of 7.07 mA would be needed. So that for a direct current of 0.3 mA through the Unknown, the alternating current required in addition will be 7.06_5 mA, which means a total supply current of 9.64 mA. An alternating current of 7.06_5 mA in the Unknown causes currents of 2.32 and 2.47 mA in the A and B and the S arms respectively, from equations (6.12) and (6.14).

(6.5)

Measurement of the Direct Measuring Current and the Alternating Loading Currents Used.

The thermocouples S.5 and S.10 of Figs. XXX11 and XXX111 were used in setting the respective supply currents in association with a Philips Millivoltmeter type 6020, and then removed from the circuit while resistance measurements were made, so as to keep the leakage resistances to earth from the junction of the A and X arms with the battery circuit and the junction of the B and S arms with the battery circuit to as nearly equal a value as possible, as discussed in section (2.4). Thermocouples were found to be necessary due to the absence of suitable meters and the fact that a loading current frequency of 160 c/s was used. They were a sealed vacuum type thermocouple, manufactured by Best Electrics Ltd. of Felixstowe Suffolk, with the heater insulated from the couple. The insulation had been tested with 100 volts D.C.applied between the heater and the couple, and they were nominally rated at 2.5 mA (S.5) and 15 mA (S.10), with a nominal couple output of 7 millivolts at rated current. The calibration of both thermocouples was carried out at D.C. against the voltage drop across a 100 ohm Cambridge standard resistor, using a method similar to that described in section (4.3.2).

From the readings of the Philips 6020 and the set current, the thermocouple constant k was calculated and the value of k plotted against heater current over the range of heater currents of interest. This, of course, assumed that the couple output voltage was related to the rms heater current by the relationship

$$E_{couple} = k\, I^2_{heater} \qquad (6.15)$$

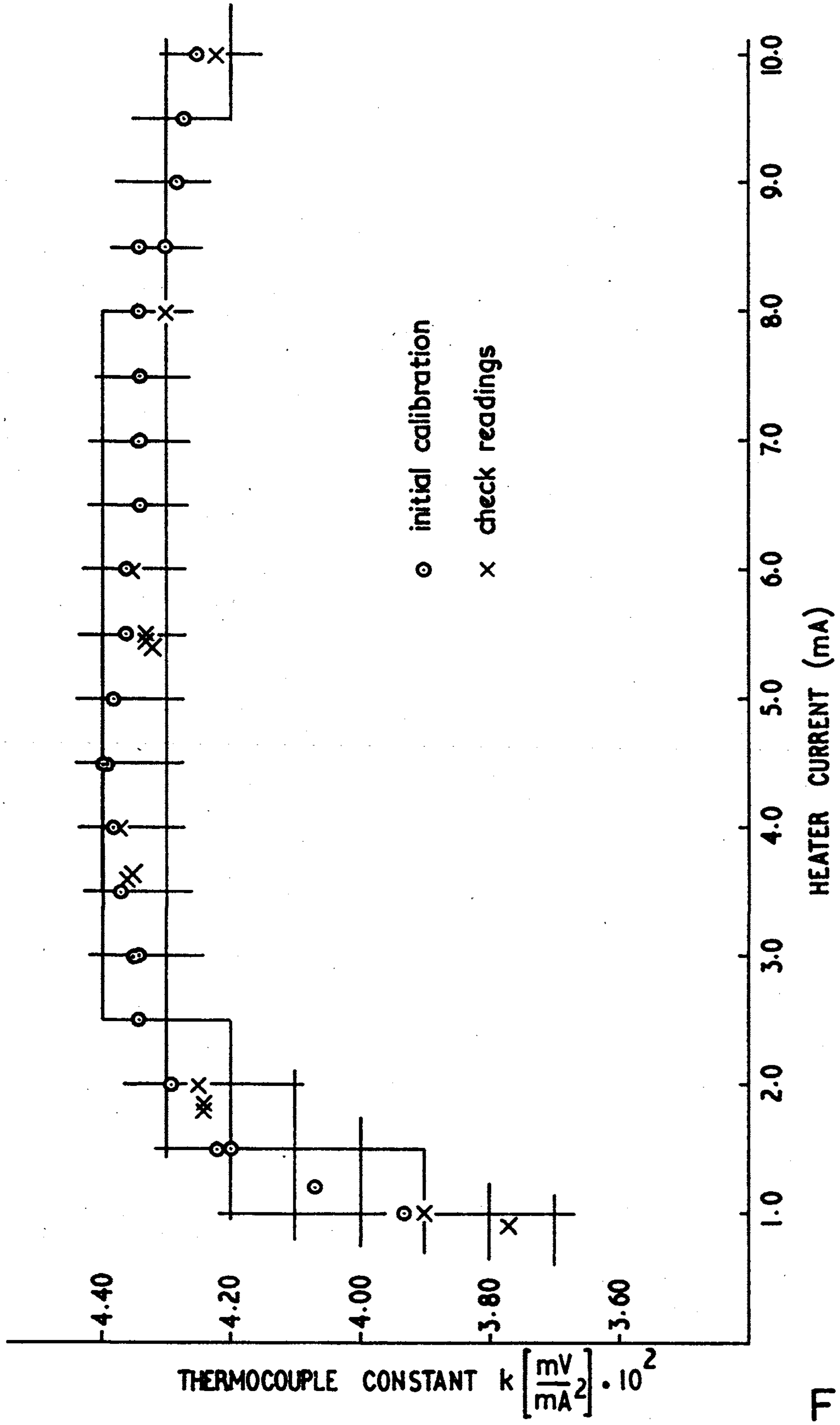


Fig XXXV.

Calibration curve obtained for the "Best" thermocouple Type S·10, Thermocouple constant k vs Heater Current.

The results of these measurements are shown for the thermocouple S.5 in Table (6.2) and for the thermocouple S.10 in Fig. XXXV.

TABLE (6.2)

S.5 Heater Current (mA)	Tinsley Potentio-meter Type 3184		Philips D.C. milli-voltmeter Type 6020		k $\left[\frac{mV}{mA^2}\right]$
	Range	Reading	Range	Reading	
0.5	0.1	0.500_0	1 mV	32.2	1.29
0.6		0.600_0		46.8	1.30
0.8		0.800_0		84.2	1.32
1.0		1.000_0	3 mV	$132._0$	1.32
	1	0.100_0		$132._0$	1.32
1.2		0.120_0		$189._0$	1.31
1.4		0.140_0		$259._0$	1.32
1.6		0.160_0	10 mV	33.7	1.32
1.8		0.180_0		42.5	1.31
2.0		0.200_0		52.4	1.31
2.2		0.220_0		63.1	1.30
2.4		0.240_0		74.7	1.30
2.5		0.250_0		80.9	1.29
1.0		0.100_0	3 mV	$131._0$	1.31
2.5		0.250_0	10 mV	81.0	1.30
1.0		0.100_0	3 mV	$131._0$	1.31

In order to pass 0.3 mA D.C. through the 1 000 ohm resistor under test corresponding to no-load in Chapters 4 and 5, the D.C. measuring current required from the battery was approximately 0.63 mA.

ALTERNATING CURRENT SUPPLY CIRCUIT
USED WHEN LOADING 1000 Ω RESISTORS.

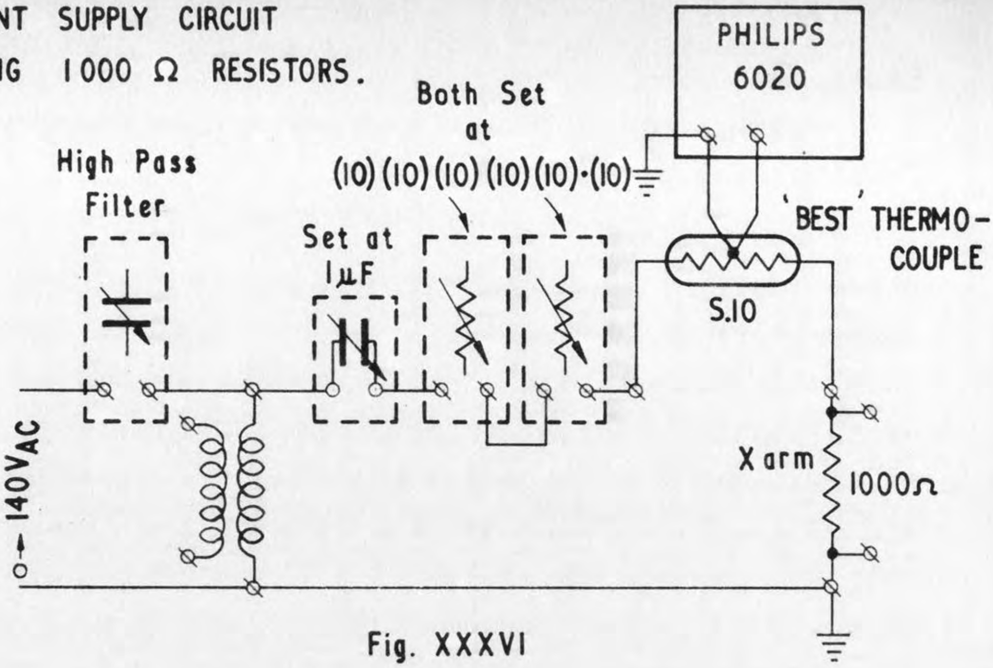
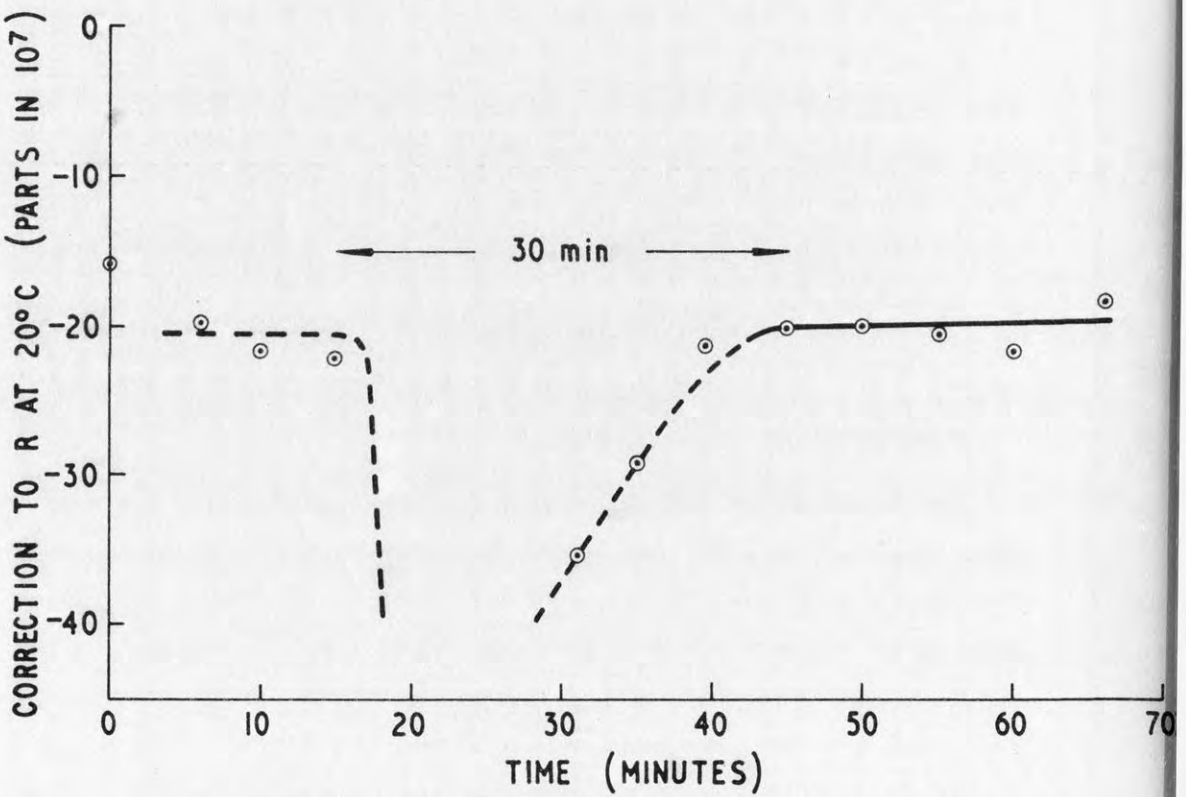


Fig. XXXVI



GRAPH OF MEASURED VALUE OF CORRECTION
TO LEEDS & NORTHRUP 1000 OHM RESISTOR
S/N 1642612 REFERRED
TO 20°C, vs TIME
FOR A STEP OF APPROX. 0·6°C IN ITS OIL BATH TEMPERATURE.

Fig. XXXVII

(6.6)

Temperature Stabilisation Time of Leeds and Northrup Rosa Construction Resistors.

It was found during preliminary measurements on the temperature coefficient of 1642612, that the period required for the temperature stabilisation of the resistance element after a change in temperature of its exterior environment, seemed to be exceptionally long, requiring anything up to 30 minutes to settle. Mr. R.C. Richardson of N.S.L. suggested that this effect is, in fact, a general property of the Leeds and Northrup Rosa type construction resistors, regardless of the resistance value. The period expected by the Author was rather less than that encountered, since the stabilisation periods of other sealed resistors used was only of the order of 15 minutes.

A curve of the correction to the Leeds and Northrup 1 000 ohm resistor 1642612 referred to a temperature of 20 °C against time, is shown in Fig. XXXV11. It can be seen from this curve that the time taken for the referred resistance correction to settle to a reasonably constant value, is about 30 minutes.

(6.7)

Measurement Procedure.

The resistor being measured was placed in an oil bath equipped with its own stirrer and 0.01 °C thermometer, while the rest of the bridge was set up in the Bridge Bath, also equipped with its own stirrer and 0.01 °C thermometer. Interconnection between the 1 000 ohm resistor under test and the rest of the bridge was made with two thick copper braid leads.

It was found during preliminary measurements, that the galvanometer fluctuations, which were evident even with the high pass filter in use on the output of the loading supply circuit, were reduced considerably if the Levell Oscillator used to drive the Savage High Power Audio Amplifier was used on its second frequency range, which had a lower limit of 150 c/s. In the light of the results of the load coefficient measurements with frequency made on the 1 ohm Cambridge L-259235 shown in Chapter 4, and the discussion on possible sources of difference between the A.C. and D.C. loading conditions in Chapter 1, it was decided that a factor of 3 increase in the frequency of the loading current would not affect the loading conditions nor therefore the load coefficients obtained. 150 c/s was considered to be a low enough frequency for stray capacitance or eddy current effects around the circuit, or frequency effects in the thermocouple used for the measurement of the loading current to produce negligible error. A loading current frequency of 160 c/s was chosen so as to be sufficiently removed from a third harmonic of the mains. The galvanometer mu-metal cored series inductor was then adjusted to parallel resonance at 160 c/s.

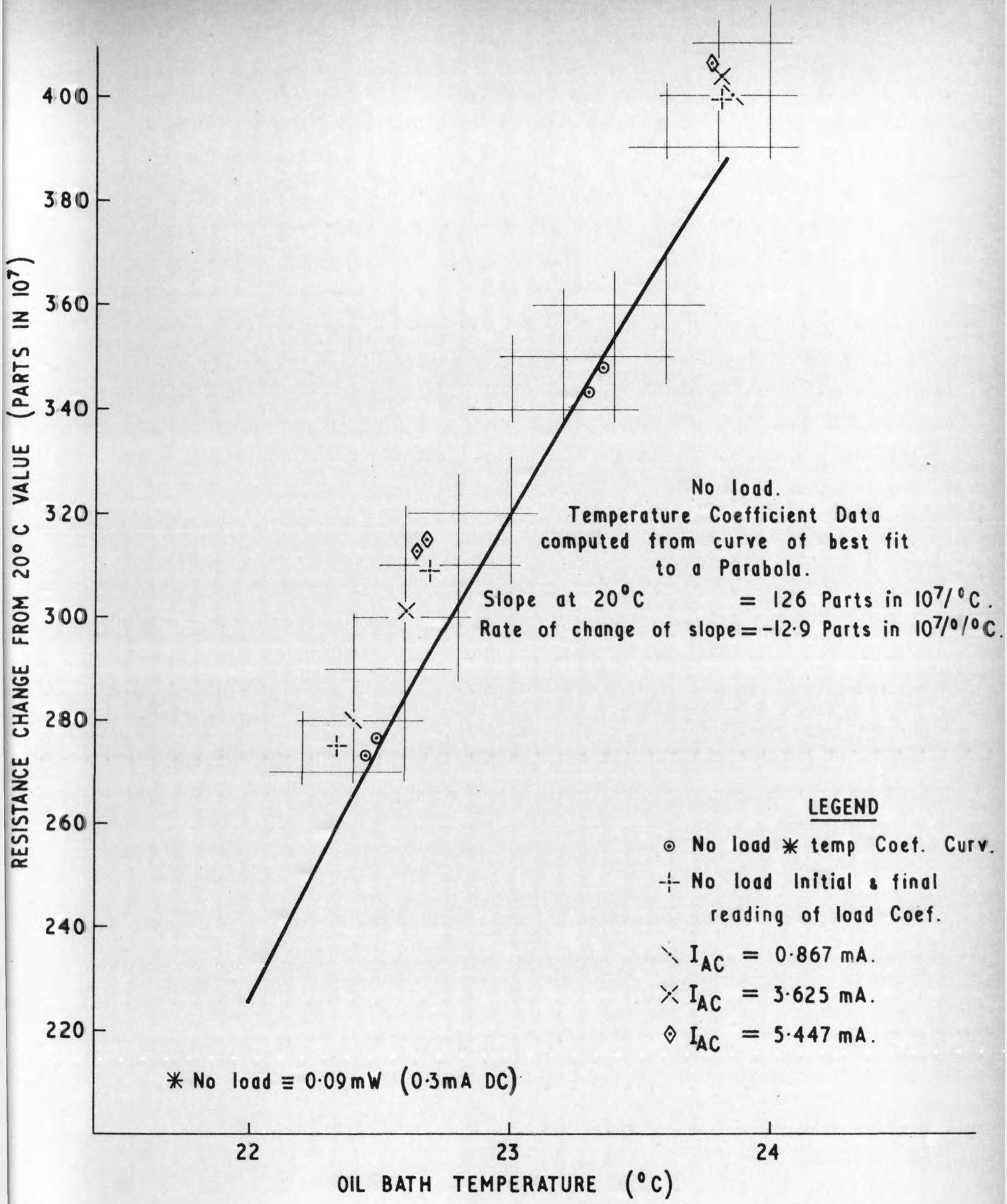
Before load coefficient measurements were made on each 1 000 ohm resistor, its temperature coefficient and that of its associated bridge circuit were found under no-load conditions (0.3 mA D.C.). The computed values of α_{20} and β for Unknown and Bridge circuit were obtained using the program TEMPCURV and later used in the processing of the results of the load coefficient measurements on the 1 000 ohm resistor under test.

(6.8)

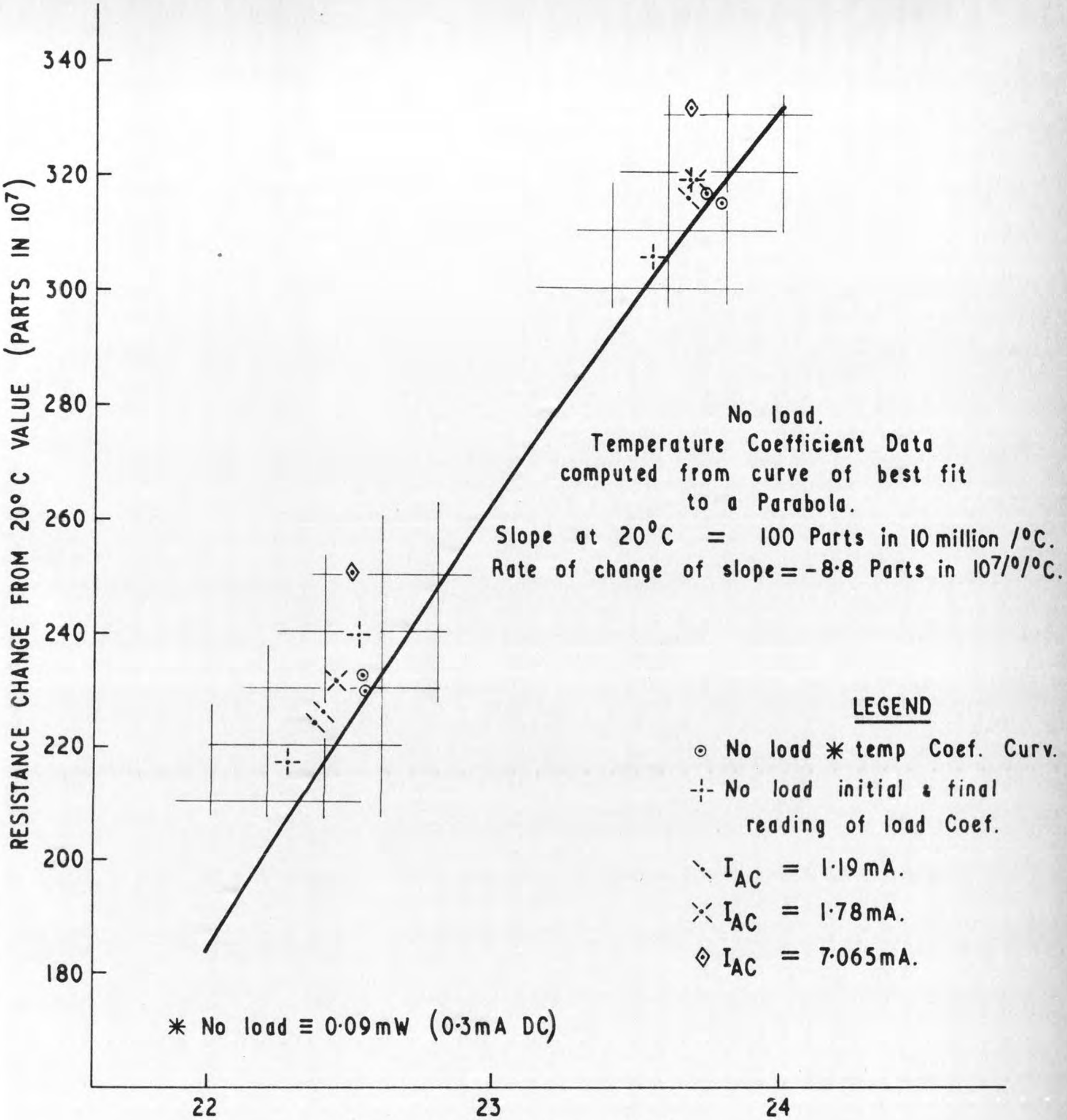
Presentation of Results.

The computed resistance changes obtained for each resistor when under load were plotted against temperature and its load coefficient calculated. The load coefficient value at 1/20 watt was used for the purpose of interpolation, so as to act as a check on the load coefficient measurements made at the lower powers in 1642613, 1642615 and B.H. No. 1 1962.

The curves obtained for each resistor are shown in Figs. XXXV111 to XL1. Only the temperatures between 22 and 24 °C are exhibited because the load coefficient measurements were made within the limits of these two temperatures. The no-load



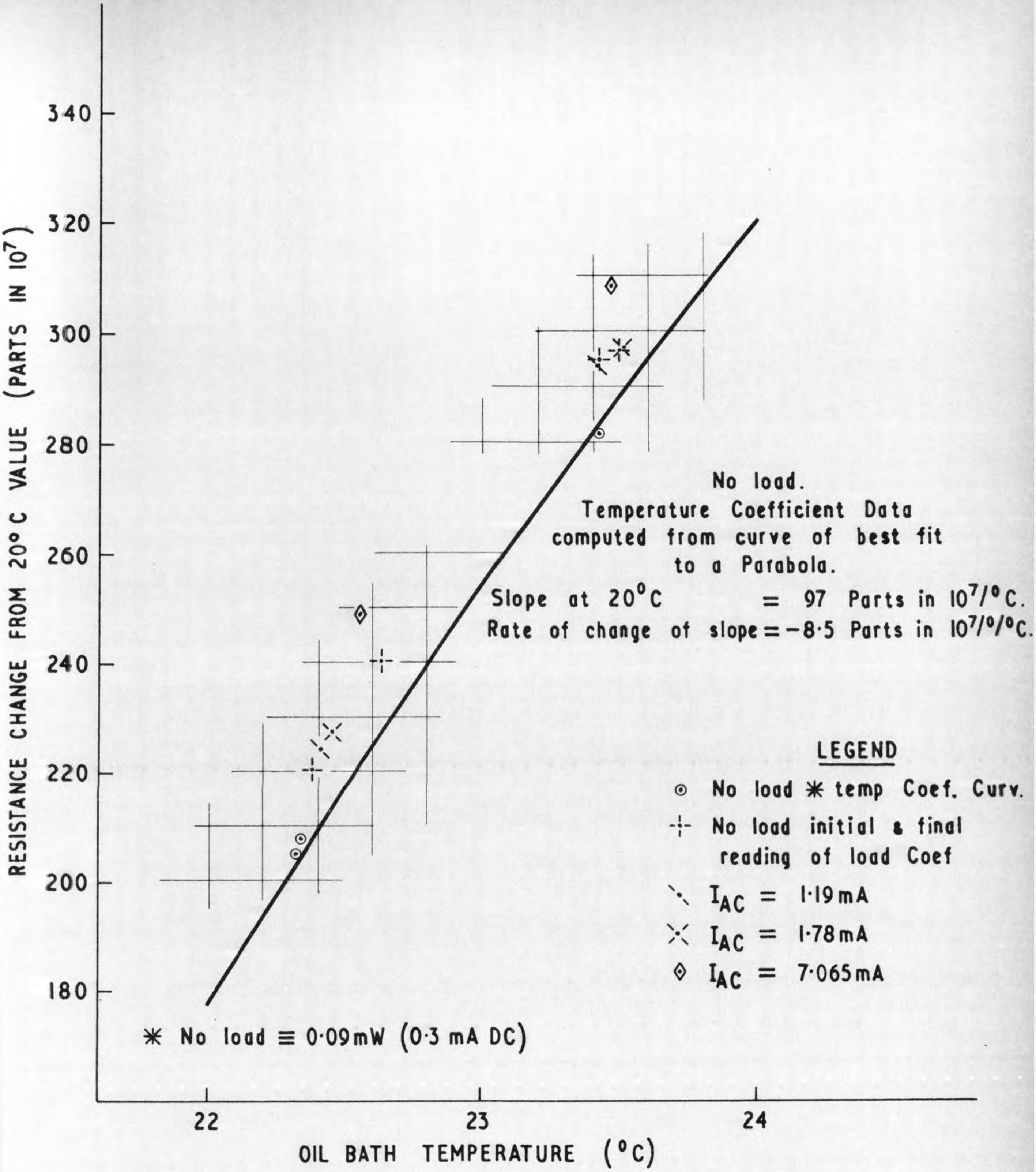
TEMPERATURE & LOAD CHARACTERISTICS OF 1000Ω L & N RESISTOR S/N 1642612. Fig. XXXVIII



OIL BATH TEMPERATURE (°C)

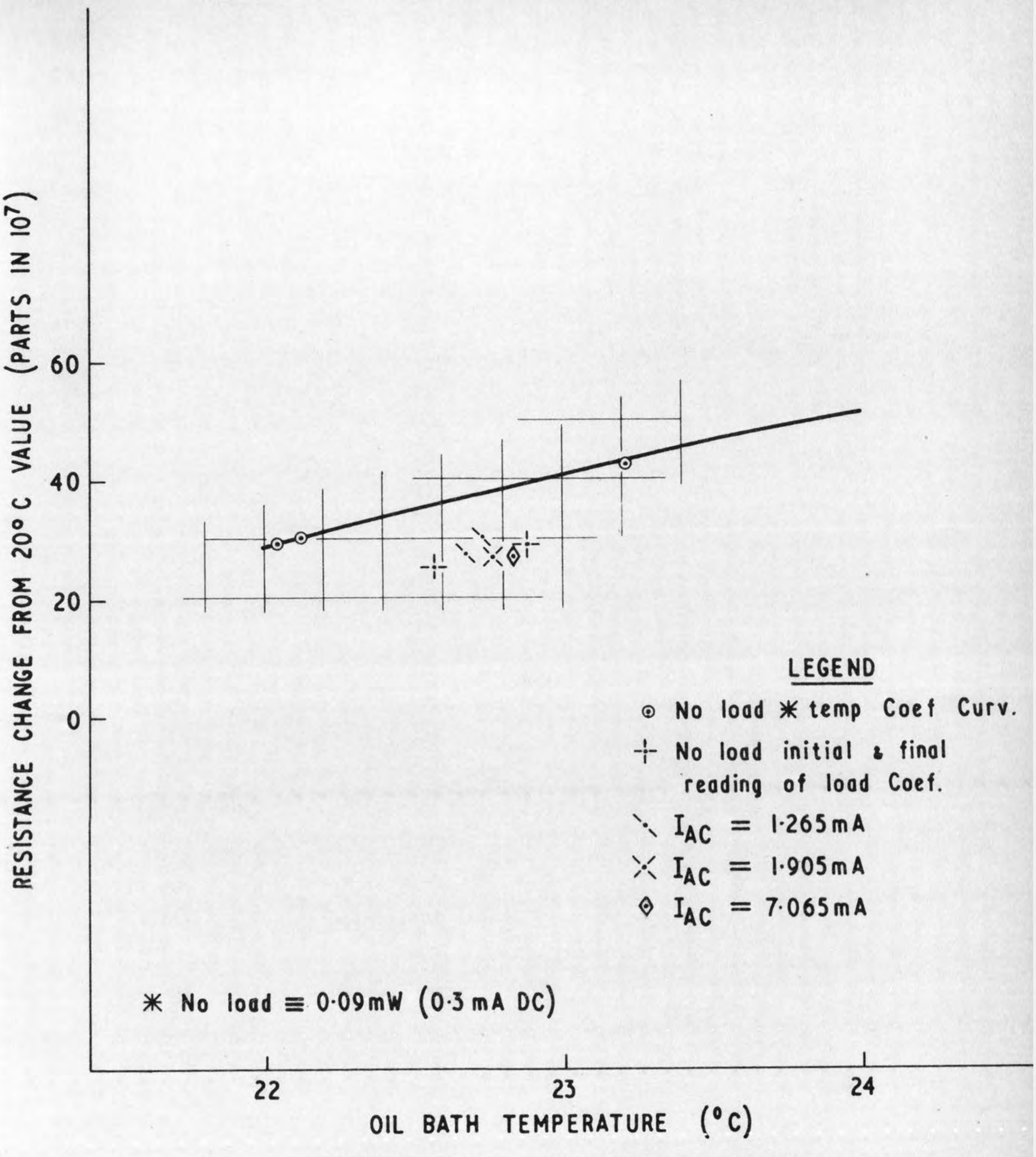
TEMPERATURE & LOAD CHARACTERISTICS
OF 1000Ω L & N STANDARD
S/N 1642613.

Fig. XXXIX



TEMPERATURE & LOAD CHARACTERISTICS OF 1000Ω L & N RESISTOR S/N 1642615.

Fig. XL



TEMPERATURE & LOAD CHARACTERISTICS
OF 1000Ω STANDARD
S/N B.H. No.1 1962.

Fig. XLI

temperature curves have been calculated from the curve of 'best fit' to the results obtained between about 19.5 and 24 $^{\circ}C$ using the method of least squares for an approximation to a parabola.

(6.9)

Assessment of Results.

Inspection of Figs. XXXV111 to XL1 indicates that the absolute correction to at least one of the bridge arms has drifted by approximately one part per million between readings made on the no-load temperature coefficient curves and the first readings of the load coefficient determination, which may be seen by comparing the no-load corrections of the load coefficient determination to the corrections obtained in the temperature coefficient determination at the same temperature of Unknown. The exact cause is unknown but it is noted that approximately 48 hours elapsed usually, between the first readings of the no-load temperature coefficient determination and those of the load coefficient determination.

In determining the load coefficients of each resistor, values were estimated from the curves of Figs. XXXV111 to XL1. As a means of checking the estimation, if it is assumed that the load coefficient estimated for the highest loading is exact, a process of interpolation may be used to find the load coefficient at smaller values of load, using the relationship

$$\ell = \frac{L\,(I_{AC}^2 + I_{DC}^2)}{I_{ACmax}^2 + I_{DC}^2} \qquad (6.16)$$

where $\sqrt{I_{AC}^2 + I_{DC}^2}$ is the effective value of the current through the resistor at the desired load and $\sqrt{I_{ACmax}^2 + I_{DC}^2}$ is the effective value of the current through the resistor at the load where the load coefficient is L. The comparison between the estimated and the check values of load coefficient is shown in Table (6.3).

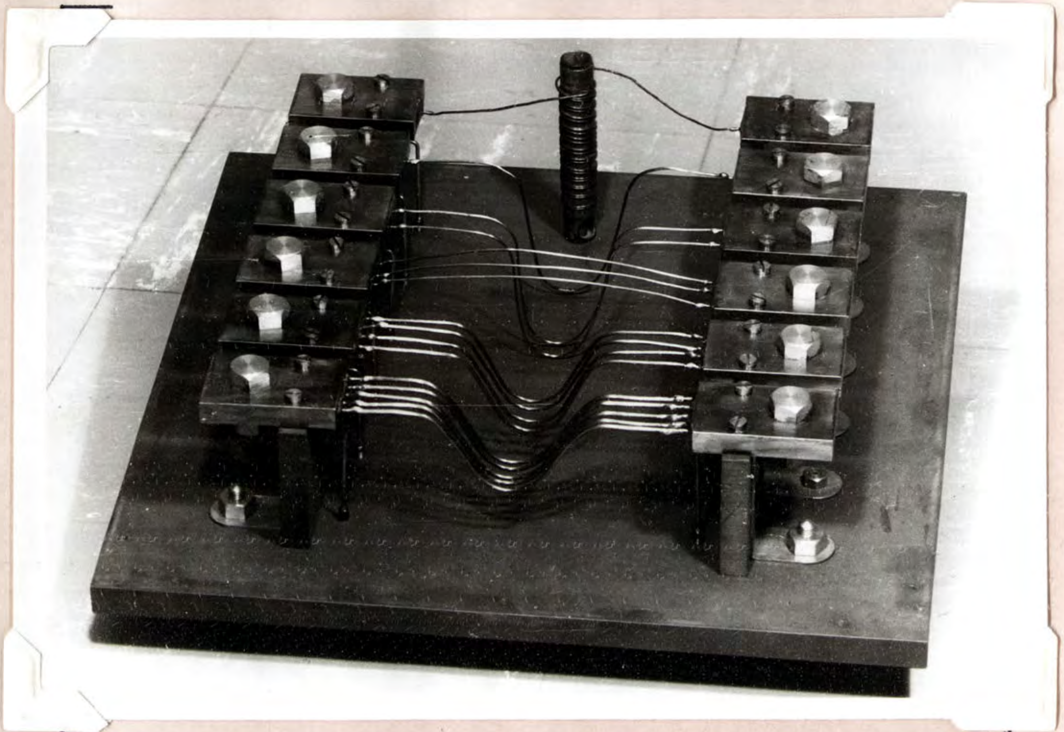
TABLE (6.3)

Resistor Under Test	Load		Load Coefficient Parts in 10^7	
	I_{AC} (mA)	I_{DC} (mA)	Estimated	Interpolated
1642612	5.45	0.3	7 ± 1	7 ± 1
	3.62	0.3	2 ± 1	3.1
	0.86_7	0.3	0	0.19
	0	0.3	0	
1642613	7.06_5	0.3	12 ± 1	12 ± 1
	1.78	0.3	0	0.7_8
	1.19	0.3	0	0.3_6
	0	0.3	0	
1642615	7.06_5	0.3	11 ± 1	11 ± 1
	1.78	0.3	0	0.7_2
	1.19	0.3	0	0.3_0
	0	0.3	0	
B.H. No.1	7.06_5	0.3	0 ± 1	0 ± 1
	1.90	0.3	0	0
	1.26	0.3	0	0
	0	0.3	0	

In obtaining the value of the estimated load coefficients, no account was taken of the loading effects of the portion of the supply current which passed through the A, B and S arms of the Wheatstone Bridge used. Inspecting Table (6.3) and referring to the end of section (6.4), it is obvious that the loading effects in other bridge arms will be finite when loading with an effective current of 7.07 mA. When testing the Leeds and Northrup resistors, the corrections to the A and B arms will generally cancel, since the A and B arms were always L and N resistors, whose load coefficients were found to approximately equal, while the S arm resistor used was B.H. No. 1 1962, whose load coefficient at the respective values of loading used was found to be negligible.

While testing B.H. No. 1 1962, 1642615 was used in the S arm. Its load coefficient at an effective current level of 2.49 mA will be approximately 1 part in 10^7 using (6.16) and Table (6.3). This resistance change is included in the estimated uncertainty in the load coefficient of B.H. No. 1 1962 and the spread of measured results shown in Fig. XL1.

Correlating the load coefficient values obtained for the three L and N 1 000 ohm standards with the measured value of their respective temperature coefficients α_{20} shows reasonable agreement with the assumptions of section (6.3). In the light of this agreement using values of α_{25} and 0.5β of + 8 parts per million / $^{\circ}$C and - 0.5 parts per million / $^{\circ}$/ $^{\circ}$C respectively, as given by the manufacturers in their initial temperature coefficient determination, the order of the resistance change in the L and N 100 ohm standard 1646270 at approximately 3.6 milliwatts power dissipation level will be approximately + 0.7 parts in 10^7. Thus the load coefficients of the ratio $^A/B$ used in A.C. load coefficient measurements on R5 at 16 and 36 watts A.C. power dissipation level will be approximately + 2 and + 6 parts in 10^7 respectively.



Photograph showing the construction of the Tinsley 0.005 ohm standard SN 101268.

Fig XLII.

CHAPTER 7

Load Coefficient Measurements on the Tinsley 4-Terminal Manganin Air Cooled 0.005 ohm Meter Shunt Type 4738 S/N 101268 Rated at 30 Amps Full Load Current

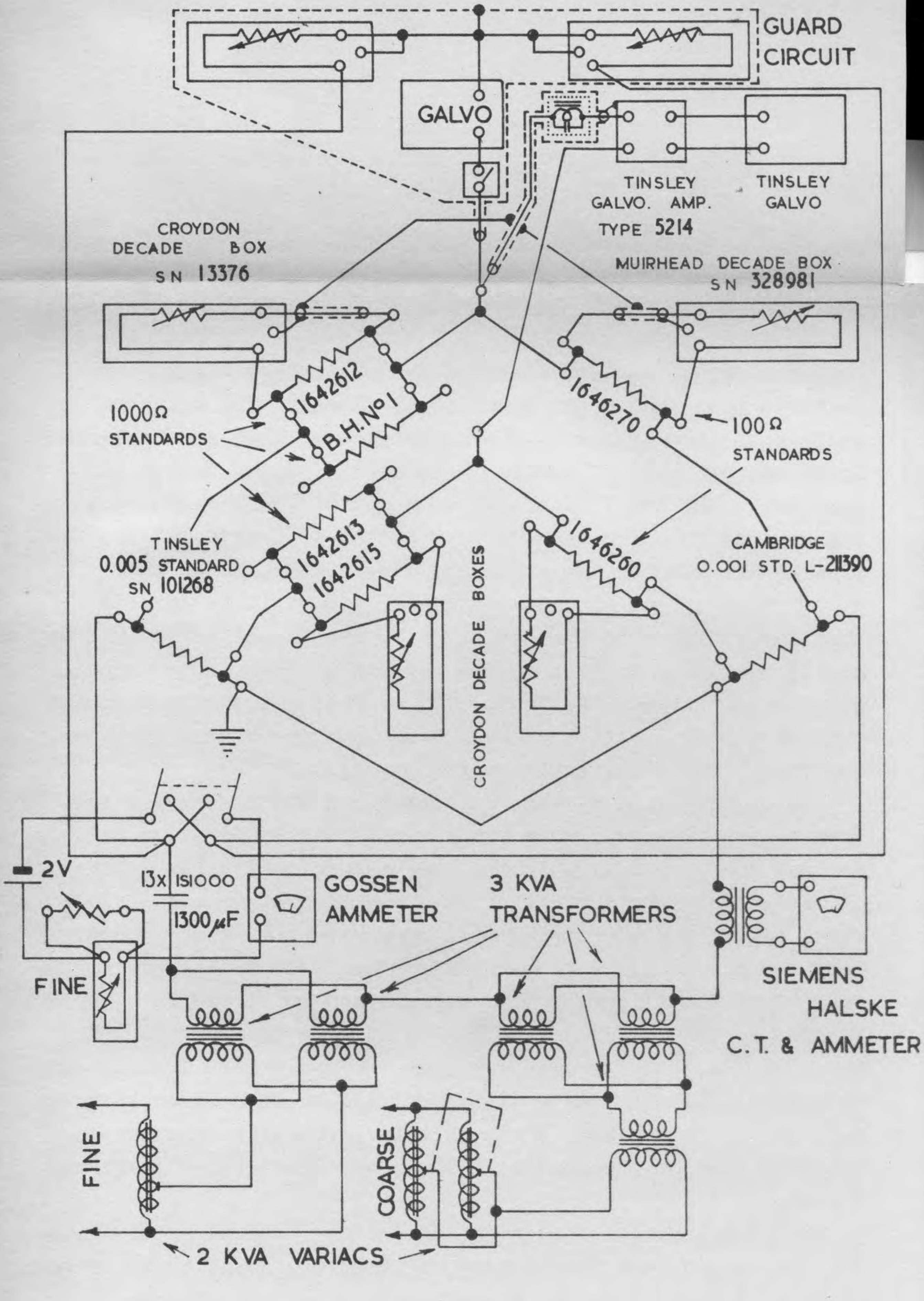
(7.1)

The resistor referred to as the Tinsley 0.005 ohm air cooled meter shunt was one of six meter shunts ranging in value from 1 ohm to 0.005 ohm and situated in a box with ventilation to the atmosphere. No provision for forced convection cooling of these shunts was provided in the box and hence in order to obtain an environment which was as constant and reproducable as possible from time to time, it was decided to enclose the box containing the meter shunts in a temperature controlled cabinet, which had been especially borrowed for this purpose from the National Standards Laboratory. The temperature control of this cabinet was such as to be capable of producing an environment which was temperature stable to within about $\pm$ 0.05 °C of a given mean in temperature over a short period, depending upon the amplitude of the external ambient temperature fluctuations with time and upon the distribution and physical size of the objects placed in the cabinet.

A photograph of the meter shunts removed from their protective box is shown in Fig. XL11 opposite. The 5 milliohm resistor is situated nearest the camera and may be seen to consist of 5 lengths of manganin wire connected in parallel between two large copper terminating blocks. Both current and potential connections are taken from these copper blocks away to terminals mounted on the plate forming the lid of the box. This plate and the stand-off spacers on which the copper blocks were mounted, were manufactured from Keramot or similar material. Close inspection of the junctions of the manganin resistance elements with the copper terminating blocks, suggested that the manganin wire had first been silver soldered at each end to copper wires of similar diameter and the copper wires soft soldered into the copper terminating blocks. The manganin section of each strand of the resistance element was measured to be approximately 7 inches in length and 0.082 inches in diameter. The copper stubs at each end of the strands were measured to be approximately 7/16 inches in length and of similar diameter to the manganin. The dimensions of the copper terminating blocks were so large that it is assumed that their effect on the overall D.C. resistance value will be negligible in comparison to the effects of the copper terminating stubs attached to them. The overall resistance included that of a small section of copper at each end of the resistor, together with two manganin-copper junctions.

The construction of the box containing the shunts was such that no provision was made for internal air temperature measurements. The temperature in the temperature controlled cabinet close to the upper surface of the Keramot plate was therefore taken as being the environmental temperature of the Unknown. Reproducable resistance measurements could be made, provided sufficient stabilisation time was allowed to elapse between changes in power dissipation levels etc. in the Unknown. The order of the time interval required for resistance stabilisation to within approximately 2 parts per million at a given load was anything up to 2 hours, although this period did depend upon the magnitude of the change in the power dissipation level concerned.

The stabilisation time required for the cabinet after changing its controlling temperature setting, was found to be of the order of 1 hour, with the distribution of objects in the temperature controlled space existing in this case. The stabilisation time of the cabinet and the uncertainty of the thermal time-constant of the Unknown indicated that an attempt at providing curves of resistance change with temperature as a function of power dissipation level would take long periods to perform at all the power dissipation levels used. This method



SCHEMATIC DIAGRAM
of the Kelvin Bridge used in
measurements made on the Tinsley 0.005 ohm resistor
SN 101268

Fig XLIII.

of presentation, although useful, is not necessary for the establishment of the A.C. loading technique. Hence the temperature stability of the temperature controlled cabinet with time was used in obtaining a curve of resistance change in the Unknown versus power dissipation level for both A.C. and D.C. loading at the arbitrary controlling temperature existing throughout the series of measurements. A temperature coefficient measurement on the Unknown was performed at 5 amps D.C. in order to provide an estimate of the effect of temperature change on the resistance value of the Unknown at higher levels of power dissipation.

It was considered doubtful that the distribution of temperature along the resistance elements of the 0.005 ohm resistor would be uniform at any power dissipation level above negligible load, owing to the presence of the large copper blocks at each end of the resistor and the shape of the manganin wires used in forming the resistance element as shown in Fig. XL11. Hence a calculable relationship between the no-load temperature coefficient curve, the load coefficient and the slope of the resistance-temperature curve would not exist at any significant power dissipation level. Even so, it was felt that the effect of the resistance changes in the copper with changes in the environmental temperature were significant in producing the measured 5 amp temperature coefficient curve obtained.

Resistance measurements were made on this 0.005 ohm resistor using a Kelvin Bridge method with a ratio X to S of 5 to 1, in order to reduce the dependency of the accuracy with which D.C. load coefficient measurements could be made, on the accuracy with which the load coefficient of the Standard was known at a particular power dissipation level in the Unknown.[3,4,5] The circuit diagram of the Kelvin Bridge used is shown in Fig. XL111. The standard resistor used as Standard throughout was a Cambridge heavy current 0.001 ohm standard resistor L-211390 with a rated full load power dissipation level of 1 000 watts. Load and temperature coefficients for this 0.001 ohm Standard were determined in later work performed on the Cambridge 0.01 Standard S-1161 discussed in Chapter 8. Temperature measurements were made on the stirred cooling oil of L-211390 using a 0.1 ^{o}C mercury-in-glass thermometer placed in the oil through the filler cap. The bridge ratio arms consisted of 1 000 and 100 ohm standard resistors, together with shunting dials of approximately 100 times their value, which were used in obtaining bridge balance. The A and P arms were each adjusted to approximately 500 ohms with the parallel connection of two 1 000 ohm standard resistors in each. When obtaining bridge balance, the A and P arm shunts only were adjusted in unison.

Corrections to the standard resistors forming the A arm with changing temperatures were found using the formula

$$\delta_A = \frac{\delta_1 + \delta_2}{2} \quad \text{proportional parts} \tag{7.1}$$

where $\delta_1 = \alpha^{1}_{20} (T - 20) + 0.5\beta^{1} (T - 20)^2$

and $\delta_2 = \alpha^{11}_{20} (T - 20) + 0.5\beta^{11} (T - 20)^2$

T being the temperature of the oil bath containing the six standard resistors forming the A, B, P and Q arms of the bridge in ^{o}C,

α^{1}_{20} and β^{1} being the temperature coefficients of one of the parallel connected standard resistors in the A arm and

α^{11}_{20} and β^{11} being the temperature coefficients of the other.

The values of α_{20} and β used for the L and N standards forming the A and B arms of the Kelvin Bridge were obtained from their original certificates of test. α_{20} and β for the 1 000 ohm standard resistor B.H. No. 1 1962 were obtained from Mr. R. E. Holmes of the N.S.L.

One set of load coefficient measurements were made at 25 c/s by obtaining the slip voltage from an induction machine whose stator windings were supplied from a 3 phase 50 c/s supply and whose rotor was driven at 25 c/s. The output voltage available from a system such as this is mainly a function of rotor frequency and hence, for a constant frequency of supply current of 25 c/s, adjustment of the alternating supply current level may only be obtained by adding series resistances to the system. These resistors were added on the high voltage - low current side of the supply transformer where necessary. The reduction of supply frequency to 25 c/s meant that the alternating voltage drop appearing across the D.C. blocking capacitor was raised to approximately 125 volts with 25 amps of alternating current passing. The alternating voltage drop across these Ducon 1S1000 100uF paper capacitors could be increased to about 150 volts at room temperature with safety provided the current carrying capacity of 5 amps was not exceeded.[25]

The guard circuit shown in Fig. XL111 was necessary since one side of the link was earthed (section 2.3).

All 50 c/s alternating currents used were derived from the mains. No problems were encountered with mains amplitude fluctuations on the zero of the galvanometer. Alternating currents flowing in the battery circuit were calculated on the assumption that the impedance around this circuit at 50 c/s is virtually equal to that encountered at D.C. Possible sources of error presented by this assumption and their effects, will be discussed in section (7.3).

Alternating currents of mains frequency were reduced below the level of detection in the galvanometer with the presence of the tuned mu-metal cored series inductor mentioned in Chapter4. The 25 c/s galvanometer alternating currents were reduced also, by retuning the above inductor. The galvanometer system used, had a maximum D.C. deflection sensitivity of 80 000 mm/uA. It was found that a bridge unbalance sensitivity equivalent to approximately $\pm$ 2 parts per million of the Unknown could be obtained with a no-load bridge supply current of 5 amps.

Measurements were made using an E.I.L. Milliohmeter to determine the total resistance present in the section of the A.C. supply circuit which was common to both A.C. and D.C. circuits. Results showed it to be 6.8 ± 0.05 milliohms, which included the resistance of the link, the current lead between the 0.005 ohm resistor and the reversing switch and the 0.005 ohm resistance of the Unknown. These results were of direct importance in calculating the amplitude of the alternating current level passing around the battery circuit and in assessing the order of importance of the correction term present in the bridge balance condition of equation (2.4). Power dissipation levels in the A and B arms of Fig. XL111 were considered to be negligible at all power dissipation levels used in the Unknown.

(7.2)

Measurement Procedure.

The no-load D.C. measuring current level used in performing temperature coefficient measurements on the 0.005 ohm resistor between 17.6 and 28.8 ^{o}C was 5 amps. The 25 c/s load coefficient determination used this value also, although an increase to 10 amps was used when making 50 c/s A.C. and direct current load coefficient measurements, in order to increase the bridge unbalance sensitivity available. The fact that 10 amps was not a negligible load was not of concern, since the equivalence of self-heating effects of alternating and direct currents were being considered only, rather than absolute load coefficient measurements.

Alternating and direct current load coefficient determinations were performed at effective loading current levels of 10, 15, 20, 25 and 30 amps. Load coefficient measurements were made such that alternating and direct current determinations followed immediately upon each other. This method allowed the effect of environmental changes in the Unknown between A.C. and D.C. determinations to be reduced to as low a level as possible.

The technique used when performing load coefficient measurements on the Unknown was to first allow several hours for the temperature of the temperature controlled cabinet to stabilise. Measurements were then made at successive power dissipation levels, changing between A.C. and D.C. values as required. The load coefficient measurements performed at 25 c/s were made after the Unknown temperature had stabilised, in a sequence of load coefficient measurements being made first at 50 c/s with 25 amps of A.C. and 5 amps of D.C. followed by a set at 25 c/s with 25 amps of A.C. and 5 amps of D.C. followed by a third set similar to the first. The level of alternating current used in this case was governed by the maximum round figure value obtainable from the 25 c/s source.

(7.3)

Supply Current Measurement.

The alternating loading current levels supplied to the Unknown in tests on the 0.005 ohm resistor at both 50 and 25 c/s, were measured with the aid of the Siemens Halske moving iron meter and its associated current transformer mentioned previously. This meter is classed as being a 0.2 per cent instrument at 50 c/s, but it had not been calibrated previously at 25 c/s.

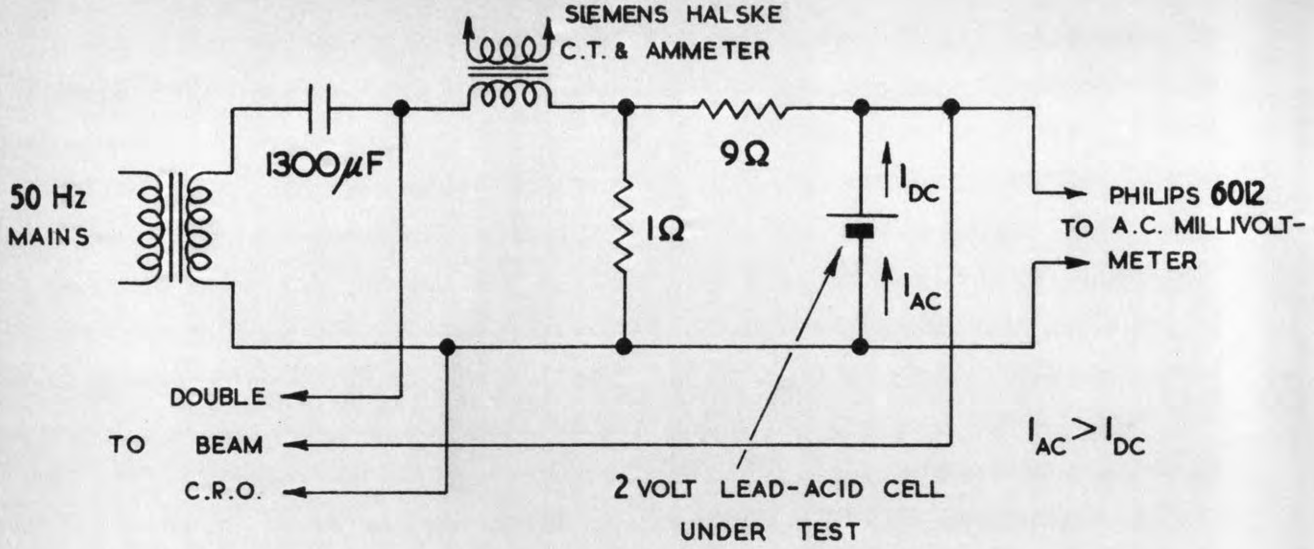
The only mechanism considered likely to produce significant changes in the meter deflection at 25 c/s from that obtained at 50 c/s for a similar level of R.M.S. alternating current, was eddy current effects. The extent of the probable error in current reading at 25 c/s may be judged with reference to typical orders of magnitude of the error in moving iron instruments from this source, as quoted by Harris.[26] This is given as between 0.5 to 1.0 per cent per kilocycle in terms of current, which will amount to a change of less than 0.1 per cent at 25 c/s.

The effect of reduced frequency on the current transformer used in association with the above moving iron ammeter, will be to increase its level of magnetisation current. As a result of this the ratio error at 50 c/s may be expected to double in moving to 25 c/s[27] for a high quality instrument, when using its rated 50 c/s burden at 25 c/s. Even so, the ratio error on the 50/5 range with approximately half rated current input only amounts to - 0.1 per cent at 25 c/s.

The measurement results obtained indicated that the resistance change in the Unknown at full load was approximately - 250 parts per million at 27.8 $^{\circ}$C. If the error in the resistance change due to uncertainties in setting the loading current levels used, is to be less than $\pm$ 2 parts per million at full load (4.5 watts), then the value of I_{EFF} must be known to better than $\pm$ 0.4 per cent of its value.

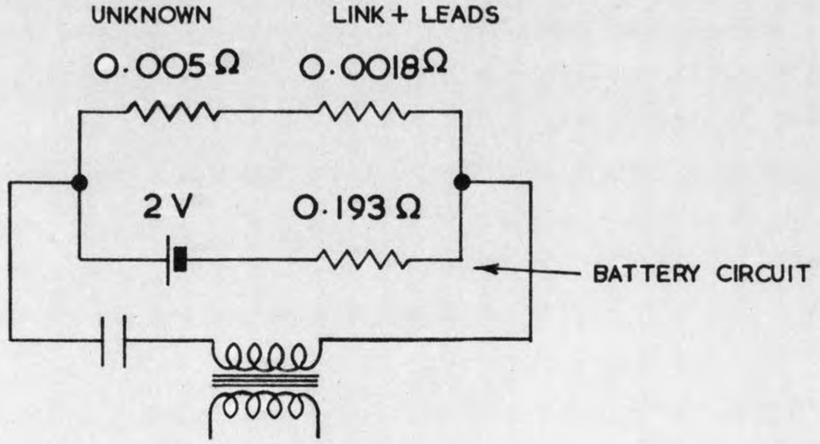
With the Siemens Halske meter and current transformer used for alternating current measurement at 50 and 25 c/s, the measurement accuracy will have been better than the $\pm$ 0.4 per cent of setting required. The 0.3 per cent Gossen moving coil direct current meter however, when used on its 60 amp range to measure 30 amps D.C., will produce a probable error of $\pm$ 0.6 per cent of reading. Uncertainties existing in the Gossen reading at 5 and 10 amps will be negligible, when the large proportion of the power dissipated in the Unknown is provided by the applied alternating current. Hence when loading with alternating current, the probable error existing in the measured value of resistance change in the Unknown due to measurement uncertainties in the level of the applied loading current, may be taken as being completely derived from the measurement uncertainties existing in the level of the applied alternating current.

The errors produced by the assumption that the alternating current level flowing in the battery circuit is calculable by assuming that the circuit impedance at 25 and 50 c/s is unchanged from its D.C. value will be negligible, provided the inductive reactance in the circuit is small compared to the resistance at 25 and 50 c/s, and provided the D.C. resistance



Circuit diagram of the measuring system used to obtain the 50 Hz impedance presented by an 'Exide' 210-240 Ampere-hour 2 Volt lead-acid cell.

Fig XLIV.



Equivalent Circuit of Fig XLIII as seen by 50 Hz loading currents.

Fig XLV.

of the 1 volt lead acid cells used does not change significantly with 50 c/s or 25 c/s alternating currents being passed through them. Measurements were made of the 50 c/s alternating voltage drop across the terminals of a fully-charged 2 volt lead acid cell with 0.5 amps of 50 c/s alternating current passing through the cell while 0.2 amps of direct current was being drawn from it, using the measuring circuit of Fig. XL1V. Results showed that the internal impedance of the 2 volt cell was approximately 0.7 milliohms resistive under these conditions, which was negligible in all cases.

Calculation of the level of alternating current flowing around the battery circuit of Fig. XL111 will show that 3.5 per cent of the alternating current level flowing through the 0.005 ohm resistor must be allowed for. This assumes that the parallel circuit as seen by the alternating loading current circuit is as shown in Fig. XLV. Hence if the alternating current level is increased by 3.5 per cent in order to account for the proportion flowing in the battery circuit of the Kelvin Bridge, the battery circuit impedance would need to be in error by approximately 12 per cent from inductive effects so as to produce an error of $\pm$ 2 parts per million in the resistance change of the Unknown at full load. Thus a ratio of X_L/R of 0.25 would be required in the battery circuit. An inductance of 150 microhenrys in the battery circuit is considered to be rather large when it is considered that the series resistor used there was air cored; fine control being effected with the aid of a high dissipation resistance box. Inductances of several orders less however would not produce any significant effect on the current distribution, hence it may be assumed that the current flowing in the battery circuit was basically as calculated.

Summarising, the probable errors existing in the current readings obtained from the ammeters used, indicate that at an effective current level of 30 amps, the corresponding uncertainty in the resistance change of the Unknown produced from this source, will be about $\pm$ 3 parts per million under direct current loading conditions and $\pm$ 2 parts per million under 25 and 50 c/s alternating current loading conditions. The uncertainties existing in the magnitude of the resistance change at lower current and power dissipation levels in the Unknown, will generally be less than at 30 amps, depending upon the respective ammeter ranges used and the value of the effective loading current.

(7.4)

Presentation of Results.

The values of α_{20} and β obtained for the 0.001 ohm Standard in Chapter 8 were 13.5 parts per million per degree C and - 1.00 parts per million per degree per degree C respectively. Its load coefficient at 30 amps was $0 {}^{+0.5}_{-0}$ parts per million and was therefore neglected completely except when assessing measurement uncertainties at full load in the Unknown.

(7.4.1)

No-Load Temperature Coefficient Measurements on the Unknown Tinsley 5 Milliohm Air Cooled Shunt S/N 101268 at 5 Amps.

The values of α_{20} and β obtained from the calculated curve of 'best fit' to the measured results were 27.4 parts per million per degree C and - 0.70 parts per million per degree per degree C respectively. The R.M.S. value of the residuals obtained between calculated and measured resistance values was 4.2 parts per million with a maximum value of - 9.5 parts per million. The curve of best fit is shown plotted in Fig. XLV1 (a), with the measured points added so as to indicate the order and placement of the residuals obtained.

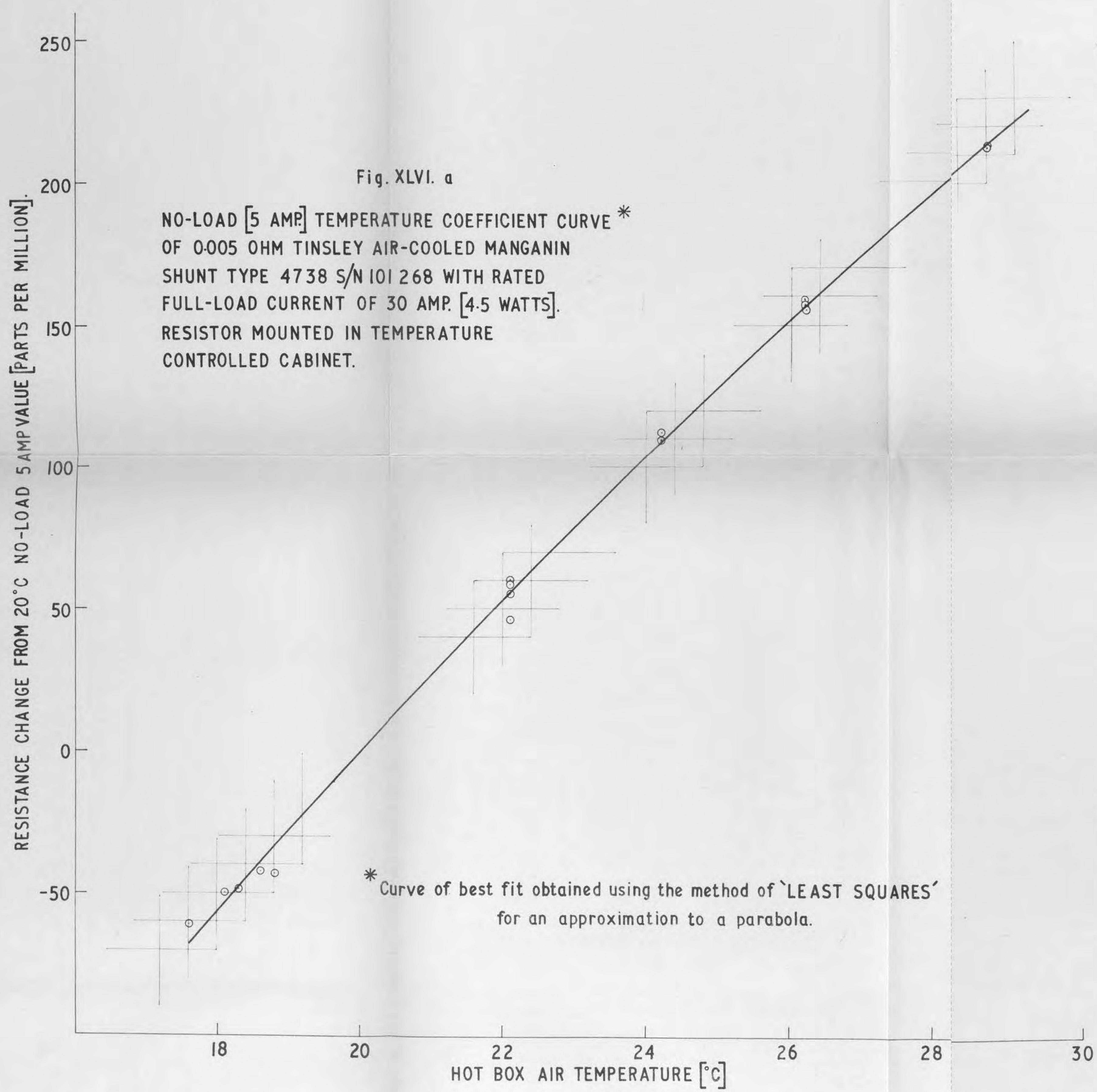
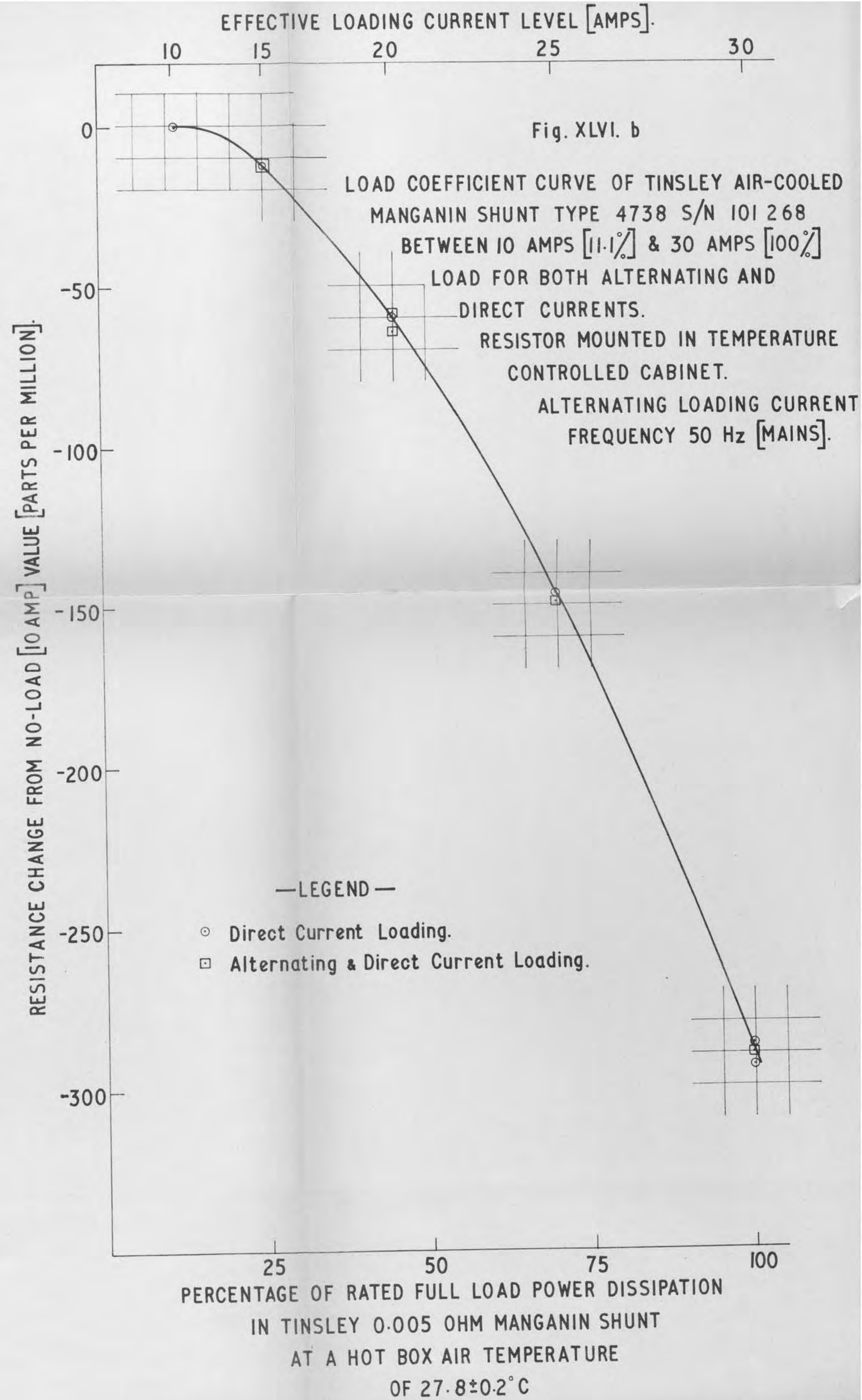


Fig. XLVI. a
NO-LOAD [5 AMP] TEMPERATURE COEFFICIENT CURVE *
OF 0.005 OHM TINSLEY AIR-COOLED MANGANIN
SHUNT TYPE 4738 S/N 101 268 WITH RATED
FULL-LOAD CURRENT OF 30 AMP. [4.5 WATTS].
RESISTOR MOUNTED IN TEMPERATURE
CONTROLLED CABINET.
* Curve of best fit obtained using the method of 'LEAST SQUARES'
for an approximation to a parabola.
RESISTANCE CHANGE FROM 20°C NO-LOAD 5 AMP VALUE [PARTS PER MILLION].
HOT BOX AIR TEMPERATURE [°C]
250
200
150
100
50
0
-50
18
20
22
24
26
28
30



EFFECTIVE LOADING CURRENT LEVEL [AMPS].
10
15
20
25
30
Fig. XLVI. b
LOAD COEFFICIENT CURVE OF TINSLEY AIR-COOLED
MANGANIN SHUNT TYPE 4738 S/N 101 268
BETWEEN 10 AMPS [11.1%] & 30 AMPS [100%]
LOAD FOR BOTH ALTERNATING AND
DIRECT CURRENTS.
RESISTOR MOUNTED IN TEMPERATURE
CONTROLLED CABINET.
ALTERNATING LOADING CURRENT
FREQUENCY 50 Hz [MAINS].
—LEGEND—
Direct Current Loading.
Alternating & Direct Current Loading.
RESISTANCE CHANGE FROM NO-LOAD [10 AMP] VALUE [PARTS PER MILLION].
0
-50
-100
-150
-200
-250
-300
25
50
75
100
PERCENTAGE OF RATED FULL LOAD POWER DISSIPATION
IN TINSLEY 0.005 OHM MANGANIN SHUNT
AT A HOT BOX AIR TEMPERATURE
OF 27.8±0.2°C

(7.4.2)

Load Coefficient Measurements on the Unknown Tinsley 5 Milliohm Air Cooled Shunt S/N 101268 at Effective Current Levels of 15, 20, 25 and 30 Amps, Relative to a No-Load Condition Using 10 Amps D.C.

The load coefficient values obtained for the Tinsley 0.005 ohm air cooled shunt are presented in the curve of Fig. XLV1 (b) for both A.C. and D.C. loading conditions. The results are presented such that all measurements were made with the air temperature of the cabinet in which the Unknown was contained, controlled at an arbitrary value of 27.8 $\pm$ 0.2 $^{\circ}$C. Resistance changes in the Unknown under load were measured relative to the resistance value at 0.5 watts or 11.1 per cent of the rated full-load power dissipation level.

(7.4.3)

Results of Comparison of Load Coefficient at 25 and 50 c/s.

The mean D.C. resistance values obtained with equal 25 c/s and 50 c/s loading were found to differ by less than 2 parts per million.

(7.5)

Assessment of the Results Obtained on the Tinsley 5 Milliohm Air Cooled Shunt S/N 101268.

(7.5.1)

Load Coefficient.

The agreement between the alternating and direct current load coefficient values obtained for the 0.005 ohm Tinsley shunt is considered to be extremely close and well within the range of probable error to be expected, when the means of environmental temperature measurement and control are considered. The range of probable error to be expected in these measurements is difficult to assess accurately, but if the temperature coefficient of the Unknown at all power dissipation levels in the Unknown at 27.8 °C may be taken as being of the same order as that obtained at the 5 amp or 2.8 per cent of rated power dissipation level, then temperature uncertainties of the order of $\pm$ 0.2 °C in the element temperature of the Unknown will cause a spread in the results obtained for the measured value of the Unknown resistance of the order of $\pm$ 5.5 parts per million at all power dissipation levels. The loading current measurement uncertainty may cause a further error of 2 or 3 parts per million at the 100 per cent rated power dissipation level, depending upon whether A.C. or D.C. loading is applied. The uncertainty of the level of the load coefficient of the Standard used at rated power dissipation level in the Unknown adds a further uncertainty of $^{+\,0.5}_{-\,0}$ parts per million to the D.C. load coefficient obtained at this power dissipation level, and uncertainties in the measured values obtained for the oil temperatures of the Bridge bath and Standard throughout may add a further uncertainty of $\pm$ 1 part per million at all power dissipation levels. The effect of laboratory air temperature changes on the resistance values of the shunted dials used in the bridge arms may be neglected for load coefficient measurements, as air temperature changes experienced were small, and the shunted dials were set at approximately 100 times the resistance value of the respective standards shunted. It is assumed that resistance instabilities may be neglected in this case, since the load coefficient measurements were made over a period of approximately 7 hours.

Summarising, it appears that the range of probable error to be expected for the load coefficients of the Unknown will be approximately $\pm$ 1 part in 10^5 for any individual measurement at rated power dissipation level reducing to approximately $\pm$ 6 parts per million at the lower power dissipation levels. Inspection of the results shown in Fig. XLV1 (b) will show that all measured values of resistance change with load were within these ranges.

(7.5.2)

Temperature Coefficient.

The residuals existing in the temperature coefficient curve of best fit to the results obtained from resistance measurements made on the Unknown at a range of 'hot box' temperatures, may be caused by effects similar to those discussed above. It may be seen by inspection of Fig. XLV1 (a) that the residuals are greatest at the low temperature end of the curve where the cabinet thermometer measured virtually the effect of laboratory air temperature fluctuations, which may or may not have been experienced by the resistance element of the Unknown. Inspection of the results obtained in Table (10.2) for the temperature coefficients of the decade resistance boxes used in the A and B arms of the Kelvin Bridge as the shunted dials, will show that the nett effect of a 1 ^{o}C rise in the temperature of both of these shunts will be to cause an error of approximately 0.6 parts per million in the resistance change obtained for the Unknown, which is unaccounted for. The effect of the laboratory air temperature change will be seen mainly in the results obtained around 17.6 and 18 ^{o}C and will be one of the causes of the large residuals experienced in this area of the curve of best fit.

(7.5.3)

Correlation of Temperature and Load Coefficient Values.

Upon inspection of the slope of the no-load temperature coefficient curve obtained in Fig. XLV1 (a) it was expected that the resistance changes with load at constant temperature would be positive, at least at the lower levels of power dissipation used. If a non-uniform heat distribution with distance along the sections of the resistance element is assumed while the resistor is under load, then an argument similar to that used in section (5.5) may be used to account for the negative load coefficient. A very significant point in this explanation, is the effect of the presence of the copper in the resistor on the temperature and load coefficients obtained.

The total effective length of copper existing in each section of the resistance element is approximately $\frac{7}{8}$ inch. If the relative resistance of similar diameter manganin and copper wire is in the ratio of 26/1[28] then the ratio of the resistance of copper to manganin in the 0.005 ohm resistor will be approximately 5.10^{-3}, if the length of each manganin section of the resistance element is taken as being 7 inches in length (section 7.1). Hence, allowing for the temperature coefficient of resistance of the copper as 4.10^{-3} proportional parts per $^{\circ}$C, it can be shown that

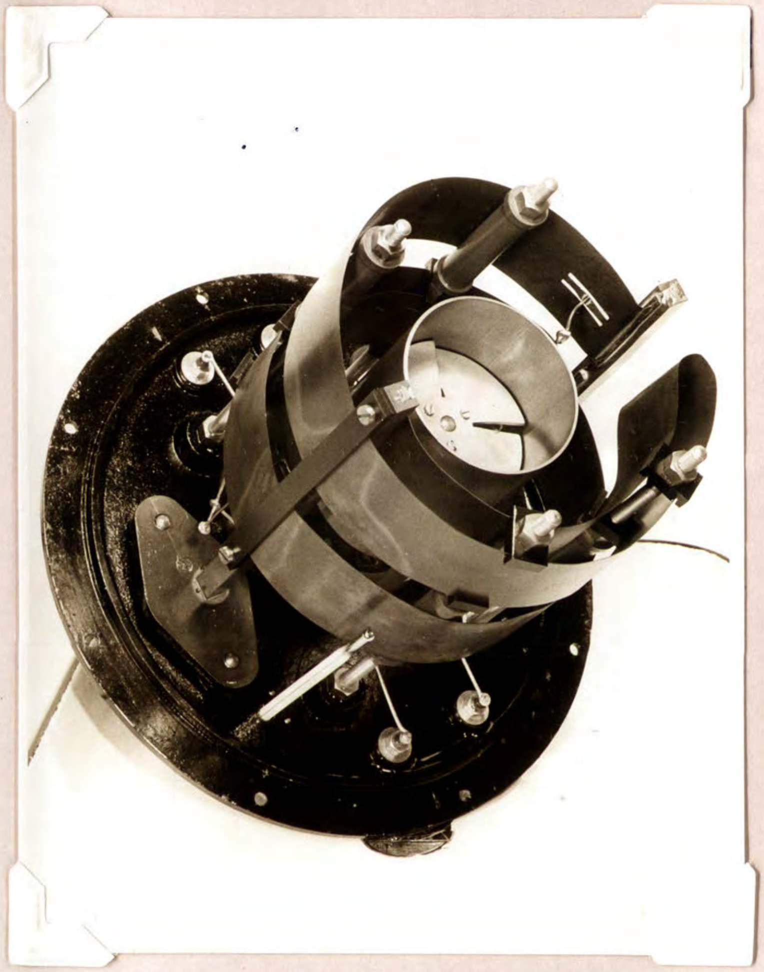
$$2.10^{-5} + \underset{\text{Manganin}}{\alpha_{20}} \simeq 27.10^{-6} \text{ proportional parts / } ^{\circ}\text{C} \tag{7.2}$$

neglecting second order effects, and

$$\underset{\text{Manganin}}{\alpha_{20}} \simeq 7 \text{ parts per million per degree C} \tag{7.3}$$

The value of β obtained from the temperature coefficient curve of Fig. XLV1 (a) may be taken as being derived completely from the presence of a significant β in a resistance-temperature relationship obtained for the manganin alone. With a greatly reduced value of α_{20} obtained for the manganin, a uniform temperature rise in the manganin of the resistance element under load would not be excessive before the nett resistance change in the manganin with load was negative. The copper terminating stubs, which produced the large value of α_{20} obtained in the composite temperature coefficient curve of Fig. XLV1 (a), are in a section of the resistor where the temperature of these stubs will not alter greatly under load, due to the influence of the large copper terminating blocks. Hence the nett result of these two effects in to produce composite temperature coefficient curves of high slope, and load coefficient values at constant temperature which may be negative.

If the temperature distribution along the resistor is non-uniform, the result would be to produce larger negative load coefficient values than would be obtained if a condition of uniform temperature distribution existed.



Photograph showing the construction of the Cambridge 0·01 ohm standard L-107021, S-1161.

Fig XLVII.

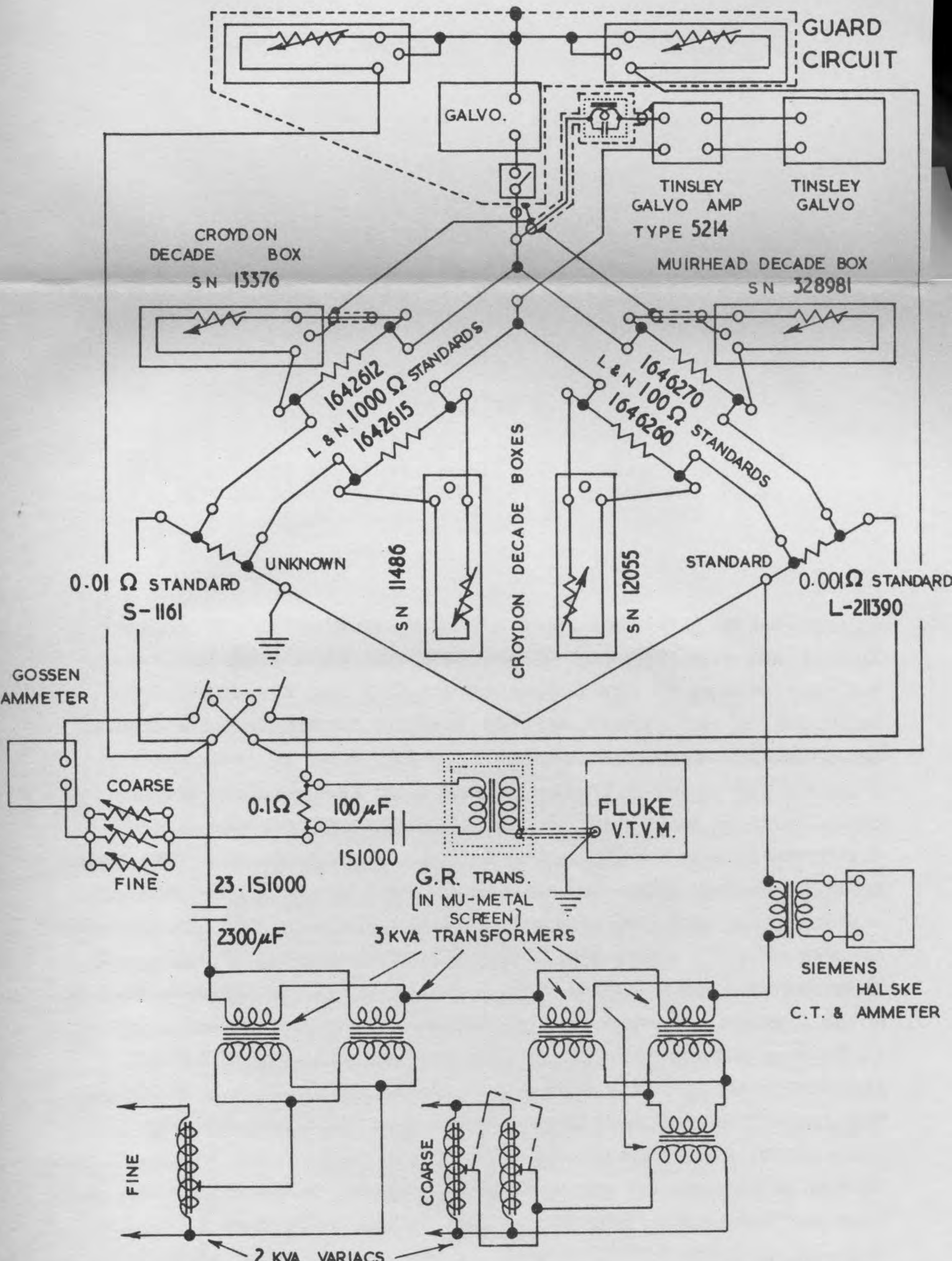
CHAPTER 8

Load Coefficient Measurements on the 0.01 ohm Heavy Current Cambridge Standard Resistor S/N L-107021 P/N S-1161

(8.1)

The heavy current 0.01 ohm Cambridge Standard resistor S-1161 was the property of the National Standards Laboratory and was loaned to the Author especially for testing under alternating and direct current loading conditions. A photograph of the resistor removed from its case is presented in Fig. XLV11, showing the arrangement of the sheet manganin resistance element, the location of the stirrer and 1/5 °C thermometer and of the potential and current leads. The water cooling jacket is not shown but it is mounted inside the oil container of which the mounting plate shown in the photograph of Fig. XLV11, forms the lid. S-1161 is one of a series of five heavy current standards, ranging in values from 0.0001 to 1 ohm, which are used by the National Standards Laboratory as reference standards during heavy current load coefficient determinations. It had however, fallen into disuse owing to its relatively high self-heating effect under load. The possession of a relatively large load coefficient by the Unknown was an advantage in any measurements made to establish the A.C. loading technique, however, since it was felt that if there were any inherent differences in the self-heating effects produced by the A.C. and D.C. loading currents used, the large value of the load coefficient might help to demonstrate it provided the effect was of sufficient magnitude not to be masked by the presence of measurement uncertainties.

A method virtually similar to that used for the presentation of the results of Chapters 4 and 5 was used to present the results of the load coefficients made here, but owing to the existence of temperature coefficient curves at constant load



SCHEMATIC DIAGRAM
of the Kelvin Bridge used in
measurements made on the Cambridge 0·01 ohm resistor
L-107021, S-1161.

Fig XLVIII.

for S-1161 produced by the D.C. section of the National Standards Laboratory during their own investigation of its change under load, it was decided to super-impose the results obtained by the Author onto these curves. The original curves for S-1161 were recalculated using the method of 'least squares' in association with a modification of the program LOADCURV presented in Chapter 12. There was a limitation on the amount of direct current capacity available for the high load measurements, load coefficient measurements could be made only up to a maximum current of 60 amps instead of up to the full load rated current of 150 amps as would have been desirable.

The Kelvin Bridge circuit was wired up as shown in the circuit diagram of Fig. XLV111. The Standard used was the 0.001 ohm Cambridge heavy current standard S/N L-211390, used in the investigation performed on the 0.005 ohm air cooled shunt discussed in Chapter 7. The bridge ratio $^{X}/_{S}$ of 10/1 was chosen so as to reduce the dependency of the direct current load coefficient measurement accuracy on the accuracy with which the load coefficient of the Standard was known.[3,4,5] The A, B, P and Q arm decade resistance shunts were set at nominally 100 times the value of the standard resistance shunted. The A and P arm shunts were adjusted in unison to obtain bridge balance.

The guard circuit shown was necessary since one side of the link was earthed.

Problems were encountered in keeping the direct current level constant between supply reversals at the higher direct current levels used. This was found to be caused by the changing series D.C. resistance contributions of the reversing switch contacts and its associated copper leads. Their resistance had become a finite proportion of the total D.C. series resistance in the supply circuit and in order to overcome this effect, large diameter supply leads were used and the supply leads to the bridge connected to the reversing switch in a fashion such that the D.C. resistance of one cross lead in the reversing switch was always present in the supply circuit. The two cross leads were adjusted to the same resistance value, within about $\pm$ 5 per cent, so as to keep resistance variation in the contacts of the reversing switch the only variable factors present when reversing the direct current. Alternating currents flowing in the battery supply circuit were determined by the alternating voltage drop across the 0.1 ohm 4-terminal non-inductive shunt as in Chapter 4.

A bridge unbalance sensitivity equivalent to $\pm$ 1 part per million of the Unknown was obtained with a no-load bridge supply current of 10 amps D.C. Measurements were made to determine the resistance value of the link L. Results showed that it was approximately equal to 0.7 milliohm of which over 50 per cent was contributed by the resistance of the current connection of S-1161, between the current terminal and the junction of the appropriate potential lead with the resistance element.

The A, B, P and Q arms standard resistors of the Kelvin Bridge were included in a stirred oil bath. Oil bath temperatures were measured with the aid of a 0.01 ^{o}C mercury-in-glass thermometer. Corrections due to changes in oil bath temperatures were applied to the respective bridge arm standards using the values of the temperature coefficients α_{20} and β provided in their original certificates of test. The level of power dissipation encountered in the A and B arm standards of the Kelvin Bridge was considered to be negligible in all cases. The temperature and load coefficients for the 1 milliohm standard however, were not known initially and had to be determined before load coefficient measurements on the 10 milliohm Unknown could commence. Oil temperatures for the 1 milliohm Standard and 10 milliohm Unknown were measured with 0.01 ^{o}C and 0.02 ^{o}C mercury-in-glass thermometers respectively. When temperature and load coefficient measurements were being made on the 1 milliohm Standard, copper-constantan thermocouples were used in association with a Tinsley portable potentiometer

Type 3184. The 0.01 and 0.02 ^{o}C mercury-in-glass thermometers available were not suitable for use on the Unknown in this case, owing to the stable temperature used in the Unknown at the time being outside their respective temperature measuring ranges.
In order to eliminate the effect of air currents on the thermal equilibrium of the Unknown while temperature and load coefficient measurements were being performed on the Standard, it was placed into a cardboard carton and surrounded with exploded polythene foam and rag waste. This increased the thermal time constant of its oil bath and hence temperature conditions could be changed in the Standard with a minimum of temperature change occurring meanwhile in the Unknown. It was found that the thermal lagging caused the stable operating temperatures reached by the Unknown to be generally higher and the heat input from the stirrer motor and mixing, sufficient to produce a stable operating temperature of approximately 40 ^{o}C at a room temperature of 21 ^{o}C, without load on the Unknown or cooling water input.

(8.2)

Measurement Procedure.

The no-load D.C. measuring current level used in all measurements made on S-1161 was 10 amps. Alternating and direct current load coefficient determinations were performed at effective loading current levels of 20, 30, 40, 50 and 60 amps.

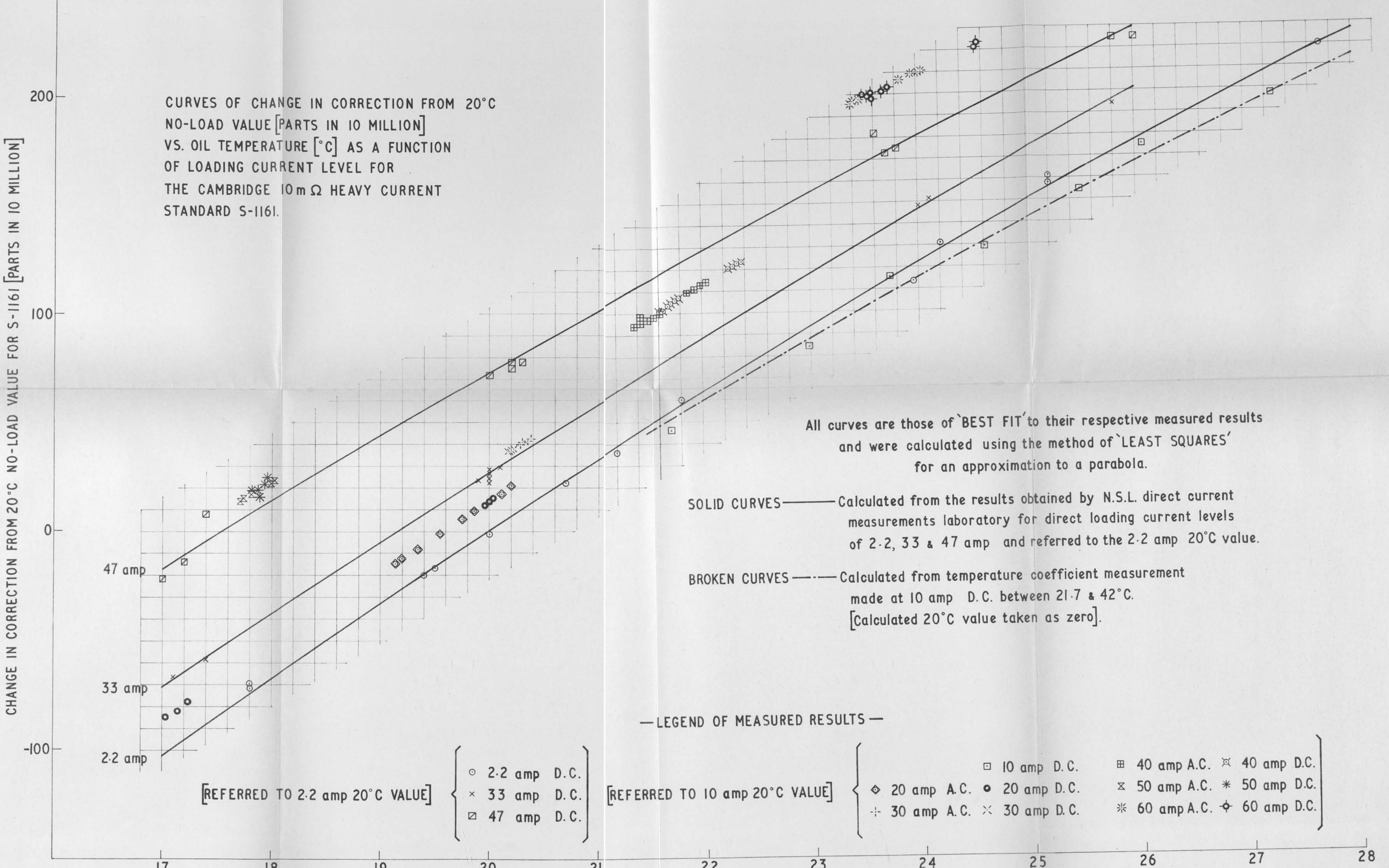
Load coefficient measurements were made such that in measurements to be performed at any particular loading current level, a series of at least 3 sets of measurements were made. e.g. If set 1 was made under D.C. loading conditions, then set 2 would be made under A.C. loading conditions and set 3 made under D.C. loading conditions again. Each set of measurements consisted of at least 5 separate bridge balances, made at approximately 5 to 10 minute intervals. Temperature coefficient measurements were made on the Standard while its oil temperature was increased in steps with the aid of a small hot-plate. Load coefficient measurements were made on the Standard using A.C. loading techniques and sufficient time was allowed for temperature stabilisation before measurements were made.

(8.3)

Supply Current Stability and Accuracy of Measurement.

The instability found to exist in the higher levels of the D.C. supply current upon reversal placed a limitation on the accuracy with which the D.C. load coefficient of the Unknown had been found. The characteristic curves recalculated for S-1161 from the results obtained by the National Standards Laboratory show that its load coefficient is approximately + 4 parts per million per watt at 20 ^{o}C. If the error in the measured value of the D.C. load coefficient due to current changes on reversal is to be less than $\pm$ 1 part per million in the worst case (60 A), then the D.C. supply current must be stable and known to better than $\pm$ 0.3 per cent of its indicated value.

Measurements made on the reproducability of the contact resistance of the reversing switch showed that as an extreme maximum, variations up to about 0.35 milliohms could be expected in the worst case on a newly cleaned and adjusted reversing switch. With 60 amps D.C. flowing from a 2 volt battery supply, the total D.C. series resistance present in the battery circuit was 33.3 milliohms. Hence the resistance variation possible in the contacts of the reversing switch under these conditions caused an uncertainty in the resulting D.C. load coefficient produced approaching $\pm$ 1 part per million. This problem ceased to exist as the level of the direct current was reduced and the total D.C. series resistance present in the D.C. supply circuit was increased. The stability of reading of the 0.2 per cent Siemens moving iron meter was better than $\pm$ 0.1 per cent of F.S.D. The ratio error of the multi-ratio Siemens current transformer used in association with this meter was of the order of 0.02 per cent and hence was negligible. It can be calculated



OIL TEMPERATURE [°C]. Fig. XLIX.

that 1/19 of the alternating current level in the Unknown passed through the battery circuit when 10 amps of direct measuring current was supplied from a D.C. battery supply of 2 volts. Therefore when the alternating current level in the Unknown was approximately equal to 60 amps, the battery circuit alternating current level was approximately equal to 3.15 amps and to cause an error of less than $\pm$ 1 part per million in the measured value of the load coefficient obtained for the Unknown at this load, the battery circuit A.C. level must have been known to within approximately $\pm$ 6 per cent of its value. This level of measurement accuracy was considered to be well within the capabilities of the measuring system used.

Direct current levels were measured on the 0.3 per cent Gossan multi-range moving coil meter. The uncertainties which existed in the Gossan reading at 10 amps were negligible when the larger proportion of power dissipation in the Unknown was provided by the applied alternating current level.

(8.4)

Presentation of Results.

(8.4.1)

No-Load Temperature Coefficients of Both the Unknown Cambridge 10 Milliohm Standard S-1161 and the Cambridge 1 Milliohm Standard.

The results of the no-load temperature coefficient measurements on both the Unknown and the Standard were processed using the computer program TEMPCURV, as shown in Chapter 12. The resulting no-load temperature coefficient curve obtained for the Unknown was found to possess a smaller slope than that of the previously obtained no-load curve at 2.2 amps over its whole range, meaning that the changes in resistance from the 20 $^{\circ}$C no-load value were generally less over the whole range. This fact is demonstrated by the portion of the 10 amp temperature coefficient curve shown in the results of Fig. XL1X. The calculated values of α_{20} and β for this 10 amp curve are 30.7

parts per million per ^{o}C and - 0.96 parts per million per degree per degree C respectively, as against 32.8 parts per million per degree C and - 1.1 parts per million per degree per degree C respectively for the 2.2 amp curve calculated from the results obtained previously by N.S.L.

The values obtained for the α_{20} and β of the Cambridge 1 milliohm heavy current standard L-211390 used as Standard were 13.5 parts per million per degree C and - 1.00 parts per million per degree per degree C respectively. These values of α_{20} and β were used during A.C. and D.C. load coefficient measurements on both the Unknown and the Standard to account for changes in temperature of the Standard between measurements, since the magnitude of its load coefficient from 10 to 60 amps was found to be quite low, as will be discussed in section (8.4.2) and hence the values of α_{20} and β at the loads experienced by the Standard could be taken to be very closely approximated by the values obtained at no-load (10 amps).

The values of the R.M.S. residual obtained in the temperature coefficient curves of S-1161 and L-211390 were 2.2 and 1.4 parts per million with maximum residuals of 7.8 and 3.0 parts per million respectively.

(8.4.2)

Load Coefficient Measurements of S-1161 at 20, 30, 40, 50 and 60 Amps Using Both D.C. and A.C. Loading Currents.

The load coefficient values obtained for the Cambridge 10 milliohm heavy current standard S-1161 are presented in Fig. XL1X for both A.C. and D.C. loading conditions. The results are presented such that its resistance value is referred to its 20 ^{o}C no-load (10 amp) condition. Several of the load coefficient curves calculated from the results obtained by N.S.L. are presented also, in order to indicate the approximate slope of the resistance - temperature relationships to be expected at the loads used. This serves as a means of correlating A.C. and D.C. load coefficient values at the same load when separated by a significant temperature range. The method used by the Author for the presentation of the results obtained by N.S.L. was to refer all resistance changes with load or temperature to the 20 °C no-load (2.2 amp) value. For convenience of presentation, the 20 °C 2.2 and 10 amp values were assumed to coincide. The maximum error expected because of this assumption was calculated from the estimated change in load coefficient versus power dissipation level at 20 °C to be approximately 4 parts per million.

The accurate calculation of the direct current load coefficient values for the Unknown depends upon a knowledge of the change in the Cambridge 1 milliohm heavy current standard resistor L-211390 employed as Standard at the direct current levels used. Only one investigation was found to be necessary, at the 60 amp level, since its load coefficient value there was found to be very close to zero parts per million, with an uncertainty of about $^{+2}_{-0}$ parts per million. This uncertainty has a significant effect at only the three highest current levels used.

(8.5)

Assessment of the Results Obtained on S-1161.

The agreement between the alternating and direct current load coefficient values obtained for S-1161 was considered to be extremely close, although several points of interest did emerge.

(8.5.1)

No-Load Temperature Coefficient Curve for S-1161.

The slope of the curve obtained by the Author at 10 amps differed significantly from that obtained by N.S.L. at 2.2 amps. The fact that the power dissipation level is greater at 10 amps than at 2.2 amps is insufficient cause, since the 4 parts per million maximum difference calculated at 20 °C in section (8.4.2) would be reduced to zero at an oil temperature of 22.2 °C. It may be significant that the majority of the measured results used in the calculation of the 10 amp curve were obtained at oil temperatures well above the highest oil temperatures used by N.S.L. in obtaining the 2.2 amp curve. Similarly, inspection of the measured points shown for the 10 amp curve in the temperature range of interest between 21 and 28 °C indicate that they could be approximated to lie on a curve which is almost parallel to the 2.2 amp curve. The measured values obtained became closer and closer together in temperature at the upper temperature end of the curve. Hence if one section of the resistance-temperature curve contains more than its proportional share of measured values, then the calculated parabola of best fit to these measured values will be influenced more by the results obtained in the temperature interval containing a disproportionately large amount of data, than by the results obtained in the temperature interval containing relatively little data. Of thirty-five readings taken on S-1161 at 10 amps in the temperature range 21.70 °C to 41.99 °C, only seven readings were taken in the temperature range from 21 to 28 °C, while eighteen readings were taken in the temperature range from 35 to 42 °C. Also the area of the maximum residuals was between 21 to 28 °C. The difference in value of R.M.S. residual between the two temperature ranges could be due both to the fact that the oil temperature was increasing relatively quickly over the temperature range between 21 and 28 °C hence producing uncertainty in the measured values obtained for the oil temperatures in this range, and to the fact that the curve of best fit was calculated from a set of results which gave emphasis to results at one end of the temperature range.

(8.5.2)

30 Amp A.C. and D.C. Load Coefficient Values Greater than those Obtained at 33 Amps by N.S.L.

Inspection of the Fig. XL1X will show that at 20 ^{o}C, the change in resistance corresponding to an increase in current from 10 to 30 amps was found to be greater than that experienced by N.S.L. for an increase in current from 2.2 to 33 amps. The cause of this is unknown but is considered most likely to be caused by possible differences existing in stirring or cooling conditions from time to time.

(8.5.3)

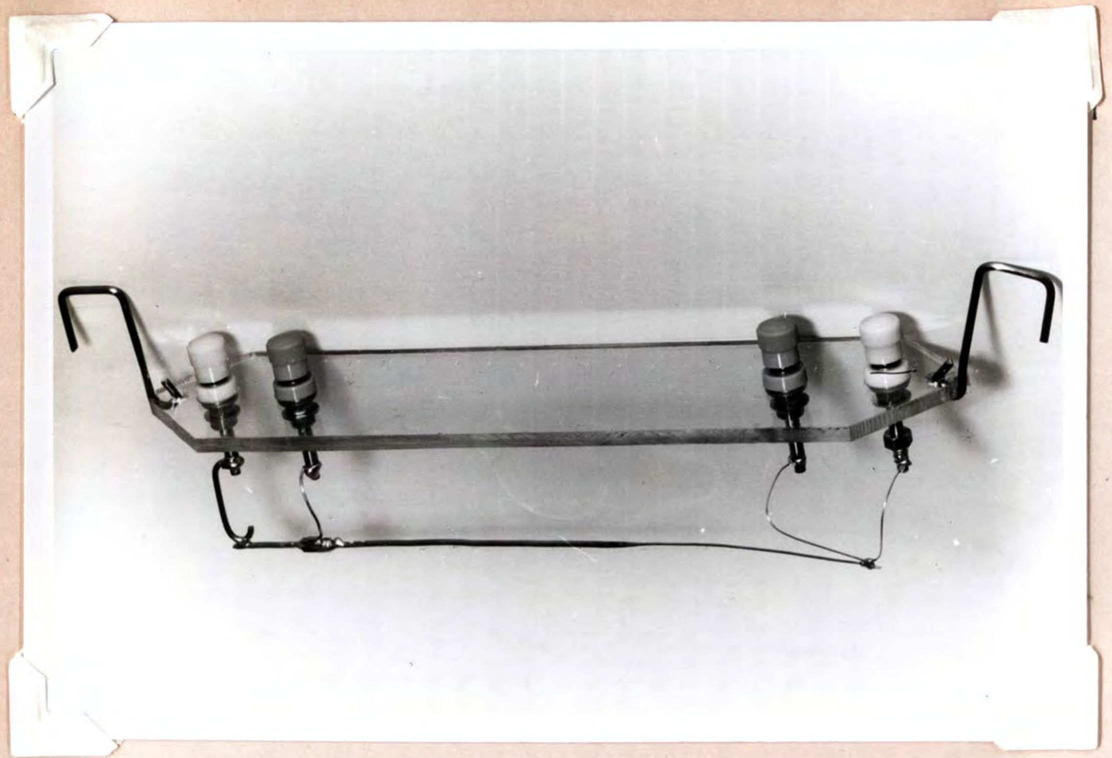
Spread of Results Obtained.

Inspection of the plotted points of Fig. XL1X obtained by the Author will show that the spread of points is generally of the order of $\pm$ 1 part per million from a mean value curve constructed by inspection.

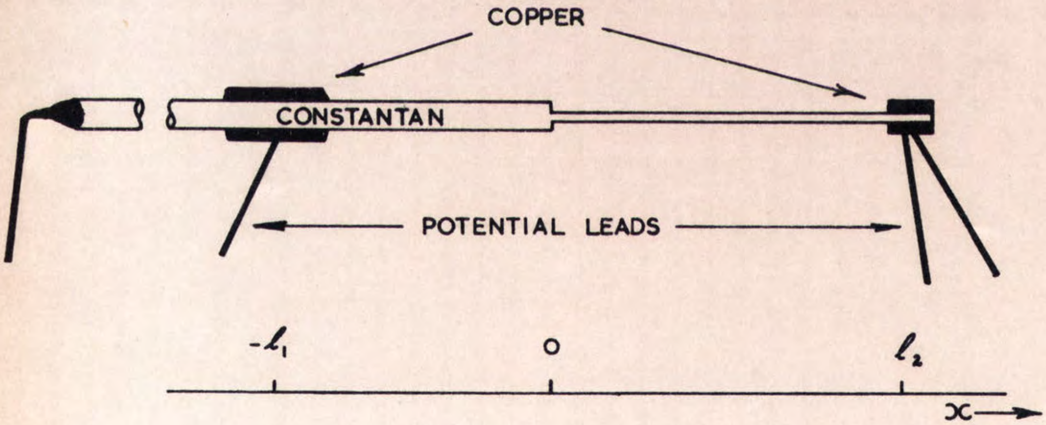
(8.5.4)

Load Coefficient Differences between Alternating and Direct Loading Current Conditions at 60 Amps.

Inspection of the results obtained in Fig. XL1X for both A.C. and D.C. loading conditions at 60 amps shows what could be taken as a difference between the two loading methods at this power dissipation level. The value of the load coefficient used for the Cambridge 1 milliohm Standard in obtaining the D.C. values here was zero parts per million. Corrections to the Standard of a Kelvin Bridge are added to the measured value obtained in order to produce the true resistance value for the Unknown and the uncertainty existing in the measured value of the load coefficient of the Standard was of such a sign as to increase the resulting direct current value of the load coefficient obtained for the Unknown at 60 amps. Therefore the small difference shown in the results of Fig. XL1X may be seen to be less than the total range of uncertainty existing in the measured values of both the A.C. and D.C. load coefficients due to all effects present, including temperature gradients etc, resistor instability, current measurement accuracy and the accuracy of knowledge of the load coefficient of the Standard.



Photograph showing the construction of the Special 0·1 ohm Constantan Resistor. (a)



Diagramatic representation of Fig L(a) above (b)

Fig L.

CHAPTER 9

Load Coefficient Measurements on a Special 0.1 ohm Constantan Resistor.

(9.1)

A special 0.1 ohm constantan resistor was constructed in an attempt to obtain a resistor in which equal effective alternating and direct loading current might produce different measured direct current resistance changes with load. This resistor is shown in the photograph of Fig. L (a). Its resistance element was constructed from a resistance material of high thermo-electric effect to copper such that its resistance value as a function of its length was non-uniform. In this way a resistor was produced in which the power dissipation per unit length throughout its length was non-uniform and a large temperature difference was experienced between each end while under load.

The resistor was constructed from a length of 0.082" diameter constantan wire which was partially drawn through successive wire drawing dies until a final diameter of 0.029" was obtained over about 3 inches at one end, leaving about 5 inches at the other end of the wire at its original diameter of 0.082". The potential leads were constructed from relatively fine copper wire. The junction of the thick end of the constantan resistance element with its corresponding copper current lead was well removed from the junction of the corresponding potential lead, while at the other end of the resistor, the copper current and potential leads were connected to the resistance element in close proximity to each other. The copper content of these junctions was kept as low as possible in order to minimise their thermal capacities and hence the duration of transient effects produced upon direct current reversal. Load coefficient measurements were made with the resistor completely submerged in stirred transformer oil, using both alternating and direct current loading techniques.

(9.1.1)

Discussion of Possible Difference Effects.

The approximate temperature distribution along a heavy current sheet manganin standard resistor in which convection cooling was efficient and in which radiation effects were negligible, was calculated in Section (1.1). From that discussion using similar assumptions, it can be shown that if the point at which the diameter of the resistor of Fig. L (b) changes abruptly is taken as x = 0, and if it may be assumed that the potential points of Fig. L (b) at $x = -\ell_1$ and ℓ_2 are isolated from each other and from x = 0, then as in equation (1.8),

$$m \simeq n \simeq \frac{Hp}{ak_{con}} \qquad (9.1)$$

where the constants have similar meanings.

At $x = -\ell_1$, area = a_1, perimeter = p_1 and

$$\theta(-\ell_1) = \theta_c \qquad (9.2)$$

The direct current from the copper potential lead to the constantan resistance element and vise versa, will be very small under all practical conditions of measurement. Hence the Peltier Effect will be small, as will be the conduction along the potential lead at $x = -\ell_1$ if it is of small diameter. The element temperature in the vicinity of $x = -\ell_1$ may therefore be taken as being equal to θ_c throughout, and from equation (1.11) provided $e^{\ell_1 \sqrt{\frac{Hp_1}{a_1 k_{con}}}}$ is large,

$$\theta_C = \theta_F + \frac{I^2_{EFF}\rho_{const}}{Hp_1\ a_1} \tag{9.3}$$

At $x = \frac{\ell_2}{2}$ area $= a_2$ perimeter $= p_2$

$$\theta(\frac{\ell_2}{2}) = \theta_H \tag{9.4}$$

Therefore from equation (1.11) provided $e^{\frac{\ell_2}{2}\sqrt{\frac{Hp_2}{a_2k_{con}}}}$ is large,

$$\theta_H = \theta_F + \frac{I^2_{EFF}\rho_{con}}{Hp_2\ a_2} \tag{9.5}$$

At $x = \ell_2$ area $= a_2$ perimeter $= p_2$

From equation (1.11) provided $e^{\ell_2\sqrt{\frac{Hp_2}{a_2k_{con}}}}$ is large,

$$\theta(\ell_2) \simeq \theta_H - \frac{q_{con,Cu}(\theta\ell_2) + \pi_{con,Cu}(^T\ell_2)\ I_{DC,con,Cu}}{\sqrt{Hp_2\ a_2\ k_{con}}} \tag{9.6}$$

The temperature difference between the points $x = \ell_2$ and $x = -\ell_1$ is given by

$$\Delta\theta_{LR} = \theta_H - \theta_C - \frac{q_{con,Cu}(\theta\ell_2) + \pi_{con,Cu}(^T\ell_2)_{LR}\ \ I_{DC}}{\sqrt{Hp_2\ a_2\ k_{con}}} \tag{9.7}$$

$$\Delta\theta_{RL} = \theta_H - \theta_C - \frac{q_{Cu,con}(\theta\ell_2) - \pi_{con,Cu}(^T\ell_2)_{RL}\ \ I_{DC}}{\sqrt{Hp_2\ a_2\ k_{con}}} \tag{9.8}$$

with the direct current flowing from left to right and right to left respectively in Fig. L (b).

If direct current reversals are used in obtaining bridge balance, the temperature difference $\Delta\theta$ of equations (9.7) and (9.8) will produce relatively large Seebeck e.m.fs which will oppose the flow of direct measuring current when it flows in one direction and which will aid the flow of direct measuring current when it is reversed. The opposing and aiding effects will not be equal however due to the dependence of q and $\pi_{con,Cu}$ on the temperatures $\theta\ell_2$ and $^T\ell_2$. The contribution of Seebeck e.m.f. to the measured resistance value obtained will correspond to that e.m.f. produced by half the difference between $\Delta\theta_{LR}$ and $\Delta\theta_{RL}$ of equations (9.7) and (9.8) above. The relevant temperature difference is therefore given by

$$\Delta\theta = \frac{q_{Cu,con}(\theta\ell_2) - q_{con,Cu}(\theta\ell_2) - \left[\pi_{con,Cu}(^T\ell_2)_{RL} + \pi_{con,Cu}(^T\ell_2)_{LR}\right]}{\sqrt{Hp_2\, a_2\, k_{con}}} \qquad (9.9)$$

which will be a complicated function of I_{DC} in general due to the relatively large Peltier Effect present in copper-constantan junctions. $q_{Cu,con}(\theta\ell_2) - q_{con,Cu}(\theta\ell_2)$ will not equal zero but will depend upon I_{DC} as will $\pi_{con,Cu}(^T\ell_2)$. Thus the thermo-electric resistance produced will be a function of I_{DC}.

When the thermo-electric circuit of Fig. L (b) is connected into its bridge measuring circuit, it has been shown[29] that the Seebeck e.m.f.

$$F_{Cu,con}(\theta\text{-}\ell_1, \theta\ell_2) = \pi_{Cu,con}(^T\text{-}\ell_1) - \pi_{Cu,con}(^T\ell_2) + \int_{\theta\ell_1}^{\theta\ell_2} \alpha_{T_{Cu}}(\theta) - \alpha_{T_{con}}(\theta)\; d\theta \qquad (9.10)$$

where

$F_{Cu,con}(\theta_{-\ell_1}, \theta_{\ell_2})$ is the Seebeck e.m.f. produced in a circuit consisting of two copper-constantan junctions at temperatures $\theta_{-\ell_1}$ and θ_{ℓ_2},

$F_{Cu,con}(\theta_{-\ell_1}, \theta_{\ell_2})$ is taken as positive if the sense of flow of direct current is from copper to constantan at the junction whose temperature is θ_{ℓ_2},

$\pi_{Cu,con}(T)$ is the Peltier coefficient at the copper constantan junction at the absolute temperature T and

$\propto T_{Cu}(\theta)$ and $\propto T_{con}(\theta)$ are the Thomson coefficients of the copper and constantan metals at a temperature θ respectively.

From (9.10) the Seebeck e.m.f. $F_{Cu,con}(\theta_{-\ell_1}, \theta_{\ell_2})$ is seen to be independent of the magnitude of any temperature gradients existing but dependent only on the temperatures of the two copper-constantan junctions at $-\ell_1$ and ℓ_2. Hence the temperature gradients existing at x = o and x = ℓ_2 will have no effect on the Seebeck e.m.f. provided the values of $\theta_{-\ell_1}$ and θ_{ℓ_2} remain constant.

The presence of Peltier cooling will produce a change in the resistivity of the element in the vicinity of the point x = ℓ_2 upon direct current reversal caused by the non-linear dependence of the resistivity on temperature. The change in resistivity will be insufficient to significantly affect the power dissipation level in the resistance element, but will cause a change in the measured value of the resistance obtained, which is dependent upon the direction of current flow. D.C. supply reversals will result in the measured value of the direct current resistance value obtained being the Arithmetic Mean of the two calculated values, corresponding to the two directions of direct current flow and corresponding temperature distributions. The Arithmetic Mean of the two calculated direct current resistance values will be very close to the true value, but there will be a difference term present which will contain expressions in both $\propto_{20}$ and β. A similar effect will occur in the vicinity of

$x = o$ due to Thomson Effect. The direct current level will alter the temperature distribution existing near $x = o$ as it passes through the temperature gradient there. The effect on the measured value of direct current resistance will be small owing to the magnitude of Thomson coefficients generally. These effects will not normally be significant in typically constructed manganin standard resistors, since symmetrical construction and convection cooling will tend to cancel the effects; the non-linearity of the resistivity - temperature curve for manganin will also be small. Even so the resistance increase at one end of a symmetrically constructed standard resistor due to a corresponding temperature increase from Peltier Effect, may be greater than the corresponding resistance fall at the other end due to Peltier Effect or vise versa. The The resistance arrangement will reverse upon direct current reversal, but the resistance value obtained will be equal to that obtained previously. The presence of Peltier Effect may therefore produce a resistance change in a normally constructed standard resistor which will be a function of the direct current level.

(9.2)

Measurement Procedure.

The no-load D.C. measuring current used in all measurements made on the special 0.1 ohm constantan resistor was 2.0 amps. Alternating and direct current loading was carried out at an effective current level of 5.0 amps only. Load coefficient measurements were made on the Unknown 0.1 ohm resistor in such a way that two bridge balances were attempted under each condition of oil bath temperature and loading current combination, separated in time by approximately five minutes. It was found that the galvanometer electrical zero was particularly unstable with time in this case and hence the detector integration time was increased somewhat with the use of a modified detection system, which will be discussed below. The cause of this

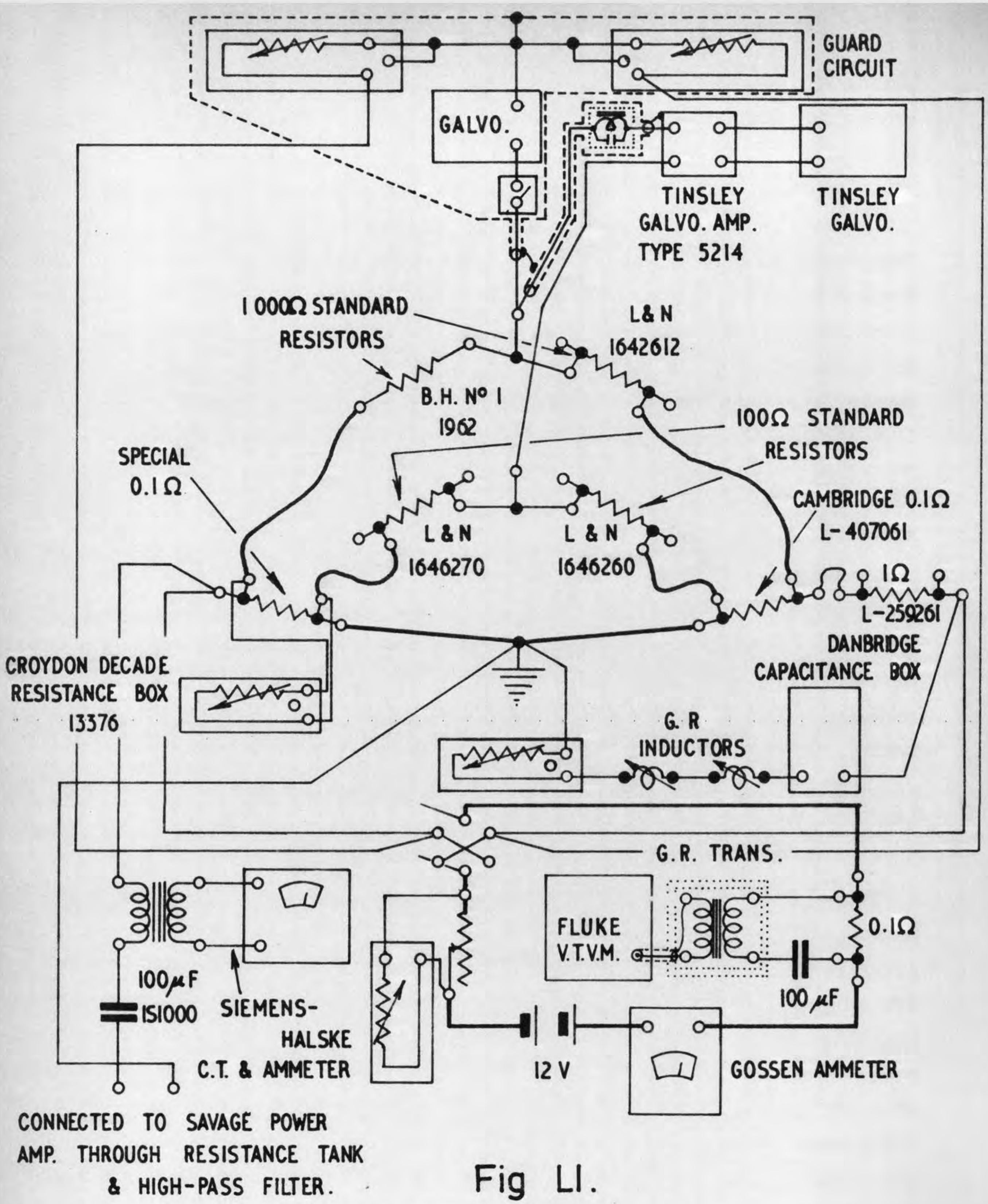


Fig L1.

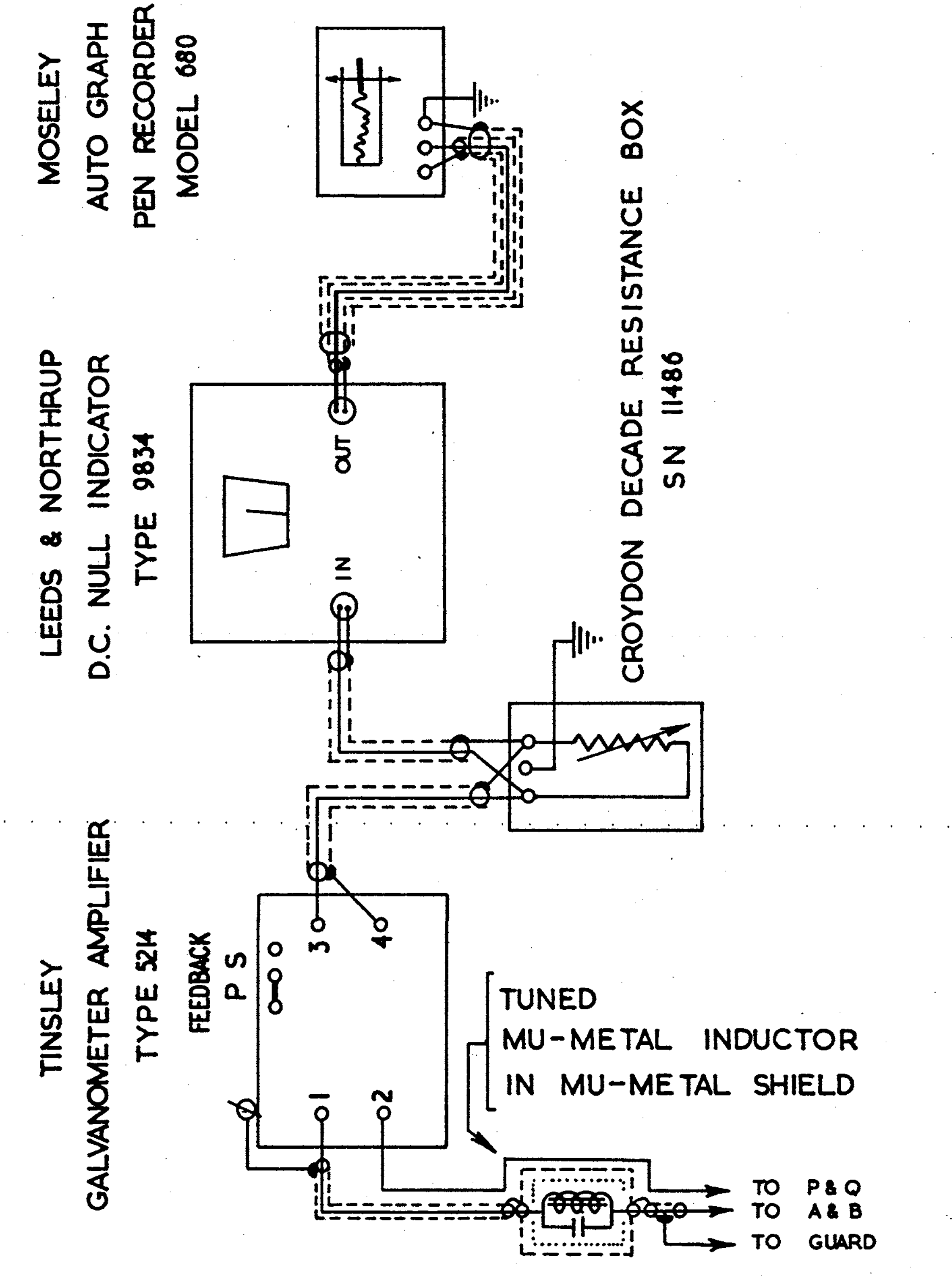
Schematic Diagram
of the Kelvin Bridge used in
measurements made on the 0.1Ω special constantan resistor.

instability was considered to be related to the thermal properties of the resistor being tested. A pair of copper-constantan junctions produce a Seebeck voltage which increases at the rate of 4.10^{-5} volts for each degree centigrade of temperature difference occurring between them at 20 ^{o}C. In the Kelvin Bridge used for resistance measurements on this resistor, the direct voltage drop across the Unknown under no-load conditions was equal to 0.2 volts. Hence, if the cooling oil possessed localised pockets which were up to one degree C higher in temperature than the average oil temperature, or if the speed of cooling oil flow past the resistance element changed sufficiently to cause a 1 ^{o}C rise in temperature at one junction relative to the other, then the changes in the D.C. voltage seen across the Unknown by the Kelvin Bridge, would have been up to 4.10^{-5} volts in 0.2 volts. In other words, a galvanometer electrical zero drift equivalent to a change of up to 4 in 10^{4} in the Unknown would have occurred. This electrical zero drift may be quite rapid depending on the thermal time constants of the two copper-constantan junctions in the Unknown.

The Kelvin Bridge used for measurements on the special 0.1 ohm constantan resistor is shown in the circuit diagram of Fig. L1. The bridge ratio, Unknown to Standard, was 1 to 1, while bridge balance was effected with the adjustment of a decade resistance box connected across the Unknown.

A guard system was considered necessary at first and hence was incorporated. But upon experiencing the electrical zero instability present, only rough adjustment was considered to be necessary. To off-set this inaccurate guard balancing procedure, the link was earthed as closely as possible to its electrical centre.

It was found upon galvanometer reversal, that the transient effects encountered were relatively insensitive to the setting of the kick correction circuit and that the thermal transients encountered appeared to swamp all others.



Bridge Balance Indicator used to provide increased integration time in measurements made on the 0·1 ohm Special constantan resistor.

Fig LII.

The integration time of the detection system used was increased with the use of a chart recorder to plot the behaviour of the spot as a function of time. The detection system used is shown in the schematic diagram of Fig. L11. The Tinsley Galvanometer Amplifier was used as the primary galvanometer, while its output current was fed into a decade resistance box, which was set at a value of 9 000 ohms and connected across a Leeds and Northrup D.C. Null Indicator Type 9834. Reduction of the input impedance was found to be necessary owing to the existence of stray capacitive coupling from output circuit to input circuit. The combination of a high input impedance in the Leeds and Northrup D.C. Null Indicator and the level of the stray capacitive coupling available from the output to input circuit, while using an open wire output lead from the Indicator, was sufficient to cause severe oscillation on the ranges of the L and N below setting 1. This capacitive coupling was reduced with the use of a shielded connecting lead between the output of the L and N to the input of the Moseley Autograph Pen Recorder Model 680.

Typical values of range settings for the components of the detection system were found to be

Tinsley Galvanometer Amplifier Type 3184 set on Max F.B.,

Croydon decade resistance box at the input to the Leeds and Northrup D.C. Null Indicator set to 9 000 ohms,

Leeds and Northrup D.C. Null Indicator set to 10^{-1}, and

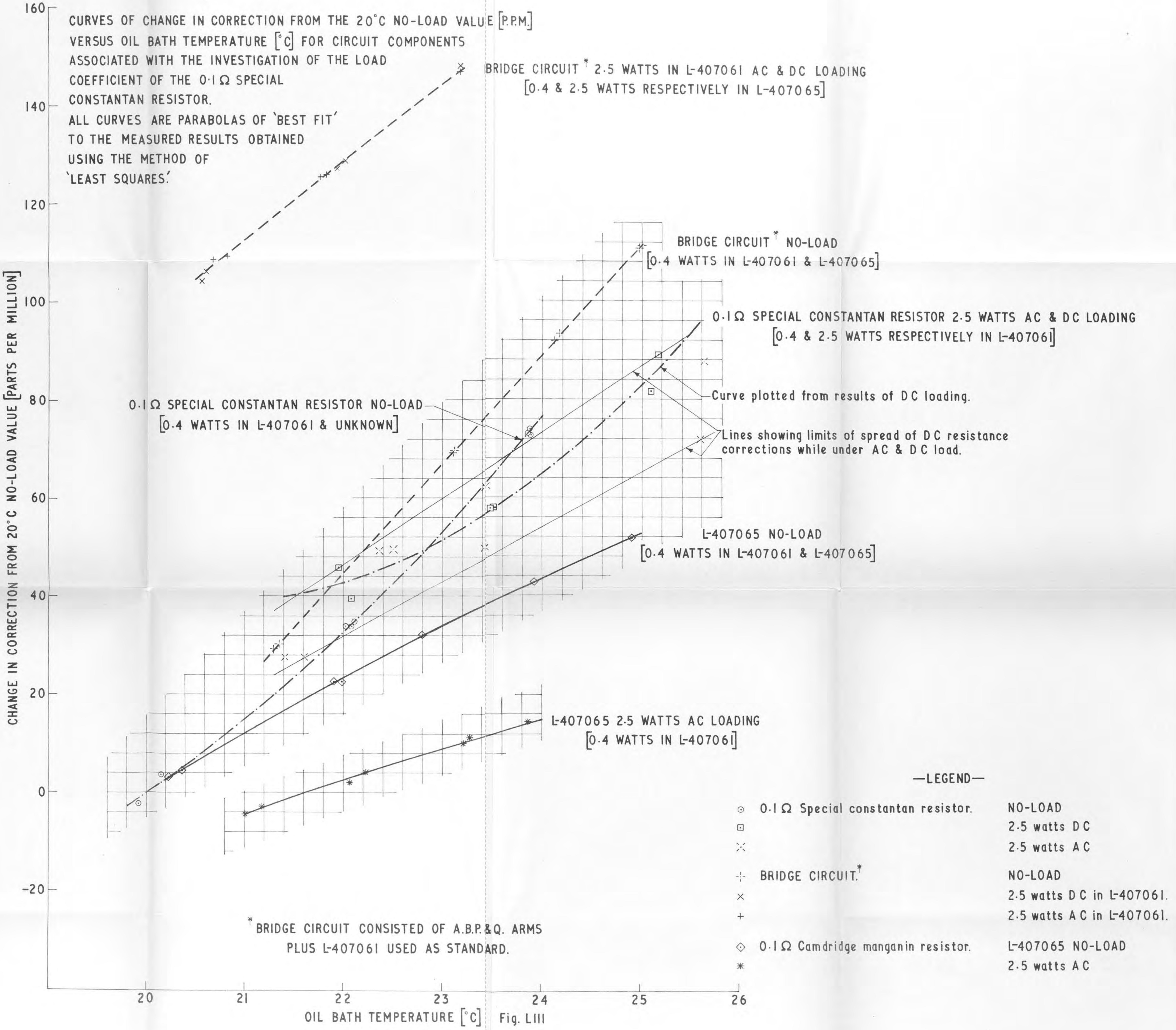
Moseley Model 680 on 1 volt range at 8"/ hour.

There was a set time used between galvanometer reversals of $\frac{3}{4}$ minute so as to allow equal time intervals for transient effects to dissipate. It was found expedient from time to time to vary these settings, but generally these give a fair idea of the range settings used in practice.

The Unknown was placed into one stirred oil bath, while the remainder of the Kelvin Bridge arms were set up in the stirred Bridge bath. Oil temperatures were monitored periodically with the aid of two 0.01 $^{\circ}$C mercury-in-glass thermometers. The direct current levels used in these measurements were measured with the 0.3 per cent Gossen moving coil Ammeter, used in the measurements of previous chapters. Alternating current levels were measured with the aid of a $\pm$ 0.5 per cent of F.S.D. Dynamometer Ammeter, borrowed from the National Standards Laboratory Power Frequency section. This was necessary owing to the absence of the Siemens Halske moving iron meter used previously while it underwent periodic calibration. Account was taken of the alternating current passing around the battery circuit with the aid of the A.C. measuring circuit described in section (4.3). Alternating loading currents were obtained from the Savage Power Amplifier through a high-pass filter. The supply frequency used was 40 c/s and the galvanometer series inductor was tuned accordingly.

The preliminary measurements on the Bridge circuit and Standard for temperature and load coefficients were carried out while using a Cambridge 0.1 ohm Standard L-407065 as a substitute Unknown. This was necessary to enable a high relative accuracy to be obtained for these measurements, since the thermal instability present while using the 0.1 ohm constantan resistor as the Unknown reduced measurement accuracy somewhat. If the Bridge circuit corrections were known to high accuracy, any difference effect noticed in tests on the special resistor would be a real effect. No-load and full load current levels in these tests were taken as 2.0 amps and 5.0 amps respectively, in order to correspond to the measurements to be made on the special resistor.

The load coefficient measurements made while using the substitute Unknown L-407065, required several modifications to the Bridge circuit of Fig. L1. These consisted of only simple rearrangements of the alternating loading current supply and capacitive kick connection circuits. D.C. and A.C. load coefficient measurements were made on the Standard L-407061 and A.C. load coefficient measurements were made on the substitute Unknown L-407065 as a means of providing a cross-check of


CURVES OF CHANGE IN CORRECTION FROM THE 20°C NO-LOAD VALUE [P.P.M.]
VERSUS OIL BATH TEMPERATURE [°C] FOR CIRCUIT COMPONENTS
ASSOCIATED WITH THE INVESTIGATION OF THE LOAD
COEFFICIENT OF THE 0·1 Ω SPECIAL
CONSTANTAN RESISTOR.
ALL CURVES ARE PARABOLAS OF 'BEST FIT'
TO THE MEASURED RESULTS OBTAINED
USING THE METHOD OF
'LEAST SQUARES.'
BRIDGE CIRCUIT * 2·5 WATTS IN L-407061 AC & DC LOADING
[0·4 & 2·5 WATTS RESPECTIVELY IN L-407065]
BRIDGE CIRCUIT * NO-LOAD
[0·4 WATTS IN L-407061 & L-407065]
0·1 Ω SPECIAL CONSTANTAN RESISTOR 2·5 WATTS AC & DC LOADING
[0·4 & 2·5 WATTS RESPECTIVELY IN L-407061]
Curve plotted from results of DC loading.
0·1 Ω SPECIAL CONSTANTAN RESISTOR NO-LOAD
[0·4 WATTS IN L-407061 & UNKNOWN]
Lines showing limits of spread of DC resistance
corrections while under AC & DC load.
L-407065 NO-LOAD
[0·4 WATTS IN L-407061 & L-407065]
L-407065 2·5 WATTS AC LOADING
[0·4 WATTS IN L-407061]
—LEGEND—
0·1 Ω Special constantan resistor. NO-LOAD
2·5 watts DC
2·5 watts AC
BRIDGE CIRCUIT.* NO-LOAD
2·5 watts DC in L-407061.
2·5 watts AC in L-407061.
0·1 Ω Camdridge manganin resistor. L-407065 NO-LOAD
2·5 watts AC
* BRIDGE CIRCUIT CONSISTED OF A.B.P.&Q. ARMS
PLUS L-407061 USED AS STANDARD.
CHANGE IN CORRECTION FROM 20°C NO-LOAD VALUE [PARTS PER MILLION]
160
140
120
100
80
60
40
20
0
-20
20
21
22
23
24
25
26
OIL BATH TEMPERATURE [°C]
Fig. LIII

results. The results obtained after calculation of the temperature coefficients for both the Bridge circuit and the substitute Unknown had been made using the computer program TEMPCURV, and the results of calculation made to obtain the load coefficients of both the Standard and the substitute Unknown at 5 amps for both A.C. and D.C. loading using the computer program LOADCURV, are presented in Table (9.1). These values were used where necessary to apply corrections to the measured results obtained from tests on the special 0.1 ohm resistor's load and temperature coefficient. The results of the temperature coefficient measurements made at constant load, (no-load and 5 amps) on the 0.1 ohm constantan resistor, are presented in graphical form in the curves of Fig. L111.

The methods used in obtaining the two sets of results necessary for calculation using TEMPCURV were basically similar to those described in similar tests made in previous chapters. Similarly, load coefficient measurements were performed as in tests described previously. Care was taken to ensure that D.C. and A.C. load coefficient measurements were performed as closely together in time as was practically possible, so that any instability encountered in the resistance value of the bridge resistors, or any modification of the cooling effects with time produced by the Unknown bath stirrer used, would have as little effect as possible on the results obtained for the A.C. and D.C. load coefficients.

(9.3)

Supply Current Measurement.

Inspection of the resulting curve of change in correction versus oil bath temperature for the Bridge circuit at 2.5 watts power dissipation in L-407061, presented in Fig. L111, will show that the load coefficient of the Bridge circuit with 2.5 watts power dissipation in the Standard is of the order of 100 parts per million at normal operating oil temperatures. Hence, the error in the resistance change, due to the measurement accuracy of the A.C. and D.C. loading current ammeters used, (0.3 per cent Gossen moving coil and Borrowed 0.5 per cent Dynamometer meter) will be ± 1 part per million at 2.5 watts A.C., and ± 0.6 parts per million at 2.5 watts D.C.

The percentage error produced in the resistance change of L-407065 due to uncertainties in the indication of the A.C. ammeter used to monitor the alternating loading current at 2.5 watts power dissipation, will be ± 1 per cent. Hence with a load coefficient of the order of - 20 parts per million in L-407065 at normal operating oil temperatures, the range of probable error in this load coefficient value from the above source, will be ± 0.2 parts per million.

The resistance change obtained in the 0.1 ohm special constantan resistor at 2.5 watts power dissipation are below 20 parts per million for the range of oil bath temperatures used in obtaining the curve of Fig. L111. Hence the range of probable error encountered in the resistance changes obtained for the 0.1 ohm special constantan resistor due to the errors of indication of the A.C. and D.C. loading current ammeters used, will be below ± 0.2 parts per million.

The accuracy with which the direct measuring current is set with the Gossen ± 0.3 per cent D.C. Ammeter, while performing A.C. load coefficient measurements on the Bridge circuit and the various resistors tested, will have a negligible effect on the measured values obtained for the resistance change under A.C. load, at 2.5 watts power dissipation. Similarly, the accuracy with which the alternating current passing around the battery circuit may be measured and allowed for, while performing A.C. load coefficient measurements, will produce a negligible effect on the uncertainty with which the value of the resistance changes of the Bridge circuit and the various resistors tested under load are known.

(9.4)

Presentation of Results.

(9.4.1)

Associated Temperature and Load Coefficient Measurements.

Measurement of Temperature and Load coefficients of the 0.1 ohm Cambridge manganin standard resistor L-407065 and of the Bridge circuit used in the measurements made on the 0.1 ohm special constantan resistor, were made with an accuracy well within that required. The values of α_{20} and β for both no-load and 2.5 watts power dissipation and the load coefficient at 20 $^{\circ}$C for 2.5 watts power dissipation in L-407065 and the Bridge circuit, are presented in Table (9.1) below.

It may be seen by inspection of Fig. L111 that the agreement between the values of the change in the correction to the Bridge circuit, obtained under A.C. and D.C. loading is within about $\pm$ 2 parts per million. This agreement is excellent, when it is realised that the range of probable error produced by the uncertainty with which the alternating and direct loading currents were set, is sufficient to account for this spread. The accuracy of the D.C. load coefficient values obtained for the Bridge circuit depends upon the accuracy with which the resistance change at 20 $^{\circ}$C and the temperature coefficient of L-407065 under load were obtained. The temperature coefficient α_{20} of a standard resistor, usually changes while under significant load. Hence if a significant power level is dissipated in the Unknown while the Bridge circuit is being loaded, then the modified value of α_{20} at the particular power dissipation level must be known, in order to correctly calculate resistance corrections for the Unknown with changing Unknown oil bath temperature.

TABLE (9.1)

Results of Measurements Made in Association With the A.C. and D.C. Load Coefficient Investigation of the 0.1 ohm Special Constantan Resistor.

	No-Load (0.4 watts)		2.5 watts		
	α_{20} pp 10^7 / °C	β pp 10^7/ °/ °C	$LC_{20°C}$ pp 10^7	α_{20} pp 10^7 / °C	β pp 10^7/ °/ °C
Cambridge 0.1 ohm L-407065	126	- 4.0	- 118	76	- 2.3
Bridge Circuit using Cambridge 0.1 ohm Standard L-407061	225	- 0.2	961	169	- 2.8

These values were computed from the curves of 'best fit' to the measured results obtained using the method of 'Least Squares'.

(9.4.2)

Temperature and Load Coefficient Measurements on the 0.1 ohm Special Constantan Resistor.

The results of temperature and load coefficients made on the 0.1 ohm special constantan resistor are shown in Fig. L111. It was mentioned in section (9.2) above that the presence of thermal effects made the stability of the galvanometer zero extremely poor; as a result of this the integration time of the detection system was increased and the bridge unbalance sensitivity decreased, in order to produce measured results with any meaning. Results showed that with full direct current loading at 2.5

watts, balance could be made to within about $\pm$ 1 in 10^5 of the Unknown after about $\frac{3}{4}$ of a minute integration time, while with alternating current loading and 2 amps of direct measuring current, balance could be made to within about $\pm$ 2 in 10^5. Hence the range of the spread of alternating and direct current load coefficient values at 2.5 watts in the 0.1 ohm special constantan resistor from this source was equivalent to 3 in 10^5 of the Unknown.

Inspection of Fig. L111 showed that all alternating and direct current load coefficient values fell within a range of about 2 in 10^5.

The balance uncertainty while making no-load (0.4 watt) measurements on the special 0.1 ohm resistor was much less, being equivalent to only about $\pm$ 2 parts per million of the Unknown.

(9.5)

Assessment of the Results Obtained.

(9.5.1)

Temperature and Load Coefficients of L-407065.

Inspection of the results of Table (9.1) shows that the value of the load coefficient of the Cambridge 0.1 ohm manganin standard resistor L-407065 is negative at 20 °C, while the slope of the curve of resistance change with temperature at 2.5 watts power dissipation is positive, although less than the slope of the curve of resistance change with temperature at no-load (0.4 watts). Assessing these points in the light of the experience gained on the high dissipation Evanohm resistor R_5 discussed in Chapter 5 and because of the presence of large lengths of copper in the resistance element, between the manganin resistance and the potential terminal junctions, it appears that the resistance change of the manganin resistance element between 0.4 and 2.5 watts is in fact negative. Hence the slope of the curve of resistance change with temperature of the manganin resistance element at 20 °C will probably be negative at this power dissipation level.

The value of β obtained at no-load and 2.5 watts should in theory be equal. The fact that the two measured values obtained differ, is not serious when the order of the values obtained for α_{20} in both cases is taken into account.

The values obtained for the load coefficient and the temperature coefficients of the Cambridge 0.1 ohm resistor L-407065 at 2.5 watts power dissipation level are used in calculating the value of the direct current load coefficient and temperature coefficients of the Bridge circuit. Hence the agreement obtained between the alternating and direct current load coefficient values of the Bridge circuit, is in fact an indication of the accuracy with which measurements have been made on L-407065 under A.C. loading conditions.

(9.5.2)

Temperature and Load Coefficients of the Bridge Circuit.

The Bridge circuit correction consisted of the contribution of three circuit components.

$$\delta_{\text{Bridge}} = \delta_A - \delta_B + \delta_S \tag{9.13}$$

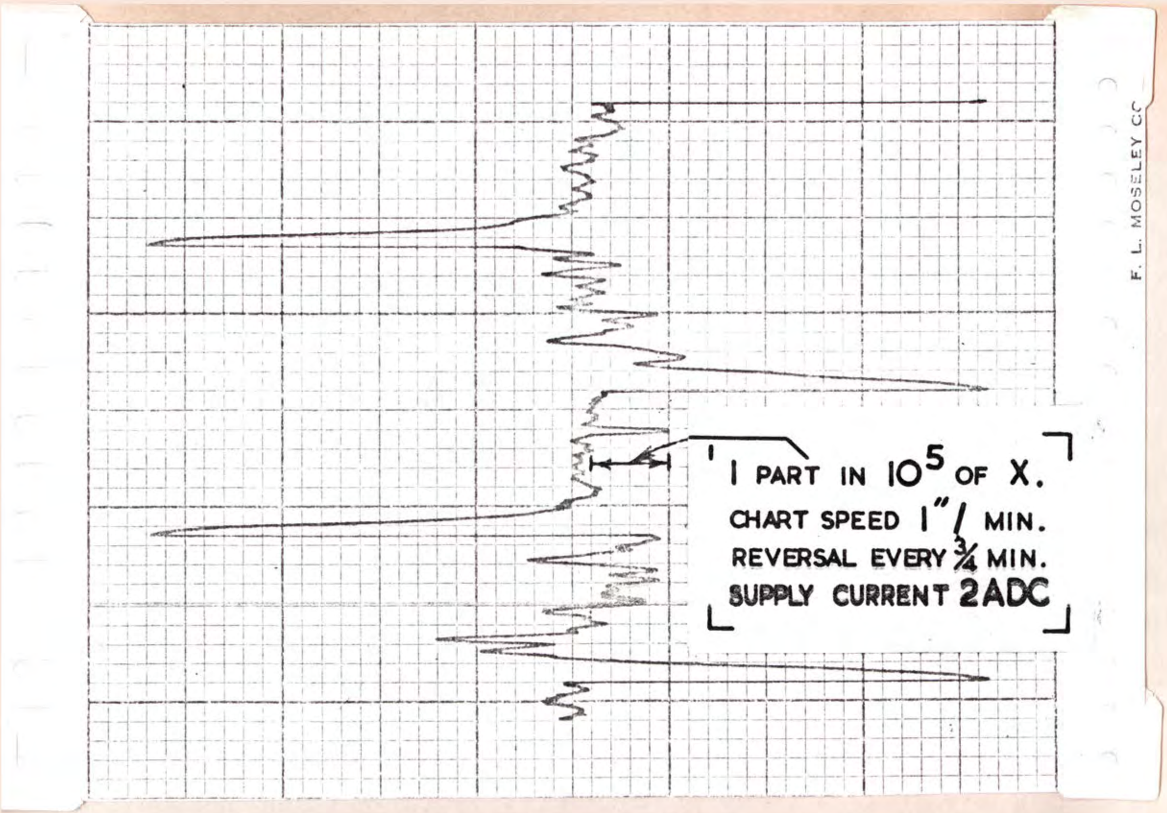
The contribution of the A and B arms were virtually independent of the level of the loading current passing through the Standard and dependent only upon oil bath temperature changes. Hence any change in the curves of resistance change with oil bath temperature for the Bridge circuit between the 0.4 and 2.5 watts power dissipation levels in the Standard L-407061, were due only to changes in the resistance value of the Standard with load. This deduction leads to the conclusion that the slope of the resistance versus temperature curve for the manganin resistance element of the 0.1 ohm Cambridge manganin standard L-407061 at 20 °C is positive, since the load coefficient of the Bridge circuit between the 0.4 and 2.5 watts power dissipation level in L-407061 is seen to be positive at 20 $^{\circ}$C in Fig. L111. The effect of the copper in L-407061 would probably be of the same order as experienced in L-407065. Hence the resistance-temperature properties of the two samples of manganin used in the construction of the two standards L-407061 and L-407065 must be quite different.

The values obtained for the load and temperature coefficients of the Bridge circuit with 2.5 watts power dissipated in L-407061 is used in calculating Bridge circuit corrections when testing the 0.1 ohm special constantan resistor under D.C. loading conditions. The accuracy obtained is of course, greater than that required, owing to the thermal instabilities experienced, but the fact that the results of the A.C. and D.C. load coefficient measurements on the Bridge circuit agree so closely, is further evidence that the A.C. loading technique is reasonable. The range of disagreement obtained in these load coefficient measurements may be accounted for completely, by the uncertainty with which the alternating and direct loading currents were set. Hence any further measurement uncertainties, such as bridge balance sensitivity or temperature gradient effects simply add to the range of probable error.

(9.5.3)

Temperature and Load Coefficients of the 0.1 ohm Special Constantan Resistor.

The relationship between the no-load (0.4 watt) and the 2.5 watt curves obtained for the 0.1 ohm special constantan resistor is difficult to explain unless the curvature of the no-load curve shown in Fig. L111 is considered to be slightly in error, it being in fact slightly negative. If this condition is allowed, then the relationship follows the classical pattern for resistors which possess non-uniform temperature distributions along their length while under load, viz the peak of the resistance-temperature curve under load is lower than the peak



Typical Detector Recording
showing the thermal fluctuations present.

Fig LIV.

of the resistance-temperature curve obtained at no-load, (see section (5.5.3)).

The spread of load coefficient values obtained in this case is insufficient to indicate any measurable difference between methods of loading used. By far the major cause of the spread of values obtained was the presence of thermal instability in the galvanometer zero. It may be seen by inspection of Fig. L1V that the thermal fluctuations obtained in the bridge balance with the current flowing in one direction, are greater than those obtained with the current direction reversed. The direction of direct current flow at the time of maximum thermal activity corresponded to the theoretical condition of maximum temperature at the point $x = \ell_2$.

CHAPTER 10

Oil Bath Temperature Gradient Measurements and Low Frequency Equivalent Circuit Determinations of Several of the Standard Resistors Used.

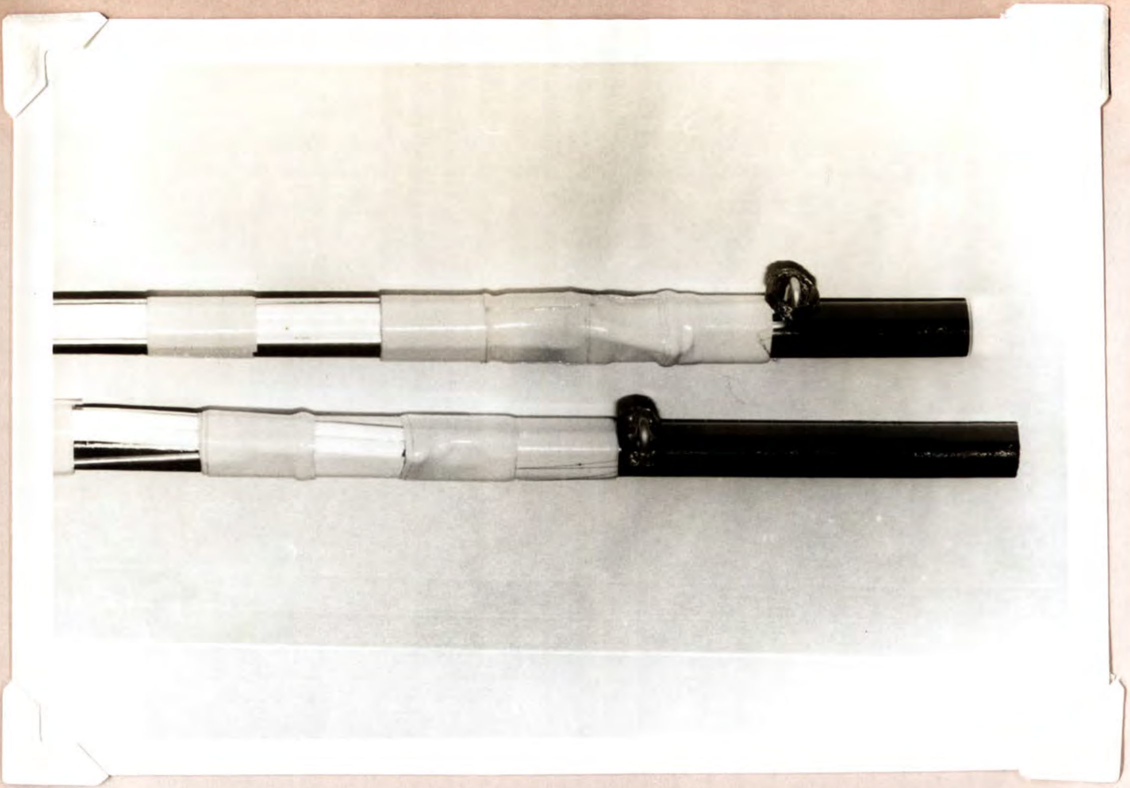
(10.1)

Temperature Gradients.

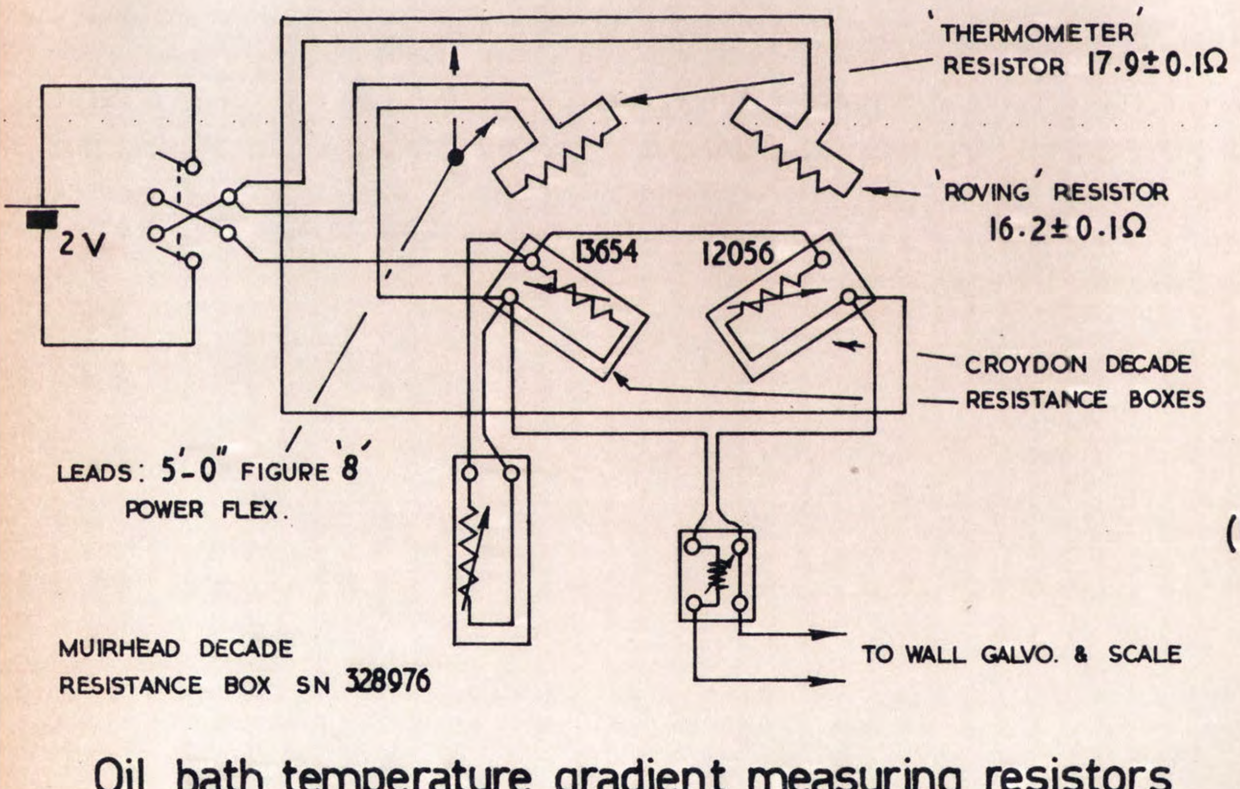
As a means of estimating the possible error contributed by Bridge and Unknown oil bath temperature gradients in the resistance measurements performed, it was decided to determine the amplitude of the temperature gradients existing between the point in the oil baths at which the respective oil temperature measuring thermometer was situated and other arbitrary points in the oil in the neighbourhood of the temperature sensitive Bridge elements. In order to obtain point measurements rather than distributed values, the temperature sensing elements used were made as small as possible in size. The smallness in size contemplated, presented the added advantage of low thermal inertia, hence reasonably fast thermal fluctuation could be detected.

For this investigation, thermocouples were considered to be unsuitable, since the significant order of magnitude of the temperature gradients sought, was 0.01 ^{o}C. The possibility existed of using a thermo-pile, but the difficulty of manufacture was considered to be too great when an alternative method existed.

Two approximately equal resistors were wound from fine copper wire, each wound to form a loosely bunched coil of about $\frac{5}{8}$" outside diameter, with a thickness of about $\frac{1}{8}$". Each was attached loosely to an ebonite rod and connected to form the ratio arms of a Wheatstone Bridge. One of the copper resistors was to be placed in the location of the mercury-in-glass thermometer used for temperature measurement in practice. This provided a system which gave results relative to this



(a)



(b)

Oil bath temperature gradient measuring resistors & Wheatstone Bridge.

Fig LV.

point and so some idea of the range of gradients encountered in practice. A photograph of these resistors and a circuit diagram of the Wheatstone Bridge used is shown in Fig. LV (a) and (b) respectively. Bridge balance to $\pm$ 1 part in 10^6 could be obtained with the adjustment of the Muirhead decade resistance box S/N 328976. In practice the bridge was balanced initially and changes in the magnitude of the temperature gradients found by reading differences in the galvanometer deflections obtained while using D.C. supply reversals. The condition of equal temperature of the 'Thermometer' and 'Roving' resistors was obtained by placing them in close proximity in the oil bath being investigated. All oil baths used were stirred similarly to the method used in practice and the locations of the standard resistors included in them were kept as closely as possible to that existing at the time resistance measurements were made. Thus a system was obtained which represented as closely as possible the temperature distributions existing in practice.

Measurements were made in general with no power dissipation in the standard resistors situated in the oil baths, but one set of measurements was made with 1 watt dissipation in the Cambridge 1 ohm resistor L-259235 situated in its Unknown oil bath.

(10.1.1)

Theory of Operation.

A change of 0.01 ^{o}C change in the temperature of a sample of copper wire at approximately 20 ^{o}C will increase its D.C. resistance by 4 parts in 10^5. The use of a copper resistance ratio in a Wheatstone Bridge circuit has the advantage of it being insensitive to changes in the mean oil bath temperature with time and sensitive only to differences in oil temperature existing between the points of the oil bath at which the ratio resistors are situated. Provided the thermal inertia of the 0.01 ^{o}C mercury-in-glass thermometer is small, it is these temperature gradients which will be the source of some of the uncertainties in the bridge balance conditions obtained in practice. The presence of small fixed temperature gradients between the thermometer and the oil in the vicinity of the bridge resistors is not serious. However if this gradient is variable, it is then that errors are introduced into the measured value of the Unknown because of the undetected changes in the temperature existing around the oil baths used. It is to be expected that the magnitude of the changes experienced in the temperature gradient existing between the thermometer and any given point in an oil bath, will be larger during periods of large oil bath temperature change with time, than during periods of relative temperature stability. Hence in assessing the possible errors obtained during any set of bridge balance conditions, the rate of change of oil temperature with time must be taken into account.

The temperature coefficient α_{20} of all of the manganin resistors incorporated in the Kelvin and Wheatstone Bridges used when making A.C. and D.C. load coefficient measurements were of the order of 6 to 10 ppm/ ^{o}C. Hence an error of 0.01 ^{o}C in the measured temperature of these resistors will have caused an error in the measured value of the bridge balance condition equivalent to 0.6 to 1 part in 10^7 of the Unknown.

The thermal time constant of the thermometer bulb of the 0.01 ^{o}C mercury-in-glass thermometers used for measurement of oil temperatures is important. Measurements were made to determine the thermal time constant of this type of thermometer.

(10.1.2)

Results Obtained.

Analysis of the results obtained for tests on the various oil baths showed that temperature gradients were generally less than 0.1 ^{o}C relative to the position of the thermometer. However a gradient of up to $\frac{1}{4}$ ^{o}C could be obtained by placing the 'Roving' temperature sensing resistor in the centre hole of any of the Cambridge N.P.L. construction resistors used. The long term stability of these gradients with time (up to 1 hour) was found to be within about 0.01 °C for a constant oil bath temperature and within 0.03 °C for an oil bath temperature increase of 0.6 °C in 90 minutes. Short term stability of the thermal gradients with time (several minutes) was within 0.01 °C in both the above cases and the direction of change was found to be random relative to the mean value.

Gradients appeared to be greater in the vertical direction at any location than at points at different locations but at the same relative height in the oil.

The establishment of a power dissipation of 1 watt in the 1 ohm Cambridge standard resistor L-259235 located in its Unknown oil bath did not appear to increase the level of the temperature gradient existing between the oil in its centre hole and the thermometer measuring the oil bath temperature, nor did the level of the short or long-term drifts appear to change.

The thermal time constants obtained for the 0.01 °C mercury-in-glass thermometers used was found to vary between values of 21 and 51 seconds, depending upon the amount of stirring present around the thermometer bulb.

(10.1.3)

Assessment of Results.

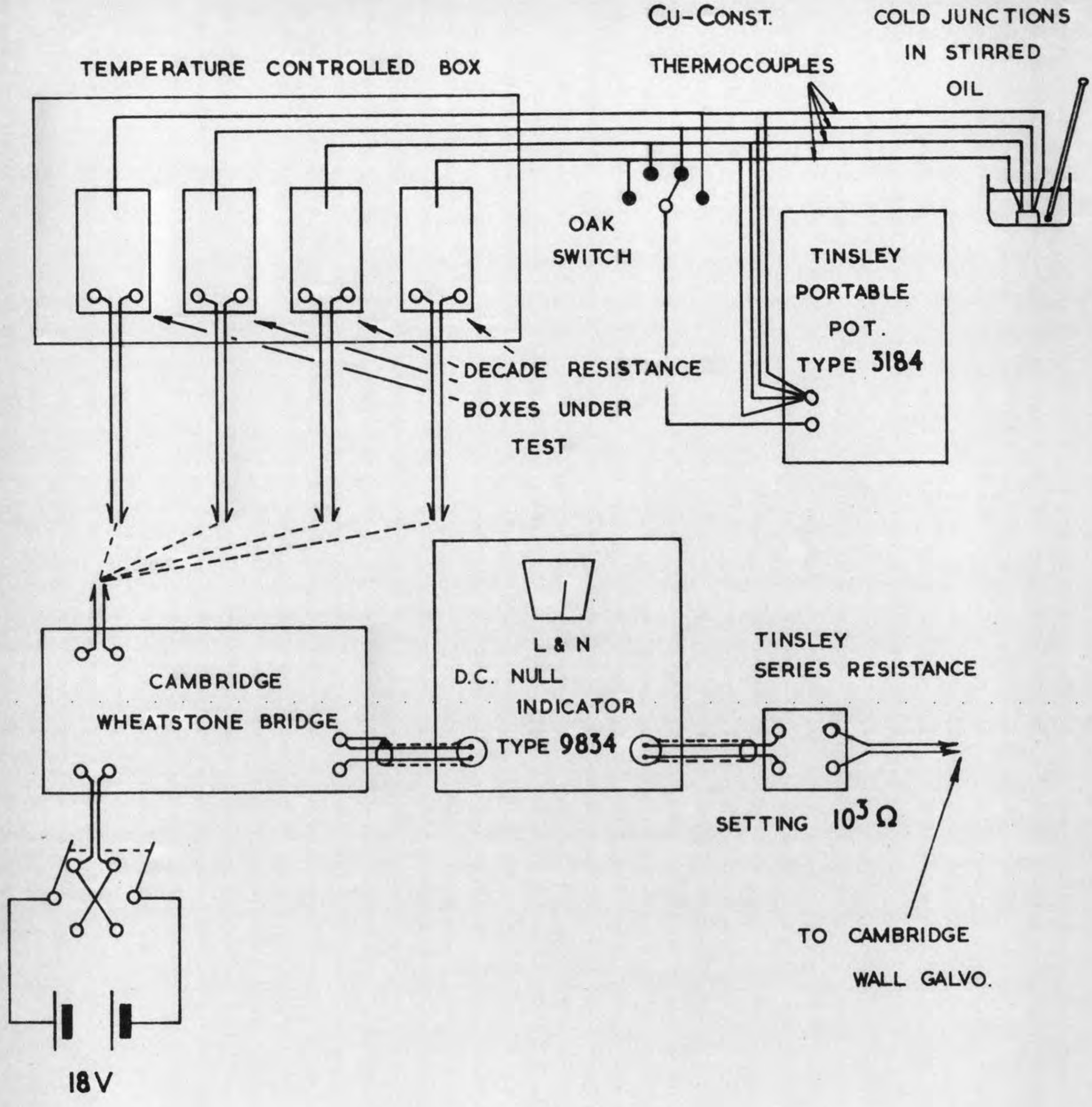
It is difficult to assess just exactly what effect the short or long-term fluctuations in the temperature gradients will have on the accuracy of the results of the measurements made on the standard resistors investigated; the greatest problem is associated with the thermal time constant of each standard resistor (see Chapter 6). If a standard resistor has a thermal time constant of the order of 10 minutes, as was found to exist in the Leeds and Northrup Rosa type resistors, most of the short-term random fluctuations in temperature around it will not be experienced by its resistance element. Small uni-directional drifts in the temperature gradient existing between the standard resistor and the temperature measuring thermometer have to be continuous for a period of time of the order of the standard resistor's time constant, in order to be significant. From the results mentioned above, it appears that most of the changes in the value of the thermal gradients with time are negligible for standard resistors with large thermal time constants.

For open-wire resistors such as R5 the problem is not as simple. The thermal inertia of R5 is such that a time constant of the order of 30 seconds would be expected. This makes the effect of changes in temperature gradient between its cooling oil and the 0.01 $^{\circ}C$ thermometer significant. There is however in this case, a saving feature, in that the temperature dependence of the resistance value of R5 at 20 $^{\circ}C$ is extremely small, α_{20} being approximately 0.7 ppm/ $^{\circ}C$.

The Cambridge heavy current standards S-1161 and L211390 present a special problem also. The resistance element of these resistors is large and hence localised temperature gradients have little effect on their resistance value. The resistance correction presented under any condition of temperature distribution would generally approach that corresponding to the mean temperature of the surface of the resistor.

If the speed of the oil, or cooling medium used, past the thermometer bulb is found to be low while the temperature of the cooling oil is changing rapidly, the thermal delay in the thermometer bulb will cause an error in the measured value obtained for the oil bath temperature. Thermal delay in the thermometer bulb may tend to slightly off-set the thermal delay of the standard resistors contained in the oil bath.

The inclusion of resistance thermometers in close proximity to the resistance element of sealed standard resistors is of definite advantage.



Schematic Diagram
of the system used to determine the temperature coefficient of the top dial of four (4) decade resistance boxes.

Fig LVI.

(10.2)

Thermal Time Constant and Temperature Coefficient Measurements on Decade Resistance Boxes.

The resistance setting of each variable shunt and the resistance value of its corresponding standard resistor, which were employed together to form the variable arms of the bridge networks used during most measurements, were nominally in the ratio of 100 to 1. Resistance ratios of this order were used so as to minimise the results of thermo-electric effects in the decade switches of the resistance shunts and the results of laboratory air temperature changes and their effects on the decade resistance coils.

(10.2.1)

Temperature Coefficient Measurements.

Several decade resistance boxes were set up with copper-constantan thermocouples mounted inside each box to sense its internal temperature. All cold junctions were connected to a 2 oz copper block which was immersed in a beaker of water at known temperature. The decade boxes were placed in a temperature controlled cabinet, and were allowed to stabilise in temperature before the first set of measurements were made two hours later. Resistance and temperature measurements were made on each box in turn, using a Cambridge Wheatstone Bridge L-316998 and a Tinsley Portable Potentiometer Type 3184, S/N 99,313 respectively. A schematic diagram of the system used is shown in Fig. LV1. After this set of readings were taken, the temperature of the temperature controlled cabinet was raised about 2 ^{o}C. The temperature inside each box was allowed to stabilise over night before resistance and temperature measurements were made again. The temperature stability of the Wheatstone Bridge L-316998 was checked against a 10 000 ohm Leeds and Northrup Standard Resistor prior to each set of measurements being performed.

(10.2.2)

Results of Temperature Coefficient Measurements.

The results of the measurements made in Section (10.2.1) are shown in Table (10.1). Four boxes were taken as typical examples and resistance measurements were made to about 1 part per million. The temperature stability of the Wheatstone Bridge L-316998 was found to be excellent; hence corrections to the resistance measurements made on the decade boxes due to temperature effects in the Bridge were negligible.

(10.2.3)

Thermal Time Constant Measurements.

It was found in practice that with correct adjustment of the heating control, the time taken for the temperature controlled cabinet used in the measurements of Section (10.2.1), to rise to within about 5 per cent of its final temperature, was only of the order of 10 minutes (final stabilisation took much longer). Hence for a system with a thermal time constant of the order of hours, an environmental thermal rise time of the order of 10 minutes would virtually be equivalent to being subjected to a step function of temperature. It was decided to perform periodic measurements on the temperature inside each resistance box and so determine the time taken for this temperature to change by 63 per cent of the temperature step in the temperature controlled cabinet.

TABLE (10.1)

Cambridge Wheatstone Bridge L-316998 Ratio	Reading	Cold Junction Temp °C	Est. Temp. of Box	Est. Temp. Coef ppm/°C
Muirhead Constantan Decade Resistance Box S/N 328981				
1 000/10000	(10)0044.5	20.2	20.7	- 16
1 000/10000	(10)0041.2	19.8	22.7	
Croydon Manganin Decade Resistance Box S/N 12058				
1 000/10000	(10)0015.6	20.2	20.7	40
1 000/10000	(10)0023.7	19.8	22.7	
Croydon Manganin Decade Resistance Box S/N 13376				
1 000/1 000	99970.3	20.2	20.7	45
1 000/1 000	99979.3	19.8	22.7	
Croydon Manganin Decade Resistance Box S/N 11486				
1 000/1 000	99970.4	20.2	20.7	44
1 000/1 000	99979.2	19.8	22.7	

(10.2.4)

Results of Thermal Time Constant Measurements.

The results obtained indicated that the time constants of all the Croydon boxes were roughly equal and approximately equal to about 40 minutes. It was found that the temperature did not increase smoothly but in small steps of the order of several hundreths of a degree C in magnitude.

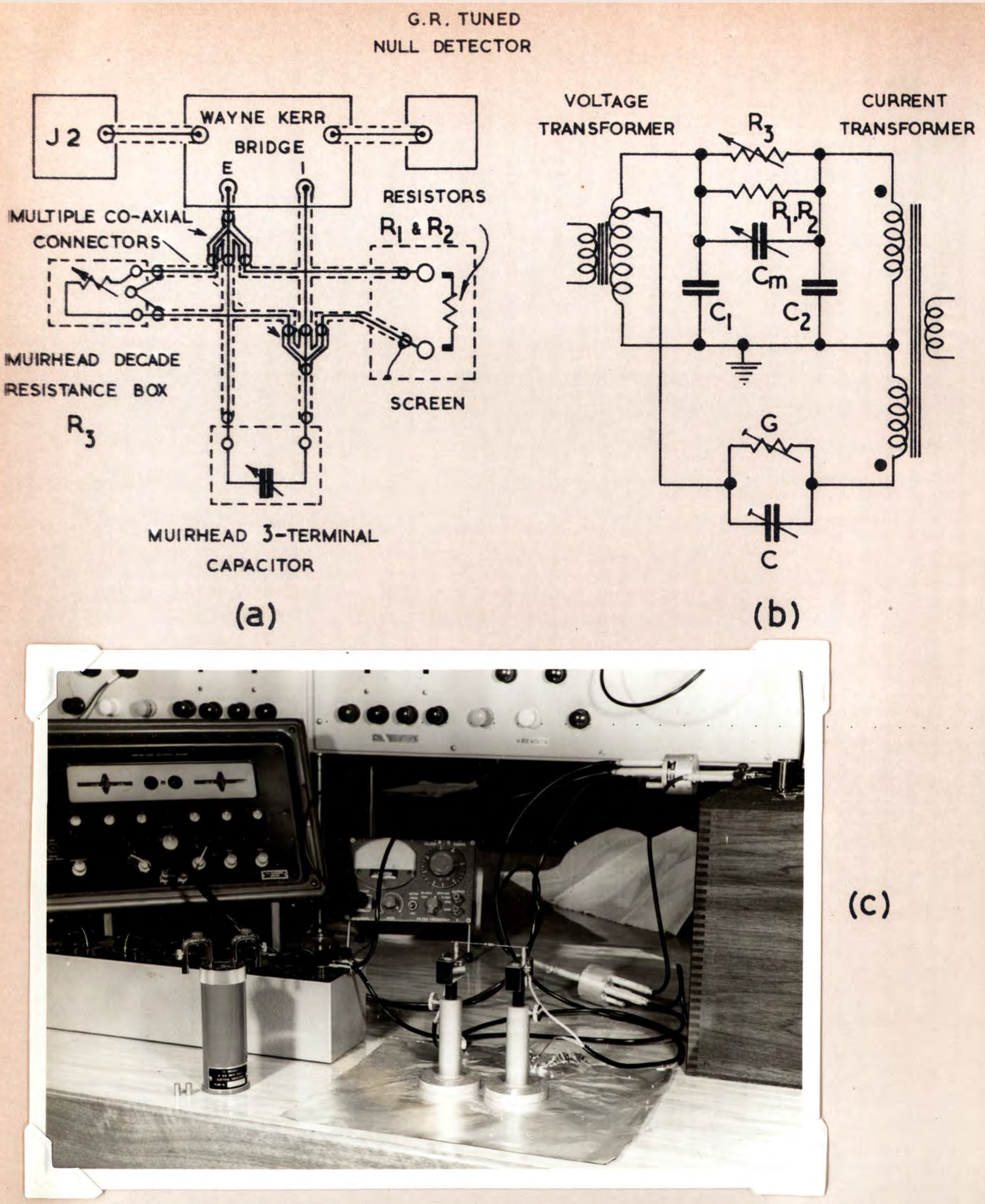
The Muirhead box, on the other hand, was found to have a substantially smaller time constant of the order of 20 minutes. The difference was thought to be due to the materials used in the construction of the boxes, the Croydon boxes being constructed basically from wood with a metal mounting plate on the top, the Muirhead boxes being constructed totally of metal.

(10.2.5)

Assessment of the Results Obtained.

The overall effect of the temperature coefficient of the shunted dials used will in certain cases be balanced out by the inclusion of one on either side of the appropriate bridge used, provided the temperature of these shunts change together and their temperature coefficients are approximately equal. If a combination of a Croydon and a Muirhead shunt is used on either side of the bridge however, as in Chapters 7 and 8, their temperature effects will be additive, because of the combination of their respective positive and negative temperature coefficients as shown in Table (10.1). A long-term change of 1 ^{o}C in air temperature will produce a change in resistance of approximately + 4 parts in 10^7 in any arm containing a Croydon shunt and a change of - 1.6 parts in 10^7 in any arm containing a Muirhead shunt. These effects are small, but since no measure was made of the internal temperature of these decade resistance boxes as bridge balances were made, they will add to the uncertainty in the measured results obtained.

Changes in the decade shunts used in the P and Q arms of any



Schematic Diagram, Circuit Diagram & Photograph of the measuring system used to determine parasitic parameters of 100 & 1 000 Ω standard resistors.

Fig LVII.

Kelvin Bridge whose link resistance is small in relation to X and S arm resistances, will have so small an effect on the bridge balance conditions obtained, that normal temperature effects in them can be neglected.

(10.3)

Low Frequency Equivalent Circuit of the Higher Valued Standard Resistors Used.

A low frequency representation for a typical standard resistor has been produced both by Johnson[30] and by Miller[1]. (see Fig. 111 Chapter 1) The equivalent circuit due to Miller[1] can, in this case, be reduced to that of Johnson[30] without significant loss of accuracy since the components neglected by Johnson have a very small effect on the level of the power dissipated in the resistor and hence on its A.C. load coefficient value.

It was assumed that typical values of parasitic circuit elements present and their effects on the power dissipation level become less as the resistance value decreases. Hence only resistors of 0.1 ohm and above were considered.

(10.3.1)

Wayne Kerr Universal Bridge Measurements Made on 1 000 and 100 ohm Standard Resistors by Substitution Against Suitable Carbon Type Radio Resistors.

The schematic and circuit diagrams of the Wayne Kerr Bridge connections used to perform measurements of the equivalent Conductance and Capacitance of the standard resistor as a function of frequency are shown in Fig. LV11 (a) and (b). A photograph of the measuring system is shown in Fig. LV11 (c).

The capacitance C and the Conductance G were the variable dials of the Wayne Kerr Universal Bridge. These were kept fixed during measurements. R_1 represents the resistor under test and $R_2 \simeq R_1$ is the carbon resistor used to standardise the system, (approximately zero capacitance and inductance[31]). C_m, C_1 and C_2 are the three capacitances of the Muirhead Variable 3-terminal capacitor. R_3 is the Muirhead Box D-825-N S/N 328981 used to trim the difference between the real components of the admittance of the resistors R_1 and R_2.

Care was taken to keep the upper dials of the Muirhead box R_3 at the same readings, so as to keep stray capacitances and inductances substantially constant. This was accomplished with the use of a carbon resistor sufficiently close to nominal that only a small amount of adjustment was required in the Muirhead decade balances between each resistance insertion.

The carbon resistor R_2 was mounted on two brass blocks whose lower surfaces had been amalgamated. Two similar brass blocks amalgamated top and bottom, were used in the mounting of R_1 into circuit so that an effective substitution was made of R_1 against R_2 only, without the added effect of the brass blocks in either case. Two resistance stands were used to mount R_1 and R_2 in reproducable positions as required. Bridge balance was indicated with the aid of a battery driven G.R. Tuned Null Detector Type 1232-A S/N 853.

Measurements were made at three bridge frequencies on four 1 000 ohm and two 100 ohm standard resistors. The bridge supply frequency before and after each series of measurements was checked to provide an accurate knowledge of ω and its change during measurements. It was assumed that the actual resistance values of the standard resistors measured were negligibly different from nominal so that the value of R_S in Johnson's equivalent circuit[30] could be taken as the nominal value. Two other circuit components remained to be found and these values were calculable from the results of two measurements at two values of ω. A third measurement at a third value of ω was made here so as to provide a means of cross-checking results.

The measurements made on the standard resistors using the method described above produced its equivalent parallel conductance and capacitance at the frequency of measurement. Analysis of Johnson's equivalent circuit shows that the relationship between its components R_S, L_S and C_p and the values of G and C obtained from the measurements made at angular frequency ω is given by

$$G = \frac{R_S}{R_S^2 + \omega^2 L_S^2}$$
$$C = C_p - \frac{L_S}{R_S^2 + \omega^2 L_S^2} \qquad (10.1)$$

The resistance value of the carbon resistor used in the substitution measurements was not accurately known. Neither was its resistance value known to be constant for any length of time and thus the conductance balance obtained was of little use. The capacitance balance however, provided an equation in two unknowns, C_p and L_S, for each value of ω.

The similtaneous solution of the two equations obtained at two different frequencies ω_1 and ω_2 in terms of C_1 and C_2 and R_S is given by

$$L_S = \sqrt[3]{\frac{R_S^4 (C_2 - C_1)}{(\omega_2 + \omega_1)(\omega_2 - \omega_1)}}$$
$$\text{and } C_p = C_1 + \sqrt[3]{\frac{C_2 - C_1}{R_S^2(\omega_2 + \omega_1)(\omega_2 - \omega_1)}} \qquad (10.2)$$

provided $R_S \gg \omega_1 L_S,\ \omega_2 L_S$

(10.3.2)

Results of Measurements Made.

The results of calculations made on the measured results obtained for the four 1 000 ohm and two 100 ohm standard resistors tested are shown in Table (10.2).

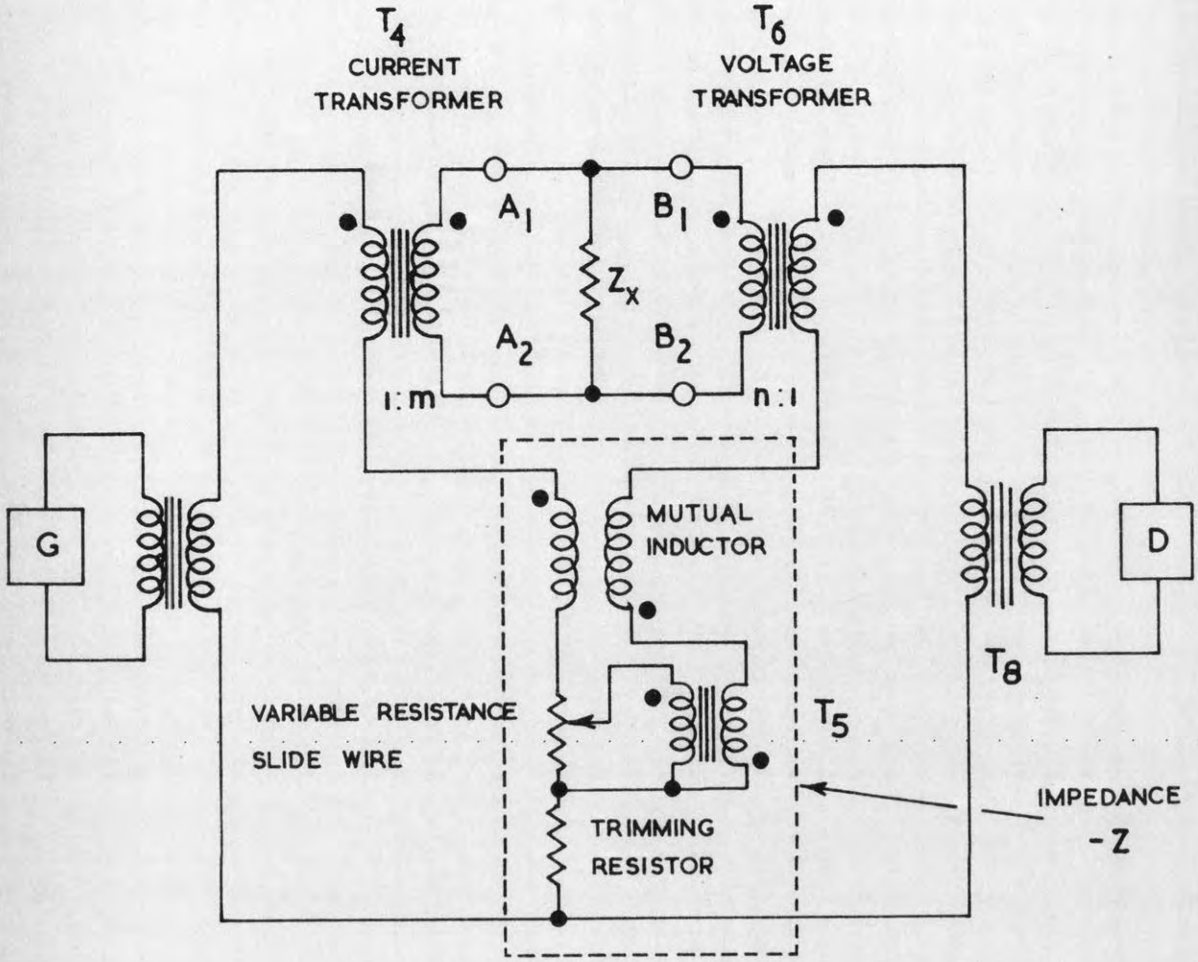
TABLE (10.2)

Calculated Values of Circuit Elements L_S and C_p

Resistor	R_S ohms	L_S (mH)	C_p (pF)
L and N S/N 1642612	1000.0	< 1.5	612 ± 5
L and N S/N 1642613	1000.0	< 1.5	622 ± 5
L and N S/N 1642615	1000.0	< 1.5	620 ± 5
B.H. No. 1 1962	1000.0	< 1.5	411 ± 5
L and N S/N 1646260	100.00	< 1.5	130 ± 5
L and N S/N 1646270	100.00	< 1.5	158 ± 5

The change in the capacitance between balances was obtained by applying the calibration of the Muirhead 3-terminal capacitor mentioned above, using the necessary interpolation required to take account of the fine subdivisions. The uncertainties in the values for C_p in Table (10.2) were derived with reference to the range of values obtained for the capacitance balance at any frequency with the carbon resistors in circuit.

Each inductance value calculated was within the range of the possible error produced by the uncertainty in reading the Muirhead capacitor, hence a maximum value only could be obtained in each case. The value of the capacitance C_p in each case could be obtained directly from the measured results without reference to equations (10.2), since the changes in the measured capacitance values with frequency for each standard



Cintel

Self & Mutual Inductance Bridge measuring system.

Fig LVIII.

were quite small and within the range of the possible error.

The calculated value of the series inductance of the carbon resistors used as substitution standards was 0.11 uH[19] (4 inches of 0.048 inch diameter wire approximately), using the assumption that the resistor was of uniform diameter throughout its length. The broader resistive portion in the centre of the resistor reduces the inductance over this length, hence the inductance value quoted above will be an upper limiting value, being possibly several per cent high.

(10.3.3)

Cintel Self and Mutual Inductance Bridge Measurements on 1 ohm and 0.1 ohm Standard Resistor made by Substitution Against a large Short Circuiting Brass Rod.

Commercially available resistance standards above about 100 ohms, generally appear capacitive in their behaviour with changing frequency, while resistance standards below 100 ohms appear inductive.[6] For this reason the parallel equivalent circuit approximation used in section (10.3.1) to represent 1 000 and 100 ohm resistance standards, was replaced by a series relationship. There were five standard resistors tested by this method in all, two 1 ohm Cambridge N.P.L. construction manganin standard resistors L-259235 and L-259261 and three 0.1 ohm Cambridge N.P.L. construction manganin standard resistors L-248968, L-407061 and L-407065.

The simplified circuit diagram[32] of the Cintel Self and Mutual Inductance Bridge used to perform measurements of the equivalent resistance and inductance of the standard resistor as a function of frequency is shown in Fig. LV111.

The principle of operation of the bridge was such that a high impedance generator G supplied a series circuit consisting of the unknown impedance Z_x and a negative impedance - Z. The value of - Z was adjusted such that the voltage appearing across the combination of Z_x and - Z in series was reduced to zero, as indicated by a null detector shown as D in Fig. LV111.

Connection to the Unknown impedance was made with two sets of leads corresponding to current and potential connections. The two current leads were twisted together throughout their length as were the two potential leads, in order to eliminate errors produced by mutual inductive coupling between the current and potential leads. The effect of mutual inductive pick-up between the current and potential circuits of the Cintel Bridge will be constant with range setting, since it will be proportional to the current passing through the Unknown and not the corresponding voltage drop produced across it. The current through the Unknown will in turn be constant with range setting since the bridge supply G is of high output impedance.

The bridge was standardised against a copper shorting link 0.501 inches in diameter and $6\frac{1}{4}$ inches in length which was connected between the two resistance stands used for mounting the Unknown. Care was taken not to alter the placement of the resistance stands or their connecting current and potential leads between inclusion of the shorting link and the Unknown into circuit. Calculation of the inductance[19] of the shorting link using equation (4.2) gave a value of approximately 0.05 uH.

Measurements were made on the supply frequency to provide an accurate knowledge of ω. It was assumed here, as in section (10.3.1), that the nominal resistance values of the standard resistors measured could be used as the value of R_S in Johnson's equivalent circuit[30] without significant error. The values of L_S and C_p to be found were calculable from the results obtained from two measurements made at two values of ω. Analysis of Johnson's equivalent circuit[30] shows that the relationship between its components R_S, L_S and C_p and the values of R and L obtained from the measurements made at angular frequency ω is given by

$$R = \frac{R_S}{(1 - \omega^2 L_S C_p)^2 + \omega^2 R_S{}^2 C_p{}^2} \qquad (10.3a)$$

$$L = \frac{L_S(1 - \omega^2 L_S C_p) - R_S^2 C_p}{(1 - \omega^2 L_S C_p)^2 + \omega^2 R_S^2 C_p^2} \qquad (10.3b)$$

If R does not alter significantly between balances at ω_1 and ω_2, then the denominator of R must be virtually constant with change in ω and equal to unity. Thus it can be shown that

$$L_S \simeq \frac{\omega_1^2 L_2 - \omega_2^2 L_1}{\omega_1^2 - \omega_2^2}$$

$$C_p \simeq \frac{(L_2 - L_1)(\omega_1^2 - \omega_2^2)}{(\omega_1^2 L_2 - \omega_2^2 L_1)^2} \qquad (10.4)$$

provided $L_S \gg R_S^2 C_p$

(10.3.4)

Results of Measurements Made.

The calculated values of L_S and C_p obtained from the measured results using equations (10.4) are presented in Table (10.3). The values of C_p for both the 1 ohm and 0.1 ohm resistors were found to be very small but roughly of the same order of magnitude.

(10.3.5)

Assessment of Results.

Inspection of the measured values obtained for L_S and C_p in the 1 000 ohm, 100 ohm, 1 ohm and 0.1 ohm standard resistors tested in this section show several trends. As the resistance values of the standards reduced, the value obtained for the shunt capacitance C_p gradually reduced. This reduction of C_p with decrease in R_S was probably a function of the winding construction of the resistance standards of lower resistance value. It is reasonable to assume that the higher the resistance value required for a particular standard resistor of a particular resistance alloy construction, the longer will be the length and the smaller will be the diameter of the resistance wire used in the resistance element. If the resistance wire is wound onto a metal former in order to improve its thermal properties[1], the increased length of element required in the higher resistance values will tend to increase the value of C_p from that existing in a low valued standard. The value of L_S will generally increase with resistance value also, since the effect of long length and small diameter on the self-inductance value of a resistance element, will be to increase it. The use of bifilar winding however, may produce a departure from this trend.

TABLE (10.3)

Calculated Values of L_S and C_p for each of the Five Cambridge N.P.L. Construction Resistors.

Resistor Under Test	L_S (uH)	C_p (pF x 10^{-5})
1 ohm L-259235	0.61 ± 0.12	0 to 3
1 ohm L-259261	0.64 ± 0.12	0 to 3
0.1 ohm L-48968	$0.30_5 \pm 0.01_5$	3.6 ± 1.2
0.1 ohm L-407061	$0.33_4 \pm 0.01_5$	2.7 ± 0.9
0.1 ohm L-407065	$0.27_6 \pm 0.01_5$	3.6 ± 1.2

It was assumed in the measurements made on the 1 000 and 100 ohm standard resistors using the Wayne Kerr Universal Bridge that the only circuit elements of Fig. LV11 (b) to alter while executing bridge balance, when the carbon resistors used as standards were substituted for the respective standard resistors under test, were the resistance R_3 and the capacitance C_m. In fact the values of C_1 and C_2 will have altered also, because of the difference in the geometry of the carbon resistors relative to that of the standard resistors tested. The changes in C_1 and C_2 may be allowed for, but upon inspection of the values of winding resistance and leakage inductance quoted in the Wayne Kerr Universal Bridge Handbook for the current and voltage transformer windings on range 6, it was found that their effect on the measured values obtained for C_p and L_S was small.

The accuracy of the measurements made on the Cambridge 1 ohm Standard resistors L-259235 and L-259261 using the Cintel Bridge was lower than those made on the three 0.1 ohm standard resistors L-248968, L-407061 and L-407065 because of the relative resistance values present; the Cintel Bridge range settings being governed mainly by the value of resistance of the Unknown in the 1 ohm cases.

The calculated value of the self-inductance of the shorting link used as a substitution standard of self inductance in the work performed using the Cintel Bridge was found to be approximately 0.05 uH in section (10.3.3). Its measured value of self-inductance was found to be of this order.

CHAPTER 11

Alternating Loading Current Supply Circuit Considerations

(11.1)

Supplying the necessary loading current required for the performance of load coefficient determinations on the wide range of standard resistor values available, presents an equally wide range of different problems. For load coefficient measurements on higher valued standard resistors, the difficulty is not one of supplying the necessary quantity of loading current, as is the case in measurements on resistors of the order of 1 milliohm, but of supplying an extremely amplitude stable loading current so as to eliminate the low frequency galvanometer fluctuations, which variations in the amplitude of the alternating loading current can produce. (Section 3.2.2). In addition, the shunting effect of the leakage resistance of the loading circuit on the measured value of the Unknown is of greater significance when the Unknown is of high resistance value.

Leakage resistances do not present as great a problem in load coefficient determinations as they do in absolute resistance measurements, since to a first approximation provided they are independent of the applied load, they will cause no error in the measured value of load coefficient obtained. If on the other hand they are low or dependent on the magnitude of the loading current applied, then they will place an ultimate limitation on the measurement accuracy.

This last point places a strict limitation on the performance of the particular capacitor chosen to act as the D.C. blocking capacitor in the alternating current supply circuit. It will be seen in the results shown in section (11.3), that the D.C. leakage current through an electrolytic capacitor with a constant polarising voltage applied to it, varied slightly with time and with the magnitude of the alternating current being passed through it. If these variations are such that the resulting uncertainty in the measured value of the load coefficient obtained is well within the uncertainty required for the measurement, then electrolytic capacitors may be used without embarrassment. This condition may be found to apply during load coefficient measurements on the lower valued standard resistors and meter shunts, where in general the permissible uncertainties in load coefficient measurements are relatively larger and the D.C. supply currents used for the no-load measuring currents are higher than in the corresponding investigations on standard resistors of the order of 1 ohm.

The only capacitors which will satisfy with certainty the strict leakage resistance requirements of the D.C. blocking capacitor found necessary in load coefficient determinations on higher valued resistors (above about 0.1 ohm), are the non-electrolytic types which employ such dielectric materials as oiled paper, mylar, polyester, mica, polystyrene, air etc. Capacitors of this type have disadvantages which are evident during load coefficient measurements on low valued resistance standards, where their large insulation resistances between plates are no longer required and compared to electrolytic capacitors with the same alternating current carrying capability, their capacitance values are lower by factors which may be of the order of 150 or greater.

As an example of the disadvantage offered by the low relative capacitance values available in non-electrolytic capacitors, consider two units with approximately the same A.C. current carrying capacity, which were available commercially. One was a Ducon electrolytic type ENP 871 with a nominal capacitance value of 15 000 uF and a D.C. working voltage rating of 25 volts. The other was a Ducon paper capacitor type 1S1000 with a capacitance value of 100 uF + 20, - 10 per cent, a D.C. working voltage rating of 125 volts and an A.C. working voltage of 75 volts. Both units had an alternating current carrying capacity at room temperature of around 5 amps.[25]

If the load coefficient of a 100 microhm 3 000 amp standard resistor is to be measured at full load using a mains supply frequency of 50 c/s, a D.C. blocking capacitor in the loading current supply circuit consisting of the parallel combination of at least 600 1S1000 units is required in order to pass the necessary loading current without overload. Corresponding to this total series capacitance and loading current frequency, an output voltage of approximately 150 volts will be required from the supply transformer at 3 000 amps. To reduce this enormous supply transformer rating, more capacitance must be added in parallel.

The largest proportion of the supply voltage will be dropped across the series capacitor, there being an alternating voltage drop of only 0.3 volts across the Unknown with possibly a further 0.3 volts drop across the associated wiring and mechanical connections of the loading circuit. Hence if the 1S1000 paper capacitors are contemplated for use as the D.C. blocking capacitor in the loading circuit, in order to dissipate approximately 2 000 watts of real power at 50 c/s, a supply transformer capable of delivering nearly 0.5 MVA is required.

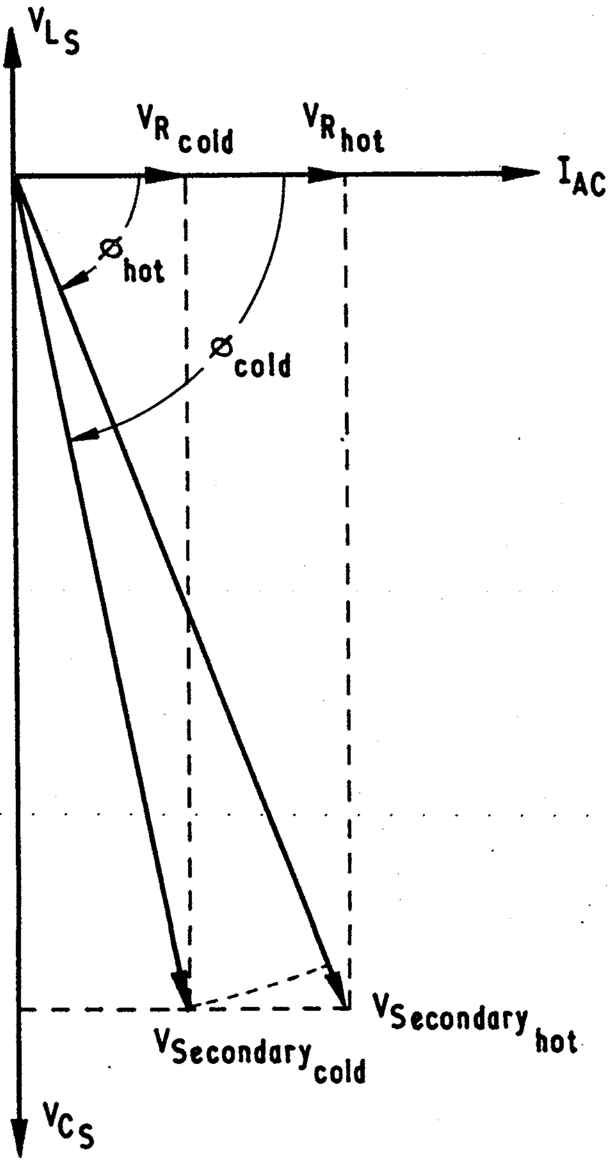
There is the possibility of reducing the series reactance of the supply circuit with the addition of a series inductor, obtaining near-resonance conditions at mains frequency. With a blocking capacitance value of 60 000 uF, the series inductance required for series resonance at 50 c/s is 1/6 of a millihenry. Inductance values of this order may be obtained using air-cored inductors and corresponding values of circuit Q around 15 or higher might be expected at 50 c/s, thus reducing the output voltage required from the supply transformer to about 10 volts.

The problem of holding the loading current constant with time once it has been set at a predetermined level can be difficult in circuits of high Q value. The Q value will decrease as the electrical resistivity increases when the copper in the series inductor, circuit wiring and transformer secondary increases in temperature due to its I^2R losses.

When using the Ducon electrolytic capacitor type ENP871 on the other hand, and applying a current rating of 5 amps A.C. at room temperature, approximately 600 will be needed also, in order to carry the necessary 3 000 amps of alternating loading current. The use of higher loading current frequencies in either of these cases will have little advantage since the current rating of the capacitors used in the D.C. blocking capacitor stipulate the number of capacitors required. The saving produced in the loading current supply transformer at higher frequencies will be off-set by the added expense of purchasing and maintaining a suitable high frequency generator.

It will be shown in section (11.3), that when employing electrolytic capacitors for use as the D.C. blocking capacitor in the alternating loading current supply circuit, the supply circuit configuration considered necessary has two D.C. blocking capacitors in series, so that the D.C. measuring current may be safely reversed when obtaining bridge balance. With the use of two series connected D.C. blocking capacitors in the loading circuit, each will be required to be capable of handling the 3 000 amps and hence each will be required to be made up of at least 600 parallel connected ENP871 units in order to pass this value of loading current without overload. As a result, the total series capacitance of the loading circuit will be approximately 4.5 farads, and the approximate alternating voltage drop across the total capacitance will be about 2.1 volts at 50 c/s.

The total loss resistance of the loading circuit is reduced from its value of 3.5 milliohms used in the series resonant case discussed above, to about 0.2 milliohm. It is made up of 0.1 milliohm from the manganin resistance element of the Unknown, a further 0.1 milliohm from the resistance of the copper interconnecting leads and mechanical junctions, and approximately 20 to 50 microhms from the reflected secondary resistance of the 50 c/s supply transformer. Assuming a reflected leakage reactance on the secondary side of the supply transformer of about



PHASOR DIAGRAM SHOWING THE RELATIVELY SMALL INCREASE REQUIRED IN SECONDARY VOLTAGE LEVEL CORRESPONDING TO A 100% INCREASE IN THE VALUE OF R, AT CONSTANT I_{AC}

Fig. LIX

0.05 milliohm, the transformer open circuited output voltage required to drive the 3 000 amps of loading current around the loading circuit would be

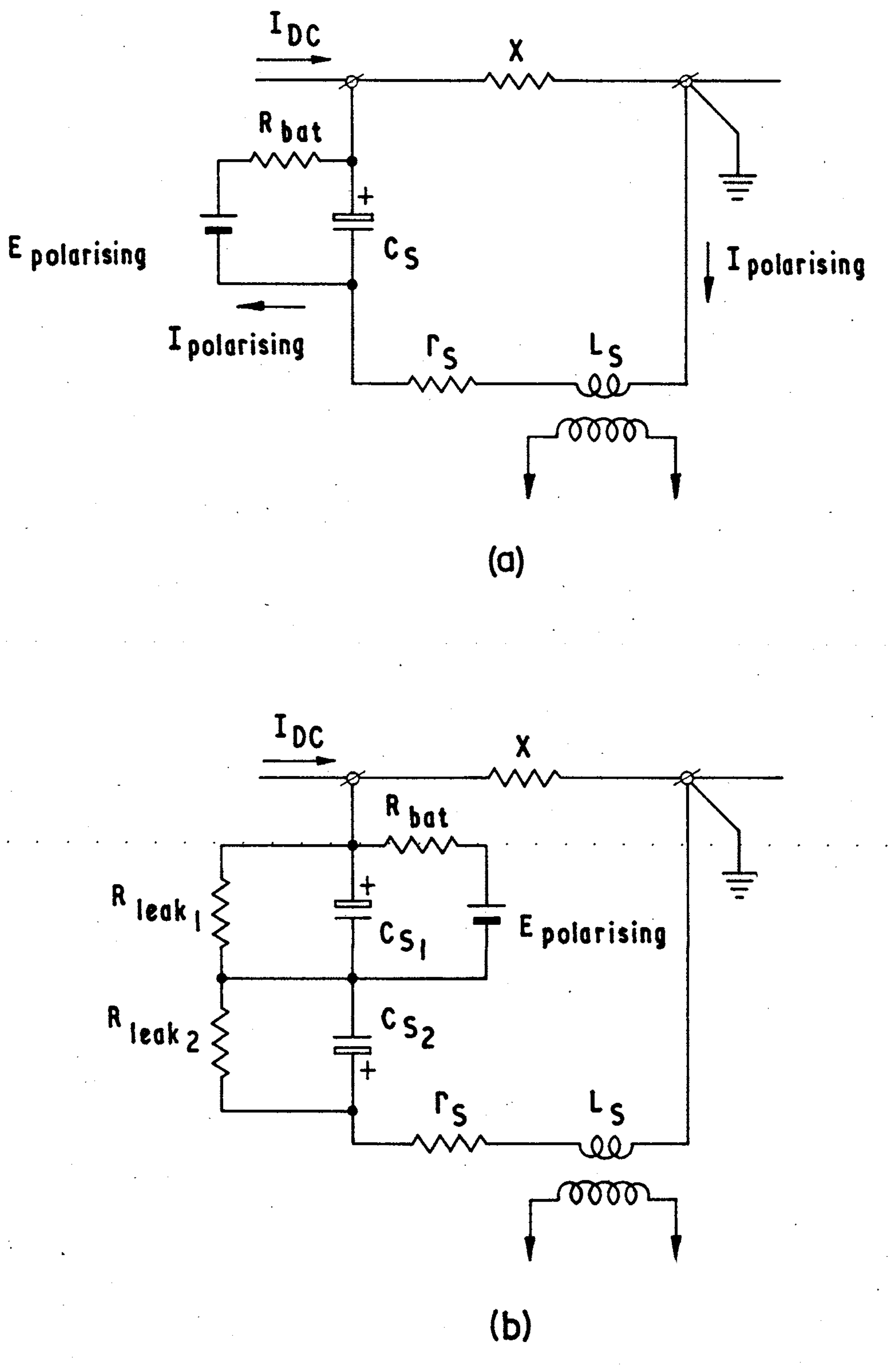
$$E_{o/c} = 3\,000 \;.\; \sqrt{0.2^2 + 0.65^2} \;.\; 10^{-3}$$

$$\simeq 2 \text{ volts} \qquad (11.1)$$

The value of the transformer secondary resistance and reactance used above were obtained using typical specifications for a heavy current low voltage transformer of this type.[27] The pertinent specifications of the transformer considered are: Leakage reactance about 7.5 per cent and efficiency about 98 per cent at unity power factor. The leakage reactance value is variable in manufacture and may be specified almost at will, but 5 to 7.5 per cent is considered to be a typical range of values, allowing in this case, a short circuit current at rated input voltage of between 60 000 and 40 000 amps respectively.[27]

The A.C. power dissipation in the loading circuit under the above conditions would be equal to about 1.8 kilowatts, but only half of this would be dissipated in copper conductors. The fact that any change in the alternating voltage drop across the resistive elements of the loading circuit has little effect on the total alternating current passed, means that the loading current should be stable with time as the copper elements of the circuit rise in temperature. A 20 per cent rise in copper resistance in this case would cause the loading current to fall only 1 per cent. This small dependence is caused by the quadrature relationship between the phasors representing the voltages across the resistive elements and the D.C. blocking capacitor as shown in Fig. L1X.

The value of capacitance required in the capacitive kick correction circuit is dependent directly on the capacitance value used in the D.C. blocking capacitor of the supply circuit.



TWO POSSIBLE ELECTROLYTIC CAPACITOR LOADING CURRENT SUPPLY CIRCUITS.

Fig. LX

Reasonable values of capacitance may be used there by choosing a suitably large value of k in equation (3.20).

(11.2)

The Use of Electrolytic Capacitors in the Alternating Loading Current Supply Circuit.

The D.C. polarising voltage across an electrolytic capacitor should be such that there is a drop in potential between the positive plate of the electrolytic and its negative plate at every instant of time, otherwise reversal of the process which produces the dielectric will result in the development of a possible short circuit. To satisfy this condition in the electrolytic D.C. blocking capacitor of the alternating loading current supply circuit of a typical Kelvin Bridge used to measure load coefficients, some type of polarising voltage supply must be used when large levels of alternating currents are to be passed through the Unknown. The D.C. voltage drop appearing across the Unknown may alter the level of the D.C. polarising voltage appearing across a single series electrolytic capacitor, possibly sending it negative.

Consider the circuit diagram appearing in Fig. LX (a). Neglecting the effect of any D.C. leakage current through the electrolytic D.C. blocking capacitor C_S and of the D.C. resistance of the supply transformer secondary and connecting leads, the polarising voltage appearing across the electrolytic capacitor C_S is given by

$$V'_{c_{DC}} = (I_{polarising} + I_{DC})\,X \qquad (11.2)$$

Consider the shunting effect of the polarising circuit of Fig. LX (a) on the measured resistance value obtained for the Unknown and on the D.C. voltage drop appearing across it. For this circuit to be a suitable practical proposition the value of $I_{polarising}$ must be very much less than I_{DC}. For load coefficient measurements, the values of R_{bat} and $I_{polarising}$ must be such that the change in them with time does not affect the measured value obtained for the load coefficient of the Unknown.

These conditions make the configuration of Fig. LX (a) unsuitable because;

(1) with $I_{polarising}$ very much less than I_{DC}, the polarising voltage $(I_{DC} + I_{polarising})X$ would not provide sufficient D.C. level across the electrolytic capacitor to cover the peak value of the instantaneous alternating voltages found across the Unknown in practical load coefficient determinations, and

(11) if the value of $(I_{DC} + I_{polarising})$ was increased to a level which was sufficient to satisfy the limitation (1), the D.C. shunting effect of the loading circuit on the Unknown would be too large.

A modification of Fig. LX (a) is shown in Fig. LX (b), where a second D.C. blocking capacitor has been added. In Fig. LX (b), neglecting any voltage drop which will occur in the resistance R_{bat} due to the D.C. leakage current being supplied by the battery $E_{polarising}$, the D.C. polarising voltages appearing across both electrolytic capacitors C_{S_1} and C_{S_2} will be given by $E_{polarising}$ and $(E_{polarising} - I_{DC} X)$ respectively. When the bridge D.C. measuring current is reversed in order to obtain bridge balance, the respective polarising voltages become $E_{polarising}$ as before and $(E_{polarising} + I_{DC} X)$.

Under most conditions of loading, the value of R_{bat} would be chosen to be substantially greater than the absolute value of the capacitive reactance of C_{S_1} at the supply frequency, while being sufficiently small enough that the D.C. voltage drop across it caused by the sum of the two leakage currents passing through C_{S_1} and C_{S_2}, produced a very small change in the effective value of $E_{polarising}$ appearing in the expression for the polarising voltage across both capacitors. The resistance R_{bat}

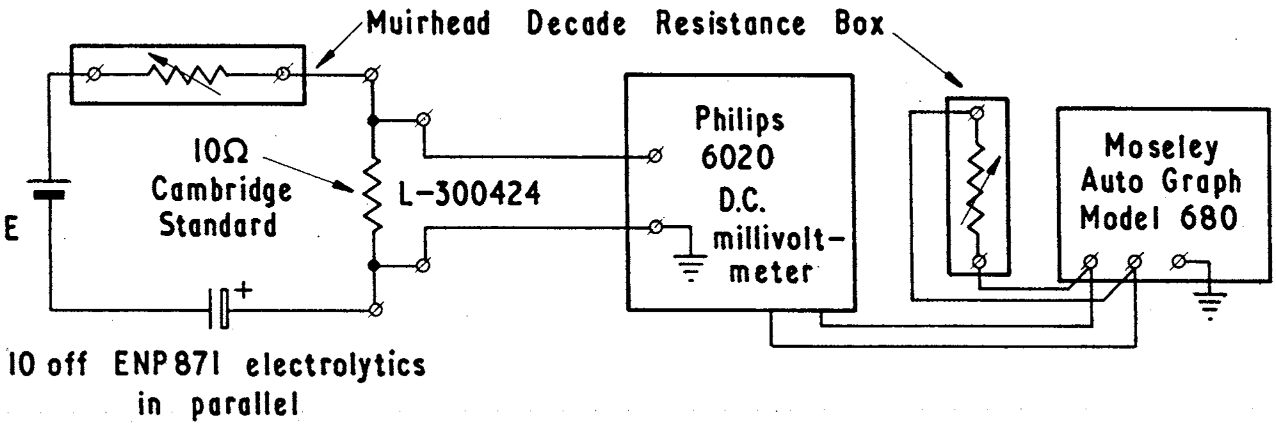
would therefore be substantially smaller than the resistance of the parallel combination of the leakage resistances R_{leak_1} and R_{leak_2}.

The polarising voltage across the capacitor C_{S_2} has two different values which depend upon the direction of flow of the D.C. measuring current in the Kelvin Bridge. In order to ensure that the instantaneous voltages appearing across both capacitors are always of correct polarity, the value of $E_{polarising}$ must generally be very much greater than the value of the voltage I_{DC} X.

The change in the steady-state leakage resistance of an electrolytic capacitor as the D.C. polarising voltage is changed does not occur instantaneously, but with an extremely long time constant. This time constant is much longer than any which can be calculated from consideration of the simple circuit conditions existing at first glance, and hence a more complicated equivalent circuit for the electrolytic must be derived in order to allow for it.

From the results obtained for the change in the value of the leakage current level of a Ducon electrolytic capacitor type ENP871 with change in level of the applied D.C. polarising voltage, the effect of the change in the level of the applied D.C. polarising voltage in a typical practical example, when reversals of the D.C. measuring current are performed to obtain bridge balance, will be small owing to the small value of the voltage I_{DC} X in relation to the required value of $E_{polarising}$. Also the value of the leakage current obtained in the D.C. blocking capacitor is less than the typical no-load current levels used in typical heavy current Unknowns. Thus, the effect of the capacitor's leakage current and its changes with changes in level of applied D.C. polarising voltage and level of alternating current passed through it, may sometimes be neglected altogether.

To cause an error of greater than 1 part in 10^5 in a



ELECTROLYTIC CAPACITOR LEAKAGE CURRENT LEVEL MEASURING CIRCUIT.

Fig. LXI

resistance measurement on a 100 microhm Unknown using a no-load D.C. level of 22 amps, the value of the leakage current through the D.C. blocking capacitor C_{S2} of Fig. LX (b) must be greater than 220 microamps. The D.C. leakage current through 600 parallel connected Ducon 15 000 uF type ENP871 electrolytic capacitors at a polarising voltage of 1.6 volts will be less than 220 microamps but of the same order, as found from measurements made on 10 ENP871 units connected in parallel. From the recordings shown in Fig. LXIV in section (11.2.2) it may be seen that a change in the level of the alternating loading current from zero to 3 000 amps will cause errors in the measured value of resistance change which are of the order of several parts per million. Thus the use of the Ducon ENP871 electrolytic capacitor would probably be suitable for use in load coefficient measurements on the 100 microhm standard resistor where the change in resistance with loading current levels up to 3 000 amps is the important parameter. For standard resistors with lower rated power dissipation levels, the number of electrolytic capacitors required in the loading circuit will be reduced, together with the required D.C. polarising voltage level which will cover the peak alternating voltage drops across the series capacitor. These reductions will tend to reduce the level of leakage current in the D.C. blocking capacitor.

(11.2.1)

Change in D.C. Leakage Current Level with Change in D.C. Polarising Voltage.

In order to find the change in the level of the D.C. leakage current through the 15 000 uF Ducon electrolytic capacitor type ENP871 mentioned in section (11.1) as the D.C. polarising voltage level was changed, the measuring system whose circuit diagram is shown in Fig. LXI was set up. The method used was to monitor the D.C. leakage current through the electrolytic by recording the D.C. voltage drop across a 10 ohm standard resistor, connected in series with the capacitor under test.

The meter of the Philips 6020 D.C. millivoltmeter, which had a 100 uA movement, was disconnected and replaced with a decade resistance box, set to the value of the resistance of the meter movement (1110. ohms). The Moseley Autograph recorder range switch was set to 100 mV F.S.D. and the deflection of the pen of the recorder could be read directly in combination with the setting of the range switch of the 6020.

In the circuit diagram of Fig. LX1, the Muirhead Decade resistance box shown in series with the battery was to provide a known time constant in the capacitor's charging and discharging circuit, which would give reasonable rates of change for the currents flowing in the circuit when the D.C. polarising voltage, represented by the battery E, was altered from one value to another. The capacitor under test consisted of ten parallel connected Ducon ENP871's in order to increase the level of the leakage currents obtained.

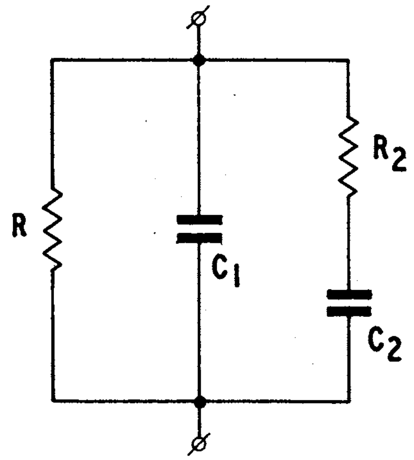
The results obtained for the leakage current level through the 10 parallel connected ENP871 electrolytic capacitors are presented in Table (11.1) for different values of applied D.C. polarising voltage ranging from an initial value of 24 volts to a final value of zero. Each value of polarising voltage was left connected for several hours so that the value of the D.C. leakage current produced had sufficient time to stabilise. The theoretical value of the time constant of the discharge circuit of the capacitor under test was found to be approximately 150 seconds and from the initial rates of change of the D.C. voltage drop across the 10 ohm standard resistor in each case, the actual time constant of the capacitor discharge circuit was found to be very close to the theoretical value.

TABLE (11.1)

Results of Measurements Made on 10 off Parallel Connected 15 000 uF Ducon Type ENP871 Electrolytic Capacitors

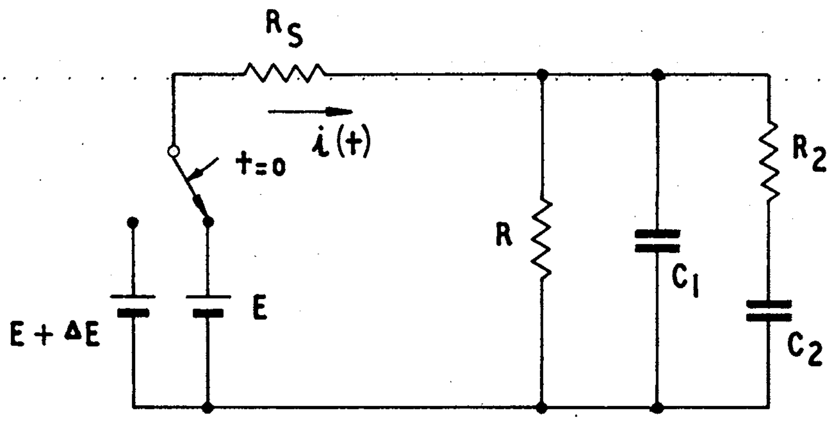
Change in the Direct Polarising Voltage			Direct Current Leakage	
From (Volts)	To (Volts)	Change (Volts)	at 600 seconds (Approx. 4 time constants) (uA)	After 24 hours (uA)
0	24	+ 24	2700	66
24	18	- 6	- 400	30
18	12	- 6	- 400	15
12	6	- 6	- 400	10
6	3	- 3	- 200	6.6
3	0	- 3	- 200	0

The results of Table (11.1) show that the final value of the D.C. leakage current changed non-linearly with the change in the value of the D.C. polarising voltage. Hence the presence of the D.C. leakage current in the electrolytic capacitor may be represented under steady state conditions by a voltage dependent resistor shunting an ideal capacitor. If the applied voltage changes in level from time to time, a more complicated equivalent circuit must be presented. During load coefficient measurements, where D.C. leakage currents or transient capacitive discharge behaviour is likely to be troublesome using the Kelvin Bridge and involving electrolytic capacitors of the order of farads, a very-low-frequency equivalent circuit of these electrolytic capacitors is required when analysing the circuit operation. The presence of the alternating loading



(a)

VERY LOW FREQUENCY EQUIVALENT CIRCUIT OF AN ELECTROLYTIC CAPACITOR. (DUCON TYPE ENP871)



Initial Conditions
$V_{C_1}(0) = V_{C_2}(0) = E$

(b)

DUCON ELECTROLYTIC CAPACITOR TYPE ENP871 IN A CHARGING CIRCUIT.

Fig. LXII

current does not present any difficulty since its frequency is generally fixed, and the transient problem presented by the occurrence of amplitude fluctuations in it may be dealt with as low frequency transient problems.

From the records obtained of the behaviour of the voltage drop across the 10 ohm standard resistor with time, after the initial discharge period of approximately four time constants of the discharge circuit had been completed following a change in the level of the applied D.C. polarising voltage across the electrolytic capacitors, it was found that the subsequent change in the level of the D.C. leakage current through the electrolytic, while approaching its final value, involved time intervals of the order of hours. This effect may be represented in the low frequency equivalent circuit of the electrolytic capacitor by a shunt circuit connected across the ideal capacitor mentioned above and consisting of the series connection of a second ideal capacitor and resistor, as shown in Fig. LX11 (a).

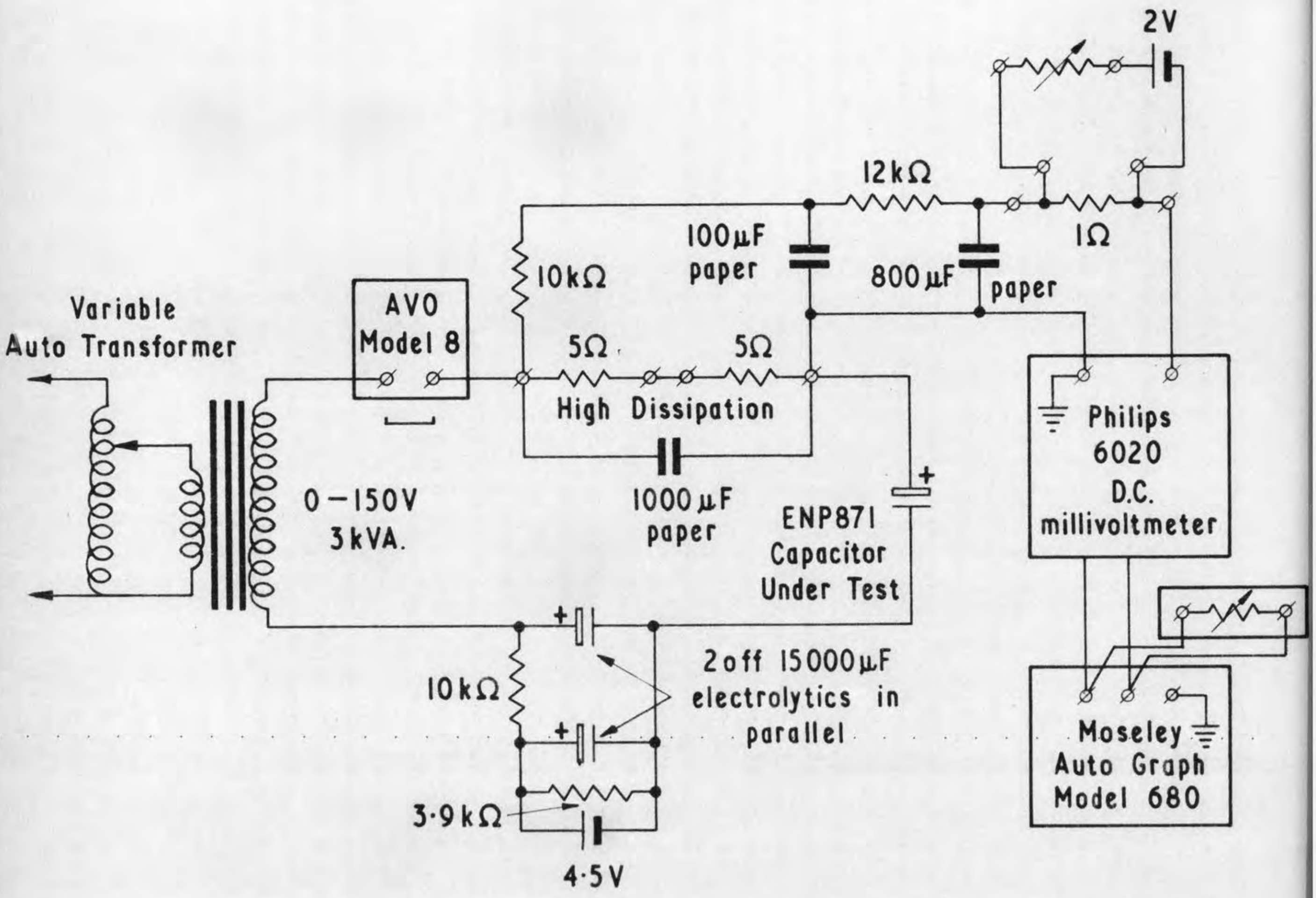
Analysing the behaviour of the equivalent circuit of Fig. LX11 (a), when a voltage E, applied initially at time $t = -\infty$, to the electrolytic capacitor through a series resistance R_S, as represented by the circuit diagram of Fig. LX11 (b), is changed suddenly at time $t = 0^+$ to a value of $(E + \Delta E)$, a current through the resistor R_S is obtained which is given by

$$i(t) \simeq \left[I_o + \frac{\Delta E}{R}\right] + \frac{\Delta E}{R_S} \; e^{-\frac{t}{R_S C_1}} + \frac{\Delta E}{R_2} \; e^{-\frac{t}{R_2 C_2}} \qquad (11.3)$$

$$\text{provided} \quad R_2\, C_2 \gg R_S\, C_1 \quad \text{and} \quad R,\, R_2 \gg R_S$$

$$\text{and where} \quad I_o \simeq \frac{E}{R}$$

In practice, the importance which must be placed on the first and third terms in the expression for i(t) of equation



MEASURING CIRCUIT USED IN THE INVESTIGATION OF THE EFFECT THAT SUPERIMPOSED 50 c/s ALTERNATING CURRENT HAD ON THE LEVEL OF THE D.C. LEAKAGE CURRENT PASSING THROUGH DUCON 15000 μF ELECTROLYTIC CAPACITORS TYPE ENP871.

Fig. LXIII

(11.3) above, will be governed by the value of leakage current through the resistor under test which will affect the resistance measurement accuracy. Under normal conditions however, the period between reversals of the direct measuring current will be small compared to the time constant $R_2 C_2$, hence the third term will appear as a resistance R_2 shunting R.

(11.2.2)

Change of Level of the D.C. Leakage Current with Variation in the Level of the 50 c/s Alternating Current Passed.

Measurements were made of the change in the level of the D.C. leakage current passing through a Ducon 15 000 uF electrolytic capacitor type ENP 871 as a function of the level of the 50 c/s alternating current passing through it. A D.C. polarising voltage of 4.5 volts was applied across the capacitor under test, measurements being made several days after its application so as to eliminate the effect of any changes in the D.C. leakage current due to the long time constant associated with the charging of these electrolytics, as found in measurements made in section (11.2.1) above.

The measuring system, whose circuit diagram is shown in Fig. LX111, was used for these measurements. The method consisted of a means of measuring the changes in the D.C. voltage drop across a resistor which was carrying the leakage current of the electrolytic under test, as different levels of 50 c/s alternating current were passed through this electrolytic. The two high dissipation 5 ohm resistors shown connected in series made up this dropping resistor, while the associated capacitors and R.C. filters were used to bypass or isolate the 50 c/s mains voltages and currents from the direct current circuits. The transformer and AVOmeter shown were used to supply and monitor the alternating current, while maintaining the D.C. continuity. The AVOmeter was short circuited once the alternating current level had been set. There was a 1 ohm standard resistor placed in series with the input circuit of the Philips 6020 across

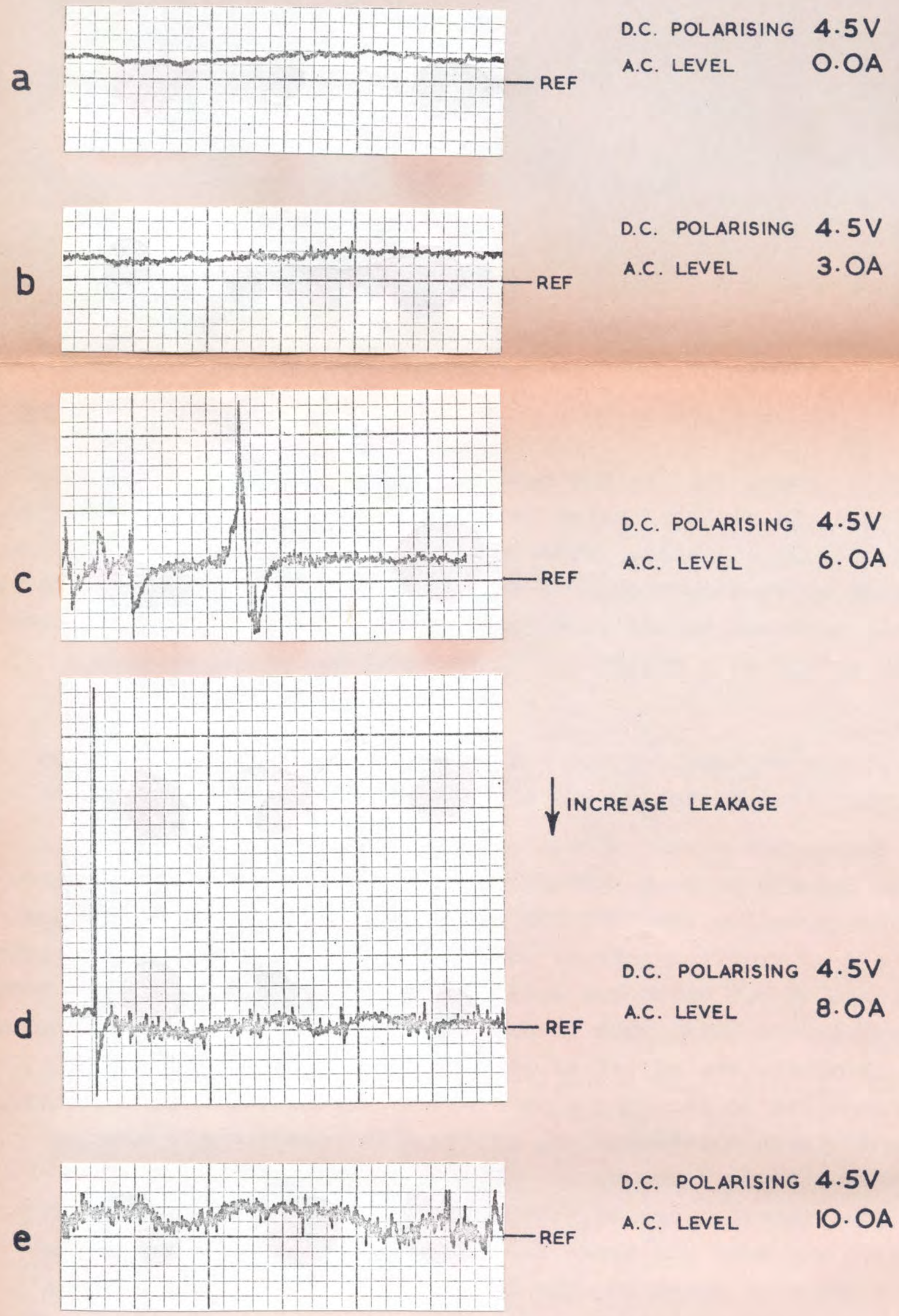


CHART SPEED, 8 INCHES / HOUR
VERTICAL DEFLECTION, 1 INCH = 2 μA CHANGE

RECORDINGS OF THE CHANGE IN THE LEVEL OF THE LEAKAGE CURRENT PASSING THROUGH A DUCON 15 000 μF ELECTROLYTIC CAPACITOR TYPE ENP871 VS TIME AS A FUNCTION OF ALTERNATING CURRENT LEVEL. [FREQUENCY = 50 Hz.]

Fig LXIV.

which a D.C. voltage was produced in order to provide an off-set of the pen of the Moseley recorder, hence enabling records of the changes in the level of the D.C. leakage current through the electrolytic under test to be made about a convenient point on the recorder chart.

The value of the corner frequency of each stage of the low-pass filter shown was chosen to eliminate the 50 c/s signal and its changes in amplitude from the Moseley. Paper capacitors were used in this filter in order to obtain low D.C. polarising effects.

Five records of the level of the D.C. leakage currents passing through the 10 ohm dropping resistor were taken altogether under various conditions of alternating loading current level between zero and 10 amps. These are shown in Fig. LX1V (a) to (e). Inspection shows that an increase in the level of the alternating current passing through the electrolytic under test did not cause the mean level of the D.C. leakage current through the electrolytic to change greatly, but that at the higher levels of alternating current the D.C. leakage current changed in random fashion to new values, subsequently returning towards its previous level.

It has been suggested by Raoult[33] that the dielectric of an electrolytic capacitor consists of O_2 adsorbed beneath a layer of Al_2O_3 rendered conducting by impregnation with electrolyte. The sudden changes in the level of the D.C. leakage current passing through the electrolytic under test appear to be a function of the level of the alternating current passed, although it was found generally, that the longer an alternating current of a certain level was applied, the smaller in value became the amplitude of these sudden steps. The periods involved in these large changes of leakage current level were too long to be caused by transient effects in the mains supply. It was found towards the end of the series of measurements on the electrolytic, that the body temperature of the electrolytic under test had risen above ambient by about 10 ^{o}C.

The largest change of D.C. leakage current shown in Fig. LX1V amounts to approximately 6 uA, whereas the mean level changed by approximately 0.4 uA. The importance of this change in mean leakage current level in practice will depend upon the number of capacitors to be connected in parallel and upon the level of no-load D.C. measuring current used. It is interesting to note that the large transient effects experienced in Fig. LX1V were not present until the rated alternating current level of 5 amps had been exceeded.

(11.3)

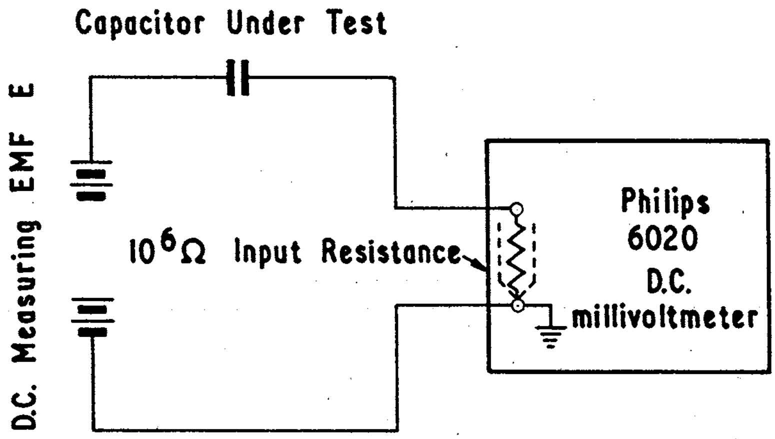
The Use of Non-Electrolytic Capacitors in the Loading Circuit.

The use of non-electrolytic capacitors becomes essential when load coefficient measurements are to be made on standard resistors of high resistance value.

The series D.C. blocking capacitors used in load coefficient measurements made by the Author were required to be of relatively large capacitance value in order to pass the levels of loading current required. Thus Ducon type 1S1000 100 uF paper capacitors were chosen.

Owing to their relatively large capacitance value per dollar (100 uF for approximately A10 dollars), it was found to be one of the few suitable types of non-electrolytic capacitors available in Australia for use in load coefficient measurements on standard resistors which have values of resistance above the range where electrolytic capacitors suffice and below the range where the requirement of very high insulation resistance restricts their use. Enquiries were made about the possibility of using Metalised Paper capacitors in their stead, but it was found that they were generally dearer and only of comparable size and rating to the Ducon 1S1000[34].

Investigations were also carried out to determine the relative magnitudes of the D.C. leakage currents which may be expected in capacitors of different capacitance value, using different dielectric materials, under two conditions of applied



CIRCUIT USED FOR THE MEASUREMENT OF D.C. LEAKAGE CURRENT THROUGH HIGH QUALITY CAPACITORS.

Fig. LXV

D.C. voltage. The measurements made were based on the requirements of British Standard 2131, 1965 para. 13.4.3,[35] and all capacitors investigated, were tested under identical conditions of applied voltage so as to obtain relative performances independent of capacitance value or voltage rating.

The circuit diagram of the measurement circuit used is shown in Fig. LXV and the results obtained are summarised in Table (11.2). The guard circuit shown connected around the voltage dropping resistor was earthed so as to bypass leakage currents from the battery supply to the voltage dropping resistor to earth. The voltage dropping resistor used consisted of the 1 megohm input resistance of the 6020 millivoltmeter on its low impedance input setting and the meter case was utilised as the guard mentioned above.

The number inserted in the column of Table (11.2) marked 'Philips 6020 RDG' was the mean of three readings obtained 1 minute $\pm$ 5 seconds after the application of the D.C. measuring voltage E to the measuring circuit. After each reading was taken, the capacitor was allowed to discharge into a short circuit for 1 minute $\pm$ 5 seconds before the next reading of the series was taken.

Inspection of Table (11.2) shows that the styroseal capacitors have smaller levels of leakage currents than do the polyester capacitors, under the same conditions of test. It may be seen though, that the D.C. voltage rating of the styroseals does not necessarily relate to the D.C. leakage current level as one might expect and in fact it is shown that a 0.12 uF styroseal capacitor rated at 400 volts D.C. WKG out-performs two styroseal capacitors with D.C. voltage ratings of 1 000 volts. The fact that this capacitor is smaller in value than either of the capacitors rated at 1 000 volts D.C., would not appear to be sufficient reason for expecting such a large drop in the D.C. leakage current level passed.

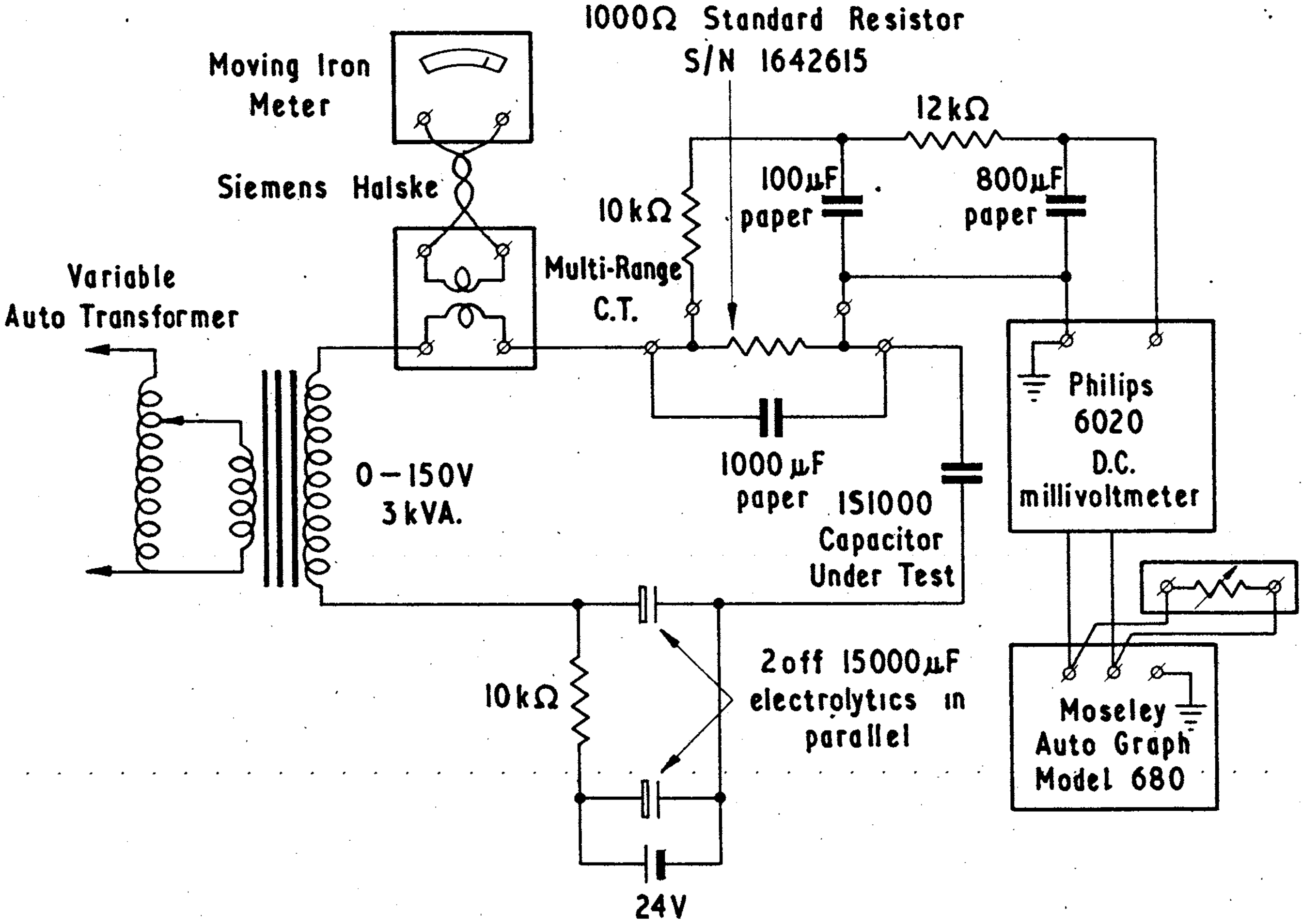
TABLE (11.2)

Relative Humidity 55%		Air Temperature 21 °C			
Capacitor Under Test		D.C. Voltage Rating	Applied D.C. Voltage E see Fig. LXV	Philips 6020	
Type	Capacitance Value			Range	RDG
Ducon Styroseal	0.47 uF	1 000 V	100 V	1 mV	40
Ducon Styroseal	0.15 uF	1 000 V	100 V	0.1 mV	10
Ducon Styroseal	0.12 uF	400 V	100 V	0.1 mV	7
Philips Polyester	0.47 uF	400 V	100 V	1 mV	40
Philips Polyester	0. 1 uF	400 V	100 V	0.3 mV	100
Ducon Styroseal	0.47 uF	1 000 V	400 V	1 mV	60
Ducon Styroseal	0.15 uF	1 000 V	400 V	1 mV	40
Ducon Styroseal	0.12 uF	400 V	400 V	0.1 mV	20
Philips Polyester	0.47 uF	400 V	400 V	3 mV	175
Philips Polyester	0. 1 uF	400 V	400 V	1 mV	45

(11.3.1)

Change in the Level of the D.C. Leakage Current Through a Ducon 1S1000 Paper Capacitor with a Change in the D.C. Polarising Voltage Level.

To investigate the effect that changes in the level of the



MEASURING CIRCUIT USED IN THE INVESTIGATION OF THE EFFECT THAT SUPERIMPOSED 50 c/s ALTERNATING CURRENT HAD ON THE LEVEL OF THE D.C. LEAKAGE CURRENT PASSING THROUGH DUCON 1S1000 100μF PAPER CAPACITORS.

Fig. LXVI

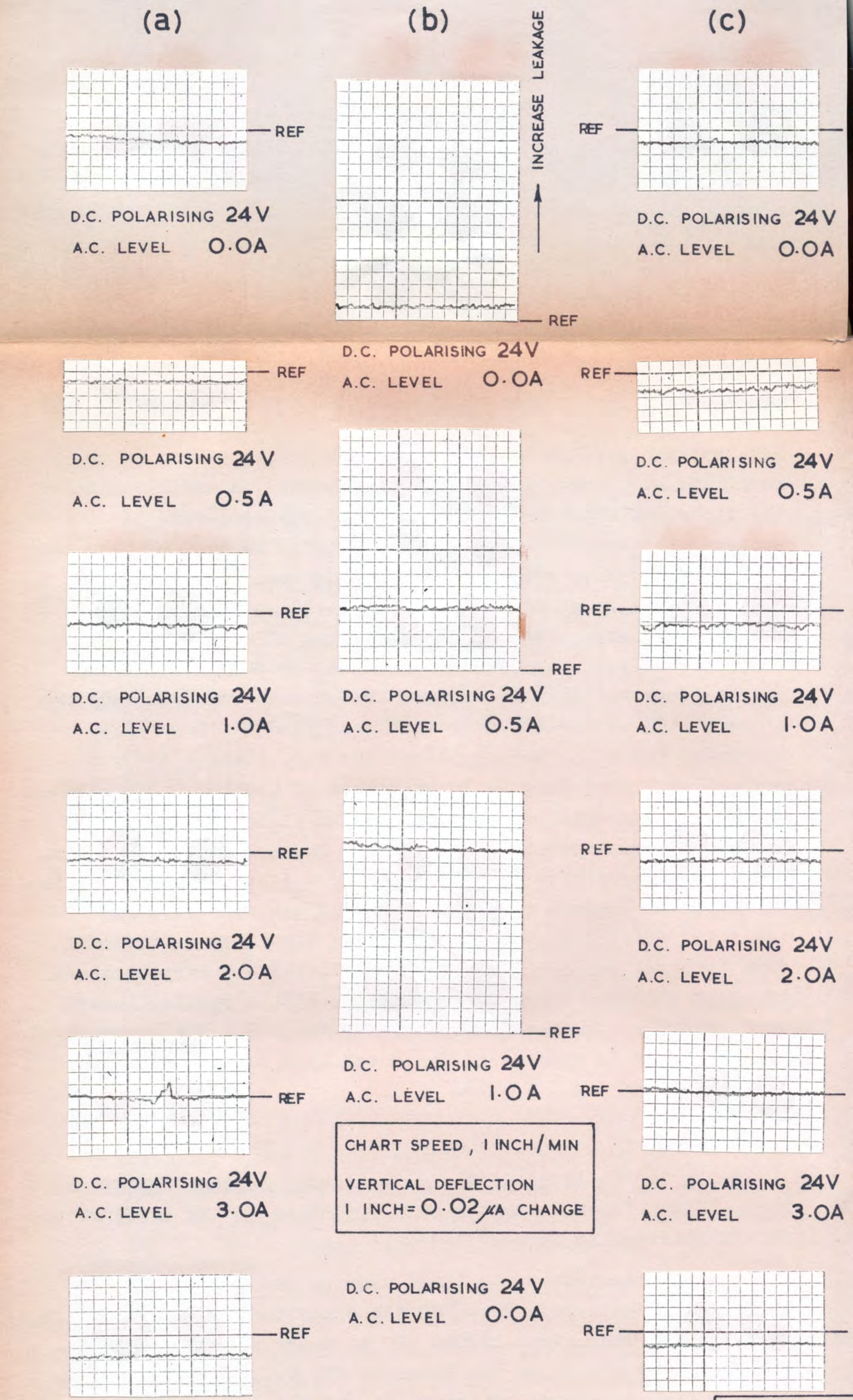
D.C. voltage applied across the 1S1000 capacitor had on the resulting D.C. leakage current being passed, a modification of the technique used in section (11.2.1) was employed. A measurement circuit was wired up similarly to that of Fig. LX1, except that the 10 ohm voltage dropping resistor connected across the input to the 6020 was replaced with a 1 000 ohm standard resistor, the 1 000 ohm setting of the decade series resistor was replaced with a setting of 99 000 ohms and the D.C. battery voltages applied to the circuit were generally higher. The capacitor under test consisted of 10 parallel connected Ducon 1S1000 capacitors and D.C. leakage current measurements were made at three values of applied D.C. voltage.

The results obtained showed that the final level of the direct leakage current through the 10 Ducon 1S1000 capacitors, changed linearly with the applied D.C. voltage, indicating that the parallel leakage resistance present was approximately 1.6×10^9 ohms. It was interesting to find that an extended time interval was required for the D.C. leakage current level to reach a steady value and it appears that a similar though smaller effect to that present in the Ducon ENP 871 electrolytic capacitor may be present in the Ducon type 1S1000 paper capacitors.

(11.3.2)

Change in the Level of the D.C. Leakage Current Through a Ducon 1S1000 Paper Capacitor with Variation in the Level of the 50 c/s Alternating Current Passed.

Similar measurements to those made on the Ducon 15 000 uF electrolytic capacitor type ENP 871 described in section (11.2.2) were made on Ducon type 1S1000 100 uF paper capacitors to investigate the change in the level of the D.C. leakage current through the capacitors, at constant applied D.C. voltage, as the level of the applied 50 c/s alternating current passed was increased from zero to a value of 3 amps. The measurement



RECORDINGS OF THE CHANGE IN THE LEVEL OF THE LEAKAGE CURRENT PASSING THROUGH 3 DUCON 100μF PAPER CAPACITORS TYPE 151000 VS TIME AS A FUNCTION OF A.C. LEVEL.

FREQUENCY = 50 Hz

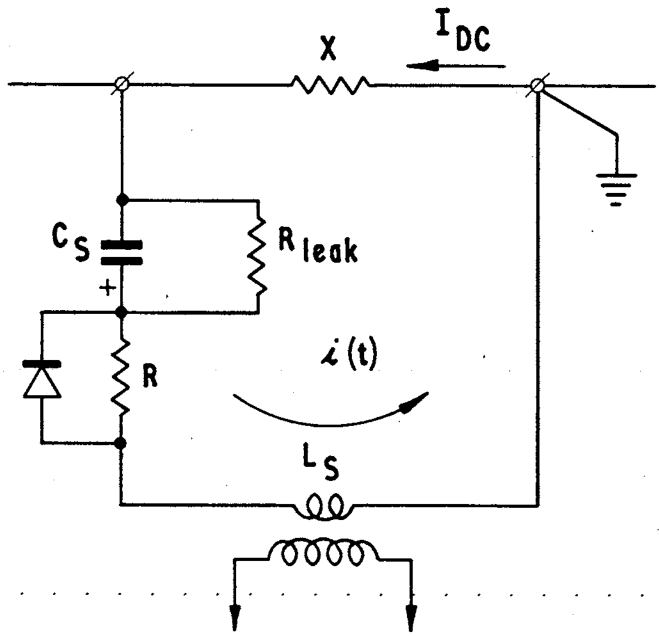
Fig LXVII

circuit used in this investigation is shown in Fig. LXV1. It is almost similar to that shown in Fig. LX111, but with modifications as shown.

The results obtained are shown in the Moseley recordings of Fig. LXV11 (a) to (c). Inspection of these shows there to be a slight increase in the level of the D.C. leakage current with an increase in the level of the alternating loading current passed, but in the case of Figs. LXV11 (a) and (c), it would be considered to be negligible under most conditions of use. The results of Fig. LXV11 (b) however, show a large relative increase in the level of the D.C. leakage current passed as the level of the alternating loading current being passed through it is increased.

It was decided to investigate whether this effect was due to the presence of a rectifying junction within the capacitor or to the heating effect of the alternating loading current being passed. For this purpose a further two measurements were made on the capacitor under discussion. One was made with the capacitor connected as before in the circuit diagram of Fig. LXV1, while the other was taken with the capacitor under test connected in reverse polarity. It was reasoned that the presence of a rectifying junction in the capacitor under test in the latter case, would be indicated by the level of the D.C. leakage current decreasing instead of increasing as before. The results obtained showed that the changes in the level of the D.C. leakage current as the level of the alternating current increased were similar in each case, indicating that the effect was probably due to self heating.

The insulation resistance of a capacitor containing a paper dielectric will decrease, with an increase in its body temperature.[35] Hence, one would expect capacitors in which the loss angle is appreciable, to exhibit a similar effect, since the heat produced as the alternating current passes through them will cause their body temperature to rise. If this process is allowed to continue unchecked at high alternating current levels



CIRCUIT SHOWING THE EFFECT OF THE PRESENCE OF A RECTIFYING JUNCTION IN THE CAPACITOR C_S.

Fig. LXVIII

and environmental temperatures, the capacitor will eventually destroy itself, and in fact it appears that the upper limit of working current for a particular value of environmental temperature is governed by this effect.

(11.4)

A Note on the Effect of the Presence of a Slight Rectifying Junction Effect in the Capacitor Used as the D.C. Blocking Capacitor.

The presence of a rectifying junction in the capacitor used as the D.C. blocking capacitor in the alternating loading current supply circuit, may be represented by the circuit diagram of Fig. LXV111. The diode depicted there would generally have a low forward conducting resistance and a higher value of reverse conducting resistance, represented by R. The plate of the capacitor C_S remote from the Unknown X will become more positive in potential relative to its other plate than would be the case if the D.C. voltage $I_{DC}X$ were the only effective D.C. voltage applied. This rise in potential will cause a uni-directional current i(t) to flow as shown, superimposed on the level of the alternating loading current being supplied to the Unknown. Hence it will cause an increase in the voltage drop across the Unknown equal to X.i(t) volts relative to earth. The effect of the extra D.C. voltage drop across the Unknown on the bridge balance condition will be removed however with the use of D.C. measuring current supply reversals, providing the mean value of the steady-state level of i(t) is independent of time and of the direction of flow of the D.C. measuring current through the Unknown. If on the other hand the value of the direct current i(t) changes with time, its change must be such as to cause negligible errors in the resistance measurement.

The presence of small rectifying junction effects in a practical bridge circuit would be difficult to detect, since they will be similar in general to effects produced by the presence of thermal emfs. The presence of a rectifying junction in the D.C. blocking capacitor of the alternating loading current supply circuit may be detected by reversing the polarity of the D.C. blocking capacitor between measurements at the same level of applied loading current and D.C. measuring current. If a significant change in the steady state value of the galvanometer electrical zero is produced, it will indicate the presence of a rectifying junction.

(11.5)
Alternating Loading Current Supply, Control and Measurement.

The loading current requirements of a laboratory equipped to carry out load coefficient measurements on D.C. standard resistors are such that current levels ranging from values of several milliamps up to very high currents of the order of 10 000 amps may be required from time to time.

(11.5.1)
Loading Current Supply Transformer Considerations.

If it is decided to use alternating current facilities to supply the required loading current levels, a multiple-secondary transformer would be required with facilities for easy interconnection of secondary tappings.

One suggested combination is shown in Table (11.3). The maximum level of loading current would be obtained in each case by connecting all transformer secondary windings in parallel, whereas maximum secondary voltage would be obtained with series connection. The possibility of using only a fraction of the available secondary windings may be considered in order to reduce the transformer secondary voltage available when high voltages are not required.

TABLE (11.3)

Heavy Current Transformer with 100 Secondary Windings, Each Rated at 3 Volts and 100 Amps. (Maximum Usable VA = 30 kVA)

Current Range (Amps)	Voltage Range (Volts)	Secondary Connections p = parallel s = series
1000 to 10 000	1 to 3	(100 - p)
100 to 1 000	3 to 30	(10 - s)-p - (10 - s)-...
10 to 100	30 to 300	(100 - s)
0 to 10	0 to 300	Use of sufficient secondaries to provide the necessary supply voltage.

The insulation resistances required between the secondary windings of the supply transformer and earth depend upon where the supply transformer is connected in the alternating loading current supply circuit. If the supply transformer secondary is connected between the earthy side of the D.C. blocking capacitor and earth, then the insulation resistance present may be low. If however it is placed on the high impedance side of the D.C. blocking capacitors, the insulation resistance must be such that it produces negligible shunting effect on the Unknown. For this reason, all supply circuit components are generally connected on the earthy side of the D.C. blocking capacitor.

(11.5.2)

Adjustment of the Alternating Loading Current Level.

Adjustment of alternating loading currents of the order of 10 000 amps would almost certainly be performed on the primary side of the supply transformer at relatively high voltage and low current levels. A system suggested by Mr. L. Medina of the National Standards Laboratory consisted of a 'Thoma Regulator' manufactured by Hochspannungsgesellschaft, Cologne, feeding the supply transformer. The input voltage to the 'Thoma Regulator' might be of the order of 11 kV, while its secondary side might be variable from 0 to 1 000 volts at 30 amps rated current. The primary of the supply transformer would be matched to the secondary of the 'Thoma Regulator'. The advantage of the 'Thoma Regulator' over the variable tapped auto-transformer is that the output voltage of the former is continuously variable, whereas that of the latter is variable in finite steps which are governed by the voltage per turn of the transformer winding.

(11.5.3)
Alternating Loading Current Measurement.

The level of typical alternating loading currents would be measured with the aid of a suitable current transformer and meter system or with an ammeter directly. The measuring system would be placed on the secondary side of the supply transformer in order to monitor the exact current level supplied to the Unknown. Current measurements made on the primary side of the supply transformer would include the magnetising current and any ratio error of the supply transformer.

(11.6)
Comparison of Relative Costs Involved in the Maintenance of Alternating and Direct Heavy Current Supplies.

The output current capacity of either a direct or an alternating current supply must be such as to provide the maximum current considered likely to be encountered in practice. The sources of heavy current considered most likely to be used in either case are (a) the parallel or series parallel combination of high capacity storage batteries through a large switchboard and (b) the combination of a series of heavy current step down transformers and a large D.C. blocking capacitor consisting of the parallel combination of many high valued good quality capacitors. The primary source of energy in (b) would most probably be the readily available 50 c/s mains.

The alternative method of providing the necessary direct loading currents from a rectified mains supply presents problems associated with the rectification and filtering required. Provided the level of ripple present on the direct current after filtering is such that the effective loading current is constant and may be determined accurately, the presence of a finite level of ripple or amplitude fluctuation will not be seen by the galvanometer in a Kelvin Bridge type of circuit.

(11.6.1)

Large Direct Currents from Storage Batteries.

One large storage battery system familiar to the Author is that possessed by the National Standards Laboratory which is used to supply heavy direct currents for the calibration of D.C. standard resistors. This system consists of 120 'Clyde' type P9 2 volt lead-acid storage cells with high discharge capacities. Typical discharge rates as obtained from 'Clyde' for their P9 cells are shown in Table (11.4)

TABLE (11.4)

Discharge Rate	Ampere - Hour Capacity
10 hours	146
5 hours	120
1 hour	74

The cells are connected such that 40 banks of 3 in series are obtained which may be inter-connected as required but which give maximum current with 6 volts between output busses. It can be seen by inspection of Table (11.4) that with an output direct current of 3 000 amps, a 1 hour discharge rate is obtained.

The life expectancy of each cell is quoted by Clyde as being approximately 15 to 20 years under good conditions. Experience obtained by N.S.L. has found this estimate to be of the correct order. The current price of replacement cells is approximately A30 dollars.

A battery system of this size requires regular maintenance in order to obtain the maximum quality of performance. The N.S.L. employs an electrician whose sole responsibility is the care and maintenance of four storage battery systems plus several dozen single 2, 6 and 12 volt cells. Approximately 20 per cent of his time might therefore be expected to be spent in attending to this heavy current battery system. The amount of floor space required in order to accommodate a battery system of this size is appreciable. The N.S.L. Heavy current battery system is stored in two layers in a special battery room over a floor area of approximately 300 square feet. The remaining space between the upper layer of cells and the roof is taken up with the necessary inter-connecting leads between the battery room and the heavy current switchboard which is situated in the direct current measurement laboratory upstairs.

Maintenance on the heavy current switchboard is minimal although water cooling must be provided to the built-in series resistors which are used to regulate the direct current flow when heavy currents are required.

(11.6.2)

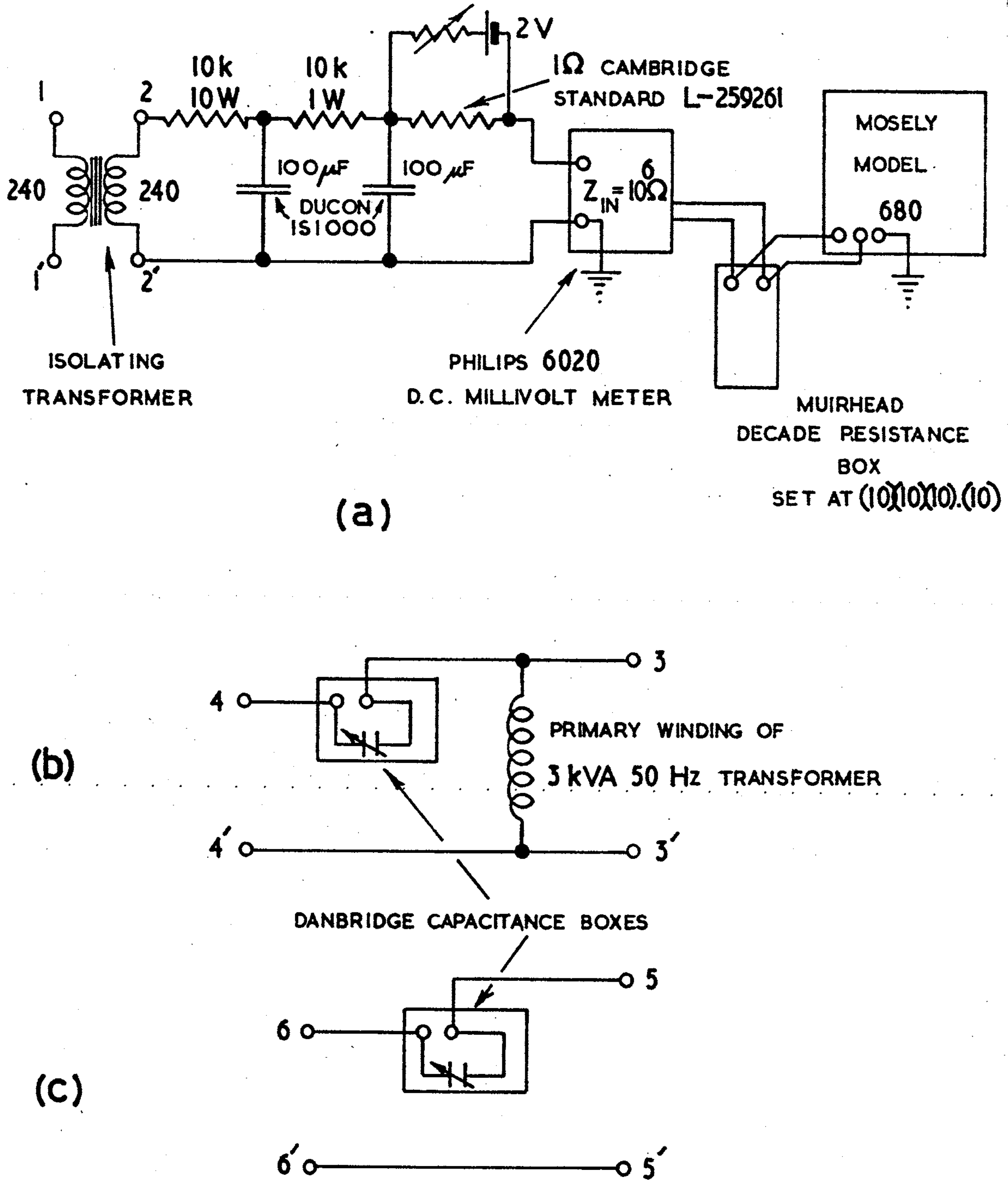
Large Alternating Currents From Transformers and Large D.C. Blocking Capacitors.

The maintenance required on typical transformers is extremely small when it is compared to that required on a typical heavy direct current storage battery system. The greatest factor of uncertainty in a typical alternating loading current supply system will be the life expectancy of the capacitors to be used in the D.C. blocking capacitor.

It was decided in section (11.1) that the electrolytic capacitor was the most reasonable solution to the problem of providing high alternating loading currents at low transformer output voltage, provided the shunt D.C. leakage resistance present in the capacitors was acceptable. Hence in high current applications it is reasonable to assume that large valued electrolytics will be used; possibly the 15 000 uF Ducon type ENP 871 which was discussed in section (11.2).

Discussion with Mr. J. Nicol of Ducon on the life expectancy of typical type ENP 871 electrolytic capacitors indicated that no detailed investigation of capacitor life expectancy had been performed, but that at 70 ^{o}C ambient temperature, a useful life expectancy of at least 2 000 hours might be expected for circuit conditions within the capacitor's ratings. Reduction of ambient temperature to the vicinity of room temperature was expected to increase this life expectancy by at least a factor of 2. The problem of electrolytic capacitor dry-out as experience in older type of electrolytic capacitors is no longer present, since modern capacitor cans are gas-tight. When storing the electrolytic capacitors, during periods of inactivity, provision of a constant D.C. polarising voltage was considered desirable, so that a period of repolarising would not be necessary before subsequent use. This type of storage procedure was considered likely to enhance capacitor life expectancy.

From the above discussion it appears that the typical time interval between replacements of electrolytic capacitors used in the D.C. blocking capacitor may be considerable, as judged from the period and type of use required of the N.S.L. heavy D.C. current supply discussed above. The floor space required



Filter Sections

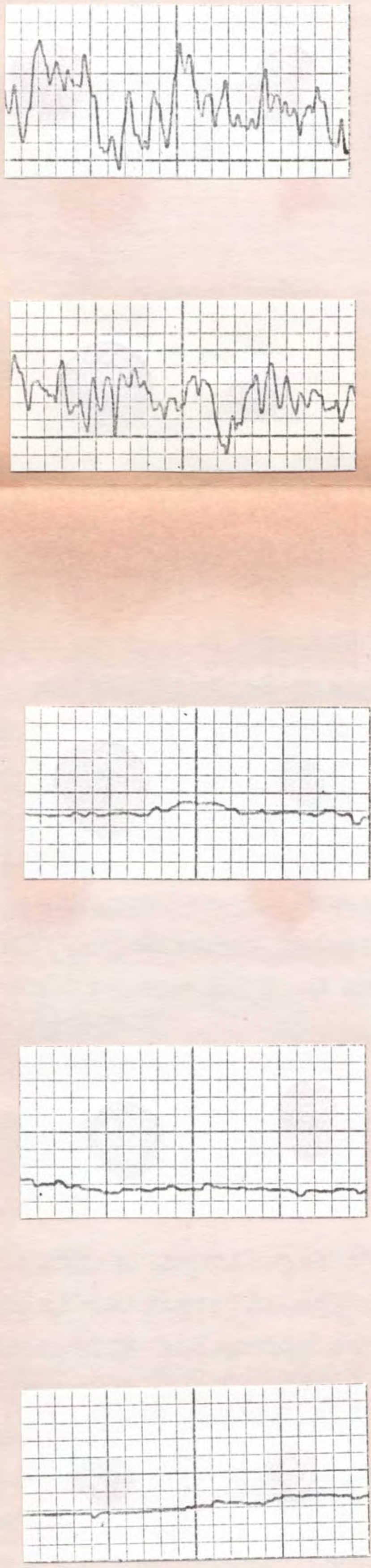
Fig LXIX.

to accommodate an alternating loading current supply of equivalent capacity to the heavy current storage battery system discussed above will be substantially less than the 300 square feet required for the battery. Hence if the installation of a new heavy current loading supply is contemplated, the money saved in floor area might counteract the added initial outlay to be expected when providing the proposed Thoma Regulator, heavy current transformer and D.C. blocking capacitors. The savings in maintenance costs would then be apparent.

(11.7)

Presentation of the Results Obtained from the Measurement of the Low Frequency Fluctuations in the Amplitude of the Alternating Loading Current Supplies.

In order to demonstrate the relative magnitudes of the fluctuations in amplitude occurring in the alternating loading currents received from different sources and under different conditions of supply, the measurement system whose circuit diagram is shown in Fig. LX1X (a) was connected. The system consisted basically of a means of eliminating most of the supply signal at fundamental supply frequency and allowing only the effects of its lower frequency amplitude fluctuations to pass to the measuring circuit. In cases where amplitude fluctuation measurements were made on the supply voltage received from the Savage 1 kW Power Amplifier used in the measurements of Chapters 4, 5, 6 and 9, the isolating transformer shown in Fig. LX1X (a) was removed and the high-pass filter shown in Fig. LX1X (b) connected in its place when desired. In one measurement on the unregulated mains, the series capacitor shown in Fig. LX1X (c) was used in producing a high pass filter from the combination of itself with the primary inductance of the isolating transformer, by connecting the circuit of Fig. LX1X (c) ahead of the isolating transformer of Fig. LX1X (a). Off-set of the pen zero of the Moseley could be obtained quite easily with the adjustment of the zero control provided, but the result of using



(a) UNREGULATED 50 Hz MAINS DIRECT.
240V INPUT

(b) REGULATED 50 Hz MAINS DIRECT.
240V INPUT

(c) UNREGULATED 50 Hz MAINS THROUGH $1 \mu F$
130V INPUT, 270V ACROSS CAP.

(d) UNREGULATED 50 Hz MAINS THROUGH $2 \mu F$
275V INPUT, 400V ACROSS CAP.

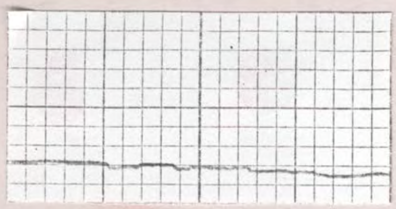
(e) UNREGULATED 50 Hz MAINS THROUGH $3 \mu F$
(1) 200V INPUT, 134V ACROSS CAP.

CHART SPEED 2 INCHES/MIN.
VERTICAL DEFLECTION 1 INCH ≡ 0·6 mV INPUT TO L.P. FILTER.

RECORDINGS OF FLUCTUATIONS AS SEEN BY THE MEASURING CIRCUIT OF FIG LXIX APPEARING IN TYPICAL ALTERNATING CURRENT SOURCES USED DURING LOAD COEFFICIENT MEASUREMENTS IN THIS THESIS.

Fig LXX (I)

(e)
(2)



UNREGULATED 50 Hz MAINS THROUGH 3μF
300V INPUT, 400V ACROSS CAP.

(f)
(1)

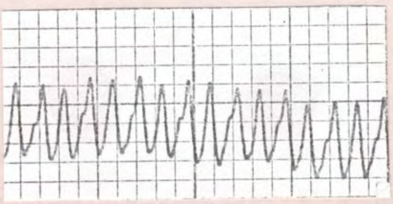
45 Hz SUPPLY FROM SAVAGE-LEVELL SYSTEM.
OUTPUT OF SAVAGE DIRECT, NO ISOLATING TRANSFORMER.

240 V INPUT.

(f)
(2)

AS IN (f)(1) ABOVE EXCEPT,
HIGH PASS FILTER OF FIG LXIX (b), SERIES
C = 5μF, NO ISOLATING TRANSFORMER.
240 V OUTPUT FROM SAVAGE,
140 V INPUT TO MEASURING CIRCUIT.

(g)
(1)



AS IN (f)(1) ABOVE EXCEPT,
SUPPLY FREQUENCY = 40 Hz.

(g)
(2)

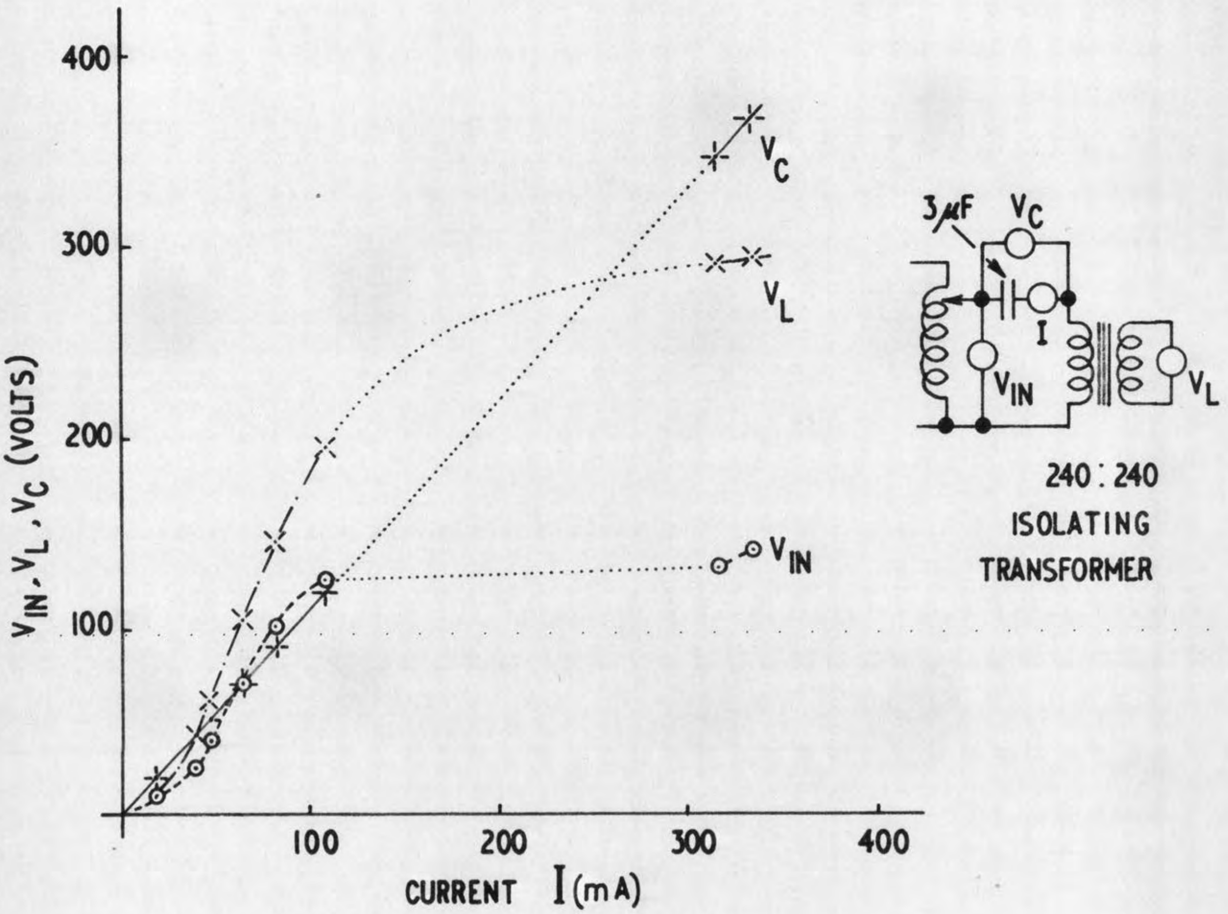
AS IN (f)(2) ABOVE EXCEPT,
SUPPLY FREQUENCY = 40 Hz,
115 V INPUT TO MEASURING CIRCUIT.

Fig LXX (II)

the measuring system described without an initial D.C. off-set of the 6020, was to provide essentially a record of the amplitude of the supply fluctuations without providing any information with regard to their sign, since the 6020 measuring system was such that both positive and negative D.C. voltages of the same amplitude gave the same meter deflection.

All recordings of the low frequency amplitude fluctuations shown in Fig. LXX were made with the recording system connected to give full-scale pen deflection with 3 mV D.C. input to the 6020, and with a recorder chart speed of 2 inches per minute. The use of corner frequencies of 1 radian per second in the low pass filter meant that only relatively low frequencies of fluctuation were passed unattenuated by the measuring system, but these were the frequencies of fluctuation most troublesome to the bridge detection system used in measurements.

The recordings shown in Fig. LXX (a) and (b) were taken of the 50 c/s mains fed straight into the primary of the isolating transformer, the former being of the unregulated mains and the latter of the mains after a commercial voltage regulation system had been employed. It will be seen that the voltage regulator caused only slight improvement in the amplitude of the random fluctuations found in the unregulated mains. The results shown in Fig. LXX (c), (d) and (e) were taken with the measuring system fed from the mains through the series capacitor shown in Fig. LX1X (c). The different traces were obtained with the capacitor set to different values of capacitance. It was found that the 50 c/s voltage drop across the capacitor caused the voltage input to the measuring system to fall below the 240 volt level used in obtaining the records of Fig. LXX (a) and (b) above, but this reduction in input voltage was found to be not in direct proportion to the capacitance value used, since the lower A.C. voltage on the iron circuit of the isolating transformer caused the transformer primary inductance to increase and so off-set somewhat the effect of the small series capacitance values used on the elimination of the low frequency



Curves of Input Voltage V_{IN} & Voltages V_L & V_C vs Current I for the high-pass filter shown.

Fig LXXI.

voltage fluctuations.

One example of the extremes of this effect was experienced in obtaining the records of Fig. LXX (e) (1) and (11), which were both obtained from the unregulated mains with the high-pass filter series capacitor of Fig. LX1X (c) equal to 3 uF, but with two different relationships between the voltage levels across the series capacitor and the isolating transformer secondary. The mechanism behind the change of relative levels of the voltage drop across the two filter components may be seen by inspection of Fig. LXX1, realising that the input voltage was the independent variable while obtaining the effect and it was approximately equal to $V_{secondary} - V_{capacitor}$. The records of Figs. LXX (c) and (d) were obtained from the unregulated mains with the high-pass filter series capacitor shown in Fig. LX1X (c) set to a value of 1 uF and 2 uF respectively.

The conclusions which may be drawn from the records of Fig. LXX (a) to (e) at this stage are that the effect of the low frequency fluctuations in the amplitude of voltages obtained from both the regulated and the unregulated mains can be reduced greatly with the use of a suitable high-pass filter.

The results from measurements made on the output voltages derived from the Savage 1 kW power amplifier, driven from its associated Levell battery powered oscillator, are shown in the records of Fig. LXX (f) and (g). The records of Figs. LXX (f) (1) and (g) (1) were obtained with the measuring circuit of Fig. LX1X (a) connected directly to the Savage and with the output voltage from the Savage adjusted to a level of 240 volts. The former record was obtained with the supply frequency set at a value of 45 c/s, while the latter was obtained at a supply frequency of 40 c/s. The results of further measurements made in order to explain the presence of the almost periodic nature of the fluctuations found in the above two records showed that the amplitude and frequency increased with supply frequency and that the source of the observed fluctuations was the Savage power amplifier itself.

The records of Fig. LXX (f) (11) and (g) (11) were made with the high-pass filter of Fig. LX1X (b) connected between the output of the savage 1 kW power amplifier and the input to the measuring circuit in place of the isolating transformer shown in Fig. LX1X (a). The input voltage to the measuring circuit, measured across the terminals 3 - 3', was 140 volts at 45 c/s and 115 volts at 40 c/s, with output voltage levels from the Savage of 240 volts in each case. The series capacitor of the high-pass filter of Fig. LX1X (b) was set to a value of 4.(10) uF, which was the value used in several of the load coefficient measurements of Chapters 4 and 5.

Comparison of the results of Fig. LXX shows that the inclusion of high-pass filter on the output of the Savage was quite successful in reducing the effect of the low frequency fluctuations in amplitude which were present in signals derived directly from the Savage.

Considering the results of all measurements made on signals derived from the three different sources used to produce the records of Fig. LXX it appears that provided there is sufficient filtering available, signals derived from any of the sources would be suitable as the alternating loading current when making load coefficient measurements on direct current resistors. The level of the rejection of the low frequency effects of fluctuations in the amplitude of the applied loading signal in any case is dependent on the value of resistance that is being loaded, the level of power dissipation required and the accuracy to which the load coefficient of the resistor under test is required to be known.

In the light of the claimed amplitude stability of $\pm$ 0.01 per cent for the Savage, it is reasonable to assume that the periodic low frequency fluctuations in the amplitude of the signal obtained from it, which were found in the records of Figs. LXX (f) (1) and (g) (1), should not have been there and were in fact due to a fault which had developed in the instrument. It is difficult to judge when the fault developed, but on account of the results obtained during the preliminary stages of the load coefficient measurements made on the Cambridge 1 ohm resistor L-259235, where the signal derived from the Savage was found to be greatly superior to that derived from the mains, even without the use of filtering, it appears that the fault developed subsequent to measurements being commenced on this resistor, since it is difficult to imagine the response of the galvanometer system used in load coefficient measurements on L-259235 being less sensitive to the type of signal shown in Fig. LXX (f) (1) than it was to the type of signal shown in Fig. LXX (b) or (a).

CHAPTER 12

Calculation Procedures and the Computer Programs Used in the Processing of the Measured Results Obtained.

(12.1)

Principle of 'Least Squares'.

It is commonly known and is demonstrated mathematically by Smart[36] that if the Arithmetic Mean of a series of measurements is accepted as being the most probable value of the quantity being measured, then the sum of the squares of the residuals taken with respect to the Arithmetic Mean will be the least possible. It is also shown[36], that the converse of the above statement applies, namely that if the sum of the squares of the residuals of a series of observations taken with respect to a certain value is the least possible, then the value will be the Arithmetic Mean.

In formulating his 'Normal Law of Errors', Gauss started with the postulate of the Arithmetic Mean, but in order to justify the use of the Normal Law of Errors as produced by Gauss, when processing the results of the load coefficient measurements made on the Standard resistors tested in this thesis, there are several conditions which must be satisfied by the results being processed. Firstly, each individual measurement must have been made independently; there is generally a tendency to 'steer' the results obtained from a particular measurement towards an acceptable value. Secondly, each measurement must have been made under comparable conditions and thirdly, each measurement must be equally trustworthy[37].

When obtaining the load coefficient measurement results of this thesis, the Author attempted to be completely impartial, so that any difference between the loading effects of the D.C. and A.C. techniques would be high-lighted.

(12.2)

'Least Squares' Approximation to a Parabola of the Corrections Obtained for a Particular Standard Resistor While Changing its Environmental Temperature, Under Constant Level of Power Dissipation.

There are many factors which will affect the resistance value obtained from a particular measurement made on a standard resistor under a controlled set of environmental conditions. Several of these, such as pressure and strain effects, are generally assumed to be constant for any series of resistance measurements made over a short time interval and under laboratory conditions. But effects such as changes in the environmental temperature of the resistor and changes in the level of its power dissipation, will produce changes in the correction to the resistor which are reproducible, provided the environment of the resistor is reproducible.

It is usual to measure the change in the resistance of a standard resistor while only one of the parameters of the resistor's environment is permitted to change at any one time. This technique results in a series of curves for the resistor under test. The resistance correction versus temperature curves are generally assumed to possess a shape which may be approximated with sufficient accuracy to a parabola over the temperature range normally encountered in the laboratory. Hence, since Gauss' Normal Law of Errors is assumed to apply to all measurements processed, the changes in correction to a particular resistor with changing temperature at constant power dissipation level were processed in order to obtain the parabola of best fit using the method of 'least squares'. The mathematical processes and manipulations required to produce the parabola of best fit using this method are presented in any text which deals with the problems of curve fitting.[38]

If it is assumed that the equation of the approximating parabola of best fit is in the form

$$\delta X = \Delta + \alpha_{20}(T - 20) + 0.5\beta(T - 20)^2 \qquad (12.1)$$

where δX is the correction in proportional parts to the Unknown X at temperature T and under a constant power dissipation level,

T is the oil bath temperature in ^{o}C and

α_{20}, β and Δ are constants, where $\Delta = \Delta_o$ when the power dissipation level in the Unknown X is at its no-load value, then it can be shown by the adaption of the formulae presented by Sokolnikoff[38], that the conditions which must be satisfied by the three constants α_{20}, β and Δ is given by the equations

$$\Delta . n + \alpha_{20}\sum_{i=1}^{n}(T_i-20) + 0.5\beta\sum_{i=1}^{n}(T_i-20)^2 = \sum_{i=1}^{n}\delta X_i$$

$$\Delta . \sum_{i=1}^{n}(T_i-20) + \alpha_{20}\sum_{i=1}^{n}(T_i-20)^2 + 0.5\beta\sum_{i=1}^{n}(T_i-20)^3 = \sum_{i=1}^{n}\delta X_i . (T_i-20)$$

$$\Delta . \sum_{i=1}^{n}(T_i-20)^2 + \alpha_{20}\sum_{i=1}^{n}(T_i-20)^3 + 0.5\beta\sum_{i=1}^{n}(T_i-20)^4 = \sum_{i=1}^{n}\delta X_i . (T_i-20)^2$$

(12.2)

The value of the load coefficient at 20 ^{o}C under any given level of power dissipation in the Unknown is found by subtracting the value Δ_o from the value obtained for Δ at this power dissipation level.

(12.3)

The Establishment of the Computer Program TEMPCURV.[39,40]

It has been mentioned in previous chapters that the method used in obtaining the temperature coefficient curves for both the Unknown and the Bridge circuit, was to take two sets of bridge balances, one set while changing one oil bath temperature in discrete steps, keeping the other bath temperature as constant as possible, and then by repeating the process on the other oil bath. The discrete changes made in either oil bath temperature were such that their values greatly exceeded any temperature change experienced from normal oil bath temperature drifts.

It was decided to use an iterative process to find the temperature coefficient curve of both the Bridge circuit and the Unknown by assuming an initial value of α_{20} and β for one of them as being equal to zero. Successive iterations were made using the value of α_{20} and β found from any one iteration on either the Bridge circuit or Unknown in order to apply corrections to the bridge balance readings obtained when calculating the temperature coefficient curve of the other bridge section. The iteration process continued until the change in the value of α_{20} obtained between two successive calculations of the temperature coefficient curve of a particular bridge section became less than a specified value.

Concentration was focussed on the values of α_{20} and β rather than the value of Δ_o, since only the relationship between the resistance change with temperature was required, rather than absolute resistance corrections.

Under constant loading conditions it may be assumed that the corrections δX and δBridge will be functions only of their temperatures. Each measured temperature of the Unknown or Bridge oil baths will be slightly different to the actual temperatures experienced by the Unknown and Bridge circuit. Hence

$$\begin{aligned} Tx_i &= Tx_{m_i} + \mathcal{E}x_i(T) \\ Tb_i &= Tb_{m_i} + \mathcal{E}b_i(T) \end{aligned} \qquad (12.3)$$

where Tx_i and Tb_i are the actual temperatures experienced by the Unknown and the Bridge circuit respectively at the ith measurement made to obtain the values of α_{20} and β for the Unknown,

Tx_{m_i} and Tb_{m_i} are the corresponding measured temperatures obtained for the Unknown and Bridge oil baths respectively, and

$\mathcal{E}x_i(T)$ and $\mathcal{E}b_i(T)$ are the corresponding temperature errors.

At the ith bridge balance, the change in proportional parts δ_i of the bridge balance condition from an arbitrary zero condition is related to the change in the Unknown and the Bridge circuit in proportional parts from this arbitrary zero condition by the relationship

$$\delta_i = \delta X_i - \delta \text{Bridge}_i \tag{12.4}$$

where

$$\begin{aligned} \delta X_i &= \Delta_x + \alpha_{20x}(Tx_i - 20) + 0.5\beta_x(Tx_i - 20)^2 \\ \delta \text{Bridge}_i &= \alpha_{20b}(Tb_i - 20) + 0.5\beta_b(Tb_i - 20)^2 \end{aligned} \tag{12.5}$$

The values of α_{20x}, β_x, α_{20b} and β_b used in the pth iterations of the iterative process will be slightly in error, hence

$$\begin{aligned} \alpha_{20x} &= \alpha_{20x_p} + \mathcal{E}_p(\alpha_{20x}) \\ \beta_x &= \beta_{x_p} + \mathcal{E}_p(\beta_x) \end{aligned} \tag{12.6}$$

$$\begin{aligned} \alpha_{20b} &= \alpha_{20b_p} + \mathcal{E}_p(\alpha_{20b}) \\ \beta_b &= \beta_{b_p} + \mathcal{E}_p(\beta_b) \end{aligned} \tag{12.7}$$

where α_{20x_p}, β_{x_p}, α_{20b_p}, β_{b_p} are the values of α_{20} and β used for the Unknown and Bridge circuit.

Substituting for δX_i, $\delta \mathrm{Bridge}_i$ in equation (12.4) gives

$$\mathcal{E}_{ip} = \delta_i - \left[\Delta X + \alpha_{20x_p}(Tx_{m_i} - 20) + 0.5\beta x_p (Tx_{m_i} - 20)^2 \right.$$
$$\left. + \alpha_{20b_p}(Tb_{m_i} - 20) + 0.5\beta b_p (Tb_{m_i} - 20)^2 \right]$$

Use of equations (12.3), (12.6) and (12.7) gives

$$\begin{aligned}
\mathcal{E}_{ip} = {} & \mathcal{E}_p(\alpha_{20x})(Tx_{m_i} - 20) + 0.5\,\mathcal{E}_p(\beta x)(Tx_{m_i} - 20)^2 \\
& + \alpha_{20x}\,\mathcal{E}x_i(T) \\
& - 0.5\beta x \left[-2\,(Tx_i - 20)\,\mathcal{E}x_i(T) + \mathcal{E}x_i(T)^2 \right] \\
& - \mathcal{E}_p(\alpha_{20b})(Tb_{m_i} - 20) - 0.5\,\mathcal{E}_p(\beta b)(Tb_{m_i} - 20)^2 \\
& - \alpha_{20b}\,\mathcal{E}b_i(T) \\
& + 0.5\beta b \left[-2\,(Tb_i - 20)\,\mathcal{E}b_i(T) + \mathcal{E}b_i(T)^2 \right]
\end{aligned} \tag{12.8}$$

The method of least squares brings $\sum_{i=1}^{n} \mathcal{E}_{ip}$ to zero for all p, hence if n is large and the errors $\mathcal{E}x_i(T)$ and $\mathcal{E}b_i(T)$ are random, then

$$\begin{aligned}
&\sum_{i=1}^{n} \mathcal{E}x_i(T) \longrightarrow 0 \\
&\sum_{i=1}^{n} \mathcal{E}b_i(T) \longrightarrow 0
\end{aligned} \tag{12.9}$$

and from equations (12.8) and (12.9)

$$\mathcal{E}_p(\alpha_{20x}) \sum_{i=1}^{n} (Tx_{m_i} - 20) + 0.5\, \mathcal{E}_p(\beta x) \sum_{i=1}^{n} \left[(Tx_{m_i} - 20)^2 \right]$$

$$- 0.5 \beta x \sum_{i=1}^{n} \left[- 2\, (Tx_i - 20)\, \mathcal{E}x_i(T) + \mathcal{E}x_i(T)^2 \right]$$

$$- \mathcal{E}_p(\alpha_{20b}) \sum_{i=1}^{n} (Tb_{m_i} - 20) - 0.5\, \mathcal{E}_p(\beta b) \sum_{i=1}^{n} \left[(Tb_{m_i} - 20)^2 \right]$$

$$+ 0.5 \beta b \sum_{i=1}^{n} \left[- 2\, (Tb_i - 20)\, \mathcal{E}b_i(T) + \mathcal{E}b_i(T)^2 \right]$$

$$= 0 \quad \text{for all } p \qquad (12.10)$$

When performing measurements on the Unknown, the range experienced by Tx_{m_i} and Tx_i is very much greater than the range experienced by Tb_{m_i} and Tb_i.

The first iteration on the measured results assumes that

$$\alpha_{20b_1} = \beta_{b_1} = 0 \qquad (12.11)$$

so that

$$\mathcal{E}_1\,(\alpha_{20b}) = \alpha_{20b}$$
$$\mathcal{E}_1\,(\beta b) = \beta_b \qquad (12.12)$$

Therefore from equation (12.10)

$$\mathcal{E}_1(\alpha_{20x}) \sum_{i=1}^{n} (Tx_{m_i} - 20) + 0.5\, \mathcal{E}_1(\beta x) \sum_{i=1}^{n} \left[(Tx_{m_i} - 20)^2 \right]$$

$$- 0.5 \beta x_1 \sum_{i=1}^{n} \left[-2\, (Tx_i - 20)\, \mathcal{E}x_i(T) + \mathcal{E}x_i(T)^2 \right]$$

$$= \alpha_{20b} \sum_{i=1}^{n} (Tb_{m_i} - 20) + 0.5 \beta_b \sum_{i=1}^{n} \left[(Tb_i - 20)^2 \right]$$

$$(12.13)$$

BLOCK FLOW DIAGRAM FOR THE PROGRAM TEMPCURV.

Fig LXXII.

and hence the magnitudes of $\mathcal{E}_1(\alpha_{20x})$ and $\mathcal{E}_1(\beta_x)$ will be small unless the magnitudes of α_{20b} and β_b are extremely large, or $\beta_{x_1}\sum_{i=1}^{n}\left[-2(Tx_i - 20)\mathcal{E}x_i(T) + \mathcal{E}x_i(T)^2\right]$ is large. Under normal conditions with sufficient temperature stabilisation time, $\beta_{x_1}\sum_{i=1}^{n}\left[-2(Tx_i - 20)\mathcal{E}x_i(T) + \mathcal{E}x_i(T)^2\right]$ will be small.

If $\mathcal{E}_1(\alpha_{20x})$ and $\mathcal{E}_1(\beta_x)$ are small in magnitude after the first iteration, then successive iterations on the results obtained for the Bridge circuit and the Unknown will cause $\mathcal{E}_p(\alpha_{20x})$ and $\mathcal{E}_p(\beta_x)$ to approach zero. If $\mathcal{E}_1(\alpha_{20x})$ and $\mathcal{E}_1(\beta_x)$ are not small in magnitude after the first iteration on the Unknown, then the magnitude of α_{20b} and β_b must be large and the first iteration on the Bridge circuit measurement results will produce values of α_{20b_2} and β_{b_2} which are extremely close to α_{20b} and β_b respectively. Successive iterations on the Unknown and Bridge circuit will thus produce values of α_{20x_p}, β_{x_p}, α_{20b_q} and β_{b_q} which approach closer and closer to α_{20x}, β_x α_{20b} and β_b respectively.

The Block Flow Diagram of the Fortran computer program 'TEMPCURV' is shown in Fig. LXX11. The process used was to calculate firstly the coefficients of Δ, α_{20} and β on the left hand side of equations (12.2), for both the Unknown and the Bridge circuit. Next, the coefficients of the right hand side of the equations (12.2) were calculated, using an initial value for the α_{20} and β for the Bridge circuit of zero. The first computed values of Δ, α_{20} and β for the Unknown were then calculated with the application of mayrix inversion to the equations (12.2). These computed values of α_{20} and β were used to produce the coefficients of the right hand side of the equations (12.2) for the Bridge circuit and its computed values of Δ, α_{20} and β calculated using mayrix inversion. The α_{20}s of

BLOCK FLOW DIAGRAM
FOR THE PROGRAM
LOADCURV

Fig LXXIII.

Unknown and the Bridge circuit were both tested to see if their changes from their last computed value were both less than a certain level, chosen arbitrarily as 0.1 parts in $10^7/\ ^oC$. If not, the above process was repeated using the new values of α_{20} and β for the Unknown and the Bridge circuit, until the change in each α_{20} after each iteration was within the required level of 0.1. At this stage, the iterative process was considered to be complete and it remained only to produce tables for the Unknown and the Bridge circuit of the points of interest at particular values of their respective oil bath temperatures. Finally, the R.M.S. value of the residuals was calculated in order to obtain an assessment of the accuracy with which the measurements were made and so assist in an assessment of the final measurement accuracy.

A protection loop was included in the program such that if the iterative process did not converge, only 100 iterations would take place and the machine would note this fact before stopping.

(12.4)

Establishment of the Computer Program 'LOADCURV'. [32,33]

Many modifications of the FORTRAN computer program LOADCURV were used in obtaining the desired parabolas of best fit to the results obtained for the appropriate Unknown under the desired conditions of load and temperature change.

The parabola of best fit to the measured results obtained was calculated at each loading condition of the Unknown using the method of 'least squares'. There was a basic similarity between LOADCURV and TEMPCURV insofar that they both produced a solution to the equations (12.2) from the measured data. TEMPCURV is more complicated however, since there is no iteration required in LOADCURV, but simply a specified number of repetitions, to cover each set of data input. The logical flow diagram of LOADCURV is shown in Fig. LXX111.

(12.5)

Other Fortran Computer Programs Used.

Other computer programs were used in processing the results of the A.C. - D.C. load coefficient measurements, but all were simplifications of sections of TEMPCURV or LOADCURV, discussed above.

CHAPTER 13

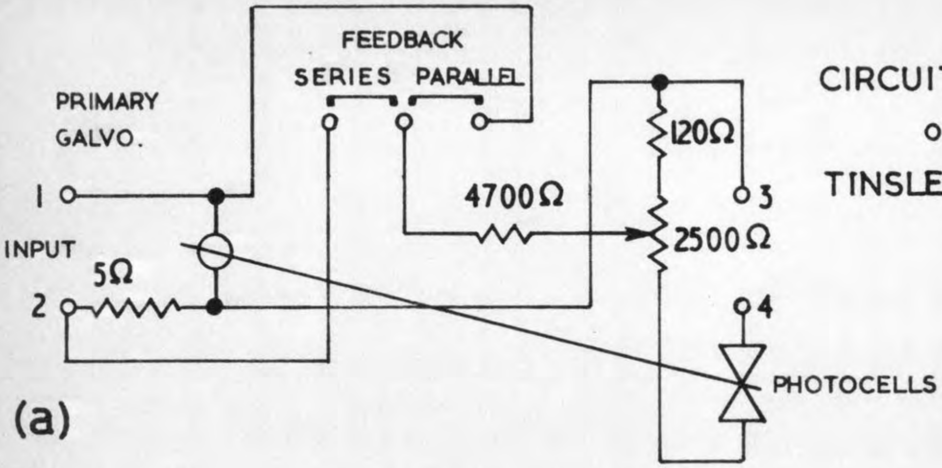
Detector Circuit Considerations, Noise and Vibration Effects.

(13.1)

The bridge detector used in the Kelvin and Wheatstone Bridges of Chapters 4 to 8 consisted of a Tinsley Galvanometer Amplifier Type 5214, followed by a Tinsley Galvanometer Type 4500. A parallel tuned mu-metal inductor was connected in series with the input circuit in order to present a high impedance to alternating currents of frequencies in the neighbourhood of the alternating loading current supply frequency. The galvanometer system used in measurements associated with the investigation of the special 0.1 ohm constantan resistor differed from the system described above in that the secondary galvanometer system used a recorder display in order to increase the integration time of the detector system.

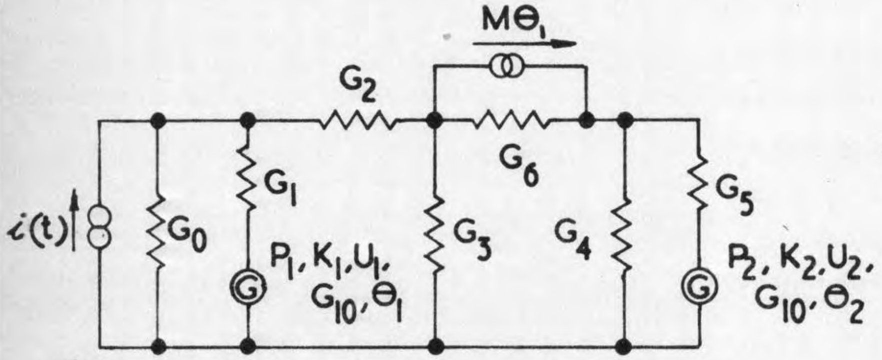
The Tinsley galvanometer amplifier was mounted in a position of minimum vibration in the laboratory during bridge measurements and leads were taken away to the Tinsley secondary galvanometer which was conveniently situated relative to the position of the variable shunted dials of the bridge and the D.C. reversing switch. As far as was possible, all leads in the galvanometer circuit were of copper, so that thermal effects were kept at a minimum.[4] The bridge balances made generally required the maximum galvanometer sensitivity available, hence the Tinsley Galvanometer Amplifier was connected and adjusted for minimum parallel feedback.

The galvanometer amplifier was basically similar to the type described by Preston[41] of the National Physical Laboratory. The use of negative feedback effectively stiffened the suspension of the primary galvanometer, although its effect on the moment of inertia and the damping constant of the suspension was negligible in practice. This meant that the free period and the



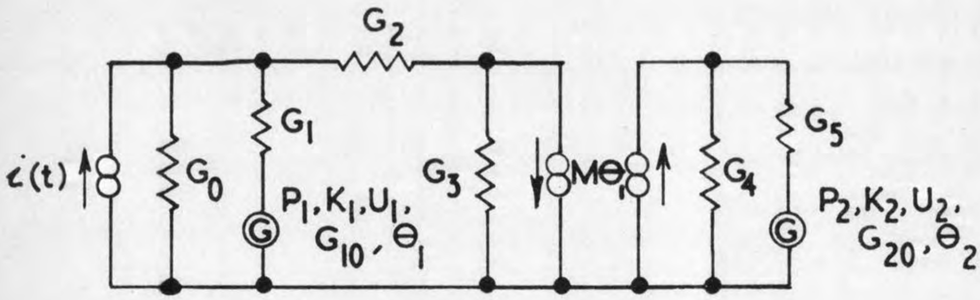
(a)

CIRCUIT DIAGRAM of the TINSLEY GALVO. AMPLIFIER TYPE 5214.



(b)

EQUIVALENT CIRCUIT of a complete galvo. system using parallel feedback.



EQUIVALENT CIRCUIT of (b) provided $G_6 \ll G_4, G_3$

(c)

Fig LXXIV.

relative damping factor of the primary galvanometer system was reduced by the application of negative feedback. The relative damping factor of the primary galvanometer suspension used in the Tinsley Galvanometer Amplifier Type 5214 was adjusted to be approximately critical without feedback, with the aid of a viscous liquid surrounding the suspension.[42]

The circuit diagram of the Tinsley Galvanometer Amplifier Type 5214 is shown in Fig. LXX1V (a). The impedance in series with the photo-electric cells is considered to be high in relation to the resistance of the secondary galvanometer used in practice. Hence the presence of amplification is represented by an ideal current generator in parallel with a large resistance. An equivalent circuit diagram of the overall galvanometer system is shown in Fig. LXX1V (b) which can be reduced to that of Fig. LXX1V (c) if the angular deflection θ_1 of the primary galvanometer suspension from its equilibrium value, is independent of the corresponding value of the back e.m.f. produced by the secondary galvanometer, and the value of

$$G_6 \ll G_4 \ , \quad G_3 \qquad (13.1)$$

G_0, G_1, G_2, G_3, G_4, G_5, G_6 are all circuit conductances, and M is the relationship between the output current of the photo-cells and θ_1. M is assumed to be constant and without phase shift for small values of θ_1 and $\frac{d\theta_1}{dt}$ and the effect of the parallel-tuned circuit appearing in the input circuit of the galvanometer system has been neglected at the frequencies of interest here. The conductance G_1 is that of the primary galvanometer coil, and the back e.m.f. e_1 from the primary galvanometer coil is assumed to be directly proportional to the angular velocity of the coil. Hence

$$e_1 = B_1 \ A_1 \ N_1 \ \frac{d\theta_1}{dt} = G_{10} \ \frac{d\theta_1}{dt} \qquad (13.2)$$

With an instantaneous current i(t) passing through the primary galvanometer coil, it can be shown upon summing the torques acting on the coil produced by (1), the restoring torque $U_1 \theta_1(t)$, (2), the damping torque $K_1 \, d\theta_1(t)/dt$, (3), the inertial torque $P(d^2\theta_1(t)/dt^2)$ and (4), the electrical motor torque $B_1 A_1 N_1 i(t)$, that the equation of motion of the galvanometer coil is given by

$$P_1 \frac{d^2 \theta_1(t)}{dt^2} + K_1 \frac{d\,\theta_1(t)}{dt} + U_1 \theta_1(t) = G_{10}\, i(t) \qquad (13.3)$$

inertia Friction suspension motor

Applying Kirchoff's Current Law at the Nodes A, B and C in Fig. LXX1V (c), using equations (13.2) and (13.3); after taking Laplace Transforms with respect to time t,

$$\bar{\theta}_1 = \frac{G_{10}\, k_1\, I_o}{P_1 S^2 + (K_1 + G_{10}^{\,2}\, G_{eq})S + (U_1 + MG_{10}\beta_1)} \qquad (13.4)$$

where

$\bar{\theta}_1$ is the Laplace Transform of $\theta_1(t)$

I_o is the Laplace Transform of $i_o(t)$

$G_{10} = B_1 A_1 N_1$

k_1 is the proportion of the input direct current which would pass through the primary galvanometer under steady-state conditions, with the current generator $M\theta_1$ open,

G_{eq} is the output conductance of the total network feeding the ideal galvanometer coil, and

β_1 is the proportion of the direct current $M\theta_1(t)$, which would pass through the primary galvanometer under steady-state conditions with the current generator $i_o(t)$ open.

Hence the equation of motion of the primary galvanometer coil when situated in the galvanometer amplifier is described by the equation

$$P_1 \frac{d^2 \theta_1(t)}{dt^2} + K_1' \frac{d\,\theta_1(t)}{dt} + U_1'\,\theta_1(t) = G_{10}\,k_1\,i_o(t) \qquad (13.5)$$

where

$$K_1' = K_1 + G_{10}^{\;2}\,G_{eq_1}$$

$$U_1' = U_1 + MG_{10}/\beta_1$$

The equation of motion of the secondary galvanometer may be obtained with reference to the above discussion as

$$P_2 \frac{d^2 \theta_2(t)}{dt^2} + K_2' \frac{d\,\theta_2(t)}{dt} + U_2\,\theta_2(t) = G_{20}\,k_2\,i_2(t) \qquad (13.6)$$

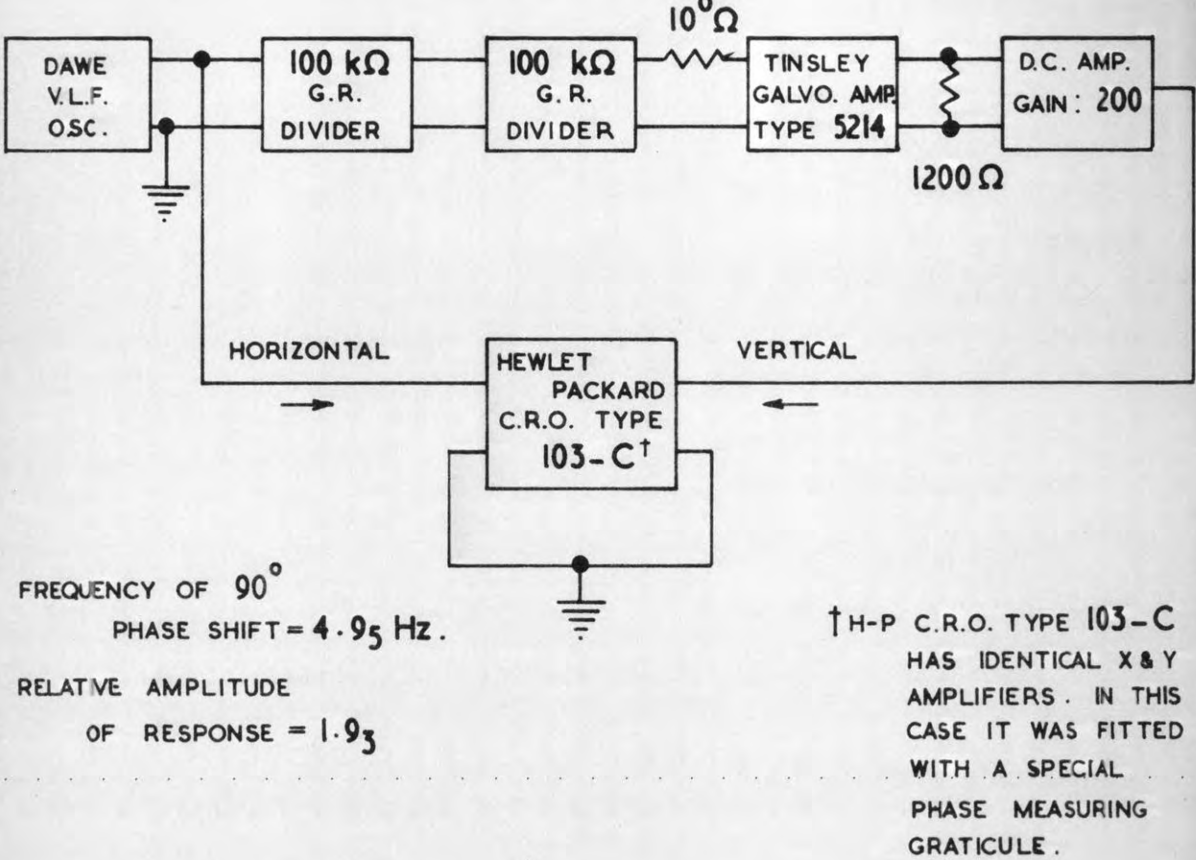
where the symbols used have similar meanings as above and

$$K_2' = K_2 + G_{20}^{\;2}\,G_{eq_2}$$

(13.2)

Noise Level Due to Brownian Motion and Thermal Resistance Noise in the Galvanometer System Used in Chapters 4 to 8.

It was found in practice that the level of noise produced in the secondary galvanometer by its damping mechanisms alone, both electrical and mechanical, was negligible. Hence the noise level in the overall galvanometer system may be assumed to have its origin in the resistances and mechanical damping of the galvanometer amplifier. The presence of negative feedback in the galvanometer amplifier has the effect of increasing its bandwidth. Hence electrical noise produced in the galvanometer amplifier will produce movement of the primary galvanometer coil



Schematic Diagram
of the measuring system used to obtain the relative frequency
response of the Tinsley Galvanometer Amplifier
Type 5214 set for minimum parallel feedback.

Fig LXX V.

with a frequency spectrum given by the frequency response curve of the galvanometer amplifier. The primary galvanometer coil movement will produce a proportional output current, which is applied directly to the secondary galvanometer. Hence the secondary galvanometer, with its narrower bandwidth, will act as a frequency selective filter, responding only to noise signal frequencies derived from the galvanometer amplifier, which are within its pass band.

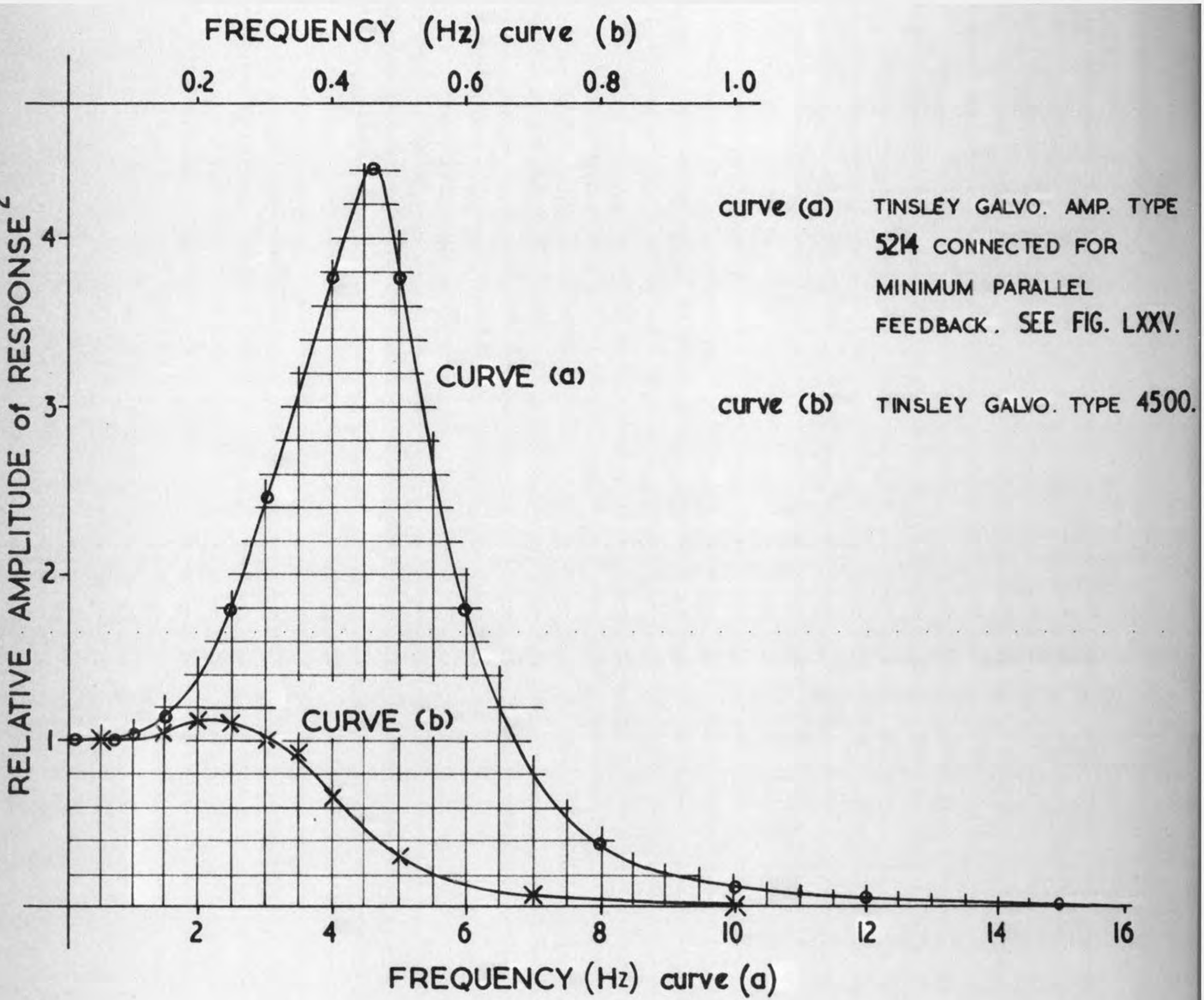
The square of the relative amplitude of the response versus frequency for both the primary and secondary galvanometer systems are obtained from equations (13.5) and (13.6) respectively to give

$$\left|\frac{A_1}{A_o}\right|^2 = \frac{1}{1 + \left[\left(\frac{K_1'}{U_1'}\right)^2 - \frac{2P_1}{U_1'}\right]\omega^2 + \left(\frac{P_1}{U_1'}\right)^2\omega^4} \tag{13.7}$$

$$\left|\frac{A_2}{A_o}\right|^2 = \frac{1}{1 + \left[\left(\frac{K_2'}{U_2}\right)^2 - \frac{2P_2}{U_2}\right]\omega^2 + \left(\frac{P_2}{U_2}\right)^2\omega^4} \tag{13.8}$$

These relationships were obtained experimentally using the set up as shown in the Fig. LXXV for measurement on the galvanometer amplifier and by measuring spot deflection as a function of frequency for the secondary galvanometer. The frequency response of the direct current amplifier shown in Fig. LXXV was tested and found to be flat throughout the frequency range used, with negligible phase shift output to input. The experimental results of these measurements are shown in the curves of Fig. LXXV1 (a) and (b).

Using the argument as presented by Smith[43] to calculate the



Curves of $\left|\frac{A}{A_o}\right|^2$ as a function of supply frequency (Hz)

Fig LXXVI.

noise level produced by the overall galvanometer system, equation (13.5) may be rewritten in the form

$$P_1 \frac{d\dot{\theta}_1}{dt} + K_1'\dot{\theta}_1 + U_1'\int\dot{\theta}_1 dt = G_{10}k_1 i_o(t) \qquad (13.9)$$

where $\dot{\theta}_1 \equiv \dfrac{d\theta_1(t)}{dt}$

Hence $Z(\omega) = R(\omega) + jX(\omega)$

$$= K_1' + j\left(\omega P_1 - \frac{U_1'}{\omega}\right)$$

which gives

$$R(\omega) = K_1'$$

$$X(\omega) = \omega P_1 + \frac{U_1'}{\omega} \qquad (13.10)$$

$$\langle\theta_1^{\,2}\rangle_{av} = 4kT \frac{R(\omega)}{\omega^2\left(R(\omega)^2 + X(\omega)^2\right)}\,\delta f' \qquad (13.11)$$

where $\delta f'$ is the bandwidth over which summation is to be made,

k is Boltzmann's Constant

and T is the Absolute Temperature.

Substituting (13.10) into (13.11)

$$\langle\theta_1^{\,2}\rangle_{av} = \frac{4kTK_1'}{2\pi U_1'^{\,2}} \int_0^{\omega_1} \frac{d\omega}{1 + \left[\left(\frac{K_1'}{U_1'}\right)^2 - \frac{2P_1}{U_1'}\right]\omega^2 + \left(\frac{P_1}{U_1'}\right)^2\omega^4} \qquad (13.12)$$

Comparison of Figs. LXXV1 (a) and (b) shows that the response of the galvanometer amplifier is flat throughout the pass band of the secondary galvanometer. Hence the mean square scale deflection produced in the secondary galvanometer due to the noise output current level produced by galvanometer amplifier is given by

$$\langle x_2^2\rangle_{av} = \frac{4kTK_1{}'M^2}{4\pi^2U_1'^2}\; Si^2_{2DC} \int_0^\infty \frac{d\omega}{1 + \left[\left(\frac{K_2'}{U_2}\right)^2 - \frac{2P_2}{U_2}\right]\omega^2 + \left(\frac{P_2}{U_2}\right)^2 \omega^4} \tag{13.13}$$

The above integral may be calculated graphically from Fig. LXXV1 (b) to be approximately equal to 3.4 radians/second.

Substituting the values of all known ratios and suspension constants in equation (13.13) from sections (13.3.1) and (13.3.2) below gives

$$\langle x_2^2\rangle_{av} \simeq 0.3 \text{ mm}^2 \tag{13.14}$$

at a temperature of approximately 300 ^{0}K

The presence of the tuned mu-metal inductor in the input circuit of the galvanometer during measurements will have little effect on the value obtained in practice for $\langle x_2^2\rangle_{av}$ above. The tuned circuit may be represented simply as a series inductor at the frequencies of interest, its series loss resistance being simply added to the output resistances of the bridge circuits used. The series impedances in this loop of Fig. LXX1V (a) is high enough at all frequencies, not to affect the values obtained for G_{eq_1}, β_1 or k_1.

(13.3)

Modified Values of Galvanometer Constants and Calculation of their Ratios.

Specifications[42] of the galvanometer movement used in the Tinsley Photocell Galvanometer Amplifier states that the damping of the primary galvanometer coil is performed with the use of a viscous liquid. Measurements were made on the frequency response of the primary galvanometer without feedback, using a high impedance constant current source. The results obtained indicated that the viscous liquid produced a relative damping factor of approximately 0.6 and that the effect of electrical damping was found to be negligible. Hence it may be assumed that, provided G_{eq_1} is small,

$$K_1 >> G_{10}^2 \quad G_{eq_1} \tag{13.15}$$

$$\text{and} \quad K_1' \simeq K_1 \tag{13.16}$$

The value of β_1 may be calculated to be approximately $\frac{1}{40}$, provided the source impedance of the current generator $i_o(t)$ is greater than about 100 ohms. It has also been calculated that the values of k_1 and k_2 are very close to unity, hence, since it was found that the direct current amplification factor of the galvanometer was very close to 40,

$$M\, G_{10}\beta_1 >> U_1 \tag{13.17}$$

$$\text{and} \quad U_1' \simeq M\, G_{10}\beta_1 \tag{13.18}$$

The amplitude of the angular swing of a galvanometer as a function of angular frequency ω, for a constant amplitude sinusoidal alternating current input, taken relative to the amplitude obtained at $\omega = 0$, is obtainable from equation

(13.5) as

$$\frac{A}{A_o} = \frac{1}{1 - \omega^2 \frac{P}{U} + j\omega\frac{K}{U}} \qquad (13.19)$$

Hence if when obtaining the curve of Fig. LXXV1 (a), the angular frequency was noted at which a 90^o phase shift occurred in A/A_o, then from this value of ω_{90} and the value of $|A/A_o|$ occurring there and from equation (13.19),

$$U_1' = \omega_{90^o}^{\ 2} P$$

$$U_1' = \omega_{90^o} K_1' \left| \frac{A}{A_o} \right|_{90^o} \qquad (13.20)$$

Substituting values obtained in practice for the galvanometer amplifier at the 90^o phase shift point,

$$U_1' = 96_5 P_1 \text{ kg m}^2 \text{ sec}^{-2}$$

$$U_1' = 60 K_1' \text{ kg m}^2 \text{ sec}^{-2} \qquad (13.21)$$

(13.3.1)

Galvanometer Amplifier Constants.

Communications were made with H. Tinsley & Co., Ltd. London, in order to obtain the moment of inertia P_1 for the liquid filled galvanometer suspension used in the galvanometer amplifier. The value of P_1 was given as 0.016 gm cm^2

$$\text{whence} \quad U_1' = 1.54 \times 10^{-6} \text{ Nm/rad} \qquad (13.22)$$

after substitution in equation (13.21a)

M was determined experimentally by passing a known direct current through the primary galvanometer and measuring the direct current output M_1 under steady-state conditions. A knowledge of the approximate direct current deflection sensitivity $Si_{o_{DC}}$ of the liquid filled galvanometer used in the galvanometer amplifier and the associated optical constant ψ_o m/rad. of the system used in obtaining this value gives

$$M = \left.\frac{I_{out}}{I_{in}}\right|_{\text{No F.B.}} \frac{\psi_o}{Si_{o_{DC}}} \quad \text{amps/radian} \qquad (13.23)$$

Substituting practical values gave

$$M = 7.0 \times 10^{-2} \quad \text{amps/radian} \qquad (13.24)$$

(13.3.2)

Secondary Galvanometer Constants.

The only information required of the secondary galvanometer is its direct current sensitivity $Si_{2_{DC}}$. This was found experimentally to be approximately $2.\ 10^9$ mm/amp.

(13.4)

Response of Galvanometer System at Alternating Loading Current Frequencies Used.

During the measurements made to determine the curves of Fig. LXXV1 (a) and (b) it was found

(a) that the presence of the secondary galvanometer in the overall system did not affect the frequency response of the galvanometer amplifier in either relative amplitude or phase in the frequency range used,

(b) that the frequency response of the galvanometer amplifier was that of a second order system, with a roll-off of 12 db/octave and a 'corner' frequency of approximately 6 c/s, and

(c) that the frequency response of the secondary galvanometer was that of a second order system, with a roll-off of 12 db/octave and a corner frequency of approximately 0.4 c/s.

Since the two galvanometer systems were found to be virtually independent, the frequency response of the overall galvanometer system was the product of the frequency responses of the galvanometer amplifier and the secondary galvanometer. Thus the relative amplitude of response of the overall galvanometer system may be calculated to be approximately - 74 db at 50 c/s, which gives a current sensitivity at 50 c/s of

$$Si_{50\ c/s} \simeq 7.1 \times 10^{-2} \text{ mm/uA peak,} \qquad (13.25)$$

for a direct current sensitivity of 80 000 mm/uA.

In the Kelvin Bridge used for A.C. load coefficient work on the respective Unknowns of Chapters 4 and 5, the alternating voltage appearing across the galvanometer terminals was approximately 0.55 volts in the worst case (36 watts in R5). Under these conditions the supply frequency of the alternating loading current was 40 c/s so that the approximate current sensitivity of the overall galvanometer system used there was

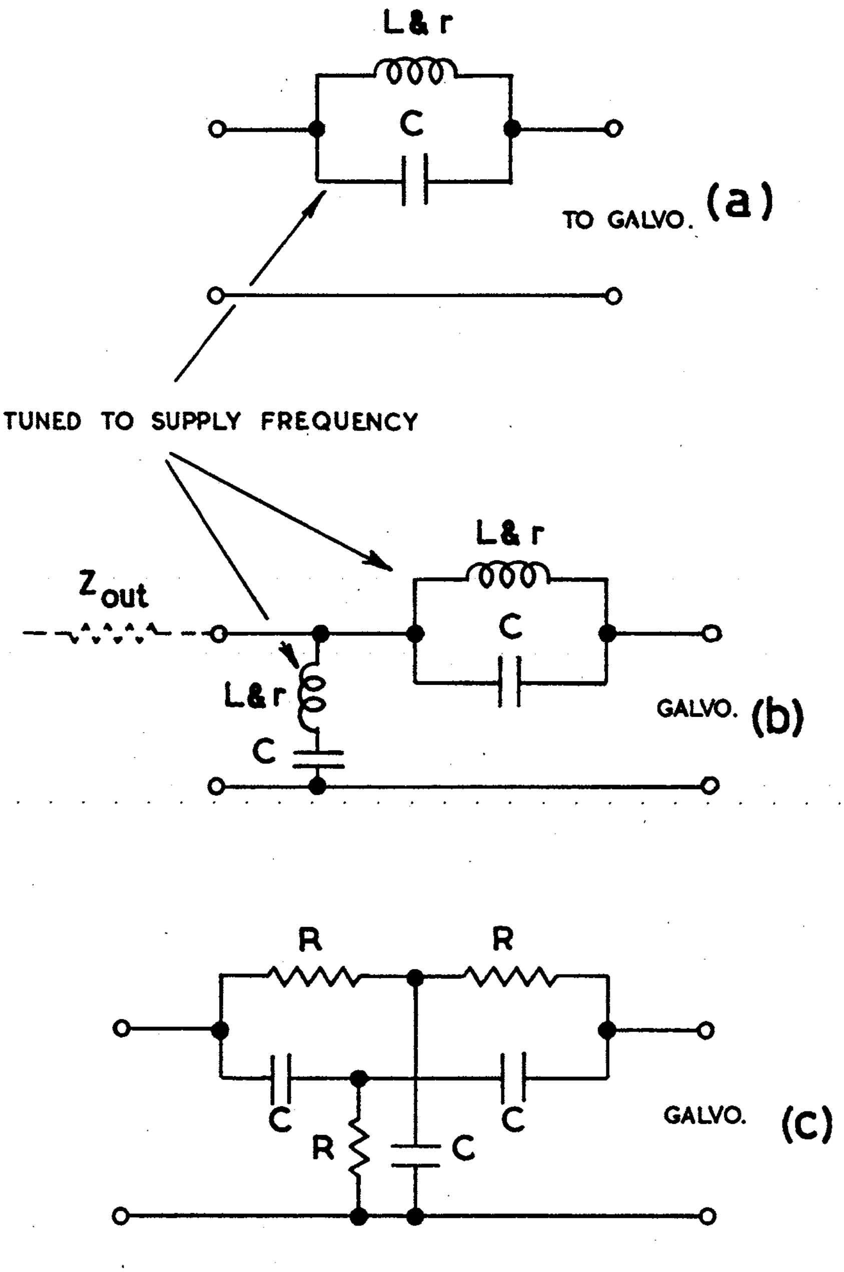
$$Si_{40\ c/s} \simeq 1.8 \times 10^{-1} \text{ mm/uA peak} \qquad (13.26)$$

It was found during load coefficient measurements on R5 that the presence of galvanometer fluctuations at 6 amps loading was becoming objectionable, although bridge balance could still be made in spite of its presence.

(13.5)

Mu-Metal Series Inductor.

The ideal specification of the transfer admittance versus frequency for any network between the galvanometer amplifier



Frequency selective 4-terminal networks.

Fig LXXVII.

input circuit and the loading current supply has a zero at all frequencies. Such a system is discussed in Chapter 14, but as an alternative, the specifications of the transfer admittance for a four terminal network to be connected between the galvanometer amplifier input circuit and the bridge detector terminals may be considered.

(a) Zero at the frequency of the alternating loading current supply and at all other frequencies except zero frequency (D.C.)

(b) Pole at zero frequency.

Similarly, the driving point admittance of this network with the galvanometer circuit removed should have a zero at zero frequency.

There are no passive element solutions, which will satisfy the above specifications, hence in practice the specifications must be relaxed and modified to include the practical difficulty of obtaining resistance-free inductors etc, while still maintaining simple passive circuit arrangements. Several different possibilities arise. Three simple arrangements are shown in Fig. LXXV11 (a) to (c). Comparison of Fig. LXXV11 (a) and (b) will show that the former is the simpler and easier to construct, it being the network used in the measurements carried out throughout the thesis. The parallel-tuned circuit is tuned to the alternating loading current supply frequency so as to present a transfer impedance of $Q_o \omega_o L$ at reasonable values of Q_o. The circuit of Fig. LXXV11 (b) however, has an extra series tuned circuit, which will act as a very low impedance at the loading current supply frequency and hence, work as an attenuator in association with the output impedance of the bridge. This arrangement gives two stages of attenuation, but the added cost and complication of winding two mu-metal inductors and of providing magnetic shields in order to isolate each inductor from 50 c/s mains pick-up and each other, is only warranted if the circuit of Fig. LXXV11 (a) is unsatisfactory.

When high-permeability magnetic materials are unavailable for use in the proposed inductors, or if the use of tuned LC filter sections are unsuitable, the circuit configuration of Fig. LXXV11 (c) may be used. The RC twin-tee may be adjusted by suitable choice of circuit component values such that there is a frequency of zero transmission at the alternating loading current supply frequency. The transfer impedance at zero frequency is finite. This transfer impedance may be made as low in value as desired, provided that capacitors of sufficient size are available. The twin-tee therefore, if properly constructed, will provide a true transmission zero at the loading current supply frequency and present a D.C. output resistance in addition to the D.C. output resistance of the bridge circuit, which may be neglected. The disadvantage of the twin-tee, is that the transmission null is very sharp, and if circuit components, or the frequency of the alternating loading current supply should change appreciably during the course of measurements, then finite transmission of the loading current may occur.

The great advantage of the R.C. twin-tee on the other hand, is the absence of any magnetic materials in the filter and hence its relative insensitivity to stray magnetic fields. Its cost may also be less, although especially wound resistors and selected capacitors may have to be used, in order to obtain the desired transmission zero at the required frequency. Any error in a circuit component value will have the effect of shifting slightly the frequency of the transmission null and of providing a transmission null at this frequency which is not a transmission zero. However the overall effect will in general be small, provided reasonable care has been taken in choosing suitable resistors and stable capacitors.

Because of the uncertainty as to exactly what loading current frequency would be used throughout load coefficient measurements made in this thesis, the circuit of Fig. LXXV11 (a) above was used. This circuit possessed the advantage that its resonant frequency could be changed relatively easily, simply by replacing the turning capacitor used.

The inductor was wound using a laminated mu-metal core. This provided a relatively high inductance value due to the large relative permeability of the core material. It is generally considered essential when winding an inductor for use in cases where any direct current will flow, to include a finite air gap in order to prevent core saturation. In this case, the direct current being passed through the inductor near bridge balance was very small and hence the resulting saturation effects were negligible.

The direct current resistance value of the inductor was required to be as small as possible, so that the direct current detector sensitivity was not greatly reduced. The design was aimed at producing an inductor whose direct current resistance was below 100 ohms. This design limitation produced an inductor with 2120 turns of 30 B & S 'Bicalex' enamel copper wire. The measured direct current resistance was 73 ± 1 ohms while the measured value of inductance was approximately 360 henrys (small signal).

The resonant series impedance of a parallel-tuned circuit consisting of an inductor of moderate Q_o shunted by a tuning capacitor, is increased above that of the inductor alone, by a factor approximately equal to Q_o. Hence as high a value of Q_o as possible was required as well as a relatively large value of series inductance. Measurements made on the parallel-tuned circuit which had been adjusted for resonance at 50 c/s, gave a series impedance value there of approximately 220 kilohms while contained in a mu-metal shield; which corresponds to a Q_o of the order of 2.3. The value of the resonant impedance will vary approximately as ω_o^2 at low frequencies, so that the transfer impedance present in Fig. LXXV11 (a) at 40 c/s would have been approximately 140 kilohms.

Calculating the value of the relative permeability obtained for the mu-metal core from the measured inductance and coil dimensions gave

$$\mu_r = 25\,000 \tag{13.27}$$

This was lower than was expected, but it was realised that no special care had been taken in producing neat fitting butt joints between the mu-metal E and I laminations. Inspection of a curve provided by the mu-metal manufacturer for their product,[44] showed that with an air gap equivalent to 0.1 per cent of the mean magnetic path length, the effective permeability was reduced to about 50 per cent of its theoretical value, for a theoretical μ_r of 1 000. The percentage reduction in effective permeability is even greater at very high values of theoretical permeability, for the same percentage air gap. 0.1 per cent of the mean magnetic path length in this case, only amounts to 0.12 mm and this order of air gap would be hard to reduce in practice, without specially machining the laminations prior to stacking.

(13.6)

The Presence of Vibration Effects and Zero Drifts in the Galvanometer Amplifier.

Vibration in the Tinsley Galvanometer Amplifier used was observed as an increase in the random noise level present on the secondary galvanometer spot.

The galvanometer amplifier was mounted on foam plastic on a bracket attached to a pillar of the outside wall of the building. This was found to be the most convenient position with an acceptable level of vibrational amplitude. Electrical connections were made to the galvanometer amplifier using loosely coiled springs, made from about 34 B & S copper wire, between the galvanometer amplifier and copper terminals mounted on a perspex sheet placed under the foam plastic. Heavier leads were taken from the copper terminals to their desired destinations.

The overall result of this type of mount was, that with the oil bath stirrers etc. in motion, there appeared to be negligible vibrational coupling between the laboratory bench and the galvanometer amplifier mounted on the wall. This fact was noticed during a course of measurements taken late at night, after normal building activities had ceased and whilst outside atmospheric conditions were calm.

During the day however, the normal level of vibration permeating through the walls of the building was substantially higher. Added to this was the fact that on blustery days with relatively high speed wind currents outside, it was found almost impossible to obtain as reliable bridge balances as were recorded during calm weather. The fact that the Measurements Laboratory, in which virtually all measurements were made, was situated on the 3rd floor, 4 levels from ground, meant that vibrational effects were magnified.

Measurements made on the short term galvanometer spot fluctuations with time, during a typical calm day, showed that the amplitude of the random vibrational movement, as observed on the secondary galvanometer spot, had a mean value of about $\pm$ 1 mm, with occasional bursts of about $\pm$ 2 mm. During blustery weather these figures could be multiplied by factors of 3 or 4. Comparison of the level of thermal motion predicted for the galvanometer system in section (13.2) with the observed level of fluctuations shows that the major proportion of this level is due to vibrational effects.

It was found while performing measurements, that the observer was able with a bit of practice, to produce a certain degree of mental integration in order to overcome these effects. Hence the presence of such relatively large vibrational effects was not as serious as might be thought. This observation is in agreement with the experience obtained by Johnson.[45]

The zero stability of the detection system was found to be excellent, even with the galvanometer amplifier parallel feedback control adjusted to its minimum value, and with the use of such a sensitive secondary galvanometer. It was found that, provided the galvanometer amplifier was allowed about 30 minutes before use after switching on, the zero of the overall galvanometer system drifted less than 10 cm in the course of a whole days measurement. There was a small amount of oscillatory drift with a period of about 1 hour, but its amplitude was relatively small and its annoyance factor minimal.

(13.7)

Detector Response Obtained While Measuring 0.1 ohm Special Constantan Resistor: Use of Integration.

A typical chart recording taken from the Moseley Autograph Pen Recorder used in the secondary galvanometer system is shown in Fig. L1V of Chapter 9. The presence of the large initial kick as the D.C. supply current is reversed, followed by the slow decay as the Peltier Thermal Effects reversed, may be seen. It will be appreciated that, with thermal time constants present of the order shown in Fig. L1V, the use of the normal secondary spot galvanometer would be almost useless, except for balances executed with a measurement accuracy of the order of 1 part in 10^4 of the Unknown.

Thermal effects experienced at 2.5 watts power dissipation in the 0.1 ohm special constantan resistor were greater in magnitude, but the thermal time constants obtained were of the same order. In all these measurements, vibration and thermal noise in the galvanometer amplifier was substantially less than thermal instability. The use of the Moseley Autograph Chart Recorder provided a continuous summation of the thermal effects, thus enabling a mean deflection to be obtained after the decay of the large initial kick experienced upon supply reversal. The recorder did not reduce the bandwidth of the detection system however, since the presence of relatively short term fluctuations may still be seen on the output trace. Its purpose was to provide a display system where low frequency effects or long term trends could be observed. The factor of improvement produced by the recording detector may be seen by inspection to be of the order of 4 or 5, since the normal limits to the improvement, such as the temperature stability of the oil baths used, bridge resistance stability and galvanometer zero stability with time were all considered to be relatively small effects in this case.

CHAPTER 14

Development of a System for the Elimination of Galvanometer A.C. and its Effects Using a Balanced Bridge.

(14.1)

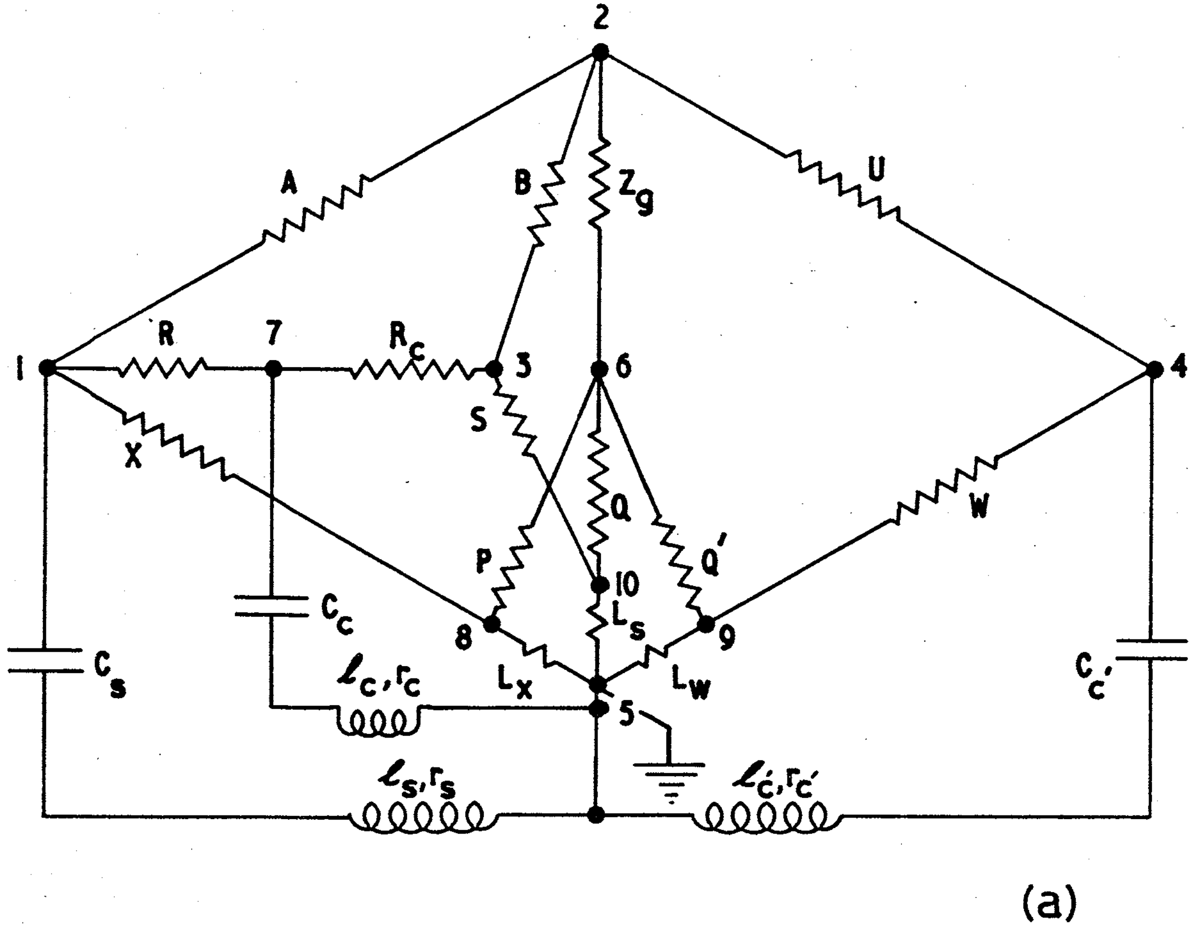
The alternating current passing through the galvanometer circuit can be reduced by using a single frequency rejection system. The more difficult in practice, is to eliminate the effect of the amplitude fluctuations in the alternating loading current on the detection system. The uncertainty in balance point detection may be reduced with the use of an integrating galvanometer, which gives the effect of reducing the bandwidth of the detecting system, but there is a practical limitation on the allowable integration times used, governed mainly by the temperature stability of the bridge environment, the stability of the circuit resistances and the stability of thermal emfs in the bridge and galvanometer circuit. The presence of amplitude fluctuations in the applied alternating loading current places a limit on the maximum possible accuracy in load coefficient measurement attainable. Thus it is important to devise a system of loading which reduces the effects of these practical limitations to a level where they do not affect the attainable accuracy of measurement.

The method of galvanometer alternating current elimination described here has several disadvantages, but theoretically eliminates all 50 c/s galvanometer current and the effect of the fluctuations in the loading current.

(14.2)

System Description.

The proposed system consists of a third pair of bridge arms which are connected to the junction of the A and B arms with



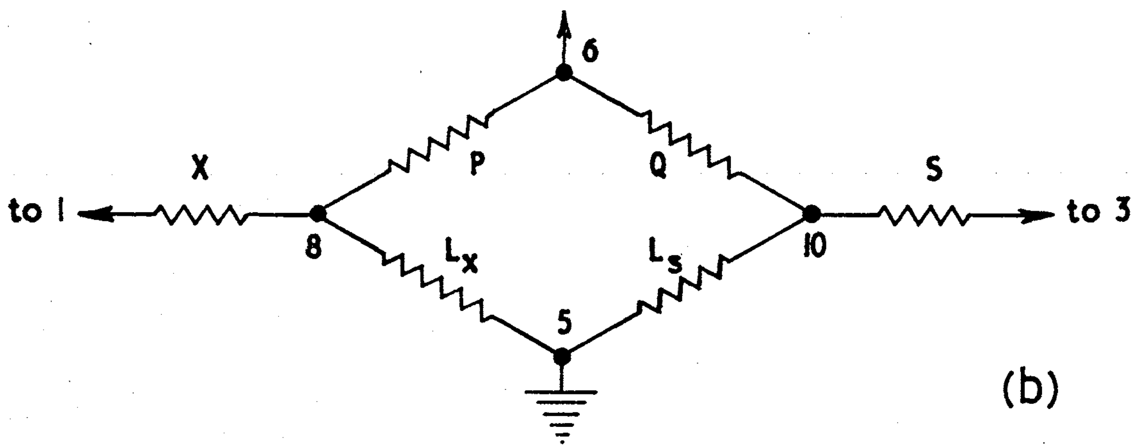


Fig LXXVIII

Circuit Diagrams

showing (a) Modifications required to Fig XIX. in order to eliminate alternating loading currents from the galvo.
(b) Link Circuit Wheatstone Bridge.

the galvanometer circuit and a point on the link of a Kelvin Bridge. The junction of these two arms is connected to an auxiliary winding on the supply transformer through a variable impedance Z_c' as shown in Fig. LXXV111 (a). The other Kelvin Bridge arms and impedances, including the capacitive kick elimination network and the galvanometer series resistance, are shown as the impedances between the A and X arms.

If the link resistances L_x, L_w and L_s are small in comparison to their corresponding resistances X, W and S, then the nodes 5, 8, 9 and 10 of Fig. LXXV111 (a) may be considered to coincide and so simplify the circuit considerably. Under these conditions it can be shown that there will be zero current flowing through the galvanometer impedance Z_g at all supply frequencies if

$$M' = \frac{A}{U} = \frac{X}{W} = \frac{r_s}{r_c'} = \frac{\ell_s}{\ell_c'} = \frac{C_c'}{C_s} = \frac{E_s}{E_c'} \qquad (14.1)$$

$$\frac{A}{B} = \frac{X}{S} = M \qquad (14.2)$$

$$\frac{r_c}{r_s} = \frac{\ell_c}{\ell_s} = \frac{C_s}{C_c} = k \qquad (14.3)$$

and

$$R_c = (kM - 1)\frac{BS}{B + S} \qquad (14.4)$$

If the link resistances L_x, L_w and L_s are not negligible in relation to the resistances X, W and S, then the conditions required for zero galvanometer alternating current are similar to those above, but given by

$$M' = \frac{A}{U} = \frac{X}{W} = \frac{P}{Q'} = \frac{L_x}{L_w} = \frac{r_s}{r_c'} = \frac{\ell_s}{\ell_c'} = \frac{C_c'}{C_s} = \frac{E_s}{E_c'} \qquad (14.5)$$

and

$$\frac{A}{B} = \frac{X}{S} = \frac{P}{Q} = \frac{L_x}{L_s} = \frac{r_s}{r_c} = \frac{\ell_s}{\ell_c} = \frac{C_c}{C_s} = 1 \qquad (14.6)$$

Condition (14.6) implies that $R_c = (kM - 1)\ \frac{BS}{B + S} = 0$ (14.7)

Equations (14.2), (14.3), (14.4), (14.6) and (14.7) may be seen to be the conditions required for correct adjustment of the capacitive kick correction circuit. Equation (14.1) may appear to present a large range of choice for the factor M', but in practice a value of unity would probably be chosen, since the equalities indicate that the currents to be supplied from the generators E_s and E_c' will be equal. The use of unity ratio also has the advantage that the supply transformer used may be wound with two identical secondary windings, making

$$\ell_s \simeq \ell_c' \quad \text{and} \quad r_s \simeq r_c' \qquad (14.8)$$

without the necessity for extra trimming components.

(14.2.1)

Disadvantages of the System.

The presence of the alternating current balancing arms produces a low resistance path to direct currents between the link and the junction of the A and B arms with the galvanometer circuit.

In order to eliminate this effect on the bridge balance condition obtained, it is necessary to obtain a relationship between the arms of the direct current Kelvin Bridge such that

$$\frac{A}{B} = \frac{X}{S} = \frac{P}{Q} = \frac{L_x}{L_s} = M \qquad (14.9)$$

If node 5 of Fig. LXXV111 (b) is adjusted such that the link resistances L_x and L_s are given by

$$L_x = L_{xo}\,(1 + \Delta) \tag{14.10}$$

$$L_s = L_{so}\,\left(1 - \frac{L_x}{L_{so}}\,\Delta\right) \tag{14.11}$$

where L_{xo} and L_{so} are the link resistances adjusted such that

$$\frac{L_{xo}}{L_{so}} = \frac{P}{Q} \tag{14.12}$$

then the presence of the network consisting of the resistances U, W, Q' and L_w between the circuit nodes 2 and 5 of Fig. LXXV111 (b) will mean that the balance condition of the direct current Kelvin Bridge circuit will be in error by an amount equivalent to ∂X_1 proportional parts of the Unknown where

$$\partial X_1 \simeq \frac{L_{xo}}{X} \cdot \frac{A}{R_{leak}} \cdot \Delta \qquad \text{proportional parts} \tag{14.13}$$

provided

$$\begin{aligned} X &\ll A \\ L_{xo} &\ll X \\ L_{so} &\ll S \\ \Delta &\ll 1 \end{aligned} \tag{14.14}$$

where R_{leak} is the resistance of the alternating current balancing arms between the circuit nodes 2 and 5, and

A is the resistance value of the A arm which gives the correct balance condition of the Kelvin Bridge with the Link Circuit Wheatstone Bridge correctly balanced.

For similar circuit inequalities, if the P arm of Fig. LXXV111

(b) is adjusted such that

$$P = P_o (1 + \delta P) \tag{14.15}$$

where P_o is the resistance of the P arm such that

$$\frac{A}{B} = \frac{X}{S} = \frac{P_o}{Q} = \frac{L_x}{L_s} \tag{14.16}$$

then the presence of the network consisting of the resistances U, W, Q' and L_W between the circuit nodes 2 and 5 of Fig. LXXV111 (b) will mean that the balance condition of the direct current Kelvin Bridge circuit will be in error by an amount equivalent to δX_2 proportional parts in the Unknown where

$$\delta X_2 \simeq -\left(1 + \frac{AB}{A + B} \cdot \frac{1}{R_{leak}} \right) \frac{L_x}{X} \; \delta P \tag{14.17}$$

proportional parts

If the resistances of the alternating current balancing arms are adjusted such that U, Q' $\gg$ W $\gg$ L_W and M' = 1

then
$$\frac{A}{R_{leak}} \simeq \frac{A}{U} = M' \tag{14.18}$$

Hence, equations (14.13) and (14.17) become

$$\delta X_1 \simeq \frac{L_{xo}}{X} \cdot \Delta \quad \text{proportional parts} \tag{14.19}$$

and
$$\delta X_2 \simeq -\left(1 + \frac{B}{A + B} \right) \frac{L_x}{X} \; \delta P \tag{14.20}$$

proportional parts respectively.

Equations (14.19) and (14.20) indicate the necessity of keeping the ratio of L_{xo}/X very small so that the presence of out-of-balances Δ proportional parts in the Link Circuit Wheatstone Bridge and δP proportional parts in the P arm will produce small effects on the bridge balance condition of the main direct current Kelvin Bridge. The presence of finite values of Δ or δP will generally have a negligible effect on the value of the resistance change obtained while performing load co-efficient measurements on the Unknown, provided they are small initially and remain constant throughout.

This limitation means that if the A arm of the direct current Kelvin Bridge is adjusted in order to obtain bridge balance, then the P arm and the link resistance L_x must be adjusted also in order to satisfy equation (14.9). Thus it appears desirable to perform bridge balance with the adjustment of the X or S arms and to keep the resistances A, B, P, Q, L_x and L_s as nearly constant as possible once they have been set initially.

The balancing of the Link Circuit Wheatstone Bridge of the direct current Kelvin Bridge is sufficiently important in keeping δX_1 of equation (14.19) small, that the inclusion of a special auxilliary galvanometer is considered desirable in any practical circuit. Thus a periodic check may be made on the Link Circuit Bridge balance. The P and Q arms must also be adjusted for correct resistance value, and this adjustment would be made using the usual method of removing and replacing the link.[4]

The use of this method in load coefficient measurements means that the alternating loading current supply circuit requirements become more complex. Firstly, an extra standard resistor of equal nominal value and power rating to the Unknown is required in order to provide the W arm. Secondly, a second set of supply capacitors is required and thirdly, the loading current supply transformer must be equipped with a second secondary winding. These points become extremely pertinent when performing load coefficient measurements on heavy current standard resistors. Here the cost of providing the extra capacitors may amount to a considerable expense and the extra transformer winding will double the size of the equivalent single winding transformer. If the Unknown and Standard resistance values are sufficiently small that the link resistances L_x, L_s and L_w become finite, calculations have shown that the capacitive kick correction circuit capacitor C_c must be equal to the capacitance of the D.C. blocking capacitor in the alternating loading current supply circuit C_s (see equation (14.6)). This requirement will add extra expense.

(14.3)

Practical Investigation.

A Kelvin Bridge was constructed from standard resistors and decade resistance boxes in order to devise a sequence of bridge balances which would provide a means of correctly adjusting the system described above. The A and B arms consisted of 1 000 ohm standard resistors, shunted by manganin decade resistance boxes set initially at (10)0 000.0 ohms. The P and Q arms consisted of 100 ohm standard resistors shunted by manganin decade resistance boxes set initially at (10) 000.0 ohms while the X and S arms consisted of 1 ohm standard resistors shunted by manganin decade resistance boxes set at 5 000 ohms. The link resistance used was designed to be finite in relation to the X and S arms and consisted of two series connected 0.1 ohm standard resistors shunted by decade resistance boxes set initially at (10) 00.0 ohms. The main bridge balance galvanometer used had a current sensitivity of 80 000 mm/uA, while the direct current supply to the bridge was from a 2 volt cell in series with an adjustable resistance.

First bridge balances were performed with the adjustment of the P arm shunt resistance while the link was open, followed by the adjustment of the A and P arm shunts in unison, while the link was closed. This procedure was repeated until no difference in bridge balance condition was noticeable with the link open or closed.

With the link circuit closed, the next balance performed was that of the Link Circuit Bridge with the adjustment of the L_s shunt resistance box. It was found that a low impedance galvanometer with a relatively high current sensitivity was required in order to reduce the damping of the circuit output resistance on the galvanometer movement and to obtain sufficient detection sensitivity. Both these features were obtained with the use of a second Tinsley Galvanometer Amplifier Type 5214.

At this stage it may be said that

$$\frac{A}{B} = \frac{P}{Q} = \frac{X}{S} = \frac{L_x}{L_s} \qquad (14.21)$$

The next step was to wire in the U, Q' and W arms and the link resistance L_w. These bridge components consisted of 1000, 100, 1.0 and 0.1 ohm standard resistors, shunted by decade resistance boxes set at (10)0 000.0 ohms, (10) 000.0 ohms, 5 000.0 ohms and (10) 00.0 ohms respectively. A check was made on the main bridge balance to ensure that the connection of these circuit elements did not upset the main bridge balance obtained previously.

The alternating loading current supply circuit, the capacitive kick correction circuit and the auxilliary alternating loading current supply circuit were connected to their appropriate bridge points. The supply transformer was fed from a variable auto transformer and contained a 52.5 - 0 - 52.5 volt centre tapped secondary winding with a 3 kVA rating. The series capacitors used were Ducon 100 uF 1S1000 paper capacitors with Danbridge decade capacitance boxes connected across each to provide a means of trimming.

The main bridge balance was then re-checked while the capacitive kick correction circuit was correctly adjusted. Following this the main galvanometer was replaced with a G.R. battery powered Tuned Null Detector Type 1232-A, S/N 853, which was tuned to mains frequency. The alternating current supply was then switched on and the loading current level in the X arm adjusted to approximately 1 amp. The tuned null detector showed a slight indication above noise level, which was reduced to noise level with the adjustment of the series capacitor of the impedance Z_s. The main bridge galvanometer was replaced and the main D.C. bridge balance checked. Special notice was taken of the absence of 50 c/s vibration and low frequency fluctuations in zero position from the secondary galvanometer spot.

(14.4)

Assessment of Performance.

The results of the measurements described in section (14.3) indicated that the proposed circuit was a practical proposition and usable in practice. The two noticeable disadvantages encountered were that the time required for setting up is considerable and that the resistances of the auxilliary alternating current supply circuit place importance on extra circuit balances.

This method of galvanometer alternating current elimination is of greatest advantage when the Unknown and Standard are equal to or greater than 0.1 ohm. Under these conditions the ratio of link to Unknown can be kept extremely small and hence the importance of the Link Circuit Wheatstone Bridge balance correspondingly reduced. Also the expense of providing suitable capacitances will be relatively small as will be that of providing a suitable supply transformer.

CONCLUSION

Effects have been discussed which were considered most likely to produce a significant difference between the two changes obtained in the D.C. resistance of a standard resistor from its no-load value when loading firstly, with direct current and secondly, with an equal effective current which was predominantly alternating current but which also employed a small direct measuring current. All these difference effects were estimated to be small, but the difficulty of calculating exact magnitudes in practical standard resistors, where complex heating and cooling mechanisms generally exist, meant that comparative load coefficient measurements were required under both loading conditions in order to conclude that the A.C. loading technique was in fact reasonable. The results of the measurements undertaken indicated no measurable difference between the resistance changes produced by either loading technique at equal effective current levels. All apparent differences could be explained with reference to the loading current measurement uncertainties and environmental effects.

Resistance changes of approximately $\pm$ 1 part in 10^7 were detectable in the 1 ohm standard resistors L-259235 and R5 when using a D.C. measuring current of 0.3 amp at no-load and under A.C. loading conditions.

A.C. and D.C. load coefficient measurements were made on the Cambridge 1 ohm manganin standard resistor L-259235 at power dissipation levels up to 5 watts. Its load coefficient was found to be approximately + 250 parts in 10^7 per watt and at 5 watts, differences between its A.C. and D.C. load coefficient values of greater than 12 parts in 10^7 would have indicated a measurable difference between the resistance changes produced by each loading technique. Differences approaching 12 parts in 10^7 were found at 5 watts, but the majority were found to be substantially lower (see Fig. XXV1). The maximum allowable difference decreased as the level of power dissipation in

L-259235 decreased, but in every case the measured differences obtained fell below the maximum allowable.

Similar load coefficient measurements were made on the 1 ohm Evanohm high dissipation resistor R5 at power dissipation levels up to 36 watts, where its load coefficient was found to be approximately - 70 parts in 10^7 at 20 °C. The maximum allowable difference between the resistance changes obtained under A.C. and D.C. loading conditions at 36 watts was approximately 6 parts in 10^7. One difference of approximately 8 parts in 10^7 was obtained but this was not considered excessive when the spread in the results obtained at 36 watts under A.C. loading conditions was observed and it was realised that the A.C. and D.C. load coefficient measurements were made with substantial time separation. The spread of the resistance changes from no-load obtained at other power dissipation levels were accounted for with reference to the current measurement uncertainties, stirrer speed fluctuations and bridge balance detection uncertainties.

Resistance changes of approximately $\pm$ 1 p.p.m. were detected in the Tinsley 0.005 ohm manganin air cooled shunt S/N 101268 and the Cambridge 0.01 ohm heavy current manganin standard resistor S-1161 using a no-load D.C. measuring current of 10 amps.

A.C. and D.C. load coefficient measurements were made on the Tinsley 0.005 ohm shunt up to a power dissipation level of 4.5 watts. Its load coefficient between 10 amps D.C. and full load was found to be approximately - 290 p.p.m. at 27.8 °C. The spread of the resistance changes obtained between 10 amps D.C. and full A.C. and D.C. loading were accounted for with reference to the temperature stability of the environment and the no-load temperature coefficient of the Unknown, the loading current measurement uncertainties and the bridge balance detection uncertainties.

A.C. and D.C. load coefficient measurements were made on the Cambridge 0.01 ohm heavy current standard resistor up to a power dissipation level of 36 watts. Its resistance change between 10 amps D.C. and 36 watts was found to be approximately + 95 p.p.m. at 24 ^{o}C. The spread of resistance changes obtained at all power dissipation levels could be explained with reference to the loading current measurement uncertainties and the bridge balance detection uncertainties.

A special 0.1 ohm constantan resistor was constructed in an attempt to obtain a resistor in which equal effective alternating and direct loading currents might produce different measured D.C. resistance changes with load. Measurement uncertainties due to large thermal effects encountered in this resistor greatly exceeded the uncertainties produced by the loading current measurement uncertainties etc. These effects necessitated a reduction in the bridge balance sensitivity and an increase in the detector integration time used. Resistance changes of approximately $\pm$ 2 p.p.m. were detectable in the special constantan resistor at a no-load D.C. level of 2 amps, using an integration time of $\frac{3}{4}$ minute between D.C. reversals. Under a direct current loading condition of 2.5 watts, changes of approximately $\pm$ 1 part in 10^5 in the Unknown were detectable, while at 2.5 watts of A.C. loading, using a D.C. measuring current of 2 amps, changes of approximately $\pm$ 2 parts in 10^5 were detectable. The differences in the measured resistance change obtained for this resistor under D.C. and A.C. loading conditions were all within the range of $\pm$ 3 parts in 10^5 of the Unknown estimated to be the maximum range of uncertainty.

A.C. load coefficient measurements were made on several other standard resistors in order to obtain the corrections to be added to account for the finite power dissipation in these standard resistors when investigating the resistance changes of the standard resistors mentioned above.

An investigation was made of the change in the load coefficient obtained for the Cambridge 1 ohm standard L-259235 at 1 watt with an increase in loading current frequency from 40 c/s to 3 000 c/s. Changes in resistance of approximately $\pm$ 1 part in 10^7 were detectable in L-259235 and the results obtained showed there to be no measurable change in load coefficient with increase in loading current frequency to 3 000 c/s.

Parallel feeding of the alternating loading current to the Unknown was considered desirable as the presence of this circuit configuration produced the lowest effect on the D.C. resistance value of the Unknown as seen by the measuring circuit. The use of A.C. loading in a Kelvin Bridge also made several circuit modifications essential in order to overcome the problems of alternating currents in the galvanometer circuit and of capacitive discharge effects in the galvanometer upon D.C. supply reversal. The necessity for earthing the measuring bridge meant that some form of guard was required around high impedance bridge points.

The use of electrolytic capacitors in the alternating loading current supply circuit of heavy current standard resistors was shown to be a practical proposition, although better quality capacitors would be required in load coefficient measurements on higher valued standards. The Thoma Regulator and the multi-secondary heavy current step-down voltage transformer overcame the problems of providing continuous adjustment of loading current level and of providing loading currents of sufficient magnitude at the required voltage level.

The relative advantages of alternating over direct current loading have been discussed. The possibility exists of obtaining the measurement accuracies of Miller's Method[1] while still obtaining the advantages of the A.C. loading technique. A V.L.F. measuring current would be used in association with a substantially higher frequency loading current. Miller's Method[1] may be used on low valued resistance standards for the measurement of resistance change, even though the presence of Thermo-electric Effects may produce errors in absolute resistance measurements.

A modification of the direct current Kelvin Bridge has been presented which theoretically eliminates all adverse effects of the alternating loading current and its changes in amplitude from the galvanometer circuit. The disadvantages of the system described and the procedure required for its setting up are discussed.

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