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FINITE ELEMENT MODELS FOR TWO-DIMENSIONAL STEADY AND TRANSIENT GROUNDWATER FLOW-PROGRAMS, CASE STUDIES, AND COMPARISON WITH FINITE DIFFERENCE METHOD

by

F. GHASSEMI

REPORT NO. 164
December 1984

THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF CIVIL ENGINEERING

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8. ABSTRACT

This report presents two computer programs called FIELS and TMATFE for simulation of two-dimensional steady and transient groundwater flow by the finite element method.

These models are sufficiently general and flexible to be applied to a wide range of problems defined by a range of characteristics including aquifer properties and boundary conditions.

The report contains the listing of both computer programs, written in FORTRAN IV, and for each model a test problem, with input data and selected outputs, is presented.

The FIELS model has been used for simulation of the Botany Basin aquifer, Sydney, Australia, assuming steady-state conditions, and the results compared with the results of a finite difference model.

The TAMTFE model has been used to predict the effects of three new pumping wells in the Botany Basin aquifer, during the next 10 years.

The report also compares the performance of the finite element and finite difference methods.

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SUMMARY

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The report contains the listing of both computer programs, written in FORTRAN IV, and for each model a test problem, with input data and selected outputs, is presented.

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The TMATFE model has been used to predict the effects of three new pumping wells in the Botany Basin aquifer, during the next 10 years.

The report also compares the performance of the finite element and finite difference methods.

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INTRODUCTION

In recent years a variety of powerful numerical techniques (Finite Difference, Finite Element, Linear Programming, Boundary Integral Equation, ...) have been developed for approximation of the partial differental equation of groundwater flow. These techniques are often used by hydrogeologists to predict the behaviour of aquifers.

Among the above mentioned methods, the finite difference method is a well known technique and published computer programs are available for this method. The finite element method however is not yet as well known to hydrogeologists.

This report is an effort to obtain, using the finite element method (Galerkin technique, with linear interpolation function and triangular elements), an approximate solution of the partial differential equations governing steady and transient flow in an aquifer with complex boundary conditions and heterogeneous properties.

The report also presents two computer programs for simulation of a two dimensional groundwater flow in steady and transient conditions. These programs are written in FORTRAN IV and are compatible with most computers. The data requirements and input format specifications are all structured in a general manner which could easily be adapted to many field problems.

1. TWO-DIMENSIONAL STEADY-STATE MATHEMATICAL MODEL BY THE FINITE ELEMENT METHOD (FEM)

1.1 Mathematical Description of the Method

The partial differential equation of two-dimensional steady groundwater flow, assuming that the Cartesian coordinate axes (x & y) are aligned with the principal components of the transmissivity tensor, can be represented as follows:

$$\frac{\partial}{\partial x}(T_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_y \frac{\partial h}{\partial y}) = Q(x,y) \qquad (1\text{-}1)$$

where:

x & y	are the cartesian coordinates	(L)
T_x & T_y	are the components of the transmissivity tensor	(L^2T^{-1})
h	is the hydraulic head	(L)
Q(x,y)	is the volumetric flux of recharge (+) or withdrawal (-) per unit area	(LT^{-1})

The problem consists of finding a function h satisfying this equation in the region R with boundary C (Fig. 1-1).

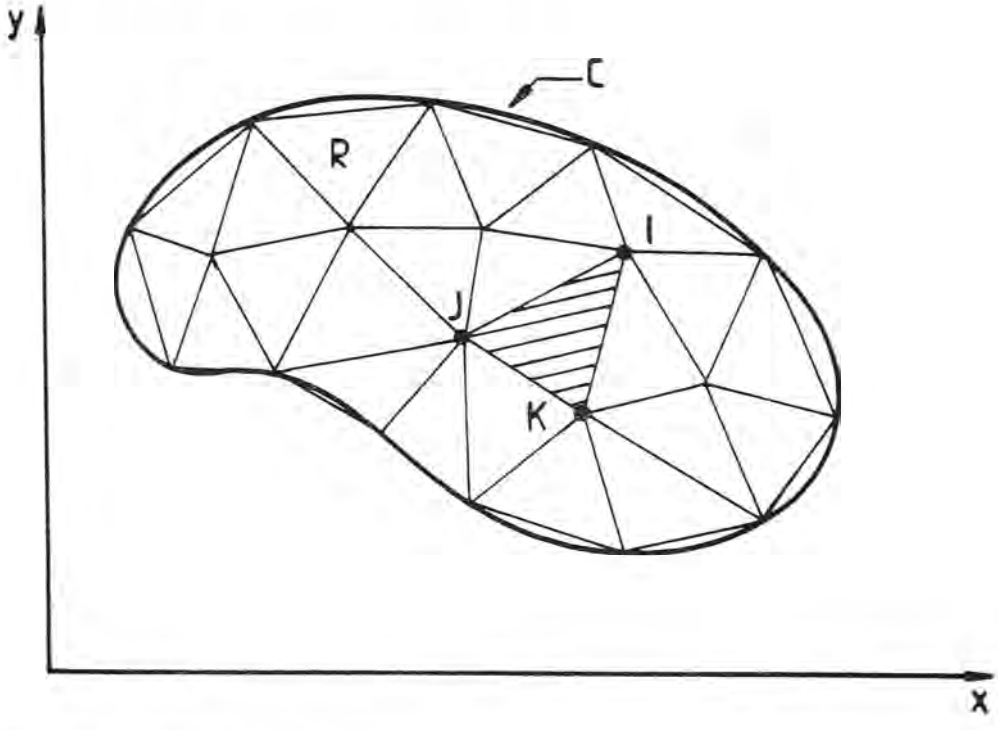


Fig. 1-1 Discretization of region (R) into triangular elements

The variational formulation of this problem requires the minimization of the functional[1]

$$F[h] = \iint_R \{\frac{1}{2}[T_x(\frac{\partial h}{\partial x})^2 + T_y (\frac{\partial h}{\partial y})^2] + Q(x,y).h\} dx.dy \qquad (1\text{-}2)$$

(1) - Remson et al (1971) and Wang et al (1982)

To solve this problem, the region R is divided into triangular elements and it is assumed that the potential h changes linearly over each element:

$$h = Ax + By + C \tag{1-3}$$

Applying this formula to the three vertices I, J and K of a triangle the following equations are obtained.

$$\begin{aligned} h_I &= AX_I + BY_I + C \\ h_J &= AX_J + BY_J + C \\ h_K &= AX_K + BY_K + C \end{aligned} \tag{1-4}$$

Solving these equations for the coefficients A, B and C the following is obtained:

$$\begin{aligned} A &= \frac{h_I(Y_I-Y_K) + h_J(Y_K-Y_I) + h_K(Y_I-Y_J)}{2\Delta} \\ B &= \frac{h_I(X_K-X_J) + h_J(X_I-X_K) + h_K(X_J-X_I)}{2\Delta} \\ C &= \frac{h_I(X_JY_K-Y_JX_K) + h_J(X_KY_I-X_IY_K) + h_K(X_IY_J-X_JY_I)}{2\Delta} \end{aligned} \tag{1-5}$$

where Δ is the area of the triangular element, and its value is:

$$\Delta = \frac{|(X_J-X_I)(Y_K-Y_I) - (Y_J-Y_I)(X_K-X_I)|}{2} \tag{1-6}$$

Substituting the values A, B and C into (1-3) yields:

$$h = N_Ih_I + N_Jh_J + N_Kh_K \tag{1-7}$$

where:

$$\begin{aligned} N_I &= [(X_JY_K-X_KY_J) + (Y_J-Y_K)x + (X_K-X_J)y]/2\Delta \\ N_J &= [(X_KY_I-X_IY_K) + (Y_K-Y_I)x + (X_I-X_K)y]/2\Delta \\ N_K &= [(X_IY_J-X_JY_I) + (Y_I-Y_J)x + (X_J-X_I)y]/2\Delta \end{aligned} \tag{1-8}$$

Each element contributes to the integral (1-2) and a set of linear equations is obtained by the conditions for minimizations

$$\frac{\partial F}{\partial h_m} = 0 \qquad \text{with } m = 1,2,3,\ldots\ldots,n \tag{1-9}$$

where n is the number of nodes, but since a node will generally be common to several elements, equation (1-9) may be written as:

$$\Sigma_e \frac{\partial F_e}{\partial h_m} = 0 \qquad \text{with } m = 1,2,3,\ldots\ldots n \tag{1-10}$$

F_e is the contribution of the eth element to the integral (1-2) and the summation is carried over all the elements having the node I in common.

Now it is necessary to develop an expression of F_e for the element IJK by substituting equation (1-7) into (1-2).

$$F_e = \iint_e \{\frac{1}{2}[T_x(\frac{\partial N_I}{\partial x} h_I + \frac{\partial N_J}{\partial x} h_J + \frac{\partial N_K}{\partial x} h_K)^2 + T_y(\frac{\partial N_I}{\partial y} h_I +$$

$$\frac{\partial N_J}{\partial_y} h_J + \frac{\partial N_K}{\partial y} h_K)^2] + Q(x,y)(N_I h_I + N_J h_J + N_K h_K)\}dx.dy \qquad (1\text{-}11)$$

The value of $\frac{\partial F_e}{\partial h}$ is to be computed for a typical I^{th} node common to several elements and ultimately the following should be computed:

$$\frac{\partial F}{\partial h_I} = \Sigma e \frac{\partial F_e}{\partial h_I} \qquad (1\text{-}12)$$

One term of this summation will yield the form of the other terms so it is only necessary to compute the contribution of one element. Differentiating equation (1-11) with respect to h_I results in:

$$\frac{\partial F_e}{\partial h_I} = \iint_e [T_x(\frac{\partial N_I}{\partial x} h_I + \frac{\partial N_J}{\partial x} h_J + \frac{\partial N_K}{\partial x} h_K).(\frac{\partial N_I}{\partial x}) +$$

$$T_y(\frac{\partial N_I}{\partial y} h_I + \frac{\partial N_J}{\partial y} h_J + \frac{\partial N_K}{\partial y} h_K) \cdot (\frac{\partial N_I}{\partial y}) + Q(x,y).N_I]dx.dy \qquad (1\text{-}13)$$

or using equation (1-8):

$$\frac{\partial F_e}{\partial h_I} = \iint_e \{T_x[(\frac{Y_J-Y_K}{2\Delta}) \cdot h_I + (\frac{Y_K-Y_I}{2\Delta}) \cdot h_J + (\frac{Y_I-Y_J}{2\Delta}) \cdot h_K] \cdot \frac{Y_J-Y_K}{2\Delta}$$

$$+ T_y[(\frac{X_K-X_J}{2\Delta}) \cdot h_I + (\frac{X_I-X_K}{2\Delta}) \cdot h_J + (\frac{X_J-X_I}{2\Delta}) \cdot h_K] \cdot \frac{X_K-X_J}{2\Delta} \qquad (1\text{-}14)$$

$$+ \frac{Q(x,y).[(X_J Y_K - X_K Y_J) + (Y_J-Y_K)x + (X_K-X_J)y]}{2\Delta}\} \cdot dx.dy$$

But as:

$$\iint_e dx.dy = \text{Area of element} = \Delta \qquad (1\text{-}15)$$

it follows that:

$$\iint_e [\frac{(X_J Y_K - X_K Y_J) + (Y_J-Y_K)x + (X_K-X_J)y}{2\Delta}] \, dx.dy =$$

$$\frac{(X_J Y_K - X_K Y_J) + (Y_J-Y_K)\overline{X} + (X_K-X_J)\overline{Y}}{2} = \frac{\Delta}{3} \qquad (1\text{-}16)$$

with:

$$\overline{X} = \frac{X_I+X_J+X_K}{3} \quad \text{and} \quad \overline{Y} = \frac{Y_I+Y_J+Y_K}{3}$$

Assuming $Q(x,y) = Q$ and Tx and Ty are constant in each element would result in:

$$\frac{\partial F_e}{\partial h_I} = M_I h_I + M_J h_J + M_K h_K + \frac{Q\Delta}{3} \tag{1-17}$$

where:

$$\begin{aligned} M_I &= \frac{1}{4\Delta}[T_x(Y_J-Y_K)^2 + T_y(X_K-X_J)^2] \\ M_J &= \frac{1}{4\Delta}[T_x(Y_K-Y_I)(Y_J-Y_K) + T_y(X_I-X_K)(X_K-X_J)] \\ M_K &= \frac{1}{4\Delta}[T_x(Y_I-Y_J)(Y_J-Y_K) + T_y(X_J-X_I)(X_K-X_J)] \end{aligned} \tag{1-18}$$

Assuming $T_x = T_y = T$ in each element would result in:

$$\begin{aligned} M_I &= \frac{T}{4\Delta}[(Y_J-Y_K)^2 + (X_K-X_J)^2] \\ M_J &= \frac{T}{4\Delta}[(Y_K-Y_I)(Y_J-Y_K) + (X_I-X_K)(X_K-X_J)] \\ M_K &= \frac{T}{4\Delta}[(Y_I-Y_J)(Y_J-Y_K) + (X_J-X_I)(X_K-X_J)] \end{aligned} \tag{1-19}$$

Equation (1-17) will be used to generate the set of linear equations, providing an approximate solution of equation (1-1). It should be mentioned that, if the term Q is defined in an element, the term $\frac{Q\Delta}{3}$ will represent the pumping or injection flow rate at each vertex of the element. On the other hand if the term $Q_I=Q$ is defined on the node I for all the neighbouring elements, the generation of the set of linear equations will be done by the algorithm:

$$\frac{\partial F_e}{\partial h_I} = M_I h_I + M_J h_J + M_K h_K \tag{1-20}$$

Then after the summation over all elements neighbouring node I, the value of Q_I will be added to the result of the summation.

It is also important to mention that equations (1-17) and (1-20) are flux equations.

To show the application of the above results, a triangular element IJK is considered (Fig. 1-2), with flux rates Q_I, Q_J and Q_K at its three vertices. For such a typical triangular element the following (1-21) set of linear equations is obtained.

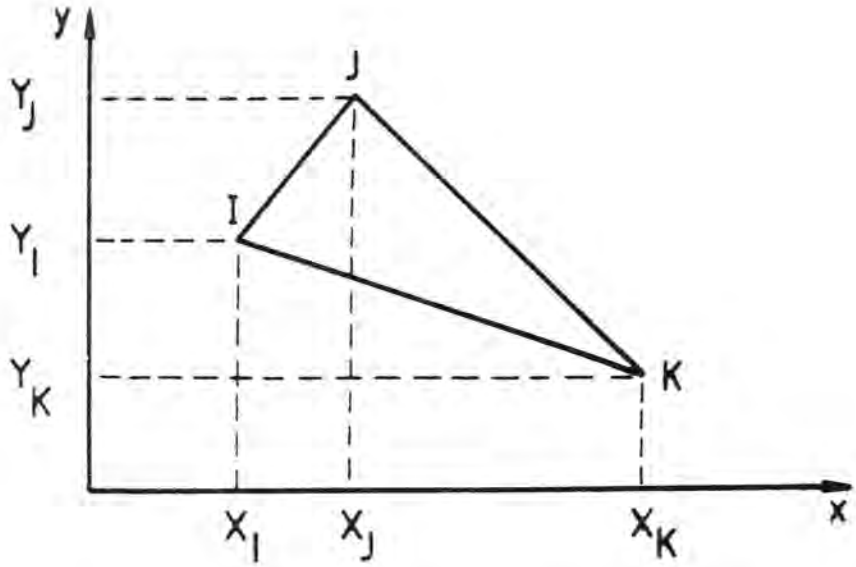


Fig. 1-2 A typical triangular region

$$\frac{1}{4\Delta}\begin{vmatrix} T_x(Y_J-Y_K)^2 + T_y(X_K-X_J)^2 & T_x(Y_K-Y_I)(Y_J-Y_K) + T_y(X_I-X_K)(X_K-X_J) & T_x(Y_I-Y_J)(Y_J-Y_K) + T_y(X_J-X_I)(X_K-X_J) \\ T_x(Y_K-Y_J)(Y_I-Y_K) + T_y(X_J-X_K)(X_K-X_I) & T_x(Y_I-Y_K)^2 + T_y(X_K-X_I)^2 & T_x(Y_J-Y_I)(Y_I-Y_K) + T_y(X_I-X_J)(X_K-X_I) \\ T_x(Y_K-Y_J)(Y_J-Y_I) + T_y(X_J-X_K)(X_I-X_J) & T_x(Y_I-Y_K)(Y_J-Y_I) + T_y(X_K-X_I)(X_I-X_J) & T_x(Y_J-Y_I)^2 + T_y(X_I-X_J)^2 \end{vmatrix} \cdot \begin{vmatrix} h_I \\ h_J \\ h_K \end{vmatrix} = \begin{vmatrix} Q_I \\ Q_J \\ Q_K \end{vmatrix} \qquad (1\text{-}21)$$

1.2 Form of the Matrix for Steady Flows

The application of the Finite Element Method (FEM) to steady groundwater flow, generates a set of linear equations as:

$$[A] \cdot [H] = [B] \qquad (1\text{-}22)$$

with:

[A] = The N x N square band matrix of coefficients for the unknown terms (N = number of unknowns), which is symmetric and diagonally dominant. For this matrix we define the following terms (Fig. 1-3).

- Width of the band = Number of elements on each side of the diagonal element = L
- Medium width of the band = L+1
- Total width of the band = 2L+1

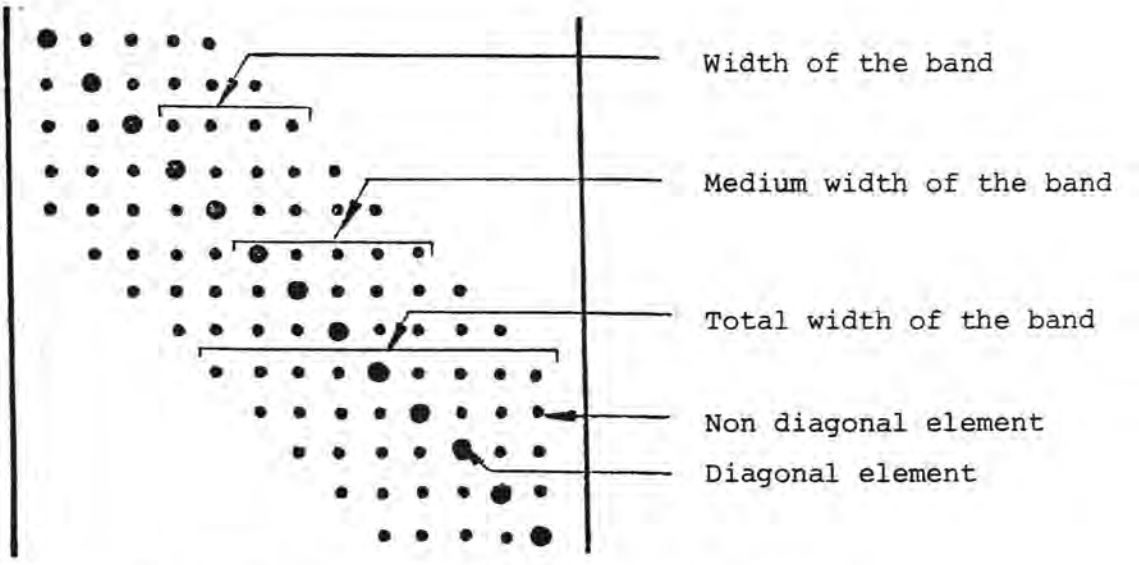


Fig. 1-3 Diagram of a square band matrix

[H] = Column matrix of unknown heads
[B] = Column matrix of known terms

It should be mentioned that this type of matrix is more complicated than the matrix generated by the Finite Difference Method (FDM). In FDM there is always a pentadiagonal matrix for a 5 point scheme, while in FEM the width of the matrix varies from problem to problem and even in a given problem will depend on the manner of numbering the nodes (see section 1.6.2).

1.3 Treatment of Boundary Conditions

Neumann boundary conditions (Impervious or constant flux boundary) do not need any special treatment because they are treated automatically like the other nodes of the mesh.

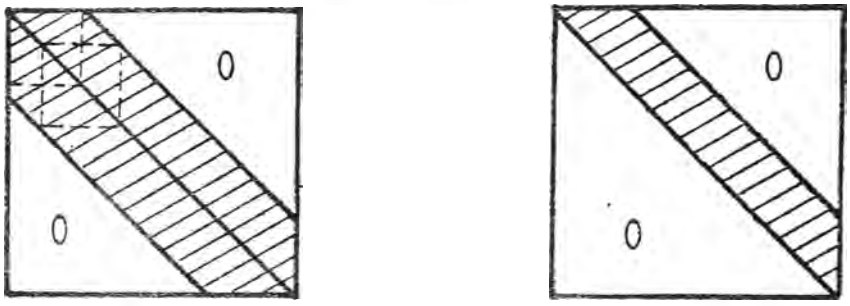
Boundary nodes with Dirichlet boundary conditions (constant head) must be coded for identification during the process of generating the linear equations. In this case the terms of $M_J h_J$ or $M_K h_K$ should be shifted to the right hand side of the equations. For example in the case of Fig. 1-2 if the node J is a node with constant head, a set of linear equations (1-23) will be generated.

$$\frac{1}{4\Delta}\begin{vmatrix} T_x(Y_J-Y_K)^2 + T_y(X_K-X_J)^2 & T_x(Y_I-Y_J)(Y_J-Y_K) + T_y(X_J-X_I)(X_K-X_J) \\ T_x(Y_K-Y_J)(Y_J-Y_I) + T_y(X_J-X_K)(X_I-X_J) & T_x(Y_J-Y_I)^2 + T_y(X_I-X_J)^2 \end{vmatrix} \cdot \begin{vmatrix} h_I \\ h_K \end{vmatrix} = \begin{vmatrix} Q_I-[T_x(Y_K-Y_I)(Y_J-Y_K) + T_y(X_I-X_K)(X_K-X_J)]\, h_J \\ Q_K-[T_x(Y_I-Y_K)(Y_J-Y_I) + T_y(X_K-X_I)(X_I-X_J)]\, h_J \end{vmatrix} \qquad (1\text{-}23)$$

It is obvious that in the above case a set of two linear equations with two unknowns (h_I and h_K) is generated since the potential of the third node (J) is known.

1.4 Solution Procedure for the Set of Linear Equations

To solve the set of linear equations (1-22), various direct methods including triangularization (upper and lower) and diagonalization, can be used. The upper triangulation procedure is used in this study. Since the matrix [A] is in band form, to accelerate the procedure of triangulation the Gauss algorithm is applied only to the elements of submatrices of the matrix A.



(A) (B)

Fig. 1-4 Diagram of band matrix (A), and an upper triangulate matrix (B)

1.5 Description of the Computer Program FIELS

The program FIELS solves the problem of two-dimensional steady-state groundwater flow by FEM. In this section the structure of the program will be briefly described.

The FORTRAN IV source program developed for this model is listed in Appendix A1 and includes almost 400 lines of statements and comments. Selected variables used in this program are defined at the beginning of the source program.

Fig. 1-5 represents a simplified flow chart of the program. The program is segmented into a main routine and seven subroutines. The succession of operations can be summarized as follows:

- SUBROUTINE REDATA, reads all the input data.
- SUBROUTINE PRDATA, prints all the input data.
- SUBROUTINE BAND, computes the width of the band matrix.
- SUBROUTINE GENER, generates the band matrix [A] and column matrix [B].
- SUBROUTINE TRIANG, upper triangulates the band matrix [A] in its compact form.
- SUBROUTINE SOLVE, solves the upper triangulated set of equations.
- SUBROUTINE PRINT prints the final results including the computed head values.

Since the subroutine GENER is the most important subroutine in this program, more details are given in the following paragraph.

Fig. 1-6 shows the simplified flow chart of subroutine GENER. In this subroutine, within the loop commencing DO 200 INO=1,INCON the index of the node INO is tested and if it is > 1 an error message is printed, requiring

corrective action (numbering the nodes correctly). If the index is < 1 (normally it will be 0), the operation is continued within another DO loop (DO 100). In this second loop the triangular elements having the nodes INO in common are searched, then for each triangle their share in the related matrix element is computed and the boundary condition is treated.

When all the elements of the line INO of matrix A have been computed, these matrix elements are transferred in compact form into the line INO of array AA (Fig. 1-7).

When the generation of all of the lines of matrix A and their transfer to array AA has been correctly performed (this operation is correct when the number of generated equations NEQ is equal to the total number of nodes with unknown heads), the total number of generated equations is printed, otherwise NEQ and the number of nodes with unknown heads will be printed.

In the next step the subroutine TRIANG upper triangulates the compact form of matrix A (Fig. 1-7a) and transforms it into the form of Fig. (1-7b). This upper triangulated matrix is then solved by subroutine SOLVE.

1.6 Data Preparation

For simulation of a field problem by program FIELS, the following steps are recommended.

1.6.1 Creating the Elements

The area of aquifer should be divided into triangular elements with special attention to the following considerations:

- The boundary of the aquifer should be approximated as closely as possible by the elements
- Smaller elements should be used in the zones of higher density of information or higher piezometric gradients.

1.6.2 Numbering the Nodes

- Firstly the nodes with unknown piezometric values should be numbered sequentially.
- Secondly the constant head nodes should be numbered.
- Numbering the nodes determines the width of the band matrix(1) which is defined as the largest difference between the numbers of the two neighbouring nodes with unknown heads. Therefore it is necessary to be careful during the numbering process. For example in Fig. 1-8a, the width of the band would be seven, while in case b it would be only four, and therefore less computer time is spent in solving the set of equations.

1.6.3 Numbering the Elements

Numbering the elements does not have any particular restrictions, but it is recommended that they be numbered sequentially (Fig. 1-8) to facilitate data preparation and control.

---

(1) - Number of terms on each side of the diagonal elements (Fig 1-3)

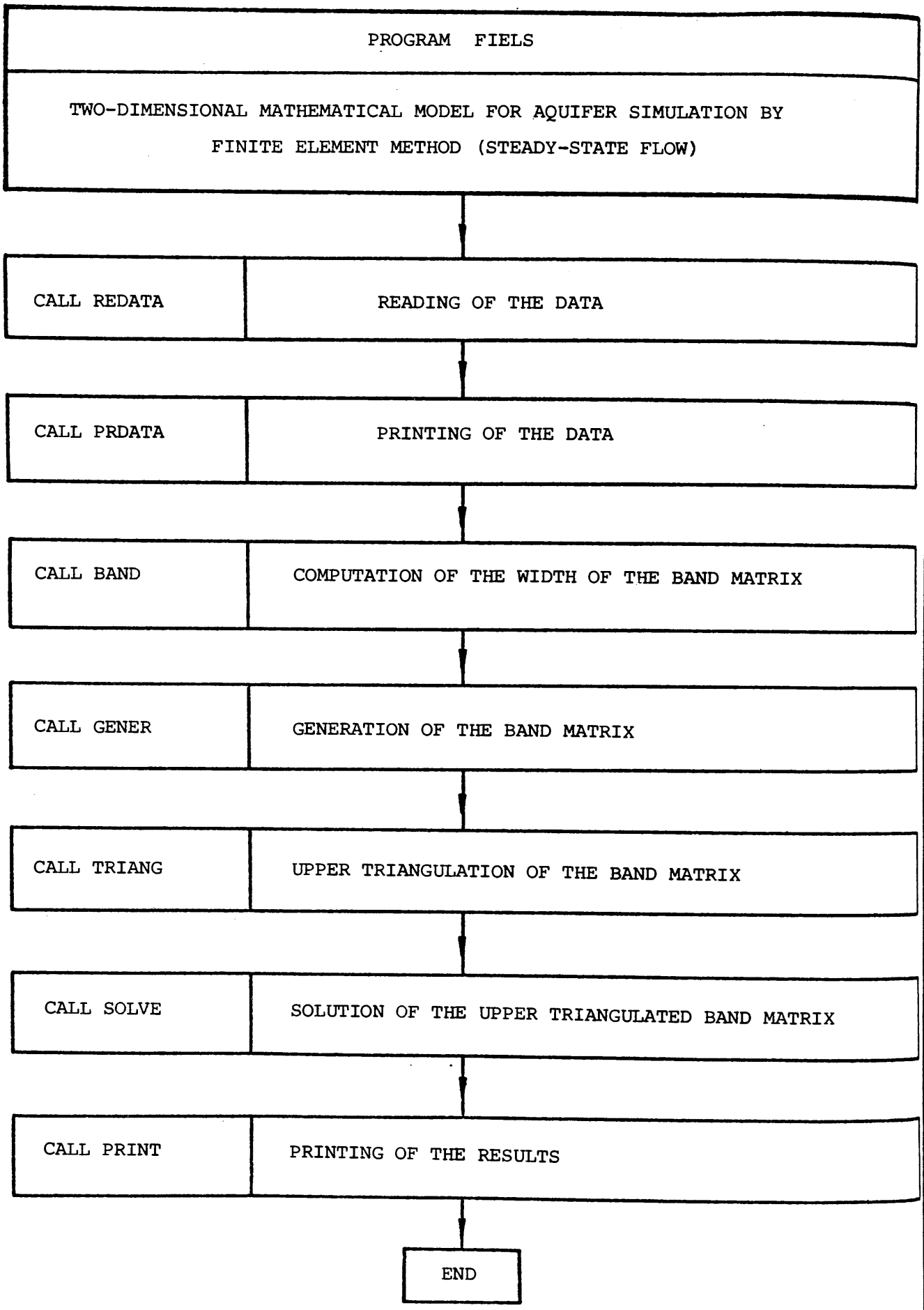


Fig. 1-5 Simplified flow chart of Program FIELS

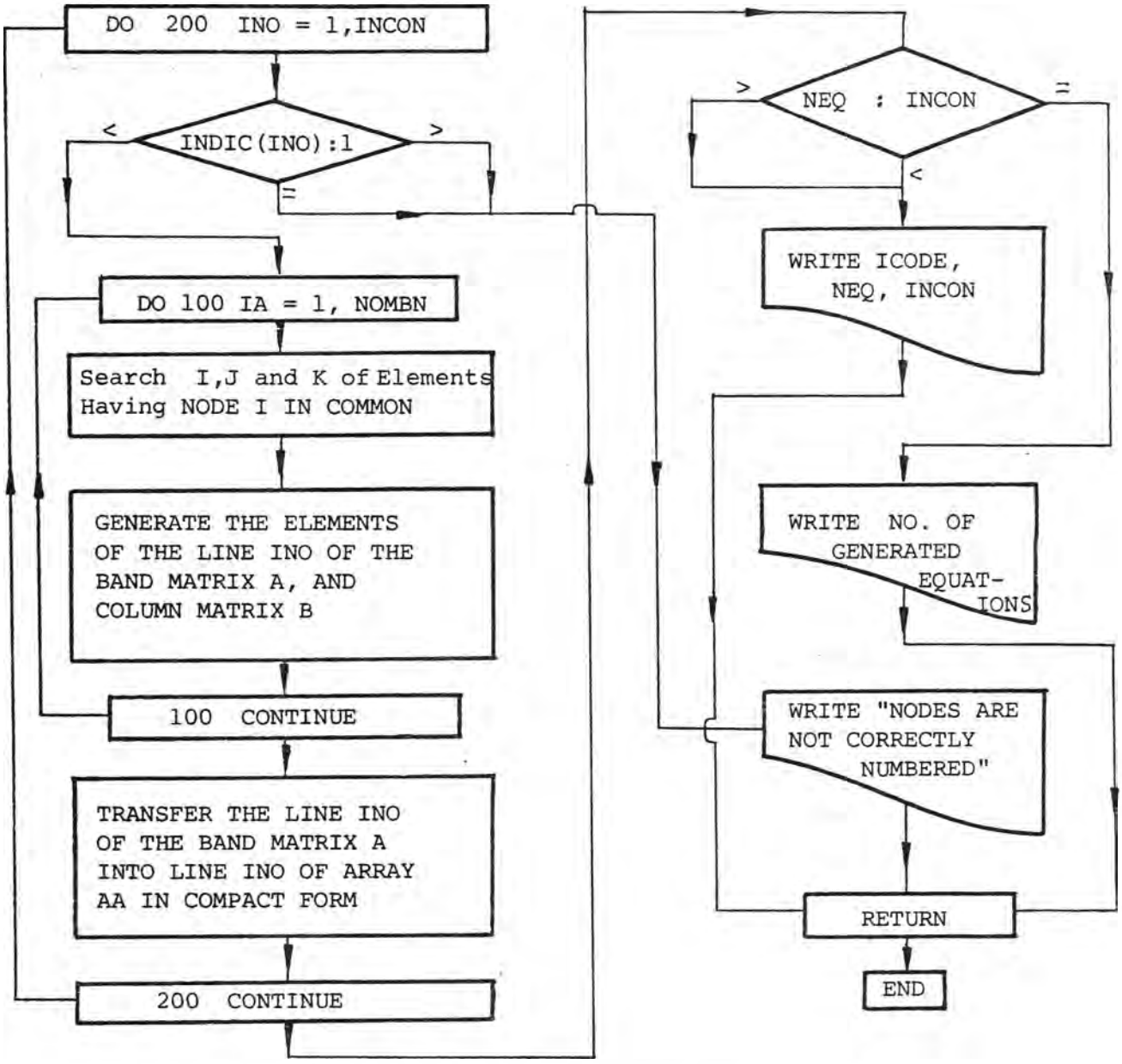


Fig. 1-6 Simplified flow chart of subroutine GENER

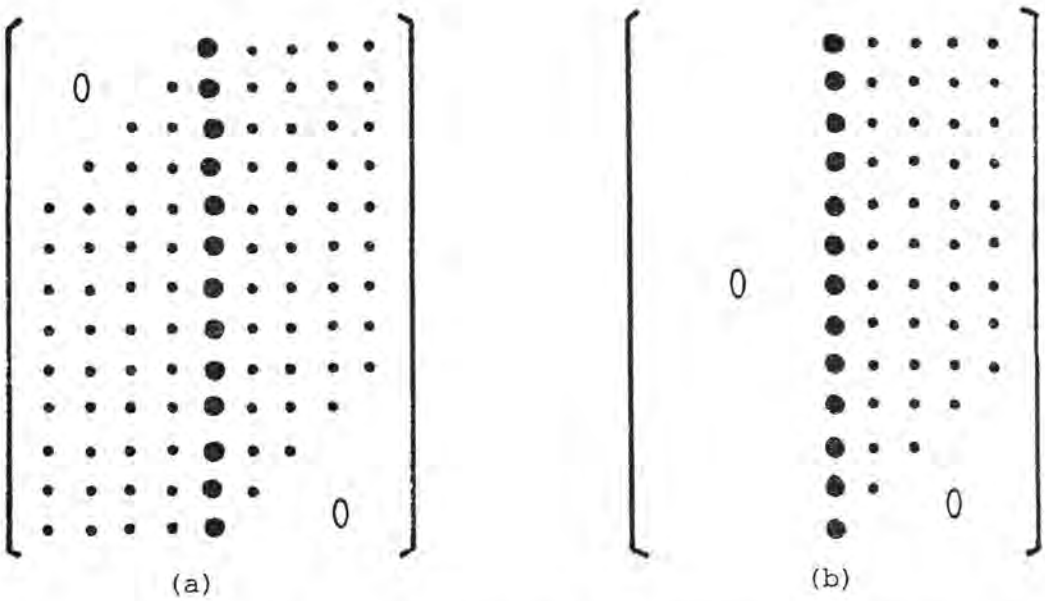


Fig. 1-7 Compact form of matrix A (a), and its upper triangulated form (b)

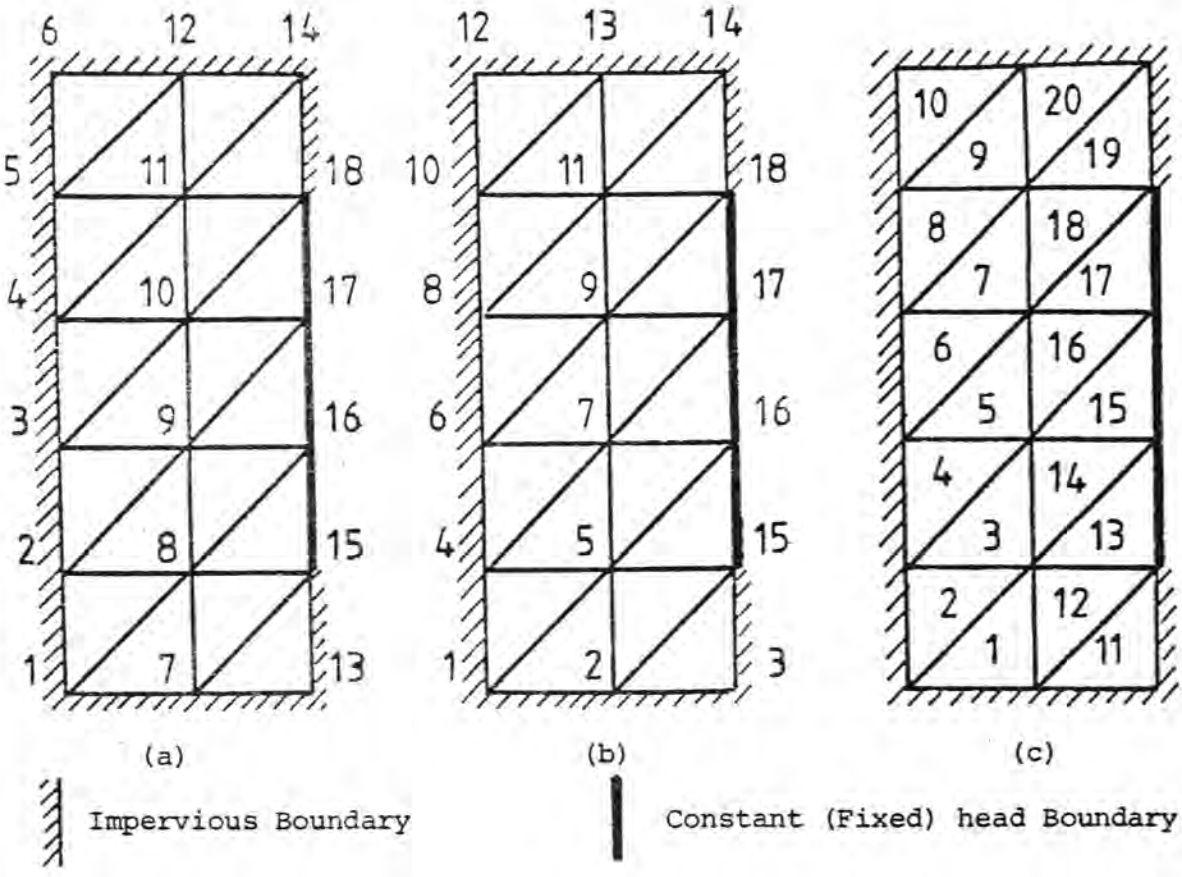


Fig. 1-8 Examples of numbering the nodes (a and b) and elements (c)

1.6.4 Input Data

After discretization of the area into triangular elements, and numbering the nodes and elements, input data should be prepared with respect to the formats presented in Appendix A2.

1.7 Advantages and Disadvantages of FEM

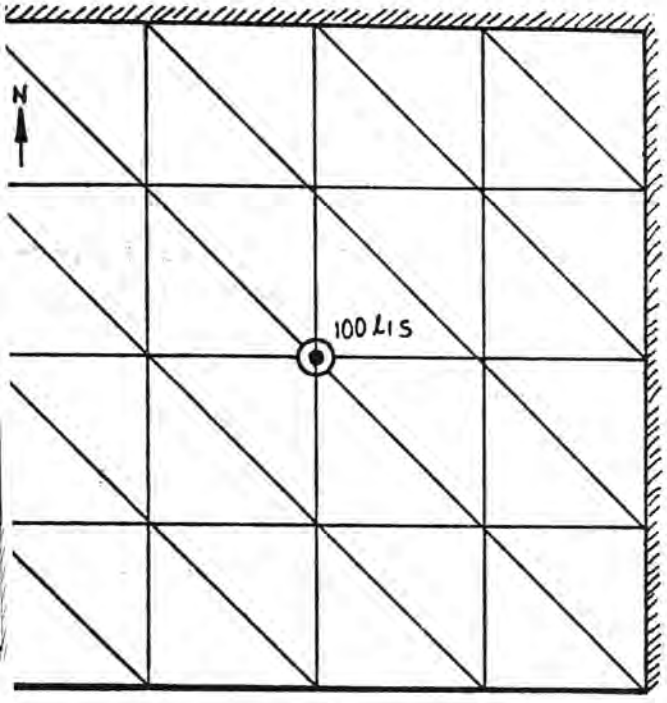
This paragraph compares the FEM with FDM and lists the advantages and disadvantages of the FEM.

1.7.1 Advantages of FEM

- Representation of the aquifer boundary is more accurate.
- It is possible to choose a variable dimension for the elements, if required.
- It is not necessary to develop special equations for the treatment of the boundary conditions.
- Solution of the generated set of linear equations is direct, so the disadvantages of the iterative methods are avoided.

1.7.2 Disadvantages of FEM

- More data elements are required, which consequently makes it more time-consuming and more vulnerable to mistakes.
- More memory locations are needed.
- The computation time is generally longer.
- The theoretical basis and programming techniques are more complicated.



g. 1-9 Discretization of Test Problem I

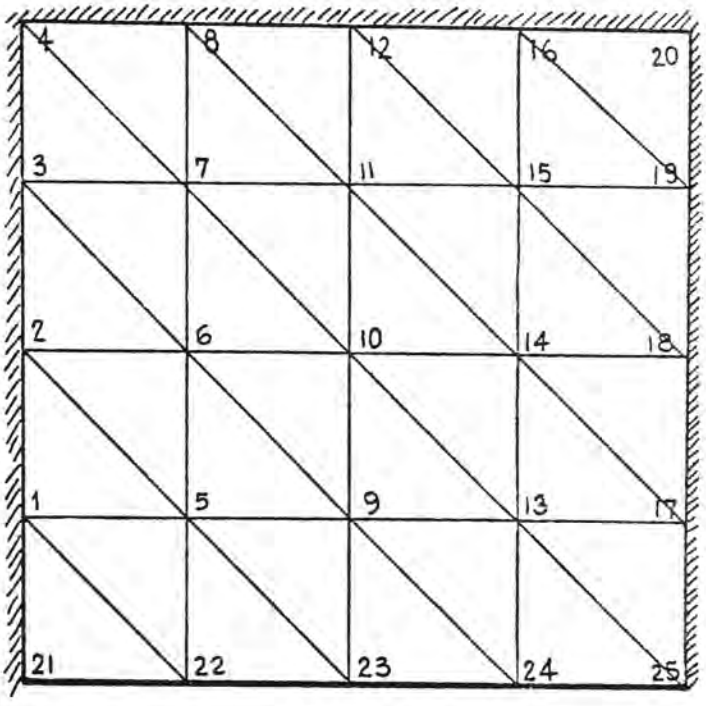
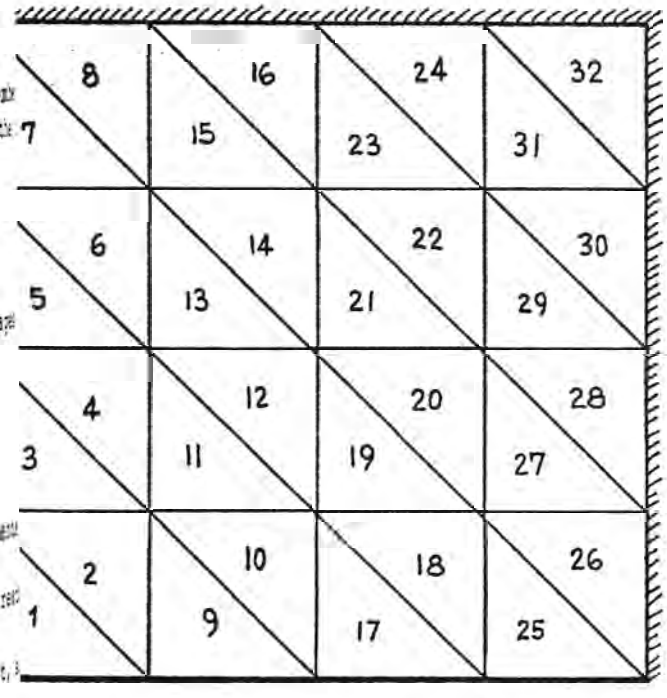


Fig. 1-10 Numbering of the nodes



j. 1-11 Numbering of the elements

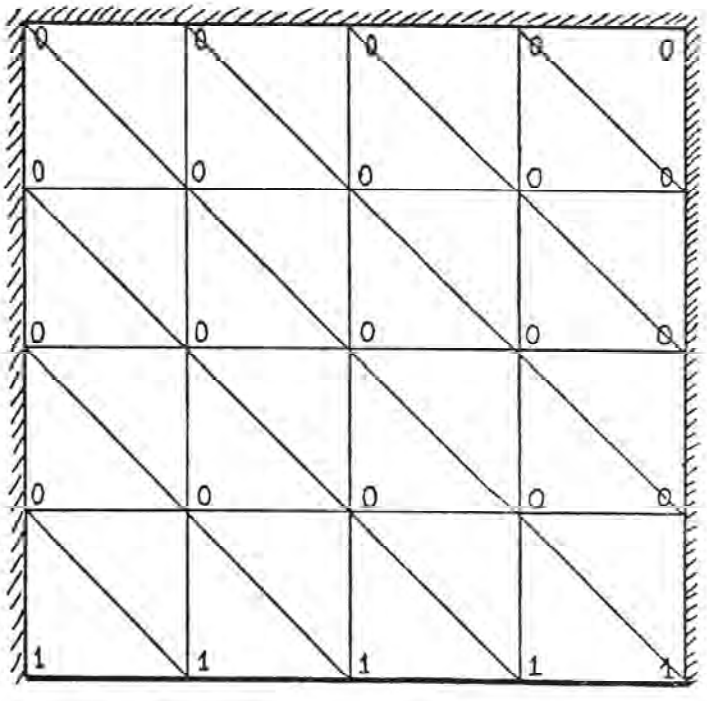
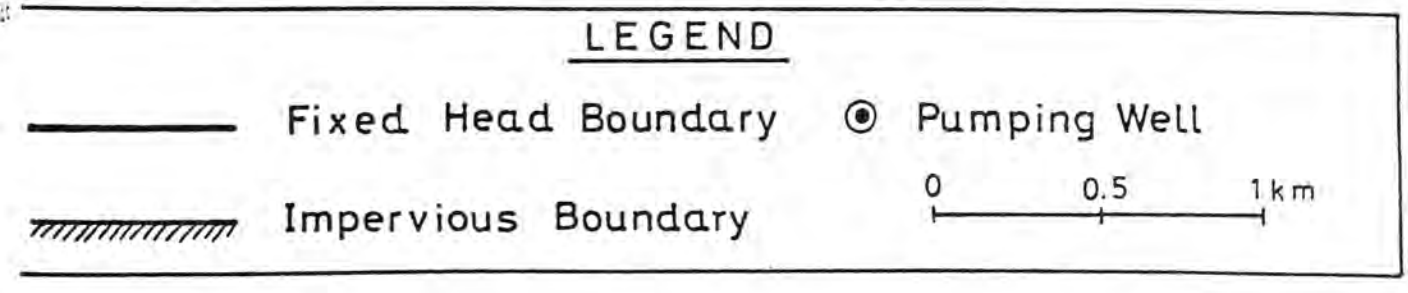


Fig. 1-12 Indexes of the nodes



1.8 Examples of Application

Two examples of application will be presented here.

1.8.1 First example (Test Problem I)

In this problem which can be used as a test problem by the users of program FIELS, a square aquifer with the following conditions is considered:

- Dimension of aquifer is 2000m x 2000m.
- Transmissivity of the aquifer is 250 m^2/d.
- Southern limit of the aquifer is a fixed head boundary with a value of 80m.
- A pumping well with a discharge rate of 100 l/sec is situated in the center of the aquifer.

It is required to compute the distribution of the heads in the aquifer under steady state conditions.

Figs. 1-9 to 1-12 respectively represent,

- Discretization of aquifer into triangular elements
- Numbering of the nodes
- Numbering of the elements
- Indexes of the nodes

Input data for this problem and outputs of Program FIELS are presented in appendix A3.

Fig. 1-13 presents the results of the computation for test problem I. This Figure shows clearly the cone of depression around the pumping well and the symmetry of the computed results.

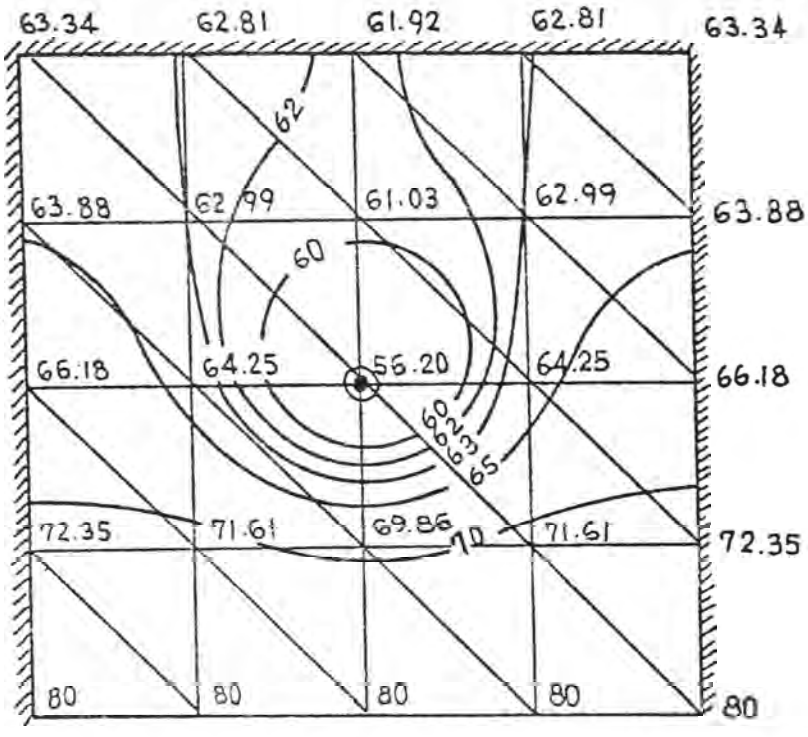


Fig. 1-13 Computed heads and contour lines (in m.) of Test Problem I (steady-state Flow)

1.8.2 Second Example (simulation of the Botany Basin Aquifer in Steady State Conditions)

The Botany Basin lies to the south of the city of Sydney. A sketch of its location is given in Fig. 1-14. In this example only the northern sector bounded by the Cooks River is included.

This aquifer was simulated by Merrick and Barratt (1981), then by Jones (1983) with the finite difference mathematical model developed by Prickett and Lonnquist (1971) for transient flow.

By simulating the Botany Basin aquifer by FEM the results of two simulations can be compared.

1.8.2.1 Geology

The Botany Basin is encircled by Hawkesbury sandstone of Triassic age. In the northwest, the sandstone is capped by Wianamatta shale. The sediments infilling the basin are of Quaternary age and range to a maximum depth of 80 metres.

Thin layers of peat, clay and cemented sand are indicative of former swampy conditions and fluctuations in water level.

The sand beds are largely unconfined, with areas of partial confinement due to lenses of peat and clay.

The geology of the basin is shown in Fig. 1-14 and is describd by Griffin (1963).

Fig. 1-15 represents the boundary conditions and discretisation of the Botany basin as it was defined by Jones (1983).

1.8.2.2 Static Water Levels

The static water level of the aquifer is not very well known. Jones (1983) adopted Fig. 1-16 as an acceptable static water level contour map of the aquifer. Therefore, this figure has been used as a reference for the comparison of the two methods.

1.8.2.3 Aquifer Transmissivities

Transmissivity values T(in m^2/day) at each node of the model were initially calculated by Jones (1983) as a product of aquifer thickness e(in m) and hydraulic conductivity K(in m/day) assuming a constant value of K = 20 m/day. Then he changed these initial values as a result of his simulation hypotheses.

Fig. 1-17 represents the final distribution of the transmissivity values accepted by M.J. Jones.

1.8.2.4 Pumping and Recharge Rates

The domain of the pumping activities is located between Lachlan Swamps and Botany Bay. Pumping rates are not well known, and controversial figures

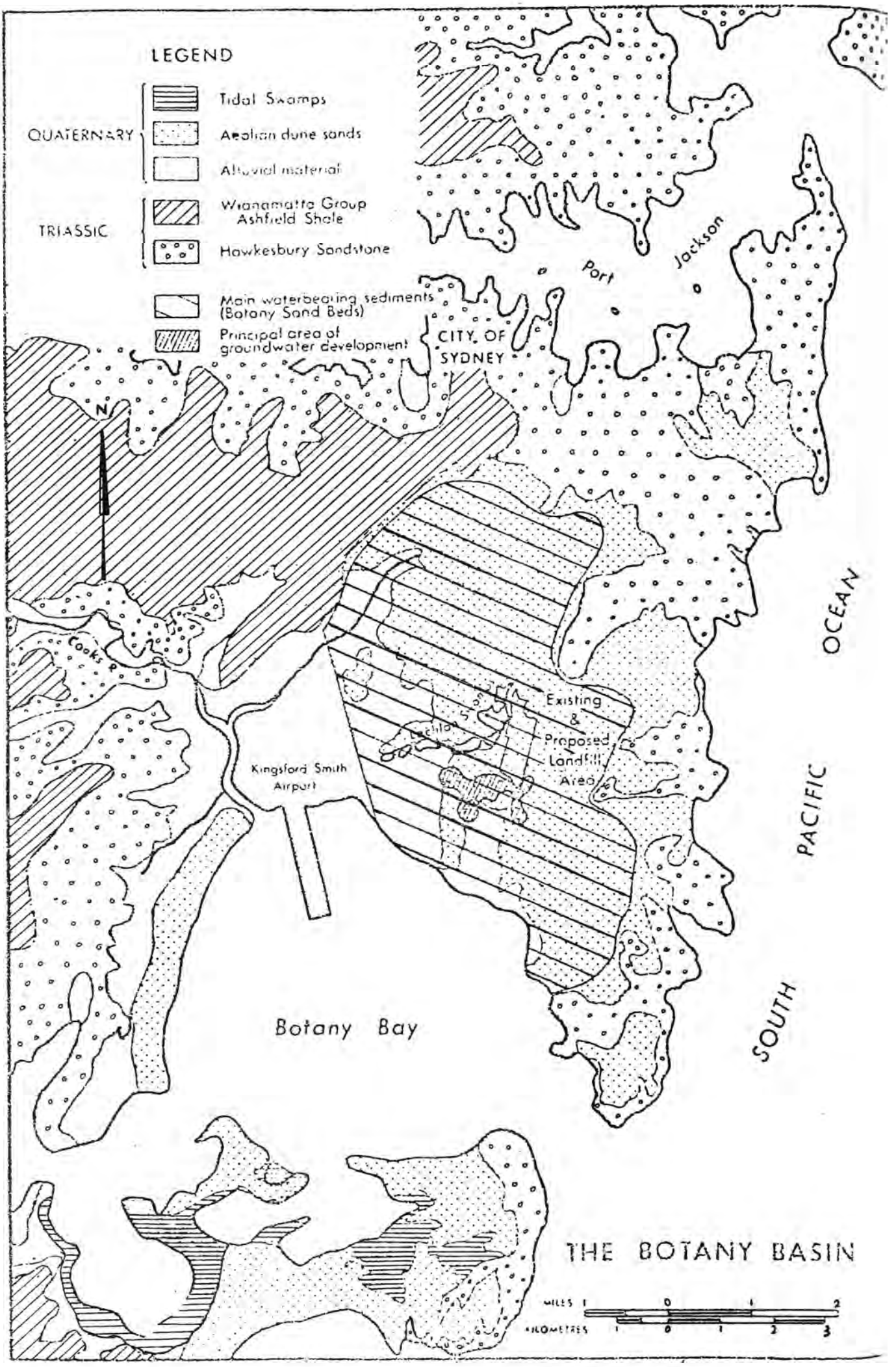


Fig. 1-14 Simplified location and geologic map of Botany Basin

are reported by different authors. Merrick and Barrett (1981) give the following estimate:

- Total pumpage from all known significant bores is of the order of 50,000 m^3/day or 579 l/sec.
- Total pumpage from non irrigation use is about 37000 m^3/day or 428 l/sec.

The major source of recharge is assumed to be from the Lachlan swamps and irrigation of parklands. It is also assumed that the water pumped from the bores located in the parklands returns to the aquifer, so these pumping wells were not simulated by Jones (1983).

Fig. 1-18 shows the final nodal distribution of withdrawals and infiltrations adopted by Jones (1983).

1.8.2.5 Result of Botany Basin Simulation by FDM

Jones (1983) simulated the Botany Basin aquifer using a modified version of the Prickett and Lonnquist(1) computer program. This Program was initially prepared for the simulation of two-dimensional transient flow, but Jones (1983) used it to simulate a steady-state flow by setting a very small storage coefficient on the nodes. The period of simulation was approximately 187 days divided into 20 time-steps, ranging from 1 to 32 days. Fig. 1-19 shows the results of his simulation and compares it with the measured static water level.

Jones (1983) results were achieved after 12 attempts at manipulation of the transmissivity values, and the recharge rates and their distributions. In spite of these efforts, since the data were not reliable the computed values were different from the measured static values, specially in the south eastern part of the area where his computations resulted in a cone of depression with negative piezometric values.

Jones (1983) came to a series of conclusions, two of which are mentioned here:

- The study found that the limitation on the amount and reliability of the data was a major problem.
- The model was not very sensitive to transmissivity variations but very sensitive to variations in the location and relative magnitude of the recharge centres.

1.8.2.6 Simulation of Botany Basin Aquifer by FEM

In this section simulation by FEM of the Botany Basin aquifer will be discussed, and the results of simulations by the FD and FE methods will be compared.

To simulate the Botany Basin by FEM, the aquifer has been discretized by regular triangular elements, to enable comparison of the results of the two procedures.

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(1) - Listing of the Program is attached as Appendix C1

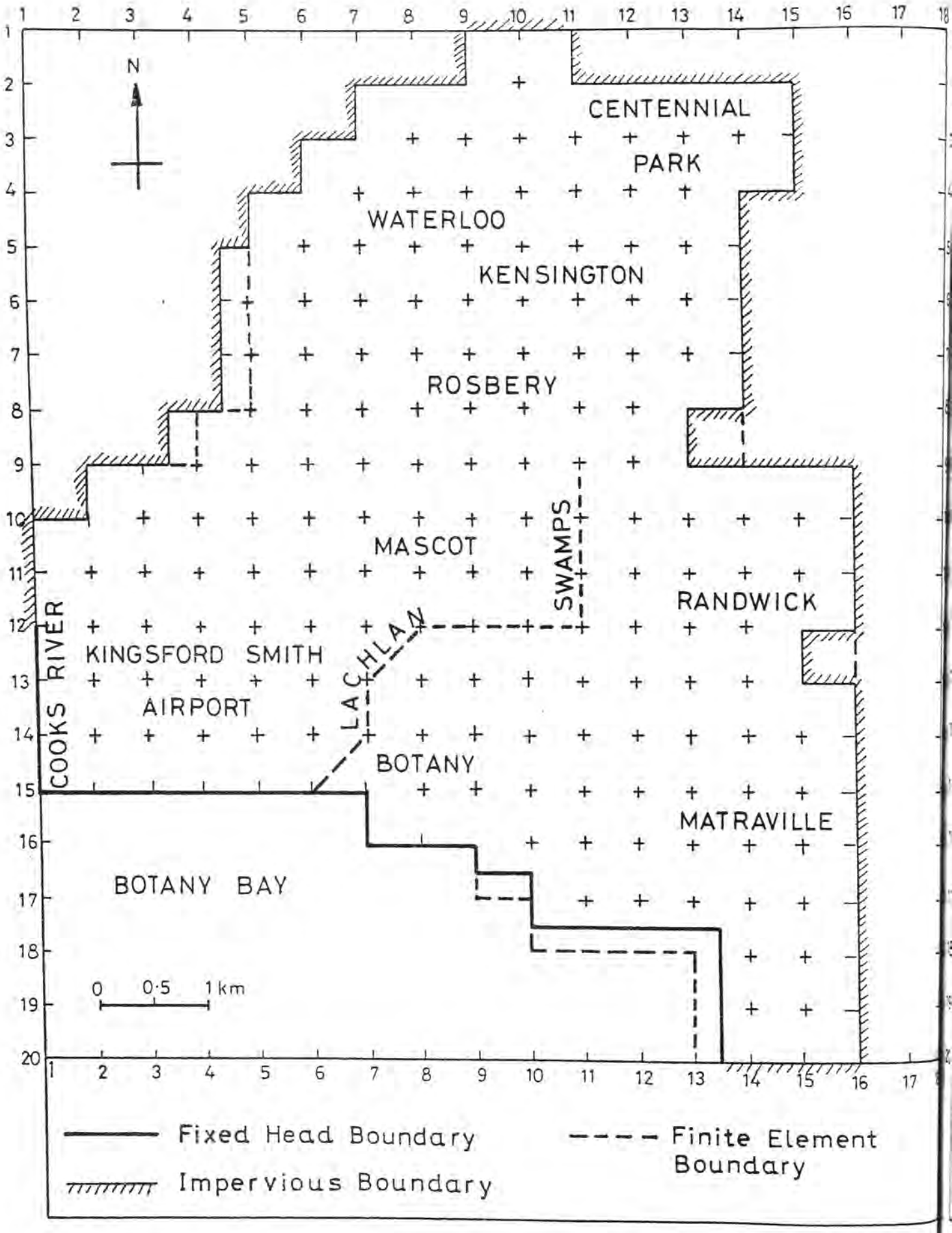

Fig. 1-15 Boundary conditions and discretization of Botany Basin aquifer by FD

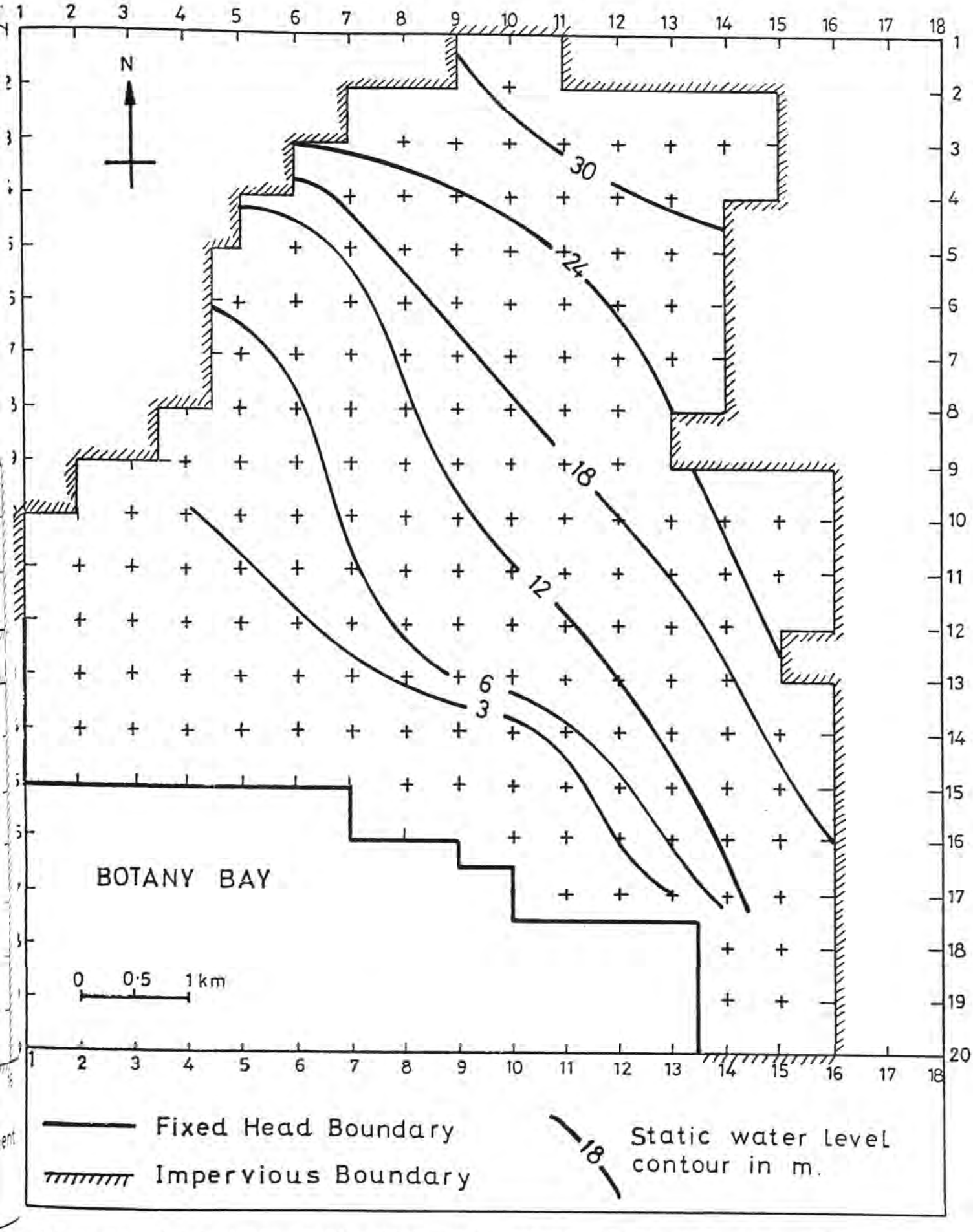

Fig. 1-16 Approximate static Water Level contours of Botany Basin Aquifer (After M.J. Jones 1983)

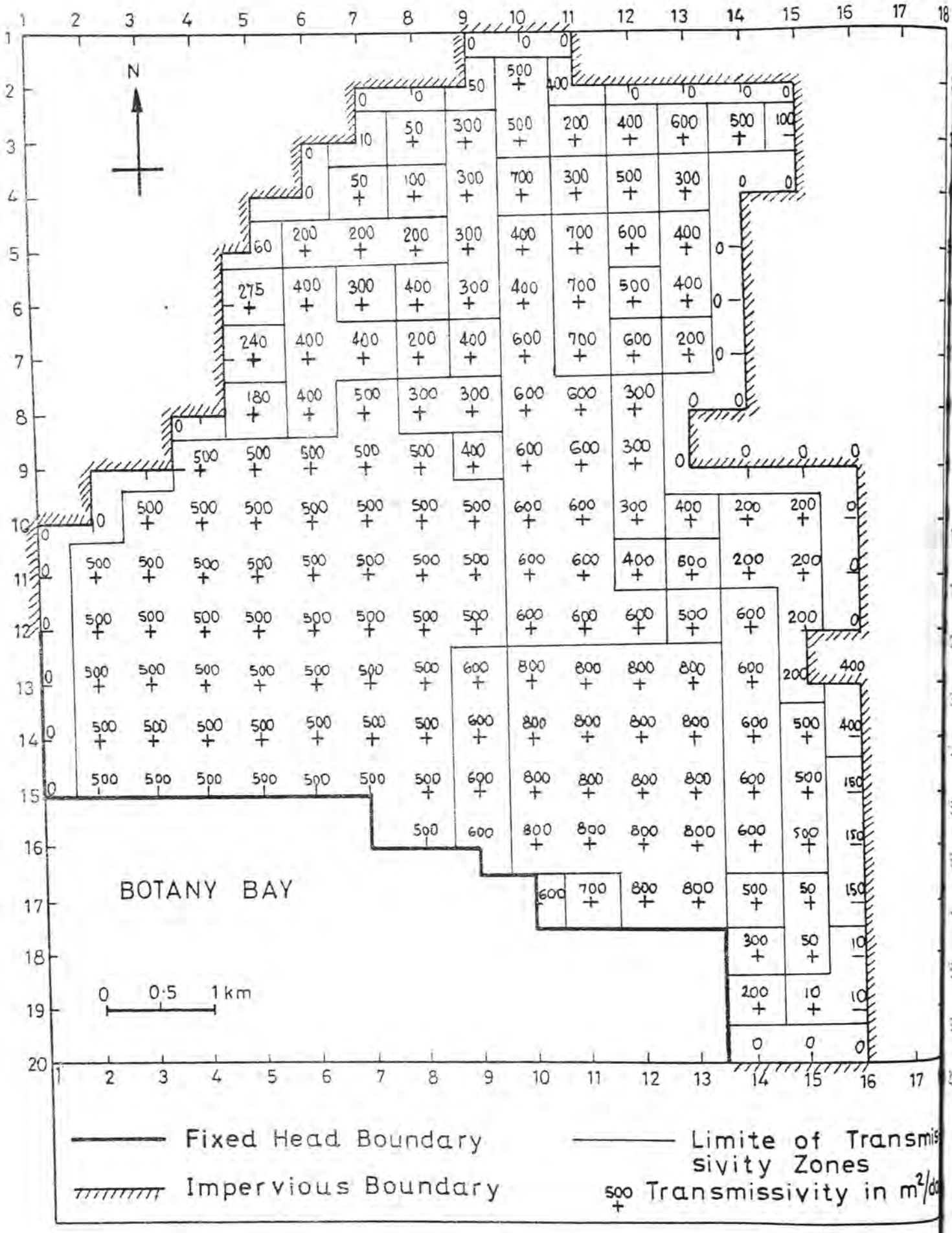

Fig. 1-17 Nodal distribution of Transmissivity values (After M.J. Jones 198

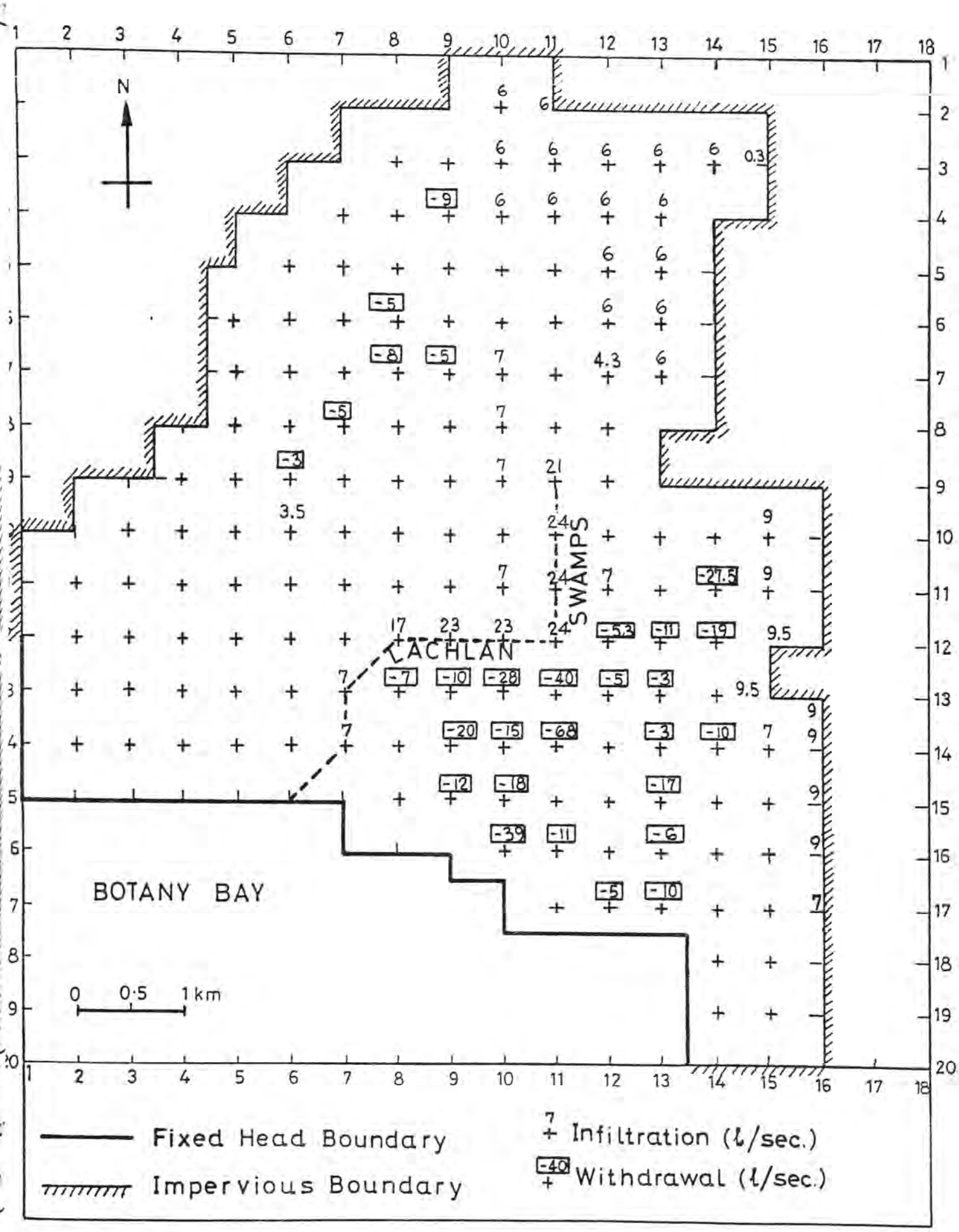

Fig. 1-18 Nodal distribution of Infiltrations and Withdrawals (After M.J. Jones)

The following figures show the data prepared for this simulation:

- Fig. 1-20 Index of the nodes
- Fig. 1-21 The boundary conditions and numbering of 218 nodes (first the nodes with unknown heads are numbered, then the fixed head nodes on Botany Bay and Cooks River).
- Fig. 1-22 The numbering of the 364 triangular elements.
- Fig. 1-23 The initial distribution of transmissivity values. These values are mainly similar to the values of Fig. 1-15, with a small difference on the boundary elements.

It should be mentioned that the distribution of infiltrations and withdrawals are the same as used in the FDM (Fig. 1-18). These values are applied to the corresponding nodes.

The results of the simulation by FEM are shown in Fig. 1-24, and are compared with the results of FDM.

The results of the two methods are very similar, and their differences can be summarized as follows:

a. The boundaries of the two models are slightly different (Fig. 1-15).

b. The transmissivity values on the boundaries of the two models show a small difference.

c. The main difference is the result of the short period of simulation (187 days). In other words, after 187 days, the aquifer has not yet reached a steady-state condition.

   The larger difference between the results of the two methods in the northern part of the aquifer is also due to the short period of simulation by FDM. Since these nodes are far from the southern fixed head boundary, they need a longer simulation period to reach a steady-state condition.

d. In the FEM simulation the heads of 7 neighbouring nodes enter into the computations (Fig. 1-25a), but in the FDM only the heads of 5 points enter the computations (Fig. 1-25b). Hence the results of compuation by FEM are more accurate.

Fig. 1-25 Neighbouring nodes in FEM (a) and FDM (b)

1.8.2.6.1 Calibration of Botany Basin model by FEM

Although the two methods of computation (FDM and FEM) have resulted in nearly similar results, but in both cases the computed static water levels are

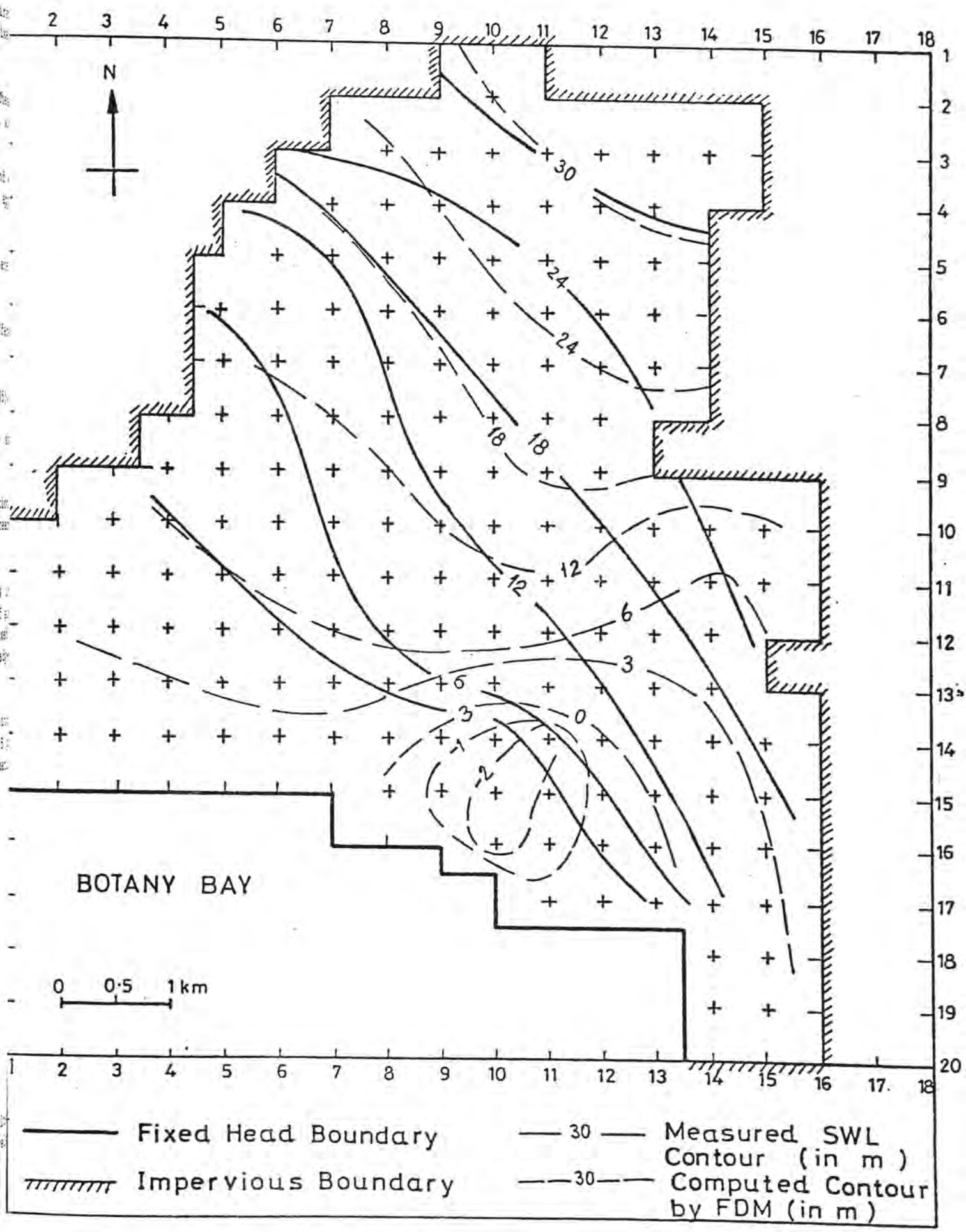

Fig. 1-19 Measured and computed static water level (SWL) contours (after M.J. Jones 1983)

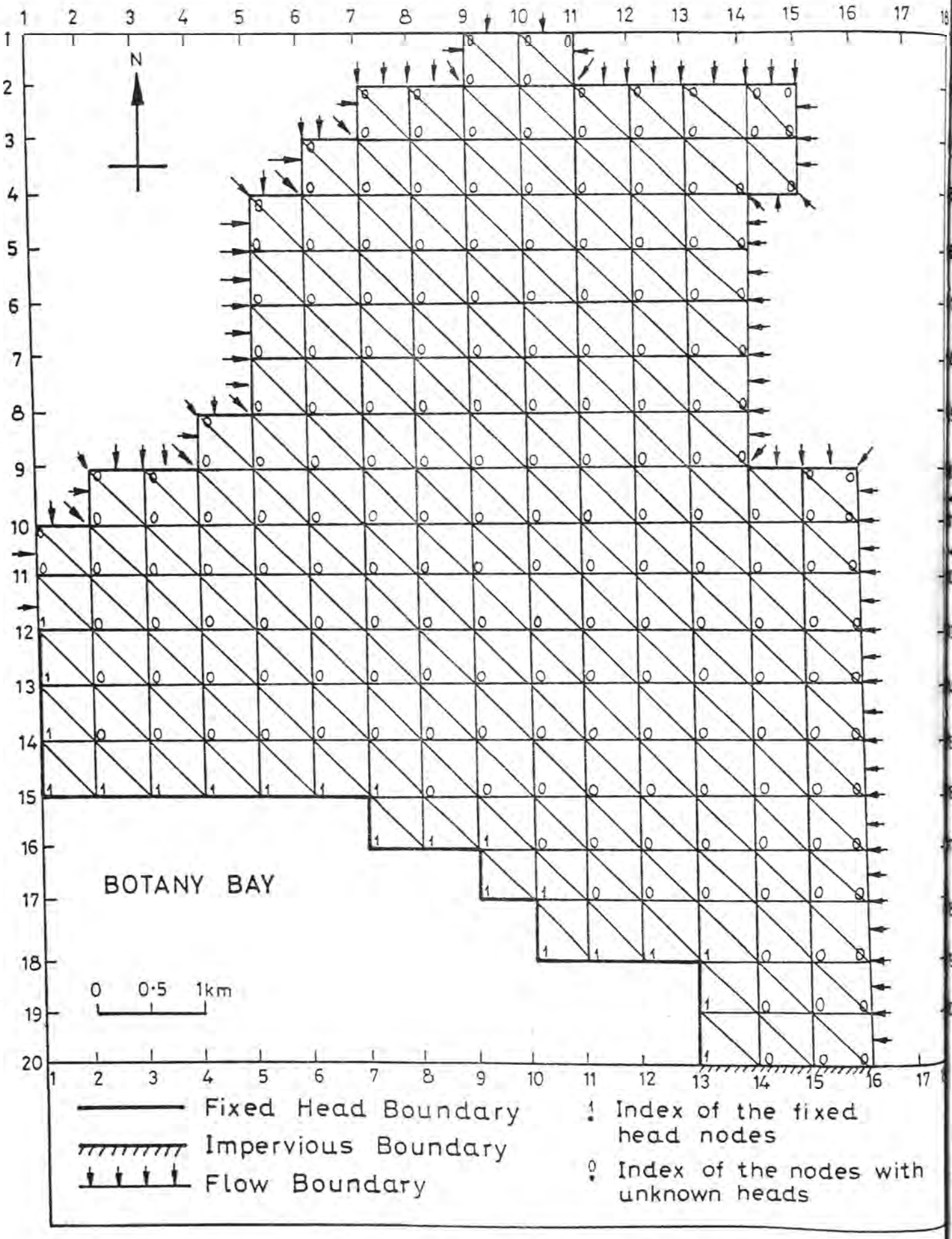

Fig. 1-20 Indexation of the nodes for the Botany Basin aquifer

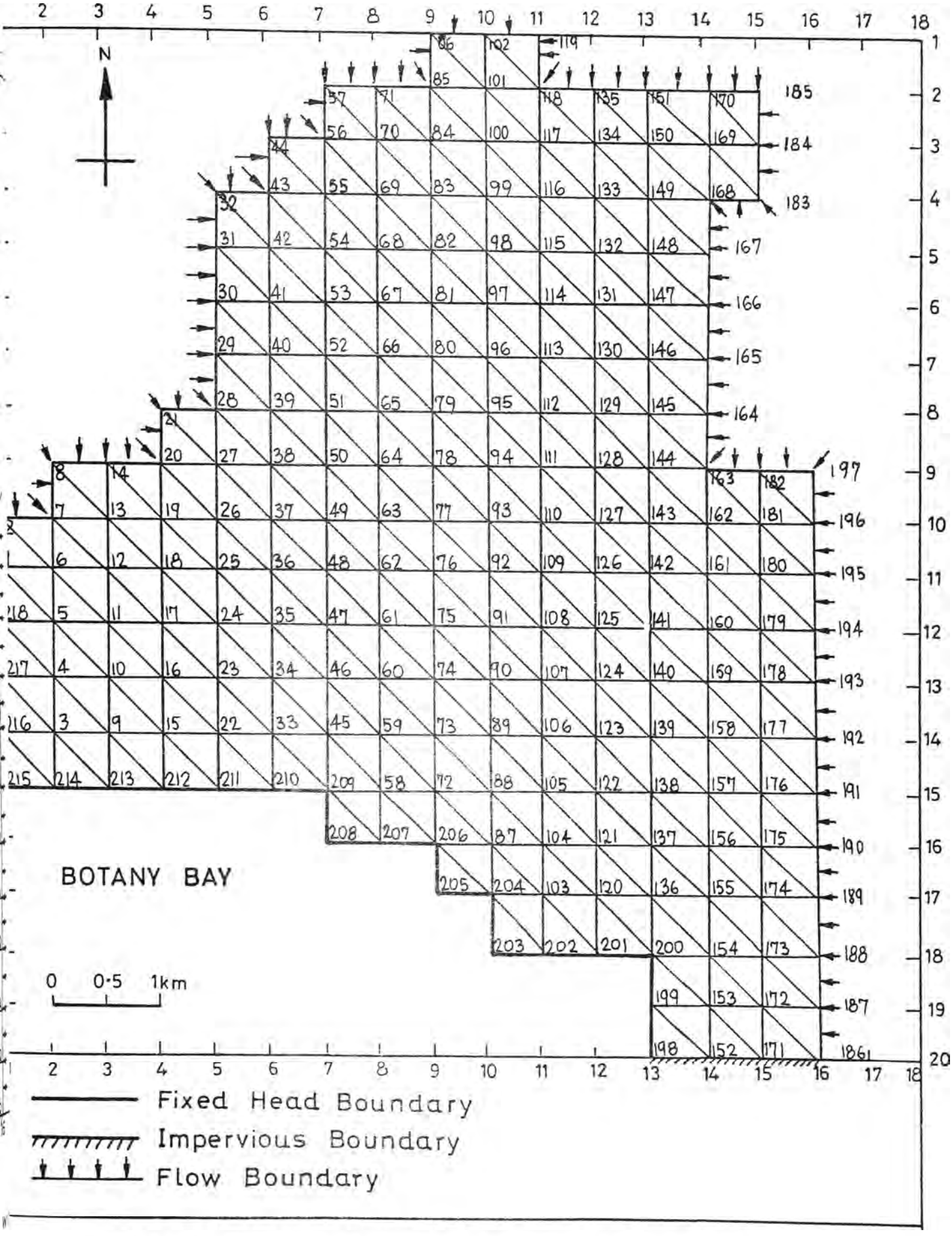

Fig. 1-21 Numbering of nodes for the Botany Basin aquifer

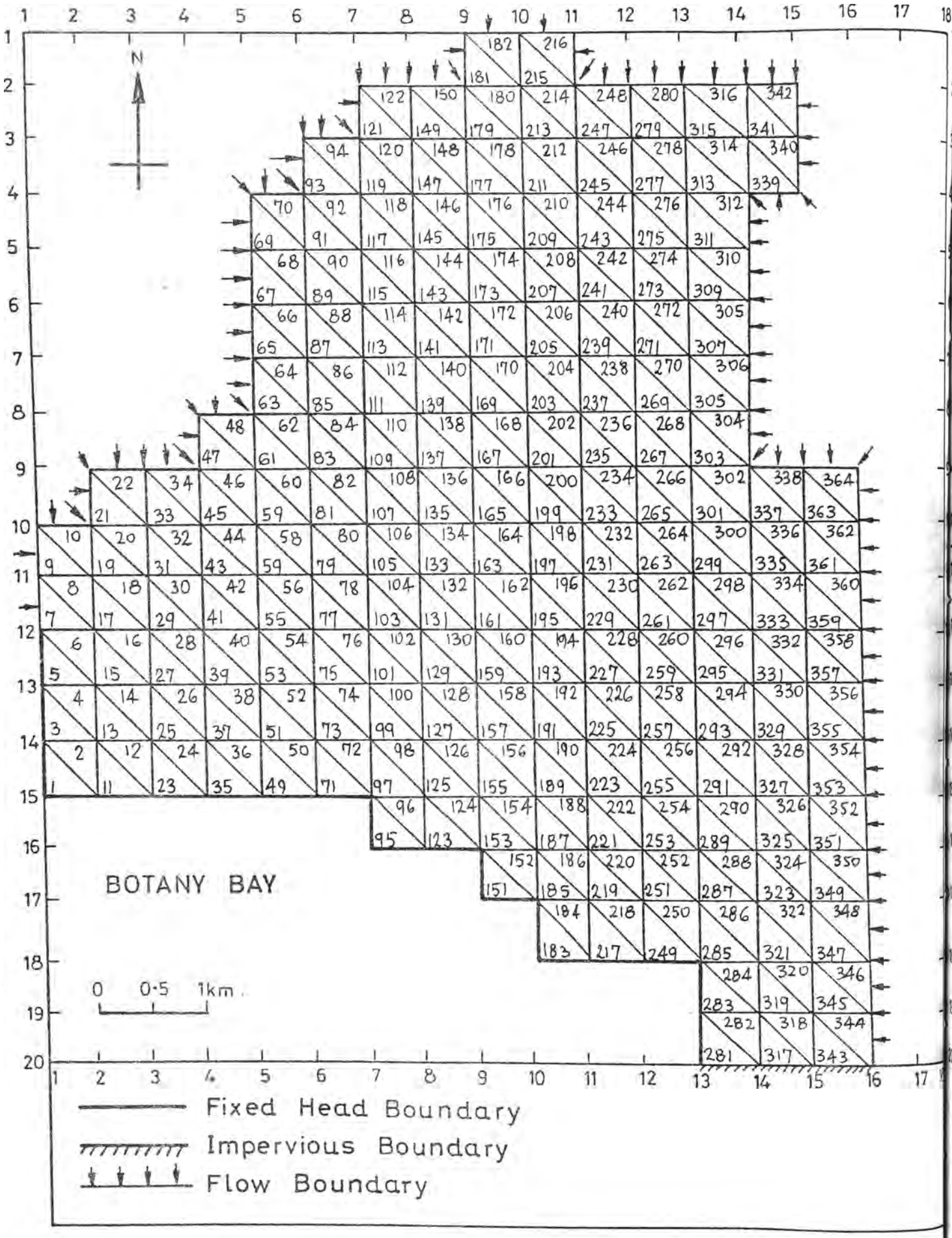

Fig. 1-22 Numbering of elements for the Botany Basin aquifer

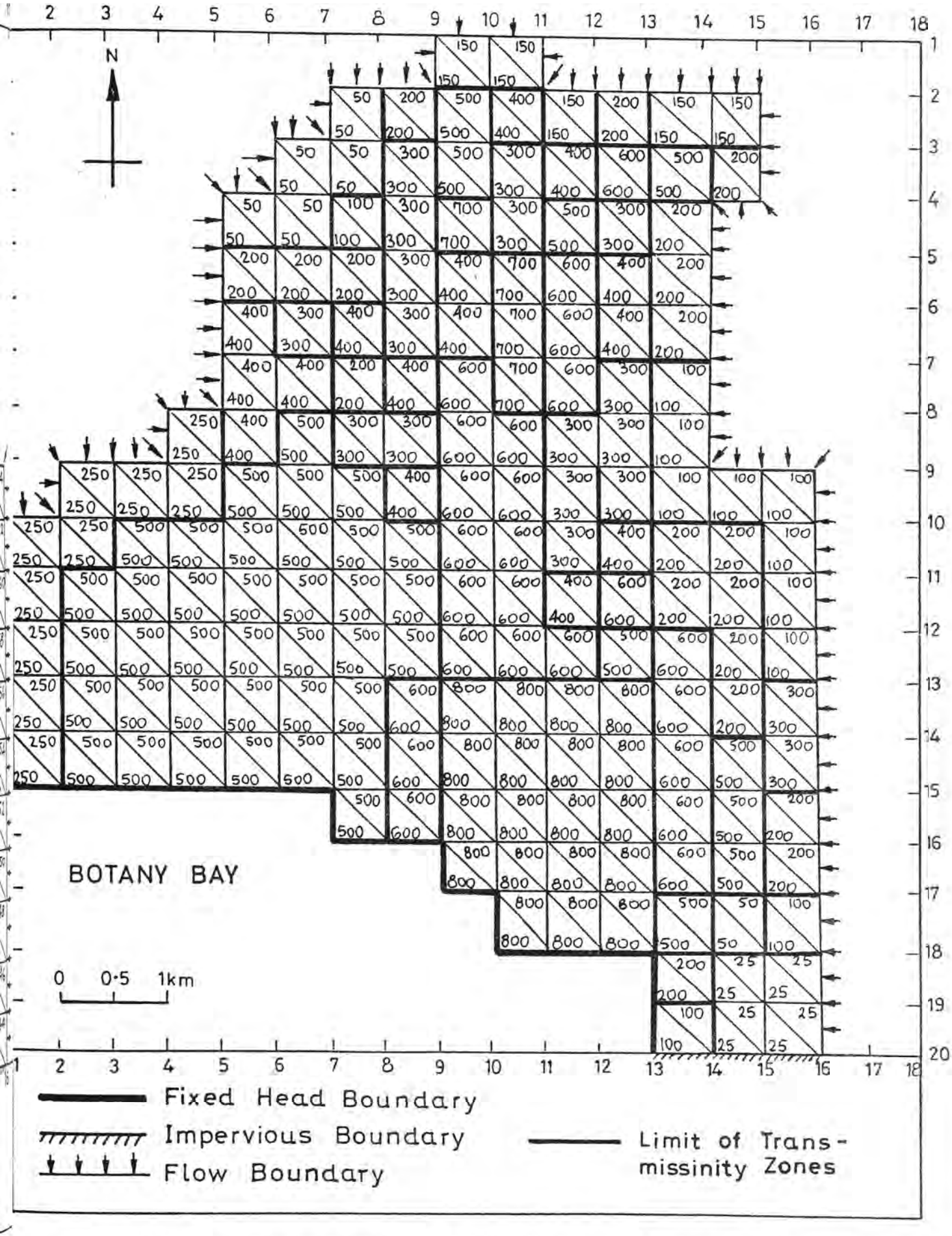

Fig. 1-23 Initial distribution of transmissivity values (in m^2/d) in elements of the Botany Basin aquifer

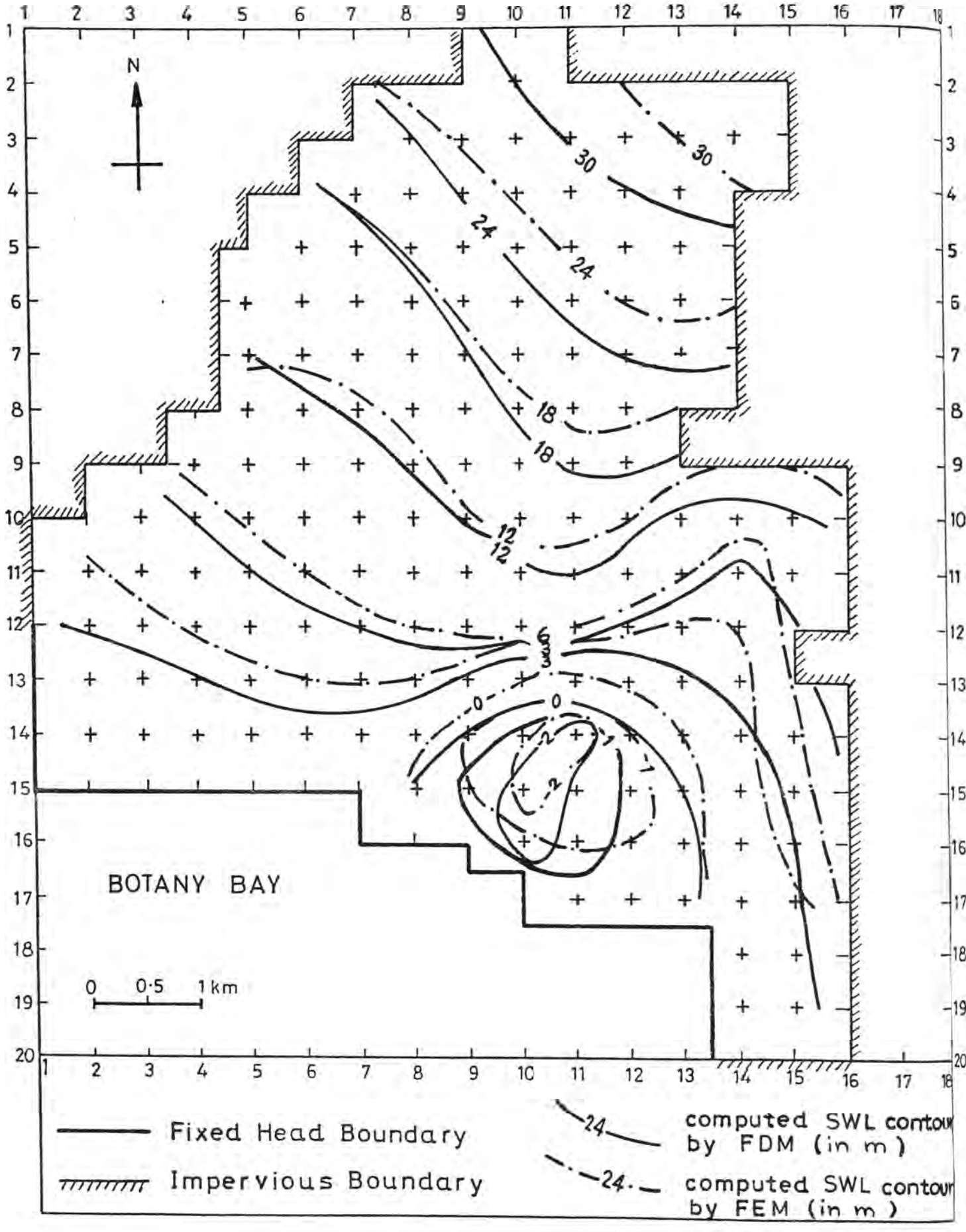

Fig. 1-24 Computed static water level (SWL) contours (in m) by FDM and FEM

different from the measured ones (Figs. 1-19 and 1-24), especially in the southeastern corner of the area. This difference shows that the input data are not reliable. Therefore to calibrate the model, it is necessary to change the input data to reproduce the measured static water levels. To achieve this objective 12 sets of input data were tested, which finally resulted in an acceptable solution (Fig. 1-28).

In the final set of input data the following assumptions were made;

a. The imprevious boundaries on the East, North and West are replaced by flow boundaries (Inflow boundary conditions on the Eastern and Northern limits, and outflow boundary conditions on the Western limit) Fig. 1-26.

b. Pumping rates on some nodes (nodes number 87, 90, 107, 160, 161, ...) are reduced by nearly 50% or more under the assumption that the amounts unaccounted for by pumping have returned to the aquifer by infiltration processes.

c. Leakage from the Lachlan Swamps to the aquifer are reduced by nearly 40% (Fig. 1-26), assuming that the clay layer under the swamp has a lower hydraulic conductivity.

d. Infiltration values are redistributed.

e. Transmissivity values are increased mainly in the South Eastern corner of aquifer in the vicinity of its eastern boundary (Fig. 1-27).

Although with the above mentioned data the measured static water level has been reproduced (by FEM), it is not a unique solution. To reach a correct solution, a new series of accurate field data should be collected, including:

- Water level measurements.
- Results of pumping tests.
- Pumping rates and their distributions.
- Infiltration rates.

1.8.2.7 Memory locations and execution time required by FEM and FDM

As mentioned in section 1.7.2 FEM requires more memory locations and longer execution time. Table 1-1 compares these parameters for FD and FE programs (modified program of Prickett - Lonnquist and Program FIELS).

Table 1-1 Memory location and execution time needed for simulation of Botany Basin by FEM and FDM

Method	FEM	FDM
Memory location used by loader	61000	24000
Execution time in CP seconds	23	35

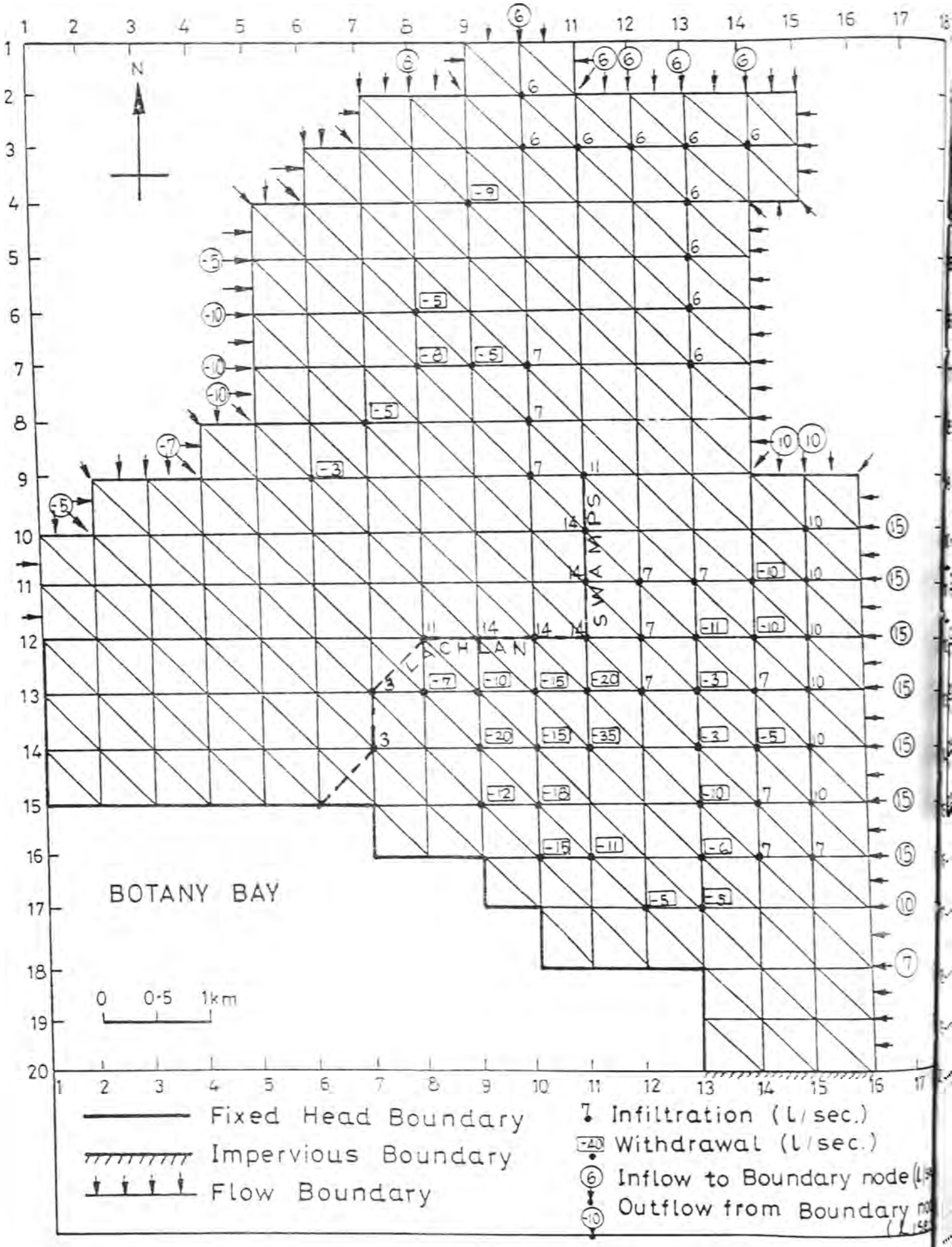

Fig. 1-26 Final distribution of infiltrations and withdrawals

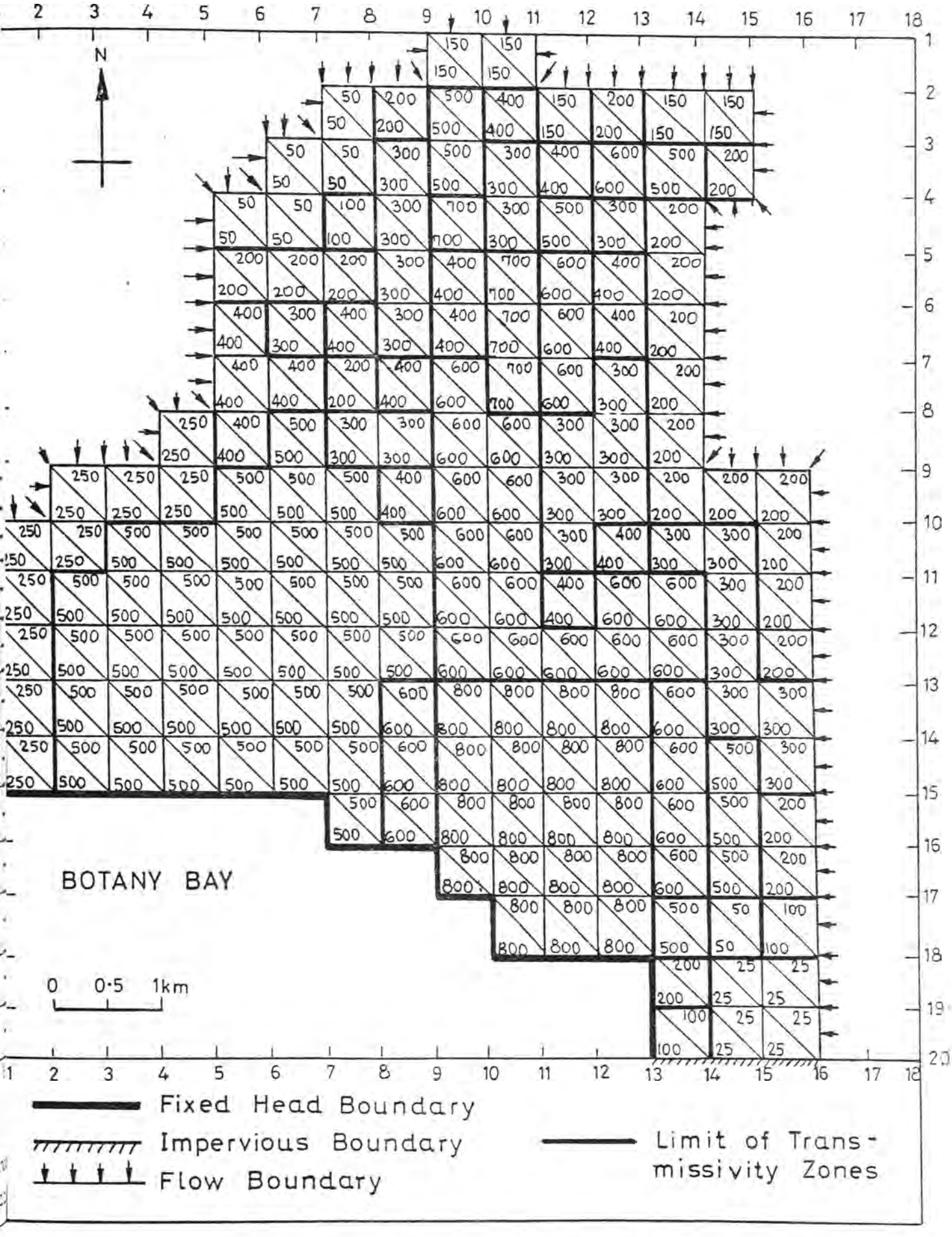

Fig. 1-27 Final distribution of transmissivity values (in m²/day) per element

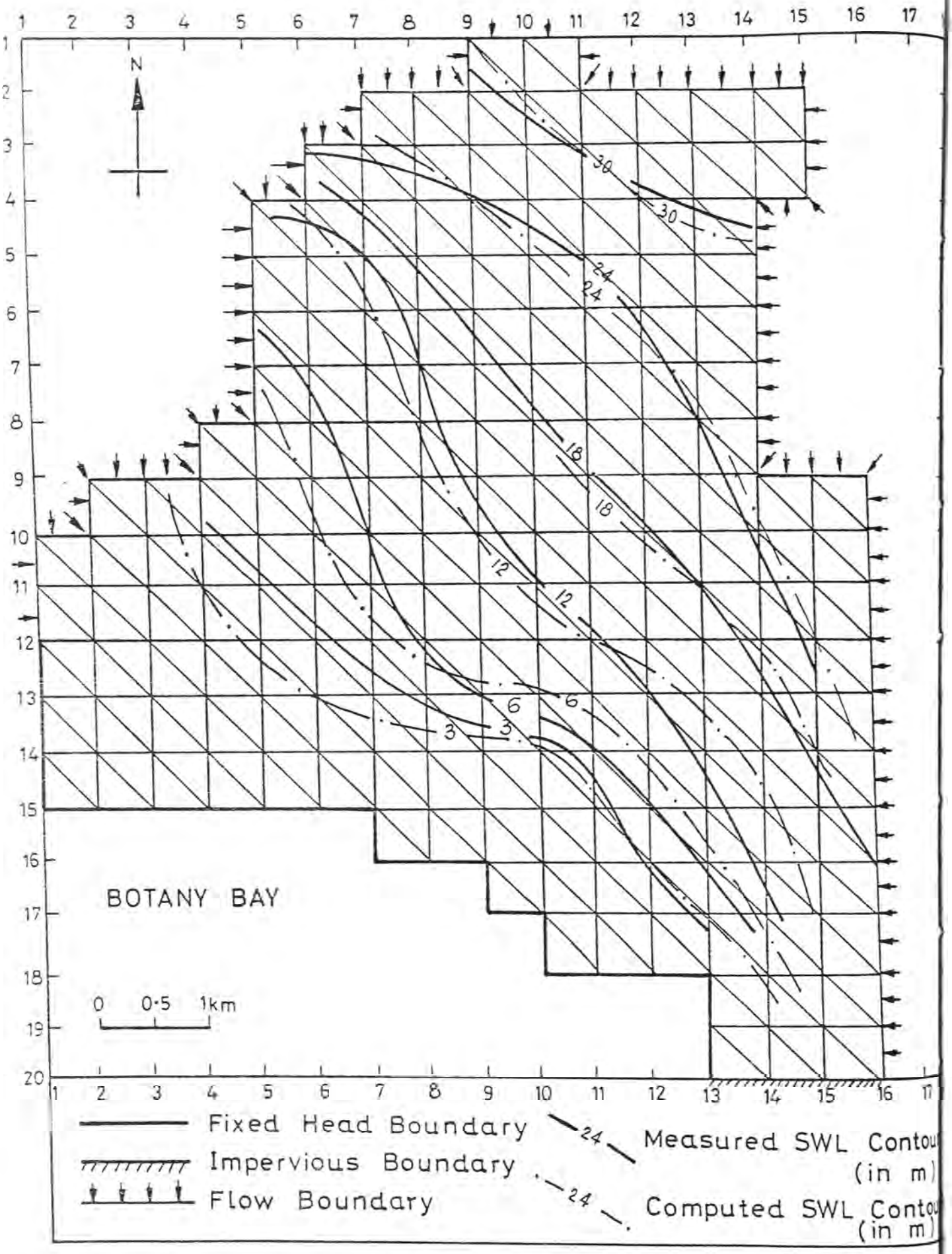

Fig. 1-28 Measured and computed (by FEM) static water level (SWL) contours

The greater number of memory locations needed by FEM is mainly due to the requirement for more data elements.

The longer execution time by FDM is due to the utilisation of a transient model to simulate a steady-state condition (20 time steps were used). If a steady-state model by FDM was used the execution time would be considerably reduced and would be smaller than the execution time of the FEM.

2. TWO-DIMENSIONAL TRANSIENT MATHEMATICAL MODEL BY THE FINITE ELEMENT METHOD (FEM)

2.1 Mathematical Description of the Method

In section 1.1, the finite element formulation for a two-dimentional steady-state flow was developed. In this section a similar formulation for a two-dimensional transient flow will be developed.

The partial differential equation of a two dimensional transient groundwater flow can be represented as follows:

$$\frac{\partial}{\partial x}(T_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_y \frac{\partial h}{\partial y}) = S(x,y)\frac{\partial h}{\partial t} + Q(x,y,t) \qquad (2\text{-}1)$$

where x, y, T_x, T_y and h have been defined in section 1-1 and:

S(x,y)	is the value of storage coefficient.
t	is the time (T).
Q(x,y,t)	is the volumetric flux of recharge or withdrawal per unit area at the moment t (LT^{-1}).

The approximate solution of equation (2-1) is the function h minimizing the functional (2-2), assuming that $\frac{\partial h}{\partial t}$ is constant over a time step.

$$F[h] = \iint_R \{\frac{1}{2}[T_x(\frac{\partial h}{\partial x})^2 + T_y(\frac{\partial h}{\partial y})^2] + [Q(x,y,t) + S(x,y)\frac{\partial h}{\partial t}].h\}dx.dy \qquad (2\text{-}2)$$

The development of equations for transient flow will be discussed in parallel with the steady-state condition. Differentiating equation (1-7) with respect to time (t) will result in:

$$\frac{\partial h}{\partial t} = N_I \frac{\partial h_I}{\partial t} + N_J \frac{\partial h_J}{\partial t} + N_K \frac{\partial h_K}{\partial t} \qquad (2\text{-}3)$$

and

$$F_e = \iint_e \{\frac{1}{2}[T_x(\frac{\partial N_I}{\partial x} h_I + \frac{\partial N_J}{\partial x} h_J + \frac{\partial N_K}{\partial x} h_K)^2 + T_y(\frac{\partial N_I}{\partial y} h_I + \frac{\partial N_J}{\partial y} h_J + \frac{\partial N_K}{\partial y} h_K)^2]$$
$$+ [Q(x,y,t) + S(x,y).(N_I \frac{\partial h_I}{\partial t} + N_J \frac{\partial h_J}{\partial t} + N_K \frac{\partial h_K}{\partial t})] . (N_I h_I + N_J h_J$$
$$+ N_K h_K)\}dx.dy \qquad (2\text{-}4)$$

consequently

$$\frac{\partial F_e}{\partial h_I} = \iint_e \{[T_x(\frac{\partial N_I}{\partial x} h_I + \frac{\partial N_J}{\partial x} h_J + \frac{\partial N_K}{\partial x} h_K).(\frac{\partial N_I}{\partial x}) + T_y(\frac{\partial N_I}{\partial y} h_I +$$
$$\frac{\partial N_J}{\partial y} h_J + \frac{\partial N_K}{\partial y} h_K).(\frac{\partial N_I}{\partial y})] + [Q(x,y,t) + S(x,y).(N_I \frac{\partial h_I}{\partial t} +$$
$$N_J \frac{\partial h_J}{\partial t} + N_K \frac{\partial h_K}{\partial t})].N_I\}dx.dy \qquad (2\text{-}5)$$

Assuming that,

$$Q(x, y, t) = Q$$
$$S(x, y) = S$$

and T_x and T_y are constant in each element, will result in:

$$\frac{\partial F_e}{\partial h_I} = M_I h_I + M_J h_J + M_K h_K + L_I \frac{\partial h_I}{\partial t} + L_J \frac{\partial h_J}{\partial t} + L_K \frac{\partial h_K}{\partial t} + \frac{Q\Delta}{3} \qquad (2\text{-}6)$$

With

$$L_I = \iint_e S\ N_I^2 dx.dy$$

$$L_J = \iint_e S\ N_I N_J dx.dy \qquad (2\text{-}7)$$

$$L_K = \iint_e S\ N_I N_K dx.dy$$

For the development of numerical algorithms for L_I, L_J and L_K the following assumptions are made:

$$\begin{array}{lll} A_I=X_JY_K-X_KY_J & A_J=X_KY_I-X_IY_K & A_K=X_IY_J-X_JY_I \\ B_I=Y_J-Y_K & B_J=Y_K-Y_I & B_K=Y_I-Y_J \\ C_I=X_K-X_J & C_J=X_I-X_K & C_K=X_J-X_I \end{array} \qquad (2\text{-}8)$$

and

$$G_X^{(1)}=\iint_e x\ dx.dy = \frac{\Delta}{3}(X_I+X_J+X_K)$$
$$G_Y^{(2)} = \iint_e y\ dx.dy = \frac{\Delta}{3}(Y_I+Y_J+Y_K)$$
$$G_{XX}^{(3)} = \iint_e x^2\ dx.dy = \frac{\Delta}{12}[X_I^2+X_J^2+X_K^2+(X_I+X_J+X_K)^2] \qquad (2\text{-}9)$$
$$G_{YY}^{(4)} = \iint_e y^2\ dx.dy = \frac{\Delta}{12}[Y_I^2+Y_J^2+Y_K^2+(Y_I+Y_J+Y_K)^2]$$
$$G_{XY}^{(5)} = \iint_e xy\ dx.dy = \frac{\Delta}{12}[X_IY_I+X_JY_J+X_KY_K+(X_I+X_J+X_K).(Y_I+Y_J+Y_K)]$$

therefore:

$$L_I = S[A_I^2\ \Delta+B_I^2\ G_{XX} + C_I^2\ G_{YY} + 2A_IB_IG_X + 2A_IC_IG_Y + 2B_IC_IG_{XY}]/(2\Delta)^2$$

$$L_J = S[A_IA_J\Delta + B_IB_JG_{XX}+C_IC_JG_{YY}+(A_IB_J+B_IA_J)\ G_X+(A_IC_J + C_IA_J)G_Y + (B_IC_J + C_IB_J)G_{XY}]/(2\Delta)^2 \qquad (2\text{-}10)$$

$$L_K = S[A_IA_K\Delta + B_IB_KG_{XX} + C_IC_KG_{YY} + (A_IB_K + B_IA_K)G_X + (A_IC_K+C_IA_K)G_Y + (B_IC_K+C_IB_K)\ .\ G_{XY}]/(2\Delta)^2$$

(1) - G_X = Area moment of element about the y axis.
(2) - G_Y = Area moment of element about the x axis.
(3) - G_{XX} = Area moment of Inertia of element about the y axis.
(4) - G_{YY} = Area moment of Inertia of element about the x axis.
(5) - G_{XY} = Area product of Inertia of element.

Now, replacing the terms $\frac{\partial h_I}{\partial t}$, $\frac{\partial h_J}{\partial t}$ and $\frac{\partial h_K}{\partial t}$ by their forward difference forms in equation (2-6) will result in:

$$\frac{\partial F_e}{\partial h_I} = M_I h_I^{t+\Delta t} + M_J h_J^{t+\Delta t} + M_K h_K^{t+\Delta t} + L_I \frac{h_I^{t+\Delta t} - h_I^t}{\Delta t} +$$

$$L_J \frac{h_J^{t+\Delta t} - h_J^t}{\Delta t} + L_K \frac{h_K^{t+\Delta t} - h_K^t}{\Delta t} + \frac{Q\Delta}{3} \qquad (2\text{-}11)$$

or:

$$\frac{\partial F_e}{\partial h_I} = h_I^{t+\Delta t}(M_I + \frac{L_I}{\Delta t}) + h_J^{t+\Delta t}(M_J + \frac{L_J}{\Delta t}) + h_K^{t+\Delta t}(M_K + \frac{L_K}{\Delta t}) -$$

$$\frac{1}{\Delta t}(L_I h_I^t + L_J h_J^t + L_K h_K^t) + \frac{Q\Delta}{3} \qquad (2\text{-}12)$$

Therefore by using (1-10) and (2-12) a set of linear equations will be generated, leading to values for $h^{t+\Delta t}$.

2.2 Form of the Matrix for Transient Flow

Application of the finite element formulation for transient flow will generate, in each time step a set of linear equations as follows:

$$[A'] \cdot [H'] = [B'] \qquad (2\text{-}13)$$

where

[A'] = The N x N band matrix of the coefficients for the unknown terms. This matrix has exactly the same properties as the matrix [A] generated in the case of steady-state flow (section 1.2).
[H'] = column matrix of unknown heads.
[B'] = column matrix of known terms.

2.3 Treatment of Boundary Conditions

The treatment of Neumann boundary conditions in transient flow, just as with steady flow does not need the development of any special equations. The treatment of Direchlet boundary conditions requires coding for identification of the nodes during the generation of the linear equations. In this case the terms of $h_J^{t+\Delta t}(M_J + \frac{L_J}{\Delta t})$ or $h_K^{t+\Delta t}(M_K + \frac{L_K}{\Delta t})$ should be shifted to the right hand side (constant terms) of the equations.

2.4 Solution of the Linear set of Equations

The solution of the linear set of equations (2-13) is similar to the solution of (1-22) which is already described in section (1-4).

2.5 Stability of the Procedure

The finite element method is a stable procedure which does not require any stability criteria.

2.6 Description of the computer Program TMATFE

The program TMATFE has been prepared to solve the problem of two-dimensional transient groundwater flow by FEM.

The FORTRAN IV source program of this model is listed in Appendix B1 and consists of almost 800 lines of instructions and comments. Selected variables used in this program are defined at the beginning of the source program.

Fig. 2-1 shows the simplified flow chart of the program. This program is segmented into a main routine and twelve subroutines. The succession of the operations which can be divided into three sections is as follows:

a. First section (Data preparations):

-SUBROUTINE TMREAD	Reads the input data.
-SUBROUTINE TMWRIT	Echoes the input data.
-SUBROUTINE TMBAND	Computes width of the band matrix.
-SUBROUTINE TMCONS	Initializes the wells and piezometers (observation wells) to their piezometric values.

b. Second section (computation in transient condition):

-SUBROUTINE TMIMPO	Imposes the boundary and internal conditions (withdrawals and injections).
-SUBROUTINE TMGENR	Generates the set of linear equations and stores the matrix A in its compact form.
-SUBROUTINE TMTRIA	Uppertriangulates the compact form of matrix A.
-SUBROUTINE TMSOLV	Solves the upper triangulated band matrix.
-SUBROUTINE TMVARI	Computes the piezometric variations in the selected wells and Piezometers.
-SUBROUTINE TMRESU	Prints the results of computation for the preselected time steps.

c. Third section (Printing of the final results):

-SUBROUTINE TMWELL	Prints the piezometric variations in the selected wells.
-SUBROUTINE TMPIEZ	Prints the piezometric variations in the piezometers (observation wells).

In this program the structure of SUBROUTINE TMGENR is similar to SUBROUTINE GENER (Section 1-5) but it uses more complicated algorithms for the generation of the band matrix.

It should also be mentioned that if in a problem the boundary condition and stresses (rates of withdrawals and infiltrations) do not change in each time step, to facilitate the task of data preparation it is possible to eliminate SUBROUTINE TMIMPO and replace its function with a few simple instructions in the main routine.

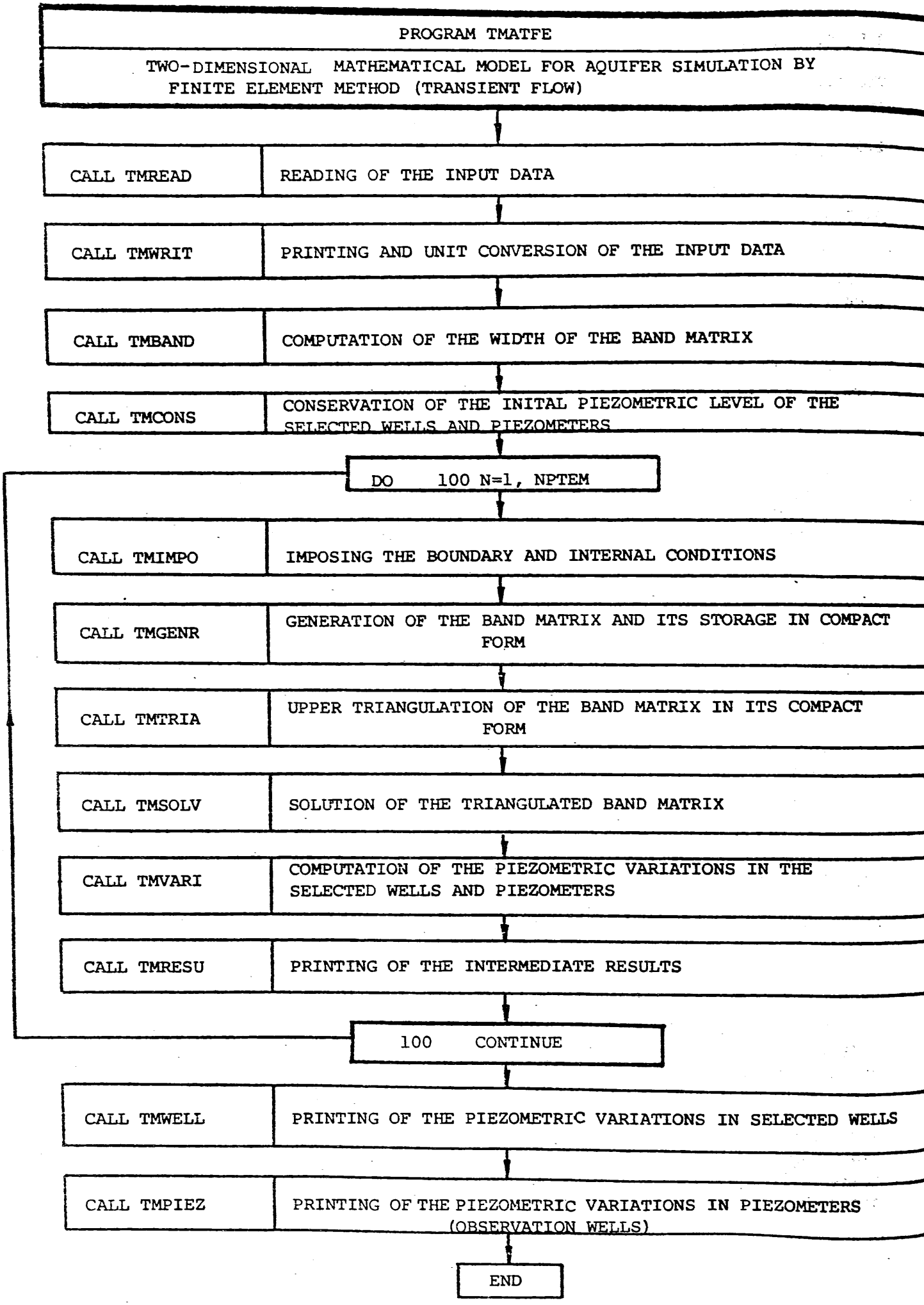

Fig. 2-1 Simplified flow chart of Program TMATFE

2.7 Data Preparation

In this procedure, creation of elements and numbering of the nodes and elements is mainly the same as with the steady flow case (Section 1.6) but more data elements are needed. Appendix B2 describes the format of the input data.

2.8 Advantages and Disadvantages of FEM in Transient Flow Simulations

The Finite Element Method simulations of two dimensional transient flow has exactly the same advantages and disadvantages described in Section 1.7 for steady flow.

2.9 Examples of Application

Two examples of application of Program TMATFE will be given in this section.

2.9.1 First Example (Test Problem II)

In this example which can be used as a test problem by the users of Program TMATFE, the square aquifer of Test Problem I (Section 1.8.1) will be simulated in transient conditions with the following assumptions:

- A single well is pumping in the centre of the aquifer (Fig. 2-2) at a rate of 100 l/sec for a period of 2 years, after which the pumping is stopped.
- Two observation wells (piezometers) are situated to the south and north of the pumping well (Fig. 2-2).

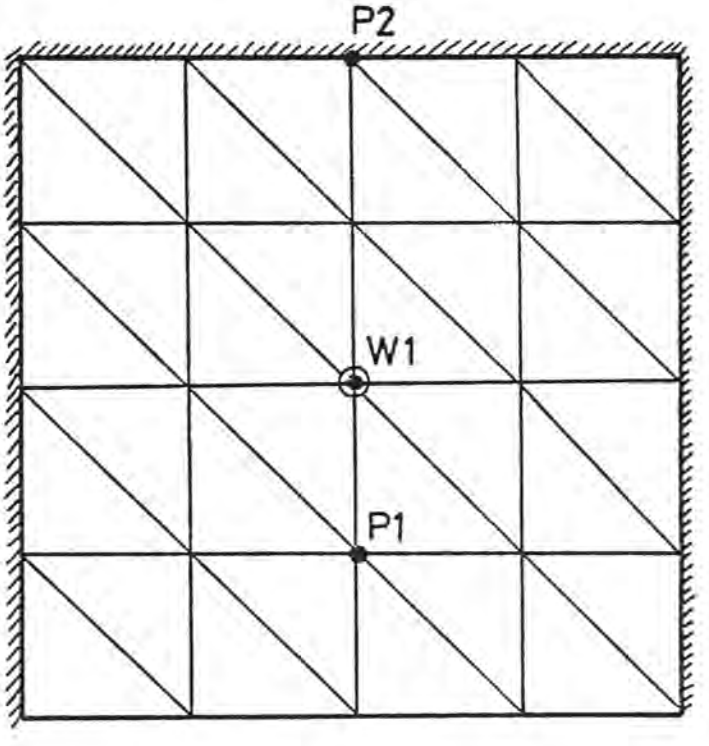

Fig. 2-2 Location of the well and Piezometers (observation wells)

- Initial heads of all the nodes is 80 m.
- Heads of the five southern boundary nodes will be 80 m in all time steps.
- Storage coefficients and transmissivity values are respectively $1x10^{-2}$

and 250 m^2/d.

- The simulation period will be 7 years divided into 15 time steps ranging from 0.5 to 360 days.

Input data for this test problem and selected output of Program TMATFE are presented in Appendix B3. Input data are prepared according to the requirements of Appendix B2. In this series, data set 8 will be read by SUBROUTINE TMIMPO. As noted, three data cards are prepared for each time step. The first two (labelled QT) are the flow rates and the third one (labelled HT) contains the fixed heads which will be imposed on the 5 nodes of the southern boundary.

Figs. 2-3, 2-4 and 2-5 represent, respectively, the piezometric variations in well No. 1 and piezometers P1 and P2. It is clear that after the start of pumping the piezometric level decreases and reaches a nearly steady state condition (after 2 years), then after stopping the pumping, the water level rises again and reaches its initial values (80 m).

Fig. 2-6 represents the piezometric map of the aquifer after 2 years of pumping. Comparing this Figure with Fig. 1-13 shows that there is a small difference between these two results, which means that, after 2 years the results are close to a steady-state condition.

Fig. 2-7 represents the piezometric map of the test aquifer after 7 years. As noted all computed values are 80 m, so that a steady-state condition similar to the initial condition has been established.

2.9.2 Second Example (Simulation of Botany Basin Aquifer in Transient Condition)

In this example the Botany Basin aquifer is simulated in transient conditions with the following assumptions;

- Storage coefficient of the aquifer is uniform and equal to 0.2.
- Period of simulation will be 10 years divided into 10 time-steps varying from 15 to 1080 days.
- Three additional pumping wells (W_1, W_2 and W_3) are located on nodes 105, 122 and 123 (Fig. 2-14).
- Three observation wells or piezometers P_1, P_2 and P_3 are located on nodes 59, 77 and 121 (Fig. 2-14).
- The computed static water level of the aquifer by FEM (Fig. 1-28) is considered as the initial condition of the problem.
- Withdrawals (except for the three new wells), infiltrations and boundary conditions are considered to be the same as in the steady-state condition.

The objective is to provide flow rates of the wells W_1, W_2 and W_3 which should not exceed 20 l/sec and should not produce the negative heads in the aquifer.

To simulate correctly such a problem, the model must be calibrated also in transient conditions. That is, the variations of the piezometric level of the aquifer during past years should be reproduced by the model. Since data were not available for this, it was assumed that the model is calibrated(1). The following alternatives were then tested.

(1) - This assumption is not far from the reality because the aquifer has been in a nearly steady state condition during the last four years.

Table 2-1 PIEZOMETRIC VARIATIONS OF WELL NO. 1

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	FLOW RATE (L/S)	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	.5	0	0	.5	-100.0	77.44	-2.56
2	1.5	0	0	2.0	-100.0	74.04	-5.96
3	5.0	0	0	7.0	-100.0	70.48	-9.52
4	13.0	0	0	20.0	-100.0	67.17	-12.83
5	20.0	0	1	10.0	-100.0	64.36	-15.64
6	40.0	0	2	0.0	-100.0	61.19	-18.81
7	90.0	0	5	0.0	-100.0	58.27	-21.73
8	120.0	0	9	0.0	-100.0	56.92	-23.08
9	180.0	1	3	0.0	-100.0	56.39	-23.61
10	270.0	2	0	0.0	-100.0	56.24	-23.76
11	360.0	3	0	0.0	0.0	77.80	-2.20
12	360.0	4	0	0.0	0.0	79.68	-.32
13	360.0	5	0	0.0	0.0	79.95	-.05
14	360.0	6	0	0.0	0.0	79.99	-.01
15	360.0	7	0	0.0	0.0	80.00	-.00

WELL NO. 1 IS LOCATED ON NODE NO. 10
INITIAL PIEZOMETRIC LEVEL OF THE WELL IS 80.00 M.
NEGATIVE FLOW RATES = WITHDRAWAL
POSITIVE FLOW RATES = RECHARGE

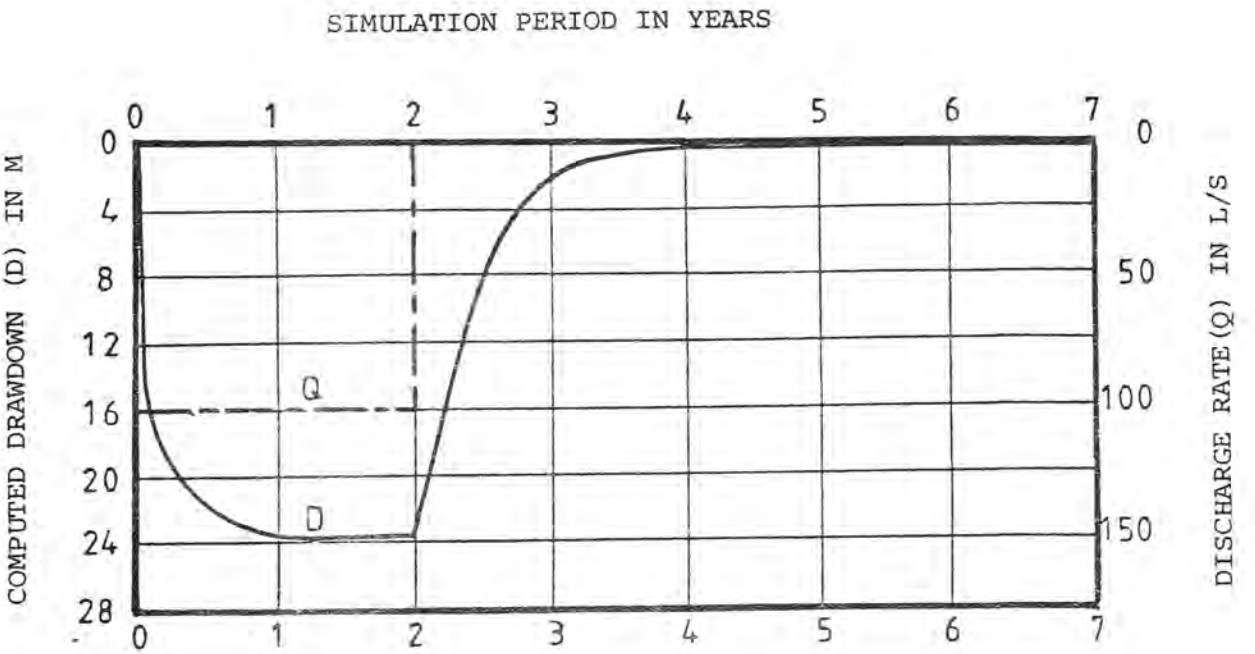

Fig. 2-3 Discharge rate and computed drawdown for well No. 1

Table 2-2 PIEZOMETRIC VARIATIONS IN PIEZOMETER NO. 1

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	.5	0	0	.5	80.10	.10
2	1.5	0	0	2.0	79.57	-.43
3	5.0	0	0	7.0	77.97	-2.03
4	13.0	0	0	20.0	75.96	-4.04
5	20.0	0	1	10.0	74.32	-5.68
6	40.0	0	2	0.0	72.56	-7.44
7	90.0	0	5	0.0	70.98	-9.02
8	120.0	0	9	0.0	70.25	-9.75
9	180.0	1	3	0.0	69.96	-10.04
10	270.0	2	0	0.0	69.88	-10.12
11	360.0	3	0	0.0	78.80	-1.20
12	360.0	4	0	0.0	79.82	-.18
13	360.0	5	0	0.0	79.97	-.03
14	360.0	6	0	0.0	80.00	-.00
15	360.0	7	0	0.0	80.00	-.00

PIEZOMETER NO. 1 IS LOCATED ON NODE NO. 9
INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS 80.00 M.

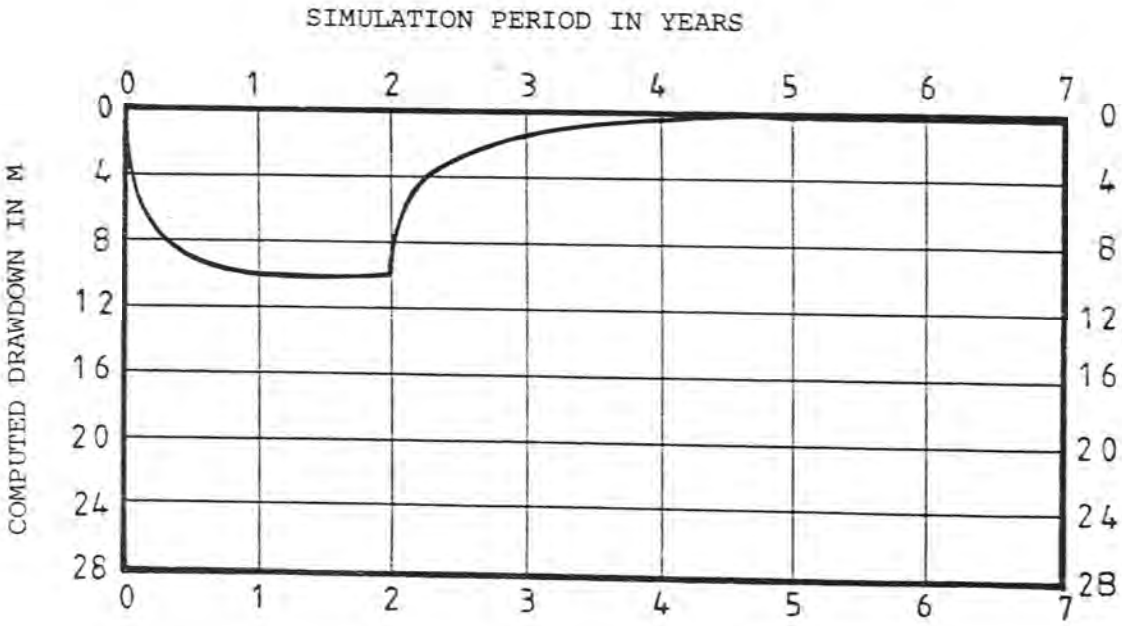

Fig. 2-4 Computed drawdown of Piezometer (observation well) No. 1

Table 2-3 PIEZOMETRIC VARIATIONS IN PIEZOMETER NO. 2

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	.5	0	0	.5	79.99	-.01
2	1.5	0	0	2.0	79.97	-.03
3	5.0	0	0	7.0	79.19	-.81
4	13.0	0	0	20.0	76.64	-3.36
5	20.0	0	1	10.0	73.27	-6.73
6	40.0	0	2	0.0	68.94	-11.06
7	90.0	0	5	0.0	64.84	-15.16
8	120.0	0	9	0.0	62.94	-17.06
9	180.0	1	3	0.0	62.18	-17.82
10	270.0	2	0	0.0	61.97	-18.03
11	360.0	3	0	0.0	77.01	-2.99
12	360.0	4	0	0.0	79.54	-.46
13	360.0	5	0	0.0	79.93	-.07
14	360.0	6	0	0.0	79.99	-.01
15	360.0	7	0	0.0	80.00	-.00

PIEZOMETER NO. 2 IS LOCATED ON NODE NO. 12
INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS 80.00 M.

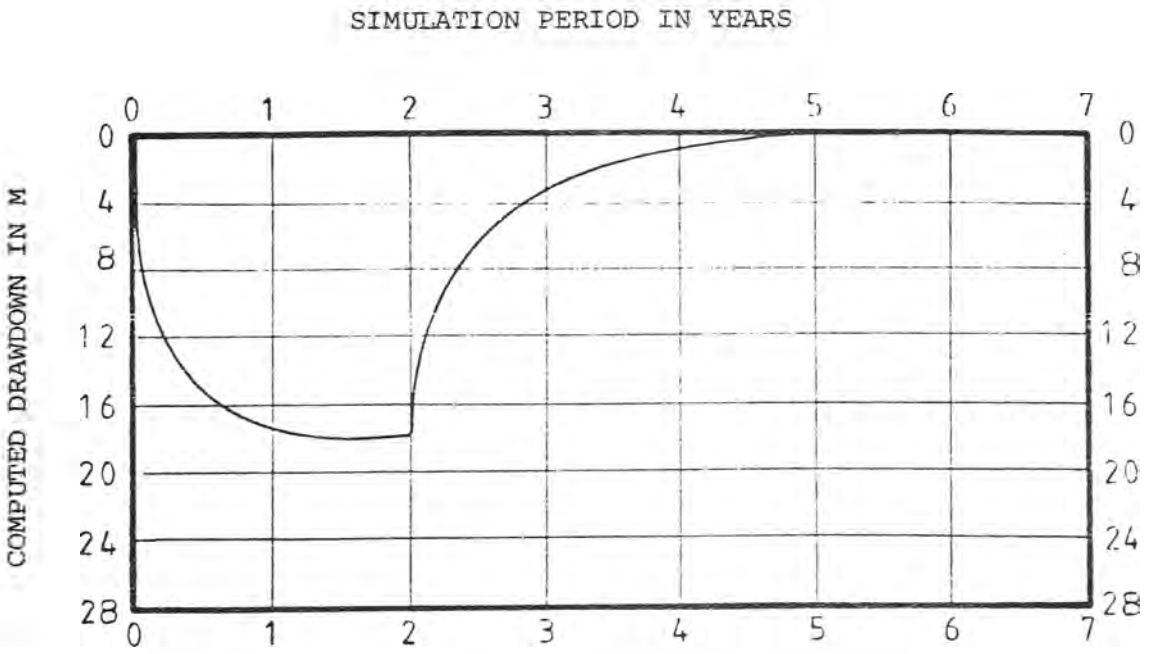

Fig. 2-5 Computed drawdown of Piezometer (observation well) No. 2

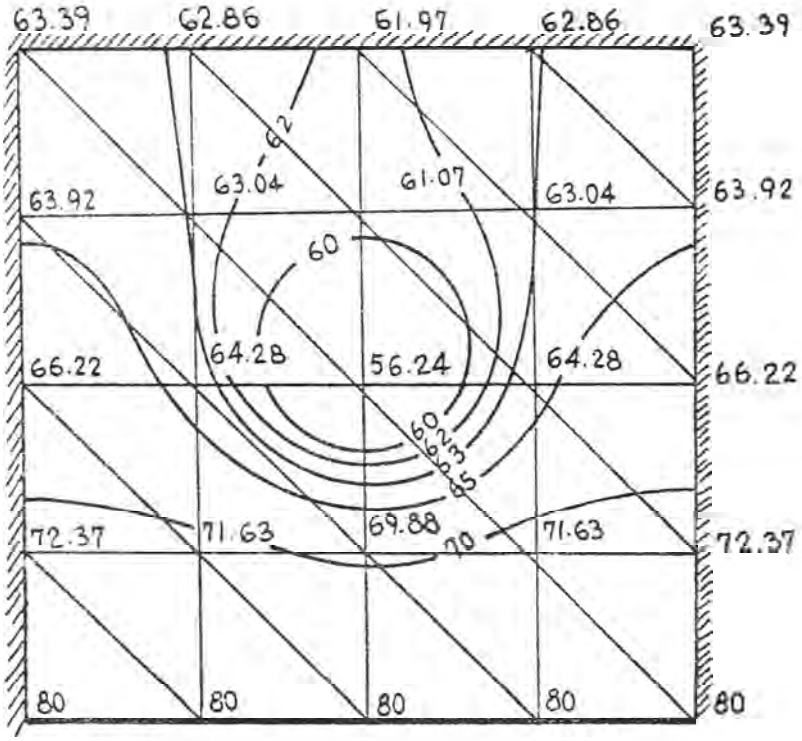

Fig. 2-6 Computed heads and contour lines (in m) for time-step No. 10 (Simulation Period = 2 years)

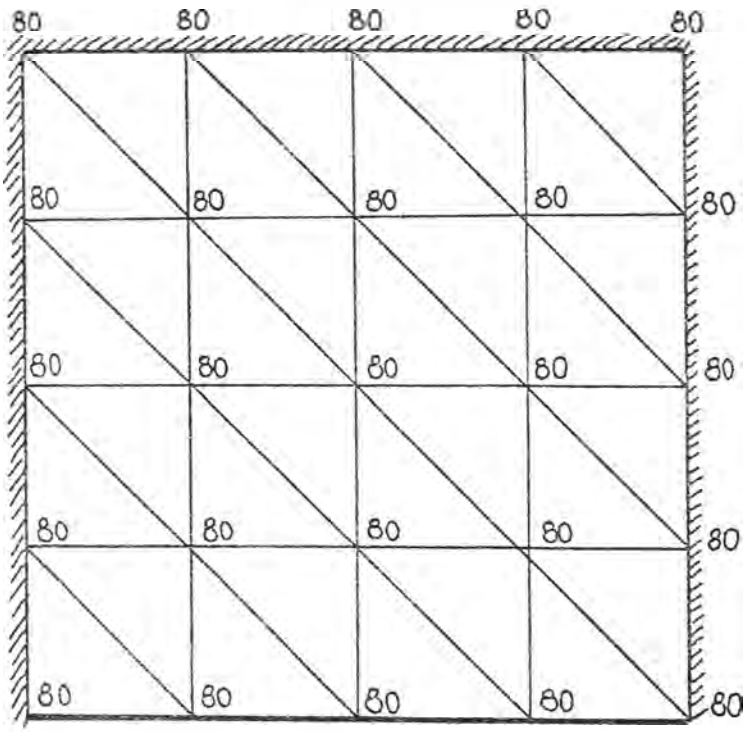

Fig. 2-7 Computed heads (in m) for time-step No. 15 (Simulation Period = 7 years)

Table 2-4 PIEZOMETRIC VARIATIONS OF WELL NO. 1

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	FLOW RATE (L/S)	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	-5.0	3.13	-.12
2	15.0	0	1	0.0	-5.0	3.05	-.20
3	60.0	0	3	0.0	-5.0	2.85	-.40
4	90.0	0	6	0.0	-5.0	2.68	-.57
5	180.0	1	0	0.0	-5.0	2.51	-.74
6	360.0	2	0	0.0	-5.0	2.35	-.90
7	540.0	3	6	0.0	-5.0	2.23	-1.02
8	540.0	5	0	0.0	-5.0	2.17	-1.08
9	720.0	7	0	0.0	-5.0	2.11	-1.14
10	1080.0	10	0	0.0	-5.0	2.06	-1.19

WELL NO. 1 IS LOCATED ON NODE NO. 105
INITIAL PIEZOMETRIC LEVEL OF THE WELL IS 3.25 M.
NEGATIVE FLOW RATES = WITHDRAWAL
POSITIVE FLOW RATES = RECHARGE

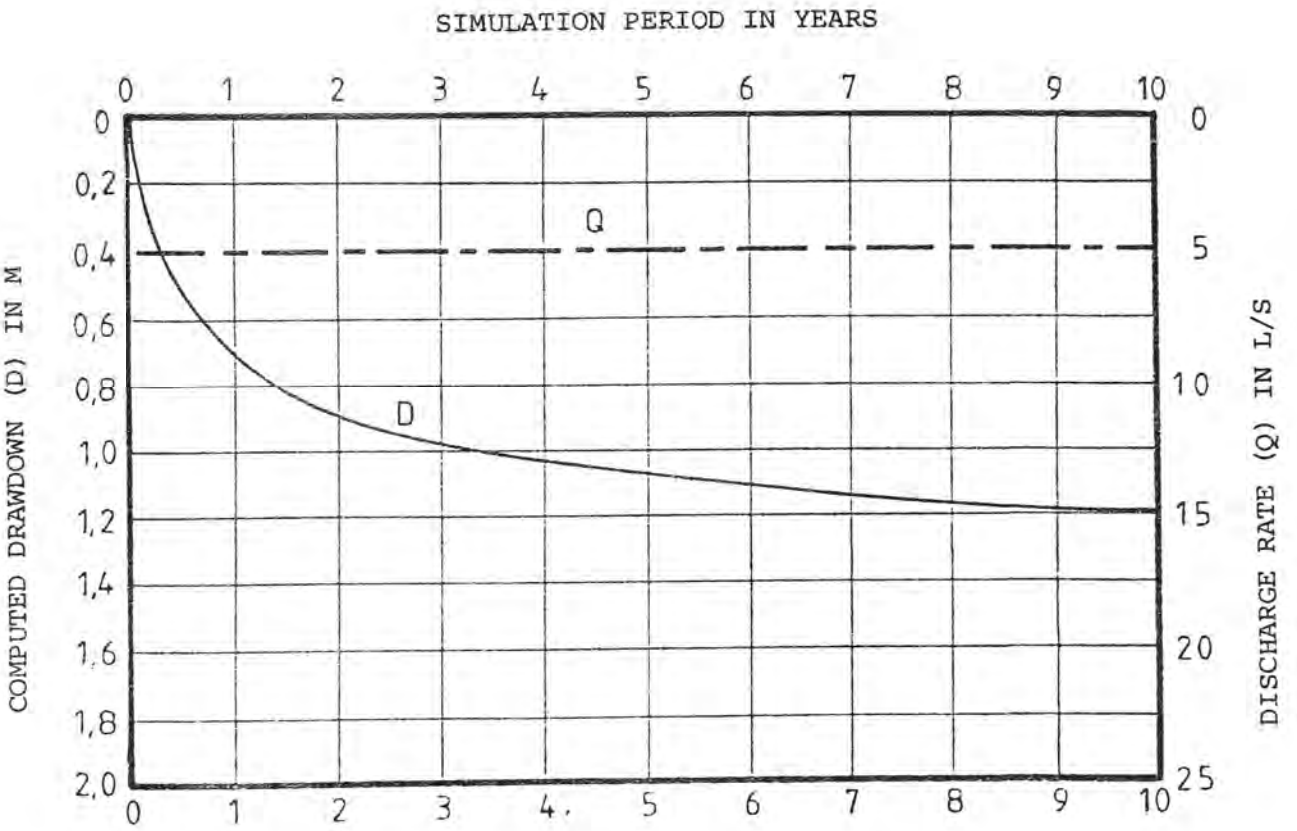

Fig. 2-8 Discharge rate and computed drawdown of well No. 1

Table 2-5 PIEZOMETRIC VARIATIONS OF WELL NO. 2

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	FLOW RATE (L/S)	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	-10.0	5.42	-.23
2	15.0	0	1	0.0	-10.0	5.27	-.38
3	60.0	0	3	0.0	-10.0	5.01	-.64
4	90.0	0	6	0.0	-10.0	4.80	-.85
5	180.0	1	0	0.0	-10.0	4.60	-1.05
6	360.0	2	0	0.0	-10.0	4.41	-1.24
7	540.0	3	6	0.0	-10.0	4.27	-1.38
8	540.0	5	0	0.0	-10.0	4.18	-1.47
9	720.0	7	0	0.0	-10.0	4.11	-1.54
10	1080.0	10	0	0.0	-10.0	4.05	-1.60

WELL NO. 2 IS LOCATED ON NODE NO. 122
INITIAL PIEZOMETRIC LEVEL OF THE WELL IS 5.65 M.
NEGATIVE FLOW RATES = WITHDRAWAL
POSITIVE FLOW RATES = RECHARGE

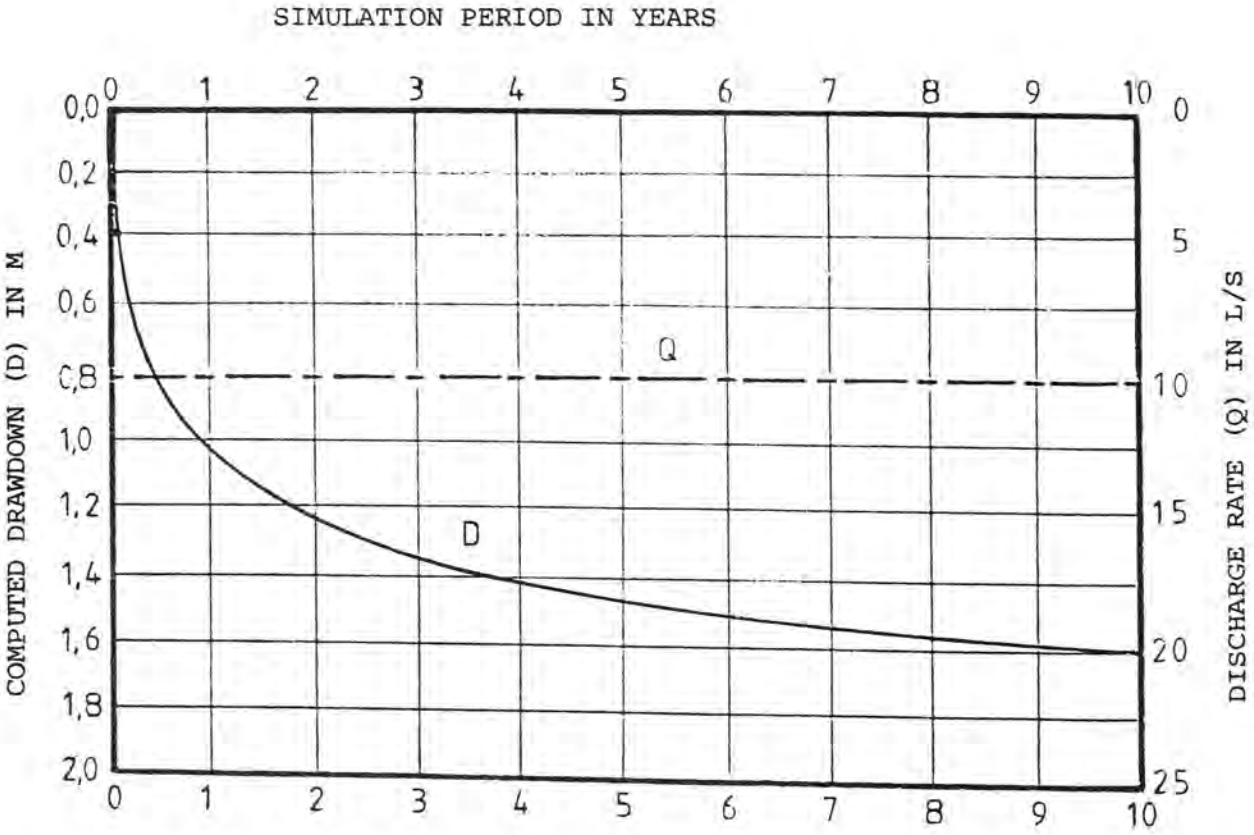

Fig. 2-9 Discharge rate and computed drawdown of well No. 2

Table 2-6 PIEZOMETRIC VARIATIONS OF WELL NO. 3

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	FLOW RATE (L/S)	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	-20.0	7.20	-.39
2	15.0	0	1	0.0	-20.0	7.01	-.58
3	60.0	0	3	0.0	-20.0	6.72	-.87
4	90.0	0	6	0.0	-20.0	6.50	-1.09
5	180.0	1	0	0.0	-20.0	6.27	-1.32
6	360.0	2	0	0.0	-20.0	6.06	-1.53
7	540.0	3	6	0.0	-20.0	5.89	-1.70
8	540.0	5	0	0.0	-20.0	5.78	-1.81
9	720.0	7	0	0.0	-20.0	5.69	-1.90
10	1080.0	10	0	0.0	-20.0	5.61	-1.98

WELL NO. 3 IS LOCATED ON NODE NO. 123
INITIAL PIEZOMETRIC LEVEL OF THE WELL IS 7.59 M.
NEGATIVE FLOW RATES = WITHDRAWAL
POSITIVE FLOW RATES = RECHARGE

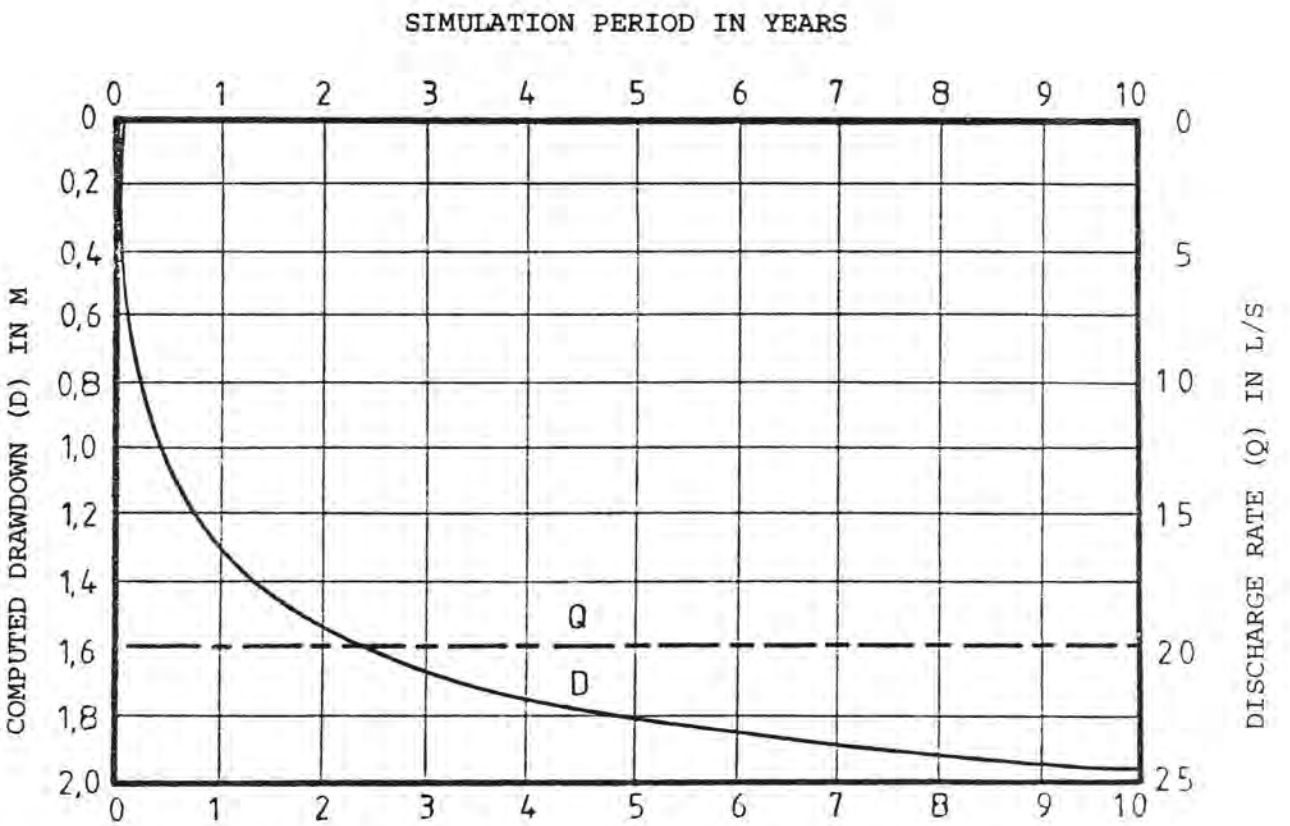

Fig. 2-10 Discharge rate and computed drawdown of well No. 3

Table 2-7 PIEZOMETRIC VARIATIONS IN PIEZOMETER NO. 1

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	2.31	-.00
2	15.0	0	1	0.0	2.31	-.00
3	60.0	0	3	0.0	2.30	-.01
4	90.0	0	6	0.0	2.27	-.04
5	180.0	1	0	0.0	2.21	-.10
6	360.0	2	0	0.0	2.14	-.17
7	540.0	3	6	0.0	2.08	-.23
8	540.0	5	0	0.0	2.04	-.27
9	720.0	7	0	0.0	2.00	-.31
10	1080.0	10	0	0.0	1.97	-.34

PIEZOMETER NO. 1 IS LOCATED ON NODE NO. 59
INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS 2.31 M.

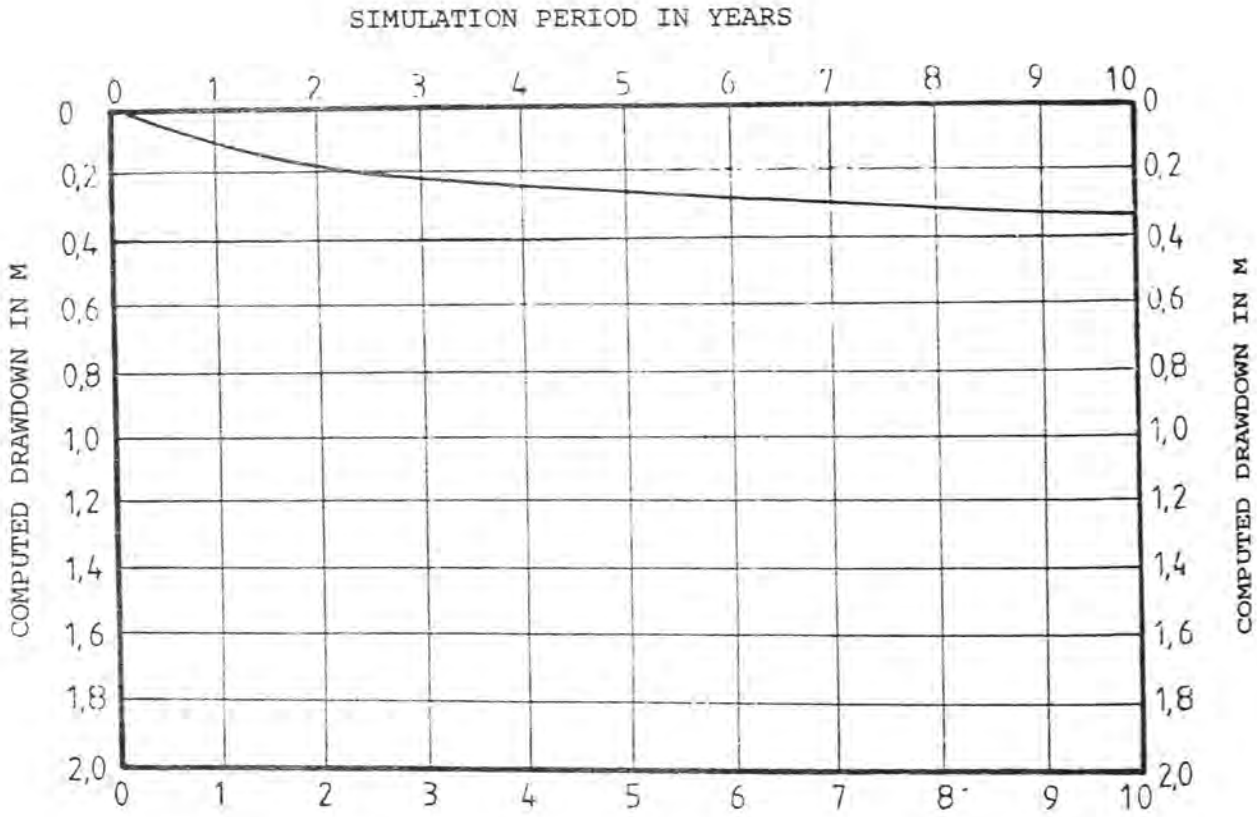

Fig. 2-11 Computed drawdown of Piezometer (observation well) No. 1

Table 2-8 PIEZOMETRIC VARIATIONS IN PIEZOMETE NO. 2

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	13.14	-.00
2	15.0	0	1	0.0	13.14	-.00
3	60.0	0	3	0.0	13.14	-.00
4	90.0	0	6	0.0	13.13	-.01
5	180.0	1	0	0.0	13.10	-.04
6	360.0	2	0	0.0	13.02	-.12
7	540.0	3	6	0.0	12.93	-.21
8	540.0	5	0	0.0	12.85	-.29
9	720.0	7	0	0.0	12.77	-.37
10	1080.0	10	0	0.0	12.68	-.46

PIEZOMETER NO. 2 IS LOCATED ON NODE NO. 77
INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS 13.14 M.

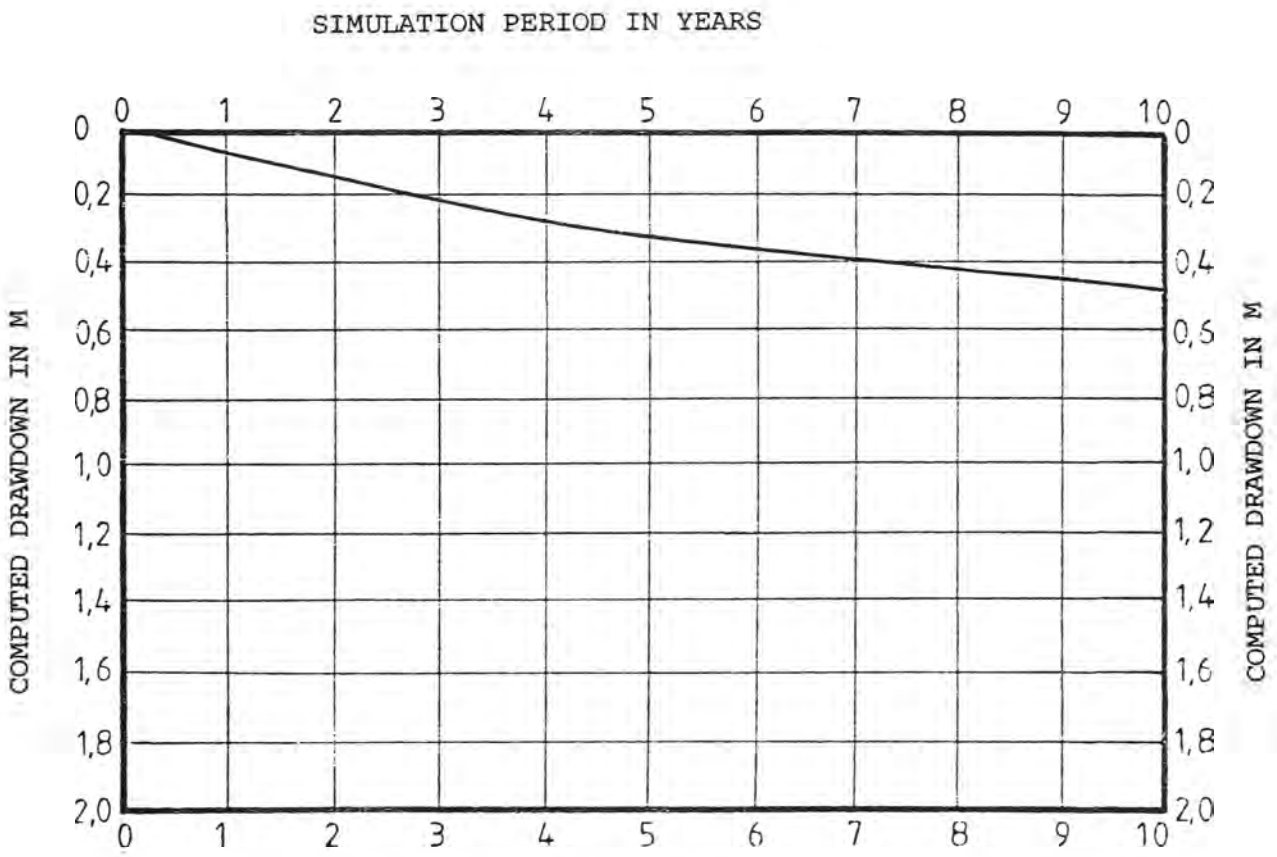

Fig. 2-12 Computed drawdown of Piezometer (observation well) No. 2

Table 2-9 PIEZOMETRIC VARIATIONS IN PIEZOMETER NO. 3

TIME STEP NO.	LENGTH OF TIME STEP (DAY)	SIMULATION PERIOD YEAR	MONTH	DAY	PIEZOMETRIC LEVEL (M)	PIEZOMETRIC VARIATIONS (M)
1	15.0	0	0	15.0	3.79	-.02
2	15.0	0	1	0.0	3.75	-.06
3	60.0	0	3	0.0	3.60	-.21
4	90.0	0	6	0.0	3.47	-.34
5	180.0	1	0	0.0	3.32	-.49
6	360.0	2	0	0.0	3.18	-.63
7	540.0	3	6	0.0	3.08	-.73
8	540.0	5	0	0.0	3.01	-.80
9	720.0	7	0	0.0	2.96	-.85
10	1080.0	10	0	0.0	2.92	-.89

PIEZOMETER NO. 3 IS LOCATED ON NODE NO. 121
INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS 3.81 M.

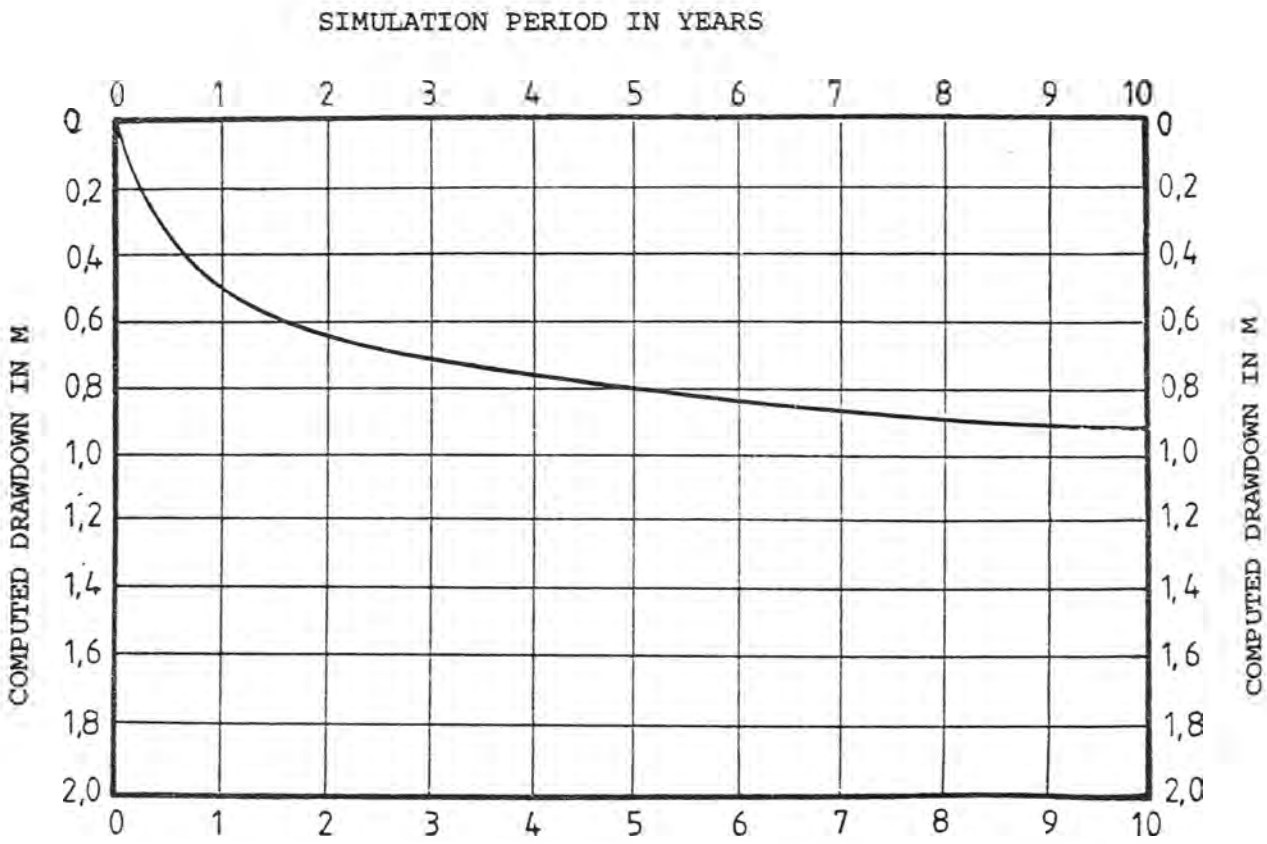

Fig. 2-13 Computed drawdown of Piezometer (observation well) No. 3

Table 2-10 COMPUTED PIEZOMETRIC VALUES FOR TIME STEP NO. 10
(YEAR= 10 MONTH= 0 DAY= 0.0)

NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1
1	1.69	2	1.65	3	.66	4	1.26	5	1.71	6	1.95	7	1.61	8	2.17	9	.79	10	1.51
11	2.11	12	2.53	13	2.70	14	2.74	15	.98	16	1.69	17	2.67	18	3.26	19	3.57	20	3.37
21	4.37	22	1.23	23	2.39	24	3.43	25	4.29	26	4.90	27	5.40	28	5.37	29	6.00	30	6.34
31	7.22	32	10.94	33	1.55	34	3.01	35	4.37	36	5.49	37	6.35	38	6.90	39	7.51	40	8.30
41	9.15	42	10.62	43	14.62	44	16.41	45	1.94	46	3.75	47	5.55	48	6.95	49	8.05	50	8.76
51	9.30	52	10.98	53	12.54	54	14.73	55	18.53	56	22.16	57	25.94	58	.58	59	1.97	60	3.89
61	7.14	62	8.71	63	10.12	64	11.42	65	13.07	66	13.95	67	15.70	68	19.14	69	22.48	70	25.73
71	29.69	72	.31	73	1.50	74	4.12	75	8.49	76	10.64	77	12.68	78	14.72	79	16.43	80	17.44
81	19.48	82	21.98	83	23.79	84	26.72	85	28.64	86	30.47	87	.03	88	.63	89	2.05	90	4.71
91	9.63	92	12.34	93	14.75	94	17.05	95	18.83	96	20.33	97	21.89	98	23.47	99	25.30	100	28.19
101	29.97	102	32.28	103	.65	104	1.12	105	2.06	106	2.96	107	5.98	108	10.95	109	14.35	110	16.93
111	18.87	112	20.26	113	21.68	114	23.26	115	24.99	116	27.50	117	29.82	118	31.29	119	31.79	120	1.49
121	2.92	122	4.05	123	5.61	124	8.87	125	12.66	126	15.67	127	18.18	128	20.18	129	21.65	130	23.06
131	24.69	132	26.59	133	29.39	134	31.49	135	34.05	136	2.95	137	5.02	138	6.67	139	8.71	140	11.38
141	14.18	142	17.31	143	19.84	144	21.99	145	23.09	146	24.86	147	26.51	148	28.59	149	31.41	150	32.83
151	35.41	152	1.64	153	1.92	154	3.23	155	6.62	156	9.19	157	11.15	158	12.51	159	15.05	160	16.95
161	19.76	162	23.51	163	25.98	164	24.36	165	25.25	166	26.05	167	29.09	168	32.31	169	33.89	170	36.32
171	6.79	172	8.15	173	12.42	174	12.23	175	13.66	176	15.97	177	18.31	178	21.30	179	23.97	180	26.77
181	29.70	182	31.82	183	33.14	184	33.95	185	35.14	186	9.42	187	11.35	188	19.83	189	19.05	190	20.24
191	21.62	192	23.28	193	26.23	194	29.99	195	32.78	196	34.00	197	33.21	198	0.00	199	0.00	200	0.00
201	0.00	202	0.00	203	0.00	204	0.00	205	0.00	206	0.00	207	0.00	208	0.00	209	0.00	210	0.00
211	0.00	212	0.00	213	0.00	214	0.00	215	0.00	216	.50	217	1.00	218	1.25				

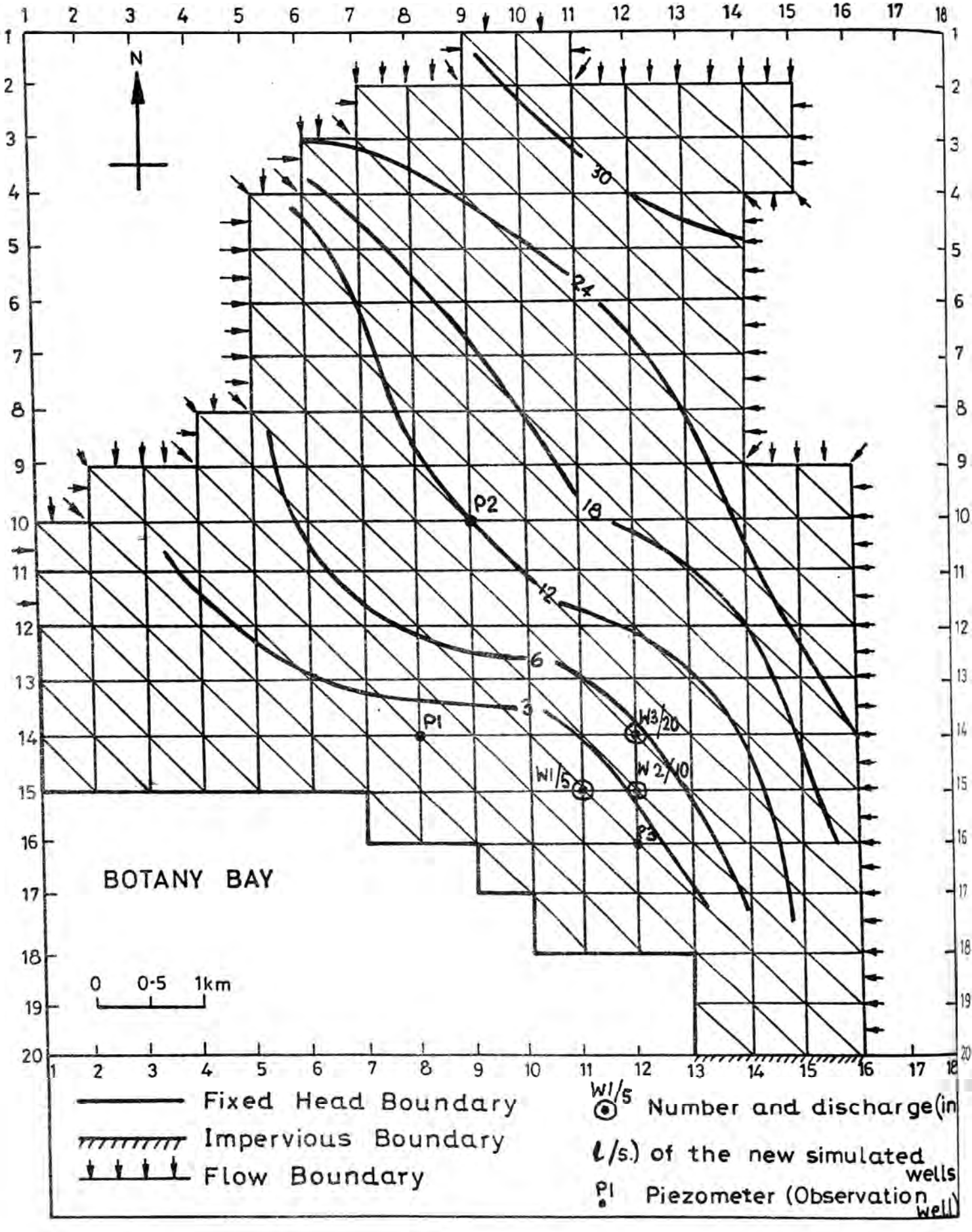

Fig. 2-14 Computed water level of Botany Basin aquifer in transient condition after 10 years

- First alternative

Discharge of W_1 = 20 l/sec
Discharge of W_2 = 20 l/sec
Discharge of W_3 = 20 l/sec

- Second alternative

Discharge of W_1 = 10 l/sec
Discharge of W_2 = 20 l/sec
Discharge of W_3 = 20 l/sec

- Third alternative

Discharge of W_1 = 5 l/sec
Discharge of W_2 = 10 l/sec
Discharge of W_3 = 20 l/sec

The first and second alternatives resulted in negative heads of -0.26 m and -0.13 m at node 87 of the model, but in the third alternative the heads at all the nodes are positive and the head at nodes 87 reaches a value of 0.03m.

Figs. 2-8 to 2-13 and tables 2-4 to 2-9 show the variation of the piezometric level in the wells and piezometers, in the case of the third alternative. It is noted that after 10 years of simulation the results are close to a steady-state condition, and the maximum value of drawdown is 1.98 m in well No. 3.

Table 2-10 represents the computed heads of the aquifer at the end of 10 years.

Fig. 2-14 shows the computed water level in the aquifer at the end of 10 years. Since the values of withdrawals and consequently the values of drawdowns are small, this Figure is similar to Fig. 1-28, representing the piezometry of the aquifer in steady-state conditions.

2.9.2.1 Memory locations and execution time of Program TMATFE

The prediction of the Botany Basin aquifer behaviour over a period of 10 years, by Program TMATFE, required 65300 memory locations and 69 cp seconds execution time on CYBER 171. These figures, along with the figures presented in section 1.8.2.7, show clearly the main disadvantages of the finite element method.

3. CONCLUSIONS

The models FIELS and TMATFE presented in this report can simulate respectively a two-dimensional steady and transient groundwater flow. These programs are general and flexible enough to be easily applied to a wide range of field problems.

Although, in the examples of applications presented in this report, regular elements have been used, the models can treat any irregular triangular elements.

In this report the performances of the finite difference, and finite element methods were compared and it was shown that:

- The mathematical formulation and programming of the FEM is more complicated than the FDM.
- The FEM requires more data elements than the FDM.
- The FEM requires more memory locations than the FDM.
- The FEM requires a longer computer execution time than the FDM.

Considering the above results, it should be concluded that the FEM should be applied only in cases in which an irregular discretization of the aquifer is necessary.

As regards the simulation of the Botany Basin aquifer, it should be concluded that the available data are not reliable. Therefore, for a correct and conclusive simulation of this aquifer a new series of accurate field data are required as follows:

- Water level measurements
- Pumping tests
- Rate of withdrawals and infiltrations and their geographic distributions.

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B - BOTANY BASIN

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APPENDICES

COMPUTER PROGRAMS AND RELATED DATA

APPENDIX A1-SOURCE PROGRAM OF FIELS

```
      PROGRAM FIELS(INPUT,OUTPUT,TAPE2,TAPE3)
C***********************************************************************
C
C
C                    UNIVERSITY OF NEW SOUTH WALES
C                   SCHOOL  OF  CIVIL   ENGINEERING
C                   DEPARTMENT OF WATER ENGINEERING
C
C
C
C     NAME OF PROGRAM          : FIELS
C     OBJECT OF PROGRAM        : TWO-DIMENSIONAL MATHEMATICAL MODEL FOR
C                                AQUIFER SIMULATION BY FINITE ELEMENT
C                                METHOD ( STEADY STATE FLOW )
C     PROGRAMING LANGUAGE      : FORTRAN IV
C     REQUIRED DEVICES         : MAGNETIC DISK OR TAPE AND LINE PRINTER
C     AUTHOR                   : DR.F.GHASSEMI (VISITING PROFESSOR)
C     VERSION DATE             : OCTOBER 1984
C     REVISED BY               :
C     COMPUTER CENTER          : COMPUTING SERVICE UNIT, NSW UNIVERSITY
C     TYPE OF COMPUTER         : CONTROL DATA (CYBER 171)
C
C
C
C***********************************************************************
C
C
C     DEFINITION OF VARIABLES:
C
C     A     =ARRAY CONTAINING ONE LINE OF THE MATRIX
C     AA    =BAND MATRIX OF THE LINEAR SYSTEM IN COMPACT FORM
C     B     =COLUMN MATRIX OF CONSTANT VALUES
C     C     =AUXILIARY ARRAY USED IN SUBROUTINE TRIANG
C     H     =GROUND WATER LEVEL IN M.
C     INDIC =INDEX OF THE NODES :
C            INDIC=1 : IF THE HEAD OF THE NODE IS KNOWN
C            INDIC=0 : IF THE HEAD OF THE NODE IS UNKNOWN
C     INCON =NUMBER OF NODES WITH UNKNOWN PIEZOMETRIC VALUES
C     IN    =LOGICAL UNIT NUMBER FOR INPUT
C     IO    =LOGICAL UNIT NUMBER FOR OUTPUT
C     IEL   =NODE NUMBER OF THE VERTEX I OF EACH ELEMENT
C     JEL   =NODE NUMBER OF THE VERTEX J OF EACH ELEMENT
C     KEL   =NODE NUMBER OF THE VERTEX K OF EACH ELEMENT
C     LARGE =WIDTH OF THE BAND MATRIX ( NUMBER OF ELEMENTS ON EACH
C            SIDE OF DIAGONAL ELEMENT
C     LT    =2*LARGE+1
C     LM    =LARGE+1
C     NOMBE =TOTAL NUMBER OF ELEMENTS
C     NOMBN =TOTAL NUMBER OF NODES
C     Q     =DISCHARGE OR RECHARGE IN L/S ( WITH NEGATIVE SIGN FOR
C            DISCHARGE AND POSITIVE SIGN FOR RECHARGE )
C     TEL   =TRANSMISSIVITY OF EACH ELEMENT IN M2/DAY
C     TITLE =PROBLEM DESCRIPTION
```

```
C     X & Y =COORDINATES OF EACH NODE IN KM.
C
C
C
C
C     DEFINITION OF FIELS:
C
C     TAPE 2 =DISK FILE OF INPUT DATA
C     TAPE 3 =DISK FILE OF OUTPUT DATA WHICH MAY BE
C            DISPOSED TO LINE PRINTER OR TERMINAL
C
C
C      REMARK :
C     ---------
C
C     ARRAYS SHOULD BE DIMENSIONED AS FOLLOWS :
C
C     DIMENSION INDIC(NOMBN),X(NOMBN),Y(NOMBN),H(NOMBN),Q(NOMBN),
C    1IEL(NOMBE),JEL(NOMBE),KEL(NOMBE),TEL(NOMBE),A(NOMBN),AA(INCON,LT),
C    2B(NOMBN),C(LT),TITLE(20)
C
C
C***********************************************************************
C                        MAIN PROGRAM
C***********************************************************************
C
      DIMENSION INDIC(218),X(218),Y(218),H(218),Q(218),IEL(364),
     1JEL(364),KEL(364),TEL(364),A(218),AA(197,39),B(218),C(39),
     2TITLE(20)
      COMMON IN,IO
      IN=2
      IO=3
C
C     ***** READING OF THE DATA
C
      CALL REDATA(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,INCON,
     1TITLE)
C
C     ***** PRINTING OF THE DATA
C
      CALL PRDATA(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,INCON,
     1TITLE)
C
C     ***** COMPUTATION OF THE WIDTH OF THE BAND MATRIX
C
      CALL BAND(INDIC,IEL,JEL,KEL,NOMBE,LARGE,LT,LM)
C
C     ***** GENERATION OF THE BAND MATRIX
C
      CALL GENER(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,NEQ,
     1INCON,ICODE,LARGE,LT,LM,A,B,AA)
      IF(ICODE)40,50,40
   40 WRITE(IO,45)
   45 FORMAT(//10X,'SIMULATION IS SUSPENDED DUE TO INCORRECT NUMBERING
     1OF THE NODES')
      GO TO 100
   50 CONTINUE
```

```
C
C     ***** UPPER TRIANGULATION OF THE BAND MATRIX
C
      CALL TRIANG(NEQ,INCON,LARGE,LT,LM,AA,B,C)
C
C     ***** SOLUTION OF THE UPPER TRIANGULATED BAND MATRIX
C
      CALL SOLVE(NEQ,INCON,LARGE,LT,LM,AA,B,H)
C
C     ***** PRINTING OF THE RESULTS
C
      CALL PRINT(INDIC,X,Y,H,Q,TITLE,NOMBN)
  100 CONTINUE
      STOP
      END
C
C**********************************************************************
      SUBROUTINE REDATA(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,
     1INCON,TITLE)
C     READING OF THE DATA
C**********************************************************************
C
      DIMENSION INDIC(1),X(1),Y(1),H(1),Q(1),IEL(1),JEL(1),KEL(1),
     1TEL(1),TITLE(20)
      COMMON IN,IO
      READ(IN,1)(TITLE(I),I=1,20)
    1 FORMAT(20A4)
      READ(IN,5)NOMBN,NOMBE,INCON
    5 FORMAT(3I5)
C
C     ***** DATA RELATED TO THE NODES
C
      DO 10 I=1,NOMBN
   10 READ(IN,20)INDIC(I),X(I),Y(I),H(I),Q(I)
   20 FORMAT(5X,I3,2X,3(F8.3,2X),F8.1)
C
C     ***** DATA RELATED TO THE ELEMENTS
C
      DO 30 I=1,NOMBE
   30 READ(IN,40)IEL(I),JEL(I),KEL(I),TEL(I)
   40 FORMAT(5X,3I5,F8.2)
      RETURN
      END
C
C
C**********************************************************************
      SUBROUTINE PRDATA(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,
     1INCON,TITLE)
C     PRINTING OF THE DATA
C**********************************************************************
C
      DIMENSION INDIC(1),X(1),Y(1),H(1),Q(1),IEL(1),JEL(1),KEL(1),
     1TEL(1),TITLE(20)
      COMMON IN,IO
      WRITE(IO,5)(TITLE(I),I=1,20)
    5 FORMAT(1H1,//20X,20A4/20X,80('*')//)
      WRITE(IO,10)NOMBN,NOMBE,INCON
```

```
   10 FORMAT(20X,'NUMBER OF NODES  =',I3//20X,'NUMBER OF ELEMENTS =',
     1I3//20X,'NUMBER OF NODES WITH UNKNOWN PIEZOMETRIC VALUES =',I3)
C
C     ***** PRINTING OF THE NODE DATA
C
      WRITE(IO,15)(TITLE(I),I=1,20)
   15 FORMAT(1H1,20X,20A4,/)
      WRITE(IO,20)
   20 FORMAT(50X,'( N O D E  D A T A )'/20X,78('-')/20X,'*',2X,'NO.',
     11X,'*',2X,'INDEX *',3X,'X  IN  KM. *',3X,'Y  IN  KM. *',3X,'H  IN
     2  M.',1X,'*',3X,'Q  IN  L/S',2X,'*'/20X,78('-'))
      DO 30 I=1,NOMBN
   30 WRITE(IO,40)I,INDIC(I),X(I),Y(I),H(I),Q(I)
   40 FORMAT(20X,'*',1X,I3,2X,'*',3X,I2,3X,'*',3X,F8.3,3X,'*',3X,F8.3,
     13X,'*',3X,F7.2,4X,'*',4X,F7.1,4X,'*'/20X,78('-'))
C
C     ***** PRINTING OF THE ELEMENT DATA
C
      WRITE(IO,50)
   50 FORMAT(1H1/27X,'( E L E M E N T  D A T A )'/20X,44('-')/20X,
     1'* NO. *',3X,'I',3X,'*',3X,'J',3X,'*',3X,'K',3X,'*',' T ( M2/D ) *
     2'/20X,44('-'))
      DO 60 I=1,NOMBE
   60 WRITE(IO,70)I,IEL(I),JEL(I),KEL(I),TEL(I)
   70 FORMAT(20X,'*',I4,' * ',I4,'  * ',I4,'  * ',I4,'  *  ',F8.2,2X,'*'
     1/20X,44('-'))
C
C     ***** UNIT CONVERSION OF THE DATA
C
      DO 80 I=1,NOMBN
      X(I)=X(I)*1000.
      Y(I)=Y(I)*1000.
   80 Q(I)=Q(I)/1000.
      DO 90 I=1,NOMBE
   90 TEL(I)=TEL(I)/86400.
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE BAND(INDIC,IEL,JEL,KEL,NOMBE,LARGE,LT,LM)
C     COMPUTATION OF THE WIDTH OF THE BAND MATRIX
C***********************************************************************
C
      DIMENSION INDIC(1),IEL(1),JEL(1),KEL(1)
      COMMON IN,IO
      LARGE=0
      DO 100 M=1,NOMBE
      I=IEL(M)
      J=JEL(M)
      K=KEL(M)
      IF(INDIC(I)-1)10,100,100
   10 IF(INDIC(J)-1)20,100,100
   20 IF(INDIC(K)-1)30,100,100
   30 ID1=IABS(I-J)
      ID2=IABS(I-K)
      ID3=IABS(K-J)
      IGDIF=ID1
```

```
      IF(IGDIF-ID2)40,50,50
   40 IGDIF=ID2
   50 IF(IGDIF-ID3)60,70,70
   60 IGDIF=ID3
   70 IF(LARGE-IGDIF)80,100,100
   80 LARGE=IGDIF
      IELME=M
  100 CONTINUE
C
      WRITE(IO,110)LARGE,IELME
  110 FORMAT(1H1,//10X,' WIDTH OF THE BAND MATRIX =',I3,' RELATED  TO
     1 THE ELEMENT NO.',I3//)
      LT=2*LARGE+1
      LM=LARGE+1
      RETURN
      END
C
C
C***********************************************************************
      SUBROUTINE GENER(INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,NOMBN,NOMBE,NEQ,
     1INCON,ICODE,LARGE,LT,LM,A,B,AA)
C     GENERATION OF THE BAND MATRIX
C***********************************************************************
C
      DIMENSION INDIC(1),X(1),Y(1),H(1),Q(1),IEL(1),JEL(1),KEL(1),
     1TEL(1),A(1),B(1),AA(INCON,LT)
      COMMON IN,IO
      DO 1 M=1,INCON
    1 B(M)=0.
      ICODE=0
      NEQ=0
      DO 200 INO=1,INCON
      IF(INDIC(INO)-1)2,260,260
    2 I=INO
      NEQ=NEQ+1
C
C     ***** GENERATION OF THE EQUATION RELATED TO NODE NO. I
C
      DO 3 M=1,INCON
    3 A(M)=0.
      DO 100 IA=1,NOMBE
      IF(INO-IEL(IA))6,5,6
    5 J=JEL(IA)
      K=KEL(IA)
      GO TO 10
    6 IF(INO-JEL(IA))8,7,8
    7 J=IEL(IA)
      K=KEL(IA)
      GO TO 10
    8 IF(INO-KEL(IA))100,9,100
    9 J=IEL(IA)
      K=JEL(IA)
   10 XI=X(I)
      YI=Y(I)
      XJ=X(J)
      YJ=Y(J)
      XK=X(K)
```

```
      YK=Y(K)
      TRANS=TEL(IA)
C
      SURFA=ABS((XJ-XI)*(YK-YI)+(YI-YJ)*(XK-XI))
      COEF=TRANS/(2.*SURFA)
      C1=COEF*((YK-YJ)*(YK-YJ)+(XK-XJ)*(XK-XJ))
      A(I)=A(I)+C1
      C2=COEF*((YK-YJ)*(YI-YK)+(XK-XJ)*(XI-XK))
      IF(INDIC(J)-1)20,15,15
   15 B(I)=B(I)-C2*H(J)
      GO TO 25
   20 A(J)=A(J)+C2
   25 C3=COEF*((YK-YJ)*(YJ-YI)+(XK-XJ)*(XJ-XI))
      IF(INDIC(K)-1)40,30,30
   30 B(I)=B(I)-C3*H(K)
      GO TO 100
   40 A(K)=A(K)+C3
  100 CONTINUE
      B(I)=B(I)+Q(I)
C
      DO 110 KK=1,LM
      K1=LARGE+KK
      K2=INO+KK-1
      IF(K2-INCON)108,108,105
  105 AA(INO,K1)=0.
      GO TO 110
  108 AA(INO,K1)=A(K2)
  110 CONTINUE
C
      DO 120 KK=1,LARGE
      K1=LARGE-KK+1
      K2=INO-KK
      IF(K2)115,115,118
  115 AA(INO,K1)=0.
      GO TO 120
  118 AA(INO,K1)=A(K2)
  120 CONTINUE
  200 CONTINUE
C
      IF (NEQ-INCON)210,240,210
  210 ICODE=1
      WRITE(IO,220)ICODE,NEQ,INCON
  220 FORMAT(//10X,'ICODE=',I2,5X,'NEQ=',I4,5X,'INCON=',I4,//)
      GO TO 300
  240 WRITE(IO,250)NEQ,NEQ
  250 FORMAT(//,10X,' SYSTEM OF LINEAR EQUATIONS IS ',I4,' X',I4,/)
      GO TO 300
  260 ICODE=1
      WRITE(IO,270)INO
  270 FORMAT(//,10X,' NODES ARE NOT CORRECTLY NUMBERED , NODE NO. '
     1,I4,' IS A NODE WITH CONSTANT HEAD')
  300 RETURN
      END
C
C
C***********************************************************************
      SUBROUTINE TRIANG(NEQ,INCON,LARGE,LT,LM,AA,B,C)
```

```
C     UPPER TRIANGULATION OF THE BAND MATRIX IN ITS COMPACT FORM
C*********************************************************************
C
      DIMENSION AA(INCON,LT),B(1),C(1)
      COMMON IN,IO
      DO 100 K=1,NEQ
      ALFA=AA(K,LM)
      DO 5 M=LM,LT
      AA(K,M)=AA(K,M)/ALFA
    5 C(M)=AA(K,M)
      B(K)=B(K)/ALFA
C
C     ***** ELIMINATION OF ONE UNKNOWN
C
      DO 50 I=1,LARGE
      IEQ=K+I
C     ***** CONTROL IN VERTICAL DIRECTION
C
      IF(IEQ-NEQ)10,10,100
   10 K1=LM-I
      K2=K1+LARGE
      IF(AA(IEQ,K1))30,50,30
   30 FACTO=AA(IEQ,K1)
      DO 40 J=K1,K2
      IJ=I+J
   40 AA(IEQ,J)=AA(IEQ,J)-FACTO*C(IJ)
      K3=K+I
      B(K3)=B(K3)-FACTO*B(K)
   50 CONTINUE
  100 CONTINUE
      WRITE(IO,150)
  150 FORMAT(///,10X,' END OF UPPER TRIANGULATION ')
      RETURN
      END
C
C
C*********************************************************************
      SUBROUTINE SOLVE(NEQ,INCON,LARGE,LT,LM,AA,B,H)
C     SOLUTION OF THE TRIANGULATED BAND MATRIX
C*********************************************************************
C
      DIMENSION AA(INCON,LT),B(1),H(1)
      COMMON IN,IO
      K1=LM+1
      NN=NEQ-1
      H(NEQ)=B(NEQ)
      DO 100 I=1,NN
      IEQ=NEQ-I
      TOT=0.
      N=0
      DO 40 J=K1,LT
      N=N+1
      N1=IEQ+N
      IF(N1-NEQ)20,20,45
   20 TOT=TOT+AA(IEQ,J)*H(N1)
   40 CONTINUE
   45 H(IEQ)=B(IEQ)-TOT
```

```
  100 CONTINUE
      RETURN
      END
C
C
C***********************************************************************
      SUBROUTINE PRINT(INDIC,X,Y,H,Q,TITLE,NOMBN)
C     PRINTING OF THE RESULTS
C***********************************************************************
      DIMENSION INDIC(1),X(1),Y(1),H(1),Q(1),TITLE(20)
      COMMON IN,IO
C
C     ***** CONVERSION OF THE INITIAL DATA
C
      DO 10 I=1,NOMBN
      X(I)=X(I)/1000.
      Y(I)=Y(I)/1000.
      Q(I)=Q(I)*1000.
   10 CONTINUE
C
      WRITE(IO,15)(TITLE(I),I=1,20)
   15 FORMAT(1H1,20X,20A4//50X,'FINAL RESULTS'//20X,78('-'))
      WRITE(IO,20)
   20 FORMAT(20X,'*',2X,'NO.',1X,'*  INDEX *',3X,'X  IN  KM. *',3X,
     1'Y  IN  KM. *',3X,'H  IN  M.',2X,'*',3X,'Q  IN  L/S',2X,'*'/20X,
     278('-'))
      DO 30 I=1,NOMBN
   30 WRITE(IO,40)I,INDIC(I),X(I),Y(I),H(I),Q(I)
   40 FORMAT(20X,'*',1X,I3,2X,'*',3X,I2,3X,'*',3X,F8.3,3X,'*',3X,F8.3,
     13X,'*',3X,F7.2,4X,'*',4X,F7.1,4X,'*'/20X,78('-'))
      WRITE(IO,50)
   50 FORMAT(20X,'INDEX 0 =NODES WITH COMPUTED PIEZOMETRIC VALUES'/,
     1/20X,'INDEX 1 =FIXED HEAD BOUNDARY NODES')
      RETURN
      END
```

APPENDIX A2 Input data formats for FIELS

CARD NO.	COLUMN	FORMAT	VARIABLE	DEFINITION
1	1 - 80	20 A4	TITLE	Description of Problem
2	1 - 5	I5	NOMBN	Total number of nodes
	6 - 10	I5	NOMBE	Total number of elements
	11 - 15	I5	INCON	Total number of nodes with unknown piezometric values

DATA SET	NUMBER OF CARDS	FORMAT	VARIABLE	DEFINITION
1	Total number of nodes (NOMBN)	5X*,I3,2X,3(F8.3,2X),F8.1	INDIC,X,Y,H,Q	Index (INDIC), coordinates (X,Y in Km) water level (H in m) and discharge or recharge values (Q in ℓ/s) for each node
2	Total number of elements (NOMBE)	5X*,3I5,F8.2	IEL,JEL,KEL,TEL	Node numbers of the three vertices I,J,K (IEL, JEL, KEL) and tranmissivity value (TEL in m^2/d) of each elment.

* The first five columns on each data card are used for the sequential numbering of the data cards to facilitate the job of data prepraration and their control. These numbers are ignored during the reading process.

APPENDIX A3-INPUT DATA AND OUTPUT FOR TEST PROBLEM I

```
CARD 1          SIMULATION OF TEST PROBLEM I (STEADY-STATE FLOW)
CARD 2      25    32    20
 ↑      N   1    0     0.          0.5        0.          0.
        N   2    0     0.          1.         0.          0.
        N   3    0     0.          1.5        0.          0.
        N   4    0     0.          2.         0.          0.
 D      N   5    0     0.5         0.5        0.          0.
        N   6    0     0.5         1.         0.          0.
 A      N   7    0     0.5         1.5        0.          0.
        N   8    0     0.5         2.         0.          0.
 T      N   9    0     1.          0.5        0.          0.
        N  10    0     1.          1.         0.       -100.
 A      N  11    0     1.          1.5        0.          0.
        N  12    0     1.          2.         0.          0.
        N  13    0     1.5         0.5        0.          0.
 S      N  14    0     1.5         1.         0.          0.
        N  15    0     1.5         1.5        0.          0.
 E      N  16    0     1.5         2.         0.          0.
        N  17    0     2.          0.5        0.          0.
 T      N  18    0     2.          1.         0.          0.
        N  19    0     2.          1.5        0.          0.
 1      N  20    0     2.          2.         0.          0.
        N  21    1     0.          0.        80.          0.
 |      N  22    1     0.5         0.        80.          0.
        N  23    1     1.          0.        80.          0.
        N  24    1     1.5         0.        80.          0.
 ↓      N  25    1     2.          0.        80.          0.
 ↑      E   1     1    21    22   250.
        E   2     1     5    22   250.
        E   3     1     2     5   250.
        E   4     2     5     6   250.
        E   5     2     3     6   250.
        E   6     3     6     7   250.
        E   7     3     4     7   250.
 D      E   8     4     7     8   250.
        E   9    22     5    23   250.
 A      E  10     5     9    23   250.
        E  11     5     6     9   250.
 T      E  12     6     9    10   250.
        E  13     6     7    10   250.
 A      E  14     7    10    11   250.
        E  15     7     8    11   250.
        E  16     8    11    12   250.
 S      E  17     9    23    24   250.
        E  18     9    13    24   250.
 E      E  19     9    10    13   250.
        E  20    10    13    14   250.
 T      E  21    10    11    14   250.
        E  22    11    14    15   250.
 2      E  23    11    12    15   250.
        E  24    12    15    16   250.
 |      E  25    13    24    25   250.
        E  26    13    25    17   250.
```

E	27	13	14	17	250.
E	28	14	17	18	250.
E	29	14	15	18	250.
E	30	15	18	19	250.
E	31	15	16	19	250.
E	32	16	19	20	250.

OUTPUT FOR TEST PROBLEM I

```
          SIMULATION OF TEST PROBLEM I (STEADY-STATE FLOW)
*****************************************************************************

NUMBER OF NODES  = 25

NUMBER OF ELEMENTS = 32

NUMBER OF NODES WITH UNKNOWN PIEZOMETRIC VALUES = 20
```

SIMULATION OF TEST PROBLEM I (STEADY-STATE FLOW)

(N O D E D A T A)

NO.	INDEX	X IN KM.	Y IN KM.	H IN M.	Q IN L/S
1	0	0.000	.500	0.00	0.0
2	0	0.000	1.000	0.00	0.0
3	0	0.000	1.500	0.00	0.0
4	0	0.000	2.000	0.00	0.0
5	0	.500	.500	0.00	0.0
6	0	.500	1.000	0.00	0.0
7	0	.500	1.500	0.00	0.0
8	0	.500	2.000	0.00	0.0
9	0	1.000	.500	0.00	0.0
10	0	1.000	1.000	0.00	-100.0
11	0	1.000	1.500	0.00	0.0
12	0	1.000	2.000	0.00	0.0
13	0	1.500	.500	0.00	0.0
14	0	1.500	1.000	0.00	0.0
15	0	1.500	1.500	0.00	0.0
16	0	1.500	2.000	0.00	0.0
17	0	2.000	.500	0.00	0.0
18	0	2.000	1.000	0.00	0.0
19	0	2.000	1.500	0.00	0.0
20	0	2.000	2.000	0.00	0.0
21	1	0.000	0.000	80.00	0.0
22	1	.500	0.000	80.00	0.0
23	1	1.000	0.000	80.00	0.0
24	1	1.500	0.000	80.00	0.0
25	1	2.000	0.000	80.00	0.0

(E L E M E N T D A T A)

NO.	I	J	K	T (M2/D)
1	1	21	22	250.00
2	1	5	22	250.00
3	1	2	5	250.00
4	2	5	6	250.00
5	2	3	6	250.00
6	3	6	7	250.00
7	3	4	7	250.00
8	4	7	8	250.00
9	22	5	23	250.00
10	5	9	23	250.00
11	5	6	9	250.00
12	6	9	10	250.00
13	6	7	10	250.00
14	7	10	11	250.00
15	7	8	11	250.00
16	8	11	12	250.00
17	9	23	24	250.00
18	9	13	24	250.00
19	9	10	13	250.00
20	10	13	14	250.00
21	10	11	14	250.00
22	11	14	15	250.00
23	11	12	15	250.00
24	12	15	16	250.00
25	13	24	25	250.00
26	13	25	17	250.00
27	13	14	17	250.00

28	14	17	18	250.00
29	14	15	18	250.00
30	15	18	19	250.00
31	15	16	19	250.00
32	16	19	20	250.00

WIDTH OF THE BAND MATRIX = 4 RELATED TO THE ELEMENT NO. 3

SYSTEM OF LINEAR EQUATIONS IS 20 X 20

END OF UPPER TRIANGULATION

SIMULATION OF TEST PROBLEM I (STEADY-STATE FLOW)
FINAL RESULTS

NO.	INDEX	X IN KM.	Y IN KM.	H IN M.	Q IN L/S
1	0	0.000	.500	72.35	0.0
2	0	0.000	1.000	66.18	0.0
3	0	0.000	1.500	63.88	0.0
4	0	0.000	2.000	63.34	0.0
5	0	.500	.500	71.61	0.0
6	0	.500	1.000	64.25	0.0
7	0	.500	1.500	62.99	0.0
8	0	.500	2.000	62.81	0.0
9	0	1.000	.500	69.86	0.0
10	0	1.000	1.000	56.20	-100.0
11	0	1.000	1.500	61.03	0.0
12	0	1.000	2.000	61.92	0.0
13	0	1.500	.500	71.61	0.0
14	0	1.500	1.000	64.25	0.0
15	0	1.500	1.500	62.99	0.0
16	0	1.500	2.000	62.81	0.0
17	0	2.000	.500	72.35	0.0
18	0	2.000	1.000	66.18	0.0
19	0	2.000	1.500	63.88	0.0
20	0	2.000	2.000	63.34	0.0
21	1	0.000	0.000	80.00	0.0
22	1	.500	0.000	80.00	0.0
23	1	1.000	0.000	80.00	0.0
24	1	1.500	0.000	80.00	0.0
25	1	2.000	0.000	80.00	0.0

INDEX 0 =NODES WITH COMPUTED PIEZOMETRIC VALUES
INDEX 1 =FIXED HEAD BOUNDARY NODES

APPENDIX B1-SOURCE PROGRAM OF TMATFE

```
      PROGRAM TMATFE(INPUT,OUTPUT,TAPE2,TAPE3)
C***********************************************************************
C
C
C                    UNIVERSITY OF NEW SOUTH WALES
C                   SCHOOL  OF  CIVIL   ENGINEERING
C                   DEPARTMENT OF WATER ENGINEERING
C
C
C
C      NAME OF PROGRAM          : TMATFE
C      OBJECT OF PROGRAM        : TWO-DIMENSIONAL MATHEMATICAL MODEL FOR
C                                 AQUIFER SIMULATION BY FINITE ELEMENT
C                                 METHOD (TRANSIENT FLOW)
C      PROGRAMMING LANGUAGE     : FORTRAN IV
C      REQUIRED DEVICES         : MAGNETIC DISK OR TAPE AND LINE PRINTER
C      AUTHOR                   : DR.F.GHASSEMI (VISITING PROFESSOR)
C      VERSION DATE             : OCTOBER 1984
C      REVISED BY               :
C      COMPUTER CENTER          : COMPUTING SERVICE UNIT, NSW UNIVERSITY
C      TYPE OF COMPUTER         : CONTROL DATA  (CYBER 171)
C
C
C***********************************************************************
C
C      DEFINITION OF VARIABLES :
C      -------------------------
C     A       = ARRAY CONTAINING ONE LINE OF THE BAND MATRIX
C     AA      = BAND MATRIX OF THE LINER SYSTEM OF EQUATIONS
C               IN ITS COMPACT FORM
C     B       = COLUMN MATRIX OF THE CONSTANT VALUES OF THE EQUATIONS
C     C       = AUXILIARY ARRAY USED IN SUBROUTINE TMTRIA
C     CODE    = CODE FOR CHECKING THE GENERATED SET OF EQUATIONS
C               CODE=0 : IF THE NUMBER OF GENERATED EQUATIONS IS
C                        EQUAL TO THE NUMBER OF UNKNOWNS(SET OF
C                        EQUATIONS IS CORRECTLY GENERATED)
C               CODE=1 : IF THE NUMBER OF GENERATED EQUATIONS IS NOT
C                        EQUAL TO THE NUMBER OF UNKNOWNS(SET OF
C                        EQUATIONS IS NOT CORRECTLY GENERATED)
C     DAY     = SEE YEAR
C     DEBIT   = FLOW RATES OF THE BOUNDARY NODES IN DIFFERENT TIME STEPS
C     DELT    = LENGTH OF EACH TIME STEP (IN DAYS)
C     DELTA   = LENGTH OF EACH TIME STEP (IN SECONDS)
C     H       = PIEZOMETRIC LEVEL AT THE BEGINNING OF EACH TIME STEP
C     H1      = PIEZOMETRIC LEVEL AT THE END OF EACH TIME STEP(IN M.)
C     HIMPO   = IMPOSED PIEZOMETRIC VALUES ON THE BOUNDARY NODES IN
C               DIFFERENT TIME STEPS
C     HINPI   = INITIAL PIEZOMETRIC VALUES OF THE PIEZOMETERS
C     HINPU   = INITIAL PIEZOMETRIC VALUES OF THE SELECTED WELLS
C     IN      = LOGICAL UNIT NUMBER FOR INPUT
C     IO      = LOGICAL UNIT NUMBER FOR OUTPUT
C     INCON   = TOTAL NUMBER OF NODES WITH UNKNOWN HEADS
C     INDIC   = INDEX OF THE NODES :
```

```
C               INDIC=1 : IF THE HEAD OF THE NODE IS KNOWN
C               INDIC=0 : IF THE HEAD OF THE NODE IS UNKNOWN
C     IEL     = NODE NUMBER OF THE VERTEX I OF EACH ELEMENT
C     JEL     = NODE NUMBER OF THE VERTEX J OF EACH ELEMENT
C     KEL     = NODE NUMBER OF THE VERTEX K OF EACH ELEMENT
C     LARGE   = NUMBER OF ELEMENTS ON EACH SIDE OF THE DIAGONAL ELEMENTS
C               OF THE BAND MATRIX = WIDTH OF THE BAND MATRIX
C     LM      = LARGE+1
C     LT      = 2*LARGE+1 = TOTAL WIDTH OF THE BAND MATRIX
C     MONTH   = SEE YEAR
C     N       = NUMBER OF EACH TIME STEP
C     NBNH    = TOTAL NUMBER OF THE BOUNDARY NODES WITH KNOWN HEADS
C     NIMPR   = TOTAL NUMBER OF INTERMEDIATE PRINTINGS OF THE COMPUTED
C               PIEZOMETRIC VALUES
C     NMEC    = MAXIMUM NUMBER OF THE ELEMENTS WITH A COMMON NODE
C     NODBH   = NUMBER OF BOUNDARY NODES WITH KNOWN HEAD
C     NODPI   = LOCATION(NODAL NUMBER)OF THE PIEZOMETERS
C     NODPU   = LOCATION(NODAL NUMBER)OF THE SELECTED WELLS
C     NOMBE   = TOTAL NUMBER OF ELEMENTS
C     NOMBN   = TOTAL NUMBER OF NODES
C     NPIEZ   = TOTAL NUMBER OF PIEZOMETERS
C     NPUIT   = TOTAL NUMBER OF SELECTED WELLS
C     NPTEM   = TOTAL NUMBER OF TIME STEPS
C     Q       = WITHDRAWAL OR RECHARGE VALUES (IN L/S)
C               WITH NEGATIVE SIGN FOR WITHDRAWAL AND
C               POSITIVE SIGN FOR RECHARGE VALUES
C      SEL    = STORAGE COEFFICIENT OF EACH ELEMENT
C      TEL    = TRANSMISSIVITY OF EACH ELEMENT(IN M2/DAY)
C      TITLE  = PROBLEM DESCRIPTION
C      TPRIH  = NUMBER OF TIME STEPS FOR PRINTING THE INTERMEDIATE
C               COMPUTED PIEZOMETRIC VALUES
C      X & Y  = COORDINATES OF EACH NODE (IN KM.)
C      YEAR,MONTH,DAY = LENGTH OF SIMULATION PERIOD(IN YEAR/MONTH/DAY)
C
C
C
C     DEFINITION OF FIELS :
C
C     TAPE 2 =DISK FILE OF INPUT DATA
C     TAPE 3 =DISK FILE OF OUTPUT DATA WHICH MAY BE
C            DISPOSED TO LINE PRINTER OR TERMINAL
C
C
C      REMARK
C      ------
C
C      ARRAYS SHOULD BE DIMENSIONED AS FOLLOWS :
C
C      DIMENSION TITLE(20),INDIC(NOMBN),X(NOMBN),Y(NOMBN),H(NOMBN),
C     1H1(NOMBN),Q(NOMBN),IEL(NOMBE),JEL(NOMBE),KEL(NOMBE),TEL(NOMBE),
C     2SEL(NOMBE),DELT(NPTEM),YEAR(NPTEM),MONTH(NPTEM),DAY(NPTEM),
C     3TPRIH(NIMPR),NODPU(NPUIT),NODPI(NPIEZ),NODBH(NBNH),HINPU(NPUIT),
C     4HINPI(NPIEZ),HIMPO(NBNH),DEBIT(NPUIT,NPTEM),II(NMEC),JJ(NMEC),
C     5KK(NMEC),ALI(NMEC),ALJ(NMEC),ALK(NMEC),AMI(NMEC),AMJ(NMEC),
C     6AMK(NMEC),A(INCON),B(INCON),AA(INCON,LT),C(LT),RABPU(NPUIT,NPTEM),
C     7RABPI(NPIEZ,NPTEM),PUNIV(NPUIT,NPTEM),PINIV(NPUIT,NPTEM)
C
```

```
C
C***********************************************************************
C                              MAIN PROGRAM
C***********************************************************************
      INTEGER YEAR,TPRIH,CODE
      DIMENSION TITLE(20),INDIC(218),X(218),Y(218),H(218),H1(218),Q(218)
     1,IEL(364),JEL(364),KEL(364),TEL(364),SEL(364),DELT(15),YEAR(15),
     2MONTH(15),DAY(15),TPRIH(5),NODPU(5),NODPI(5),NODBH(30),HINPU(5),
     3HINPI(5),HIMPO(30),DEBIT(3,15),II(10),JJ(10),KK(10),ALI(10),
     4ALJ(10),ALK(10),AMI(10),AMJ(10),AMK(10),A(197),AA(197,39),B(197),
     5C(39),RABPU(3,15),RABPI(3,15),PUNIV(3,15),PINIV(3,15)
C
      COMMON IN,IO
      IN=2
      IO=3
C
C     ***** READING OF THE INPUT DATA
C
      CALL TMREAD(TITLE,NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,NMEC,
     1NBNH,INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,SEL,DELT,YEAR,MONTH,DAY,TPRIH,
     2NODPU,NODPI,NODBH)
C
C     ***** PRINTING AND UNIT CONVERSION OF THE INPUT DATA
C
      CALL TMWRIT(TITLE,NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,NBNH,
     1NMEC,INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,SEL,DELT,YEAR,MONTH,DAY,TPRIH,
     2NODPU,NODPI,NODBH)
C
C     ***** COMPUTATION OF THE WIDTH OF THE BAND MATRIX
C
      CALL TMBAND(INDIC,IEL,JEL,KEL,NOMBE,LARGE,LT,LM)
C
C     ***** CONSERVATION OF THE INITIAL PIEZOMETRIC LEVEL OF THE
C           SELECTED WELLS AND PIEZOMETERS
C
      CALL TMCONS(NPUIT,NPIEZ,NODPU,NODPI,HINPU,HINPI,H)
C
      DO 100 N=1,NPTEM
      DELTA=DELT(N)*86400.
C
C     ***** IMPOSING THE BOUNDARY AND INTERNAL CONDITIONS
C
      CALL TMIMPO(NPUIT,NODPU,Q,DEBIT,INDIC,H,H1,NOMBN,N,NPTEM,NBNH,
     1NODBH,HIMPO,INCON)
C
C     ***** GENERATION OF THE BAND MATRIX AND ITS CONSERVATION IN
C           COMPACT FORM
      CALL TMGENR(INDIC,X,Y,IEL,JEL,KEL,SEL,TEL,NOMBE,NEQ,INCON,
     1CODE,II,JJ,KK,ALI,ALJ,ALK,AMI,AMJ,AMK,H,H1,Q,A,B,AA,LARGE,LT,LM,
     2DELTA,N)
      IF(CODE)42,50,42
   42 CONTINUE
      WRITE(IO,46)
   46 FORMAT(//10X,'SIMULATION IS SUSPENDED DUE TO INCORRECT NUMBERING
     1OF THE NODES')
      GO TO 200
   50 CONTINUE
```

```
C
C     ***** UPPER TRIANGULATION OF THE BAND MATRIX IN ITS COMPACT FORM
      CALL TMTRIA(NEQ,INCON,LARGE,LT,LM,AA,B,C)
C
C     ***** SOLUTION OF THE TRIANGULATED BAND MATRIX
C
      CALL TMSOLV(NEQ,INCON,LARGE,LT,LM,AA,B,H1)
C
C     ***** COMPUTATION OF THE PIEZOMETRIC VARIATIONS IN THE SELECTED
C           WELLS AND PIEZOMETERS , ALSO TRANSFER OF ARRAY H1 IN
C           ARRAY H
      CALL TMVARI(H,H1,NPUIT,NPIEZ,NODPU,NODPI,HINPU,HINPI,RABPU,
     1RABPI,N,NPTEM,PUNIV,PINIV,NOMBN)
C
C     ***** PRINTING OF THE INTERMEDIATE RESULTS
C
      DO 60 M=1,NIMPR
      IF(N-TPRIH(M))60,55,60
   55 CALL TMRESU(H1,N,NOMBN,YEAR,MONTH,DAY)
   60 CONTINUE
      WRITE(IO,80)N
   80 FORMAT(/,10X,'END OF TIME STEP NO.',I3,//)
  100 CONTINUE
C
C     ***** PRINTING OF THE PIEZOMETRIC VARIATIONS IN SELECTED WELLS
C
      CALL TMWELL(DEBIT,DELT,PUNIV,RABPU,NODPU,HINPU,NPUIT,NPTEM,
     1YEAR,MONTH,DAY)
C
C     ***** PRINTING OF THE PIEZOMETRIC VARIATIONS IN PIEZOMETERS
C
      CALL TMPIEZ(DELT,PINIV,RABPI,NODPI,HINPI,NPIEZ,NPTEM,
     1YEAR,MONTH,DAY)
C
  200 STOP
      END
C*********************************************************************
      SUBROUTINE TMREAD(TITLE,NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,
     1NMEC,NBNH,INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,SEL,DELT,YEAR,MONTH,DAY,
     2TPRIH,NODPU,NODPI,NODBH)
C     READING OF THE INPUT DATA
C*********************************************************************
      INTEGER YEAR,TPRIH
      DIMENSION TITLE(1),INDIC(1),X(1),Y(1),H(1),Q(1),IEL(1),JEL(1),
     1KEL(1),TEL(1),SEL(1),DELT(1),YEAR(1),MONTH(1),DAY(1),TPRIH(1),
     2NODPU(1),NODPI(1),NODBH(1)
      COMMON IN,IO
C
C     ***** GENERAL DATA
C
      READ(IN,5)(TITLE(I),I=1,20)
    5 FORMAT(20A4)
      READ(IN,10)NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,NMEC,NBNH
   10 FORMAT(5X,9I5)
C
C     ***** NODE DATA
C
```

```
      DO 20 I=1,NOMBN
   20 READ(IN,30)INDIC(I),X(I),Y(I),H(I),Q(I)
   30 FORMAT(5X,I3,2X,3(F8.3,2X),F8.1)
C
C     ***** ELEMENT DATA
C
      DO 40 I=1,NOMBE
   40 READ(IN,50)IEL(I),JEL(I),KEL(I),TEL(I),SEL(I)
   50 FORMAT(5X,3I5,F8.2,2X,F9.7)
C
C     ***** LENGTH OF THE TIME STEPS AND THE SIMULATION PERIODS
      DO 60 I=1,NPTEM
   60 READ(IN,70)DELT(I),YEAR(I),MONTH(I),DAY(I)
   70 FORMAT(5X,F10.1,2I5,F5.1)
C
C     ***** NUMBERS OF TIME STEPS FOR WHICH THE RESULTS WILL BE PRINTED
C
      READ(IN,80)(TPRIH(I),I=1,NIMPR)
   80 FORMAT(5X,15I4)
C
C     ***** LOCATION(NODE NUMBER)OF THE SELECTED WELLS AND PIEZOMETERS
C
C
      READ(IN,80)(NODPU(I),I=1,NPUIT)
      READ(IN,80)(NODPI(I),I=1,NPIEZ)
C
C     ***** NODE NUMBERS OF THE BOUNDARY NODES WITH FIXED HEADS
C
      READ(IN,80)(NODBH(I),I=1,NBNH)
      RETURN
      END
C***********************************************************************
      SUBROUTINE TMWRIT(TITLE,NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,
     1NBNH,NMEC,INDIC,X,Y,H,Q,IEL,JEL,KEL,TEL,SEL,DELT,YEAR,MONTH,DAY,
     2TPRIH,NODPU,NODPI,NODBH)
C     PRINTING AND UNIT CONVERSION OF THE INPUT DATA
C***********************************************************************
      INTEGER YEAR,TPRIH
      DIMENSION TITLE(20),INDIC(1),X(1),Y(1),H(1),Q(1),IEL(1),JEL(1),
     1KEL(1),TEL(1),SEL(1),DELT(1),YEAR(1),MONTH(1),DAY(1),TPRIH(1),
     2NODPU(1),NODPI(1),NODBH(1)
      COMMON IN,IO
      WRITE(IO,10)(TITLE(I),I=1,20)
   10 FORMAT(1H1/20X,20A4/20X,80('*'))
      WRITE(IO,20)NOMBN,NOMBE,INCON,NPTEM,NIMPR,NPUIT,NPIEZ,NBNH,NMEC
   20 FORMAT(/30X,'NOMBN=',I4,15X,'NOMBE=',I4,15X,'INCON=',I4//30X,
     1'NPTEM=',I4,15X,'NIMPR=',I4,15X,'NPUIT=',I4//30X,'NPIEZ=',I4,
     215X,'NBNH =',I4,15X,'NMEC =',I4)
C
C     ***** PRINTING OF THE NODE DATA
C
      WRITE(IO,25)(TITLE(I),I=1,20)
   25 FORMAT(1H1,15X,20A4,/)
      WRITE(IO,30)
   30 FORMAT(44X,'( N O D E  D A T A )'//20X,68('-')/20X,'*',2X,'NO.',
     11X,'*',' INDEX * X  IN  KM. * Y  IN  KM. * H  IN  M.  * Q  IN  L/S
     2 *'/20X,68('-'))
```

```
      DO 40 I=1,NOMBN
   40 WRITE(IO,50)I,INDIC(I),X(I),Y(I),H(I),Q(I)
   50 FORMAT(20X,'* ',I3,2X,'*',2X,I2,3X,'*',1X,F8.3,3X,'*',1X,F8.3,3X,
     1'*',1X,F7.2,4X,'*',1X,F8.1,3X,'*'/20X,68('-'))
C
C     ***** PRINTING OF THE ELEMEMT DATA
C
      WRITE(IO,60)
   60 FORMAT(1H1/33X,'( E L E M E N T   D A T A )'//20X,56('-')/20X,
     1'* NO. *',3X,'I',3X,'*',3X,'J',3X,'*',3X,'K',3X,'*',' T ( M2/D ) *
     2',5X,'S',5X,'*'/20X,56('-'))
      DO 70 I=1,NOMBE
   70 WRITE(IO,80)I,IEL(I),JEL(I),KEL(I),TEL(I),SEL(I)
   80 FORMAT(20X,'*',I4,' * ',I4,'  * ',I4,'  * ',I4,'  *  ',F8.2,2X,
     1'*',1X,F9.7,1X,'*'/20X,56('-'))
C
C     ***** PRINTING THE LENGTH OF THE TIME STEPS AND SIMULATION PEIODS
C
      WRITE(IO,90)
   90 FORMAT(1H1/14X,'TIME STEPS AND SIMULATION PERIODS'/10X,41('-')/,
     110X,'*',,5X,'*',1X,'TIME STEP',1X,'*',2X,'SIMULATION PERIOD',2X,
     2'*'/10X,'*',1X,'NO.',1X,'*',11X,23('-')/10X,'*',5X,'*',2X,
     3'IN DAY',3X,'*',1X,'YEAR * MONTH *  DAY *'/10X,41('-'))
      DO 100 I=1,NPTEM
  100 WRITE(IO,110)I,DELT(I),YEAR(I),MONTH(I),DAY(I)
  110 FORMAT(10X,'*',1X,I3,1X,'*',2X,F7.1,2X,'*',1X,I3,2X,'*',1X,I3,3X,
     1'*',1X,F4.1,1X,'*'/10X,41('-'))
C
C     ***** PRINTING THE NUMBERS OF THE TIME STEPS FOR WHICH THE RESULTS
C           WILL BE PRINTED
C
      WRITE(IO,120)
  120 FORMAT(1H1//13X,'RESULTS WILL BE PRINTED FOR THE'/19X,
     1'FOLLOWING TIME STEPS'/10X,40('*'))
      DO 130 I=1,NIMPR
  130 WRITE(IO,140)TPRIH(I)
  140 FORMAT(21X,'TIME STEP NO. ',I2/)
C
C     ***** PRINTING THE LOCATIONS OF THE WELLS AND PIEZOMETERS
C
      WRITE(IO,150)
  150 FORMAT(1H1//17X,'WELLS ARE LOCATED ON THE'/21X,'FOLLOWING NODES'/,
     116X,26('*'))
      DO 160 I=1,NPUIT
  160 WRITE(IO,170)I,NODPU(I)
  170 FORMAT(16X,'WELL NO.',I3,3X,'NODE NO.',I4,/)
      WRITE(IO,180)
  180 FORMAT(1H1//18X,'PIEZOMETERS ARE LOCATED ON THE'/25X,
     1'FOLLOWING NODES'/17X,32('*'))
      DO 190 I=1,NPIEZ
  190 WRITE(IO,200)I,NODPI(I)
  200 FORMAT(17X,'PIEZOMETER NO.',I3,3X,'NODE NO.',I4,/)
C
C     ***** PRINTING THE NUMBERS OF THE BOUNDARY NODES
C           WITH FIXED HEAD VALUES
C
      WRITE(IO,210)
```

```
  210 FORMAT(1H1/18X,'BOUNDARY NODES WITH'/23X,'FIXED HEAD'/15X,26('*'))
      DO 220 I=1,NBNH
  220 WRITE(IO,230)I,NODBH(I)
  230 FORMAT(18X,I3,' =NODE NO. ',I4,/)
C
C     ***** UNIT CONVERSION OF THE INPUT DATA
C
      DO 240 I=1,NOMBN
      X(I)=X(I)*1000.
      Y(I)=Y(I)*1000.
  240 Q(I)=Q(I)/1000.
      DO 250 I=1,NOMBE
  250 TEL(I)=TEL(I)/86400.
      RETURN
      END
C***********************************************************************
      SUBROUTINE TMBAND(INDIC,IEL,JEL,KEL,NOMBE,LARGE,LT,LM)
C     COMPUTATION OF THE WIDTH OF THE BAND MATRIX
C***********************************************************************
      DIMENSION INDIC(1),IEL(1),JEL(1),KEL(1)
      COMMON IN,IO
      LARGE=0
      DO 100 M=1,NOMBE
      I=IEL(M)
      J=JEL(M)
      K=KEL(M)
      IF(INDIC(I)-1)10,100,100
   10 IF(INDIC(J)-1)20,100,100
   20 IF(INDIC(K)-1)30,100,100
   30 ID1=IABS(I-J)
      ID2=IABS(I-K)
      ID3=IABS(K-J)
      IGDIF=ID1
      IF(IGDIF-ID2)40,50,50
   40 IGDIF=ID2
   50 IF(IGDIF-ID3)60,70,70
   60 IGDIF=ID3
   70 IF(LARGE-IGDIF)80,100,100
   80 LARGE=IGDIF
      IELME=M
  100 CONTINUE
      WRITE(IO,110)LARGE,IELME
  110 FORMAT(1H1,//10X,'WIDTH OF THE BAND MATRIX=',I3,
     1'  WHICH IS RELATED TO THE ELEMENT NO.',I3//)
      LT=2*LARGE+1
      LM=LARGE+1
      WRITE(IO,120)LM,LT
  120 FORMAT(10X,'LM=',I3,5X,'LT=',I3//)
      RETURN
      END
C***********************************************************************
      SUBROUTINE TMCONS(NPUIT,NPIEZ,NODPU,NODPI,HINPU,HINPI,H)
C     STORAGE OF THE INITIAL PIEZOMETRIC LEVELS OF THE SELECTED
C     WELLS AND PIEZOMETERS
C***********************************************************************
      DIMENSION NODPU(1),NODPI(1),HINPU(1),HINPI(1),H(1)
      COMMON IN,IO
```

```
      DO 10 L=1,NPUIT
      NPU=NODPU(L)
      HINPU(L)=H(NPU)
   10 CONTINUE
      DO 20 L=1,NPIEZ
      NPI=NODPI(L)
      HINPI(L)=H(NPI)
   20 CONTINUE
      RETURN
      END
C*********************************************************************
      SUBROUTINE TMIMPO(NPUIT,NODPU,Q,DEBIT,INDIC,H,H1,NOMBN,N,NPTEM,
     1NBNH,NODBH,HIMPO,INCON)
C     IMPOSING THE BOUNDARY AND INTERNAL CONDITIONS
C*********************************************************************
      DIMENSION NODPU(1),Q(1),DEBIT(NPUIT,NPTEM),INDIC(1),H(1),H1(1),
     1NODBH(1),HIMPO(1)
      COMMON IN,IO
C
C     ***** READING AND PRINTING OF THE FLOW RATES (IN L/S)
C
      READ(IN,20)(Q(I),I=1,INCON)
   20 FORMAT(5X,10F7.2)
      WRITE(IO,25)N
   25 FORMAT(1H0/10X,'DATA RELATED TO TIME STEP NO.',I4/9X,35('*')//,
     115X,'1=FLOW RATES IN L/S'/17X,17('-')/)
      WRITE(IO,30)(I,Q(I),I=1,INCON)
   30 FORMAT(5(2X,'I=',I3,3X,'Q=',F7.1,' ,'))
      DO 40 I=1,INCON
   40 Q(I)=Q(I)/1000.
C
C     ***** STORAGE OF THE DISCHARGE RATES OF THE SELECTED WELLS
C
   45 DO 50 I=1,NPUIT
      NW=NODPU(I)
   50 DEBIT(I,N)=Q(NW)
C
C     ***** READING AND PRINTING THE CONSTANT HEADS
C
      READ(IN,60)(HIMPO(I),I=1,NBNH)
   60 FORMAT(5X,10F7.2)
      WRITE(IO,70)
   70 FORMAT(//,15X,'2=CONSTANT HEAD VALUES IN M.'/17X,26('-')/)
      WRITE(IO,80)(I,NODBH(I),HIMPO(I),I=1,NBNH)
   80 FORMAT(4(1X,'I=',I3,2X,'NODBH=',I3,2X,'H=',F7.2,' ,'))
      DO 90 I=1,NBNH
      NH=NODBH(I)
   90 H1(NH)=HIMPO(I)
      RETURN
      END
C*********************************************************************
      SUBROUTINE TMGENR(INDIC,X,Y,IEL,JEL,KEL,SEL,TEL,NOMBE,NEQ,
     1INCON,CODE,II,JJ,KK,ALI,ALJ,ALK,AMI,AMJ,AMK,H,H1,Q,A,B,AA,LARGE,
     2LT,LM,DELTA,N)
C    GENERATION OF THE BAND MATRIX AND ITS STORAGE IN COMPACT FORM
C*********************************************************************
      INTEGER CODE
```

```
      DIMENSION INDIC(1),X(1),Y(1),IEL(1),JEL(1),KEL(1),SEL(1),TEL(1),
     1II(1),JJ(1),KK(1),ALI(1),ALJ(1),ALK(1),AMI(1),AMJ(1),AMK(1),H(1),
     2H1(1),Q(1),A(1),B(1),AA(INCON,LT)
      COMMON IN,IO
      DO 1 M=1,INCON
    1 B(M)=0.
      NEQ=0
      CODE=0
      DO 200 INO=1,INCON
      IF(INDIC(INO)-1)2,260,260
    2 I=INO
      NEQ=NEQ+1
C
C     ***** II,JJ,KK,ALI,ALJ,ALK,AMI,AMJ,AMK OF THE NODES AROUND THE
C           NODE I=INO
C
      NE=0
      DO 100 IA=1,NOMBE
      IF(INO-IEL(IA))6,5,6
    5 J=JEL(IA)
      K=KEL(IA)
      GO TO 10
    6 IF(INO-JEL(IA))8,7,8
    7 J=IEL(IA)
      K=KEL(IA)
      GO TO 10
    8 IF(INO-KEL(IA))100,9,100
    9 J=IEL(IA)
      K=JEL(IA)
C
   10 NE=NE+1
      II(NE)=I
      JJ(NE)=J
      KK(NE)=K
C
      XI=X(I)
      YI=Y(I)
      XJ=X(J)
      YJ=Y(J)
      XK=X(K)
      YK=Y(K)
      TOTX=XI+XJ+XK
      TOTY=YI+YJ+YK
C
      AI=XJ*YK-XK*YJ
      BI=YJ-YK
      CI=XK-XJ
      AJ=XK*YI-XI*YK
      BJ=YK-YI
      CJ=XI-XK
      AK=XI*YJ-XJ*YI
      BK=YI-YJ
      CK=XJ-XI
C
      SURFA=ABS((XJ-XI)*(YK-YI)+(YI-YJ)*(XK-XI))
      TRIAN=SURFA/2.
      TRI12=TRIAN/12.
```

```
C
      C1=TEL(IA)/(2.*SURFA)
      C2=SEL(IA)
      C3=SURFA*SURFA
C
C     GX=INTEGRAL OF      X*DX*DY
C     GY=INTEGRAL OF      Y*DX*DY
C     GXX=INTEGRAL OF X**2*DX*DY
C     GYY=INTEGRAL OF Y**2*DX*DY
C     GXY=INTEGRAL OF  X*Y*DX*DY
C     SURFACE OF INTEGRATION =TRIANGLE IJK
C
      GX=(TRIAN*TOTX)/3.
      GY=(TRIAN*TOTY)/3.
      GXX=TRI12*(XI*XI+XJ*XJ+XK*XK+TOTX*TOTX)
      GYY=TRI12*(YI*YI+YJ*YJ+YK*YK+TOTY*TOTY)
      GXY=TRI12*(XI*YI+XJ*YJ+XK*YK+TOTX*TOTY)
C
      ALI(NE)=C2*(AI*AI*TRIAN+BI*BI*GXX+CI*CI*GYY+2.*AI*BI*GX+
     12.*AI*CI*GY+2.*BI*CI*GXY)/C3
      ALJ(NE)=C2*(AI*AJ*TRIAN+BI*BJ*GXX+CI*CJ*GYY+(AI*BJ+BI*AJ)*GX+
     1(AI*CJ+CI*AJ)*GY+(BI*CJ+CI*BJ)*GXY)/C3
      ALK(NE)=C2*(AI*AK*TRIAN+BI*BK*GXX+CI*CK*GYY+(AI*BK+BI*AK)*GX+
     1(AI*CK+CI*AK)*GY+(BI*CK+CI*BK)*GXY)/C3
C
      AMI(NE)=C1*((YJ-YK)*(YJ-YK)+(XK-XJ)*(XK-XJ))
      AMJ(NE)=C1*((YK-YI)*(YJ-YK)+(XI-XK)*(XK-XJ))
      AMK(NE)=C1*((YI-YJ)*(YJ-YK)+(XJ-XI)*(XK-XJ))
  100 CONTINUE
      DO 105 M=1,INCON
  105 A(M)=0.
C
      DO 150 M=1,NE
      I=II(M)
      J=JJ(M)
      K=KK(M)
      B(I)=B(I)+(ALI(M)*H(I)+ALJ(M)*H(J)+ALK(M)*H(K))/DELTA
      A(I)=A(I)+AMI(M)+ALI(M)/DELTA
      IF(INDIC(J)-1)120,115,115
  115 B(I)=B(I)-H1(J)*(AMJ(M)+ALJ(M)/DELTA)
      GO TO 125
  120 A(J)=A(J)+AMJ(M)+ALJ(M)/DELTA
  125 IF(INDIC(K)-1)140,130,130
  130 B(I)=B(I)-H1(K)*(AMK(M)+ALK(M)/DELTA)
      GO TO 150
  140 A(K)=A(K)+AMK(M)+ALK(M)/DELTA
  150 CONTINUE
      B(I)=B(I)+Q(I)
C
C     ***** TRANSFER OF ARRAY A INTO ARRAY AA
C
      DO 170 KA=1,LM
      K1=LARGE+KA
      K2=INO+KA-1
      IF(K2-INCON)160,160,155
  155 AA(INO,K1)=0.
      GO TO 170
```

```
  160 AA(INO,K1)=A(K2)
  170 CONTINUE
C
      DO 190 KA=1,LARGE
      K1=LARGE-KA+1
      K2=INO-KA
      IF(K2)180,180,185
  180 AA(INO,K1)=0.
      GO TO 190
  185 AA(INO,K1)=A(K2)
  190 CONTINUE
C
  200 CONTINUE
C
      IF(N-1)205,205,300
  205 IF(NEQ-INCON)210,240,210
  210 CODE=1
      WRITE(IO,220)CODE,NEQ,INCON
  220 FORMAT(//10X,'CODE=',I2,5X,'NEQ=',I4,5X,'INCON=',I4,//)
      GO TO 300
  240 WRITE(IO,250)NEQ,NEQ
  250 FORMAT(//10X,'SYSTEM OF LINEAR EQUATIONS IS',I4,'X'I4,//)
      GO TO 300
  260 CODE=1
      WRITE(IO,270)INO
  270 FORMAT(//,10X,'NODES ARE NOT CORRECTLY NUMBERED , NODE NO.',
     1I4,'IS A NODE WITH CONSTANT HEAD')
  300 RETURN
      END
C***********************************************************************
      SUBROUTINE TMTRIA(NEQ,INCON,LARGE,LT,LM,AA,B,C)
C     UPPER TRIANGULATION OF THE BAND MATRIX IN ITS COMPACT FORM
C***********************************************************************
      DIMENSION AA(INCON,LT),B(1),C(1)
      COMMON IN,IO
      DO 100 K=1,NEQ
      ALFA=AA(K,LM)
      DO 5 M=LM,LT
      AA(K,M)=AA(K,M)/ALFA
    5 C(M)=AA(K,M)
      B(K)=B(K)/ALFA
C     ***** ELIMINATION OF ONE UNKNOWN IN OTHER EQUATIONS
      DO 50 I=1,LARGE
      IEQ=K+I
C     ***** CONTROL IN VERTICAL DIRECTION
      IF(IEQ-NEQ)10,10,100
   10 K1=LM-I
      K2=K1+LARGE
      IF(AA(IEQ,K1))30,50,30
   30 FACTO=AA(IEQ,K1)
      DO 40 J=K1,K2
      IJ=I+J
   40 AA(IEQ,J)=AA(IEQ,J)-FACTO*C(IJ)
      K3=K+I
      B(K3)=B(K3)-FACTO*B(K)
   50 CONTINUE
  100 CONTINUE
```

```
      RETURN
      END
C*********************************************************************
      SUBROUTINE TMSOLV(NEQ,INCON,LARGE,LT,LM,AA,B,H1)
C     SOLUTION OF THE TRIANGULATED BAND MATRIX
C*********************************************************************
      DIMENSION AA(INCON,LT),B(1),H1(1)
      COMMON IN,IO
      K1=LM+1
      NN=NEQ-1
      H1(NEQ)=B(NEQ)
      DO 100 I=1,NN
      IEQ=NEQ-I
      TOT=0.
      N=0
      DO 40 J=K1,LT
      N=N+1
      N1=IEQ+N
      IF(N1-NEQ)20,20,45
   20 TOT=TOT+AA(IEQ,J)*H1(N1)
   40 CONTINUE
   45 H1(IEQ)=B(IEQ)-TOT
  100 CONTINUE
      RETURN
      END
C*********************************************************************
      SUBROUTINE TMVARI(H,H1,NPUIT,NPIEZ,NODPU,NODPI,HINPU,HINPI,RABPU,
     1RABPI,N,NPTEM,PUNIV,PINIV,NOMBN)
C     COMPUTATION OF THE PIEZOMETRIC VARIATIONS IN THE WELLS AND
C     PIEZOMETERS , ALSO TRANSFER OF ARRAY H1 INTO ARRAY H
C*********************************************************************
      DIMENSION H(1),H1(1),NODPU(1),NODPI(1),HINPU(1),HINPI(1),
     1RABPU(NPUIT,NPTEM),RABPI(NPIEZ,NPTEM),PUNIV(NPUIT,NPTEM),
     2PINIV(NPIEZ,NPTEM)
      COMMON IN,IO
C     ***** PIEZOMETRIC VARIATIONS IN THE WELLS
      DO 10 L=1,NPUIT
      NPU=NODPU(L)
      PUNIV(L,N)=H1(NPU)
      RABPU(L,N)=H1(NPU)-HINPU(L)
   10 CONTINUE
C     ***** PIEZOMETRIC VARIATIONS IN THE PIEZOMETERS
      DO 20 L=1,NPIEZ
      NPI=NODPI(L)
      PINIV(L,N)=H1(NPI)
      RABPI(L,N)=H1(NPI)-HINPI(L)
   20 CONTINUE
C     ***** TRANSFER OF ARRAY H1 INTO ARRAY H
      DO 30 I=1,NOMBN
   30 H(I)=H1(I)
      RETURN
      END
C*********************************************************************
      SUBROUTINE TMRESU(H1,N,NOMBN,YEAR,MONTH,DAY)
C     PRINTING OF THE INTERMEDIATE RESULTS
C*********************************************************************
      INTEGER YEAR
```

```
      DIMENSION H1(1),YEAR(1),MONTH(1),DAY(1)
      COMMON IN,IO
      WRITE(IO,5)N,YEAR(N),MONTH(N),DAY(N)
    5 FORMAT(1H1,/30X,'  COMPUTED PIEZOMETRIC VALUES FOR TIME STEP NO.',
     1I3/36X,'( YEAR=',I3,5X,'MONTH=',I3,5X,'DAY=',F4.1,' )'/30X,
     252('*')/)
      WRITE(IO,10)
   10 FORMAT(10(1X,'NO.',4X,'H1',2X,),/120('*')/)
      NL=NOMBN/10+1
      DO 100 I=1,NL
      J1=10*I-9
      J2=10*I
      JJ=J2
      IF(J2-NOMBN)60,60,20
   20 DO 50 L=1,10
      IF(L-10)30,100,100
   30 J2=JJ-L
      IF(J2-NOMBN)60,60,50
   50 CONTINUE
   60 WRITE(IO,70)(J,H1(J),J=J1,J2)
   70 FORMAT(10(1X,I3,2X,F6.2),/)
  100 CONTINUE
      WRITE(IO,150)
  150 FORMAT(1H1)
      RETURN
      END
C***********************************************************************
      SUBROUTINE TMWELL(DEBIT,DELT,PUNIV,RABPU,NODPU,HINPU,NPUIT,NPTEM,
     1YEAR,MONTH,DAY)
C     PRINTING OF THE PIEZOMETRIC VARIATIONS IN SELECTED WELLS
C***********************************************************************
      INTEGER YEAR
      DIMENSION DEBIT(NPUIT,NPTEM),DELT(1),PUNIV(NPUIT,NPTEM),
     1RABPU(NPUIT,NPTEM),NODPU(1),HINPU(1),YEAR(1),MONTH(1),DAY(1)
      COMMON IN,IO
      DO 10 I=1,NPUIT
      DO 10 J=1,NPTEM
   10 DEBIT(I,J)=DEBIT(I,J)*1000.
      DO 200 NP=1,NPUIT
      WRITE(IO,20)NP
   20 FORMAT(1H1/22X,'PIEZOMETRIC VARIATIONS OF WELL NO.',I4//10X,
     168('-')/10X,'* TIME  * LENGTH *',5X,'SIMULATION',6X,'*',8X,'*',9X,
     2'*',' PIEZO- *'/10X,'*',7X,'*','   OF   *',7X,'PERIOD',8X,'*',
     3'  FLOW  * PIEZO-  * METRIC *')
      WRITE(IO,30)
   30 FORMAT(10X,'*','  STEP *  TIME  ',23('-'),'  RATE  * METRIC  * VAR
     1IA- *'/10X,'*',7X,'*',2X,'STEP',2X,'*',6X,'*',7X,'*',6X,'*',8X,'*'
     2,' LEVEL   * TIONS  *'/10X,'*',3X,'NO.',' * (DAY)  * YEAR * MONTH
     3*  DAY *  (L/S) *  (M)',4X,'*  (M)   *'/10X,68('-'))
      DO 100 K=1,NPTEM
      WRITE(IO,50)K,DELT(K),YEAR(K),MONTH(K),DAY(K),DEBIT(NP,K),
     1PUNIV(NP,K),RABPU(NP,K)
   50 FORMAT(10X,'*',2X,I3,2X,'*',F7.1,1X,'*',2X,I2,2X,'*',2X,I2,3X,'*',
     11X,F4.1,1X,'*',F7.1,1X,'*',F8.2,1X,'*',1X,F6.2,1X,'*'/10X,68('-'))
  100 CONTINUE
      WRITE(IO,150)NP,NODPU(NP),HINPU(NP)
  150 FORMAT(/10X,'WELL NO.',I3,' IS LOCATED ON NODE NO. ',I3/10X,
```

```
     1'INITIAL PIEZOMETRIC LEVEL OF THE WELL IS ',F7.2,' M.')
      WRITE(IO,160)
  160 FORMAT(10X,'NEGATIVE FLOW RATES = WITHDRAWAL'/10X,'POSITIVE FLOW R
     1ATES = RECHARGE')
  200 CONTINUE
      RETURN
      END
C***********************************************************************
      SUBROUTINE TMPIEZ(DELT,PINIV,RABPI,NODPI,HINPI,NPIEZ,NPTEM,
     1YEAR,MONTH,DAY)
C     PRINTING OF THE PIEZOMETRIC VARIATIONS IN THE PIEZOMETERS
C***********************************************************************
      INTEGER YEAR
      DIMENSION DELT(1),PINIV(NPIEZ,NPTEM),RABPI(NPIEZ,NPTEM),
     1NODPI(1),HINPI(1),YEAR(1),MONTH(1),DAY(1)
      COMMON IN,IO
      DO 200 NPI=1,NPIEZ
      WRITE(IO,20)NPI
   20 FORMAT(1H1,/15X,'PIEZOMETRIC VARIATIONS IN PIEZOMETER NO.',I3,
     1//10X,59('-')/10X,'*  TIME * LENGTH *',5X,'SIMULATION',6X,'*',9X,
     2'* PIEZO- *'/10X,'*',7X,'*   OF   *',7X,'PERIOD',8X,'* PIEZO-  * M
     3ETRIC *')
      WRITE(IO,30)
   30 FORMAT(10X,'*  STEP *  TIME  ',23('-'),1X,'METRIC',2X,'* VARIA- *'
     1/10X,'*',7X,'*  STEP  *',6X,'*',7X,'*',6X,'*',' LEVEL   * TIONS  *
     2'/10X,'*',3X,'NO.',' * (DAY)  * YEAR * MONTH *  DAY *  (M)    *  (
     3M)   *'/10X,59('-'))
      DO 100 K=1,NPTEM
      WRITE(IO,50)K,DELT(K),YEAR(K),MONTH(K),DAY(K),PINIV(NPI,K),RABPI(N
     1PI,K)
   50 FORMAT(10X,'*',2X,I3,2X,'*',F7.1,1X,'*',2X,I2,2X,'*',2X,I2,3X,'*',
     1F5.1,1X,'*',F8.2,1X,'*',F7.2,1X,'*',/10X,59('-'))
  100 CONTINUE
      WRITE(IO,150)NPI,NODPI(NPI),HINPI(NPI)
  150 FORMAT(/10X,'PIEZOMETER NO.',I3,' IS LOCATED ON NODE NO. ',
     1I3/10X,'INITIAL PIEZOMETRIC LEVEL OF THE PIEZOMETER IS ',F7.2,
     2' M.')
  200 CONTINUE
      RETURN
      END
```

APPENDIX B2 Input data formats for TMATFE

CARD NO.	COLUMN	FORMAT	VARIABLE	DEFINITION
1	1 - 80	20 A4	TITLE	Description of Problem
2	1 - 5	5X	-	-
	6 - 10	I5	NOMBN	Total number of nodes
	11 - 15	I5	NOMBE	Total number of elements
	16 - 20	I5	INCON	Total number of nodes with unknown heads
	21 - 25	I5	NPTEM	Total number of time steps
	26 - 30	I5	NIMPR	Total number of intermediate printings of computed Piezometric values
	31 - 35	I5	NPUIT	Total number of selected wells
	36 - 40	I5	NPIEZ	Total number of selected piezometers (observation wells)
	41 - 45	I5	NMEC	Maximum number of elements with a common nodes
	46 - 50	I5	NBNH	Total number of boundary nodes with constant heads

APPENDIX B2 (continued)

DATA SET	NUMBER OF CARDS	FORMAT	VARIABLE	DEFINITION
1	Total number of nodes (NOMBN)	5X*,I3,2X,3(F8.3,2X),F8.1	INDIC,X,Y,H,Q	Index (INDIC), coordinates (X,Y in km) water level (H in m.) and discharge or recharge values (Q in ℓ/s) of each node
2	Total number of elements (NOMBE)	5X*,3I5,F8.2,2X,F9.7	IEL,JEL,KEL,TEL, SEL	Node number of the three vertices I,J,K (IEL, JEL, KEL), transmissivity (TEL in m^2/d) and storage coefficient (SEL) of each elements
3	Total number of time steps (NPTEM)	5X*,F10.1,2I5,F5.1	DELT, YEAR, MONTH, DAY	Length of time steps (DELT in days) and simulation period in years, months and days
4	Total number of intermediate printings/15 (NIMPR/15)	5X*, 15I4	TPRIH	Numbers of time steps for which the results will be printed
5	Total number of selected wells/15 (NPUIT/15)	5X*, 15I4	NODPU	Location (node number) of the selected wells
6	Total number of selected Piezometers/15 (NPIEZ/15)	5X*, 15I4	NODPI	Location (node number) of the selected Piezometers (observations wells)

APPENDIX B2 (continued)

DATA SET		NUMBER OF CARDS	FORMAT	VARIABLE	DEFINITION
7		Total number of boundary nodes with constant heads/15 (NBNH/15)	5X*, 15I4	NODBH	Node numbers of the boundary nodes with constant (fixed) heads
8**	a	Total number of nodes with unknown heads/10 (INCON/10)	5X*, 10F7.2	Q	Recharge or withdrawal values (in ℓ/s with + sign for recharges and - sign for withdrawals) of the nodes with unknown heads
	b	Total number of boundary nodes with constant heads/10 (NBNH/10)	5X*, 10F7.2	HIMPO	Head values (in m.) of the nodes with constant heads

* The first five columns on each data card are used for the sequential numbering of the data cards to facilitate the job of data preparation and their control. These numbers are ignored during the reading Process.

** Data set 8(a & b) should be prepared for each time step and placed sequentially

APPENDIX B3-INPUT DATA AND OUTPUT FOR TEST PROBLEM II

```
CARD 1          SIMULATION OF TEST PROBLEM II (TRANSIENT FLOW)
CARD 2 GEN        25    32    20    15     2     1     2     6     5
       N   1  0      0.          0.5        80.           0.
       N   2  0      0.          1.         80.           0.
       N   3  0      0.          1.5        80.           0.
       N   4  0      0.          2.         80.           0.
  D    N   5  0      0.5         0.5        80.           0.
       N   6  0      0.5         1.         80.           0.
  A    N   7  0      0.5         1.5        80.           0.
       N   8  0      0.5         2.         80.           0.
  T    N   9  0      1.          0.5        80.           0.
       N  10  0      1.          1.         80.        -100.
  A    N  11  0      1.          1.5        80.           0.
       N  12  0      1.          2.         80.           0.
       N  13  0      1.5         0.5        80.           0.
  S    N  14  0      1.5         1.         80.           0.
       N  15  0      1.5         1.5        80.           0.
  E    N  16  0      1.5         2.         80.           0.
       N  17  0      2.          0.5        80.           0.
  T    N  18  0      2.          1.         80.           0.
       N  19  0      2.          1.5        80.           0.
  1    N  20  0      2.          2.         80.           0.
       N  21  1      0.          0.         80.           0.
       N  22  1      0.5         0.         80.           0.
       N  23  1      1.          0.         80.           0.
       N  24  1      1.5         0.         80.           0.
       N  25  1      2.          0.         80.           0.
       E   1     1   21    22  250.      .01
       E   2     1    5    22  250.      .01
       E   3     1    2     5  250.      .01
       E   4     2    5     6  250.      .01
       E   5     2    3     6  250.      .01
       E   6     3    6     7  250.      .01
  D    E   7     3    4     7  250.      .01
       E   8     4    7     8  250.      .01
  A    E   9     5   22    23  250.      .01
       E  10     5    9    23  250.      .01
  T    E  11     5    6     9  250.      .01
       E  12     6    9    10  250.      .01
  A    E  13     7    6    10  250.      .01
       E  14     7   10    11  250.      .01
       E  15     7    8    11  250.      .01
  S    E  16     8   11    12  250.      .01
       E  17     9   23    24  250.      .01
  E    E  18     9   24    13  250.      .01
       E  19     9   10    13  250.      .01
  T    E  20    10   13    14  250.      .01
       E  21    10   11    14  250.      .01
  2    E  22    11   14    15  250.      .01
       E  23    11   12    15  250.      .01
       E  24    12   15    16  250.      .01
       E  25    13   24    25  250.      .01
       E  26    13   17    25  250.      .01
```

```
        E  27   13   14   17  250.      .01
        E  28   14   17   18  250.      .01
        E  29   14   15   18  250.      .01
        E  30   15   18   19  250.      .01
        E  31   15   16   19  250.      .01
        E  32   16   19   20  250.      .01
D       D   1        .5    0    0    .5
        D   2       1.5    0    0   2.
A       D   3        5.    0    0   7.
        D   4       13.    0    0  20.
T       D   5       20.    0    1  10.
        D   6       40.    0    2   0.
A       D   7       90.    0    5   0.
        D   8      120.    0    9   0.
S       D   9      180.    1    3   0.
        D  10      270.    2    0   0.
E       D  11      360.    3    0   0.
        D  12      360.    4    0   0.
T       D  13      360.    5    0   0.
        D  14      360.    6    0   0.
3       D  15      360.    7    0   0.
SET 4   TPRIH  10  15
SET 5   LWELL  10
SET 6   LPIEZ   9  12
SET 7   NODBH  21  22  23  24  25
        QT  1   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  1   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  1  80.    80.    80.    80.    80.
        QT  2   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  2   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  2  80.    80.    80.    80.    80.
        QT  3   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  3   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  3  80.    80.    80.    80.    80.
D       QT  4   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  4   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
A       HT  4  80.    80.    80.    80.    80.
        QT  5   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
T       QT  5   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  5  80.    80.    80.    80.    80.
A       QT  6   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  6   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  6  80.    80.    80.    80.    80.
S       QT  7   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  7   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
E       HT  7  80.    80.    80.    80.    80.
        QT  8   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
T       QT  8   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  8  80.    80.    80.    80.    80.
8       QT  9   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT  9   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT  9  80.    80.    80.    80.    80.
        QT 10   0.     0.     0.     0.     0.     0.     0.     0.     0.  -100.
        QT 10   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        HT 10  80.    80.    80.    80.    80.
        QT 11   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
        QT 11   0.     0.     0.     0.     0.     0.     0.     0.     0.     0.
```

HT 11	80.	80.	80.	80.	80.					
QT 12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
QT 12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
HT 12	80.	80.	80.	80.	80.					
QT 13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
QT 13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
HT 13	80.	80.	80.	80.	80.					
QT 14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
QT 14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
HT 14	80.	80.	80.	80.	80.					
QT 15	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
QT 15	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
HT 15	80.	80.	80.	80.	80.					

SELECTED OUTPUT FOR TEST PROBLEM II

SIMULATION OF TEST PROBLEM II (TRANSIENT FLOW)

****

NOMBN= 25	NOMBE= 32	INCON= 20
NPTEM= 15	NIMPR= 2	NPUIT= 1
NPIEZ= 2	NBNH = 5	NMEC = 6

SIMULATION OF TEST PROBLEM II (TRANSIENT FLOW)

(N O D E D A T A)

NO.	INDEX	X IN KM.	Y IN KM.	H IN M.	Q IN L/S
1	0	0.000	.500	80.00	0.0
2	0	0.000	1.000	80.00	0.0
3	0	0.000	1.500	80.00	0.0
4	0	0.000	2.000	80.00	0.0
5	0	.500	.500	80.00	0.0
6	0	.500	1.000	80.00	0.0
7	0	.500	1.500	80.00	0.0
8	0	.500	2.000	80.00	0.0
9	0	1.000	.500	80.00	0.0
10	0	1.000	1.000	80.00	-100.0
11	0	1.000	1.500	80.00	0.0
12	0	1.000	2.000	80.00	0.0
13	0	1.500	.500	80.00	0.0
14	0	1.500	1.000	80.00	0.0
15	0	1.500	1.500	80.00	0.0
16	0	1.500	2.000	80.00	0.0
17	0	2.000	.500	80.00	0.0
18	0	2.000	1.000	80.00	0.0
19	0	2.000	1.500	80.00	0.0
20	0	2.000	2.000	80.00	0.0
21	1	0.000	0.000	80.00	0.0
22	1	.500	0.000	80.00	0.0
23	1	1.000	0.000	80.00	0.0
24	1	1.500	0.000	80.00	0.0
25	1	2.000	0.000	80.00	0.0

(E L E M E N T D A T A)

NO.	I	J	K	T (M2/D)	S
1	1	21	22	250.00	.0100000
2	1	5	22	250.00	.0100000
3	1	2	5	250.00	.0100000
4	2	5	6	250.00	.0100000
5	2	3	6	250.00	.0100000
6	3	6	7	250.00	.0100000
7	3	4	7	250.00	.0100000
8	4	7	8	250.00	.0100000
9	5	22	23	250.00	.0100000
10	5	9	23	250.00	.0100000
11	5	6	9	250.00	.0100000
12	6	9	10	250.00	.0100000
13	7	6	10	250.00	.0100000
14	7	10	11	250.00	.0100000
15	7	8	11	250.00	.0100000
16	8	11	12	250.00	.0100000
17	9	23	24	250.00	.0100000
18	9	24	13	250.00	.0100000
19	9	10	13	250.00	.0100000
20	10	13	14	250.00	.0100000
21	10	11	14	250.00	.0100000
22	11	14	15	250.00	.0100000
23	11	12	15	250.00	.0100000
24	12	15	16	250.00	.0100000
25	13	24	25	250.00	.0100000

26	13	17	25	250.00	.0100000
27	13	14	17	250.00	.0100000
28	14	17	18	250.00	.0100000
29	14	15	18	250.00	.0100000
30	15	18	19	250.00	.0100000
31	15	16	19	250.00	.0100000
32	16	19	20	250.00	.0100000

TIME STEPS AND SIMULATION PERIODS

NO.	TIME STEP IN DAY	SIMULATION PERIOD YEAR	MONTH	DAY
1	.5	0	0	.5
2	1.5	0	0	2.0
3	5.0	0	0	7.0
4	13.0	0	0	20.0
5	20.0	0	1	10.0
6	40.0	0	2	0.0
7	90.0	0	5	0.0
8	120.0	0	9	0.0
9	180.0	1	3	0.0
10	270.0	2	0	0.0
11	360.0	3	0	0.0
12	360.0	4	0	0.0
13	360.0	5	0	0.0
14	360.0	6	0	0.0
15	360.0	7	0	0.0

RESULTS WILL BE PRINTED FOR THE
FOLLOWING TIME STEPS
****

TIME STEP NO. 10

TIME STEP NO. 15

WELLS ARE LOCATED ON THE
FOLLOWING NODES
****

WELL NO. 1 NODE NO. 10

PIEZOMETERS ARE LOCATED ON THE
FOLLOWING NODES
****

PIEZOMETER NO. 1 NODE NO. 9

PIEZOMETER NO. 2 NODE NO. 12

BOUNDARY NODES WITH
FIXED HEAD
****

1 =NODE NO. 21

2 =NODE NO. 22

3 =NODE NO. 23

4 =NODE NO. 24

5 =NODE NO. 25

```
WIDTH OF THE BAND MATRIX=  4  WHICH IS RELATED TO THE ELEMENT NO.  3

LM=  5     LT=  9

DATA RELATED TO TIME STEP NO.   1
*********************************

     1=FLOW RATES IN L/S
     -------------------

I=  1  Q=    0.0 ;  I=  2  Q=    0.0 ;  I=  3  Q=    0.0 ;  I=  4  Q=    0.0 ;  I=  5  Q=    0.0 ;
I=  6  Q=    0.0 ;  I=  7  Q=    0.0 ;  I=  8  Q=    0.0 ;  I=  9  Q=    0.0 ;  I= 10  Q= -100.0 ;
I= 11  Q=    0.0 ;  I= 12  Q=    0.0 ;  I= 13  Q=    0.0 ;  I= 14  Q=    0.0 ;  I= 15  Q=    0.0 ;
I= 16  Q=    0.0 ;  I= 17  Q=    0.0 ;  I= 18  Q=    0.0 ;  I= 19  Q=    0.0 ;  I= 20  Q=    0.0 ;

     2=CONSTANT HEAD VALUES IN M.
     ----------------------------

I=  1  NODBH= 21  H=  80.00 ; I=  2  NODBH= 22  H=  80.00 , I=  3  NODBH= 23  H=  80.00 , I=  4  NODBH= 24  H=  80.00 ,
I=  5  NODBH= 25  H=  80.00 ; I=

SYSTEM OF LINEAR EQUATIONS IS  20X  20

END OF TIME STEP NO.  1

DATA RELATED TO TIME STEP NO.   2
*********************************

     1=FLOW RATES IN L/S
     -------------------

I=  1  Q=    0.0 ;  I=  2  Q=    0.0 ;  I=  3  Q=    0.0 ;  I=  4  Q=    0.0 ;  I=  5  Q=    0.0 ;
I=  6  Q=    0.0 ;  I=  7  Q=    0.0 ;  I=  8  Q=    0.0 ;  I=  9  Q=    0.0 ;  I= 10  Q= -100.0 ;
I= 11  Q=    0.0 ;  I= 12  Q=    0.0 ;  I= 13  Q=    0.0 ;  I= 14  Q=    0.0 ;  I= 15  Q=    0.0 ;
I= 16  Q=    0.0 ;  I= 17  Q=    0.0 ;  I= 18  Q=    0.0 ;  I= 19  Q=    0.0 ;  I= 20  Q=    0.0 ;

     2=CONSTANT HEAD VALUES IN M.
     ----------------------------

I=  1  NODBH= 21  H=  80.00 ; I=  2  NODBH= 22  H=  80.00 , I=  3  NODBH= 23  H=  80.00 , I=  4  NODBH= 24  H=  80.00 ,
I=  5  NODBH= 25  H=  80.00 ; I=

END OF TIME STEP NO.  2
```

COMPUTED PIEZOMETRIC VALUES FOR TIME STEP NO. 10
(YEAR= 2 MONTH= 0 DAY= 0.0)

NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1
1	72.37	2	66.22	3	63.92	4	63.39	5	71.63	6	64.28	7	63.04	8	62.86	9	69.88	10	56.24
11	61.07	12	61.97	13	71.63	14	64.28	15	63.04	16	62.86	17	72.37	18	66.22	19	63.92	20	63.39
21	80.00	22	80.00	23	80.00	24	80.00	25	80.00										

COMPUTED PIEZOMETRIC VALUES FOR TIME STEP NO. 15
(YEAR= 7 MONTH= 0 DAY= 0.0)

NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1	NO.	H1
1	80.00	2	80.00	3	80.00	4	80.00	5	80.00	6	80.00	7	80.00	8	80.00	9	80.00	10	80.00
11	80.00	12	80.00	13	80.00	14	80.00	15	80.00	16	80.00	17	80.00	18	80.00	19	80.00	20	80.00
21	80.00	22	80.00	23	80.00	24	80.00	25	80.00										

APPENDIX C1 - MODIFIED PROGRAM OF T.A. PRICKETT AND C.G. LONNQUIST

```
      PROGRAM MAIN(INPUT=80,OUTPUT=140,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
C     BASIC AQUIFER SIMULATION PROGRAM                VERSION OCTOBER, 1982
C
C
C     THIS PROGRAM IS ESSENTIALLY THE SAME AS THAT DOCUMENTED BY
C     T.A. PRICKETT AND C.G. LONNQUIST OF THE ILLINOIS STATE WATER
C     SURVEY.  CHANGES HAVE BEEN MADE TO PROVIDE AN ECHO CHECK OF
C     INPUT DATA, AND SOME MINOR CHANGES HAVE BEEN MADE TO THE
C     FORMAT STATEMENTS. MATRICES ARE STORED IN (BLANK) COMMON
C     BLOCK TO REDUCE COMPUTER STORAGE USED.
C
C
C
C     DEFINITION OF VARIABLES
C
C     HO(I,J)-----HEADS AT START OF TIME
C                 INCREMENT (I,J)
C     H(I,J)------HEADS AT END OF TIME
C                 INCREMENT (M)
C     SF1(I,J)----STORAGE FACTOR FOR
C                 ARTESIAN CONDITIONS
C                 (M2)
C     Q(I,J)------CONSTANT WITHDRAWAL
C                 RATES(M3/DAY)
C     T(I,J,1)----AQUIFER TRANSMISSIVITY
C                 BETWEEN I,J AND I,J+1
C                 (M3/DAY/M)
C     T(I,J,2)----AQUIFER TRANSMISSIVITY
C                 BETWEEN I,J AND I+1,J
C                 (M3/DAY/M)
C     AA,BB,CC,DD-COEFFICIENTS IN WATER
C                 BALANCE EQUATIONS
C     NR----------NO OF ROWS IN MODEL
C     NC----------NO OF COLUMNS IN MODEL
C     NSTEPS------NO. OF TIME INCREMENTS
C     DELTA-------TIME INCREMENTS (DAYS)
C     HH,S1,QQ,TT-DEFAULT VALUES
C     I-----------MODEL COLUMN NUMBER
C     J-----------MODEL ROW NUMBER
C
C
C
      INTEGER OUT
C
      DIMENSION H(30,30),HO(30,30),
     1SF1(30,30),Q(30,30),T(30,30,2),
     2B(30),G(30),DL(30,30)
C
      COMMON H,HO,SF1,Q,T,B,G,DL
      DATA SUMQ/0.0/
C
```

```
C       TURN OFF UNDERFLOW TRAP
        CALL ERRSET(KOUNT,100)
C
C
C
C       DEFINE INPUT AND OUTPUT DEVICE NUMBERS
C
        IN  = 5
        OUT = 6
C
C       READ PARAMETER CARD AND
C       DEFAULT VALUE CARD
        READ(IN,10) NSTEPS , DELTA , ERROR ,
       1NC , NR , TT , S1 , HH , QQ
     10 FORMAT(I6,2F6.1/2I6,4F6.0)
C
C       ECHO CHECK OF INPUT DATA
C
       WRITE(OUT,15) NSTEPS,DELTA,ERROR,NC,NR
     15 FORMAT(1H1,'NSTEPS=',I3,' DELTA=',F6.2,' ERROR=',F5.2,' COLS=',I3,
      1 ' ROWS=',I3)
       WRITE(OUT,16) TT,TT,S1,HH,QQ
     16 FORMAT(1H ,'TRANSMISSIVITY J&I (M3/D/M)=',2F8.2,' S/FACTOR (M2)=',
      1 E12.4,' DATUM HEAD (M)=',F6.2,' FLOW (M3/D)=',F8.2)
C
C       FILL ARRAYS WITH DEFAULT VALUES
C
        DO 20 I=1,NC
        DO 20 J=1,NR
        T(I,J,1)=TT
        T(I,J,2)=TT
        SF1(I,J)=S1
        H(I,J)=HH
        HO(I,J)=HH
        DL(I,J)=0.0
     20 Q(I,J)=QQ
C
C       READ NODE CARDS
C
        WRITE(OUT,25)
     25 FORMAT(1H0,'NODAL DATA:')
     30 READ(IN,40)I,J,T(I,J,1),
       1T(I,J,2),SF1(I,J),H(I,J),Q(I,J)
     40 FORMAT(2I3,5F6.0)
        SUMQ = SUMQ + Q(I,J)
        IF(EOF(5).NE.0) GO TO 50
C
C       ECHO CHECK OF NODEL DATA
C
        WRITE(OUT,45) I,J
     45 FORMAT(1H0,'COL=',I3,' ROW=',I3)
        WRITE(OUT,16)  T(I,J,1),T(I,J,2),SF1(I,J),H(I,J),Q(I,J)
        GO TO 30
C
C       START OF SIMULATION
C
C
```

```
   50 TIME=0
      WRITE(OUT,1000) SUMQ
 1000 FORMAT(//' ****** SUM OF FLOWS IS ',F10.0//)
      DO 320 ISTEP=1,NSTEPS
      TIME=TIME+DELTA
C
C     PREDICT HEADS FOR NEXT
C     TIME INCREMENT
C
   53 DO 70  I=1,NC
      DO 70  J=1,NR
      D=H(I,J)-HO(I,J)
      HO(I,J)=H(I,J)
      F=1.0
      IF(DL(I,J).EQ.0.0) GO TO 60
      IF (ISTEP.GT.2)  F=D/DL(I,J)
      IF(F.GT.5.) F=5.0
      IF(F.LT.0.0) F=0.0
   60 DL(I,J)=D
   70 H(I,J)=H(I,J)+D*F
C
C     REDEFINE ESTIMATES OF HEADS BY IADI METHOD
C
      ITER=0
   80 E=0.0
      ITER=ITER+1
C
C     COLUMN CALCULATIONS
C
      DO 190 II=1,NC
      I=II
      IF(MOD(ISTEP+ITER,2).EQ.1) I=NC-I+1
C
C
      DO 170 J=1,NR
C     CALCULATE B AND G ARRAYS
C
      BB=SF1(I,J)/DELTA
      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)
      AA=0.0
      CC=0.0
      IF(J-1)90,100,90
   90 AA=-T(I,J-1,1)
      BB=BB+T(I,J-1,1)
  100 IF(J-NR)110,120,110
  110 CC=-T(I,J,1)
      BB=BB+T(I,J,1)
  120 IF(I-1)130,140,130
  130 BB=BB+T(I-1,J,2)
      DD=DD+H(I-1,J)*T(I-1,J,2)
  140 IF(I-NC)150,159,150
  150 BB=BB+T(I,J,2)
      DD=DD+H(I+1,J)*T(I,J,2)
  159 IF(J-1) 160,161,160
  160 W=BB-AA*B(J-1)
      G(J)=(DD-AA*G(J-1))/W
      GO TO 162
```

```
  161 W=BB
      G(J)=DD/W
  162 B(J)=CC/W
  170 CONTINUE
C
C     RE-ESTIMATE HEADS
      E=E+ABS(H(I,NR)-G(NR))
      H(I,NR)=G(NR)
         NRM1=NR-1
         DO 190 K=1,NRM1
            N=NR-K
            HA=G(N)-B(N)*H(I,N+1)
            E=E+ABS(HA-H(I,N))
            H(I,N)=HA
  190 CONTINUE
C
C     ROW CALCULATIONS
C
      DO 300 JJ=1,NR
      J=JJ
      IF(MOD(ISTEP+ITER,2).EQ.1)  J=NR-J+1
      DO 280 I=1,NC
      BB=SF1(I,J)/DELTA
      DD=HO(I,J)*SF1(I,J)/DELTA-Q(I,J)
      AA=0.0
      CC=0.0
      IF(J-1) 200,210,200
  200 BB=BB+T(I,J-1,1)
      DD=DD+H(I,J-1)*T(I,J-1,1)
  210 IF(J-NR) 220,230,220
  220 DD=DD+H(I,J+1)*T(I,J,1)
      BB=BB+T(I,J,1)
  230 IF (I-1)240,250,240
  240 BB=BB+T(I-1,J,2)
      AA=-T(I-1,J,2)
  250 IF(I-NC)260,269,260
  260 BB=BB+T(I,J,2)
      CC=-T(I,J,2)
  269 IF(I-1) 270,271,270
  270 W=BB-AA*B(I-1)
      G(I)=(DD-AA*G(I-1))/W
      GO TO 272
  271 W=BB
      G(I)=DD/W
  272 B(I)=CC/W
  280 CONTINUE
C
C     RE-ESTIMATE HEADS
C
      E=E+ABS(H(NC,J)-G(NC))
      H(NC,J)=G(NC)
         NCM1=NC-1
         DO 300 K=1,NCM1
            N=NC-K
            HA=G(N)-B(N)*H(N+1,J)
            E=E+ABS(H(N,J)-HA)
            H(N,J)=HA
```

```
  300 CONTINUE
      IF(E.GT.ERROR) GO TO 80
C
C     PRINT RESULTS
C
      WRITE(OUT,310) ISTEP,TIME,E,ITER
  310 FORMAT(/,1H0,I4,' TIME =',F7.2,' DAYS     E =',E13.7,' METRES',5X,
     1 'NO. ITERATIONS =',I5/)
      DELTA=DELTA*1.2
      DO 320 J=1,NR
  320 WRITE(OUT,330) J,(H(I,J),I=1,NC)
  330 FORMAT(I3,1X,16F8.2)
C
      STOP
      END
```