

## An econometric analysis of interest rate forecasts

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# AN ECONOMETRIC ANALYSIS OF INTEREST RATE FORECASTS

#### BY DAYNE KELLY

## MASTER OF COMMERCE (HONS) 10<sup>TH</sup> NOVEMBER 1998

#### **Declaration**

I hereby declare that this submission is my own work and to the best of my knowledge it contains no material previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where acknowledgment is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except to
the extent that assistance from others in the project's design and conception or in style,
presentation and linguistic expression is acknowledged.
Simular Management

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#### Introduction

Over the last few decades some econometricians have been concerned with the question of whether forward (exchange or interest) rates are optimal predictors of future spot rates. The forward interest rate being the optimal predictor of the future spot interest rate is in fact known as the Expectations Theory of the Term Structure of interest rates. The alternative is that the discrepancy in forward and future spot interest rates is the result of a Term Premium. When these econometricians tested the assumption that forward rates were optimal predictors of future spot rates, an important realisation occurred. That was, if the evidence could not support the Expectations Hypothesis then the joint assumption that interest rate market participants are risk neutral and endowed with Rational Expectations, also could not be supported (Taylor, 1989, p.142). So there was a shift in focus in the literature and survey data were utilised as it allowed the concept of the rationality of expectations to be tested directly without the imposition of any assumptions about risk (MacDonald, 1990, p.230). This thesis makes use of survey data of the expected spot interest rate coupled with actual spot interest rates to ascertain whether forecasters are Rational.

The survey data were provided by *Consensus Economics* and was reported monthly. Data from Japan, Canada, Australia, France, Germany, Italy, the U.K. and the U.S.A. were utilised. With greater emphasis placed on the U.S. data. Graphs of the data can be found in chapters two and four. The qualities that ensure the accuracy of forecasters are those qualities that secure an efficient market and they are the essential underlying assumptions in the Expectations Theory of the Term Structure of Interest Rates. Hence, literature concerning the Term Structure of Interest Rates is surveyed.

In constructing the survey data, agents were asked for their expectations of the spot rate and these were compares with actual outcomes. Overlapping data problems arose because the frequency of collection was less than the length of the forecast. This problem is not uncommon when utilising survey data. The literature pointed to three methods of coping with the issue of overlapping data. Firstly, a non-overlapping data series can be constructed by imposing a prior restriction of synchrony between the sampling period and the contract period (Canarella and Pollard, 1986, p.576). Secondly, the error structure can be corrected by

imposing a second order moving average structure. Thirdly, Hansen and Hodricks Generalised Method of Moments (GMM) technique can be utilised as it modifies the standard errors to provide a consistent estimate of the coefficient covariance matrix. All three of these methods are employed in Chapters two (for the U.S. data) and four (for data from seven other countries) and the resultant outcomes compared.

Both the Sampling and moving average estimation techniques make use of Ordinary Least Squares (OLS) for which stationarity is a necessary assumption. As, if the data is nonstationarity sample moments of the data may not settle down to their population value and despite their being no relation between the series, the regression method could pick up spurious correlations. Further, conventional test statistics are not valid as they assume a normal distribution and as mentioned nonstationary data does not have a constant mean and variance. The GMM also assumes stationarity.

The assumption of stationarity was questioned by Nourzad and Grennier (1995) and Choi and Wohar (1991). Via the use of the Dickey Fuller test these authors found evidence of a unit root in interest rate data. This testing procedure was also used in this thesis on the *Consensus Economics* survey data. The results of these tests can be found in Chapter three (U.S. data only) and Chapter four.

Johansens Maximum Likelihood approach was then used. This technique yields estimates of the cointegration vectors, and the number of such cointegrating relationships can be determined via a likelihood ratio test. In the presence of a cointegrating relationship a further estimation technique was utilised. A dynamic regression equation implicitly performs the same corrections, in theory, as those achieved by the use of non-parametric correction terms (Banerjee et al., 1993, p.240). This method has in the past, however, failed to achieve full efficiency. Phillips and Loretan suggested including leads in the regression and this aided in the estimating and eliminating of the effects of long-run feedback between the errors. The Phillips and Loretan estimator is known as Dynamic OLS. The estimated coefficients obtained from the regressions using the *Consensus Economics* data are tabulated in Chapters three and four.

Chapter four plays an important role in determining whether the results posted in Chapter three were particular to the United States, or whether patterns emerge in the data or, at most, the results are uniform. The seven countries used in Chapter four are Japan, Canada, Germany, France, Italy, Australia and the U.K.

Interestingly the Dickey Fuller test results were inconclusive. This does not defy intuition as interest rates are more likely to deviate from the mean for prolonged periods of time but will eventually revert back. Interest rates could not accelerate without check for economic and political reasons. Hence, the concept of local to unity (first considered in this context by Stock, 1995) was introduced.

Chapter five sees the construction of a data generating process where all variables can be controlled using Monte Carlo simulation. This enables the determination of how assumptions about the generation of the data affects the estimates obtained using OLS with MA(2) errors, Johansen's Maximum Likelihood estimation of an error-correction model and DOLS. Of particular interest is; how enforcing a local to unity root affects both the coefficients and the accuracy of forecasters. Confidence regions are constructed and each technique's coverage rate is determined.

The conclusion points to a general acceptance of the Rationality and accuracy of forecasters. However, whether interest rate data does exhibit a local to unity root cannot, as yet, be assessed. If the presence of such a root is suspected, then the Monte Carlo simulation indicates that the econometrician is better off using simple OLS techniques rather than cointegrating techniques.

#### Chapter 1

#### Term Structure of Interest Rates, Rationality and Efficiency

This thesis is predominantly concerned with the accuracy of forecasters. For a forecaster to be accurate s/he needs to form expectations in a Rational (unbiased and efficient) manner. If agents form their expectations Rationally then their expectation of the future spot interest rate will be the best predictor of the actual future spot rate. Furthermore, if in forming these expectations the agent does not rely on past errors the Efficiency criteria is met. The presence of the two properties of Rationality and Unbiasedness is also referred to as Speculative Efficiency. In short, for forecasters to exhibit Speculative Efficiency there must be, a matching of expectations and actual interest rates, no autocorrelation in the errors and previous forecast errors should have no bearing on current forecasts.

This concept of Rationality (Unbiasedness and Efficiency) also forms the basis of all explanatory theories of the Term Structure of Interest Rates. It is for this reason that the large body of literature which deals with the issue of the Term Structure of Interest Rates should be examined.

To begin the discussion of the literature the Term Structure of Interest Rates needs to be defined. The Term Structure of Interest Rates is the analysis of the yield differentials of securities that are homogeneous in all other respects except their term to maturity (Juttner, 1994, p.469). There are a number of different theories which claim to explain the Term Structure of Interest Rates. This thesis makes reference to only the Expectations Theory, the Pure Expectations Theory and the Risk Premium Model.

#### 1.1 The Expectations Theory

The Expectations Theory of the Term Structure of Interest Rates proffers that the relationship between the rates of return on short-term and long-term bonds can be explained by using expectations about future levels of interest rates. This theory depends on five assumptions:

- 1) All securities are riskless regarding the payment of interest and the payment of principal, and therefore are perfect substitutes
- 2) Market participants expect a set of short-term future interest rates and their expectations are uniform and unbiased.
- 3) Transaction costs are assumed to be zero.
- 4) Investors are profit maximisers.
- 5) The coupon rate is zero; interest on a security is paid when it matures.

  (Juttner, 1994, p.478)

The Expectations Theory implies that, regardless of whether an investment is made on, say, a one year or a two year note, the return or yield at the end of the first year will be the same. If this was not the case, investors would move out of the lower yielding note to the higher yielding note, driving down the yield of the latter until the two yields are equal. Inherent in the discussion of this theory are two different interest rates - the spot and forward rates. In order to determine the forward rate two spot rates need to be known. The spot rate on the one year note, which is known at time t and is assumed to prevail during the first year, and the corresponding spot rate on the two year note (which is also known at the time t). These two rates imply a forward rate. In other words, the spot rate is the interest rate determined in the current market pertaining to the current period. The forward rate is determined at the present time for future periods and is written with two time subscripts; one for the period in which they are determined, the other for the period ahead when delivery is to be made. Forward rates are not directly observable. They are implied by spot rates and can be calculated from them. For example, Hicks in his 1939 paper constructed forward rates for a one period debt via the formula  $I_1 = \frac{(1+S_2)^2}{(1+S_1)} - 1^1$ , where  $S_1$  and  $S_2$  are the currently prevailing spot interest

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<sup>&</sup>lt;sup>1</sup> Notation has been changed so as to avoid confusion with the notation used in this thesis.

rates on one period and two period debts respectively. Hence  $I_1$ , the implied forward rate, can be calculated from the yield curve and indicates the markets expectation of future spot rates.

#### 1.2 Pure Expectations Theory

The Pure Expectations Theory postulates that the forward rate applicable to the period t+k on a one period security as implied by the yield curve at time t, t+k t, equals the markets expectation in period t of the corresponding spot rate  $E_t(t+k)$ . Or t+k t, t =  $E_t(t+k)$  (Juttner, 1994, p.482). In other words, the Forward Premium, or Risk Premium, are zero. The Risk Premium is the difference between the implied forward interest rate and the expected future spot rate. This theory has been attributed to Fisher (1930) or Lutz (1940). As additional terms and interest rates have been introduced by the Pure Expectations Hypothesis, they need to be defined as follows:

Future interest rates are time subscripted the same way as forward rates but are determined in the futures market. Futures rates reflect the opinion of what the market rate will be at the delivery date of the futures contract. Similarly, the future spot interest rate, used in this thesis, reflects the opinion of agents of what spot rates will be in the future. These rates are not directly observable and exist only in the minds of the market participants. A note as to the relationship between the future, forward and expected spot rates is that at the delivery date of the futures contract, the future rate becomes the spot rate. So if expectations are rational, future and expected rates should only differ by chance (Juttner, 1994, p.535-542). Essentially forward and futures rates should not differ as they express essentially the interest rate expectations for the same period ahead. However in practice, they do differ due to transactions costs. (Juttner, 1994, p. 542).

#### 1.3 Risk Premium Model

With an understanding of the above definitions, the Risk Premium Model can now be defined. The Risk Premium Model of the Term Structure of Interest Rates states that, contrary to the Pure Expectations Theory, the forward rates are biased estimates of the expected future spot rates due to the presence of a risk premium. The risk premium is said to compensate

holders of longer term bonds for the risk of unanticipated bond price fluctuations. For this model, in equilibrium, the implied forward rate (which includes the risk premium) will lie above the expected short rates. This theory was introduced by Hicks and is also known as the Liquidity Preference Theory (Juttner, 1994, p.486).

#### 1.4 The Modelling of the Term Structure of Interest Rates

An influential survey by Melino (1988) catalogued evidence regarding the existence of Term Premia in the Term Structure of Interest Rates. In short, Melino highlighted two major developments in relations to the Term Premia: whether the Term Premia were zero or not and after the broad acceptance of rational expectations later research considered whether the Term Premia were time invariant (Melino, 1988, p.337).

Melino commented that early empirical work focused on the accuracy of forward rates as predictors of subsequent spot rates. For instance Macauley, 1938, found little evidence of successful forecasting using short term securities, and Kessel, 1965, constructed a series of implied forward rates and found they consistently over-predicted subsequent spot rates. Thus Kessel presumed that forward rates should be seen as the market's expectation of the spot rate - plus a Term Premium, which varies positively with the level of the spot rate (Melino, 1988, p.346-347).

In 1962 Meiselman published a paper in defence of the Pure Expectations Hypothesis. He developed an Error Learning Model which postulated that implied forward rates evolved according to the rule:

$$I_t(n) - I_{t-1}(n+1) = \alpha_n + \beta_n(S_t(1) - I_{t-1}(1))$$
  $n = 1, 2, 3....$ 

where I represents the forward rates, S the spot rates and  $\alpha$  the term Premia. From this model he concluded that the term premium was zero and that the forward rate behaved as expectations should. Both of these notions were found to be invalid. Melino claimed the strongest criticism being that it was hard to see what was learnt from the significant correlation between the forward rate changes and innovations in the spot rate. In fact, if rational expectations held, the question that should have been asked was, are the correlations between the forward rate changes and innovations in the spot rate consistent with the

stochastic properties of the latter, as opposed to whether these correlations were non-zero? (Melino, 1988, p.348).

Modigliani and Sutch (1966,1967) followed on from Meiselman's paper as far as how best to model expectations. They believed that in their long run interest rate ( $R_t$ ) expression;

$$R_{t}(n) = \sum_{i=0}^{L} \beta_{j} S_{t-j}(1) + Z_{t} \delta + \varepsilon_{t}$$

the effects of expectations were portrayed by the distributed lag. Where S represented spot rates,  $Z_t$  represented the term Premia and  $\beta$  and  $\delta$  were parameters (Melino, 1988, p.350). A subsequent paper by Modigliani indicated this research was misspent. In their 1973 paper, Modigliani and Shiller showed the awkwardness of the hypothesis - that only the past history of a process was useful in predicting its future values. Modigliani and Shiller found that the rate of inflation actually helped improve the prediction of the spot rates (Melino, 1988, p.352). Further problems followed, as it was impossible to distinguish between the effects of variables on expectations and the term premium, when faced with such a reduced form equation as above. This problem was solved with the advent of rational expectations, which by its nature contains agents expectations.

The widespread adoption of rational expectations in the term structure literature, sparked many papers. Concentrating on the empirical research, Roll (1970) invoked rational expectations which allowed him to estimate historical Term Premia by adapting the Martingale model of Samuelson (1965) to the forward rate. Melino reports that Roll found that the Term Premia were usually positive and tended to increase with maturity. Fama (1984) supported this view and extended the findings to securities with longer maturities.

Melino noted that once the existence of the Term Premium was accepted but the debate switched to whether or not it varied over time. Most of the empirical research in the 1970's and 80's centred around the null hypothesis of rational expectations and time invariant Term Premium. Some findings are worth noting; Fama, 1976, found that the forecast implicit in the term structure for short rates, generated serially correlated forecast errors. Campbell and Schoenholtz, 1983, and Fama (1984) all found that at short maturities the term Premia had varied, and that expectations alone could not explain movements in yields (Melino, 1988, p.356).

In the long term securities arena, it was reputedly much more difficult to reject the expectations hypothesis. Melino noted that it proved very difficult to predict the first difference in long rates using only the information contained in lagged long rates. More generally, most of the observed variance of the change in long rates could not be predicted exante. Hence there appeared to be evidence to support the expectations hypothesis.

Pesando (1978) followed on from the findings in the Modiglianni-Sutch and Modiglianni-Shiller papers by regressing the change in the long rate on a distributed lag of the changes in the short rate and then on distributed lags of the changes in both the short rate and the inflation rate. He found that only the contemporaneous change in the short rate mattered, and hence concluded that changes in the long rate were unpredictable. In his review, Melino retorted that the inability to predict more than modest changes in the long rate does not mean that time varying term Premia are small and unimportant, as the expectations models said nothing about the per cent value of the variation of the change in the long rate. Just because these values are small does not discredit the model. (Melino, 1988, p.355).

Prior to 1988, empirical research relied on more robust measurement techniques. Melino noted that Sargent (1979) assumed that the first differences of short and long rates followed a fourth order bivariate autoregressive process. Allowing for this process, Sargent then found that the restrictions implied by the expectations hypothesis were satisfied. Melino had pointed out in an earlier paper that Sargent did not impose the full set of restrictions and that the first difference representation and the restrictions implied by the expectations model were awkward. The two could only hold simultaneously if the data displayed singularity. Melino noted this was rectified in a paper by Hansen and Sargent, 1981, where the full restrictions were imposed and the Generalised Method of Moments technique was used to correct the serial correlation in the errors. In this case the restrictions were rejected.

#### 1.5 Efficiency

The differing assumptions and estimating techniques highlighted in the summary of Melino's paper above will be analysed in this thesis with respect to whether they shed any new light on the question of the existence of an efficient interest rate market. The word efficient is used frequently in the literature on the Term Structure of Interest Rates and has come to mean different things. When making reference to the Efficient Market Hypothesis in this paper it is referring to that hypothesis that has been used in financial literature. There are three forms of this hypothesis, namely, the weak form of the Efficient Market Hypothesis which states that prices (interest rates) are assumed to reflect any information that may be contained in the past history of the price (interest rate) itself. The semi-strong form; which assumes all publicly available information is reflected in prices and the strong form which presumes all information, be it public or private, is reflected in prices so there is no opportunity to gain superior returns (Haugen, 1993, p.635). The term Efficiency is also used in reference to Rational Expectations, which is in turn a necessary assumption in the particular type of efficiency analysed in this thesis. When expectations are rational, they are uniform, unbiased and efficient. By unbiased it is meant that there is a matching of expectations and actual interest rates. Efficiency, in this case, implies that there is no autocorrelation in the errors and previous forecast errors have no bearing on current forecasts.

#### 1.6 Rational Expectations

The existence of Rational Expectations has often been taken for granted in the analysis of the Efficient markets hypothesis. As Taylor (1989) noted, previous tests of whether the forward foreign exchange rates act as optimal predictors of the future spot exchange rate have been a test of the joint null hypothesis, that the market participants are both risk neutral and possess Rational Expectations. He claimed that people had absorbed the Rational Expectations hypothesis into their maintained hypothesis, and had gone off in search of the Term Premia. With the use of survey data Taylor wished to rectify this situation. Survey data supposedly enabled him to "apportion the blame for the non-optimality of the forward rate as a spot predictor, between the failure of the risk neutrality assumption, and the failure of the rational expectations assumption" (Taylor, 1989,p.142). To apportion the blame, Taylor defined the

Risk Premium as  $p_t = (s_{t+n}^e - i_t) / s_t$  where s was the spot exchange rate and i the forward exchange rate and e denoted the agents expectations formed at time t. Equivalently the premium could be written as  $p_t = [(s_{t+n}^e - s_t) / s_t] - [(i_t - s_t) / s_t]$ 

noting that the realised rate of appreciation will differ from the expected rate via a forecast error  $u_{t+n}$  and invoking Rational Expectations, which would imply that the forecasting errors would be orthogonal to the information set at time t, the regression model estimated became:

$$(s_{t+n} - s_t) / s_t = \alpha + \beta (i_t - s_t) / s_t + \mu_{t+n}$$

The joint hypothesis of risk neutrality and rational expectations translated into a test of  $\alpha=0$   $\beta=1$  (Taylor, 1989, p.143).

To work out which of the two effects was predominant in the rejection of the Pure Expectations Hypothesis the author noted that the regression coefficient;

$$\beta = \frac{\text{cov}\{(s_{t+n} - s_t) / s_t (i_t - s_t) / s_t\}}{\text{var}\{(i_t - s_t) s_t\}}$$

via some algebraic manipulation could be split up into

$$\beta_{RE} = -\frac{\text{cov}\{\mu_{t+n}(i_t - s_t) / s_t\}}{\text{var}\{(i_t - s_t) / s_t\}} \text{ and } \beta_{RN} = 1 - \frac{\text{cov}\{(s_{t+n}^e - s_t) / s_t, (i_t - s_t) / s_t\}}{\text{var}\{(i_t - s_t) / s_t\}}.$$

Under rational Expectations  $\beta_{RE}=0$  and under Risk Neutrality  $\beta_{RN}=0$  (Taylor, 1989, p.144). Taylor found that there was evidence of either risk-averse or non-rational behaviour, though, neither of these models could be said to be entirely at fault.

This paper was useful in its acknowledgement of Rational Expectations, in its attempt to model them and in its use of survey data which had the benefit of containing additional information, namely, expectations. However there is another way of testing for the existence of Rational Expectations which involves asserting whether the underlying assumption hold, ie, Unbiasedness and Efficiency. MacDonald (1990) used survey data, this time on the foreign exchange market expectations, to test whether foreign exchange market participants are rational. He did this by estimating the equations

(1) 
$$\Delta s_{t+k} = \rho + \delta \Delta s_{t+k}^e + \varphi_{t+k}$$

and

(2) 
$$s_{++}^e - s_{++} = \Phi_0 + \Phi_1 X_e + \zeta_{++}$$

Where s denotes the spot interest rate, e agents expectations and X any additional variable that may have contributed additional information to the obtaining of the forecast error, such as, lagged forecasts from other interest rate markets. For the Unbiased property MacDonald postulated that if agents formed optimal forecasts of the future spot rate then  $\rho=0$  and  $\delta=1$ . For the error orthogonality property  $\Phi_0=\Phi_1=0$ . Furthermore, if the data was non-overlapping then the error terms should be serially uncorrelated (MacDonald, 1990, p.230).

MacDonald tied this recent interpretation of the Efficient Market Hypothesis Structure back to previous work. He noted that the main conclusion to emerge from this work was that the unbiased proposition failed, because  $\delta$  deviated from unity and because of the existence of a constant Risk Premium. The Term (Risk) Premium having been the main topic of concentration in the previous Efficient Market literature (MacDonald, 1990, p.230).

MacDonald included lagged forecast errors and the Forward Premium from the foreign exchange market and lagged forecast errors from other markets as a means to test the efficiency principal of Rational Expectations. All of these variables should have no additional information and should hence have no bearing on the forecast if the market is efficient. The null hypothesis was, therefore, that the coefficients were jointly zero.

The equations estimated were (MacDonald, 1990,p. 238-239);

$$s_{t+k}^{e} - s_{t+k} = \alpha_0 + \sum_{j=0}^{l} \alpha_j (s_{t-j}^{e} - s_{t-j}) + \upsilon_{t+k}$$

$$s_{t+k}^{ei} - s_{t+k}^{i} = \alpha_0^{i} + \sum_{i=1}^{m} \sum_{j=0}^{l} \alpha_{ij} (s_{t-j}^{ei} - s_{t-j}^{i}) + \upsilon_{t+k}^{i}$$

$$s_{t+k}^{e} - s_{t+k} = \alpha_0 + \sum_{t+k} \alpha_i (fp_{t-j} - s_{t-j}) + \upsilon_{t+k}^{i}$$

Where the spot rate and expected spot rate are denoted as before and fp represents the forward premium. The hypothesis failed to be accepted, in that the expectational forecast error had no additional information which could be exploited by agents, but the Forward Premium did. Overall MacDonald found that the survey expectational series was a biased predictor for future exchange rates. This paper highlighted methods of testing for Unbiasedness, Efficiency and Rationality and used data similar to that used in this thesis.

#### 1.7 Speculative Efficiency

The concept of Rationality is an important assumption which forms the basis of a particular type of efficiency - Speculative Efficiency - which is the focus of this thesis. This form of efficiency expressly states that, when expectations of future spot rates are formed rationally in an unbiased and efficient manner, and economic agents are risk neutral, the forward rate will turn out to be the best predictor of the expected future spot rate (Juttner, 1992, p.55). Canarella and Pollard (1986) also made reference to the speculative efficiency hypothesis which was to them simply the unbiased predictor hypothesis renamed. The latter definition shall be the one adhered to in this thesis and, hence, the terms Unbiasedness and Speculative Efficiency shall be used interchangeably.

#### 1.8 Purpose

This thesis attempts to introduce the model by which speculative efficiency will be analysed, and to look at this model in the context of the development of interest rate modelling as outlined in the Term Structure literature. This thesis follows recent interest rate literature which has used survey data in order to measure agent expectations. The Data used is obtained from *Consensus Economics* and consists of both the spot and expectations of what the spot interest rate will be three months ahead. The time period being from May, 1990 to September 1996, for all countries except Australia where the data ceases from September 1995. The second variable will be termed the forecast of the spot interest rate, and hence will be represented by an (f). This forecast variable is taken from the mean forecast variable posted in *Consensus Economics*. Averaging accounted for the problems of forecasters differing from time to time, and occasionally failing to report their expectations of the future spot interest rate.

The basic models from which these issues will be discussed are:

(a) 
$$s_{t+3} = \alpha_0 + \beta f_t + \varepsilon_{t+3}$$

(b) 
$$s_{t+3}-f_t=\alpha_0+\alpha_1(s_{t-1}f_{t-3})+\alpha_2(s_{t-1}-f_{t-4})+\varepsilon_{t+3}$$

Where  $s_{t+3}$  is the spot interest rate at time t+3 and  $f_t$  is the mean forecast of the interest rate determined at time t to eventuate at time t+3. In other words, it is the agent's expectation of what the spot rate will be in three months.

Model (a) has been used in the literature to test unconditional bias. DeBondt and Bange (1992) used a version of this model in relation to inflation data. In this model, if the constant term and forecast coefficient are insignificantly different from zero, and there is no serial correlation, then the forecasts are not subject to systematic bias. Therefore, the market is found to be efficient. MacDonald and Taylor (1988) included lagged forecast errors in their model, as in model (b). In other words, this equation allows for past forecast errors to have a bearing on the present forecast error. This is in line with an Efficient Market in which asset prices should reflect all available information and therefore no other information should be used in predicting future prices (MacDonald, 1990, p.235). Model (b) was also used by MacDonald (1990) with the test for efficiency being that all coefficients were jointly zero meaning that the agents were only exploiting information available to them at the present time t. Canarella and Pollard (1986) included in their analysis of data from the London Metal Exchange, a model similar to (b), as they wished to analyse Speculative Efficiency, and so tested whether past forecast errors had any explanatory power in relation to the current forecast error. Their paper differed as it concerned the testing of future and spot prices, not interest rates and expectations. Other models have been used in relation to the question of forecaster accuracy, many including inflation data. However, as the primary concern in this paper is the modelling of interest rate survey data the two models, (a) and (b), are best suited to the data.

The following chapter seeks to model the US data using the original techniques outlined in the Term Structure literature, namely, OLS, GMM and OLS with moving average errors (or GLS). Data from one country is concentrated on initially for simplicity. A study of the nature of the data is undertaken, though at this stage it is constrained to the assumption of stationarity world. The results from the tests of Unbiasedness and Efficiency are then compared. The U.S. data was chosen, as opposed to the Australian data, as the full data set was available and this allowed for more meaningful comparisons with the data from the seven other countries in Chapter four.

#### Chapter 2

#### **Stationarity**

As the Term Structure literature has spanned a large time period it has been associated with major developments in econometrics, both in terms of theory and estimation techniques. Hence, this literature provides a framework in which the question of whether the results of the tests of Unbiasedness and Efficiency, or Rationality, change when assumptions about the data change and alternative estimates are used. These assumptions concern the presence of stationarity, nonstationarity or a local to unity root. Different estimation techniques are used, as a point of comparison, with the relaxation of these assumptions. Further, as the data is overlapping, the three methods of correcting for this problem are compared.

This chapter begins the analysis with the assumption that the data is stationary. Therefore the point of comparison centres on the three techniques, utilised in the Term Structure Literature, which coped with the overlapping data problem. This problem is inherent in most survey data.

#### **2.1 Data**

The data used in this thesis consists of monthly expected and actual spot interest rates for eight countries, namely, the United States; the United Kingdom; Canada; Japan; France; Germany and Italy tabulated in the monthly survey publication *Consensus Economics*, for the period February 1990 to September 1996, excepting Australia, for which the period is November 1990 to March 1996. The spot rate is the interest rate taken on the day of the date of the publication; for this paper it is the three month short term interest rate. Survey participants contributing to *Consensus Economics* are required to predict this three month interest rate, three months ahead, and the mean of this prediction across forecasters will be termed the forecast rate from here on. The investigation will begin with the data from the United States in this chapter with a summary of the results from other countries in chapter 4.

Graph 2.1 displays the actual and predicted spot rates for the U.S. (Note that the spot rate has been adjusted so that the forecast and actual rate are in line). In other words, the spot rate for January has been aligned with the forecast for January that was made three months earlier. At a glance the average forecasts of the spot rate, represented by the dotted line, are relatively close to the actual rates. Indicating that forecast errors are small. However, the sign of the forecast error changes relatively infrequently.

Graph 2.1 U.S. Actual & Forecast Spot Interest Rates

The estimation methods that have evolved throughout the course of the Term Structure of Interest Rates literature will now be now be discussed. Key articles which used these techniques will be summarised and their findings noted. The *Consensus Economics* data will then be used to estimate the two equations (a) and (b) using each of the techniques commonly used in the literature. The problems associated with each of these estimation techniques will be discussed in relation to the results obtained from the *Consensus Economics* data.

#### 2.2 Ordinary Least Squares (OLS)

Originally the theory that forward rates are unbiased predictors of future spot rates (ie, the pure expectations theory of the term structure of interest rates) was frequently tested using a simple linear equation,  $s_{t+3} = \alpha + \beta i_{t+3} + \varepsilon_{t+3}$  where s is the spot rate, i the forward rate and  $\alpha$  the often mentioned term premium. A version of this simple model can be found in Friedman's 1979 paper.

Friedman sought to test the hypothesis about the relationship between the forward rate and the interest rate expectations; a form of the spot and forward model mentioned earlier. Friedman did this through the use of survey data which sampled market participants expectations. The data consisted of thirty-one quarterly observations beginning in September 1969. The discrepancy between these the forward rate and interest rate expectations was termed the Term premium. In order to determine whether this premium existed, the author ran the regression;  $I_i = \alpha + \beta s^{Ei} + \mu_i$ , where  $s^{Ei}$  was the expectation of the future spot rate, I the forward rate. Tests for the Pure Expectations theory corresponded to the null hypothesis that  $\alpha=0$  and  $\beta=1$ . The results either rejected the joint null hypothesis, or the individual hypothesis of  $\alpha=0$  and  $\beta=1$  (Freidman, 1979, p.968).

Fama (1984) also tackled the issue of the forecast ability of forward rates. Fama stated that preceding literature found that forward rates were poor forecasts of future spot rates, and that his papers' point of distinction was that it examined a new approach to measuring the information in forward rates about premiums and future spot rates. Or in other words, he found a new way of writing the original equation mentioned above.

Defining  $V1_t = \exp(-s_{t+1})$ , the one month return on a bill with  $\tau$  months to maturity as  $H\tau_{T+1} \equiv \ln(V(\tau-1)_{t+1} / V\tau_t)$  and the premium  $P\tau_{T+1}$  as the difference between the return, defined above, and the one month spot rate, Fama considered two regressions;

(1) 
$$P\tau_{t+1} = \alpha_1 + \beta_1(i\tau_{t+1} - s_{t+1}) + \varepsilon_{t+1}$$

(2) 
$$s_{t+\tau} - s_{t+1} = \alpha_2 + \beta_2 (i\tau_t - s_{t+1}) + \eta_{t+\tau-1}$$

 $V_{\pi}$  is the price of a discount bond that matures at time  $t+\tau$ ,  $s_{t+1}$  is the one month spot interest rate and i is the forward interest rate. Equations (1) and (2) therefore represented

regressions of the future premium,  $P\tau_{T+1}$ , and the change in the spot rate on the current forward-spot differential. They aim to tell us whether the current forward-spot differential had power as a predictor of either the future premium or the future change in the one month spot rate (Fama, 1984, p.510-511). Any evidence that  $\beta_l$  was positive indicated that the forward rate, observed at time t, contained information about the premium to be observed at time t+1, if  $\beta_2$  was significantly positive. This meant that the forward rate, at time t, had the power to predict the one month spot rate. Under the Pure Expectations Hypothesis the  $\beta_l$  coefficient should be 0 and  $\beta_2$  should be 1. Fama found no evidence for autocorrelation in his models.

A further OLS regression of the change in the one month spot rate  $s_{t+2} - s_{t+1}$  on the forward spot differential  $f2_t - s_{t+1}$  indicated that the one month forward rate f2, observed at t, had power as a forecast of the one month spot rate  $s_{t+2}$ , as the slope coefficient was significantly different from zero in this regression. As already stated, Fama found no evidence for autocorrelation in his data and hence found no reason against the use of OLS estimation techniques.

The early work using OLS is replicated with the new data obtained from *Consensus Economics*. The two equations (a) and (b) are also used to test for Speculative Efficiency in the interest rate market. The results of this regression will be used to highlight the problems associated with using this technique. Moreover, the resultant estimation coefficient will be used as a basis for comparison with results from other techniques.

The results from the two OLS regressions are as follows:

(a) 
$$s_{t+3} = 0.27080 + 0.92739f_t + \varepsilon_{t+3}$$
 D.W. = 0.4628

(b) 
$$s_{t+3}-f_t = -0.23222 + 0.12234(s_t-f_{t-3}) - 0.06395 + \varepsilon_{t+3}$$
 D.W. = 0.5468

The Durbin Watson statistics indicate that these models are misspecified. Before this problem is addressed though, a test of the Speculative Efficiency of the forecasts is undertaken to show how the results are affected when the underlying model assumptions are not in line with the data (as was customary in the papers published in the 1970's and early 1980's). The hypothesis of Speculative Efficiency states that the coefficients in both

regressions should be  $\alpha_0=0$  and  $\alpha_1=1$ , or  $\alpha_0=\alpha_1=\alpha_2=0$ , respectively. The joint test is rejected for model (a) with the test statistic at 3.82 exceeding the critical value of 3.11. The null hypothesis of efficiency is not rejected for equation (b) with the test statistic, 1.84, lying below the critical value of 2.72. The results of these joint tests are displayed once again in tables 2.5 and 2.6 and are followed by a more detailed discussion of the ensuing results with reference to the other modelling techniques utilised later on in this chapter.

Intuitively, the reason for the misspecification, as highlighted in the Durbin Watson statistic, lies in the nature of the data. That is the data are overlapping as the forecasters are predicting three months ahead but reporting monthly which means that two months of information becomes available before the next forecast is required. This results in serial correlation in the errors which will bias the OLS standard errors downward and hence the t-statistics will be overestimated. A number of ways to deal with this issue have been introduced in the literature. These shall now be discussed.

#### 2.3 Sampling

One way of dealing with the overlapping data problem, whilst still using OLS, was discussed in Canarella and Polland (1986). Here they sampled out every third observation which resulted in three sub samples of non-overlapping data. As previously mentioned, although they too were concerned with the issue of Speculative Efficiency, they sought to model spot and future prices for metals traded on the London Metal Exchange.

The sub-samples in this thesis can be estimated using OLS, as this method reduces the sample size to roughly 26 data points so as not to lose too many degrees of freedom, only model (1),  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$  will be fitted to the data. For evidence speculative efficiency we would expect  $\alpha_0$  to equal zero and  $\alpha_1$  to equal one. This hypothesis can be verified via a joint test, the F-test. See table 2.1.

**Table 2.1** Sampling Regression Results for U.S. Data

	α <sub>0</sub>	$\alpha_1$	R <sup>2</sup>	D-W	F/Test
Sample 1	0.28	0.92	0.90	1.22	1.28
	$(0.32)^{(i)}$	(0.06)			(3.40)
Sample 2	0.35	0.92	0.92	1.39	1.08
	(0.30)	(0.06)			(3.40)
Sample 3	0.19	0.93	0.90	1.38	1.55
	(0.33)	(0.06)			(3.40)

(i) Data in parenthesis are standard errors

The coefficients are not only similar in each sub-sample, but are also similar to the original OLS regression of the full period. However the joint test, the F test, indicates that the speculative efficiency hypothesis is now not rejected in each sub-sample as the 5% critical value is 3.40. The Durbin Watson statistic bounds, at 1% significance, are (1.06, 1.21). This indicates that apparently the autocorrelation problem is not present here. However, due to limited data, which results in information loss and an accompanying loss of efficiency, this sampling method is not the optimal solution.

#### 2.4 Seemingly Unrelated Regression (SUR)

The sub-sample three equations used in the analysis in table 2.1 will now be estimated using the SUR technique. Using the sampling technique outlined above, three data sets result of the form  $(x_1, x_4, ...), (x_2, x_5, ...), (x_3, x_6, ...)$ . Hence the correlation that is exploited by the SUR technique is due to serial correlation in the disturbances. This is captured by the sampling procedure that realigned the data. The SUR technique allows fro a more general covariance structure than that of the GLS or the OLS with MA(2) errors. By comparing the standard errors of these regressions, it can be seen whether the three sub-sample are yielding similar results, and are representative of the whole data set.

**Table 2.2** Seemingly Unrelated Regression Results for U.S. Data

	$\alpha_0$	$\alpha_1$	R <sup>2</sup>	D-W
Sample 1	0.40	0.90	0.90	1.18
	(0.32)	(0.06)		
Sample 2	0.58	0.87	0.91	1.28
	(0.29)	(0.06)		
Sample 3	0.51	0.87	0.90	1.24
	(0.32)	(0.06)		

(i) Data in parenthesis are standard errors

The Durbin Watson upper and lower bounds for one regressor and 25 observations is (1.06,1.21). The first sample lies within the uncertain region. However, the standard errors reported for these regressions are very similar to those of the OLS standard errors which indicates that either there is such a small level of correlation in the error terms so as to not bias the standard errors, or there is no correlation at all. Alternatively, there is a close relationship between the SUR technique with constrained slopes and GLS specified with MA(2) errors. If the MA parameter estimates are highly significant, then it is possible that the similarity in results is as a result of there being strong correlation between errors but little efficiency gain, as the regressors in each of the three equations are highly correlated.

Imposing the restriction that all the slope coefficients are equal and all the intercept terms are equal, the SUR estimated coefficients become;

Table 2.3 Restricted SUR Results For U.S. Data

	$\alpha_0$	$\alpha_1$	$\mathbb{R}^2$	D-W
Sample 1	0.46	0.89	0.90	1.16
	(0.27)	(0.05)		
Sample 2	0.46	0.89	0.91	1.31
	(0.27)	(0.05)		
Sample 3	0.46	0.89	0.90	1.27
	(0.27)	(0.05)		

(i) Data in parenthesis are standard errors

This is the most efficient estimate available using this method as it makes use of all available data and enforces only one estimate of the slope and intercept variable. As opposed to six different coefficients when in reality there should only be two. To test whether this restriction is valid the likelihood ratio test is undertaken. This involves comparing the log likelihood's (LL) of the restricted and unrestricted system of equations via the formula; 2\*(LL) Unrestricted – LL Restricted). This comes to 4.203 and is distributed as a  $\chi^2$ , with the degrees of freedom equivalent to the number of restrictions imposed. There are 4 restrictions in this case, with the constant term and slope coefficients restricted in each equation. Hence the critical value is 9.49, indicating that the null cannot be rejected. This indicates that the model is stable across the three data sub-samples.

To test the Speculative Efficiency hypothesis in this system of equations, the assumption that all intercept terms are zero and all slope coefficients are one, is presumed to be true, and the equations are restricted likewise. The results are recorded in table 2.4.

Table 2.4 Restricted SUR Results For U.S. Data

	α	$\alpha_1$	$\mathbb{R}^2$	D-W
Sample 1	-0.00	1.00	0.90	1.18
	(0.00)	(0.00)		
Sample 2	-0.00	1.00	0.91	1.38
Sample 3	-0.00	1.00	0.89	1.32

<sup>(</sup>i) Data in parenthesis are standard errors

Once again the likelihood ratio test was performed yielding a statistic of 8.4402. The critical value is, once again, 9.49, indicating that the null cannot be rejected. This is in line with the single equation OLS result.

### 2.5 Moving Average errors

The overlapping data problem can also be dealt with, by correcting for the specific form of the autocorrelation. In this case, the nature of the data indicates that the errors follow a second order Moving Average (MA) process due to the presence of new information that becomes

available within the contract interval. The finding of serially correlated errors and the subsequent correction for it was the next step in the development of the interest rate literature. Gregory and Voss (1991) addressed certain problems with interest rate data techniques overlooked by Friedman in his 1979 paper. The authors considered three models (Gregory and Voss, 1991, p.926),

$$(1) s_{1t+1} - i_{1t} = a + \varepsilon_{t+1}$$

(2) 
$$s_{1t+1} - i_{1t} = a + b(i_{1t} - s_{1t}) + \varepsilon_{t+1}$$

(3) 
$$s_{lt+1} - i_{lt} = a + b(s_{lt} - i_{lt-1}) + \varepsilon_{t+1}$$

Where  $s_t$  is the spot rate and  $i_t$  the forward interest rate. They found fourth order serial correlation in all equations with the use of Godfrey's 1978 LM test. Gregory and Voss find fourth order serial correlation, as opposed to the usual second order, due to their time interval of a quarter.

Froot (1989), in his analysis of the Expectations Hypothesis of the Term Structure of Interest Rates, used survey data ranging from 1969-1986 which consisted of financial market participants expectations of 3 month Treasury Bills and Eurodollar deposits; 12 month Treasury Bills and the Bond Buyer Index and the 30 year mortgage rate. The respondents were required to post expectations of this rate for 3-6 months.

Using this data and applying the regression equation;

$$i_{t+j} - i_t = \alpha + \beta f p_t + \eta_{t+j}$$

where fp was the forward premium  $fp_t=f_t-i_t$ , or in words, the forward rate less the short rate (i) (Froot, 1989, p.286), Froot noted that the Expectations Hypothesis and Rational Expectations together, would imply that the forward premium was an efficient forecaster of the future interest rate changes, ie,  $\alpha=0$  and  $\beta=1$ . He also noted that previous studies had found the estimate of  $\beta$  statistically less than one, and for shorter maturities not significantly different from zero, meaning that the forward premium was of no help in forecasting future changes in the short rate. Froot made allowances for an MA(1) process at the 6 month forecast horizon, he also made use of Hansen and Hodricks GMM, to take care of the overlapping data problem, which in this thesis will be looked into in more detail later. Nevertheless the more general error structure did not alter the finding of no evidence to support the null hypothesis. Hence Froot ran the regression  $s_{t+i} = \alpha_2 + \beta_2 fp_t + \varepsilon_t$  where the null hypothesis was that

 $\alpha_2=0$  and  $\beta_2=1$  and the error term was random (Froot, 1989, p.292). If  $\beta_2=1$  then he could not reject the hypothesis that the variance in the term premium was zero as the probability limit of  $\beta_2$  is equal to  $1+\beta_{tp}$ , where  $\beta_{tp}$  is the time varying parameter. To construct the standard errors for this regression, the author used the covariance matrix estimator suggested by Newey and West (1987) which is appropriate for hypothesis testing when the disturbances are autocorrelated and the estimator used is GMM. For instruments with a duration of one year or less,  $\beta_2$  was statistically less than one, and hence once again the expectations hypothesis failed. That is, Froot found evidence of a statistically significant time varying premium.

With regards to the literature which sought to test whether expectations are biased and which utilised the Moving Average error process to curb the overlapping data problem, MacDonald (1990) found that the observational frequency of his data was less than the forecast horizon, and hence there was a moving average error structure in the data. This is in line with the problems associated with the Consensus Survey data. To ascertain whether this error structure adequately fits the US data the OLS regressions of (a) and (b) were run, and the errors were saved. The autocorrelation and partial autocorrelation functions of these errors were then graphed. This method of testing for determining the error structure was used as it gives a clear indication as to whether the errors follow a moving average or autoregressive function. In this case the autocorrelation functions both exhibited significant spikes for the first two lags, whilst the partial autocorrrelation functions, declined exponentially. This confirms the intuitive assumption that the errors follow a MA(2) process. Equations (a) and (b) are therefore re-estimated taking into account the MA error process. The estimated coefficients were as follows;

(a) 
$$s_{t+3} = 0.19302 + 0.94943f_t + 0.678\epsilon_{t-1} + 0.5286\epsilon_{t-2}$$
 D.W. =1.62  
(b)  $s_{t+3} - f_t = -0.27857 + 0.08512(s_t - f_{t-3}) - 0.03865 + 0.881\epsilon_{t-1} + 0.5754\epsilon_{t-2}$  D.W. =1.92

The standard errors are recorded in tables 2.5 and 2.6. On both models the moving average coefficients are highly significant. Once again the joint tests were undertaken and in this case both equations failed to reject the null hypotheses of Unbiasedness and Efficiency. Equation (a) yielded a joint test statistic of 0.90, well below the critical value of 3.11. Whilst the

efficiency test statistic obtained from equation (b) was 1.09 which was below the critical value of 2.72. Tables 2.5 and 2.6. replicate these findings but with the inclusion of the standard errors. A comparison of findings follows these tables.

#### 2.6 Generalised Method of Moments (GMM)

A popular technique frequently used in financial literature is the Generalised Methods of Moments. This technique deals with the overlapping data problem found when using survey data of the same nature of that which is used in this thesis. Hansen and Hoderick (1980) proposed this technique which estimates the coefficients consistently with OLS procedures, but is able to use the entire data set. The Generalised Method of Moments procedure used in this thesis modifies the standard errors to provide a consistent estimate of the coefficient variance-covariance matrix. With respect to the Expectations Theory of the Term Structure Literature, this technique was first discussed by Shiller, Campbell and Schoenholtz (1983).

Shiller et al. assumed that expectations were rational, which implied that the error terms were uncorrelated in a regression of  $s_{t+3} - s_t$  on a constant and  $i_t - s_t$  where i was the forward rate and s the spot. Shiller et al. interpreted the intercept as a constant Risk Premium. Once again the hypothesis implied that the slope coefficient was 1. Shiller et al. adopted the GMM which allowed them to use the full sample of monthly data, by correcting the error term for serial correlation. They reduced the sample period to eliminate the effect of the introduction of new Federal Reserve operating procedures. Recent observations were also dropped due to their highly influential nature. However the slope coefficient remained insignificantly different from zero.

Tease (1988) applied the Hansen and Hodrick technique to weekly overlapping data on Australian interest rates. Tease noted that if the expectations theory was valid, and expectations were formed rationally then the forward rates, implicit in the yield curve, should be unbiased predictors of the future spot interest rates. In other words, in the simple linear regression  $s_{t+1} = \alpha + \beta i_{t+1} + v_{t+1}$ ,  $\alpha = 0$  and  $\beta = 1$  (Tease, 1988, p.122). Although similar to regression (a) in this thesis, this paper once again dealt with forward rates (i). Interestingly Tease did not make reference to the fact that implicit in the testing of the Pure Expectations

Hypothesis, is the assumption of Rational Expectations. Tease found, for the Australian data, that the joint restriction  $\alpha=0$  and  $\beta=1$  could not be rejected for the period 4 January 1980 to 21 March 1986, however for the period 16 December 1983 to 21 March 1986 this was not the case. Testing the less restrictive criterion, namely  $\beta=1$ , resulted in the expectations criteria not being able to be rejected.

Tease indicated that there was evidence of instability with respect to the short-term interest rates being serially correlated, which would bias the estimate of  $\beta$  towards one. To eliminate this possible problem, Tease decided to subtract  $s_t$  from both sides. Hence a test of the predictive performance of the forward spot rate differential was undertaken. The results indicated that prior to December 1983, pre float of the Australian dollar, the strong expectations hypothesis was rejected, however this was not the case in the full sample, or post float period. Therefore, Tease claimed that Australian data supported the expectations hypothesis particularly in the presence of a floating exchange rate, which went against previous papers findings.

MacDonald and MacMillan, (1994) sought to model data from the same survey (*Consensus Economics*) as utilised in this thesis in order to test the validity of the Pure Expectations Hypothesis. However the data was from a different time period and MacDonald and MacMillan decided to construct forward rates. Once again the link between expectations being formed rationally and the Pure Expectations Hypothesis was made. That is, if expectations are formed rationally then the authors claimed that  $s_{t+k} - s_t = s_{t+k}^e + \varepsilon_{t+k}$  and a regression equation

$$s_{t+3} - s_t = \alpha_2 + \beta_2(i_t - s_t) + v_t \tag{1}$$

could be estimated, where i is the forward rate. So the test of  $\alpha=0$  and  $\beta=1$  became a joint test of the Pure Expectations Hypothesis and the assumption that agents formed their expectations rationally.

MacDonald and MacMillan noted that equation (1) could also be written as

$$s_{t+3}^s - s_t = \alpha_4 + \beta_4(i_t - s_t) + v_{t,4}$$
 or

$$s_{t+3} - s_{t+3}^s = \alpha_5 + \beta_5 (i_t - s_t) + v_{t,5}$$
 or

$$i_t - s_{t+3}^s = \alpha_3 + \beta_3(i_t - s_t) + v_{t,3}$$

where s<sup>s</sup> was the survey consensus measure of expectations. They concluded that if  $\beta_3=0$  or  $\beta_4=1$  then the Expectations Hypothesis, allowing for a Term Premium would not be rejected, and, if  $\alpha_5=\beta_5=0$ , then the hypothesis that agents do not make systematic forecast errors could not be rejected.

Due to overlapping data, the GMM estimation method was used. However there was mention that, due to the cross sectional nature of the data, it would be more efficient to use feasible GLS which amounted to a SURE-GMM estimator. The authors noted that this would improve efficiency if the independent variable was different for each panel member. As this was not the case, the SURE-GMM estimator would be equivalent to the GMM estimator and hence the simple GMM was used (MacDonald and MacMillan, 1994, p.1075). In conclusion, MacDonald and MacMillan found that the pooled data could not support the Pure Expectations Hypothesis and rationality.

With respect to the literature which tried to test the assumption of rationality expressly, MacDonald (1990) found evidence of MA(2) errors in the exchange rate data and in order to rectify this situation, used the GMM. De Bondt and Bange (1992) in an attempt to test the efficiency of forecasters, used consensus data (a mean of 52 individual predictors) and Hansen and Hodricks GMM to take care of overlapping data in the estimation of  $\pi_{t,k} = \alpha + \beta \pi_{t,k}^e + \mu_{t,k}$  where  $\pi_{t,k}^e$  was the forecast at time t of the inflation rate between the months of t and t+k.

De Bondt and Bange set out various equations which sought to model these expectations. They were all based on the equation  $\pi^e_{t,k-1} = \theta z + (1-\theta)\pi_{t-r,k}$  (1) where z represents either a twice lagged inflation rate (which results in extrapolative expectations) the lagged forecast  $\pi^e_{t-k,k}$  (adaptive expectations), or a constant equilibrium rate of inflation  $\pi$  (regressive expectations). For a random walk process to result from this equation, the restriction  $\theta$ =0 has to be applied. The authors also noted that by subtracting  $\pi_{t-k,k}$ , the most recently observed inflation rate, from both sides of the above equation under each expectations regime, three equations could be estimated. Furthermore, by subtracting  $\pi_{t,k}$ , the actual inflation rate, from

one, yields three more equations with the inflation forecast error as a dependent variable (DeBondt and Bange, 1992, p.483).

This paper also made use of Hansen and Hodrick's technique (GMM) for the estimation of (1) due to the overlapping data problem. To test for the presence of Rational Expectations all coefficients in the above equation should equal zero. The results from the joint hypothesis,  $\alpha$ =0 and  $\beta$ =1, indicated that the null hypothesis could not be rejected and the presence of positive intercepts indicated that that the forecasters were systematically too low. With respect to the Rationality tests, the null of all coefficients being zero was rejected and there was evidence that the surveyed agents gave too much weight to inflation in the distant past. Also the presence of serial correlation between adjacent forecast errors pointed to expectations being insufficiently adaptive.

Recalling that one of the purposes of this thesis is to compare and contrast the estimation techniques utilised by related literature, the GMM technique is used to model equations (a) and (b). Due to the presence of MA(2) errors the weighting matrix is that which Newey and West (1987) proposed. Table 2.5 contains the estimated coefficients for the three techniques used in the estimation of equations (a) whilst table 2.6 displays the corresponding coefficients for equation (b). They also contain the tests for Unbiasedness (table 2.5) and Efficiency (table 2.6). These table enables a comparison of the techniques to be made.

Table 2.5 Estimation of  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$  and Tests for Unbiasedness

	α <sub>0</sub>	$\alpha_1$	Errors	Errors	F/ Wald Test
			$\theta_1$	$\theta_2$	$\alpha_0=0$ $\alpha_1=1$
GMM	-0.27	0.93			5.43
	$(0.20)^{(i)}$	(0.04)			(5.99) (ii)
OLS	0.27	0.93			3.82
	(0.18)	(0.03)			(3.11) (iii)
OLS	0.19	0.95	0.67	0.53	0.90
+MA(2)	(0.25)	(0.05)	(0.10)	(0.10)	(3.11)

<sup>(</sup>i) Data in parenthesis are standard errors

<sup>(</sup>ii) Data in parenthesis is the Chi squared 5% critical value with 2 Degrees of freedom

<sup>(</sup>iii) Data in parenthesis is the F-test 5% critical value with 2 and 80 Degrees of freedom

Table 2.6 Estimation of  $s_{t+3}$ - $f_t$ =  $\alpha_0 + \alpha_1 (s_1$ - $f_{t-3}) + \alpha_2 (s_{t-1}$ - $f_{t-4}) + \varepsilon_{t+3}$  and Efficiency Test Results

	α <sub>0</sub>	$\alpha_1$	$\alpha_2$	Errors	Errors	F/Wald Test	F/ Wald Test
				$\theta_1$	$\theta_2$	$\alpha_1=\alpha_2=0$	$\alpha_0=\alpha_1=\alpha_2=0$
GMM	-0.20	0.14	-0.07			4.82	9.78
	$(0.11)^{(i)}$	(0.07)	(0.06)			(5.99) <sup>(ii)</sup>	$(7.82)^{(iv)}$
OLS	-0.23	0.12	-0.06			0.41	1.84
	(0.11)	(0.14)	(0.12)			(3.11) <sup>(iii)</sup>	$(2.72)^{(V)}$
GLS	-0.28	0.08	-0.04	0.88	0.58	0.78	1.09
	(0.19)	(0.07)	(0.08)	(0.10)	(0.18)	(3.11)	(2.72)

The first column of tests relates to the weaker hypothesis which allows for a constant term. The second column assumes that forecast errors are purely random

- (i) Data parenthesis are standard errors
- (ii) Data in parenthesis is the Chi squared 5% critical value with 2 Degrees of freedom
- (iii) Data in parenthesis is the F-test 5% critical value with 2 and 80 Degrees of freedom
- (Iv) Data in parenthesis is the Chi squared 5% critical value with 3 Degrees of freedom
- (v) Data in parenthesis is the F-test critical value with 3 Degrees of freedom

The GMM estimates are very similar to the OLS estimates which is to be expected as OLS estimation are embedded in the GMM framework as a special case. If the system is just identified then OLS estimates are GMM estimates. The GMM estimation technique used in this thesis does alter the covariance matrix of the estimated coefficients so the coefficients are not going to be identical to the OLS coefficients. Comparing coefficients, firstly, with respect to the Unbiasedness Regressions, table 2.5. The GMM coefficient for the constant term is negative but insignificant. OLS and OLS correcting for MA errors, GLS, have a positive intercept term but, they too, are insignificant. The slope coefficients are all extremely significant and close to one. For the GMM and OLS regression they deviate from one by only 0.07. For the moving average regression this is reduced to 0.05. It would be expected in light of these findings that these regressions would accept the null of Unbiasedness. This is not the case in the OLS model, which indicates that the constant term has a significantly large t- ratio (small standard error) to warrant it not being considered zero. This could be due to the serial correlation in the error term which would inflate the t-statistics. This appears to be the case, as the regression corrected for MA errors rectifies the problem and finds evidence to support the Unbiasedness of forecasters.

For efficiency, estimation of equation (b) indicates that for the GMM, forecaster's present errors cannot be explained by their past errors. Results from the GLS indicate that the previous forecast error has some weight in the presence of the present forecast error. For the GMM and GLS regressions, the constant terms have a negative effect on the forecast error which would imply that the expected spot rate was overestimated. As well, the lagged forecast error was negative meaning that as more information became available, agents revised their estimates downward. However, this variable was not significant.

In summary, a review of key interest rate literature was undertaken in this chapter. What was of particular interest in this literature was the estimation technique utilised by the author(s) in their analysis. These techniques were then used in the estimation of the *Consensus Economics* data and their results compared. The results indicated that both OLS estimates confer with the Speculative Efficiency hypothesis, but the GMM estimates result in a Wald statistic that exceeds the critical value, and hence the null hypothesis of forecaster efficiency cannot be accepted. This is only true for the strong form of the argument,  $\alpha_1 = \alpha_2 = \alpha_3 = 0$ , column eight in table 2.6. The weak argument  $\alpha_1 = \alpha_2 = 0$ , column seven, is accepted in all cases. The MA estimates find efficiency when lagged forecast errors are included, but in the simple model they find consistent bias in the forecasters predictions.

Although the GMM and GLS regressions should yield equivalent results as they both correct for the overlapping data problem, the GMM and OLS estimates did not support the hypothesis of Unbiasedness, whilst the GLS regression did. The discrepancy in these results could be explained by the fact that one important assumption underlying the GMM estimation technique, is that the data is strictly stationary (Ogaki, 1992, p.12). So a test of the data needs to be undertaken before these results can be relied upon. This is done via the Dickey Fuller unit root test in the next chapter. Once again for simplicity the U.S. data analysed first with a summary of equivalent tests and procedures for the other seven countries reported in chapter 4.

### **Chapter 3**

# **Implications of Non-Stationarity & Unit Roots**

The papers by Gregory and Voss, De Bondt and Bange, Froot, MacDonald, Shiller et al. and Tease, summarised in the previous chapter, addressed the issue of overlapping data and serial correlation in the error terms. This they achieved either by imposing a moving average structure or using OLS, but making appropriate modifications in the estimated asymptotic covariance matrix, that is, they utilised the GMM technique. Both of these methods will eliminate the problems associated with OLS estimates which bias the standard errors downwards. However, as stated in Hansen and Hodrick, (1980) one of the key assumptions needed to justify the GMM procedure, is that the time series are jointly stationary. Furthermore, as mentioned by MacDonald (1990), a precondition to any test of the efficient market hypothesis is that the variables are covariance stationary. Therefore tests of the stationarity of the interest rates are required in order to determine whether these techniques are best suited to interest rate data, or whether the data should be modelled in a nonstationary or cointegrating framework. However, it should be noted that interest rates cannot actually be nonstationary as this lack of certainty would damage the economy and persistence is needed in interest rates in order for them to permeate throughout the economy and hence be effective economic tools.

Nourzad and Grennier (1995) found their interest rate time series exhibited nonstationarity and the implications of this finding was outlined in their paper. It can be recalled that the expectations theory, implies, that the Term Premium is zero and hence short-term bonds yield the same expected return as long-term bonds. The alternative hypothesis is that lenders require a Premium as compensation for the risk of holding long-term financial assets.

Nourzad and Grennier stated that the empirical literature put forward by Cook and Hahn (1990), Dua (1991), Fama (1990), Freidman (1979), Kane (1983) Mankiw and Miron (1986) and Van Horne (1965) found little support for the Pure Expectations Theory. These articles tested this hypothesis, using the single equation approach and hence, as Nourzad and Grennier pointed out, neglected to take into account the interaction between the forward and expected spot rates. Moreover, these previous studies did not distinguish between the short-

run disequilibrium dynamics and long-run equilibrium relationships between forward rates and expected spot rates.

The most important issue raised in Nourzad and Grennier's paper was the fact that the variables used to test the Expectations Hypothesis may not be stationary and therefore the empirical results may be spurious. Nourzad and Grennier therefore re-examined the Expectations Hypothesis of the term structure using a Maximum Likelihood approach to estimation and Johansen's inference on cointegration and error-correction. They believed that their methodology was particularly useful in light of the fact that "the Expectation Hypothesis is essentially a long-run equilibrium proposition, that allows for short-run deviations of the forward rate from its equilibrium relation with the expected spot rate" (Nourzad and Grennier, 1995, p.282).

In discussing the previous analysis of this problem, Nourzad and Grennier noted that by defining the Term Premium as the difference between the forward rate and the corresponding expected future spot rate, one has to adopt a hypothesis on the formation of expectations. Then one has to test the Expectations Hypothesis of the term structure and the hypothesis of the expectation formation jointly. The joint null hypothesis will be rejected when either the hypothesis governing the formation of expectations is not consistent with the data, or the Expectation Hypothesis of the term structure is rejected, or both. This leads to the problem that the test cannot distinguish between the three possible causes of the rejection. Nourzad and Grennier suggested that a solution to this problem was to use survey data on expectations of future interest rates, so that the test equation no longer depends on the hypothesis governing the formation of expectations.

In their paper the authors found, by recalling the simple regression equation

$$i_r = \mu + \theta f_3 + \vartheta_r$$

that a test of the pure expectations hypothesis assumes  $\theta=1$  and  $\mu=0$  (Nourzad and Grennier, 1995, p.284). Where  $f_t$  is the expected future spot rate three months ahead and  $i_t$  is the forward rate. The strong form of this hypothesis predicts that the constant term in the above equation, which represents the Term Premium, is not zero. The weak form of the hypothesis suggests that the Term Premium varies over time. In a criticism of the standard approach to testing the Expectations Hypothesis, the authors claimed that interactions between the forward rate and

the expected spot rate were ignored, and a distinction between the short-run deviations of this rate from the long-run equilibrium relationship between them was not made. Furthermore, results from this regression may be misleading if the variables are integrated and if they are cointegrated, merely differencing them, will not suffice. Instead an error-correction model would be a more appropriate specification for modelling the process. Before undertaking these tests Nourzad and Grennier also pointed out that as they were using a survey based measure of the expected future interest rate they needed to test whether expectations were unbiased. This was achieved by regressing the spot rate, on the expected spot rate, and undertaking the joint hypothesis of a zero constant term and a coefficient on the expected spot rate insignificantly different from 1. That is, the regression was run and the joint test of  $\Theta_0 = 0$ ,  $\Theta_1 = 1$  was undertaken.

The joint test was accepted at high levels of confidence.

$$s_{t+3} = \Theta_0 + \Theta_1 f_t + v_T$$

Using the Augmented Dickey Fuller unit root test to test for nonstationarity in the data and testing down from two roots, Nourzad and Grennier (1995) found that the forward (f) and expected future spot rate (s) were I(1). That is, the nonstationarity could be removed by first differencing the variables. Hence they moved on to testing for cointegration with the use of Johansens (1988) cointegration test. Both the Trace and Maximal Eigenvalue tests found support for the hypothesis that there was at least one cointegrating vector. Hence the authors tested the Expectations Hypothesis using the estimate of the cointegrating eigenvector. The results published were that the estimated eigenvectors were 1.000, -1.141, and -0.727. These coefficients are negative as they were reported as they would appear on the left-hand side of the estimated equation. That is, the regression equation was  $i_t = 0.727 + 1.14f_t$ . The authors stated that they performed a joint test on these variables and found that the individual significance of the intercept term, Risk Premium, more than offset the fact that the slope coefficient was not statistically different from 1. In other words the hypothesis is rejected and the Term Premium was not zero. However, unlike other authors they claimed that this was consistent with the strong form of the Expectations Hypothesis. (Nourzad and Grennier, 1995, p.291).

Combining the findings above and the previous articles concerning the use of weekly data Choi and Wohar (1991) offer a concise examination of the topic in the presence of nonstationary time series. They used econometric techniques which had previously not been used to test the Expectations Theory, and they claimed that their results were subsequently in contrast to prior work. Evidence of the predictive power of the yield curve was tested using weekly and monthly overlapping data and quarterly non-overlapping data. The employment of weekly and monthly data supposedly led to results that contrasted previous authors, in that they support the weak form of the Expectations Hypothesis in which the yield curve has substantive predictive power. Choi and Wohar also considered the possibility of a long run relationship between the three and six month rates by employing cointegration tests.

The authors concentrated on the version of the Expectations Hypothesis that postulates that, the expected return from investing in an n-period bond should equal the expected return from investing in a one period bond over n successive periods. This coupled with Rational Expectations, yielded the following estimating equation

$$i_{t+1} - i_t = \alpha_0 + \pi (R_t - i_t) + e_{t+1}$$

where  $R_t$  and  $i_t$  are per-period yields on a two period and one period bond in period t respectively (Choi and Wohar, 1991, p.85). For this equation, the authors noted that if the Term Structure had predictive power then  $\pi$  should be significantly different from zero. The primary concern of Choi and Wohar's paper was the predictive power of the yield curve. The efficient use of information was only briefly considered.

Choi and Wohar mentioned that overlapping data results in serially correlated error terms. The weekly data followed a MA(12) process whilst the monthly data followed a MA(2) process. This was because of the fact that the errors realised in weeks t+1 through to t+12 were not in the information sets of the agents, at week t, whose forecast horizon was week t+13. Citing Hansen and Hodrick (1980), which noted that using GLS will result in inconsistent estimators, Choi and Wohar used the GMM process which was described by Hansen and Hodrick. As well, due to the volatility of interest rates, the error terms were considered heteroscedastic. A Newey and West procedure to produce a consistent variance - covariance matrix was also used. This involved obtaining a consistent estimator of the long run variance using the following equation

$$S_{Tl}^{2} = T^{-1} \sum_{t=1}^{T} \hat{u}_{t}^{2} + 2T^{-1} \sum_{j=1}^{l} w_{l}(j) \sum_{t=j+1}^{T} \hat{u}_{t} \hat{u}_{t-j} \text{ where } w_{l}(j) = 1 - j(l+1)^{-1}.$$

Choi and Wohar drew on the findings of Engle and Granger (1987) and stated that if the long and short interest rates had stochastic trends, and moved together, then the short and long

rates should be cointegrated. This proposition could be assessed by testing whether the estimated residuals, from a regression of the long rate on the short rate, had a unit root. If the residuals did not have a unit root, while each of the regressors did, then the null hypothesis of no cointegration was rejected in favour of the variables being cointegrated. Dickey Fuller and Augmented Dickey Fuller tests were employed to determine the existence of unit roots.

If they found evidence of a cointegrating relationship between the long and the short rate then Choi and Wohar proposed the error-correction model;

$$(i_t - i_{t-1}) = B_1(L)(R_t - R_{t-1}) + dz_{t-1} + w_t$$

Where  $B_1(L)$  and  $B_2(L)$  are polynomials and L was the lag operator (Choi and Wohar, 1991, p.88). Choi and Wohar noted that they were looking for a significant d parameter estimate which supported the premise of a long run relationship between the interest rates.

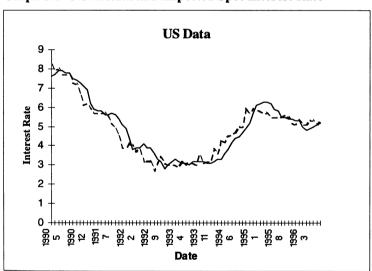
As far as the results were concerned, using weekly and monthly data, the authors found evidence to support the expectations hypothesis for the period 1910-1914. However, for the periods after the founding of the Federal reserve, the strong expectations hypothesis could not be supported, yet the weaker version could for 1919-1933 and 1959-1978. Quarterly data yielded results consistent with Mankiw and Miron, that is the Expectations Hypothesis was rejected, except for the period 1934-1959 when the Federal reserve was pegging interest rates. Interestingly when weekly and monthly data is used Mankiw and Miron's results could not be supported.

Choi and Wohar stated that Dickey Fuller tests indicated that the three and six month rates possessed a Unit Root. As well, there was evidence of a cointegrating relationship between these variables for all four sub-periods, 1910-1914, 1919-1933, 1959-1978 and 1979-1989. Using the Engle and Granger (1987) finding that: if cointegration exists then short-term fluctuations in the deviation of short rates from their long-run value are stationary and actual changes in the short rate could satisfy an error-correction mechanism. Choi and Wohar fitted their error-correction model to each of the four data periods. Analysing these error-correction models indicated that the coefficient on the error-correction term, z, were significantly different from zero for all sub-periods, except 1979-1989 (Choi and Wohar, 1991, p.88-90). This added more weight to the premise that there was a long run relationship between the three and six month rates. The authors noted that the error-correction coefficient was small in

value, which indicated that once shocked the spread only gradually returned to equilibrium. This model also indicated that the spread between the long and short rates has explanatory power for predicting changes in short-term interest rates.

Encompassing the recent literature findings of unit roots and cointegrating relationships in interest rate data, an Augmented Dickey Fuller Unit Root test on the spot and expected future spot rate was undertaken for the U.S. *Consensus Economics* data. The absolute value of the test statistic for a constant and no trend in the data was 2.15 for the spot rate and 2.52 for the expected future spot rate both of which do not exceed the critical value of 3.13. Hence, the null of a unit root is accepted.

Recalling the graph of the data in chapter 1, the US data exhibited nonstationary behaviour, in that the data did not seem to be oscillating around a mean, and the fact that both the spot and the expected spot rate moved together, indicates that these variables are cointegrated.



Graph 3.1 U.S. Actual and Expected Spot Interest Rate

Testing this intuitive assumption statistically, Johansen's Maximum Likelihood procedure was used to test for a cointegrating relationship.

Unit Roots	Max Root	Critical Values	Trace Test	Critical Values
		5%		5%
n-r=2	64.48	14.90	58.38	17.95
n-r=1	5.89	8.18	5.33	8.18

As the above results indicate the hypothesis that there are two unit roots (n-r)=2 can be rejected in favour of one unit root at the five percent level using both statistics. The hypothesis of one unit root (and therefore one cointegrating vector) cannot be rejected in favour of no unit root, hence there is evidence for a cointegrating vector.

A problem arises with this technique, as the Johansen approach is designed for vector autoregressive systems, and the errors in the spot and forward regression follow a moving average process. Phillips and Ouliaris (1990), provide a solution to this problem. They recommend the Phillips cointegrating test for models with positive MA errors. The Phillips test for cointegration regresses the errors, obtained from the original regression, as follows;

$$\hat{u}_t = \hat{o}\hat{u}_{t-1} + \hat{k}_t$$

This is followed by the computation of the test statistic which has a non-parametric correction  $\frac{T}{T}$ 

for correlation;

Where

$$\hat{z}_{\alpha} = T^{-1}(\hat{\alpha} - 1) - (\frac{1}{2})(s_{Tl}^2 - s_k^2)(T^{-2}\sum_{t=1}^{T}\hat{u}_{t-1}^2)^{-1}$$

$$s_k^2 = T^{-1}\sum_{t=1}^{T}\hat{k}_t^2$$

$$s_{Tl}^2 = T^{-1} \sum_{t=1}^{T} \hat{k}_t^2 + 2T^{-1} \sum_{t=1}^{l} w_{sl} \sum_{t=1}^{T} \hat{k}_t \hat{k}_{t-s}$$

For a choice of lag such as  $w_{sl}=1-s/(l+1)$ .

The Phillips test statistics, under the hypothesis of no cointegration and for any general specification of the error process, has a distribution which is the same as the Dickey Fuller test. The Phillips test has the additional benefit of being independent of nuisance parameters due to the non-parametric correction. As well the Phillips test is not affected by the presence of moving average errors. (Phillips and Ouliaris, 1991, p. 175)

The results from the Phillips test on the U.S. data is that the test statistic at -3.62 is below the critical value at -3.04 so once again there is evidence for cointegration.

It is evident that the U.S. data is cointegrated, therefore simple techniques used in chapter two are not valid. Two techniques, which are designed for cointegrating equations, are now estimated and the results recorded in tables 3.2 and 3.5. These techniques are Maximum Likelihood and Dynamic OLS (Phillips and Loretan, 1991, p.408).

### 3.1 Maximum Likelihood

An error-correction model (ECM) of the form;

$$\Delta f(t) = \mu_1 + \alpha_1 z_{t-3} + \Gamma_{11}(i) \Delta f(t-1) + \Gamma_{12} \Delta f(t-2) + \Gamma_{13} \Delta s_{t+3}(t-1) + \Gamma_{14} \Delta s_{t+3}(t-2) + \varepsilon_1$$

and

$$\Delta s_{t+3}(t) = \mu_2 + \alpha_2 z_{t-3} + \Gamma_{21} \Delta f(t-1) + \Gamma_{22} \Delta f(t-2) + \Gamma_{23} \Delta s_{t+3}(t-1) + \Gamma_{24} \Delta s_{t+3}(t-2) + \varepsilon_2$$

where 
$$z_{t-3} = \beta' y_{t-3}$$
 and  $y_{t-3} = \begin{bmatrix} s_{t+3-2} \\ f_{t-2} \end{bmatrix}$ .

was estimated using the maximum Likelihood procedure. As the data was monthly and overlapping with agents predicting three months ahead, a lag length of three was chosen.

**Table 3.2 Johansen Maximum Likelihood Estimation Of the ECM** 

	$\Delta f$	$\Delta s_{t+3}$
Constant	0.16	0.05
	$(0.02)^{(i)}$	(0.05)
Adjustment Matrix	-1.13	-0.50
	$(0.12)^1$	(0.27)
Short Run Parameter on $\Delta s_{t-1}$	0.04	-0.13
	(0.05)	(0.12)
Short Run Parameter on $\Delta s_{t-2}$	0.18	0.25
	(0.05)	(0.12)
Short Run Parameter on $\Delta f_{t-1}$	-0.28	-0.19
	(0.10)	(0.23)
Short Run Parameter on $\Delta f_{t-2}$	-0.28	-0.10
	(0.09)	(0.20)

<sup>(</sup>i) Data in parenthesis are standard errors

The estimated Eigenvector was 0.481 and -0.452 corresponding to a cointegrating relationship of  $0.481f_t$  - 0.452  $s_{t+3}$ . The Eigenvector  $\beta$  was taken from the Maximum Likelihood regression, normalised with respect to  $f_t$ ,, the expected future spot rate, and used in the estimation of the error-correction models. The coefficient on  $z_{t-3}$ ,  $\alpha$ , in table 3.3 is then the normalised adjustment parameter. The Adjustment parameters indicate what proportion of the disequilibrium, in each of the dependent variables, are corrected in the next period. Table 3.2 records the Eigenvectors, adjustment matrix and short run parameter estimates obtained from the error-correction model.

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<sup>&</sup>lt;sup>1</sup> This table displays the non-normalised eigenvectors  $\beta$  whereby the estimated equation is;  $\Delta f_t = \frac{\alpha}{\gamma} (f_t - \beta y_{t-3})$ 

**Table 3.3 Normalised Error-Correction Model** 

	Δf	$\Delta s_{t+3}$
Constant	0.02	0.22
	(0.03)	(0.02)
Adjustment Matrix, α	-0.09	-0.72
	(0.07)	(0.05)
Short Run Parameter on $\Delta s_{t-1}$	0.04	-0.13
	(0.05)	(0.12)
Short Run Parameter on $\Delta s_{t-2}$	0.18	0.25
	(0.05)	(0.12)
Short Run Parameter on $\Delta f_{t-1}$	-0.28	-0.19
	(0.10)	(0.23)
Short Run Parameter on $\Delta f_{t-2}$	-0.28	-0.10
	(0.09)	(0.20)

<sup>(</sup>i) Data in parenthesis are standard errors

This model indicates that the error-correction parameter was not utilised when predicting the future spot rate.

Once again a test for the hypothesis of Unbiasedness will be undertaken. This is done by imposing a linear restriction on the Beta coefficients and by using the likelihood ratio test ascertaining whether this restriction is valid (Johansen, 1988, p.236). The linear restriction in this case is that  $\beta = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ . The Likelihood ratio statistic yielded from this test is 8.817 which is greater than the critical value of 5.99. Hence the null hypothesis of Speculative Efficiency cannot be accepted.

#### 3.2 Dynamic OLS (DOLS)

The second estimation technique that will be used in the presence of cointegrating equations is DOLS. In the I(1) case with a single cointegrating vector, this method involves regressing one of the variables onto contemporaneous levels of the remaining variables, leads and lags of

their first differences and a constant (Stock and Watson, 1993, p.784). Thus giving the equation a dynamic structure. This estimation technique allows for the estimation of a stable coefficient on the forecast rate. However in order to use the estimates for testing purposes, the standard errors needs to be corrected via the Newey West procedure. This involves obtaining a consistent estimator of the long run variance using the following equation

$$S_{Tl}^{2} = T^{-1} \sum_{t=1}^{T} u_{t}^{2} + 2T^{-1} \sum_{j=1}^{l} w_{l}(j) \sum_{t=j+1}^{T} u_{t}^{2} u_{t-j} \text{ where } w_{l}(j) = 1 - j(l+1)^{-1}.$$

An application of this technique yielded the results which can be found in table 3.3. The Speculative Efficiency Hypothesis was tested using a Wald Test. This technique was originally discussed in Phillips and Loretan (1991)<sup>2</sup> however the name Dynamic OLS, or DOLS, comes from Stock and Watson (1993).

Phillips and Loretan assumed the generating mechanism for the variable  $y_t$  was the cointegrating system

$$y_{1t} = \beta' y_{2t} + \mu_{it}, \Delta y_{2t} = \mu_{2t}$$
 (1)

Phillips and Loretan showed that a single equation model could be built to estimate the cointegrating relationship. They achieved this by either running an OLS regression on the equation

$$y_{1t} = \beta' y_{2t} + d_1(L)(y_{1t} - \beta' y_{2t}) + d_2(L)' \Delta y_{2t} + \eta_t$$
 (2)

where

$$d_1(L) = b_{11}(L) - \sigma'_{21} \sum_{1}^{-1} b_{21} - 1 = \sum_{j=1}^{\infty} d_{1j} L^j$$

and

$$d_2(L) = b_{12}(L) - \sigma'_{21} \sum_{i=0}^{-1} B_{22}(L) = \sum_{i=0}^{\infty} d_{2j} L^i.$$

Supposing that the polynomials in (2) are of finite degree with

Then (2) becomes;  $d_1(L) = d_{11}(L), d_2(L) = d_{20} + d_{21}L$ 

$$y_{1t} = \beta' y_{2t} + d_{11}(y_{1t-1} - \beta' y_{2t-1}) + d_{20}\Delta y_{2t} + d_{21}\Delta y_{2t-1} + \eta$$
 (3)

and can be estimated via Non-linear Least Squares (NLS).

<sup>&</sup>lt;sup>2</sup> Phillips and Loretan conceded that the inclusion of leads as a means of achieving valid conditioning was also advanced by Stock and Watson (1991). However they became aware of this paper only when their work had been completed. Stock and Watson developed DOLS further by allowing for a higher order of integration, I(d) in 1993.

Taking the initial condition t=0 and  $y_0=0$  and writing;

$$y_{1t} = \sum_{k=1}^{t-1} \Delta y_{1t-k}, y_{2t-1} = \sum_{k=1}^{t-1} \Delta y_{2t-k}$$

then equation (3) can be written as;

$$y_{1t} = \beta y_{2t} + \sum_{k=1}^{t-1} f_{1k} \Delta y_{1t-k} + \sum_{k=0}^{t-1} f_{2k} \Delta y_{2t-k} + \eta_t.$$

Assuming that terms in the above equation are truncated at a single lag, Phillips and Loretan found that the fitted regression on this equation was,

$$y_{1t} = \hat{\beta}y_{2t} + \hat{f}_{11}\Delta y_{1t-1} + \hat{f}_{20}\Delta y_{2t} + \hat{f}_{21}\Delta y_{2t-1} + \hat{\eta}_t$$
 (4).

Next they assumed that the true error structure was  $B(L)\mu_1 + \varepsilon_1$  with a diagonal autoregressive operator and a scalar error covariance matrix (Phillips and Loretan, 1991, p.421-422). Using equations (1), (3) and (4) Phillips and Loretan discussed the asymptotic results of an OLS regression on equation (1), NLS on equation (3) and OLS on equation (4). The discussion of the asymptotics of these regressions results in the development of DOLS.

NLS was used as equation (3) contained lagged equilibrium relationships. These relationships were used as the lags of  $\Delta y_I$  were not an adequate proxy for the past history of  $u_I$ . This was due to the persistence in the effects of the innovations that arose from the presence of unit roots in the system. Thus, Phillips and Loretan stated that asymptotic theory favoured the use of NLS. When comparing the asymptotics of applying an OLS regression on equations (1) and (4) and a Non-linear Least Squares regression on equation (3) the authors found that OLS was equivalent to Maximum Likelihood on equation (1), despite the fact that the OLS regression ignored the dynamic error structure. On the other hand Non-linear Least Squares did fit the error dynamics, but this did not improve the efficiency of the estimator. However, the benefit of using Non-linear Least Squares was that the t-ratios were asymptotically normal, not so with the OLS regression. Estimation of (4) lead to a second order bias effect. That is; the explanatory variable may have been generated by a process that displayed autocorrelation and the error process may not have been a Martingale difference sequence. The effect is that in finite samples the coefficient estimates have non-zero means. This is also known as endogeneity bias.

If the assumption of no autocorrelation was relaxed, Phillips and Loretan found that the asymptotics of the Non-linear Least Squares regression of (3) contained nuisance parameters in the limit distribution. This was due to the fact that there was feedback in the error terms. To eliminate this problem and once again obtain an asymptotically efficient estimator, the leads of  $\Delta y_{2t}$  were included in the regression. Thus, equation (2) becomes,

$$y_{1t} = \beta' y_{2t} + d_1(L)(y_{1t} - \beta' y_{2t}) + d_2(L')\Delta y_{2t} + d_3(L^{-1})'\Delta y_{2t} + v_t$$
 and d<sub>1</sub>(L) and d<sub>2</sub>(L) are as before in (2).

$$d_3(L^{-1}) = \sum_{k=1}^{\infty} d_{3k} L^{-k}$$

So in short, with feedback in the error terms, leads and lags should be included in the regression (Phillips and Loretan, 1991, p.424-426). Or leads should be included when the regressors are not strongly exogenous. Stock and Watson also discovered the benefits of including leads in a linear regression of I(1) variates. However, they also found that either OLS or GLS can be used to estimate this revised equation with the results being asymptotically equivalent to the Johansen estimator. The OLS estimation technique is chosen estimation technique for all DOLS regressions reported in this thesis.

In this thesis the two estimation techniques mentioned in this paper will be used. That is, the linear regression equation will include both the leads and lags in one instance, and the regression with only lags will also be estimated. The later equation is estimated, despite the fact that it is asymptotically inefficient, for comparative reasons.

A small experiment will be undertaken to determine whether the inclusion of leads in a regression that contains lagged variables affects the outcome of tests for Unbiasedness and Efficiency. Table 3.4 contains the results from the regression which contained the lags of the variables. The residuals in the regression were then tested to see if they exhibited any serial correlation. This was done by plotting their partial autocorrelation and correlation functions. The plot of the residuals revealed that the model with just lags, exhibits an autocorrelation function that broke the bounds on the first two lags and a partial autocorrelation function that deteriorated rapidly. This points to an MA(2) error structure. Remodelling the lagged regression correcting for the misspecification yields the results displayed in the second row of table 3.4;

Table 3.4 OLS with Lags, OLS with Lags corrected for MA(2) Errors

	α	f <sub>t</sub>	$\Delta f_t$	$\Delta f_{t-1}$	$\theta_1$	$\theta_2$	Wald
						·	
Coefficient	0.40	0.90	0.16	0.09			6.76
Uncorrected t-ratio's	2.29	26.68	2.87	1.62			$(4.22)^{(i)}$
Corrected t-ratio's	(1.43)	(16.65)	(1.79)	(1.01)			
Wald Critical Value							5.99
Coefficient	0.41	0.90	0.10	0.05	0.63	0.54	6.11
Uncorrected t-ratio's	1.63	18.24	2.91	1.45	6.56	5.64	(3.80) <sup>(i)</sup>
Corrected t-ratio's	(1.01)	(11.34)	(1.81)	(0.90)	(4.08)	(3.51)	
Wald Critical Value							5.99

(i) Data in parenthesis are corrected Wald statistics

t-ratios and the Wald test for Unbiasedness were corrected via the Newey West procedure as the t-ratios were not normal. The joint hypothesis of the constant term being zero and the slope coefficient being 1, now yields a statistic of 3.80 which is below the critical value of 5.99. This means that the hypothesis of Speculative Efficiency cannot be rejected. Following Phillips and Loretan (1991) it is known that this method of modelling does not yield asymptotically efficient estimators, so the leads are included in the regression the standard errors are, once again, corrected via the Newey West procedure. The coefficients and adjusted t-ratios (in parenthesis) are displayed in table 3.5 (overleaf).

Table 3.5 DOLS Regression Coefficients and Unbiasedness Test Results

	α	f <sub>t</sub>	$\Delta f_t$	$\Delta f_{t-1}$	$\Delta f_{t+1}$	$\Delta f_{t+2}$	Wald
Coefficient	-0.05	1.00	-0.25	0.002	0.91	1.19	1.60
Uncorrected t-ratio's	0.44	44.41	1.50	0.07	5.08	7.00	(0.88) <sup>(iii)</sup>
Corrected t-ratio's	(0.24)	(24.54)	(0.83)	(0.04)	(2.81)	(3.87)	
Wald Critical Value							5.99

(iii) Data in parenthesis are corrected Wald statistics

Comparing the two techniques the single equation estimator was found to be greatly improved by adding the leads, as it better modelled the information set and rectified the problems that arose if the error terms were not uncorrelated at all lags. In this case the inclusion of leads affected the coefficient on  $f_t$  resulting in the estimated coefficient being closer to one. To test the Unbiasedness hypothesis on the DOLS estimates, Wald tests were undertaken. Once again these had to be adjusted by the ratio of variances;

$$Wald \times \left(\frac{\hat{\sigma}_{sr}^2}{\hat{\sigma}_{Lr}^2}\right)^{\frac{1}{2}}$$

Where the long run variance is approximated by

$$S_{Tl}^{2} = T^{-1} \sum_{t=1}^{T} \hat{u}_{t}^{2} + 2T^{-1} \sum_{j=1}^{l} w_{l}(j) \sum_{t=j+1}^{T} \hat{u}_{t} \hat{u}_{t-j}.$$

The null hypothesis remained that the constant was zero and the coefficient on the forward rate was not significantly different from 1. The Wald test corrected via the Newey West procedure became 1.60 for the DOLS estimation, which was clearly less than the Chi-squared critical value at two degrees of freedom of 5.99, and 4.22 for the lag regression, which also does not exceed the critical value of 5.99. Hence the null of efficiency could not be rejected when the estimation technique was DOLS. A comparison of this result and the Maximum Likelihood, error-correction model result, with the results from chapter 2, should give an indication of how changing the underlying assumptions of data and hence changing the modelling technique, alters the hypothesis test results.

This comparison concerns the GMM and OLS with MA(2) errors estimates from Chapter 2. These two models will be considered, due to the fact that they both corrected for the overlapping data problem whilst using the full sample period and hence make a more meaningful comparison.

Table 3.6 Comparison of Unbiasedness Test Results

	Wald/F-Test
OLS and MA(2) errors	0.90
F-test critical value	3.11
GMM	5.43
OLS (lagged)	4.22
DOLS	0.88
Johansen	8.817
Wald test critical value	5.99

The GMM and OLS MA(2) found support for the assumption of Unbiasedness, however, the GMM could not accept the notion of Efficiency were as the OLS, MA(2) could. In comparison both the lagged and leads regression and the simple lagged regression supported the hypothesis of Unbiasedness. However the Johansen procedure rejected the notion of Unbiasedness which is at odds with the other estimation techniques.

Overall it appears that altering the underlying assumptions of the models did alter the conclusion, depending on which estimation technique was used. In short, there was support for the accuracy, Efficiency and Unbiasedness of forecasters, which is fortunate, as it implies that they are rational which is an assumption at the basis of the Term Structure Literature, when DOLS was used. Further, the DOLS estimates resulted in a slope coefficient of 1.00, which was the restriction that has been tested throughout this thesis, and hence modelling the data using DOLS techniques has lead to an even stronger support for the assumption of rational forecasters. However, the same cannot be said for the Johansen technique. This could be due to the presence of moving average errors which can affect the Johansen procedure.

All these results pertain to US data, hence it should be ascertained as to whether these results are true for other countries and in particular whether the Johansen technique consistently fails to accept the hypothesis of Unbiasedness. The next chapter uses the same estimation techniques described in chapters 2 and 3 on data from the United Kingdom, Canada, Germany, France, Italy, Japan and Australia. This enables a comparison across countries and will provide a forum for a better assessment of whether the results of the hypothesis tests are sensitive to the different estimation techniques.

### Chapter 4

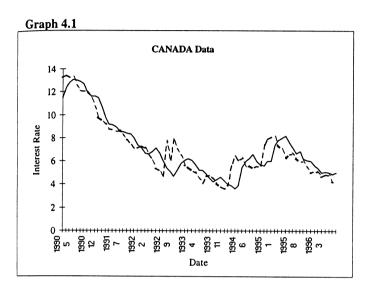
#### **International Comparison**

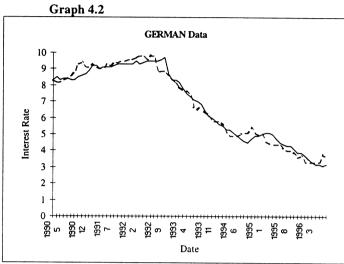
The findings reported in Chapters 2 and 3 relate purely to the United States. As *Consensus Economics* provides the same data for several other countries, it allows for the re-estimation of the equations outlined in the previous chapters. It can then be assessed whether patterns emerge in the data. This chapter covers both the stationary and nonstationary framework with the data originating from seven countries. These are; Japan, Canada, Germany, France, Italy, the United Kingdom and Australia. These countries were chosen, as being highly developed first world countries, the expectation of Efficient Markets should not be considered illogical.

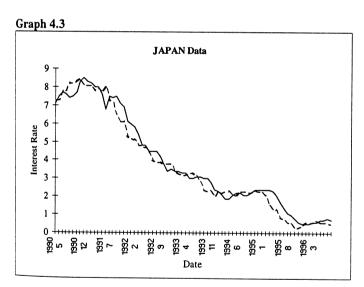
As previously mentioned there are fewer data points for Australia with only 68 observations compared with the 77 observations recorded for the other participating nations. The data was obtained from *Consensus Economics* for the time period, May, 1990 to September, 1996. As this is the same time period as that of the U.S. data, a feasible comparison between countries can be made. The Australian time period differs slightly with the first recorded forecast in November 1990 and the last March 1996.

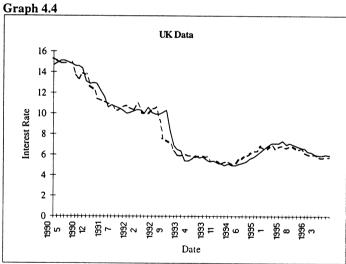
Graphs of the data are provided overleaf. Graph 4.1 to 4.7 display the actual and predicted spot rates. As with the U.S. data, the spot rate has been adjusted so that the forecast and actual rate are in line. Or in other words, the spot rates for January are aligned to the forecast for January that was made three months earlier. The average forecasts of the countries spot rate are represented by the dotted line.

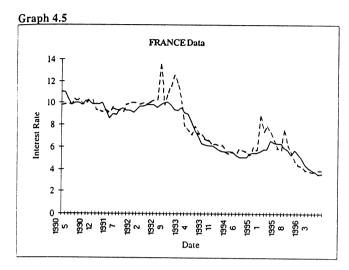
## **Graphs of the Spot and Forecasts**

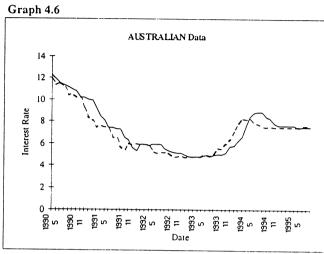


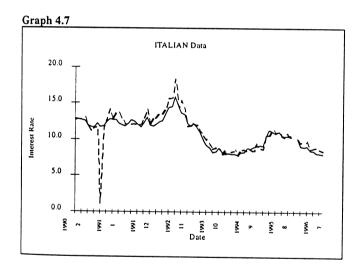


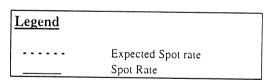












The original equations forming the basis of the hypothesis testing are, as before;

(a) 
$$s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$$

(b) 
$$s_{t+3}-f_t = \alpha_0 + \alpha_1(s_{t-1}f_{t-3}) + \alpha_2(s_{t-1}-f_{t-4}) + \varepsilon_{t+3}$$

These will be referred to frequently throughout this chapter as equations (a) and (b). To begin with all previous assumptions, developed throughout chapters 2 &3, relating to the data will be relaxed and the international comparison shall commence in the stationary world using simple OLS techniques.

### **4.1 OLS**

Table 4.1 displays the results, from all countries, obtained via an OLS regression on equations (a) and (b). The f-test displayed in the first column relates both to the weak form of the Efficiency Hypothesis,  $\alpha_1 = \alpha_2 = 0$  and for the test for Unbiasedness,  $\alpha_0 = 0$  and  $\alpha_1 = 1$ , depending on the equation estimated.

In general with regards to regression (b) the coefficients are expected to be zero (hence have relatively large standard errors) whilst for equation (a) the intercept term is also expected to be zero but the slope coefficient is anticipated to be 1, or significantly different from zero, hence exhibit a relatively low standard error.

Table 4.1 OLS Regression Results for  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$ 

Country	$\alpha_0$	$\alpha_1$	R <sup>2</sup>	D.W.	F/Test
Japan	-0.19	1.01	0.97	0.58	4.96
	(0.09) <sup>(i)</sup>	(0.02)			
Canada	0.54	0.91	0.83	0.64	2.24
	(0.36)	(0.05)			
United Kingdom	0.32	0.95	0.96	0.72	4.14
	(0.20)	(0.02)			
Germany	-0.10	1.02	0.98	0.54	0.43
	(0.13)	(0.02)			
France	0.40	0.99	0.84	1.0	3.94
	(0.39)	(0.05)	,		
Italy	0.77	0.93	0.52	1.30	0.23
	(1.14)	(0.10)			
Australia	0.83	0.85	0.89	0.42	12.38
	(0.28)	(0.04)			
F-test Critical Value					3.13 <sup>(ii)</sup>

(i) Data in parenthesis are standard errors

(ii)..." are F-test 5% critical values for 2 and 70 Degrees of Freedom

The test results for the hypothesis of Unbiasedness are recorded in table 4.1. They indicate that on average, Unbiasedness cannot be accepted with Japan, the United Kingdom, France and Australia failing to accept the hypothesis that the intercept term is insignificantly different from zero. These findings were in line with those obtained from the U.S. data. The Durbin Watson Upper and Lower bounds for equation (a) are (1.45-1.50). Evidently, for all countries, the regressions produced Durbin Watson statistics that fell below these upper and lower bounds. This indicates that the model is misspecified.

For the test of strong efficiency recorded in table 4.2, all coefficients should be insignificantly different from zero. Most countries could not reject this premise, except for Japan. As Japan fails to reject the weaker form of the theory,  $\alpha_1 = \alpha_2 = 0$ . The failure to accept the strong

efficiency assumption can be apportioned to the significance of the constant term. Hence, forecasters perceive that there is a Term Premium and incorporate this assumption into their estimation of future spot rates.

Table 4.2 OLS Regression Results for  $s_{t+3}$ - $f_t$ =  $\alpha_0 + \alpha_1(s_{t-1}f_{t-3}) + \alpha_2(s_{t-1}-f_{t-4}) + \varepsilon_{t+3}$ 

Country	$\alpha_0$	$\alpha_1$	$\alpha_2$	R <sup>2</sup>	D.W.	F/Test	F/Test
							$\alpha_0 = \alpha_1 = \alpha_2 = 0$
Japan	-0.17	0.01	0.02	0.01	0.58	0.37	4.26
	$(0.05)^{(i)}$	(0.07)	(0.06)				
Canada	-0.24	0.11	-0.07	0.01	0.64	0.51	1.33
	(0.14)	(0.11)	(0.77)				
United	-0.28	0.002	0.02	0.003	0.58	0.11	1.62
Kingdom	(0.12)	(0.09)	(0.07)				
Germany	-0.07	-0.03	0.005	0.002	0.63	0.09	0.39
	(0.09)	(0.09)	(0.08)				
France	0.27	-0.09	-0.008	0.02	0.85	1.64	0.96
	(0.13)	(0.10)	(0.10)				
Italy	-0.19	0.02	-0.04	0.003	1.02	0.10	0.31
	(0.12)	(0.12)	(0.12)				
Australia	-0.46	0.02	0.002	0.002	0.52	0.05	2.19
	(0.18)	(0.14)	(0.11)				
Critical Values						(3.13) <sup>(ii)</sup>	(2.74) <sup>(iii)</sup>

(i) Data in parenthesis are standard errors

(ii)..." are F-test 5% critical values for 2 and 70 Degrees of Freedom

(iii).." for 3 and 70 Degrees of Freedom

However, as with the U.S., weight cannot be given to these findings due to the equation misspecification which is reflected in the Durbin Watson Statistics. For equation (b) the Upper and Lower bounds are (1.42-1.53). As can be seen the statistics reported in table 4.2 are well below these. The reason for this misspecification is that the data is overlapping, as was the case with the U.S. data. This results in serial correlation in the errors which will bias the OLS standard errors downward and lead to overestimated t-statistics. The methods used in Chapter 2 to rectify this problem, with regards to the U.S. data, shall now be applied to the

data from the seven countries. These methods are; sampling, allowing for moving average errors and GMM.

## **4.2 OLS Sub-Sample**

Following the example of Canarrella and Polland (1986) outlined in chapter 2, three subsamples were taken from each data set. That is, every third observation was sampled out which resulted in three sub samples of non-overlapping data. A simple OLS regression of the equation;  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$  which allows for the testing of the Unbiasedness hypothesis, was run separately on these sub-samples. These results are recorded in table 4.3 (overleaf).

Table 4.3 OLS Sub-Sample Regression Results for  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$ 

Country	Sample	$\alpha_0$	$\alpha_1$	D-W	F/Test <sup>(i)</sup>
	3	0.95	0.86	2.20	1.42
		(0.66)	(0.09)		
Canada	2	0.53	0.91	2.40	0.65
		(0.63)	(0.08)		
	1	0.16	0.96	1.94	0.44
		(0.60)	(0.08)		
	3	-0.20	1.01	2.03	1.08
		(0.19)	(0.04)		
Japan	2	-0.19	1.01	1.63	1.92
		(0.13)	(0.03)		
	1	-0.18	1.01	1.36	2.01
		(0.15)	(0.03)		
	3	-0.22	1.03	1.61	0.54
		(0.21)	(0.03)		
Germany	2	-0.04	1.01	1.64	0.10
		(0.15)	(0.03)		
	1	-0.03	1.01	1.56	0.14
		(0.23)	(0.03)		
	3	0.03	1.05	1.82	2.26
		(0.74)	(0.09)		
France	2	0.60	0.96	2.13	1.24
		(0.67)	(0.08)		
	1	0.50	0.96	2.58	0.80
		(0.65)	(0.08)		
	3	0.22	0.96	1.85	1.85
		(0.26)	(0.03)		
United Kingdom	2	0.38	0.94	2.04	1.69
		(0.38)	(0.04)		
	1	0.37	0.95	2.22	0.90
		(0.39)	(0.04)		
	3	0.33	0.98	1.57	0.15
		(1.65)	(0.15)		
Italy	2	-0.21	1.03	1.57	0.28
		(1.30)	(0.12)		
	1	1.67	0.86	1.12	0.85
<del></del>		(1.28)	(0.12)		
F-test Critical Value		0.5-			3.40
	3	0.95	0.84	1.79	3.16
   A	•	(0.60)	(0.08)	1	
Australia	2	0.87	0.85	1.52	4.94
		(0.46)	(0.06)	1.55	
	1	0.71	0.87	1.23	3.94
E Transfer in 1971		(0.47)	(0.06)		2.40
F-Test Critical Value		L	sion equation s.= 0	L	3.49

<sup>(</sup>i) The Joint test is for  $\alpha_0 = \alpha_1 = 0$  in the regression equation  $s_i = \alpha_0 + \alpha_1 f_i + \varepsilon_i$ 

Based on these results, all countries excepting Australia, do not reject the null hypothesis of Unbiasedness for all three sub-samples. For Australia sub-samples two and one, reject this hypothesis, due to the presence of a significant constant term. It is possible that as sub-sample three fails to reject the premise of Unbiasedness, forecasters became more accurate as additional information became available to them.

Recalling the Durbin Upper and Lower bounds for one regressor and 25 observations, (1.06,1.19) and for the presence of negative autocorrelation (2.81,2.94), these statistics indicate that sampling the data has rectified the misspecification problem, except for the first Italian sub-sample. This anomaly could possibly be due to the presence of contemporaneous correlation.

#### **4.3 SUR**

To take into account the serial correlation in the disturbances whilst allowing for a more general covariance structure, the sub-samples were estimated using the Seemingly Unrelated Regression technique. As in Chapter 2 the standard errors of each of these regression are reported. Likelihood ratio tests of the Unbiasedness restriction,  $\alpha_0=0$  and  $\alpha_1=1$ , were undertaken for each of the seven countries. These results are displayed in table 4.4.

Table 4.4 SUR Regression Results for  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$ 

Country	Sample	$\alpha_0$	$\alpha_1$	D-W	Standard Error of	Likelihood Ratio Test
	3	0.93	0.86	2.34	Regression 1.09	
	3	$(0.57)^{(i)}$	(0.08)	2.34	1.09	
Canada	2	0.86	0.87	2.59	1.11	4.84
Callada	2	(0.56)	(0.07)	2.57	1.11	(9.49) <sup>(ii)</sup>
	1	0.40	0.92	2.15	1.06	(2.42)
	1	(0.55)	(0.07)	2.13	1.00	
	3	-0.09	0.99	1.94	0.52	
		(0.18)	(0.04)	1.7.	0.02	
Japan	2	-0.10	0.99	1.54	0.39	4.32
oup un		(0.14)	(0.03)			
	1	-0.13	0.99	1.34	0.42	
		(0.16)	(0.03)			
	3	-0.10	1.02	1.71	0.32	
		(0.19)	(0.03)			
Germany	2	-0.05	1.01	1.69	0.39	1.51
•		(0.23)	(0.03)			
	1	-0.13	1.02	1.58	0.32	
		(0.18)	(0.02)			
	3	0.16	1.03	1.82	1.00	
		(0.63)	(0.08)		:	
France	2	0.64	0.95	2.11	0.98	5.90
		(0.62)	(0.08)			
	1	0.46	0.97	2.59	0.95	
		(0.60)	(0.08			
	3	0.37	0.94	1.14	0.46	
	_	(0.26)	(0.03)			
United	2	0.56	0.92	1.97	0.71	7.69
Kingdom		(0.39)	(0.04)	2.16	0.72	
	1	0.55	0.93	2.16	0.72	
		(0.40)	(0.04)	1.50	0.65	
	3	-0.37	1.06	1.52	0.65	
Teal-	2	(0.46)	(0.04)	1.20	0.49	0.00
Italy	2	-0.42	1.07	1.30	0.48	9.82
	1	(0.36)	(0.03)	1.52	0.45	
	1	0.17 (0.33)	1.01 (0.03)	1.53	0.45	
	3	1.54	0.75	1.58	0.76	
		(0.58)	(0.08)	1.56	0.70	
Australia	2	1.26	0.79	1.41	0.61	16.03
1 rusti arra		(0.46)	(0.06)	1.71	0.01	10.03
	1	0.97	0.83	1.19	0.64	
		(0.47)	(0.06)	1.17	0.07	
(I) G	lard errors	1 (0.17)	(0.00)	I	L	I

<sup>(</sup>i) Standard errors

<sup>(</sup>ii) Critical value, chi squared with 4 degrees of freedom

Table 4.4 illustrates that the slope coefficients, in all cases, were clearly significantly different from zero, whilst the intercept term in all cases, except Australia, were not significantly different from zero. In order to test the Unbiasedness property, the system was estimated once again with the restrictions that  $\alpha_0=0$  and  $\alpha_I=I$  imposed. The log likelihood (LRR) was recorded and compared to the log likelihood of the unrestricted SUR regression (LRU) using the likelihood ratio test,  $2*(LRU-LRR)\sim\chi^4$ 

The restrictions could not be rejected for all countries excepting, Italy and Australia. This is consistent with the previous findings.

The standard errors of the regressions recorded in table 4.4, are intended to show that each sub-sample is estimating the same data set. Or, in other words, the data subsets originate from the same data source. This appears to be true in all cases as the standard errors, obtained for each sub-sample, are relatively close to those standard errors obtained from the other regressions within the system. From this it can be concluded the data division was valid.

Table 4.5 displays the SUR regressions coefficients that result from the estimation of the same sub-samples with the restriction that all the slope coefficients are equal and all the intercept coefficients are equal imposed. This is the most efficient method of estimating the coefficients, using the SUR technique on the three sub-samples, as it enforces only one coefficient to be estimated for each variable. Therefore, the whole data set was utilised to obtain an estimate for  $\alpha_0$  and  $\alpha_1$ . To test whether the imposition of this restriction was valid a likelihood ratio test was undertaken for each country. The test statistics are recorded in table 4.5 and are distributed as a  $\chi^2$ , with the degrees of freedom equivalent to the number of restrictions imposed. There were 4 restrictions in this case, with the constant term and slope coefficients restricted in each equation. Hence the critical value is 9.49, indicating that the null could not be rejected by any country. This indicates that, for each of the seven countries, the model was stable across the three data sub-samples.

Table 4.5 Restricted SUR Regression Results for  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$ 

Country	Sample	$\alpha_0$	$\alpha_1$	D-W	Likelihood Ratio Test
	3	0.77	0.88	2.23	
	ł	(0.48)	(0.06)		
Canada	2	0.77	0.88	2.0	2.35
		(0.48)	(0.06)		
	1	0.77	0.88	1.76	
		(0.48)	(0.06)		
	3	-0.11	0.99	1.30	
		(0.14)	(0.03)		
Japan	2	-0.11	0.99	1.56	0.23
		(0.14)	(0.03)		
	1	-0.11	0.99	1.96	
		(0.14)	(0.03)		
	3	-0.21	1.03	1.77	
		(0.16)	(0.02)		
Germany	2	-0.21	1.03	1.76	1.02
		(0.16)	(0.02)		
	1	-0.21	1.03	1.61	
		(0.16)	(0.02)		
	3	0.41	0.99	2.55	
		(0.53)	(0.07)		
France	2	0.41	0.99	2.13	2.01
		(0.53)	(0.07)		
	1	0.41	0.99	1.76	
		(0.53)	(0.07)		
	3	0.36	0.95	1.15	
		(0.25)	(0.03)		
United	2	0.36	0.95	2.04	2.71
Kingdom		(0.25)	(0.03)		
	1	0.36	0.95	2.21	
		(0.25)	(0.03)		
	3	2.20	0.81	1.48	
		(1.15)	(1.03)		
Italy	2	2.20	0.81	0.86	3.47
		(1.15)	(1.03)		
	1	2.20	0.81	1.35	
		(1.15)	(1.03)		
	3	1.11	0.81	1.74	
		(0.42)	(0.05)		
Australia	2	1.11	0.81	1.48	2.39
		(0.42)	(0.05)		
	1	1.11	0.81	1.14	
		(0.42)	(0.05)		
Critical Value					9.49 <sup>(ii)</sup>

<sup>(</sup>i) Standard errors

<sup>(</sup>ii) Critical value, chi squared with 4 degrees of freedom

Although this technique does rectify the problem of overlapping data, a more optimal solution to the problem is to utilise the full data set. For this reason the OLS with MA(2) errors and the GMM estimation techniques were considered.

### 4.4 OLS with MA(2) Error Process

A moving average error structure was applied to the data in order to use the full data set, that is the 77 observations as opposed to 25 in each sub-sample. Once again both equations (a) and (b) are used so that the notions of Unbiasedness and Efficiency can be tested. The results of this analysis is shown in tables 4.6 and 4.7

Table 4.6 OLS with MA(2) errors for  $s_{t+3} = \alpha_0 + \beta f_t + \epsilon_{t+3}$ 

Country <sup>(i)</sup>	$\alpha_0$	$\alpha_1$	$\theta_1$	$\theta_2$	F/Test
· · · · · · · · · · · · · · · · · · ·					
Japan	-0.13	1.00	0.75	0.66	1.50
	$(0.14)^{(i)}$	(0.03)	(0.08)	(0.09)	
Canada	0.51	0.92	0.68	0.52	0.63
	(0.53)	(0.07)	(0.10)	(0.10)	
Germany	-0.02	1.00	0.78	0.43	0.07
	(0.19)	(0.03)	(0.10)	(0.10)	
Italy	0.90	0.92	0.28	0.25	0.17
	(1.55)	(0.14)	(0.11)	(0.11)	
France	0.73	0.94	0.55	0.44	1.39
	(0.61)	(0.07)	(0.10)	(0.10)	
U.K.	0.25	0.96	0.94	0.60	0.65
	(0.32)	(0.03)	(0.09)	(0.09)	
Australia	0.61	0.89	0.90	0.84	5.98
	(0.36)	(0.04)	(0.07)	(0.07)	
Critical Value					3.15 <sup>(ii)</sup>

(i) Data in Parenthesis are standard errors

(iii) ... " are F-test 5% critical values for 2 and 60 Degrees of Freedom

Table 4.7 OLS Regression on  $s_{t+3}$ - $f_t$ =  $\alpha_0 + \alpha_1(s_t$ - $f_{t-3}) + \alpha_2(s_{t-1}$ - $f_{t-4}) + \varepsilon_{t+3}$  with MA(2) errors

Country <sup>(i)</sup>	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\theta_1$	$\theta_2$	F/Test	F/Test
							$\alpha_0 = \alpha_1 = \alpha_2 = 0$
Japan	-0.18	0.03	-0.04	0.80	0.70	1.02	2.27
_	$(0.08)^{(ii)}$	(0.03)	(0.04)	(0.08)	(0.08)		
Canada	-0.28	0.08	0.04	0.71	0.64	1.20	1.14
	(0.23)	(0.52)	(0.07)	(0.09)	(0.10)		
Germany	-0.08	-0.02	-0.01	0.96	0.26	0.15	0.35
-	(0.12)	(0.05)	(0.04)	(0.11)	(0.13)		
Italy	-0.27	-0.04	-0.01	0.46	0.31	0.10	0.27
·	(0.39)	(0.10)	(0.01)	(0.11)	(0.13)		
France	0.22	-0.06	-0.04	0.63	0.45	0.692	0.532
	(0.22)	(0.07)	(0.07)	(0.11)	(0.12)		
U.K.	-0.52	0.008	0.009	1.08	1.00	0.25	1.89
	(0.24)	(0.04)	(0.01)	(0.04	(0.04)		
Australia	-0.67	0.002	-0.002	0.94	1.00	0.004	3.02
	(0.32)	(0.02)	(0.08)	(0.13)	(0.14)		
Critical Values						3.15 <sup>(iii)</sup>	2.76 <sup>(iv)</sup>

(i) Regression equation is

$$s_{t+3} - f_t = \alpha_0 + \alpha_1 (s_t - f_{t-3}) + \alpha_2 (s_{t-1} - f_{t-4}) + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2}$$

(ii) Data in Parenthesis are standard errors

(iii) ... " are F-test 5% critical values for 2 and 60 Degrees of Freedom

(iv) ..." for 3 and 60 Degrees of Freedom

Regarding equation (b), in table 4.7, and the joint test for strong efficiency, Australia was once again the only country to find evidence against this hypothesis. Weak efficiency is supported by all countries. For Unbiasedness, Australia alone failed to accept this hypothesis. The presence of a large and significant constant term was the reason behind the failure of Efficiency and Unbiasedness for Australia. Note that for all countries, the coefficients on the moving average error terms are significantly different from zero.

Comparing these OLS estimates for Australia with those displayed in table 4.1, it can be seen that the constant terms reduce in magnitude when a moving average error structure is applied, the presence of moving average error terms in the original OLS regression would have biased the standard errors downward. However correcting for MA(2) errors does not alter the conclusion that the constant terms were indiscriminate from zero. As well, much less weight is placed on the previous forecast error when attempting to explain the present forecast error. Yet, the standard errors indicate that in both cases these coefficients are indeterminate from zero. In other words, the misspecification alone does not account for Australia's rejection of

the properties of Unbiasedness and Efficiency, as it has with Japan. So due to the presence of a significant constant term, there is evidence supporting a Term Premium when forecasting interest rates in Australia.

### **4.5 GMM**

Another method of correcting for the overlapping data problem, inherent in the interest rate data, is the Hansen and Hodrick (1980) GMM technique. This resulted in the following estimates for equation (a), table 4.8 and equation (b) table 4.9;

Table 4.8 GMM for  $s_{t+3} = \alpha_0 + \beta f_t + \varepsilon_{t+3}$ 

Country	α <sub>0</sub>	$\alpha_1$	Wald
Japan	-0.19	1.01	7.71
	(0.09)	(0.02)	
Canada	0.54	0.91	3.16
	(0.51)	(0.06)	
Germany	-0.10	1.02	0.46
	(0.18)	(0.02)	
Italy	0.77	0.93	0.97
	(0.94)	(0.10)	
France	0.40	0.99	6.22
	(0.41)	(0.06)	
U.K.	0.32	0.95	4.77
	(0.17)	(0.02)	
Australia	0.83	0.85	14.68
	(0.32)	(0.04)	
Wald test Critical Value			5.99 <sup>(i)</sup>

<sup>(</sup>i) Data in parenthesis is the 5% Chi squared critical value with 2 Degrees of Freedom

Table 4.9 GMM Regression for  $s_{t+3}$ - $f_t$ =  $\alpha_0 + \alpha_1(s_t$ - $f_{t-3}) + \alpha_2(s_{t-1}$ - $f_{t-4}) + \epsilon_{t+3}$ 

Country	$\alpha_0$	$\alpha_1$	$\alpha_2$	Wald	Wald
					$\alpha_0=\alpha_1=\alpha_2=0$
Japan	-0.18	0.06	0.004	3.90	10.72
	(0.06)	(0.06)	(0.04)		
Canada	-0.28	0.06	-0.07	0.56	4.35
	(0.17)	(0.09)	(0.09)		
Germany	-0.08	-0.03	0.01	0.63	4.61
	(0.07)	(0.06)	(0.03)		
Italy	-0.24	-0.03	-0.01	0.17	1.12
	(0.29)	(0.11)	(0.05)		
France	0.29	-0.07	0.007	1.43	4.69
	(0.15)	(0.07)	(0.05)		
U.K.	-0.27	-0.007	0.02	1.11	3.94
	(0.14)	(0.03)	(0.02)		
Australia	-0.55	0.04	0.006	2.57	18.51
	(0.17)	(0.05)	(0.03)		
Wald test critical Values				5.99 <sup>(i)</sup>	7.81 <sup>(ii)</sup>

(i) Data in parenthesis is the 5% Chi squared critical value with 2 Degrees of Freedom (ii)" with 3 Degrees of Freedom

Comparing the two tables with the results obtained from the previous regressions, it is evident that the GMM results are far less conclusive with regards to the hypotheses of Efficiency and Unbiasedness. This is similar to the U.S. findings recorded in chapter 2. Japan and Australia could not accept the null hypothesis of Unbiasedness, as seen in table 4.8, which is in line with previous findings. That is, Australia could not accept the notion of Unbiasedness when the equation,  $s_{t+3} = \alpha_0 + \beta f_t + \varepsilon_{t+3}$ , was estimated using OLS, SUR and when allowing for moving average errors. Japan failed to accept Unbiasedness on only one prior occasion, that is, when using the OLS estimation technique. When using the GMM estimation technique France also could not accept the null hypothesis of Unbiasedness. France, had no prior rejection record.

Japan and Australia could also not support the notion of efficiency as tested via the regression  $s_{t+3}$ - $f_t$ =  $\alpha_0 + \alpha_1(s_t$ - $f_{t-3}) + \alpha_2(s_t$ - $f_{t-4}) + \varepsilon_{t+3}$ , using a Wald joint test, table 4.9. Both of these countries could not accept this hypothesis on one previous occasion; Japan when the OLS estimation technique was used and Australia when moving average errors were allowed for. Interestingly Japan only failed to accept the notion of Efficiency previously when the OLS technique was used. It has been shown that this technique showed significant bias and therefore the test results could not be considered accurate. So, the GMM technique yielded an unexpected result from Japan.

The coefficients have not been radically altered in comparison to the OLS and OLS with MA(2) errors, with the use of GMM. The standard errors have changed, and this has resulted in the more frequent rejection of the null hypotheses. The change in the standard errors is to be expected, as the GMM estimation technique expressly alters the standard errors, as mentioned in Chapter 2. Interestingly, it can be attained from the comparison of the GMM and OLS & MA(2) estimates, that although both these techniques correct for the overlapping data problem, for relatively small samples they do not yield the same results.

#### 4.6 Nonstationarity & Cointegration

As mentioned in chapter 2, both the theory of Speculative Efficiency and the use of the above estimation techniques, assumed that the variables in question were stationary. Previous literature has disputed this assumption, and the U.S. data failed to reject the hypothesis that the data contained a unit root. Hence, Augmented Dickey Fuller tests on the spot and expected spot interest rate were undertaken for all seven countries. These were followed by two tests for cointegrating relationships - the Johansen test and the Phillips and Perron test, which allows for moving average error terms. These results are shown in tables 4.10, 4.11 and 4.12 respectively.

**Table 4.10** 

	Augmented Dickey Fuller	Augmented Dickey Fuller
	Unit Root Test	Unit Root Test
	$S_{t+3}$	$f_t$
Australia	-2.56	-1.89
Canada	-2.58	-2.55
Germany	-2.74	-2.62
France	-2.34	-1.91
U.K.	-1.17	-1.32
Japan	-1.62	-1.46
Italy	-2.76	-1.60

The critical value for the Dickey Fuller test with a constant and trend is -3.13. This means that in all cases, when the absolute value is taken, the critical value is not exceeded by the test statistic at the 10% level and hence the null of a unit root cannot be rejected. Recalling the Graphs 4.2 to 4.7, the data series, for each country, do move together which indicates that they might be cointegrated. The presence of a long-run, cointegrating, relationship is important as it allows the expression of a equilibrium relationship between two variables that may be individually nonstationary.

Testing for cointegrating relationships using Johansens procedure, and allowing for a lag length of two, resulted in the hypothesis of there being two unit roots (two cointegrating vectors) being rejected in favour of one unit root for all countries (see table 4.11). The hypothesis of one cointegrating vector (one unit root in the errors) could not be rejected in favour of no unit roots for all cases, as the 5% critical value for the Johansen test is 8.18.

**Table 4.11 Johansen ML Cointegration Test Statistics** 

	Ma	x Root	Trace Test			
Australia	62.376	7.165	69.541	7.165		
Canada	62.376	6.545	68.921	6.545		
Germany	43.295	0.018	43.313	0.018		
France	30.362	0.042	30.404	0.042		
U.K.	67.543	4.700	72.243	4.700		
Japan	51.441	1.717	53.158	1.717		
Italy	75.055	1.773	76.828	1.773		
Critical Value	14.90 <sup>(i)</sup>	8.18 (ii)	17.95	8.18		

<sup>(</sup>i) 5% Critical Value for 2 Unit Roots

As the Johansen approach is designed for vector autoregressive systems, and the errors in the spot and forward regression follow a moving average process, the Phillips cointegrating test for models with positive MA errors is used once again. This involves regressing the errors, obtained from the original regression, as follows;

and computing the test statistic which has a non-parametric correction for correlation.

 $\hat{u}_t = \hat{o}\hat{u}_{t-1} + \hat{k}_t$ 

Where

$$\hat{z}_{\alpha} = T^{-1}(\hat{\alpha} - 1) - (\frac{1}{2})(s_{Tl}^2 - s_k^2)(T^{-2}\sum_{t=1}^{T}\hat{u}_{t-1}^2)^{-1}$$

$$s_k^2 = T^{-1}\sum_{t=1}^{T}\hat{k}_t^k$$

For a choice of lag such as  $w_{sl}=1-s/(l+1)$ .

$$s_{Tl}^2 = T^{-1} \sum_{1}^{T} \hat{k}_t^2 + 2T^{-1} \sum_{s=1}^{T} w_{sl} \sum_{t=s+1}^{T} \hat{k}_t \hat{k}_{t-s}$$

Using the better suited Phillips and Perron test, the presence of a cointegrating vector could not be rejected by any of the countries under observation, as the 10% critical value was -3.04.

<sup>(</sup>ii) 5% Critical Value for 1 Unit Root

**Table 4.12 Phillips and Perron Test Statistics** 

	Phillips and Perron Test for a Cointegrating
	Vector
Australia	-3.07
Canada	-4.00
Germany	-3.12
France	-5.11
U.K.	-4.24
Japan	-3.80
Italy	-5.92
Critical Value	-3.04

In light of this finding for a cointegrating relationship for all countries, two further equations will be estimated. OLS with lags and OLS with leads and lags, or Dynamic OLS, which in the presence of moving average error terms, yields more efficient estimates. The coefficients from these regressions are recorded in tables 4.14 and 4.15. The Maximum Likelihood estimates of the error correction model coefficients are recorded in table 4.13.

#### 4.7 Maximum Likelihood

The Maximum Likelihood estimation technique is specifically designed for data that exhibits a cointegrating relationship. Using the Beta (or Cointegrating Matrix) estimated using Johansen's procedure, the error-correction equations were estimated and the standard errors of the short run and error-correction parameters obtained.

The error correction model was of the form;

$$\Delta f(t) = \mu_1 + \alpha_1 z_{t-3} + \Gamma_{11}(i) \Delta f(t-1) + \Gamma_{12} \Delta f(t-2) + \Gamma_{13} \Delta s_{t+3}(t-1) + \Gamma_{14} \Delta s_{t+3}(t-2) + \varepsilon_1$$
 and

$$\Delta s_{t+3}(t) = \mu_2 + \alpha_2 z_{t-3} + \Gamma_{21} \Delta f(t-1) + \Gamma_{22} \Delta f(t-2) + \Gamma_{23} \Delta s_{t+3}(t-1) + \Gamma_{24} \Delta s_{t+3}(t-2) + \varepsilon_2$$
where  $s_{t+3-2}$ 

where 
$$z_{t-3} = \beta' y_{t-3}$$
 and  $y_{t-3} = \begin{bmatrix} s_{t+3-2} \\ f_{t-2} \end{bmatrix}$ .

The coefficients and associated standard errors are reported in Table 4.13.

Table 4.13 Johansen Maximum Likelihood Estimates of the ECM

Country		Constant	β <sup>(ii)</sup>	$\boldsymbol{lpha^{(i)}}$	$\Delta f_{t-1}$	$\Delta f_{t-2}$	$\Delta s_{(t+3)-1}$	$\Delta s_{(t+3)-2}$	LR
•			•				(0.5) 1	(113)-2	Test
Australia	$\Delta f_t$	0.11	0.37	-1.67	-0.42	-0.34	0.13	0.14	0.520
		(0.02)		(0.15)	(0.10)	(0.08)	(0.06)	(0.06)	
	$\Delta s_{t+3}$	-0.03	-0.37	0.07	-0.009	0.20	0.22	0.25	
		(0.05)		(0.34)	(0.22)	(0.19)	(0.13)	(0.13)	
Canada	$\Delta f_t$	0.09	0.20	-2.47	-0.34	-0.38	0.05	-0.38	4.191
		(0.03)		(0.24)	(0.10)	(0.08)	(0.04)	(0.08)	
	$\Delta s_{t+3}$	-0.20	-0.19	1.12	0.06	0.25	-0.12	0.17	
		(0.09)		(0.64)	(0.28)	(0.12)	(0.12)	(0.22)	
Germany	$\Delta f_t$	-0.06	0.52	-1.01	-0.16	-0.38	0.16	0.16	0.142
		(0.02)		(0.13)	(0.11)	(0.10)	(0.07)	(0.07)	
	$\Delta s_{t+3}$	-0.04	-0.51	-0.04	0.16	0.10	0.06	-0.05	
		(0.03)		(0.24)	(0.20)	(0.18)	(0.13)	(0.13)	
France	$\Delta f_t$	-0.12	0.19	-1.14	-0.04	-0.12	0.10	0.15	0.167
		(0.04)		(0.26)	(0.12)	(0.11)	(0.04)	(0.04)	
	$\Delta s_{t+3}$	0.09	-0.18	2.83	0.39	0.90	-0.42	-0.22	
		(0.11)		(0.81)	(0.36)	(0.35)	(0.12)	(0.14)	
U.K.	$\Delta f_t$	0.03	0.34	-2.45	-0.54	-0.63	0.04	0.35	3.227
		(0.03)		(0.22)	(0.10)	(0.09)	(0.58)	(0.07)	
	$\Delta s_{t+3}$	-0.07	-0.34	-0.19	0.20	0.02	0.16	-0.12	
		(0.05)		(0.39)	(0.18)	(0.15)	(0.12)	(0.12)	
Japan	$\Delta f_t$	0.04	0.46	-1.41	-0.47	-0.47	0.13	0.18	9.271
		(0.02)		(0.16)	(0.10)	(0.10)	(0.08)	(0.08)	
	$\Delta s_{t+3}$	-0.07	-0.44	0.07	-0.04	0.11	0.02	0.33	
		(0.03)		(0.21)	(0.14)	(0.13)	(0.11)	(0.11)	
Italy	$\Delta f_t$	0.40	0.16	-3.70	-0.45	-0.68	0.10	0.11	6.493
		(0.05)		(0.32)	(0.09)	(0.09)	(0.04)	(0.05)	
	$\Delta s_{t+3}$	0.05	-0.15	-0.83	-0.15	-0.25	-0.23	0.11	
		(0.15)		(0.87)	(0.24)	(0.25)	(0.12)	(0.13)	

<sup>(</sup>i) Data in parenthesis are standard errors

Table 4.14 shows the re-estimated error-correction model results with the normalised cointegrating Eigenvector (B). These Eigenvectors are normalised with respect to their associated expected future spot rates.

<sup>(</sup>ii)  $\alpha$  represents the Adjustment Matrix

<sup>(</sup>iii)  $\beta$  represents the cointegrating eigenvector

Table 4.14 Normalised Maximum Likelihood Estimates of the ECM

Country	Dependent	Constant	B <sup>(ii)</sup>	$\alpha^{(i)}$	$\Delta f_{t-1}$	$\Delta f_{t-2}$	$\Delta s_{(t+3)-1}$	$\Delta s_{(t+3)-2}$	LR
	Variable								Test
Australia	$\Delta f_t$	0.002	1	-0.01	-0.42	-0.34	0.13	0.14	0.520
		(0.03)		(0.09)	(0.10)	(0.08)	(0.06)	(0.06)	
	$\Delta s_{t+3}$	0.06	-1	-0.73	-0.009	0.20	0.22	0.25	
		(0.02)		(0.06)	(0.22)	(0.19)	(0.13)	(0.13)	
Canada	$\Delta f_t$	0.03	1	0.02	-0.34	-0.38	0.05	-0.38	4.191
	·	(0.05)		(0.07)	(0.10)	(0.08)	(0.04)	(0.08)	
	$\Delta s_{t+3}$	0.11	-0.95	-0.80	0.06	0.25	-0.12	0.17	
:		(0.05)		(0.06)	(0.28)	(0.12)	(0.12)	(0.22)	
Germany	$\Delta f_t$	0.03	1	-0.08	-0.16	-0.38	0.16	0.16	0.142
	_	(0.02)		(0.06)	(0.11)	(0.10)	(0.07)	(0.07)	
	$\Delta s_{t+3}$	-0.001	-0.98	-0.70	0.16	0.10	0.06	-0.05	
		(0.02)		(0.07)	(0.20)	(0.18)	(0.13)	(0.13)	
France	$\Delta f_t$	-0.05	1	0.006	-0.04	-0.12	0.10	0.15	0.167
		(0.04)		(0.05)	(0.12)	(0.11)	(0.04)	(0.04)	
	$\Delta s_{t+3}$	-0.10	-0.95	-0.85	0.39	0.90	-0.42	-0.22	
		(0.07)		(0.09)	(0.36)	(0.35)	(0.12)	(0.14)	
U.K.	$\Delta f_t$	-0.05	1	0.11	-0.54	-0.63	0.04	0.35	3.227
		(0.05)		(0.10)	(0.10)	(0.09)	(0.58)	(0.07)	
	$\Delta s_{t+3}$	-0.11	-1	-0.77	0.20	0.02	0.16	-0.12	
		(0.03)		(0.06)	(0.18)	(0.15)	(0.12)	(0.12)	
Japan	$\Delta f_t$	-0.07	1	0.15	-0.47	-0.47	0.13	0.18	9.271
_		(0.04)		(0.10)	(0.10)	(0.10)	(0.08)	(0.08)	
	$\Delta s_{t+3}$	0.03	-0.96	-0.58	-0.04	0.11	0.02	0.33	
		(0.02)		(0.06)	(0.14)	(0.13)	(0.11)	(0.11)	
Italy	$\Delta f_t$	-0.03	1	-0.08	-0.45	-0.68	0.10	0.11	0.146
		(0.06)		(0.04)	(0.09)	(0.09)	(0.04)	(0.05)	
	$\Delta s_{t+3}$	0.17	-0.97	-0.91	-0.15	-0.25	-0.23	0.11	
		(0.11)		(0.07)	(0.24)	(0.25)	(0.12)	(0.13)	

<sup>(</sup>I) Data in parenthesis are standard errors

The coefficients on the lagged differences represent the short run parameters. The  $\alpha$  coefficient represents the adjustment parameter, and indicates the speed of the adjustment back to equilibrium. The error-correction term plays a significant role in determining the change in the expected spot rate  $(\Delta f_t)$  for the United Kingdom, Japan and Italy only.

To test for Unbiasedness, which interestingly is a necessary assumption for the Maximum Likelihood technique (as pointed out in Nouzard and Grennier (1995)), a linear restriction is placed on the  $\beta$  coefficients. The likelihood ratio statistic is compared to the Chi squared

<sup>(</sup>ii) α represents the Adjustment Matrix

<sup>(</sup>iii)  $\beta$  represents the normalised cointegrating eigenvector

statistic with two degrees of freedom, as there are two restrictions imposed on the system. The results fail to reject the null hypothesis for all countries except Japan.

Therefore, by relaxing the assumption of stationarity and modelling the Consensus Economics data in a cointegrating framework using Johansen's Maximum Likelihood technique, the test for Unbiasedness has yielded results which differ from the those results posted in tables 4.2 to 4.9. In particular, the Australian data now fails to reject the Unbiasedness hypothesis.

Chapter 3 introduced another technique (DOLS) which yields efficient coefficients in the cointegrating framework. Hence, the data will be estimated using this technique. As well OLS with lags will be estimated, and coefficients recorded for comparative purposes.

#### 4.8 DOLS & OLS with Lags

The data was estimated via the OLS with Lags (table 4.15) and DOLS (table 4.16) methodology as discussed in Phillips and Loretan (1991). The estimated coefficients from these regressions can be found in Tables 4.14 and 4.15 respectively, for all seven countries. In both the lags and leads (DOLS) and lags regression the critical value for the Wald test was 5.99. The tables also contain the t-tests and Wald tests altered using the Newey West procedure,

which involves multiplying each of these test statistics with the ratio of variances;

Where the long run variance is approximated by 
$$t_{\beta} \times \left(\frac{\hat{\sigma}_{sr}^2}{\hat{\sigma}_{Lr}^2}\right)^{\frac{1}{2}}$$

$$S_{Tl}^2 = T^{-1} \sum_{t=1}^{T} u_t^2 + 2T^{-1} \sum_{j=1}^{l} w_l(j) \sum_{t=j+1}^{T} u_t u_{t-j}.$$

As experienced beforehand, using the U.S. data, the estimated slope coefficients resulting from the DOLS regressions, were very close to the desired value of 1.00.

Table 4.15 OLS estimation of  $s_{t+3} = \alpha + \beta_1 f_t + \beta_2 \Delta f_t + \beta_3 \Delta f_{t-1} + \varepsilon$ 

Country	α	$\mathbf{f_t}$	$\Delta f_t$	$\Delta f_{t-1}$	Wald
Australia	1.00	0.83	0.08	0.06	15.82
	$(1.87)^{(i)}$	(11.58)	(0.86)	(0.60)	
United Kingdom	0.41	0.94	0.07	0.04	6.40
	(1.19)	(24.30)	(0.98)	(0.63)	
France	0.17	1.02	-0.13	-0.13	4.64
	(0.25)	(11.83)	(0.90)	(0.93)	
Italy	0.95	0.91	0.11	0.03	0.20
	(0.48)	(5.11)	(0.49)	(0.48)	
Germany	-0.11	1.02	0.007	-0.04	0.56
	(0.44)	(30.34)	(0.10)	(0.47)	
Canada	0.77	0.87	0.21	0.10	4.64
	(1.24)	(10.58)	(1.44)	(0.65)	
Japan	-0.18	1.01	0.05	-0.01	5.39
	(1.12)	(28.52)	(0.52)	(0.10)	
Wald test Critical Value					5.99 <sup>(ii)</sup>

<sup>(</sup>i) Data in parenthesis are the Newey West corrected t-ratios

When looking at the coefficients in table 4.15, it should be noted that the hypothesis that  $\alpha_0=0$  and  $\alpha_1=1$ , Unbiasedness, is rejected for the Australian and U.K data. Before placing too much emphasis on these results it is necessary to ascertain whether the disturbances are serially correlated. This is important as if the data is serially correlated then there will be feedback in the error terms. This will bias the estimation which will have to be rectified by including leads in the regression (Phillips and Loretan, 1991, p.426).

To test for serial correlation the partial autocorrelation and autocorrelation functions where graphed for each of the regressions in table 4.15. This detection method was used, as it does not discriminate between moving average and autocorrelated error terms, and indicates the order of correlation in either case. In all cases the autocorrelation function displayed significant spikes, whilst the partial autocorrelation function declined, indicating that there

<sup>(</sup>ii) Data in parenthesis is the 5% Chi squared critical value with 2 Degrees of Freedom

were moving average error structures. Moving average errors of order two were found for most countries, however, the U.K., France and Canada indicated MA(1) error. In any case as this estimation technique, of OLS with lags, does not yield efficient results in the presence of a moving average error structure, these results cannot be seen to carry much weight. The leads were included in the regression equations yielding the DOLS estimation results posted in table 4.16.

Table 4.16 DOLS:  $s_{t+3} = \alpha + \beta_t f_t + \beta_2 \Delta f_t + \beta_3 \Delta f_{t-1} + \beta_4 \Delta f_{t+1} + \beta_5 \Delta f_{t+2} + \varepsilon$ 

Country	α	$\mathbf{f_t}$	$\Delta f_t$	$\Delta f_{t-1}$	$\Delta f_{t+1}$	$\Delta f_{t+2}$	Wald
Australia	0.12	0.97	-0.39	0.004	1.06	1.20	5.88
	$(0.45)^{(i)}$	(26.10)	(1.68)	(0.10)	(4.26)	(4.92)	
United Kingdom	0.09	1.00	-0.21	0.005	0.65	1.00	1.09
	(0.48)	(45.60)	(1.26)	(0.13)	(3.74)	(5.84)	
France	0.34	1.02	-0.14	0.05	1.09	1.02	5.17
	(0.58)	(14.05)	(0.26)	(0.43)	(2.06)	(1.93)	
Italy	0.42	0.97	0.45	-0.004	1.16	0.64	0.38
	(0.25)	(6.33)	(0.84)	(0.02)	(2.18)	(1.22)	
Germany	0.04	1.01	-0.02	-0.05	0.60	0.98	9.00
	(0.23)	(46.61)	(0.06)	(1.09)	(2.11)	(3.68)	
Canada	0.007	1.01	-0.41	0.002	1.12	1.43	1.27
	(0.02)	(20.54)	(1.41)	(0.02)	(3.57)	(4.83)	
Japan	-0.11	1.03	-0.15	-0.05	1.02	0.94	4.22
	(1.20)	(51.90)	(0.72)	(0.89)	(4.58)	(4.39)	
Wald test Critical Value							5.99 <sup>(ii)</sup>

<sup>(</sup>i) Data in parenthesis are the Newey West corrected t-ratios

For the leads and lags regression, table 4.16, the null hypothesis could not be rejected in all cases, except Germany. This is very surprising, as the German data had been, up until this point in the research, well behaved. Further, the coefficients and individual t-statistics clearly indicate that the constant term is not significantly different from zero whilst the Slope is. This was also estimated to be numerically close to 1.00 at 1.01.

<sup>(</sup>ii) Data in parenthesis is the 5% Chi squared critical value with 2 Degrees of Freedom

In Summary, if the data can be considered nonstationary then Germany and Australia fail to accept the premise of Unbiasedness. If, however, the data is considered stationary there is evidence that at least two countries do not possess Unbiased and Efficient forecasters, namely, Japan and Australia, as they consistently failed to accept the null hypothesis. This could be due to individual agents making absurd forecasts and hence altering the mean, or agents perceiving the market to be inefficient and/ or, including a Term Premia in the determination of the interest rates as a reward for loss of liquidity. Australia does not have a well developed long security market, and hence perhaps long run analysis, cointegration, is not suited to the data. Nevertheless, Australian forecasters do appear to improve in Efficiency when more information becomes available to agents, as highlighted by the acceptance of the principle of Unbiasedness in sub-sample 3.

Intuitively interest rate data should not be nonstationary, as this would imply that they could run at levels of positive or negative infinity. Rather, it would be expected that interest rates may deviate from the mean for prolonged periods of time, but that they would eventually move back in line with a mean. For political, social and economic reasons, it is obvious that interest rates cannot run at excessively high or low rates for a considerable period of time. Hence, Chapter 5 looks at an alternative assumption about the generation of interest rate data and how best to model this data.

# **Chapter 5**

### **Monte Carlo Simulation**

The previous chapters have assumed that the interest rate data was either stationary or contained a unit root. While the assumption of the data being stationary was dismissed, intuitively the alternate of there being an exact unit root may also be incorrect. In fact, it is more likely that the data contains a local to unity root. That is, the data will return to equilibrium eventually. Stock (1995) highlighted the problems associated with using I(0) or I(1) approaches when, in fact, the data contained a local to unity root. This chapter seeks to determine how the presence of a local to unit root affects the performance of the OLS, DOLS and Johansen Maximum Likelihood estimators. As well as how the hypothesis of Unbiasedness is affected. The data generating process is constructed to ensure that its exact nature is known. Further, the data is constrained to be overlapping so that it is equivalent to the *Consensus Economics* data used throughout this thesis.

With the data generating process established, the comparative stability of the three estimation techniques, OLS with MA(2) errors, Johansen Maximum Likelihood and DOLS estimation, can be determined via a Monte Carlo experiment. These three techniques were utilised as they have been used in conjunction with different data generating assumptions. The OLS with MA(2) errors regression assumes that the underlying data process is stationary, whilst the Johansen and DOLS procedures assume an I(1) process.

The simple equation,  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$ 

was estimated using OLS correcting for MA(2) errors and this process was replicated 1000 times. The coefficients were collected and then averaged with the average coefficient values recorded. This process was repeated using the DOLS and Johansen Maximum Likelihood estimation techniques. To begin with the correct data generating assumptions for each of these estimation techniques were enforced. That is, the data was assumed to be I(0) when the equation was estimated using OLS and when the DOLS and Johansen procedures were used the data was I(1). This was for the purpose of yielding a control variable, which formed the basis for comparison when these data generating assumptions were relaxed.

Additionally the tests for Unbiasedness were repeated over the 1000 replications. A count was made of the amount of times the test statistic exceeded the critical value and this percentage was also recorded. Through the use of this Monte Carlo experiment, it should become evident how incorrectly modelling the data affects the percentage of times the Unbiasedness hypothesis is rejected.

Originally, two stationary data generating processes were considered. One modelled the moving average error process directly,

(1) 
$$s_{t+3} = \rho * f_t + \mu_{t+3}$$
$$\mu_{t+3} = \mu + \theta_1 \mu_{t+2} + \theta_2 \mu_{t+1}$$

the other indirectly through the construction of an overlapping data problem. This was achieved by setting,

$$f_t = \rho * f_{t-1} + \varepsilon_t$$

and then generating;

$$s_{t+3} = f_t + \mu_t$$

Using the data generated from (2) the equation,

$$s_{t+3} = \alpha_0 + \alpha_1 f_t + \varepsilon_{t+3}$$

Was estimated using Ordinary Least Squares corrected for MA(2) errors. The coefficients of these moving average error processes were graphed, Graphs 5.1 and 5.2, and revealed that the most frequently estimated error coefficients were 0.5 in value.

Graph 5.1 Frequency Histogram for  $\theta_1$ 

Grapn	3.1	Fraquency histogram for 01
HISTOGR.	AM -	BOLS
PCT.	N	
0.385	385	I
0.369	369	I XXXXXXXXX
0.353	353	I XXXXXXXXX
0.337	337	I XXXXXXXXX
0.321	321	I XXXXXXXXXXXXXXX
0.305	305	I XXXXXXXXXXXXXXX
0.289	289	I XXXXXXXXXXXXXXXXXX
0.273	273	I XXXXXXXXXXXXXXXXX
0.257	257	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.241	241	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.225	225	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.209	209	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.193	193	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.177	177	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.161	161	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.145	145	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.129	129	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.113	113	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.097	97	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.081	81	I XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.065	65	
0.049	49	T XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.033	33	TXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.017	17	TXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.001	1	III IXXXXXXXXXXXXXXXXXXXXXXXX
1		0.610 0.65
L		0.125 0.249 0.372 0.495 0.619 0.742 0.865

Graph 5.2 Frequency Histogram for  $\theta_{2}$ 

HISTOGRAM -	- BOLS	
PCT.	N	
0.361 363	1 I	
0.346 34	6 I	XXXXXXXXX
0.331 333	1 I	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
0.316 31	6 I	xxxxxxxxxxxxxxxx
0.301 30	1 I	xxxxxxxxxxxxxxxx
0.286 28	6 I	xxxxxxxxxxxxxxxx
0.271 27	1 I	xxxxxxxxxxxxxxxx
0.256 25	6 I	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
0.241 24	1 I	xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
0.226 22	6 I	xxxxxxxxxxxxxxxx
0.211 21	1 I	xxxxxxxxxxxxxxxx
0.196 19	6 I	XXXXXXXXXXXXXXXXXX
0.181 18	1 I	XXXXXXXXXXXXXXXXXX
0.166 16	6 I	xxxxxxxxxxxxxxxx
0.151 15	1 I	XXXXXXXXXXXXXXXXXX
0.136 13	6 I	XXXXXXXXXXXXXXXXXX
0.121 12	1 I	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.106 10	6 I	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
	1 I	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.076 7		XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.061 6		XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
	6 I	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
	1 I	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
		XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
0.001		XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
[		II
1	-0.5	36E-01 0.779E-01 0.209 0.341 0.472 0.604 0.735

Graphs, 5.1 and 5.2, were taken from the estimation with a sample size of 77 which is in line with the actual data set, as is the assumption that both error coefficients equal 0.5. For comparative reasons error coefficients equalling 0.6 and 0.3 were deemed appropriate from the frequency histogram. Altering the error coefficients enables the stability of the model to be assessed as does altering the sample size. The sample size was chosen to be 77, for the reason mentioned previously, and as well a value of 200 was decided on as this is a reasonably large sample size when dealing with monthly data. The data generating process used to determine the value of the moving average coefficients, ie that process which sought to measure the moving average process indirectly, exhibited extreme sensitivity to changing assumptions. As well it yielded F-test statistics that were at odds with those found using the actual data sets and those found using the data generating process (2). The problem lay in the fact that process (1) was not invertible. Therefore, for the remainder of this chapter the data is generated with the moving average coefficients imposed directly, that is process (2) was used.

With the Data Generating Process being;

$$f_{t} = \rho * f_{t-1} + \varepsilon_{t}$$

$$s_{t+3} = f_t + \mu_{t+3}$$

Equation (3) was estimated under the assumption of stationarity,  $\rho$ =0, using OLS with MA(2) errors and this estimation was replicated 1000 times. The sample size and moving average error coefficients were altered with the equation being re-estimated following each alteration. The average values of these estimated coefficients are exhibited in graph 5.3.

Next a unit root was imposed on the data generating process by constraining  $\rho=1$  in the DGP.  $f_t = \rho * f_{t-1} + \varepsilon_t$ 

The following equation was then estimated;

(4) 
$$s_{t+3} = \alpha + \alpha_1 f_t + \alpha_2 \Delta f_t + \alpha_3 \Delta f_{t-1} + \alpha_4 \Delta f_{t+1} + \alpha_5 \Delta f_{t+2} + \varepsilon$$

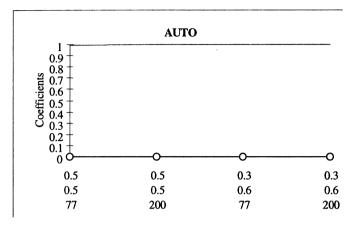
which is otherwise known as Dynamic OLS. The average values of these estimated coefficients were collected and displayed table 5.2.

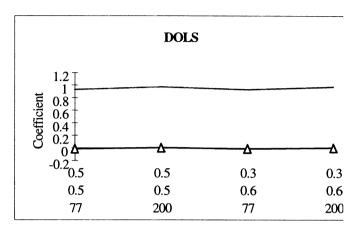
These values, table 5.1 and 5.2, are the efficient values as the correct estimation techniques are used for the underlying data structure. That is, the data was made stationary for the

moving average estimation, and I(1) for the DOLS estimation. In fact, for the stationary process, the extreme case of a white noise process was enforced on the data.

**Graph 5.3 Estimated Moving Average Regression Coefficients** 

**Graph 5.4 Estimated DOLS Regression Coefficients** 

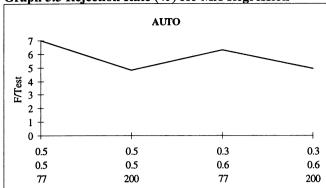




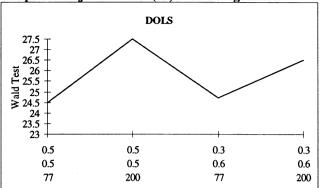
The line dotted with symbols represents the coefficients in the moving average and DOLS regressions when the sample size and the error coefficients are altered. The line marked by the circles and triangles represents the intercept term in each of these regressions. These graphs show that the equations are very stable as the coefficients are not greatly affected by either altering the sample size or the moving average coefficients. Further, the coefficients are close to the desired levels of zero and one.

The system also reported the number of times the F-test statistic (or Wald for the DOLS regression) exceeded the critical value of 3.15 (5.99), when testing the joint hypothesis that  $\alpha_0=0$  and  $\alpha_1=1$ . In other words, a count was made of how many times the hypothesis of Unbiasedness was rejected over the 1000 replications. For the varying sample sizes and moving average coefficients, the percentage of times that the F-Test (or Wald test) exceeded the critical value(s) is recorded in tables 5.1 and 5.2 respectively. A graphical representation of these percentages is recorded in graphs 5.5 and 5.6.

Graph 5.5 Rejection Rate (%) for MA Regression



Graph 5.6 Rejection Rate (%) DOLS Regression



The percentage of times the F-test statistic exceeded the critical value, or the percentage of times the hypothesis of Unbiasedness was rejected, was extremely low for the moving average regression. The recorded percentage levels were very close regardless of the sample size or value of the moving average coefficients. The DOLS regression also accepted the null hypothesis of Unbiasedness the majority of the time. The rejection levels are considerably higher, when using the DOLS model, due to the intercept term being significant on occasion during the 1000 replications. Given the value of the intercept coefficients obtained using the *Consensus Economics* data, this is not unusual. The models appear relatively stable in their rejection of the null, even though the DOLS model seems sensitive to the change in sample size the actual percentage values are very close to each other. The standard errors have been corrected in relation to the Wald test. As these counts record how many times the null is rejected and considering the fact that these percentages are low, there is evidence that the hypothesis of Unbiasedness is supported by the data over the 1000 replications.

These experiments assumed that the models contained either a unit root or were stationary and hence assessed how well the equations coped under these assumptions. Next the supposition that this root is exactly one is relaxed and the models are tested under the local to unity methodology, a concept introduced in the previous chapter. Formally a process is nearly integrated if it satisfies  $x = \alpha_T x_{t-1} + u_t$ , where  $u_t$  is a stationary process and  $\alpha_T = 1 - \frac{c}{T}$  here c is a constant and it is obvious that as T goes to infinity,  $\alpha_T$  approaches one. Therefore, for very large samples  $x_t$  is very close to a unit root process. What the existence of such a process may do to the estimators used throughout this chapter is uncertain. A recent manuscript delivered by Stock (1995) outlines the problems associated with local to unity roots with

respect to three common econometric questions. He pointed out that econometricians commonly have evidence that they have a large autoregressive root but that they are unsure that this is exactly, rather than nearly, one. In his article Stock analyses the properties associated with the presence of local to unity roots with respect to three practical problems. One of these is closely related to the modelling issue addressed in this thesis and shall therefore be concentrated on. This problem concerns tests of the hypothesis that  $x_{t-1}$  does not predict  $y_t$ , and  $x_t$  is serially correlated and possibly has a unit autoregressive root. The leading example given by Stock is tests in linear rational expectations models with highly autoregressive regressors, for example, tests of whether stock returns can be predicted by lagged variables, such as, the dividend yield.

If in this problem  $x_t$  is I(1),  $(\alpha=1)$  then the methods of stationary time series econometrics will be inappropriate and this problem should be handled using the technology of integration and cointegration. However, if  $\alpha_T$  is large but not exactly one, then both the I(0) approach and I(1) approach can produce systematic errors in inference. The distribution depends on the nuisance parameter,  $\alpha_T$ .

If 
$$T^{\frac{1}{2}}x_{[TS]}$$

Converges to a Brownian motion under the hypothesis that  $x_t$  is I(1), then Stock found that the asymptotic power of the test for an autoregressive unit root with fixed size against the alternative of;  $\overline{\alpha}_T = 1 + \frac{\overline{c}}{T}$ 

was strictly less than one. Further, Stock stated that it followed that if the critical value tended to infinity without the sample size (which is what is needed if the test is to distinguish between I(1) and I(0) processes consistently) then the asymptotic power of the test against the local to unity alternative was zero. In other words it is measurable to distinguish between I(1) and local to unity.

To consider, in more detail, the effects of a local to unity root on hypothesis testing and the OLS estimator, Stock considered the bivariable model

$$x_{t} = \mu_{t} + v_{t}, (1 - \alpha L)v_{t} = u_{1t}, u_{1t} = a(L)^{-1} \varepsilon_{1t}$$

$$y_{t} = \mu_{y} + \gamma x_{t-1} + u_{2t}, u_{2t} = \varepsilon_{2t} \text{ where } \alpha_{T} = 1 - \frac{c}{T} \text{ (Stock, 1995, p.8)}.$$

Suppose the econometrician wants to test whether  $\gamma = \gamma_0$ , in the linear application of the Rational Expectations hypothesis  $\gamma=0$ . Stock found that the limiting distribution of the t-statistic when testing  $\gamma = \gamma_0$  depended on the nuisance parameter c and  $\gamma$ . This led to a non-standard asymptotic distribution of the  $t_{\gamma}$  and the asymptotic inference could not, in general, rely on simply substituting a suitable estimator for c when selecting critical values for the tests of  $\gamma_0$ .

Stock used a Monte Carlo simulation to determine whether this issue was important in sample sizes typically encountered in econometrics. The autoregressive process in this simulation consisted of three equations,

$$\Delta y_{t} = 0.4 \widetilde{\Delta} y_{t-1} + \varepsilon_{2t}$$

$$c_{t} = \theta y_{t} + \varepsilon_{1t}$$

$$r_{1t} = \varepsilon_{3t}$$

where  $\tilde{\Delta} = 1 - \alpha L$ . For T=100 the values of  $\alpha$  were (1,0.95,0.90). These corresponded to the arbitrary choice of the rate of c being; 0, -5 and -10 (Stock, 1995, p.12).

Testing the hypothetical problem, mentioned earlier, coincided to the testing whether  $y_{t-1}$  forecasts  $r_t$  at the 5% significance level in a regression of  $r_t$  onto  $y_{t-1}$ . Analysing this problem under two hypothesises of I(0) and I(1) roots, Stock found that when alpha equalled one, the I(0) methodology did poorly with large size distortions and prediction intervals which had low coverage. The I(1) methodology, in this instance, did well. However, when the root was slightly less than one, the I(1) methodology failed. In this chapter Monte Carlo analysis will also be used to determine which methodology, I(0) or I(1), fairs better when the value of alpha deviates from one. Additionally, the estimated mean coefficients will be monitored so as to determine whether the model is stable and not adversely affected by adjustments to the generation of the data.

In summary, Stock noted that the untestable presence of a local to unity root could produce substantial distortions in inference, at least when a regressor is endogenous (Stock, 1995, p.23). With reference to the equations to be estimated in this thesis the Johansen and DOLS estimates meet this criteria and hence the presence of a local to unity root can drastically altered inferences.

At present, the existence of a local to unity root cannot be determined outright. As well, it is not possible to estimate the value of the nuisance parameter, rather, these are decided arbitrarily beforehand. For this thesis the values for c are; 0, 13, 25 and 77. When c=77 the process is stationarity, in fact it is white noise, and when c=0 the data contains a unit root. These are the two extreme cases. The Monte Carlo is devised to allow for these roots to be reflected in the future spot rate variables.

That is, when generating the expected spot rate variables, the process allows for

$$f_t = \alpha_T * f_{t-1} + \varepsilon_t.$$

Where  $\alpha_T = 1 - \frac{c}{T}$ .

This expected spot rate is then fed into the equation,

where

$$s_{t+3} = 1 * f_t + \mu_{t+3}$$
  
$$\mu_{t+3} = \mu + \theta_1 \mu_{t+2} + \theta_2 \mu_{t+1}$$

The Moving Average, DOLS and Error Correction Models (ECM) where estimated using the data generated from the process outlined above. The sample size, moving average coefficients and the rate of c was altered. The average coefficient values for each of these regressions are displayed in tables. 5.1 to 5.3. The percentage of times the test for Unbiasedness failed to be accepted is also recorded in these tables. As well the tables record the percentage of times the t-statistic on the expected spot variable exceeded the critical value of 2.

Note that the DOLS standard errors, T-statistics and Chi Squared statistics are corrected using Newey West's procedure which involves multiplying every one of these test statistics with the ratio of variances;  $t_{\beta} \times \left(\frac{\hat{\sigma}_{sr}^2}{\hat{\sigma}_{Lr}^2}\right)^{\frac{1}{2}}$ 

Where the long run variance is approximated by  $S_{Tl}^2 = T^{-1} \sum_{t=1}^{T} u_t^2 + 2T^{-1} \sum_{j=1}^{l} w_l(j) \sum_{t=j+1}^{T} u_t^2 u_{t-j}$ .

Table 5.1 Moving Average Regression of  $s_{t+3} = \alpha_0 + \alpha_1 ft + \varepsilon_{t+3}$ 

	Sample	MA Coefficient	MA Coefficient				T/Test Rejection Rate	F/Test Rejection Rate
	size	$\theta_1$	$\theta_2$	С	α <sub>0</sub> .	$\alpha_1$	$\alpha_1=0$	$\alpha_0=0 \alpha_1=1$
							%	%
AUTO	77	0.5	0.5	77	0.0010	0.996	100	7.0
		0.5	0.5	0	0.0080	0.997	100	7.0
		0.5	0.5	13	0.0007	0.998	100	6.0
		0.5	0.5	25	0.0008	0.998	100	6.0
AUTO	200	0.5	0.5	77	0.0000	1.000	100	4.8
1		0.5	0.5	0	-0.0040	0.999	100	5.3
l		0.5	0.5	13	0.0007	1.000	100	4.3
		0.5	0.5	25	0.0000	1.000	100	4.8
AUTO	77	0.6	0.3	77	0.0005	0.997	100	6.3
		0.6	0.3	0	0.0000	0.999	100	5.7
	ļ	0.6	0.3	13	-0.0002	0.998	100	5.4
		0.6	0.3	25	0.0002	0.998	100	5.9
AUTO	200	0.6	0.3	77	0.0000	1.000	100	4.9
		0.6	0.3	0	-0.0004	0.999	100	5.4
		0.6	0.3	13	0.0007	1.000	100	4.5
1		0.6	0.3	25	0.0000	1.000	100	4.4

Table 5.2 DOLS estimation;  $s_{t+3} = \alpha_0 + \alpha_1 f_t + \alpha_2 \Delta f_t + \alpha_3 \Delta f_{t-1} + \alpha_4 \Delta f_{t+1} + \alpha_5 \Delta f_{t+2} + \varepsilon$ 

	Sample										T/Test	Wald
	Size	$\theta_1$	$\theta_2$	c	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	α4	$\alpha_5$	$\alpha_1=1$	$\alpha_0=0$ $\alpha_1=1$
											%	%
DOLS	77	0.5	0.5	77	-0.020	-0.09	0.0020	0.000	-0.05	-0.02	0.0	53.4
		0.5	0.5	0	-0.015	0.93	0.0130	0.009	0.92	0.94	99.8	24.5
ŀ		0.5	0.5	13	-0.022	0.76	0.0280	0.020	0.77	0.80	95.4	33.8
		0.5	0.5	25	-0.021	0.6	0.0240	0.014	0.62	0.65	68.2	39.5
DOLS	200	0.5	0.5	77	-0.007	-0.05	0.0210	0.007	-0.02	-0.01	0.5	92.6
		0.5	0.5	0	0.002	0.97	0.0010	-0.002	0.98	0.97	100.0	27.5
		0.5	0.5	13	0.009	0.91	0.0070	0.003	0.92	0.91	100.0	37.4
		0.5	0.5	25	0.009	0.85	0.0080	0.003	0.86	0.86	100.0	49.5
DOLS	77	0.6	0.3	77	-0.020	-0.09	0.0200	0.000	-0.05	-0.02	7.1	55.5
		0.6	0.3	0	-0.014	0.93	0.0120	0.010	0.92	0.94	99.9	24.7
		0.6	0.3	13	-0.020	0.76	0.0270	0.020	0.77	0.80	96.5	34.2
		0.6	0.3	25	-0.020	0.60	0.0200	0.010	0.62	0.64	70.7	40.9
DOLS	200	0.6	0.3	77	-0.007	-0.05	0.0200	0.007	-0.02	-0.014	08.1	93.7
ŀ		0.6	0.3	0	-0.001	0.97	0.0007	-0.002	0.98	0.97	100.0	26.5
		0.6	0.3	13	-0.009	0.91	0.0060	0.004	0.92	0.91	100.0	37.3
L		0.6	0.3	25	-0.009	0.85	0.0070	0.004	0.86	0.86	100.0	50.3

As is evident the DOLS estimates are very similar for the nonstationary and local to unity scenarios but not so for the stationary, white noise, state. This is also reflected in the Wald joint test for Unbiasedness and the t-test for the significance of the slope coefficient. In both cases the percentage of rejections is drastically altered when the data is assumed to be white noise. The same can also be said for the Johansen Maximum Likelihood estimation of the ECM;

$$\Delta f(t) = \mu_1 + \alpha_1 z_{t-3} + \Gamma_{11}(i)\Delta f(t-1) + \Gamma_{12}\Delta f(t-2) + \Gamma_{13}\Delta s_{t+3}(t-1) + \Gamma_{14}\Delta s_{t+3}(t-2) + \varepsilon_1$$

and

$$\Delta s_{t+3}(t) = \mu_2 + \alpha_2 z_{t-3} + \Gamma_{21} \Delta f(t-1) + \Gamma_{22} \Delta f(t-2) + \Gamma_{23} \Delta s_{t+3}(t-1) + \Gamma_{24} \Delta s_{t+3}(t-2) + \varepsilon_2$$
where  $z_{t-3} = \beta' y_{t-3}$  and  $y_{t-3} = \begin{bmatrix} s_{t+3-2} \\ f_{t-2} \end{bmatrix}$ .

This technique is normally used to estimate data that is I(1) and is cointegrated.

Table 5.3 Johansen Maximum Likelihood Estimation of the ECM

Table		en Maximu					ECM					
	Sample	$\theta_1$	$\theta_2$	С	Constant	$\beta^1$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	LR/Test
	Size				0.010	0.000	0.50	0.10	0.66	0.17	0.26	11.60
$\Delta f_t$	77	0.5	0.5	77	-0.010	-0.003	-2.53	-0.10	-0.66	-0.17	-0.36	11.60
$\Delta s_{t+3}$		0.5	0.5	77	-0.010	0.109	-6.70	-0.57	-0.33	-0.41	-0.41	
$\Delta f_t$		0.5	0.5	0	0.005	0.119	0.18	-0.01	-0.02	-0.02	-0.02	0.02
$\Delta s_{t+3}$		0.5	0.5	0	-0.004	-0.124	5.07	-0.54	0.51	-0.35	0.30	
$\Delta f_t$		0.5	0.5	13	-0.008	0.116	0.25	-0.01	-0.10	-0.02	-0.08	0.00
$\Delta s_{t+3}$		0.5	0.5	13	-0.010	-0.124	5.17	-0.55	0.42	-0.36	0.23	
$\Delta f_t$		0.5	0.5	25	-0.008	-0.107	-0.48	-0.02	-0.18	-0.04	-0.13	0.00
$\Delta s_{t+3}$		0.5	0.5	25	-0.010	0.123	-5.37	-0.55	0.32	-0.37	0.16	
$\Delta f_t$	200	0.5	0.5	77	-0.004	-0.048	-7.68	-0.09	-0.86	-0.14	-0.76	37.80
$\Delta s_{t+3}$		0.5	0.5	77	-0.002	0.068	-11.00	-0.43	-0.63	-0.15	-0.99	
$\Delta f_t$		0.5	0.5	0	-0.003	0.071	0.19	-0.01	-0.00	-0.01	-0.00	0.09
$\Delta s_{t+3}$		0.5	0.5	0	-0.004	-0.073	8.04	-0.53	0.51	-0.31	0.29	
$\Delta f_t$		0.5	0.5	13	-0.003	0.066	0.47	-0.01	-0.03	-0.02	-0.02	0.03
$\Delta s_{t+3}$		0.5	0.5	13	-0.003	-0.071	8.47	-0.54	0.48	-0.32	0.26	
$\Delta f_t$		0.5	0.5	25	-0.003	0.064	0.66	-0.02	-0.06	-0.03	-0.04	0.04
$\Delta s_{t+3}$	•	0.5	0.5	25	-0.003	-0.071	8.62	-0.54	0.44	-0.32	0.23	
$\Delta f_t$	77	0.6	0.3	77	-0.010	0.040	2.25	-0.07	-0.64	-0.15	-0.29	3.30
$\Delta s_{t+3}$		0.6	0.3	77	-0.010	0.100	7.34	-0.74	-0.07	-0.44	-0.21	
$\Delta f_t$		0.6	0.3	0	-0.004	0.126	0.00	0.00	-0.02	-0.01	-0.03	0.00
$\Delta s_{t+3}$		0.6	0.3	0	-0.008	-0.132	4.69	-0.71	0.69	-0.35	0.32	
$\Delta f_t$		0.6	0.3	13	-0.008	0.120	0.33	-0.01	-0.09	-0.03	-0.08	0.00
$\Delta s_{t+3}$		0.6	0.3	13	-0.008	-0.131	4.91	-0.72	0.59	-0.37	0.24	
$\Delta f_t$		0.6	0.3	25	-0.010	-0.107	-0.61	-0.02	-0.18	-0.06	-0.12	0.00
$\Delta s_{t+3}$		0.6	0.3	25	-0.008	0.129	-5.09	-0.72	0.49	-0.39	0.16	
$\Delta f_t$	200	0.6	0.3	77	-0.003	0.101	-7.96	-0.05	-0.93	-0.11	-0.86	51.90
$\Delta s_{t+3}$		0.6	0.3	77	-0.002	0.017	-9.76	-0.58	-0.48	-0.06	-1.06	
$\Delta f_t$		0.6	0.3	0	-0.003	0.075	-0.02	-0.00	-0.01	-0.00	-0.01	0.00
$\Delta s_{t+3}$		0.6	0.3	0	-0.003	-0.077	7.16	-0.70	0.70	-0.31	0.30	
$\Delta f_t$		0.6	0.3	13	-0.003	0.070	-0.03	-0.00	-0.04	-0.00	-0.04	0.00
$\Delta s_{t+3}$		0.6	0.3	13	-0.003	-0.075	7.15	-0.70	0.66	-0.31	0.27	
$\Delta f_t$		0.6	0.3	25	-0.003	0.067	-0.03	-0.00	-0.07	-0.00	-0.07	0.00
$\Delta S_{t+3}$		0.6	0.3	25	-0.003	-0.075	8.16	-0.70	0.63	-0.31	0.24	
	l	<u> </u>	0.5		0.005	0.075		1 0.70	0.05	1 0.51		L

<sup>1</sup> This table displays the non-normalised eigenvectors  $\beta$  whereby the estimated equation is;  $\Delta f_t = \frac{\alpha}{\gamma} (f_t - \beta y_{t-3})$ 

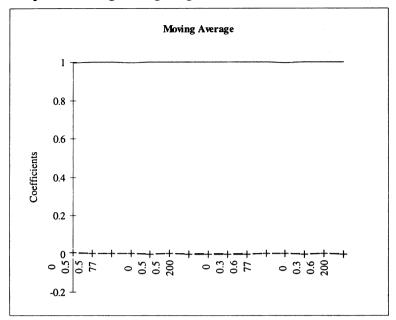
Table 5.3 exhibits the results from the Johansen procedure. Both the Johansen and DOLS techniques do not fair well when their fundamental assumption of nonstationarity is relaxed. The coefficients are so different that they shall not be viewed in graphical form, rather for the DOLS and Johansen technique comparisons will only be made between the local to unity and I(1) data generating processes. The likelihood ratio test which tests the restriction of the beta coefficient being [1, -1] found that this restriction could not be rejected in the majority of the time, and hence found evidence for Unbiasedness. However, when the data was made white noise, this estimation technique yielded vastly different coefficients and in one case found the restrictions to not be valid 51.9 percent of the time.

A graphical analysis of the mean coefficient values will now be undertaken. The X axis records the assumptions that underlie the generation of the data, such as, the sample size, the moving average error coefficients and the value of c. For the Moving Average and DOLS regressions the desired coefficient values are; a constant term close to zero and an intercept term close to one.

### **5.1 The Coefficients**

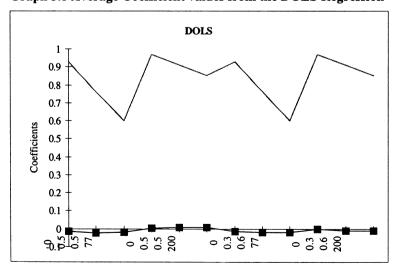
Graphs 5.7 to 5.10 exhibit the mean values of the coefficients on the constant and expected spot rate under the different scenarios. That is, for a unit root, c=0 or  $\alpha_T=1$ , and for the two local to unity values for c=13 and c=25. For the moving average estimation (graph 5.7) the stationary coefficients are also included (c=77). The dotted line, in this graph, represents the intercept (constant) term and the full line the slope coefficient. Regardless of the underlying data process, both coefficients remained very stable and are in line with the values recorded throughout this thesis.

**Graph 5.7 Moving Average Regression Coefficients** 

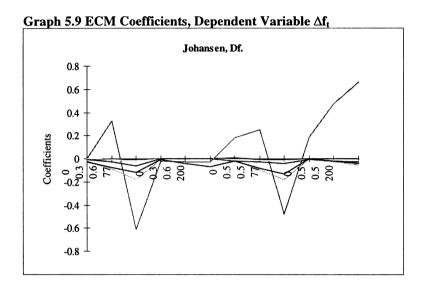


The DOLS, graph 5.8, estimates did not fare so well. The slope coefficient, represented by the full line, moved further from the desired level of one the further the root moved from unity. However, the fall was not as drastic when the sample size increased to 200 data points, as the value of  $\alpha$  did not deviate so far from one. The smallest root being 0.88. The intercept term, the line dotted with squares, interestingly remained uniform no matter the underlying assumptions.

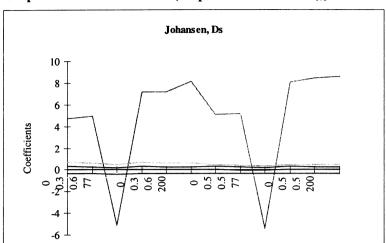
**Graph 5.8 Average Coefficient values from the DOLS Regression** 



Graphs 5.9 and 5.10 display the movements of all coefficients in the Johansen Maximum Likelihood estimation of the ECM. The coefficient displaying the most alteration is that of the adjustment coefficient. This alters radically the further the process moves from a unit root, as the adjustment coefficient needs to work harder to bring the equation into equilibrium. Interestingly, this coefficient is very stable when the sample size increases and when the moving average error coefficients are 0.6 and 0.3 respectively. However, the same cannot be said when the moving average error term coefficients are both 0.5 in value.



Graph 5.10 relates to the corresponding equation estimated by Johansen process with the differenced spot rate as the dependent variable, as opposed to the differenced forecast rate. Once again the adjustment coefficient shows the most movement, particularly as we move further from a unit root. When the sample size is increased the root becomes very close to unity and the adjustment coefficient remains fairly consistent in size. This is true regardless of the error process.

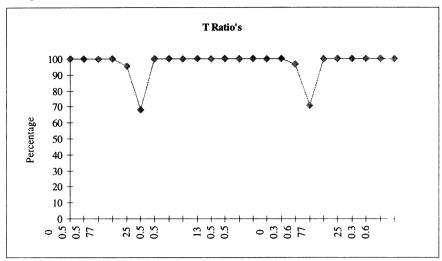


Graph 5.10 ECM Coefficients, Dependent Variable Δs<sub>t+3</sub>

#### **5.2 Hypothesis Tests**

In relation to the hypothesis tests, the acceptance of the premise that the slope coefficient was one could not be rejected on most occasions. The moving average regression supported the hypothesis of the slope coefficient 100% of the time regardless of the moving average coefficients or the value of the roots. For the DOLS equation when the sample size was relatively small, that is there were 77 observations, the value of the local to unity root,  $\alpha_T$ , greatly affected the percentage of t-ratios that were significant. With fewer coefficients being significantly different from zero the further  $\alpha_T$  moved from 1. When the sample size increased and hence the effect of the changing c dissipated, all coefficients were reportedly significantly different from zero. This can be seen clearly in Graph 5.11.

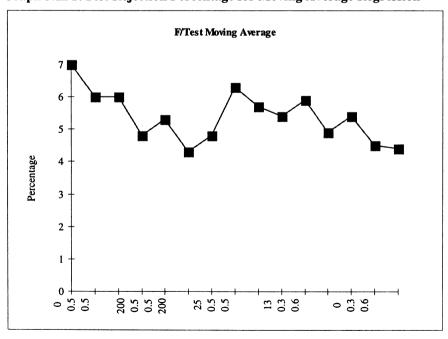
**Graph 5.11 T-Test Rejection Rate** 



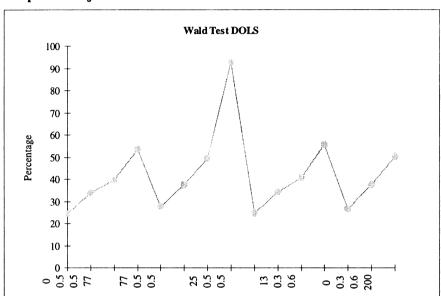
With regards to the more important and informative joint tests, for the moving average regression the rejection levels were extremely low regardless of the underlying data assumptions, as displayed in graph 5.12. As the null is true,  $\alpha_0=0$  and  $\alpha_1=1$ , we would expect rejection probabilities of near 5%. The GLS or MA(2) regression does well. In fact rejection levels move closer to 5% the further  $\alpha_T$  moved from one. The joint test was, once again,  $\alpha_0=0$  and  $\alpha_1=1$  for the regression,

$$s_{t+3} = \alpha_0 + \alpha_l f_t + \varepsilon_{t+3}.$$

Graph 5.12 F/Test Rejection Percentage for Moving Average Regression



For the DOLS rejection levels alter quite significantly depending on the value of  $\alpha_T$ . With the highest levels being recorded the further  $\alpha_T$  is from 1. Interestingly when the sample size increases the rejection rate does not reduce, in fact it increases. These percentages are far from the desired level of 5%.



Graph 5.13 Rejection Rates for the Wald test

Graph 5.13 plots the rejection rates of the joint test for Unbiasedness, the Wald test. It appears that the larger the value of c, or the smaller the value of  $\alpha_T$ , the greater the number of times the null hypothesis cannot be accepted. In short the further the data moves from an I(1) process the more times the DOLS technique cannot support the notion of Speculative Efficiency. Even for the larger sample size the DOLS estimates and test results were affected quite markedly when the data moved from an I(1) process.

As far as the Unbiasedness hypothesis is concerned when the correct estimation techniques are used for the underlying data structure, the Monte Carlo simulations indicate that the hypothesis cannot be rejected the majority of the time.

#### 5.3 Coverage

To further ascertain the ability of the three estimation techniques, confidence intervals were constructed and a count was made for each time the relevant estimated coefficients lay within this region. For a sample size of 77 the moving average regression was estimated with the data being white noise. This yields efficient estimates and allows normal tables to be used in the construction of the confidence region using  $\hat{\alpha}_1 = \pm t_{\alpha/2} se(\hat{\alpha}_1)$ . This 95% confidence interval was then used when the true process was made to be I(1) but the estimation technique was reliant on the assumption of stationarity. The true data process was then altered to reflect a local to unity root with c=25. A similar technique was then used for the two I(1) regressions and the counts were recorded in table 5.4.

Table 5.4 Coverage Percentages, Sample size 77

DGP/ Estimation	I(0)	I(1)	I(α)
T=77			c=25
I(0)	95%	98%	89.8%
I(1) DOLS	0.4%	95%	19.3%
I(1) Johansen	11.4%	95%	9.4%

Table 5.5 shows the percentage of times the coefficients lay within the desired confidence region when the sample size was increased to 200.

Table 5.5 Coverage Percentages, Sample size 200

DGP/ Estimation	I(0)	I(1)	I(a)	Ι(α)
T=200			c=13	c=25
I(0)	95%	99.6%	97%	94.9%
I(1) DOLS	0.2%	95%	38%	17.7%
I(1) Johansen	6.7%	95%	11.5%	5.1%

Evidently the I(1) regressions behaved poorly in both the white noise and local to unity cases and interestingly the larger the sample size the fewer the times the coefficients lay within the

confidence interval. However when the value of c was reduced to 13 and therefore the local to unity root was very close to 1, the DOLS technique improved quite substantially as did the Johansen procedure though from a lower base. In all cases the moving average regression performed very well and the larger the sample size the higher the count.

In summary, the Monte Carlo Simulation revealed that the Moving Average regression produced, on average, coefficients that were close to the desired levels of zero and one. The experiment also indicated that the moving average regression failed to reject the concept of Unbiasedness the majority of the time. Both of these results occurred regardless of the sample size, moving average error coefficients or value of  $\alpha_T$ . This is in line with the previous findings in this thesis, when the *Consensus Economic* data was used the coefficients reported in tables 2.5 and 4.6 are close to zero and one. As well, the joint test results indicated that the hypothesis of Unbiasedness could not be rejected for all countries except Australia.

The DOLS regression also yielded coefficients close to zero and one in value. However, the joint test of this hypothesis resulted in percentages that varied greatly when assumptions about the data changed. Also, DOLS exhibited low coverage when the data process strayed from the I(1) methodology. The DOLS results obtained from the use of the Consensus data showed this technique to yield slope coefficients very close to the desired one. The hypothesis of Unbiasedness could not be accepted on only one occasion, for the German data. Clearly, this technique relies heavily on the presence of a Unit root in the data and does not perform well when this assumption is relaxed.

The Johansen procedure performed badly on all accounts. It yielded very low coverage rates when the assumption of a unit root was relaxed. Apparently, if the data is believed to contain a local to unity root the best estimation technique to use is a simple OLS regression.

## Chapter 6

### **Conclusion**

The main objectives of this thesis were to;

- (1) Empirically assess the sensitivity of the results of the tests of Unbiasedness and Efficiency of forecasters to different estimation techniques and
- (2) By imposing a local to unity root evolve the previous literature, which had moved from assumptions of stationarity to finding that the interest rate data was I(1).

In both cases the estimation techniques used were those which had frequently been utilised in the related Term Structure of Interest Rate literature. These being OLS corrected for moving average errors, the GMM, Johansen's Maximum Likelihood estimation of an error-correction model (ECM) and Dynamic Ordinary Least Squares (DOLS).

The data came from a monthly publication *Consensus Economics* and covered the period, February 1990 to September 1996 for all countries except Australia, which had fewer data points with the period ranging from November 1990 to March 1996. Due to the fewer data points available for Australia, the United States data was utilised for the first two chapters. The data from the seven other countries namely, Japan, Canada, Australia, Italy, France, Germany and the United Kingdom was utilised in Chapter 4 to determine whether the previous findings were particular to the U.S. data.

Chapter 2 assumed, as did the early Term Structure literature, that the data was stationary. This being the case, the only problem that needed to be addressed when it came to estimation of the equation;  $s_{t+3} = \alpha_0 + \beta f_t + \varepsilon_{t+3}$ , was that the data was overlapping. The three methods of correcting for this occurrence were undertaken. That is; the data was split into three subsamples, thus sampling out the overlapping problem, correcting for the moving average error structure and using the GMM technique. Although the estimated coefficients obtained from the estimation of the equation, utilising the techniques mentioned above, were similar the test results were not. The GMM equations found that the hypothesises of Unbiasedness and Efficiency could not be supported whilst the moving average regression found the opposite.

This discrepancy could have been the result of assuming stationarity when in fact it was not present. Hence, chapter 3 began by testing the assumption of stationarity.

An Augmented Dickey Fuller unit root test was performed on the U.S. data and it was found that the data contained a unit root. Johansen's procedure was then used to test for a cointegrating relationship. Having found such a relationship the data was estimated using firstly, Johansen's Maximum Likelihood estimation of the ECM and secondly, Dynamic OLS (DOLS).

Once again the test results for Unbiasedness were not conclusive. The Johansen Maximum Likelihood estimation of the ECM could not accept the hypothesis whilst the DOLS regression could not reject it. To assess whether this inconclusive outcome was merely a U.S. phenomenon the data from the seven other developed countries was used and the equations were re-estimated.

Comparing the Rationality test results, firstly in relation to the GMM and moving average regressions, all countries excepting Australia could not reject the null of rational forecasters. In both cases Australia rejected the null of either Efficiency or Unbiasedness. The GMM procedure yielded slightly different results with Australia being joined by Japan in their rejection of the null of Rationality. Furthermore, France could not accept the null of Unbiasedness when the GMM estimation technique was used.

The spot and expected future spot interest rate data from all eight countries was shown to contain a unit root when the Augmented Dickey Fuller Unit Root test was applied. In light of this result, the full data sets, from all eight countries, were estimated using the Johansen and DOLS techniques. For the Johansen Maximum Likelihood estimation of the ECM, only Japan and Italy could not accept the null of Unbiasedness. For the DOLS regression all countries except Germany could not reject the null. Interestingly no country consistently rejects the null of Speculative Efficiency.

Chapter 5 began by creating a data generating process that exhibited a moving average error structure equivalent to that found in the actual data. Then the data was constrained to be stationary and a moving average regression was replicated 1000 times using Monte Carlo

simulation. The average value of the coefficients was recorded as was the percentage of time the hypothesis of Unbiasedness could not be accepted. This process was repeated with the data being I(1) and the estimation technique being DOLS. In both cases the slope coefficient was very close to the desired level of one and the hypothesis was accepted the majority of the time. These results could be considered as the benchmark. That is, if the data is correctly modelled then the results of the tests should clearly indicate that the forecasters behaved in a rational manner.

Chapter 5 then introduced the concept of local to unity roots. The assumptions of the data being stationarity and or I(1) was then relaxed and the roots were allowed to deviate from one. The three estimation methods, DOLS, OLS corrected for MA(2) errors and Johansen's Maximum Likelihood, were used once again to re-estimate the equations with the data moving from an I(1) process to a stationary process. The null hypothesis rejection percentage was also recorded. The mean coefficients recorded in tables 5.1 and 5.2 indicated that the Johansen procedure did not perform well when the assumption of an I(1) process was relaxed. the DOLS technique did not perform too badly for minor deviations from a unit root whilst the moving average simple linear regression yielded consistent results no matter the value of the root. The rejection rate of the hypothesis of rationality was also consistent for the moving average regression.

Finally, confidence intervals were constructed for the DOLS and Johansen regressions when the actual data was constrained to an I(1) process and for the moving average regression when the data process was white noise. The data was then estimated using these techniques when the process deviated from the necessary truth. The percentage of times the coefficients fell within the confidence bounds was recorded. As was expected, given the results above, the moving average regression performed exceedingly well whilst the DOLS regression did not yield many coefficients that lay within the 5% confidence interval.

Unfortunately, as there is as yet no estimation technique designed to specifically cope with local to unity roots, this final analysis was rather limited. It would have been useful to see how well such a technique coped with stationary and I(1) data. The development of such a technique is an area for future research. However, given the current techniques available to econometricians this chapter indicates that if a local to unity process is suspected then using a stationary, as opposed to a non-stationary, or I(1) technique, is advisable.

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