The SNR penalty for partial response systems operated above the Nyquist rate

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Declaration

I hereby declare that this submission is my one work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of higher degree or diploma of an university or other institute of higher learning, except where due acknowledgement is made in the text.

Wulin Hu

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Abstract

Partial Response Systems (PRS) are useful tools to achieve frequency efficiency, especially the improved PRS model. In these systems digital signals can be transmitted above the Nyquist rate which once was thought to be impossible. The frequency efficiency of PRS systems is achieved at the expense of a deterioration of the system performance. Such a deterioration can be expressed as an SNR (signal to noise ratio) penalty. The SNR penalty of Duobinary (class 1) or, 1 + D PRS system, in terms of the system polynomial version was evaluated by Zakarevicius and Feher under some assumptions. There two questions remained: what is the validity of the assumptions used earlier and can we find some other PRS systems which have better performance in high speed transmission?

By using computer simulation some PRS systems categorised by Kabal and Pasupathy [6] have been evaluated by the Author. The results of the project show:

1) The superiority of the improved PRS systems to conventional PRS systems is generally true.

2) The assumption that an alternating error pattern
would produce the worst SNR penalty should be modified when the length of the error sequence is relatively large.

3) Some of the PRS systems have good speed tolerance, especially the improved PRS system $1+2D+D^2$ (class 2) which has about 0.02 dB

SNR penalty when operated at a speed 10% above the Nyquist rate.

Chapter 1 of the this project report is used to introduce the PRS systems and their the system polynomial expressions. Chapter 2 introduces the method of SNR penalty computation. Chapter 3 presents the improved PRS system version. The SNR penalty calculations for some PRS systems are demonstrated in chapter 4. The explanation of the reason for the good speed tolerance in appropriate cases is given in chapter 5. The detailed data of SNR penalties for these PRS systems appear in the Appendix of the this project report.

Contents

Introduction

In digital communications, PCM (Pulse Code Modulation) and PAM (Pulse Amplitude Modulation) are both useful tools. The constraint on permissible PAM and PCM signal waveforms is that they should not cause intersymbol interference (ISI). However the inevitable timing errors and incompatibilities with some channel characteristics cause signal design based on this criterion to become very difficult. The use of partial-response signalling (PRS) can alleviate the constraints on waveforms. The basic idea behind the PRS is to introduce some controlled amount of ISI into the data stream, rather than try to eliminate it completely, since the intersymbol interference is then known. The controlled intersymbol interference introduced can shape the system spectrum and this spectrum shaping can make the system less sensitive to timing errors. This allows practical PRS systems to transmit at a speed equal to or even faster than the Nyquist rate.

The partial response systems introduced by Lender [1] are capable of being operated at or even above the Nyquist rate. Let us call it conventional model PRS. Conventional model PRS systems are speed tolerant, but this speed tolerance is quite limited. A new partial response system model discovered by Wu and Feher [2] demonstrated greater speed tolerance than conventional model partial response systems in the computer simulations and experimental hardware results. Let us call it improved partial response system.

When transmission speed goes up, especially when the Nyquist rate is exceeded, the system performance goes down. The increased spectral efficiency obtained by PRS signalling above the Nyquist rate will be at the expense of the deterioration of the system performance. This performance deterioration can be conveniently expressed as a certain signal_to_noise ratio (SNR) penalty [3,4]. Partial response system introduces a controlled amount of intersymbol interference (ISI) by itself to achieve spectral efficiency. Operating above the Nyquist rate then introduces further ISI, which is undesired and yields the SNR penalty.

Kretzmer categorised the characteristics of several PRS schemes [5]. Kabal and Pasupathy presented their study of PRS on the comparison of different PRS schemes on their paper 'Partial_Response Signalling' [6]. Zakarevicius and Feher evaluated the SNR penalty in one of the partial response systems, the version 1+D (after the terminology of Kabal and Pasupathy), both in conventional and improved versions with MLSE [3],[4]. By introducing a signalling technique of subsequent bandwidth expansion at the signal destination Court has discussed the high speed property of PRS class 4 [13]. In this project the SNR penalties for some types of PRS systems have been evaluated by using the principles used by Zakarevicius and Feher.

The results of the project show that 1) the superiority of the improved

PRS system versions to conventional versions is generally true, 2) the assumption made by Zakarevicius and Feher of that the worst SNR penalty would occur when the error pattern is alternating in sign should be modified when the length of the error sequence is large, 3) some of the improved PRS systems show very good speed tolerance, especially the improved PRS system version $1+2D+D^2$ which has about 0.02 dB SNR penalty at 10% above the Nyquist rate and less than 1 dB at 45% above the Nyquist rate.

This project report consists of 5 chapters, a short conclusion and an appendix. Chapter 1 is used to introduce the PRS systems and their polynomial expressions. Chapter 2 introduces the method of the SNR penalty computation. Chapter 3 presents the improved PRS systems. The SNR penalty calculations for some PRS are demonstrated in chapter 4 and the explanation of good speed tolerance for some improved PRS is given in chapter 5. The detailed result data are listed in the appendix.

Chapter 1. GENERALIZED PRS SYSTEMS

The general baseband model of a synchronous data communications system has been discussed by several authors [5], [6]. Fig.1.1. shows a block diagram of a typical baseband digital communication system. The model's transfer function $H(\omega)$ encompasses the transmitter filter (baseband modulator or signal generator characteristic), the equivalent baseband channel, and the receiving filter (which may include an equalizer).

The sample sequence of the impulse response $h(t)$ can characterize an ideal, noiseless digital system (no distortion due to channel imperfections or sampler offsets in the system). Fig 1.2 shows a typical system, where the system consists of a tapped delay line with coefficients $\{f_n\}$. Let N be the smallest number of contiguous nonzero samples of the impulse response $h(t)$ and $\{f_n\}$, n = 0, 1, 2, ..., N-1, be these N sample values, Then the system polynomial $F(D)$ can be expressed as

$$F(D) = \sum_{n=0}^{N-1} f_n D^n \tag{1.1}$$

where D is the delay operator. The output sample sequence $\{y_n\}$ of the

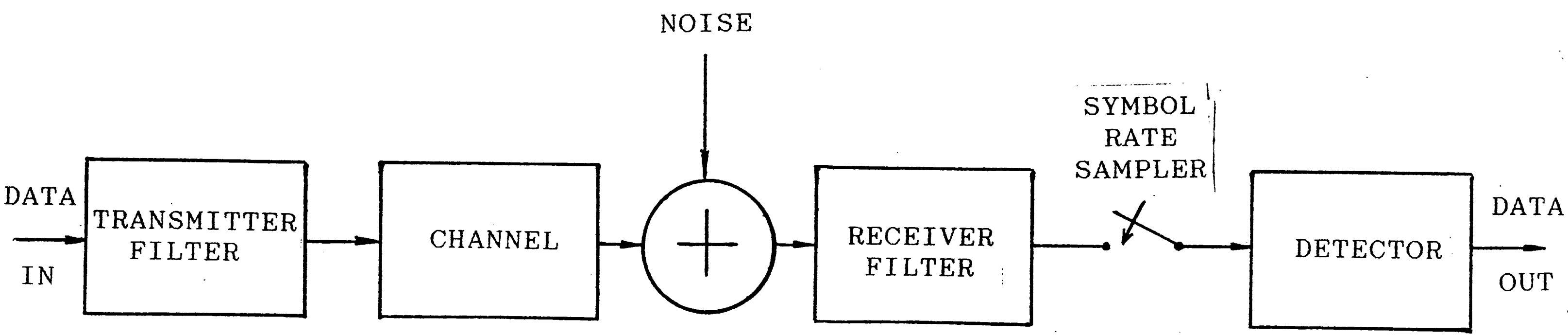

Fig. 1.1 A model for synchronous data communication system (redrew from [6]).

system is then expressed in terms of the input sample sequence $\{x_n\}$ and the PRS system polynomial as

$$Y(D) = X(D)F(D) \tag{1.2}$$

where

$$\begin{aligned} X(D) &= \sum_{n=0}^{\infty} x_n D^n, \\ Y(D) &= \sum_{n=0}^{\infty} y_n D^n. \end{aligned} \tag{1.3}$$

The $\{x_n\}$ is assumed to be an independent m-ary symbol sequence which has the equally likely values {-(m-1),-(m-3),... (m-3),(m-1)}.

While Fig 1.2 naturally suggests a digital system, an analog approach has its advantages in some cases. For this reason let us consider PRS systems in the frequency domain by evaluating the PRS system function $H(\omega)$ which can give an insight into the frequency domain properties of PRS systems.

In Fig. 1.2, the system has a tapped delay line with coefficients $\{f_n\}$ and a cascaded filter with frequencyresponse $G(\omega)$. The frequency response ($\mathscr{F}(\omega)$) of the delay line or the transversal filter is periodic and the period is $2\pi/T$, where T is the symbol duration. The Fourier transform $\mathscr{F}(\omega)$ of the first part of the system is given as

$$\begin{aligned} \mathscr{F}(\omega) &= F(D)\big|_{D=\exp(-j\omega T)} \\ &= \sum_{n=0}^{N-1} f_n \exp(-j\omega n T) \end{aligned} \tag{1.4}$$

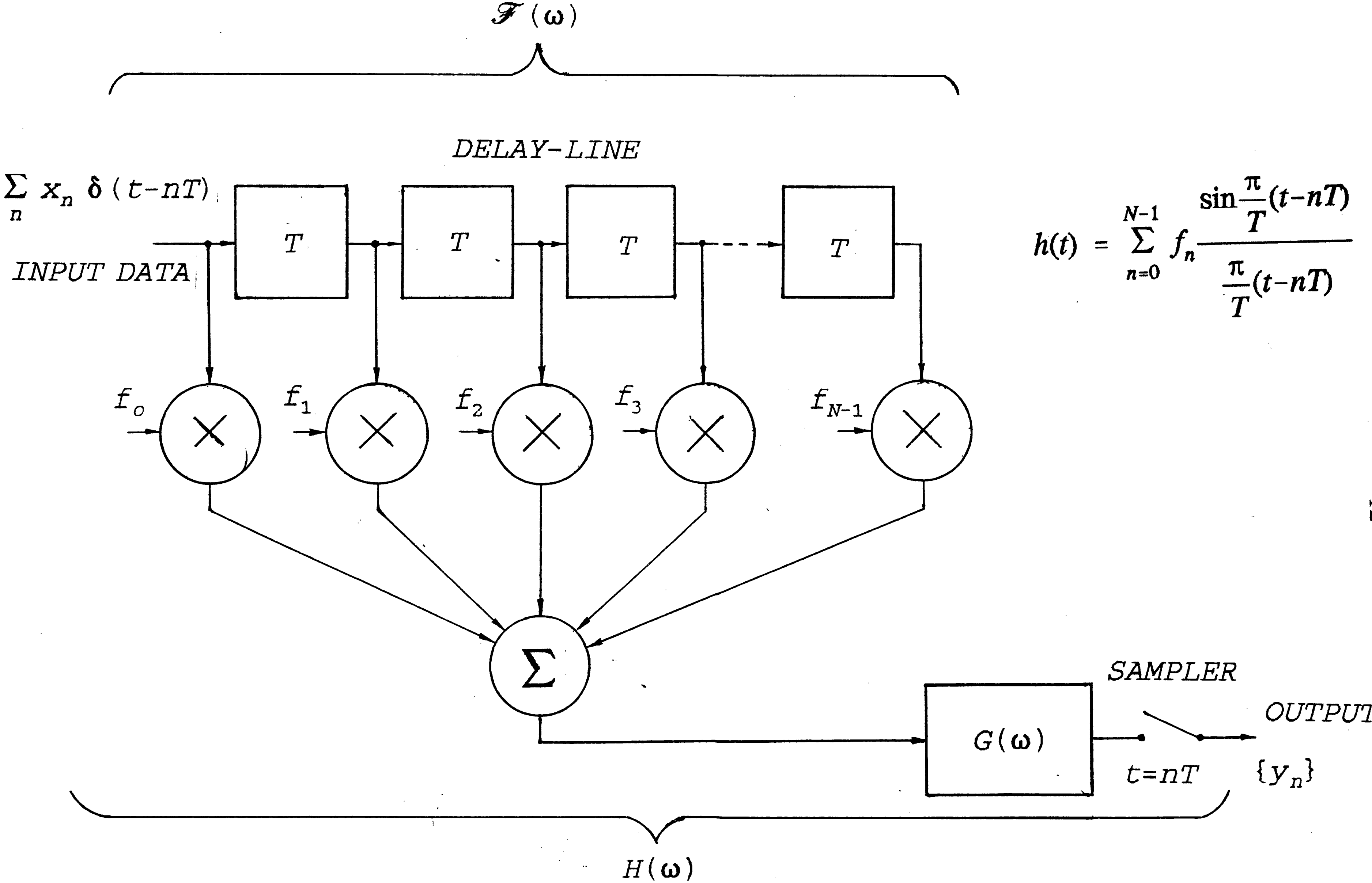

Fig. 1.2 A general partial response system model (redrew from[6]).

The entire system's impulse response $h(t)$ will then be equal to $\{f_n\}$ if and only if $G(\omega)$ satisfies Nyquist's first criterion, that is all but one of the samples of the impulse response of $g(t)$ be zero, i.e., $g(nT) = 0$, for $n \neq 0$, and $g(0) = 1$ [7]. That is

$$h(t) = \{f_n\}, \quad when \begin{cases} g(0) = 1 \\ g(nT) = 0 \end{cases}, \quad n = 0,1,2,...$$

By now we have artificially separated the PRS system into two parts: $\mathscr{F}(\omega)$ and $G(\omega)$ as shown in Fig. 1.2. $\mathscr{F}(\omega)$ is used to produce the desired periodical sample values, while $G(\omega)$ preserving the sample values may be used to band-limit the resulting system function. This separation has its advantage in the study of different system functions. When system polynomial $F(D)$ is given, one can choose different $G(\omega)$ satisfying Nyquist's first criterion to get different system functions $H(\omega)$, while all of them can have identical sampled responses. If one goes further to study the various desirable properties of $H(\omega)$ and how some of these properties, such as spectral nulls, are affected by the choice of $G(\omega)$ it can be found that this separation will give more convenience.

Within Nyquist's first criterion, to maximize the data rate in the available bandwidth, one may choose the PRS systems which occupy the bandwidth that can support the transmission without undesired intersymbol interference, i.e., $H(\omega) = 0$ when $|\omega| > \pi/T$. Equivalently it can be expressed as

$$G(\omega) = \begin{cases} T, & |\omega| \leq \pi/T \\ 0, & elsewhere \end{cases}. \tag{1.6}$$

The corresponding system impulse response is given by

$$h(t) = \sum_{n=0}^{N-1} f_n \frac{\sin\frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)} \tag{1.7}$$

The PRS systems which satisfy this criterion were called minimum bandwidth systems [6].

Some further studies have shown that at the expense of a degradation of the error performance the use of even narrower bandwidth for PRS systems is still possible [2], [3]. There are still other choices for $G(\omega)$ (occupying a larger bandwidth) that allow the use of system polynomials (such as $1 - D$) but which are unsuitable for so-called minimum band-width systems.

The use of system polynomial $F(D)$ gives great convenience in studies in PRS systems. With a particular $F(D)$ given one can write out the corresponding system impulse response easily. When the system polynomials have the two factors $1 + D$ and $1 - D$, some of system's important properties such as spectral null can be deduced out. With combinations of just the two factors $1 + D$ and $1 - D$, most of the common partial-response systems can be developed. Table 1.1 (introduced from [6]) shows a number of PRS system polynomials, the corresponding $|H(\omega)|$, $h(t)$ and the output levels L. The class designations of the table come from Kretzmer [5]. In the general formula for $h(t)$ (1.7), the time origin is chosen at sample f_0. However shifting the time zero often simplifies the resulting expression. In Table 1.1, the time origin of $h(t)$ has been changed to the center of the nonzero samples to simplify the exprssions, that is $t = (N - 1)T/2$.

The first entry in the table, version $1 + D$, or duobinary PRS, being the typical PRS system, has been discussed in details by many authors [1], [6], [8]. Not only does this PRS system satisfy Nyquist's first criterion, it can also satisfy Nyquist's second criterion, i.e., that the pulsewidth should be undistorted. The second system $1 - D$ which has a discontinuity in the system function $H(\omega)$ at $\omega = \pi/T$ is not practical in the minimum bandwidth. Modified duobinary, the next entry, has both a dc null and a null at $\omega = \pi/T$. Those features make it practicable in minimum bandwidth with reduced low-frequency components in spectrum that are desirable in systems such as transformer coupled circuits, dc powered cables, SSB modems, and carrier systems with carrier pilot tones. The fourth entry, $1 + 2D + D^2$, which has the same response as a full raised cosine characteristic [8] but sampled at twice the usual rate, has a very good performance when operated on improved version at a speed above the Nyquist rate. It has been found that the SNR penalty is less than 1 dB while the operating speed has exceeded the Nyquist rate 45%. In the last chapter, this PRS system will be discussed in detail. The next 5 entries were used to demostrate the fact that any appropriate polynomial in D may be used to modify the basic polynomials $1 \pm D$. These entries' high speed performance will be evaluated as well. The number of output levels L for a PRS system with M nonzero pulse samples lies in the range

TABLE 1.1
Characteristics of Minimum Bandwidth Partial-Response Systems

System Polynomial $F(D)$	Frequency Response $H(\omega)$ for $\lvert\omega\rvert \leq \pi/T$	Impulse Response $h(t)$	No. of output levels L
$1+D$ *duabinary* *class* 1	$2T\cos\frac{\omega}{2}T$	$\frac{4T^2}{\pi}\frac{\cos(\pi t/T)}{T^2-4t^2}$	$2m-1$
$1-D$ *dicode*	$j2T\sin\frac{\omega}{2}T$	$\frac{8T\,t}{\pi}\frac{\cos(\pi t/T)}{4t^2-T^2}$	$2m-1$
$1-D^2$ *modified* *dubinary*	$j2T\sin\omega T$	$\frac{2T^2}{\pi}\frac{\sin(\pi t/T)}{t^2-T^2}$	$2m-1$
$1+2D+D^2$ *class* 2	$4T\cos^2\frac{\omega}{2}T$	$\frac{2T^3}{\pi t}\frac{\sin(\pi t/T)}{T^2-t^2}$	$4m-3$
$1+D-$ $-D^2-D^3$	$j4T\cos\frac{\omega T}{2}\sin\omega T$	$-\frac{64T^3t}{\pi}\frac{\cos(\pi t/T)}{(4t^2-9T^2)(4t^2-T^2)}$	$4m-3$
$1-D-$ $-D^2+D3$	$-4T\sin\frac{\omega T}{2}\sin\omega T$	$\frac{16T^2}{\pi}\frac{\cos(\pi t/T)(4t^2-3T^2)}{(4t^2-9T^2)(4t^2-T^2)}$	$4m-3$
$1-2D+D^4$ *class* 5	$-4T\sin^2\omega T$	$\frac{8T^3}{\pi t}\frac{\sin(\pi t/T)}{t^2-4T^2}$	$4m-3$
$2+D-D^2$ *class* 3	$T+T\cos\omega T+$ $+j3T\sin\omega T$	$\frac{T^2}{\pi t}\sin(\pi t/T)(\frac{3t-T}{t^2-T^2})$	$4m-3$
$2-D^2-D^4$	$-T+T\cos 2\omega T+$ $+j3T\sin 2\omega T$	$\frac{2T^2}{\pi t}\sin(\pi t/T)\,(\frac{2T-3t}{t^2-4T^2})$	$4m-3$

$$M(m - 1) + 1 \leq L \leq m^M$$

for a m-ary input signal. The minimum value can be obtained when the pulse samples have the same magnitude. The limitations of the number of output levels for a practical PRS system are the complexity of implementation and the inevitable distortions present in real systems. For a PRS system with a large number of output levels the error performance tends to degrade at a given SNR.

Chapter 2. SNR PENALTY OF PRS WITH MSLE

In digital communication systems, one way to achieve good spectral efficiency is to increase the signalling rate to above the Nyquist rate. The expense of doing so is the deterioration of system performance. One convenient way to express this deterioration is to treat it as a certain signal-to-noise ratio penalty.

Partial response systems are known to have speed tolerance [1], [8]. Some articles have addressed that not only can PRS systems be operated at the Nyquist rate, but they can also work at a speed faster than the Nyquist rate as well, which once was thought to be a impossible [2], [3]-[4],[9]-[11].
The principle behind partial response systems is the introduction of a controlled amount of intersymbol interference (ISI). Operating above the Nyquist rate then introduces further ISI, which is undesired and it can be expressed as the SNR penalty.

The SNR penalty is the increase in SNR that is required in the presence of the undesired ISI to achieve the same bit error rate that would have been present if the undesired ISI had been absent. The introduction of the improved efficiency partial response systems largely reduces the undesired ISI, especially for modest increases in signalling speed above the Nyquist rate [2], [3].

MLSE can be employed to minimise the PRS systmes SNR penalty. The derivation for the SNR penalty after MLSE has been presented by Zakarevicus and Feher [4]. The SNR penalty for a certain PRS system, version $1 + D$, after MLSE was computed as a function of the signalling rate in excess of the Nyquist rate, both in the conventional and the improved efficiency types.

One of the jobs for digital communication systems is to estimate the transmitted symbols from the information provided by the received wave form. Maximum Likelihood Sequence Estimation (MLSE) is a way of doing this estimation which minimises the probability of error.

MLSE can be summarised as follows: If the information symbols are $a_1, a_2, ..., a_N$ which are transmitted by modulating a train of pulses $g(t)$ then the transmitted waveform produced is

$$s(t) = \sum_{i=1}^{N} a_i g(t-iT) \tag{2.1}$$

where T is the symbol duration and respectively 1/T is the signalling rate. When $s(t)$ is the input to the channel, then the output of the channel will be

$$r(t) = \sum_{i=1}^{N} a_i h(t-iT) + n(t) \tag{2.2}$$

where $h(t)$ is the convolution of $g(t)$ and the channel impulse response, and $n(t)$ is white Gaussian noise. For simplicity we can include $g(t)$ within the channel impulse response without loss of generality; i.e. from now on we will treat $g(t)$ as an impulse and simply call $h(t)$ the channel impulse response.

Let the output signal sequence from the system be $\hat{a}_1, \hat{a}_2, ..., \hat{a}_N$, then in

MLSE the selected sequence will minimise [12]

$$\int_{-\infty}^{\infty} \left| r(t) - \sum_{i=1}^{N} \hat{a}_i h(t-iT) \right|^2 dt \tag{2.3}$$

If the channel were noiseless or $n(t)$ were zero, and there is not any undesirable ISI in the channel, the integral (2.3) would be zero for the sequence that was actually transmitted. It is the presence of noise and the undesirable ISI, if there is any, which make (2.3) non-zero for the transmitted sequence and allows the possibility that the sequence $\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N$ is not in fact the sequence which was transmitted, i.e. at least one error has been made in the estimation. The MLSE procedure is used to minimise this error. To calculate the error possibility for such a system one may compute (2.3) for every possible transmitted sequence. 2^N computations would be involved if $\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N$ were only a binary sequence and M^N computations for the case of M levels. Clearly this is not an easy task. Using a dynamic programming approach the Viterbi algorithm can greatly reduce the number of computations required to a much lower number. Since in practice the Viterbi algorithm or a variant of it will invariably be used in MLSE, we can simply refer to MLSE as Viterbi detection.

A digital signal or a pulse sequence (possibly multi-level) distorted by the addition of noise and intersymbol interference (ISI) after having passed through a channel is estimated by MLSE in an optimum fashion, but a certain probability of error remains.

Consider an error sequence $[e]$ which is the differences between the true channel input and the estimated channel input after MLSE at the sampling instants. This error sequence $[e]$ can be written as

$$[e] = [e_1, e_2, e_3, \dots] \tag{2.4}$$

where e_i is assumed to have a value between M to 0 and M is an integer. This value represents the error between the true channel input and the estimated channel input at that instant. For example, 0 corresponds to no error at that instant, ± 1 corresponds to an error between immediately adjacent levels, ± 2 corresponds to an error between levels separated by an intervening level and so on. Thus ± 1 is much more likely than ± 2 and so on.

If we let $[e]$ be the input to the channel, with the channel characterised by its discrete-time response $[h]$ then we have a corresponding (discrete-time) channel output $[q]$. Clearly $[q]$ is given by

$$[q] = [e] \otimes [h] \tag{2.5}$$

where the symbol $\otimes$ stands for discrete convolution. This $[q]$ represents the differences between the true output due to the correct input sequence and the output due to the erroneous input sequence as deduced by the MLSE detector, at the output side of the channel. The ISI due to the channel is included in both output sequences (due to the true and erroneous inputs).

Let the symbol Q be introduced for the energy content of the output in Fig.2.1, thus

Fig. 2.1 Output due to an input error sequence (redrew from [4]).

$$Q = [q]\,[q]' \tag{2.6}$$

where $[q]'$ represents the transpose of $[q]$.

To evaluate the SNR penalty of a PRS system, we may want to compare only the performance of the system with or without undesirable ISI. According to [4] we can, without loss of generality, make

$$[h]\,[h]' = 1 \tag{2.7}$$

since only the relative magnitudes of the $[h]$ components are of significance. When there is no undesirable ISI, $[h]$ will have no sidelobe component. If a single error of magnitude 1 occurs in the sequence $[e]$, we will have $Q = 1$. The evaluation of the probability of error p_e can be best illustrated by computing the probability of error p_e for a particular sequence $[e]$ at a particular signal-to-noise ratio (SNR), or the probability of occurrence of a particular sequence $[e]$. Let us first compute the special error sequence

$$[1, 0, 0, ..., 0]$$

which corresponds to a single error, still in the presence of undesirable ISI. To find the SNR penalty for such a special case, one then needs to compare the p_e with ISI to the p_e with no ISI.

That an error occurs means the MLSE has selected the wrong input sequence. The condition for this to happen is that the channel noise is such that expression (2.3) is smaller for the wrong sequence than for the true sequence. In the discrete time case, the probability of selecting the wrong sequence becomes the probability that

$$\begin{gathered} n_1^2 + n_2^2 + ... > \\ (n_1 - q_1)^2 + (n_2 - q_2)^2 + ... \end{gathered} \tag{2.8}$$

where n_1, n_2, ... stand for the noise components at the appropriate sampling instants. (2.8) can be condensed to

$$n > A \tag{2.9}$$

where

$$A = \frac{q_1^2 + q_2^2 + ...}{2} = \frac{Q}{2} \tag{2.10}$$

and

$$n = q_1 n_1 + q_2 n_2 + \ldots, \tag{2.11}$$

and, further, the variance σ^2 of the random noise n (i.e. noise power) can be written as

$$\sigma^2 = (q_1^2 + q_2^2 + \ldots) N_n = Q N_n \tag{2.12}$$

where N_n represents the noise power (variance) associated with each of $n_1, n_2, \ldots$. After the normalisation of $[e]$ and $[h]$ as disussed earlier, the signal level has then been fixed, $1/N_n$ is thus proportional to SNR. We know that the probability that $n > A$ or the probability of the MLSE selects a wrong input sequence is proportional to

$$erfc(A/\sqrt{2}\sigma) = \frac{2}{\sqrt{\pi}} \int_{A/\sqrt{2}\sigma}^{\infty} \exp(-u^2)du \tag{2.13}$$

and from (2.10) and (2.12) we can get

$$\frac{A^2}{\sigma^2} = \frac{Q}{4N_n} \tag{2.14}$$

By putting (2.14) into (2.13), we can find that the p_e for a particular $[e]$ is determined by the SNR and $[e]$, but only through its contribution to Q. How the q's are distributed is irrelevant. That is to say, for a given SNR, p_e will be the same for all combinations of ISI and $[e]$, provided which result in the same Q. In particular, if there is no undesirable ISI and meanwhile there is only one single error, Q will be replaced by 1. Thus (2.14) will become

$$\frac{A^2}{\sigma^2} = \frac{1}{4N_n'} \tag{2.15}$$

where N_n' is proportional to the original SNR (the case with no SNR penalty). In the presentation of the undesirable ISI, Q will no loger be equal to 1. In this case, to keep p_e unchanged, we should change N_n so as

$$\frac{1}{4N_n'} = \frac{Q}{4N_n} \tag{2.16}$$

From (2.16) we can find

$$\frac{1}{Q} = \frac{N_n}{N_n'} \tag{2.17}$$

Clearly Q represents the change of SNR which results p_e unchanged and since typically $Q < 1$ thus $1/Q$ expressed in dB represents the SNR penalty due to undesirable ISI after MLSE.

So far only some specific error sequences have been considered. To compute the total probability of error, we should included all possible error sequences into the consideration and the number of errors in each sequence has to be taken into account in obtaining even the probability of a single symbol error. However, the fact that the exponential dependence of p_e on SNR, which appears as the so called rain fall curve, reminds us that for large enough SNR the p_e of one sequence, for which Q is minimum, will dominate the overall p_e. Here let us denote it as $Q_{\min}$.

Chapter 3. THE IMPROVED PRS SYSTEMS

In the PRS systems showed in chapter 1 it was found that they were speed tolerant in some degree [1] [8]. Later on Feher et al demonstrated a new PRS model which has a greater speed tolerance than the older model [2] [3]. Here following the terminology of Feher let us denote the new PRS model as Improved version and the others as Conventional version. In the case of signalling at the Nyquist rate the signalling rate f_s is at double the Nyquist bandwidth f_N;i.e.,

$$f_s = 2f_N = 1/T \quad ,$$

where T is the symbol duration. If f_s were to be increased to above $2f_N$, the system would have operation above the Nyquist rate. There is an alternative and equivalent way of looking at the operation above the Nyquist rate used by [2] and [3], that is to keep f_s the same, but reduce the bandwidth to less than f_N. To keep consistency with [2], [3], the latter way will be adopted here. The increase above the Nyquist rate can be expresed by using a parameter ξ such that the reduced bandwidth comes to be as $f_N/(1 + \xi)$.

When the conventional PRS systems are operating at a speed above the Nyquist rate, only the frequency scale is changed, which results in all frequency components suffering distortion as illustrated in Fig. 3.1 (a,b) where the cosine partial response case (or the $1 + D$) is used. For an improved version of such a PRS system as in Fig. 3.1 (c) the frequency response is simply "chopped off" at $f_N/(1+\xi)$. In this case most of the original frequency components thus remain, the only frequency components suffered are whose which have been cut off. Such a PRS system can be considered as a cosine response PRS system followed by a brick wall filter.

The impulse response $h(t)$ of a PRS system consists of some SINC pulses separated by a certain delay time. This time delay is related to the original frequency transfer function $H(\omega)$. Ordinarily it corresponds with the spectral null at $\omega = \pi/T$. The width of the SINC pulses is controlled by the bandwidth. The difference in the impulse response $h(t)$ between a conventional PRS system and a improved PRS system can be illustrated by Fig. 3.1 where a PRS system version $1 + D$ is used as an example. Fig. 3.1 (a) shows a conventional model at Nyquist rate scheme, in which the cosine response null is at f_N, that makes the delay between the two SINC functions equal to the symbol period T. The bandwidth is also f_N, so that

$$h(t) = f(t) + f(t-T) \tag{3.1}$$

where

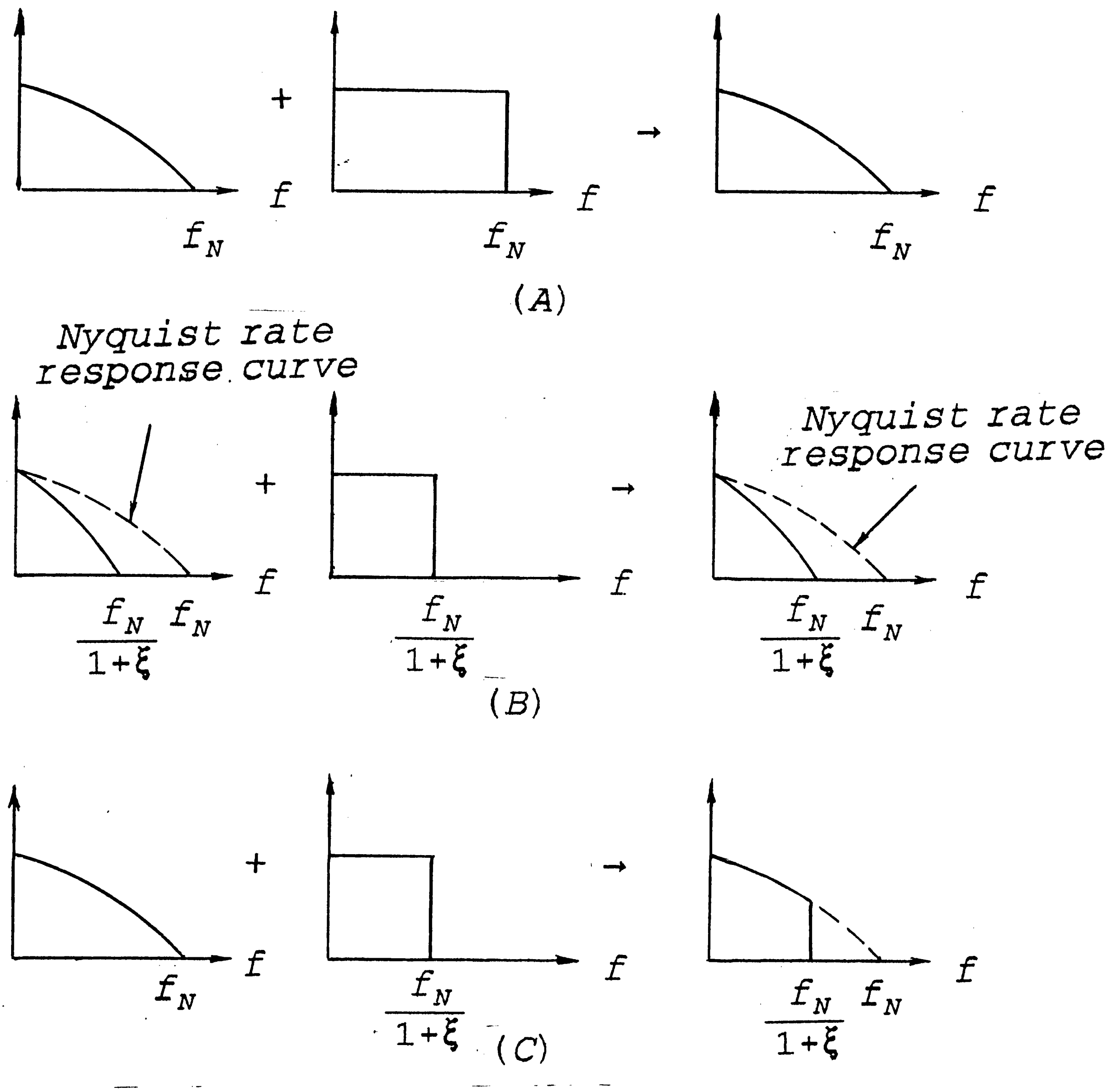

Fig. 3.1 The illustration of improved PRS system (redrew from [3]).
(A) Operation at or below thw Nyquist rate. (B) Conventional mofel operating above the Nyquist rate where all frequency components are suffering. (C) The Operation of an impeoved model above the Nyquist rate where high frequency components are chopped off whlist the frequency components below $f_N/(1+\xi)$ are kept undistorted.

$$f(t) = \frac{\sin(2\pi f_N t)}{2\pi f_N t}. \tag{3.2}$$

The system's behaviour of time domain in this situation is shown in Fig. 3.2. When operating at a speed above the Nyquist rate for the same scheme, f_N is simply changed to $f_N/(1 + \xi)$ and T to $T(1 + \xi)$, where ξ represents the fractional increase above the Nyquist rate. as showing in Fig. 3.1 (b). Fig. 3.3 shows the corresponding SINC functions, where we can see the curves of the SINC functions become wider and the second one shifts to right as the operating speed goes up. On the other hand, for an improved model in Fig. 3.1 (c), f_N goes to $f_N/(1 + \xi)$ but T is kept unchanged. In Fig. 3.1 (c) it is clearly shown that the frequency components below $f_N/(1+\xi)$ are kept undistorted. The related SINC functions are illustrated in Fig. 3.4 where the curve of the SINC functions are wider as in Fig. 3.3 but the second one is kept unshifted. It is this fact that makes the sidelobes, clearly they stand for the ISI, smaller than the conventional model.

The system impulse response expressed for computing purpose of such schemes can be written as

$$h_n = f_n + \hat{f}_{n-1} \tag{3.3}$$

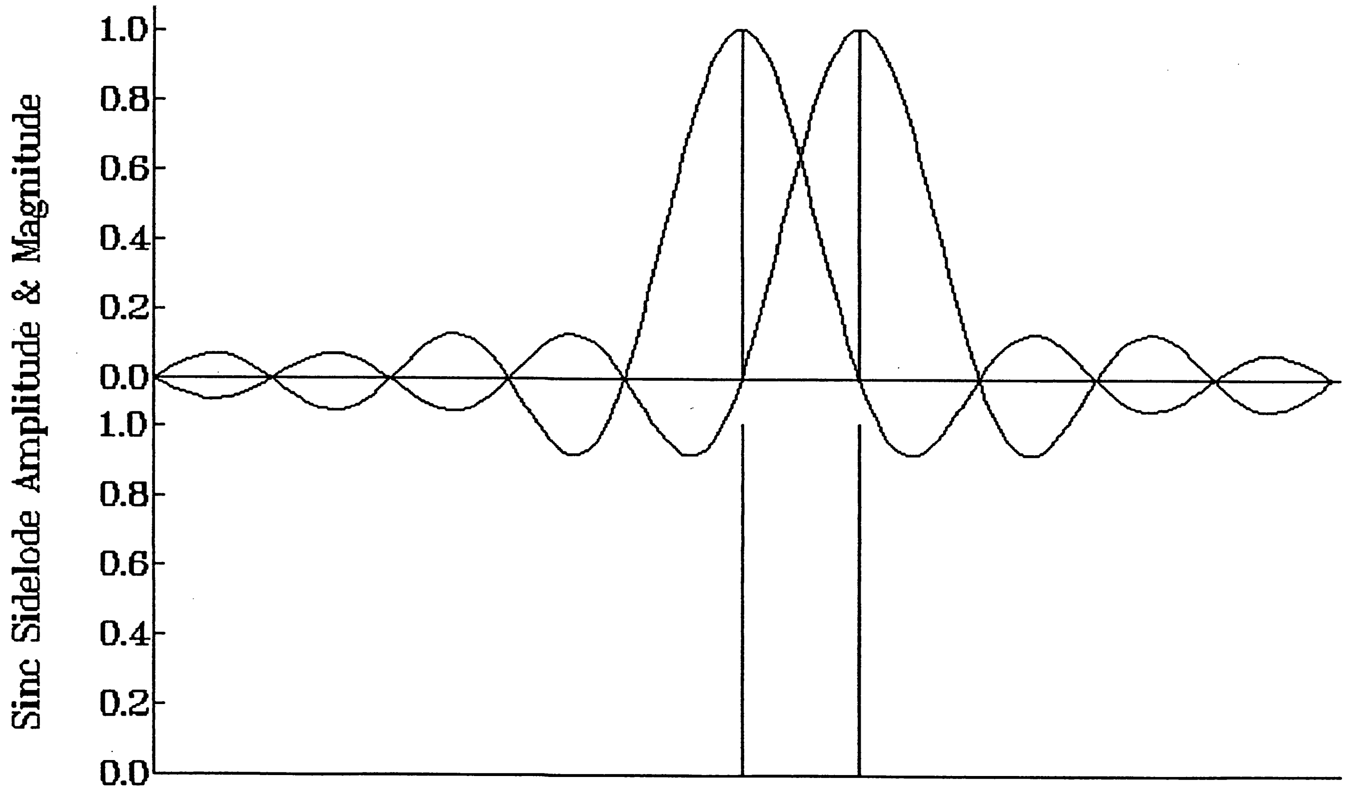

Fig. 3.2 The time domain behaviour of the PRS version 1+D operating at the Nyquist rate.

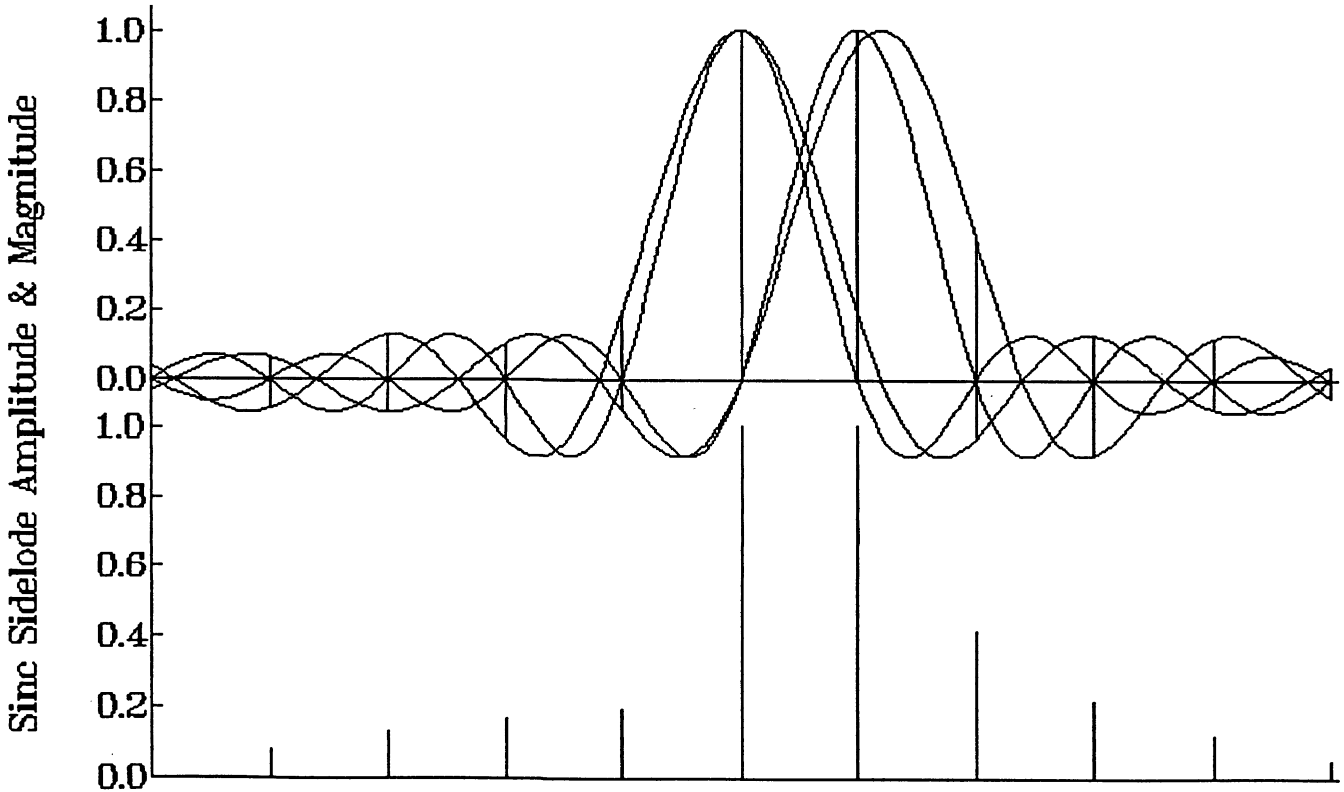

Fig. 3.3 The time domain behaviour of the conventional PRS version 1+D operating above the Nyquist rate.

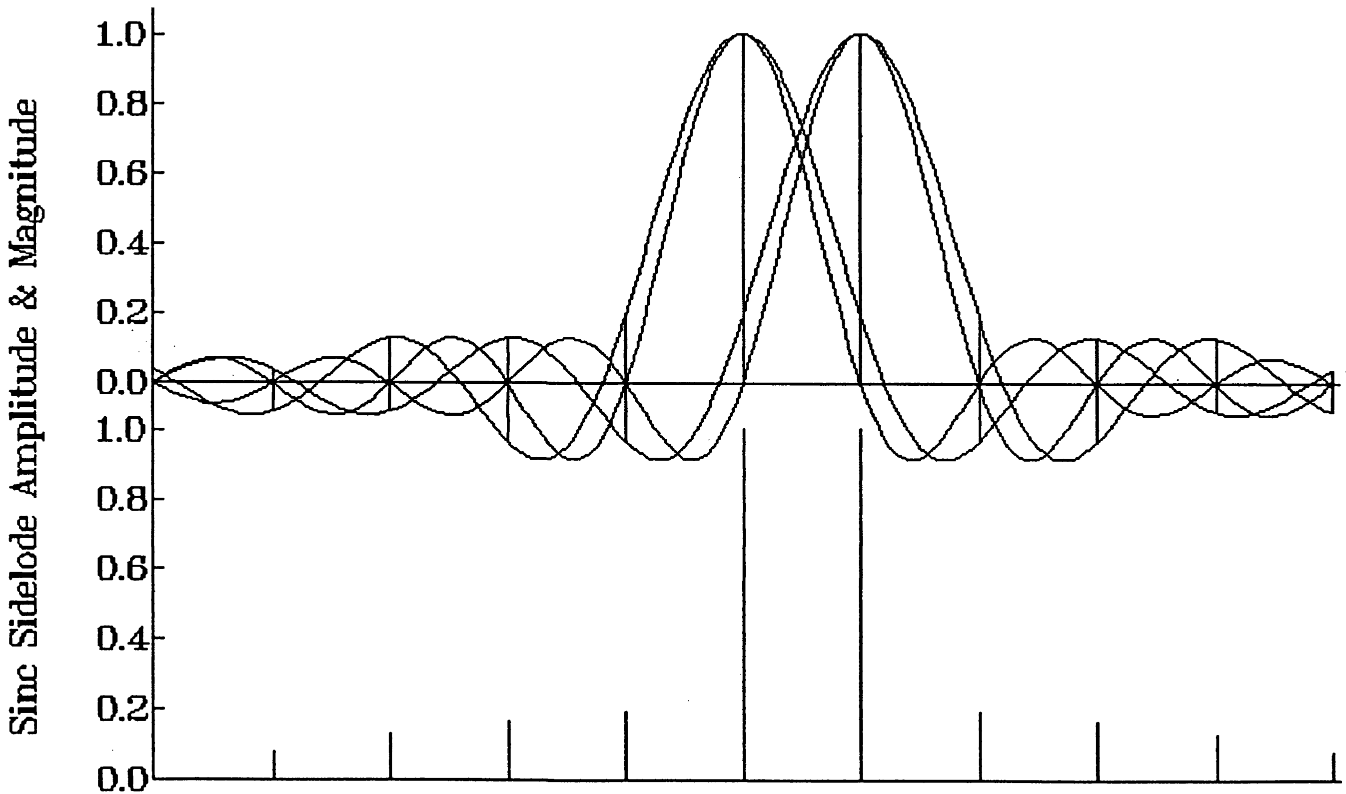

Fig. 3.4 The time domain behavior of the improved PRS version 1+D operating above the Nyquist rate.

$$h_n = f_n + f_{n-1} \tag{3.4}$$

where $f_n = \dfrac{\sin(nx)}{nx}$, $\hat{f}_n = \dfrac{\sin(n-\xi)x}{(n-\xi)x}$ and

$x = \dfrac{\pi}{1+\xi}$. (3.3) applies to the conventional case while (3.4) is used for the improved case.

Chapter 4. THE SNR PENALTY COMPUTATIONS

So far we have discussed Q and found that $1/Q$ is the SNR penalty. Also have we introduced the conception of $Q_{\min}$. The interest in computation of $Q_{\min}$ lies on what is the best that one can do in terms of SNR penalty, when the Nyquist rate is exceeded, especially in the improved efficiency systems [2,3].

To find $Q_{\min}$, strictly speaking, one have to compute all possible $[e]$, that is to calculate all possible $[e]$ lengths as well as a function of the increase above the Nyquist rate. Clearly this is a formidable task. Zakarevicius and Feher did this by using some reasonable simplifications. Since the likelihood of the errors between adjacent levels are much larger than errors between levels separated by an intervening level, they assume that the $[e]$ components are 0 or ± 1 only. Furthermore, after computing Q for all possible $[e]$ at a fixed rate above the Nyquist rate and for a constant number of elements in $[e]$, they found that worst $Q_{\min}$ is produced by an alternating pattern of +1 and -1, i.e. with

$$[e] = [+1,\ -1,\ +1,\ -1,\ \dots\ ] \tag{2.22}$$

They then assume that the error pattern in (4.1) will give $Q_{\min}$ for all signalling speeds and all $[e]$ length.Using the formulas for $[h]$ as shown earlier and the assumption they then computed SNR penalty for PRS system version $1 + D$ and the result is shown in [4].

Now the raising questions are: can we find some other PRS systems which are better in high speed transmission and what the validity of the assumption used earlier is? By using the principles discussed earlier and used by Zakarevicius and Feher [4], the SNR penalties for most of the PRS systems introduced in Chapter 1 are computed by the author. The method of calculation used can be best illustrated by using the familiar PRS system version $1 + D$, or duobinary system, as the example which follows.

The impulse response $[h]$ of the PRS system can be found by using (3.3) and (3.4) for a given ξ. For the convenience of the computation, 2 data files of $[h]$ (for improved and conventional models respectively) are created for some typical points of ξ from 0.01 to 0.1. Fig.4.1 shows the sidelobes of $[h]$ at the point of $\xi = 0.1$. From Chapter 2 we know that

$$Q = [q][q]'$$

and

$$[q] = [e] \otimes [h],$$

where $\otimes$ stands for discrete convolution and $[q]'$ for the transpose of $[q]$.

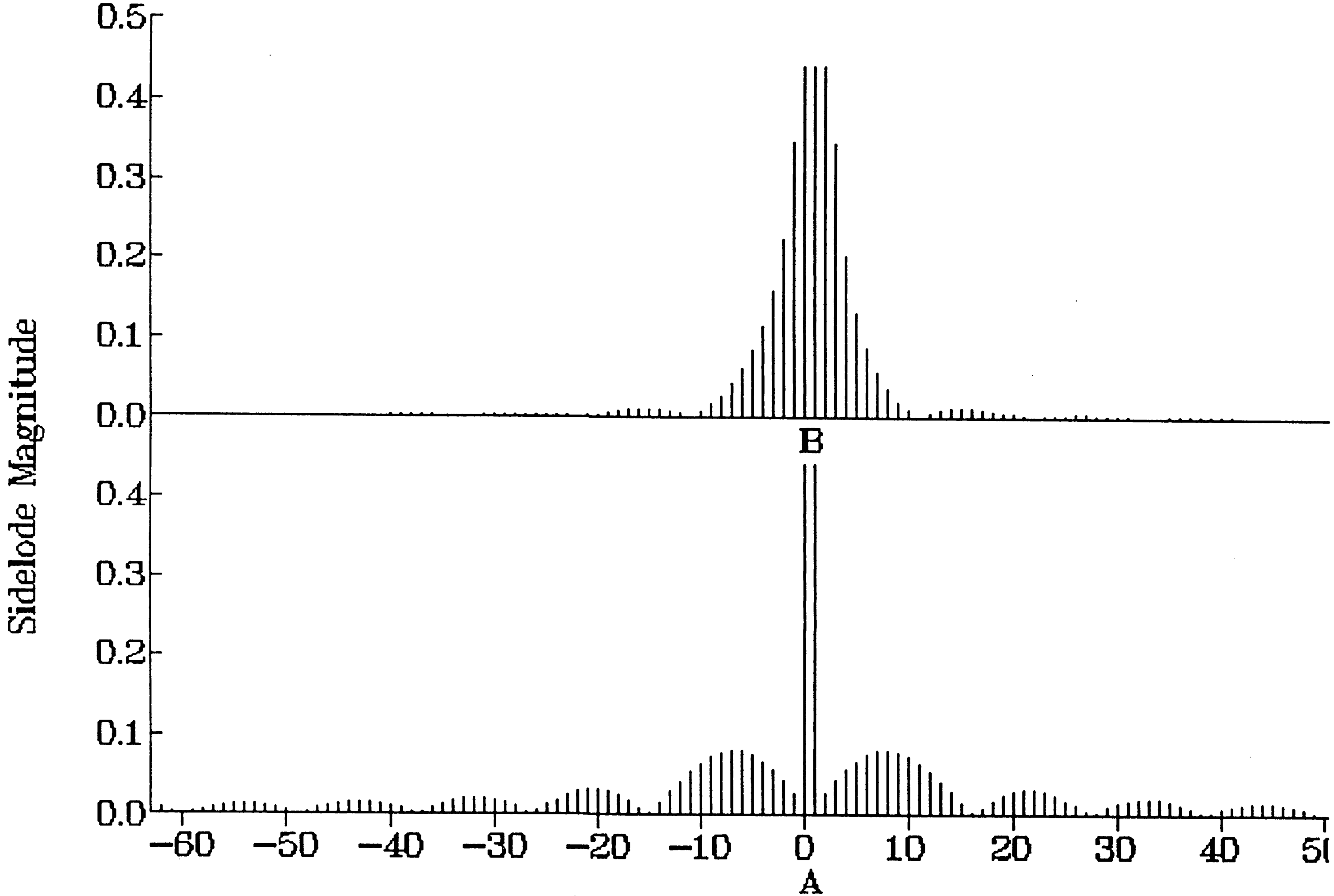

Fig. 4.1 The sidelobe magnitudes of PRS $1+D$, $\xi=0.1$.
(A): Improved version, (B): Conventional version.

Using a computer the discrete convolution can be done by doing the shifting and adding recurrently. This is the first principle of the discrete convolution, that's first moltiplying $[h]$ by e_n then shifting $[h]$ to the right by n, then adding the items at corresponding position together to find q_n. Suppose we have a error sequence $[e] = [1, -1, 1, -1, ...]$, then table 4.1 shows the method of the convolution.

After having found Q, the calculation of SNR penalty becomes simple and it can be written out as

$$Penalty = 10\times\log(1/Q)\ dB. \tag{4.1}$$

By using the method intruduced by Zakarevicius and Feher and discused in Chapter 2 and the assumption made by them, which states that an alternating pattern of $[e]$ produces the smallest Q or $Q_{\min}$ for PRS system version $1 + D$, a $Q_{\min}$ for such a system was computed out by the author and the results are plotted in Fig. 4.2 which is similar with Zakarevicius and Feher [4].

In Fig. 4.2 we can find that there are differences between the lines that represent 11 and 22 error items in $[e]$ respectively. The differences here indicate that the number of error items is a factor in the computation of SNR penalty. Theoretically speaking, the number of error items in $[e]$ to be considered in MLSE should be infinite as well as h_n. Clearly these are

Table 4.1

	$[e]$	*when n goes larger* $[h]$ *shefts toward right*						
$n=0$	$e_0=1$	h_0	h_1	h_2	h_3	h_4	h_5	...
$n=1$	$e_1=-1$	$-h_{-1}$	$-h_0$	$-h_1$	$-h_2$	$-h_3$	$-h_4$	...
$n=2$	$e_2=1$	h_{-2}	h_{-1}	h_0	h_1	h_2	h_3	...
$n=3$	$e_3=-1$	$-h_{-3}$	$-h_{-2}$	$-h_{-1}$	$-h_0$	$-h_1$	$-h_2$	...
				. . .				
				+				
				↓				
+		q_0	q_1	q_2	. . .			
				$Q = q_0^2 + q_1^2 + q_2^2 + \ldots$				

Where h_n and q_n stand for nth item of $[h]$ and $[q]$ respectively.

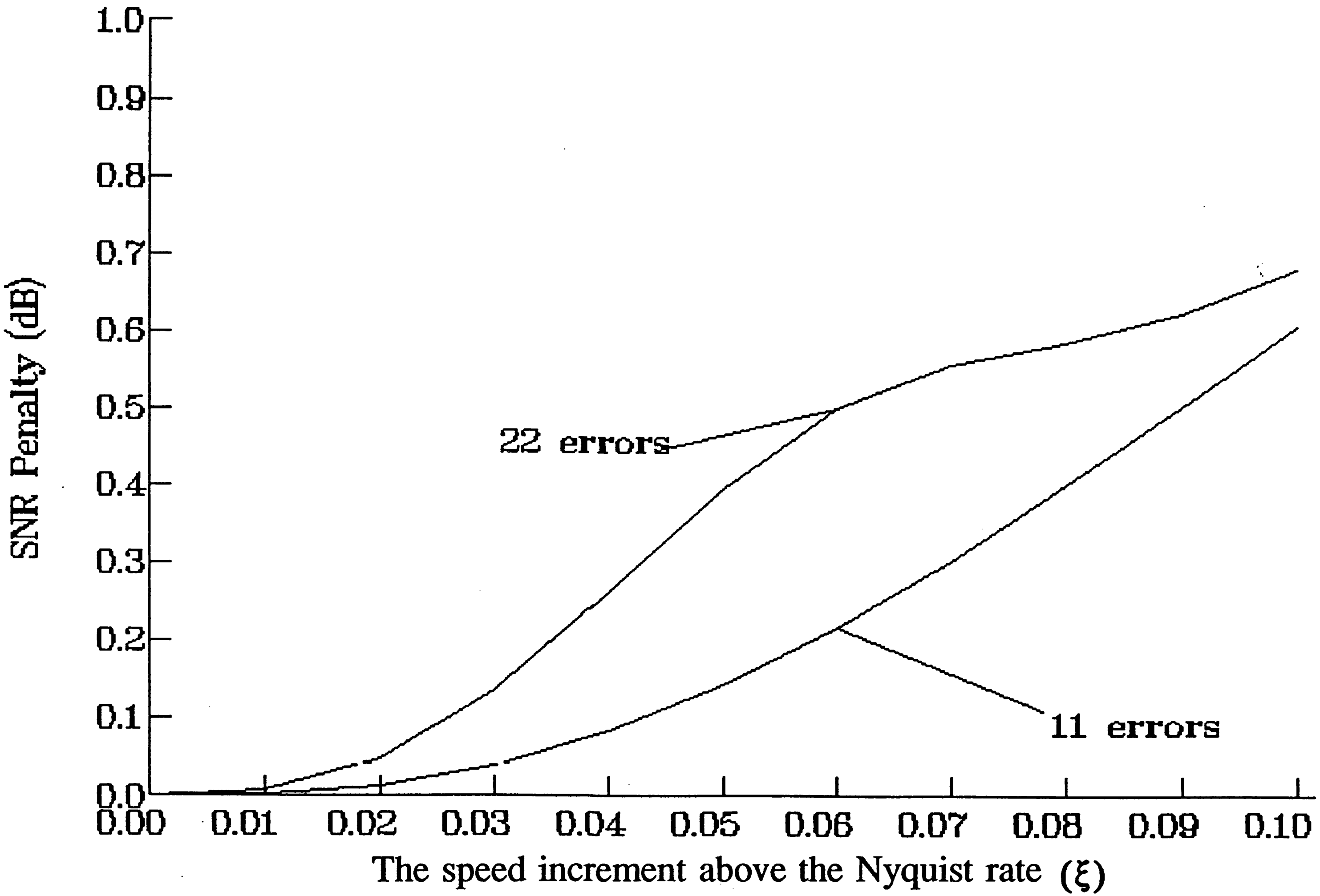

Fig. 4.2 The SNR penalties for improved PRS version $1+D$, alterlating error pattern.

impossible at present situation. However the number of error items to be considered is a function of the number of $[h]$ of which are significantly non-zero. Thus what we can do is to evaluate the effect of the number of error items as well as h_n upon the computation of the SNR penalty and find out when the number of error items and h_n to be considered is large enough in present circumstance. The SNR penalties corresponding to the [e]'s from 11 error item to 22 items are computed and the results are plotted in Fig. 4.3 for this purpose.

In Fig. 4.3 one can find that after the error items calulated are large than about 10 to 15 the SNR penalties become roughly steady. Thus 22 error items is probably a sufficiently large enough number in the computation. For the number of h_n to be computed the situation is similar since the magnitudes of the sidelobes of h_n decline very rapidly for small speed increase above the Nyquist rate. In the SNR computations the number of h_n used is 30. This number may be possibly sufficient in present situation.

The second question is the validity of the assumption made by Zakarevicius and Feher. They assumed that the worst SNR penalty would be produced by an alternative pattern of $[e]$. To verify this assumption, one has to check all possible error pattern for all length of $[e]$ as well as a function of all possible ξ. This is a forbidable task in present circumstance. What has been done by the author is test all possible combinations of $[e]$ for a length of 8 in $[e]$ and

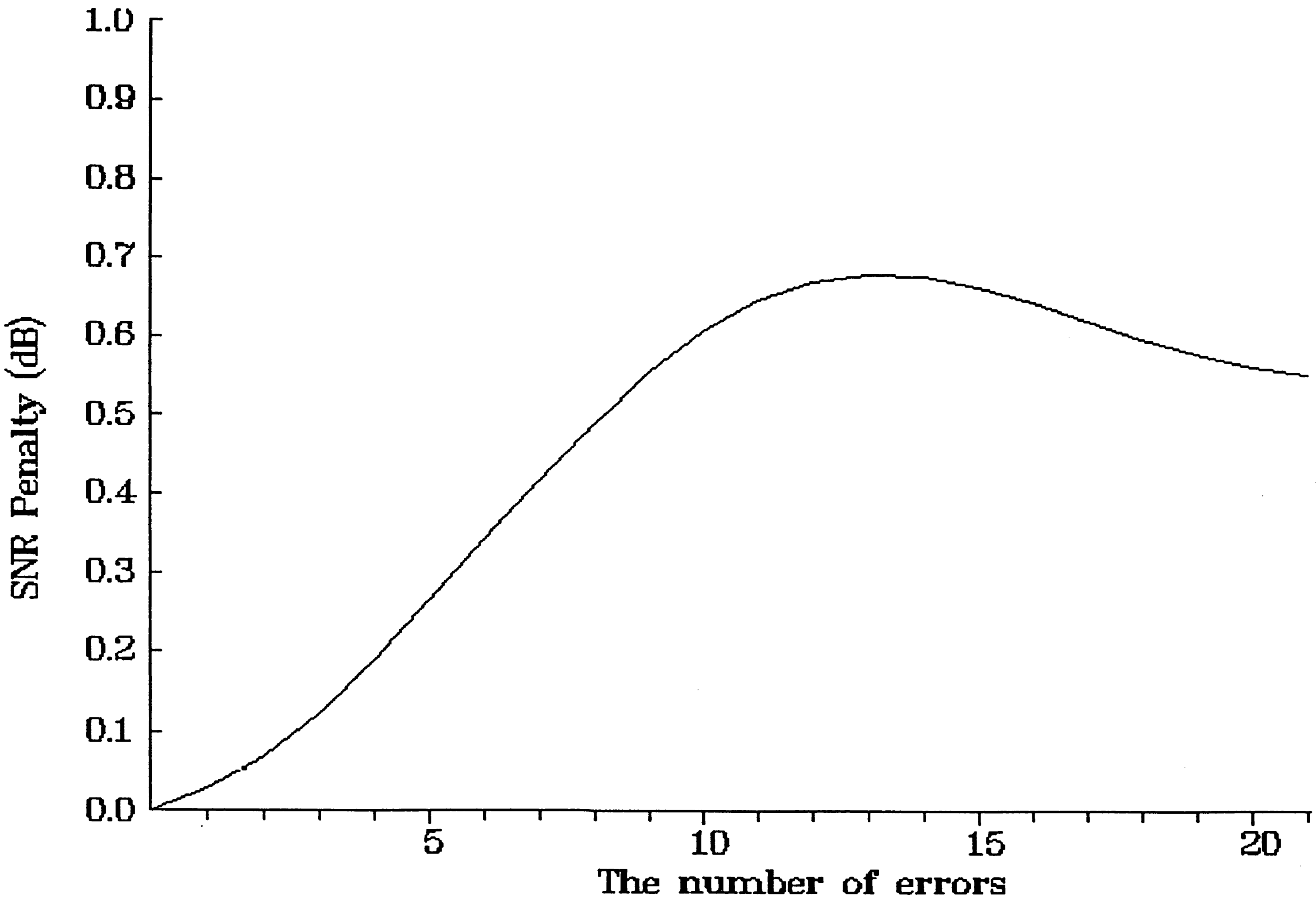

Fig. 4.3 SNR penalty vs the number of error items.
Improved PRS version $1+D$, $\xi=0.1$, alterlating error pattern.

11 values of ξ from 0.00, 0.01 ..., to 0.1 for the PRS version $1 + D$. The way of doing the test is to sort a file of Q which contains all calculated Q for the PRS version by using a computer to find the smallest Q or Q_{min}. The number of the error patterns associated with only one ξ value tested here is

$$\begin{aligned} No. &= 2^0 \times C_8^1 + 2^1 \times C_8^2 + 2^2 \times C_8^3 + \ldots + 2^6 \times C_8^7 + 2^7 \times C_8^8 = \\ &= 1 \times 8 + 2 \times 28 + 4 \times 56 + \ldots + 64 \times 8 + 128 \times 1 = \\ &= 3280. \end{aligned}$$

For all 11 values of ξ the computations involved is $11 \times 3280 = 36080$. The result shows that for small ξ, alternating pattern of $[e]$ produces Q_{min} but the bigger ones don't. This result leads us back to the old question of what pattern of $[e]$ will produce Q_{min}. At present situation we can not increase the length of $[e]$ because the file to sorted will become too large for the computer used. However by analysing the sorted file we can find that only a certain number of $[e]$ series can possibly produce Q_{min}. By carefully choosing candidate $[e]$ series we then can greatly reduce the size of the file of Q which is going to be sorted.

To find out the candidate $[e]$ series for the PRS systems, all possible Q for the systems with error items limited to 6 has been sorted. The $[e]$ patterns which give small Q are then chosen as the candidate patterns and rearranged for large number of error items, here are 11 and 22 error items in agreement with Zakarevicius and Feher [4]. Clearly such an arrangement is still another assumption for $[e]$. However the results of the computations shows that there are no significant differences between the Q values in the first group of the sorted file for those candidate patterns. From this fact we further assume that the difference between $Q_{\min}$, if it is not produced by one of those candidate patterns, and the smallest Q produced by those candidate patterns is not of significanceso that we can use this smallest Q in the place of $Q_{\min}$. The candidate patterns chosen are shown in table 4.2.

The $[h]$ for the PRS systems introduced in Chapter 1 are given in table 4.3 except version $1 - D$ or Dicode PRS system since this PRS system has a unique $|H(\omega)|$ form which is unsuitable for narrow bandwidth systems. The computations of $Q_{\min}$ for those PRS Systems are similar with the computation for version $1 + D$ except that sometimes Q values are larger than 1. This is because we are computing multi-error case instead of single error case. It shall not be a serious problem because what we are computing is the SNR penalty, that is a comparison for the systems with or without ISI, so that we can use Q without ISI to normalize the result, in this case, it is the Q for $\xi = 0$.

The final results for seven improved PRS versions are plotted in Fig. 4.4 where we can find that some of the PRS systems have good high speed properties and some have not. In Fig. 4.4 the length of the error sequences are chosen as 22 to be consistent with Zakarevicius and Feher. Since the SNR

Table 4.2

No	Candidate Error Pattern
1	1 -1 1 -1 1 -1 1 -1 1 -1 1
2	1 -1 1 -1 1 -1 1 -1 1 -1 0
3	0 0 1 -1 1 -1 1 -1 1 -1 1
4	0 1 -1 1 -1 1 -1 1 -1 1 0
5	1 -1 1 -1 1 -1 1 -1 1 0 0
6	0 0 0 1 -1 1 -1 1 -1 1 -1
7	0 0 1 -1 1 -1 1 -1 1 -1 0
8	0 1 -1 1 -1 1 -1 1 -1 0 0
9	1 -1 1 -1 1 -1 1 -1 0 0 0
10	0 0 0 0 1 -1 1 -1 1 -1 1
11	0 0 0 1 -1 1 -1 1 -1 1 0
12	0 0 1 -1 1 -1 1 -1 1 0 0
13	0 1 -1 1 -1 1 -1 1 0 0 0
14	1 -1 1 -1 1 -1 1 0 0 0 0
15	0 1 0 1 -1 1 -1 1 -1 1 -1
16	0 1 -1 1 -1 1 0 1 -1 1 -1
17	0 1 -1 0 -1 1 -1 1 -1 1 -1
18	1 0 1 -1 1 -1 1 0 1 -1 1
19	1 0 1 0 1 0 1 0 1 0 1
20	1 0 1 -1 1 -1 1 -1 1 -1 1
21	0 1 -1 1 -1 0 -1 1 -1 1 -1
22	0 1 -1 1 0 1 -1 1 -1 1 -1
23	1 0 0 -1 1 -1 1 -1 1 -1 1
24	1 -1 0 1 -1 0 1 -1 0 1 -1
25	0 1 -1 0 1 -1 0 1 -1 0 1
26	1 -1 1 0 -1 1 -1 0 1 -1 1
27	1 -1 1 -1 0 0 -1 1 -1 1 -1
28	1 -1 1 -1 ... 1 1 -1 1 -1
29	1 -1 1 -1 ... -1 1 -1 1 0

30	0 0 1 -1 1 -1 ... 1 -1 1 -1
31	0 1 -1 1 -1 1 .... 1 -1 1 -1 0
32	1 -1 1 -1 1 ... 1 -1 1 -1 0 0
33	0 0 0 1 -1 1 -1 ... -1 1 -1 1
34	0 0 1 -1 1 -1 1 ... -1 1 -1 1 0
35	0 1 -1 1 -1 ... -1 1 -1 1 0 0
36	1 -1 1 -1 1 ... -1 1 -1 1 0 0 0
37	0 0 0 0 1 -1 1 -1 ... 1 -1 1 -1
38	0 0 0 1 -1 1 -1 1 ... 1 -1 1 -1 0
39	0 0 1 -1 1 -1 1 ... 1 -1 1 -1 0 0
40	0 1 -1 1 -1 1 ... 1 -1 1 -1 0 0 0
41	1 -1 1 -1 1 ... 1 -1 1 -1 0 0 0 0
42	0 0 0 0 0 1 -1 1 -1 ... -1 1 -1 1
43	0 0 0 0 1 -1 1 -1 1 ... -1 1 -1 1 0
44	0 0 0 1 -1 1 -1 1 ... -1 1 -1 1 0 0
45	0 0 1 -1 1 -1 ... 1 -1 1 -1 1 0 0 0
46	0 1 -1 1 -1 1 ... 1 1 -1 1 0 0 0 0
47	1 -1 1 -1 1 ... -1 1 -1 1 0 0 0 0 0
48	0 0 0 0 0 0 1 -1 1 -1 ... 1 -1 1 -1
49	0 0 0 0 0 1 -1 1 -1 1 ... 1 -1 1 -1 0
50	0 0 0 0 1 -1 1 -1 ... 1 -1 1 -1 0 0
51	0 0 0 1 -1 1 -1 ... 1 -1 1 -1 0 0 0
52	0 0 1 -1 1 -1 ... 1 -1 1 -1 0 0 0 0
53	0 1 -1 1 -1 ... 1 -1 1 -1 0 0 0 0 0
54	1 -1 1 -1 ... 1 -1 1 -1 0 0 0 0 0 0
55	1 0 1 0 1 0 ... 1 0 1 0 1 0 1 0 1 0
56	1 -1 0 1 -1 0 0 0 ... 0 0 0 0 0 0 0
57	1 -1 0 0 1 -1 0 0 ... 0 0 1 -1 0 0 1 -1
58	1 0 1 -1 1 -1 ... 1 -1 1 -1 1 -1 1
59	1 -1 1 -1 1 ... 1 -1 1 -1 1 0 1

Table 4.3:

version	*improved*	*conventional*
$1 + D$	$f_n + f_{n-1}$	$f_n + \hat{f}_{n-1}$
$1 - D^2$	$f_n - f_{n-2}$	$f_n - \hat{f}_{n-2}$
$1 + 2D + D^2$	$f_n + 2f_{n-1} + f_{n-2}$	$f_n + 2\hat{f}_{n-1} + \hat{f}_{n-2}$
$1 + D - D^2 - D^3$	$f_n + f_{n-1} - f_{n-2} - f_{n-3}$	$f_n + \hat{f}_{n-1} - \hat{f}_{n-2} - \hat{f}_{n-3}$
$1 - D - D^2 + D^3$	$f_n - f_{n-1} - f_{n-2} + f_{n-3}$	$f_n - \hat{f}_{n-1} - \hat{f}_{n-2} + \hat{f}_{n-3}$
$1 - 2D^2 + D^4$	$f_n - 2f_{n-2} + f_{n-4}$	$f_n - 2\hat{f}_{n-2} + \hat{f}_{n-4}$
$2 + D - D^2$	$2f_n + f_{n-1} - f_{n-2}$	$2f_n + \hat{f}_{n-1} - \hat{f}_{n-2}$
$2 - D^2 - D^4$	$2f_n - f_{n-2} - f_{n-4}$	$2f_n - \hat{f}_{n-2} - \hat{f}_{n-4}$

where

$$f_n = \frac{\sin nx}{nx}, \quad \hat{f}_{n-i} = \frac{\sin(n-i\xi)x}{(n-i\xi)x}, \quad x = \frac{\pi}{1+\xi}.$$

penalty isdependent on the error length as shown in Fig 4.3 as well as the number of h_n, the values of the SNR penalty shown in Fig. 4.4, in Fig. 4.5 and 4.6 are used to illustrate the tendency rather than the precision. Fig. 4.5 and 4.6 are plotted in different vertical scale to suit the PRS versions which have extreme values of SNR penalty. The comparison between Improved PRS system version and conventional version is shown in Fig. 4.7 where the superiority of improved version can be seen very clearly.

List 4.1 gives some sample data of the results in columns of SNR penalty in dB, the values of ξ and the corresponding error patterns $[e]$.
The results here illustrate that there are three improved versions of PRS systems, $1 + 2D + D^2$, $1 + D - D^2 - D^3$ and $1 - 2D^2 - D^4$ (see Fig. 4.5, and appendix) which have very good performance when operating above Nyquist rate, especially version $1 + 2D + D^2$ which has only about 0.012 dB and 0.02 dB SNR penalties for 11 error and 22 error items at a speed 10 percent above Nyquist rate. The reason will be explained in next chapter.

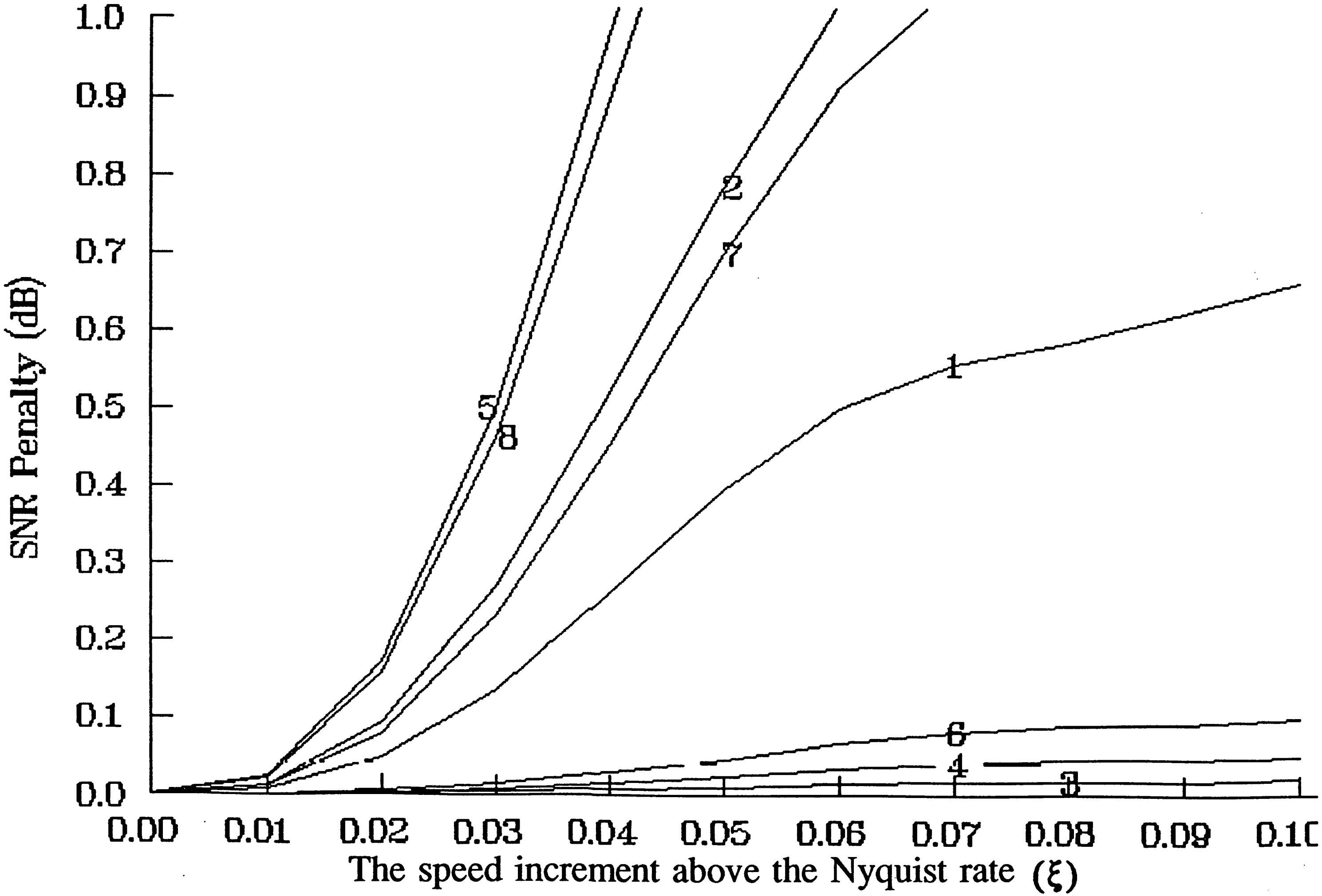

Fig.4.4 The SNR penalties for 8 improved PRS systems. The length of [e] is 22. Line 1 represents version $1+D$, line 2 for $1-D^2$, line 3 $1+2D+D^2$, line 4 $1+D-D^2-D^3$, line 5 $1-D-D^2+D^3$, line 6 $1-2D^2+D^4$, line 7 $2+D-D^2$, line 8 $2-D^2-D^4$.

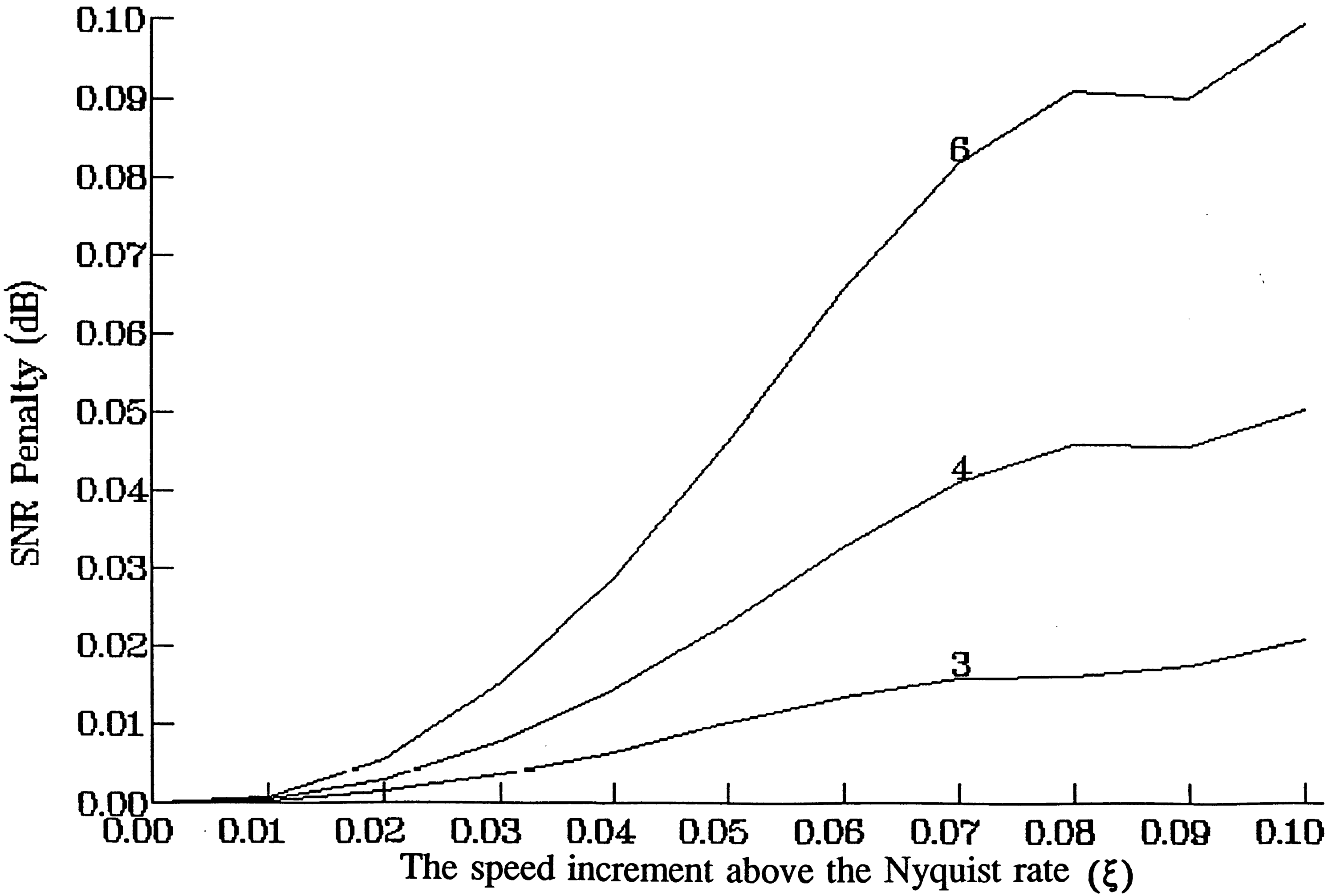

Fig. 4.5 The SNR penalties for some improved PRS systems. The length of [e] is 22. Line 3 represents version $1+2D+D^2$, line 4 $1+D-D^2-D^3$, line 6 $1-2D^2-D^4$.

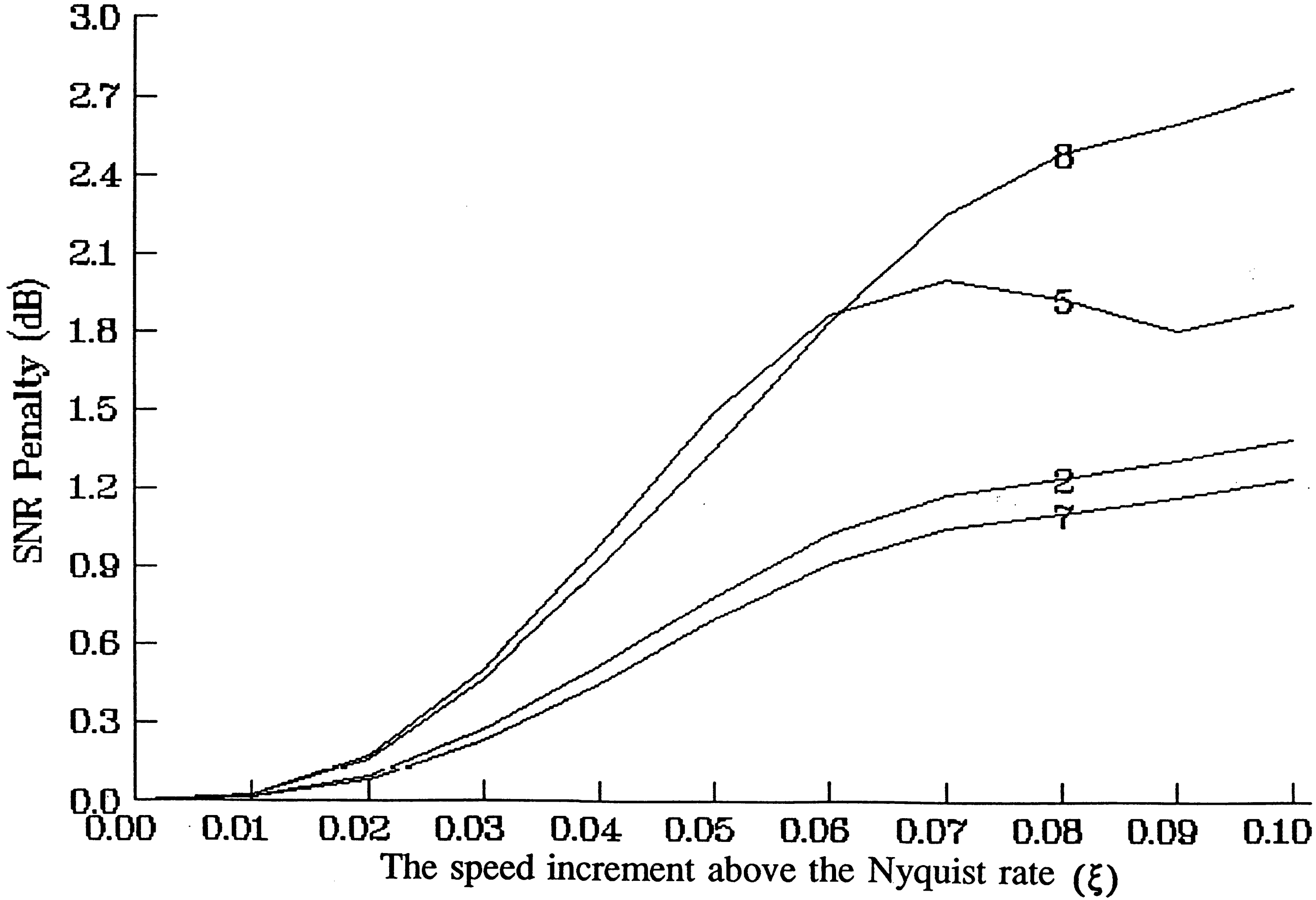

Fig. 4.6 The SNR penalties for some improved PRS systems. The length of [e] is 22. Line 2 represents version $1-D^2$, line 5 $1-D-D^2+D^3$, line 7 $2+D-D^2$, line 8 $2-D^2-D^4$.

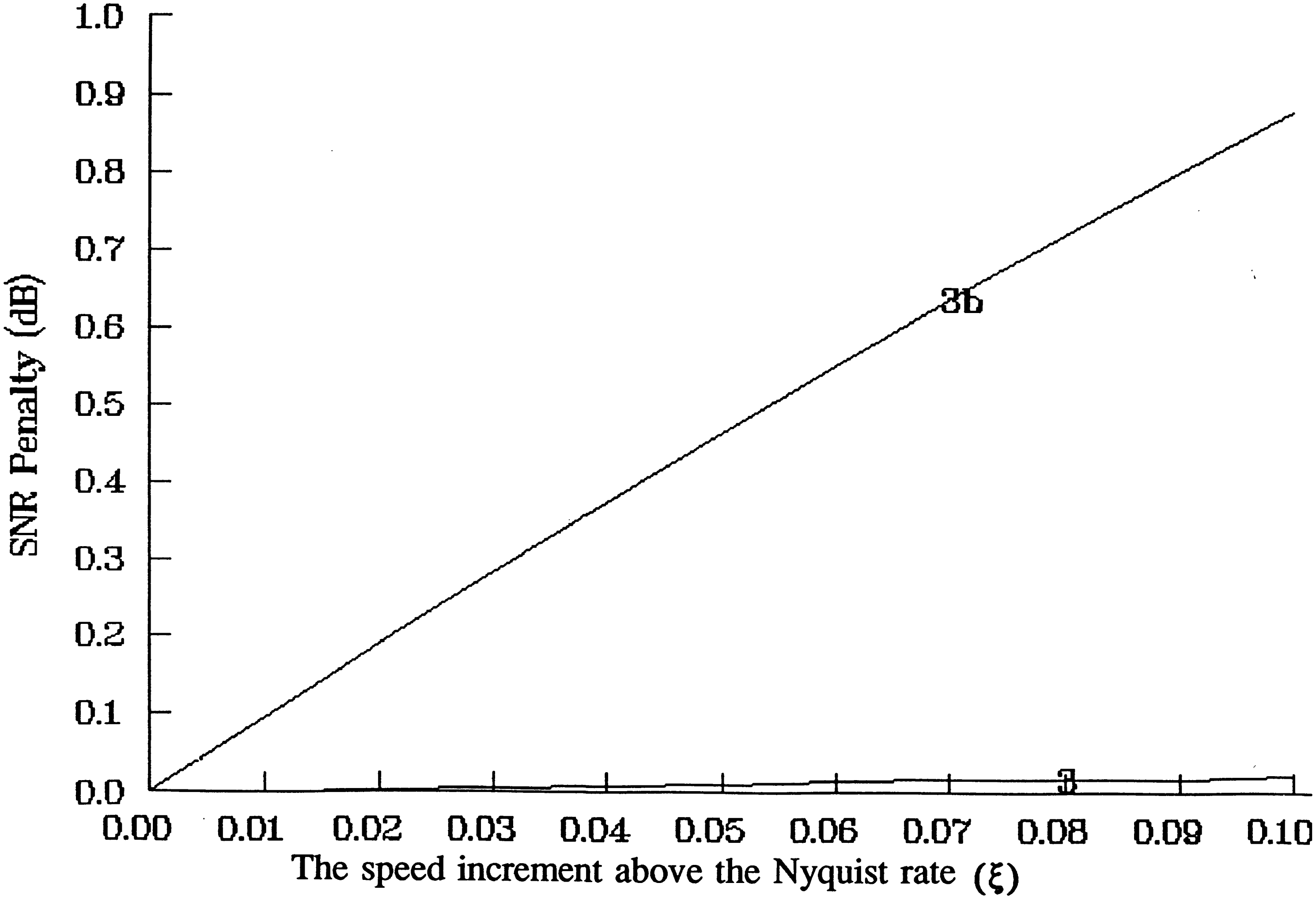

Fig. 4.7 The SNR Penalty comparison of improved and conventional PRS version $1+2D+D^2$. The length of [e] is 22. line 3 for improved version and line 3b for conventioal one.

List 4.1

The SNR Penalties (dB), the speed increments ξ and the error sequences $[e]$ for Improved Version $1+D$. The number of the error sequences is 22.

PENALTY (dB)	ξ	$[e]$
6.539×10^{-3}	0.01	1 -1 1 -1 ... 1 -1 1 -1
4.718×10^{-2}	0.02	1 -1 1 -1 ... 1 -1 1 -1
1.363×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
2.622×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 -1
3.941×10^{-1}	0.05	1 -1 1 -1 ... 1 -1 1 -1
4.984×10^{-1}	0.06	1 -1 1 -1 ... 1 -1 1 -1
5.563×10^{-1}	0.07	1 -1 1 -1 ... 1 -1 1 -1
5.849×10^{-1}	0.08	1 -1 1 ... 1 -1 1 0 0 0
6.232×10^{-1}	0.09	1 -1 1 ... 1 -1 1 0 0 0 0 0 0
6.628×10^{-1}	0.10	1 -1 1 ... 1 -1 1 0 0 0 0 0 0

The same items while the number of the error sequences is 11.

PENALTY (dB)	ξ	$[e]$
1.652×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.245×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.843×10^{-2}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.138×10^{-2}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.402×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.131×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.992×10^{-1}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.965×10^{-1}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.011×10^{-1}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.062×10^{-1}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

The SNR Penalties (dB), the speed increments ξ and the error sequences $[e]$ for Improved Version $1+2D+D^2$. The number of the error sequences is 22.		
PENALTY (dB)	ξ	$[e]$
1.957×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 -1
1.353×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 -1
3.626×10^{-3}	0.03	1 -1 1 -1 ... 1 -1 1 -1
6.514×10^{-3}	0.04	0 0 0 0 1 -1 1 ... -1 1 -1
1.062×10^{-2}	0.05	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.365×10^{-2}	0.06	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.588×10^{-2}	0.07	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.606×10^{-2}	0.08	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.752×10^{-2}	0.09	1 -1 1 -1 ... 1 -1 0 1
2.086×10^{-2}	0.10	1 -1 1 -1 ... 1 -1 0 1
The same items while the number of the error sequences is 11.		
PENALTY (dB)	ξ	$[e]$
4.521×10^{-7}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.281×10^{-5}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.661×10^{-5}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.257×10^{-4}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.810×10^{-4}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.913×10^{-3}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.538×10^{-3}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.804×10^{-3}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.704×10^{-3}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.220×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

The SNR Penalties (dB), the speed increments ξ and the error sequences $[e]$ for Improved Version $1+D-D^2-D^3$. The number of the error sequences is 22.		
PENALTY(dB)	ξ	$[e]$
3.948×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 -1
2.792×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 -1
7.769×10^{-3}	0.03	1 -1 1 -1 ... 1 -1 1 -1
1.439×10^{-2}	0.04	0 0 0 1 -1 1 ... 1 -1 1 -1 1
2.309×10^{-2}	0.05	0 0 0 0 0 0 1 -1 1 ... -1 1 -1
3.278×10^{-2}	0.06	0 0 0 0 0 0 1 -1 1 ... -1 1 -1
4.100×10^{-2}	0.07	0 0 0 0 0 1 -1 ... 1 -1 0
4.580×10^{-2}	0.08	0 0 0 0 0 1 -1 ... 1 -1 0
4.569×10^{-2}	0.09	0 0 0 0 0 1 -1 ... 1 -1 0 0
5.032×10^{-2}	0.10	0 0 1 -1 ... 1 -1 0 0 0 0 0
The same items while the number of the error sequences is 11.		
PENALTY (dB)	ξ	$[e]$
1.802×10^{-6}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.102×10^{-5}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.442×10^{-4}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.291×10^{-3}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.489×10^{-3}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.583×10^{-3}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.405×10^{-2}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.310×10^{-2}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.464×10^{-2}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.848×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

The SNR Penalties (dB), the speed increments ξ and the error sequences $[e]$ for Improved Version $1-2D^2+D^4$. The number of the error sequences is 22.

PENALTY (dB)	ξ	$[e]$
7.563×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 0
5.404×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 0
1.531×10^{-2}	0.03	1 -1 1 -1 ... 1 -1 1 0
2.875×10^{-2}	0.04	0 0 1 -1 ... 1 -1 1 0
4.608×10^{-2}	0.05	0 0 0 0 0 1 -1 ... -1 1 -1 0
6.544×10^{-2}	0.06	0 0 0 0 0 1 -1 ... -1 1 -1 0
8.176×10^{-2}	0.07	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.090×10^{-2}	0.08	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.002×10^{-2}	0.09	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.948×10^{-2}	0.10	0 1 -1 1 ... 1 -1 0 0 0 0 0

The same items while the number of the error sequences is 11.

PENALTY (dB)	ξ	$[e]$
3.596×10^{-6}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.016×10^{-4}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.840×10^{-4}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.560×10^{-3}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.913×10^{-3}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.504×10^{-2}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.793×10^{-2}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.601×10^{-2}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.907×10^{-2}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.655×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

Chapter 5. PRS system

$1+2D+D^2$

The results of chapter 4 indicate that there are three types of PRS Systems which have very good speed tolerance when operating in the improved system version, especially improved version $1+2D+D^2$. In this chapter the explanation of why this can happen and how fast the system can run above the Nyquist rate are presented.

The research of Zakarevicius and Feher shows that in the improved PRS version when ξ (the fractional increase above the Nyquist rate) is small, the sidelobes of sinc function are approximately equal but with alternative sign. The use of the improved version $1+D$ results in the cancellation of the sidelobe [3].

The mathematical analysis shows as:

$$\underset{n\neq 0}{f(nT)} = \sin\frac{\left(\dfrac{n\pi}{1+\xi}\right)}{\left(\dfrac{n\pi}{1+\xi}\right)} = \xi\,\frac{\sin\left(\dfrac{n\pi}{1+\xi}\right)}{\left(\dfrac{n\pi\xi}{1+\xi}\right)} \,. \qquad (5.1)$$

Because

$$\begin{aligned} \sin\left(\frac{n\pi}{1+\xi}\right) &= \sin\left(n\pi - \frac{n\pi\xi}{1+\xi}\right) \\ &= (-1)^{n+1}\sin\left(\frac{n\pi\xi}{1+\xi}\right), \end{aligned} \qquad (5.2)$$

so we have

$$f_{n\neq 0}(nT) = (-1)^{n+1}\xi \frac{\sin\left(\frac{n\pi\xi}{1+\xi}\right)}{\left(\frac{n\pi\xi}{1+\xi}\right)} \tag{5.3}$$

When $\xi \ll 1$ we can get $1 + \xi \approx 1$ so that

$$f_{n\neq 0}(nT) \approx (-1)^{n+1}\xi \frac{\sin(n\pi\xi)}{n\pi\xi} \quad .$$

Furthermore if n is relatively small so as $n\pi\xi \ll 1$, then we can have

$$\begin{aligned} f_{n\neq 0}(nT) &\approx (-1)^{n+1}\xi \frac{\sin(n\pi\xi)}{n\pi\xi} \\ &\approx (-1)^{n+1}\xi \frac{n\pi\xi}{n\pi\xi} \approx (-1)^{n+1}\xi . \end{aligned} \tag{5.4}$$

Equ.(5.4) shows us that all the sidelobes of $f(nT)$ $(n\neq 0)$ have the magnitude ξ weighted by a sinc function. When $\xi \ll 1$ and n is small then the sidelobes are independent of n and all have the same magnitude ξ . This analysis was used by Zakarevicius and Feher in their computation of the improved version $1+D$. But the second condition of $n\pi\xi \ll 1$ is quite restrictive. If $\xi = 0.1$, only for $n < 1/\pi < 1$ can we get $n\pi\xi < 0.1$. For

$\xi = 0.01$ we need $n < 10/\pi \approx 3$ to yield the same result. Clearly such a small n is not sufficient and consequently for a relatively large ξ the remaining sidelobes of h_n for the improved version $1+D$ could be comparatively large and thus the SNR penalties are also large.

From (5.3) we know that the sidelobes of $f(nT)$ $(n \neq 0)$ have the magnitude ξ weighted by a SINC function. For such a SINC function, if n is a continuous variable, zero points can be found at the points

$$\frac{n\pi\xi}{1+\xi} = \pm N\pi, \quad N = 0, \pm 1, \pm 2, \ldots$$

or

$$0n = \frac{1+\xi}{\xi} \times N, \; N = \pm 1, \pm 2, \ldots$$

For a discrete variable n, there should be about $(1 + \xi)/\xi$ sidelobes between any of the two zero points. When ξ is small the number of the sidelobes should be quite large. For example, when $\xi = 0.1$ there are about 11 sidelobes within one span of the SINC function and we may expect more sidelobes for smaller ξ. Clearly for such a ξ the curve of the SINC function between the adjacent sidelobes can be well linearly approximated and the smaller ξ is, the more the sidelobes are within one span, and the more accurate the approximation is.

For a PRS system if which can linearly cancel the SINC function sidelobes then it will have better speed tolerance than Duobinary or $1+D$ PRS system. This is the case of improved version $1+2D+D^2$. This particular version adds the two next adjacent sidelobes together and then adds them to the middle one which has been multiplied by two. Because the sidelobes of SINC function are alternating in sign, to add to the middle one means to cancel them. The use of three adjacent sidelobes in the cancellation is a use of linear approximation. In this PRS system even ξ is relatively large, for $\xi = 0.1$ the cancellation is still nearly complete.

Fig.5.1 shows the variation of the sinc function with the improved brick-wall type low-pass filter is implemented and the absolute values of the sidelobes, with $\xi = 0.1$. Here we can see the variation of sidelobes is approximately linear.

There is another way of looking at this PRS system which is to note that

$$\begin{aligned} &1 + 2D + D^2 \\ &= (1+D) + D(1+D) \end{aligned} \tag{5.5}$$

This equation shows us that the $1 + 2D + D^2$ system can be thought of as two $1 + D$ systems, one delayed by D, combined together. Since the sidelobes of the impulse response $[h]$ in the

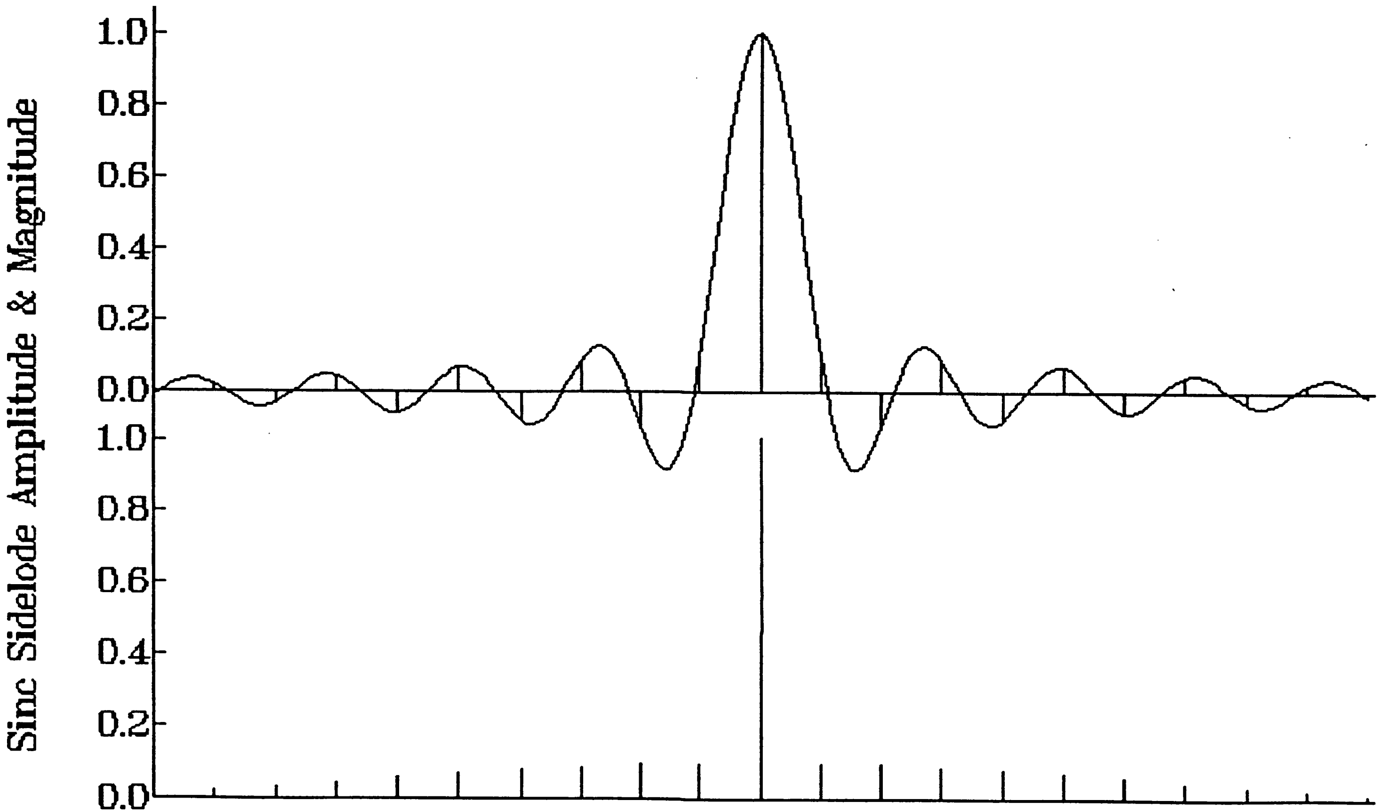

Fig. 5.1 The SINC function the sample values for improved PRS version, while $\xi=0.1$.

$1+D$ are of alternating sign and adjacent sidelobes are of approximately equal magnitude when ξ is small as shown in Fig 5.2, combining itself after a delay of D can cause further sidelobe cancellation. Let us extend this point of view further to combine two PRS system $1+2D+D^2$, one delayed by D to form a new PRS system. Similarly it can be written as

$$
\begin{aligned}
&(1+2D+D^2)+D\,(1+2D+D^2) \\
&\quad = (1+D)\,(1+2D+D^2) \\
&\quad = 1+3D+3D^2+D^3
\end{aligned}
$$

Using the same calculation method used by [4] and chapter 3, the SNR penalties for the improved PRS version $1+2D+D^2$ and the new PRS system version $1+3D+3D^2+D^3$ are computed. The results are plotted in Fig.5.4 and 5.5 while in Fig. 5.5 ξ is taken as 0.10,0.15, 0.20 to 0.55. As we can see the curves indicate the new version has better speed tolerance as we expected. In Fig. 5.5 The curves show that even when $\xi = 0.45$ the SNR penalty is still less than 1 dB. This means a very good speed tolerance indeed.

Fig.5.3 shows the comparison of the impulse response $[h]$ of two improved PRS versions $1+2D+D^2$ and $1+D$. For other two improved PRS systems, $1+D-D^2-D^3$ and $1-2D^2-D^4$, similar linear sidelobe cancellation can be found.

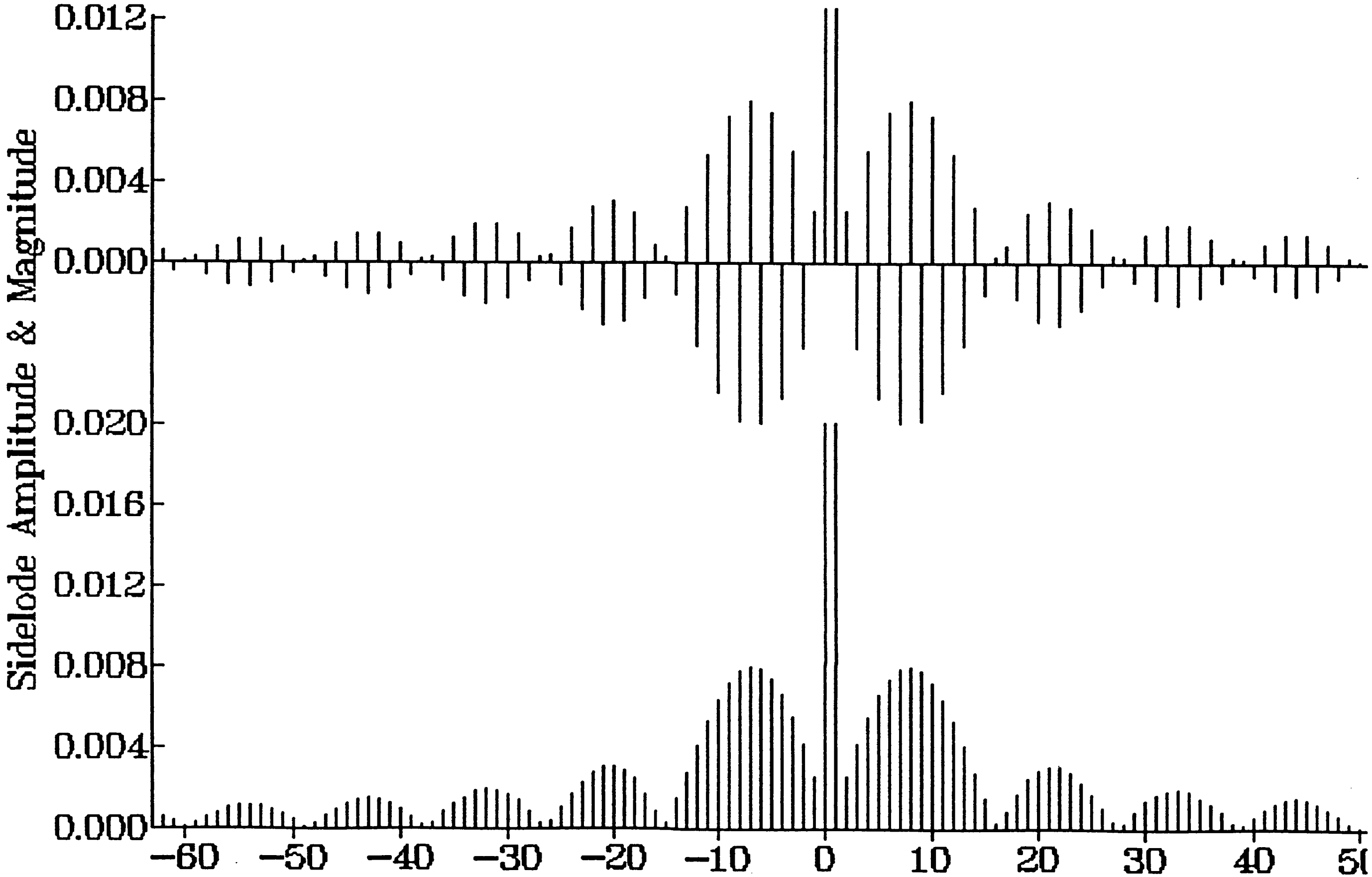

Fig. 5.2 The sidelobes of $[h]$ for improved PRS version $1+D$, while $\xi=0.1$.

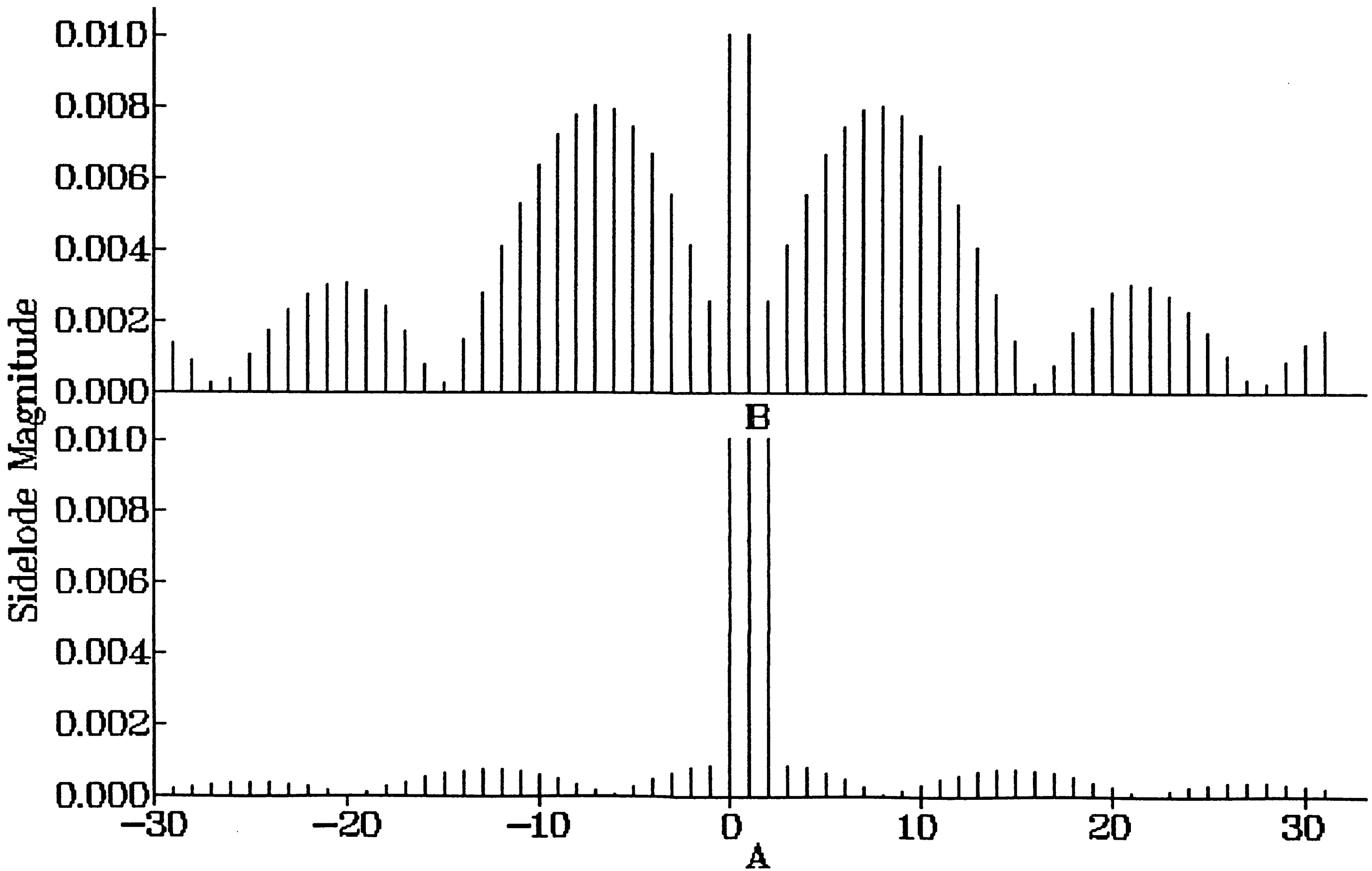

Fig. 5.3 The comparison of the sidelobes of $[h]$ for two improved PRS versions, (A): $1+2D+D^2$ and (B): $1+D$ while $\xi=0.1$.

Table 5.1

Row	Class	dB
1	$1 + D$	0.66
2	$1 - D^2$	1.39
3	$1 + 2D + D^2$ $= (1+D) + D(1+D)$	0.02
4	$1 + D - D^2 - D^3$ $= (1+D) - D^2(1+D)$ $= (1-D^2) + D(1-D^2)$	0.05
5	$1 - 2D^2 + D^4$ $= (1-D^2) - D^2(1-D^2)$	0.10
6	$1+3D+3D^2+D^3$ $= (1+D)(1+2D+D^2)$	0.003

(In all cases the SNR penalty is for the improved version)

A comparison for a number of the improved PRS systems ars shown in Table 5.1 where the frequency increment above the Nyquist rate $\xi=0.1$. In the case of $1-D^2$, raw two, a degree of sidelobe cancellation is found. Since after two period of delay the sidelobes of the SINC fuction are now in phase. The subtraction of them means cancellation. The result of this cancellation is not as good as for the case of $1+D$ in raw 1, because after two period shift the equality of the SINC fuction sidelobes is not as good as after one period shift. But If we examine the case of improved PRS version $1+D-D^2-D^3$, row 4, we can find that it is a combination of two $1-D^2$ one shifted by

D . Among the more complicated systems in raw 3 to 6 the $1+2D+D^2$ gives the second best performance and It is the simplest, requiring the smallest number of output levels.

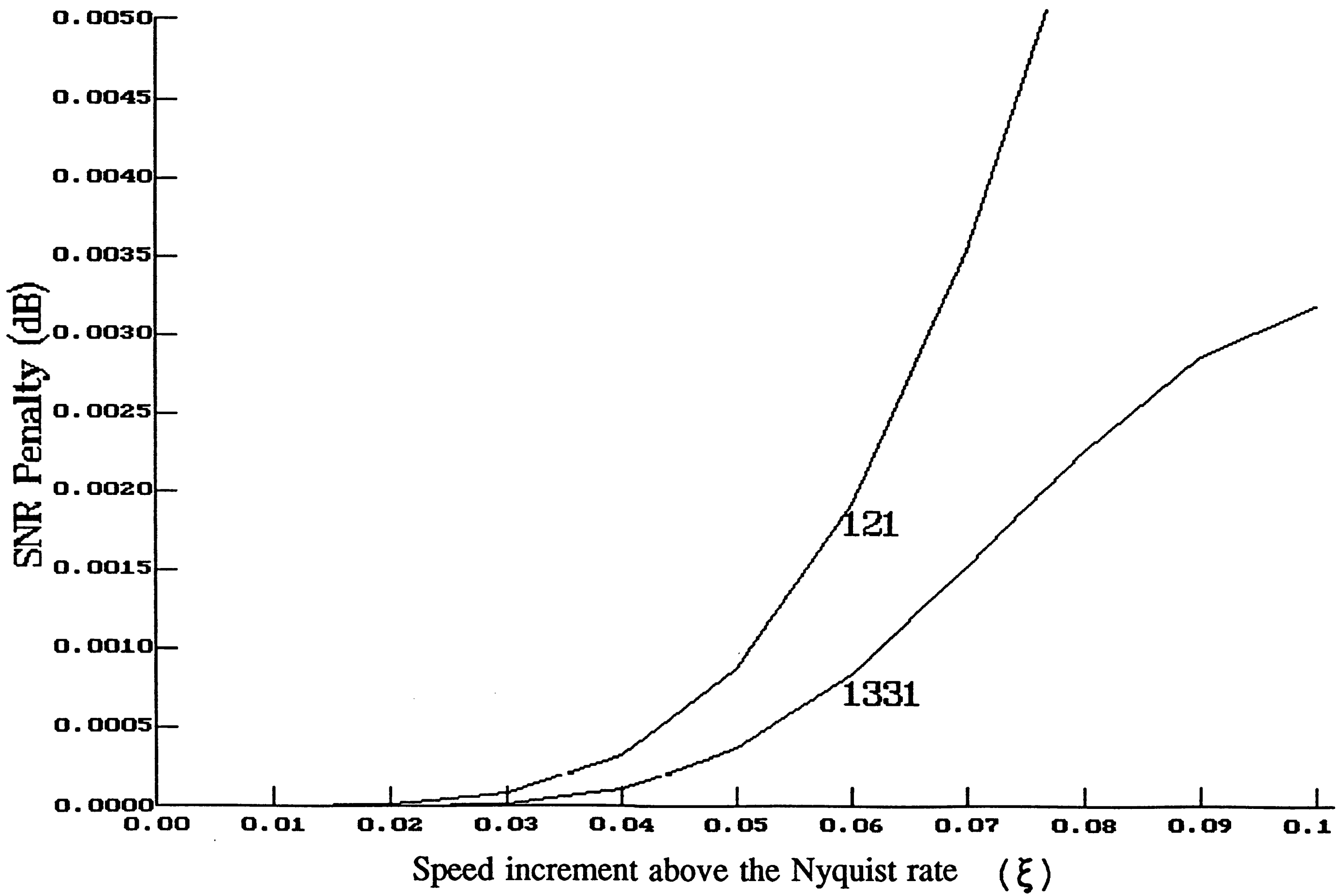

Fig. 5.4 The SNR penalties for improved PRS version $1+2D+D^2$ and $1+3D+3D^2+D^3$

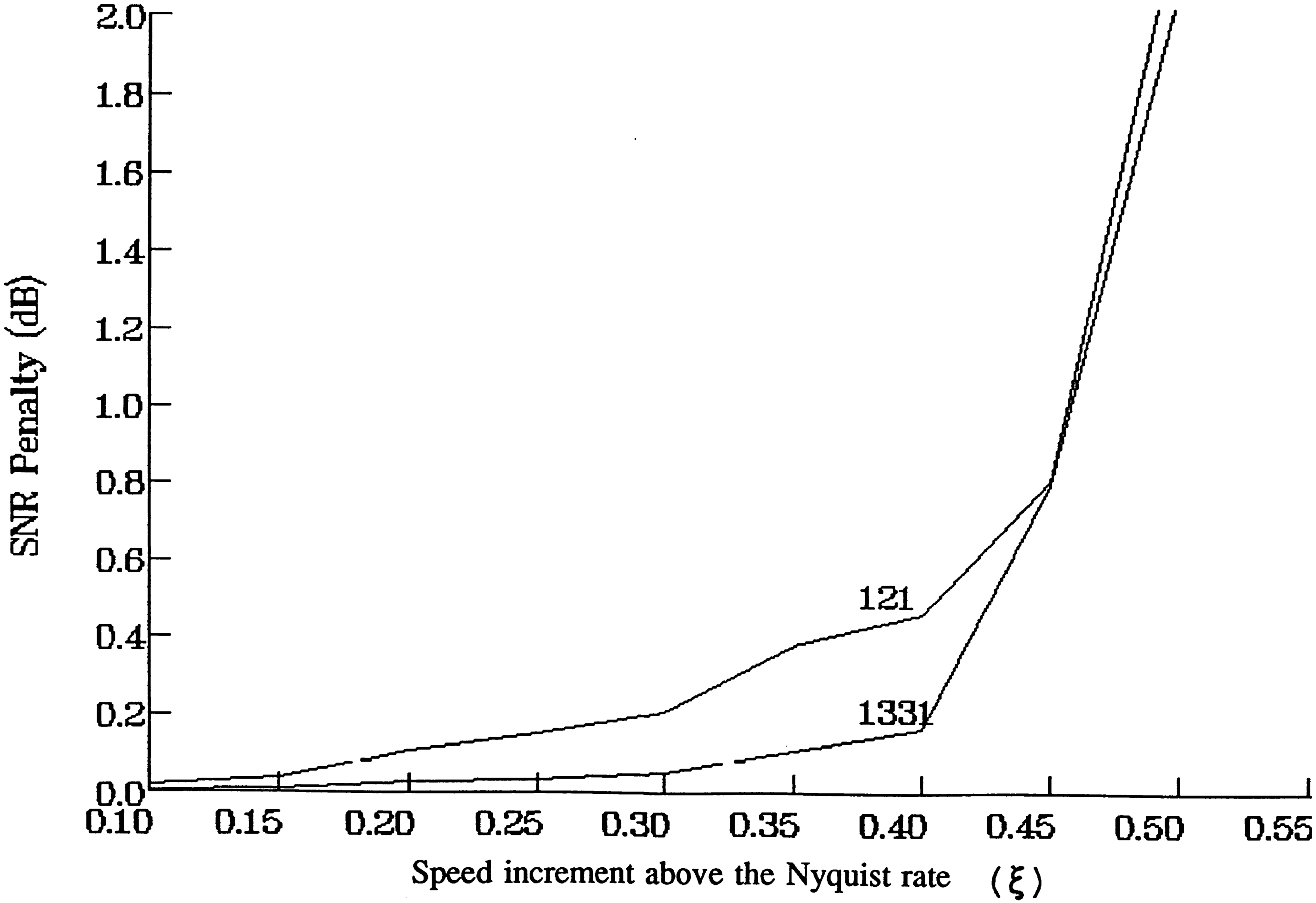

Fig. 5.5 The SNR penalties for improved PRS version $1+2D+D^2$ and $1+3D+3D^2+D^3$, while $\xi=0.1-0.55$.

Conclusion

Partial Response Systems (PRS) are speed tolerant. In these systems digital signal can be transmitted at a speed faster than the Nyquist rate. The price of doing so is a deterioration of the system performance. This system performance reduction can be expressed as an SNR (signal-to-noise ratio) penalty. The performance of conventional versions of PRS systems deteriorate rapidly when the Nyquist rate is exceeded. In this sense their speed tolerance is quite limited. The improved version of PRS systems introduced by Wu and Feher shows much better high speed performance.

Zakarevicius and Feher in a computer simulation have evaluated the SNR penalty for Duobinary or, in the system polynomial version, 1+D PRS system with MLSE under some assumptions both in conventional and improved versions. The raising questions here are: are there any other PRS systems which have better speed properties and what is the validity of the assumptions made by Zakarevicius and Feher? Following the method introduced by Zakarevicius and Feher, the author of this project report evaluated the validity of the assumption used earlier and calculated the SNR penalties for some other PRS systems with MLSE.

The results show that the assumption of that the alternating error pattern would produce the worst SNR penalty should be modified when the length of the error sequence is relatively large. By carefully sorting the error patterns a group of candidate error patterns were then chosen.

The computations of SNR penalties for those PRS systems indicate that the superiority of the improved PRS system version to the conventional PRS system version is generally true.

The results show that some improved PRS systems versions have much better speed property than the improved version of Duobinary, or 1+D PRS system, such as the improved PRS version $1+D-D^2-D^3$, $1-2D^2-D^4$ and $1+2D+D^2$, especially the improved PRS version $1+2D+D^2$ (class 2) which shows about 0.02 dB and 1 dB SNR penalty at 10% and 45% above the Nyquist rate. The reason is that under the linear approximation the sidelobe cancellation which occurs in the improved version is more complete than the one used earlier.

For the purpose of comparison a new improved PRS system version $1+3D+3D^2+D^3$ was introduced and its SNR penalty was computed. The high speed performance of this newly introduced version is even better than the improved PRS version $1+2D+D^2$ as we expected. From the implementation point of view, the improved PRS version $1+3D+3D^2+D^3$ is not very valuable because of its complexity. However the most valuable one hence should be the improved PRS system version $1+2D+D^2$ rather than any other PRS systems. Its system performance is better than all other PRS systems which have been computed except the PRS system $1+D$ whilst its complexity is same as or better than all other PRS systems.

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Appendix

The SNR penalties for some PRS systems (both improved and conventional), corresponding $[e]$ patterns and ξ (fractional increase above Nyquist rate) are listed below:

PRS Improved Version $1+D$, 22 error sequence.		
PENALTY (dB)	ξ	$[e]$
6.539×10^{-3}	0.01	1 -1 1 -1 ... 1 -1 1 -1
4.718×10^{-2}	0.02	1 -1 1 -1 ... 1 -1 1 -1
1.363×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
2.622×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 -1
3.941×10^{-1}	0.05	1 -1 1 -1 ... 1 -1 1 -1
4.984×10^{-1}	0.06	1 -1 1 -1 ... 1 -1 1 -1
5.563×10^{-1}	0.07	1 -1 1 -1 ... 1 -1 1 -1
5.849×10^{-1}	0.08	1 -1 1 ... 1 -1 1 0 0 0
6.232×10^{-1}	0.09	1 -1 1 ... -1 1 0 0 0 0 0 0
6.628×10^{-1}	0.10	1 -1 1 ... -1 1 0 0 0 0 0 0

PRS Improved Version $1+D$, 11 error sequence.		
PENALTY (dB)	ξ	$[e]$
1.652×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.245×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.843×10^{-2}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.138×10^{-2}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.402×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.131×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.992×10^{-1}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.965×10^{-1}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.011×10^{-1}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.062×10^{-1}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $1-D^2$, 22 error sequence.

PENALTY (*dB*)	ξ	[*e*]
1.286×10^{-2}	0.01	1 -1 1 -1 ... 1 -1 1 -1
9.296×10^{-2}	0.02	1 -1 1 -1 ... 1 -1 1 -1
2.690×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
5.174×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 -1
7.838×10^{-1}	0.05	1 -1 1 -1 ...1 -1 1 0
1.025×10^{0}	0.06	1 -1 1 -1 ...1 -1 1 0
1.177×10^{0}	0.07	1 -1 1 -1 ...1 -1 1 0
1.242×10^{0}	0.08	1 -1 1 ... 1 -1 1 0 0 0
1.312×10^{0}	0.09	1 -1 1 ... 1 -1 0 0 0 0 0 0
1.392×10^{0}	0.10	1 -1 1 ... 1 -1 0 0 0 0 0 0

PRS Improved Version $1-D^2$, 11 error sequence.

PENALTY (*dB*)	ξ	[*e*]
3.283×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.482×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.692×10^{-2}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.698×19^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.836×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.330×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.100×10^{-1}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.118×10^{-1}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.032×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.258×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $1+2D+D^2$, 22 error sequence.		
PENALTY (dB)	ξ	[*e*]
1.957×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 -1
1.353×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 -1
3.626×10^{-3}	0.03	1 -1 1 -1 ... 1 -1 1 -1
6.514×10^{-3}	0.04	0 0 0 0 1 -1 1 ... -1 1 -1
1.062×10^{-2}	0.05	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.365×10^{-2}	0.06	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.588×10^{-2}	0.07	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.606×10^{-2}	0.08	0 0 0 0 0 0 1 -1 ... -1 1 -1
1.752×10^{-2}	0.09	1 -1 1 -1 ... 1 -1 0 1
2.086×10^{-2}	0.10	1 -1 1 -1 ... 1 -1 0 1

PRS Improved Version $1+2D+D^2$, 11 error sequence.		
PENALTY (dB)	ξ	[*e*]
4.521×10^{-7}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.281×10^{-5}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.661×10^{-5}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.257×10^{-4}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.810×10^{-4}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.913×10^{-3}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.538×10^{-3}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.804×10^{-3}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.704×10^{-3}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.220×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $1+D-D^2-D^3$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
3.948×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 -1
2.792×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 -1
7.769×10^{-3}	0.03	1 -1 1 -1 ... 1 -1 1 -1
1.439×10^{-2}	0.04	0 0 0 1 -1 1... 1 -1 1 -1 1
2.309×10^{-2}	0.05	0 0 0 0 0 0 1 -1 ... -1 1 -1
3.278×10^{-2}	0.06	0 0 0 0 0 0 1 -1 ... -1 1 -1
4.100×10^{-2}	0.07	0 0 0 0 0 1 -1 ... 1 -1 0
4.580×10^{-2}	0.08	0 0 0 0 0 1 -1 ... 1 -1 0
4.565×10^{-2}	0.09	0 0 0 0 0 1 -1 ... 1 -1 0
5.032×10^{-2}	0.10	0 0 1 -1 ... 1 -1 0 0 0 0
PRS Improved Version $1+D-D^2-D^3$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.802×10^{-6}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.102×10^{-5}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.442×10^{-4}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.291×10^{-3}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.489×10^{-3}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.583×10^{-3}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.405×10^{-2}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.310×10^{-2}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.464×10^{-2}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.848×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $1-2D^2+D^4$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
7.563×10^{-4}	0.01	1 -1 1 -1 ... 1 -1 1 0
5.404×10^{-3}	0.02	1 -1 1 -1 ... 1 -1 1 0
1.531×10^{-2}	0.03	1 -1 1 -1 ... 1 -1 1 0
2.875×10^{-2}	0.04	0 0 1 -1 ... 1 -1 1 0
4.608×10^{-2}	0.05	0 0 0 0 0 1 -1 ... -1 1 -1 0
6.544×10^{-2}	0.06	0 0 0 0 0 1 -1 ... -1 1 -1 0
8.176×10^{-2}	0.07	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.090×10^{-2}	0.08	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.002×10^{-2}	0.09	0 0 0 0 0 1 -1 ... -1 1 -1 0
9.948×10^{-2}	0.10	0 1 -1 1 ... 1 -1 0 0 0 0 0
PRS Improved Version $1-2D^2+D^4$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
3.596×10^{-6}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.016×10^{-4}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.840×10^{-4}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.560×10^{-3}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.913×10^{-3}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.504×10^{-2}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.793×10^{-2}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.601×10^{-2}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.907×10^{-2}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.655×10^{-2}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $1-D-D^2+D^3$, 22 error sequence.		
PENALTY (dB)	ξ	$[e]$
2.332×10^{-2}	0.01	1 0 1 -1 ... 1 -1 1 -1
1.699×10^{-1}	0.02	1 0 1 -1 ... 1 -1 1 -1
4.994×10^{-1}	0.03	1 0 1 -1 ... 1 -1 1 -1
9.802×10^{-1}	0.04	1 0 1 -1 ... 1 -1 1 -1
1.490×10^{0}	0.05	1 0 1 -1 ... 1 -1 1 -1
1.865×10^{0}	0.06	1 0 1 -1 ... 1 -1 1 -1
2.001×10^{0}	0.07	1 0 1 -1 ... 1 -1 1 -1
1.928×10^{0}	0.08	1 0 1 -1 ... 1 -1 1 -1
1.808×10^{0}	0.09	1 -1 1 ... -1 1 0 0 0 0 0
1.906×10^{0}	0.10	0 0 0 0 1 -1 ... 1 -1 0 0

PRS Improved Version $1-D-D^2+D^3$, 11 error sequence.		
PENALTY (dB)	ξ	$[e]$
4.347×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.297×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.025×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.194×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.825×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.868×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.285×10^{-1}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.105×10^{0}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.410×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.735×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $2+D-D^2$, 22 error sequence.		
PENALTY (dB)	ξ	[*e*]
1.075×10^{-2}	0.01	1 -1 1 -1 ... 1 -1 1 -1
7.816×10^{-2}	0.02	1 -1 1 -1 ... 1 -1 1 0
2.295×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 0
4.522×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 0
6.995×10^{-1}	0.05	1 -1 1 -1 ... 1 -1 1 0
9.117×10^{-1}	0.06	1 -1 1 -1 ... 1 -1 1 0
1.046×10^{0}	0.07	1 -1 1 -1 ... 1 -1 1 0
1.104×10^{0}	0.08	1 -1 1 -1 ... -1 1 0 0 0
1.170×10^{0}	0.09	1 -1 1 ... 1 -1 0 0 0 0 0 0
1.243×10^{0}	0.10	1 -1 1 ... 1 -1 0 0 0 0 0 0
PRS Improved Version $2+D-D^2$, 11 error sequence.		
PENALTY (dB)	ξ	[*e*]
2.961×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.237×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.923×10^{-2}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.472×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.546×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.886×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.474×10^{-1}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.283×10^{-1}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.252×10^{-1}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.126×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Improved Version $2-D^2-D^4$, 22 error sequence.		
PENALTY (dB)	ξ	$[e]$
2.174×10^{-2}	0.01	1 -1 1 -1 ... 1 -1 1 -1
1.578×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 -1
4.603×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
8.944×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 -1
1.344×10^{0}	0.05	1 -1 1 -1 ... 1 -1 1 -1
1.842×10^{0}	0.06	1 -1 1 -1 ... -1 1 0 0 0
2.254×10^{0}	0.07	1 -1 1 -1 ... -1 1 0 0 0
2.489×10^{0}	0.08	1 -1 1 -1 ... -1 1 0 0 0
2.599×10^{0}	0.09	1 -1 1 ... -1 1 0 0 0 0 0
2.736×10^{0}	0.10	1 -1 1 ... 1 -1 0 0 0 0 0 0
PRS Improved Version $2-D^2-D^4$, 11 error sequence.		
PENALTY (dB)	ξ	$[e]$
5.853×10^{-3}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.444×10^{-2}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.384×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.966×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.171×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.945×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.125×10^{0}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.506×10^{0}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.930×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.379×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Conventional Version $1+D$, 22 error sequence.

PENALTY (dB)	ξ	[*e*]
2.673×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 -1
5.066×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 -1
7.073×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
8.695×10^{-1}	0.04	1 -1 1 -1 ... 1 -1 1 -1
1.001×10^{0}	0.05	1 -1 1 -1 ... 1 -1 1 -1
1.111×10^{0}	0.06	1 -1 1 -1 ... 1 -1 1 -1
1.210×10^{0}	0.07	1 -1 1 -1 ... 1 -1 1 -1
1.305×10^{0}	0.08	1 -1 1 -1 ... 1 -1 1 -1
1.399×10^{0}	0.09	1 -1 1 -1 ... 1 -1 1 -1
1.494×10^{0}	0.10	1 -1 1 -1 ... 1 -1 1 -1

PRS Conventional Version $1+D$, 11 error sequence.

PENALTY (dB)	ξ	[*e*]
2.088×10^{-1}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.019×10^{-1}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.762×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.315×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.686×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.885×10^{-1}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.130×10^{0}	0.07	1 -1 1 0 -1 1 -1 0 1 -1 1
1.305×10^{0}	0.08	1 -1 1 0 -1 1 -1 0 1 -1 1
1.483×10^{0}	0.09	1 -1 1 0 -1 1 -1 0 1 -1 1
1.664×10^{0}	0.10	1 -1 1 0 -1 1 -1 0 1 -1 1

PRS Conventional Version $1-D^2$, 22 error sequence.		
***PENALTY* (*dB*)**	ξ	[*e*]
2.530×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 0
5.210×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 0
8.048×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 0
1.090×10^{0}	0.04	1 -1 1 -1 ... 1 -1 1 0
1.352×10^{0}	0.05	1 -1 1 -1 ... 1 -1 1 0
1.567×10^{0}	0.06	1 -1 1 -1 ... 1 -1 1 0
1.726×10^{0}	0.07	1 -1 1 -1 ... 1 -1 1 0
1.834×10^{0}	0.08	1 -1 1 -1 ... 1 -1 1 0
1.915×10^{0}	0.09	0 0 0 0 0 1 -1 1 ... 1 -1 0
2.016×10^{0}	0.10	0 0 0 0 0 1 -1 1 ... 1 -1 0
PRS Conventional Version $1-D^2$, 11 error sequence.		
***PENALTY* (*dB*)**	ξ	[*e*]
1.928×10^{-1}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
3.871×10^{-1}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
5.834×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
7.806×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.770×10^{-1}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.172×10^{0}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.364×10^{0}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.551×10^{0}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.731×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.898×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Conventional Version $1+2D+D^2$, 22 error sequence.		
PENALTY (dB)	ξ	[*e*]
1.323×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 -1
2.338×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 -1
3.341×10^{-1}	0.03	1 -1 0 1 -1 0 ... 1 -1 0 1
4.456×10^{-1}	0.04	1 -1 0 1 -1 0 ... 1 -1 0 1
5.564×10^{-1}	0.05	1 -1 0 1 -1 0 ... 1 -1 0 1
6.657×10^{-1}	0.06	1 -1 0 1 -1 0 ... 1 -1 0 1
7.734×10^{-1}	0.07	1 -1 0 1 -1 0 ... 1 -1 0 1
8.794×10^{-1}	0.08	1 -1 0 1 -1 0 ... 1 -1 0 1
9.843×10^{-1}	0.09	1 -1 0 1 -1 0 ... 1 -1 0 1
1.088×10^{0}	0.10	1 -1 0 1 -1 0 ... 1 -1 0 1
PRS Conventional Version $1+2D+D^2$, 11 error sequence.		
PENALTY (dB)	ξ	[*e*]
9.517×10^{-2}	0.01	1 -1 0 1 -1 0 1 -1 0 1 -1
1.892×10^{-1}	0.02	1 -1 0 1 -1 0 1 -1 0 1 -1
2.820×10^{-1}	0.03	1 -1 0 1 -1 0 1 -1 0 1 -1
3.732×10^{-1}	0.04	1 -1 0 1 -1 0 1 -1 0 1 -1
4.628×10^{-1}	0.05	1 -1 0 1 -1 0 1 -1 0 1 -1
5.506×10^{-1}	0.06	1 -1 0 1 -1 0 1 -1 0 1 -1
6.362×10^{-1}	0.07	1 -1 0 1 -1 0 1 -1 0 1 -1
7.196×10^{-1}	0.08	1 -1 0 1 -1 0 1 -1 0 1 -1
8.002×10^{-1}	0.09	1 -1 0 1 -1 0 1 -1 0 1 -1
8.779×10^{-1}	0.10	1 -1 0 1 -1 0 1 -1 0 1 -1

PRS Conventional Version $1+D-D^2-D^3$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.309×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 -1
2.319×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 -1
3.057×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 -1
3.672×10^{-1}	0.04	0 0 0 0 0 0 1 -1 1 ... 1 -1
4.208×10^{-1}	0.05	0 0 0 0 0 0 1 -1 1 ... 1 -1
4.667×10^{-1}	0.06	0 0 0 0 0 0 1 -1 1 ... 1 -1
5.131×10^{-1}	0.07	1 -1 0 1 -1 0 ... 1 -1 0 1
5.903×10^{-1}	0.08	1 -1 0 1 -1 0 ... 1 -1 0 1
6.683×10^{-1}	0.09	1 -1 0 1 -1 0 ... 1 -1 0 1
7.473×10^{-1}	0.10	1 -1 0 1 -1 0 ... 1 -1 0 1
PRS Conventional Version $1+D-D^2-D^3$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.014×10^{-1}	0.01	1 -1 1 0 -1 1 -1 0 1 -1 1
2.021×10^{-1}	0.02	1 -1 1 0 -1 1 -1 0 1 -1 1
3.018×10^{-1}	0.03	1 -1 1 0 -1 1 -1 0 1 -1 1
4.001×10^{-1}	0.04	1 -1 1 0 -1 1 -1 0 1 -1 1
4.968×10^{-1}	0.05	1 -1 1 0 -1 1 -1 0 1 -1 1
5.913×10^{-1}	0.06	1 -1 1 0 -1 1 -1 0 1 -1 1
6.833×10^{-1}	0.07	1 -1 1 0 -1 1 -1 0 1 -1 1
7.723×10^{-1}	0.08	1 -1 1 0 -1 1 -1 0 1 -1 1
8.581×10^{-1}	0.09	1 -1 1 0 -1 1 -1 0 1 -1 1
9.405×10^{0}	0.10	1 -1 1 0 -1 1 -1 0 1 -1 1

PRS Conventional Version $1-D-D^2+D^3$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
2.033×10^{-1}	0.01	1 0 1 -1 ... 1 -1 1 -1
4.741×10^{-1}	0.02	1 0 1 -1 ... 1 -1 1 -1
8.390×10^{-1}	0.03	1 0 1 -1 ... 1 -1 1 -1
1.263×10^{0}	0.04	1 0 1 -1 ... 1 -1 1 -1
1.659×10^{0}	0.05	1 0 1 -1 ... 1 -1 1 -1
1.932×10^{0}	0.06	1 0 1 -1 ... 1 -1 1 -1
2.038×10^{0}	0.07	1 0 1 -1 ... 1 -1 1 -1
2.074×10^{0}	0.08	0 1 -1 1 ... -1 1 0 0
2.197×10^{0}	0.09	0 0 0 0 1 -1 ... 1 -1 0 0
2.306×10^{0}	0.10	0 0 0 0 1 -1 ... 1 -1 0 0

PRS Conventional Version $1-D-D^2+D^3$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.299×10^{-1}	0.01	1 0 1 -1 1 -1 1 -1 1 -1 1
2.734×10^{-1}	0.02	1 0 1 -1 1 -1 1 -1 1 -1 1
4.384×10^{-1}	0.03	1 0 1 -1 1 -1 1 -1 1 -1 1
6.264×10^{-1}	0.04	1 0 1 -1 1 -1 1 -1 1 -1 1
8.349×10^{-1}	0.05	1 0 1 -1 1 -1 1 -1 1 -1 1
1.061×10^{0}	0.06	1 0 1 -1 1 -1 1 -1 1 -1 1
1.304×10^{0}	0.07	1 0 1 -1 1 -1 1 -1 1 -1 1
1.562×10^{0}	0.08	1 0 1 -1 1 -1 1 -1 1 -1 1
1.831×10^{0}	0.09	1 0 1 -1 1 -1 1 -1 1 -1 1
2.101×10^{0}	0.10	1 0 1 -1 1 -1 1 -1 1 -1 1

PRS Conventional Version $1-2D^2+D^4$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.449×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 0
2.673×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 0
3.675×10^{-1}	0.03	0 0 0 0 0 1 -1 ... -1 1 0
4.534×10^{-1}	0.04	0 0 0 0 0 1 -1 ... -1 1 0
5.230×10^{-1}	0.05	0 0 0 0 0 1 -1 ... -1 1 0
5.790×10^{-1}	0.06	0 0 0 0 0 1 -1 ... -1 1 0
6.248×10^{-1}	0.07	0 0 0 0 0 1 -1 ... -1 1 0
6.642×10^{-1}	0.08	0 0 0 0 0 1 -1 ... -1 1 0
7.004×10^{-1}	0.09	0 0 0 0 0 1 -1 ... -1 1 0
7.371×10^{-1}	0.10	1 -1 1 -1 ... 1 -1 1 0
PRS Conventional Version $1-2D^2+D^4$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
8.601×10^{-2}	0.01	1 -1 1 -1 0 0 -1 1 -1 1 -1
1.684×10^{-1}	0.02	1 -1 1 -1 0 0 -1 1 -1 1 -1
2.470×10^{-1}	0.03	1 -1 1 -1 0 0 -1 1 -1 1 -1
3.220×10^{-1}	0.04	1 -1 1 -1 0 0 -1 1 -1 1 -1
3.938×10^{-1}	0.05	1 -1 1 -1 0 0 -1 1 -1 1 -1
4.629×10^{-1}	0.06	1 -1 1 -1 0 0 -1 1 -1 1 -1
5.301×10^{-1}	0.07	1 -1 1 -1 0 0 -1 1 -1 1 -1
6.030×10^{-1}	0.08	1 -1 1 0 -1 1 -1 0 1 -1 1
6.787×10^{-1}	0.09	1 -1 1 0 -1 1 -1 0 1 -1 1
7.542×10^{-1}	0.10	1 -1 1 0 -1 1 -1 0 1 -1 1

PRS Conventional Version $2-D^2-D^4$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
2.637×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 0 0 0
5.800×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 0 0 0
9.733×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 0 0 0
1.434×10^{0}	0.04	1 -1 1 -1 ... 1 -1 1 0 0 0
1.917×10^{0}	0.05	1 -1 1 -1 ... 1 -1 1 0 0 0
2.358×10^{0}	0.06	1 -1 1 -1 ... 1 -1 1 0 0 0
2.696×10^{0}	0.07	1 -1 1 -1 ... 1 -1 1 0 0 0
2.906×10^{0}	0.08	1 -1 1 -1 ... 1 -1 1 0 0 0
3.001×10^{0}	0.09	1 -1 1 -1 ... 1 -1 1 0 0 0
3.093×10^{0}	0.10	0 0 0 1 -1 1 ... 1 -1 0 0 0
PRS Conventional Version $2-D^2-D^4$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
1.980×10^{-1}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.132×10^{-1}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.546×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
9.241×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.219×10^{0}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.537×10^{0}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.877×10^{0}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.235×10^{0}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.605×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
2.972×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1

PRS Conventional Version $2+D-D^2$, 22 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
2.990×10^{-1}	0.01	1 -1 1 -1 ... 1 -1 1 0
5.945×10^{-1}	0.02	1 -1 1 -1 ... 1 -1 1 0
8.780×10^{-1}	0.03	1 -1 1 -1 ... 1 -1 1 0
1.137×10^{0}	0.04	1 -1 1 -1 ... 1 -1 1 0
1.361×10^{0}	0.05	1 -1 1 -1 ... 1 -1 1 0
1.541×10^{0}	0.06	1 -1 1 -1 ... 1 -1 1 0
1.681×10^{0}	0.07	1 -1 1 -1 ... 1 -1 1 0
1.790×10^{0}	0.08	1 -1 1 -1 ... 1 -1 1 0
1.880×10^{0}	0.09	1 -1 1 -1 ... 1 -1 1 0
1.968×10^{0}	0.10	1 -1 1 -1 ... 1 -1 1 0

PRS Conventional Version $2+D-D^2$, 11 error sequence.		
PENALTY (*dB*)	ξ	[*e*]
2.302×10^{-1}	0.01	1 -1 1 -1 1 -1 1 -1 1 -1 1
4.560×10^{-1}	0.02	1 -1 1 -1 1 -1 1 -1 1 -1 1
6.754×10^{-1}	0.03	1 -1 1 -1 1 -1 1 -1 1 -1 1
8.862×10^{-1}	0.04	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.087×10^{0}	0.05	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.278×10^{0}	0.06	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.458×10^{0}	0.07	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.625×10^{0}	0.08	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.779×10^{0}	0.09	1 -1 1 -1 1 -1 1 -1 1 -1 1
1.917×10^{0}	0.10	1 -1 1 -1 1 -1 1 -1 1 -1 1