Soliton WDM system using channel-isolating notch filters

P. L. Chu[1,a], Boris A. Malomed[b] and G. D. Peng[a]

[a] Optical Communications Group, School of Electrical Engineering
The University of New South Wales, Sydney 2052, Australia

[b] Department of Interdisciplinary Studies, Faculty of Engineering,
Tel Aviv University, Tel Aviv 69978, Israel

ABSTRACT

We propose a WDM system of sech or Gaussian type of optical pulses using a set of fixed- or sliding- frequency notch filters for channel-channel isolation. We have first demonstrate that a soliton trapped in a channel between two notches is very robust. We have also predicted *optimum ratios* between the channel separation and the soliton's spectral width for both the sech and Gaussian pulses. We find that the Gaussian pulses admit ~70 % more channels than sech pulses for the same bandwidth. We investigated suppression of effects of interchannel collisions. We demonstrate that the effects can be largely eliminated by notch filters, while requiring an additional gain ~100%, and only ~5%, respectively for the sech and Gaussian pulses, of the basic gain for compensating the fiber loss,

1. INTRODUCTION

The wavelength-division multiplexing (WDM), i.e., use of many channels in one fiber, with the separation $\delta\lambda \sim 1$ nm between their carrier wavelengths, has attracted a great deal of interest in optical communications (see Refs. [1]-[6] and references therein). The most serious problem in using WDM in the soliton mode is timing jitter induced by collisions of pulses belonging to different channels. Recently, the interchannel collisions were considered in many theoretical works [2]-[4]. It particular, a detailed study [4] shows that the collision-induced jitter can be quite effectively reduced by means of strong dispersion management. To further reduce the interaction between solitons of different channels in the WDM system, *isolating* optical filters, based on the Fabry-Perot étalons or various fiber-grating schemes have been proposed [6]. The objective of this work is to study, analytically and numerically, the propagation, stability, and interchannel interactions of sech and Gaussian pulses in a WDM systems in which the channels are separated by narrow *notch filters*, and to find an optimum ratio of the channel separation to the spectral width of soliton. We will consider both the fixed-frequency and frequency-sliding cases. The analysis will assume distributed filtering and extra gain.

The existing channel-insulating filters [6], based on fiber gratings, may be considered in some approximation as a system of notches. Before proceeding to the analysis of the model, we would like to mention that it is possible to build nearly ideal comb-like notch filters, using altogether different physics, namely, *quantum dots* where very deep potential wells of a small size a would trap electrons. The quantization produces, for a large quantum number N, the following oscillation frequency of an electron in the discrete state inside the quantum dots: $\omega_N = \pi \hbar N/ma$, m being the electron's mass, which provides for an equidistant spectrum of the absorption lines. Because the spectral width $\Delta\omega$ corresponding to $\delta\lambda \sim 1$ nm is about 100 GHz (and the spectral width of the soliton with the temporal duration ~30 ps is on the same order of magnitude, which makes WDM appropriate for the use with the solitons), one can immediately conclude, from the above expression for ω_N, that quantum dots with $a \sim 100$ nm can be used as the notch filters mentioned above.

2. SOLITON IN NOTCH FILTERS

[1] Correspondence: email: P.CHU@unsw.edu.au, telephone: 61-2-93855304, fax: 61-2-93855993

Part of the SPIE Conference on Optical Devices for Fiber Communication
Boston, Massachusetts • September 1999 SPIE Vol. 3847 • 0277-786X/99/$10.00

Analytically, we consider a channel of the spectral width $\Delta\omega \equiv \omega_1 - \omega_2$, isolated by two infinitely narrow notches (which are described by the δ-functions in the spectral representation). It is easy to see that the corresponding Nonlinear Schroedinger equation in the temporal domain is

$$iu_z - (1/2)\beta(z)u_{\tau\tau} + |u|^2 u = i\gamma u - i\Gamma \sum_{n=1,2} e^{-i\omega_n t} \int_{-\infty}^{+\infty} e^{i\omega_n \tau'} u(\tau')d\tau', \tag{1}$$

where $\Gamma > 0$ and $\gamma > 0$ are respectively the filtering strength and compensatory extra gain. The dispersion coefficient $\beta(z)$ is variable in the case of dispersion management, and the other notation is standard [7].

Treating the terms on r.h.s. of (1) as small perturbations, and using the commonly known technique of the energy- and momentum-balance, one can easily derive, in a general form, evolution equations for the energy $E \equiv \int_{-\infty}^{+\infty} |u(\tau)|^2 d\tau$ and central frequency Ω of an arbitrary pulse $u_0(\tau)$ (including the case of a vibrating dispersion management pulse, see, e.g., [8, 9]):

$$\frac{dE}{dz} = 2\gamma E - 2\Gamma \sum_{n=1,2} \left| \int_{-\infty}^{+\infty} d\tau e^{i(\omega_n - \Omega)\tau} u_0(\tau) \right|^2, \tag{2}$$

$$E\frac{d\Omega}{dz} = 2\Gamma \sum_{n=1,2} (\Omega - \omega_n) \left| \int_{-\infty}^{+\infty} d\tau e^{i(\omega_n - \Omega)\tau} u_0(\tau) \right|^2. \tag{3}$$

It is straightforward to insert into Eqs.(2) and (3) the usual soliton $u_0 = \tau_0^{-1}\mathrm{sech}(\tau/\tau_0)e^{i\phi(z)}$ with the temporal width τ_0 ($\beta \equiv -1$), which yields

$$\frac{d\tau_0}{dz} = -2\gamma\tau_0 + \pi^2\Gamma\tau_0^2 \sum_{n=1,2} \mathrm{sech}^2\left(\frac{1}{2}\pi\tau_0 (\Omega - \omega_n)\right), \tag{4}$$

$$\frac{d\Omega}{dz} = \pi^2\Gamma\tau_0 \sum_{n=1,2} (\Omega - \omega_n)\, \mathrm{sech}^2\left(\frac{1}{2}\pi\tau_0 (\Omega - \omega_n)\right). \tag{5}$$

It is obvious that Eqs. (4) and (5) have a fixed point with $\Omega = \frac{1}{2}(\omega_1 + \omega_2)$, corresponding to a soliton frequency-locked at the channel's center. Now the necessary value of the compensating gain following from Eq. (4):

$$\gamma = (\pi^2\Gamma\tau_0)\,\mathrm{sech}^2 W, \quad W \equiv (\pi/4)\tau_0\Delta\omega, \tag{6}$$

W being the *relative width* of the channel. Further analysis of Eqs. (4) and (5) reveals that this fixed point is stable provided that $W\tanh W > 1/2$, i.e., $W > W_{\min} = 0.77$. This implies that stable propagation of soliton is possible if the channel is sufficiently wide. The same conclusion has been obtained when a soliton propagates in a Fabry-Perot étalon filter [5].

3. DISPERSION MANAGEMENT WITH NORTH FILTERS

In the case of *strong* dispersion management, the pulse assumes a nearly Gaussian form [8, 9]. Following the work [9], it can be conveniently represented in the form

$$u_0 = \tau_0 \sqrt{\frac{P}{\tau_0^2 + 2i\Delta(z)}} \exp\left[-\frac{\tau^2}{\tau_0^2 + 2i\Delta(z)}\right], \tag{7}$$

where $\Delta(z) \equiv -\int_0^z \beta(z')\,dz' + \Delta_0$ is the accumulated dispersion, assuming that, in the lowest approximation, the average dispersion is zero, and the constants P, τ_0, and Δ_0 determine the pulse's peak power, width, and chirp. Substitution of this into (2) and (3) leads to

$$\frac{d}{dz}(P\tau_0) = 2\gamma P\tau_0 - 2\sqrt{\pi}\Gamma P\tau_0^2 \sum_{n=1,2} \exp\left[-\frac{1}{2}\tau_0^2(\Omega - \omega_n)^2\right], \tag{8}$$

$$\frac{d\Omega}{dz} = 2\sqrt{\pi}\Gamma P\tau_0 \sum_{n=1,2} (\Omega - \omega_n)\exp\left[-\frac{1}{2}\tau_0^2(\Omega - \omega_n)^2\right]. \tag{9}$$

A remarkable property of these equations is that, although the Gaussian pulse (7) is strongly pulsating, Eqs. (8) and (9) contain no explicit z-dependence. The fixed point with $\Omega = \frac{1}{2}(\omega_1 + \omega_2)$ demands the compensating gain (cf. Eq. (6):

$$\gamma = 2\sqrt{\pi}\Gamma\tau_0 \exp\left(-2W^2\right), \; W \equiv \frac{1}{2}\Delta\omega\tau_0. \tag{10}$$

This point is stable provided that: $W \geq W_{\min} = 1$.

For both the sech and Gaussian pulses considered here, the fixed point exists for any value of the relative width W. Therefore, a relevant issue is to find its *optimal* value that provides for the strongest stability of the trapped soliton. Straightforward analysis of Eqs. (4) through (9) shows that the fastest suppression of the soliton's frequency fluctuations is attained at

$$W = W_{\text{sech}}^{(\text{opt})} \approx 2.169, \;\text{ and }\; W = W_{\text{Gauss}}^{\text{opt}} = \sqrt{3}, \tag{11}$$

for the sech and Gaussian pulses, respectively (the former value is a root of the equation $W = 2(\tanh W + W\text{sech}^2 W)$). This, in turn, translates into the ratio of the optimal channel separation $\Delta\omega$ between the sech and Gaussian pulses as

$$\frac{\Delta\omega_{\text{sech,opt}}}{\Delta\omega_{\text{Gauss,opt}}} = \frac{2.169\sqrt{2\ln 2}}{\sqrt{3}\ln\left(\sqrt{2}+1\right)} \approx 1.67. \tag{12}$$

This result shows that Gaussian pulses allow 70% more channels than the sech solitons.

The above consideration can be generalized to the case of frequency-sliding filters for which we replace the constant $\omega_{1,2}$ by $\omega_{1,2}(z) = \omega_{1,2}^{(0)} + \chi z$, where χ is the sliding rate. The fixed point solutions can now be found by putting $\Omega = \Omega^{(0)} + \chi z$ into Eqs. (5) and (9). As a result, the l.h.s. of Eqs. (5) and (9) is χ, rather than 0, and $\Omega - \omega_{1,2}$ is replaced by $\Omega^{(0)} - \omega_{1,2}^{(0)}$. In this case, the fixed point is chatacterized by the *bias* parameter

$$b \equiv \left[2\Omega^{(0)} - \left(\omega_1^{(0)} + \omega_2^{(0)}\right)\right] / \left(\omega_1^{(0)} + \omega_2^{(0)}\right), \tag{13}$$

which is a function of the normalized sliding rate, $\lambda \equiv \chi/(2\pi\Gamma)$. The final equations relating b and λ for the sech and Gaussian pulses take the form, respectively,

$$\lambda = \sum_{+,-} W(b \pm 1)\mathrm{sech}^2\left(W(b \pm 1)\right), \tag{14}$$

$$\lambda = \pi^{-1/2} \sum_{+,-} W(b \pm 1)\exp\left(-\frac{1}{2}W^2(b \pm 1)^2\right). \tag{15}$$

In this case, Eqs. (6) and (10) should be modified in an obvious way.

A solution for the bias parameter exists provided that the normalized sliding rate does not exceed a maximum value $\lambda_{\max}$. The dependence of $\lambda_{\max}$ on W, found numerically from Eqs. (14) and (15) for both types of the pulses, is displayed in Figure 1. In particular, it is easy to show that, in both cases, $\lambda_{\max}(W)$ vanishes exactly at the above-mentioned stability boundary $W_{\min}$ for the non-sliding solution, and $\lambda_{\max}(W = \infty) = 1/\sinh(2W_{\min})$ for the sech pulse, and $(\pi e)^{-1/2}$ for the Gaussian. It is also easy to prove that fixed point for the sliding case remains stable as long as it exists.

The above analytical results were checked by numerical simulations. Here, we will only display the numerical results obtained for the sech pulses in the model with constant dispersion ($\beta \equiv -1$) and without frequency sliding. In the simulation, the filtering term in Eq. (1) is replaced by four uniformly separated notches of small but finite width (no more than 1/10 of the soliton's spectral width), as shown in Figure 2a. A sech soliton was initially launched into one of the channels with width given by Eq.(11) and gain given by Eq.(6). As shown in Figures 2b and 2c, the soliton remains practically intact with a slight oscillation in amplitude. To test the robustness of this soliton pulse, we also ran the simulations in which the initial pulse is contaminated by noise, as shown in Figure 3. Here, we introduced a background noise of considerable amplitude (5% of the peak amplitude of soliton) corresponding to a 10noise with regard to the soliton's peak power. From the spectrum of the soliton (dark solid line) in Figure 3a, we now see that the noise gives rise to a significant spike around the centre frequency. However, from Figures 3b and 3c, it can be seen that the noise has very little impact on the soliton propagation although the oscillation of its amplitude is somewhat more prominent than the case without noise contamination (Figures 2b and 2c). Hence we can conclude that the soliton predicted by the theoretical analysis is fairly accurate.

The stabilizing effect of the filter on soliton propagation such as the suppression of interchannel jittering [10] can be studied by introducing a parameter, "friction coefficient", α into the following equations governing the deviation $\delta\Omega$ of the soliton centre frequency as it propagates in the channel:

$$\frac{d(\delta\Omega)}{dz} = -\alpha\delta\Omega + F(z),\ \frac{dT}{dz} = \beta\,\delta\Omega(z), \tag{16}$$

where $F(z)$ is an external distrurbance (e.g., the effective random force corresponding to the Gordon-Haus jitter [10], or an effective interaction force induced by the collision with a soliton in an adjacent channel, see below). Linearizing Eqs. (5) and (9), we find the values of α for both types of the pulses in the optimum cases:

$$\alpha_{\mathrm{sech}} = 2\pi^2\Gamma\tau_0\left[2W_{\mathrm{opt}}\tanh(W_{\mathrm{opt}}) - 1\right]\mathrm{sech}^2(W_{\mathrm{opt}}),\ \alpha_{\mathrm{Gauss}} = 8\sqrt{\pi}\exp(-3/2)\Gamma\tau_0\,. \tag{17}$$

Further consideration shows that, for the intrachannel stabilization, the notch filters prove to be neither better nor worse than the usual parabolic filters.

For the sech solitons, description of the collision by Eq. (16) is justified if the collision is fast enough, i.e., the attraction of the two solitons due to the cross-phase modulation does not essentially change their relative inverse group velocity δ. For this case, the function $F(z)$ was found in [3]. An estimate shows that, with the wavelength separation between the channels ~ 1 nm, and irrespective of the actual value of the dispersion, a relative collision-induced perturbation of δ is $\sim 1/\tau_0$[ps], where τ_0[ps] is the value of the soliton's temporal width in ps. Thus, for all the cases of practical interest for WDM, the collision is indeed fast. Continuing the estimate and assuming dense pulse stream, with the temporal separation between the solitons in each channel $\sim 3\tau_0$, one can easily conclude that the relative (divided by $3\tau_0$)) average collision-induced shift (jitter) accumulated over the transmission distance L is $\sim LD/10\tau_0^2$.

This estimate was obtained for "complete collisions" [4], i.e., those in which a soliton overtakes and passes another one. Because the function $F(z)$ in Eq. (16) is odd [3], the complete collisions give rise (in the first approximation) to position shifts of the solitons without frequency shift and the same result can be obtained for the Gaussians [4]. An "incomplete collision", which may happen at the input point between initially overlapping solitons [4], does give rise to a random frequency shift, i.e., to a velocity shift, which results in a position shift linearly growing with z. An estimate shows that an additional contribution of the incomplete collision into the relative jitter is $\sim LD/3\tau_0^2$. For $L \sim 10,000$ km, and with $\beta \sim -2$ ps^2/km in dispersion shifted fibres and $\tau_0 \sim 30$ ps, we conclude, from the above estimates, that the collision-induced jitter is really serious, producing the accumulated relative temporal shift ~ 10, which implies complete destruction of the information content carried by the pulses.

If sufficiently strong filters are present, the situation is altogether different. We call the filters strong if the relaxation length α^{-1}, see Eq. (16), is much smaller than the collision length, τ_0/δ. In this case, an approximate solution to Eq. (16) for $\delta\Omega$ becomes very simple, $\delta\Omega = F(z)/\alpha$, and then we obtain the net temporal shift,

$$T = (\beta/\alpha)\int_{z_0}^{\infty} F(z)dz \equiv (\beta/\alpha)\Delta_0\Omega\,, \tag{18}$$

$\Delta_0\Omega$ being the net collision-induced frequency shift in the absence of the filters ($\alpha = 0$). The expression (18) immediately tells us that the complete collisions will produce no net effect at all, while the incomplete collision will result in a negligible accumulated relative temporal shift, $\sim 1/30\tau_0$[ps].

As it follows from (17), the underlying condition, $1/\alpha \ll \tau_0/\delta$, suggests to take $\Gamma \sim 10\delta/\tau_0^2$. What is really important is the size of the extra gain that must balance the filtering as per Eq. (6). With this estimate for Γ, we obtain $\gamma \sim \delta/\tau_0$. In the above-mentioned case ($\beta \sim -2$ ps^2/km, $\tau_0 \sim 30$ ps), this finally leads to an estimate ~ 0.025 dB/km for the necessary extra power-gain. Comparing it to the basic gain necessary to compensate the fiber loss, ~ 0.02 dB/km, we conclude that the extra gain, roughly, doubles the net in-line gain.

To check these conclusion, we simulated the collisions between the solitons in two channels. A typical example, displayed in Figure 4, illustrates the suppression of the collision effects by the notches in the most critical case of the *in*complete collision. For the situation of "incomplete collision" with and without the notch Filters, solitons can emergy from the collision essentially 'intact'. However, the effects on the velocity of individual soliton in the two cases are quite different. In the case without the notch filters, the solitons engaged in the collision process experienced a significant 'slowing down'. Whereas with the filters in, the impact of 'incomplete collision' on the soliton velocity is substantially reduced. This is obvious from the comparison of soliton trajectories when colliding without filter (thick solid lines) and when colliding with filters (thin solid lines), with the trajectories of free soliton (dashed lines). The collision induced time jittering of soliton is certainly detrimental to optical communication systems.

For the Gaussian pulses in the strong dispersion management, similar analysis can be developed by replacing β in Eq.(18) with a small average dispersion. We conclude that the collision effects for the Gaussian pulses can be completely suppressed by notch filters, requiring merely an extra gain of only $\sim 5\%$ to compensate for the fiber loss. This is another significant advantage offered by the strong dispersion management regime.

4. CONSLUSIONS

In conclusion, we have proposed a WDM system for sech and Gaussian pulses based on a set of fixed- or sliding-frequency notch filters separating the channels in the frequency domain. We have demonstrated that a soliton trapped in a channel by just two notches is very robust, and we have found optimum ratios between the channel's width and the soliton's spectral width for the sech and Gaussian pulses. The Gaussian pulses allow about 70% more channels. We have demonstrated that the filters can essentially eliminate the effects of interchannel collisions between the pulses.

5. ACKNOWLEDGMENT

B.A.M. appreciates support from the School of Electrical Engineering at the University of New South Wales.

6. REFERENCES

[1] L.F. Mollenauer, S.G. Evangelides, and J.P. Gordon, J. Lightwave Technol. **9**, 362 (1991); M.J. Ablowitz, G. Biondini, S. Chakravarty, R.B. Jenkins and J.R. Sauer, Opt. Lett. **21**, 1646 (1996); P.V. Mamyshev and L.F. Mollenauer, Opt. Lett. **21**, 1659 (1996).

[2] S. Wabnitz, Optics Lett. **21**, 638 (1996); S. Kumar, Y. Kodama, and A. Hasegawa, Electron. Lett. **33**, 459 (1997); J.F.L. Devaney, W. Forysiak, A.M. Niculae, and N.J. Doran, Opt. Lett. **22**, 1695 (1997); T. Hirooka and A. Hasegawa, Opt. Lett. **23**, 768 (1998).

[3] M.J. Ablowitz, G. Biondini, S. Chakravarty, and R.L. Horne, Opt. Comm. **150**, 305 (1998).

[4] D.J. Kaup, B.A. Malomed, and J. Yang, Opt. Lett., in press.

[5] E.A. Golovchenko, A.N. Pilipetskii, and C.R. Menyuk, Opt. Lett. **21**, 195 (1996).

[6] S.S. Orlov, A. Yariv, and S. Van Essen, Opt. Lett. **22**, 688 (1997); X.J. Gu, Opt. Lett. **23**, 509 (1998).

[7] G.P. Agrawal. *Nonlinear Fiber Optics* (Academic Press: San Diego, 1995).

[8] J.N. Kutz, P. Holmes, S.G. Evangelides, and J.P. Gordon, J. Opt. Soc. Am. B **15**, 87 (1998); A. Berntson, N.J. Doran, W. Forysiak, and J.H.B. Nijhof, Opt. Lett. **23**, 900 (1998); S.K. Turitsyn, I. Gabitov, E.W. Laedke, V.K. Mezentsev, S.L. Musher, E.G. Shapiro, T. Schäfer, and K.H. Spatschek, Opt. Comm. **151**, 117 (1998); S.K. Turitsyn, Phys. Rev. E **58**, R1256 (1998).

[9] T. I. Lakoba, J. Yang, D.K. Kaup, and B.A. Malomed, Opt. Comm. **149**, 366 (1998).

[10] A. Mecozzi, J.D. Moores, H.A. Haus, and Y. Lai, Opt. Lett. **16**, 841 (1991); Y. Kodama and A. Hasegawa, Opt. Lett. **17**, 31 (1992).

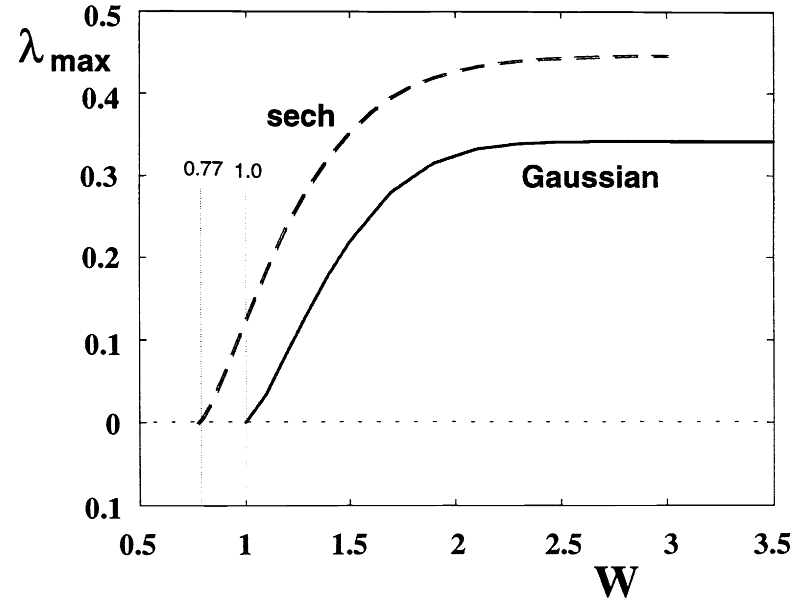

Fig.1 The maximum normalized frequency-sliding rate, admitting a stationary solution for the sech (dashed line) and Guassian (solid line) pulses trapped by two frequency-sliding notch filters, vs. the relative frequency separation between the notches.

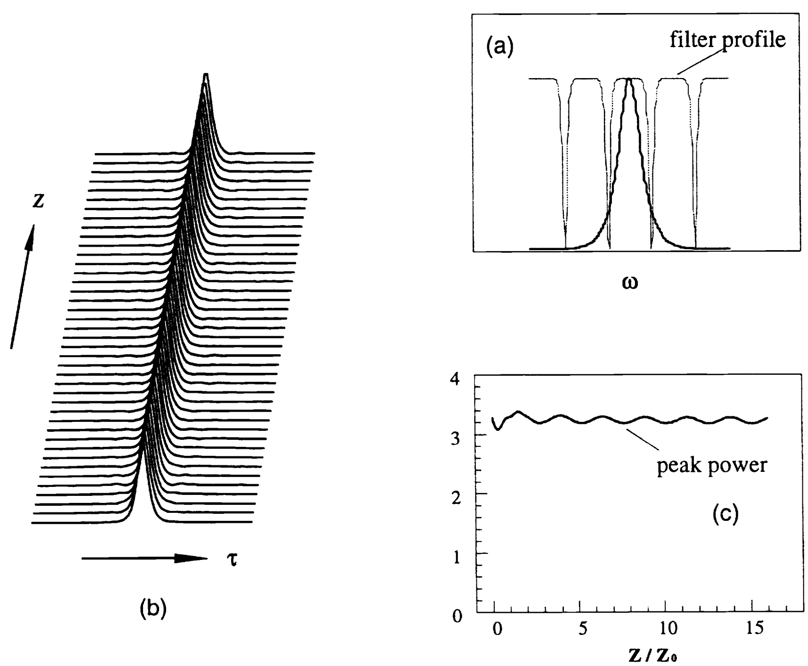

Fig.2 Numerical simulations of the model with four narrow notches: (a) the notches and the soliton in the frequency domain; (b) an example of evolution starting with the sech pulse predicted by the analytical approximation in the optimum case (Eq.11), with τ_0=0.55, the notch-filtering strength Γ=0.1, and the extra gain γ=0.0276, predicted by Eq.6; (c) the evolution of the soliton's peak power.

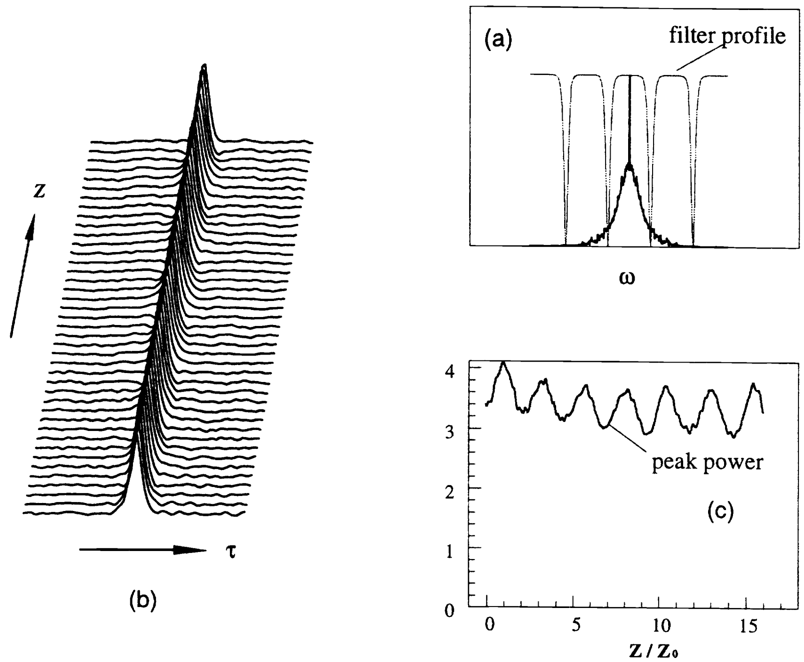

Fig.3 The same as in Fig. 2 for the case when the initial state contains a random-noise component, with the intensity equal 10% of the soliton's peak power.

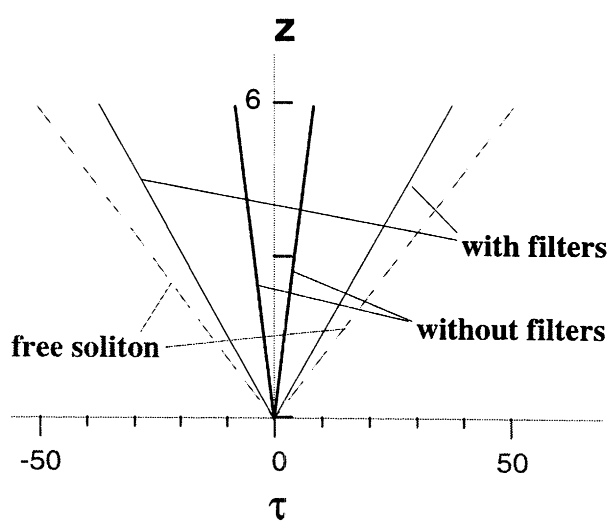

Fig.4 A typical example of the (most dangerous) incomplete collision. The trajectories of the solitons' centers are shown with and without the filters and extra gain. The dashed lines show the trajectories of each soliton in the absence of the collision. Here, τ_0=0.5, $\Delta\omega$=2π, Γ=1/3, and γ =0.2.