Linear approximation of irreversible processes at laser interaction with plasma

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LINEAR APPROXIMATION OF IRREVERSIBLE PROCESSES AT LASER INTERACTION WITH PLASMA.

Thesis for the Degree of Master in Science

by Reynaldo Castillo.

-1979-

I hereby certify that the work contained in this thesis has not been submitted for a higher degree to any other university or institution.

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Date..................

CONTENTS

SUMMARY:

The question of ponderomotive or of the more general non-linear forces and absorption processes in a plasma in presence of an external electromagnetic field, e.g. of laser radiation , are considered from a statistical point of view.

First the stress tensor for the system matter-field is studied in relation with the concept of local equilibrium , which permits to obtain the expression first proposed by Helmholtz and used by Landau and Lifshitz later , for the ponderomotive force.

Then the properties of an absorptive-dispersive medium are discussed Here we come to the conclusion that for the considered medium , the expression for the stress tensor and ponderomotive force is still an open question.

Only for the case of a medium whose response is linear in the applied field , the generated heat could be written as a function of the imaginary part of the dielectric constant.

The discussion of the method proposed by Pekar for determining the expressions for energy density and generated heat in the electrodynamics of a dispersive and absorbing medium , is discussed . Here we come to the conclusion that this method is questionable.

A discussion of the mechanism of entropy production by relaxation and absorption , is done. The absorption properties of a plasma are studied considering three different approaches : 1.- A special model is derived using a set of oscillators . 2.-The hydrodynamic two fluid plasma is used , and 3.-The linear response theory is used.

The role of the Onsager coefficients is evaluated , and it is demonstrated that the determination of the absorption constant in each of the three approaches can be considered as an application of the fluctuation dissipation theorem.

The theory used here is based in the theory of fluctuations for systems near equilibrium and it presents a unifiying point of view to study the non-equilibrium process in a plasma without utilizing the

kinetic equations approach.

It enables us to discuss the energy relation for a plasma in an external field which is not only useful for understanding the mechanism and the character of the absorption and relaxation phenomena , but it is also used in the calculation of the "energy-velocity" , ie. the rate of energy transfer in electromagnetic waves propagating in an absorptive plasma.

An extension of the theory in order to include more general cases is discussed.

CHAPTER 1

1). INTRODUCTION

The interaction of laser light and plasma has intrinsic theoretical interest ,both as a general topic in the wide field of interaction of electromagnetic fields and matter ,and specially also for its present practical interest due to its application to the problem of laser-induced thermonuclear fusion.Together with magnetic-confinement fusion, laser-induced fusion has been the subject of intensive studies in leading centers in Japan , the Soviet Union, United States and Europe, as a principal prospect for generating controlled thermonuclear power.(1).

Appart from this very direct application ,there are several other processes which arise when intense laser radiation interacts with solids ,the key idea in all the processes is to irradiate a solid with sufficient intense laser beam ,in such a way that the solid will perform a rapid transition to a plasma state; there the absorption mechanism, both in the plasma state and in the initial solid state ,is a very important question whose solution is basic for the insight in the dynamics of the interaction.(2).

For high laser intensities the nonlinear effects have a dominanating influence in the structure of the plasma flow ; there we have to differentiate between the thermo-kinetic forces due to gas dynamics pressures, and the forces of electrodynamic origin ,even though both effects will be present in the macroscopic description of the "dielectric" properties of the plasma. (3).

The intensity dependent collision frequency will set a criterium for the predominance of the so called non-linear forces of electromagnetic origin over the thermokinetic forces. These non-linear forces are related to the processes such as :generation of [KeV] ions, self focusing, production of magnetic fields and in relation with the problem of controlled thermonuclear fusion , the non-linear force could be the principal mechanism to compress plasma in order to produce exothermal nuclear reactions.

There are two different schemes proposed to compress plasma :the gas dynamic scheme and the optical compression scheme based in non-linear forces.

In the gas dynamic scheme for compressed plasma , the outer corona of the plasma pellet absorbs the laser energy which is transported to the core by hot electrons producing the heating and later the blow-off of the outer layer of the core. The resulting reaction forces will compress the inner part of the core to high densities.

On the other hand in the optical scheme very short and high intensity laser pulses are applied , in this way thermalization is avoided and fast, cold , thick plasma blocks are produced due to non-linear forces , these blocks will play the role of compressing material in order to get the conditions demanded by controlled thermonuclear fusion. If this optical compression scheme is used ,since the transfer of laser energy directly to plasma kinetic energy is a low entropy process (Isentropic in ideal conditions), due to the negligible heating it will provide the same plasma compression than the gas dynamics compression scheme, with less laser energy (10 less laser energy) for the same reaction gain , the later being defined as the ratio $G = \frac{\epsilon_F}{E_o} = f(n,T,\sigma)$ where ϵ_F is the reaction energy and E_o is the input energy (2 ,3).

Also the optical scheme does not need the tayloring of the laser pulse in order to achieve isentropic conditions, as required by the gas dynamic scheme, where the undesirable effects of entropy production due to generation of shocks are always present.Further there are complications with the meaning of the collision frequency , which decreases rapidly at high laser irradiances.(3).

We can formulate the two central questions in laser-plasma interaction: a)the absorption of energy , and b) the forces acting on the plasma. Many theories have been developed to explain such interaction , each of them facing one or both central questions.The following descriptions or theories can be mentioned:

1) The hydrodynamical description of laser plasma interaction, where the absorption processes are associated with the introduction of an effective collision frequency , that includes all the mechanisms with which the external field is pumping energy to the plasma.

2)The kinetic description:Here it is assumed that the interaction is a slow irreversible process ,described by a Fokker-Planck type equation.

3) The statistical description , where the same results of the kinetic theory can be obtained, but using here the linear response theory , with the advantage that in this case it is not necessary to introduce an arbitrary cut-off parameter, as required by the Fokker-Planck equation.In the statistical description , the absorption processes will be described by the introduction of the so called "generalized susceptibility".(5).

After the absorption processes are described by one of the above mentioned theories , we turn our attention to the balance equations for the plasma particles, which , in combination with the Maxwell equations for plasma , will give us explicit expressions for the non-linear force. In order to see the principal features of this problem , we begin summarizing some of the plasma properties. First the plasma parameters are introduced statistica-llyand thair relation with the concept of thermal equilibrium is studied; then the classical description of the non-linear forces in transparent plasma is given and finally the hydrodynamical description of the plasma is studied. (6).

2.-GENERALIZED DESCRIPTION OF THE PLASMA

Adopting the definition of H.Hora (7), we understand for plasma a state of matter characterized by a high electrical conductivity and mostly gaseous mechanical properties.Historically the first studies in the modern Plasma Physics were realized in connection with the phenomena of gas discharges, that is the production of ionized gas by application of electric fields.

Large regions of the Universe are made up of plasma , with the consequence that until recent years the study of strongly ionized gases has been connected to Astrophysics .Nuclear explotions ,however, produce almost completely ionized matter , whose connection with the production of fully ionized plasma in the laser plasma interaction has revived the interest in this field with the aim of obtaining controlled nuclear fusion.

A fully ionized plasma can be considered as a mixture of charged particles which can be described by the Hamiltonian(8):

2.1)- $$H = \sum_{\mu=1}^{s} \sum_{j=1}^{N_\mu} \frac{p_{j\mu}^2}{2m_\mu} + e^2 \sum_{\nu,\mu=1}^{s} \sum_{j<n} V_{jn}^{(\mu\nu)}$$

where

2.2).- $$V_{jn}^{(\mu\nu)} = z_\mu z_\nu \left\{ \frac{1}{x_{j\mu} - x_{n\nu}} \right\}$$

The index μ,ν are characterizing the kind of particle, the index j is characterizing a particular particle, N_μ is the number of particles of the type μ ,which are electrically characterized by a charge $z_\mu e$ and dynamically by a mass m_μ , a position vector $x_{\mu j}$,and a momentum $p_{j\mu}$.Now in order to avoid the complications that arise when work with the mixture of charged particles, the model of one component plasma moving in a background oppositely charged is adopted, the Hamiltonian for this system being:

2.3).- $$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + e^2 \sum_{j<m} \sum V_{jm}\left(|x_j - x_m|\right)$$ where:

2.4) $$V_{jm} = \frac{1}{|x_j - x_m|}$$

This Hamiltonian describes an assembly of particles interacting through two body forces, deriving from a potential $e^2 V_{jm}$ (we are assuming particles of valency one), in the absence of any external fields, and it contains all the information about the plasma .The Hamilton canonical equations are deduced from the Hamiltonian in the form:

2.5) $$\frac{\partial H(x,p)}{\partial \vec{x}_j} = -\dot{\vec{p}}_j \qquad \frac{\partial H(x,p)}{\partial p_j} = \dot{\vec{x}}_j$$

But better than that is to follow the mechanical description of the system. As usual we are interested in the statistical description .In order to obtain such description ,a Gibbsian ensamble in the phase space is introduced.A particular system in a given state of motion will be described by a point in the phase space, the ensamble representing the real system will correspond to a cloud of points whose density will be given in the space by the function :

2.6) $$f_N(x_1, x_2, \ldots, p_1, \ldots p_p, t)$$

called the N-particle distribution function; f_N evolves in time according to the Liouville theorem;

2.7) $$\frac{\partial f_N}{\partial t} + \sum_{j=1}^{N}\left\{\frac{\partial f_N}{\partial \vec{x}_j}\dot{\vec{x}}_j + \frac{\partial f_N}{\partial p_j}\cdot\dot{\vec{p}}_j\right\} = 0$$

which combined with the Hamilton canonical equations 2.5) , gives:

2.8) $$\frac{\partial f_N}{\partial t} + [f_N, H] = 0$$

where the Poisson bracket is defined as:

2.9) $$[a,b] = \sum_j\left(\frac{\partial a}{\partial \vec{x}_j}\frac{\partial b}{\partial p_j} - \frac{\partial a}{\partial p_j}\frac{\partial b}{\partial \vec{x}_j}\right)$$

Now since the variable v_i ,the velocity , will be more connected to the relation between microscopic and macroscopic quantities (point that it will be developed later), we will write the N-particle distribution function as:

2.10) $$f_1(\vec{x}_1, \vec{x}_2, \ldots, \vec{v}_1, \vec{v}_2, \ldots \vec{v}_N, t)$$

the change from $\vec{p}$ to $\vec{v}$ being a trivial one.

As long as magnetic fields are absent and relativistic effects are not taken into account, then the Liouville equation becomes:

2.11) $$\frac{\partial f_N}{\partial t} + \sum \vec{v}_j \circ \vec{\nabla}_j f_N = \frac{e^2}{m}\sum_{j<m}\sum(\vec{\nabla}_j V_{jm})\cdot \partial_{jm} f_N$$

Now without entering into the discussion about the Ergodic hypothesis, we adopt as valid the point of view that in introducing a macroscopic description ,the observable value of any macroscopic dynamical quantity is the average value weighed with f_N ,of the corresponding microscopic quantity, then:

2.12). $$M(t) = \int (d\vec{x})^N (d\vec{v})^N f_N(\vec{x}, \vec{v}; t)\, M(\vec{x}, \vec{v})$$

and since f_N is a kind of probability function , it verifies:

2.13)
$$\int (d\vec{x})^N (d\vec{v})^N f_N = 1 \quad , \qquad f_N \geq 0$$

Now since all macroscopic quantities are functions of the coordinates of only a small number of particles , say one , two , therefore the weighing function in equation 2.11) is actually the integral of f_N over all particles except those on which M depends ; these integrals are called reduced distribution functions of s particles and they will be defined as:

2.14)
$$f_s(\vec{x}_1, \ldots, \vec{x}_s, \vec{v}_1, \ldots \vec{v}_s ; t) = \frac{N!}{(N-s)!} \int (d\vec{x})^{N-s} (d\vec{v})^{N-s} f_N$$

in terms of these functions the more important macroscopic quantities are defined as follows:

2.15) Density at point $\vec{x}$:
$$n(\vec{x};t) = \int d\vec{x}_1 d\vec{v}_1 \, \delta(x - x_1) f_1(\vec{x}_1, \vec{v}_1, t)$$

2.16) Local hydrodynamics velocity:
$$u(\vec{x};t) = [n(\vec{x};t)]^{-1} \int d\vec{x}_1 d\vec{v}_1 \vec{v}_1 \, \delta(\vec{x} - \vec{x}_1) f_1(\vec{x}_1, \vec{v}_1; t)$$

2.17) Local energy density at point $\vec{x}$:
$$E(\vec{x};t) = [n(\vec{x};t)]^{-1} \Big\{ \int d\vec{x}_1 d\vec{v}_1 \tfrac{1}{2} m \vec{v}_1^{\,2} \, \delta(\vec{x} - \vec{x}_1) f_1(\vec{x}_1, \vec{v}_1; t) + \tfrac{1}{2} e^2 \int dx_1 dx_2 dv_1 dv_2 \, V_{12}(\vec{x}_1 - \vec{x}_2) \, \delta(x - x_1) f_2(x_1, x_2, \vec{v}_1, \vec{v}_2, t) \Big\}$$

2.18) Density correlation between points x, x' :
$$g(\vec{x}, \vec{x}'; t) = \int d\vec{x}_1 d\vec{x}_2 \, d\vec{v}_1 d\vec{v}_2 \, \delta(\vec{x}_1 - x) \delta(\vec{x}_2 - x') \left[f_2(\vec{x}_1, \vec{x}_2, \vec{v}_1, \vec{v}_2; t) - f_1(\vec{x}_1, \vec{v}_1; t) f_1(\vec{x}_2, \vec{v}_2; t) \right]$$

Some points which characterize the statistical description have to be understood .First there is the point that , unlike the mechanical description, here the momentum or velocity and the position are not regarded as functions of the time , in a statistical theory the evolution of the system is described by the change in time of the density at a given point ($\vec{x}, \vec{v}$) in the phase space; equilibrium statistical mechanics makes a "a priori" hypothesis about the form of f_N , and arrives at time-independent solutions for the Liouville equations (9), from which expectation values are computed, but no such simple functional exist in the non-equilibrium situation and in order to handle this situation the kinetic theory is introduced as the theory of the processes evolving to equilibrium.

Summarizing , the method operates in the following way: if we multiply the Liouville equation by $N!/(N-s)!$ and we integrate over $\int dx_{s+1} \cdots dx_N$ under the following hypothesis:

2.19) i.- $f_N(\vec{x}_1, \ldots, \vec{x}_i, \ldots \vec{x}_j, \ldots \vec{x}_N) \doteq f_N(\vec{x}_1, \ldots, \vec{x}_j, \ldots x_i, \ldots, \vec{x}_N)$

ii).- $f_N = 0$ outside some arbitrary large but finite region of the phase space.

then after the indicated operations , the equation 2.11). becomes:

2.20) $$\partial_t f_s(1, \ldots, s) + \sum_{j=1}^{s} \vec{v}_j \cdot \nabla_j f_s = \frac{e^2}{m} \sum_{j=1}^{s} \int d\vec{x}_{s+1}\, d\vec{v}_{s+1}\, (\vec{\nabla}_j V_{j,s+1})\, \partial_{v_j} f_{s+1}(1, \ldots, s+1)$$

In order to avoid being concerned with boundary effects , one passed to the thermodynamics limit $\begin{cases} N \to \infty \\ V \to \infty \\ N/V = c \end{cases}$, where N is the number of particles, the volume enclosing them and c the average density. Physically it means that in a real gas all local properties such as hydrodynamical variables ,intensive thermodynamics parameters , etc , must have finite values which are independent of the size of the system.

The chain of equations represented by letting s=1,2,3,. ....,in equation 2.20), is the so called B.B.G.K.Y. hierarchy.It has the noteworthy property of giving $\partial f_s/\partial t$ in terms of f_{s+1} , thus if we want to know f_1 ,to get it from 2.20) we need to know f_2 and f_3 and so forth.Clearly we have to make some simplifications: this is done assuming some approximate expression for f_2 in terms of f_1 . If that expression exist , then the s=1 equation will be:

2.21)
$$\partial_t f_1 + \vec{v}_1 \nabla_1 f_1 = \frac{e^2}{m} \int d\vec{x}_2\, d\vec{v}_2\, (\vec{\nabla}_1 V_{12}) \cdot \partial_{v_1} f_2(f_1)$$

It would amount to an equation which would govern the time development of f_1 ,without knowing the higher f_s.Such equation as 2.21 giving $\frac{\partial f_1}{\partial t}$ in terms of f_1 alone is called a kinetic equation; the various evolution equations so obtained correspond to different approximations and are called:

i). The Boltzmann equation ,with no second term or the single particle Liouville equation.

ii). The Vlasov equation.

iii).The Boltzmann equation.

iv). The Landau equation.

v). The Fokker-Planck equation.

A last statement about the physical meaning of the functions f_1 , f_s has to be made now , this is in relation with the fact that over f_1 ,unlike the case of f_s, $s \neq 1$,we have the possibility of external control.A system could be prepared with a given velocity distribution by putting together several streams of particles with various velocities, also on the system several inhomogeneities can be created, imposing over it mechanical or thermal constraints, but we have no control over the correlations; these are produced by the molecular interactions and adapt themselves to the instantaneous microscopic state , according to the laws of dynamics.This different behaviour of f_1 in relation with the other functions f_s, $s \neq 1$,allow us , according to R. Balescu (5) , to set a criterion with respect to the order of magnitude of the physical parameter of the system.Refering again to R. Balescu (5) , we ask for an initial condition which guarantee that the correlation appearing in the time zero has been produced and has the same order of magnitud that the ones appearing in later times , even if by an extremely improbable fluctuation , a correlation of radical different order of magnitud appears at given times , it will decay very rapidly and it will be replaced by another one of the normal order of magnitude , which it will be set by the size of the correlations in a system in thermodynamical equilibrium.

For an inhomogeneous system , that is in particular having in mind the laser-plasma interactions, a system in which the inhomogeneity is crated by an external field , due to the macroscopic origin of the perturbation , the scale length over which the local macroscopic quantities (which are functionals of f_1) vary , is usually long compared with the microscopic lenght. This knowledge allows us to consider a region large compare to microscopic length in which f_1 is the same that the one in equilibrium.This hypothesis of local equilibrium is stated more precisely as the condition $L_m \ll L_h$, where L_m is the largest microscopic parameter (as the mean free path or the range of the interactions) and L_h is the hydrodynamical length, defined as:

$$L_h = \text{minimun}\left(\left|\frac{f_1}{\nabla f_1}\right|\right)$$

This length will fix the size of each cell in which the condition of local equilibrium is valid for f_s , s= 1, 2, 3,..... .Some of these ideas will be applied in the determination of the plasma parameters.(10).

3). PLASMA PARAMETERS.

Let us assume a plasma fully ionized in thermodynamic equilibrium , for which the temperature is high enough to produce the ionization, but not high enough to produce relativistic effects.Let us suppose also that the plasma behaves classically, the quantum effects are not important; also due to the long range Coulomb forces among the plasma particles ,there are correlations. Assume for such a plasma that the correlations are weak , in order to make a linear approximation in the B.B.G.K. (2.20) hierarchy of kinetic equations J.Ivon (11) proved that the correlation is given by

3.1) $$\mathcal{E} = \frac{r_0}{r}\, e^{-\lambda_D r}$$

where: $r_0 = \dfrac{e^2}{\kappa T}$

and: $\lambda_D = \left(\dfrac{4\pi n_e e^2}{\kappa T}\right)$ is the Debye length.

e=1.602 10^{-19} [coulomb] , is the elementary charge , n_e is the electron density , κ is the Boltzmann constant , T is the temperature.

Note : The number of elementary charges for the ions is z=1.

A method like this one allows us to study the thermodynamics of a gas of charged particles and of course since it takes into account the molecular structure of the system , it is more general than the one proposed by Debye and Huckel (1923) for electrolytes.The importance of this method rests in its statistical nature and second in the fact that it is based in the study of correlations, concept that it will be used later to study the idea of local equilibrium.The key concept that we get from the theory is the Debye length , defined in equation (3.1).

The Debye length is the characteristic parameter that limits the collective behaviour of the phenomena , due to the electrostatic interactions among the plasma particles collective oscillations are produced, from there it is easy to see that under the influence of an external perturbation with wave length $\lambda < \lambda_D$ it is impossible for the plasma to react collectively.

Considering then the mean square root velocity μ_o of the electrons in terms of the temperature , we can define the frequency of the electrostatic oscillations of the electrons :

3.2). $$\omega_p = \left(\frac{\lambda_o}{\mu_o}\right)^{-1} = \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2}$$

where m_e is the electron mass.

As we can see, the two plasma parameters λ_D and ω_p are functions of the four quantities e^2 , m_e , n_e and $\mathcal{L}=(\gamma T)^{-1}$. With this quantities , for a classical plasma we can construct the non-dimensional parameter $\gamma = e^2 n_e^{\frac{1}{3}} \mathcal{L}$

Taken into account that:

$\langle \frac{1}{2} m v_e^2 \rangle^{-1} \sim \mathcal{L}$ and $r_{12} \sim n_e^{-\frac{1}{3}}$

we can see that the parameter γ is proportional to the ratio of the average interaction energy of the two particles to their average kinetic energy.In a classical plasma we assume $\gamma \ll 1$ (12).

Now in relation to the four quantities e^2 , m_e , n_e and $\mathcal{L}$, we can define two characteristic times for the classical plasma . Let us first consider the plasma time:

3.3) $t_p = \left(\frac{m_e}{e^2 n_e}\right)^{\frac{1}{2}}$

t_p is proportional to the square root of the mass and is independent of the particle velocity.It is analogous to the period of oscillation of a particle subject to an elastic force with a spring constant equal to $e^2 n_e$. This is not just a coincidence , it express a fundamental property of the plasma behaviour.Since $t_p \propto \omega_p^{-1}$ then t_p is a time which is reflecting the behaviour of the plasma particles as an harmonic oscillator , due to Coulomb interactions .

The second time that it will be defined is a relaxation time :

3.4). $t_r = \left(e^4 n_e\right)^{-1} \left(m_e \alpha^{-3}\right)^{\frac{1}{2}}$

Another time that we will consider is the "duration of the collision", t_c , that is the time which a particle moving with the average velocity spends in the sphere of influence of another particle. This time which has a precise meaning in a gas , due to the long range of coulomb interactions in a plasma , it losts its meaning; each plasma particle is interacting simultaneously with a large number of other particles , but due to its definition , we can consider a time $t_c \propto t_p^{-1}$ as this characteristic time .

Algebraically it is easily proved that the relation among the two characteristic times of a plasma is:

$$\frac{t_p}{t_r} = \alpha^{\frac{3}{2}}$$

therefore the scale of characteristic time for a plasma is $t_r \gg t_p$, $\alpha^{3/2} \ll 1$
We note that this ratio is given by many authors (8) as the fundamental parameter of the plasma.

Finally we introduce the expression for the collision frequency in the model of a perfect Lorentz gas :

3.5) $$\nu = \frac{\sqrt{2}}{\gamma_E(z)} t_r^{-1} \ln\left[\left(4\sqrt{\pi}\,\alpha^{3/2}\right)^{-1}\right]$$

where $\gamma_E(z)$ is Spitzer's correction factor for electron-electron collision and $\ln\left[\left(4\sqrt{\pi}\,\alpha^{3/2}\right)^{-1}\right]$ is the Coulomb logarithm.

Now in order to set a criterium for the strength of the field, let us consider the following: an electron "quivers" in the field of a wave $E_0 e^{-iwt}$, with a velocity :

$$|\vec{v}_E| = \left|\frac{eE_0}{m\omega}\right| \quad ; \quad v_E = 22\,\lambda\sqrt{I}\ [cm\,sec^{-1}]$$

where λ is in $[\mu m]$ and I in $[W cm^{-2}]$. The electron thermal velocity is of the order of $v_T = \left(\frac{\chi T_e}{m_e}\right)^{1/2}$, this two magnitudes are defining the field strength parameter η :

3.6)
$$\eta = \frac{v_E}{v_T}$$

When η =1 , $v_E = v_T$ and the field intensity can be measured by introducing the characteristic frequency w_E as: $\omega_E = \frac{eE_0}{m v_e} = \frac{eE_0}{m v_T} = eE_0\sqrt{\frac{\alpha}{m}}$

then the limit for a critical field E_c which does not disturb appreciably the electron velocity distribution during a period of the plasma oscillation, (that is , during an effective duration of the collision) is given by:

$$\frac{\omega_E}{\omega_p} = \frac{1}{2} E\sqrt{\frac{\alpha}{nc}} \ll 1$$

In the limit $w_E / w_p = 1$,from there we obtain the expression for the critical field $E_c = 2\sqrt{\frac{\pi n}{\alpha}}$ .For example for a typical thermonuclear field

$$T \leq 3\cdot 10^{7}\,{}^{\circ}K \qquad ; \qquad E_c \sim 10^{10}\left[\frac{Volt}{cm}\right]$$
$$n = 10^{18}\ [cm^{-3}]$$

In the region of strong fields one actually expects , taking into account our previous considerations , that the "collision operator " $\mathcal{C}$ depends on the electric field .We are talking here of a collision operator , assuming that there exist a kinetic equation describing the evolution of such plasma. The limit η =1 is setting also the optimun irradiance I_{opt} at which the power absorbed by the plasma due to inverse Bremsstrahlung will be maximum , that is $I_{op} \approx 3\times 10^{12}\left[\frac{T_e}{\lambda^2}\right][W cm^{-2}]$ with T_e in $[eV]$ and λ in $[\mu m]$.

Now in order to connect the plasma parameters with the macroscopic refractive index $\tilde{n}$, we recall (as it will be proved in chapter 2), that

that the same expressions can be obtained from the model of a plasma , as a set of independent oscillators described by the equations:

$$3.7) \qquad \ddot{\vec{r}}_k + \nu_k \dot{\vec{r}}_k + \omega_k^2 \vec{r}_k = \frac{e_k}{m_k} \qquad k = 1, 2, \cdots$$

where for a plasma $\omega_k = 0$

This one is also a macroscopic description , but it is mentioned since in it is clearly exposed the collective behaviour of the plasma. And , at least for a "collisionless " plasma , that is for example in the case of a plasma under the influence of a high frequency field , the same expression is obtained by the Linear Response theory. (13).

Finally a last consideration for the following: when the quantum effects are important , that is when we are talking about plasmas at low temperatures and of high densities , the non-dimensional parameter of the plasma must be modified , in order to take into account the statistical behaviour of the particles . Apart from the characteristic parameters $e^2, m_e, n_e, \mathcal{L}$,the quamtum character is introduced with $\hbar = \frac{h}{2\pi}$ where h is the Planck constant; under this conditions the non-dimensional parameter is: $\alpha' = \hbar^2 n^{\frac{2}{3}} \mathcal{L} m^{-1}$ which measures the importance of quantum statistical effects or the "degree of degeneration " of the system . Now in order to measure simultaneously the effects of interactions and quantum statistic , the parameter

$$\alpha'' = \frac{\alpha}{\alpha'} = e^2 m \hbar^{-2} n^{-\frac{1}{3}}$$

is introduced , then for a degenerate electron gas $\alpha'' \ll 1$.

4). CLASSICAL EXPRESSIONS FOR THE PONDEROMOTIVE FORCE.

As we mentioned in the first paragraphs , the ligth absorpion processes and the forces acting in the plasma medium are the central questions in the following discussion about the laser-plasma interaction. We will discuss now till what extent the expressions for the forces as given by Landau and Lifshitz (14) could be applied to plasma-laser interaction.

The expressions for ponderomotive forces acting in a dielectric medium in the presence of a constant external electric field were obtained by the mentioned authors , based in energy considerations in connection with the thermodynamics of a deformation. The basic equations are:

4.1) i.- $$F = F_0(T,\rho) - \varepsilon(\rho,T)\frac{\vec{E}^2}{8\pi}$$

ii.- $$\vec{D} = \varepsilon(\rho,T)\vec{E}$$

where F is the free energy for given values of temperature and density per unit volume of the dielectric thermally isolated, which is adiabatically deformed under the action of a static field , F_0 is the free energy in absence of the field , and the linear relation between the electric field $\vec{E}$ and the electric induction $\vec{D}$ is determined by the "dielectric constant " $\varepsilon(\rho,T)$, which depends only on the density and on the temperature T. Combining the expressions 4.1), with the thermodynamical expressions for the deformation :

4.2). i.- $$\sigma_{ik} = \left[F - \rho\left(\frac{\partial F}{\partial \rho}\right)_{E,T} \right]\delta_{ik} + \frac{E_i D_k}{4\pi}$$

ii.- $$f_i = \frac{\partial \sigma_{ik}}{\partial x_k}$$

where σ_{ik} is the stress tensor and f_i the force acting on the dielectric we obtain finally the expression for the force in the isothermal case:

4.3). $$f_E = -\mathrm{grad}\ p_0(\rho,T) + \frac{1}{8\pi}\left[\vec{E}^2\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right] - \frac{\vec{E}^2}{8\pi}\mathrm{grad}\,\varepsilon$$

where the second and third terms in this expression are called the Helmholtz ponderomotive force.

The calculations for the case where a magnetic field is present are similar to the electric case , as far as the thermodynamical energy relations are concerned , but due to the presence of conduction currents , another term appears. Taking this ideas into account, the expression for the force due to a magnetic field is:

4.4) $$\vec{f} = -\operatorname{grad} p_0(\rho,T) + \frac{1}{4\pi}\operatorname{grad}\left[\vec{H}^2\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T\right] - \frac{\vec{H}^2}{8\pi}\operatorname{grad}\mu + \frac{\mu}{c}\vec{J}\times\vec{H}$$

In a medium where μ is different from the unity , all the terms in (4.4) are approximatelly of the same order . But if , as it usually happens, $\mu \approx 1$, then the last term will be the dominant one and the expression for the force will become: $$f_M \approx \frac{1}{c}\vec{J}\times\vec{H}$$

The assumptions that were made in the obtainment of equations 4.1 and 4.3 limit drastically the application of the theory.These limitations are connected with a question until now open in Physics , which is : what is the form of the macroscopic energy-momentum tensor of the electromagnetic field in ponderable matter?.Problems related with statistical considerations as well as relativistic generalizations will show us that the application of expressions like those ones to laser plasma interactions has to be examined more carefully.

Relativity requieres that energy and momentum conservation laws are written as :

4.5) $$\partial_\beta T_\alpha{}^\beta = 0$$

$\alpha = 0$ for energy , $\alpha = 1,2,3$ for momentum .

In the case of a system made up of charged particles and a electromagnetic field , the tensor can be split in a part corresponding to matter ${}^{(m)}T_\alpha{}^\beta$ and one corresponding to the field ${}^{(f)}T_\alpha{}^\beta$. Then defining $\partial_\beta{}^{(m)}T_\alpha{}^\beta = f_\alpha$ as the ponderomotive four-force due to the splitting of the tensor , we get:

4.6) $$\partial_\beta{}^{(m)}T_\alpha{}^\beta = f_\alpha = -{}^{(f)}T_\alpha{}^\beta$$

Now different splitting of the $T_\alpha{}^\beta$ will give different expressions for the force and there is where the Abraham-Minkovski controversy arises (16).

De Groot has proposed the expressions:

4.7). $$ {}^{(f)}T_{\alpha}{}^{\beta} = F_{\alpha\gamma} H^{\beta\gamma} - \frac{1}{4} F_{\gamma\delta} F^{\gamma\delta} \delta_{\alpha}{}^{\beta} + c^{-2}\left[F_{\alpha\gamma} M^{\gamma\delta} - M_{\alpha\gamma} F^{\gamma\varepsilon}\right] U_{\delta} U^{\beta} - c^{-4} U_{\alpha} U^{\beta} U^{\gamma} F_{\gamma\delta} M^{\delta\varepsilon} U_{\varepsilon} $$

where $F^{\alpha\beta}$ are the macroscopic fields , $M^{\alpha\beta}$ the polarization , and U^{α} the macroscopic four-velocity . Also is defined $H^{\alpha\beta} = F^{\alpha\beta} - M^{\alpha\beta}$ .On the other hand, the material part of the tensor is :

4.8). $$ {}^{(m)}T_{\alpha}{}^{\beta} = \rho U_{\alpha} U^{\beta} + \sigma_{\alpha}{}^{\beta} $$

where ρc^2 is the rest mass and internal energy density in the rest frame , and $\sigma_{\alpha}{}^{\beta}$ contains fluctuations and correlations . The space-space components of $\sigma_{\alpha}{}^{\beta}$ are the relativistic generalizations of the pressure tensor.In the three dimensional notation , the field tensor become:

4.9). $$ {}^{(f)}T^{\alpha\beta} = \begin{pmatrix} \frac{1}{2}(\vec{E}^2 + \vec{B}^2) & (E \times H)^i \\ (E \times H)^i & -E^i D^j - H^i B^j + (\frac{1}{2}E^2 + \frac{1}{2}B^2 - B\cdot M) g^{ij} \end{pmatrix} ; \quad g^{ij} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

This tensor agrees with the one proposed by Einstein and Lamb (11) in several terms and it is used when is applied to obtain the expression for the ponderomotive force.It results in:

4.10). $$ f = (\vec{\nabla} \vec{E})\cdot \vec{P} + (\nabla B)\cdot M + \partial_0 (\vec{P} \times \vec{B}) - \vec{M} \times \vec{B}) $$

which is a relativistic generalization of the expression proposed for the force by Kelvin.(18).

If we adopt now the point of view of Minkowski-Abraham in the split of the total Energy-Momentum tensor , that is the hypothesis which says that the material part of the tensor corresponds to the one in absence of any field , we obtain again:

4.11). $$ {}^{(m)}_{(0)}T_{\alpha}{}^{\beta} = \rho_{(0)} U_{\alpha} U^{\beta} + \sigma_{(0)\alpha}{}^{\beta} $$

If we postulate also:

4.12)i.- $\vec{P} = \kappa\,(\rho,T)\,\vec{E}$

$\vec{M} = \chi\,(\rho,T)\,\vec{B}$

ii.-Thermodynamical equilibrium and non relativistic behaviour in the rest frame.

It is possible then to obtain for the field tensor:

4.13).

$$ {}^{(f)}_{(0)}T^{\alpha\beta} = \begin{pmatrix} \frac{1}{2}\vec{D}\cdot\vec{E} + \frac{1}{2}\vec{B}\cdot\vec{H} + \frac{1}{2}\vec{E}^2 T\frac{\partial\kappa}{\partial\rho} + \frac{1}{2}\vec{B}^2 T\frac{\partial\chi}{\partial\rho} & c\,(\vec{E}\times\vec{H})^i \\ \frac{1}{c}\,(\vec{E}\times\vec{H})^i & -E^iD^j - H^iB^j + \frac{1}{2}\left(D\cdot E + B\cdot H - E^2\rho\frac{\partial\kappa}{\partial\rho} - B^2\rho\frac{\partial\chi}{\partial\rho}\right)g^{ij} \end{pmatrix} $$

As we can see , these expressions will result in the case where the susceptibilities are independent of the temperature in a expression for the energy:

4.14)i.- $${}^{(f)}_{(0)}T^{00} = \frac{1}{4\pi}\left(D\cdot E + B\cdot H\right)$$

Also if the system remains in statistical equilibrium , under the assumption of: $\vec{P} = \kappa\,(\rho)\,\vec{E}$

$\vec{M} = \chi\,(\rho)\,\vec{B}$

only in this case we have for the electric field part of the field tensor:

4.14).ii.- $$\sigma_{ik} = -\frac{\vec{E}^2}{8\pi}\left[\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)\right]\delta_{ik} + \frac{\varepsilon E_i E_k}{4\pi}$$

which is the expression obtained by Landau and Lifshitz . Other considerations have to be taken into account thinking about the laser plasma interaction in relation with the material part of the tensor . These authors reduced the material part of the tensor to: $${}^{(m)}_{\;0}T_{ik} = -p(\rho,T)\,\delta_{ik}$$

which corresponds to the energy-momentum tensor for a fluid in rest, whose particles are not in interaction, in other words it behaves as a perfect gas. They did not considered correlations or colective behaviour , which are principal plasma characteristics.

Summarizing one can say that an expression as (14) proposed by Landau and and Lifshitz , based in considerations of Abraham and Minkowski (19) ,due to the severe restrictive hypothesis we have to make , that we have:

1).-Statistical equilibrium , non-relativistic behaviour in the rest system.

2).-Linear susceptibilities , depending only on density.

3).-Non interacting particles.

Then we only have a limited range of applicability and the conditions required are not always fulfilled by the parameters characterizing the laser plasma. interaction.

There are other questions associated with the energy-momentum tensor in the closed system field-matter , when the fields are time dependent (20), these are related to the absorption and dispersion processes that become dominant now. But there is a domain of frequencies in which a macroscopic description is possible , in this case due to the dependance on time of the Hamiltonian (energy), the susceptibilities become frequency dependent. The relation between $\vec{D}$ and $\vec{E}$ is summarized in this case by $\vec{D} = \varepsilon(\omega)\vec{E}$, where the "dielectric constant" depends on the frequency and on the properties of the body, the details of this formulation will be given later , but for now on it will be assumed that the relation exists. In that case, for a weak field we assume:

4.15). $$\mathcal{U} = \frac{1}{4\pi}(\vec{E}\cdot\vec{D} + H\cdot B)$$

This then is the expression for the energy in the static field , taking into account the relationships:

$$\vec{D} = \varepsilon(\omega)\vec{E}$$
$$\vec{H} = \vec{B}$$

we will get for the energy in a transparent medium , that is a medium where absorption is not taken into account , the following expression for the average energy:

4.16). $$\mathcal{U} = \frac{1}{16\pi}\left[\frac{d(\omega\varepsilon)}{d\omega}\vec{E}^2 + \vec{H}^2\right]$$

Using (4,16) a basic relation in the thermodynamics of a deformation in a transparent medium , P. Pitaevskii (21) was able to show that the form of the stress tensor remains the same as the one deduced for a static field for a medium in thermodynamical equilibrium. In this case:

$$\sigma_{ik} = -p_0(\rho,T)\,\delta_{ik} - \frac{\vec{E}^2}{8\pi}\left[\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]\delta_{ik} + \varepsilon\,\frac{E_i E_k}{4\pi}$$

is valid, even in the presence of dispersion.

There it remains the question of the validity of the equation as the expression for the energy in a phenomenon as the laser plasma interaction , where absorption and fluctuations are important . It is clear that the problem must be analyzed more carefully. (22)

5. HYDRODYNAMICAL DESCRIPTION OF PLASMAS.

Following H.Hora (4) , the plasma as a continuum will be described by functions of the density $n(\vec{r},t)$,velocity $\vec{v}(\vec{r},t)$,temperature $T(\vec{r},t)$, which are continuous functions of the coordinates and time , and the energy exchange of the plasma.

Considering the plasma as composed by a fluid of ions and one of electrons , its evolution is given by the Euler equation of motion:

5.1).

$$n_i m_i \frac{d\vec{v}_i}{dt} = n_i e\vec{E} + Z n_i \frac{e}{c}\,\vec{v}_i \times \vec{H} + n_e m_e \nu(\vec{v}_i - \vec{v}_e) + \vec{\nabla} K n_i T_i + \vec{\chi}_i$$

5.2.).

$$n_e m_e \frac{d\vec{v}_e}{dt} = -e\vec{E} - \frac{e}{c}\,\vec{v}_e \times \vec{H} - n_e m_e \nu(\vec{v}_i - \vec{v}_e) + \vec{\nabla} K n_e T_e + \vec{\chi}_e$$

where equal density was assumed for electrons and ions , $\vec{\chi}_i$; $\vec{\chi}_e$ are the external forces acting on the plasma , $\vec{v}_i$ and $\vec{v}_e$ are the velocity fields of the fluids , the term ν is the collision frequency which is given by a statistical theory.

The equations of motion are complemented by the equations of continuity:

5.3) i.- $$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i\vec{v}_i) = 0$$

ii.- $$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e\vec{v}_e) = 0$$

which are expressing the mass balance, and the energy balance;

5.4) $$\frac{\partial}{\partial t}\left(\frac{m_i n_i}{2}\vec{v}_i^{\,2} + 2n_i KT\right) + W_i = \frac{\partial}{\partial t}U_i$$

5.5) $$\frac{\partial}{\partial t}\left(\frac{m_e n_e}{2}\vec{v}_e^{\,2} + 2n_e KT\right) + W_e = \frac{\partial}{\partial t}U_e$$

where W_e , W_i is the emitted or absorbed energy.

Now , it has been shown by Schluter (3) , that the substraction of the equations of motion 5.1 and 5.2 results in a generelized Ohm's Law for plasma:

5.6) $$\frac{m_e}{e^2 n_e}\left(\frac{\partial\vec{J}}{\partial t} + \nu\vec{J}\right) = \vec{E} + \vec{v}\times\vec{H} + \frac{c}{e n_e}\vec{J}\times\vec{H} + \frac{c}{e n_e}\nabla p_e$$

neglecting the non-linear terms and considering the definition of plasma frequency , which defines the electrostatic oscillation of the electrons as follows:

5.7) $$\omega_p^2 = \frac{4\pi e^2 n_e}{m_e}$$

then the equation 5.6 becomes:

5.8) $$4\pi\left(\frac{\partial\vec{J}}{\partial t} + \nu\vec{J}\right) = \omega_p^2\vec{E}$$

This equation plus the Maxwell equation for the electromagnetic fields of the plasma:

5.9) $$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial H}{\partial t}$$

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

it results for the periodic fields in a wave equation:

5.10) $$\Delta\vec{E} + \frac{1}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2(1+i\frac{\nu}{\omega})}\right]\frac{\partial^2}{\partial t^2}\vec{E} = 0$$

where the complex refractive index , given by:

5.11) $$\tilde{n} = n + ik = \left(1 - \frac{\omega_p}{\omega^2 \left(1 + i\frac{\nu}{\omega}\right)} \right)^{\frac{1}{2}}$$

is introduced , in order to describe the optical properties of the plasma (23). The real part of n , called the refractive index is given by:

5.12) $$n = \left[\frac{1}{2} \left\{ \left[\left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right)^2 + \left(\frac{\nu}{\omega} \frac{\omega_p^2}{\omega^2+\nu^2}\right)^2 \right]^{\frac{1}{2}} + \left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right) \right\} \right]^{1/2}$$

The imaginary part , called the absorption coefficient , is given by:

5.13). $$k = \left[\frac{1}{2} \left\{ \left[\left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right) + \left(\frac{\nu}{\omega} \frac{\omega_p^2}{\omega^2+\nu^2}\right)^2 \right]^{\frac{1}{2}} - \left(1 - \frac{\omega_p^2}{\omega^2+\nu^2}\right) \right\} \right]^{\frac{1}{2}}$$

As we can see , the complex refractive index is determined by the dispersion relation of the electromagnetic waves in the plasma. The macroscopic properties of the medium are determining ω_p through the extensive variables n , m , and the irreversible character of the phenomenon is given by the collision frequency ν , which is also a function of the temperature and which was introduced macroscopically as a friction coefficient between the fluids.

Now , from the equations (5.1) and (5.2) and considering the expression for the refractive index , it results in an expression for the ponderomotive force , with which the electromagnetic field act on the plasma medium(4):

5.14) $$\vec{f} = -\nabla \vec{p} + \nabla \cdot \left[\overline{\overline{U}} + \frac{1}{4\pi} \left(\tilde{n}^2 - 1\right) \vec{E}\,\vec{E} \right] - \frac{\partial}{\partial t} \frac{\vec{E} \times \bar{H}}{4\pi c}$$

where U is the Maxwellian stress tensor:

$$U = \frac{1}{4\pi} \begin{bmatrix} \frac{1}{2}\left(E_x^2 - E_y^2 - E_z^2 + H_x^2 - H_y^2 - H_z^2\right) & E_x E_y + H_x H_y & E_x E_z + H_x H_z \\ E_x E_y + H_x H_y & \frac{1}{2}\left(-E_x^2 + E_y^2 - E_z^2 - H_x^2 + H_y^2 + H_z^2\right) & E_y E_z + H_y H_z \\ E_x E_y + H_x H_z & E_y E_z + H_y H_z & \frac{1}{2}\left(-E_x^2 - E_y^2 + E_z^2 - H_x^2 - H_y^2 + H_z^2\right) \end{bmatrix}$$

and $\vec{p}$ represents the total gas dynamics pressure

In difference to the former subsection , this force 5.14) is automatically valid for the dispersive plasma , even with dissipation . (3).

Before the discussion of the microscopic description of the plasma , we need , in order to show its importance , to consider the basic hypothesis underlying in all the macroscopic description , that is the hypothesis of local equilibrium.

In the previous discussions we did not take into account the molecular structure of the systems. We considered them as a continuous medium , which could be divided into small but still macroscopic elements to assure that each element contained many particles and we were there assuming that each small element could be assigned ordinary thermodynamic character.Each of them had definitive temperature , mean density and thermodynamic potential.In relation with this , the dependance of thermodynamics quantities on coordinates and time is to be understood as a variation of the local equilibrium characteristics. For such small elements , the local equilibrium is established extremely rapidly for the majority of the systems even when the systems as a whole remain in a state of non-equilibrium, but in doing so , we can formulate the law of change of states for a system in incomplete equilibrium. In order to set the size of the small element or cell in local equilibrium , we have to get a characteristic parameter of the system , then we are sure that there is local equilibrium in the cell of volume l^3 when $\left.\frac{\partial P}{\partial x}\right|_{-\frac{1}{2}}^{1/2} \approx 0$,where P is a thermodynamical variable (24).

CHAPTER II

ENTROPY PRODUCTION

1).ENTROPY PRODUCTION:

In chapter 1 we discussed some expressions for the force-densities acting in plasmas based on relations of the energy and momentum balance equations. The irreversible character of the interaction of a system with an external field (in particular laser - plasma interaction), was stressed making the hypothesis of local equilibrium , which allows us to formulate the first Law of Thermodynamics locally.

Now we will procede to discuss the second Law of thermodynamics , which in the form of the so called entropy balance equation plays a central role in the whole theory of non-equilibrium thermodynamics . This equation expresses the fact that the entropy of a volume element changes with time due to two reasons . First it changes because entropy flows into the volume element , second because there is an entropy source due to irreversible phenomena inside the volume element . This entropy source is always a non-negative quantity , since entropy can only be created , never destroyed. For reversible transformations the entropy source vanishes. (1).

These statements for systems whose properties are continuous functions of space coordinates and of time can be formulated as:

1.1).

$$S = \int_{\Omega} \rho s \, d\Omega$$

1.2). $$\frac{\partial}{\partial t}\left(\rho s + \operatorname{div}^{(s)}\vec{J}_{total} - \sigma\right)d\Omega = 0$$

where is the density ρ, s is the entropy per unit mass ; ${}^{(s)}\vec{J}_{total}$ is the total entropy flow density per unit area and unit time , σ is the entropy source strength per unit volume and unit time or entropy production due to irreversible processes.

In a more familiar form with the identities :

1.3). $$S = \int_{\Omega} \rho s \, d\Omega$$

1.4). $$\frac{dS_e}{dt} = -\int_{\Sigma(\Omega)} {}^{(s)}\vec{J}_{total} \cdot d\vec{\Sigma}$$

1.5). $$\frac{dS_i}{dt} = \int \sigma \, d\Omega$$

the second law takes the form:

1.6). $$dS = dS_e + dS_i$$

where, dS_e is the entropy supplied to the system by its surroundings and dS_i is the entropy produced inside the system, dS have the properties:

1.7). $$dS_i \begin{cases} = 0 & \text{for reversible processes} \\ \geq 0 & \text{for irreversible processes} \end{cases}$$

which is one of the formulations of the second law of the Thermodynamics Now , in order to relate the variations in the properties of the systems to the rate of change of entropy , we will consider the following relations , all of them valid in a cell in local equilibrium , for a system of n components among which r chemical reactions are possible:

1.8).Gibbs's relation:

$$T ds = du + p dv - \sum_{K=1}^{n} \mu_K dc_K$$

where p is equilibrium pressure, $\mathcal{U}$ is internal energy , v is specific volume , μ_k is chemical potential, c_k is the mass fraction.

1.9).Balance of mass:

$$\rho \frac{dc_k}{dt} = - \operatorname{div} J_k + \sum_{J=1} \nu_{kJ} J_J \qquad k = 1, 2, \ldots, m$$

where J_J is the diffusion flow, $\nu_{kJ} J_J$ is the rate of production of k per unit volume in the J chemical reaction.

1.10) Balance of energy:

$$\frac{du}{dt} = \frac{dq}{dt} - p\frac{dv}{dt} - v\,\bar{\bar{\sigma}} : \operatorname{grad}\vec{v} + v \sum_k \vec{J}_k \cdot \vec{F}_k$$

where:

1.11). $$\bar{\bar{P}} = \mathcal{U} - \bar{\bar{\sigma}}$$

is the total pressure tensor, dq is the heat, F_k force per unit mass. Then , taking into account 1.8), 1.9) and 1.10), plus the equations:

1.12) $$\rho \frac{dq}{dt} = - \operatorname{div} J_q$$

where J_q is the heat flow.

and the definition of chemical affinities:

$$A_J = \sum_{k=1}^{m} \nu_{kJ} \mu_k \qquad J = 1, \ldots r$$

we come to the balance of entropy:

1.13) $$\rho \frac{ds}{dt} = - \left[\frac{J_q - \sum_k \mu_k J_k}{T} \right] - \frac{1}{T^2} J_q \cdot \operatorname{grad} T - \frac{1}{T} \sum_{k=1}^{m} J_k \cdot \left[T \operatorname{grad} \frac{\mu_k}{T} - F_k \right]$$
$$- \frac{1}{T} \sigma : \operatorname{grad} v - \frac{1}{T} \sum_{J=1}^{r} J_J A_J$$

From this equation we arrive to the equation for the entropy flux:

1.14) $$J_s = \frac{1}{T} \left[J_q - \sum_{k=1}^{m} \mu_k J_k \right]$$

and entropy production:

1.15)

$$\sigma = \frac{1}{T^2}\vec{J}_q \cdot \mathrm{grad}\, T - \frac{1}{T}\sum_{K=1}^{n} \vec{J}_K \cdot \left(T\, \mathrm{grad}\, \frac{\mu_K}{T} - \vec{F}_K\right) - \frac{1}{T}\overline{\overline{P}} : \mathrm{grad}\, v - \frac{1}{T}\sum_{\mathfrak{J}=1} J_{\mathfrak{J}} A_{\mathfrak{J}}$$

We can see that the structure of the expression for σ is that of a bilinear form. It consists of a sum of products of two factors. One of these factors in each term is a flow quantity (heat flow $\vec{J}_q$, diffusion flow $\vec{J}_K$, momentum flow or viscous pressure tensor $\overline{\overline{P}}$ and the chemical reaction rate $A_{\mathfrak{J}}$), the other factor in each term is related to a gradient of an intensive state combined with the external force when there is one. These quantities are called "thermodynamic forces" or affinities. (2).

In general the entropy source strength , as we saw an example before , could be written as:

1.16)

$$\sigma = \sum_{a=1}^{n} J_a X_a \geq 0$$

that is a product of affinities and the conjugate flow term

The products of conjugated thermodynamic forces and the effect of their actions must be a scalar , and hence are the products of two scalars, the dot products of two vectors , or double dot products of two tensors of rank two, that is:

1.17)

$$\sigma = \sum_{a=1}^{n_0} J_a X_a + \sum_{a=1}^{n_1} \vec{J}_a \cdot \vec{X}_a + \sum_{a=1}^{n_2} \overline{\overline{J}}_a : \overline{\overline{X}}_a$$

where n_0 number of scalars , n_1 is the number of vectorial, and n_2 the number of tensorial (rank two)thermodynamic forces.

We noted also that the local formulation of the second law of the Thermodynamics in the form (1.6) , allows irreversible phenomena , called "coupled phenomena" , which entail a decrease in entropy to occur at some place, provided that concurrently occurring at the same place are irreversible phenomena , called "coupling phenomena" , which result in such a considerable production of entropy that the ultimate overall entropy increment is positive.

Also we have to take into account that σ can be factorized in a different way corresponding to determinated elections of χ_a and J_a. This choice must accomplish the condition that in equilibrium state, when $\chi_a = 0$, then $\sigma = 0$.

Other property of σ is that it has to be invariant Galilei, since the irreversibility or reversibility of the phenomena must be invariant under that transformation.

Finally it must be noted that in contradiction to the entropy, the entropy source strength is not a function state, since it depends on the mode of change between given states.

2. LINEAR PHENOMENOLOGICAL EQUATIONS.

There are a set of relationships among affinities and the fluxes in the expression for the entropy source strength $\sigma = \sum_a J_a \chi_a$ that can be deduced from the phenomenological linear laws of irreversible Thermodynamics, for instance the Fick law of diffusion, the Ohm law, etc. (3) or from a statistical theory taking into account the microscopic stucture of the system. These are called the "linear phenomenological laws", we shall summarize the principal features of them, without going into too much detail at this stage.

We know that the generalized flows, as functions of the affinities, are depending on all forces, as expressed by:

2.1). $$J_a = J_a(\chi_b) \qquad b = 1, 2, \cdots n$$

from that expression, using a Maclaurin series expansion, we get:

2.2). $$J_a = J_a(\chi_b = 0) + \sum_{b=1}^{n} \frac{\partial J_a}{\partial \chi_b} \chi_b + \frac{1}{2!} \sum_{b,c=1}^{n} \frac{\partial^2 J_a}{\partial \chi_b \partial \chi_c} \chi_b \chi_c + \cdots$$

If $X_b=0$, the system goes into an equilibrium state , for which $J_a=0$, then:

2.3). $$J_a\left(X_b=0\right)=0$$

In first approximations , that is for states not very far from equilibrium , we can confine ourselves to linear relationships called linear phenomenological equations of the type:

$$J_a=\sum_{b=1}^{n} L_{ab}X_b$$

in which:

2.4). $$L_{ab}=\left(\frac{\partial J_a}{\partial X_b}\right)_{X_c}=\left(\frac{J_a}{X_b}\right)_{X_c=0} \qquad c\neq b$$

The physical meaning of the coefficients depend on the specific applications of the theory , at this point it can be said in general terms that these coefficients are not functions of the thermodynamical forces, hence also they are not functions of the effects , that is the fluxes.On the other hand they can depend on the parameters of the local state of a substance or depend on the kind of substance . Also coefficients of the type L_{aa} (the same index) relate the conjugate forces and fluxes ,and coefficients of the type L_{ab} are concern with cross effects.

The linear phenomenological law:

2.5). $$J_a=\sum_{b=1}^{n} L_{ab}X_b$$

allows us to write for the entropy source:

2.6) $$\sigma=\sum_a J_aX_a$$

and the expression:

2.7) $$\sigma=\sum_{a,b=1}^{n} L_{ab}X_aX_b$$

Finally , due to spatial and time symmetries , the phenomenological coefficients satisfy the so called:

a).- Spatial symmetries ; (Curie Principle) : Quantities whose tensorial character differs by an odd number of rank, cannot interact in an isotropic medium.

b). Onsager Reciprocal Relations (Time Symmetries). With a proper choice of thermodynamical forces and generalized flows , the phenomenological coefficients are related by:

2.8) $$L_{ab} = \varepsilon_a \varepsilon_b L_{ba}$$

if there are no forces related by a vector product.

The coefficients are given by:

2.9) $$L_{ab}(\bar{B}, \omega) = \varepsilon_a \varepsilon_b L(-\bar{B}, -\omega)$$

if there are forces related by a vector product. The index $\varepsilon_a = 1$ characterizes the so called α -variables and $\varepsilon_a = -1$ for β variables. The results of chapter one will be reset in this more general frame.

3. ENTROPY PRODUCTION IN DIELECTRIC SYSTEMS.

The question of irreversible processes in a polarizable or dielectric system in presence of an external electromagnetic field has been the subject of studies for several years , we have for example the discussion in the last century between Helmholtz and Kelvin about the expressions for the density of ponderomotive forces with which a static field acts in a liquid dielectric medium (4) .Of course this problem was not formulated in the frame of non equilibrium Physics , but since we are dealing there with the phenomena of dielectric relaxation , it fits perfectly with the current subject of studying irreversible Thermodynamics. On the other hand , the balance equations , in particular the balance of energy in plasmas has been a subject for controversy in the last years , here the point of controversy is the erroneous interpretation of some terms in the balance energy equation , without taking into account its statistical character . (5) .

Also it will only be mentioned the related problem of the definition of the "material" energy momentum tensor , having in mind that in a thermodynamical theory , dealing both with the electromagnetic field and the material system , Minkowski or Abraham's definition may be equivalent , but since some studies analizing the problem from different points of view have solved the problem in favour of Abraham's formulation, we will adopt that definition here. (1).

In order to discuss the irreversible phenomena due to electromagnetic forces , we have to formulate the basic conservation laws of energy and momentum, taking into account the presence of an electromagnetic field, the evolution of the fields will be determined by the Maxwell's equations, summarized as follows in the Gauss system:

3.1) $$\vec{\nabla} \cdot \vec{D} = \rho e$$

3.2) $$\vec{\nabla} \cdot \vec{B} = 0$$

3.3) $$\frac{\partial \vec{D}}{\partial t} - c\, \vec{\nabla} \times \vec{H} = -\vec{I}$$

3.4) $$\frac{\partial \vec{B}}{\partial t} + c\, \vec{\nabla} \times \vec{E} = 0$$

here , ρ is the density of mass , e the electric mass per unit mass, $\vec{I}$ is the electric current density, the quantities $\vec{D}$ and $\vec{H}$ are the electric and magnetic displacement vectors , and in a system at rest they are connected with the fields $\vec{E}$ and $\vec{B}$ by the relations:

3.5). $$\vec{D} = \hat{\varepsilon} \cdot \vec{E}$$

3.6). $$\vec{H} = \hat{\mu}^{-1} \cdot \vec{E}$$

called the constitutive equations.

In an isotropic system we have:

3.7). $$\hat{\varepsilon} = \varepsilon\, \hat{\mathcal{U}}$$

3.8). $\hat{\mu} = \mu \hat{\mathcal{U}}$

$\hat{\mathcal{U}}$ where is the unity tensor , and ε is the dielectric constant and μ the magnetic permeability.

Now we shall define :

3.9). $$\vec{P} = \vec{D} - \vec{E} = \left(\hat{\varepsilon} - \hat{\mathcal{U}}\right) \cdot \vec{E} = \hat{\varkappa} \cdot \vec{E}$$

where $\vec{P}$ is the polarization and $\hat{\kappa}$ is the electric susceptibility tensor.

In a similar way:

3.10). $$\vec{M} = \left(\hat{\mu} - \hat{\mathcal{U}}\right) \cdot \vec{H} = \hat{\chi} \cdot \vec{H}$$

where $\vec{M}$ is the magnetic polarization and $\hat{\chi}$ the magnetic susceptibility tensor .As before in an isotropic system $\hat{\varkappa} = \varkappa \hat{\mathcal{U}}, \hat{\chi} = \chi \hat{\mathcal{U}}$.

We recall that in all these relations $\varkappa$ and χ depend only on the thermodynamical variables characterizing the local equilibrium state of the system ; in doing that we will consider then the same expression for the Gibbs's Law as in the case of reversible Thermodynamics. (6). The restriction that this hypothesis of local equilibrium is imposing in the theory , has a clear meaning. The thermodynamical theory that it will be developed in the following could be applied to the case of weak fields , for which the system remains also in a state of polarization equilibrium . (Stationary field and polarization (7).Let us consider first the balance equations for the momentum density of the electromagnetic field.Following Abraham 's definition and equations (3.9) and (3.10) we get:

3.11) $$\frac{1}{c} \frac{\partial}{\partial t}\left[\vec{E} \times \vec{H}\right] = \frac{1}{c} \frac{\partial}{\partial t}\left[\left(\vec{D} \times \vec{B}\right) - \left(\vec{P} \times \vec{B}\right) - \left(\vec{E} \times \vec{M}\right)\right]$$

On the other hand from the Maxwell's equations (3.1)......(3.4) we have:

3.12) $$\frac{1}{c} \frac{\partial}{\partial t}\left(\vec{D} \times \vec{B}\right) = div\left[\vec{D}\vec{E} + \vec{B}\vec{H} - \left(\frac{1}{2}\vec{E}^2 + \frac{1}{2}\vec{B}^2 - \vec{M} \cdot \vec{B}\right)\vec{\mathcal{U}}\right]$$
$$- \left(grad \vec{E}\right) \cdot \vec{P} - \left(grad \vec{B}\right) \cdot \vec{M} - \rho e \vec{E} - \frac{1}{c} \vec{I} \times \vec{B}$$

and from (3.11) and (3.12) , we get:

3.13) $$\frac{1}{c}\frac{\partial}{\partial t}\left[\vec{E}\times\vec{H}\right] = \mathrm{div}\left[\vec{D}\vec{E}+\vec{B}\vec{H}-\frac{1}{2}\left[\vec{E}^2+\frac{1}{2}\vec{B}^2-\vec{M}\vec{B}\right]\hat{\mathcal{U}}\right]-\rho e\vec{E}-\frac{1}{c}\vec{I}\times\vec{B}$$
$$-\left(\mathrm{grad}\,\vec{E}\right)\cdot\vec{B}-\left(\mathrm{grad}\,\vec{B}\right)\cdot\vec{M}-\frac{1}{c}\frac{\partial}{\partial t}\left[\vec{P}\times\vec{B}+\vec{M}\times\vec{B}\right]$$

Let us consider now the balance of mass :

3.14) $$\frac{\partial\rho}{\partial t} = -\,\mathrm{div}\,\rho\vec{v}$$

where ρ is the total density $\rho=\sum_{k=1}^{n}\rho_k$ and $\vec{v}$ the barycentric velocity:

3.15) $$\vec{v} = \sum_{k=1}^{n}\frac{\rho_k v_k}{\rho},$$

and the definition of the barycentric substantial time derivative:

3.16). $$\frac{d}{dt} = \frac{\partial}{\partial t}+\vec{v}\cdot\mathrm{grad}$$

From (3.14) and (3.16) we can derive the following equations:

3.17) Balance of polarization P:

$$\frac{\partial\vec{P}}{\partial t} = -\,\mathrm{div}\left(\vec{v}\vec{P}\right)+\rho\frac{d\vec{p}}{dt}$$

3.18) Balance of Magnetization M:

$$\frac{\partial\vec{M}}{\partial t} = -\,\mathrm{div}\left(\vec{v}\vec{M}\right)+\rho\frac{d\vec{m}}{dt}$$

with $\vec{P}=\rho\vec{p}$ and $\vec{M}=\rho\vec{m}$ and since:

$$\frac{\partial\vec{B}}{\partial t} = \frac{d\vec{B}}{dt}-\vec{v}\cdot\mathrm{grad}\,\vec{B} \qquad \frac{\partial\vec{E}}{\partial t} = \frac{d\vec{E}}{dt}-\vec{v}\cdot\mathrm{grad}\,\vec{E}$$

we get the equation:

3.19) $$\frac{1}{c}\frac{\partial}{\partial t}\left[\vec{E}\times\vec{H}\right] = \mathrm{div}\,\bar{\bar{\sigma}}-\vec{F}$$

where $\bar{\bar{\sigma}}$ is the stress tensor given by:

3.20) $$\bar{\bar{\sigma}} = \left(\vec{D}\vec{E} + \vec{B}\vec{H}\right) + \frac{v}{c}\left(\vec{P}\times\vec{B}\right) - \frac{v}{c}\left(\vec{M}\times\vec{E}\right) - \frac{1}{2}\left(\vec{E}^2 + \vec{B}^2 - 2\vec{M}\cdot\vec{B}\right)\vec{U}$$

and F the volume force, given by:

3.21) $$\vec{F} = \rho e\vec{E} + \frac{1}{c}\vec{J}\times\vec{B} + \left(\text{grad}\,\vec{E}\right)\cdot\vec{P} + \left(\text{grad}\,\vec{B}\right)\cdot\vec{M} + \frac{\rho}{c}\frac{d}{dt}\left[\vec{p}\times\vec{B} - \vec{m}\times\vec{E}\right]$$

Since the momentum of the field and matter must be conserved , we have the balance of momentum equation:

3.22) $$\frac{\partial}{\partial t}\left(\rho\vec{v} + \frac{1}{c}\vec{E}\times\vec{H}\right) = -\,\text{div}\left(\rho\vec{v}\vec{v} + \mathcal{P} - \bar{\bar{\sigma}}\right)$$

where $\mathcal{P}$ is the pressure tensor , from where we get , considering equation (3.20), the motion equation:

3.23) $$\rho\frac{d\vec{v}}{dt} = -\,\text{div}\,\mathcal{P} + \vec{F}$$

We recall that the quantities that have a well defined meaning in the thermodynamics of a polarizable medium are div T - F (4) , and $\mathcal{P} - \bar{\bar{\sigma}}$ due to the modification of the pressure tensor $\mathcal{P}$,for the presence of the electromagnetic field.

Now , from equation (3.23) , multiplying by $\vec{v}$, we obtain:

3. 24). $$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho\vec{v}^2\right) = -\,\text{div}\left(\frac{1}{2}\rho\vec{v}^2\vec{v} + \mathcal{P}\cdot\vec{v}\right) + \tilde{\mathcal{P}}:\text{grad}\,\vec{v} + \vec{F}\cdot\vec{v}$$

balance of kinetic energy , where is the trasposed tensor.Then considering the Poynting theorem , derived from the Maxwell 's equations (3.1)......(3.4) ,we get :

3.25) $$\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t} = -\,\text{div}\left(\vec{E}\times\vec{H}\right) - \vec{J}\cdot\vec{E}$$

After considering the expression 3.24) for the volume force, we get the energy balance equation:

3.26) $$\frac{\partial}{\partial t}\left\{\frac{1}{2}\rho\vec{v}^2 + \frac{1}{2}\vec{E} + \frac{1}{2}\vec{B}^2 - \vec{M}'\cdot\vec{B} + c^{-1}\vec{E}'\cdot\left(\vec{v}\times\vec{M}'\right)\right\} = -\,\text{div}\left\{\frac{1}{2}\rho\vec{v}^2\vec{v} + \right.$$
$$\left.\mathcal{P}\cdot\vec{v} - \left(\vec{p}'\cdot\vec{E} + \vec{M}'\cdot\vec{B}\right)\vec{v} + c\,\vec{E}\times\vec{H}\right\} + \tilde{\mathcal{P}}:\text{grad}\,\vec{v} - i\vec{E}' - \rho\vec{E}'\cdot\frac{d\vec{p}'}{dt} - \rho\vec{B}'\cdot\frac{d\vec{m}'}{dt}$$

In 3.26) , the dashed quantities are the magnitudes measured by an observer moving with the velocity $\vec{v}$ of the medium .There the terms of the order $\left(\frac{\vec{v}}{c}\right)^2$ are neglected in a non-relativistic approximation.

Now . if we consider the balance of total energy of matter and field, we get:

3.27). $$\frac{\partial \mathcal{U}}{\partial t} = -\operatorname{div} \vec{J}_u$$

from there is deduced the balance of internal energy :

3.28)

$$\rho u = \mathcal{U}_t - \left\{ \frac{1}{2}\rho \vec{v}^2 + \frac{1}{2}\vec{D}'\cdot\vec{E}' + \frac{1}{2}\vec{B}'\cdot\vec{H}' - \frac{1}{2}\vec{P}'\cdot\vec{E}' - \frac{1}{2}\vec{M}'\cdot\vec{B}' + 2c^{-1}\vec{v}\cdot\left(\vec{E}'\times\vec{H}'\right)\right\}$$

Now , in equation 3.28), if we consider the relationships:

3.29)

$$\vec{D} = \vec{D}' - \frac{1}{c}\vec{v}\times\vec{H}'$$

$$\vec{E} = \vec{E}' - \frac{1}{c}\vec{v}\times\vec{B}'$$

$$\vec{P} = \vec{P}' + \frac{1}{c}\vec{v}\times\vec{M}'$$

$$M = \vec{M}' - \frac{1}{c}\vec{v}\times\vec{P}'$$

$$\vec{H} = \vec{H}' + \frac{1}{c}\vec{v}\times\vec{D}'$$

$$\vec{B} = \vec{B}' + \frac{1}{c}\vec{v}\times\vec{E}'$$

which are the relationships for the fields $\vec{E}'$,$\vec{D}'$,$\vec{B}'$,$\vec{H}'$ and $\vec{M}'$ measured by an observer moving with velocity $\vec{v}$ of the medium, then we get the equation:

3.30) $$\rho u = \mathcal{U}_t - \left\{\frac{1}{2}\left[\rho\vec{v}^2 + \vec{D}'\cdot\vec{E}' + \vec{B}'\cdot\vec{H}' - \vec{P}'\cdot\vec{E}' - \vec{M}'\cdot\vec{B}'\right] + 2c^{-1}\vec{v}\cdot\left(\vec{E}'\times\vec{H}'\right)\right\}$$

then if the heat flux is defined as:

3.31). $$\vec{J}_q = \vec{J}_e - \left\{\frac{1}{2}\rho\vec{v}^2\vec{v} + \rho\mathcal{U}\vec{v} + \bar{\bar{P}}\cdot\vec{v} - \left(\vec{P}'\cdot\vec{E} + \vec{M}'\cdot\vec{B}\right)\vec{v} + c\left(\vec{E}\times\vec{H}\right)\right\}$$

from equations 3.26) , 3.27) , 3.30) and 3.31) , we get the balance of internal energy as:

3.32).

$$\rho \frac{du}{dt} = -\operatorname{div} \vec{J}_q - \bar{\bar{\mathcal{P}}} : \operatorname{grad} \vec{v} + \vec{J} \cdot \vec{E}' + \rho \vec{E}' \cdot \frac{d\vec{p}'}{dt} + \rho \vec{B}' \cdot \frac{d\vec{m}'}{dt}$$

now, for a reversible (equilibrium)transformation , we have the relations:

3.33)

$$\operatorname{div} \vec{J}_q = -\rho \frac{dq}{dt}$$
$$\vec{E}_{eq} = \rho \bar{\bar{K}}^{-1} \cdot \vec{p}'$$
$$\vec{B}' = \rho (\bar{\bar{\chi}} + \bar{\bar{u}}) \bar{\bar{\chi}}^{-1} \cdot \vec{m}'$$

where $\bar{\bar{\kappa}}^{-1}$ and $\bar{\bar{\chi}}$ are the electric and magnetic susceptibility tensor , depending only on the thermodynamic variables characterizing the (local) equilibrium state of the system.Now , taking into account 3.32) , 3.33) , and 1.15) , we come to the expression of balance for the entropy:

3.34)

$$\rho \frac{ds}{dt} = -\operatorname{div}\left(\frac{J_q - \sum_k \mu_k J_k}{T}\right) - \frac{1}{T^2} \vec{J}_q \cdot \operatorname{grad} T - \frac{1}{T} \sum_{k=1}^{n} J_k \cdot \left\{ T \operatorname{grad} \frac{\mu_k}{T} - z_k \vec{E}' \right\}$$
$$- \frac{1}{T} \bar{\bar{\Pi}} : \operatorname{grad} \vec{v} - \frac{\rho}{T} \frac{dp'}{dt} \cdot \left(\vec{E}_{eq} - E'\right) - \frac{\rho}{T} \frac{dm'}{dt} \cdot \left(\vec{B}_{eq} - \vec{B}'\right)$$

where $\tilde{\bar{\Pi}} = \bar{\bar{P}} - p\bar{\bar{u}}$ is the viscous pressure tensor , then from this last expression , the entropy flux is given by:

3.35)

$$\vec{J}_s = \frac{1}{T}\left(J_q - \sum_{k=1}^{n} \mu_k J_k \right)$$

and the entropy source strength is given by:

3.36)

$$\sigma = \frac{1}{T^2} \vec{J}_q \cdot \operatorname{grad} T \quad \frac{1}{T} \sum_{k=1}^{n} \vec{J}_k \cdot \left\{ T \operatorname{grad} \frac{\mu_k}{T} - z_k \vec{E}' \right\} - \frac{\tilde{\bar{\Pi}}}{T} : \operatorname{grad} \vec{v}$$
$$- \frac{\rho}{T} \frac{d\vec{p}'}{dt} \cdot \left(\vec{E}'_{eq} - \vec{E}' \right) - \frac{\rho}{T} \frac{d\vec{m}'}{dt} \cdot \left(\vec{B}_{eq} - \vec{B} \right)$$

As a difference with other systems , in a polarizable one ,in the expression that describes the entropy source , two terms appear ,which(8) are related to that phenomenon. Let us consider then separately this phenomenon , analizing just the case of electric relaxation , since in a plasma the magnetic case is irrelevant. From 3.36) , we have the expression:

3.37). $$\sigma_p = \frac{\rho}{T}\frac{d\vec{p}'}{dt}\cdot\left(\vec{E}_{eq} - \vec{E}\right) \geq 0$$

For the term that describes the entropy source associated with the polarization in a system at rest , $\vec{v}= 0$. Then we get:

3.38). $$\sigma_p = \frac{1}{T}\frac{\partial\vec{P}}{\partial t}\cdot\left(\vec{E}_{eq} - \vec{E}\right)$$

Now , if we consider that 3.38) satisfies the conditions established in (2.5) , the linear phenomenological law associated with 3.38) is:

3.39). $$\frac{\partial\vec{P}}{\partial t} = \frac{L}{T}\cdot\left(\vec{E}_{eq} - \vec{E}\right)$$

where L is the phenomenological coefficient connected to relaxation phenomena. An integral of 3.39) is:

3.40). $$\vec{P}(t) = \varkappa\vec{E}\left(1 - e^{\frac{t}{\tau}}\right)$$

For an isotropic system ,where the equation 3.39) becomes $\bar{\bar{\varkappa}} = \hat{U}K$ we have:

3.41) $$\frac{\partial\vec{P}}{\partial t} = \frac{L}{T\kappa}\left(\vec{P} - \varkappa\vec{E}\right)$$

There the relaxation time is given by:

3.42). $$\tau = \frac{\varkappa T}{L}$$

For time dependent fields $\vec{E}$ (t) , a Fourier expansion integral for $\vec{E}$ (t) and $\vec{P}$ (t) gives the relation:

3.43). $$\vec{P}(\omega) = \hat{\varkappa}(\omega)\vec{E}(\omega)$$

where $$\hat{\kappa}(\omega) = \frac{\varkappa}{1 - i\omega\tau}$$

and where :

3.44)
$$K'(\omega) = \frac{\varkappa}{1+\omega^2\tau^2} \qquad K''(\omega) = \frac{\omega\tau K}{1+\omega^2\tau^2}$$

satisfying the Kramers-Kroning relations.

From the last considerations it is clear that only when $\omega = 0$ we have that $\hat{K}(0) = K$.(statistic equilibrium). Only in this case the value of the expressions(I;4.14.i) for the energy or(I;4.14.ii)for the stress tensor have a clear thermodynamical meaning .When the relaxation phenomena are present , the expression I;4.3 (the Helmholtz force) can be considered valid for a frequency which is small compared with the frequencies that characterized the set up of the electric and magnetic polarization of the material. If the frequency is such that allows the dispersion to appear, we have then to consider another approach in order to study the phenomenon of relaxation. (9). Since we cannot define the usual thermodynamic functions in the classical sense , a possible approach could be the statistical one , using for example the fluctuation dissipation theorem, that relates the correlation function of spontaneous fluctuations for the stationary process $\mathcal{L}(t)$, to the relaxation function $\varkappa(t)$ which contains the susceptibility matrix , and it is related to the dissipation (or entropy production) of the system under the influence of time dependent driving forces and will give us also a criterium to establish local equilibrium (11) . Also it gives a precise meaning to the concept of relaxation time.

Without going into all the details of the kinetic desciption, the principal features of the dispersion phenomenon will be discussed in the following paragraphs.

4.- DISPERSION RELATIONS

In our discussion in subsections 1 , 2 , and 3 , the medium has been assumed to be non-dispersive .In that case , without taking into account absorption , all the terms in the balance equations can be interpreted unambigously (3) , so as it was established in equations 3.33), in subsection 3), for a linear medium at rest , non changing in time , non-absorptive , non-dispersive and isotropic .In such case the constitutive equations are:

4.1)
$$\vec{D} = \hat{\varepsilon}\vec{E} \qquad \hat{\varepsilon} = \hat{\mathcal{U}} - \hat{\varkappa}$$
$$\vec{H} = \hat{\mu}^{-1}\vec{B} \qquad \hat{\mu} = \mathcal{U} - \hat{\chi}$$

In this case $\hat{\varepsilon}$, $\hat{\mu}$ are real quantities and under the assumption of local equilibrium , they depend only on the thermodynamics variables .Considering 4.1) , we get the Poynting theorem:

4.2).
$$\vec{E}\cdot\frac{\partial}{\partial t}\left(\hat{\varepsilon}\cdot\vec{E}\right) + \vec{H}\cdot\frac{\partial}{\partial t}\left(\hat{\mu}\cdot\vec{H}\right) = -\operatorname{div} c\left(\vec{E}\times\vec{H}\right) - \vec{I}\cdot\vec{E}$$

But since $\hat{\varepsilon}$ and $\hat{\mu}$ are depending only on the thermodynamical variables , 4.2 takes the form:

4.3).
$$\frac{\partial}{\partial t}\left(\varepsilon\vec{E}^{2} + \mu\vec{H}^{2}\right) + \operatorname{div} c\left(\vec{E}\times\vec{H}\right) + \vec{I}\cdot\vec{E} = 0$$

In this case , with the restrictions we have already mentioned , the quantity:

4.4)
$$\mathcal{U} = \left(\varepsilon\vec{E}^{2} + \mu\vec{H}^{2}\right)$$

is inmediately identified with the energy density , and the vector given by:

4.5)
$$S = c\left(\vec{E}\times\vec{H}\right)$$

is the total energy flux.

Now , if we consider electromagnetic fields whose frequencies are not small compared with the frequencies that are characterizing the set up of electric and magnetic polarizations , then expression 4.4). will have not a clear meaning . In the general case , in presence of disper - sion , we cannot define $\mathcal{U}$ as the thermodynamic value of the energy . This conclusion comes from the fact that the dispersion is always given simultaneously with the dissipation of the energy . This means that a dispersive medium is at the same time an absorptive one . (11). Thinking in terms of a plasma , even a collisionless one will have to some degree Landau damping , such that a certain amount of dissipation will always occur.

The question of the energy relations in a dispersive medium has been considered in several monographs , nevertheless , in considering the question of energy relations in an absorptive medium , we can say that the proposed answers are insufficiently clear, as indicated for example by the appearance of several articles in the current literature, so we consider necessary to discuss the subject again in order to find the connection with the fluctuation theory , which it will be used in the next sections.

From the Maxwell's equations , we arrived before to the Poynting theorem :

4.6). $$\vec{E}\cdot\frac{\partial}{\partial t}\left(\hat{\varepsilon}\circ\vec{E}\right)+H\cdot\frac{\partial}{\partial t}\left(\vec{\mu}\circ\vec{H}\right)=-\operatorname{div} c\left(\vec{E}\times\vec{H}\right)-I\cdot\vec{E}$$

where:

4.7). $$\vec{D}=\hat{\varepsilon}\circ\vec{E}$$

and

4.8) $$\vec{H}=\hat{\mu}^{-1}\cdot\vec{B}$$

Here $\hat{\varepsilon}$ and $\hat{\mu}$ depend on the temperature and density in the case of local thermodynamic equilibrium. The Poynting theorem (4.2) remains valid in a dispersive medium , but in this case , for a weak field (that is the case it will be considered first) , $\vec{D}$ is a linear functional of $\vec{E}$, and we can write:

4.9)
$$D_i(\vec{r},t) = \int_0^t d\tau_1 \int d^3 r_0 \, \varepsilon_{ij}(\tau_1, \vec{r}_0, \vec{r}, t) \, E_j(\tau_1, \vec{r}_0)$$

Looking at this equation , we can see that the value of $\vec{D}(\vec{r},t)$ is determined by the field $\vec{E}(r_0,\tau_1)$ at times $\tau_1 \leq t$ and for points $\vec{r}_0$ situated in a neighborhood of $\vec{r}$. The generalizations about μ are the same ,but considering that $\mu \simeq 1$, even below the optical frequencies , we shall consider only ε for which a macroscopic description is possible , in a certain range of frequencies ω_i for which :

$$\lambda_i \sim \frac{c}{\omega_i} \gg a$$

where a is the atomic dimension.

For a medium homogeneous and not changing in time , let us consider the Fourier transform :

4.10)
$$E_i(\vec{k},\omega) = \frac{1}{(2\pi)^4} \int E_i(\vec{r},t) \, e^{-i(\vec{k}\cdot\vec{r} - \omega t)} \, d^3 r \, dt$$

and similarly for D we get from equation 4.9). the relationship:

4.11).
$$D_i(\vec{k},\omega) = \varepsilon_{ij}(\omega,\vec{k}) \, E_j(\omega,\vec{k})$$

The dependance of ε_{ij} in ω is related with the frequency dispersion and the dependance on $\vec{k}$ correspond to spatial dispersion but since we are considering a wavelegth $\lambda_{i} \gg a$, we will consider in the following only frequency dispersion due to the fact that the field strength may be looked upon as a slowly varying function of the coordinates since the field strength varies only a little over a distance equal to a. For example , for an electromagnetic ligth wave in a dielectric we have:

$$\omega \sim 10^{15}\,[sec^{-1}] \quad , \lambda \sim 5\times10^{-5}\,[cm] \quad a \sim 10^{-8}\,[cm]$$

Now , in order to explain the basic features of the problem of dispersion and absorption of electromagnetic waves in a dielectric medium , we will consider a very general model .It will be assumed that the matter consists of atoms having one electron each.If the electrons are displaced from the equilibrium position , an elastic force appears which is proportional to the displacement, also it will be considered that the main mass of the atom is connected with its positively charged part. When a monochromatic electromagnetic wave is passing through the dielectric medium, an alternating electric field appears in each point , given by the expression:

4.12)

$$\vec{E}(t) = \vec{E}_0\, e^{-i\omega t}$$

as a result of the field variation , the electrons have a periodic motion and become itself a source of electromagnetic waves , which leads to the electron losing energy, that is , it will be considered that when an electron moves , it is acted upon by a frictional force, proportional to the electron velocity . Then , for a medium consisting of a set of oscillators with masses m_γ , free eigen - frequencies ω_k and friction coefficients γ_k with the mentioned properties , we will have the equation of motion:

4.13)

$$\ddot{\vec{r}}_k + \gamma_k \dot{\vec{r}}_k + \omega_k^2 \vec{r}_k = \frac{e_k}{m_k}\vec{E}(t)$$

where e_k is the charge and $e_k \vec{r}_k$ is the dipole moment of the osci -

llator , $\vec{E}$ (t) is identified with the averaged macroscopic field , for a plasma $\omega_k = 0$ .Anyway this assumption is completely justified.

n_k it will be used to denote the concentration of the oscillators of the type k , then the polarization is given by:

$$\vec{P} = \sum_k e_k \vec{\Pi}_k n_k$$

now , from 4.13). , we get:

4.14).
$$\ddot{\vec{P}}_k + \gamma_k \dot{\vec{P}} + \omega_k^2 P_k = \frac{e_k^2 N_k}{m_k} \vec{E}(t)$$

the solution of 4.14) is given by:

4.15).
$$\vec{P}_k(t) = \vec{P}_k^0(t) + \vec{P}_k^{(E)}(t)$$

where :

4.16).
$$\vec{P}_k^0(t) = e^{-\gamma_k t}\left[\vec{A}_1 \cos\omega_1 t + \vec{A}_2 \operatorname{sen}\omega_1 t\right]$$

and: $\omega_1^2 = (\omega_k^2 - \gamma_k^2)$

This part of the solution does not depend on $\vec{E}(t)$ and the constants A_1 and A_2 are determined by the initial values of the polarization vector and its derivative. $\vec{P}_k^{(E)}(t)$ is given on the other hand by:

4.17)
$$\vec{P}_k^{(E)}(t) = \frac{e_k^2 n_k}{m_k}\int_0^t e^{-\gamma_k(t-t')} \sin\omega_1(t-t')\,\vec{E}(t')\,dt'$$

As we can see , there is no dependance on the initial data.

If we write: $\vec{D} = \vec{E} + 4\pi\vec{P}$

for the induction vector $\vec{D}$, in order to intoduce later the plasma frequency, we get from 4.16) and 4.17) :

4.18).

$$\vec{D}(t) = \vec{E}(t) + \sum_k \left\{ \frac{4\pi e^2 m_k}{m_k} \int_0^t e^{-\nu_k (t-t')} \sin \omega_\perp (t-t') \vec{E}(t') dt' + 4\pi \vec{P}_k^{\circ}(t) \right\}$$

defining:

4.19).

$$f(t-t') = \frac{4\pi e^2 m_k}{m_k} e^{\nu_k (t-t')} \sin \omega_\perp (t-t')$$

we get:

4.20).

$$\vec{D}(t) = \vec{E}(t) + \sum_k \left[\int_0^t f(t-t') \vec{E}(t') dt' + 4\pi P_k^{\circ}(t) \right]$$

If $(t - t_\circ)$, (where $t_\circ$ is the initial time), satisying:

$$(t - t_\circ) \gg \frac{1}{\nu_k}$$

where ν_k^{-1} is the time required for the establishement of steady conditions . We can consider the asymptotic expression of 4.16) with the initial time approach to $\rightarrow -\infty$, then:

4.21).

$$\vec{D}(t) = \vec{E}(t) + \sum_k \int_{-\infty}^t f(t-t') \vec{E}(t') dt'$$

which is giving the connection between $\vec{D}$ and $\vec{E}$ for a steady process.

Now as it is customary , it will be defined:

4.22)

$$\varepsilon(t-t') = \delta(t-t') + f(t-t')$$

and then , we get:

4.23).

$$\vec{D}(t) = \int_{-\infty}^{t} \varepsilon(t-t')\vec{E}(t')dt'$$

where for simplicity , the plasma is assumed to be a one-compo - nent one and the index k is omitted.

The Fourier transform of 4.23) will give as before:

4.24)

$$D(\omega) = \varepsilon(\omega) E(\omega)$$

and

4.25).

$$\varepsilon(\omega) = 1 + \int_{0}^{\infty} f(t) e^{i\omega t} dt = 1 + f(\omega)$$

In this equation $\varepsilon(t-t')$ is the dielectric constant , and $\varepsilon(\omega)$, the Fourier component of the dielectric constant.

In general $\varepsilon(\omega)$ is complex :

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$$

and from 4.21) , we get the relationships:

4.26)

$$\varepsilon(-\omega) = \varepsilon^{*}(\omega)$$

$$\varepsilon'(\omega) = \varepsilon'(-\omega)$$

$$\varepsilon''(\omega) = -\varepsilon''(-\omega)$$

so $\varepsilon'(\omega)$ is an even function of ω , and $\varepsilon''(\omega)$ is an odd one.

From the relations 4.2) , we are considering that in linear elec - trodynamics the loss density $\overline{Q}$ is given by:

$$\overline{Q} = \frac{\omega\varepsilon''}{8\pi}|\vec{E}_0^2|$$

Some authors have been trying to get a general criterium for the determination of the expression for the energy density and evolved heat in electrodynamics , for example quoting the work of Pekar (12), we have:

4.27).
$$\left(\frac{d\mathcal{U}}{dt}\right) = \frac{1}{4\pi}\frac{\partial}{\partial t}\left(\vec{D}_{even}\cdot\vec{E}\right) + \frac{1}{4\pi}\frac{\partial}{\partial t}\left(\vec{B}\cdot\vec{B}\right)$$

4.28)
$$\mathcal{Q} = \frac{1}{4\pi}\frac{\partial D_{odd}}{\partial t}\cdot E$$

where $\mathcal{U}$ is the energy density , and Q is the loss density and:

4.29)
$$\vec{D}_{even} = \sum_p \varepsilon_{2p}\frac{d^{2p}}{dt^{2p}}\vec{E}(t) \qquad \vec{D}_{odd} = \sum_p \varepsilon_{2p+1}\frac{d^{2p+1}}{dt^{2p+1}}E(t)$$

$$\mathcal{E}(\omega) = \sum_n \varepsilon_n(-i\omega)^n$$

But this method, as it has been discussed in the literature from a different point of view , is erroneous because of general considerations and also from particular applications. (9).

In relation with this problem , we think that it needs a more precise investigation , since it is related with the very basics of non- equilibrium Physics . In order to clarify this view , we shall write some expressions of the linear response theory .(13). It is well known that the relation:

4.30).
$$\langle A\rangle = \langle A\rangle_0 + \int_{-\infty}^{t}\frac{1}{i\hbar}\langle[A(t), H^1_{t'}(t')]\rangle_0\, dt'$$

describes the retarded response of the average values of an operator A to the switching on of a perturbation $H^1_{t'}(t')$, for a quantum statistical ensemble .The classical equations have the same form , except that the quantic Poisson bracket is replaced by a classical Poisson bracket , in 4.30).

In the Heinssenberg picture , the operator A(t) is given by:

4.31)

$$A(t) = e^{\frac{iHt}{\hbar}} A e^{-\frac{iHt}{\hbar}}$$

and: $$\langle \ldots \rangle_0 = Tr[\rho_0 \ldots]$$

represents an average with the equilibrium statistical operator, since the perturbation $H_t^1(p,q)$ can often be represented in the form:

$$H_t^1 = -\sum_j B_j(p,q) F_j(t)$$

Here $F_j(t)$ are representing the externally driven forces, which are functions of time , and $B_j(p,q)$ are the dynamic variables conjugate to the fields and are not explicily time dependent. The connection between the driving force F(t) and the response $\Delta A(t)$ of the system is given by the linear integral relation:

4.32)

$$\Delta A = \langle A \rangle - \langle A \rangle_0 = \int_{-\infty}^{t} L(t-t') F(t') dt'$$

Now , the symmetric relations 4.26) are valid for:

4.33)

$$L(t-t') = -\langle\langle A(t) B(t') \rangle\rangle$$

which are related to the non-thermodynamic fluctuations of the quantity A in the presence of an external field and therefore they are related to the "Onsager principle of Kinetic coefficients", but it has nothing to do with the time symmetry of the electromagnetic quantities $\vec{E}$ and $\vec{D}$. These are macroscopic expressions obtained either by the traditional method of Lorenz or by the method of phased averaged procedures , assuming in both cases that the fluctuations of the quantities are irre - levant.

Now , coming back to the expression :

$$\varepsilon(\omega) = 1 + \int_0^\infty f(t)\, e^{-i\omega t} dt$$

and using the theory of complex variables , it is proved , as it has been done elsewhere (6), that $\varepsilon'(\omega)$, $\varepsilon''(\omega)$ satisfy the relations:

4.33).

$$\varepsilon'(\omega') - 1 = \frac{1}{\pi} P \int \frac{\varepsilon''\mu}{\mu - \omega'} d\mu \;;\quad \varepsilon''(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\varepsilon'\mu - 1}{\mu - \omega'} d\mu$$

where the sign P indicates that we are taking the principal va - lue of the integral.

Considering now again the case of the harmonic oscillators , substi- tuting the expression (4.19) in (4.25) and integrating with respect to t , we find for $\omega' = 0$ and $\omega = \omega'$ that:

4.34).

$$\varepsilon(\omega) = 1 - \sum_k \frac{\omega_{(p)k}^2}{(\omega^2 - \omega_k^2) + i\nu_k\omega} \;;\quad \omega_{(p)k} = \frac{4\pi e_k^2 n_k}{m_k}$$

For a one component plasma (we do not consider the question of the background of positive ions maintaining the quasi-neutrality of the medium) we get:

4.35)

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\nu} \;;\quad \omega_p^2 = \frac{4\pi e^2 n}{m}$$

4.36).

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} \;;\quad \varepsilon''(\omega) = \left(\frac{\nu}{\omega}\right)\frac{\omega_p^2}{\omega^2 + \rho^2}$$

which are the same expressions for the "dielectric constant " of the plasma as they are obtained from the macroscopic two fluids theory of the plasma , for example . (14).

In equations 4.35) and 4.36) , the quantity associated with non-equilibrium properties is the friction coefficient ν (14). In the ordinary hydrodynamical theory of plasma , ν is assumed to be given as collision frequency and it is responsible for all the absorption processes in the plasma. In order to clarify its meaning , we will consider first the spontaneous electric moment appearing in our system in thermodynamic equilibrium , due to the fluctuation in the position and the movement of the charges.

From the equation (4.14) , considering just the one component model , we get:

4 .37)

$$\ddot{P} + \nu \mathring{P} + \omega_0^2 P = \frac{e^2 n}{m} y(t)$$

where $y(t)$ is a random force , and ω_0 is the free eigen frequency.

If we consider 4 .37) simultaneously with the system:

4.38).

$$\mathring{P} = -(0)\left(\frac{m\omega_0^2 P}{}\right) - (-T)\left(\frac{p}{mT}\right)$$

$$\mathring{p} = -(T)\left(\frac{m\omega_0^2 P}{T}\right) - (\nu T)\left(\frac{p}{mT}\right)$$

where : $\dot{P} = emr$
$p = em\dot{r}$

are given in 4.38) by the usual canonical equations for a damped oscillator ; that way of writing 4.38) makes more transparent the symmetry of the kinetic coefficients . Now conside - ring the Linear phenomenological law (2.5) of this chapter , written in tha general form:

4.39)

$$\dot{x}_i = - \gamma_{ik} X_k$$

we see that the rol of the fluxes is given in equations 4.38) by $\dot{P}$ and $\dot{p}$ and the corresponding generalized forces are given by:

and $$\left(\frac{m\omega_0^2 P}{T}\right) ; \left(\frac{p}{mT}\right)$$

where the temperature T appears from the relation:

$$\Delta S = - \frac{R_{min}}{T}$$

where ΔS is the variation of the entropy in the fluctuations, and R_{min} is the minimun work necessary to produce a reversible chan - ge in the thermodynamic parameters . (15).

In this formulation γ_{ik} are playing the role of the Onsager coeffi - cients, then we can write:

$$\gamma_{11} = 0 \quad , \gamma_{12} = -\gamma_{21} = -T \quad , \gamma_{22} = \nu T$$

So , the friction coefficient in this elementary formulation as in the general case , is an Onsager coefficient , closely related to the fluc- tuations in the system. From 4.37) it is possible to prove that for the Fourier component of the fluctuations of P , we have the expression:

4.40)

$$(P^2)_\omega = \frac{\nu T e^2 n m^{-1}}{\pi\left[(\omega^2 - \omega_0^2) + \omega^2\nu^2\right]}$$

and the expression:

4.41).

$$(P^2)_\omega = \frac{T}{\pi}\,\varepsilon''(\omega)$$

This result agrees with the expression for $(P^2)_\omega$ obtained in the quantum mechanical case.

This relationship between ν and the fluctuations in linear i - rreversible thermodynamics , will be deduced now using the theory of li - near response.

As we are also interested in comparing the different macroscopic descriptions of irreversible phenomena in plasma , we will briefly discuss the conclusions reached by E. Schmutzer and B. Wilhelmi (16).Reading these authors it can be seen that the connection between and fluc - tuations appears very transparent in a general way.

A medium consisting of N different kinds of particles , for example a plasma moving under the influence of arbitrary external gravitational and electromagnetic fields , is considered , without taking into account boundary effects and irreversible cross effects as thermoelectricity or others. The equation of motion for a particle of the kind k is given as:

4.42)

$${}^{(k)}m\left[\ddot{\vec{r}} + \mathrm{grad}\,\phi + {}^{(k)}\nu\left({}^{(k)}\dot{\vec{r}} - {}^{(k)}\vec{v}\right)\right] = {}^{(k)}e\,{}^{(k)}F_L$$

where ${}^{(k)}e$ is the electric charge , ${}^{(k)}m$ is the mass , ${}^{(k)}\dot{\vec{r}}$ is the re - lative velocity of the particles in relation with the medium , ${}^{(k)}\nu$ is the atomistic friction coefficient , $\vec{v}$ is the bulk velocity of the medium, ${}^{(k)}\vec{F}_L$ is the Lorentz force given by :

4.43)

$${}^{(k)}F_L = \vec{E} + \frac{1}{c}\,{}^{(k)}\dot{\vec{r}} \times \vec{B}$$

where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field , and ϕ is a Newtonian gravitational potential.

Then we introduce the coefficients:

a).-

$$^{(K)}\sigma_0 = \frac{1}{1 + \frac{^{(K)}e^2 \vec{B}^2}{c^2\, ^{(K)}\gamma^2}}$$

b).-

$$^{(K)}\sigma_1 = \frac{^{(K)}m\, ^{(K)}\sigma_0}{^{(K)}\gamma}$$

c).-

$$^{(K)}\sigma_2 = \frac{^{(K)}m\, ^{(K)}e\, ^{(K)}\sigma_0}{c\, ^{(K)}\gamma^2}$$

d).-

$$^{(K)}\sigma_3 = \frac{^{(K)}e^2\, ^{(K)}m\, ^{(K)}\sigma_0}{c^2\, ^{(K)}\gamma^3}$$

e).-

$$^{(K)}\sigma_4 = \frac{^{(K)}e\, ^{(K)}\sigma_0}{^{(K)}\gamma}$$

f).-

$$^{(K)}\sigma_5 = \frac{^{(K)}e^2\, ^{(K)}\sigma_0}{c\, ^{(K)}\gamma^2}$$

g).-

$$^{(K)}\sigma_6 = \frac{^{(K)}e^3\, ^{(K)}\sigma_0}{c^2\, ^{(K)}\gamma^3}$$

(The meaning of this coefficients appears in a clear way when values are given to the variables they depend on .This coefficients were introduced by Schmutzer (16)).

With the coefficients and equation 4.42), we get:

4.44)

$$ {}^{(k)}\vec{v} + {}^{(k)}\sigma_1 {}^{(k)}\dot{\vec{v}} + {}^{(k)}\sigma_2 {}^{(k)}\dot{\vec{v}} \times \vec{B} + {}^{(k)}\sigma_3 \vec{B}\left[{}^{(k)}\dot{\vec{v}} \cdot B\right] = {}^{(k)}\sigma_4 \left[1 + \frac{1}{c} \vec{v} \times \vec{B} - {}^{(k)}\sigma_1 \vec{a} \right. $$

$$ \left. {}^{(k)}\sigma_6 \vec{B}\left[\vec{E}\vec{B} - \frac{{}^{(k)}m}{{}^{(k)}e} \vec{a}\,\vec{B} \right] + {}^{(k)}\sigma_5 \left[\vec{E}\times\vec{B} - \frac{1}{c}\vec{B}\times(\vec{v}\times\vec{B})\right] - {}^{(k)}\sigma_2 \vec{a}\times\vec{B} \right] \equiv {}^{(k)}\vec{F} $$

where : $\vec{a} = \dot{\vec{v}} + \operatorname{grad}\phi$

is the bulk acceleration of the medium.

The equation 4.44) is then applied to the optical case for a medium at rest , in the presence of an external electromagnetic field. Then in 4.44) the following restrictions are imposed :

$$ \vec{v} = 0 \qquad \phi = 0 $$

$$ \omega \gtrsim {}^{(k)}\Gamma $$

where ω is the angular frequency of the applied fields , written as:

$$ \vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t) \quad , \quad \vec{B} = \vec{B}_0 \cos(\vec{k}\cdot\vec{r} - \omega t) $$

and

$$ {}^{(k)}\Gamma = \frac{1}{{}^{(k)}\sigma_1} = \frac{{}^{(k)}\nu}{{}^{(k)}m} \; \frac{1}{{}^{(k)}\sigma_0} $$

is related to the collision frequency.

As it is well known for the linear polarized and monocromatic wave crossing the medium , we have the linear relationships:

4.45 . a.-

$$ \vec{B}_0 = \frac{c}{\omega} \vec{k} \times \vec{E}_0 $$

b).-

$$\vec{E}\cdot\hat{k} = \vec{B}\circ\hat{k} = 0$$

c).-

$$\vec{E}\circ\vec{B} = 0$$

d).-

$$k^2 = \frac{\varepsilon\mu}{c^2}\omega^2 = \frac{\omega^2}{c_{ph}^2}$$

e).-

$$\vec{S} = \frac{c^2 E_0^2}{8\pi\mu\omega}\hat{k}\left[1+\cos 2(\vec{k}\cdot\vec{r}-\omega t)\right]$$

where $\vec{S}$ is the Poynting vector .

It is necessary to recall that the equations:

4.46) . $\vec{D} = \hat{\varepsilon}\vec{E}$; $\vec{B} = \hat{\mu}\vec{H}$

are being used.

These equations (the constitutive equations) that in view of our dis - cussion are valid only in a very special case , are certainly not valid here , where we have $\frac{\omega}{^{(k)}\gamma} \approx 100$; nevertheless the other considerations in Schmutzer 's article are very useful , so following with it , we have, taking into account the restrictions 4.44') , the equation of motion for the particle of the kind (k) becomes:

4.47). $${}^{(k)}m^{(k)}\dot{\vec{v}} + {}^{(k)}\gamma^{(k)}\vec{v} = {}^{(k)}e^{(k)}\vec{F}_L$$

which is the equivalent of 4.10). for the case of a plasma .

The solution of 4.47) , is given after the descomposition:

4.48).

$$ {}^{(k)}\vec{v} = \hat{e}_1 {}^{(k)}v_1 + \hat{e}_2 {}^{(k)}v_2 + \hat{e}_3 {}^{(k)}v_3 $$

where :

$$ \hat{e}_1 = \frac{\vec{k}}{|\vec{k}|} , \quad \hat{e}_2 = \frac{\vec{E}}{|\vec{E}|} , \quad \hat{e}_3 = \hat{e}_1 \times \hat{e}_2 $$

then , putting 4.48) into 4.47) , we get the system:

4.49)

$$ \boxed{\begin{aligned} {}^{(k)}m\, {}^{(k)}\dot{v}_1 + {}^{(k)}\nu\, {}^{(k)}v_1 &= \frac{{}^{(k)}e}{c}\, {}^{(k)}v_3\, B \\ {}^{(k)}m\, {}^{(k)}\dot{v}_2 + {}^{(k)}\nu\, {}^{(k)}v_2 &= {}^{(k)}e\, E - \frac{{}^{(k)}e}{c}\, {}^{(k)}v_1\, B \\ {}^{(k)}m\, {}^{(k)}\dot{v}_3 + {}^{(k)}\nu\, {}^{(k)}v_3 &= 0 \end{aligned}} $$

The last equation in 4.49) gives :

$$ {}^{(k)}v_3 = {}^{(k)}v_3\, e^{-\frac{{}^{(k)}\nu}{{}^{(k)}m} t} $$

and because as $t \to \infty$, ${}^{(k)}v_3 \to 0$,this equation is not taken into account any further . Solving the system for ${}^{(k)}v_1$, ${}^{(k)}v_2$ and assuming that:

$$ \sigma_0 = \frac{1}{1 + \frac{{}^{(k)}e^2 B^2}{c^2\, {}^{(k)}\nu^2}} \approx 1 $$

we get :

4.50).

$$ {}^{(k)}\langle \overline{v_1} \rangle = \frac{E_0 B_0\, {}^{(k)}\sigma_0}{2\, {}^{(k)}\sigma_1^2 ({}^{(k)}\Gamma^2 + \omega^2)} $$

$$ {}^{(k)}\langle \overline{v_2} \rangle = 0 $$

Here the symbol $\overline{<>}$ stands for time average. Then considering 4.50) the solution of 4.49 is given by:

4.51).

$$\overline{<{}^{(K)}\vec{v}>} = \frac{4\pi e^2}{c^{2\,(K)}m^2}\ \frac{1}{{}^{(K)}\Gamma^2 + \omega^2}\ \hat{e}_1$$

With this value of ${}^{(K)}\vec{v}$, the expression for the average electric current in the direction of $\vec{S}$:

$$\overline{<\vec{J}>} = \sum_K {}^{(K)}e^{(K)}N^{(K)}v$$

is given by:

4.52).

$$\overline{<\vec{J}>} = \frac{4\pi}{c^2}\sum_K \frac{{}^{(K)}N^{(K)}e^3}{{}^{(K)}\nu^2\left(1 + \frac{\omega^2}{{}^{(K)}\Gamma^2}\right)}$$

and in the same way, for the diffusion current:

4.53).

$$\overline{<{}^{(K)}\vec{j}>} = {}^{(K)}N^{(K)}v = \frac{4\pi\,{}^{(K)}N^{(K)}e^2}{c^{2\,(K)}\nu^2\left(1 + \frac{\omega^2}{{}^{(K)}\Gamma^2}\right)}$$

where ${}^{(K)}N$ is the particle density. If we call:

$${}^{(K)}\hat{R}(\omega) = \frac{4\pi\,{}^{(K)}N^{(K)}e^2}{c^{2\,(K)}\nu^2\left(1 + \frac{\omega^2}{{}^{(K)}\Gamma^2}\right)}$$

then we have :

$$\overline{<{}^{(K)}j>} = {}^{(K)}R(\omega)\,\hat{e}_1$$

$$\overline{<\vec{J}>} = \sum {}^{(K)}J^{(K)}e\,\hat{e}_1$$

As we can see the relationship between the expressions 4.52) and 4.53) , and the equations 4.35) and 4.36) of the first part , is clear and is based in a function ${}^{(k)}\hat{R}(\omega)$ which is connected with fluctuations . Also we would like to mention that equation 4.51) was used by Schmutzer and Wilhelmi in order to explain the apparition of a strong magnetic field . The average current $\overset{\frown}{J}$ will be the cause of this strong field (17) On the other hand Schmutzer has considered also a general relativistic equation of motion for a continuum , introducing in it the irreversible effect , but this equation is only applied to a case of weak external fields , that is , the expression for the transport coefficients is studied considering the dynamical constrains imposed over the system. Then as it has already been shown , in equations 4.52) and 4.53) , the relation between the electric and diffusion current is established :

4.54).
$$\overline{\langle \vec{J} \rangle} = \sum_{k} {}^{(k)}e \, \overline{\langle {}^{(k)}\vec{J} \rangle}$$

This relationship is not surprising but it corresponds to the group of general relations existing between the response of the system to a mechanical perturbation and some of the coefficients appearing in the transport equations (18), or in a more precise way , between the Onsager coefficients and the response of the system.If we consider the equation (4.14) for the harmonic oscillator or the equation (4.42) for the k- particles , as a Langevin equation , and on the other hand the expre - ssions (4.52) and (4.53) , it can be proved that they are related to the linear susceptibility matrix . This connection is summarized in the equation:

$$\mu(\omega) = \frac{1}{m(i\omega + \nu)}$$

This relationship is easily proved , taking into account the expression (4.53) for $\vec{J}$, relating electric response and diffusion.

At this stage , we come back to our question of entropy production, since in order to get an expression for the entropy production when dispersive processes are present , it is necessary to know the susceptibility matrix. In the case of a "monocromatic " driven field , oscillating with a frequency ω_0 , we can write for the field:

4.55). $$\vec{F}(t) = \frac{1}{2}\left[F(t)\, e^{-i\omega_0 t} + F^*(t)\, e^{i\omega_0 t}\right]$$

In a period $2\pi/\omega_0$ the entropy production in the system will be given by:

4.56). $$\int_{-\frac{2\pi}{\omega_0}}^{\frac{2\pi}{\omega_0}} \frac{dS}{dt} = \frac{i\pi}{2T}\, \vec{F} \left\{ \hat{R}^*(\omega_0) - \hat{R}(\omega) \right\} \vec{F}^*$$

where $\hat{R}(\omega)$ is the susceptibility for the process. (15).

The details of this deduction and an evolution of the entropy production will be discussed in the next chapter , since they are not just based in linear phenomenological equations as in the case of irreversible thermodynamics , but in a statistical description of irreversibility.

CHAPTER 3

FLUCTUATIONS

1.- STATISTICAL METHODS

In the precedings chapters we analized different expressions for the forces acting on the plasma and considered some macroscopic descriptions of the absorption processes in it . In particular we compared the expressions for the dielectric constant from the two fluids theory with the expressions based on considerations of the plasma as a set of osci - llators . Also it was mentioned the relationship of relevant physical magnitudes with fluctuations . In this chapter the same questions and problems studied in the other chapters are going to be considered , but in the frame of statistical physics.

In non- equilibrium physics , a theoretical investigation of the electromagnetic properties of the plasma can be done , setting up the so called kinetic equation or transport equation for the particle distribution function . This is the traditional microscopic theory of transport coefficients ; in this theory the kinetic equation describes the long time behaviour of the system , and is then solved for stationary or periodic conditions . It is necessary however to have in mind that the kinetic equation is itself an approximation and cannot be derived without some restrictive conditions (1), these restrictions introduce some difficulties which arise when the method of kinetic equations is applied to the description of plasmas. The more important difficulties are (2):

a).-A rigorous molecular definition of a thermal constraint, in other words - what is the form of an effective Hamiltonian which describes the thermodynamical flows? - . The problem does not exist for dynamical constraints , which can be represented adding a perturbation term in the Hamiltonian.

b).-The correlation part of the transport coefficients: this problem is connected with the fact that in general we have to consider two contributions to the transport coefficients ; one is the contribution of the velocity distribution and the other is the contribution of the correlation function. For example for the case of a plasma in an external field , we have to take into account when discussing the conductivity that the field detected by the particles is not really the external field , but an effective field , including polarization effects which are due to intermolecular correlations (3).

In general also the heat flow is defined more completely as the transport of kinetic energy and potential energy , the last one connected with the correlation function.This concept cannot be discussed with a kinetic equation which only gives the velocity distribution in the stationary state .

Some other problems have already been mentioned in the discussion about plasma parameters , that is the problem of a strong field , and the dependance of the fields on time . In the study of the behaviour of a plasma in a oscillating electromagnetic field of very high frequency $\omega \approx \omega_p$ the long time approximation $t \gg \omega_p^{-1}$ wipes out all the details of the evolution processes. In order to consider these details we have to introduce a kinetic equation not just valid for the "long time approximation", but valid as well on the whole range of time associated with this generalization , then we have to consider a non -Markoffian equation valid for short times (4).

On the other hand , there is a method from statistical mechanics which allows us to consider the non-equilibrium processes in a system with imposed mechanical constraints , without some of the restrictions of the kinetic equation (5) , that is , the "Theory of Linear Response of Classical and Quantum systems to Mechanical Perturbations " . Eventhough the formulation of the theory by R. Kubo (1957) and other authors has been criticized (6) , it is clear that it is particularly useful in the study of the high frequency properties of the plasma , without the utilization of the kinetic equation . (7).

For strong fields , due to the fact that strong mechanical constraints arise in the system , thermal constraints cannot be included as a pertur - bative term in the Hamiltonian , the theory in that case is not well founded as in the case of a weak field , eventhough attempts to extend the theory to both cases , that is , strong fields (8) and thermal constraints (9) have been proposed . Here , better than discussing the difficulties of the theory , we will study how it can be used in order to describe the dielectric properties of a plasma from a unifying point of view , that embrace the different models proposed.

With this purpose in mind , the central idea is the following: the linear response theory resulted in a proof of the fluctuation - dissipa - tion theorem , which states that the linear response of a given system to an external perturbation is expressed interms of fluctuation proper - ties in thermal equilibrium . On the other hand , this theorem may be represented by a stochastic equation describing the fluctuation equation that corresponds to a generalization of the familiar Langevin equation of the theory of Brownian movement, the generalization allows us to introduce both random and friction frequency dependent forces which are connec - ted at the same time for a fluctuation dissipation theorem .

These extentions of the Langevin theory of Brownian motion became

, after a extended period of time, a general theory of non-equilibrium processes . The early attemps along the lines of the fluctuation-dissipation theorem by Einstein and Nyquist (10), were later developed by Uhlenbeck and Orstein (11), which applied Langevin's theory to the harmonic oscillator . In the same way Onsager and Machlup were able to establish a stochastic foundation for irreversible thermodynamics (12), using as base a extension of Langevin's theory .Subsequently , Callen , Welton (13) and Kubo , among others, gave general formulations of the fluctuation-dissipation theorem . These results were used by Landau and Lifshitz when they extended the theory in order to include equations for hydrodynamic fluctuations and for electromagnetic fluctuations (14) . Other developments and discussions about the domain of validity of the theory can be seen in the articles of Zwanzing (15) or Fox and Uhlenbeck . (16).

2.- LANGEVIN'S EQUATION.

The classical description of the Brownian motion it is based in the phenomenological stochastic equation :

3.1). $$m\dot{\mu}(t) = -m\nu\mu(t) + R(t)$$

where $m\nu\mu$ is the frictional force exerted by the medium , and $R(t)$ is the random force , whose average value is zero due to the random collisions of the surrounding molecules.

In order to simplify the model , the following two assumptions are made about $R(t)$ (17). :

3.1'). i.- $R(t)$ describes a Gaussian process.

ii.- The time correlation of $R(t)$ is infinitely short , that is :

$$\langle R(t_1) R(t_2) \rangle = 2\pi C \delta(t_1 - t_2)$$

where C is a constant and $\langle\ldots\rangle$ stand for statistical average.

The model described in that way, is suitable for the description of a Brownian particle, having a mass much bigger than the mass of the colliding molecules, because the motion is then disturbed by a **great** number of successive collisions which remain correlated only over the time scale of the molecular motion, which is much shorter than the time scale of the Brownian motion, justifying then the two assumptions i) and ii).

On the other hand, the consideration of these two assumptions, will determine the properties of the stochastic process $\mu(t)$. It was proved by Wang and Uhlenbeck (1945), (17), that due to the hypothesis i) and ii), $\mu(t)$ is Gaussian by i), and a Markoffian process by ii), then all the information about $\mu(t)$ can be obtained from the transition probability:

3.2).
$$W(\mu_0,t_0;\mu,t_0) = \delta(\mu-\mu_0)$$

which is a solution of the Fokker - Planck equation:

3.3).
$$\frac{\partial}{\partial t} W = \frac{\partial}{\partial \mu}\left[D(\mu)\frac{\partial}{\partial \mu} + A(\mu)\right] W$$

$W(\mu_0,t_0;\mu,t)\,d\mu_0\,d\mu$ is defined as the joint probability of finding μ at the range $(\mu_0, \mu_0+d\mu)$ in t_0 and in the range $(\mu, \mu+d\mu)$ in the time t.

The coefficients in the Fokker - Planck equation are deffined as:

3.4).
$$A(\mu) = \lim_{\Delta t\to 0} \frac{\langle\Delta\mu\rangle}{\Delta t}$$

$$D(\mu) = \lim_{\Delta t\to 0} \frac{\langle\Delta\mu^2\rangle}{\Delta t}$$

which can be computed from the Langevin 's equation (3.1) in the following way: integrating 3.1 over a short time Δt we get:

$$\Delta u = -\gamma u \, \Delta t + \frac{1}{m} \int_t^{t+\Delta t} R(t')\,dt'$$

therefore:

3.5).
$$\Delta(u) = \lim_{\Delta t \to 0} \frac{\langle \Delta u \rangle}{\Delta t} = -\gamma u$$

since $\langle R(t) \rangle = 0$

then , since

$$\langle \Delta u^2 \rangle = \gamma^2 u^2 \Delta t^2 + \frac{1}{m^2} \iint_t^{t+\Delta t} dt'\,dt'' \langle\langle R(t')R(t'')\rangle\rangle$$

we have for $D(u)$:

$$D(u) = \lim_{\Delta t \to 0} \frac{\langle \Delta u^2 \rangle}{\Delta t} = \frac{1}{m^2} \iint_t^{t+\Delta t} \langle R(t')R(t'')\rangle \, dt'\,dt''$$

Since

$$\langle R(t_1)\, R(t_2) \rangle = 2\pi C \delta(t_1 - t_2)$$

we get:

3.6).
$$D(u) = \frac{2\pi C}{m^2}$$

The expression .3.5) can be written in a more general form as:

3.7).

$$D(u) = \frac{1}{m^2}\int_0^\infty \langle R(t_0)R(t_0+t)\rangle dt$$

and since we are assuming that the Brownian motion is taking place in a medium in thermal equilibrium , we have:

3.8).

$$W(u_0,t_0;u,t) = A\exp\left(-\frac{1}{2}\frac{mu^2}{kT}\right)$$

where K is the Boltzmann constant and T the temperature, which requires that the Einstein relation must be valid between $D(u)$ the diffussion constant and γ the friction constant , in the form:

3.9)

$$D(u) = \frac{\gamma}{m} KT$$

Combining this equation with equation 3.1.6) , we get :

3.10).

$$m\gamma = \frac{m^2 D}{KT} = \frac{1}{KT}\int_0^\infty \langle R(t_0)R(t_0+t)\rangle dt$$

Equation 3.9). is in this way written as an expression for the fluctuation - dissipation theorem , which states that the systematic part of the microscopic force which appears as the friction $m\gamma$ in a forced motion is actually determined by the correlation in equilibrium of the random forces.

The extension of equation 3.1 for the case of a harmonically bound particle , was established by Ornstein (17) in 1919 , while studying the microscopic bases of Smoluchowski 's equation . He considered the system of equations :

3.11).

$$\frac{dx}{dt} = \mu(t)$$

$$\frac{d\mu}{dt} + \beta\mu = A(t) + \frac{1}{m}K(x)$$

For this process , the information is given by the Fokker - Planck equation :

3.12).

$$\frac{\partial W}{\partial t} = -\frac{1}{f}\frac{\partial}{\partial x}\left(K(x)W\right) + D\frac{\partial^2 W}{\partial x^2}$$

where f and D are defined by:

3.13). i.-

$$\frac{1}{\beta m} = f$$

ii.-

$$\Delta x^2 = \frac{2KT}{m\beta}\Delta t = 2D\Delta t$$

For the particular case of a harmonically bound particle , for which : $\frac{1}{m}K(x) = -\omega^2 x$

where ω is the frequency , equation 3.11) becomes:

3.14).

$$\frac{\partial W}{\partial t} = \frac{\omega^2}{\beta}\frac{\partial}{\partial x}\left[xW\right] + D\frac{\partial^2 W}{\partial x^2}$$

whose solution is given by:

3.15).

$$W(x_o, x; t) = \left[\frac{\omega^2}{2\pi\beta\left(1 - e^{\frac{2\omega^2}{\beta}t}\right)}\right]\exp\left[-\frac{\omega^2}{2\beta D}\frac{\left(x - x_o e^{-\frac{\omega^2}{\beta}t}\right)}{1 - e^{\frac{2\omega^2}{\beta}t}}\right]$$

that gives :

3.16). i.-

$$\langle x \rangle = x_0 e^{-\frac{\omega^2}{\beta}t}$$

ii.-

$$\langle x^2 \rangle = \frac{KT}{m\omega^2} + \left[x_0^2 - \frac{KT}{m\omega^2} \right] e^{-\frac{2\omega^2}{\beta}t}$$

This result is valid for times $t \gg \beta^{-1}$ .Now , for the parti - cular case of $K(x) = -m\omega^2 x$ the system becomes:

3.17).

$$\frac{d^2x}{dt^2} + \beta \frac{dx}{dt} + \omega^2 x = A(t)$$

(which is the same equation (4.37 considered in chapter 2, as a model for the matter , as for the forces in such equation (4.37) , they are assumed random forces $\eta(t)$ with the properties 3.1)i and 3.1)ii.)

With the initial conditions:

3.18). i.-

$$x(t=0) = x_0$$

ii.-

$$u = \left(\frac{dx}{dt} \right)_{t=0} = u_0$$

the solution of equation 3.17) is given by:

3.19). i.-

$$u = -\frac{2\omega^2 x_0 + \beta u_0}{2\omega_1} e^{-\frac{\beta t}{2}} \sin \omega_1 t + u_0 e^{-\frac{\beta t}{2}} \cos \omega_1 t +$$
$$\frac{1}{\omega_1} \int_0^t A(\eta) e^{-\frac{\beta(t-\eta)}{2}} \left[-\frac{\beta}{2} \sin \omega_1 (t-\eta) + \omega_1 \cos \omega_1 (t-\eta) \right] d\eta$$

ii.-

$$x = \frac{\beta x_0 + 2u_0}{2\omega_1} e^{-\frac{\beta t}{2}} \sin \omega_1 t + x_0 e^{-\frac{\beta t}{2}} \cos \omega_1 t +$$
$$\frac{1}{\omega_1} \int_0^t A(\eta) e^{-\frac{\beta(t-\eta)}{2}} \sin \omega_1 (t-\eta) \, d\eta$$

where :

$$\omega_1 = \omega^2 - \frac{\beta^2}{4}$$

Then considering that for the condition : $\langle A(t) \rangle = 0$

equation 3.19) ii , gives :

3.20).

$$\langle x \rangle_{(x_0, u_0)} = \frac{\beta x_0 + 2u_0}{2\omega_1} e^{-\frac{\beta t}{2}} \sin \omega_1 t + x_0 e^{-\frac{\beta t}{2}} \cos \omega_1 t$$

This equation gives the mean value of x , in a canonical ensamble of the harmonic oscillators for which , at t = 0 , we pick a sub - ensamble of oscillators , which have a deviation and velocity x_0, u_0 respectively . If we pick an ensamble for which the deviations x_0 at t= 0 are the same , but the velocities are arbitrary , since in that ensamble the velocities and deviations are not correlated., that is :

3.21). i .-

$$\langle u_0 \rangle_{x_0} = 0$$

ii .-

$$\langle u^2 \rangle_{x_0} = \frac{\kappa T}{m}$$

we then get for the average of x :

3.22).

$$\langle x \rangle_{x_0} = x_0 e^{-\frac{\beta t}{2}} \left(\frac{\beta}{2\omega_1} \sin \omega_1 t + \cos \omega_1 t \right)$$

Now , considering the condition 3.1).ii , or $\langle A(t_1)A(t_2)\rangle = \phi(t_1-t_2)$ where $\phi(x)$ is an even function with a sharp maximun at x = 0 , we get:

3.23).
$$\langle x^2\rangle_{(x_0,u_0)} = \left(\frac{\beta x_0 + 2u_0}{2\omega_1} e^{-\frac{\beta t}{2}} \sin\omega_1 t + x_0 e^{-\frac{\beta t}{2}} \cos\omega_1 t\right)^2 + $$
$$+ \frac{\tau_1}{2\omega_1^2\beta}\left(1-e^{-\beta t}\right) - \frac{\tau_2}{8\omega^2\omega_1^2}\left(\beta-\beta e^{-\beta t}\cos 2\omega_1 t + 2\omega_1 e^{-\beta t}\sin 2\omega_1 t\right)$$

where :

3.24). i.-
$$\tau_1 = \int_{-\infty}^{\infty} \phi(w)\cos\omega_1 w \, dw$$

ii .-
$$\tau_2 = \int_{-\infty}^{\infty} \phi(w)\, dw$$

which are conditions about the form of $\phi(w)$. Due to the fact that it is a function with a sharp maximun , we can make use of $\cos\omega w \approx 1$, then we get :

3.25).
$$\tau_1 = \tau_2 = \frac{2\beta KT}{m} = \int_{-\infty}^{\infty} \phi(w)\, dw$$

with
$$\langle A(t_1)A(t_2)\rangle = \phi(t_1-t_2)$$

that gives :

3.26).
$$\langle x^2\rangle_{x_0} = \frac{KT}{m\omega^2} + \left(x_0^2 - \frac{KT}{m\omega^2}\right) e^{-\beta t}\left(\cos\omega_1 t + \frac{\beta}{2\omega_1}\sin\omega_1 t\right)$$

Actually, more important for us than the obtention of averages values , is the relation 3.25), which appears again as a manifestation of the fluctuation -dissipation theorem. There the microscopic friction force is determined by the correlation of the random force in the same way that is was done in the preceding paragraphs.

From the fluctuation theory we also have , in order to get other demonstration of 3.25) or 3.10 , the relation:

3.27).

$$\left(y_i \, y_k \right)_\omega = \frac{1}{2\pi} \left(\gamma_{ik} + \gamma_{ki} \right)$$

where y_i is the random force and γ_{ik} is an Onsager coefficient From equations 4.37 and 4.38 (chapter 2) , we have for the Onsager coefficient :

$$\gamma_{22} = \nu T$$

from where :

3.28).

$$\left(y^2 \right) = \frac{\nu T}{\pi}$$

which is basically the same relation (3.25) , but in a different notation.

Summarizing , we then have the relations: $m\,\nu = \frac{1}{KT} \int_0^\infty \langle R(t) R(t+t_0) \rangle \, dt$

For the Langevin equation:

$$\frac{2\beta KT}{m} = \int_{-\infty}^{\infty} \phi(w)\, d\omega \qquad \phi(t_1 - t_2) = \langle A(t_1) A(t_2) \rangle$$

For harmonic oscillators :

$$\left(y^2 \right)_\omega = \frac{\nu T}{\pi}$$

where νT is an Onsager coefficient.

Einstein relation:

$$D = \frac{\nu}{m} KT$$

All these equations lead to the prototype Fluctuation Dissipation Theo - rem :

3.29).

$$\langle F(t_1) F_2(t_2) \rangle = 2 K T \nu \, \delta(t_1 - t_2)$$

This is not surprising , since all of the above equations are des - criptions of a Gaussian Markov process .

3. LINEAR RESPONSE.

Now we will discuss the relation 3.29) from a statistical point of view , which allows its extension to non - Markoffian systems (19). The following discussion does not pretend to be a complete exposition of the subject ; it is just a summary of the ideas relevant to this work . Landau and Lifshitz 's work (14) will be followed in this discussion:

Let us consider the quantity $\vec{x}(\vec{r},t)$ and its random fluctuations, $\vec{x}(\vec{r},t)$ has to be understood as a real quantity , for which the mean value is zero in the absence of external effects . The deviations of the average value of $\vec{x}$ in that way defined , are characterizing the non-thermodynamic fluctuations of $\vec{x}$.

In the quantum mechanical case , the operator associated with the quantity $\vec{x}(\vec{r},t)$ will be $\hat{x}$. Let us define the space-time **Fourier** components of the operator $\hat{x}(\vec{r},t)$ by means of the equations:

3.30).

$$x_{k\omega} = \int d\vec{r}\, dt\, e^{-i(\omega t + \vec{k}\cdot\vec{r})} \vec{x}(\vec{r},t)$$

$$\vec{x}(\vec{r},t) = \frac{1}{(2\pi)^4} \int d\vec{k}\, d\omega\, e^{-i(\omega t - \vec{k}\cdot\vec{r})} x_{k\omega}$$

We now consider the correlation function defined as the mean value of the product of the fluctuations of $\vec{x}$ at different points of space at different times , in order to describe the characteristics of the fluctuations .The average is carried out on both , the quantum-mechanical state of the system and on the statistical distribution of the various quantum mechanical states of the system. If the medium is spatially homogeneous and only stationary states of the system are considered , the correlation will take the form:

3.31)

$$\langle x_i(\vec{r}_1,t_1)\, x_j(r_2,t_2)\rangle \overset{def}{\equiv} \langle x_i x_j\rangle_{rt}$$

where :

$\vec{r} = \vec{r}_2 - \vec{r}_1$

$t = t_2 - t_1$

that is , it will depend only on the relative distance and the absolute value of the time segment between the points of which the fluctuations are analized .

The spectral distribution of the correlation function will be:

3.32)

$$\langle x_i x_j\rangle_{k\omega} = \int d\vec{r}\, dt\; e^{-i(\vec{k}\cdot\vec{r}-\omega t)} \langle x_i x_j\rangle_{rt}$$

with (3.30) and (3.32) we deduce the relation:

3.33)

$$\langle x_i^{+}(\vec{k},\omega)\, x_j(\vec{k}',\omega')\rangle = (2\pi)^4\, \delta(\vec{k}-\vec{k}')\, \delta(\omega-\omega')\langle x_i x_j\rangle_{k\omega}$$

where x_i^{+} is the Hamiltonian conjugate of x_i .

Now the general relationship between the correlation function and the dissipative properties of the system will be discuss . This is the central concept in the fluctuation - dissipation theorem .

Let us first calculate the mean value of the product of $x_i^+(\vec{k},\omega)$ and $x_j(\vec{k}',\omega')$. If the system is in a definite stationary state n , then the quantum mechanical average value is defined as the corresponding diagonal matrix element of the operator :

3. 34).

$$(x_i^+(\vec{k},\omega)\, x_j(\vec{k}',\omega'))_{nn} = \sum_m x_i^+(\vec{k},\omega)_{nm}\, x_j(\vec{k}',\omega')_{mn}$$

Since x is time dependent , the evaluation of the matrix elements must be done considering the wave functions of the stationary states , then:

3.35).

$$\left(x_{k\omega}\right)_{nm} = 2\pi\delta(\omega+\omega_{nm})(x_k)_{nm}$$

where:

$$\omega_{mn} = \frac{E_m - E}{\hbar}$$

is the frequency of the transition from the stationary state n to the stationary state m , and $(x_k)_{nm}$ is the matrix element independent of time . Putting 3.35) into 3. 34) anagously for $(\vec{x}^+_{j\omega})_{nm}$ and taking the statistical average , we get:

3. 36).

$$\langle x_i^+(\vec{k})\, x_j(\vec{k}')\rangle_\omega = 2\pi\sum_{n,m} f(E_n)\, x_i^+(\vec{k})_{nm}\, x_j(\vec{k}')_{mn}\, \delta(\omega-\omega_{mn})$$

where $f(E_n)$ is the statistical distribution function of the various quantum mechanical states of the system, $f(E_n)$ for the case of statistical equilibrium is defined by the Gibbs distribution :

3.37).

$$f(E_n) = \exp (F - E_n)/T$$

where F is the free energy and T is the temperature.

Let us suposse now that a periodic perturbation with frequency ω is acting over the system , if then we write the perturbation as the poten - tial $\vec{A}$ ($\vec{r}$,t) , the energy operator of the perturbation may be written as :

3.38).

$$\hat{V} = -\int d\bar{r}\, \bar{A}(\bar{r},t)\, \bar{x}(\bar{r},t)$$

Transforming to Fourier components in this equation , we get:

3.39).

$$V = -\frac{1}{2} \mathrm{Re} \sum_{k} A_k(t)\, x_k^{+}(t)$$

where:

$$A_k(t) = A_{k\omega}\, e^{-i\omega t}$$

Under the influence of the perturbation,transitions between diffe- rent states of the system are possible . Considering 3.38) and 3.39) they give for the matrix element of the transition of the system $n \Rightarrow m$

3.40).

$$V_{nm} = -\pi \sum_{k} \left\{ A_{k\omega} (x_k^{+})_{nm}\, \delta(\omega - \omega_{mn}) + A_{k\omega}^{*} (x_k)_{nm}\, \delta(\omega + \omega_{nm}) \right\}$$

From this equation we obtain the following expression for the transition probability of the system per unit time:

3.41).

$$w_{nm} = \frac{\pi}{2\hbar^2} \sum_{\vec{K},\vec{K}'} A_i(\vec{K},\omega) A_j^*(\vec{K}',\omega) \left\{ x_i^\dagger(\vec{K})_{mn} x_j(\vec{K}')_{nm} \delta(\omega + \omega_{nm}) + x_i^\dagger(\vec{K})_{nm} x_j(\vec{K}')_{mn} \delta(\omega - \omega_{nm}) \right\}$$

In each transition $n \rightarrow m$ the system absorbs the energy quantum $\hbar\omega_{mn}$ whose source is the external perturbation , hence the energy absorbed per unit time equals :

3. 42).

$$Q_n = \sum_m w_{nm} \hbar\omega_{mn}$$

Averaging this equation over all the stationary states n , we get:

3.43).

$$Q = \sum_{m,n} f(E_n) w_{nm} \hbar\omega_{mn}$$

then putting 3.41) into 3.43) , we get :

3.44).

$$Q = \frac{\pi\omega}{2\hbar} \sum_{\vec{K}',\vec{K}} A_i(\vec{K},\omega) A_j^*(\vec{K}',\omega) \sum_{m,n} \left\{ f(E-\hbar\omega) - f(E_n) \right\} x_i^\dagger(\vec{K})_{nm} x_j(\vec{K}')_{mn} \delta(\omega - \omega_{nm})$$

then taking into account 3.36 we have finally:

3.45).

$$Q = \frac{\omega}{4K} \sum_{K,K'} A_i(\vec{K},\omega) A_j^*(\vec{K}',\omega) \left\{ \langle x_i^\dagger(\vec{K}) x_j(\vec{K}') \rangle_\omega^{\hbar\omega} - \langle x_i^\dagger(\vec{K}) x_j(\vec{K}') \rangle_\omega^{\hbar\omega} \right\}$$

where the symbol $\langle \cdots \rangle_\omega^{\hbar\omega}$ stands for the average calculated with

$f(E_n-\hbar\omega)$.

The relation 3.45), is in that way connecting the mean energy absorbed by the system per unit time , to the correlation function of the fluctuating quantities $\vec{x}_i$.

On the other hand , considering the definition of $\vec{x}$ as a quantity whose average value is different from zero only when the perturbation is present , we can write :

3.46)

$$\langle x_i \rangle = \hat{\mathcal{L}}_{ij} A_j$$

where $\mathcal{L}_{ij}$ is a linear space time integral operator .(These considerations are valid for a field weak enough to have just a linear response). Transforming to Fourier components we have:

3.47).

$$x_i(\vec{K},\omega) = \mathcal{L}_{ij}(\vec{K},\omega)\, A_j(\vec{K},\omega)$$

where $\mathcal{L}_{ij}(\vec{K},\omega)$ are macroscopic coefficients characterizing the dissipative properties of the system.

The relation 3.47) is then characterizing a so called Linear Dissipative process. For this process , the absorbed energy Q is expressed directly in terms of the coefficients $\mathcal{L}_{ij}$. In order to see that , let us consider a change in the mean internal energy of the system . As we know this change equals the mean value of the partial derivative of the Hamiltonian of the system with respect to the time .Since in the Hamiltonian only V depends on time , we have :

3.48).

$$\frac{\partial \mathcal{U}}{\partial t} = \int dr\, \dot{A}(\vec{r},t) \langle \vec{x}(\vec{r},t) \rangle$$

Averaging 3.48) with respect to the period of the external perturbation and taking Fourier components , considering 3.47) , we have :
3.49).

$$Q = \frac{\omega}{4} i \sum_{k} \left(\alpha_{if}^{*} - \alpha_{fi} \right) A_i(\vec{k},\omega) A_f^{*}(\vec{k};\omega)$$

Taking into account the expression (3.36) for the average of the correlation , and comparing with (3.44) , we get the expression:
3.50).

$$\langle x_i x_f \rangle_{k\omega}^{\hbar\omega} - \langle x_i x_f \rangle_{k\omega} = \hbar i \left\{ \alpha_{if}^{*}(\vec{k},\omega) - \alpha_{fi}(\vec{k},\omega) \right\}$$

Fundamental equation that establishes the general connection bet - ween the correlation function of the fluctuating quantities and the dissipative properties of the system characterized by the coefficients α_{if} .

The classical case is simplified by using the expansion of the dis - tribution function $f(E-\hbar\omega)$ in $\langle x_i x_f \rangle_{k\omega}^{\hbar\omega}$ in a power series of . In the limit as $\hbar \rightarrow 0$, we have:
3.51).

$$\frac{\partial}{\partial E} \langle x_i x_f \rangle_{k\omega} = i \left\{ \alpha_{fi}(\vec{k},\omega) - \alpha_{if}^{*}(\vec{k};\omega) \right\}$$

with :
3.52)

$$\frac{\partial}{\partial E} \langle x_i x_f \rangle_{k\omega} \delta(\vec{k}-\vec{k}') \equiv (2\pi)^{-2} \sum_{m,n} \frac{\partial f(E_m)}{\partial E_m} x_i^{+}(\vec{k})_{nm} x_f(\vec{k}') \delta(\omega-\omega_{nm})$$

Then , just considering equilibrium distributions and taking into account:

$$\langle x_i x_j \rangle^{\hbar\omega}_{h\omega} = e^{\frac{\hbar\omega}{T}} \langle x_i x_j \rangle_{k\omega}$$

we get:

3.53).

$$\langle x_i x_j \rangle_{k\omega} = \frac{}{\exp(\hbar\omega/T)-1}\, i \left\{ \alpha^{*}_{ij}(\vec{k},\omega) - \alpha_{ji}(\vec{k},\omega) \right\}$$

formula obtained in 1951 by Callen and Welton (19) ,which connect the fluctuations of the quantities in the system with the dissipative properties of it.

The equation 3.53) may be modified by considering that the fluctuation of is due to a random potential R (t) , for which :

3.54).

$$R_i = \alpha^{-1}_{ij} x_j$$

then 3.53) could be applied in the form:

3.54').

$$\langle R_i R_j \rangle_{k\omega} = \frac{\hbar}{\exp(\hbar\omega/T)-1}\, i \left\{ \alpha^{-1}_{ji}(\vec{k},\omega) - \alpha^{-1*}_{ij}(\vec{k},\omega) \right\}$$

The equations 3.54) and 3.54') are known as the fluctuation - dissipation theorem and expressed in this form they are especially suitable for our considerations.

Considering the definition of a symmetrized space-time correlation function :

3.55).

$$\langle x_i x_j \rangle^{(s)}_{rt} = \frac{1}{2} \langle x_i(\vec{r}_1,t) x_j(\vec{r}_2,t_2) + x_j(\vec{r}_2,t_2) x_i(\vec{r}_1,t) \rangle$$

the spectral distribution of the symmetrized correlation function is determined by the expression :

3.56)

$$\langle x_i x_j \rangle^{(\omega)}_{\vec{k}\omega} = \frac{\hbar}{2} \coth \frac{\hbar\omega}{T} i \left\{ \alpha^{*}_{ij}(\vec{k},\omega) - \alpha_{ji}(\vec{k},\omega) \right\}$$

for sufficiently high temperatures $T \gg \hbar\omega$ 3.56) becomes:

3.57).

$$\langle x_i x_j \rangle_{k\omega} = \frac{T}{\omega}\, i \left\{ \alpha^{*}_{ij}(\vec{k},\omega) - \alpha_{ji}(\vec{k},\omega) \right\}$$

The properties of the tensor $\alpha_{ij}(\vec{k},\omega)$ are deduced in the general case from the properties of the correlation function $\langle x_i x_j \rangle^{(\omega)}_{\vec{k}\omega}$. We just mention this properties which have been demonstrated (14). In general α_{ij} is complex. Let us write :

$$\alpha_{ij} = \alpha'_{ij} + \alpha''_{ij}$$

then :

3.58) i.-

$$\alpha_{ij}(\vec{k},\omega) = \alpha^{*}_{ij}(-\vec{k},-\omega)$$

$$\alpha'_{ij}(\vec{k},\omega) = \alpha'_{ij}(-\vec{k},-\omega)$$

$$\alpha''_{ij}(\vec{k},\omega) = -\alpha''_{ij}(-\vec{k},-\omega)$$

in presence of an external field $\vec{B}_o$ (a magnetic field), we have :

ii.-

$$\alpha_{ij}(\omega,\vec{k},\vec{B}_o) = \alpha_{ji}(-k,\omega,-B_o)$$

iii.-

$$\alpha'_{ij}(\vec{k},\omega) - \alpha_{ij}(\vec{k},\infty) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\alpha''_{ij}(\vec{k},\omega')}{\omega'-\omega}\, d\omega'$$

$$\alpha''_{ij}(\vec{k},\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\alpha_{ij}(\vec{k},\infty) - \alpha'_{ij}(\vec{k},\omega')}{\omega'-\omega}\, d\omega'$$

Let us now apply the general theory of fluctuations exposed in the previous sections to the analysis of electromagnetic fluctuations in a medium with time dispersion. Here temporal fluctuations are understood as temporal oscillations of physical quantities averaged in volumes physically infinitesimal. The essential result does not change if we consider the quantities as classical magnitudes .

Due to fluctuation in position and velocity of the charges of the medium , spontaneous electric and magnetic momentum appear in the medium, let us call them $\hat{e}$ and $\hat{m}$ respectively , refered to the unit volume . These fields are connected to the induction and intensity of the magnetic field by the relations:

3.59).

$$D_i = \hat{\varepsilon}_{ik} E_k + e_i$$

$$B_i = \hat{\mu}_{ik} H_k + m_i$$

In Fourier components these equations become:

3.60).

$$D_{i\omega} = \varepsilon_{ik}(\omega) E_{k\omega} + e_{i\omega}$$

$$B_{i\omega} = \mu_{ik}(\omega) H_{k\omega} + m_{i\omega}$$

The Maxwell's equations are then:

3.61). i.-

$$\left(\mathrm{rot}\, E_\omega\right)_i = \frac{i\omega}{c}\left(\mu_{ik} H_{k\omega} + m_{i\omega}\right)$$

ii.-

$$\left(\mathrm{rot}\, H_\omega\right)_i = -\frac{i\omega}{c}\left(\varepsilon_{ik} E_{k\omega} + e_{i\omega}\right)$$

Then considering the balance of energy , as it is deduced from the Maxwell 's equations

3.62).

$$\int \frac{1}{4\pi}\left\{\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right\} dV = -\frac{c}{4\pi}\oint(\vec{E}\times\vec{H})\cdot d\vec{\Sigma}$$

after the replacement of 3.59) in 3.62) , we get:

3.63).

$$\int \frac{1}{4\pi}\left\{E_i\frac{\partial}{\partial t}(\hat{\varepsilon}_{ik}E_k) + H_i\frac{\partial}{\partial t}(\hat{\mu}_{ik}H_k)\right\} dV = -\frac{c}{4\pi}\oint(\vec{E}\times\vec{H})\cdot d\vec{\Sigma} - \frac{1}{4\pi}\int\left(E\cdot\frac{\partial e}{\partial t} + H\cdot\frac{\partial\hat{m}}{\partial t}\right) dV$$

from where we deduce thet the variation of the energy connected with the exterior action , in this case the random fields $\hat{e}$ and $\hat{m}$, is given by:

3.64).

$$-\frac{1}{4\pi}\int\left(\vec{E}\cdot\frac{\partial\hat{e}}{\partial t} + \hat{H}\cdot\frac{\partial\hat{m}}{\partial t}\right) dV$$

Comparing with (3.38) , then we can see the correspondence :

3.65). i.- $\vec{A}(\vec{r},t) \longrightarrow \hat{e}$ or $\hat{m}$

ii.- $\vec{x}(\vec{r},t) \longrightarrow \vec{E}$ or $\vec{H}$

Also the relation (3.38) , expressed in Fourier components as:

3.66).

$$A_{\mu\omega} = \sum_{\eta} \alpha^{-1}_{\mu\eta}(\omega)\, x_{\eta\omega}$$

is here replaced by the Maxwell equations (3.61), and then we have:

3.67). i.-

$$e_{i\omega} = -\varepsilon_{ik}E_{k\omega} + \frac{ic}{\omega}(\mathrm{rot}\, H_\omega)_i$$

ii.-

$$m_{i\omega} = -\mu_{ik}H_{k\omega} - \frac{ic}{\omega}(\mathrm{rot}\, E_\omega)_i$$

Taking into account (3.66) and (3.54) , we can see that the coefficients $\mathcal{L}^{-1}_{\mu\eta}$ relating for example $E_{k\omega}$ and $e_{i\omega}$ are equal to $-\varepsilon_{ik}$, if we consider $\hat{e}_\omega$ and $\vec{E}_\omega$ at the same point , otherwise they are zero. So we have finally:

3.68).

$$(e_i^{(1)} e_k^{(2)})_\omega = i\hbar(\varepsilon^*_{ki} - \varepsilon_{ik})\,\delta(r_1 - r_2)\cot g\,\frac{\hbar\omega}{2T}$$

Analogously , according to (3.61) we have :

3.69).

$$(m_i^{(1)} m_k^{(2)})_\omega = i\hbar(\mu^*_{ki} - \mu_{ik})\,\delta(r_1 - r_2)\cot g\,\frac{\hbar\omega}{2T}$$

Now , considering that:

$$\varepsilon_{ik} = (\varepsilon_{ki}^{-1})^* \quad ; \quad \varepsilon_{ik} = \varepsilon_{ik}' + \varepsilon_{ik}''$$

finally we have:

3.70).

$$\langle e_i^{(1)} e_k^{(2)}\rangle_\omega = 2\hbar\,\varepsilon''_{ik}\,\delta(r_1 - r_2)\operatorname{cotanh}\frac{\hbar\omega}{2T}$$

$$\langle m_i^{(1)} m_k^{(2)}\rangle_\omega = 2\hbar\,\mu''_{ik}\,\delta(r_1 - r_2)\operatorname{cotanh}\frac{\hbar\omega}{2T}$$

These equations have been obtained in different ways by several authors (22). The important point for us is their connection with the imaginary part of the "dielectric constant " .

For low frequency and for temperatures $KT \gg \hbar\omega$, considering the relations :

3.71). i.-

$$\varepsilon_{ij} = \frac{4\pi\sigma_{ik}}{\omega}$$

ii.-

$$\hat{J}_\omega = -\frac{i\omega}{4\pi}\hat{K}_\omega$$

σ_{ik} where is the conductability and $\hat{J}$ a fluctuational current, we get : (22).

3.72).

$$\langle J_i^{(1)} J_k^{(2)} \rangle = \frac{T}{\pi} \sigma_{ik} \, \delta(r_1 - r_2)$$

Now we are in condition to come back to the problem of the Langevin equation , in order to see the relation between both theories: Let us consider first the definition of the mobility μ , as it is given by the Einstein relation:

3.73).

$$\mu = \frac{D}{kT} = \frac{1}{m\gamma}$$

Considering now the definition of the diffusion constant D:

3.74).

$$D = \lim_{t \to \infty} \langle \{ x(t) - x(0) \}^2 \rangle$$

since : $$x(t) - x(0) = \int_0^t u(t')\,dt'$$

we have :

3.75).

$$D = \lim_{t \to \infty} \frac{1}{2t} \int_0^t dt_1 \int_0^t dt_2 \langle u(t_1) u(t_2) \rangle$$

$$= \lim_{t \to \infty} \frac{1}{2t} \int_0^t dt_1 \int_0^{t-t_1} dt' \langle u(t_1) u(t_1 + t') \rangle$$

if again we impose the condition:

$$\lim_{t \to \infty} \langle u(t_0) u(t_0 + t) \rangle = 0$$

we have :

3.76).

$$\mu = \frac{1}{m\nu} = \frac{D}{kT} = \frac{1}{kT}\int_0^\infty \langle \mu(t_0)\mu(t_0+t)\rangle dt$$

that is the mobility is connected to the correlation function of μ .

Now , considering the inversion of the fluctuation - dissipation theorem (7) , in the form:

3.77).

$$\frac{\omega}{T}\langle x^2\rangle_\omega = -\mathcal{L}''(\omega)$$

with: $\langle x_\omega x_{\omega'}\rangle = (x^2)_\omega\, \delta(\omega+\omega')$

simultaneously with the Langevin equation:

3.78).

$$\mathring{u}(t) = -\int_{t_0}^{t} \gamma(t-t')\, u(t')dt' + \frac{1}{m}R(t) + \frac{1}{m}K(t)$$

where: $K(t) = K_0 \cos \omega t \qquad , -\infty < t$

is a periodical external force , and R(t) is a random force for which we assume :

3.79). i.- $\langle R(t)\rangle = 0$

ii .- $\langle u(t_0)R(t)\rangle = 0 \qquad t > t_0$

the identification of $A_j \rightarrow K$ and $x_i \rightarrow \mu$ in expression (3.46) gives:

$$\langle u(t) \rangle = \mathrm{Re}\left[\mu(\omega) K_0 e^{i\omega t} \right]$$

where $\mu(\omega)$ is a complex linear operator playing the rol of $\mathcal{L}_{ij}$ in the general formulation.

Now $\mu(\omega)$ is evaluated from the equation (3.78), considering the conditions (3.79) and the Fourier transform:

$$\nu(\omega) = \int_0^{\infty} e^{-i\omega t} \nu(t) dt$$

so, we have:

3.80).

$$\mu(\omega) = \frac{1}{m} \frac{1}{i\omega + \nu(\omega)}$$

On the other hand, from the equation (3.77), we have:

3.81).

$$\mu''(\omega) = \frac{\omega}{T} \langle u^2 \rangle_\omega$$

and considering also the expression for the fluctuation-dissipation theorem :

$$\langle R^2 \rangle_\omega = \frac{\mathcal{L}''}{\omega |\alpha^2|}$$

in the particular case of (3.78), when K(t)=0 . it will give:

3.82).

$$\langle R^2 \rangle_\omega = \frac{T \mu''}{\omega |\mu|^2}$$

then from the equations 3.81) and 3.82) we get the equation:

3.83).

$$\langle u^2 \rangle_\omega = \frac{1}{m^2} \frac{1}{|\mu|^2} \langle R^2 \rangle_\omega$$

This equation is connecting the spectrum of u and the random force R(t).

Now , since the correlation is an even function in t and considering 3.80) and 3.83), and the relation (3.73) . we have the relationships:

3.84). i.-

$$\mathrm{Re}[\mu(\omega)] = \frac{1}{KT} \langle u^2 \rangle_\omega$$

ii.-

$$m \,\mathrm{Re}[\nu(\omega)] = \mathrm{Re}[\mu(\omega)^{-1}]$$

Result which is a generalization of (3.10).

Another example in which the fluctuation -dissipation theorem is used in connection with the Brownian motion , is in the determination of dynamic friction and diffusion coefficients in a plasma. Slow irreversible processes for which the relaxation time considerably exceeds the time of the particle mean free path are possible in a plasma , in a non- equilibrium state , due to the long range character of the Coulomb forces . Long range collisions for which deflections of the colliding particles occur only at a small angle with little change of velocity ,play a principal rol in the evolution of the plasma. That kind of processes are described by the Fokker-Planck equation in which the effect of the collisions is reduced to particle diffusion into the velocity space. The equation that we will consider is :

3.85)

$$\frac{\partial W}{\partial t} = - \frac{\partial}{\partial v_i}(\nu_i W) + \frac{1}{2} \frac{\partial^2}{\partial v_i \partial v_j}(D_{ij} W)$$

where W is the distribution function of an isolated group of particles due to the diffusion into the velocity space. In this equation ν_i and D_{ij} are defined as :

3.86). i.-

$$\nu_i = \frac{\langle \Delta v_i \rangle}{\Delta t}$$

ii.-

$$D_{ij} = \frac{\langle \Delta v_i \Delta v_j \rangle}{\Delta t}$$

In the general case ,the friction coefficient ν_i and the diffusion coefficient D_{ij} depend on both the distribution function of an isolated group of particles , called "test particles ", and on the distribution functio of the remaining particles of the plasma.

Considering the motion of the individual test particles in the plasma and assuming the other particles are in thermal equilibrium, the coefficients of friction and diffusion can be evaluated expressing them as functions of the spectral distribution of random electric fields in the medium .Several authors proposed such a method of determining the coefficients (23)

The calculations in this approach are based in the equation:

3.87).

$$\dot{v}(t) = \frac{q}{m} E_i[\vec{r}_0(t),t] + \frac{q^2}{m^2} \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' E_j[\vec{r}_0(t''),t''] \frac{\partial}{\partial x_{0j}} E_i[r_0(t),t]$$

with: $r_0(t) = r_0 + v(t-t_0)$

$t = t + \Delta t$

where q is the charge of the particle and m is the mass.

$E_i[\vec{r}(t),t]$ is a random fluctuating field.The time in which the integration is done is much bigger than the period of the fluctuating field .

Averaging 3.87) with respect to the fluctuations , it gives :
3.89).

$$\nu = \frac{q^2}{2\pi m v} \operatorname{Im} \int d\vec{k} \frac{\omega}{k^2 \varepsilon(\vec{k},\omega)} + \frac{q^2}{16\pi^3 m^2 v} \int d\vec{k}\, \omega \frac{\partial}{\partial\omega} \langle E^2 \rangle_{k\omega}$$

$$D_{if} = \frac{q^2}{8\pi^3 M^2} \int d\vec{k} \frac{k_i k_f}{k^2} \langle E^2 \rangle_{k\omega} \qquad ; \qquad \vec{k}\vec{v} = \omega$$

Which are essentially the same expressions that we analized before. In equation 3.89) , the fluctuation fields $\vec{E}$ are evaluated considerring the fluctuation - dissipation theorem , in order to express them as a function of the "dielectric constant" .

In this way the theorem appears as a powerful method to calculate the fricction coefficient responsable for the absorption in a forced motion, as a function of the properties of the system or in another way , as a function of the fluctuating random forces in equilibrium.

In this chapter we have shown that essentially the same method is applied to different models of a plasma . In this analysis we were trying to present a unifying methodology in which all the processes of absorption in plasmas are based . The extension to include a gravitational field is a trivial one and it was already considered in the discussion of the equation (3.78) with the inclusion of the potential field K(x) . On the other hand , if we consider the generalized Ohm's Law from the Schluter equation (1;5.8) , in the linear approximation with the inclussion of a random force field R (t) :

3.90).

$$\frac{d\vec{J}}{dt} + \nu \vec{J} = \frac{\omega_p^2}{4\pi} E(t) + R(t)$$

where $\vec{J}$ is the current; we can see that it corresponds at least formally also with the type of equation (3.1) , or following Schluter , we call 3.90) a Diffusion Equation .The coefficient ν that appears in this equation is anyway , undoubtly determined by the random forces in the medium.

Summarizing , we can say that the determination of the absorption constant in all the different models that we discussed is based in different formulations of the fluctuation -dissipation theorem.

CONCLUSIONS

In this thesis we discussed some problems associated with the expressions of the forces acting in a plasma in an external electromagnetic field , especially a laser field, and the related question of entropy production. The central idea was to use a simple method to embrace other apparently unrelated fundamental plasma-physics problems. Here the external perturbation was included as a perturbative term in the Hamiltonian in order to apply the Linear-response Theory as it was formulated by Landau and Lifshitz, for electromagnetic fluctuations . Considering this theory it was possible to relate a coefficient characterizing the forced evolution of the system by the friction coefficient to the randomly fluctuating electromagnetic fields in an equilibrium system.

This result , formulated in that way , appears as an application of the fluctuation dissipation theorem.

In chapter I , the first Law of Tharmodynamics was discussed in connection with two different phenomena: first the relaxation in a polarizable system , that leads for "small frequencies" to Helmholtz ' ponderomotive force , and secondly , we discussed the dispersion phenomena for a model of matter as a set of oscillators.

For the two fluids model of a plasma , from this work , two principal conclusions arised: first , taking into account dispersion, it resulted in a frquency dependent dielectric constant.From there it is clear that we do not have any reason to assume that the stress tensor and then the ponderomotive force will have the same form in a dispersive medium than it has in a medium in local equilibrium , besides, from here we can see that the domain of validity for the expression of the ponderomotive force proposed by Abraham and by Minkowski,is reduced just for the case of a static field.

In chapter II, the question of entropy production is discussed for the same models. The expression for entropy production due to relaxation is discussed and especially the entropy production in an absorptive medium was considered . In relation with this an equation was proposed in order to describe the regression of the fluctuating polarization. From there it is derived that the Onsager coefficient is of value νT where ν is the"friction coefficient" and T is the temperature.

Also the balance of energy is discussed in relation with the electro - magnetic fields in macroscopic electrodynamics . There we make clear the differences between the average macroscopic fields of classical electro - dynamics calculated in thermal equilibrium with the quantities charac - terizing a non-equilibrium state.

In chapter III , the theory of fluctuations is developed in order to unify the different approaches . From that analysis we came to the conclusion that as far as we limit ourselves to a linear theory , we can always use the fluctuation-dissipation theory in order to evaluate the absorption constants in a non-equilibrium plasma.

Eventhough the theory of the Fokker-Planck equation was used before , to study the diffusion phenomena in a plasma , as far as we know it was not connected to other models of laser-plasma interactions , as the two fluid theory or our new model of oscillators . Here we have shown that in all those models , the absorption processes could be studied by different formulations of the fluctuation dissipation theorem.

The limitations of the theory in that way formulated came from the basic hypothesis we made: a) The theory is valid for a system which is closed to equilibrium (linear approximation). b) The correlation time for the fluctuating forces is considerable less than the relaxation time for all the systems . This is expressed by a fluctuating force function , involving a Dirac delta function of the time variables.

The basic hypothesis we made are showing us the possible future extensions of the theory , in order to study more generel cases: First we have to consider the theory of fluctuations for a system far from equilibrium, a situation that will arise for a system in the presence of a strong external electromagnetic field .Here we have to consider , appart from the mechanical perturbation , the thermal constraint acting on the system due to the non linear interaction with the field.

Secondly , the case of a strong high frequency field has to be analized , here the problems associated with the determination of the plasma polarization are caused by the nonstationarity of the fluctuation process connected with the time dependance of the field.

These two characteristic of the field , that is high intensity and frequency will result in a change of the expression for the diffusion and friction coefficient of the plasma.

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