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COMPUTER-AIDED OPTIMIZATION OF MULTIPLE CONSTRAINT SINGLE PASS FACE MILLING OPERATIONS

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ABSTRACT

Constrained optimization analyses, strategies and CAM software for single pass face milling on CNC and conventional machine tools are outlined and discussed. The constrained optimization is based on criteria typified by the maximum production rate and allows for a range of machine tool constraints. The component surface roughness requirements are also taken into account for finish face milling operations. It is shown that the optimization approach involving mathematical analyses of constrained economic trends and their graphical representation on the feed-speed domain provides clearly defined strategies which guarantee the global optimum solutions and the software developed is suitable for on-line applications. Numerical simulation studies have verified the optimization strategies and computer software, and shown the economic superiority of optimization over handbook recommendations. The numerical studies have also highlighted the increased economic benefit of using optimization in process planning for modern computer-based manufacturing.

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NOMENCLATURE

a_a, a_r	axial and radial depths of cut, respectively
A, G, G_1, α, β, ε, δ	constants in cutting force, torque and power equations
C_T	average cost (excluding material) per component
C_{To}	optimum C_T
C_{Tr}	handbook recommended C_T
D	cutter diameter
f_z, f_{zopt}	feed per tooth, optimum feed per tooth
f_{zmin}, f_{zmax}	min. and max. permissible f_z due to machine tool feed and speed constraints
f_{zFf}	feed per tooth limit due to feed force constraint
f_{zFTR}	feed per tooth limit due to combined feed force, torque and surface roughness constraints
$f_{zF}(V_{min})$	f_z on the $\partial T_T/\partial f=0$ locus when $V=V_{min}$
$f_{zP}(V_{min})$	f_z on the P_{max} locus when $V=V_{min}$
f_{zRt}	feed per tooth limit due to surface roughness constraint
f_{zsj}	feed per tooth step at v_{fsj} for given z and V
f_{zTq}	feed per tooth limit due to torque constraint
F_{fmax}	machine tool maximum feed force constraint
K, n, n_1, n_2, n_3, n_4, n_5	tool-life equation constants
ℓ	length of workpiece milled
N_{min}, N_{max}	machine tool min. and max. spindle speed constraints
N_{si}	ith available spindle speed step on machine tool
P	machining power
P_{max}	machine tool maximum power constraint
r	nose radius of cutting tool or insert
R_{tmax}	max. surface roughness (peak-to-valley) constraint
T	tool-life in time units
T_{ac}	actual cutting time
T_c	cutting (feed engagement) time
T_L	loading/unloading and idle time per component
T_{qmax}	machine tool maximum torque constraint
T_R	cutter replacement time per failure
T_T	average total production time per component
T_{To}	optimum T_T
T_{Tr}	handbook recommended T_T
v_f	feed speed
v_{fmax}, v_{fmin}	machine tool max. and min. feed speed constraints
v_{sj}	jth available feed speed step on machine tool
V, V_{opt}	cutting speed ($=\pi DN$), optimum cutting speed
V_{max}, V_{min}	machine tool max. and min. cutting speed limits for a given D
V_{si}	ith available cutting speed step on machine tool for a given D
x	labour and overhead cost rate
y	cutter cost per failure
z	number of teeth on milling cutter
μ	constant of proportionality ($= 1/\pi$)

1. INTRODUCTION

The selection of cutting conditions is an important step in process planning of machining operations. The notion of an optimum speed was recognized by Taylor [1] early this century. He realised that increasing the cutting speed in single pass turning would reduce the actual cutting time per component but the production interruption time due to more frequent tool failure and replacement would increase so that a compromise or optimum speed should be selected.

Traditionally, economic optimization of machining involves the selection of feed and cutting speed according to a variety of economic criteria such as the minimum cost per component, maximum production rate or maximum profit rate [2-5]. A realistic optimization study should also allow for the many technological and practical constraints which limit the feasible domain for the selection of optimum cutting conditions. However, this work has proved to be surprisingly difficult requiring intricate mathematical analyses and computer assistance, depending on quantitatively reliable mathematical functions of the various machining performance measures and detailed specifications of the machine tools, cutting tools and components which act as constraints on the permissible feeds and speeds [3,4,6-8]. As a consequence of this difficulty, progress in developing realistic optimization strategies for selecting the economic cutting conditions in turning operations has been slow, let alone milling and other operations [3]. Therefore, the selection of cutting conditions in machining operations has often relied on experience, 'rules of thumb' and handbook recommendations, some of which have been captured in computer packages in keeping with the trend towards computer-aided manufacturing. Such approximate practices may have been claimed to be justified in conventional manufacturing systems where it was estimated that only about 5% of the total available production time is spent on machining components. However, this percentage has been forecast to increase to over 70% in modern computer based manufacturing involving CNC machine tools [9]. Therefore, the optimization of cutting conditions in process planning can no longer be ignored if maximum economic benefits are to be achieved.

While promising progress has been made in the optimization of turning operations since Taylor's work [1] whereby various practical and technological constraints have been considered and well defined optimization strategies and computer software have been developed [10-16], the optimization of the various types of milling operations has received relatively little attention and often relied on the use of available computer-aided mathematical programming and numerical search techniques in attempts to provide the optimum feed and speed [17-20]. These 'computer packaged strategies' neither guarantee global optimum solutions nor provide clearly defined economic characteristics and solution strategies which allow for the ready identification of trends in the way in which the optimum solution can change with alternative constraints. More recently, computer aided optimization strategies which can show the effects of the different constraints, guarantee a global optimum solution and be suitable for on-line applications have been developed for single and multi-pass peripheral and end-milling operations [6-8, 21] although further research is required for face milling and other machining operations used in practice. It is also interesting to note that some recent attempts have been made to use multiple-criterion optimization strategies rather than the single economic criterion optimization traditionally used in ‘economics of machining’ studies [22,23].

This paper forms part of an ongoing research program at the University of Melbourne and a recent collaboration between the University of Melbourne and Queensland University of Technology. The optimization analyses, strategies and software module for the selection of economic cutting conditions in single pass face milling on CNC and conventional machine tools are outlined. The analysis presented is based on the popular economic criterion of minimum production time (or maximum production rate) while the strategy developed applies for the minimum cost per component criterion due to the proven mathematical similarity of the two objective functions [6-8, 21]. The constraints considered are the machine tool feed and speed limits (boundaries and steps), machine tool maximum feed force, spindle torque and power constraints as well as the component surface roughness constraint when finish milling is considered. The optimization strategies are assessed by a numerical simulation to compare the optimized results with those based on the recommendations of a comprehensive machining data handbook [24]. This simulation study also highlights the importance of using optimization in process planning and reducing the non-productive time in machining operations.

2. OBJECTIVE FUNCTIONS AND CONSTRAINTS

For the optimization of single pass face milling operations, the well-known objective functions for the popular minimum production time and minimum production cost per component criteria [3, 6] have been employed, i.e.

$$T_T = T_L + T_c + T_R \frac{T_{ac}}{T} \tag{1}$$

and

$$C_T = x\left(T_L + T_c + T_R^{'} \frac{T_{ac}}{T} \right) \tag{2}$$

where $T^{'}_{\mathrm{R}} = (T_R + y/x)$ and T, T_c, T_{ac}, T_L, T_R, x and y are as defined in the nomenclature.

The tool-life equation is taken from one of the very few comprehensive machining handbooks [24] and is given by

$$T = \frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} \tag{3}$$

Some of the process variables for single pass face milling are illustrated in Fig. 1. This handbook also provides the values of the empirical constants in the ‘extended Taylor-type’ tool-life equation for a range of tool-workpiece material combinations. The cutting time T_c for face milling of a workpiece of length ℓ can be approximately expressed as:

$$T_c \approx T_{ac} = \frac{\ell\, D}{\mu V\, f_z\, z} \tag{4}$$

Since the labour and overhead cost rate, x, and cutter cost per failure, y, are constant, although these should be minimized through good management and purchasing policy, equations (1) and (2) are mathematically similar. Hence, the characteristics and strategies for minimizing

T_T and C_T are similar although it is expected that the optimum feed and speed for the two criteria are not necessarily the same under the same constraint conditions. Thus, only the analysis for the minimum time per component T_T equation will be explicitly presented in this paper although both criteria have been analysed and will be considered in the numerical simulation study below.

The fundamental form of the objective function to be optimized can be obtained by substituting equations (3) and (4) into equation (1), that is

$$T_T = T_L + \frac{\ell D}{\mu V f_z z} + \frac{\ell T_R}{\mu K} V^{1/n-1} f_z^{1/n_1-1} a_r^{1/n_2} a_a^{1/n_3} z^{1/n_4} D^{1-1/n_5} \tag{5}$$

It is expected that in single pass face milling optimization, the loading/unloading time T_L and cutter replacement time T_R have been minimized using work study techniques and well-designed handling devices while the milling cutter (D, z) has been selected a priori for the given workpiece dimensions (ℓ, a_r and a_a) and tool-work material combination. Thus, only the cutting speed V and feed per tooth f_z in equation (5) have to be optimized for the minimum T_T (or C_T) value without violating a number of constraints, such as those imposed by the machine tool, which in fact limit the feasible domain of speed V and feed per tooth f_z and result in a constrained optimum T_T or C_T. The machine tool constraints considered in the present study include the limiting feed force, F_{fmax}, spindle torque, T_{qmax}, maximum power, P_{max}, as well as the feed and spindle speed boundaries (v_{fmin}, v_{fmax}, N_{min}, N_{max}). When milling on conventional machine tools, the additional machine tool discrete feed and speed step constraints have to be taken into account. Likewise, the component surface roughness requirement has also to be included in the study for finish milling operations. Based on a detailed study [6], drawing on the empirical force and power equations in the comprehensive handbook [24], the mathematical expressions for these constraints are summarized as follows:

(i) ***Feed force constraint***

$$f_z \le f_{zFf} = \left(\frac{F_{f\,max}}{A a_a^{\beta} a_r^{\varepsilon} z D^{-\delta}} \right)^{1/\alpha} \tag{6}$$

(ii) ***Spindle torque constraint***

$$f_z \le f_{zTq} = \left(\frac{2 T_{q\,max}}{G_1 a_a^{\beta} a_r^{\varepsilon} z D^{1-\delta}} \right)^{1/\alpha} \tag{7}$$

(iii) ***Machine tool maximum power constraint***

$$P = G V f_z^{\alpha} a_a^{\beta} a_r^{\varepsilon} z D^{-\delta} \le P_{max} \tag{8}$$

(iv) ***Machine tool speed and feed boundary constraints***

$$\pi D N_{min} = V_{min} \le V \le V_{max} = \pi D N_{max} \tag{9}$$

$$\frac{\pi D v_{f\,min}}{z V} = f_{z\,min} \le f_z \le f_{z\,max} = \frac{\pi D v_{f\,max}}{z V} \tag{10}$$

(v) ***Machine tool speed and feed step constraints***

$$V \in \{V_{si}\} = \{\pi D N_{si}\} \qquad i\text{=1 to } I \text{ steps} \tag{11}$$

$$f_z \in \{f_{zsj}\} = \left\{\frac{\pi D v_{fsj}}{z V}\right\} \qquad j\text{=1 to } J \text{ steps} \tag{12}$$

(vi) ***Component surface roughness constraint***

An extensive survey [6] has revealed that although some theoretical surface roughness equations have been reported, there is a general lack of published data to support these equations. Thus this constraint cannot be accurately allowed for in the machining optimization until reliable surface roughness equations and associated data become available. For the purpose of this study, the "ideal" peak-to-valley height equations found in reference [3] have been adopted and the resulting constraint expression for sharp cornered teeth or inserts is given by

$$f_z \le f_{zRt} = (\tan C_s + \cot C_e) R_{t\,\max} \tag{13}$$

For large nose radius teeth or inserts, Eq. (13) becomes

$$f_z \le f_{zRt} \approx (8 r R_{t\,\max})^{0.5} \tag{13a}$$

The other "ideal" peak-to-valley height equation for small nose radius teeth where the straight major cutting edge and nose radius are engaged in cutting can be found in [3].

The empirical constants A, G, G_1, α, β, ε and δ in the above constraint equations are dependent on the tool-work material combination, cutter geometry and the units used. These constants can be found in the comprehensive machining data handbook [24]. In addition, F_{fmax}, T_{qmax}, P_{max}, N_{min}, N_{max}, v_{fmin} and v_{fmax} are constraints that have been specified in the nomenclature and whose values can be found from the machine tool specifications included in the handbook [24], R_{tmax} is the maximum surface roughness (peak-to-valley) limit, and C_s and C_e are respectively the approach angle and minor cutting edge angle of a face milling cutter tooth.

It is evident that the magnitudes of these constraints limit the permissible domain for the optimization of cutting speed V and feed per tooth f_z in equation (5). Furthermore, the maximum F_{fmax}, T_{qmax} and R_{tmax} constraints, which only limit the permissible feed per tooth f_z and are mutually exclusive, can be generalised by a feed per tooth limit f_{zFTR}, i.e.

$$f_z \le f_{zFTR} = min\{f_{zFf}, f_{zTq}, f_{zRt}\} \tag{14}$$

For rough milling, the surface roughness requirement and constraint can be ignored so that Eq. (14) can be simplified as

$$f_z \le f_{zFT} = min\{f_{zFf}, f_{zTq}\} \tag{15}$$

It has been found from a quantitative study of the machining performance data in the handbook [24] that the values of the exponents in the 'empirical' tool-life and power equations lie in the ranges $1/n>1/n_1>0$, $1/n>1$ and $1>\alpha>n/n_1$ while $1/n_1$ can take any value greater than zero. The optimization strategy will be developed below based on these common ranges of the machining performance data.

3. DEVELOPMENT OF OPTIMIZATION STRATEGY AND SOFTWARE

For a global minimum time per component, the cutting speed and feed per tooth should be found by simultaneously solving the partial derivative equations of the objective function in equation (5) with respect to the cutting speed and feed equated to zero, i.e.

$$\frac{\partial T_T}{\partial V} = \frac{\ell\, D}{\mu V^2\, f_z\, z}\left[\frac{T_R}{T}\left(\frac{1}{n}-1\right)-1\right] = 0 \qquad (16)$$

$$\frac{\partial T_T}{\partial f_z} = \frac{\ell\, D}{\mu V\, f_z^2\, z}\left[\frac{T_R}{T}\left(\frac{1}{n_1}-1\right)-1\right] = 0 \qquad (17)$$

From these the ‘economic tool-life’ equations with respect to the cutting speed and feed per tooth can be found and are respectively given by

$$\frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{\,1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} = T_R\left(\tfrac{1}{n}-1\right) = T_V \qquad (18)$$

$$\frac{K\, D^{1/n_5}}{V^{1/n}\, f_z^{\,1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} = T_R\left(\tfrac{1}{n_1}-1\right) = T_F \qquad (19)$$

Since $n \neq n_1$ for the common tool-work material combinations, as mentioned earlier, equations (18) and (19) cannot be simultaneously satisfied for a minimized T_T and hence a unique pair of V and f_z does not exist for a global ‘turning point’ minimum T_T. Therefore it is necessary to study the T_T characteristics in order to establish a strategy for selecting the V and f_z such that the production time per component is minimized.

Fig. 2(a) illustrates the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci in the f_z-V domain. It has been proved [6] that for the usual values of the empirical tool-life equation exponents $1/n>1/n_1>1$, the $\partial T_T/\partial f_z=0$ curve is above and to the right of the $\partial T_T/\partial V=0$ curve on the f_z-V diagram. Further, a local ‘turning point’ optimum T_T with respect to V always exists for a given f_z, since $1/n>1$, which can be found on the curve described by equation (18), i.e. the $\partial T_T/\partial V=0$ curve. Similarly, the f_z for a local ‘turning point’ optimum T_T can be obtained from equation (19) (on the $\partial T_T/\partial f_z=0$ curve) for a given cutting speed V when $1/n_1>1$. The time per component T_T characteristics along the $\partial T_T/\partial V=0$ locus can be found by substituting equation (18) in equation (5) from which

$$T_T = T_L + \frac{\ell}{\mu}\left[\frac{T_R}{n\,K}\right]^n (1-n)^{n-1}\, f_z^{\,n/n_1-1} a_r^{n/n_2}\, a_a^{n/n_3}\, z^{n/n_4-1} D^{1-n/n_5} \qquad (20)$$

Thus, since $n/n_1<1$, T_T will decrease along the $\partial T_T/\partial V=0$ curve as f_z increases or V decreases, as indicated by the arrowheads in Fig. 2(a). It can also be proved that similar T_T characteristics exist along the $\partial T_T/\partial f_z=0$ locus (when $1/n_1>1$), as shown in Fig. 2(a).

By contrast, when $1/n>1$ but $1/n_1\leq 1$, $\partial T_T/\partial f_z$ in equation (17) is always negative and equation (19) does not apply so that the necessary condition for a local 'turning point' minimum (i.e. $\partial T_T/\partial f_z=0$) can never be satisfied and the minimum T_T for any given V will occur when f_z is as high as possible [6], as shown in Fig. 2(b). Since equations (16) and (18) still apply, it can be shown again that the T_T value decreases along $\partial T_T/\partial V=0$ locus as f_z increases or V decreases.

It is interesting to note that the above T_T characteristics are similar to those found in earlier optimization studies for turning operations [3, 4] which led to the popular strategy of selecting V and f_z in the 'high feed-low speed' region in the vicinity of the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ (when $1/n_1>1$) loci. These T_T (or C_T) characteristics stem from the speed and feed exponents n and n_1 values in the tool-life equations which lie in similar ranges for turning and milling operations. However, in practice a number of constraints as noted earlier have to be satisfied and, as such, this popular strategy is not always valid in selecting the optimum cutting speed and feed per tooth. The effect of related sets of constraints on the selection of optimum cutting conditions are considered separately below before developing the optimization strategy for optimum speed and feed per tooth allowing for the combined effect of all the sets of constraints.

3.1. Effects of machine tool feed and speed constraints

When the machine tool feed speed v_f and spindle speed N limits are considered, it is found that these constraints define an available f_z-V domain with the upper curve occurring at v_{fmax} and the lower boundary at v_{fmin}, while the maximum and minimum spindle speeds (for a given cutter diameter D) will establish the cutting speed boundaries as vertical lines on the f_z-V diagram, as shown in Fig. 3. The feed per tooth can be expressed in terms of the feed speed v_f and cutting speed V, which are normally the specified quantities (for a given D) in a machine tool, as follows:

$$f_z = \frac{v_f}{zN} = \frac{Dv_f}{\mu zV} \tag{21}$$

By comparing the slopes of the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci with that of the constant v_f curve at their intersections, it has been proved [6] that these curves cross in the way shown in Fig. 3 for the common range of machining performance exponents noted above, i.e the slopes of $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci are steeper than those for the constant v_f curves.

The T_T trends along a constant v_f curve can be found by substituting equation (21) into equation (5), which gives

$$T_{Tvf} = T_L + \frac{\ell}{v_f} + \frac{T_R \ell}{K\mu^{1/n_1}} v_f^{(1/n_1-1)} a_r^{1/n_2} a_a^{1/n_3} z^{(1/n_4-1/n_1)} D^{(1/n_1-1/n_5)} V^{(1/n-1/n_1)} \tag{22}$$

Since $1/n > 1/n_1$, the T_T value decreases as V decreases, as indicated by the arrowheads on the v_f as well as v_{fmin} and v_{fmax} curves in Fig. 3.

The T_T characteristics on the constant V lines can be established readily by superimposing the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ curves on an f_z-V diagram. The direction of reducing T_T is again shown by the arrowheads on the constant cutting speed boundaries. It is apparent that the time per component T_T characteristics along the limiting V lines depend on the relative positions of the $\partial T_T/\partial f_z=0$ (when $1/n_1>1$) locus with respect to the V_{min} and V_{max} boundaries, so does the optimum speed and feed per tooth solution. From a detailed study of the T_T characteristics when only the feed and cutting speed boundary constraints are considered, three possible optimum solutions, all on the V_{min} limit, have been identified and the corresponding limiting conditions established, as shown in Figs. 3(a), (b) and (c) where the feed per tooth f_{zFVn} at the intersection of the $\partial T_T/\partial f_z=0$ locus and V_{min} limit can be found from equation (19) with $V=V_{min}$, that is

$$f_{zFVn} = \left(\frac{K\ D^{1/n_5}}{T_R(1/n_1 - 1)V_{min}^{1/n}\ a_r^{1/n_2}\ a_a^{1/n_3}\ z^{1/n_4}} \right)^{n_1} \tag{23}$$

For example, when $1/n>1/n_1>1$ and $f_{zFVn}>f_{zmax}$, the optimum solution is at $f_{zopt}=f_{zmax}$ and $V_{opt}=V_{min}$, as shown in Fig. 3(a). The corresponding optimum T_T (i.e. T_{To}) is found by substituting f_{zopt} and V_{opt} into equation (5). It should be noted that the location of the $\partial T_T/\partial V=0$ curve on the f_z-V diagram does not affect the optimum solutions. When $0<1/n_1\leq 1$, the $\partial T_T/\partial f_z=0$ locus does not exist, as noted earlier, and the optimum solution is always at $f_{zopt}=f_{zmax}$ and $V_{opt}=V_{min}$, as shown in Fig. 3(d), which is coincident with the solution shown in Fig. 3(a).

3.2. Effects of machine tool feed force, torque and power and component surface roughness constraints

From the above analysis, the machine tool feed force, spindle torque and the component surface roughness constraints have been generalised and represented by an upper feed per tooth limit f_{zFTR}. By contrast, the machine tool maximum power constraint will come into play in the 'high' speed region of the machine tool operating range and will limit both the feed and cutting speed from which a pair of constrained optimum feed and speed can be selected. This is due to the fact that in the ‘low’ speed region, the machine tool maximum power may not be permitted since this will involve an excessive spindle torque. As such, the more severe spindle torque constraint should be considered in this ‘low’ speed region in the optimization study.

In order to establish an optimization strategy, it is again necessary to study the T_T trends on the f_z-V diagram while considering the effects of these constraints. As shown in Fig. 4, the maximum power P_{max} and the $\partial T_T/\partial V=0$ curves intersect such that for the usual set of exponent values, $1>\alpha>n/n_1>0$, the slope of the P_{max} curve is less than that of the $\partial T_T/\partial V=0$ locus at the point of intersection and so the curves cross in the way shown in the figure. It has been proven [6] that the maximum power constraint curve intersects the $\partial T_T/\partial f_z=0$ curve in the same manner as the $\partial T_T/\partial V=0$ curve on the fz-V graph.

Using the same procedure in arriving at equation (22) for the T_T trends along the constant v_f curve, the T_T trend along the P_{max} locus can be found by substituting V from equation (8) (with P=P_{max}) into equation (5), which shows that T_T decreases along the P_{max} locus as f_z increases for the common conditions of $1>\alpha>n/n_1$ and $1/n>1/n_1>0$. Fig. 4 illustrates the decreasing T_T trends by the arrowheads on the P_{max} and $\partial T_T/\partial V=0$ curves in the F_z-V domain. Based on the T_T trends discussed earlier, the optimum will lie on the $\partial T_T/\partial V=0$ curve if it is below the P_{max} curve, or on the P_{max} curve if the $\partial T_T/\partial V=0$ curve is above the P_{max} curve. This is because in the latter case, the cutting conditions above the P_{max} curve are not feasible and the T_T trends suggest that the optimum be selected on the P_{max} curve at its intersection with the highest possible feed. The portions of the P_{max} and $\partial T_T/\partial V=0$ curves on which the optimum point is likely to lie are shown by solid lines in Fig. 4.

When both the machine tool maximum power P_{max} limit and the generalised feed per tooth f_{zFTR} constraint (due to the feed force, torque and surface roughness) are considered jointly, the T_T characteristics can be found by superimposing the loci of these constraints on the f_z-V diagram as shown in Fig. 4. This shows that the optimum solution will be on the generalised f_{zFTR} constraint, at its intersection with either the P_{max} curve or the $\partial T_T/\partial V=0$ curve, depending on which intersection is at a lower speed. The two cutting speeds where f_{zFTR} intersects with the P_{max} and $\partial T_T/\partial V=0$ curves can be found by substituting f_{zFTR} into equations (8) and (18), respectively, and given by

$$V_{FP} = \frac{P_{max}\, D^{\delta}}{G\, f_{zFTR}^{\alpha}\, a_r^{\varepsilon}\, a_a^{\beta}\, z} \tag{24}$$

$$V_{TV} = \frac{K\, D^{1/n_5}}{T_R\left(1/n-1\right) f_{zFTR}^{1/n_1}\, a_r^{1/n_2}\, a_a^{1/n_3}\, z^{1/n_4}} \tag{25}$$

Depending on the relative positions of the $\partial T_T/\partial V=0$ curve and the constraint loci on the f_z-V diagram, these two constrained optimum V and f_z are possible and can be determined using the strategy suggested above. For example, if $V_{TV}<V_{FP}$, the optimum (f_{zopt}, V_{opt}) solution is at the intersection of f_{zFTR} and the $\partial T_T/\partial V=0$ locus as in Fig. 4(a).

3.3. Constrained optimization strategy and CAM software

The economic characteristics and associated optimization strategies have been presented above when separately considering related sets of constraints. In practice, the economic trends and the combined effects of all the constraints must be considered jointly in the selection of cutting conditions. In addition, the discrete feed and speed steps will need to be taken into account for milling on conventional machine tools where only the feed and speed step combinations are available for selection. This results in more complex strategies which benefits greatly from computer assistance for their implementation after the various possible constrained optimum solutions and the corresponding limits identifying these solutions are established.

By applying the above analyses, the economic trends for the combined effects of the various machine tool and component surface roughness constraints can be found by superimposing the upper feed per tooth limit, f_{zFTR}, and the P_{max} limiting speed-feed curve together with the feed speed and cutting speed boundaries as well as the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ loci on the f_z-V diagram. Since the relative positions of these curves on the f_z-V diagram can vary depending on the magnitudes of the constraints allowed by the machine tool specification, the machining performance data for the given tool-workpiece material combination, the cut and cutter geometry, and the time (and cost) factors, the 'active' constraints on which the optimum conditions may lie can also vary. To establish the optimization strategy, it is necessary to identify the various constrained optimum solutions on the 'active' constraints, and the corresponding limiting conditions for identifying each optimum solution.

For CNC machine tools with continuously variable feeds and speeds within the boundaries, a detailed analysis of the T_T trends in the f_z-V domain has resulted in 11 distinctly different solutions being identified for a total of 22 different limiting conditions or cases representing all possible relative positions of the $\partial T_T/\partial V=0$ and $\partial T_T/\partial f_z=0$ curves and the various constraints' loci [6]. These are shown in Fig. 5 where the arrowheads indicate the T_T decreasing direction and the 'dot' highlights the optimum feed and speed. The limiting conditions for identifying each constrained optimum f_z and V solution are given in the captions of each diagram in Fig. 5. Since every possible case can be identified explicitly, the analysis assisted by the f_z-V graphical approach has provided a means of guaranteeing and identifying the various possible global optimal solutions. A computer-aided strategy and software for arriving at the required constrained global optimum f_z and V solution as well as the optimum T_T (or C_T) when milling on a CNC machine has been developed involving a simple sequential testing of the limiting conditions for identifying the appropriate global solution. A simplified flow chart is given in Fig. 6 to illustrate the procedure in implementing the optimization strategy to arrive at the global optimum solution. The program will first check the feasibility of a single pass operation and if it is not feasible, the program will be terminated; otherwise, sequential checking of the limiting conditions is carried out to identify the optimum solution. For example, if $f_{zFTR}\geq f_{z1}$ (the maximum f_z at the v_{fmax} and V_{min}), the program will check the following conditions for the global optimum solution: if $1/n_1>1$, $f_{zF}(V_{min})\geq f_{z1}$ and $f_{zP}(V_{min})\geq f_{z1}$ or $1\geq 1/n_1>0$ and $f_{zP}(V_{min})\geq f_{z1}$, the optimum solution is at $V_{opt}=V_{min}$ and $f_{zopt}=f_{zmax}$. If any of these conditions is not satisfied, the procedure will be directed towards checking the limiting conditions for other possible solutions following a logical sequence. As soon as the limiting conditions for identifying the final global optimum solution are satisfied, the optimum V_{opt} and f_{zopt} and T_{To} (or C_{To}) are evaluated and the program is terminated. Thus the program needs not check the limiting conditions for all the possible global optimum solutions each time it is run. Furthermore, this deterministic mathematical approach to the selection of the optimum cutting conditions leads to very fast computer processing time suitable for on-line applications.

It should be noted that among the 11 possible solutions, there is one representing the situation where single pass face milling is not feasible since at least one of the practical constraints will be violated for the 'input' conditions so that multipass milling operations or an alternative machine tool must be considered. Interestingly, in only four of the 22 different cases do the solutions match with the popularly acclaimed strategy of selecting the largest possible feed with the speed being found from $\partial T_T/\partial V=0$ curve of equation (18).

For conventional milling machines, only discrete feed and speed step combinations are available for the selection of optimum cutting conditions. The task to locate the optimum solution for this situation has been found to be very complex. This is especially so because no generalised expression for the machine tool feed and speed step spacings can be established, though they are usually in geometrical progressions, and the objective function in equation (5) is non-linear so that there are no universal patterns for the behaviour of the objective function between the discrete steps. Specifically, the strategy is to start with the appropriate 'continuous feed and speed' solution found by using the strategy for CNC machine tools and then to perform numerical comparisons of the T_T (or C_T) values at the permitted discrete feeds and speeds surrounding the 'continuous' solution. A strategy for reducing the 'search domain' based on the T_T (or C_T) trends established in the optimization analyses has been developed to minimize the number of T_T (or C_T) comparisons in arriving at the final discrete feed and speed step solution [6, 8]. Once the optimum feed per tooth and speed steps have been found, these are substitued in equation (5) to evaluate the corresponding optimum T_T (or C_T)

Thus, despite the complexity of the constrained optimization analyses and strategies requiring computer assistance, the approach explicitly identifies the various possible solutions and associated limiting conditions resulting in clearly defined strategies which ensure a global optimum solution. In addition, the active constraints associated with each global solution can be identified. For conventional machine tools, the computer program for CNC machines has been extended to incorporate the strategy discussed above to determine the optimum feed and speed step combination.

4. ASSESSMENT OF THE OPTIMIZATION STRATEGIES

While many machining data handbooks [25] only provide recommendations for the selection of some of the cutting conditions, such as the feed per tooth and cutting speed in milling, irrespective of the machine tool (constraints) used, one metal cutting conditions handbook [24] provides not only complete tool-life, force, torque and power equations for a range of tool-work material combinations, but also detailed information and specifications for a number of milling machines. Furthermore, this handbook provides a methodology for selecting a standard cutting tool as well as the feed and speed for single pass rough face milling such that the conventional machine tool constraints are not violated. A study of this methodology clearly indicated that the handbook solutions were feasible, but not necessarily optimal [6]. Nevertheless, this handbook provides a unique opportunity to assess the developed optimization strategies and software as the optimized times and costs should always be equal or superior to those from the handbook. Further, the benefits of using optimization strategies over handbook recommendations can be evaluated by a numerical simulation study.

For the purposes of this numerical simulation study, rough face milling of S1214 free machining steel on a conventional universal milling machine quoted in the handbook [24] with a spindle motor power of 4.5 KW and 16 discrete steps for both the feed speed and spindle speed was considered. A CNC machine tool was also simulated by ignoring the discrete feed and speed steps, thus enabling the comparison of the solutions for conventional and CNC machine tools to be made. A high speed steel (HSS) face milling cutter recommended in this handbook of 160 mm diameter and with 16 teeth was selected to perform these 'operations'. The reason for choosing an HSS cutter is that its complete machining performance data for cutting force, power, torque and tool-life when machining the free machining steel are available in the handbook although carbide is more commonly used than HSS for face milling cutters. The relevant constant and exponent values of the empirical tool-life, force, torque and power equations for HSS face milling cutters cutting S1214 free machining steel work material are given in Table 1. According to the handbook [24], when the tool-work material combination is changed, the tool-life equation in Eq. (3) still applies but with different values of constant and exponents. It has been found that for other common tool materials, such as carbide, the relationships between the exponents in the tool-life as well as the force and power equations are in the range considered in this study and the developed optimization strategy will still apply.

Three levels of radial and axial depths of cut spanning the handbook recommended ranges and three component lengths ℓ (200, 400 and 600 mm) were tested at three levels of loading/unloading time T_L (1.2, 1.6 and 2.0 min.) and one level of cutter replacement time T_R (2 min.). When the minimum cost per component criterion was considered, the labour and overhead cost rate x of \$0.60/min. and the cutter cost per failure y of \$15.00 were selected. Thus 81 combinations were considered for each criterion and each machine tool in the simulation study.

An examination of the optimum solutions has revealed that for all cases studied the optimum feeds per tooth and cutting speeds were considered to be feasible and the optimum T_T and C_T for the simulated CNC machine tool were always less than or equal to those of the conventional machine. While this was anticipated from the optimization analysis on the basis that more constraints are involved when the discrete feed and speed steps are considered, the result is heartening in as much as it provides further verification that the optimization strategy is correct. It was also found that there were 18 out of the 81 cases where single pass was not feasible, due to at least one of the machine tool constraints being violated, so that multipass operations may have to be considered. This again confirmed the optimization analysis and the subsequent optimization strategies and computer software.

Quantitative comparisons between the handbook [24] recommended and optimized solutions have been carried out to assess the penalty of using the handbook recommendations rather than the optimum conditions. For this purpose, the results for conventional machine tool were compared based on the percentage increase in the handbook time (or cost) per component over the corresponding optimized solution, that is

$$\frac{\Delta T_T}{T_{To}}100 = \left[\frac{T_{Tr} - T_{To}}{T_{To}}\right]100 = \left[\frac{T'_{Tr} - T'_{To}}{T'_{To}}\right]\left[1 - \frac{T_L}{T_{To}}\right]100 \tag{26}$$

$$\frac{\Delta C_T}{C_{To}}100 = \left[\frac{C_{Tr} - C_{To}}{C_{To}}\right]100 = \left[\frac{C'_{Tr} - C'_{To}}{C'_{To}}\right]\left[1 - \frac{xT_L}{C_{To}}\right]100 \tag{27}$$

where the T'_T and C'_T are the time and cost per component when T_L is zero.

From the linearity of equations (26) and (27), the maximum penalties of using handbook recommendations (or maximum benefits of using the optimum conditions) will occur when the loading/unloading and idle time T_L is as small as possible (ideally zero). Also these penalties reduce linearly to zero as T_L/T_{To} and xT_L/C_{To} increase to 1. Thus these equations highlight the need to continually reduce the non-productive times and costs. It also follows that in modern flexible manufacturing, where the non-productive times and costs are minimized and are small proportions of the total production times and costs, the use of optimization strategy becomes more important than in the past.

Fig. 7 shows the overall economic benefits of using optimization over handbook recommendation found in this simulation study for alternate loading/unloading times or costs according to equations (26) and (27). Based on the ideal loading/unloading time of zero, the overall results show that the use of handbook recommendations will lead to an average increase of 31% in T_T over the optimized times with a range of 0% to 93%. The corresponding values for the minimum C_T criterion are an average increase of 52% and a range of 0% to 115%. It is apparent that these penalties are significant and emphasize the need to use optimization strategy for efficient and economic production.

The T_T and C_T penalties of using handbook rather than optimized cutting conditions for the non-zero non-productive times (T_L) considered in this study are shown by histograms in Fig. 8. It is noted that the average percentage time penalty is about 21% with a range of 0% to 73% while the corresponding cost penalties are about 38% on average, ranging from 0% to 105%. It is again apparent that even for $T_L \neq 0$ the penalties of using handbook cutting conditions are considerable, so that in general substantial benefits would be gained if these cutting conditions were optimized using the above strategies.

During the course of implementing the optimization strategies, computer processing times have been recorded as a criterion to assess the developed software. When the program was run on an IBM personal computer, the processing (excluding input and output) times for most combinations of the selected conditions and for the time and cost criteria were about 0.03 second for the simulated CNC machine tool with only a few cases where the computer processing times were 0.04 to 0.05 second. When the strategy for conventional machine tools was tested, all the processing times were within 0.08 second. Therefore, the developed deterministic, rather than numerical search, optimization strategies and software module are suitable for on-line applications in computer-aided manufacturing.

5. CONCLUSIONS

Realistic and clearly defined optimization strategies and the associated CAM software for single pass face milling on CNC and conventional machine tools have been presented based on the criteria typified by the minimum production time and cost per component and allowing for the many practical constraints. Despite the complexity, the detailed optimization analysis assisted by the feed-speed diagrams has provided a deeper understanding of the economic characteristics and the influence of machining performance data and constraints imposed by the machine tool design specification. It has also provided a means of guaranteeing a global optimum solution.

The simulation study has shown the substantial economic benefits of using optimization strategies over the handbook recommended cutting conditions in rough face milling operations, and highlighted the increased benefits of using optimization strategies in process planning in modern computer controlled and automated manufacture where the proportions of non-productive times and costs are low and continually being improved. This study has also shown that the developed optimization strategies and software are suitable for on-line CAM applications.

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Table 1. Machining performance data for HSS face milling cutters milling S1214 free machining steel.

Tool-life equation	Force, torque and power equations
$K=0.916$, $n=0.2$, $n_1=0.5$, $n_2=2.0$, $n_3=1.33$, $n_4=2.0$, $n_5=0.8$	$A=278.3$, $G=0.773$, $G_1=0.773$, $\alpha=0.8$, $\beta=0.95$, $\varepsilon=1.1$, $\delta=1.1$

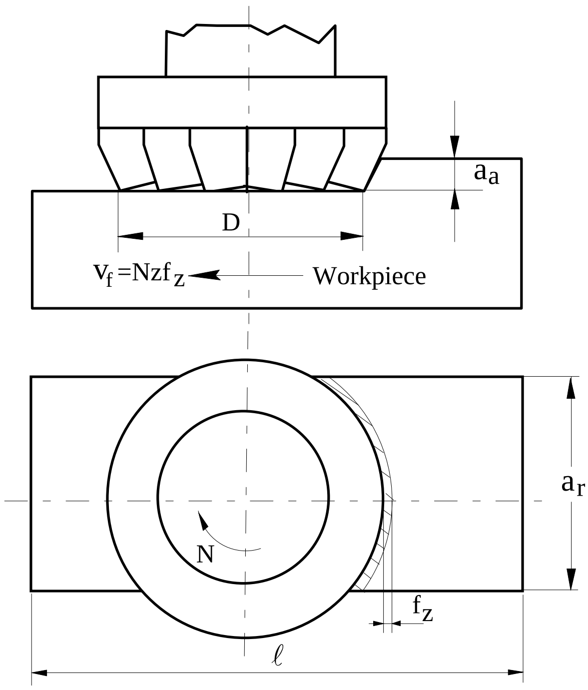
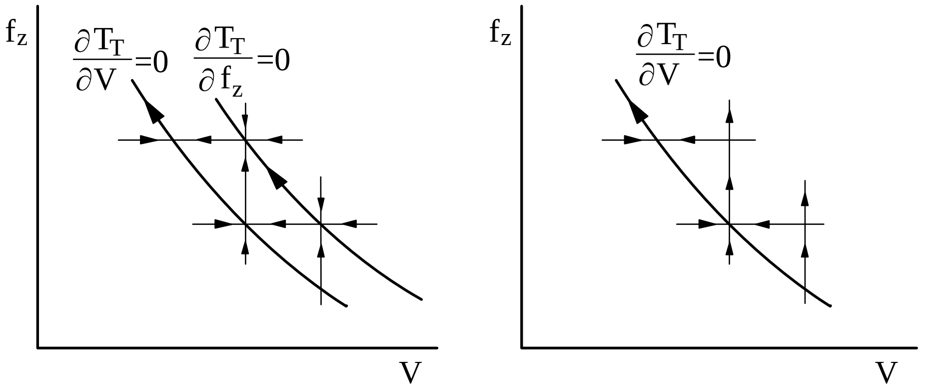
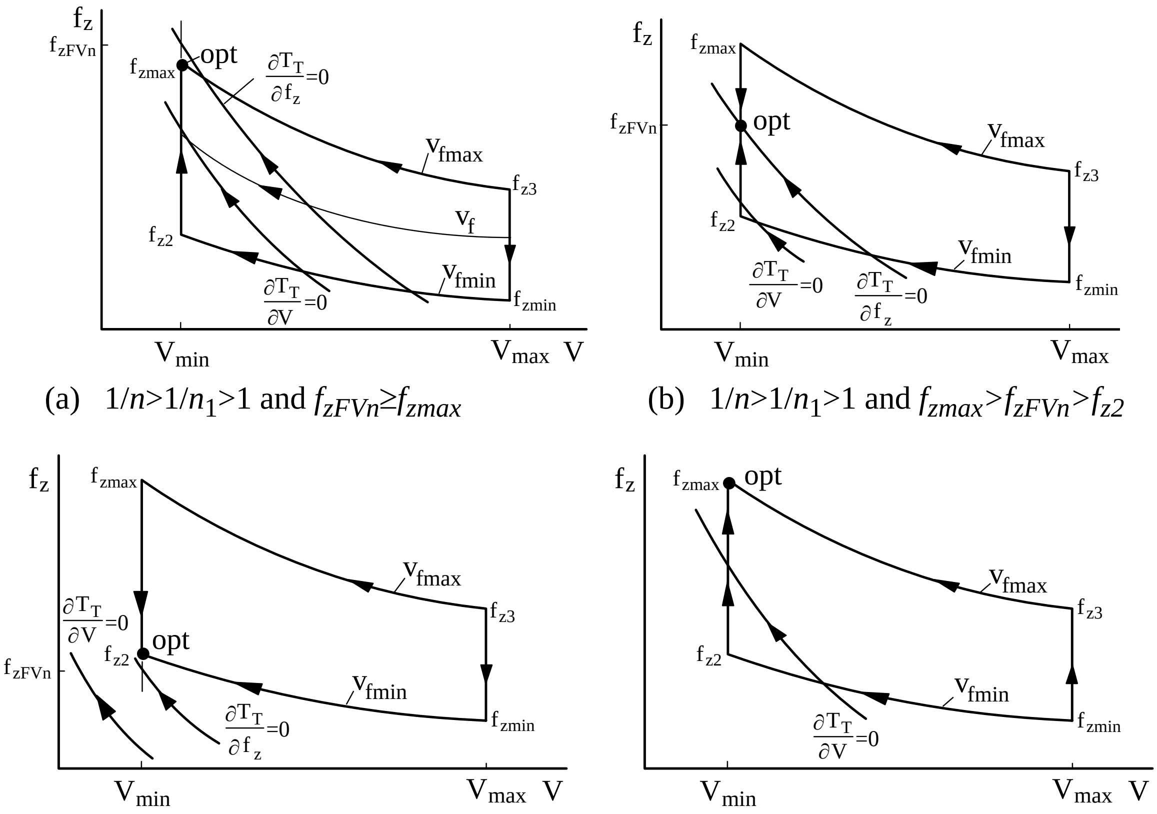


Fig. 1. Some process variables in single pass face milling operations.



(a) $1/n>1/n_1>1$ (b) $1/n>1\geq1/n_1>0$

Fig. 2. Diagrammatic presentation of time per component characteristics.



(a) $1/n>1/n_1>1$ and $f_{zFVn}\geq f_{zmax}$ (b) $1/n>1/n_1>1$ and $f_{zmax}>f_{zFVn}>f_{z2}$

(c) $1/n>1/n_1>1$ and $f_{zFVn}\leq f_{z2}$ (d) $1/n>1\geq1/n_1>0$

Fig. 3. Production time per component characteristics, feed and speed boundary constraints and the various constrained optimum V and f_z.

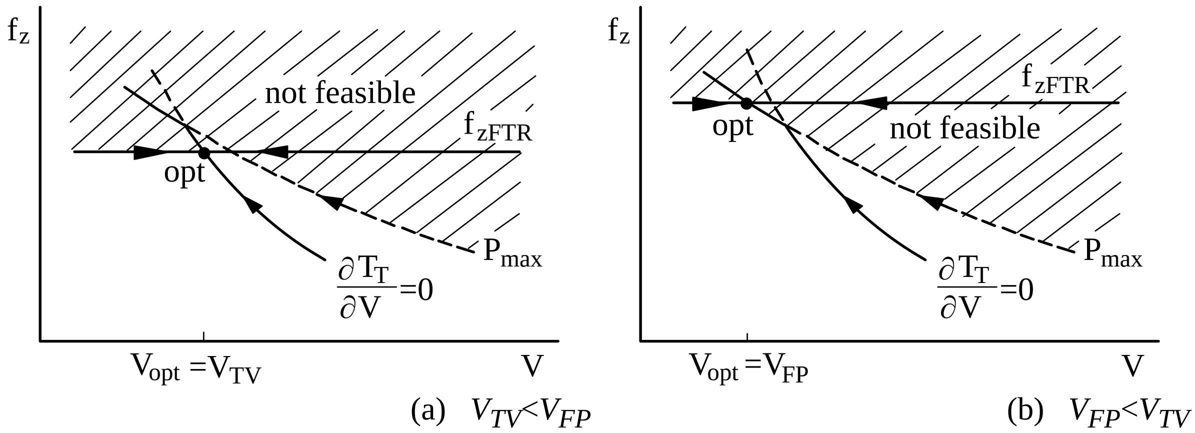


Fig. 4. Production time per component characteristics, the feed force, torque, power and surface finish constraints, and the various constrained optimum V and f_z.

Fig. 5 (e-copy not available)

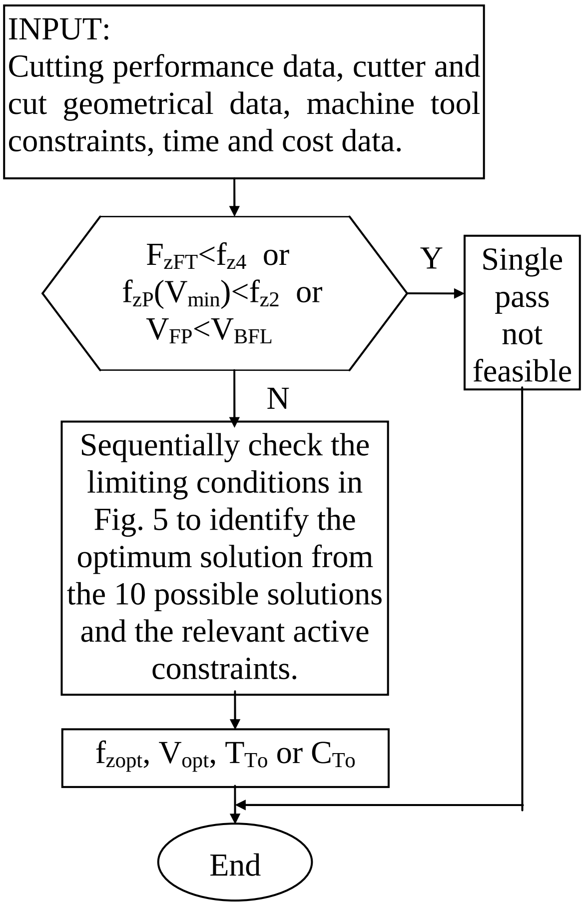


Fig. 6. A simplified flowchart showing the optimization procedure.

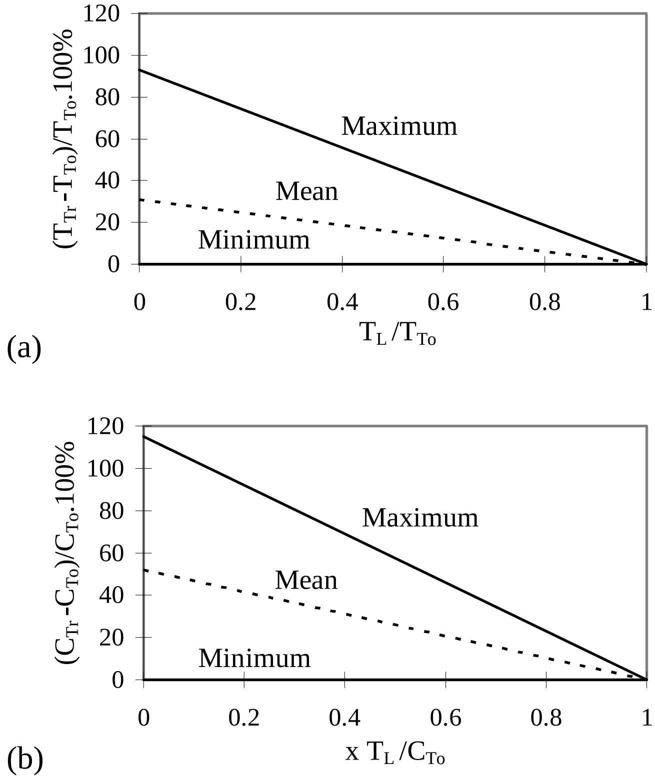


Fig. 7. Time and cost penalties for alternative non-productive loading/unloading times.

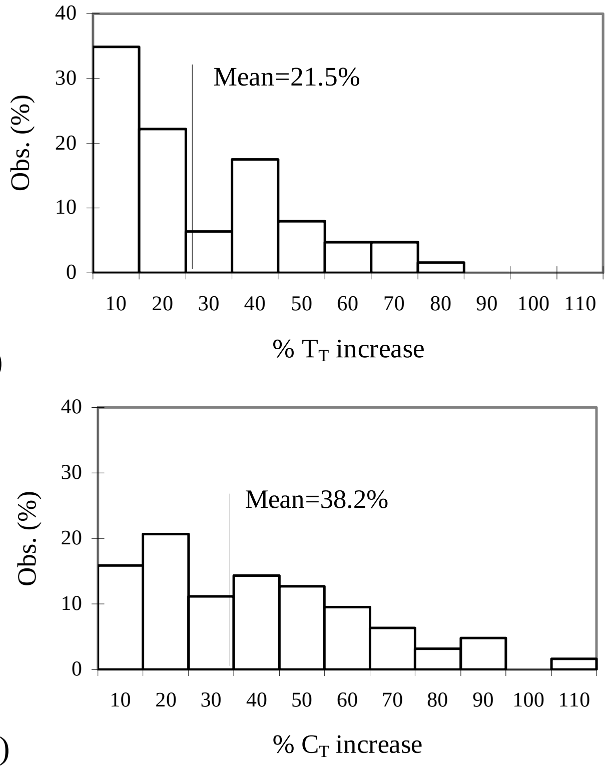


Fig. 8. Time and cost per component penalties of using handbook recommendations over optimization.