ON THE PROBABILITY OF FAILURE BY YIELDING OF HULL GIRDER MIDSHIP SECTION

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LIST OF SYMBOLS

A	Cross section of the midship section
a_s	Cross section of a stiffener $(a_{s_1} - a_{s_5})$
B	Breadth of the midship section
b	Flange width of the stiffener
c.o.v.	Coefficient of variance = standard deviation/mean value
D	Depth of the midship section
D_x	Variance of a random variable, x
$f_x(x)$	Probability density function of the random variable, x
$F_x(x)$	Distribution function of the random variable, x
h	Double bottom height of the midship section; also stiffener web height
I_x	Moment of inertia of the midship section about X-axis; also used for stiffener
I_{NA}	Moment of inertia of the midship section about neutral axis
I_Y	Moment of inertia of the stiffener about Y-axis
I_{XY}	Product moment of inertia of the stiffener about XY-axis
i_{x1}	Moment of inertia of the stiffener about centroidal x_1-axis
i_{y1}	Moment of inertia of the stiffener about centroidal y_1-axis
i_{x1y1}	Centroidal product moment of inertia
i_s	Centroidal moment of inertia of stiffener about a horizontal axis (Figure D1)
ℓ	Subscript for the lower limit of a random variable
M_S	Still water bending moment amidships
M_W	Wave induced bending moment amidships
M_T	Total bending moment amidships = M_S+M_W
R	Resistance moment of the midship section = σ_Y.Z
S	Load effect moment of the midship section = M_T
S_X	Static moment of a cross section about X-axis
S_Y	Static moment of a cross section about Y-axis
T	Exposure time following the coating breakdown, years
t	Shell plate thickness, t_1-t_5 respectively at deck, tanktop, outer bottom, side shell and bilge corner
t_f	Flange thickness of stiffener, t_{f1}-t_{f5}
t_w	Web thickness of stiffener, t_{w1}-t_{w5}
u	Subscript for the upper limit of a random variable
V_x	First moment of area of the midship section about X-axis

X	Vector of basic variables, $(x_1, x_2, \ldots)^T$
x	Value of a basic variable
x_c	Distance of stiffener centroid from Y-axis (x_{c1}-x_{c5})
y_c	Distance of stiffener centroid from X-axis (y_{c1}-y_{c5})
y_d	Distance of the midship section neutral axis from deck = $\bar{y}$
y_k	Distance of the midship section neutral axis from keel = $D - \bar{y}$
Z	Elastic section modulus of the midship section at the deck level
δ	Average annual corrosion wear (mm/yr)
δ_T	Total corrosion wear over a period T, years
σ	Standard deviation of a random variable
σ_Y	Yield strength of the material
θ	Angle between x_1-axis and the horizontal axis through the centroid of a stiffener
ϕ	Angle between two adjacent stiffeners in the bilge (Figure A2)

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1. General Introduction

In recent years significant progress has been made in the computation of loads that act on a structure and the strength of an as-built structure. It has long been known that both of these quantities are random variables. Therefore, the problem of structural design is essentially a problem which should involve the theory of probability at the design stage. In the fields of aerospace and civil engineering the methodology was proposed by Pugsley (1955) and Freudenthal (1956) during the mid-fifties. Following these works a number of researchers in the field of structural design including naval architects contributed towards the probabilistic methods of design.

However, in the routine design of structures a simple, deterministic approach is still popular and useful. For example, ship structural design is based on classification society rules. One of the main aspects of ship structural design is to determine the required mid-ship section modulus to prevent yielding against the applied loading. Implicit in this method are two quantities. The total bending moment amidships, M_T with two components, still water bending moment, M_S, and the wave-induced bending moment, M_W. It is obvious that the wave-induced bending moment is a random variable (St Denis and Pierson 1953), but the still water bending moment is also a random variable although it does not seem so obvious (Hughes, 1988). On the other hand, the resistance bending moment is the product of material yield strength, σ_Y and the elastic section modulus, Z, both of which are also random variables. For the sake of simplicity we assume that a random variable may be defined by its mean and variance. It may be mentioned that although a normal or Gaussian probability distribution is directly defined by mean and variance, it is not the only possible type of distribution. The other types such as log-normal and Type I asymptotic extreme value are also defined by these two parameters (Ochi, 1990).

Other probability distributions might be more appropriate, even if only the mean and variance are known (Melchers 1999). The philosophy in the classification society rule is to magnify the total bending moment by a ‘factor’ and reduce the material yield strength by another ‘factor’ to come up with an allowable stress. And then obtain the required elastic section modulus amidships simply by dividing the former by the latter.

However, essentially the factors are to be applied on the mean values of the variables because the actual value of a random variable, by definition, is never known. Therefore, in this approach of design the mean strength of the hull girder is set to higher magnitude than the mean load effect. But in real life the actual strength of some ships may be much smaller than the mean value predicted at the design stage; particularly when aged. On the other hand the wave-induced component of bending moment may be larger than predicted at the design stage. This is true if the value used in design is based on a period lesser than the design life of the vessel. For example, a ship operating along the North Atlantic route is likely to experience a probable extreme significant wave height of 19.17m over an exposure period of 10 years. Whereas, the same ship is likely to experience a significant wave height of 20.20m over an exposure of 20 years which may be its design life (Ochi

1978). Essentially, the load effect increases with time whereas the strength decreases with time due to corrosion and other environmental degradation of the structure among other factors.

Consequently, a ship designed on this basis may be quite safe when built and put into service, but not so safe when aged, or may be even unsafe. Therefore, it is necessary to investigate the total bending moment and the resistance moment as functions of time.

In this work it is intended to develop a procedure or a working tool to derive the probability distribution of the resistance moment as a function of age taking into consideration of the corrosion wastage. A similar approach to derive the probability distribution of the load effect or total bending moment as a function of age may be derived but will not be elaborated in the present work.

It may be mentioned that the actual failure of a ship will involve progressive failure of components which are also affected by age but the detailed calculations are too complicated at design stage.

2. Formulation of the Overall Problem

The overall problem here is to define statistically the probability of failure, p_f, of the hull girder midship section by yielding at any given age, taken into consideration of the corrosion wastage.

At this point it is desirable to distinguish between two types of variables. The variables such as principal dimensions of ships, plate thicknesses, stiffener scantlings, material yield strength etc. will be termed as Basic Variables, $x_i; i = 1,2,\ldots,n$. On the other hand, still water bending moment, M_S wave-induced bending moment M_W, midship section modulus, Z etc. will be termed as Design Variables which are, in fact, functions of two or more basic variables.

In its most generalised form the overall problem may be expressed as

$$p_f = \Pr[G(\mathbf{X}) \le 0] = \int \ldots \int_{G(x)\le 0} f_x(\mathbf{X})dx \tag{2-1}$$

where

$\mathbf{X} = (x_1, x_2, \ldots, x_n)^T$; vector of n basic variables

and

$G(\mathbf{X}) = 0$ is the limit state equation;
$G(\mathbf{X}) \le 0$ indicates unsafe domain in the n-dimensional basic variable space.

$f_x(\mathbf{X})$ is the joint probability density function (pdf) for the n-dimensional vector **X** of basic variables.

It may be pointed out that $G(\mathbf{X})$ is almost always a nonlinear function of x_i. There are two levels of serious difficulties in finding p_f from this formulation in any realistic structural problem having ten or more basic variables. First difficulty is to derive the joint pdf, $f(\mathbf{X})$ especially allowing for the statistical dependency of variables and the second difficulty is to carry out the multiple integration in a highly nonlinear domain. Limited success has been achieved with relatively small number of basic variables (Melchers, 1999).

Therefore, we would concentrate our efforts on a somewhat restricted formulation in terms of Design variables:

Let us define two design variables:

S = Load effect
R = Structural resistance

In the present problem

$$S = M_T = M_S + M_W \tag{2-2}$$

and

$$R = M_R = Z.\sigma_Y \tag{2-3}$$

Therefore, the limit state equation becomes

$$G(R,S) = R - S \tag{2-4}$$

The integration domain is now a simple linear function in 2-dimensional space.

The probability of failure is now defined as

$$\begin{aligned} p_f &= \Pr[G(R,S) \le 0] \\ &= \Pr[(R-S) \le 0] \\ &= \iint_{(r-s)\le 0} f_{rs}(r,s)\,dr\,ds \end{aligned} \tag{2-5}$$

Strictly speaking the random design variables R and S are not statistically independent; at least the principal dimensions are common basic variables in the calculations of M_S, M_W and Z. But the variance of these basic variables are quite small and consequently dependence of R and S is rather weak.

We hereby assume R and S statistically independent to re-arrange Equation (2-5) as follows

$$p_f = \int_{s=-\infty}^{\infty} \boxed{\int_{r=-\infty}^{r\le s} f_R(r)\,dr}\, f_s(s)\,ds = \int_{r=-\infty}^{\infty} \boxed{\int_{s=-\infty}^{s\ge r} f_S(s)\,ds}\, f_R(r)\,dr$$

or $$p_f = \int_{s=-\infty}^{\infty} F_R(s) f_S(s)\,ds = \int_{r=-\infty}^{\infty} [1 - F_S(r)]\, f_R(r)\,dr \tag{2-6}$$

These are the well-known convolution integrals established long ago in the field of structural reliability analysis.

3. Evaluation of the Probability of Failure, p_f

3.1 Integration of the Convolution Integrals

For a few types of distributions $f_R(r)$ and $f_S(s)$ it is possible to carry out the integration analytically. For example, either both $f_R(r)$ and $f_S(s)$ normal or both log-normal. In case both are normal it may be shown (Melchers 1999) that:

$$p_f = \Phi\left[\frac{-(\mu_R - \mu_S)}{\left(\sigma_R^2 + \sigma_S^2\right)^{1/2}}\right] = \Phi(-\beta) \tag{3-1}$$

where $\Phi(\quad)$ is the standard normal distribution function extensively tabulated in texts.

Here μ_R, σ_R^2 and μ_S, σ_S^2 are the mean and variance of the resistance, R and load effect, S respectively and β is called the safety index and is defined as:

$$\beta = \frac{\mu_R - \mu_S}{\left(\sigma_R^2 + \sigma_S^2\right)^{1/2}} \tag{3-2}$$

Nevertheless, if the distributions of R and S are available the integration of Equation (2-6) may be carried out numerically (Dahlquist and Björk, 1974; Davis and Rabinowitz, 1975) for all types of distribution. Analytical solutions are only possible for normal and log-normal distributions.

3.2 Transformation of the pdfs of R and S into pdfs of more usual Design Variables

In Equation (2-2) load effect, S is shown to be the sum of two other more common design variables M_S and M_W for which it is assumed that the pdf's are available. Then by using the theory of transformation of variables (Ochi, 1978) it can be shown that:

$$f_S(s) = \int_0^\infty f_{M_s}(s-y) f_{M_w}(y)\,dy \quad = \int_0^\infty f_{M_s}(x) f_{M_w}(s-x)\,dx \tag{3-3}$$

where x, y are the dummy variables.

Therefore, once $f_{M_s}(\quad)$ and $f_{M_w}(\quad)$ are derived we may use Equation (3-3) to form $f_S(s)$ to be used in the convolution integral.

Similarly from Equation (2-3) where R is expressed as a product of two common design variables Z and σ_Y we may write:

$$f_R(r) = \int_0^\infty \frac{1}{|y|} f_Z(r/y) f_{\sigma_Y}(y)\,dy \quad = \int_0^\infty \frac{1}{|x|} f_Z(x) f_{\sigma_Y}(r/x)\,dx \tag{3-4}$$

Again x, y are the dummy variables and $f_R(r)$ may be found from Equation (3-4) provided $f_Z(\quad)$ and $f_{\sigma_Y}(\quad)$ are available. Usually the integrations are to be carried out numerically even if $f_{\sigma_Y}(\quad)$ and $f_Z(\quad)$ are normal. Derivation of $f_{\sigma_Y}(\quad)$ and $f_Z(\quad)$ are shown in Section 4.

3.3 Sample Space of a Random Variable

In the present problem if only the magnitudes of still water and wave-induced bending moments are considered irrespective of sagging and hogging conditions then all basic and design variables are non-negative quantities. The sample space is the range of real numbers which the random variable may ever assume. Fortunately, most continuous distribution types useful in the structural reliability analyses have sample spaces $0 \le x < \infty$. Examples include log-normal, Rayleigh, Weibull etc. But some other useful distribution types have the sample space $-\infty < x < \infty$. This category includes the Gumbel's asymptotic Extreme Value distribution Type I and most important of all the Normal or Gaussian distribution.

To avoid x taking an unrealistic negative value it is necessary to use a Truncated distribution for the second category.

3.4 Truncated Distributions

Let the original distribution be:

$$f_X(x) \quad ; \quad -\infty < x < \infty$$

Let the truncated distribution be:

$$f_{X_*}(x) \quad ; \quad x_\ell \le x_* \le x_u$$

where x_u and x_ℓ are the upper and lower limits of the subspace x_*.

By definition of the probability-density-function:

$$\int_{x_\ell}^{x_u} f_{X_*}(x)dx = 1 = \int_{-\infty}^{\infty} f_X(x)dx = \int_{-\infty}^{x_\ell} f_X(x)dx + \int_{x_\ell}^{x_u} f_X(x)dx + \int_{x_u}^{\infty} f_X(x)dx \qquad \text{(a)}$$

or $$\int_{x_\ell}^{x_u} \frac{f_X(x)dx}{F(x_u) - F(x_\ell)} = \int_{x_\ell}^{x_u} f_{X_*}(x)dx = 1 \qquad \text{(b)}$$

where $F_X(\quad)$ is the cumulative distribution function of x.

Finally from (b)

$$\boxed{f_{X_*}(x) = \frac{f_X(x)}{F(x_u) - F(x_\ell)}} \qquad \text{(3-5)}$$

3.5 A Special Case – Truncated Normal Distribution: $0 \le x < \infty$

By introducing a linear transformation of variables, namely $Y = (X - \mu)/\sigma$, it can be easily shown that

$$f_{x_*}(x) = \frac{f_x(x)}{\Phi(\mu/\sigma)} = \frac{1}{\Phi(\mu/\sigma)\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad (0 \le x < \infty) \tag{3-6}$$

where $\Phi(\quad)$ is the standard normal distribution and in the present context $\Phi(\mu/\sigma)$ is quite close to unity.

In Section 4, Equation (3-6) will be used to define the probability density function of the midship section modulus, Z.

4. <u>Derivation of the Probability Density Functions</u> $f_{\sigma_Y}(\quad)$ <u>and</u> $f_Z(\quad)$

To carry out the integrations in Equation (3-4) it is necessary to form the probability density functions $f_{\sigma_Y}(\quad)$ and $f_Z(\quad)$. For the material yield strength it is assumed to be a log-normal distribution given as

$$f_{\sigma_Y}(\sigma_Y) = \frac{1}{\sigma\sqrt{2\pi}\sigma_Y} \exp\left[-\frac{(\ell n\sigma_Y - \mu)^2}{2\sigma^2}\right] \quad (0 \le \sigma_Y \le \infty) \tag{4-1}$$

Here the logarithm of the random variable σ_Y is normally distributed with mean $\mu = E(\ell n\sigma_Y)$ and $\sigma^2 = \mathrm{var}(\ell n\sigma_Y)$.

Unlike $f_{\sigma_Y}(\quad)$ for which μ and σ are readily available, the mean and variance are to be computed to form $f_Z(\quad)$ because $Z = Z(X)$ is a function of a number of basic variables, such as midship section geometry, plate thicknesses and stiffener scantlings. Moreover, $Z(X)$ is a nonlinear function of these variables making it difficult to compute the mean μ_Z and variance σ_Z^2. In the following section approximate expressions are derived for μ_Z and σ_Z^2 to form $f_Z(\quad)$.

4.1 Approximation of the Function $Z(X)$ and its Mean and Variance

The derivation of an accurate expression of $Z(X)$ for a typical midship section where $X = (x_1, x_2, \ldots x_n)^T$ is given in Appendix A.

By a Taylor series expansion of a general multivariate function about the mean values of the independent variables, x_i –

$$Z(x) = Z(\bar{x}) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial Z}{\partial x_i}\Big|_{\bar{x}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} (x_i - \bar{x}_i)(x_j - \bar{x}_j) \frac{\partial^2 Z}{\partial x_i \partial x_j}\Big|_{\bar{x}} + \text{higher order terms} \tag{4-2}$$

This series is approximated by its linear form as

$$Z(x) \cong Z(\bar{x}) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial Z}{\partial x_i}\Big|_{\bar{x}} \tag{4-3}$$

Ivanov (2001a) and the present author noted that the contributions of the second and third terms in subsequent calculation of mean and variance of Z is well below 1%.

From this approximate expression of Z the mean and variance are:

$$\text{Mean of } Z = \mu_Z = Z(\bar{x}) \tag{4-4}$$

and

$$\text{Variance of } Z = \sigma_Z^2 = \sum_{i=1}^{n} a_i^2 \sigma_{x_i}^2 \tag{4-5}$$

where $a_i = \frac{\partial Z}{\partial x_i}\Big|_{\bar{x}} =$ partial derivative of Z with respect to x_i and evaluated at the mean value of $x_i = \bar{x}_i$. It is to be noted that Eqn (4-5) is valid only if all x_i are statistically independent, which is assumed in this analysis.

Eqns (4-4) and (4-5) are commonly known as First Order Second Moment (FOSM) approximation and is an accepted method. To evaluate the variance, σ_Z^2 it is necessary to calculate the partial derivatives $\frac{\partial Z}{\partial x_i}\Big|_{\bar{x}}$ with respect to all x_i.

A method has been given in Appendix B to derive expressions for these derivatives.

Ivanov (1982, 1984, 1987, 1991) has shown that the shipbuilding structural profiles follow normal distribution. It is assumed that the midship section modulus also follow the normal distribution. The truncated version is:

$$f_z(z) = \frac{1}{\Phi(\mu_z / \sigma_z)\sqrt{2\pi}\sigma_z} \exp\left[-\frac{(z - \mu_z)^2}{2\sigma_z^2} \right] \quad (0 \le z \le \infty) \tag{4-6}$$

Since the midship section modulus is a function of over 20 other random variables, this assumption is also validated by the Central Limit theorem.

Finally, substituting Equations (4-1) and (4-6) in Equation (4-5) the integration may be carried out numerically to obtain $f_R(r)$, the probability density function for the resistance moment, R.

Similarly, knowing the pdfs of $f_{M_s}(\quad)$ and $f_{M_w}(\quad)$ it is possible to complete the integration in Equation (3-4) to derive $f_S(s)$, the probability density function for the load effect or total bending moment amidships.

And finally, Equation (2-6) may be used to estimate the probability of failure, p_f of the hull girder midship section by yielding.

5. <u>Effect of Corrosion Wastage</u>

After the breakdown of protective coating the effect of corrosion is to reduce the thicknesses of shell plates and stiffeners. This will, in turn, alter the Equations (4-4) and (4-5) which are based on mean and variance of the basic variables x_i.

The rates of corrosion losses are discussed in <u>Appendix C</u>. The corrosion rates which are expressed as its mean and variance are used to calculate the time dependent mean and variance of x_i as follows.

5.1 Mean and Variance of Shell Plate Thicknesses

Based on empirical corrosion model where the mean and standard deviation are given as average annual rates (mm/yr) the time-dependent values of the deck, tanktop and outer bottom plating, t_i, are:

Mean: $$t_i(T) = t_i(0) - \delta_{V,T} \qquad (5\text{-}1)$$

where i = 1,2,3 represent deck, tanktop and outer bottom respectively, and

T = exposure time in years after coating breakdown

$$\delta_{V,T} = T \cdot \delta_{V,a} \qquad (5\text{-}2)$$

$\delta_{V,a}$ is the average corrosion loss per year in the vertical direction and $\delta_{V,T}$ is the total loss over T years.

Variance: $$\sigma^2_{t_i}(T) = \sigma^2_{t_i}(0) + (\sigma_{V,T})^2 \qquad (5\text{-}3)$$

where $$\sigma_{V,T} = T \cdot \sigma_{V,a} \qquad (5\text{-}4)$$

and $\sigma_{V,a}$ and $\sigma_{V,T}$ are the standard deviations of corrosion loss per year and total over T years respectively. Also $t_i(0)$ and $\sigma^2_{t_i}(0)$ are the mean and variance of t_i at T=0. The corresponding equations for the side shell are:

$$t_4(T) = t_4(0) - \delta_{W,T} \tag{5-5}$$

and $$\sigma_{t_4}^2(T) = \sigma_{t_4}^2(0) + (\sigma_{W,T})^2 \tag{5-6}$$

again $$\delta_{W,T} = T \cdot \delta_{W,a} \tag{5-7}$$

and $$\sigma_{W,T} = T\sigma_{W,a} \tag{5-8}$$

The subscript 'w' indicates corrosion loss in horizontal direction. In this formulation distinction is made for corrosion losses in the vertical and horizontal direction (Ivanov 2001a, b) but this distinction may be ignored. Then, Equations (5-1) to (5-4) will be valid for all plating including the bilge corner which is part vertical, part horizontal.. Alternatively, based on Melchers phenomenological corrosion model which gives mean and variance directly as functions of time the following changes are to be made:

$$\delta_{V,T} = \delta_{W,T} = c_i(t) \tag{5-9}$$

where c_i; i=1-4 is explained in Appendix C.

Similarly, $$\sigma_{V,T} = \sigma_{W,T} = \sigma_c(t) \tag{5-10}$$

Notice that $t \equiv T$ is the exposure period in years followed by the coating breakdown.

In the following it is assumed that $\delta_{V,a} = \delta_{W,a} = \delta_a$ and $\sigma_{V,a} = \sigma_{W,a} = \sigma_a$.

5.2 Mean and Variance of Four Basic Variables related to Stiffeners

In the present work the stiffeners are defined by four 'basic' variables: cross-sectional area a_s, centroidal moment of inertia i_s, and location of the centroid x_c and y_c. The number of stiffeners n are treated as fixed. But strictly speaking these geometric properties are not the 'basic' variables for the stiffeners and each one is a function of other variables. Those are, in fact, web height and thickness, flange width and thickness and various fillet radii of the rolled sections. Ivanov (2001a, 2003b) from the American Bureau of Shipping gave a detailed account of accurately computing all geometric properties of various shipbuilding structural profiles. Ivanov also presented a method of calculating the time-dependent mean and variance of these geometric properties based on empirical values for corrosion losses. More recently Chowdhury (2006) has revisited Ivanov's method to predict probability of reduction of geometric properties of structural profiles as the ship ages. In addition to Ivanov's empirical values of corrosion loss based on past measurements, Chowdhury has introduced the more recent corrosion models of Paik et. al. (2003) and Melchers (2001). Readers should consult these references for details. However, for the sake of completeness the procedure to calculate time dependent mean and variance of a_s, i_s, x_c, y_c, of inverted angle or L-profiles is given in Appendix D. Appendix E gives the time dependent values of the stiffener basic variables.

5.3 Summary of Calculation Procedures for $f_z(Z)$

1. Read the initial (T=0) inputs: the mean values and variances or coefficients of variances (c.o.v) of the geometric variables (B, D, h), the shell plate thicknesses (t_1-

t_5) and six scantling variables (h, b, t_w, t_f, R_1 and R_2) for each of the five sets of angle bar stiffeners.

2. Modify the above mean values and variances by incorporating the corrosion losses. For shell plate thicknesses use the equations developed in section 5.1 and for stiffeners use Appendix E. At this stage ignore reduction in the geometric variables (B, D, h) due to corrosion wear.

3. Calculate the stiffener geometric properties and their mean values and variances using Appendix D and Appendix E.

4. Calculate the midship section modulus and other geometric properties together with their partial derivatives using Appendix A and Appendix B respectively.

5. Calculate the mean and variance of the section modulus using Equations (4-4) and (4-5). Finally construct the time-dependent probability density function of the midship section modulus using Equation (4-6).

6. By using the time dependent probability density functions, $f_Z(Z)$, it is possible to predict the probabilities of reduction of midship section modulus at any period of exposure, T.

The procedures may be repeated for any value of T.

Sample calculations are summarised in Appendix F.

6. <u>Results and Discussion</u>

The results of a sample calculation are given in Appendix F. The entire calculations are very conveniently carried out by Excel spreadsheets. Once the formulas developed in the test are entered into the spreadsheet for a selected midship section it is only necessary to change the exposure period, T, for any chosen corrosion model. We used T = 5, 10, 15 and 20 years as exposure period. In this exercise we tried three corrosion models in order of severity; Paik et al. 'severe', Ivanov and Melchers' model. The mean values and the standard deviations of the midship section modulus are presented in Table F2.

It is observed that almost all geometric properties such as cross-sectional area, first and second moment of areas and section modulus are *linear* functions of the exposure period, T. This is true both for the stiffeners cross-section and the midship section. However, the standard deviations of these geometric properties are *not linear* functions of T; instead it increases more rapidly with exposure period.

Although the data in Table F2 are useful in writing the truncated normal distribution of Z as required in Eqn (4-6) we went one step further. Using the truncated distribution it is easy to predict the probabilities or *'likelihood'* of reduction of magnitudes of midship section modulus with exposure period, T. These results are presented in Table F3. For practicing engineers, for which this methodology is proposed, it is considered more convenient to make this prediction in qualitative terms, such as 'unlikely', 'likely' or 'most likely' scenarios.

In future, it will be worth finding empirical relationships between the geometrical properties and the corrosion data and period of exposure. The main issue here will be to investigate whether the scantlings of the midship section affect this relationship.

In addition, it should also be useful to examine the issues of accuracy of this model against more accurate nonlinear models. But with any such nonlinear model there will be difficulties in terms of numerical solution of constrained optimisation problems with too many variables (Melchers, 1999).

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APPENDIX A

Derivation of the Elastic Section Modulus, Z

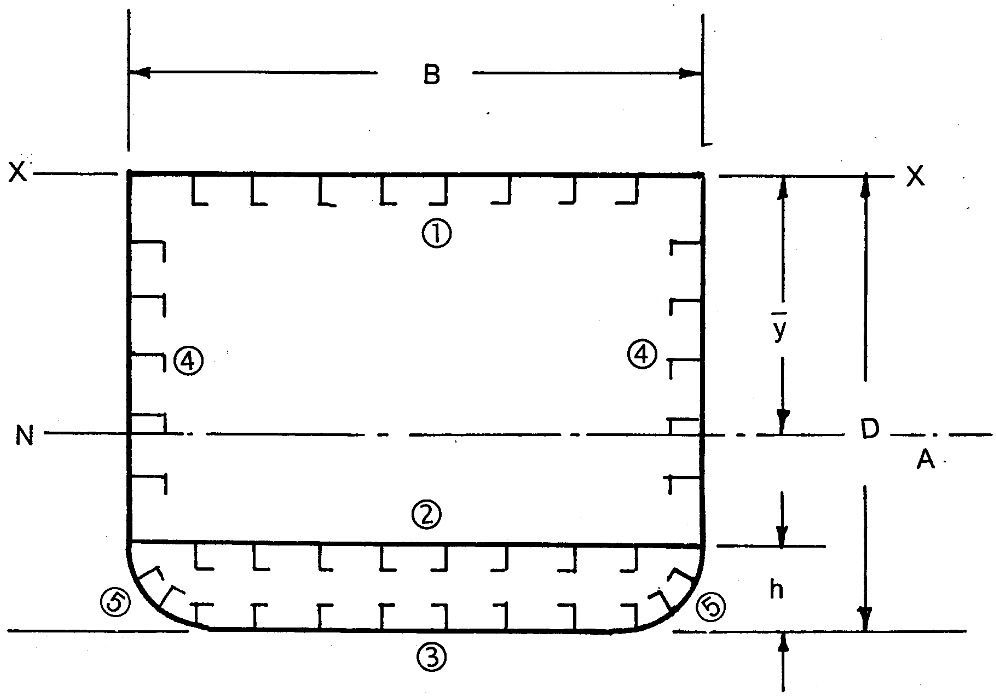

Figure A1: Idealized midship section of a ship

The above figure represents a simplified midship section of a ship showing the plates and stiffeners. The bilge corner is idealised as quarter circles with radius, h (Figure A2).

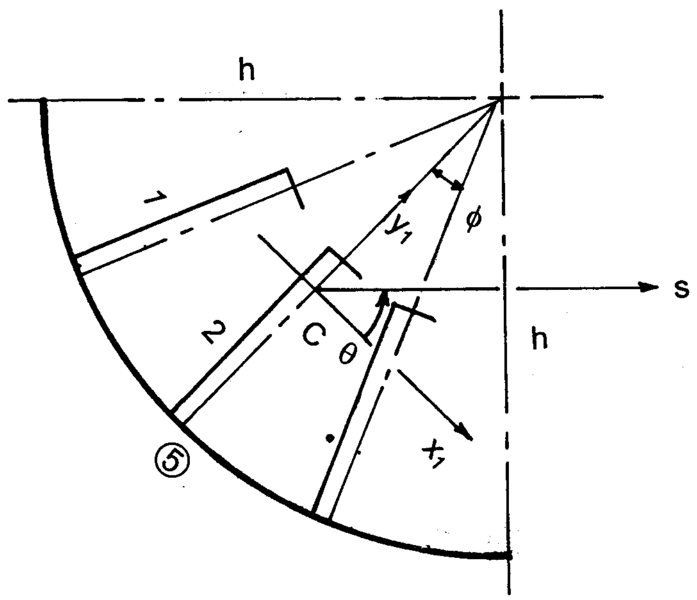

Figure A2: Stiffeners at the bilge corner

We define the basic variables, x_i as follows:

Geometric variables: $B=x_1$, $D=x_2$, $h=x_3$.

We now label the deck, inner bottom/tanktop, outer bottom, two side shells and two bilge corners as regions 1, 2, 3, 4 and 5 respectively. These region numbers will be used as subscripts for identification purposes.

Plate thickness: $t_1=x_4$, $t_2=x_5$, $t_3=x_6$, $t_4=x_7$ $t_5=x_8$

The geometric properties of an inverted angle section is derived in Appendix D.

Stiffeners cross sectional areas: $a_{s1}=x_9$, $a_{s2}=x_{10}$, $a_{s3}=x_{11}$, $a_{s4}=x_{12}$, $a_{s5}=x_{13}$

Referred to Figure D1 in Appendix D:

Stiffener centroidal locations: $y_{c1}=x_{14}$, $y_{c2}=x_{15}$, $y_{c3}=x_{16}$, $x_{c4}=x_{17}$, $y_{c5}=x_{18}$

Note: x_{c4} will be considered negative if the flange tip of the shell stiffeners are pointing upward [Equation (A-2) and (A-3)].

Stiffener centroidal moments of inertia: $i_{s1}=x_{19}$, $i_{s2}=x_{20}$, $i_{s3}=x_{21}$, $i_{s4}=x_{22}$, $i_{s5}=x_{23}$

Here i_{s5} is the average value of the centroidal moments of inertia of stiffeners at the bilge corner.

In addition, we also specify the number of stiffeners in each region as n_1, n_2, n_3, n_4 and n_5 respectively on deck, tanktop, inner bottom, <u>one</u> side shell and <u>one</u> bilge corner respectively. In the present formulation $i = 1-23$, altogether 23 basic variables defining the midship section modulus.

<u>Midship Cross-Sectional Area, A</u>

$$\begin{aligned} A &= B(t_1 + t_2 + t_3) + 2Dt_4 - 2h(t_3 + t_4) + \pi h t_5 \\ &\quad + n_1 a_{s1} + n_2 a_{s2} + n_3 a_{s3} + 2n_4 a_{s4} + 2n_5 a_{s5} \end{aligned} \tag{A-1}$$

<u>First Moment of Area about X-X</u>

The X-X axis passes through the centre of deck plating (Figure A1)

$$\begin{aligned} V_X &= Bt_2(D-h) + Dt_3(B-2h) + (D-h)^2 t_4 \\ &\quad + \pi h t_5\left[D - h\left(1 - \frac{2}{\pi}\right)\right] + n_1 a_{s1} y_{c1} + n_2 a_{s2}(D - h + y_{c2}) \\ &\quad + n_3 a_{s3}(D - y_{c3}) + 2n_4 a_{s4}\left[\frac{D-h}{2} + x_{c4}\right] \\ &\quad + 2n_5 a_{s5}\left[(D-h) + (h - y_{c5}) f_1(\phi)\right] \end{aligned} \tag{A-2}$$

where

$$f_1(\phi) = \frac{1}{n_5}\sum_{j=1}^{n_5} \mathrm{Sin}\, j\phi; \quad \phi = \frac{90^\circ}{n_5 + 1} \tag{A-3}$$

Moment of Inertia about X-X Axis, I_x

$$
\begin{aligned}
I_x = {} & \frac{Bt_1^3}{12} + \left\{\frac{Bt_2^3}{12} + Bt_2(D-h)^2\right\} + \left\{\frac{(B-2h)t_3^3}{12} + (B-2h)t_3D^2\right\} \\
& + \frac{2}{3}(D-h)^3 t_4 + \left\{\pi h^3 t_5\left(\frac{1}{2} - \frac{4}{\pi^2}\right) + \pi h t_5\left[D - h\left(1 - \frac{2}{\pi}\right)\right]^2\right\} \\
& + n_1\left(i_{s1} + a_{s1}y_{c1}^2\right) + n_2\left[i_{s2} + a_{s2}(D - h + y_{c2})^2\right] \\
& + n_3\left[i_{s3} + a_{s3}(D - y_{c3})^2\right] + 2n_4\left[i_{s4} + a_{s4}\left\{\frac{2n_4+1}{6(n_4+1)}(D-h)^2\right.\right. \\
& \left.\left. + (D-h)x_{c4} + x_{c4}^2\right\}\right] + 2n_5\left[i_{s5} + a_{s5}\left\{(D-h)^2\right.\right. \\
& \left.\left. + 2(D-h)(h - y_{c5})f_1(\phi) + (h - y_{c5})^2 f_2(\phi)\right\}\right]
\end{aligned}
\tag{A-4}
$$

where

$$
i_{s5} = i_{x1}f_2(\phi) + i_{y1}f_3(\phi) - i_{x1y1}f_4(\phi) = \text{Average of } n_s \text{ stiffeners} \tag{A-5}
$$

also

$$
f_2(\phi) = \frac{1}{n_5}\sum_{j=1}^{n_5} \mathrm{Sin}^2 j\phi, \quad f_3(\phi) = \frac{1}{n_5}\sum_{j=1}^{n_5} \mathrm{Cos}^2 j\phi, \quad f_4(\phi) = \frac{1}{n_5}\sum_{j=1}^{n_5} \mathrm{Sin}\ 2j\phi, \tag{A-6}
$$

Location of the Neutral Axis, NA

$$
\bar{y} = y_d = \frac{V_x}{A} \quad \text{and} \quad y_k = D - \bar{y} \tag{A-7}
$$

Moment of Inertia about the Neutral Axis, I_{NA}

$$
I_{NA} = I_x - A\bar{y}^2 = I_x - \frac{V_x^2}{A} \tag{A-8}
$$

Midship Section Modulus, Z

There are two values of Z corresponding to y_d and y_k; but in practice deck-side section modulus is usually smaller of the two. Hence we choose

$$
Z_d = Z = \frac{I_{NA}}{y_d} = \frac{AI_x}{V_x} - V_x \tag{A-9}
$$

where A, V_x and I_x are all defined in terms of the basic variables.

APPENDIX B

Partial Derivatives of Z With Respect to Basic Variables, x_i

Differentiating Equations (A-9) partially with respect to any basic variable, x_i we obtain:

$$\frac{\partial Z}{\partial x_i} = Z_i' = \frac{1}{V_x}\left(AI_{x,i}' + I_x A_i'\right) - cV_{x,i}' \quad \text{(B-1)}$$

where

$$c = 1 + \frac{AI_x}{V_x^2} \quad \text{(B-2)}$$

and the prime indicates partial differentiation with respect to a basic variable, x_i.

Therefore, in addition to the three geometric properties A, V_x, I_x all we need are their partial derivatives with respect to any basic variable, x_i to calculate Z_i. These derivatives are listed below.

Partial Derivatives of A

$$\frac{\partial A}{\partial B} = A_1' = t_1 + t_2 + t_3 \quad \text{(B-3)} \qquad \frac{\partial A}{\partial D} = A_2' = 2t_4 \quad \text{(B-4)}$$

$$\frac{\partial A}{\partial h} = A_3' = -2(t_3 + t_4) + \pi t_5 \quad \text{(B-5)} \qquad \frac{\partial A}{\partial t_1} = A_4' = B \quad \text{(B-6)}$$

$$\frac{\partial A}{\partial t_2} = A_5' = B \quad \text{(B-7)} \qquad \frac{\partial A}{\partial t_3} = A_6' = B - 2h \quad \text{(B-8)}$$

$$\frac{\partial A}{\partial t_4} = A_7' = 2(D - h) \quad \text{(B-9)} \qquad \frac{\partial A}{\partial t_5} = A_8' = \pi h \quad \text{(B-10)}$$

$$\frac{\partial A}{\partial a_{s1}} = A_9' = n_1 \quad \text{(B-11)} \qquad \frac{\partial A}{\partial a_{s2}} = A_{10}' = n_2 \quad \text{(B-12)}$$

$$\frac{\partial A}{\partial a_{s3}} = A_{11}' = n_3 \quad \text{(B-13)} \qquad \frac{\partial A}{\partial a_{s4}} = A_{12}' = 2n_4 \quad \text{(B-14)}$$

$$\frac{\partial A}{\partial a_{s5}} = A_{13}' = 2n_5 \quad \text{(B-17)} \qquad \frac{\partial A}{\partial y_{c1}} = A_{14}' = 0 \quad \text{(B-16)}$$

Similarly,

$$A_{15}' = 0 \quad \text{(B-17)} \qquad A_{16}' = 0 \quad \text{(B-18)} \qquad A_{17}' = 0 \quad \text{(B-19)}$$

$$A_{18}' = 0 \quad \text{(B-20)}$$

$$\frac{\partial A}{\partial i_{s1}} = A_{19}' = 0 \quad \text{(B-21)}$$

$A'_{20} = 0$ (B-22) $A'_{21} = 0$ (B-23) $A'_{22} = 0$ (B-24)

$A'_{23} = 0$ (B-25)

Partial Derivatives of V_X

$$\frac{\partial V_X}{\partial B} = V'_{X,1} = (D-h)t_2 + Dt_3 \quad \text{(B-26)}$$

$$\frac{\partial V_X}{\partial D} = V'_{X,2} = Bt_2 + (B-2h)t_3 + 2(D-h)t_4 + \pi h t_5 + n_2 a_{s2} + n_3 a_{s3} + n_4 a_{s4} + 2n_5 a_{s5} \quad \text{(B-27)}$$

$$\frac{\partial V_X}{\partial h} = V'_{X,3} = -[Bt_2 + 2Dt_3 + 2(D-h)t_4] + \pi t_5\left[D - 2\left(1-\frac{2}{\pi}\right)h\right] - [n_2 a_{s2} + n_4 a_{s4} + 2n_5 a_{s5}\{1 - f_1(\phi)\}] \quad \text{(B-28)}$$

$$\frac{\partial V_X}{\partial t_1} = V'_{X,4} = 0 \quad \text{(B-29)} \qquad \frac{\partial V_X}{\partial t_2} = V'_{X,5} = B(D-h) \quad \text{(B-30)}$$

$$\frac{\partial V_X}{\partial t_3} = V'_{X,5} = D(B-2h) \quad \text{(B-31)} \qquad \frac{\partial V_X}{\partial t_4} = V'_{X,7} = (D-h)^2 \quad \text{(B-32)}$$

$$\frac{\partial V_X}{\partial t_5} = V'_{X,8} = \pi h\left[D - h\left(1-\frac{2}{\pi}\right)\right] \quad \text{(B-33)}$$

$$\frac{\partial V_X}{\partial a_{s1}} = V'_{X,9} = n_1 y_{c1} \quad \text{(B-34)} \qquad \frac{\partial V_X}{\partial a_{s2}} = V'_{X,10} = n_2(D - h + y_{c2}) \quad \text{(B-35)}$$

$$\frac{\partial V_X}{\partial a_{s3}} = V'_{X,11} = n_3(D - y_{c3}) \quad \text{(B-36)} \qquad \frac{\partial V_X}{\partial a_{s4}} = V'_{X,12} = 2n_4\left(\frac{D-h}{2} + x_{c4}\right) \quad \text{(B-37)}$$

$$\frac{\partial V_X}{\partial a_{s5}} = V'_{X,13} = 2n_5[(D-h) + (h - y_{c5})f_1(\phi)] \quad \text{(B-38)}$$

$$\frac{\partial V_X}{\partial y_{c1}} = V'_{X,14} = n_1 a_{s1} \quad \text{(B-39)} \qquad \frac{\partial V_X}{\partial y_{c2}} = V'_{X,15} = n_2 a_{s2} \quad \text{(B-40)}$$

$$\frac{\partial V_X}{\partial y_{c3}} = V'_{X,16} = -n_3 a_{s3} \quad \text{(B-41)} \qquad \frac{\partial V_X}{\partial x_{c4}} = V'_{X,17} = 2n_4 a_{s4} \quad \text{(B-42)}$$

$$\frac{\partial V_X}{\partial y_{c5}} = V'_{X,18} = -2n_5 a_{s5} f_1(\phi) \quad \text{(B-43)} \qquad \frac{\partial V_X}{\partial i_{s1}} = V'_{X,19} = 0 \quad \text{(B-44)}$$

Similarly,

$$V'_{X,20} = 0 \qquad \text{(B-45)} \qquad V'_{X,21} = 0 \qquad \text{(B-46)}$$

$$V'_{X,22} = 0 \qquad \text{(B-47)} \qquad V'_{X,23} = 0 \qquad \text{(B-48)}$$

Partial Derivatives of I_X

$$\frac{\partial I_X}{\partial B} = I'_{X,1} = \frac{1}{12}\left(t_1^3 + t_2^3 + t_3^3\right) + (D-h)^2 t_2 + D^2 t_3 \qquad \text{(B-49)}$$

$$\begin{aligned}\frac{\partial I_X}{\partial D} = I'_{X,2} &= 2Bt_2(D-h) + 2(B-2h)Dt_3 + 2(D-h)^2 t_4 \\ &+ 2\pi h t_5\left[D - h\left(1-\frac{2}{\pi}\right)\right] + 2n_2 a_{s2}(D-h+y_{c2}) \\ &+ 2n_3 a_{s3}(D-y_{c3}) + 2n_4 a_{s4}\left\{\frac{2n_4+1}{3(n_4+1)}(D-h) + x_{c4}\right\} \\ &+ 4n_5 a_{s5}\{(D-h) + (h-y_{c5})f_1(\phi)\}\end{aligned} \qquad \text{(B-50)}$$

$$\begin{aligned}\frac{\partial I_X}{\partial h} = I'_{X,3} &= -2(D-h)Bt_2 - 2\left(\frac{t_3^3}{12} + D^2 t_3\right) - 2(D-h)^2 t_4 \\ &+ \pi t_5\left\{3\left(\frac{1}{2}-\frac{4}{\pi^2}\right)h^2 + \left[D^2 - 4\left(1-\frac{2}{\pi}\right)Dh + 3\left(1-\frac{2}{\pi}\right)^2 h^2\right]\right\} \\ &- 2n_2 a_{s2}(D-h+y_{c2}) - 2n_4 a_{s4}\left\{\frac{2n_4+1}{3(n_4+1)}(D-h) + x_{c4}\right\} \\ &+ 4n_5 a_{s5}\{(D-2h+y_{c5})f_1(\phi) + (h-y_{c5})f_2(\phi) - (D-h)\}\end{aligned} \qquad \text{(B-51)}$$

$$\frac{\partial I_X}{\partial t_1} = I'_{X,4} = \frac{1}{4}Bt_1^2 \qquad \text{(B-52)} \qquad \frac{\partial I_X}{\partial t_2} = I'_{X,5} = B\left[\frac{t_2^2}{4} + (D-h)^2\right] \qquad \text{(B-53)}$$

$$\frac{\partial I_X}{\partial t_3} = I'_{X,6} = (B-2h)\left[\frac{t_3^2}{4} + D^2\right] \qquad \text{(B-54)} \qquad \frac{\partial I_X}{\partial t_4} = I'_{X,7} = \frac{2}{3}(D-h)^3 \qquad \text{(B-55)}$$

$$\frac{\partial I_X}{\partial t_5} = I'_{X,8} = \pi h\left[\left(\frac{1}{2}-\frac{4}{\pi^2}\right)h^2 + \left\{D - h\left(1-\frac{2}{\pi}\right)\right\}^2\right] \qquad \text{(B-56)}$$

$$\frac{\partial I_X}{\partial a_{s1}} = I'_{X,9} = n_1 y_{c1}^2 \qquad \text{(B-57)} \qquad \frac{\partial I_X}{\partial a_{s2}} = I'_{X,10} = n_2(D-h+y_{c2})^2 \qquad \text{(B-58)}$$

$$\frac{\partial I_X}{\partial a_{s3}} = I'_{X,11} = n_3 \left(D - y_{c3}\right)^2 \qquad \text{(B-59)}$$

$$\frac{\partial I_X}{\partial a_{s4}} = I'_{X,12} = 2n_4 \left\{ \frac{2n_4 + 1}{6(n_4 + 1)} (D - h)^2 + (D - h) x_{c4} + x_{c4}^2 \right\} \qquad \text{(B-60)}$$

$$\frac{\partial I_X}{\partial a_{s5}} = I'_{X,13} = 2n_5 \left\{ (D - h)^2 + 2(D - h)(h - y_{c5}) f_1(\phi) + (h - y_{c5})^2 f_2(\phi) \right\} \qquad \text{(B-61)}$$

$$\frac{\partial I_X}{\partial y_{c1}} = I'_{X,14} = 2n_1 a_{s1} y_{c1} \qquad \text{(B-62)}$$

$$\frac{\partial I_X}{\partial y_{c2}} = I'_{X,15} = 2n_2 a_{s2} \left(D - h + y_{c2}\right) \qquad \text{(B-63)}$$

$$\frac{\partial I_X}{\partial y_{c3}} = I'_{X,16} = -2n_3 a_{s3} \left(D - y_{c3}\right) \qquad \text{(B-64)}$$

$$\frac{\partial I_X}{\partial x_{c4}} = I'_{X,17} = 2n_4 a_{s4} \left(D - h + 2x_{c4}\right) \qquad \text{(B-65)}$$

$$\frac{\partial I_X}{\partial y_{c5}} = I'_{X,18} = -4n_5 a_{s5} \left[(D - h) f_1(\phi) + (h - y_{c5}) f_2(\phi) \right] \qquad \text{(B-66)}$$

$$\frac{\partial I_X}{\partial i_{s1}} = I'_{X,19} = n_1 \qquad \text{(B-67)}$$

$$\frac{\partial I_X}{\partial i_{s2}} = I'_{X,20} = n_2 \qquad \text{(B-68)}$$

$$\frac{\partial I_X}{\partial i_{s3}} = I'_{X,21} = n_3 \qquad \text{(B-69)}$$

$$\frac{\partial I_X}{\partial i_{s4}} = I'_{X,22} = 2n_4 \qquad \text{(B-70)}$$

$$\frac{\partial I_X}{\partial i_{s5}} = I'_{X,23} = 2n_5 \qquad \text{(B-71)}$$

APPENDIX C

Probabilistic Models for Corrosion Loss

For time-dependent reliability assessment of ship structures it is essential to have mathematical models which provide mean and standard deviation of corrosion losses as function of exposure time.

Work on corrosion loss prediction may be broadly classified into two groups:

(1) Empirical model – based on past data or measurements of corrosion wastage.
(2) Phenomenological model – derived with some physical basis of the corrosion mechanisms involved.

Some early empirical models include Southwell and Alexander (1970) and Reinhart and Jenkins (1972). But the most recent work which is highly relevant to this study is due to Paik, et al. (2004) and is adopted here.

Early phenomenological models include Evans (1966), Cernov and Ponomarenko (1991), among others. Most recently Melchers (2003a and 2003b) suggested a more refined model to take into account all probable corrosion phases – kinetic, diffusion, and two anaerobic corrosion processes. In this paper Melchers also discussed all known factors that affect corrosion behaviour but in the end retained the effect of seawater temperature as the most predominant factor. This model is more complicated and include nonlinear nature of corrosion loss versus exposure time in phases 2 and 3.

Empirical Corrosion Wastage Model

Paik et al., 2004 proposed a model based on extensive measurements of seawater ballast tanks of large oil tankers and bulk carriers. In this model it was assumed that corrosion begins after the break-down of protective coating. According to this source a 5-year coating life is considered to represent undesirable situation, that is, poor handling and maintenance. More realistic coating life may be around 10 years. But accurate estimate of coating life is not considered here; the main issue is the period of exposure, T years following the coating breakdown.

After analysing data collected from nearly 2000 locations the final recommendations are as follows:

The average or most probable values are:

Coating Life, T_c (yr)	Mean Thickness Loss (mm/yr)	Standard Deviation (mm/yr)
5	0.0466	0.0378
7.5	0.0579	0.0479
10	0.0823	0.0758

The upper bound representing 95% band the possible severe corrosion values are:

Coating Life, T_c (yr)	Thickness Loss (mm/yr)	Standard Deviation (mm/yr)
5	0.1469	0.0314
7.5	0.1938	0.0426
10	0.2894	0.0644

A few observations about this empirical model.

(1) In this model the corrosion loss was mostly measured by the technique of ultrasonic thickness measurements. This implies that the measurements were made at several points within a single plating and the average corrosion loss is recorded.

(2) The measured data permitted a linear model implying that the annualised corrosion rate is constant and not a function of exposure period as is the case for the other model.

(3) It is observed that the average corrosion rate approximately follows Weibull distribution.

(4) Some classification societies nowadays specify the Nominal Design Corrosion Values (NDCV). For example, ABS (2000) suggests that the nominal design corrosion margin for coated seawater ballast tank plates needs to be in the range of 1.0-1.5mm for a 20-25 year service life, with routine coating maintenance assumed. This corresponds to the most probable value proposed here. But for severe corrosion cases, NDCV may need to be in the range of 3.5-4.5mm as reflected in the values given above for this condition.

<u>Phenomenological Corrosion Wastage Model</u>

Melcher's model is proposed for general corrosion of mild and low alloy steels under fully aerated at-sea immersion conditions as measured by weight loss. The work reported herein is based entirely on corrosion weight losses reported for coupons and may not be true indication of corrosion in ship structures. Moreover, the numerical coefficients are derived on the basis of a limited number of observations over a short time period. As such, this model is not appropriate in the present context.

However, there are not many published data on the true phenomenological corrosion models which are readily usable. Therefore, this model will be used with some modifications as shown below.

The notion of coating protection period will be maintained and use the data from this model for the entire design life of the vessel. This is not unreasonable because within a short (3-4 year) period the corrosion loss stabilises to a uniform rate under anaerobic condition.

The corrosion loss-exposure time model proposed is shown in the figure below.

The model is defined by six parameters: three slopes r_o, r_a and r_s; period of transition from aerobic to anaerobic phase and two loss measurements c_a and c_s. These six parameters are sufficient to uniquely define the curve, but additional equations had to be derived to calculate the corrosion loss at any exposure period.

In this model phases are: 1-kinetic, 2-oxygen diffusion, 3-initial SRB (sulphate reducing bacteria) and 4-final and steady state SRB controlled phase.

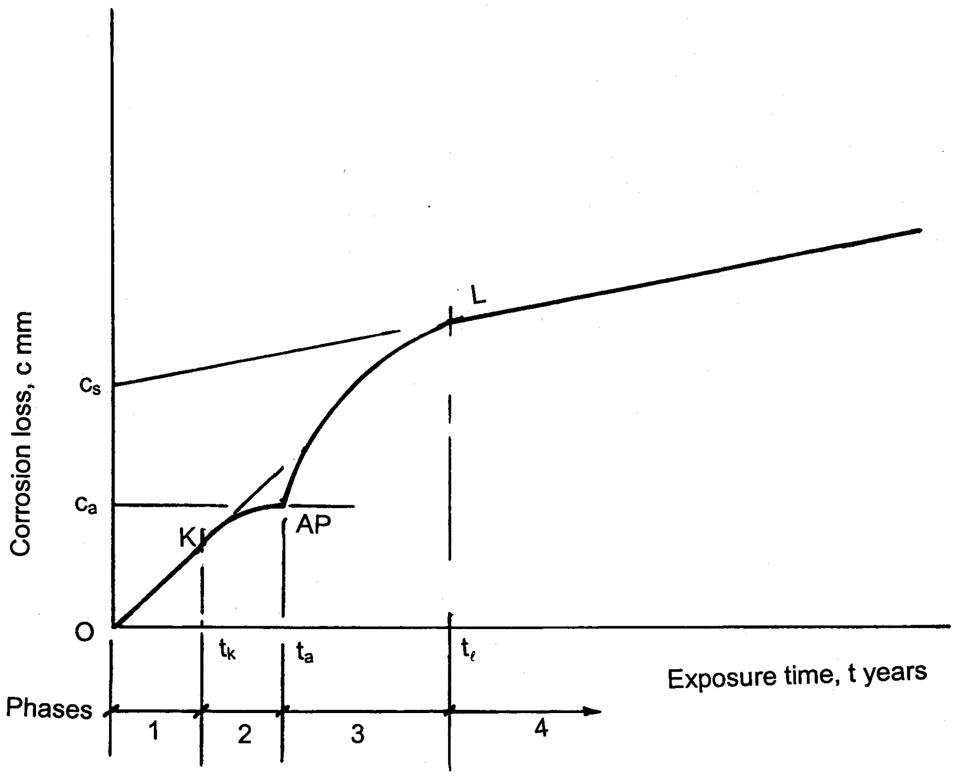

Figure C1: Phenomenological corrosion model (Melchers 2003a)

By analysing the limited amount of available data and some field measurements Melchers gave the following expressions to determine the six parameters:

$$\left.\begin{array}{ll} t_a = 6.61\exp(-0.088T) & ; \quad c_a = 0.32\exp(-0.038T) \\ r_o = 0.076\exp(0.054T) & ; \quad r_a = 0.066\exp(0.061T) \\ c_s = 0.075 + 5{,}678T^{-4} & ; \quad r_s = 0.045\exp(0.017T) \end{array}\right\} \qquad \text{(C-1)}$$

where T=seawater temperature, °C.

But as mentioned above these equations are not sufficient to calculate the mean (total) corrosion loss, c(t) mm, at any exposure period t, year. The present author has developed the additional equations on the assumption that second order polynomials may be used to represent nonlinear phases 2 and 3.

<u>Summary of Calculations</u>

After calculating the six parameters for any given seawater temperature, T°C the mean corrosion losses may be calculated from the following equations.

$$
\left.\begin{array}{ll}
c_1(t)=r_0 t\,; & 0 \le t \le t_k \\
c_2(t)=c_a-\left(\dfrac{t_a}{t_a-t_k}\right)\left[1-2\dfrac{t}{t_a}+\dfrac{t^2}{t_a^2}\right]\dfrac{r_0 t_a}{2}\,; & t_k \le t \le t_a \\
c_3(t)=c_a+\left(\dfrac{t_a}{t_\ell-t_a}\right)\left[2\bar{r}\left(\dfrac{t}{t_a}-1\right)-(r_a-r_s)\left(\dfrac{t^2}{t_a^2}-1\right)\right]\dfrac{t_a}{2}\,; & t_k \le t \le t_\ell \\
c_4(t)=c_s+r_s t\,; & t \ge t_\ell
\end{array}\right\} \tag{C-2}
$$

Here the subscripts on c(t) indicate the corrosion phase. In the above equations the expressions for the intermediate quantities in terms of six parameters are:

$$
\left.\begin{array}{l}
t_k = 2\dfrac{c_a}{r_0}-t_a \\
t_\ell = \left(\dfrac{t_a}{r_a-r_s}\right)\left[(c_s-c_a)\dfrac{2}{t_a}+(r_a+r_s)\right] \\
\bar{r} = \dfrac{t_\ell}{t_a} r_a - r_s
\end{array}\right\} \tag{C-3}
$$

The standard deviation as a function of time is given by:

$$
\sigma_c(t)=(0.006+0.0003T)\frac{t}{t_a} \tag{C-4}
$$

In all these equations, t = net exposure period in year after coating breakdown.

It should be noted that in the main text ‘T’ is used to represent exposure time in years not ‘t’ as is used here.

The main difference of this model with the empirical model is that in phases 2 and 3 the corrosion rates (mm/yr) are not constant. Moreover, the phenomenological model suggests much larger rates in phase 1 and at the beginning of phase 3; onset of anaerobic phase. But the rate is stabilised to a constant rate after phase 3 and continues indefinitely.

It is interesting to make some comparison of the two models. Assuming the seawater temperature to be 15°C, the coating life 5 years and the ship design life 25 years:

	Empirical	Phenomenological
The mean corrosion loss	0.93 mm (most probable) 2.94 mm (severe)	1.35 mm
The standard deviation	0.756 mm (most probable) 0.628 mm (severe)	0.12 mm

It appears that the phenomenological loss prediction is about 50% more than the most probable value of the empirical model. On the other hand, the standard deviations are much higher in the empirical model.

APPENDIX D

Geometric Properties of an Inverted Angle

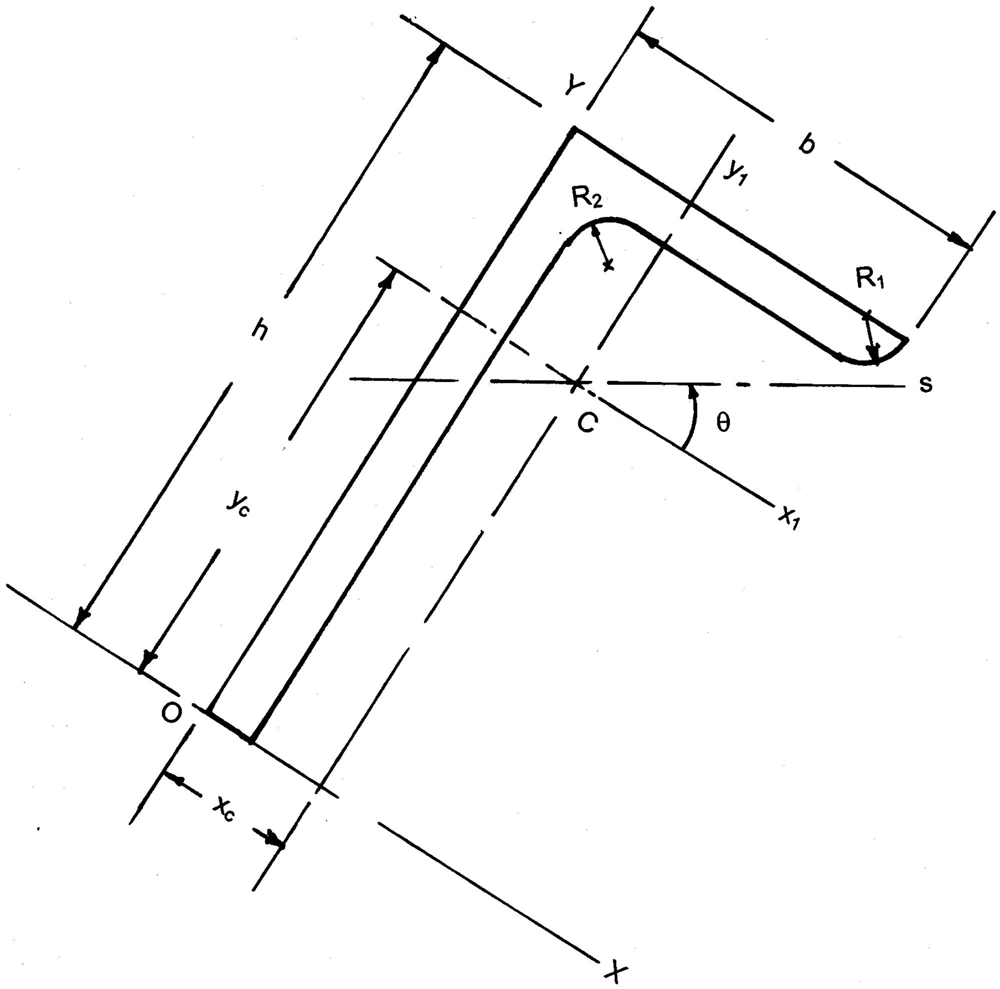

Figure D1: An inverted angle stiffener section (Ivanov 2001a)

This section is defined by six basic variables; h, b, t_w, t_f, R_1 and R_2. The following geometric properties are based on Ivanov (2001a).

Cross-Section, a_s

$$a_s = bt_f + (h - t_f)t_w + \left(1 - \frac{\pi}{4}\right)\left(R_2^2 - R_1^2\right) \tag{D-1}$$

Static Moment about X-axis, S_x

$$\begin{aligned} S_x = &\frac{1}{2}\left\{h^2 t_w + (b - t_w)(2h - t_f)t_f\right\} \\ &+ \left(1 - \frac{\pi}{4}\right)(h - t_f)\left(R_2^2 - R_1^2\right) - \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(R_2^3 + R_1^3\right) \end{aligned} \tag{D-2}$$

Hence,

$$y_c = \frac{S_x}{a_s} \tag{D-3}$$

Static Moment about Y-axis, S_Y

$$\begin{aligned} S_Y &= \frac{1}{2}\left\{b^2 t_f + (h - t_f)t_w^2\right\} \\ &\quad + \left(1 - \frac{\pi}{4}\right)\left(t_w R_2^2 - bR_1^2\right) + \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(R_2^3 + R_1^3\right) \end{aligned} \tag{D-4}$$

Hence,

$$x_c = \frac{S_Y}{a_s} \tag{D-5}$$

Moment of Inertia about X-axis, I_x

$$\begin{aligned} I_x &= \left\{\frac{h^3 t_w}{3} + (b - t_w)t_f\left[h^2 - \left(h - \frac{t_f}{3}\right)t_f\right]\right\} \\ &\quad + \left(1 - \frac{5\pi}{16}\right)\left(R_2^4 - R_1^4\right) + \left(1 - \frac{\pi}{4}\right)(h - t_f)^2\left(R_2^2 - R_1^2\right) \\ &\quad\quad - \left(\frac{5}{3} - \frac{\pi}{2}\right)(h - t_f)\left(R_2^3 + R_1^3\right) \end{aligned} \tag{D-6}$$

Hence, the centroidal moment of inertia

$$i_{x1} = I_x - a_s y_c^2 = I_x - \frac{S_x^2}{a_s} \tag{D-7}$$

Moment of Inertia about Y-axis, I_Y

$$\begin{aligned} I_Y &= \frac{1}{3}\left\{h t_w^3 + \left(b^3 - t_w^3\right)t_f\right\} + \left(1 - \frac{5\pi}{16}\right)\left(R_2^4 - R_1^4\right) \\ &\quad + \left(1 - \frac{\pi}{4}\right)\left(t_w^2 R_2^2 - b^2 R_1^2\right)^2 + \left(\frac{5}{3} - \frac{\pi}{2}\right)\left(t_w R_2^3 + bR_1^3\right) \end{aligned} \tag{D-8}$$

Hence, the centroidal moment of inertia

$$i_{y1} = I_Y - a_s x_c^2 = I_Y - \frac{S_Y^2}{a_s} \tag{D-9}$$

Product Moment of Inertia, I_{XY}

$$I_{XY} = \frac{1}{2}\left\{\frac{h^2 t_w{}^2}{2} + \left(b^2 - t_w^2\right)\left(h - \frac{t_f}{2}\right)t_f\right\} - \left(\frac{19}{24} - \frac{\pi}{4}\right)\left(R_2^4 - R_1^4\right)$$
$$+\left(1 - \frac{\pi}{4}\right)\left(h - t_f\right)\left[\left(t_w R_2^2 - bR_1^2\right) + \left(R_2^3 + R_1^3\right)\right] \qquad \text{(D-10)}$$
$$-\left(\frac{5}{6} - \frac{\pi}{4}\right)\left(t_w R_2^3 + bR_1^3\right)$$

Hence, the centroidal product moment of inertia

$$i_{x1y1} = I_{XY} - a_s x_c y_c = I_{XY} - \frac{S_X S_Y}{a_s} \qquad \text{(D-11)}$$

Centroidal Moment of Inertia about a Horizontal Axis

For deck, inner bottom and outer bottom where the stiffener webs are vertically orientated, $i_s = i_{x1}$. Similarly for the side shells where the stiffener webs are horizontally oriented, $i_s = i_{y1}$. But in the bilge corners the stiffener webs are neither vertical nor horizontal and the general expression for the centroidal moment of inertia about Cs-axis (see Figure D1) may be given as

$$i_{s,j} = i_{x1}\sin^2 j\phi + i_{y1}\mathrm{Cos}^2 j\phi - i_{x1y1}\mathrm{Sin}2j\phi \qquad \text{(D-12)}$$

where

$$\phi = \frac{90^\circ}{n_s + 1} \quad ; \quad \theta_j = 90^\circ - j\phi \qquad \text{(D-13)}$$

and $j = 1$ indicate the bilge stiffener closest to tank top.

Mean and Variance of x_c, y_c, and i_s

Mean of a_s : $\bar{a}_s = a_s\left(\bar{x}\right)$; Eqn. (D – 1) (D-14)

Variance of a_s : $\sigma_{a_s}^2 = \sum_{j=1}^{6}\left\{\left.\frac{\partial a_s}{\partial x_j}\right|_{\bar{x}} \cdot \sigma_{xj}\right\}^2$ (D-15)

Mean of x_c : $\bar{x}_c = x_c\left(\bar{x}\right)$: Eqn.(D – 5) (D-16)

Variance of x_c : $\sigma_{x_c}^2 = \sum_{j=1}^{6}\left\{\left.\frac{\partial x_c}{\partial x_j}\right|_{\bar{x}} \cdot \sigma_{xj}\right\}^2$ (D-17) where $\frac{\partial x_c}{\partial x_j} = \frac{1}{a_s^2}\left[a_s S_Y^{'} - S_Y a_s^{'}\right]$ (D-18)

and the prime indicate partial differentiation with respect to $x_j : j = 1-6$.

Similarly,

Mean of y_c : $\bar{y}_c = y_c(\bar{x})$: Eqn.(D − 3) (D-19)

Variance of y_c : $\sigma_{y_c}^2 = \sum_{j=1}^{6}\left\{\left.\frac{\partial y_c}{\partial x_j}\right|_{\bar{x}} \cdot \sigma_{xj}\right\}^2$ (D-20)

where $\frac{\partial y_c}{\partial x_j} = \frac{1}{a_s^2}\left[a_s S_X^{'} - S_X a_s^{'}\right]$ (D-21)

Mean of i_s : $\bar{i}_s = i_s(\bar{x})$: Eqn.(D − 12) (D-22)

Variance of i_s : $\sigma_{i_s}^2 = \sum_{j=1}^{6}\left\{\left.\frac{\partial i_s}{\partial x_j}\right|_{\bar{x}} \sigma_{x_j}\right\}^2$ (D-23)

where $\frac{\partial i_s}{\partial x_j} = \frac{\partial i_{x1}}{\partial x_j}\mathrm{Cos}^2\theta - \frac{\partial i_{y1}}{\partial x_j}\mathrm{Sin}^2\theta - i_{x1y1}\mathrm{Sin}2\theta$ (D-24)

also $\frac{\partial i_{x1}}{\partial x_j} = I_X^{'} - \frac{1}{a_s^2}\left(2a_s S_x S_x^{'} - S_x^2 a_s^{'}\right)$ (D-25)

$\frac{\partial i_{y1}}{\partial x_j} = I_Y^{'} - \frac{1}{a_s^2}\left(2a_s S_Y S_Y^{'} - S_Y^2 a_s^{'}\right)$ (D-26)

$\frac{\partial i_{x1y1}}{\partial x_j} = I_{XY}^{'} - \frac{1}{a_s^2}\left[\left(S_X^{'} S_Y + S_X S_Y^{'}\right)a_s - S_X S_Y a_s^{'}\right]$ (D-27)

Derivatives with respect to the basic variables

There are six cross-sectional properties, a_s, S_X, S_Y, I_X, I_Y and I_{XY} expressed as functions of six basic variables: $h=x_1$, $b=x_2$, $t_w=x_3$, $t_f=x_4$, $R_1=x_5$ and $R_2=x_6$. Therefore, $6\times6=36$ partial derivatives are required to evaluate the mean and variance of a_s, i_s, x_c and y_c.

These derivatives are listed below.

Derivatives of a_s; $a'_{s,j} = \frac{\partial a_s}{\partial x_j}$; j=1 to 6

$$a'_{s,1} = \frac{\partial a_s}{\partial x_1} = \frac{\partial a_s}{\partial h} = t_w \tag{1,1}$$

$$a'_{s,2} = \frac{\partial a_s}{\partial b} = t_f \tag{1,2}$$

$$a'_{s,3} = \frac{\partial a_s}{\partial t_w} = (h - t_f) \tag{1,3}$$

$$a'_{s,4} = \frac{\partial a_s}{\partial t_f} = (b - t_w) \tag{1,4}$$

$$a'_{s,5} = \frac{\partial a_s}{\partial R_1} = -2\left(1 - \frac{\pi}{4}\right)R_1 \tag{1,5}$$

$$a'_{s,6} = \frac{\partial a_s}{\partial R_2} = 2\left(1 - \frac{\pi}{4}\right)R_2 \tag{1,6}$$

Derivatives of S_X

$$S'_{x,1} = \frac{\partial S_x}{\partial h} = \{ht_w + (b - t_w)t_f\} + \left(1 - \frac{\pi}{4}\right)\left(R_2^2 - R_1^2\right) \tag{2,1}$$

$$S'_{x,2} = \frac{\partial S_x}{\partial b} = \frac{1}{2}(2h - t_f)t_f = \left(h - \frac{t_f}{2}\right)t_f \tag{2,2}$$

$$S'_{x,3} = \frac{\partial S_x}{\partial t_w} = \frac{1}{2}\left\{h^2 - (2h - t_f)t_f\right\} = \left\{\frac{h^2}{2} - \left(h - \frac{t_f}{2}\right)t_f\right\} \tag{2,3}$$

eg. $$S'_{x,4} = \frac{\partial S_x}{\partial t_f} = (b - t_w)(h - t_f) - \left(1 - \frac{\pi}{4}\right)\left(R_2^2 - R_1^2\right) \tag{2,4}$$

$$S'_{x,5} = \frac{\partial S_x}{\partial R_1} = -\left[2\left(1 - \frac{\pi}{4}\right)(h - t_f)R_1 + 3\left(\frac{5}{6} - \frac{\pi}{4}\right)R_1^2\right] \tag{2,5}$$

$$S'_{x,6} = \frac{\partial S_x}{\partial R_2} = \left[2\left(1 - \frac{\pi}{4}\right)(h - t_f)R_2 - 3\left(\frac{5}{6} - \frac{\pi}{4}\right)R_2^2\right] \tag{2,6}$$

Derivatives of S_Y

$$S'_{Y,1} = \frac{\partial S_Y}{\partial h} = \frac{1}{2} t_w^2 \tag{3,1}$$

$$S'_{Y,2} = \frac{\partial S_Y}{\partial b} = bt_f - \left(1 - \frac{\pi}{4}\right) R_1^2 \tag{3,2}$$

$$S'_{Y,3} = \frac{\partial S_Y}{\partial t_w} = (h - t_f) t_w + \left(1 - \frac{\pi}{4}\right) R_2^2 \tag{3,3}$$

$$S'_{Y,4} = \frac{\partial S_Y}{\partial t_f} = \frac{1}{2}\left(b^2 - t_w^2\right) \tag{3,4}$$

$$S'_{Y,5} = \frac{\partial S_Y}{\partial R_1} = \left(1 - \frac{\pi}{4}\right)(-2bR_1) + \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(3R_1^2\right) \tag{3,5}$$

$$S'_{Y,6} = \frac{\partial S_Y}{\partial R_2} = \left(1 - \frac{\pi}{4}\right)(2t_w R_2) + \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(3R_2^2\right) \tag{3,6}$$

Derivatives of I_X

$$\begin{aligned} I'_{X,1} = \frac{\partial I_X}{\partial h} &= h^2 t_w + (b - t_w)(2h - t_f) t_f \\ &- \left(\frac{5}{3} - \frac{\pi}{2}\right)\left(R_2^3 + R_1^3\right) \\ &+ \left(1 - \frac{\pi}{4}\right) 2(h - t_f)\left(R_2^2 - R_1^2\right) \end{aligned} \tag{4,1}$$

$$I'_{X,2} = \frac{\partial I_X}{\partial b} = \left\{h^2 - \left(h - \frac{t_f}{3}\right) t_f\right\} t_f \tag{4,2}$$

$$I'_{X,3} = \frac{\partial I_X}{\partial t_w} = \frac{h^3}{3} - \left\{h^2 - \left(h - \frac{t_f}{3}\right) t_f\right\} t_f \tag{4,3}$$

$$\begin{aligned} I'_{X,4} = \frac{\partial I_X}{\partial t_f} &= (b - t_w)\left\{h^2 - 2ht_f + t_f^2\right\} + \left(\frac{5}{3} - \frac{\pi}{2}\right)\left(R_2^3 + R_1^3\right) \\ &- \left(1 - \frac{\pi}{4}\right) 2(h - t_f)\left(R_2^2 - R_1^2\right) \end{aligned}$$

eg. $$I'_{X,4} = (b - t_w)(h - t_f)^2 + \left(\frac{5}{3} - \frac{\pi}{2}\right)\left(R_2^3 + R_1^3\right) - \left(1 - \frac{\pi}{4}\right) 2(h - t_f)\left(R_2^2 - R_1^2\right) \tag{4,4}$$

$$I'_{X,5}=\frac{\partial I_X}{\partial R_1}=-\left(1-\frac{5\pi}{16}\right)\left(4R_1^3\right)-\left(\frac{5}{3}-\frac{\pi}{2}\right)\left(h-t_f\right)\left(3R_1^2\right)-\left(1-\frac{\pi}{4}\right)\left(h-t_f\right)^2\left(2R_1\right) \tag{4,5}$$

$$I'_{X,6}=\frac{\partial I_X}{\partial R_2}=\left(1-\frac{5\pi}{16}\right)\left(4R_2^3\right)-\left(\frac{5}{3}-\frac{\pi}{2}\right)\left(h-t_f\right)\left(3R_2^2\right)+\left(1-\frac{\pi}{4}\right)\left(h-t_f\right)^2\left(2R_2\right) \tag{4,6}$$

<u>Derivatives of I_Y</u>

$$I'_{Y,1}=\frac{\partial I_Y}{\partial h}=\frac{1}{3}t_w^3 \tag{5,1}$$

eg. $$I'_{Y,2}=\frac{\partial I_Y}{\partial b}=b^2t_f+\left(\frac{5}{3}-\frac{\pi}{2}\right)R_1^3-\left(1-\frac{\pi}{4}\right)\left(2bR_1^2\right) \tag{5,2}$$

$$I'_{Y,3}=\frac{\partial I_Y}{\partial t_w}=t_w^2\left(h-t_f\right)+\left(\frac{5}{3}-\frac{\pi}{2}\right)\left(R_2^3\right)+\left(1-\frac{\pi}{4}\right)\left(2t_wR_2^2\right) \tag{5,3}$$

$$I'_{Y,4}=\frac{\partial I_Y}{\partial t_f}=\frac{1}{3}\left(b^3-t_w^3\right) \tag{5,4}$$

$$I'_{Y,5}=\frac{\partial I_Y}{\partial R_1}=-\left[\left(1-\frac{5\pi}{16}\right)\left(4R_1^3\right)-\left(\frac{5}{3}-\frac{\pi}{2}\right)\left(3bR_1^2\right)+\left(1-\frac{\pi}{4}\right)\left(2b^2R_1\right)\right] \tag{5,5}$$

$$I'_{Y,6}=\frac{\partial I_Y}{\partial R_2}=\left[\left(1-\frac{5\pi}{16}\right)\left(4R_2^3\right)+\left(\frac{5}{3}-\frac{\pi}{2}\right)\left(3t_wR_2^2\right)+\left(1-\frac{\pi}{4}\right)\left(2t_w^2R_2\right)\right] \tag{5,6}$$

<u>Derivatives of I_{XY}</u>

$$\begin{aligned}I'_{XY,1}=\frac{\partial I_{XY}}{\partial h}&=\frac{1}{2}\left\{ht_w^2+\left(b^2-t_w^2\right)t_f\right\}\\&+\left(1-\frac{\pi}{4}\right)\left[\left(t_wR_2^2-bR_1^2\right)+\left(R_2^3+R_1^3\right)\right]\end{aligned} \tag{6,1}$$

$$I'_{XY,2}=\frac{\partial I_{XY}}{\partial b}=b\left(h-\frac{t_f}{2}\right)t_f+\left(1-\frac{\pi}{4}\right)\left(h-t_f\right)\left[-R_1^2\right]-\left(\frac{5}{6}-\frac{\pi}{4}\right)\left(R_1^3\right) \tag{6,2}$$

$$I'_{XY,3}=\frac{\partial I_{XY}}{\partial t_w}=\frac{1}{2}\left\{h^2t_w-2t_w\left(h-\frac{t_f}{2}\right)t_f\right\}+\left(1-\frac{\pi}{4}\right)\left(h-t_f\right)R_2^2-\left(\frac{5}{6}-\frac{\pi}{4}\right)\left(R_2^3\right) \tag{6,3}$$

$$I'_{XY,4}=\frac{\partial I_{XY}}{\partial t_f}=\frac{1}{2}\left\{\left(b^2-t_w^2\right)\left(h-t_f\right)\right\}-\left(1-\frac{\pi}{4}\right)\left[\left(t_wR_2^2-bR_1^2\right)+\left(R_2^3+R_1^3\right)\right] \tag{6,4}$$

$$I'_{XY,5} = \frac{\partial I_{XY}}{\partial R_1} = \left(\frac{19}{24} - \frac{\pi}{4}\right)\left(4R_1^3\right) + \left(1 - \frac{\pi}{4}\right)\left(h - t_f\right)\left[3R_1^2 - 2bR_1\right]$$
$$- \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(3bR_1^2\right) \tag{6,5}$$

$$I'_{XY,6} = \frac{\partial I_{XY}}{\partial R_2} = -\left(\frac{19}{24} - \frac{\pi}{4}\right)\left(4R_2^3\right) + \left(1 - \frac{\pi}{4}\right)\left(h - t_f\right)\left[2t_w R_2 + 3R_2^2\right]$$
$$- \left(\frac{5}{6} - \frac{\pi}{4}\right)\left(3t_w R_2^2\right) \tag{6,6}$$

APPENDIX E

Time Dependent Mean and Variance of Stiffener Basic Variables

Ivanov (2001a) has discussed the complexity of shape of a shrunk cross section of a profile.

However, based on the assumption of uniform corrosion it is possible to write the basic variables as a function of time T and corrosion wear δ. Maintaining continuity at the boundary edges these expressions are:

$$
\begin{aligned}
h(T) &= h(0) - \frac{1}{2}\delta_{V,T} \\
b(T) &= b(0) - \frac{1}{2}\left(\delta_{V,T} + \delta_{W,T}\right) \\
t_w(T) &= t_w(0) - \delta_{W,T} \\
t_f(T) &= t_f(0) - \delta_{V,T} \\
R_1(T) &= R_1(0) - \frac{1}{2}\delta_{V,T} \qquad \text{and} \\
R_2(T) &= R_2(0)
\end{aligned}
\tag{E-1}
$$

where T is the exposure period and $\delta_{V,T}$ and $\delta_{W,T}$ are the total corrosion loss over T years, along vertical and horizontal direction respectively.

Using Equation (E-1) the mean and variance of the stiffener basic variables are:

Means:

$$
\begin{aligned}
\bar{h}(T) &= \bar{h}(0) - \frac{1}{2}\delta_{V,T} \\
\bar{b}(T) &= \bar{b}(0) - \frac{1}{2}\left(\delta_{V,T} + \delta_{W,T}\right) \\
\bar{t}_w(T) &= \bar{t}_w(0) - \delta_{W,T} \\
\bar{t}_f(T) &= \bar{t}_f(0) - \delta_{V,T} \\
\bar{R}_1(T) &= \bar{R}_1(0) - \frac{1}{2}\delta_{V,T} \qquad \text{and} \\
\bar{R}_2(T) &= \bar{R}_2(0)
\end{aligned}
\tag{E-2}
$$

where a bar implies the mean value of the basic variables.

Variances:

$$\begin{aligned}
\sigma_h^2(T) &= \sigma_h^2(0) + \frac{1}{4}\cdot\sigma_{V,T}^2 \\
\sigma_b^2(T) &= \sigma_b^2(0) + \frac{1}{4}\left(\sigma_{V,T}^2 + \sigma_{W,T}^2\right) \\
\sigma_{tw}^2(T) &= \sigma_{tw}^2(0) + \sigma_{W,T}^2 \\
\sigma_{tf}^2(T) &= \sigma_{tf}^2(0) + \sigma_{V,T}^2 \\
\sigma_{R1}^2(T) &= \sigma_{R1}^2(0) + \frac{1}{4}\sigma_{V,T}^2 \qquad \text{and} \\
\sigma_{R2}^2(T) &= \sigma_{R2}^2(0)
\end{aligned} \tag{E-3}$$

As usual, $\delta_{V,T} = T\delta_{V,a}$, $\sigma_{V,T} = T\sigma_{v,a}$, $\delta_{W,T} = T\delta_{w,a}$ and $\sigma_{W,T} = T\sigma_{w,a}$.

APPENDIX F

Sample Calculations

The following calculations are based on an idealised midship section described below.

Scantling Data (Mean Values)

(a) Midship section geometric variables:

$B = 20\text{ m}, \quad D = 10\text{ m}, \quad h = 1\text{m}$

(b) Shell plate thicknesses:

$t_1 = t_2 = t_3 = t_4 = t_5 = 16\text{ mm}$

(c) Inverted angle stiffeners:

$h = 20\text{ cm}, \quad b = 8\text{ cm}, \quad t_w = 15\text{ mm}$

$t_f = 15\text{ mm}, \quad R_1 = 15\text{ mm}, \quad R_2 = 18\text{ mm}$

All 5 sets of stiffeners are assumed to be of same scantlings given above.

(d) Number of stiffeners:

$n_1 = 9, \quad n_2 = 9, \quad n_3 = 8, \quad n_4 = 4, \quad n_5 = 2$

Notice that the subscripts indicate the regions: 1, 2, 3, 4 and 5 for deck, tank-top, outer bottom, side shell and the bilge corner respectively. n_4, n_5 are for one side only.

(e) *In the following calculations, 3 girders were also assumed to be placed in between the tank-top and outer bottom. No new scantling variable was introduced by assuming all three girders are of symmetric I-shape with overall depth = h (double-bottom height) and web thickness = t_3 (thickness of outer bottom). Also flange width = c_1h and flange thickness = c_2t_3; where $c_1 = 0.40$ and $c_2 = 1.5$ are the adjustable inputs. These girders are not shown in Figure A1.*

Scantling Standard Deviations

Geometric variables : $\sigma_B = 0.1\text{ m}, \quad \sigma_D = 0.05\text{ m}, \quad \sigma_h = 0.005\text{ m}$

Shell plates : $\sigma_t = 0.40\text{ mm}$ for all.

Stiffener scantlings : $\sigma_h = 0.1\text{ cm}, \quad \sigma_b = 0.05\text{ cm}, \quad \sigma_{t_w} = 0.40\text{ mm},$

$\sigma_{t_f} = 0.20\text{ mm}, \quad \sigma_{R_1} = \sigma_{R_2} = 0.0$

Corrosion Data for Loss of Thickness

Three alternative sources were considered. Ivanov's arbitrary values, Paik's 'severe' values with intermediate coating life $IT_c = 7.5$ years) and Melchers' phenomenological data. These data are given in Table F1.

TABLE F1

Corrosion model	Corrosion wear mm/year	Standard deviation mm/year
Paik et al. 'severe'	$\delta_{v,a} = 0.1938$	0.0426
Ivanov	$\delta_{v,a} = 0.10$ $\delta_{w,a} = 0.10$	$\sigma_{v,a} = 0.04$ $\sigma_{w,a} = 0.04$
Melchers*	$T = 5 \quad : \quad \delta_{v,a} = 0.0954$ $T = 10 \quad : \quad \delta_{v,a} = 0.0767$ $T = 15 \quad : \quad \delta_{v,a} = 0.0747$ $T = 20 \quad : \quad \delta_{v,a} = 0.06735$	$\sigma_{v,a} = 0.006$

* In this model the **total** corrosion losses were found for T = 5, 10, 15 and 20 years using the appropriate equations given in Appendix C. The equivalent average annual value is then taken as inputs shown above.

TABLE F2
Mean values and standard deviations of the midship section modulus, Z

Years of exposure, T	Parameters	Paik et al. 'severe'	Ivanov	Melchers
T = 0	Mean of Z	4.1712 m^3	4.1712 m^3	4.1712 m^3
	σ_Z	0.0798 m^3	0.0798 m^3	0.0798 m^3
T = 5	Mean of Z	3.9169 m^3	4.040 m^3	4.0460 m^3
	σ_Z	0.0884 m^3	0.0876 m^3	0.0796 m^3
T = 10	Mean of Z	3.6625 m^3	3.9087 m^3	3.9699 m^3
	σ_Z	0.1113 m^3	0.1084 m^3	0.0800 m^3
T = 15	Mean of Z	3.4083 m^3	3.7775 m^3	3.8937 m^3
	σ_Z	0.1417 m^3	0.1362 m^3	0.0808 m^3
T = 20	Mean of Z	3.1541 m^3	3.6463 m^3	3.8176 m^3
	σ_Z	0.1758 m^3	0.1677 m^3	0.0819 m^3

Comment: The section modulus and most other section properties are near-perfect linear functions of T in any corrosion model where the annual average wear is constant, as in the cases of Ivanov and Paik. In the case of Melchers, Z tends to be linear after $T \approx 10$ years. However, the standard deviation, σ_Z is always non-linear with respect to T.

Estimation of Probabilities of Reduction in Z

The objective here is to predict the probability of Z falling below a specified percentage at any given period of exposure, T. For example, after 10 years of exposure what is the probability that Z will fall, say, below 95% of its original value?

The relevant equation is:

$$f_z(Z) = \frac{1}{\Phi(\mu_z/\sigma_z)\sqrt{2\pi}\,\sigma_z} \exp\left[-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right] \tag{4-6}$$

Noting that in Table F2 the minimum value of $\dfrac{\mu_z}{\sigma_z} = \dfrac{Z}{\sigma_z} \cong 18$ (for T = 20 in Paik's model), which corresponds to $\Phi(\mu_z/Z) = 1.0$. Hence Eqn (4-6) in this situation reduces to truncated normal distribution:

$$f_z(Z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} \exp\left[-\frac{(Z-\mu_z)^2}{2\sigma_z^2}\right] \tag{E-1}$$

Now we non-dimensionalise Z as follows:

$$\bar{z} = \frac{Z(T)}{Z(0)} \quad \text{and} \quad \sigma_z = \frac{\sigma_Z(T)}{Z(0)} \tag{E-2}$$

Then introduce the following well-known linear transformation:

$$y = \frac{z_k - \bar{z}}{\sigma_z} \tag{E-2}$$

where $\bar{z}, \sigma_z$ = non-dimensionalised mean and standard deviation at any T
z_k = specified fraction of Z.

Finally

$$\Pr\{z \le z_k\} = \Phi(y) \tag{E-2}$$

where $\Phi(y)$ =standard normal distribution tabulated extensively for positive values of y.

If y is negative, $\Phi(-y) = 1 - \Phi(y)$.

<u>A Sample Calculation</u>

Take T = 10 yr in Ivanov's model. It is required to estimate the probability of Z falling to **95% or below** of its original value. Note that here $z_k = 0.95$.

From Table F2 : $\bar{z} = \dfrac{Z(10)}{Z(0)} = \dfrac{3.9087}{4.1712} = 0.937$ and

$$\sigma_z = \frac{0.1084}{4.1712} = 0.026$$

Then standardised normal variable, $y = \frac{z_k - z}{\sigma_z}$

$$= \frac{0.95 - 0.937}{0.026} = 0.500$$

Hence $\Pr\{z \le 0.95\} = \Phi(0.50) = 0.6915$.

Therefore, after 10 years of coating breakdown the probability of the section modulus, Z falling below 95% of its original value is 69.15%. Repeating the calculation based on Paik's 'severe' corrosion model, this probability shoots from 69.15% to 99.65%; almost a certainty.

Probability estimates based on all three models are briefly summarised below.

TABLE F3
Probability of midship section modulus falling below a certain percentage of the original value

Exposure period, T	$z(T) \le 95\%$			$z(T) \le 90\%$			$z(T) \le 85\%$		
	Paik	Ivanov	Melchers	Paik	Ivanov	Melchers	Paik	Ivanov	Melchers
5	0.5080	0.1894	0.1475	0.0330	0.0060	0.0	0.0	0.0	0.0
10	0.9965	0.6915	0.4640	0.7950	0.0778	0.004	0.147	0.0	0.0
15	1.0	0.9131	0.8325	0.9927	0.4320	0.042	0.834	0.045	0.0
20	1.0	0.9706	0.962	0.9997	0.7400	0.219	0.987	0.277	0.005

The results given in Table F3 above are also presented graphically in Figures F1-F3.

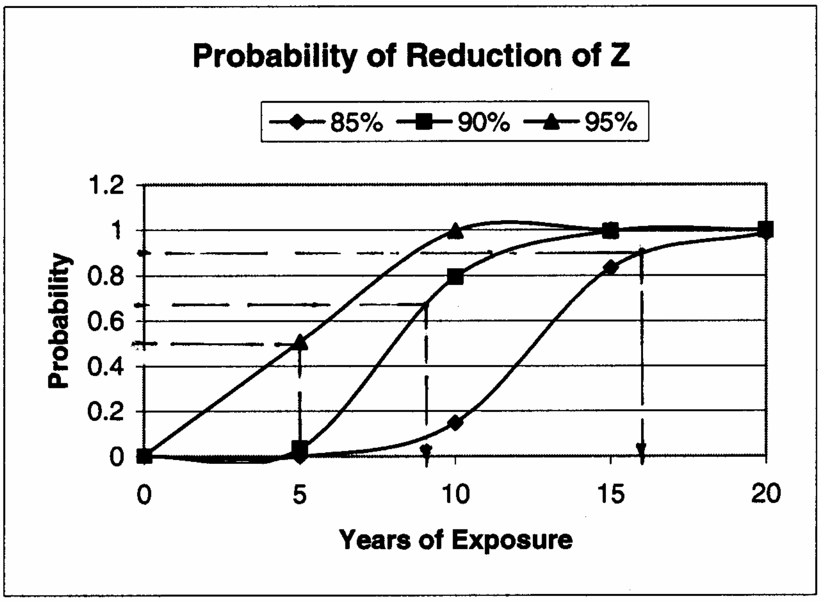

Figure F1: Paik et al. severe corrosion model

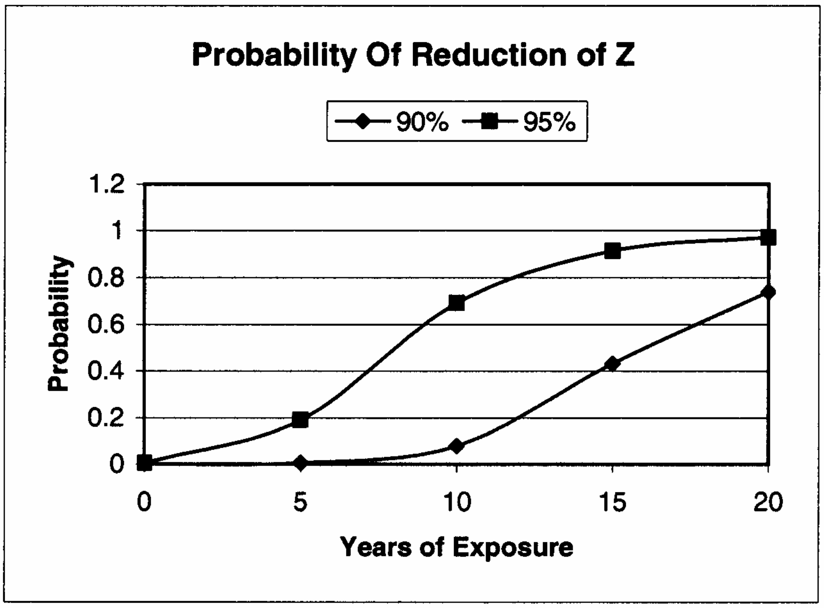

Figure F2: Ivanov's empirical corrosion model

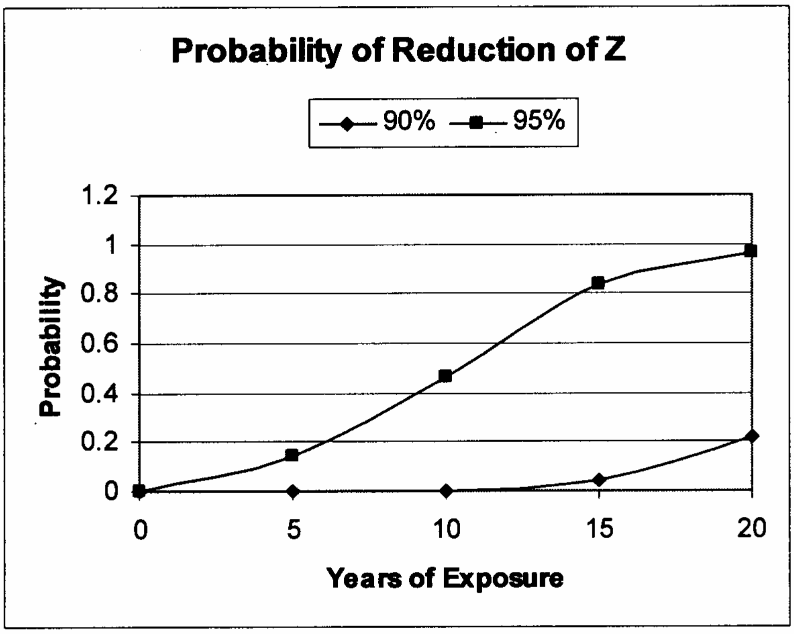

Figure F3: Melchers' phenomenological corrosion model

An Alternative Way of Interpreting Probability

It is agreed that the probabilities may be qualitatively described as follows.

Less than 5%	=	*'unlikely'*
50% or more	=	*'may be'*
66% or more	=	*'likely'*
90% or more	=	*'most likely'*
About 100%	=	*'certainly'*

Then the probabilities in Table F3 will transform into Table F4.

TABLE F4
Qualitative probability of reduction of midship section modulus

Exposure period, T	$z(T) \leq 95\%$			$z(T) \leq 90\%$			$z(T) \leq 85\%$		
	Paik	Ivanov	Melchers	Paik	Ivanov	Melchers	Paik	Ivanov	Melchers
5	maybe	unlikely	unlikely	unlikely	unlikely	*unlikely*	unlikely	*unlikely*	*unlikely*
10	certainly	likely	"	likely	"	"	"	"	"
15	"	most likely	likely	certainly	"	"	likely	"	"
20	"	"	most likely	"	likely	"	most likely	"	"

Note: Using Melchers' corrosion model it is 'unlikely' that the section modulus will fall below 90% at T = 20 years. Similarly, it is 'unlikely' that the section modulus will drop below 85% at T = 20 using the Ivanov model.