The role of term structure based interest rate forecasts in risk management : empirical evidence from the SFE ten year Commonwealth government bond futures and options markets

Author:
McInnes, Ross W.

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THE ROLE OF TERM STRUCTURE BASED INTEREST RATE FORECASTS IN RISK MANAGEMENT: Empirical evidence from the SFE ten year Commonwealth government bond futures and options markets

Ross W. McInnes

A thesis submitted in partial fulfilment of the requirements of the degree of Master of Commerce (Honours) to the University of New South Wales
1997

CERTIFICATION

I certify that this thesis has not been previously submitted for a University degree or any similar Award.

Ross McInnes

22/10/97

CERTIFICATE OF ORIGINALITY

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ABSTRACT

This paper presents the empirical development of an interest rate risk pricing model that has at its foundations, the arbitrage free certainty forward term structure first discussed by Ho and Lee (1986). Three central hypotheses were used to test the proposition that term structure based interest rate forecasts have a role to play in risk management. The first hypothesis stated that the Australian market term structure was derivable from the observed Commonwealth government coupon bond yield curve. The second stated that contained in this term structure was the market's implied forecasts of *N*-period forward interest rates. Due to these forecasts being made under uncertainty they were assumed to have significant errors. The third stated that the distribution of these forecasting errors could be used to approximate the stochastic behaviour of the term structure. Over the 01/01/94 to 31/12/94 sample trading period investigated, there were four main results. The derived term structures were found to be consistent with the pricing assumptions of the market. They priced SFE ten year bond futures accurately to within 0.2 of one basis point. Next, the errors in the market's term structure based interest rate forecasts were found to be statistically significant. The distribution of these errors formed the basis of the so called Yield Error Margin database. Thirdly, the error database was seen to provide a reasonable guide to the pricing behaviour of the SFE ten year bond futures option market although there was some bias present. ITM put options were priced more reliably than the other option series included in the pricing analysis. Finally, the hedging simulation conducted reaffirmed the rewards from utilising interest rate insurance. The naive strategy of

purchasing put options to protect the value of a fixed interest portfolio was shown to successfully reduce the probability of large negative returns. However in a high volatility environment the effectiveness of this strategy was shown to be restricted. Overall the research conducted in this paper provided some important insights into the stochastic behaviour of the Australian market term structure that supported the proposition that the market's implied *N*-period forward interest rate forecasts have an important role in risk management.

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CHAPTER 1

Introduction

The objective of this paper was to determine if term structure based interest rate forecasts have a role to play to play in risk management. An interest rate risk pricing model was empirically developed that attempted to replicate the Australian interest rate market's futures, volatility and option pricing assumptions. The risk model based its methodology on the arbitrage free certainty forward term structure first presented by Ho and Lee (1986) and more recently discussed by Heath, Jarrow and Morton (1992).[1]

What differentiated this paper from previous empirical work was that it tested three central hypotheses. The first hypothesis was that the Australian market term structure could be constructed directly from the observable Commonwealth government coupon bond yield curve.[2] In an empirical sense the zero coupon curve that was derived from this process was assumed to be the closest measure of the "true" term structure. This zero coupon yield to maturity relationship was seen to measure the riskless rate of return available on Australian market securities from one day to ten years. From a theoretical perspective this curve was equivalent to Ho and Lee's (1986) so called "initial" discount function. [3]

The second hypothesis was that the derived term structure contained the market's forecast of interest rates in N - periods of time. For the purposes of this paper

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[1] The term structure relates to the absolute time value of money. The implied certainty N - period forward term structure equated to the interest rate curve derived from today's initial term structure.
[2] Where the yield curve was defined as that observed relationship between yield and time to maturity. It was constructed by placing bonds and discount securities of equivalent credit quality in order according to time to maturity.
[3] Ho and Lee, (1986) 1013.

the "market" was defined as those individual traders, hedgers and institutions i.e. Banks who participate in forming the consensus view on yields. The constructed implied N - period forward term structure, equivalent to Ho and Lee's (1986) so called arbitrage free certainty case, reflected today's best estimate of the level of interest rates at the N - period forward date.[4]

The major point to make, about today's N - period forward term structure, was that it would only be equivalent to the actual term structure that eventuated in N - periods of time if the present forward view of interest rates was without error. This was not assumed to be the general case. Market participants faced uncertainty when making decisions about the future level of yields. There was seen to exist a high probability of unforeseen shocks, between today and the N - period forward date, that would influence the level of interest rates.[5] Thus the market's current forecasts were assumed to have significant errors.

This discussion leads to the third and final core hypothesis of the paper. It was proposed that the distribution of the market's forecasting errors could be used to model the stochastic behaviour of the term structure. For this purpose Ho and Lee (1986) proposed a so called perturbation function and Heath, Jarrow and Morton (1992) a stochastic process with a risk neutral drift term. In this paper, the N - period certainty forward term structure was modified by the market's forecasting errors. Individual errors were measured by the Yield Error Margin between today's implied N - period certainty forward term structure and the actual term structure that eventuated in N - periods of time.

[4] Ho and Lee, (1986) 1017.
[5] Shocks take the form of economic i.e. unexpected changes to inflation and or political where there was an abrupt change to government policy.

Applying this methodology to every trading day in a given sample period was assumed to produce a so called Yield Error Margin database. The individual error outcomes that made up the database were assigned a sample consistent probability weighting. For example, if the market underestimated N - period forward yields by 0.10 % on fifteen occasions out of a sample of two hundred and fifty, then it could be stated that there was a 6.00 % (15 / 250) chance of this outcome. This approach it was assumed had the ability to produce a probability weighted distribution of forward term structures which could be used to generate an array of forward security prices. Therefore the interest rate risk associated with a security could be calculated.

The interest rate risk pricing model presented in this paper was used to replicate the observed prices of Sydney Futures Exchange (SFE here after) bond futures and option contracts as well as to hedge a fixed interest portfolio, containing the Commonwealth government coupon bonds in the SFE bond futures pricing basket, over the 01/01/94 to 31/12/94 sample period. The data for this empirical modelling was taken from both physical and derivative markets. End of business day prices were recorded for bank bills, short dated interest rate swaps, Commonwealth government coupon bonds and the ten year bond futures/options contracts traded on the SFE.

Studying the behaviour of the Australian interest rate market allowed the development of some important insights into an area that has undergone rapid change in the last ten to fifteen years. The deregulation of these markets in the early to mid - 1980's, followed by the massive fall in yields in the early 1990's, has resulted in a growing sophistication in these markets. In light of these observations, the 1994 sample period provided an excellent opportunity to test the pricing behaviour of the Australian interest rate market. The sample period was correlated with a time when, due to some "early" US Federal Reserve tightenings of monetary policy, there was a dramatic surge in both the level of bond yields and volatility.[6]

In empirical analysis the model builder often faces a direct tradeoff between the complexity of the assumptions underlying the model and the practical cost of efficiently implementing the proposed pricing structure. This has especially characterised the research that has applied models based on the stochastic behaviour of the term structure to the pricing of interest rate risk. Recent published empirical work has tended to concentrate on the short end of the yield curve. Ronn and Sias (1991), Flesaker (1993) and Amin and Morton (1994) all used a term structure based framework to price contingent claims on either the US Treasury bill or Eurodollar markets. Bond market pricing analysis was seen as too costly because it would have involved the estimation of the dynamics of the entire term structure.[7]

In line with these type of issues, the current paper focused its attention on the distributional characteristics of one particular segment of the term structure. Although the entire term structure was used to price the bonds in this study, it was only the forecasting errors associated with the four to five Commonwealth government coupon bonds making up the SFE ten year bond futures pricing basket, that were recorded in the Yield Error Margin databases. For similar reasons of tractability, the market's interest rate forecasts were recorded for only four forward periods: two weeks, one month, two months and three months forward. Thus, to explain the volatility

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[6] The US Federal Reserve increased the Fed funds rate for the first time in five years in early February 1994 . It occurred at a time when bond markets around the world appeared "comfortable" with the inflation / interest rate outlook. For example Australian ten year bond yields in January 1994 were at a fifteen year low of 6.35 %. By June 1994 the closing monthly yield was 9.65 % . See Table F2 in the Reserve Bank of Australia January 1995 Bulletin.
[7] Amin and Morton, (1994) 142.

assumptions of the Australian bond futures option market the analysis was restricted to four Yield Error Margin distributions.

The overall objective of the research was to determine the role of the market's term structure based implied N - period forward interest rate forecasts in risk management. In pursuing an answer to this question the research was attempting to move beyond the large body of work, such as that by Bloch (1974) and Cuthbertson (1996), that concentrated solely on the accuracy of the market's term structure based interest rate forecasts. In this paper a risk pricing model was presented that approximated the stochastic behaviour of the term structure, over some forward time period, by the distribution of errors in the market's interest rate forecasts. In effect the model was attempting to provide us with information about the likely variations from today's implied N - period forward term structure. The resultant probability weighted distributions of the forward term structure allowed interest rate risk to be estimated and hence contingent claims to be priced.

The remainder of the paper will be organised in the following way : Chapter 2 presents a literature review covering term structure and interest rate risk pricing issues. Chapter 3 details the methodology and data sources used to develop the interest rate risk pricing model. Chapter 4 discusses the derivation of the Australian market term structure and whether it was consistent with the pricing assumptions of the market. Chapter 5 outlines the distributional characteristics of the Yield Error Margin databases. Chapter 6 determines the role of the Yield Error Margin distributions in the pricing of contingent claims on fixed interest securities. Chapter 7 examines the rewards from hedging a fixed interest portfolio. Chapter 8 summarises the research results and states its conclusions.

CHAPTER 2

Literature Review and Modelling Background

2.1 Introduction

The model presented in this paper attempted to approximate the stochastic behaviour of the term structure. The objective was to determine an empirical mechanism that could be used to reliably generate a distribution of term structures at some forward date. The model therefore had two key parts. The first dealt with the estimation of the initial term structure and the second aimed to measure the likely variability of the implied N - period forward term structure. These two parts were applied to the pricing of fixed interest futures and options as well as to the hedging of fixed interest portfolios.

The traditional theories of the term structure attempted to explain what factors influenced the observed shape and level of the term structure. Three general hypotheses about the formation of the term structure have emerged. Firstly, the market segmentation theory proposed that institutional factors explained the present orientation of the term structure. Secondly, the liquidity premium hypothesis stated in simplest terms that longer dated securities must offer higher returns to induce investors to give up the greater liquidity of short term securities. The final traditional proposition was the expectation hypothesis which assumed that the shape and position of the term structure depended strictly on the market's expectation of future rates.

However, these classic term structure theories were static in nature. They were only concerned with the characteristics of the current term structure. The modern dynamic term structure theories go a step further. They consider not only the orientation of the currently observed term structure but also deal with the evolution of the term structure through time. These modern theories have a theoretical background based on the modelling first introduced by the Black and Scholes (1973) stock option pricing model and its binomial equivalents. This option pricing model was essentially the first area of finance theory to utilise the higher level of mathematics associated with the Physics and Engineering disciplines. This increased level of sophistication was directed at modelling the distributional characteristics of share prices.

This type of analysis became increasingly relevant to the interest rate securities markets. In the late Seventies in the US and in the early Eighties in Australia, public policy shifted from the strict regulation of these markets to a belief in the benefits of the free market[8]. The newly established price variability of the debt markets made it a natural place for academics and practitioners alike to apply the stochastic modelling associated with stock option pricing theory.

In more recent times this type of analysis has been extended to study the behaviour of the overall yield curve. Instead of looking at one particular point on the curve, such as the ninety day bank bill, the dynamics of the whole yield curve were directly investigated. If the stochastic behaviour of the underlying term structure could be estimated, then at some forward date a likely distribution of term structures could be generated. This distribution, it can be suggested, allowed contingent claims on securities of all maturity and coupon structures to be accurately priced. A generalised methodology of pricing interest rate risk was the ultimate objective of these modelling strategies. In this modelling "nirvana" the same model could be used to price both ninety day bank bill and ten year coupon bond derivative securities.

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[8] See November 1985 Reserve Bank of Australia bulletin. For example in June 1982 the Reserve Bank of Australia replaced the existing "tap" system with a tender system for bonds. Under the new sale process the price of Commonwealth government coupon bonds was set by market demand rather than by the government.

To demonstrate this theoretical background and its development over time, Chapter 2 was broken into five sections. The first gave a brief overview of the traditional theories of the term structure. The second discussed the single variable stochastic process incorporated into the Black and Scholes option pricing model. The third examined the so called short rate term structure models. In these models the "short rate" was seen to drive both the shape of the current term structure and its likely dynamics. The fourth discussed the stochastic term structure models. These models attempted to generate a framework that measured the stochastic behaviour of the entire term structure. The final section presented the results of recent empirical research into the pricing behaviour of term structure based risk models.

2.2 Traditional Theories of the Term Structure

Shiller (1990) and Bierwag's (1989) review of the traditional theories of the term structure attempted to summarise the vast amount of work done in the area. This reflected the fact that financial economists have been involved in this field of research since the 1930's and 1940's.[9] The objective of this research was to determine what market based forces were responsible for the varying shapes of the term structure. In its purest form the term structure was defined as that relationship between yield and term to maturity on default free debt. Thus, the study of the term structure may be regarded as the analysis of the market price of time or alternatively as the absolute time value of money.

The market segmentation hypothesis in its simplest form proposed that the formation of the term structure was directly influenced by the underlying institutional environment. The fact that financial institutions operated in particular maturity environments was seen to determine the ultimate shape of the term structure. Banks

[9] For example see Lutz, (1940).

were seen to have a bias to invest in short to intermediate securities because by nature the bulk of their deposits were in short end maturities. In contrast, Life insurance companies were seen to operate in the longer end of the term structure because their liabilities were longer dated. Bierwag (1989) suggested, that in its most extreme form the market segmentation hypothesis implied that the interest rate for any given maturity was determined solely by the demand and supply for securities of that maturity.[10] Securities of different maturities were not substitutes because market participants only had one investment habitat.

The limitation, with any institutional based explanation of the term structure, is that the deregulation process mentioned in Section 2.1 has introduced dramatic changes to the World and Australian financial system. There has been as a result an increasing blurring of function in the financial system. For example banks now offer insurance products and insurance companies now provide deposit / lending facilities. In the modern financial system it would be hard to find support for the hypothesis that key market participants were confined to specific maturity environments.[11]

In contrast the liquidity premium hypothesis focused on the higher levels of risk associated with holding longer dated securities to explain the observed orientation of the term structure. Long term bond prices were assumed to fluctuate more on average than short term security prices. Investors in long term securities were therefore more likely to experience large capital gains and losses. Shiller (1990) suggested this hypothesis was supported if we used the concept of duration as the index of interest rate risk.[12] Since coupon bonds could be regarded as portfolios of discount or zero

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[10] Bierwag, (1989) 86.

[11] This process of change has continued with the recent publication of the Wallis Australian Financial System inquiry in late March 1997. The last major inquiry was the Martin Review group in May 1983.

[12] Shiller, (1990) 637.

coupon bonds, it may be more useful to describe bonds by the weighted average of the terms of the constituent discount bonds, rather than by the term of the longest bond in the portfolio. The relationship of price risk to duration, in the discrete case, could be shown by Equation (2.1).[13] For a given change in yield the bond with the longest duration would experience the greatest variation in price.

$$\frac{\Delta B}{B} = - Du \, \Delta y \tag{2.1}$$

Where ;

B = The bond price.

Du = The duration of the bond.

y = The trading yield of the bond.

Δ= The discrete change in the variable.

In order to attract investors long term securities must trade in the market at higher yields. Intuitively, this suggests that the liquidity premium must always be positive. This statement, on first perusal, would appear to have empirical support. As noted by McEnally and Jordan (1995) in the US markets over the last thirty years short dated yields have tended to be below long term yields. The term structure has generally been "normal" in shape which supported the existence of a positive liquidity premium.

The problem faced by the liquidity premium hypothesis was that there have been times when the term structure was inverse in nature.[14] Short term yields were higher than long term yields. In these cases the term premium must be assumed to be zero. No liquidity premium or any other additional compensation was needed to induce

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[13] Hull, (1993) 100.

[14] McEnally and Jordan, (1995) 802. For example in February 1981 the US curve was inverse in nature.

investors to hold long term securities. Recent empirical research also suggested that a simple belief in the liquidity premium hypothesis was unlikely to fully explain the observed term structure relationships. Ilmanen (1996) conducted a study into the rewards available to investors from holding longer dated securities. World bond index returns, for maturities > five years, were compared to the returns on one month Eurodollar deposits for the period January 1978 to June 1993. This comparison showed that the market offered meagre reward, if any, for bearing additional interest rate risk. The average additional return over the fifteen year period was only 0.73 % (9.55 % - 8.82 %) which was not found to be statistically significant.[15]

Under the expectations hypothesis of the term structure, the shape and position of the term structure depended solely on the market's expectations of future yields. This theory proposed that if investors and borrowers expect future yields to be different from their current levels, and they act on these expectations, then the observed term structure would not remain in its present state. For example, if the currently observed term structure was flat at 10.00 % and the market expects the one year yield to be 12.00 % in one years time, then the curve will steepen with long yields moving above short yields.

This adjustment occurs because the investor who purchased today's two year bond at 10.00 % would be disadvantaged if the higher expectations were realised in one years time. The second investor who invested for one year at 10.00 % and then invested in one year's time at 12.00 %, would assuming annual compounding, earn a higher total two year return of 11.00 %. Investors in this stylised environment would sell current two year bonds until they reached a yield of 11.00 %. This ensured that in

[15] Ilmanen, (1996) 53.

the new equilibrium that investors returns matched the revised interest rate expectations whatever investment strategy was pursued.

This result would only hold strictly under the condition of certainty. It assumed that today's expected forward one year yield in one years time would actually be the yield that eventuated in the market in the future. Under these conditions if the implied forward differed from this expected yield then arbitrageurs could extract riskless profits by combining the one year and two year bonds currently trading in the market. This approach formed the basis of Ho and Lee's (1986) so called certainty case to be discussed in Section 2.5.

Under the more general condition of uncertainty, these results would have to be modified. Today's expected yields could not be guaranteed to equal the actual yields that prevail in one years time. The expected 12.00 % one year forward one year yield represented only one potential future outcome. Other possibilities had a positive probability of occurring. The departure of the actual yield from the expected yield represented an error in expectations.

The role of expectations under the condition of uncertainty was clarified by Cox Ingersoll and Ross (1981). Their so called local expectations hypothesis stated that the instantaneous expected yield on bonds was equal to the prevailing spot yield.[16] Bierwag (1989) suggested that this hypothesis could be equated with the proposition that future expected bond yields equalled the implied forward yields derived from the term structure.[17] Therefore the market's forecasts of future yields was equivalent to the implied N - period forwards of the current term structure.

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16 Cox, Ingersoll and Ross, (1981) 795.

17 Bierwag, (1989) 89.

In summary, the traditional theories of the term structure suggested that the level and shape of the observed term structure, on any given day, was determined by those agents who entered their preferences into the interest rate market. In practice, these agents were likely to be the individual traders, hedgers and institutions such as banks or insurance companies who participated in forming the consensus view on yields. Participants' expectations, it can be stated, reflected a range of economic, political and institutional influences. Therefore the term structure summarised a range of factors that cannot generally be estimated into an observed relationship between default free yields and time to maturity.

2.3 Single Variable Stochastic Process

The Black and Scholes (1973) option pricing model provided a breakthrough in the modelling and pricing of security price risk. The individual investor's level of risk aversity was removed from the contingent claim valuation equation. Black and Scholes relied upon two assumptions to price call options on non - dividend paying stocks in this efficient way. Firstly, that it was possible to construct an instantaneously riskless hedge portfolio and secondly, that the underlying stock price followed a geometric Brownian process.

The hedge portfolio contained two securities, a long underlying stock position and to offset the risk of this security a short call option position.[18] To keep this portfolio riskless, the number of short calls was assumed to be dynamically adjusted with changes in the stock price. It was proposed that the hedge portfolio over a given

[18] "Long" in this context means that the investor had purchased the security and "short" that the investor had net sold the security.

trading interval must earn the risk free rate or otherwise arbitrage opportunities existed. Today's hedge portfolio value was seen to be given by Equation (2.2).

$$S\text{-}C\Big/\frac{dc}{ds} \tag{2.2}$$

Where ;

S = Today's stock price.

C = Today's call option price.

$\frac{dc}{ds}$ = Delta hedge ratio which measured the change in the value of the call option relative to the change in the underlying stock price.

The change in the hedge portfolio's value over a short interval of time was assumed to be evaluated by Equation (2.3).

$$\Delta S - \Delta C\Big/\frac{dc}{ds} \tag{2.3}$$

This introduced the second fundamental assumption of the Black and Scholes model. Changes in the underlying stock price were seen to follow a geometric Brownian stochastic process. It was assumed that the stock price followed a Markov process where only its current value was relevant to predicting its future. The past history of the stock price and the way in which the present had emerged from the past were irrelevant. This implied that the stock returns across different intervals of time were independent. Equation (2.4) described the evolution of the stock price through time.

$$\Delta S = \mu S\,\Delta t + \sigma S\,\Delta z \tag{2.4}$$

Where ;

ΔS = Change in the stock price over the discrete time interval Δt.

μ = Constant drift return component of the underlying stock price change.

σ = Constant standard deviation or volatility associated with the return of the stock.

$\Delta z = \varepsilon \sqrt{\Delta t}$. The random noise term component of the expected return. The randomness was assumed to be drawn from a normal distribution.

Δt = The discrete time interval over which the share price was assumed to change.

The stock price change over the discrete time interval was assumed to be made up of two components: a constant positive drift term and a stochastic component that was related directly to stock price variance and time. Consistent with modelling under the condition of uncertainty, the amount of potential variance in the stock price increased with the length of the time interval. This form of stochastic process has been integrated into many risk pricing models including Heath, Jarrow and Morton's (1992) forward rate process, to be discussed in Section 2.5.

Black and Scholes's two key assumptions implied that the stock call option price was a function of the share price and time $C = f(S, T)$. Incorporating Ito's Lemma into the analysis allowed an expression for ΔC in Equation (2.3) to be developed and ultimately to the derivation of the Black and Scholes differential equation.

$$yC = \frac{dc}{dt} + yS\frac{dc}{ds} + \frac{1}{2}\sigma^2 S^2 \frac{d^2c}{ds^2} \qquad (2.5)$$

Where ;

y = The risk free yield.

To find a solution to this differential equation a set of boundary conditions was established , $C = S - X$ when $S > X$ and $C = 0$ when $S < X$. To solve Equation (2.5) subject to these boundary conditions Black and Scholes utilised a heat transfer equation from the Physics discipline.[19] The result of this transformation was the well known Black and Scholes non - dividend stock European call option pricing model.

$$C = SN(d1) - Xe^{-y(T-t)}N(d2) \qquad (2.6)$$

$$d1 = \frac{Ln(S/X) + (y + 0.5\sigma^2)(T - t)}{\sigma\sqrt{(T - t)}}$$

$$d2 = \frac{Ln(S/X) + (y - 0.5\sigma^2)(T - t)}{\sigma\sqrt{(T - t)}}$$

Where ;

C, S = As defined in Equation (2.2).

X = The exercise/strike price associated with the call option.

σ^2 = Variance of the underlying stock.

$N(d)$ = The cumulative normal density function for a standardised normal variable. This measured the probability that a standardised normal variable was less than d.

T = Maturity / expiration date of the stock option.

t = Today's trading date.

T- t = Remaining life of the option measured in days.

y = As defined in Equation (2.5).

[19] This heat transfer equation has entered the folklore of the financial markets.

Cox and Rubinstein's (1985) analysis provided important insights into the distributional assumptions underlying Equation (2.6).[20] Cox and Rubinstein suggested that the central part of any option pricing model were its assumptions about the likely future distribution of the underlying security price. They demonstrated that it was possible to use the distributional characteristics of the binomial process to replicate Black and Scholes's stock option pricing model.

Cox and Rubinstein introduced a game of chance that had a binomial based payoff to highlight the proposition being presented. The gamble had the following six parts ;

1. There were N successive draws.
2. The player was able to draw from an urn containing 100 balls.
3. Out of the 100 balls K were black and 100 - K were red.
4. After each draw the ball that was drawn from the urn was replaced so that each draw was an independent event. The probability of drawing a black or red ball remained constant across each of the N successive draws.
5. The player could only make a bet at the start of the game.
6. The bet's payoff was calculated the following way. For every \$1 bet at the start of the game the participant received, for the N draws made, an accumulated value. This value was determined by the number of black balls drawn vis - a - vis the number of red. The black ball paid \$ u and the red ball paid \$ d. It was assumed that \$ $u >$ \$ d.

The characteristics of the bet's payoff could be summarised by Pascal's Triangle in Table 2.1..

[20] See also Cox, Ross and Rubinstein (1979).

Table 2.1:

Pascal's Triangle

(N = trials, rows and j = outcomes, column)

$N\backslash j$	0	1	2*	3	4	5	6
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4*	1	4	6*	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

There were two points to make about this payoff table. Firstly, note how as the number of draws (trials) increased the potential outcomes become more normally distributed. Secondly, the numbers in Pascal's Triangle represented the relative probability of a particular outcome for a given number of trials;

$$\frac{N!}{j!(N-j)!} \tag{2.7}$$

Where ;

$N! = N \times (N\text{-}1) \times (N\text{-}2) \ldots\ldots 3 \times 2 \times 1.$

A particular outcome in the table was defined by $X_N = u^j d^{N\text{-}j}$ for $j = 0, 1, 2, \ldots, N$. For $N = 4, j = 2$ or u^2d^2 there were six ways for this outcome to eventuate over the four trials.[21] Since the value of X_N was not known in advance of the bet we assume that X_N followed a stochastic process.

21 For trial $N = 4$ and outcome $j = 2$ from the Pascal Triangle: The relative probability of two black and two red.

$$6 = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \ (2 \times 1)} \quad \text{(As per Pascal's table)}$$

Within the structure of the bet there was a second probability factor that had to be measured. Following on from point three of the bet's characteristics, the probability of drawing a black ball was seen to equal $q = K / 100$. For the red ball the associated probability was $1 - q = (100 - K) / 100$. In general the probability of any one outcome containing j drawings of black balls and $N - j$ drawings of red balls was set by $q^j(1 - q)^{N-j}$. Combining this result with Equation (2.7) allowed the game participant to determine the probability of particular bet outcomes. Cox and Rubinstein took this result one step further by asking the question, for an outcome $j = a$ what was the probability that $X_N > u^a d^{N-a}$?

$$\phi[a; N, q] = \sum_{j=a}^{N} \left(\frac{N!}{j!(N-j)!}\right) q^j (1-q)^{N-j} \tag{2.8}$$

Equation (2.8) represented the so called complementary binomial distribution function. In probability theory the standard assumption was that the sum of all the individual outcome probabilities equalled one. From this assumption it was possible, using Equation (2.8), to calculate the likelihood of the bet payoff exceeding a certain level. This type of approach was obviously closely related to the risk - reward involved in option trading. In pricing stock options, the major concern was over the probability of an option being in the money at expiration given that today's stock price was S and the strike was X.

Identifying this linkage, Cox and Rubinstein applied the concepts developed in the gamble directly to the pricing of stock options. The central assumption was that the stock price, as per Equation (2.8), followed a multiplicative binomial process. As a consequence the stock price either rose or fell in each discrete time interval. The stock offered two rates of return, u with probability q and d with probability $1 - q$. To prevent the opportunity of riskless arbitrage within this process it was assumed that $u > y > d$. Where y, as defined in Equation (2.5), was the risk free rate of return. If this did not hold and instead $u > d > y$ then investors could make a certain profit by borrowing at y and buying the stock. [22]

<u>Figure 1:</u> The call option payoff associated with the one period case

$Cu = max\ [\ uS - X\ ,\ 0\ ]$ *with probability q*

C

$Cd = max\ [\ dS - X\ ,\ 0\ ]$ *with probability 1- q*

To price the call option in this one period case a Black and Scholes inspired hedge portfolio was constructed. The hedge portfolio contained two securities Θ shares of stock, purchased at the current stock price of S and $\$B$ invested in riskless bonds. Today's value of the portfolio was equal to $S\Theta + B$. At the end of the single period the portfolio, like the call option, had two potential values. The immediate objective of the modelling was to ensure that the hedge portfolio replicated the call option payoffs. To price the call option in this way we needed to find those weightings of Θ and B that were consistent with the call option values at the end of the single period. To ensure the appropriate portfolio weightings were obtained the following equation system was constructed.

$$\begin{aligned} uS\Theta + yB &= Cu \\ dS\Theta + yB &= Cd \end{aligned} \qquad (2.9)$$

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[22] Cox and Rubinstein, (1985) 40. The authors defined y = 1+ rate of interest on a default free loan over a given period. u = 1+ rate of return if the share price rises and d = 1+ rate of return if the share price falls.

Solving for Θ and B and substituting these results into today's call value, set as $C = S\Theta + B$, generated Equation (2.10). The probability weighted call value in one period's time was established inside the square brackets. This value was then discounted by y, the risk free rate for one period, to determine today's call option price.

$$C = [pCu + (1-p)Cd\,] \,/\, y \tag{2.10}$$

Where ;

$p = (y - d) / (u - d)$ and $1 - p = (u - y) / (u - d)$

Equation (2.10) could be generalised to the case where N the number of periods was large. Combining Equation (2.8), which detailed the complementary binomial probabilities, with Equation (2.10) produced Equation (2.11). Equation (2.11) followed a recursive procedure to price the call. By starting at the expiration date and working backwards, today's price of the call option could be evaluated. In effect, the pricing methodology treated the problem as a series of single period models bound together. This recursive procedure forms the basis of most current binomial / lattice based risk pricing models, such as the Black, Derman and Toy (1990) model to be discussed in Section 2.4..

$$C = \left\{ \sum_{j=0}^{N} \left(\frac{N!}{j!(N-j)!} \right) p^{j} (1-p)^{N-j} \max[u^{j} d^{N-j} (S - X), 0] \right\} \Big/ y^{N} \tag{2.11}$$

Where ;

N = The total number of discrete time periods within the overall time interval.

This pricing model could be further refined when it was recognised, over the N - intervals, that part of the binomial lattice would have zero value due to outcomes where $S < X$. At expiration there will be some percentage of the outcomes that will be irrelevant to the payoff of the call option. Cox and Rubinbstein attempted to remove these outcomes from the valuation equation by determining the minimum number of up moves necessary for the call to finish in the money. To determine this, as per Equation (2.8), they set variable a to equal the smallest non-negative number such that $u^a d^{N-a}\ S > X$.[23] Incorporating this reduced payoff structure and rearranging Equation (2.11) into two parts, led to Equation (2.12).

$$C = S\left[\sum_{j=a}^{N}\left(\frac{N!}{j!(N-j)!}\right)p^j(1-p)^{N-j}\left(\frac{u^j d^{N-j}}{y^n}\right)\right] - X\,y^{-N}\left[\sum_{j=a}^{N}\left(\frac{N!}{j!(N-j)!}\right)p^j(1-p)^{N-j}\right] \tag{2.12}$$

Equation (2.12) stated that today's value of the stock call option was calculated by the difference between S × (the expected real return over the N - time periods remaining in the life of the option) and X × (discounted to today's value at the N - period riskless yield) × (the overall probability associated with the expected real return). In the limit, when N the number of trading intervals ⇒ ∞ the binomial model, due to the characteristics of Pascal's Triangle, approached a continuous normal distribution. Therefore it could be shown that the binomial model under such conditions approximated the Black and Scholes stock option pricing formula in Equation (2.6). The complementary binomial probabilities in the square brackets in Equation (2.12) were replaced by the assumption that the return on the stock was taken from a standard normal distribution.

The Black and Scholes option pricing model provides a benchmark for all empirical modelling in the finance discipline. As Rubinstein (1994) stated, it is viewed

[23] $a > \ln(X / Sd^N) / \ln(u / d)$. If $a > N$ then the value of the call = 0

as one of the most successful models in the social science area.[24] To price call options on non-dividend paying stocks, Equation (2.6) required only five relatively easily approximated inputs: the current stock price S, the strike price X, the time to expiration $T - t$, the risk free rate of interest y and the only non - observable input the level of stock price volatility σ. The Black and Scholes pricing model can for a low degree of effort be set up on any basic spreadsheet or even programmable calculator. The attraction of the Black and Scholes model was that it can be efficiently implemented because its pricing inputs were easily measured and its pricing structure could be activated at a low cost.

These comments suggest that all empirical models should aim to have these characteristics. In applied modelling there always exists tensions between theoretical rigour, the successful estimation of inputs and the subsequent cost of implementing the pricing assumptions. This conflict represents the tradeoff between the complexity of the underlying pricing model, its pricing accuracy and the practical concerns of the model builder i.e. such as the speed of the pricing algorithm. The successful resolution of this conflict ultimately determines how wide spread the acceptance of any newly proposed model will be.[25]

2.4 Short Rate Term Structure Models

Cox, Ingersoll and Ross (1985) proposed that the term structure was the endogenous product of a general equilibrium model. This approach attempted to incorporate the three traditional theories of the term structure discussed in

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24 Rubinstein, (1994) 772.

25 This type of constraint was present in Gagnon's (1990) empirical analysis. Gagnon applied a modified Black and Scholes model to the pricing of Canadian bond options. The modification was that the bond's duration was used as the volatility input in the pricing model. The problem faced by this approach was that a closed form solution was not available and that it also assumed that bond yields were constant over the life of the option . Gagnon suggested that the model was only suitable for short dated options because of these assumptions, which limited its wider acceptance.

Section 2.2.. Anticipation of futures events (Expectation hypothesis), risk preferences (Liquidity premium hypothesis) and the timing of investment/consumption alternatives (Market segmentation hypothesis) were all included in this modelling framework. What differentiated Cox, Ingersoll and Ross's model from these traditional theories was that the derived term structure was used to price interest rate risk.

Underlying the general equilibrium model was a complete intertemporal description of a continuous time competitive economy.[26] The representative individual had to decide on the optimal level of consumption and investment to partake in over their expected life. However, Cox, Ingersoll and Ross modified the "classic" intertemporal maximisation problem. In the Cox, Ingersoll and Ross model the representative individual had to make an additional decision. This extra decision determined how much wealth should be directed towards contingent claims, given the consumption and investment decisions that were made.

Cox, Ingersoll and Ross developed their single factor model of the term structure from this background. The central assumption of this model was that the state of technology, R, determined the production opportunities available in the economy. The return from these production opportunities was also seen to be function of R. Developments in R were assumed to follow a stochastic process whose dynamics were demonstrated in Equation (2.13)

$$dR\,(t) = [\,\xi R + \zeta]\,dt + \varsigma\sqrt{R}\,dw\,(t) \tag{2.13}$$

Where ;

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[26] The general equilibrium model was consistent with Debreu. For example 1. There was seen to be one good which acted as the numeraire. 2. Production consisted of a set of n-linear activities 3. The vector of expected returns on activities was α. 4. The covariance rates of returns was GG'. 5. R represented the state of technology.

ξ, ζ = constants $\zeta > 0$ and ς was a $1 \times (n+k)$ vector, each of whose components was the constant ς_o.

These changes in the state of technology were also assumed to influence the equilibrium short interest rate y. y was seen to follow the stochastic process detailed in Equation (2.14).

$$dy = \Psi(\theta - y)\, dt + \sigma\sqrt{y}\, dz_1 \tag{2.14}$$

Where ;

Ψ = The speed of adjustment parameter.

θ = The long term value or central location of y.

$\Psi\,(\theta - y)$ = Mean - reverting behaviour. With Ψ, $\theta > 0$. [27]

$\sigma\sqrt{y}\, dz_1 \equiv \varsigma\, \sqrt{R}\, dw\,(t)$. The noise term as in Equation (2.4).

Equation (2.14) had a similar structure to the stock price dynamics presented in Section 2.3.. The fundamental difference was that the constant drift term in Equation (2.4) was replaced by a function that was mean - reverting. The equilibrium short interest rate was assumed to follow a continuous time first-order autoregressive process, where it was elastically pulled back towards θ. Ψ determined how fast y moved back towards θ.

The problem with this stochastic process was that it did not rule out the positive probability of negative interest rates.[28] Cox, Ingersoll and Ross neutralised this problem in their model by assuming that $2\Psi\theta \geq \sigma^2$. The implied "pull" of the drift

[27] Vasicek, (1977) 185. The author incorporated a similar mean reverting process.

[28] From late 1995 through to late 1997, the Japanese interest rate structure has gone close to testing the zero interest rate assumption. 90 day Yen rates have traded below 0.50 % or 1/2 % as a result of the severe deflationary forces in the economy.

rate was seen to preclude zero and negative interest rates. This assumption had important ramifications for the distributional characteristics of y.

Firstly, y was seen to have a probability distribution that was conditional on its value at current t. Cox, Ingersoll and Ross assumed that the absolute level of volatility increased when y increased which was consistent with the square root volatility term in Equation (2.14). [29] Secondly, as a result of these assumptions y was seen to follow a non - central Chi - square distribution. This distribution starts at the origin and skews to the right with a long "tail". This part of the modelling, as per the lognormal assumption incorporated in the Black and Scholes model, ensured that the underlying variable could not become negative which was consistent with empirical observation.

Cox, Ingersoll and Ross utilised these assumptions, regarding the dynamics of the equilibrium short rate y, to construct the entire term structure of interest rates. Each point on the term structure was assumed to equal the average expected yield of the short rate for the period T - t. In economic terms, this suggested that the observed three year bond yield should reflect the market's view of the average official cash rate over the next three years.

$$B(t, T) = e^{-Y(T-t)} \tag{2.15}$$

Where ;

$B(t, T)$ = The bond price for today's date t of a bond maturing at date T.

$T - t$ = The time to maturity of the bond in question.

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[29] This assumed an asymmetry in the volatility of interest rates. Empirical testing of this hypothesis on the Australian interest rate markets would certainly be interesting given recent history. In January 1994 10 year bond yields were at fifteen year lows of 6.35 %. Over the subsequent six months 10 year bond yields retraced a fall in yields that had taken eighteen months to complete. Volatility at these historically low yields was very high.

$Y(t, T)$ = The yield on the bond was set by the average expected short rate y.

Cox, Ingersoll and Ross suggested that this version of the single factor term structure model would only hold if individuals were assumed to be characterised by constant relative risk aversion. This was defined as a situation where individuals were indifferent to a proportional loss of wealth, even though the absolute size of the loss increased with the level of wealth. This scenario was equivalent to the risk neutral assumptions that were used in the Black and Scholes modelling framework.

Cox, Ingersoll and Ross proposed that outside this risk neutral environment that additional factors, such as the market price of risk λ, would enter the derivation of the equilibrium term structure. Bond prices under this alternative scenario were determined by Equation (2.16).

$$B(t, T) = A(t, T)\, e^{-G(t,T)\,y} \tag{2.16}$$

Where ;

$A(t, T)$, $G(t, T)$ were complex functions of five variables Ψ, λ, θ, σ, as defined before and γ which was a measure of the level of risk aversion in the individual's utility function.[30]

Applying Cox, Ingersoll and Ross's term structure approach to bond option pricing produced a valuation equation consistent with the structure of Equation (2.6). In this case, as per the short rate y in Equation (2.14), the bond price was assumed to follow a non - central Chi-square distribution. The price of a bond call option was calculated by Equation (2.17).

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[30] Cox, Ingersoll and Ross, (1985) 393. The yield to maturity of a given bond on the term structure was given by $Y(t, T) = [\,y\,G(t, T) - \ln A(t, T)\,] / (T - t)$. At the limit of the term structure the yield was determined by a distinct relationship. $Y(t, \infty) = 2\,\Psi\,\theta \;/\; \gamma + \Psi + \lambda$.

$$C = B(t,T)\chi^2(.) - XB(t,t^*)\chi^2(.) \tag{2.17}$$

Where ;

$B(t,T) =$ The bond price as before.

$\chi^2(.) =$ Non - central Chi - square distribution.

$XB(t,t^*) =$ The strike price was discounted by the bond price associated with the option's maturity date t^*.

The Cox, Ingersoll and Ross model of the term structure highlighted the tradeoffs involved with developing empirical risk pricing models. The advantage of using the Cox, Ingersoll and Ross model was that it completely specified the construction of an equilibrium term structure. The disadvantage was that in its non - risk neutral form the Cox, Ingersoll and Ross model required the estimation of six parameters: y the short rate, Ψ the speed of adjustment parameter, θ the central location of y, σ the volatility of the short rate, λ the market price of risk and γ which was a measure of risk aversion. Obviously the majority of these parameters were not directly observable.

To apply the Cox, Ingersoll and Ross model to a practical pricing problem, it was inevitable that certain simplifying assumptions would have to be made to measure the model inputs. As suggested by Chirarella, Lo and Pham (1989) in their empirical test of the Cox, Ingersoll and Ross model on the Australian market, the potential for pricing errors was from three areas; model specification, econometric estimation techniques and data collection.[31]

The Black, Derman and Toy (1990) term structure model, like the Cox, Ingersoll and Ross model relied on the dynamic process of the so called short rate to price

[31]Chiarella, Lo and Pham, (1989) 15.

interest rate risk. In contrast to Cox, Ingersoll and Ross, the term structure was not seen to be the endogenous product of the Black, Derman and Toy model. Instead the "true" term structure was assumed to be measured by the yields on zero coupon US Treasury bonds of varying maturities observed in the fixed interest market.[32] This approach was consistent with the term structure derivation presented in Chapter 3 of this paper.

The central proposition put forward by Black, Derman and Toy was that the forward short rate, like the Cox and Rubinstein option pricing model in Section 2.3, evolved in a binomial lattice. The forward short rates were assumed to be derived in such a way that they were consistent with both the observed zero coupon yield curve and the volatilities associated with individual maturity points along the curve. Black, Derman and Toy, as per the direction taken by this paper, used the information contained in today's term structure to generate the dynamics in the model. Once the forward short rate binomial lattice had been constructed bond and bond option prices could be calculated.

The risk pricing model was based on five main assumptions. Firstly, the forward value of the "short rate", defined as the annualised one period interest rate, determined the prices of all other securities. Secondly, there existed two input arrays that described the model's term structure. The yields on zero coupon treasury bonds for various maturities formed the model yield curve. The volatilities associated with the different maturities along the yield curve formed the so called volatility array.

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32 Gregory and Livingston, (1992) 68. Since 1982 US Treasury coupon bonds have been "stripped" into their par bond and coupon components. The authors state that the market for these securities was strong.

Thirdly, it was assumed that changes in bond yields across the yield curve were perfectly correlated. This implied that only parallel shifts in the term structure were considered. This assumption suggested that all interest rate securities must have the same expected return over a given period which was basically a restatement of the local expectations hypothesis discussed in Section 2.2.. Fourthly, it was assumed that the price of a fixed interest security had an equal probability of moving up or down. Black, Derman and Toy assumed a risk neutral environment where there was a 50 / 50 chance of favourable changes in bond prices. Finally, the short rate was assumed to have a lognormal distribution when the number of forward periods was large which was consistent with the Black and Scholes model in Section 2.3..

Black, Derman and Toy used a "dummy set" of yield and volatility data to illustrate the workings of their model. Table 2.2 displayed this dummy set of data. To implement their pricing analysis Black, Derman and Toy, as per Shiller (1990), viewed a US Treasury coupon bond as a portfolio of individual zero coupon bonds. The coupons to be paid by the bond and the final principal value associated with the bond were treated as distinct cash flows related to different zero coupon bonds. For example a US Treasury bond with three years to maturity which paid a 10 % coupon and had a \$100 face value was assumed to represent a portfolio of three underlying zero coupon securities. A one year zero bond with a \$10 face value, a two year zero bond with a \$10 face value and a three year zero coupon bond with a \$110 face value. Appendix A displayed the three sub-portfolios that made up the three year Treasury coupon bond.

Table 2.2: BDT Dummy Term Structure

Maturity Years	Yield (%)	Yld/ Vol. (%)
1	10	20
2	11	19
3	12	18
4	12.5	17
5	13	16

Sub-portfolio one could be priced under the condition of certainty. The cash flow of \$10 associated with this portfolio was to be received in one years time. The short rate in this case was known. The one year yield in Table 2.2 was recorded as 10.00 %. Today's price of sub-portfolio one was given by $\$9.09 = (0.5 \times \$10 + 0.5 \times \$10) / 1.10$.

Uncertainty entered the valuation of sub-portfolio two. The cash flow in this case was not to be received for two years. The value of this portfolio was determined by two yields, the current one year yield and the one year forward one year yield. Under the condition of certainty the one year forward one year yield could be calculated directly from the yield curve displayed in Table 2.2. The implied one year forward yield in this case, assuming annual compounding, would be $(1.11)^2/(1.10) = 12.00$ %. As with the analysis in Section 2.2, there existed no guarantees that this yield would actually materialise in one years time.

Black, Derman and Toy assumed that the one year forward yield evolved in a binomial structure. This suggested that instead of the certainty yield of 12.00 %, that there would be two one year forward one year yields in the pricing model. These two yields captured the impact of uncertainty on forward yields. To ensure these yields were consistent with the current term structure two constraints were imposed. The first constraint was today's price of the two year zero coupon bond. The current two year yield was 11.00 %. The price of a two year zero coupon bond with a face value of $10 was $8.12 = $10 / (1.11) ^{2}. The second constraint was the level of recorded volatility associated with the two year part of the yield curve. In Table 2.2 two year volatility was quoted as 19.00 %.

Black, Derman and Toy proposed that an iterative process should be used to solve this problem. Two yields would be chosen at random until the correct yields were found. The two one year forward one year yields that were consistent with the two constraints was shown to equal 14.32 % and 9.79 %. Discounting the two year forward price of $10 by these forward yields, generated two potential prices for the one year forward one year zero coupon bond of $8.75 and $9.11. Discounting these prices by today's one year rate of 10.00 % produced the current market price of the two year zero (0.5 × $ 8.75 + 0.5 × $ 9.11) / 1.10 = $8.12. The volatility constraint was also met where σ^2 = ln (14.33 / 9.79) / 2 = 19 %.

To price sub-portfolio three the same process was implemented. Find those two year forward one year rates consistent with today's value of the three year zero coupon bond trading at 12.00 %, $78.29 = $ 110 / (1.12)^3, and the current level of three year volatility quoted at 18.00 %. There was one additional assumption that had to be made in this case. Two years out there were assumed to be three two year forward one year rates.

These represented three unknowns *yuu*, *yud* and *ydd*.[33] To find a unique solution, given that there were only two constraints, Black, Derman and Toy utilised their assumption that the short rate was lognormally distributed. This meant that volatility was dependent only on time. This assumption implied that in one period of time when the short rate was 14.32 %, volatility was calculated by 0.5 × Ln (*yuu*/*yud*)

[33] This assumed, as will be discussed in Section 2.5 that the binomial tree was recombining.

and when the short rate was 9.79 %, the volatility was given by $0.5 \times \text{Ln}\ (yud/ydd)$. These volatilities must be equal so that $yuu\ /\ yud = yud\ /\ ydd$ or $yud = \sqrt{yuu \times ydd}$. For each iteration it was only necessary to choose *yuu* and *yud*. With two unknowns and two constraints a unique solution was available. The two year forward one year rates consistent with this analysis were found to be 19.42 %, 13.77 % and 9.76 %. Following this approach, for all the inputs in Table 2.2, allowed a market consistent forward one year interest rate lattice to be constructed.

This interest rate lattice could be used to price contingent claims. Adding the three sub-portfolio's together produced a forward bond pricing lattice for today's three year US Treasury coupon bond. These forward bond prices determined the payoff associated with bond options. For example the payoff of a two year call option, with a strike price X = \$95, was set by the two year forward coupon adjusted price of the bond. The coupons were removed from this pricing analysis under the standard assumption that the holder of the derivative forwent the right to receive these payments. Appendix A demonstrated that in two years time the forward one year zero coupon bond price had three values, \$92.11, \$96.69 and \$100.22. The expiration value of the call option related to these bond prices was set by $[\ B - X\ ,\ 0\ ]$. The backward recursive procedure described in Section 2.3 was used to determine today's price of the call which equalled \$ 1.77.

Black, Derman and Toy acknowledged that this pricing model would have to reworked if it was to be used in a practical setting. As presented, the model was limited to what they called "coarse steps" i.e. one year steps. For accuracy they suggested that daily steps should be used to price longer dated bonds. The constraint faced by this approach was that it would require a vast amount of computer power and time to solve the pricing algorithm. To combat this onerous task Black, Derman and

Toy proposed a two stage pricing strategy. Firstly, a "coarse tree" was constructed with enough steps to adequately value the US Treasury coupon bond from its maturity date back to today. Secondly, a more detailed "fine tree" was set up for the period between today and the option expiration to value the bond option. To capture the timing of coupon payments it may be necessary to interpolate from the "coarse tree" to the "fine tree".

2.5 Stochastic Term Structure Models

Ho and Lee (1986), in contrast to the single factor models presented in Section 2.4, attempted to model arbitrage free stochastic movements in the entire term structure. Consistent with the Black, Derman and Toy (1990) approach the initial term structure was taken as given and the dynamics were modelled within a binomial framework. Ho and Lee's overall objective was to use the term structure based model to price interest rate contingent claims.

In the Ho and Lee model the so called discount function formed the basis of the term structure analysis. The discount function was defined as the inverse of the term structure. It described the relationship between a zero coupon bond's price and its time to maturity. The discount curve was assumed to be continuous with a zero coupon bond for every possible maturity date.[34]

At initial time 0 there was certainty with an observable discount function described by $B(\,.\,) = B_0^{(0)}(\,.\,)$. For example $B_0^{(0)}(1)$ defined today's price of a bond with one year to maturity. Due to the presence of uncertainty the discount function in one period of time, $N = 1$, was specified by two possible functions $B_0^{(1)}(\,.\,)$

[34] The underlying bond market was assumed to be in equilibrium at all discrete points in time. For state i, that eventuated at period N, the equilibrium price of the bond with maturity T was $= B_i^{(N)}(T)$.

= downstate or $B_1^{(1)}(.)$ = upstate. The stochastic nature of the term structure was captured by a binomial process. This assumption when applied to the general case ,where $N > 1$, led to the construction of a binomial based discount function lattice from which forward bond prices could be calculated.

The focus of Ho and Lee's analysis was on the factors that determined the spread between the downstate and upstate discount functions within the binomial lattice. Under conditions of certainty, as discussed in Section 2.2, the one period forward discount function must be equivalent to today's implied one period forward discount function. Today's expected discount function would be the actual discount function that eventuated in one period's time. Equation (2.18) defined $F_i^{(N)}(T)$ as the arbitrage free N - period forward discount function under certainty.

$$F_i^{(N)}(T) = B_i^{(N+1)}(T) = B_{i+1}^{(N+1)}(T) = \frac{B_i^{(N)}(T+1)}{B_i^{(N)}(1)} \qquad (2.18)$$

In practice agents face uncertainty when making decisions. There existed no guarantee that today's implied N - period forward discount function would be equivalent to the observed discount function in N - periods of time. The aim was to evaluate how perturbed (different) today's implied certainty forward $F_i^{(N)}(T)$ was from the actual discount function at N+1. For this role Ho and Lee proposed two so called perturbation functions. These functions were responsible for producing the upstate and downstate discount curves in the binomial bond pricing lattice.

$$B_{i+1}^{(N+1)}(T) = \frac{B_i^{(N)}(T+1)}{B_i^{(N)}(1)} \times h(T) \quad \text{Upstate} \qquad (2.19)$$

$$B_i^{(N+1)}(T) = \frac{B_i^{(N)}(T+1)}{B_i^{(N)}(1)} \times h^*(T) \quad \text{Downstate}$$

Where ;

$h(T)$, $h^*(T)$ were the perturbation functions that generated the difference between upstate and downstate discount curves.

Three restrictions were imposed on the perturbation functions to ensure that the forward discount functions followed a stochastic process that was arbitrage free. Firstly, it was assumed that falls in bond prices were matched by rises in bond prices. This implied that bonds across the term structure experienced the same weighted average movements in their price. There were no arbitrage opportunities because all bonds earned the same one period return. This was equivalent in nature to Cox, Ingersoll and Ross's (1981) local expectation hypothesis. The first restriction was represented by Equation (2.20)

$$\pi h(T) + (1-\pi) h^*(T) = 1 \quad \text{for } N, i > 0 \tag{2.20}$$

Where;

$\pi = y - d / u - d$ = "risk neutral" probability as per Section 2.3.
y, u, d = bond returns associated with upstate / downstate and the risk free yield.

Rewriting the functions in Equation (2.19) in terms of $h(T)$ and $h^*(T)$ and substituting them into Equation (2.20) produced the second restriction.

$$B_i^{(N)}(T) = [\, \pi B_{i+1}^{(N+1)}(T-1) + (1-\pi)\; B_i^{(N+1)}(T-1)\,] \times B_i^{(N)}(1) \tag{2.21}$$

To rule out arbitrage opportunities the average expected forward bond price, discounted by the prevailing one period bond return, should equal the original bond price from which the forwards were derived. The uncertainty weighted forwards should agree with the original discount function from which they were produced. In effect this construction replicated the arbitrage free conditions associated with the certainty case in Equation (2.18). Therefore the dynamics in the Ho and Lee model, as per Black, Derman and Toy (1990), were made consistent with today's observed market term structure.

The final restriction was that the stochastic movements in the discount function were assumed to be path independent.[35] In the constructed binomial lattice, an upward movement followed by a down move, equalled a down move followed by an up move. Ho and Lee's binomial risk pricing lattice recombined which meant the number of pricing nodes did not follow the function 2^N. This characteristic decreased the computational effort of using the Ho and Lee model as the number of nodes does not become explosive as N, the number of forward periods, increased.

Ho and Lee combined these three restrictions to derive a unique solution for the perturbation functions. The Ho and Lee model via the perturbation functions detailed in Equation (2.22) was able to develop an arbitrage free stochastic process that was compatible with the initial market term structure. The evolution of the term structure was set in a binomial framework. This allowed bond options to be priced via the backward recursive process discussed in Section 2.3. Today's current bond option value could be established by starting at the expiration date and working back along the forward discount function lattice.

$$h(T) = \frac{1}{\pi + (1 - \pi)\delta^T} \qquad (2.22)$$

$$h^*(T) = \frac{\delta^T}{\pi + (1 - \pi)\delta^T}$$

[35] Ho and Lee, (1986) 1019. Path independence was equivalent to the Markov property mentioned in Section 2.3..

Where ;

δ = The spread parameter between the two perturbation functions.
$\delta = 1$ was the certainty case and $\delta \Rightarrow 0$ implied high term structure volatility.

The Ho and Lee term structure model was seen to have three weaknesses. Firstly, volatility was assumed in restriction one, to be constant along the entire term structure. In practice, as mentioned by Hull (1989), the level of volatility often varied with time to maturity.[36] Secondly, this constant volatility assumption implied that only parallel shifts in the term structure were contemplated by the model's dynamics. Other type of shifts in the term structure were not considered. Thirdly, Ho and Lee did not address the issue of negative interest rates in their model. In cases where δ the spread parameter was close to 0 and bond prices were high (yields low), there does appear to have existed the positive probability of negative interest rates. As has been a theme in Chapter 2, modelling uncertainty often involves tradeoffs between theoretical rigour and practical application.

Consistent with these types of problems Heath, Jarrow and Morton (1992) attempted to provide a unifying contingent claim pricing theory of which all arbitrage pricing models were seen as special cases. Their overall approach was to draw upon the assumptions of the likes of Cox, Ingersoll and Ross and Ho and Lee whilst attempting to avoid some of the limitations inherent in these models. Heath, Jarrow and Morton stated that their basic model building objective was; [37]

" Given an initial forward rate curve and a mechanism which describes how it fluctuates, ... develop an arbitrage pricing model which yields contingent claim valuations which do not explicitly depend on the market price of risk ".

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[36] Hull, (1989) 274.
[37] Heath, Jarrow and Morton, (1992) 98.

Heath, Jarrow and Morton's term structure based model, unlike the Ho and Lee approach, was developed in continuous rather than discrete time. The bond market, as in the Ho and Lee model, was seen to be complete with bond prices along the term structure being defined by $B\ (\ t\ ,\ T\ )$ the time t price of a bond maturing at time T. Heath, Jarrow and Morton analysed the dynamics of the overall term structure in terms of the so called instantaneous forward rate. This was defined as the yield that could be contracted for at time t, on a riskless loan that began at date T and was returned an instant later.

$$f(\,t\,,T\,)\ =\ -\,\partial \log B\,(\,t\,,T\,)\,/\ \partial\,T \qquad (2.23)$$

Heath, Jarrow and Morton proposed that the forward yield followed a stochastic process that had similar characteristics to the Black and Scholes Equation (2.4). Equation (2.24) demonstrated that changes in the forward yield were described by a drift and a volatility term. The significance of Equation (2.24) was that it applied generally to the evolution of all forward interest rates of all maturities.

$$f(\,t\,,T\,)-f(\,0\,,T\,)=\int_o^t \alpha\,(\,\upsilon\,,T\,,\omega\,)\,\mathrm{d}\upsilon+\sum_{i=1}^{N}\int_o^t \sigma_i(\,\upsilon\,,T\,,\omega\,)\,dW_i \qquad (2.24)$$

Where ;

$f(\,0\,,T\,)=$ was a fixed non - random initial forward yield curve which was measurable as $f(\,0\,,.)$. Equivalent to today's known implied forward term structure and Ho and Lee's certainty forward discount function described in Equation (2.18).

$\int_o^t \alpha\,(\,\upsilon\,,T\,,\omega\,)\,\mathrm{d}\upsilon =$ The drift rate of the forward curve.

$\sum_{i=1}^{N}\int_o^t \sigma_i\,(\,\upsilon\,,T\,,\,\omega\,)\,dW_i =$ The volatility component .

Heath, Jarrow and Morton stated that the so called spot yield $y(t)$ was a special case of $f(t, T)$. It was defined as the instantaneous forward yield at time t for date t, $f(t, t)$. The spot yield was seen to follow a stochastic process equivalent to Equation (2.24) except that $f(0, T)$ was replaced by $f(0, t)$. This spot stochastic process was input directly into the dynamics of bond prices. Equation (2.25) portrayed the stochastic process followed by the bond price.

$$dB(t, T) = [y(t) + b(t, T)]B(t, T)dt + \sum_{i=1}^{N} a_i(t, T)B(t, T)dW_i \tag{2.25}$$

Where ;

$[y(t) + b(t, T)]B(t, T)dt$ = The drift of the bond price stochastic process.

$\sum_{i=1}^{N} a_i(t, T)B(t, T)dW_i$ = The noise / volatility term.

Heath, Jarrow and Morton suggested that the bond price dynamics developed in Equation (2.25) faced two major constraints. Firstly, changes in the bond price were seen to be non-Markov because of the way the drift and volatility coefficients had been specified. The stochastic bond price process was therefore not independent of its history.[38] Hull (1993) stated that this characteristic of the Heath, Jarrow and Morton model made it difficult to implement in practice.[39] In terms of a binomial pricing lattice this implied, unlike the Ho and Lee model, that the Heath, Jarrow and Morton "tree" would not be recombining. An upmove followed by a downmove does not necessarily lead to the same term structure as a downmove followed by an up

[38] See Jeffrey (1995) and Li, Ritchken and Sankarasubramanian (1995) for Heath, Jarrow and Morton based models that introduced Markov properties to the dynamic framework.
[39] Hull, (1993) 401.

movement. In general after N - steps there were 2^N pricing nodes. Thus the user of the model faces an explosive number of computations as N increased.

The second constraint was that the drift term in Equation (2.25) contained, via the definition of $b(t, T)$, an excess return or market price of risk parameter. Any pricing model derived from this stochastic process would therefore contain expectational variables. To eliminate this problem, whilst maintaining an arbitrage free environment, Heath, Jarrow and Morton introduced a Martingale probability measure of bond returns. It was assumed that there existed a unique Martingale probability of return for every bond. This factor replaced the market price of risk in the dynamics of the bond price and simultaneously removed arbitrage opportunities from bond price movements. The Martingale probability measure was related to the fair game analysis in the market efficiency literature. A fair game was one where on average, across a large number of trials, the expected return of the gamble equalled its actual return.[40]

To highlight this constraint the $b(t, T)$ drift factor in Equation (2.25) could be re-written in terms of the market price of risk. There was assumed to be a separate market price of risk for every bond in the term structure.

$$b(t, T) = \sum_{i=1}^{N} a_i(t, T)(-\gamma_i(t; B_1, \ldots, B_N)) \tag{2.26}$$

$b(t, T)$ = Instantaneous excess expected return on a T- maturity bond at time t.

$\sum_{i=1}^{N} a_i(t, T) = -\int_t^T \sigma_i(t, T)\, dW_i(t)$ The covariance between the T - maturity bond's return and the ith random factor for $i = 1$ to N.
The amount of risk.

[40] Copeland and Weston, (1988) 347.

$(-\gamma_i(t; B_1,\dots, B_N))$ = For a bond of T - maturity the market price of risk due to its association with the random factors $W_i(t)$ for $i = 1$ to N. The price of risk.

$B_1,\dots, B_N$ = The vector of bonds chosen.

Heath, Jarrow and Morton substituted a vector of Martingale probability measures $\tilde{Q}_{(B1,\dots,BN)}$, one for each bond, into Equation (2.26). The replacement of the market price of risk factors by the Martingale probability measures modified the bond drift term in Equation (2.25).[41] A bond pricing equation, without expectational variables, could now be applied to the valuation of fixed interest contingent claims.

To demonstrate this pricing approach Heath, Jarrow and Morton presented a continuous time version of the Ho and Lee model. The authors assumed that this version of the model equated to the situation where there was only one source of uncertainty i.e. one single Brownian motion and that the volatility term was a positive constant. The forward rate stochastic process, consistent with these assumptions, was given by Equation (2.27). Equation (2.28) applied the dynamics in Equation (2.27) to the pricing of bonds.

$$f(t, T) = f(0, T) + \sigma^2 t(T - t/2) + \sigma \tilde{W}(t) \qquad (2.27)$$

$$B(t, T) = [B(0, T) / B(0, t)]\, e^{-(\sigma^2/2)Tt(T-t) - \sigma(T-t)\tilde{W}(t)} \qquad (2.28)$$

---

41 New drift $\int_o^t \alpha(v, t)\, dv = - \sum_{i=1}^{N} \int_o^t \sigma_i(v, t)\, \phi_i(v)\, dv + \sum_{i=1}^{N} \sigma_i(v, t) \int_v^t \sigma_i(v, y) dy\, dv$

where: $\phi_i(t)$ = Martingale probability measure and $\int_o^t \sigma_i(v, y)\, dy$ = The "term structure of volatilities".

A European call option on the bond $B(t, T)$ with an expiration date t^*, where $0 \leq t \leq t^* \leq T$, was assumed to be priced by Equation (2.29). Assuming that the underlying distribution was normal, this was basically equivalent to Black and Scholes's Equation (2.6). The modification, in this case, was that the level of volatility was not measured by changes in the spot bond price $B(t, T)$ but rather by the variance of the forward bond price $B(t, T) / B(t, t^*)$.

$$C(t) = B(t, T)\ N(d1) - XB(t, t^*)N(d1 - \sigma(T - t^*)\sqrt{(t^* - t)}) \qquad (2.29)$$

Where ;

$$d1 = [\log B(t, T) / XB(t, t^*)) + (0.5)\, \sigma(T - t^*)^2 (t^* - t)]$$

$$/\ \sigma(T - t^*)\sqrt{(t^* - t)}$$

The constraint faced by this model was that the forward rate process in Equation (2.27) did not preclude the positive probability of negative interest rates. To solve this problem, Heath, Jarrow and Morton proposed a so called bi-modal volatility approach where the volatility parameter had a specific relationship with the current level of forward rates. The assumptions made were that when forward rates were "small", the forward process was based on proportional volatility and when they were viewed as "large", volatility switched to a constant level. The aim of this approach was to ensure non - negative as well as non - explosive rate structures within the risk pricing model. The tradeoff with these assumptions was that it reinforced the path dependency or non-Markov nature of the dynamics of the pricing model.

2.6 Empirical tests of the Term Structure Models

The discussion in Sections 2.4 and 2.5 suggested that the term structure based risk pricing models faced a number of constraints in any practical applications. There existed the strong possibility of conflicts between the complexity of the underlying theoretical model, its pricing accuracy and the practical concerns of the model builder. Analysis of the Heath, Jarrow and Morton paper highlighted these type of concerns. Heath, Jarrow and Morton attempted to present a general model of interest rate risk. In doing so, they incorporated into their model three key assumptions. Firstly, the stochastic nature of the forward rate process was constrained so that it was arbitrage free. Secondly, the model removed expectational variables and the market price of risk from the pricing of contingent claims. The third assumption aimed to constrain the dynamics of the model so that negative interest rates had a non - positive probability.

While all this was achieved the resulting model relied upon a stochastic process that was both path dependent and computationally difficult to implement. With generality comes complexity and therefore high cost. Reflecting these type of problems, the recent empirical literature published in the major journals has tended to apply the term structure based models to the pricing of shorter dated securities. As mentioned in Chapter 1, Amin and Morton (1994) suggested that empirical bond market pricing analysis was too costly because it would have involved estimation of the dynamics of the entire term structure.

Chan, Karolyi, Longstaff and Sanders (1992) empirically tested the performance of a wide variety of term structure models. The models' ability to forecast changes in one month US Treasury bills was examined. Their approach exploited the fact that many term structure models incorporated a dynamic process, for the short - term riskless rate y, that could be nested within the stochastic process in Equation (2.30).

$$dy = (\alpha + \beta y)\, dt + \sigma y^{\gamma}\, dz \qquad (2.30)$$

The various term structure models could be obtained from Equation (2.30) by placing the appropriate restrictions on the four parameters α , β , σ and γ. For example the Cox, Ingersoll and Ross single factor general equilibrium model discussed in Section 2.4, see Equation (2.14), could be obtained by setting $\gamma = 1/2$.[42]

The parameters in Equation (2.30) were estimated via the Generalised Method of Moments technique for the period June 1964 to December 1989. The research covered n = 307 monthly observations in total. The authors found γ was the most important feature differentiating the different term structure models. Those models which specified γ to be ≥ 1 captured the dynamics of the short term interest rate better than those that required $\gamma < 1$. This suggested that the relationship between interest rate volatility and the level of y was the most important feature of any dynamic model of the short term interest rate.

However, in terms of the models' outright forecasting ability, the results were less powerful. The models' were used to forecast both the future level and the volatility of the one month US Treasury bill yields. The coefficient of determination or R^2 of these tests suggested only a low level of explanatory power. For example, the models only explained between 5.46 % to 20.49 % of the volatility in one month US treasury bill yields.[43]

42 Chan, Karolyi, Longstaff and Sanders, (1992) 1211. The other models tested included Vasicek among others.

43 Note more recent research by Brenner, Harjes and Kroner (1996) suggested that Chan, Karolyi, Longstaff and Sanders may have overstated the power of their findings. The authors claim that the level of y was not the only major factor determining the level of volatility. Modelling volatility in terms of interest rate shocks i.e. unexpected information was also seen as an appropriate strategy. As will be discussed in Chapter 5 of this paper volatility price makers sometimes responded irrationally to large sudden moves in the level of y. There is also the hypothesis to be tested that in the 1990's, as a consequence of the sustained low level of world inflation, that there have been structural changes in the behaviour of key interest rate markets.

Ronn and Sias (1991) applied a Ho and Lee based model to the pricing of US thirteen week T-Bill options. Their approach combined the perturbation functions of the Ho and Lee model with the additional assumption that the level of volatility was non-stationary. This assumption relaxed the requirement that term structure shifts must be equal in magnitude irrespective of the level of interest rates. Ronn and Sias suggested that this allowed them, as per Chan, Karolyi, Longstaff and Sanders's findings, to align interest rate volatility with the level of the short rate. It was assumed that the δ (spread) and π (probability) parameters in Equation (2.22) were linearly dependent on the three month interest rate y.

$$\pi(y) = \pi_0 + \pi_1 y \text{ and } \delta(y) = \delta_0 + \delta_1 y \tag{2.31}$$

Ronn and Sias's second major assumption was that the so called yt statistic was the empirical equivalent of the theoretical perturbation functions in Equation (2.22). The yt statistic was a ratio that measured the actual thirteen week T-bill price in one months time versus today's implied one month forward thirteen week T-bill price. The yt statistic provided a historical record of how accurate today's implied certainty one month forwards had been. $yt < 1$ ($yt > 1$) suggested that yields had risen (fallen) unexpectedly over the month.[44]

$$yt = \frac{B_{t+1mth}(13week)}{\left[B_t(13week + 1mth) \Big/ B_t(1mth) \right]} \tag{2.32}$$

Where ;

$B_{t+1mth}(13week)$ = The actual 13 week T-bill price observed in one months time.

---

44 Equation (2.32) was equivalent to re-arranging Equation (2.19) in terms of $h(T)$ and $h^*(T)$. $yt < 1$ ($yt > 1$) were equivalent to the downstate $h^*(T)$ (upstate $h(T)$).

$$\left[B_t \ (13week + 1mth) \Big/ B_t \ (1mth) \right] = \text{Implied one month 13 week T-bill forecast.}$$

Ronn and Sias utilised this empirical measure of Ho and Lee's perturbation functions and the non - linear least squares procedure to estimate the parameters contained in Equation (2.31), for the period November 1979 to December 1988. In effect, what Ronn and Sias had produced were volatility estimates for the implied one month forward thirteen week T-bill price. Ronn and Sias applied these volatility estimates to the pricing of thirteen week T- bill options.

To calculate the option price there were two required inputs, today's one month implied forward thirteen week T-bill price and the current three month rate which was substituted into Equation (2.31). Consistent with its binomial framework the implied forward, when modified by the estimated perturbation functions produced two possible values for the expiration date of the option. Ronn and Sias used Equation (2.10) and the calculated π probabilities to calculate the price of a one month option on a thirteen week T-bill.

Although the results of this option pricing analysis were reasonable the Ronn and Sias approach had one major weakness. The research only provided volatility information on a very specific scenario, the perturbations of the one month implied forward thirteen week T-Bill prices.[45] The empirical results did not provide volatility information for other periods such as six weeks before expiration. This suggested that the *yt* statistic would have to be calculated for a whole range of other scenarios, if the methodology was to be applied to more general pricing problems.

---

[45] This was why Ronn and Sias only had a small sample of model option prices. The authors volatility database only provided information for those dates $\approx$ thirty days before expiration.

Amin and Morton (1994) used the Heath, Jarrow and Morton model to price Eurodollar future options over the period 1987 - 1992. Amin and Morton compared Heath, Jarrow and Morton's approach to the Black and Scholes model. Both methodologies were seen to price contingent claims via volatility parameters and not through drift terms or risk premia. The major difference between the models was that, while the Black and Scholes model only needed a single scalar to provide all the relevant volatility information, the Heath, Jarrow and Morton model needed volatility information to describe the stochastic evolution of the entire term structure. It was by definition a difficult model to efficiently implement.

Amin and Morton in their study tested a broad class of path dependent Heath, Jarrow and Morton based models.[46] They suggested that advances in computing technology and numerical techniques allowed this type of model to value options effectively despite the fact that the computational effort grows exponentially with the number of steps. Amin and Morton focused on the stochastic process presented in Equation (2.33).

$$df(t,T) = \alpha(t,T,\cdot)\,dt + \sigma(t,T,f(t,T))\,dW(t) \tag{2.33}$$

Equation (2.33) specifies the evolution of forward interest rates as in Equation (2.24). The two key assumptions in Equation (2.33) was that there was only one source of uncertainty $W(t)$ i.e. single factor model and that the choice of $\sigma(\,.\,)$ completely determined all contingent claim prices. The drift terms in Equation (2.33) were seen to be uniquely determined by the risk neutral no-arbitrage Martingale

[46]As mentioned in Footnote 38 there is currently a body of work emerging that presents the Heath, Jarrow and Morton model with path independent qualities. For example Li, Ritchken and Sankarasubramanian (1995) present a class of volatilities for forward rates that make the dynamics of the term structure Markovian with respect to two variables. They suggest that empirical research in this area has potentially important consequences for all term structure modelling.

conditions discussed in Section 2.5.. Amin and Morton tested six versions of Equation (2.33), each one specifying a different $\sigma(\,.\,)$ functionality. When $\sigma(\,.\,) = \sigma_0$ then the stochastic model was seen to represent the so called absolute version which was equivalent to a continuous time Ho and Lee model.

To numerically apply this model to the pricing of Eurodollar futures options, Amin and Morton used a binomial tree lattice. Equation (2.34) was the discrete version of the stochastic differential process presented in Equation (2.33).

$$
\begin{aligned}
& f(t+h_i, t+t_j) - f(t, t+t_j) && (2.34)\\
&= \alpha(t, t+t_j, \cdot)\, h_i + \sigma(t, t+t_j, f(t, t+t_j))\sqrt{h_i} \text{ with prob. } 0.5 \\
&= \alpha(t, t+t_j, \cdot)\, h_i - \sigma(t, t+t_j, f(t, t+t_j))\sqrt{h_i} \text{ with prob. } 0.5
\end{aligned}
$$

The increment to the forward rate of maturity $t+t_j$ over the interval $(t+h_i, t+t_j)$ had a mean drift term $\alpha(t, t+t_j, \cdot)\, h_i$ and a standard deviation term $\sigma(t, t+t_j, f(t, t+t_j))\sqrt{h_i}$. It was worth noting that the drift terms were restricted, as per Ho and Lee's Equation (2.21), to be consistent with the original security prices. Amin and Morton stated that generally fewer then ten steps were needed to accurately price Eurodollar future options. This meant that the final step in the non-recombining tree contained 1024 pricing nodes. This was seen to adequately model the true underlying distribution of the Eurodollar futures prices.

The option pricing model on average was successful, producing option prices that were within 1.5 to 2 yield basis points i.e., 10.02 % - 10.00 % = 2 yield basis points, of those prices observed in the market. Amin and Morton considered that the model provided a good fit of the futures option market, given that it generally had a one basis point bid - ask spread. The most important part of the results was that the Ho and Lee constant volatility model provided the most stable option prices. The assumptions underlying the Ho and Lee model were seen to be the most relevant in a practical pricing situation.

2.7 Closing Comments on the Literature Review

The term structure has been the focus of research for a significant period of time. This was because it summarised market expectations and the factors that formed these views into an observed yield to maturity relationship. These non-observable influences were given economic meaning by the level and shape of the term structure. The three main traditional term structure theories focused on the anticipation of future events, risk preferences and the type of investments made to explain the formation of the interest rate curve. The modern dynamic theories generally assumed that the term structure was exogenously given and attempted to model the potential for change in this construction.

The Black and Scholes (1973) model and its binomial equivalents provided a breakthrough for the dynamic modelling of security prices. The stochastic process presented for stock prices relied on two components, a mean drift term and a volatility / noise term. This approximation of the likely distribution of security prices has become incorporated into most modern contingent claim models. The dynamics took the form of a generalised Wiener process where parameter a was the drift and b was the volatility term.

$$dx = a\,dt + b\,dz \tag{2.35}$$

With the major changes wrought in the financial system over the last fifteen to twenty years we have witnessed the systematic removal of restrictions on the stochastic behaviour of interest rates. Accompanying this new found price variability has been an ever increasingly sophisticated range of interest rate risk models. The current state of the art was represented by the Heath, Jarrow and Morton (1992) model. The authors attempted to provide a unifying theory of interest rate risk. Its assumptions which aimed to model the stochastic behaviour of the entire forward term structure were still based on the generalised Wiener process presented in Equation (2.35).

The other theme of the literature survey was that there often existed direct conflicts between the complexity of the underlying theoretical assumptions and their empirical application. To efficiently implement these intricate models it was often necessary to make comprises. As was discussed in relation to the Heath, Jarrow and Morton model, the three constraints that it tried to satisfy; no risk preferences, no arbitrage and no negative interest rates: created other problems such as the path dependence of interest rates. One can suggest that this fact limits the appeal of the Heath, Jarrow and Morton model, particularly in certain long term option applications. Therefore there are still many issues to be resolved in the term structure area which provides the impetus for continued theoretical and empirical research.

CHAPTER 3

Methodology and Data

3.1 Introduction

As outlined in Chapter 1 the objective of this paper was to determine if term structure based interest rate forecasts had a role to play in risk management. Three central hypotheses were constructed to test this proposition. The first hypothesis stated that the Australian market term structure was derivable from the Commonwealth government coupon bond yield curve. The second hypothesis stated that contained in this term structure was the market's implied forecasts of N-period forward interest rates. These implied forecasts in reflecting decisions made under uncertainty were assumed to have errors. The third hypothesis proposed that these forecasting errors could be used to approximate the stochastic behaviour of the term structure.

These three central hypotheses were broken up into a total of eleven hypotheses which were tested over the sample period 01/01/94 to 31/12/94. The data for this empirical research was taken from both physical and derivative markets. End of business day prices were recorded for bank bill, Commonwealth government coupon bond, interest rate swap and the ten year bond futures/options contracts traded on the Sydney Futures Exchange. As will be discussed in Chapters 4 to 7, the chosen sample period proved interesting because it coincided with a time when the market's interest rate expectations repeatedly failed to keep pace with the actual change in the level of yields.

Chapter 3 was divided into five sections. Section (3.2) dealt with the derivation of the Australian market term structure from the observed Commonwealth government bond coupon curve. Section (3.3) discussed the development of the so called Yield Error Margin database. Section (3.4) presented an approach to pricing fixed interest options that relied on the implied certainty forward and the Yield Error Margin database of Section (3.3). Section (3.5) constructed a "naive" hedging strategy for fixed interest portfolios and Section (3.6) summarised the hypotheses to be examined in Chapters 4 to 7.

3.2 Derivation of the Australian Market Term Structure

The first major hypothesis of this paper stated that the Australian market term structure was derivable from the observed Commonwealth government coupon bond curve.[47] In keeping with Chapter 2's definition of the term structure, as the market's absolute value of time, the zero coupon equivalent of the Commonwealth government coupon bond curve (zero coupon government bond curve here after) was assumed to provide the best measure of the riskless rate of return in the Australian market place. The securities issued by the Australian Commonwealth government were seen to be the least likely to default amongst all the Australian securities available and thereby provided a credit quality benchmark.

This approach was consistent with Black, Derman and Toy (1990) who used the yields on zero coupon US Treasury bonds of different maturities to construct the US market term structure.[48] The constructed zero coupon government bond curve was also comparable to what Hull (1993) called the spot rate curve because it measured the riskless rate of return demanded by the market for holding a security that paid no

[47] The Commonwealth government coupon bond curve was also known in the market as the Treasury curve. To avoid confusion with US based research, this paper refers to it at all times as the Commonwealth government coupon bond curve, to denote government debt issued by the Australian government.
[48]Black, Derman and Toy, (1990) 33. Note that there exists no active Australian market in zero coupon Commonwealth government bonds. A zero coupon curve had to be derived from the observed Commonwealth government bond curve.

coupons for N - periods.[49] Therefore the zero coupon government bond curve was assumed to provide the closest empirical approximation of the "true" term structure.

The methodology used to transform the Commonwealth government coupon bond curve into the zero coupon government bond curve had two aspects. The first adopted the assumptions of Shiller (1990) and Black, Derman and Toy (1990) where the individual coupon bonds that made up the Commonwealth government coupon bond curve were seen to represent a portfolio of underlying zero coupon bonds.[50] Each coupon cashflow and the principal repayment at maturity associated with the coupon bond were treated as distinct cashflows that related to individual zero coupon securities. The corollary of this was that the current market price of the coupon bond could be set equal to the sum of its zero coupon bond components.

This approach fitted neatly into the institutional arrangements of the Australian fixed interest market. The observed market prices of the bonds that made up the Commonwealth government coupon bond curve were calculated according to the conventions contained in the Reserve Bank of Australia (RBA here after) bond pricing formula. [51]

RBA bond pricing Formula:

$$B_t = V^{f/d}(\, g\,(\, 1 + a_N\,)\,) \; + 100\, V^N \tag{3.1}$$

---

[49]Hull, (1993) 81. The zero coupon government bond curve was also free of what Fabozzi (1995) called the "coupon effect". This effect measures the sensitivity of the traded yield of a bond to different cashflow characteristics. For example, for two bonds with the same maturity date but different size coupons, the bond with the lower coupon would trade at a higher yield.

[50]Shiller, (1990) 634. Black , Derman and Toy, (1990) 36.

[51]See Attachment 1 which details the RBA bond pricing conventions.

Ex - Interest : Seven days or less before coupon:

$B_t = V^{f/d}(g\, a_N) + 100\, V^N$

Where ;

Settlement date = $t + 3$ business days from today's transaction date t.

B_t = The bond price as at today's date t. Price quotations per \$100 face value.

$V = 1 / (1 + Y)$

Y = The current market yield of the bond divided by 200.

f = Number of days from the date of settlement to the next coupon payment date.

d = Number of days between the last coupon date and the next coupon payment date.

g = Half yearly rate of coupon payment per \$100 face value.

N = The number of half year periods from the next interest payment date to maturity.

$a_N = V + V^2 + \ldots\ldots\ldots V^N = (1 - V^N) / Y$

The most important assumption underlying Equation (3.1) was that each of the cashflows associated with the coupon bond was discounted at the prevailing market yield of the bond.[52] The construction in Table 3.1 highlights the significance of this key assumption. To determine the price of the 15/01/98 bond as at 01/11/92 the discount factor $1 / (1 + Y)^{f/d+N}$ was applied to each of bond's eleven cashflows. The objective of the zero coupon government bond curve derivation was to replace

[52]The current yield to maturity was assumed to be the rate at which all the coupon payments of the bond were re-invested over its time to maturity. The RBA in Attachment 1 identified that in practice this was unlikely to hold as the level of yields changed over the life of the bond. The aim of the RBA bond pricing formula was to ensure that buyers and sellers used the same basis when pricing bonds in the market place.

Table 3.1: Pricing the 15/01/98 bond as at 01/11/92

Where ; $Y =$ 8.30/200 = 0.0415

$f/d =$ 75/184 = 0.4076087

No. Cashflow	Cashflow	Date	Discount Factor	PV
1.	$6.25	15/01/93	$1/(1.0415)^{0.407}$	6.14726512
2.	$6.25	15/07/93	" "1.407	5.90231888
3.	$6.25	15/01/94	" "2.407	5.66713287
4.	$6.25	15/07/94	" " 3.407	5.44131816
5.	$6.25	15/01/95	" "4.407	5.22450136
6.	$6.25	15/07/95	" " 5.407	5.01632392
7.	$6.25	15/01/96	" " 6.407	4.81644159
8.	$6.25	15/07/96	" " 7.407	4.62452385
9.	$6.25	15/01/97	" "8.407	4.44025334
10.	$6.25	15/07/97	" "9.407	4.26332533
11.	$106.25	15/01/98	$(1/1.0415)^{10.407}$	69.58860364

Total Bond Price	121.132

$1 / (1 + Y)^{f/d+N}$ with discount factors based on market consistent zero coupon yields. Each of the eleven cashflows would be discounted by their associated N-period zero coupon yield to attain the current market bond price.

This discussion leads to the second aspect of the zero coupon government bond curve derivation. To generate zero coupon yields from the Commonwealth government coupon bond curve the so called "bootstrapping" technique, discussed by Kawaller and Marshall (1996) and Hull (1993), was utilised.[53] The methodology relied on three factors: The current market prices of the coupon bonds that made up the Commonwealth government coupon bond curve, the coupon / principal cashflows associated with these bonds and the assumption that short dated risk free discount

[53] Kawaller and Marshall, (1996) 52.Hull, (1993) 81- 84.

securities could be used as the initial anchor points for the construction of the zero coupon government bond curve.

The bootstrapping technique was used to construct the Australian market term structure in the following way. Y_1 denoted the yield of the single period risk free discount security consistent with the observed Commonwealth government coupon bond curve. Y_1 was seen to represent the first zero coupon yield in the term structure. Assume that as part of the Commonwealth government coupon bond curve that there existed a two period bond with coupon payments c_2 and a closing market price of B_2. The aim was to determine the two period zero coupon yield Y_2 that was compatible with Y_1, c_2 and B_2. Equation (3.2) demonstrated the problem at hand.

$$B_2 = \frac{c_2}{(1 + Y_1)} + \frac{c_2}{(1 + Y_2)^2} \tag{3.2}$$

Solving for Y_2, as the only unknown in Equation (3.2), allowed the two period zero coupon yield to be calculated. To generate the $N = 3$ zero coupon rate, an identical calculation process could be carried out with B_2, c_2 replaced by B_3, c_3 and an additional discounting term added to Equation (3.2). Y_3 would be the unknown in this equation.

In general terms this step by step iterative bootstrapping based process could be represented by Equation (3.3). Assuming that N-1 zero coupon yields were known the Nth zero coupon yield could be calculated.

$$B_N = \sum_{i=1}^{N-1} \frac{c_N}{(1 + Y_i)^{T_i}} + \frac{c_N}{(1 + Y_N)^{T_N}} \tag{3.3}$$

Where ;

Y_i = The zero coupon yields associated with the i to N - 1th periods.

T_i = The term to maturity of the ith zero coupon yield.

B_N = The observed price of the Nth - period coupon bond.

c_N = The coupon associated with the Nth - period coupon bond.

Y_N = The unknown zero coupon yield of the Nth period.

T_N = The term to maturity of the Nth - period coupon bond.

The central problem with using the bootstrapping technique described in Equation (3.3) was that it assumed that there were no "gaps" in the underlying Commonwealth government coupon bond curve. The Commonwealth government coupon bond curve may in fact at times have "gaps" in the sense that there can exist large maturity differences between the coupon bonds observed along the curve.

Such a scenario can be illustrated in Table 3.1.. Assume that the last bond on the coupon curve was the 15/01/98 bond and that the second last bond was the 15/07/96 series. Under these circumstances Equation (3.3) would not be able to find the zero coupon yield for the 15/01/98 date because there were three unknowns: Firstly, the zero coupon yield for the 15/01/97 date, secondly the zero coupon yield for the 15/07/97 date and finally the zero coupon yield associated with the 15/01/98 date. This suggests that if the N-1th zero coupon yield was not available then the Nth zero coupon yield could not be found. There may not always exist the necessary number of anchor points to build a complete zero coupon government bond curve.[54]

Hull (1993) suggested using the well known linear interpolation technique to overcome this yield curve "gap" problem.[55] This approach involved reducing n zero

[54] Note that all Commonwealth coupon bonds are issued with a 15th of the month maturity date and that all coupons are also paid on this date.
[55] Hull, (1993) 84 - 86.

coupon yield unknowns into an expression where there was only one unknown. For the purposes of this paper Equation (3.4) was used to linearly interpolate between dates without a known zero coupon yield.

$$Y_{N-1} = Y_{N-2} + [((T_{N-1} - T_{N-2}) / (T_N - T_{N-2})) \times (Y_N - Y_{N-2})] \qquad (3.4)$$

Where ;

Y_{N-2} = The last known derived zero coupon yield. The N-2th yield.

T_{N-2} = The term to maturity of the N-2th zero coupon yield.

Y_{N-1} = The zero coupon yield to be calculated. The N-1th yield.

T_{N-1} = The term to maturity of the N-1th zero coupon yield.

Y_N = The longest dated unknown zero coupon yield. The Nth yield.

T_N = The term to maturity of the Nth zero coupon yield.

$((T_{N-1} - T_{N-2}) / (T_N - T_{N-2}))$ = Weighting ratio applied to the yield difference between $Y_N - Y_{N-2}$ to determine the level of Y_{N-1}.

In Equation (3.4) Y_{N-1} and Y_N were assumed to be unknown but Y_{N-2} and the weighting ratio were known variables. Re-arranging these known and unknown factors allowed Y_{N-1} to be made a function of Y_N.[56] With Y_N, the Nth zero coupon yield, the only unknown it was possible to use Equation (3.3) to complete the construction of the zero coupon government bond curve.

In practice a five part bootstrapping based procedure was used to generate the zero coupon government bond curve. The first part of the zero coupon bond curve derivation involved recording the four interest rate market data series that were used

56 Assume in Equation (3.4) that $Y_{N-2} = 6$ and that the weighting ratio was = 0.5. After substitution $Y_{N-1} = 3 + 0.5Y_N$.

as daily inputs into Equation (3.3). Series one was the RBA determined official overnight cash yield. This overnight yield was 4.75 % between 01/01/94 and 16/08/94. The RBA increased the official rate by 0.75 % to 5.50 % on 17/08/94, 1.00 % to 6.50 % on 24/10/94 and a further 1.00 % to 7.50 % on 14/12/94.[57]

Series two was the Australian bank bill security data, for maturities of one to six months, taken from the Reuters page BBSW. This page was available at 10.15 a.m. on every trading day. The page served as a daily rate setting mechanism for the floating side of swap contracts and short term corporate loan/drawdown facilities. The BBSW bank bill yields were used in place of the Commonwealth government treasury note data series because these yields were more consistently available. There were certain periods of time when treasury notes of particular maturity dates were not issued leaving gaps in the data set.[58] To modify the BBSW bank bill series so that it better approximated the equivalent riskless Commonwealth treasury note yield series, 0.02 % was deducted from the quoted mid-bank bill rate.[59]

Series three was the closing 4.30 p.m. one year swap yields taken from the Telerate Tactician fixed interest service. The main reason for the inclusion of the swap series was that Commonwealth government coupon bonds were not actively traded once they became a short dated security. With one to two coupons remaining in their schedule there were few markets made in these type of bonds. They were generally regarded as longer dated Commonwealth treasury notes that would be held to maturity. To counter this market characteristic the one year swap rate series was used

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57 RBA bulletin, (January 1995) 27.

58 Treasury notes were issued at weekly tenders and were used mainly for money market management by the RBA.

59 This credit spread between Commonwealth government and bank issued paper obviously fluctuated depending on market conditions and the relative supply of the two securities. 0.02 % on average over a full trading year was seen as a reasonable assumption.

to approximate the trading yield of a one year Commonwealth government security. To compensate for the credit differentials between bank and government backed paper 0.10 % was deducted from the swap yield series. This reduction in the yield, as per the bank bill series, was seen to adequately capture the average level of the spread between one year bank and government paper over the sample period.

The fourth data series was the so called "Hot stock" bonds which were assumed to represent the observed Commonwealth government coupon bond curve. The term "Hot stock" was used by the market to describe those Commonwealth government coupon bonds, from > twelve months to ten years in maturity, that had the greatest level of turnover, liquidity and marketability.[60] They were the benchmark risk free bonds from which the Australian market term structure could be developed. This assumption was supported by the fact that these "Hot stocks" were used by the Sydney Futures exchange (SFE here after) to price their three year and ten year bond futures contracts. The daily closing 4.30 p.m. yields of the "Hot stock" bond category were taken, like the one year swap series, from the Telerate Tactician fixed interest service.

Appendix B summarised the interest rate market data needed for the derivation of the zero coupon government bond curve on a given trading day. The four data series can be split into two groups. The RBA official cash rate, the bank bill and the one year swap yields formed the short dated "anchor" data series. The "Hot stocks" that made up the Commonwealth government coupon bond curve represented the

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60 The "Hot stock" bond category changes over time when individual bonds fall to less than one year in maturity and when the Australian treasury issuing programme decides to concentrate on different parts of the curve. For example the Australian treasury may issue beyond ten years when there was demand for this type of maturity from the market. This will also obviously depend on the Budget deficit and general financing requirements. At any point in time which bonds were classified as "Hot stocks " was available from the market's price makers. The Reuters page GLTT supplied by Australian Gilts featured the current trading yields of this bond category.

Australian yield curve from one year to ten years. Combining these two groups of market data provided the base from which to construct the zero coupon government bond curve.

The second part of the zero coupon government bond curve derivation developed the short dated part of the term structure. The short dated "anchor" yields, from one day to one year, were converted to discount factors via Equation (3.5). These were viewed as the initial zero coupon discount factors from which to start the bootstrapping algorithm.

$$Zd_{Sa-t} = 1 / (1 + (Y_{Sa-t} / 100))^{((Sa-t) / 365)} \tag{3.5}$$

Where ;

Zd_{Sa-t} = The zero coupon discount factor associated with the short dated "anchor" security of days to maturity $Sa - t$ for trading date t.
Y_{Sa-t} = The yield associated with the short dated "anchor" security of days to maturity $Sa - t$ for trading date t.

$Sa - t$ = The days to maturity of the short dated "anchor" security as at trading date t. This was set with reference to the $t + 3$ bond settlement date and not the actual trading day date i.e. 04/01/94 was the trading day but 07/01/94 was the bond settlement date. See Equation (3.1).

The third part of the process involved the first bond in the "Hot stock" series. As per the bond market conventions covered in Table 3.1 this bond's coupon payments fell on known dates. Using the linear interpolation technique introduced in Equation (3.4) it was possible to calculate the zero coupon discount factors, for those coupon dates less then one year in maturity, from the short dated "anchor" curve.

The fourth part of the construction substituted these known interpolated discount factors into the cashflow structure of the shortest maturity coupon bond. This allowed

Equation (3.3) to calculate the unknown zero coupon discount factor that was associated with the maturity date of the bond.[61] Thus the zero coupon bond coupon curve could be extended beyond the one year boundary set by the short dated "anchor" curve. Appendix C illustrated how parts three and four of the derivation were used to generate the zero coupon discount factors for the 15/02/95 Commonwealth coupon bond on the 04/01/94 trading date.

The final part of the derivation repeated steps three and four for the remaining bonds in the "Hot stock" series. Applying the bootstrapping methodology across the entire bond maturity spectrum allowed a complete zero coupon government bond curve to be generated. This measure of the Australian market term structure, from one day to ten years, was constructed for every full trading day in the sample period 01/01/94 to 31/12/94. Half trading days such as 30/12/94 were excluded from the sample because they suffered from abnormally low levels of liquidity and turnover. On these days the market was not fully operational with many market participants absent from trading. Thus $n = 248$ model term structures were produced.

To assess whether the derived zero coupon government bond curve was consistent with the pricing assumptions of the Australian fixed interest rate market it was used to evaluate the ten year Commonwealth government bond futures contract traded (ten year bond futures contract here after) on the SFE. This security was chosen for this role because it provided the key pricing benchmark for all the longer dated swaps, issuing and investment transactions that occurred in the interest rate market.[62] As was assumed with the zero coupon government bond curve the ten year

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[61] The zero coupon discount factors could be converted to an equivalent zero coupon yield by rearranging Equation (3.5). The zero coupon yield = $[(1/ (Zd_{Sa-t}{}^{(365/Sa-t)}) -1] \times 100$.

[62] For example most issuing done by Australian state governments or other non - Commonwealth names was quoted as a yield spread over the ten year bond futures contract.

bond futures contract was traded on the basis that its yield represented the return from a risk free security that carried negligible default risk.

To ensure the absence of riskless arbitrage the observed price of the ten year bond futures contract should agree with the Cash and Carry fair value price.[63] Equation (3.6) demonstrated that this was equivalent to the average forward yield of a basket of SFE specified Commonwealth government coupon bonds.[64] Appendix D highlighted the workings of the Cash and Carry pricing approach for the 04/01/94 trading date.

$$F_{(CAC)t} = 100 - \left(\left(\sum_{i=1}^{n} YB_{(CAC)fi} \right) / n \right)_t \tag{3.6}$$

Where;

$F_{(CAC)t}$ = The Cash and Carry fair value ten year bond futures price at trading date t.

$YB_{(CAC)fi}$ = The forward yield YB of the ith Commonwealth government coupon bond that was incorporated into the SFE ten year bond futures contract pricing basket. The ith bond's forward yield was determined by the bond's forward price at the ten year bond futures cash settlement date ft. This forward bond price was set equal to $B_{(CAC)ft}$. Using RBA Equation (3.1) a bond yield was found that was consistent with $B_{(CAC)ft}$.

$$B_{(CAC)ft} = B_t \times (1 + (Y_{ft-t+3} \times (ft\text{-}t\text{+}3 / 365))) - c \times (1 + (I_t[Y_{ft-ct}] \times (ft\text{-}ct / 365)))$$

B_t = The Commonwealth government coupon bond price set at date t but received/paid at t+3. This price was calculated by Equation (3.1).

Y_{ft-t+3} = The market yield for days to maturity $ft - t + 3$.

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63 The prices were called Cash and Carry because the observed futures price could be replicated by purchasing the underlying portfolio of bonds (Cash instrument equivalent) and holding them (Carry) at a cost until the ten year bond futures contract settlement date.

64 See Attachment 2 and 3. Attachment 2 details the SFE specification of the ten year bond futures contract. Attachment 3 records what Commonwealth government coupon bonds were used in the futures pricing basket for the 1994 trading sample.This covered the March 1994, June 1994, September 1994, December 1994 and March 1995 ten year bond futures contracts.

$ft - t+3$ = The number of days between date $t+3$ and the ten year bond futures cash settlement date ft.

c = The coupon paid by the ith Commonwealth government bond for the period between date $t+3$ and the ten year bond futures settlement date ft. The coupon was subtracted from the forward bond price $B_{(CAC)ft}$ because the holder of the ten year bond futures contract forwent the right to this payment.

$I_t[Y_{ft-ct}]$ = The implied forward yield of days to maturity ft-ct as at today's trading date t.

ft-ct = The number of days between the coupon payment date ct and the ten year bond futures cash settlement date ft.

n = The number of Commonwealth government coupon bonds in the SFE ten year bond futures contract pricing basket. For the sample period 01/01/94 to 31/12/94 n equalled either four or five.

The zero coupon government bond curve based (Model here after) ten year bond futures prices were produced by implied forward term structures. The zero coupon discount factors associated with today's trading date were used to calculate implied forward discount factors for the ten year bond futures cash settlement date ft.

$$Zd_{m-ft} = \frac{Zd_{m-t+3}}{Zd_{ft-t+3}} \tag{3.7}$$

Where ;

Zd_{m-ft} = The implied forward zero coupon discount factor associated with the SFE ten year bond futures cash settlement date ft of days to maturity $m - ft$.

Zd_{m-t+3} = The zero coupon discount factor associated with today's term structure of days to maturity m-t+3. Where the date $m = ft+1day, ft+2day, \ldots\ldots, ft+jthday$. The jth day was the longest dated zero coupon discount factor associated with today's term structure.

Zd_{ft-t+3} = The zero coupon discount factor associated with today's term structure of days to maturity $ft - t+3$.

The calculated Zd_{m-ft} formed an implied forward term structure for the ten year bond futures cash settlement date *ft*. These implied forward zero coupon discount factors were subsequently substituted into the cashflow structure of the individual bonds that made up the SFE ten year bond futures pricing basket. In keeping with Equation (3.6) these forward bond prices were converted to implied forward yields which allowed a model futures price to be generated. Appendix E demonstrated the model pricing methodology for the 04/01/94 trading date.

$$F_{(Model)t} = 100 - \left(\left(\sum_{i=1}^{n} YB_{(Model)ft} \right) / n \right)_t \tag{3.8}$$

Where ;

$F_{(Model)t}, YB_{(Model)ft}$ = Equivalent to the definitions in Equation (3.6) except that $B_{(Model)ft}$ was produced by the implied forward zero coupon discount factors of Equation (3.7) instead of the Cash and Carry methodology.

Three separate measures were used to investigate the performance of the model pricing methodology. The first measure was the average pricing error statistic. The pricing error was calculated as the difference between the observed 4.30 p.m. closing ten year bond futures price reported in yield terms and the model generated closing futures price reported in yield terms. This pricing error, Market yield - Model yield was calculated for each full trading day in the 01/01/94 to 31/12/94 sample.

$$Y_{(Fut\ error)t} = (100 - F_t) - \left(\left(\sum_{i=1}^{n} YB_{(Model)ft} \right) / n \right)_t \tag{3.9}$$

Where ;

$Y_{(Fut\ error)t}$ = The pricing error between the ten year bond futures equivalent yield observed in the market and those produced by the term structure based model for trading date *t*.

F_t = The 4.30 p.m. closing SFE ten year bond futures price for trading date t. The price was converted to an equivalent yield by the relationship $(100 - F_t)$.

$\left(\left(\sum_{i=1}^{n} Y_{B(Model)ft} \right) / n \right)_t$ = The model average implied forward yield at ft of the Commonwealth government coupon bonds that were incorporated into the SFE ten year bond futures pricing basket for today's trading date t.

Consistent with the analysis of Flesaker (1993) and Amin and Morton (1994) the pricing error in Equation (3.9) was reported in terms of yield basis points.[65] For example if the market yield was observed as 6.85 % and the model yield was 6.84 % the pricing error would be reported as 6.85 % - 6.84 % = one basis point rather than 0.01 %. This approach was pursued throughout Chapter 3 because the Australian interest market uses the basis point as its basic unit of measurement.

Hypothesis 1: The average difference between the SFE ten year bond futures prices and the term structure model prices = 0

Hypothesis one examined the proposition that the model prices were reliable estimates of the market. The objective was to determine if on average the model pricing errors were not significantly different from zero. The test statistic used for this research was drawn from Kenkel (1989).[66] The statistic in Equation (3.10) depended on the random variable $(\bar{x}_1 - \bar{x}_2)$. This variable provided an average sample measure of $Y_{(Fut\ error)}$, the model pricing error introduced in Equation (3.9). The test allowed the analysis to determine if the difference between the two population means, market versus model, was consistently around zero.

65 Flesaker, (1993) 492 and Amin and Morton, (1994) 148 and 162. Both papers investigated the characteristics of Eurodollar futures and futures options markets. In these markets, like the SFE based contracts, the price change was quoted in so called basis points. The minimum change in the quoted futures was given as 0.01 which corresponded to a basis point or tick. Note in 1997 the Eurodollar contract has been changed so as to allow half or 0.005 point bid - ask spreads.

66 Kenkel, (1989) 462.

To be accurate the test statistic in Equation (3.10) relied on the two price series being normally distributed and independent of each other. These assumptions were seen as reasonable, given that the sample sizes were set equal to 248 and that the model prices were not derivative of those observed in the market but were instead generated by the methodology introduced in Equation (3.7).

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\sigma_1^2/n + \sigma_2^2/n}} \tag{3.10}$$

Where ;

Z = The observed value of the test statistic.

$\bar{x}_1$ = Sample mean of the market observed ten year bond futures yield series.

σ_1^2 = Sample variance of the market observed ten year bond futures yield series.

$\bar{x}_2$ = Sample mean of the model produced ten year bond futures yield series.

σ_2^2 = Sample variance of the model produced ten year bond futures yield series.

n = The number of full trading days in the sample. For the 01/01/94 to 31/12/94 sample, n was set equal to 248.

The average pricing error approach was repeated for the Cash and Carry futures pricing series. Equation (3.6) was used to calculate Cash and Carry ten year bond futures prices for every day in the trading sample. They were viewed as a set of control prices from which the model pricing methodology could be compared. The Cash and Carry futures price series also provided an opportunity to assess the efficiency of the ten year bond futures contract. As mentioned earlier the observed ten year bond futures price should reflect the forward value of the underlying physical bond portfolio.

Hypothesis 2: The average difference between the SFE ten year bond futures prices and the Cash and Carry model prices = 0 [67]

Ordinary Least Squares analysis provided the second measure of the model's ten year bond futures pricing performance. Following the approach of Ronn and Sias (1991) and Flesaker (1993) a simple regression relationship was used to examine the proposition that the model produced unbiased estimates of the futures prices observed in the market.[68] This regression was estimated for the 01/01/94 to 31/12/94 trading period.

Hypothesis 3: The term structure model produced unbiased estimates of the ten bond futures prices observed in the market.

$$(100 - F_t) = \alpha + \beta \left(\left(\sum_{i=1}^{n} YB_{(Model)ft} \right) / n \right)_t + \varepsilon_t \tag{3.11}$$

Where ;

F_t , $\left(\left(\sum_{i=1}^{n} YB_{(Model)ft} \right) / n \right)_t$ = As defined in Equation (3.9).

ε_t = The error or disturbance term of the stochastic relationship.

The model prices were seen to be unbiased if $\alpha = 0$ and $\beta = 1$. If the regression intercept and slope parameter in Equation (3.11) were found to have these values then it could be stated that the model perfectly predicted the ten year bond futures prices observed in the market.

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67 Hypothesis two used the same test statistic as hypothesis one. The sample mean and variance of the Cash and Carry price series were used in the testing of hypothesis two.

68 Ronn and Sias, (1991) 100. Flesaker, (1993) 491. Equation (3.11) was a standard test of the relationship between model and market prices. As Flesaker suggested the OLS methodology does have its merits when it comes to providing feedback from model testing.

The third measure of model performance complemented the results associated with Hypothesis three. It can be suggested that the ultimate practical test of any pricing model lies in its capacity to accurately predict the value of the targeted variable. The model's ability to forecast ten year bond futures prices was tested on the out of sample January 1995 period. The regression estimated for the 1994 trading sample in Equation (3.11) was used to generate ten year bond futures prices for every full trading day between 01/01/95 and 31/12/95. The accuracy of the forecasted prices was measured by the error statistic in Equation (3.12). If the average pricing error for the January 1995 period was found to be similar to the original trading sample then this would support the hypothesis that the term structure based model had accurately captured the pricing assumptions of the Australian interest rate market.

$$Y_{(Forecast\ error)\,t} = (100 - F_t) - \left(\left(\sum_{i=1}^{n} YB_{(Forecast)ft} \right) / n \right)_t \qquad (3.12)$$

Where ;

$Y_{(Forecast\ error)\,t}, F_t, YB_{(forecast)ft}$ = Equivalent to the definitions in Equation (3.9) except that the regression in Equation (3.11) was used to generate the forward bond yields.

3.3 Calculation of the Yield Error Margin database

Section 3.3 detailed the development of the so called Yield Error Margin database. This market based construction was used to test the two key remaining hypotheses associated with this paper. The second major hypothesis stated that the market's current best forecast of the future level of interest rates was equal to today's implied N-period forward term structure. These implied forecasts were seen to have errors because they reflected decisions made under uncertainty. The third major hypothesis proposed that over a given sample period the distributional characteristics of these forecasting errors could be used to measure the stochastic behaviour of the term structure.

In Section 3.2 implied forward term structures were used to price SFE ten year bond futures contracts. In the language of Ho and Lee (1986) these model based futures prices represented implied certainty forwards.[69] Today's observed futures prices should reflect the information contained in today's term structure. If this was not the case it would be possible to construct a replicating bond portfolio that made risk free profits.

Market traded futures and forward contracts allow agents to lock into today's best estimate of the underlying securities price at some given N-period forward date. The futures and forward contracts provide certainty for market participants. However, they do not provide the actual price of the underlying security at the forward contract settlement date. Agents operate under uncertainty so that there exists no guarantee that today's current forecast of the security price need actually eventuate in N-periods of time. The current estimate of the security price at the N-period forward date represents only one of a range of potential outcomes that have a positive probability of occurring.

Consistent with the aims of Ho and Lee (1986) the current paper was focused on determining the discrepancies between today's implied N-period forward term structures and the actual term structures that eventuated on the N-period forward date. Ho and Lee's upstate and downstate theoretical perturbation functions specified the deviations of the actual term structure from the implied certainty forward functions. As discussed in Chapter 2, Ronn and Sias (1991) provided an interesting empirical interpretation of the perturbation functions. Ronn and Sias's yt statistic measured the term structure perturbations via expression (3.13).[70]

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69 The futures prices were equivalent to the implied certainty forward generated by Equation (2.18).
70 Ronn and Sias, (1991) 93.

$$yt = \frac{Actual\ yield\ in\ N - Periods\ of\ time}{Today's\ implied\ forward\ yield\ in\ N - Periods\ of\ time} \tag{3.13}$$

The Yield Error Margin statistic like the yt ratio attempted to generate a historical database that recorded the errors in the market's term structure based interest rate forecasts. Using the assumptions presented in Equation (3.7) implied N-period forward term structures were constructed from today's derived zero coupon government bond curve. Equation (3.14) replaced the ten year bond futures cash settlement date ft with the zero coupon discount factors associated with the N-period forward date.

$$Zd_{m-N} = \frac{Zd_{m-t+3}}{Zd_{N-t+3}} \tag{3.14}$$

Where ;

Zd_{m-N} = The implied forward zero coupon discount factor associated with the N-period forward date of days to maturity m-N.

Zd_{m-t+3} = The zero coupon discount factor associated with today's term structure of days to maturity m-t+3. Where m was any trading date = $N+1day, N+2day, \ldots\ldots, N+jthday$. The jth day was the longest dated zero coupon discount factor associated with today's term structure.

Zd_{N-t+3} = The zero coupon discount factor associated with today's term structure of days to maturity $N - t+3$.

Equation (3.14) allowed the market's implied N-period forward forecasts of yields to be calculated. The accuracy of these implied N-period forward yields was assessed by comparing them with the actual term structure that was observed on the N-period forward date. The error between today's N-period forward forecasted yields and the actual yields in N-periods of time was defined as the Yield Error Margin statistic.

To allow the empirical development of the Yield Error Margin statistic the following methodology was pursued. In the interests of tractability today's term structure was only used to construct four implied N-period forward term structures: two weeks, one month, two months and three months forward.[71] Consistent with the analysis in Section 3.2 each of the four implied forward term structures was used to produce forward prices for the physical Commonwealth government coupon bonds that made up the SFE ten year bond futures pricing basket. These forward bond prices were converted to yields from which subsequently the arithmetic mean was calculated. This mean represented the implied N-period forward average basket yield. The actual term structures observed on these forward dates were also used to construct average futures basket yields.

The empirical Yield Error Margin statistic was calculated by the relationship in Equation (3.15). In keeping with the error assumptions presented in Section 3.2 the Yield Error Margin statistic was measured in yield basis point terms. A positive forecasting error indicated that today's expectation of the level of yields had been too high. A negative Yield Error Margin suggested that the market had underestimated the actual increase in yields for the period between today and the N-period forward date.

$$\mathrm{YEM}_t = \left(\left(\sum_{i=1}^{n} YB_{IFNt} \right) / n \right)_t - \left(\left(\sum_{i=1}^{n} YB_{ANt} \right) / n \right)_{Nt} \qquad (3.15)$$

Where ;

[71] Implied N-period forwards could have been calculated for 1 day, 2 days, 3 days...... 2000 days forward. The two week, one month , two month and three month forward periods were chosen because they enabled the Yield Error Margin assumptions to be applied to the pricing of shorter dated options such as those traded on the SFE.

YEM_t = The calculated Yield Error Margin associated with the implied N-period forward term structure, either two weeks, one month, two months or three months forward for today's trading date t.

YB_{IFNt} = Today's implied N-period forward bond yield of the ith Commonwealth government coupon bond that was incorporated in the SFE ten year bond futures pricing basket. As per Equation (3.7) the implied forward bond prices were converted to a market consistent bond yield via RBA Equation (3.1).

YB_{ANt} = The actual bond yield observed at the N-period forward date of the ith Commonwealth government coupon bond that was incorporated in the SFE ten year bond futures pricing basket.

n = The number of Commonwealth government coupon bonds in the SFE ten year bond futures pricing basket. The date of today's trading date t determined which futures contract pricing basket was used for the construction of YEM_t. For example on 04/01/94 the March 1994 ten year bond futures contract was used. This was known as the so called "nearby" futures contract. It was the closest contract currently trading.

Four Yield Error Margin statistics; representing the two week, one month, two month and three month implied forward forecasting errors were produced for every full trading day in the 01/01/94 to 31/12/94 sample. Appendix F demonstrates the calculation of the two week Yield Error Margin statistic for the 04/01/94 trading day.

The actual sample sizes of the four Yield Error Margin databases varied because of data constraints. For example on the December 29th 1994 trading date it was not possible to calculate the Yield Error Margin statistic for either the two month or three month Yield Error Margin databases. Yield curve data had not been obtained for the period beyond 31/01/95.[72] As a consequence the two week and one month Yield Error Margin databases had the full number of observations n = 248. The two month

[72] Yield curve data for January 1995 was incorporated into the ten year bond futures forecasting research discussed in Section 3.2.

and three month databases were limited to n = 230 and n = 208 observations respectively.

The empirical accuracy of today's term structure based implied N-period forward interest rate forecasts has been tackled by a range of researchers including Bloch (1974), Fama and Bliss (1987) and Cuthbertson (1996).[73] The current paper utilised the testing methodology put forward in hypotheses one and two. Hypothesis four examined the proposition that the average difference between the forecasted average ten year bond futures basket yields and the actual observed ten year bond futures basket yields was equal to zero. In other words we were testing whether the market's forecasting errors were statistically significant over the 01/01/94 to 31/12/94 sample period. This analysis was performed on each of the four Yield Error Margin databases.

Hypothesis 4: The average difference between today's implied forecasts and the actual observed yields = 0 [74]

Hypothesis four investigated the market's average implied N-period forward forecasting error. The average error represented only one aspect of the four Yield Error Margin databases. The Yield Error Margin samples also contained important information on the distribution of the market's forecasting errors over the 1994 trading period. The overall objective of the research was to determine if the distributional characteristics of the forecasting errors could be used to measure interest rate risk.

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[73] Bloch tested this hypothesis on the Australian markets, Fama and Bliss American and Cuthbertson on British interest rate markets.

[74] Equation (3.10) was used for Hypothesis 4. $\bar{x}_1$ in this case was equal to the implied N- period forward forecast of the average SFE ten year bond basket yield. Where the implied N- period forecast was either two week, one month, two month or three months forward. $\bar{x}_2$ was equal to the mean of the actual average basket yield observed on the forward date.

As already discussed in Section 3.3 the observed price of the SFE ten year bond futures contract was equivalent to today's implied certainty forward. To evaluate interest rate risk, for applications such as option pricing, these certainty forwards had to be modified by a function that was related to the volatility of the term structure. For this purpose Ho and Lee (1986) proposed the perturbation function and Heath, Jarrow and Morton (1992) a stochastic process with a risk neutral drift term.[75] In the current paper the distributional characteristics of the Yield Error Margin databases were seen to define the stochastic behaviour of the term structure. The Yield Error Margin databases could be used to modify today's implied certainty forward term structure so that a distribution of *N*-period forward term structures was produced. This it was assumed generated a distribution of forward bond prices from which interest rate risk could be measured.

To highlight the assumptions being presented Table 3.2 incorporated a simulated Yield Error Margin database. Column one titled "YEM pts" recorded the forecasting errors as per Equation (3.15) in yield basis points terms. It demonstrated for the sample period that the forecasting errors were distributed over a sixty basis point range between negative thirty basis points and positive thirty basis points. This range of errors suggested that the *N*- period forward term structure was likely to be spread over a sixty point range.

To construct column two, titled "Frequency outcome" in Table 3.2, the Yield Error Margin outcomes were sorted into a number of ten basis point classes. For example, the positive thirty basis point class interval included all those Yield Error Margin observations that were in the range plus thirty basis points to plus forty basis

[75] Ho and Lee, (1986) 1017 and Amin and Morton, (1994) 145.

Table 3.2: Relative frequency table for simulated Yield Error Margin database

YEM Pts.	Frequency Outcome	Relative Frequency
30	6	0.06
20	9	0.09
10	22	0.22
0	27	0.27
-10	21	0.21
-20	12	0.12
-30	3	0.03
Total	100	1.00

Figure 2: Relative frequency histogram of Table 3.2

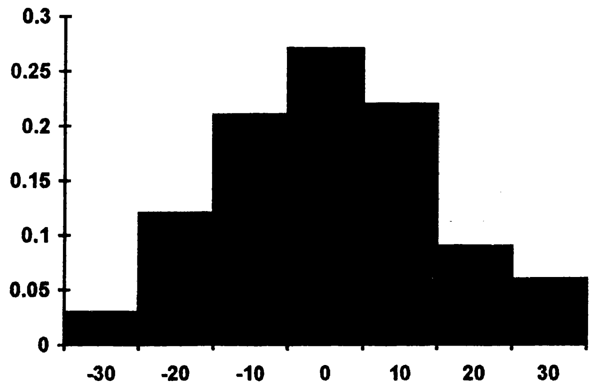
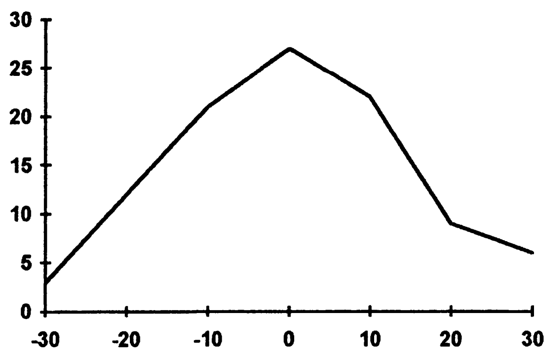


Figure 3: Approximate density function of Table 3.2



points. Column two also showed that there was a total of one hundred observations, n = 100, in the dummy Yield Error Margin database. Column three in Table 3.2 calculated the relative frequency of each of the ten basis point classes. If the ith class contained f_i observations then the relative frequency of the ith class was given by f_i / n, where n denoted the total number of observations in the sample. This construction aimed to assign a probability to the individual forecasting errors.

Figures 2. and 3. summarised the significance of Table 3.2.. The histogram in Figure 2. plotted the relative frequency of each of the Yield Error Margin ten basis point classes. It illustrated the general characteristics of the simulated Yield Error Margin distribution. For example, it demonstrated that the N-period implied forwards were without error twenty seven percent of the time. Out of one hundred observations twenty seven of the Yield Error Margin statistics were in the zero error basis point class (f_i = 27 / n =100). In contrast, Figure 2. showed that the market's implied forecasts were thirty basis points above the actual observed yields only six percent of the time (f_i = 6 / n =100). The Yield Error Margin distribution assigned a relatively low probability to larger forecasting errors.

In the case where the number of observations and intervals was large the outline of the relatively frequency histogram would begin to approach a smooth curve. This smooth curve was called the density function of a random variable. For a continuous variable the area under the curve equals one which represents the sum of all the probabilities of the individual outcomes in the distribution. Thus, although Figure 3. did not graph a continuous variable the probability assumptions were seen to be similar. Consistent with the methodology of Cox and Rubinstein (1985) Table 3.2 could be used to approximate the cumulative probability of a particular forecasting outcome.[76] For example there existed a ninety four percent chance (1- 0.06) that the Yield Error Margin was less than positive thirty basis points. As will be discussed in Section 3.4 this approach can be applied to option pricing because it allowed the calculation of expected payoffs.

In the literature survey in Chapter 2 the assumptions made about the distributional characteristics of the underlying variable were seen to be central to the risk pricing models. A variety of distributions were examined including binomial and Chi - square. Essentially these were all variations on the general assumption that the random variable followed a normal distribution. As Davidson and Mackinnon (1993) suggested the normal distribution was commonly applied in modelling because it was computationally convenient with well defined properties.[77] As was well known when a random variable was normally distributed it was completely described by its mean and variance.

Hypothesis 5 : The four Yield Error Margin and the SFE ten year bond futures distributions were normally distributed.

Hypothesis five used the SFE ten year bond futures contract as a benchmark from which to compare the four Yield Error Margin databases. The SFE ten year bond futures contract frequency distribution was constructed from daily yield basis point changes. The closing SFE ten year bond futures price for today's trading date *t* was converted to a yield and then compared to the closing bond futures equivalent yield for trading day $t+1$. This calculation was performed for every full trading day in the sample 01/01/94 to 31/12/94.

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[76] Cox and Rubinstein, (1985) 170.
[77] Davidson and Mackinnon, (1993) 61.

$$F_{y\Delta t+1} = (100 - F_{t+1}) - (100 - F_t) \qquad (3.16)$$

$F_{y\Delta t+1}$ = The daily yield basis point change in the closing SFE ten year bond futures price for trading day $t+1$.

F_{t+1} = The closing 4.30 p.m. ten year bond futures price for trading day $t+1$.

F_t = The closing 4.30 p.m. ten year bond futures price for today's trading date t.

Following Heynen, Kemna and Vorst's (1994) study of stock market returns the Kolmogorov - Smirnov test was used to examine the five distributions for normality.[78] This normality test was based on the observed D statistic which measured the maximum difference between the normal cumulative distribution function and the Yield Error Margin / ten year bond futures cumulative distributions. This test was one sided, with the decision rule that if observed $D >$ critical D then Ho was rejected.[79]

$$D = \text{Sup} \mid F_o(x) - S(x) \mid \qquad (3.17)$$

Where ;

D = The observed test statistic which measured the absolute difference between the normal and the Yield Error Margin / ten year bond futures distributions.

$F_o(x)$ = Normal cumulative distribution function for all values $\leq x$.

$S(x)$ = Denotes the proportion of sample observations less than or equal to value x.

Sup = The symbol " sup " means the supremum or maximum of all possible values.

Hypothesis six provided an alternative test of normality. In this case the focus was on whether the Yield Error Margin and ten year bond futures distributions were characterised by skewness or kurtosis. Skewness describes the situation where the

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78 Heynen, Kemna and Vorst, (1994) 41.

79 The test was taken from Hoel, Port and Stone, (1971) 168 -169.

distribution was not symmetrical and the most frequent outcome does not occur at the mean. Kurtosis deals with a distribution that has "fat" tails, where a high percentage of outcomes lie at the extremes of the distribution. The distribution as a result has a high level of variance relative to the benchmark normal distribution. Appendix G contained a set of frequency distributions that highlight the significance of these two non - normal characteristics.

Hypothesis 6 : The Yield Error Margin and ten year bond distributions were not characterised by significant Skewness or Kurtosis.

The observed test statistics for skewness and kurtosis were taken from Davidson and Mackinnon (1993).[80]

Skewness: Calculated skewness / $\sqrt{6/n}$ (3.18) (a)

Kurtosis: Calculated kurtosis / $\sqrt{24/n}$ (3.18) (b)

Where ;

$$\text{Calculated Skewness} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{\sigma}\right)^3$$

$$\text{Calculated Kurtosis} = \left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{\sigma}\right)^4\right\} - \frac{3(n-1)^2}{(n-2)(n-3)}$$ [81]

n = The number of observations in the Yield Error Margin / ten year bond futures frequency distribution.

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80 Davidson and Mackinnon, (1993) 568 - 569.

81 x_i was defined as the individual Yield Error Margin/ futures outcome in the database and $\bar{x}$ was the mean of the Yield Error Margin / futures database. σ = The standard deviation of the Yield Error Margin / futures database. Equation (3.20) details the formula for calculating the standard deviation statistic.

The Yield Error Margin distributions also provided the paper with the opportunity to develop a market consistent measure of volatility. As noted by Campa and Chang (1995), the volatility quotes provided by option price makers summarised the market's assessment of the future distribution of the underlying security price.[82] In the financial markets the term "volatility" was used to describe the movement of an assets price or return over a given unit of time. The main constraint facing practitioners was that the volatility of a security price was not directly observable. To overcome this problem option price makers were assumed to rely on two measures of volatility, both based on the standard deviation of the change in the security's price, to guide their decisions on the appropriate level of traded volatility. Traded volatility was defined as that level of volatility actually input into market maker's option pricing models.

The level of implied volatility was seen to be commonly consulted by option price makers. The price maker uses his/her assumed correct option pricing model to determine what level of volatility was consistent with the observed option prices. To generate an implied volatility series for this paper an approach similar to Jorion (1995) was adopted.[83] The SFE modified Black (1976) model for European options on futures contracts was used to invert the level of implied volatility from At-the-money (ATM here after) call ten year bond futures option settlement prices.[84] Unlike the option pricing models discussed in Chapter 2 the SFE pricing methodology did not discount the expected expiration value of the option contract because premiums on SFE options do not have to be paid up front.

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82 Campa and Chang, (1995) 534.

83 Jorion, (1995) 509. The author admitted that any Black and Scholes based model has some limitations in this role. However, for At - the - money options the Black and Scholes model was seen to generate reasonably reliable estimates of implied volatility.

84 Note that as shown in Attachment four the option settlement prices reflected the closing volatilities supplied by price makers in the trading pit. The aim of the study was to determine these trader quoted implied volatilities.

The volatility parameter in the SFE option pricing Equation (3.19) was as usual denoted by σ. The objective of the implied volatility research was to find, via trial and error, the level of σ ; given the time to maturity $T-t$, the underlying ten year bond futures price F, and the designated ATM strike X, that was consistent with the 4.30 p.m. call option settlement price recorded by the SFE. This strategy was applied to every full trading day in the 01/01/94 to 31/12/94 sample. Appendix H presented an implied volatility calculation using Equation (3.19).

$$C = FN(d1) - XN(d2) \tag{3.19}$$

Where ;

C = The 4.30 p.m. ATM call option settlement price for the SFE ten year bond futures contract on trading date t.

F = The 4.30 p.m. closing SFE ten year bond futures price converted to a SFE By - Laws bond price on trading date t.

SFE By - Laws bond price $= 1000 \times [\, 6\,(\,1 - V^{20}\,)\,/\,Y + 100V^{20}\,]$

As per the RBA bond formula in Equation (3.1) ;

$V = 1/(\,1+Y\,)$.

$Y =$ (100 - 4.30 p.m. closing SFE ten year bond futures price) / 200

$X =$ The ATM strike price.

ATM = The SFE arranges ten year bond futures option strikes in 25 basis point intervals. The ATM strike was defined as that option strike $\pm$ 12.5 basis points from the current closing futures price. Note that the strike price X was converted as per F to a SFE By - Laws consistent Commonwealth government coupon bond price.

$$d1 = \frac{Ln(F/X) + (\,0.5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d2 = \frac{Ln(F/X) - (\,0.5\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}$$

$\sigma =$ The level of implied volatility associated with the 4.30 p.m. ATM call option settlement price for the SFE ten year bond futures contract on trading date t.

$T - t =$ Time to expiry of the ATM call option on the SFE ten year bond futures contract. For example on trading day 04/01/94 the expiry date of the March 1994 ten year bond futures option contract was used to calculate the number of days $T - t$.

$N(d_1), N(d_2) =$ The cumulative probability associated with the standardised normal variables d_1 and d_2.

A historical measure of volatility was also assumed to be consulted by option price makers. As its name suggests this methodology attempts to approximate the level of market volatility by calculating the standard deviation of the average daily change in the securities closing price over some fixed historical interval of trading time.[85] To construct a historical volatility series for the 01/01/94 to 31/12/94 sample period a four part process was utilised. Firstly, the 4.30 p.m. closing ten year bond futures prices for the thirty trading days preceding today's trading date t were converted to an implied yield and then to a SFE By - Laws bond price as introduced in Equation (3.19).[86] This approach limited the calculated historical ten year bond futures volatility series to $n = 218$. The first thirty business days of the sample period 01/01/94 to 31/12/94 were used to construct the first entry in the historical volatility series.

Secondly, the log of the ratio $F_{t+1 \text{ (By-Laws equivalent)}} / F_{t \text{ (By-Laws equivalent)}}$ was calculated for each of the preceding thirty trading days up to and including the current trading day t. This series represented the daily percentage change in the closing ten year bond

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85 Yield volatility can also be derived and quoted. For the purposes of the current research only price volatility was considered.

86 In the research consulted no one particular optimal historical period was seen to better approximate the level of market volatility. The calculated thirty day historical volatility represented approximately six trading weeks and was therefore seen to give a reasonably relevant guide to the level of market volatility.

futures price over the last thirty trading days. The third step of the process calculated the standard deviation of this price change series via Equation (3.20).

$$\sigma_{H30} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \tag{3.20}$$

Where ;

σ_{H30} = The standard deviation of the daily percentage change in the closing ten year bond futures prices over the last thirty trading days.

x_i = The ith individual log ratio F_{t+1}/F_t. For i =1 to 30 trading days.

$\bar{x}$ = The mean of the log ratios F_{t+1}/F_t over the last thirty trading days.

n = The thirty trading days included in the calculation.

Equation (3.20) measured the average level of daily volatility over the last thirty trading days. The final part of the process converted this approximation of daily volatility to a market consistent annualised quote by multiplying it by $\sqrt{250}$. Where two hundred and fifty days was assumed to represent the number of trading days over a given year. Appendix I demonstrated the application of this methodology to the July to August 1995 period.

Hypothesis 7 : The average difference between the thirty day historic volatility series and the implied volatility series = 0

Hypothesis seven examined the proposition that there existed a strong relationship between implied volatility and historical volatility. The observed test statistic was taken from Equation (3.10). $\bar{x}_1$ in this case represented the average of the implied volatility series and $\bar{x}_2$ the average of the thirty day historical volatility series. Acceptance of the null hypothesis would support the hypothesis that SFE ten year bond futures option price makers used the historical volatility series to approximate the appropriate level of traded volatility.

To determine whether the Yield Error Margin distributions could be used to generate a market consistent measure of volatility, the following approach was pursued. As discussed, the Yield Error Margin databases were constructed by comparing today's implied N-period forward average ten year bond futures basket yield and the actual average ten year bond futures basket yield observed on the N-period forward date. The discrepancy between the two yields represented an error in today's interest rate forecasts.

This forecasting error was assumed to measure the propensity of yields to change over a given discrete time interval.[87] This change in yield it can be suggested provided an alternative approximation of interest rate volatility. To allow the comparison of this model based volatility measure with the market's historic volatility approach, the implied forward average bond futures basket yield and the actual average bond futures basket yield were converted to SFE By - Law bond prices as detailed in Equation (3.19). Table 3.3 used the two week forecasting error calculated in Appendix F to demonstrate the assumptions being put forward.

Table 3.3: Volatility approximation : Error in Implied vs Actual

Impl. Av. Yield	Actual Av. yield
6.8241875	6.5625
Implied Price	Actual Price
$137,074.65	$139,416.1
Log Impl./ Act.	-0.01693732
Error	-1.70%

87 This assumption gained further support when it was remembered that the forecasting errors reflect the fact that new information has come into the market and changed the level and shape of the term structure.

In Table 3.3 today's forecast of the ten year bond futures pricing basket was shown as \$137,074.65. On the actual forward date the ten year bond futures pricing basket was valued at \$139,416.10. Yields over the two week interval unexpectedly fell by 26.2 basis points. The forecasting error in percentage terms could be measured by the log of the ratio Implied futures basket price / Actual futures basket price. This error represented one input in the two week Yield Error Margin volatility database. The standard deviation calculation incorporated in Equation (3.20) could therefore be applied to the Yield Error Margin volatility databases.

To allow the direct comparison of model and market measures of annualised volatility the standard deviations of the Yield Error Margin volatility databases were multiplied by $\sqrt{d_{(year)}/n_{(N-t)}}$ rather than $\sqrt{250}$. Where $d_{(year)}$ was the number of days in a full calendar year and $n_{(N-t)}$ was the number of days associated with the implied N-period forward. For example, for the two week Yield Error Margin database each volatility input would be multiplied by $\sqrt{26}$ to denote the number of fortnights in a year.

Hypothesis 8 : The Yield Error Margin volatility database produced unbiased estimates of historic volatility.

Hypothesis eight replicated the approach taken in Hypothesis three for the 01/01/94 to 31/12/94 sample period. As in Equation (3.11), Ho the null hypothesis was accepted when $\alpha = 0$ and $\beta = 1$. Under these conditions it could be stated that the Yield Error Margin distributions produced volatility quotations that were unbiased predictors of the market's historical volatility series. For the regression analysis presented in Equation (3.21) $n = 218$. As mentioned the thirty day historical volatility series was restricted to this size sample because of the way it was calculated.

$$H_{30vt} = \alpha + \beta YEM_{vt} + \varepsilon_t \qquad (3.21)$$

Where ;

H_{30vt} = The calculated thirty day historical volatility series.

YEM_{vt} = The Yield Error Margin N- period forward volatility series.

ε_t = The error or disturbance term of the stochastic relationship.

3.4 Pricing Fixed Interest Contingent Claims

Section 3.4 applied the interest rate risk pricing assumptions introduced in Sections 3.2 and 3.3 to the pricing of SFE ten year bond futures options. In Section 3.2 the Australian market term structure was derived from the observed Commonwealth government bond coupon curve. The constructed zero coupon government bond curves were seen to measure the Australian market's risk free value of time for one day to ten years. To determine whether the model term structure was consistent with the pricing assumptions of the market it was used to evaluate the SFE ten year bond futures contract.

To prevent arbitrage it was stated that the ten year bond futures contract should reflect the average forward yield of a basket of SFE specified Commonwealth government coupon bonds. An implied forward term structure associated with the ten year bond futures cash settlement date ft was used to generate these forward bond yields. The word "implied" in this context denoted the fact that the N-period forward curve was constructed from information contained in today's term structure. The ten year bond futures contract was therefore viewed as a certainty forward that allowed agents to lock into today's best estimate of future long term interest rates.

The central assumption of Section 3.3 was that market participants faced uncertainty when making decisions. There existed no guarantee that today's best forecast of the security's price would be the actual price observed N- periods forward. New information or shocks that eventuate between today and the N- period forward date would change the level and shape of the term structure. The current implied forward interest rate forecasts represent only one of a range of potential prices that have a positive probability of occurring at the N- period forward date.

The primary focus of the analysis was to determine the errors in today's implied N- period forward interest rate forecasts. This was measured empirically by the so called Yield Error Margin statistic. The statistic calculated the difference between today's implied N - period forward forecast of the ten year bond futures average basket yield and the actual ten year bond futures average basket yield observed on the N - period forward date. For each full trading day in the 01/01/94 to 31/12/94 sample four average basket futures yield forecasts were produced for two weeks, one month, two months and three months forward. This approach created four Yield Error Margin databases that recorded the errors in the market's interest rate forecasts.

The Yield Error Margin databases were assumed to provide important interest rate risk management information. As well as defining the market's average forecasting error the databases also contained historical evidence on the distribution of these errors. Table 3.2 and Figures 2. and 3. summarised the significance of these distributions. The central assumption was that the relative frequency analysis allowed both the range and the probability of the forecasting errors to be measured. This aspect of the methodology was assumed to approximate the stochastic behaviour of the implied forward term structure.

The Cox and Rubinstein (1985) binomial based valuation model discussed in Chapter 2 was seen to best underline the essential "risk-reward" of trading option securities. The fundamental premise was that the price of an option contract should reflect the probability of the underlying security price being in the money at expiration. As in Figure 3. it was possible to calculate the cumulative probability of the security price being at the strike price X at expiration. This probability ultimately reflected the volatility assumption of the option price maker. The level of assumed volatility determined the likely range of the security price over the remaining life of the option and the relative probability of particular outcomes in this distribution. Thus the price maker on every trading day must be sure that his/her option price reflects, given time to maturity and today's underlying security price, an appropriate view of market volatility.

In Appendix I thirty day historical volatility for the July to August 1995 period was calculated. For this trading interval the standard deviation of the daily percentage change in the closing ten year bond futures price was measured as 0.55 %. If price makers thought that this level of volatility was relevant for the next thirty days then a particular assumption about the likely distribution of the ten year bond futures price was being incorporated into the observed futures option prices.

The distribution of ten year bond futures prices over the next thirty days was being approximated by $0.0055277612 \times \sqrt{30} = 3.027$ %. If the daily price changes were normally distributed, then with 95 % confidence it could be stated that over the next thirty days that the bond price was going to be within two standard deviations of today's observed ten year bond futures price. For example if the current ten year bond futures price was 90.90 or in terms of the SFE By - Laws bond price \$118,780.05, the range of the bond price over the next thirty days was assumed to be \$118,780.05 ± 6.054 %.

Price terms	\$111,589.11	<	\$118,780.05	<	\$125,970.99
Yield terms	10.13 %	>	9.1 %	>	8.15 %

In utilising this volatility quote the option price maker was suggesting that there existed only a five percent chance that bond yields would increase (decrease) by more than one hundred basis points over the next thirty days. By definition smaller changes in bond yields such as twenty basis points were given a much higher probability. Consistent with the approach of Rubinstein (1994) observed option prices were seen to summarise the probabilistic assumptions of the market's price makers. [88]

The objective of Section 3.4 was to use the implied certainty forwards of Section 3.2 and the Yield Error Margin distributions of Section 3.3 to replicate the observed SFE ten year bond futures option prices for the 01/01/94 to 31/12/94 trading period. Applying these distributions to the implied certainty forwards was seen to generate an array of probability weighted ten year bond futures prices that were associated with the expiration date of the option. This forward array of prices was assumed to represent the final step or distributional product of the "true" stochastic process followed by the term structure between today and the option maturity date. In contrast, Amin and Morton's (1994) binomial Heath, Jarrow and Morton (1992) based Eurodollar option pricing model contained ten steps in its pricing structure.[89] The ultimate efficacy of this modelling approach depends on the ability of the four Yield

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[88] Rubinstein, (1994) 772.
[89] Amin and Morton, (1994) 179.

Error Margin databases to match the distributional assumptions of the option price makers. [90]

Equation (3.22) summarised the assumptions associated with the option pricing model. At today's trading day t an implied N-period forward term structure as per Equation (3.7) could be constructed. The Yield Error Margin databases provided a historical record of the errors in these market based interest rate forecasts. The distribution of these errors was assumed to approximate the stochastic behaviour of the term structure. Equation (3.22) combined these two aspects of the methodology to produce a distribution of ten year bond futures average basket yields for the option expiration date *ot*.

$$\left(\left(\sum_{i=1}^{n} YB_{(distn.)ot}\right)/\,n\,\right)_{jt} = \left(\left(\sum_{i=1}^{n} YB_{(model)ot}\right)/\,n\,\right)_{t} \pm YEM_{jt} \tag{3.22}$$

Where ;

$\left(\left(\sum_{i=1}^{n} YB_{(distn.)ot}\right)/\,n\,\right)_{jt}$ = The jth ten year bond futures average basket yield in the distribution associated with the SFE ten year bond futures option expiration date *ot* for today's trading date t.[91] For $j = 1$ to k which, as per Table 3.2, was determined by the number of classes in the individual Yield Error Margin database.

[90] The model option pricing methodology was seen to have a number of limitations. The characteristics of the constructed Yield Error Margin distributional databases were seen to be dependent on the volatility of the underlying sample period. The size and frequency of the individual forecasting errors obviously depended on the number and type of information shocks received over the trading sample. The option pricing methodology could not be seen as a general one because only four distributions were calculated. As a result, the research was aimed at assessing whether the distribution of the market's forecasting errors could provide a useful approximation of term structure volatility over a given sample period.

[91] Where *ot* was set by the last trading day of the ten year bond futures contract.

$\left(\left(\sum_{i=1}^{n} YB_{(model)\,ot} \right) / n \right)_t =$ The model ten year bond futures average basket yield associated with the SFE ten year bond futures option expiration date *ot* for today's trading date *t*.These yields were consistent with Equation (3.8).

YEM_{jt} = The *j*th calculated Yield Error Margin associated with the implied *N*- period forward Yield Error Margin database. Where *N* consistent with Equation (3.15) was either two weeks, one month, two months or three months forward. For $j = 1$ to k.

$n =$ As defined in Equations (3.6) and (3.15).

Consistent with the first step of Appendix H the distribution of average basket yields at *ot* were converted to SFE By - Law bond prices. As discussed with regards to Figure 2. each of the bond prices in the distribution could be assigned a unique probability that was equivalent to its relative frequency in the sample. These probability weightings allowed the expected payoff of a ten year bond futures option with strike price X to be calculated.

$$C_{(model)t} = \sum_{j=1}^{k} p_j \times (B_j - X)_{ot} \qquad (3.23)$$

$$P_{(model)t} = \sum_{j=1}^{k} p_j \times (X - B_j)_{ot}$$

Where ;

$C, P_{(model)t}$ = The replicated SFE ten year bond futures call (put) 4.30 p.m. settlement option price with strike price X as at today's trading date t.

p_j = The probability assigned to the *j*th bond price in the distribution. For $j = 1$ to k.

B_j= The *j*th SFE By - Laws bond price in the bond price distribution associated with the option expiration date *ot*. For j = 1 to *k*. Where *k*, as in Equation (3.22), was determined by the number of basis point classes in the underlying Yield Error Margin database.

X = The strike price associated with the SFE ten year bond futures option.

$(B_j - X)_{ot}$, $(X - B_j)_{ot}$ = The payoff associated with the jth bond price in the bond price distribution for the option expiration date *ot*. As per Equation (3.19) the expiration value of the option was not discounted because SFE option premiums were not paid up front.

Appendix J incorporated the simulated Yield Error Margin database in Table 3.2 to demonstrate the workings of Equation (3.23). The robustness of the option pricing model was tested by the same three performance measures used in the futures pricing analysis in Section 3.2.. Equation (3.23) was used to replicate the 4.30 p.m. option settlement prices recorded by the SFE. The use of settlement option prices was assumed to eliminate what Heynen, Kemna and Vorst (1994) called the non-simultaneous pricing problem of futures and option trading.[92]

If closing option prices were used then additional errors could have been introduced to the performance study. The closing option prices record the last trade in the ten year bond futures option market. This may have occurred earlier in the trading day when the underlying futures price was trading at a different price to the closing futures price. The closing ten year bond futures option price will therefore not necessarily correspond to the closing ten year bond futures price. In contrast the settlement option prices, as detailed in Attachment four, reflect the closing price of the ten year bond futures contract and the closing volatilities supplied by the traders in the option price making pit.

The average pricing error statistic measured the basis point discrepancy between the SFE ten year bond futures option settlement prices and the replicating model

[92] Heynen, Kemna and Vorst, (1994) 40. Whaley, (1986) 135. This was also known as the contemporaneous pricing problem.

option prices produced by Equation (3.23). SFE ten year bond futures option prices as discussed in Appendix H were quoted in terms of yield basis points.

$$Y_{(opt.\ error)t} = C, P_{(model)\,t} - C, P_{(market)\,t} \qquad (3.24)$$

Where ;

$Y_{(opt.\ error)t}$ = The calculated yield basis point difference between the model produced option price and the SFE recorded ten year bond futures option settlement price for today's trading date t.

$C, P_{(model)\,t}$ = The call (put) model produced settlement option price for trading date t.

$C, P_{(market)\,t}$ = The SFE call (put) settlement option price for trading date t. Only the option series associated with the nearby ten year bond futures contract were considered. For example for the 04/01/94 trading date only options on the March 1994 ten year bond futures contract were examined. Equation (3.15) defined the term "nearby".

Equation (3.24) recorded the pricing errors not only for At - the - money (ATM here after) calls and puts but also for those strikes twenty five basis points In-the-money (ITM here after) and Out-the-money (OTM here after).[93] Following Cakici, Chatterjee and Wolf (1993) this research aimed to determine if Equation (3.23) suffered from any systematic biases relating to the moneyness of the option strike. [94]

Hypothesis 9: The average difference between SFE ten year bond futures option prices and model prices = 0

93 The ATM strike was defined in Equation (3.19). The ITM call (OTM put) strike was equal to the ATM strike - 25 basis points. The OTM call (ITM put) strike was equal to the ATM strike +25 basis points.

94 Cakici, Chatterjee and Wolf, (1993) 5. Time to maturity bias was not considered.

Hypothesis nine was therefore tested on six option pricing series. The statistic introduced in Equation (3.10) was used to test the proposition that on average the model pricing error was equal to zero. The sample size of this study was limited to n = 217. Options prices with less than fifteen days to maturity were excluded from the analysis. This was done, as per Rubinstein (1985), to decrease the chances of any price distortions associated with option expiration being included in the sample.[95] For example OTM options close to expiry might be next to worthless in real terms but actually be offered for sale by price makers at two to three basis points.

As with hypothesis three in Section 3.2 Ordinary Least Square analysis was used to provide the second measure of the model's pricing performance. Following Amin and Morton (1994) a simple regression relationship was run to test the proposition that the term structure based option pricing model had produced unbiased estimates of the SFE ten year bond futures option settlement price series.[96] The regression in Equation (3.25) was used to test the unbiased hypothesis on each of six option series. The unbiased hypothesis was tested by the null hypothesis that $\alpha = 0$ and $\beta = 1$. If the coefficients in Equation (3.25) were shown to equal these values then it could be stated that the model perfectly replicated the ten year bond futures option settlement prices recorded by the SFE.

Hypothesis 10: The Yield Error Margin model produced unbiased estimates of those futures option settlement prices recorded by the SFE.

$$C, P_{(market)\,t} = \alpha + \beta\, C, P_{(model)t} + \varepsilon_t \tag{3.25}$$

[95]Rubinstein, (1985) 466. In this study options with less than 21 days to maturity were omitted from the sample.
[96]Amin and Morton, (1994) 163.

Where ;

$C, P_{(market)\,t}$, $C, P_{(model)\,t}$ = As defined in Equation (3.24) for all six option series.

ε_t= As defined in Equation (3.11).

The last measure of model performance used the regression estimated in Equation (3.25), for the 1994 sample period, to forecast ten year bond futures option settlement prices for the 01/01/95 to 31/01/95 trading interval. The accuracy of the forecasted prices was measured by the error statistic in Equation (3.26). As with the bond futures pricing analysis in Section 3.2, if the average option pricing error for this out of sample period was similar to the original sample, then this would lend support to the hypothesis that the model methodology had adequately captured the pricing assumptions of the price makers in the option trading pit.

$$Y_{(Forecast\ error)t} = C, P_{(Forecast)\,t} - C, P_{(market)\,t} \tag{3.26}$$

Where ;

$Y_{(Forecast\ error)t}$ = Measured the error in yield basis points between the forecasted option price and that recorded by the SFE over January 1995.

$C, P_{(Forecast)\,t}$ = The forecasted option price generated by Equation (3.25).

$C, P_{(market)\,t}$= The ten year bond futures option settlement prices recorded by the SFE over January 1995.

3.5 Rewards from Hedging Fixed Interest Portfolios

Section 3.5 investigated the impact of interest rate hedging on the performance of fixed interest portfolios. The implied certainty forwards of Section 3.2 were combined with the Yield Error Margin distributions of Section 3.3 to implement a "naive" fixed interest portfolio hedge. The rewards from pursuing this strategy were determined by whether on average the hedged portfolio out-performed an unhedged portfolio over the 1994 sample period.

The representative fixed interest portfolio used in the hedging simulation was assumed to contain the Commonwealth government coupon bonds that were incorporated in the SFE ten year bond futures pricing basket. The current value of the portfolio was set equal to the sum of the market prices of the constituent bonds. One million dollars was seen to be invested in each of the bonds. For today's trading date t the value of the fixed interest portfolio was calculated by Equation (3.27).[97]

$$Vp_t = \sum_{i=1}^{n} B_t \times \$\ 1{,}000{,}000 \tag{3.27}$$

Where;

Vp_t = The value of the representative fixed interest portfolio at today's trading date t.

B_t = The current market price of the ith Commonwealth government coupon bond incorporated in the nearby ten year bond futures pricing basket. The market price was set by the RBA bond pricing formula in Equation (3.1).

n = The number of Commonwealth government coupon bonds in the nearby ten year bond futures pricing basket. Equation (3.15) defined "nearby".

The focus of the hedging research was on the performance of the representative fixed interest portfolio over a given holding period. The investment horizon used in the simulation was assumed to be the period between today's trading date t and the cash settlement date of the nearby ten year bond futures contract ft. It was also assumed

[97]The ultimate dollar value of the fixed interest portfolio was determined by whether or not the bonds in the portfolio, on a particular day, were trading at a discount or premium to par. A bond was at a premium > par if the current trading yield < its coupon and at a discount < par if the current trading yield > its coupon.

that the minimum realistic investment period was one month in duration. Those trading days, less than one month before a SFE ten year bond futures cash settlement date were excluded from the analysis. The hedging simulation was limited to n = 163 observations.

The objective of the fixed interest manager was to maximise the value of the portfolio at the end of horizon trading date ft. The methodology associated with Equation (3.7) allowed the manager to forecast the expected value of the fixed interest portfolio at ft. The expected portfolio value had two components that both relied on today's implied N-period forward term structure. Firstly, the implied forward price of the underlying bonds at ft and secondly, the implied forward value of the reinvested coupons received during the holding period at ft. Equation (3.28) recorded today's best estimate of the value of the fixed interest portfolio at the end of the investment horizon ft.

$$I_t[Vp_{ft}] = \sum_{i=1}^{n} B_{(Model)\,ft} \times \$1{,}000{,}000 + \sum_{i=1}^{n} c \times (\,1 + (\,I_t[Y_{ft-ct}] \times (ft - ct\,/\,365\,)\,)\,) \tag{3.28}$$

Where ;

$I_t[Vp_{ft}]$ = The implied forward value of the representative portfolio at ft given the information contained in the term structure at today's trading date t.

$B_{(Model)\,ft}$ = Today's implied forward price at ft of the ith Commonwealth government coupon bond included in the representative fixed interest portfolio. This was consistent with the valuation in Equations (3.7) and (3.8).

c = The coupon paid by the ith Commonwealth government coupon bond over the holding period ft - t.

$I_t[Y_{ft-ct}]$, $ft - ct$ = As specified in Equation (3.6).

The expected holding period return of the portfolio was computed by the annualised percentage change between Vp_t and $I_t\,[Vp_{ft}]$. Fabozzi (1995) called this measure of portfolio performance the arbitrage free total return which was consistent with the implied certainty forward analysis in Section 3.3.[98] Using the information contained in today's term structure the manager of the fixed interest portfolio was able to determine the expected performance of the fund over the investment horizon $ft - t$.

$$I_t[Yp_{ft-t}] = [\, I_t[Vp_{ft}] \,/\, Vp_t - 1 \,] \times (36500 \,/\, ft - t) \qquad (3.29)$$

$I_t[Yp_{ft-t}]$ = The expected return of the representative fixed interest portfolio over the investment horizon $ft - t$ as at today's trading date t.

$I_t[Vp_{ft}]$, Vp_t = Defined in Equations (3.28) and (3.27) respectively.

$ft - t$ = The number of days between today's trading date and the ten year bond futures cash settlement date ft.

A central theme of Chapter 3 has been that there will exist errors in today's interest rate forecasts because they reflect decisions made under uncertainty. This proposition can be extended to the return generated in Equation (3.29). $I_t[Yp_{ft-t}]$ represented today's implied forecast of the return generated by the fixed interest portfolio over the investment horizon $ft - t$. In practice $I_t[Yp_{ft-t}]$ was only one of a number of possible return outcomes that had a positive probability of occurring. The Yield Error Margin analysis in Section 3.3 suggested that the fixed interest manager would face a distribution of returns over the holding period. It was likely that a subset of this distribution would contain returns that were highly negative.

98Fabozzi, (1995) 76.

This part of the distribution was assumed to be correlated with a significant increase in the level of yields over the investment horizon. An increase in the underlying level of interest rates decreases the present value of the cashflows associated with the bonds in the portfolio. As demonstrated by Equation (3.2) when Y increases the value of B decreases. The end of period portfolio value Vp_{ft} would be below Vp_t and as a consequence the return would be negative. Therefore the key risk to the performance of the fixed interest portfolio over the holding period was a rise in the level interest rates. [99]

To protect the portfolio value against adverse interest rate moves over the holding period *ft-t,* the fixed interest manager was assumed to have implemented a maximum loss hedging strategy. The objective of this strategy, as per the analysis of Figlewski, Chidambaran and Kaplan (1993) and Beighley (1994), was to limit the loss of the portfolio to a fixed percentage value.[100] For this hedging simulation the maximum loss over the investment horizon was set at -5.00 %. The fixed interest portfolio manager was prepared to risk losing a maximum of 5.00 % of the portfolio value over the holding period.

To implement this maximum loss hedging strategy the portfolio manager was assumed to purchase SFE ten year bond futures put options. This was called a "naive" hedging strategy for two reasons. Firstly, it was based on a "set and forget" policy. The ten year bond futures options were purchased at the start of the investment period on today's trading date t and held until their expiration date at ot. The hedge in this

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[99] Other potential risks such as credit were not an issue in this simulation. As discussed in Section 3.2 the bonds held in the portfolio were issued by the Commonwealth government of Australia. Credit risk was assumed to be negligible in this case.

[100] Figlewski, Chidambaran and Kaplan, (1993) 47. The fixed percentage strategy was the second hedging policy studied. Beighley, (1994) 71. Set his maximum loss at -4.00 %.

context can be viewed as simple interest rate insurance. Secondly, the hedging strategy was implemented on the basis of a hedge ratio of one. Today's value of the fixed interest portfolio Vp_t was hedged by an equal dollar value of put options. For example if the current portfolio was worth \$4,000,000 then forty ten year bond futures put option contracts, worth \$100,000 each, were purchased.

The Yield Error Margin analysis introduced in Section 3.3 was used to target the appropriate ten year bond futures put option strike. The Yield Error Margin distributions were used to modify the implied certainty forward bonds yields associated with the expected end of horizon portfolio value in Equation (3.28). The role of the Yield Error Margins was to adjust the implied forward yields of the bonds in the portfolio until a -5.00 % annualised expected holding period return was recorded. The yield point move consistent with this maximum loss target was used to determine the correct put option strike. As mentioned in Section 3.3 and 3.4 the SFE arranges the ten year bond futures option strikes in discrete twenty five basis point intervals. The "naive" hedge strategy strike was defined in Equation (3.30) as that put option strike $\pm$ 12.5 basis points from the adjusted ten year bond futures price.

$$X_{hedge} = (F_t - Y_{ML}) \pm 12.5 \text{ basis points} \qquad (3.30)$$

Where ;

X_{hedge} = The "naive" hedge strategy SFE ten year bond futures put option strike.

F_t = The closing SFE ten year bond futures price for today's trading date t.

Y_{ML}= The yield basis point move associated with the expected value of the representative fixed interest portfolio registering a -5.00 % return for the holding period *ft-t*. The basis point move consistent with the maximum loss hedging strategy.

The maximum loss hedging strategy could now be activated, as the correct number of put options and the required strike price were known. The essential risk-reward of this "naive" hedge was that it allowed the fixed interest portfolio manager to trade off a fixed percentage of his/her upside return for a lower probability of large negative returns.[101] Three performance measures were used to determine whether the manager would have benefited from the "naive" hedging strategy over 01/01/94 to 31/12/94. The focus of this research was on comparing the relative performance of the unhedged portfolio versus the hedged portfolio over the trading sample. The returns of the unhedged and the hedged portfolios for the holding period *ft-t* were calculated by the assumptions contained in Equation (3.31) (a) and Equation (3.31) (b).[102]

$$Yp_{ft-t} = [\ Vp_{ft}\ /\ Vp_t - 1\ ] \times 36500\ /\ ft - t \qquad (3.31)\ (a)$$

$$Yp_{(hedge)ft-t} = [\ (\ Vp_{ft} + ((\ X_{hedge} - F,\ 0) - Put_{cost}\ )\ )\ /\ Vp_t - 1\ ] \times\ 36500\ /\ ft - t \qquad (3.31)\ (b)$$

Where ;

Yp_{ft-t} , $Yp_{(hedge)ft-t}$ = The unhedged and hedged annualised return recorded for the representative fixed interest portfolio for the holding period $ft - t$.

Vp_t = As defined in Equation (3.27).

Vp_{ft} = Consistent with the assumptions of Equation (3.28) except that $B_{(Model)ft}$ was replaced by B_{ft} the actual *i*th Commonwealth government coupon bond prices observed at *ft*.

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101 As mentioned by Beighley, (1994) 69. The combination of a long portfolio position and a long put position created a synthetic long call position.
102 Note that the quoted portfolio returns ignored the transaction costs of purchasing options i.e. brokerage / settlement costs and taxation issues.

$(X_{hedge} - F, 0)$ = The expiration value of the hedge strategy option strike purchased at the start of the investment period on trading day t.

Put_{cost} = The total cost of purchasing the interest rate insurance. The cost was determined by the number of puts purchased × cost in basis points × dollar value of a one basis point move in the SFE By - Laws bond price at the hedge strategy strike. See Appendix H.

The annualised returns in Equations (3.31) were computed for every trading day in the hedging simulation sample n = 163. This process generated the two portfolio return series that formed the basis of the performance testing. The first criteria used to compare the hedged and unhedged series was their respective sample mean returns and standard deviations. The mean was calculated by the simple average return while the standard deviation was produced by the assumptions incorporated in Equation (3.20).[103] A priori the hedged portfolio return series should demonstrate a lower degree of variance relative to the hedged series. As mentioned the put options should limit the number of large negative returns recorded for the hedged portfolio series over the sample period.

The second portfolio performance test was incorporated into Hypothesis eleven. Following Hancock and Weise's (1994) analysis of hedging strategies on the S&P 500, the null hypothesis examined the proposition that the average difference between the two return series was equal to zero.[104] Rejection of the null hypothesis would support the hypothesis that the "naive" hedging strategy had, on a statistical basis, significantly altered the return characteristics of the representative fixed interest portfolio. In contrast to the similar tests in Sections 3.2 to 3.4 the two portfolio return series were not assumed to be independent. The hedged portfolio return series was seen to be a modified version of the unhegded portfolio return series whose value was similarly

[103]Mean portfolio return $\Sigma x_i / n$ where x_i = The ith annualised return associated with the hedged and unhedged portfolio return series. These were the variables substituted into Equation (3.20) to calculate the return standard deviation.
[104]Hancock and Weise, (1994) 429 - 430.

affected by changes in long term interest rates. In this case the statistical test focused on the mean and variance of the spread between the hedged and unhedged return series. This modified test statistic was taken from Kenkel (1989).[105]

Hypothesis 11: The average difference between the hedged and unhedged portfolio returns = 0

$$\tau = \frac{\bar{d} - 0}{\sigma_d / \sqrt{n}} \tag{3.32}$$

Where ;

τ = The calculated test statistic taken from the student t distribution.
d = The difference between the ith unhedged - hedged returns.
Where i = 1 to 163.

$\bar{d}$ = The mean of the difference in the two return series. See Footnote 103.

σ_d = Standard deviation of the difference in the two return series.

n = The number of observations in the two return series. Where n = 163.

The third performance test applied the relative frequency distribution analysis, introduced in Table 3.2. and displayed in Figure 2., to the two portfolio return series. The individual hedged and unhedged return series outcomes were sorted into a range of annualised return classes. The aim of this construction was to highlight the distributional characteristics of the hedged and unhedged portfolio returns over the 1994 trading sample. It was assumed, as suggested by Beighley (1994), that the distribution of the hedged portfolio returns should be truncated around the maximum loss target of -5.00 %.[106] Consistent with the lower variance proposition, if the "naive" hedging policy has been correctly implemented, then the hedged portfolio return series should display a distribution that has a relatively low number of large

[105]Kenkel, (1989) 471.
[106]Beighley, (1994) 69.

negative return outcomes, with a concentration of returns around the -5.00 % return area.

3.6 Summary of the Hypotheses to be tested

The objective of the current research was to determine whether term structure based interest rate forecasts have a role to play in risk management. Three core hypotheses were used to test this proposition. The first stated that the Australian market term structure could be constructed from the observed Commonwealth government coupon bond curve. The second stated that contained in today's term structure was the market's current best estimate of interest rates *N*- periods forwards. The market traded futures and forward contracts should reflect these implied forecasts or otherwise riskless arbitrage was available. It was assumed however that because agents make decisions under uncertainty that today's so called certainty forwards only represented one possible path for interest rates. There would be errors in these forecasts, as measured by the actual term structure that was observed on the *N*-period forward trading date. This led to the third central hypothesis which proposed that the distribution of these forecasting errors could be used to approximate the stochastic behaviour of the term structure between today and a given forward date.

This three part approach to the analysis of interest rate risk was applied to the pricing of ten year bond futures and option contracts traded on the SFE, as well as to the hedging of fixed interest portfolios, over the 01/01/94 to 31/12/94 calendar period. Hypotheses one to eleven were all aimed at testing the proposition that the term structure based interest rate risk model could provide a framework from which to replicate the market's underlying futures, volatility and option pricing assumptions. The market data used to test these hypotheses was summarised in Appendix K. The results of this research was presented in Chapters 4 - 7.

CHAPTER 4

Deriving the Australian Market Term Structure

4.1 Introduction

The first major hypothesis of this paper stated that the Australian market term structure was derivable from the observed Commonwealth government coupon bond curve. This zero coupon yield to maturity relationship was seen to measure the riskless rate of return available on Australian market securities from one day to ten years. This so called "initial" term structure formed the basis of the Yield Error Margin risk pricing model empirically developed in Chapters 5, 6 and 7.

To determine whether the model term structure was consistent with the pricing assumptions of the Australian interest rate market it was used to evaluate Sydney Futures Exchange (SFE here after) ten year bond futures contracts over the 01/01/94 to 31/12/94 sample period. Three separate measures were used to investigate its pricing performance. Firstly, the ability of the constructed term structure to generate realistic futures prices was examined. Its accuracy in this role was measured by the yield basis point spread between market and model prices. Secondly, a simple regression relationship was estimated to determine support for the hypothesis that model prices were unbiased predictors of market prices. Finally, the regression equation and model prices were combined to forecast ten year bond futures prices for the outside sample period 01/01/95 to 31/01/95. The results of this analysis suggested that the model term structure was suitable for pricing interest rate securities traded in the Australian market place.

4.2 Pricing Performance of the Term Structure Model

As discussed in Section 3.2 the model term structure was derived from an approach that took its pricing inputs from the short dated bank bill swap curve and the "Hot stock" Commonwealth government coupon bond curve. The short dated bank bill discount securities, after the necessary credit risk adjustment, were assumed to make up the first twelve months of the Australian market term structure. These physical market rates were viewed as the anchor points for the rest of the term structure derivation. A so called bootstrapping based procedure combined the anchor yields and the "Hot stock" bond prices to generate the remaining term structure from twelve months to ten years.

This modelling approach produced market based term structures for every full trading day over calendar 1994. In total n = 248 model term structures were produced for the sample period. In a practical sense these model term structures represented zero coupon bond curves. Each discount factor associated with individual points on the term structure was equivalent to a zero coupon yield to maturity. In being based on the Commonwealth government coupon bond curve these zero coupon yields were seen to represent the riskless (default free) time value of money for all maturity dates in the Australian market context.

The term structure, constructed for the first trading day of the sample 04/01/94, was presented in Table 4.1..[107] As mentioned in Attachment 1 by the Reserve Bank of Australia (RBA here after) bonds were quoted on the basis of a semi - annual yield. This quotation method was consistent with the semi - annual coupon structure of the

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[107]It was important to note that the model term structures could also be represented by a zero coupon discount function curve as per the assumptions of Ho and Lee (1986).Instead it was decided to present them on a yield basis so that some form of comparison could take place between the Commonwealth government bond coupon curve and the model term structure.

Table 4.1:

Zero Coupon Curve 04/01/94

Maturity[a]	Coupon	Yield	Price	*Zero*	Zero - Mkt.[b]
15/02/95	13	5.2	113.387	*5.21*	1.00
15/04/95	12.5	5.15	111.785	*5.17*	2.00
15/09/95	10.5	5.35	111.485	*5.36*	1.00
15/07/96	13	5.71	123.112	*5.77*	6.00
15/03/97	12.5	5.92	122.763	*5.99*	7.00
15/09/97	12.5	6.06	124.901	*6.14*	8.00
15/01/98	12.5	6.14	128.338	*6.24*	10.00
15/08/98	7	6.26	105.671	*6.33*	7.00
15/03/99	6.25	6.4	101.304	*6.47*	7.00
15/07/99	12	6.41	131.39	*6.55*	14.00
15/04/00	7	6.53	103.988	*6.62*	9.00
15/07/00	13	6.53	140.132	*6.70*	17.00
15/11/01	12	6.65	134.059	*6.82*	17.00
15/10/02	10	6.75	123.535	*6.92*	17.00
15/08/03	9.5	6.78	122.704	*6.94*	16.00
15/09/04	9	6.84	119.003	*7.00*	16.00
15/07/05	7.5	6.88	108.462	*7.03*	15.00
15/11/06	6.75	6.91	99.628	*7.04*	13.00

(a) The maturity dates associated with the "Hot stock" Commonwealth bonds. The coupon, yield and price of these bonds on the 04/01/94 trading date was also included.
(b) The yield basis point difference between the derived zero coupon curve and the observed coupon curve.

majority of bonds traded in the Australian market. In respect of this the Zero column in Table 4.1 recorded the zero coupon yields on a semi-annual basis.[108]

The main observation that can be made from Table 4.1 was that the model derived zero coupon yields were greater than the underlying bond coupon market yields. There existed a positive spread between model and market yields. The positive spread also generally increased with time to maturity. These characteristics were found to hold generally for the entire sample period.

[108] This involved the transformation of the zero yields generated by the formula in Footnote 61. The semi-annual zero coupon equivalent yield was calculated by $(((1+(Zy/100))^{0.5})-1)\times 200$.

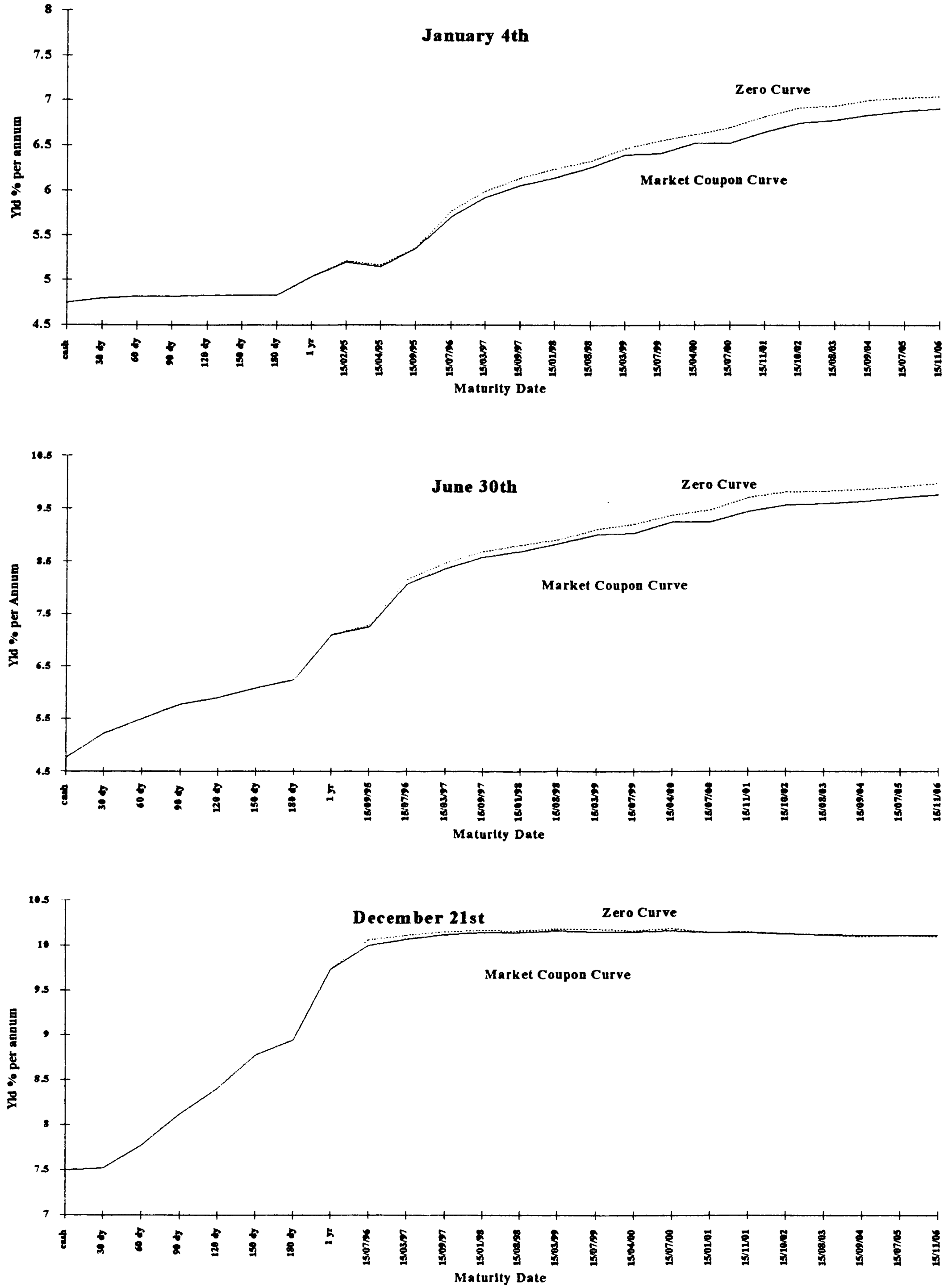
Over the 1994 sample period the yield curve was characterised by two factors, a general increase in the level of interest rates and a marked steepening of the yield curve. Over the sample period the longer dated part of the term structure responded more vigorously and well ahead of the response in the shorter dated anchor yields. In the period, January 1994 to June 1994, ten year bond yields increased from 6.80 % to 9.80 %, +300 basis points, whilst ninety day bank bill rates over the same period increased by only +100 basis points from 4.80 % to 5.80 %. As a result of this uneven dynamic relationship the yield curve steepened dramatically with short dated yields well below the longer dated maturities.

Figure 4. summarised this change in the structure of the yield curve. In response to these moves, the yield point spread between the derived zero coupon curve and the coupon bond curve increased especially for the > five year part of the curve. This change in magnitude was illustrated by the 15/08/03 bond where the spread moved from +16 basis points to +23 basis points over the January to June 1994 period. The increase in yields and the more normal shape of the yield curve reinforced the fact that the zero coupon yields were above their equivalent coupon bond counterparts.

The yield point spread increasing as a function of the steepening yield curve was consistent with the theoretical term structure analysis of Hull (1993).[109] This functionality was related to the coupon effect mentioned in Footnote 49. The trading yield of a coupon bearing bond was affected by the fact that the investor gets some part of the bond cashflows before the maturity date of the bond. With a normal yield curve these earlier payments would be discounted at lower yields relative to the

[109]Hull, (1993) 82 - 83.

Figure 4: Zero coupon curve vs observed bond curve



to the maturity date of the bond. In this environment the coupon bond will trade at a higher price (lower yield) than a zero coupon bond of the same maturity that has only one final future payment. This tendency for higher relative zero coupon yields would obviously be accentuated by a steeper yield curve where short dated yields were lower relative to those with longer maturities.

The only exception to this positive spread pattern was the mid to late December 1994 part of the sample. The yield curve from two years to ten years was virtually flat and actually was slightly inverse for the five to ten year part of the yield curve. The zero coupon curve in line with the above comments was at first above the coupon curve before moving below (-1 basis point) the observed market coupon yield structure. The third chart in Figure 4. demonstrated this behaviour.

The derived zero coupon government bond curves presented in Figure 4. were assumed to approximate the "true" Australian market term structure. It was proposed that if these curves were consistent with the pricing assumptions of the market place then they should be able to evaluate market traded securities that have similar default free risk characteristics. In line with this assumption the derived term structures were used to price the benchmark ten year Commonwealth government coupon bond futures contract traded on the SFE. To prevent arbitrage the observed futures price should reflect the average forward yield of a set of SFE pre-determined Commonwealth government coupon bonds called the basket.

Equation (3.7) was used to construct implied forward zero coupon discount curves to price the ten year bond futures contracts. These forward discount factors were the product of today's initial term structure and the riskless rate that applied for the period between today and the ten year bond futures cash settlement date *ft*. This

N-period forward zero coupon curve was used to obtain a forward price for each of the Commonwealth government coupon bonds that made up the ten year bond futures contract pricing basket. These forward bond prices were then converted to forward dated yields by RBA Equation (3.1). The arithmetic average of these forward yields represented the model average basket yield / futures price.

To provide a futures pricing benchmark the Cash and Carry model presented in Equation (3.6) was used to create an alternative ten year bond futures contract pricing series. The Cash and Carry pricing methodology used the prevailing money market rate associated with the futures cash settlement date to generate a forward price for the coupon bonds in the futures contract basket. Any coupon payments made during the holding period were subtracted from the individual forward bond prices. These coupon adjusted forward bond prices were used to construct an average basket yield so that a Cash and Carry derived futures price could be calculated.

The results of this research were presented in Table 4.2. The average spread column was the key performance indicator in this Table. This statistic measured the average pricing error of the model.[110] The error was calculated by Equation (3.9) as the difference between the observed closing futures equivalent yield and the synthetic futures equivalent yield generated by the term structure model. This pricing error, Market yield - Model yield, was recorded for each full trading day in the sample. Appendix L recorded these daily results. Table 4.2 summarised these daily pricing errors into monthly and All sample results. The entries in the table should be read in terms of yield basis points. The January 1994 average spread result was therefore

[110]Flesaker (1993) adopted a similar approach when reporting on the Eurodollar futures and options market. The average basis point error was deemed the appropriate test given that the market was quoted in yield points. Gay and Manaster (1991) reported their model pricing errors in terms of $ dollar difference. The US bond market was quoted in price terms.

Table 4.2:

Term Structure Model vs Futures Market prices

Month	*Av. spr.* [(a)]	*Max.*	*Min.*	*Std. dev.* [(b)]	*Correl. Cx.* [(c)]
Jan-94	-0.60597	0.662675	-2.2463	0.770872	0.99925638
Feb-94	-0.97783	0.2363	-0.98238	0.603876	0.99981216
Mar-94	-0.01973	2.171622	-0.01973	0.740616	0.99962882
Apr-94	-0.30639	0.86306	-1.35268	0.546946	0.99937566
May-94	0.097256	0.959394	-0.61702	0.4975	0.99952473
Jun-94	0.277234	2.04012	-2.04404	0.946406	0.99984201
Jul-94	0.686631	2.57656	-0.83464	0.824182	0.998188
Aug-94	0.119849	1.19592	-1.00414	0.627817	0.99858157
Sep-94	0.719616	1.98375	-0.3998	0.634993	0.99987094
Oct-94	0.794349	1.95750	-0.68652	0.772268	0.99789796
Nov-94	1.24935	3.29500	0.55625	0.614278	0.99869817
Dec-94	0.201704	1.34850	-1.23925	0.692009	0.9985878
ALL SAMPLE	*0.204894*	*3.295*	*-2.2463*	*0.909077*	*0.99998555*

Cash and Carry Model vs Futures Market prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	-0.39466	0.844115	-2.07639	0.789703	0.99925179
Feb-94	-0.87855	0.294589	-1.98389	0.578676	0.99982733
Mar-94	0.18868	2.5232	-0.6725	0.772496	0.99957846
Apr-94	-0.09414	1.090171	-1.07656	0.551798	0.99939731
May-94	0.205751	1.114717	-0.5886	0.518259	0.99948352
Jun-94	0.532181	2.42133	-1.68115	0.988632	0.99982249
Jul-94	0.944428	2.960016	-0.72276	0.867511	0.997977
Aug-94	0.259151	1.349672	-0.9006	0.654141	0.99847207
Sep-94	1.01659	2.443308	-0.34364	0.724287	0.99986278
Oct-94	1.177565	2.38582	-0.31121	0.764206	0.99798087
Nov-94	1.463582	3.59681	0.77723	0.623414	0.99891555
Dec-94	0.64441	2.19412	-1.45090	0.995304	0.99709601
ALL SAMPLE	*0.437608*	*3.596809*	*-2.07639*	*0.978043*	*0.99998405*

(a) The spread in Table 4.2 was set by Market yield - Model yield = pricing error for trading day t. Average spread was the simple daily average for trading month. These pricing errors should be read in yield point terms i.e. 0.3 = 3/10th of 1 basis point (b) The standard deviation of the pricing error for the trading month. (c) The correlation coefficient. It determined to what extent the model and observed yields were correlated. Measured by $\rho_{x,\,y} = Cov\,(X\,,\,Y)\,/\,\sigma_x \times \sigma_y$. Where a correlation of 1 implied a perfect positive linear association and 0 meant no linear association.

equivalent to - 0.6 of 1 basis point or the difference in yield between 10.00 % and 10.006 %.

For the All sample period the average yield basis point pricing error was only 0.2 of 1 basis point. The dollar value of 2/10th of 1 basis point on one $100,000 face value ten year bond futures contract, trading at 92.00 or 8.00 %, was equal to AUD $16. To put this error into perspective, for the sample period the average brokerage and settlement charge per contract was estimated to have equalled $10 a round trip i.e. buying and selling the futures contract.

Of equal interest was the fact that the average pricing error was stable across all the trading months. The average monthly pricing error ranged between +1.2 basis points in November 1994 to -0.9 of 1 basis point in January 1994. The standard deviation of the pricing errors (All sample 0.9 of 1 basis point) and the correlation coefficients (All sample 0.99998555) of the model versus the market were also relatively constant among the trading months. All three summary statistics suggested that model prices were consistent with those observed in the market over the sample period.

Appendix M included two graphs that provided support for the results in Table 4.2.. In Figure (i) in Appendix M the calculated basis point spread or pricing error was plotted against each day in the trading sample. It was shown earlier in Figure 4. that the term structure experienced both an overall increase in yields and more importantly a radical change in shape during the sample period. The long end of the term structure increased in yield four to five months before the short end responded. Figure (i) in Appendix M attempted to determine whether there was a relationship between the changing shape of the term structure and the model pricing errors. Any observable time related trend in the pricing errors would expose biases or weaknesses in the pricing methodology.

The term structure as previously mentioned steepened dramatically between January and June 1994. The model pricing errors displayed in Figure (i) did not change in behaviour or develop any obvious trends during this period. The errors remained randomly distributed during the sub-sample. The market - model spread was also unaffected by other events that determined the slope of the term structure. The three tightenings of monetary policy by the RBA on the 17th August, 24th October and December 14th 1994 had no obvious impact on the pricing errors of the model.[111]

Other factors, outside developments in the shape of the term structure, also had little influence on the pattern of pricing errors. The basis point spread remained unperturbed by either the ten year bond futures contract rollovers, dates: March 15, June 15, September 15 and December 15 or the significant increase in interest rate volatility that occurred during the sample period.[112] On the basis of the visual evidence in Figure (i) Appendix M there was support for the view that the model pricing errors were uncorrelated with any one particular underlying characteristic of the interest rate market or event that took place over 1994.

In terms of a potential bias in the term structure pricing model there existed one area of concern. In the second half of the sample, from June to December 1994, the pricing errors had a tendency towards being positive where market yields were > model yields. The average spread in this period was 0.6 of one basis point which was greater than the overall sample average of 0.2 of one basis point. Despite this slight

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[111] Reserve Bank of Australia, (January 1995 Bulletin) 27.
[112] Chapters 5, 6 and 7 deal with this issue in great depth.

pricing bias, the errors never became explosive, with only one outlier occurring in the subsample of +3.295 basis points on 08/11/94. Whether this bias in the model was anything more significant was examined in the Ordinary Least Square analysis in Section 4.3..

Figure (ii) in Appendix M reaffirmed that the futures pricing model was generally accurate. Figure (ii) displayed the relative frequency of individual pricing errors as per Figure 2.. What can be stated after studying Figure (ii) was that the bulk of the pricing errors were in the range $\pm$ 1 basis point. 197 (79.4 %) of the sample pricing errors were in this range while there were only five ($\approx$2.00 %) observations > 2 basis points and one observation > 3 basis points. Amin and Morton (1994) suggested that the significance of model pricing errors was related to the bid - ask spread of the underlying market.[113] Under this criteria, with the ten year bond futures generally having a one basis point bid - ask spread, the model prices would appear highly robust.

Hypothesis one statistically tested the strength of the relationship between market and model futures prices. The test, as per Gay and Manaster's (1991) and Bhattacharya's (1987) analysis of the US Treasury bond futures market, examined the null hypothesis that the average difference between the observed ten year bond futures equivalent yields and the term structure model equivalent yields was equal to zero. Assuming that the two series were independent and that they were normally distributed the critical test statistic at 5.00 %/1.00 % significance levels was set for n = 248 at $\pm$ 1.960 / $\pm$2.576. The observed value for this test was 0.0124 so that the null hypothesis could not be rejected at either level of significance.[114] On average the model prices were not significantly different from those observed in the market.

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[113]Amin and Morton, (1994) 161.
[114]The test was taken from Equation (3.10). $\bar{x}_1$= 8.99754 $\bar{x}_2$ = 8.99549. $\sigma_1{}^2$ = 1.8487, $\sigma_2{}^2$ = 1.834 and n = 248.

Table 4.2 also incorporated the pricing results of the Cash and Carry model. This standard futures pricing model produced good estimates of the observed market yields. The average futures pricing error for the sample trading period was 0.437 of 1 basis point. This relationship, as in the term structure model case, was stable across the twelve months included in the sample. Figures (iii) and (iv) in Appendix N, which replicated the analysis in Appendix M, demonstrated that the Cash and Carry model produced futures equivalent yields were generally consistent with the term structure results.

The results of Hypothesis two provided further support for the accuracy of the Cash and Carry pricing series. On the basis of the test statistic introduced in Equation (3.10), the calculated Cash and Carry value of 0.0264762 led to the acceptance of the null hypothesis.[115] On average the Cash and Carry model produced pricing errors that were not statistically different from zero. The stability of the Cash and Carry results suggested that a simple replicating physical bond portfolio strategy would have adequately priced the ten year bond futures contract over the 1994 trading period.

There was seen to exist no ideal way to compare which of the two pricing models best explained the observed futures prices.[116] In this case the term structure model and the Cash and Carry model were compared on the basis of the summary statistics in Table 4.2 and the studies in Appendix M and N. The average pricing error of the term structure pricing model was lower 0.204894 versus 0.437608 of 1 basis point and the term structure pricing standard deviation was also lower at 0.91 versus 0.98 of 1 basis

[115]In the Cash and Carry example: $\bar{x}_1$= 8.99754, $\bar{x}_2$ = 8.99316425. $\sigma_1{}^2$= 1.8487, $\sigma_2{}^2$= 1.8325442, n = 248.

[116]It actually would be a worthwhile research topic to discuss what was the best criteria to measure and compare model performance. None of the papers consulted discussed this topic in any great depth.

point. Appendix N Figure (iv) supported the view that the Cash and Carry model was less accurate than the term structure pricing model. The percentage of pricing errors in the $\pm$ 1 point range was 70 % in the case of the Cash and Carry model vs 79 % in the term structure case. Added to this the Cash and Carry model produced eleven observations > $\pm$ 2 basis points versus five observations by the term structure model. This comparison criteria does suggest that on average the term structure model produced more reliable estimates of the future prices observed in the market.

4.3 Regression and Forecasting analysis

The term structure model was found to outperform the Cash and Carry pricing model. Hypothesis three used a simple regression relationship to test the proposition that the term structure model had produced unbiased estimates of those futures equivalent yields observed in the market. The unbiased hypothesis was tested by Equation (3.11) in Chapter 3 ;

$$(100 - F_t) = \alpha + \beta \left(\left(\sum_{i=1}^{n} YB_{(Model\,)ft} \right) / n \right)_t + \varepsilon_t$$

Where the model prices were unbiased if $\alpha = 0$ and $\beta = 1$. In practical terms if the parameters were equal to these test values then it could be concluded that the term structure model produced prices that perfectly predicted those observed in the futures market. The estimate of α determined the degree of pricing bias whilst β measured the degree of inefficiency in the regression equation.[117]

The estimated regression was shown in Table 4.3. The R^2 or coefficient of determination confirmed the results in Table 4.2.. The very high level R^2 = 0.9997 implied that the estimated regression equation was a good fit. The R^2 in this context

117Whaley, (1982) 43.

Table 4.3:

OLS estimation bond futures yields 01/01/94 to 31/01/94 n =248

Market futures yields	α	$\beta \times$ *Model*
(100 - F_t)	-0.033857	1.0039916
Std. errors of est.	0.00313	0.000344
Observed t statistic	-10.82**	2917**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.9997

Serial Correlation Tests	Rho	DW
	0.45974	1.0766

** Denotes significance at the 5.00 % and 1.00 % levels

was a ratio that measured the proportion of explained variation in market prices to total variation in market prices. In being close to 1 the term structure model prices provided a high level of explanatory power with respect to market prices. The conclusions that can be drawn from this apparently excellent result were tempered to some degree when it was remembered that the R^2 ratio was only a descriptive statistic. In practice it measured the extent of correlation between variables rather then proving the existence of any direct linkage between model and market prices.

This linkage was better measured by the significance of the β coefficient in the regression equation. In Table 4.3 the model coefficient β was seen to be a highly significant explanatory variable. The observed t-ratio of 2719 was an extremely high value for this type of test and thereby supported the hypothesis that model prices accurately reflected those observed in the market. The β coefficient estimate of

1.0039916 was consistent with the results in Section 4.2.. The positive >1 coefficient implied that model yields were on average marginally lower than those observed in the market.

The constant α term in the regression was also found to be significantly different from 0. However the estimate of - 0.033857 indicated the existence of a potential bias in the relationship between market and model yields. To predict market yields, the intercept term indicated that three basis points should be deducted from model yields. As was demonstrated in Appendix M Figure (ii) a 3 basis point pricing error was a rare event. This finding combined with the fact that β was shown to be significantly different from 1 (t-statistic = 11.60), led to the rejection of the null hypothesis. The model yields were not perfectly unbiased predictors of markets yields over the sample trading period.

The power of any statements drawn from the estimated regression were also possibly reduced by the low Durbin and Watson statistic. The calculated 1.0766 level put it below the lower bound test statistic 1.63 at the 5.00 % significance level. There was evidence that the residuals suffered from autocorrelation. The Rho statistic = 0.45974 meant that, $\varepsilon_t = 0.46 \times \varepsilon_{t-1}$ so that the residuals were positively correlated. There existed the tendency for positive residuals to follow positive residuals and negative residuals to follow negative residuals. With Rho > 0 successive values of the residuals were not independent of one another and therefore a key assumption underlying the Ordinary Least Squares approach was breached.[118] The real impact of autocorrelation was that it reduced sampling variances, decreasing the efficiency of the β estimate and potentially introduced errors into the calculated R^2 and t - statistics.

[118]The OLS estimation technique assumed that the residuals of the regression were independent. Rho = 0.

To correct for autocorrelation the standard procedure was to re-estimate the original regression by transforming all variables by ρ-differencing. Where ρ = Rho in Table 4.3.. The technique required regressing $y_t - \rho y_{t-1}$ on $x_t - \rho x_{t-1}$. Maddala's (1992) analysis suggested that because the estimation of ρ was subject to sampling errors that this methodology should only be applied under specific circumstances. The rough rule of thumb was that the original regression equation should be transformed into a first difference equation whenever the Durbin and Watson statistic was < R^2.[119] This decision making criteria did not hold in the present case. The autocorrelation present may not have been significant enough to impact on the regression results in Table 4.3..

To access the merits of this proposal more closely and following Barnhill's (1990) response to autocorrelation in his US Treasury bond futures study, a Generalised Least Squares estimation that assumed the presence of first order autocorrelation AR (1) was run.[120] The results of this work encouraged the view that the autocorrelation present did not fundamentally change the results of Table 4.3.. For example, in the corrected regression the β coefficient was still of the same general magnitude 1.0037 and significance.

The regression and average error pricing results suggested that the model had a slight tendency to underestimate market yields. In Chapter 3 the ultimate practical test of any pricing model was seen to rest with its ability to accurately predict the targeted variable. To examine the robustness of the term structure model in this forecasting

[119]Maddala, (1992) 232.

[120]Barnhill, (1990) 80. The generalised least squares estimation was run using the Autocorrelation command in Shazam Version 7.0. It assumed AR (1) where $\varepsilon_t = \rho\varepsilon_{t-1} + vt$. The results were similar to Table 4.3: (100- F_t) = -0.0311 + 1.0037 Model_t DW = 2.0715 rho = -0.03662 Durbin h = -1.1420.

role, the estimated regression relationship in Table 4.3 was used to forecast ten year bond futures equivalent yields for the out of sample period 01/01/95 to 31/01/95. The results of this study were presented in Table 4.4.

The forecasting results supported the hypothesis of a strong relationship between the term structure pricing model and the futures prices observed in the market. The average forecasting error over January 1995 was - 0.75 of 1 basis point. The forecasted yields were on average 0.75 of 1 basis point higher than the market yields. This was higher and of a different sign to the sample average of 0.2 of 1 basis point but it still represented a consistent level of pricing accuracy. 70 % of the pricing errors were in the range $\pm$ 1 basis point and there were no outliers above 3 basis points. The standard deviation of 0.67 of 1 basis point meant that the forecasted prices generally ranged between 0.1 of a 1 basis point to 1.4 basis points above the market. The forecasted prices were a good indicator of the value of the targeted SFE ten year bond futures contract. The pricing bias in the model does not appear to have any great practical significance.

4.4 Implications of the Term Structure Model Results

Two main conclusions can be drawn from the results presented in Chapter 4. Firstly, the results provided support for the hypothesis that the derived term structures were consistent with the pricing assumptions of the market. The constructed zero coupon government bond curves were able to price a market traded security with a similar risk profile to a high degree of accuracy. Although it was too strong to suggest that the model prices were perfect unbiased predictors of the market, the ability of the model to successfully forecast SFE ten year bond futures prices supported the claims of a strong economic relationship. It appears that the Australian market term structure

Table 4.4:

Forecasted 10 year bond futures yields 01/01/95 to 31/01/95

Date	Fut. Price[a]	Fut. Yield[b]	Forecast [c]	Spread [d]
3/01/95	89.885	10.115	10.126914	-1.19140
4/01/95	89.77	10.23	10.225451	0.45492
5/01/95	89.635	10.365	10.385982	-2.09822
6/01/95	89.595	10.405	10.426264	-2.12641
9/01/95	89.52	10.48	10.493087	-1.30873
10/01/95	89.53	10.47	10.485771	-1.57707
11/01/95	89.575	10.425	10.437037	-1.20370
12/01/95	89.64	10.36	10.365063	-0.50633
13/01/95	89.53	10.47	10.473946	-0.39462
16/01/95	89.65	10.35	10.351753	-0.17529
17/01/95	89.66	10.34	10.3429	-0.29002
18/01/95	89.66	10.34	10.347903	-0.79026
19/01/95	89.61	10.39	10.393338	-0.33382
20/01/95	89.53	10.47	10.475638	-0.56379
23/01/95	89.38	10.62	10.625323	-0.53230
24/01/95	89.44	10.56	10.560061	-0.00611
25/01/95	89.56	10.44	10.440953	-0.09525
27/01/95	89.715	10.285	10.289443	-0.44427
30/01/95	89.785	10.215	10.22532	-1.03202
31/01/95	89.56	10.44	10.448139	-0.81391

(a) Consistent with the sample methodology these were the closing SFE ten year bond futures prices for 01/01/95 to 31/01/95 $n = 20$. (b) The futures equivalent yield = 100 - futures price. (c) The forecasted yields were derived from regression $(100 - F_t) = -0.033857 + 1.0039916 \times$ Model yield (d) Spread = Market - Forecast yield. The average was - 0.75 of 1 basis point. The standard deviation 0.676 of 1 basis point and the correlation coefficient equalled 0.9984.

could be successfully derived from the observed Commonwealth government bond coupon curve.

The strength of the term structure methodology gives it the potential to be applied to other markets and applications. In Chapter 5 the model term structure formed the basis of a measure of interest rate volatility that was subsequently used to price interest rate options. It may also have a role to play in other fixed interest markets such as the US, Germany or UK once adjusted for their relevant institutional factors. This would allow the term structure model to be applied to other applications such as the pricing of longer dated forward foreign exchange contracts. A reliable market consistent term structure, in calculating the true time value of money for a given context, has theoretical and empirical ramifications for a range of traded markets.

Secondly, the pricing results indicated that the SFE ten year bond futures market over the sample period was fairly priced and did not provide opportunities for riskless arbitrage. Both modelling approaches, the term structure based Model and Cash and Carry, suggested that in general the ten year bond futures market was priced $\pm$ 1 basis point from fair value. This implied, given a bid - ask spread of 1 basis point, that it was not possible using currently available yield curve information to make abnormal returns from the pricing of the SFE ten year bond futures market. This would seem consistent with a mature market that has been trading since December 1984.

While published research, dealing specifically with the SFE ten year bond futures market was limited, the related US Treasury bond futures market has been the subject of much interest. Barnhill (1990) in support of the Cash and Carry model results, found that the simple minimum forward price, i.e. the Cash and Carry price, explained between 78 to 99 % of the variation in the observed Treasury bond futures price.[121] The so called naive model provided a strong foundation from which to price and trade bond futures contracts.

Other studies of the US Treasury bond futures market provide the impetus for new research into the term structure based pricing of the SFE ten year bond futures contracts. The central difference between the two futures contracts was that the US Treasury bond contract was physically settled rather than cash settled. The physical settlement procedure provides for two basic options: which particular bond was cheapest to deliver and the timing of that delivery.[122] Gay and Manaster (1991) reported that these implicit delivery options, available to the holder of a short futures position, were not always fully factored into the observed Treasury bond futures prices. The Treasury bond futures prices were seen to be trade too high relative to the equilibrium price of the implicit delivery options. The market was to some degree giving shorts a small free option.

This type of pricing error has relevance for the present paper. Throughout 1995 the SFE proposed the introduction of physical delivery for the settlement of the ten year and three year bond futures contract. The central objective of this change, according to the SFE discussion paper dated August 1995, was to bring the Australian contract into line with the main offshore bond futures contracts and thereby increase its international acceptance.

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[121] Barnhill, (1990) 80.

[122] Hull (1993) 91- 92. The cheapest to deliver bond was that bond which minimised the calculation, quoted physical bond price - (quoted futures price × bond conversion factor). There can be up to 30 bonds considered for Treasury bond delivery a characteristic that would not exist in the Australian market.

The ability of the term structure model to price the SFE ten year bond futures contract under any new settlement specification is an area of great interest. This type of analysis is especially critical in the early stages of trading in a new contract design. It is likely that the market may take time to come to terms with the new specification details and therefore pricing inefficiencies may exist. The determination of the Commonwealth bond that is cheapest to deliver would likely form the central part of any research into the physical delivery issue.

CHAPTER 5

Characteristics of the Yield Error Margin Databases

5.1 Introduction

Chapter 5 presented the results of the Yield Error Margin research. These market based constructions were used to test the two key remaining hypotheses associated with this paper. The second major hypothesis stated that the market's current best forecasts of interest rates was equal to today's implied *N*-period forward term structure. In Chapter 4 the implied *N*-period forward term structure was used to evaluate the Sydney Futures Exchange (SFE here after) ten year bond futures contract. The *N*-period forward date in this case was the ten year bond futures cash settlement date. The results of this analysis suggested that the observed futures prices were consistent with the interest rate forecasts contained in today's term structure. The ten year bond futures prices were equivalent to the implied certainty forwards detailed in Equation (2.18).

In practice market participants face uncertainty when making decisions. The bond futures contracts provide a degree of certainty by allowing agents to lock into today's best forecast of the underlying Commonwealth government coupon bond basket. What these securities did not provide was the actual average bond basket price on the contract settlement date. There exists no guarantee that today's best forecast of the security prices would be the actual price observed *N*-periods forward. The current interest rate forecasts represent only one of a range of possible prices that have a positive probability of occurring on the *N*-period forward date.

The role of Section 5.2 was to report on the accuracy of today's implied *N*-period forward interest rate forecasts. This was measured empirically by the Yield Error

Margin statistic. This statistic calculated the discrepancies between today's implied N-period forward forecast of the ten year bond futures average basket yield and the actual ten year bond futures average basket yield observed on the N-period forward date. For each full trading day over the 01/01/94 to 31/12/94 sample period four average basket yield forecasts were generated; two weeks, one month, two months and three months forward. This process created four Yield Error Margin databases that recorded the errors in the market's interest rate forecasts.

This leads to the third and final core hypothesis which proposed that the Yield Error Margin databases could be used to approximate the stochastic behaviour of the term structure. It was assumed that the databases provided important interest rate risk management information. As well as defining the market's average forecasting error the databases were also seen to contain historical evidence on the distribution of these errors. To determine the efficacy of this hypothesis Section 5.3 focused on the distributional characteristics of the Yield Error Margin databases whilst Section 5.4 assessed whether the Yield Error Margin databases could be used to develop a market consistent measure of interest rate volatility.

5.2 The Calculated Yield Error Margin databases

Section 5.2 reported the results associated with Equation (3.15). The Yield Error Margin statistic was based on the hypothesis that today's implied N-period forward term structure represented the market's best estimate of the level of interest rates at the given N-period forward date. These constructed forward term structures would only be equivalent to the actual term structures observed on these forward dates if today's forecasts were without error. Consistent with modelling under the presence of uncertainty this was not seen to hold in the general case. It was highly probable that there would be errors between today's implied forecasts and the actual interest rate structure that occurred on the *N*-period forward date. This forecasting error for a given trading day represented an individual entry in the Yield Error Margin databases.

Today's initial term structure was used to construct via Equation (3.14) four implied *N*-period forward term structures: two weeks, one month, two months and three months forward. The four implied forward term structures were used to price the physical Commonwealth government coupon bonds that made up the SFE ten year bond futures pricing basket. These implied forward bonds prices were subsequently converted to yields by Equation (3.1). The arithmetic mean of these forward yields was calculated to generate an average bond futures basket yield for the four forward horizons.

The actual term structures observed on these forward dates were also used to construct average futures basket yields. The four Yield Error Margin statistics were therefore calculated by the relationship: Today's implied *N*-period forward average basket yield - actual average basket yield observed on the *N*-period forward date. Consistent with Chapter 4 the forecasting error was measured in yield basis point terms, with a positive number indicating that the forecasts had been too high and a negative number that they had been too low.

This method of analysis was carried out for each day in the sample 01/01/94 to 31/12/94. The Yield Error Margin results for the first trading day in the sample, 04/01/94, were displayed in Table 5.1.. In line with the normal shape of the underlying yield curve and the expectation hypothesis, introduced in Chapter 2, the implied forward bond yields were found to be higher than those initially observed on 04/01/94.

Table 5.1:

Yield Error Margin results 04/01/94

Basket Yield Implied vs Actual 2week Forward
Date: 18/01/94

Bond [(a)]	Impl. Yld.	Act. Yld.	Pts. Diff.
15/10/02	6.76272635	6.51	25.2726354
15/08/03	6.79174425	6.54	25.1744246
15/09/04	6.85170023	6.58	27.1700234
15/07/05	6.8905595	6.62	27.0559495
Average[(b)]	*6.82418258*	*6.5625*	*26.1682582*

Basket Yield Implied vs Actual 1mth. Forward
Date: 04/02/94

Bond	Impl. Yld.	Act. Yld.	Pts. Diff.
15/10/02	6.7801232	6.46	32.0123196
15/08/03	6.80793458	6.49	31.7934576
15/09/04	6.86761395	6.53	33.7613949
15/07/05	6.90507048	6.55	35.5070482
Average	*6.84018555*	*6.5075*	*33.2685551*

Basket Yield Implied vs Actual 2mth. Forward
Date: 04/03/94

Bond	Impl. Yld.	Act. Yld.	Pts. Diff.
15/10/02	6.8059063	7.39	-58.40937
15/08/03	6.83171376	7.41	-57.8286244
15/09/04	6.89134885	7.45	-55.8651151
15/07/05	6.92649998	7.47	-54.3500021
Average	*6.86386722*	*7.43*	*-56.6132779*

Basket Yield Implied vs Actual 3mth. Forward
Date: 05/04/94

Bond	Impl. Yld.	Act. Yld.	Pts. Diff.
15/10/02	6.83481105	8.19	-135.518895
15/08/03	6.8583609	8.21	-135.16391
15/09/04	6.91535159	8.23	-131.464841
15/07/05	6.95014427	8.27	-131.985573
Average	*6.88966695*	*8.225*	*-133.533305*

(a) These were the Commonwealth government coupon bonds that were incorporated in the pricing basket of the nearby March 1994 10 year bond futures contract. (b) The average basket yield of the Commonwealth government coupon bonds was used to compare today's market forecast with the actual yields observed on the forward date. The hatched areas represented the results associated with Equation (3.15).

The simple equation $(1 + y_{1/100})^{T_1/365} \times (1 + y_{2/100})^{T_2/365} = (1 + y_{3/100})^{T_3/365}$ determined that if $y_1 < y_3$ then $y_2 > y_3$ assuming $T_1 < T_2 < T_3$ and $T_1 + T_2 = T_3$. Where y_2 was assumed to be the implied forward bond yield derived from y_1 / y_3 the short / long yields taken from today's initial yield curve. This pricing must hold in today's observed physical market or otherwise it would be possible to construct an arbitrage portfolio from combining the securities trading at yields y_1 / y_3 with maturities of time T_1 / T_3. This construction represented a restatement of the certainty forward analysis associated with Ho and Lee (1986).

These higher implied forward bond yields were confirmed when Table 4.1. and Table 5.1. were compared. Table 4.1. displayed the yields of the Commonwealth government coupon bonds associated with the initial 04/01/94 yield curve in the "Yield" column. The average futures basket yield in this case was 6.8125 %. In Table 5.1., the average basket yield was 6.824 % for the 2 week implied forward, 6.84 % for the 1 month forward, 6.863 % for the 2 month forward and 6.89 % for the 3 month forward. The fact that on 04/01/94 the short anchor rates equivalent to y_1, were trading at around 4.80 % and the longer dated, equivalent to y_3, were trading at around 6.80 % means that by definition the implied forward rates, equivalent to y_2, were calculated to trade at higher yields. This positive spread relationship between the implied forward bond yields and the initial bond yields held in varying degrees for the entire sample. As mentioned in Chapter 4 the yield curve was normal in shape for the entire sample period 01/01/94 to 31/12/94.

Table 5.1. demonstrated that the accuracy of the market's forecasts on the 04/01/94 trading date had been mixed for the four forward periods. For example on 04/01/94 the 2 week forward forecast of the average basket yield was 6.824 %. On the actual 2 week forward date, 18/01/94, the observed term structure produced an

average basket yield of 6.5625 %. The forecasted yield was 26.2 basis points higher than was actually realised. The 04/01/94 forecast one month forward was similarly 33.3 basis points too high. For the longer dated forwards the situation was reversed with the 2 month and 3 month forecasts underestimating the actual increase in yields by 56.5 basis points and 133.5 basis points respectively. These four forecasting errors represented the inputs into the Yield Error Margin databases for the 04/01/94 trading date.

The four Yield Error Margin forecasting errors associated with each trading day in the 1994 sample were recorded in Appendix O. Table 5.2. summarised these errors into monthly and All sample results. The entries in Table 5.2. represented the average monthly implied forward pricing error. As in Table 5.1. the Yield Error Margin results were recorded in yield basis points. From the All Sample results it was obvious that on average the market had underestimated the increase in longer dated yields. Across each of the four forecasting horizons the average Yield Error Margin was negative. The implied forecasts were generally too low over the sample period.

The Yield Error Margin results, particularly in the case of the 2 month and 3 month horizons, could be broken into three periods. The largest negative errors in the implied forecasts were associated with the first third of the sample, January to April 1994. In the middle part of the sample, from May to September 1994, the forecasting errors decreased in size and stabilised to some degree. In the last part of the sample, October to December 1994 the errors started to show signs of reversal with the errors on average becoming positive (Implied > Actual). The implied forecasts started to overestimate the actual level of yields.

Table 5.2:

Summary of Monthly Yield Error Margin results

Month	*2 week* (a)	*1 month*	*2 month*	*3 month*
Jan-94	**8.574574947**	**-18.1979742**	**-89.9400551**	**-156.342216**
Feb-94	**-38.820134**	**-70.5023448**	**-141.058111**	**-190.813822**
Mar-94	**-32.11447**	**-67.43182**	**-113.59364**	**-163.15312**
Apr-94	**-29.3295273**	**-46.3935101**	**-99.1046894**	**-123.417723**
May-94	**1.263504045**	**-50.5937125**	**-78.532611**	**-53.0300964**
Jun-94	**-33.7160913**	**-29.1850091**	**-5.56696338**	**-56.9660779**
Jul-94	**13.98955**	**25.30914**	**-29.40070**	**-51.05388**
Aug-94	**-11.1508341**	**-56.3294449**	**-78.2008118**	**-103.494443**
Sep-94	**-20.2279062**	**-23.7295246**	**-51.0772492**	**-3.12709273**
Oct-94	**-17.78645**	**-27.32095**	**24.9176**	**10.1195**
Nov-94	**18.78101909**	**47.62952773**	**36.58167045**	**N/A (b)**
Dec-94	**10.44930**	**-6.37294**	**N/A**	**N/A**

Summary statistics All Sample 01/01/94 to 31/12/94

***All Sample* (c)**	***-10.93***	***-27.06****	***-56.22*****	***-88.17*****
Std. Dev.	**31.70**	**46.54**	**63.13**	**70.40**
Max. Spr.	**70.41**	**70.03**	**81.02**	**52.14**
Min. Spr.	**-114.55**	**-129.17**	**-176.04**	**-229.81**
Range Spr. (d)	**184.97**	**199.20**	**257.05**	**281.95**

(a) Average monthly error measured in basis point terms: Implied Forward - Actual observed yield for each of the four forward horizons. (b) The 2 week and 1 month sample sizes $n = 248$. Due to constraints with the yield data the 2 month sample size $n = 230$ and the 3 month sample size $n = 208$. (c) The average All Sample results for the four forward time horizons. (d) Determined the range of the implied forecasting errors. Range = Maximum spread - Minimum spread. * Denotes the implied forecasting error was significant at the 5.00 % level.** Denotes the error was significant at the 1.00 % level.

These three phases in the forecasting errors reflected changes in the market's interest rate expectations. Figure 4. in Chapter 4 summarised these developments. The relatively small amount of slope in the 04/01/94 yield curve reflected the market view that yields would rise only marginally in the future. These forecasts of stability were subsequently proven wrong by the marked steepening of the yield curve over the first half of 1994. As discussed in Chapter 4 the initial dramatic increases in yield were concentrated in the longer dated part of the curve.

The stabilisation of the Yield Error Margin's in the middle part of the sample was correlated with a time when longer dated yields traded in a 70 basis point range. Over the period 30/06/94 to 08/09/94 ten year bond yields traded between 9.90 % and 9.20 %. It can be suggested that after a significant 350 basis point increase in yields in four months that the market required additional information to further sell bonds and thereby increase the level of longer dated yields. Finally, the Yield Error Margin's turned positive in the later part of the sample in response to the market's expectation of a significant near term tightening of monetary policy. The curve became "flat" with yields from one year to ten years trading around the 10.00 % level. It should be noted that this expectation of 10.00 % official cash rates in December 1994 was proven wrong in 1995. The Reserve Bank of Australia held cash rates steady at 7.50 % throughout 1995.

This pattern in the recorded Yield Error Margins implied that the market's expectations over the sample period could be defined by three words ; Shock (large negative errors), Recovery (errors stabilised) and finally Overshooting (positive errors). Therefore, there was evidence of lags and persistence in the market's expectational behaviour. This performance, as will be discussed in Sections 5.3 and 5.4, had important implications for the volatility characteristics of the SFE ten year bond futures contract. [123]

The final observation to make about the Yield Error Margin results was that they reaffirmed the relationship between time and uncertainty. The size of the average All Sample error increased with the forecasting horizon. The 2 week average error was

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[123] This style of market behaviour, caution, reaction and overshooting would appear to be a consistent characteristic of the Australian financial markets. The Chart in Attachment 6 was taken from the February 1997 RBA bulletin. It demonstrated that the market had not been fully prepared for two of the three easings that occurred over the second half of 1996. It can be suggested that the overshooting observed in January 1997 reflected the market's fear of missing the fourth easing. The 90 day Bank Bill yield in late February 1997 subsequently moved back above the official cash rate.

-10.9 basis points, the 1 month error was - 27.1 basis points, the 2 month error - 56.2 basis points and the three month error was - 88.2 basis points. The higher standard deviation (2 week 31.70 basis points vs 3 month 70.40 basis points) and the larger absolute error range (2 week 184.97 basis points vs 3 month 281.95 basis points) of the longer dated Yield Error Margin's databases reinforced the linkage between a longer horizon and greater uncertainty. The economic significance of this result was that the accuracy of market forecasts decreased as the forward time horizon increased. The potential for unforeseen shocks and events was clearly correlated with the period of time for which we looked forward.

This proposition was also supported by the results associated with Hypothesis four which tested the statistical significance of the market's average forecasting errors. The null hypothesis examined the proposition that the average difference between the forecasted yields and the actual observed yields was equal to zero. The critical test statistic at the 5.00 % / 1.00 % significance levels was set for $n = 248$ (2 week and 1 month), 230 (2 month), 208 (3 month) $\pm$ 1.96 / 2.576. The observed test values derived from Equation (3.10) for the 2 week Yield Error Margin database were - 0.93938160, for the 1 month - 2.38571893, for the 2 month -5.14910703 and the 3 month Yield Error Margin sample -8.44094688.[124]

For the two week sample the null hypothesis could not be rejected. The opposite was the case for the longer dated horizons, 1 month to 3 months, with the null hypothesis being rejected. This rejection increased in strength with the Yield Error

[124] For the two week sample, $\bar{x}_1$=Average implied forecast = 8.950326137, $\bar{x}_2$= Average actual observed yield = 9.059652823 $\sigma_1{}^2$= 1.801467189, $\sigma_2{}^2$= 1.62291076 the variances of the respective series. n = 248. For the 1 month sample $\bar{x}_1$ = 8.9975592409, $\bar{x}_2$ = 9.246191532, $\sigma_1{}^2$ = 1.817741457 , $\sigma_2{}^2$= 1.37281139, n =248. For the 2 month sample $\bar{x}_1$= 8.919666591, $\bar{x}_2$ = 9.481821739, $\sigma_1{}^2$ = 1.848194352 ,$\sigma_2{}^2$= 0.893229529, n =230. For the 3 month sample $\bar{x}_1$ = 8.919666591, $\bar{x}_2$ = 9.656790865,$\sigma_1{}^2$= 1.697994843, $\sigma_2{}^2$= 0.570174122, n =208.

Margin time horizon. In Table 5.2 the errors in the 2 month and 3 month Yield Error Margin databases were found to be significant at both the 5.00 % and 1.00 % levels. Therefore it can be concluded that the market's forecasting errors were generally significant for the sample period 01/ 01 / 94 to 31 /12 / 94.

These findings were consistent with other research that had tested the implied forecasts of the term structure. Fama and Bliss (1987), Froot (1989) and De-Bondt and Bange (1992) all found that implied forward rates provided little power for forecasting the actual short term changes in yields.[125] Fama and Bliss followed an approach similar to the present paper by using a term structure derived from the market to forecast changes in yields. The key test was whether the spread between today's implied *N*-period forward spot rate and today's spot rate could predict the spread between the observed realised spot rate in *N*-periods of time and today's spot rate. The results of Fama and Bliss's Ordinary Least Square analysis suggested that the implied forward spot spread did not successfully predict changes in yields over a one year horizon.[126]

De-Bondt and Bange's (1992) study was worthy of further review because it looked for the source of these expectational errors. With relevance to this papers sample period the authors noted, as per the well known Fisher effect, that market participants were slow to adjust their forward views of inflation. They weighted past inflation performance very highly when making decisions about the current environment.

125For example Froot, (1989) 304. Implied forecasts were unable to predict changes in long rates because of systematic expectational errors.

126Fama and Bliss, (1987) 684. The regression run was of the form: (Actual spot rate at forward date *N* - today's spot rate) = $\alpha + \beta$ (today's implied *N*-period forward spot rate - today's spot rate) + ε

This type of behaviour, it can be suggested, had a role to play in explaining the magnitude of the forecasting errors in the 01/01/94 to 31/12/94 sample. The period between 1990 and 1993 had been characterised by a protracted Australian economic slowdown. As a result, Australian headline and underlying inflation moved from its late 1980's level of 8.00 % - 9.00 % to thirty year lows of around 1.5 % in late 1993.[127] Ten year bond yields over this corresponding period dropped from 14.00 % to 6.5 %. Appendix P incorporated a graph of the period 1990 to 1996 that highlighted this change in the level of the Australian long term rates.

In January 1994, as discussed earlier, the market appeared to have been comfortable with the current inflation environment. The normal yield curve at the time had a "gentle" slope. This observed behaviour continued over January despite persistent rumours of a possible tightening of US monetary policy. However, this complacency in expectations was changed in February 1994 when, for the first time in three years, the US Federal reserve actually tightened monetary policy.

he Australian bond market, as already detailed, performed poorly over the subsequent three to four months. The holder of bonds would have earned significant losses over this period. Market participants appear to have taken the view that if the Federal Reserve was tightening monetary policy then the world economy must be headed for a strong sustained recovery. As a consequence Australian inflation would move back towards its pre-recession levels. It can be proposed that the market was looking to past experience, when Australia was classed as a "high inflation country",

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[127] See the Reserve Bank Bulletin Table G.1 and Australian Bureau of Statistics publication series 6401.

to justify the higher yields witnessed in 1994.[128] De-Bondt and Bange's hypothesis was therefore likely to find some support in the Australian interest rate market.

5.3 Distributional Characteristics of Yield Error Margin Databases

Section 5.3 focused on the distributional characteristics of the Yield Error Margin databases. The first part of this research involved constructing relative frequency distributions from each of the four underlying Yield Error Margin databases. Table 5.3. displayed the relatively frequency table associated with the 2 week Yield Error Margin database. Figure 5. converted this information into a relative frequency histogram and an approximate density function. The relative frequency results for the 1 month, 2 month and 3 month Yield Error Margin databases were presented in Appendix Q.

The "YEM pts." column in Table 5.3 accounted for the forecasting errors in the implied 2 week forward average bond futures basket yields. As per the analysis in Table 3.2 each of the individual n = 248 forecasting errors were sorted into a number of ten basis point intervals. For example the forecasting error of -12.61 basis points on 05/12/94 was placed in the -10 basis point error category. This particular interval covered forecasting errors in the range of -10 to -19.999 basis points.

The "Frequency Outcome" column in Table 5.3 recorded the number of times that individual forecasting errors were found to correspond with a particular 10 basis point class. Of central interest was the conversion of these frequency outcomes into

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128Note that the Australian 10 year bond - US 30 year bond spread moved from 0 basis points in January 1994 to 227 basis points in June 1994. US bond rates only increased by 100 basis points in this period. Australian bonds in a strong world growth phase were seen as relatively risky assets. Data sourced from Datastream International. Appendix P demonstrated that these fears were not supported by developments in 1995. Bond yields through 1995 actually declined by 200 basis points, towards 8.00 %, as inflation fears subsided.

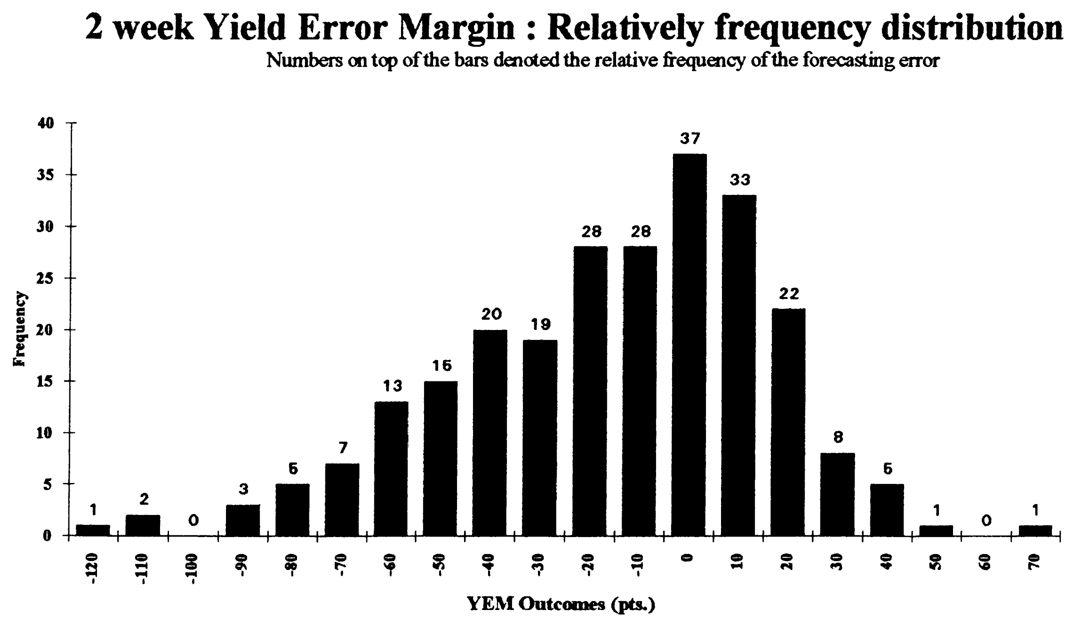
Table 5.3:

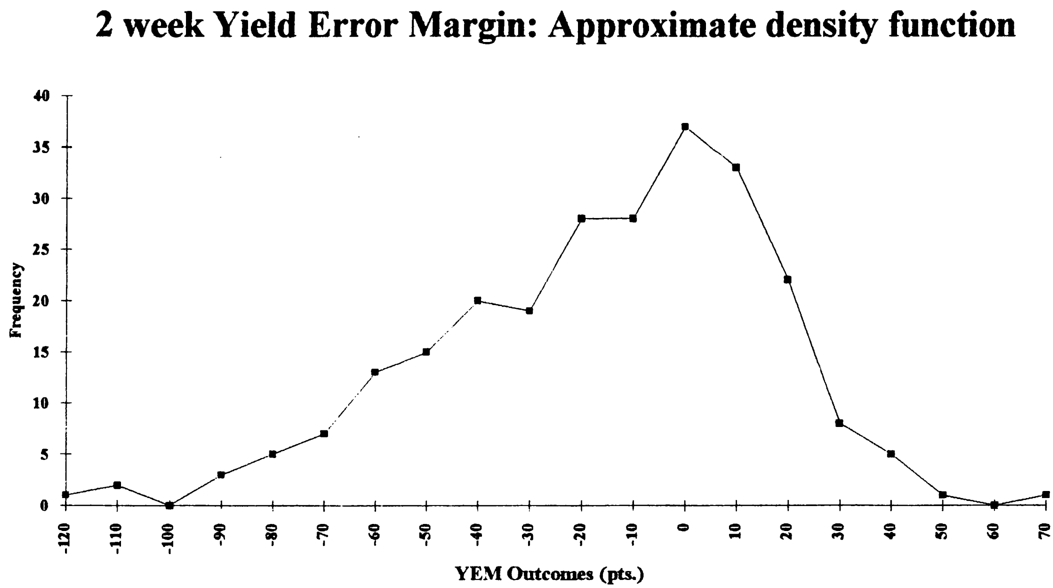
2 week Yield Error Margin database: Relative frequency distribution

YEM [a] pts.	Frequency [b] Outcome	Relative [c] Frequency
-120	1	0.4
-110	2	0.8
-100	0	0.0
-90	3	1.2
-80	5	2.0
-70	7	2.8
-60	13	5.2
-50	15	6.0
-40	20	8.1
-30	19	7.7
-20	28	11.3
-10	28	11.3
0	37	14.9
10	33	13.3
20	22	8.9
30	8	3.2
40	5	2.0
50	1	0.4
60	0	0.0
70	1	0.4
Total	248	100

(a)Yield Error Margin = Forecasting error in the implied 2 week forward average bond futures basket yield - actual observed 2 week forward average bond futures basket yield. Each error was allocated to one of the 10 basis point intervals. (b) The Frequency Outcome column recorded the number of times a particular error category occurred. (c) Established the frequency of a particular error outcome as a proportion of the total sample. This measure was equivalent to the probability of a particular outcome. As per the standard assumption in probability analysis the sum of the individual probabilities = 1. In this case the individual probabilities have been multiplied by 100 to record the probabilities directly as percentages.

Figure 5:


2 week Yield Error Margin : Relatively frequency distribution
Numbers on top of the bars denoted the relative frequency of the forecasting error
Frequency
40
35
30
25
20
15
10
5
0
1
2
0
3
5
7
13
15
20
19
28
28
37
33
22
8
5
1
0
1
-120
-110
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
YEM Outcomes (pts.)


2 week Yield Error Margin: Approximate density function
Frequency
40
35
30
25
20
15
10
5
0
-120
-110
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
YEM Outcomes (pts.)

measurable probabilities. This was done by dividing the raw frequency associated with a particular error class by $n = 248$. This generated relative frequencies which were assumed to equal the probability of a particular size forecasting error in the 2 week Yield Error Margin database.

For example, in Table 5.3. it could be stated that there existed ≈ a 15.00 % (1 in 7) chance that today's 2 week implied forward would perfectly predict the actual observed level of yields in 2 weeks times. The probability of a zero forecasting error was set equal to 15.00 %. Consistent with general probability analysis these relative frequencies summed to one. This relative frequency analysis described the 2 week Yield Error Margin distribution in two ways: Firstly, it highlighted the absolute range of the forecasting errors and secondly, it calculated the likelihood of particular errors occurring over the sample period. These weightings were critical to the option pricing analysis in Chapter 6.

Figure 5. summarised the distributional information contained in Table 5.3. The relative frequency histogram and the approximate density function suggested that there was a degree of bias in the 2 week Yield Error Margin volatility database. From Figure 5. it was clear that there existed a greater probability of negative errors than positive ones. The two week forecasts, 57 % (141 / 248) of the time underestimated observed yields and only 28 % (70 / 248) of the time overestimated yields. There was an observed negative skewness in the 2 week Yield Error Margin database.

This bias of underestimating the actual level of yields was a characteristic shared with the longer dated Yield Error Margin databases. Visual inspection of the relative frequency histograms in Appendix Q supported this view. In this regard the 3 month Yield Error Margin volatility distribution was particularly extreme. 83.00 % of the forecasting errors were negative whilst 10.00 % were positive.[129] The nature of the forecasting bias in the longer dated Yield Error Margin distributions was fundamentally different to the 2 week example. As discussed in Section 5.2 the longer dated distributions had larger negative mean errors. The distributions around these negative means were less peaky or skewed than was the case with the 2 week distribution. Error outcomes in these longer dated distributions were weighted more equally than would have been the case if they were drawn from a normal distribution. These distributions were seen to look more uniform than normal in character.

This initial analysis suggested that the Yield Error Margin distributions were not normally distributed. As mentioned in Chapter 3 the advantage of the normal distribution was that it was computationally convenient. The underlying random variable was completely described by its mean and variance. Hypotheses five and six formally tested the Yield Error Margin distributions for normality.

In Hypothesis five the Kolmogorov-Smirnov D statistic, associated with Equation (3.17), was used to examine the null hypothesis that the individual Yield Error Margin outcomes were selected from a normal distribution whose cumulative distribution function was given by $F_o(x)$. This test was one sided with the decision rule that if D observed > D critical then Ho was rejected. Hypothesis six provided an alternative test of normality. It used Equations (3.18) (a) and (b) to determine whether the Yield Error Margin distributions suffered from kurtosis or skewness. For both tests of normality the SFE ten year bond futures contract was used as the benchmark distribution from which to compare the results of the four Yield Error Margin

[129]In the 1month case 67.0 % of the errors were negative, 26.9 % were positive. In the 2 month case 74.6% of errors were negative, 20.8 % were positive.

databases. The SFE ten year bond futures frequency distribution was constructed by Equation (3.16) with the results displayed in Appendix R.

The results of Hypothesis five and six were presented in Table 5.4.. The statistical tests generally supported the visual observation that the Yield Error Margin distributions were not normally distributed. The 2 week, 1 month, 2 month and ten years bond futures distributions all had observed D statistics greater than the critical D statistic. The null hypothesis was rejected, so that the distributions were found to be significantly different from a normal distribution. The 3 month Yield Error Margin was the only distribution in the set that accepted H_0.

In the results from Hypothesis six the two week Yield Error Margin distribution was the only distribution affected by skewness. The problem of kurtosis was more widespread. The 2 week Yield Error Margin was the only distribution that did not have significant levels of kurtosis. The 1 month Yield Error Margin was found to have a flat so called platykurtic distribution at the 1.00 % level. The 2 month and 3 month distributions were found to be platykurtic at only the 5.00 % significance level. The ten years bond futures distribution in contrast to the Yield Error Margin distributions was found to have a peaked distribution that was leptokurtic.

These findings were significant for two reasons. Firstly, the distributional results associated with the ten year bond futures contract were consistent with other studies in this area. Heynen, Kemna and Vorst (1994) and Venkateswaran, Brorsen and Hall (1993) found that asset returns were often leptokurtic. The return distributions had higher peaks and thicker tails then would be present under a normal distribution.

Table 5.4:

Hypothesis 5 : The Kolmogorov - Smirnov normality test

Distribution	Sample n	Critical D (a)	Observed D (b)	Accept Ho if Obs.D < Crit. D
2 week	248	0.10351	0.1054	Reject Ho
1 month	248	0.10351	0.1503	Reject Ho
2 month	230	0.10747	0.1630	Reject Ho
3 month	208	0.11302	0.089448	Accept Ho
10 year fut.	247	0.1037	0.1200	Reject Ho

(a) Critical D was approximated at the 1.00% significance level by $163/\sqrt{n}$. See Hoel et. al., (1971)
(b) Observed D was calculated by Equation (3.17).

Hypothesis 6 : Kurtosis normality test

Distribution	Sample n	Calc. Kurt. (a)	Obs. Kurt. (b)	Comments
2 week	248	0.17	0.55	None
1 month	248	-1.02	-3.27 **	flat/platykurtic
2 month	230	-0.81	-2.51*	flat/platykurtic
3 month	208	-0.84	-2.47*	flat/platykurtic
10 year fut.	247	1.14	3.66**	peak/leptokurtic

(a) Calculated kurtosis was derived from Equation (3.18) (b).
(b) Observed kurtosis = calculated kurtosis / $\sqrt{24/n}$. The critical test statistic for kurtosis at the 5.00 % / 1.00 % level for n = 248 , 247 , 230 and 208 was given by $\pm$ 1.956 / $\pm$ 2.576.
* Denoted kurtosis was significant at 5.00% ** Denoted kurtosis was significant at 1.00% .

Hypothesis 6 : Skewness normality test

Distribution	Sample n	Calc. Skew (a)	Obs. Skew. (b)	Comments
2 week	248	-0.56	-3.60 **	Left -ve bias
1 month	248	0.07	0.45	None
2 month	230	0.26	1.61	None
3 month	208	-0.03	-0.17	None
10 year fut.	247	0.24	1.54	None

(a) Calculated skewness was derived from Equation (3.18) (a).
(b) Observed skewness = calculated skewness / $\sqrt{6/n}$. The critical test statistic for skewness at the 5.00 % / 1.00 % level for n = 248 , 247 , 230 and 208 was given by $\pm$ 1.956 / $\pm$ 2.576.
* Denoted skewness was significant at 5.00%. ** Denoted skewness was significant at 1.00% .

The ten year bond futures approximate density function in Appendix R captured these distributional traits. The peaked nature of the distribution was summarised by the fact that, for the 1994 sample period, 51.00 % (127 / 247) of the time the daily yield change from one close to the other was only $\pm$ 10 basis points. Although this obviously missed the potential for large intra - day moves in yield, it does suggest that for a significant proportion of the time the futures market was actually trading in tight ranges.

In contrast to this, the ten year bond futures distribution in Appendix R was also seen to have relatively "fat" tails. During the trading sample there were significantly more shocks or large basis point moves than would have occurred if the return structure was truly normal This type of price action suggests that market behaviour can be broken into two parts. Quiet, lull type trading periods where the market searches for new information and direction. The behaviour in this phase was characterised by range trading. The second part of the market's price action was a more violent trading period where the market attempted to factor in the meaning of "powerful" new information. The market breaks the existing price bracket and attempts to form a new one at a different central price. The impact of this two part trading behaviour on implied volatility was discussed in Section 5.4.[130]

The results were also important because they demonstrated that the Yield Error Margin distributions statistically biased relative to the normal distribution. The results in Section 5.2 showed that the Yield Error Margin distributions were centred around negative means. The market's implied forecasts underestimated the change in yields.

[130] These results also suggested a potential future area of research. To test a trading strategy that buys and sells "breaks" of established ranges that form in markets. In other words trade with the volatility. Obviously the central mathematical part of this test would centre on defining exactly what a "range" was and secondly what was defined as a "break" of this defined range.

The distributional results in Section 5.3 suggested that the Yield Error Margin distributions were flatter than would have been the case if they were normally distributed. Too much weight was given to observations away from the mean and correspondingly not enough to those close to the mean. The conclusion that can be drawn from these results was that the Yield Error Margin distributions would likely produce biased option and interest rate risk prices. Section 5.5 introduced an additional assumption that attempted to resolve this issue.

5.4 The Yield Error Margin Volatility Measure vs the Market

The objective of Section 5.4 was to determine if the Yield Error Margin distribution methodology could be used to produce market consistent measures of volatility. In Chapter 3 it was established that the market relied on implied and historical measures of volatility to approximate the appropriate level of trading volatility for option pricing. As its name suggests, implied volatility was determined directly from the option prices observed in the market. Historical volatility, like the Yield Error Margin databases was derived from changes in the level of market yields.

Equation (3.19) calculated an implied volatility series for the sample period from SFE ten year bond futures option settlement prices. The results of this study were presented in Table 5.5. and Figure 6.. The All Sample results demonstrated that there was an 11.00 % range in the level of implied volatility during 1994. The maximum of 17.50 % was quoted in June 1994 and the minimum of 6.45 % was quoted during January 1994. These two volatility quotes suggested that very different distributional assumptions were being made by the market about the path of the ten year bond futures price.

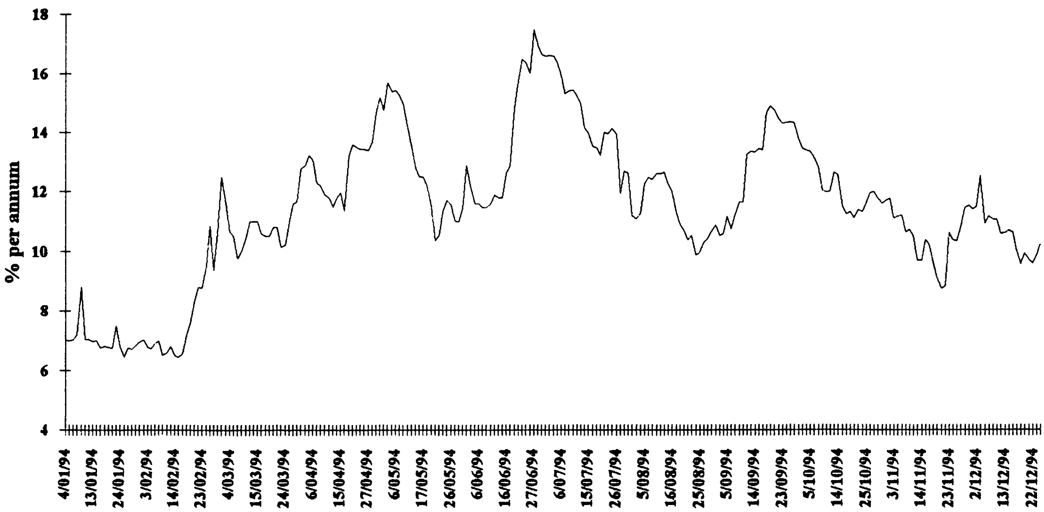
Table 5.5:

Implied SFE 10 year bond futures option volatility

Month	Av. Impl. Vol. (a)	Max. Vol. (b)	Min. Vol.	Range
Jan-94	7.01	8.80	6.45	2.35
Feb-94	7.56	10.86	6.45	4.41
Mar-94	10.98	12.88	9.75	3.13
Apr-94	12.78	14.67	11.36	3.31
May-94	12.94	15.69	10.35	5.34
Jun-94	13.85	17.50	11.47	6.03
Jul-94	14.45	16.62	11.95	4.67
Aug-94	11.31	12.65	9.86	2.79
Sep-94	13.07	14.87	10.54	4.33
Oct-94	12.04	13.40	11.10	2.30
Nov-94	10.47	11.78	8.73	3.05
Dec-94	10.62	12.52	9.56	2.96
All Sample (c)	11.47	17.5	6.45	11.05

(a) The monthly average of daily implied volatility quotes. (b) Maximum and minimum implied volatility quoted over the month. The Range statistic was the difference between max. and min.. (c) All Sample results for implied volatility.

Figure 6: Daily implied volatility 01/01/94 to 31/12/94



Assuming the ten year bond futures contract was trading at a 9.00 % yield, a 17.50 % volatility quote implied that ten year bond yields were likely to trade in nearly a 6.00 % range, 6.45 % < 9.00 % < 12.25 %, over the next year. Appendix P provided some important historical perspective for these market based distributional assumptions. Over the six year period covering 1990 - 1996 there was an 8.00 % range in the ten year bond yield, between a low of 6.35 % and a high of 14.25 %. Added to this, the average annual yield range for the six year period was calculated to be only 2.31 %. The market was clearly factoring in a unique period of extremely high volatility.

By contrast, the 6.45 % volatility quote implied a much smaller yield range distribution. The range of yields projected on the basis of this quote was only 2.00 % (8.00 % to 10.00 %). This sample low in implied volatility was recorded in the January 1994 period. Volatility price makers were assuming that the market would be relatively quiet over coming months. Consistent with the yield curve analysis in Section 5.2 these low volatility forecasts were subsequently proven to be incorrect.

For the sample period there does appear to have been a correlation between the increases in yields and the level of implied volatility derived from the market. In a broad sense, the dramatic surge in yields from February 1994 to June 1994 was accompanied by a similar style rise in the level of calculated implied volatility. A small event study was used to highlight this relationship. In the period from 15/06/94 to 21/06/94 the level of implied volatility increased from 11.8 % to 15.75 %. Over the same time yields increased by 97 basis points (0.97 %). Yields increased nearly 20 basis points a day for five days in a row. Obviously the market's volatility price makers responded rationally to this extraordinary change in the environment by increasing the price of risk.

What was more important for the analysis, was that the peak in the level of sample implied volatility on 27/06/94 of 17.5 % occurred only six days later. In this week implied volatility jumped by a further 1.75 % despite the fact that yields had only increased by a fairly modest 17 basis points (0.17 %). A suggested explanation for this behaviour was that the 10.15 % yield recorded on the 27/06/94 trading date, represented a new 1994 high. It can be proposed that the volatility price makers, in fearing a repeat of the spectacular price action of a week earlier, responded to this new threat by raising quoted volatility to a new higher level.

However, the 27/06/94 yield level represented a three month high in yields. Ten year bond yields subsequently, into early September 1994, traded in a new 70 basis point range. Implied volatility in response fell from its peak of 17.5 % to a low of 9.86 % on the 24/08/94. What was significant about this behaviour was that it suggested, in contrast to Jorion's (1995) analysis of the foreign exchange markets, that implied volatility responded to changes in yield rather than providing reliable forecasts of future yield volatility. These comments have important ramifications for the interest rate hedging simulation conducted in Chapter 7.

In terms of modelling this style of implied volatility behaviour, current research has focused on the so called Generalised Autoregressive Conditional Heteroskedasticity or GARCH group of models. As presented by Heynen, Kemna and Vorst (1994), the basic hypothesis associated with this approach was that the current level of observed implied volatility was a function of the volatility one period earlier plus the shock that occurred during that t - 1 period. Equation (5.1) summarised these assumptions.

$$\sigma^2_t = \alpha + \beta_1 \, \xi_{t-1}\sigma^2_{t-1} + \beta_2 \, \sigma^2_{t-1} \qquad (5.1)$$

Where ;

t = Today's trading date.

$t - 1$ = The trading date associated with the previous period.

ξ = The shock from the previous period.

σ^2 = The level of traded volatility.

The conclusion that can be drawn from this mathematical relationship was that there were going to be "clusterings" of high volatility periods and "clusterings" of low volatility periods. There existed, as in the June 1994 event study, periods of persistence in quoted volatility. The volatility price makers, as in De - Bondt and Bange's (1992) analysis introduced in Section 5.2, relied on previous experience to make decisions. They would generally await clarification of a situation before adjusting their pricing expectations.[131]

Of more immediate interest was the level of historical volatility recorded over the sample period. The thirty business day historical volatility results were presented in Table 5.6. and Figure 7.. The sample size of the historical volatility series was reduced to n = 218 because the first thirty sample futures observations were used to create the first entry of this series.

The historical volatility series was found to possess similar properties to the implied volatility series. In line with the sharp increases in yields the calculated average of these changes also increased. In fact, in common with the implied series, the peak in historical volatility of 16.26 % occurred on the 29/06/94, only two days after the peak

[131] In the equities area there has been some success in applying the so called exponential GARCH model to explain the behaviour of implied volatility. See Hentschel (1995). Volatility was seen to be best modelled in the equities market by an asymmetric relationship, where volatility increased more when the market sold off. This was associated with the concept of financial leverage.

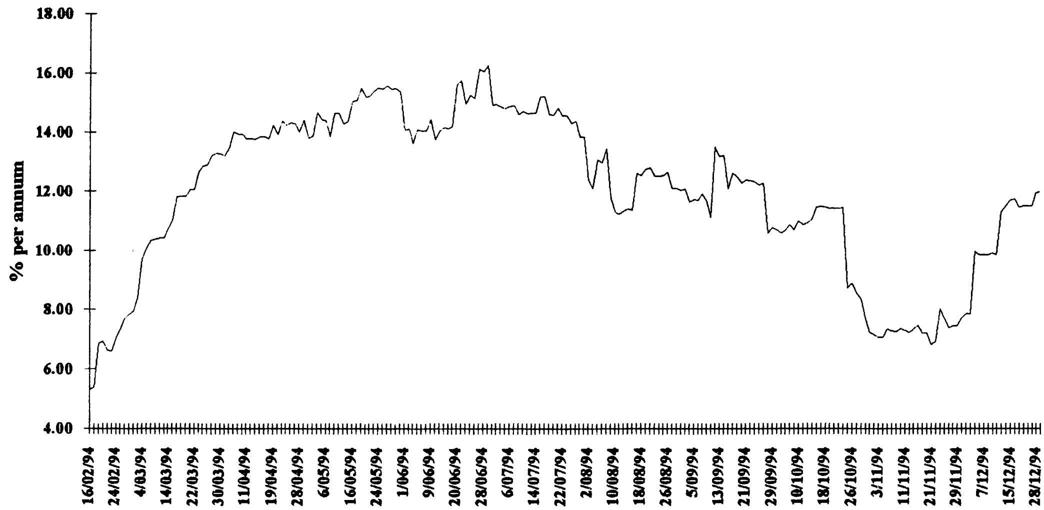
Table 5.6:

30 day historical SFE 10 year bond futures volatility (a)

Month	Av. Hist. Vol.	Max. Vol.	Min. Vol.	Range
Feb-94	6.63	7.68	5.31	2.38
Mar-94	11.23	13.29	7.84	5.46
Apr-94	14.00	14.41	13.46	0.95
May-94	14.87	15.56	13.80	1.76
Jun-94	14.71	16.26	13.64	2.63
Jul-94	14.64	15.22	13.84	1.37
Aug-94	12.26	13.45	11.24	2.20
Sep-94	12.05	13.51	10.61	2.90
Oct-94	10.45	11.49	7.67	3.82
Nov-94	7.32	8.03	6.81	1.22
Dec-94	10.62	11.99	7.85	4.14
All Sample	*11.97*	*16.26*	*5.31*	*10.96*

(a) The numbers in the Table were defined in Table 5.5. $n = 218$ in this case.

Figure 7: Daily historical volatility 16/02/94 to 31/12/94



in implied volatility. Added to this observation the average spread between the two series for the period 16/02/94 to 31/12/94 was only 0.13 % (Implied - Historic). There does appear to have been a reasonable relationship between calculated and quoted volatility.

This conclusion was further strengthened by the results associated with Hypothesis seven. The null hypothesis tested that on average the difference between the historical and implied volatility series was zero. With an observed t - statistic of 0.56265380 it could be stated that on average the difference between the two series was negligible at both the 5.00 % and 1.00 % significance levels.[132] It can be suggested that the historical volatility series served as a base from which price makers set their level of traded volatility over the sample period.

To compare the Yield Error Margin databases with the market's measure of volatility the following methodology was adopted. The Yield Error Margin databases, like the historical volatility series, were calculated directly from the changes in the closing prices observed in the market. The 2 week Yield Error Margin database was assumed to have, despite its known negative skewness, the closest overall distributional characteristics to the underlying SFE ten year bond futures series. To test the relationship between model and market volatility the recorded 2 week Yield Error Margin database and the thirty day historical volatility series were compared for the period 16/02/94 to 31/12/94.

[132] Test statistic taken from Equation (3.10) . $\bar{x}_1$ = The average implied volatility 16/02/94 to 31/12/94 = 12.097. $\bar{x}_2$ = The average historic volatility = 11.97. $\sigma_1{}^2 = 1.96487$. $\sigma_2{}^2 = 2.68954571$. $n = 218$. This result does obscure the fact that the level of historic and implied volatility were quite different at certain times over the sample. There may exist profitable trading strategies that exploit large spreads in the two series.

To allow this comparison the 2 week Yield Error Margin database was transformed to make it more compatible with the historical volatility series. The 2 week Yield Error Margin statistic calculated the errors in the market's short term interest rate forecasts. This, it was assumed, was equivalent to the potential for yields to change over a two week period. The individual percentage errors recorded in the 2 week Yield Error Margin database were multiplied by a factor of $\sqrt{26}$. Consistent with the market approach of quoting volatility on an annualised basis, 26 was used because it equated to the number of fortnights in a year. This was seen to generate a 2 week Yield Error Margin volatility series that could be directly compared with the thirty day historical volatility series presented in Figure 7. The results of this transformation were reported in Appendix S.

Hypothesis eight used the simple regression in Equation (3.21) to test the hypothesis that the converted 2 week Yield Error Margin volatility series had produced unbiased estimates of the calculated thirty day historical volatility series. The null hypothesis was accepted if $\alpha = 0$ and $\beta = 1$. Under these conditions it could be stated that the 2 week Yield Error Margin volatility measure was a perfect predictor of calculated historic market volatility. The results of this analysis were presented in Table 5.7..

Table 5.7 demonstrated that the 2 week Yield Error Margin volatility series only had a low level of explanatory power, especially when viewed against the SFE ten year bond futures pricing results discussed in Chapter 4. For example the regression R^2 was calculated as 0.16689827 versus Table 4.3 where the R^2 was shown to be 0.9997. From this first level of testing it was obvious that the transformed Yield Error Margin volatility series possessed only a modest capacity to explain the variations in the thirty day historical volatility series.

Table 5.7:

OLS estimation historic volatility 16/02/94 to 31/01/94 n=218

30 day Historic Volatility	α	$\beta \times YEM_{vt}$
$H30_{vt}$ =	8.633520362	0.399724503
Std. errors of est.	0.527245761	0.059939832
Observed t statistic	16.37**	6.67 **

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.166898276

** Denotes significance at the 5.00 % and 1.00 % levels

This finding was supported by the size of the intercept term which was estimated at α = 8.633520362. Its statistical significance suggested that two statements could be made. Firstly, that the null hypothesis was rejected, the model volatility measure was not a perfect predictor of market volatility $\alpha \neq 0$ and $\beta \neq 1$(t-statistic -10.015). Secondly, that the model significantly underestimated thirty day historic volatility. The results in Appendix S added weight to this conclusion. The average annual volatility of the converted 2 week Yield Error Margin database was 8.35 % versus 11.97 % for the historical series over the period 16/02/94 to 31/12/94.

As Jorion (1995) stated, faulty test procedures can be due to three factors: measurement errors, inappropriate statistical inferences or the use of the wrong pricing model. In the present case better volatility forecasting results may have been available if the 2 week Yield Error Margin database had been transformed by another conversion methodology. This question awaits further research to be answered.[133]

5.5 Consequences of the Yield Error Margin database results

There were five main results associated with Chapter 5. Firstly, that over the sample period the term structure based implied N - period forward interest rate forecasts were found to have significant errors. Only the 2 week implied forward forecasts were found to have an acceptable level of error. Market expectations were generally unable to keep pace with the substantial increase in yields that occurred over the sample period.

These type of findings were consistent with other research that had tested the forecasting power of term structure based implied N - period forwards. The most interesting of these was De - Bondt and Bange's (1992) study of inflation expectations. The authors suggested that errors occur because agents were reticent to change there view quickly and instead preferred to wait until the real impact of shocks had become more evident. It was reasonable to assume that there existed lags and persistence in the market's expectational behaviour.

The second result was that changes in the SFE ten year bond futures closing prices were not found to be normally distributed over the sample period. Instead the futures distribution was seen to be leptokurtic in nature. The futures return distribution, as was a common finding in the literature, was found to be peaky and to have relatively "fat" tails. This suggests that the market trading behaviour could be broken into two parts. The first, where futures prices trade in tight ranges as the market searches for new signals and a second where the market, on the back of

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[133] Jorion, (1995) 526.

powerful new information, attempts to move to a completely new price level. This observation provides the incentive for new research which examines the profitability of a trading strategy that buys and sells the "breaks" of identified trading ranges.

The third finding was that increases in the level of implied ten year bond futures volatility was generally correlated with increases in the yields of the underlying bonds. In common with the interest rate forecasting results there was evidence of persistence in the behaviour of the market's volatility price makers. A case study was presented where market makers responded rationally to a strong increase in yields by raising the price of interest rate risk i.e. the level of volatility. What was less consistent was their response, a week later, to a new high in yields. Despite less violent price action being associated with this move in yields, market makers actually quoted volatility at a new sample high.

It appears price makers looked to the previous week's shock to guide their current decision making. However, this new yield level turned out to be a three month high. Implied volatility subsequently declined over the next two to three months. This analysis tends to suggest that implied volatility may not be an accurate indicator of forward market volatility. More focused research is needed to confirm whether this hypothesis holds more generally for the Australian financial futures markets.

Consistent with this proposition there was evidence that market makers used historical volatility as a base for their price making activities. There was a reasonable degree of correlation between the two series. It was found that on average the difference between the two series was not statistically significant. This suggests that the GARCH volatility models may improve their fit of financial futures behaviour if a moving average volatility term was incorporated into their design.

The fourth result was related to the distributional characteristics of the Yield Error Margin databases. These findings were critical because these distributions were assumed to have estimated the stochastic behaviour of the term structure, for a variety of horizons, over the sample period 01/01/94 to 31/12/94. From an interest rate risk pricing perspective the calculated Yield Error Margin distributions were seen to be biased for two reasons. Firstly, all four of the Yield Error Margin data bases were found to have negative means. This reflected the well documented surge in the level of yields over the sample period. The Yield Error Margin risk pricing methodology, as presented, was sensitive to the underlying dataset.

The second form of bias was related to the actual shape of the Yield Error Margin distributions. The Yield Error Margin distributions on the basis of the Kolmogorov-Smirnov, Kurtosis and Skewness tests of normality, were generally found to be platykurtic in nature. This type of distribution places a lot of emphasis on "outside" observations in comparison to the benchmark normal case. These two forms of bias suggest that the calculated Yield Error Margin distributions would most likely misprice interest rate risk in their present form.

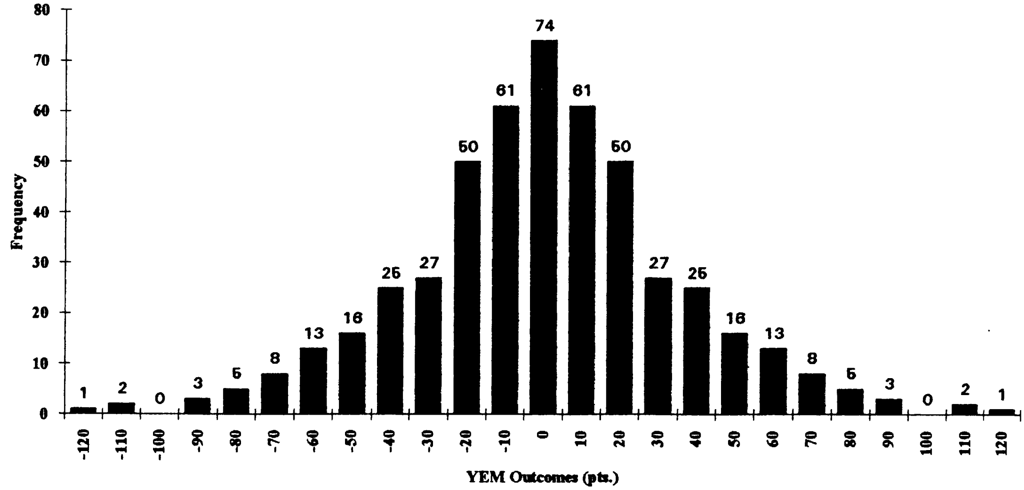
The final major result associated with Chapter 5 was that the model volatility measure was only able to explain with a low level of power the historical volatility series calculated from the changes in the ten year bond futures closing prices. The model measure of volatility was shown to systematically underestimate the level of market volatility. Further research was required to determine if an alternative conversion methodology, that annualised model volatility differently, would have produced more accurate volatility forecasts.

It would appear from the results discussed in Chapter 5 that the calculated Yield Error Margin distributions would have to be modified if they were to be used to replicate the market's interest rate risk pricing assumptions. An additional assumption was introduced to the analysis to overcome this problem. This proposal was driven by the uniqueness of the sample period incorporated in this paper. In Appendix P it was demonstrated that the 1994 period was correlated with a time of extreme change in the level of the ten year bond yield. It has been shown that the market repeatedly failed to adequately adjust its interest rate expectations in line with these changes in yield.

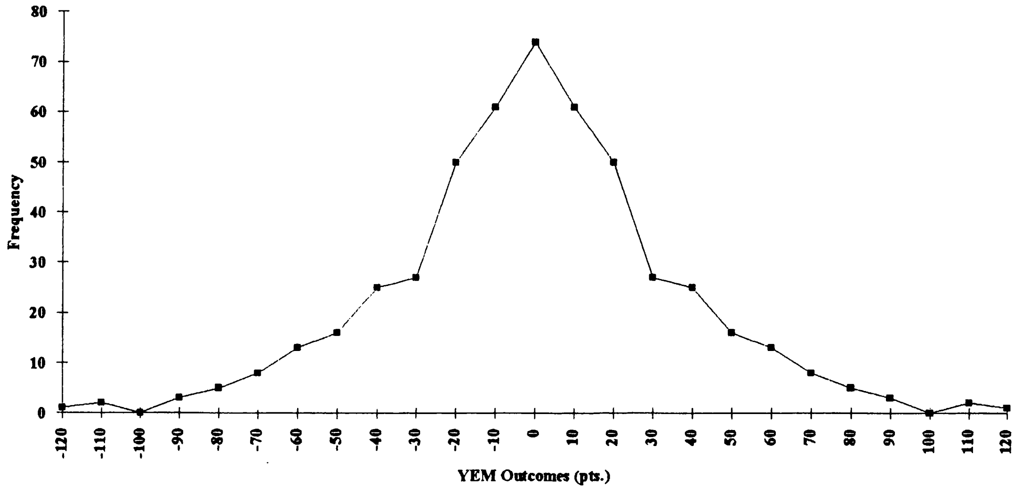
In response it was proposed that there existed the positive probability of another random sample period, of similar length, where expectations in the term structure over estimated the changes in yields with an equal magnitude to the underestimation recorded in this study. This was assumed to allow the generation of four so called "Mirror" Yield Error Margin databases. Overlaying these databases on the "Original" Yield Error Margin databases was seen to construct a set of so called "Composite" Yield Error Margin databases. This transformation was seen to centre the Yield Error Margin distributions whilst incorporating the probability characteristics of the original underlying databases. Figure 8. and Appendix T presented the results of this transformation for the 2 week Yield Error Margin database. This composite distribution could be compared directly with Figure 5. in Section 5.3..

Figure 8: Composite 2 week Yield Error Margin: Relative frequency distribution

Number on top of the bars denoted the relative frequency of the forecasting error



Composite 2 week YEM: Approximate density function



CHAPTER 6

Pricing Fixed Interest Contingent Claims

6.1 Introduction

Chapter 6 applied the interest rate risk pricing assumptions associated with Chapters 4 and 5 to the valuation of Sydney Futures Exchange (SFE here after) ten year bond futures options. The model ten year bond futures prices generated in Chapter 4 were modified by the "Composite" Yield Error Margin databases constructed in Chapter 5. Combining these two aspects of the methodology was assumed to generate an array of ten year bond futures prices. Probability weightings, or relative frequencies, were attached to each of the different outcomes within the constructed bond price distribution. These weightings allowed the model to determine the expected payoff of options on the SFE ten year bond futures contract. This modelling approach was used to replicate the ten year bond futures option settlement prices recorded by the SFE over the trading period 01/01/94 to 31/12/94.

The option pricing analysis was broken into three parts. The first part of the research investigated the ability of the model to accurately replicate SFE ten year bond futures option settlement prices. The accuracy of the model was measured by the yield point spread between model generated prices and those observed in the market. The model option prices were produced daily for six different series; namely At-the-money (ATM here after), 25 basis point In-the-money (ITM here after) and 25 basis point Out-the-money (OTM here after) for both calls and puts. Secondly, regressions were run on each of the six series to test the proposition that the model had produced prices that were unbiased estimates of those observed in the market. The final part of the testing examined whether the "best" of the regression series could be used to forecast the ten year bond futures option prices for the out of sample period 01/01/95 to

31/12/95. The results of this study suggested that the model was able to generate bond futures option prices that were consistent with those observed in the market. However, there was some bias in the model pricing performance. Puts and particularly those ITM were more accurately valued than the other option series.

6.2 Option Pricing Performance of the Term Structure Model

In Chapter 6 the interest rate risk pricing assumptions underlying the three core hypotheses of this paper were used to replicate the ten year bond futures option settlement prices recorded by the SFE. The first hypothesis proposed that a market consistent term structure could be derived from the observed Commonwealth government coupon bond yield curve. In Chapter 4 this proposition was tested by the ability of the model term structure to accurately price the SFE ten year bond futures contract. The constructed term structures were found to produce futures prices that were robust estimates of market prices. The model term structures had adequately captured the pricing assumptions of the Australian interest rate market over the 01/01/94 to 31/12/94 sample period.

In Chapter 5 the second and third key hypotheses associated with this paper were empirically tested. The second core hypothesis stated that the market's current best forecasts of interest rates was represented by today's implied N-period forward term structure. To keep this analysis consistent with Chapter 4 these forecasting errors were calculated by the difference between today's implied N-period forward average ten year bond futures contract basket yield and the actual average basket yield that was observed N-periods forward. These forecasts, in reflecting decisions made under uncertainty, were generally found to contain statistically significant errors.

The third core hypothesis of this paper proposed that the distribution of these forecasting errors could be used to measure the volatility of the term structure. In Chapter 5 the Yield Error Margin distributions were found to be biased for the sample period 01/01/94 to 31/12/94. This reflected the market's underestimation of the strong rise in yields that occurred over the sample period. So called "Composite" Yield Error Margin distributions were constructed to correct for this error bias. These distributions were central to the model's option pricing performance because they were assumed to empirically approximate the stochastic behaviour of the implied forward term structure.

To replicate the ten year bond futures option settlement prices recorded by the SFE over the 1994 sample period, the implied certainty forward term structures of Chapter 4 were modified by the "Composite" Yield Error Margin distributions associated with Chapter 5. In Chapter 6 what was ultimately being tested was the distributional characteristics of the four "Composite" Yield Error Margin databases. To accurately price ten year bond futures options the model must be able to generate distributions of the underlying ten year bond futures price that were compatible with the market's view of interest rate volatility.

The model's risk pricing assumptions were summarised by Equations (3.22) and (3.23). The ATM put option pricing result for trading day 04/01/94 was displayed in Appendix U. In the first table of Appendix L the closing ten year bond futures price for 04/01/94 was recorded as 93.13 (6.87 %). An ATM put for the 04/01/94 trading date was defined by Equation (3.19) as the 93.25 strike. Also displayed in Appendix L was the model's ten year bond futures equivalent yield. This yield of 6.869 % represented the implied certainty forward for the 04/01/94 trading date. The price reflected the information contained in today's model term structure.

This implied certainty forward yield was modified by the Yield Error Margin distribution that most closely matched the market's current volatility view. In Chapter 5 the January 1994 period was noted as a low volatility period. For the 04/01/94 trading date the 1 month Yield Error Margin distribution was assumed to give the best volatility / distribution fit for the six option series being evaluated. Applying the 1 month "Composite" Yield Error Margin database to the 6.869 % futures equivalent yield generated the bond yield distribution shown in Appendix U. The bond yields in the distribution were subsequently converted to the SFE By - Laws bond prices introduced in Equation (3.19).

The distribution of bond prices was compared with the ten year bond price associated with the ATM option strike. For the 04/01/94 trading date the 93.25 strike implied a yield of 6.75 % or an equivalent SFE By - Laws bond price of $137,733.07. The put option for settlement date 15/03/94 gave the holder the right to sell a ten year bond futures contract over the intervening period, between 04/01/94 and 15/03/94, at a yield of 6.75 % or for the $100,000 face value of the futures contract at a price of $137,733.07. The futures contract assumed a 12.00 % coupon for the representative bond so that in this example, where the current yield was less than the coupon, the contract value was greater than par. This was consistent with the comments made in footnote ninety seven.

The strike bond price, X, determined the ultimate payoff of the put option. The payoff was set by the relationship [X - B, 0]. If at expiry yields had moved above 6.75 % (bond prices lower) then the put would register a positive payoff equal to X - B. In contrast, if yields (bond prices higher) had moved below 6.75 % then the payoff of the option would equal 0.[134] For the case where yields did not change from today's forecasted level, the put option payoff at expiry would be 6.75 % / \$137,733.0 - 6.869 % / \$136,678.97 which equalled \$1054.10. On the 04/01/94 trading day the ATM put was a slightly ITM strike.

The next step in the pricing process involved weighting the expiration payoffs by their individual Yield Error Margin assigned probabilities. For example in the Yield Error Margin database the zero yield change outcome was assigned a 6.5 % chance. Its part in today's ATM put option price was equal to \$1054.10 times by its probability 0.06451613 = \$68.01. Consistent with the ATM put option being slightly ITM (\$137,733.07 - \$136,678.97) on 04/01/94 it was seen to have ≈ 60.70 % probability of remaining ITM come option expiry. This probability weighted calculation was performed on each of the bond prices in the distribution.

Following Equation (3.23) the probability weighted payoffs were summed together to value the ATM put option price for 04/01/94 at \$2577.20. As detailed in Appendix H the option price makers quoted the SFE bond futures option contracts in terms of yield basis points. Attachment 5 contained a Reuters market pricing page for 21/08/95 that demonstrated this practice. The dollar value of the option price was converted to a yield basis point quote by dividing the dollar price by the dollar value of ± 1 basis point change in yield at the strike price. At 6.75 % a 1 basis point change in yield was worth \$89.07. The model ATM put option price for 04/01/94 was quoted as 28.9 basis points. In comparison, the ATM put option price recorded by the SFE to settle open option positions on 04/01/94 was 25.5 basis points. Equation (3.24)

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[134] This of course ignored the opportunity for early exercise. The SFE bond contracts were American options.

calculated that the pricing error for the ATM put, Model quote - Market quote, was equal to 3.4 basis points.

This methodology was used to evaluate SFE ten year bond futures option settlement prices for every full trading day in the sample 01/01/94 to 31/12/94. In total six option prices were generated for each day in the sample. For both ten year bond futures option calls and puts; ATM, 25 basis points ITM and 25 basis point OTM prices were calculated. The size of each of the six samples was $n = 217$. These daily results were shown in Appendix V. Table 6.1 summarised these daily errors into monthly and All Sample results.

The average spread column was the key performance indicator in Table 6.1. It recorded the average model pricing error for the sample period. In keeping with Chapter 4 the average error was measured in yield basis point terms. The All Sample average pricing error statistic suggested that the model prices were a reasonable approximation of those observed in the market. For the six option series investigated, the average error ranged between +0.61 (Model > Market) of one basis point for the ATM / ITM calls to -0.21 (Model < Market) of one basis point for ITM puts. There was a tendency for the term structure based model to underprice put options and overprice calls.

This small average pricing error was consistent with the futures pricing results in Chapter 4 and was also comparable with other empirical option pricing research. Amin and Morton (1994) and to a lesser degree Flesaker (1993) had similarly accurate average pricing errors. For example Amin and Morton (1994) found that their term

Table 6.1:

Term Structure Model vs OTM Call option prices

Month	*Av. spr.* [a]	*Max.*	*Min.*	*Std. dev.* [b]	*Correl. Cx.* [c]
Jan-94	-0.29	5.1	-4.2	2.710	0.636
Feb-94	0.88	2.1	-0.9	0.899	0.863
Mar-94	5.32	8.6	1.5	2.010	0.615
Apr-94	4.08	7.1	-5.1	3.677	0.767
May-94	1.25	8.7	-3.6	3.943	0.952
Jun-94	0.28	3.4	-2.7	1.776	0.969
Jul-94	0.69	5.9	-6.7	4.151	0.818
Aug-94	-0.32	4.7	-2.3	1.866	0.891
Sep-94	-2.75	5.1	-6.5	3.971	0.629
Oct-94	-0.94	3.73	-5.3	3.734	0.914
Nov-94	0.00	4.8	-2.8	2.333	0.887
Dec-94	-1.82	6.4	-6.3	5.167	0.830
ALL SAMPLE	*0.59*	*8.7*	*-6.7*	*3.672*	*0.946*

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.

Term Structure Model vs ATM Call option prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	-1.17	5.7	-4.9	3.562	0.757
Feb-94	0.45	2.5	-1.7	1.267	0.950
Mar-94	5.93	9.4	2	1.990	0.741
Apr-94	4.86	8.8	-6.1	4.166	0.774
May-94	1.03	8.5	-4.6	4.621	0.957
Jun-94	0.64	4.1	-2.4	1.802	0.972
Jul-94	0.95	6.5	-6.4	4.371	0.809
Aug-94	-0.55	4.2	-3.8	2.289	0.911
Sep-94	-2.49	5.4	-6.1	4.081	0.651
Oct-94	-0.96	5.4	-5.3	4.418	0.904
Nov-94	0.17	5.7	-3.9	2.943	0.893
Dec-94	-1.97	6.7	-7.1	5.837	0.813
ALL SAMPLE	*0.61*	*9.4*	*-6.4*	*4.177*	*0.947*

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.
(a)The spread in Table 6.1 was calculated by Model - Market price = Average pricing error for the month. These pricing errors should be read in yield point terms i.e. 0.3 = 3/10th of 1 basis point.
(b) Standard deviation of the pricing error for the trading month. (c) The correlation coefficient. It measured the correlation between model and market prices. Where $\rho_{X,Y} = \mathrm{Cov}(X, Y) / \sigma_X \times \sigma_Y$.

Table 6.1 Continued:

Term Structure Model vs ITM Call option prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	-1.21	5.9	-4.3	3.421	0.874
Feb-94	0.075	2.6	-1.7	1.305	0.975
Mar-94	5.747	8.8	2.5	1.736	0.857
Apr-94	4.128	7.4	-5.7	3.781	0.818
May-94	1.495	9	-3.9	4.40	0.947
Jun-94	0.492	4.5	-3.6	2.417	0.962
Jul-94	0.533	6.8	-7.1	4.399	0.777
Aug-94	-0.545	4.1	-3.5	1.880	0.957
Sep-94	-2.579	5.4	-6.2	4.281	0.672
Oct-94	-0.89	5.6	-6.2	4.281	0.907
Nov-94	0.93	5.7	-2.2	2.552	0.92
Dec-94	-1.16	7.1	-6.3	5.111	0.839
ALL SAMPLE	*0.61*	*9*	*-6.2*	*3.959*	*0.949*

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.

Term Structure Model vs OTM Put option prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	-1.13	3.8	-4.6	2.990	0.654
Feb-94	0.40	2.45	-1.8	1.072	0.798
Mar-94	2.16	5.4	-1.1	1.838	0.829
Apr-94	3.2	6.8	-0.3	1.981	0.798
May-94	-0.4	5.9	-4.1	3.114	0.959
Jun-94	-4.74	-1.5	-8.3	2.014	0.942
Jul-94	-0.01	6.8	-6.3	3.388	0.914
Aug-94	-1.57	3.1	-3.9	2.035	0.924
Sep-94	0.3	4.5	-2.8	1.988	0.919
Oct-94	-1.545	5.9	-6.8	4.474	0.778
Nov-94	-0.65	5.3	-4.2	2.763	0.755
Dec-94	3.32	5.6	0.2	2.14	0.574
ALL SAMPLE	*-0.19*	*6.8*	*-8.3*	*3.276*	*0.955*

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.

Table 6.1 Continued:

Term Structure Model vs ATM Put option prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	**-0.82**	**3.8**	**-4.7**	**3.103**	**0.819**
Feb-94	**0.87**	**2.7**	**-1.6**	**1.08**	**0.957**
Mar-94	**2.32**	**5.7**	**-1.3**	**2.10**	**0.866**
Apr-94	**3.77**	**7.1**	**-0.4**	**1.925**	**0.867**
May-94	**-0.582**	**6**	**-6**	**3.50**	**0.960**
Jun-94	**-4.64**	**-1.6**	**-8.2**	**1.842**	**0.956**
Jul-94	**0.310**	**2.5**	**-5.1**	**3.713**	**0.909**
Aug-94	**-1.759**	**3.7**	**-4.2**	**2.241**	**0.950**
Sep-94	**0.393**	**4.1**	**-2.3**	**1.878**	**0.948**
Oct-94	**-1.31**	**6.7**	**-6.8**	**4.844**	**0.780**
Nov-94	**-1.70**	**4.3**	**-5.1**	**2.812**	**0.861**
Dec-94	**3.54**	**5.8**	**0.2**	**2.192**	**0.654**
ALL SAMPLE	***-0.13***	***7.1***	***-8.2***	***3.515***	***0.957***

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.

Term Structure Model vs ITM Put option prices

Month	*Av. spr.*	*Max.*	*Min.*	*Std. dev.*	*Correl. Cx.*
Jan-94	**-0.105**	**3.9**	**-3.5**	**2.22**	**0.936**
Feb-94	**1.22**	**2.5**	**-1.2**	**0.915**	**0.987**
Mar-94	**1.44**	**5.3**	**-1.8**	**2.066**	**0.900**
Apr-94	**2.769**	**6.3**	**-1.4**	**1.943**	**0.899**
May-94	**-0.166**	**6.7**	**-4.7**	**2.88**	**0.961**
Jun-94	**-5.04**	**-2.0**	**-8.4**	**1.908**	**0.954**
Jul-94	**-0.095**	**6.2**	**-5.7**	**3.468**	**0.920**
Aug-94	**-1.27**	**3.4**	**-4.2**	**2.134**	**0.969**
Sep-94	**-0.093**	**3.1**	**-3.2**	**1.803**	**0.963**
Oct-94	**-1.565**	**5.7**	**-4.5**	**4.162**	**0.825**
Nov-94	**-1.35**	**4.5**	**-4.9**	**2.778**	**0.869**
Dec-94	**2.71**	**5.6**	**0.2**	**1.779**	**0.829**
ALL SAMPLE	***-0.21***	***6.7***	***-8.4***	***3.112***	***0.995***

*The average pricing error was not found to be statistically significant at either 5.00 % or 1.00%.

structure based option pricing model had produced an average absolute error of 1.5 to 2.0 basis points when applied to the Eurodollar futures option market.[135]

However, this conclusion of robust model prices was tempered to some extent when the other summary statistics in Table 6.1. were analysed. In particular the standard deviation of the errors implied that the model prices were not as accurate as the average error statistic had suggested. For the ATM call option series the standard deviation of the sample pricing error was measured in Table 6.1. at 4.177 basis points. This implied that generally the model's pricing errors for ATM calls were in a range 0.61 points $\pm$ 4.177 basis points. The model ATM call prices were likely to be -3.57 basis points to + 4.79 basis points away from the market recorded price.

The model ATM call prices were clearly distributed in a greater range than the model futures prices in Chapter 4. What introduced some perspective to this result was that the SFE bond futures options market was normally quoted with a three basis point bid - ask spread. This characteristic was highlighted in Attachment 5. Amin and Morton (1994) proposed that the significance of pricing errors was related to the bid - ask spread of the underlying market. They suggested that their average pricing error of two basis points was reasonable given that the Eurodollar futures option market had a bid - ask spread of around 1 basis point. Under this rule of thumb the model option prices can be viewed as satisfactory approximations of market option prices.

The model ITM put option price series supported this view. In this option series the standard deviation of the model error was only 3.1 basis points. The general range

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135 Amin and Morton, (1994) 141. Flesaker, (1993) 492. His model produced slightly larger average errors of around 3.5 basis points.

for the model ITM put option pricing errors was -0.21 $\pm$ 3.1 basis points over the sample period. The model ITM put prices were usually only a market spread away from the option prices observed in the market. This result did raise the issue of whether the model evaluated put options more reliably than calls over the sample period. To help answer this question the distribution of the model option pricing errors was investigated.

The distributional analysis presented in Appendix M for the model futures prices was replicated for the ATM call option and the ITM put option pricing series. The results of this research were presented in Appendix W. This work supported the view that the model more accurately priced puts than calls. It was assumed that $\pm$ 3.5 basis points was an acceptable level of error given the standard market bid - ask spread. Under this criteria of performance, it was obvious that the model had produced ITM put prices with more accuracy than the ATM calls. 77.0 % or 167 / 217 of the ITM put errors were in this range while only 51 % or 111 / 217 of the ATM call pricing errors fell into this category.

Hypothesis nine tested whether this observed pricing bias had wider significance. The null hypothesis examined the proposition that the average difference between the model and market option pricing series was equal to zero basis points. With n = 217 the critical test statistic for the 5.00 % / 1.00 % significance level was set at $\pm$ 1.960 / $\pm$ 2.576. The observed values of this test were calculated by Equation (3.10). For each of the six option series the observed values were well inside the null hypothesis acceptance range. For the two series already discussed, the ATM call observed value was 0.5206 and for the ITM put the value was -0.197.[136] It could be stated that on

[136]For the ATM put the observed statistic = - 0.114. OTM put the observed statistic = 0.571178. OTM call the observed statistic = 0.571178. ITM call the observed statistic = 0.53316.

average the model option prices were not significantly different from those observed in the market.

These findings bring into the discussion the often problematic area of statistical significance versus economic significance. As Rubinstein (1985) stated, the significance of a given option pricing error can depend on the particular perspective of the agent.[137] To a price maker or trader a five basis point discrepancy between the model and the market prices would appear to be a trading opportunity.[138] To others such as the hedger of a mortgage deal the differential may be of little overall concern. The regression analysis in Section 6.3 attempted to clarify this issue.

6.3 Option Pricing Regression and Forecasting Analysis

Hypothesis ten evaluated the proposition that the term structure based model had produced unbiased estimates of the settlement option prices recorded by the SFE. Equation (3.25) was estimated for each of the six option series over the 1994 trading sample.

$$C, P_{(market)t} = \alpha + \beta\, C, P_{(model)t} + \varepsilon_t$$

The unbiased hypothesis as in Chapters 4 and 5 was tested via the null hypothesis that $\alpha = 0$ and $\beta = 1$. The results of the six regressions were presented in Table 6.2.. The R^2 in Table 6.2 represented the ability of the model prices to explain variations in market option prices. This basic descriptive statistic suggested that the model provided a good approximation of prices observed in the market. The R^2

137Rubinstein, (1985) 479. Rubinstein made reference to the economic versus the statistical significance of a 2.00 % model pricing error.

138See Appendix W. 23.9 % of the ATM call errors were > $\pm$ 5 basis points.

Table 6.2:

OLS estimation OTM Call option prices 01/01/94 to 31/01/94 n=217

Market OTM Call prices	α	$\beta \times$ *Model*
C_t =	1.754923	0.871129
Std. errors of est.	0.437687	0.020453
Observed t statistic	4.009536**	42.59096**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.894036

Serial Correlation	Rho	DW
Tests	0.68424	0.6295*

OLS estimation ATM Call option prices 01/01/94 to 31/01/94 n=217

Market ATM Call prices	α	$\beta \times$ *Model*
C_t =	3.719633	0.850558
Std. errors of est.	0.622276	0.019626
Observed t statistic	5.977467**	43.33738**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.897283

Serial Correlation	Rho	DW
Tests	0.70340	0.5863*

** Denoted significance at the 5.00 % and 1.00 % level.
* Denoted significance at the 5.00 % level.

Table 6.2 Continued:

OLS estimation ITM Call option prices 01/01/94 to 31/01/94 n=217

Market ITM Call prices	α	$\beta \times$ *Model*
C_t =	5.16945	0.869134
Std. errors of est.	0.903315	0.019674
Observed t statistic	5.722789**	44.17727**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.900768

Serial Correlation	Rho	DW
Tests	0.68603	0.6211*

OLS estimation OTM Put option prices 01/01/94 to 31/01/94 n=217

Market OTM put prices	α	$\beta \times$ *Model*
P_t =	1.570465	0.926527
Std. errors of est.	0.427306	0.019642
Observed t statistic	3.67527**	47.17162**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.911891

Serial Correlation	Rho	DW
Tests	0.73316	0.5334*

** Denoted significance at the 5.00 % and 1.00 % level.
* Denoted significance at the 5.00 % level.

Table 6.2 Continued:

OLS estimation ATM Put option prices 01/01/94 to 31/01/94 *n*=217

Market ATM Put prices	α	$\beta \times$ *Model*
P_t =	2.465275	0.919241
Std. errors of est.	0.596383	0.019032
Observed t statistic	4.133713**	48.2999**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
$\alpha = 0$, $\beta = 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$	No	No

R^2	0.915616

Serial Correlation Tests	Rho	DW
	0.71584	0.5665*

OLS estimation ITM Put option prices 01/01/94 to 31/01/94 *n*=217

Market ITM put prices	α	$\beta \times$ *Model*
P_t =	1.55041012	0.96863086
Std. errors of est.	0.84054914	0.01904658
Observed t statistic	1.84452051	50.855881**

Significance Tests	5.00%	1.00%
Test statistic	1.96	2.576
Only $\beta \neq 0$	No	No

Unbiased Tests	5.00%	1.00%
$\beta = 1$ **	Yes	Yes

R^2	0.92325049

Serial Correlation Tests	Rho	DW
	0.68928	0.6203*

** Denoted significance at the 5.00 % and 1.00 % level.
* Denoted significance at the 5.00 % level.

ranged from 0.894 for OTM calls to 0.923 for the ITM puts. The tight spread of the R^2 indicates that there was no notable pricing biases in the model.

In support of the relationship between market and model prices the β coefficients were all highly significant in Table 6.2. The observed t-statistics for the test $\beta = 0$ ranged from 42.59 for OTM calls to 50.86 for ITM puts. The null hypothesis was rejected at both the 1.00 % and 5.00 % significance levels. Under this test criteria the model prices were seen to be significant explanatory variables of market prices. The actual β coefficients ranged between 0.8506 for ATM calls to 0.9686 for the ITM put model prices.

In contrast, the estimates of the intercept term α provided evidence of a model put option pricing bias as discussed in Section 6.2. The size of the α term ranged from a low of 1.5504 for ITM puts to a high of 5.169 for ITM calls. This indicated that the relationship between model and market prices was variable between the put option and call option series. For ITM puts only 1.55 basis points was added to the model price to ensure it matched the market. The ITM call option pricing series in contrast needed 5.169 basis points on average added to its market forecasts. As was suggested in Section 6.2, an error of this size approaches economic if not statistical significance.

This analysis suggested that the model methodology had more accurately priced put options and especially ITM put options over the sample period. This hypothesis gained further support when the ITM put option regression results were studied in greater detail. The α estimate in contrast to the other five option series was found to be insignificantly different from zero. The observed t-statistic was calculated to equal 1.8445. The null hypothesis $\alpha = 0$ was accepted at both the 5.00 % and 1.00 %

significance level. The β coefficient was also uniquely seen to be insignificantly different from one. The null hypothesis $\beta = 1$ could not be rejected with the observed test statistic equal to -1.646970. In the other model series the observed test statistics for this test were all > 3.5.[139] The null hypothesis was rejected in these cases.

These results suggested that the term structure based model had produced unbiased estimates of the market's ITM put option price series. It could be stated that the model had perfectly predicted the ITM put option settlement prices recorded by the SFE. To determine the strength of this claim an F-test was run. It tested the joint null hypothesis that $\alpha = 0$ and $\beta = 1$. The critical value of this test was set at the 5.00 % level as $F_{0.05,2,214} = 3.00$.[140] With the observed statistic equal to 1.8548344 the null hypothesis could not be rejected at either the 5.00 % or 1.00 % significance level. This type of finding was rare in the literature. Term structure based model option prices have generally been found to be biased estimators of the market.[141]

The low Durbin and Watson (DW here after) statistic recorded for the ITM put regression decreased the validity of this statement. The observed DW of 0.6203 was < critical DW lower bound = 1.63. This low level of the DW statistic was common to all the regression results in Table 6.2. The DW statistic ranged from 0.5334 for the OTM puts to 0.6295 for OTM calls.[142] As discussed in Chapter 4 the impact of

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[139]The observed $\beta = 1$ test statistic ranged from -3.74 for the OTM put series to -7.61 for the ITM call series.

[140]The statistic for this test was taken from Pindyck and Rubinfield, (1981) 81. The critical value was set by F (k , n - k -1).Where k was equal to the number of coefficients in the null hypothesis and n = sample size. The observed F was calculated via the Shazam version 7.0 F- test statistic. The 1.00 % critical test F- statistic = 4.61.

[141]Amin and Morton, (1994) 164 -165. Despite the low level of error that characterised their pricing results the null hypothesis of unbiasedness was rejected.

[142]Maddala (1992) DW < R^2 decision rule. The rough rule of thumb decision criteria discussed in Chapter 4 was breached in this case. 0.6203 < 0.92325. The autocorrelation appeared more significant than was the case with the ten year bond futures results in Chapter 4.

autocorrelation was that it reduced sampling variances and therefore potentially introduced errors into the regressions R^2 and t-statistics.

Following the methodology introduced in Chapter 4 a Generalised Least Squares regression was run to determine the power of the autocorrelation in the ITM put option series. The results of this analysis suggested that the level of autocorrelation present in the residuals had definitely influenced the acceptance of the unbiased null hypothesis.[143] The β coefficient decreased in the new regression from 0.96 to 0.92 and the α intercept increased from 1.5504 to 3.4090. The $\beta = 1$ test was also found to reject the null hypothesis. Therefore in this case acceptance of the unbiasedness hypothesis should be treated with a high degree of caution.

To determine whether the original ITM put option regression in Table 6.2 had any economic significance it was used to forecast ITM put option prices for the out of sample period 01/01/95 to 31/01/95. The estimated regression equation in Table 6.2. was combined with model prices to produce the forecasting results displayed in Table 6.3..

The forecasting results were consistent with the 01/01/94 to 31/12/94 All Sample results. They effectively summarised the overall option pricing performance of the model's interest rate risk pricing assumptions. The key result was that the standard deviation of the forecasting errors was equal to 3.9 basis points. In a general sense the model prices were a satisfactory approximation of those observed in the market. However, the model did have the capacity to generate pricing errors that were economically significant. The forecasting error on 20/01/95 of -5.7 basis points

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[143]Using the Auto command in Shazam version 7.0 the generalised regression result was ITM Put = 3.4090 + 0.92362 Model. Durbin h = -3.3021 suggests that higher levels than AR(1) were present. The observed test statistic for Ho: $\beta = 1$ was -3.33.

Table 6.3:

Forecasted ITM put option prices 01/01/95 to 31/01/95

Date	Forecast (a)	Market (b)	Spr.(c)
3/01/95	55.1157	51	4.115697
4/01/95	47.26979	44.5	2.769787
5/01/95	55.60001	52.5	3.100012
6/01/95	44.84821	41	3.84821
9/01/95	47.85097	44	3.850965
10/01/95	47.7541	46	1.754102
11/01/95	45.33252	42	3.332525
12/01/95	54.43766	51	3.437655
13/01/95	46.20429	42	4.204293
16/01/95	43.10467	47.5	-4.39533
17/01/95	42.5235	47.5	-4.9765
18/01/95	42.91095	47.5	-4.58905
19/01/95	30.60934	35	-4.39066
20/01/95	35.25876	41	-5.74124
23/01/95	54.92197	51.5	3.42197
24/01/95	51.4349	46.5	4.934899
25/01/95	33.22464	37	-3.77536
27/01/95	39.32701	41.5	-2.17299
30/01/95	35.25876	37	-1.74124
31/01/95	33.61209	36	-2.38791

(a) Forecasted ITM put option prices. Produced by the regression equation ITM put = 1.5504 + 0.9686 $\times$ Model.(b) ITM put option settlement prices recorded for the period 01/01/95 to 31/01/95 (c) Forecast - Observed = pricing error. The average error was 0.23 of one basis point. The standard deviation was 3.89 basis points and the correlation coefficient was 0.893.

fits into this category. Overall the model provided a reasonable base from which to replicate the pricing assumptions of the SFE ten year bond futures option market but it could not be considered a first best pricing methodology at this stage of its development.

6.4 Option Pricing Results : A Perspective

The results reported in Chapter 6 raised a number of issues regarding the effectiveness of the model option pricing approach. On average the model pricing errors were not found to be statistically significant. It could be suggested that the Yield Error Margin distributions had provided a reasonable base from which to replicate the volatility and option pricing assumptions of the market. Across the six option series the calculated average pricing error ranged between -0.21 of one basis point to 0.61 of one basis point. This level of average error was comparable to the futures pricing results in Chapter 4 and to other term structure based option pricing research.

However, the standard deviation of the model pricing errors reduced the strength of these positive findings. For example, the standard deviation of the ATM call pricing errors for the sample period was 4.177 basis points. This suggests that the ATM call option pricing errors fell into a range of between - 3.57 basis points and + 4.79 basis points. The distributional analysis in Appendix W also found that 23.9 % of the ATM call pricing errors were greater than $\pm$ 5 basis points. For a market where the bid - ask spread was generally 3 basis points, it was reasonable to assume that a 5 basis point error was economically significant. A discrepancy of this magnitude would be considered a trading opportunity by option price makers.

The regression analysis in Table 6.2 also supported the view that the model had more accurately priced puts then calls. In particular the ITM put option series was seen to have the highest R^2 and the most significant model based coefficients. The initial OLS analysis in Table 6.2 suggested that the model ITM put option prices were unbiased perfect predictors of the market. This conclusion was subsequently treated with caution when it was found that the regression residuals suffered from a fair degree of autocorrelation.

The forecasting study done, using the original ITM put regression, effectively summarised the overall pricing performance of the model. For the out of sample period 01/01/95 to 31/01/95 the forecasting errors were generally distributed between -3.66 basis points and + 4.1 basis points. Nine out of the twenty errors in this sample were $< \pm 3.5$ basis points. Therefore the model can be seen to generally generate a satisfactory level of pricing error. However, there existed the positive probability that the pricing errors would occasionally be economically significant. The model in a practical sense definitely has room for improvement.

The nature of these pricing results provided an interesting perspective on the development and testing of term structure based risk pricing models. As stated in Chapter 1, the attraction of using the term structure to model interest rate risk was that it summarised all current interest rate expectations. The observed level and shape of today's initial yield curve was assumed to reflect the complete information set available to market participants.

Against this background it was proposed that a model which combined the implied N-period forward term structure with a set of assumptions that described its stochastic behaviour could be used to evaluate contingent claims on securities of all maturity dates and coupon structures. A generalised methodology of pricing interest rate risk was the ultimate objective of this modelling strategy. In this modelling "nirvana" the same assumptions could be used to simultaneously price both ninety day bank bill and ten year coupon bearing bond option contracts.

The downside of pursuing this approach, as mentioned in Chapter 2, was that with the objective of generality comes complexity and therefore high cost. This issue has been a factor in the development of all empirical term structure based option pricing models. For example Amin and Morton (1994) stated that the lack of recent empirical work investigating the Heath, Jarrow and Morton contingent claim pricing model was directly attributable to the difficulty (cost) of implementing its underlying assumptions.[144] Consistent with this comment the authors limited their discussion to short dated Eurodollar futures options because of the assumed high cost of estimating the dynamics of the entire term structure. The US Treasury bond futures options contract was seen to present too many problems for an empirical study.

In line with these type of empirical issues the current paper focused its attention on the distributional characteristics of a segment of the term structure. Although the entire term structure was used to price the bonds in the study, it was only the market forecasting errors involved with four to five bonds that were recorded. The Yield Error Margin distributions that were constructed from this approach were assumed represent the last or end step of an equivalent binomial or lattice based option pricing structure. In the case of the "Composite" 2 week Yield Error Margin database, see Figure 8. in Chapter 5, this "final step" distribution was 240 basis points wide and contained 25 pricing nodes. In contrast Amin and Morton's binomial model had 10 steps in its structure and 2^{10} or 1024 pricing nodes in its final step.[145] This was seen to give a good approximation of the true underlying continuous Eurodollar futures distribution.

[144] Amin and Morton, (1994) 142.
[145] Amin and Morton, (1994) 179.

This pricing structure raises two points that may have had some bearing on the empirical accuracy of the model. The SFE ten year bond futures options contracts investigated in Chapter 6 were American style options. They can be exercised at any time over the life of the option contract. The model as presented was only able to generate a futures price distribution for the expiration date of the option i.e. the final step. Therefore the model assumptions could not value the benefit of early exercise to the holder of the option.

In practice this factor was likely to have only a marginal impact on the short dated options valued in this paper. In Chapter 6 the maximum time to maturity of the options priced was approximately ninety days. The premium charged for an American option for shorter dates especially those with less than thirty days to expiry was probably very small. As Flesaker (1993) found in his study of Eurodollar futures options the early exercise premium associated with American options was probably worth less than 1 basis point.[146]

Further perspective of this issue was provided by Cakici, Chatterjee and Wolf's (1993) tests of different option pricing models on the US Treasury bond futures contract during 1987. Their central finding was that Black's European futures option pricing model produced prices as accurate as the more complex American option pricing model sourced from Barone-Adesi and Whaley.[147] It was unlikely that the European characteristics of the term structure based pricing model were a major source of bias over the sample period.

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[146]Flesaker, (1993) 485.
[147]Cakici, Chatterjee and Wolf, (1993) 12.

The second point was related to the key issue of generating a reliable pricing distribution. Over the sample period the market consistently adjusted its volatility view. This factor in combination with the changing time to maturity of options meant that everyday a new underlying distribution was being factored into the pricing of the bond futures options. The Yield Error Margin distributions as mentioned had a relatively finite set of pricing points available when compared to Amin and Morton's binomial based model. This implied that the discrete nature of the Yield Error Margin databases may have constrained the models ability to "fine tune" the underlying futures pricing distribution.[148] A new version of the current model would potentially involve incorporating the 2 week Yield Error Margin distributions into a binomial structure. For pricing a 2 month option there would be four steps with sixteen distinct Yield Error Margin distributions related to the final node. A more precise bond futures distribution was likely to result.

These new versions of the model could be used to test the efficiency of the SFE bond futures option market. This question was not dealt with in Chapter 6. The interest in this topic for future research was driven by a number of factors that characterised this market. The three basis point spread in this market in conjunction with a relatively low level of trading volume suggests that there may exist arbitrage opportunities on an intra - day basis. The volume of the ten year bond futures options market was relatively small, representing only $\approx$ 12 % of the futures volume in 1994 and 1995.[149] This observation implies that the bond futures option market may

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[148]The put option pricing bias of the model may have been related to the convexity of the yield/price relationship of bonds. The fact that bond prices change proportionally less when yields rise suggested that there may have been so called "pricing compression" that favoured the pricing of puts.
[149]In 1994 ten year bond futures options traded a daily average of 3,141 contracts versus 23,697 contracts for ten year futures. In 1995 ten year bond futures options traded a daily average of 2,282 contracts versus 19,799 contracts for ten year futures. Note also that the highest average option daily volume of 5,542 contracts was recorded for the February 1994 period. This was correlated with the start of the jump in yields that occurred in early 1994. Chapter 7 discusses this observation in greater depth. Source SFE statistics department.

become inefficient at times when large discrete transactions attempt to use the market. There may be excess positive returns accruing to price makers who provide liquidity to this market. The most effective way to test this proposition would be to use "Time and Sales" data for the futures and options contracts rather than the business close data used in the analysis in Chapter 6. This would enable individual trades to be analysed for potential arbitrage opportunities.

CHAPTER 7

Rewards From Hedging Fixed Interest Portfolios

7.1 Introduction

In Chapter 7 the implied *N*-period forward term structures constructed in Chapter 4 were combined with the Yield Error Margin distributions of Chapter 5 to implement a "naive" fixed interest portfolio hedge. The rewards from pursuing this strategy were determined by whether on average a hedged portfolio out - performed an equivalent unhedged portfolio over the sample period 01/01/94 to 31/12/94.

The hedging simulation in Chapter 7 had the following structure. The implied certainty forwards derived in Chapter 4 were used to determine the expected end of horizon return on the nominated fixed interest portfolio. The portfolio was assumed to consist of the Commonwealth government coupon bonds that made up the Sydney Futures Exchange (SFE here after) ten year bond futures pricing basket. The Yield Error Margin distributions were then applied to the certainty forwards to demonstrate what yield point increase was necessary for the expected portfolio return to reach a targeted - 5.00 %. Once this was established, SFE ten year bond futures put options were purchased at a strike price set by the closing futures price minus the yield point move equated with the - 5.00 % return. A hedged portfolio was constructed with its end of period return compared to the unhedged portfolio.

The effectiveness of this hedging strategy was measured three ways. Firstly, the mean and variance of the two portfolio returns were investigated. A priori the hedged portfolio return should have shown a lower degree of variance vis - a - vis the unhedged portfolio. Secondly, a statistical test was run to determine if the hedging strategy had significantly altered the return characteristics of the fixed interest portfolio. Finally, the return distributions of the unhedged and hedged portfolio were compared to ascertain the true risk - reward of the hedging policy.

The results of this research suggested that the manager of a fixed interest portfolio would have been rewarded for pursuing the "naive" hedging strategy over the sample period. However, it appeared that the effectiveness of the "naive" hedging strategy was dramatically reduced in high volatility environments. Other hedging techniques should be contemplated under such circumstances.

7.2 Portfolio Performance : Hedged versus Unhedged

To illustrate the workings of the hedging simulation the results for the 04/01/94 trading day were presented in Appendix X. There were five parts to the hedging process. The first step involved Equation (3.27) which calculated today's fixed interest portfolio value. Four Commonwealth government coupon bonds were incorporated in the pricing basket of the March 1994 bond futures contract. Each of the observed physical bond market prices on 04/01/94 was multiplied by $1,000,000. The total purchase price of the fixed interest portfolio was therefore set at $4,737,040.00. This was the benchmark portfolio value used to calculate the unhedged and hedged portfolio returns.

The second part of the simulation used Equation (3.28) to determine the expected investment horizon value of the portfolio. The investment horizon in this case was the nearby ten year bond futures cash settlement date 16/03/94. The portfolio manager was assumed to hold the portfolio for seventy one days between 04/01/94 and 16/03/94. The implied seventy one day forward term structure suggested that bond yields would rise over the investment period by a modest five to six yield basis points. For example, comparing the yield of the 15/07/05 bond in Table 4.1 with its implied forward yield in Appendix X it was noted that the market was expecting a yield increase, from 6.88 % to 6.931%, of ≈ 5 basis points.[150]

The fall in the capital value of the portfolio associated with this increase in yield was offset by the receipt of coupon payments over the investment horizon. Coupons were received from three of the bonds in the portfolio.[151] The total contribution of the coupons to the end period portfolio value was after reinvestment $130,482.01. In Table 2 of Appendix X the expected end period value of the fixed interest portfolio was given as $ 4,778,991.10. Despite the forecasted rise in yields the expected end period return was recorded, consistent with Equation (3.29), as + 4.553 %.

The third part of the process involved the Yield Error Margin distributions. Based on the expected end period portfolio value it was found in Table five of Appendix X that an increase in yield of thirty basis points produced a negative investment horizon return of - 5.41 %. To determine the appropriate SFE ten year bond futures option put strike thirty basis points was deducted from the closing ten year bond futures price recorded on 04/01/94. The adjusted futures price was calculated as 93.13 - 0.30 = 92.83. Applying the ± 12.5 basis point decision rule incorporated in Equation (3.30) to the adjusted futures price was seen to generate the so called "hedging strategy strike". The SFE ten year bond futures 92.75 March 1994 put option fulfilled this role.

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[150] If the four yields in the second table in Appendix X were summed together and then divided by four a futures average basket yield was derived. The average was 6.869 % which was the model futures equivalent yield derived in Chapter 4.

[151] In the first table in Appendix X note that the 15/08/03 bond paid a coupon on 15/02/94 worth $47,500. The 15/09/04 bond paid a coupon on 15/03/94 worth $45,000 and the 15/07/05 bond paid a coupon worth $37,500 on 15/01/94.

The fourth part of the analysis activated the hedging strategy. To match the initial value of the portfolio forty seven 92.75 ten year bond futures put options were purchased. The cost of each individual put was 5.9 yield basis points with the total cost of the hedge set equal to $23,845.69. This hedging cost, as will be discussed in more detail later in Section 7.2, was significantly increased by the high level of volatility associated with the May to July 1994 trading period.

The final step of the simulation compared the hedged portfolio return to the unhedged portfolio return. By the end of the investment horizon on 16/03/94 bond yields had actually increased by around forty five basis points. The unhedged portfolio on 16/03/94 was worth only $4,650,382.01. As a consequence of this fall in value Equation (3.31) (a) calculated the unhedged portfolio return as -9.40 %. In comparison the hedged portfolio benefited from incorporating the 92.75 strike put option. At expiration the ten year bond futures price was 92.595. The put option expired In-the-money (ITM here after) by 15.5 basis points versus its original cost of 5.9 basis points. The net payoff associated with the put option was $38,799.77. Therefore the hedged value of the portfolio equalled the unhedged portfolio value $4,650,382.01 plus the net put payoff $38,799.77 = $4,689,181.78. Equation (3.31) (b) calculated that the portfolio manager was able to restrict the loss of the portfolio over the investment horizon by purchasing put options to - 5.19 %.

The hedging simulation was run for every trading day in the reduced sample where $n = 163$. In Chapter 3 the minimum realistic investment horizon was assumed to be one month in duration. Those trading days less than one month before a SFE ten year bond futures cash settlement date were excluded from the sample. The daily results of the hedging strategy were recorded in Appendix Y. The average monthly and average All Sample results were displayed in Table 7.1..

Table 7.1:

Unhedged Fixed Interest Portfolio Returns

Month	*Av. Ret.* [(a)]	*High*	*low*	*Std. dev.* [(b)]
Jan-94	**-24.08**	**-9.4**	**-42.98**	**9.076**
Feb-94	**-44.72**	**-37.14**	**-56.67**	**6.402**
Mar-94	**-33.11**	**-27.6**	**-38.8**	**3.611**
Apr-94	**-28.12**	**-17.1**	**-36.7**	**5.769**
May-94	**-12.45**	**1.24**	**-18.1**	**8.297**
Jun-94	**-2.16**	**11.7**	**-17.87**	**9.514**
Jul-94	**-3.79**	**3.3**	**-13.3**	**5.265**
Aug-94	**-19.60**	**-11.5**	**-27.7**	**5.057**
Sep-94	**9.31**	**14.1**	**2.96**	**2.861**
Oct-94	**12.81**	**27.2**	**6.4**	**5.813**
Nov-94	**36.32**	**43.11**	**29.88**	**5.232**
ALL SAMPLE[(c)]	*-9.79***	*43.11*	*-56.67*	*22.09*

Hedged Fixed Interest Portfolio Returns

Month	*Av. Ret.*	*High*	*Low*	*Std. dev.*
Jan-94	**2.73**	**18.8**	**-5.19**	**5.741**
Feb-94	**11.11**	**19.66**	**6.2**	**3.943**
Mar-94	**-11.68**	**-6.9**	**-16.1**	**2.72**
Apr-94	**-23.8**	**-18.34**	**-32.9**	**3.581**
May-94	**-24.09**	**-10.8**	**-34.8**	**7.724**
Jun-94	**-5.29**	**0.03**	**-7.98**	**2.829**
Jul-94	**-7.31**	**-0.8**	**-10.3**	**2.136**
Aug-94	**-3.44**	**0.35**	**-4.4**	**2.906**
Sep-94	**1.39**	**5.09**	**-4.45**	**2.560**
Oct-94	**4.99**	**20.77**	**-2.07**	**5.679**
Nov-94	**28.68**	**35.03**	**22.22**	**5.161**
ALL SAMPLE	*-3.06***	*35.03*	*-34.8*	*14.08*

** The average difference between the two return series was statistically significant at the 5.00%/1.00% level.(a) The average monthly return measured in % per annum terms.(b) The standard deviation of the return for the month. (c) The All sample results for average return, high, low and standard deviation.

The average All Sample unhedged portfolio return of -9.79 % reflected the discussion in Chapter 4 and particularly Chapter 5. The strong rise in long bond yields in the first four months of the sample led to large negative annualised returns. The largest annualised loss of - 56.67 % was recorded on 14/02/94. As was discussed in Chapter 5, it appeared in early 1994 that investors looked to the high inflation past of Australia when making their portfolio decisions.

Consistent with the analysis of Ilmanen (1996) it can be suggested that Australian long bond yields increased in response to two factors: expectations of future increases in cash rates and secondly a rise in the Australian bond market risk premium.[152] Footnote 128 Chapter 5 noted this increased risk premium demanded by investors. The spread between the Australian ten year bond and the "bellwether" US thirty year bond (AUD 10 year - US 30 year) increased from 0 basis points in January 1994 to +227 basis points in June 1994. Market participants who purchased Australian ten year bonds in the first three to four months of the sample would have recorded negative returns for the 1994 trading period.

The unhedged monthly average returns paralleled the behaviour of the Yield Error Margin statistics calculated in Chapter 5. The major negative returns reported in the early part of the sample gave way to a mid-year phase where returns were less negative. After their late June interim peak, bond yields range traded between 9.20 % and 9.90 %. As a result portfolio returns stabilised. The large negative unhedged return recorded in August reflected the renewed surge of ten year bond yields in early to mid - September 1994. In contrast, the large positive unhedged bond portfolio returns reported in the later part of the sample reflected the fact that bond yields had peaked for 1994 around the 10.60 % area. In late November and early December

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[152] Ilmanen, (1996) 55.

yields traded between 10.00 % and 10.30 %. Purchasing fixed interest portfolios at this time proved to be a successful investment strategy.

The hedged portfolio return series demonstrated the impact of pursuing the "naive" hedging strategy. The SFE ten year bond futures put options generally offered interest rate insurance when it was needed most. In the two phases when yields increased strongly, January - April and August - September 1994 the hedged portfolio was protected from significant losses. However, this protection came at a cost. In the September - November period the unhedged fixed interest portfolio outperformed the hedged portfolio. With yields peaking the cost of the put option insurance became an unnecessary "drag" on the performance of the fixed interest portfolio.

Comparing the All Sample results of the hedged and unhedged portfolios reinforced the trade-offs involved with naively hedging a fixed interest portfolio. The hedged portfolio for the sample period recorded both a lower average level of loss, - 3.1 % vs - 9.79 %, and a lower level of return variance, -14.1 % vs - 22.1 %. For the manager who adopted a "naive" hedging strategy the rewards were a less negative average return and less risk. This more risk averse strategy proved to be the best approach for the 01/01/94 to 31/12/94 sample period.

The All Sample results suggested that the "naive" hedging strategy had modified the return characteristics of the underlying fixed interest portfolio. Hypothesis eleven used the statistical test in Equation (3.32) to determine the power of this observation. The null hypothesis tested the proposition that on average the difference between the two return series was equal to zero. In contrast to the tests in Chapters 4 to 6 the return series were not assumed to be independent. In this case the focus was on the mean and variance of the spread between the hedged and unhedged series.[153] The critical value for this test at the 5.00 % / 1.00 % significance level for $n = 163$ was $\pm$ 1.960 / $\pm$ 2.576. The observed value of the test was calculated as 4.379. The null hypothesis could therefore be rejected. It could be stated, on the basis of this test, that the "naive" hedging strategy had significantly modified the mean return of the fixed interest portfolio over the sample period.

The distributional analysis introduced in Chapter 4 generated further support for this hypothesis. Figure 9. displayed the relative frequency histograms associated with the unhedged return series and the hedged return series for the 1994 sample period. As suggested by Beighley (1994), the distribution of hedged portfolio returns was truncated around the maximum loss target of - 5.00 %.[154] 50.9 % (83/163) of the time the hedged portfolio returns were distributed between 0 and - 10.00 %. For the unhedged return the equivalent statistic was only 16.5 % (27 /163). The hedging strategy had compressed the returns of the fixed interest portfolio.

However, Figure 9. also demonstrated in keeping with the analysis of Figlewski, Chidambaran and Kaplan (1993) that while the put option strategy reduced the chances of big losses it did not always eliminate them.[155] In the unhedged distribution there existed a 44.2 % chance that the portfolio return would exceed a 15.00 % loss. In fact there was nearly a 1 in 5 chance, 19.6 % (32/163), that the unhedged return would record a loss greater than 30.00 %. In the hedged portfolio case the probabilities of these events were lowered to 17.8 % and 2.45 % respectively. It can be stated that despite the presence of interest rate insurance that there still existed the

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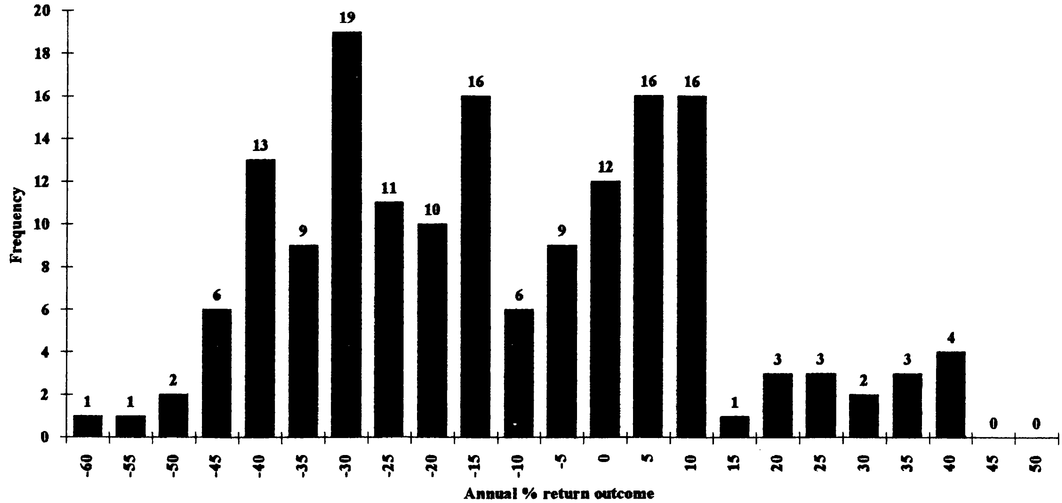
[153]The statistic in Equation (3.32) was utilised in preference to Equation (3.10) because the hedged series was seen to be a modified version of the unhedged series. $\bar{d} = 6.73$, $\sigma_d = 19.63$.
[154]Beighley, (1994) 69.
[155]Figlewski, Chidambaran and Kaplan, (1993) 51.

Figure 9:

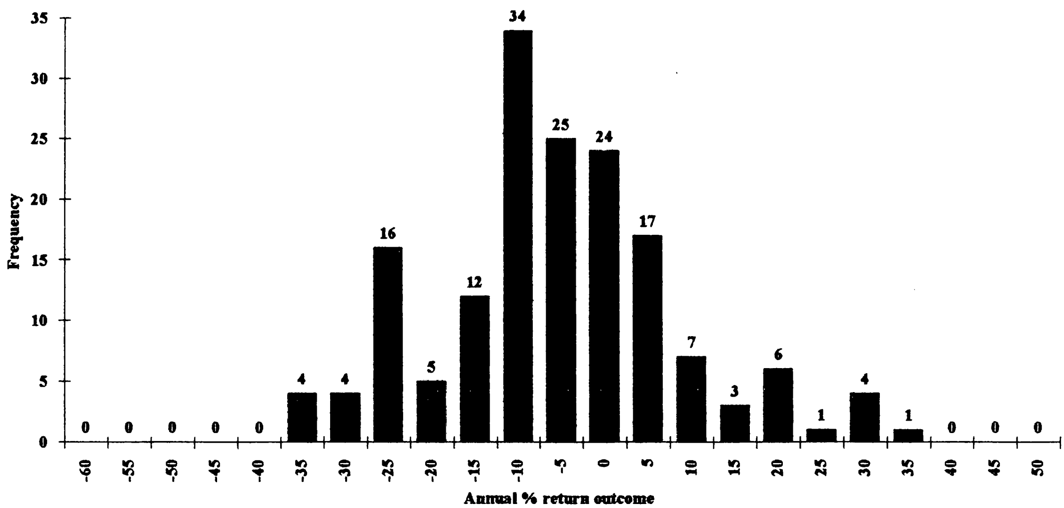
Unhedged Portfolio Returns: Relative frequency distribution

Number on top of the bars denoted the relative frequency of a particular % return n = 163



Hedged Portfolio Returns: Relative frequency distribution

Number on top of the bars denoted the relative frequency of a particular % return n = 163



the positive probability of large negative returns in the hedged fixed interest portfolio.

This observation was related to two influences. These factors contributed to the average monthly hedged returns being more negative than the unhedged results over the May to July period. The first was the level of traded volatility. As discussed in Chapter 5, the level of both implied and historical volatility increased strongly during the middle months of 1994. In the 04/01/94 hedging example presented in Appendix X it was clear that the low cost of the put options had improved the ability of the hedge to minimise the loss of the overall portfolio. In January 1994 volatility was at a sample low, averaging 7.01 %. In this case it can be assumed that the cost of interest rate insurance was relatively inexpensive.

In contrast, see Table 5.5. in Chapter 5, in July 1994 the average level of implied bond price volatility was at a sample high of 14.45 %. The impact that this higher level of volatility had on the effectiveness of the "naive" hedging strategy can be demonstrated by analysis of the 07/07/94 trading day. Like the January 04/01/94 example there was a seventy one day investment horizon between 07/07/94 and 16/09/94. The Yield Error Margin distribution once again suggested buying SFE ten year bond futures put options thirty basis points below the current closing futures price. The closing futures price for 07/07/94 was 90.265 so that the hedging strike was the 90.00 series put option. On 07/07/94 the level of quoted volatility had more than doubled from its January levels of 7.00 % to 15.50 %. In this environment the 90.00 put option strike cost 33.5 basis points.

Over the investment period ten year bond yields increased by more than thirty five basis points. The unhedged portfolio as a result recorded a -1.20 % return. In comparison the hedged portfolio performed poorly recording a negative return of -8.4 %. Despite the fact that the put options expired ITM (90.00 - futures close 89.875 = +12.5 basis points) there was no benefit to the portfolio. The effective payoff associated with the put options was actually a 21 basis point loss. This loss of $67,217 was incurred because the original cost of the 90.00 strike put option of 33.5 basis points was greater than its maturity value of 12.5 basis points.

The 15.50 % volatility quote reflected the market's expectations of large swings in yields. At this level of volatility, yields would have had to increase by 60 - 70 basis points for the put option to have provided any degree of portfolio protection.[156] A lower level of volatility would have clearly improved the success of the hedging strategy. For example if volatility had been quoted at 7.00 %, as on 04/01/94, the return on the hedged portfolio would have been significantly improved. In this case the 90.00 strike put option would have cost only 9.96 basis points. The net payoff of the put option would have been + 2.54 basis points instead of the loss of - 21.0 basis points originally recorded. In response the hedged return would have been - 0.29 % or basically breakeven. This example highlighted the fact that the effectiveness of the "naive" hedging strategy was significantly reduced by a high volatility environment.

The second factor that had the potential to downgrade the hedged portfolio returns was the presence of some "slippage" in the hedging structure.[157] There existed a slight mismatch between the end of the investment horizon and the maturity date of the hedging instrument. The 02/05/94 trading day illustrated this point. On this trading

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[156]Figlewski, Chidambaran and Kaplan (1993) 53. The authors studied hedging strategies in equities. They suggested at a 10 % volatility quote that a - 4.6 % move in the underlying stock would be viewed as a disaster. If volatility was quoted at 30 % such a move would be seen as an expected event.

[157]" Slippage " in the hedging strategy was also related to the discrete nature of the put option strikes. See Equation (3.19).As in the 04/01/94 example the - 5.00 % target return was set at the 92.83 futures price. To actually implement the hedging transaction a 92.75 strike put option had to be purchased. There was an eight basis point differential between the optimal level and the actual hedging level. This obviously modified the performance of the hedge.

day the investment horizon was a forty five day run until the 16/06/94. The put option protecting the fixed interest portfolio expired on 15/06/94. There was a one day gap in the hedging structure. This gap proved very costly in this case.

Reflecting the high volatility of mid - June 1994 there was a thirty basis point increase in yield in the twenty four hours between the close of business on 15/06/94 and the close on 16/06/94. On 15/06/94 the put option, as occurred in the 07/07/94 example, had finished 11 basis points ITM but its original cost had been 27.5 basis points. The put option thereby contributed a 16.5 basis point loss to the hedged portfolio. The 02/05/94 hedged and unhedged portfolio returns were - 32.9 % and - 22.6 % respectively. If the put option had matured one day later it would have been able to provide insurance against the overnight thirty basis point increase in yields. The return of the hedged portfolio under this alternative scenario would have been - 14.1 %.

This case study reinforced the point that when attempting to hedge interest rate risk that efforts should be made to measure all the potential residual risks associated with a particular strategy. Without this scrutiny the performance of a given hedging strategy can be severely impaired. In the 02/05/94 example a one day residual risk equated to nearly a 19.00 % variance in the return of the fixed interest portfolio.

7.3 Confirmation of some Common Hedging Principles

The hedging simulation run for the 01/01/94 to 31/12/94 sample period reaffirmed some common hedging principles. The average All Sample hedged fixed interest portfolio return was - 3.10 %. In comparison the unhedged portfolio recorded an average return of - 9.8 %. As expected the standard deviation of the hedged return series was lower than the unhedged return series. The "naive" hedging strategy achieved its goal of providing interest rate insurance when it was needed most. The fixed interest portfolio manager who followed this more risk averse strategy over the sample period would have been rewarded by attaining lower losses at an overall reduced level of risk. The put option strategy swapped a fixed amount of upside return potential for the lower probability of large negative returns.

The results associated with Hypothesis 11 demonstrated that the hedging strategy had significantly altered the return characteristics of the fixed interest portfolio. The mean return of the two series was shown to be statistically different at both the 5.00 % and 1.00 % levels. This finding was supported by the relative frequency distributions displayed in Figure 9.. 50.9 % of the hedged portfolio sample returns were distributed in the interval 0 to - 10 %. The equivalent statistic for the unhedged return series was only 16.5 %. The returns of the hedge portfolio were truncated around the maximum loss - 5.00 % target level.

The other major point of the distributional analysis was that while the hedging strategy largely reduced the probability of large negative returns it did not totally eliminate them. There existed nearly a 1 in 5 chance that the unhedged portfolio recorded a loss of more than 30.00 %. In comparison the hedged portfolio only had a 2.5 % probability of the same event.

The propensity of the hedged portfolio to record large negative returns was influenced by two factors. Firstly, the level of traded volatility was seen to be a critical determinant of the performance of the hedged portfolio. It was demonstrated via a case study that the high level of market quoted volatility, in the period May to July 1994, had decreased markedly the effectiveness of the hedging strategy. There were trading days when the put option would finish ITM at the end of the investment horizon but was unable to enhance the return of the portfolio. The very high premium paid for interest rate insurance negated the potential benefits to the hedged portfolio. This suggests that under these conditions other strategies, such as using futures, should be considered when hedging fixed interest portfolios.

Secondly, there was also a degree of "slippage" between the end of the investment horizon and the expiration of the hedging structure. In the 02/05/94 example there was a thirty basis point increase in yield between the put option maturity date on 15/06/94 and the end of the horizon period on 16/06/94. Despite the fact that a one day gap may usually be considered innocuous, in this case it was correlated with a large adverse move in yields. As a result the hedged portfolio performed poorly recording a return of - 32.9 %. This result highlighted the fact that all potential residual risks should be considered when implementing a hedging strategy. Without this analysis the protection offered by a hedge strategy can be severely impaired.

The hedging simulation in Chapter 7 supported the results discussed in Chapters 4 and 5. The 01/01/94 to 31/12/94 sample was a unique period that was characterised by a strong rise in both yields and volatility. Attachment 7 reinforced this claim. The 1994 year was the only twelve month period over the last ten years to have recorded a negative bond index return. The - 4.7 % return compared unfavourably to the average 1985 - 1995 annual return of +13.9 %. Although this indexed based return, which covered bonds from three to ten years, was not directly comparable to the - 9.7 % unhedged return recorded in Table 7.1 the result does suggest that naively hedged bond portfolios outperformed unhedged bond portfolios over 1994.[158]

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[158]This negative bond market result caused problems for the Australian Superannuation industry. Even some so called "Capital stable" funds recorded negative results over 1994. In the wake of this poor performance there were calls for a review of Superannuation management practices.

This result had economic significance for the market structure of the still emerging Australian Superannuation Industry. Attachment 8 demonstrated the dynamic growth of funds under management in Australia over the last ten years. US based equity mutual fund research, such as that by Brown and Goetzmann (1995), provided an interesting perspective on the potential merits of hedging portfolios.[159] They confirmed the expectation that poorly performing funds did disappear more frequently than stronger performers. There were definite rewards accruing to those funds that outperformed their peers, especially in terms of market share. The attraction of hedging, given these type of findings, was that it may directly affect the survival and ultimately the prosperity of an individual fund.

The "naive" hedge presented in Chapter 7 may provide the funds manager with a way of not becoming a "loser". Footnote 149 in Chapter 6 supported this hypothesis. Over the period 1992-1995 the highest volume of trading in the ten year bond futures options market occurred during February and March 1994. This coincided, as has already been well documented, with a period when bond yields increased significantly. This provided initial evidence that funds and institutions were attempting to lower their chance of negative portfolio returns by buying interest rate insurance.[160]

This type of observed behaviour opens the way for future research into the Australian Superannuation industry. The focus of this analysis would be on the type of hedging policies implemented by funds managers. A potential hypothesis would be one that tests the proposition that there existed a correlation between hedging policy and

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[159] Brown and Goetzmann, (1995) 680.

[160] The managers may also have remembered the lessons from the 1987 stockmarket crash. Those funds that had reduced their exposure to the stockmarket prior to the late October 1987 crash, carried this performance advantage forward through many years of higher average annual returns. It was clear that this advantage was used by fund managers to market their services and increase their market share in the late 1980's.

the survival of individual funds. The results displayed in Attachment 7 suggest that an "active" style of hedging would probably be the most effective over time. In the "good" years a "passive" hedging policy that was always implemented, may prove to be an unnecessary drag on return performance i.e. as per the last three months of the 1994 sample. The best portfolio strategy maybe to use the trading strategy that trade "breaks" in market ranges, as introduced in Chapter 5, as a way of activating the "naive" hedging structure.

CHAPTER 8

Conclusions

8.1 Introduction

The objective of this paper was to determine if term structure based interest rate forecasts had a role to play in risk management. Three central hypotheses were constructed to test this proposition. These tests had at their foundations the arbitrage free certainty forward term structure first presented by Ho and Lee (1986). The first hypothesis stated that the Australian market term structure was derivable from the Commonwealth government coupon bond yield curve. This zero coupon yield to maturity relationship was assumed to measure the riskless rate of return available on Australian market securities from one day to ten years. The second hypothesis stated that contained in this term structure was the market's implied forecasts of N-period forward interest rates. Due to these forecasts being made under uncertainty they were assumed to have significant errors. These errors were measured empirically by the so called Yield Error Margin statistic. The third hypothesis proposed that these forecasting errors could be used to approximate the stochastic behaviour of the term structure.

This three part approach to the analysis of interest rate risk was applied to the pricing of ten year bond futures and options contracts traded on the Sydney Futures Exchange (SFE here after), as well as to the hedging of fixed interest portfolios, over the 01/01/94 to 31/12/94 calendar period. For this empirical research the three central hypotheses were broken up into a total of eleven sub - hypotheses. These hypotheses were all aimed at testing the proposition that the term structure based interest rate risk model had provided a framework from which to replicate the market's underlying futures, volatility and option pricing assumptions. Appendix Z summarised the results associated with this research.

8.2 Deriving the Australian Market Term Structure

Hypothesis one was empirically tested in Chapter 4. It proposed that the Australian market term structure was derivable from the observed Commonwealth government coupon bond curve. This measure of the Australian market term structure, from one day to ten years, was constructed for every full trading day in the sample period. These zero coupon government bond curves were found to be generally above, in yield terms, the observed Commonwealth government coupon bond yield curve. This reflected the normal shape of the yield curve over the 1994 trading period where short term rates were significantly lower than the longer end of the curve.

To determine if these approximations of the "true" term structure were consistent with the pricing assumptions of the Australian interest rate market they were used to price the SFE ten year bond futures contracts. Implied forward model term structures were used in this role. It was found that the model was able to price these market traded securities to a high degree of accuracy. On average, the term structure based model produced prices that were within 0.2 of 1 basis point of the market. Although it was too strong to suggest that the model prices were perfect unbiased predictors of the market, the ability of the model to successfully forecast SFE ten year bond futures prices supported the hypothesis that there existed a strong economic relationship.

8.3 Characteristics of the Yield Error Margin Databases

Chapter 5 empirically developed the so called Yield Error Margin database. This market based construction was used to test the two key remaining hypotheses associated with this paper. Hypothesis two focused on the performance of the market's interest rate forecasts. These forecasts were derived from the model term structures constructed in Chapter 4. Consistent with the aims of Ho and Lee (1986) the calculated Yield Error Margin statistics measured the discrepancies between today's implied N-period forward term structures and the actual term structures that eventuated on the N-period forward date.

For the sample period the forecasting errors were calculated for four discrete periods: 2 weeks, 1 month, 2 months and 3 months forward. These results were used to construct the Yield Error Margin databases. The market's forecasts were generally found to have statistically significant errors. Consistent with decision making under uncertainty the magnitude of these errors increased with the length of the forecasting horizon.

Hypothesis three proposed that the stochastic behaviour of the term structure could be approximated by the distributional characteristics of the Yield Error Margin databases. Over the 1994 trading period these databases were found to have two forms of bias. Firstly, the Yield Error Margin distributions all had negative means. The market's forecasts were on average below the actual yields that eventuated on the forward dates. The Yield Error Margin distributions were also found to have a second form of bias. They were generally found to be platykurtic which meant, relative to a normal distribution, that they placed too much weight on "outside" observations.

To counter these biases the so called "Mirror" distribution was introduced to modify the sample Yield Error Margin distributions. The reason for its inclusion was that the market had consistently underestimated the increase in yields over the 1994 trading period. Therefore it was assumed that their existed the positive probability of an equivalent trading period where yields were consistently overestimated. This

"Mirror" distribution was subsequently overlaid on the original dataset to form the "Composite" Yield Error Margin database. This transformation centred the Yield Error Margin distributions, whilst maintaining the probability structure of the original Yield Error Margin databases. It was these "Composite" Yield Error Margin databases that were applied to the SFE ten year bond futures option pricing in Chapter 6.

8.4 Pricing Fixed Interest Contingent Claims

Chapter 6 applied the interest rate risk pricing results associated with Chapters 4 and 5 to the valuation of SFE ten year bond futures options. The implied certainty forwards of Chapter 4 were modified by the "Composite" Yield Error Margin databases constructed in Chapter 5. Combining these two aspects of the model was assumed to generate a probability weighted distribution of ten year bond futures prices. These weightings allowed the model to determine the expected payoff of options on the SFE ten year bond futures contract.

The results of this research were presented for six different option pricing series ; At - The - Money puts and calls as well as those 25 basis points In - The - Money and Out - The - Money. On average, the model pricing errors were not found to be statistically significant. Across the six option series the average pricing errors ranged between - 0.21 of one basis point to 0.61 of one basis point. The regression analysis supported the view that the model more accurately priced puts than calls. In particular the In - The - Money put option series was seen to have the highest R^2 and the most significant model based coefficients.

The forecasting study done, using the estimated It - The - Money put regression relationship, effectively summarised the overall pricing performance of the model. For the out of sample period 01/01/95 to 31/01/95 the forecasting errors were generally distributed between - 3.66 basis points and + 4.1 basis points. Nine out of the twenty errors in this sample were < $\pm$ 3.5 basis points. Therefore the model could be seen to generally generate a satisfactory level of pricing error. Despite this there still existed the positive probability that these errors would occasionally be economically significant.

8.5 Rewards from Hedging Fixed Interest Portfolios

The "naive" hedging strategy in Chapter 7 had the following structure. The implied certainty forwards derived in Chapter 4 were used to determine the expected end of horizon return on a given fixed interest portfolio. The Yield Error Margin distributions constructed in Chapter 5 were then applied to the certainty forwards to determine what yield point increase was necessary for the expected portfolio return to reach a targeted - 5.00 %. Once this was established, SFE ten year bond futures put options were purchased at a strike price set by the closing futures price - the yield point move equated with the - 5.00 % return.

The hedging simulation reaffirmed the rewards from utilising interest rate insurance. The average All Sample hedged fixed interest portfolio return was - 3.1 %. In comparison the unhedged portfolio recorded an average return of - 9.8 %. As expected the variance of the hedged return series was lower than the unhedged portfolio return series. Over the sample period the fixed interest portfolio manager who followed this more risk averse strategy would have been rewarded by attaining lower losses at an overall reduced level of risk.

The statistical test run on the two return series supported the hypothesis that the "naive" hedging strategy had significantly altered the return characteristics of the fixed interest portfolio. The hedged portfolio returns were found to be truncated around the maximum loss - 5.00 % target return. The other major point associated with the distributional analysis was that while the hedging strategy largely reduced the probability of large negative returns it did not totally eliminate them.

The high level of market quoted volatility, in the period May to July 1994, was shown to have decreased the effectiveness of the hedging strategy. There were trading days when the put option would finish In - The - Money at the end of the investment horizon but was unable to enhance the return of the portfolio. The high cost of the interest rate insurance had negated the potential benefits of the "naive" hedge. This suggested that under these circumstances other strategies, such as using futures, should be considered when hedging fixed interest portfolios.

8.6 Avenues for Future Research

From the analysis in this paper there were seen to exist five new avenues for research. The first topic was related to the term structure based ten year bond futures pricing model presented in Chapter 4. Throughout 1995 the SFE had been proposing the introduction of physical delivery for the settlement of the ten year and three year bond futures contracts. The ability of the term structure model to price the SFE ten year bond futures contract, under any new settlement specification, was an area of great interest. This type of analysis was especially critical in the early stages of trading in a new contract design. It was probable that the market may take time to come to terms with the new information and therefore pricing inefficiencies may initially exist. The determination of the Commonwealth government coupon bond that was cheapest to deliver would likely form the central part of the research into this issue.

The second and third new research areas were concerned with the characteristics of the forecasting errors discussed in Chapter 5. Over the 01/01/94 to 31/12/94 sample period the market consistently underestimated the propensity of yields to rise. The pattern of these forecasting errors implied that the market's expectations could be defined by three words: Shock (large negative errors), Recovery (errors stabilised) and finally Overshooting (positive errors). The strong surge in ten year bond yields over the sample period, 6.35 % to 10.60 %, frustrated the expectations of market participants. It also suggested that there was evidence of lags and persistence in the market's forecasts. The market delayed changing its behaviour until it was clear that the interest rate shocks were permanent.

This type of behaviour was common to other market activities. In the SFE ten year bond futures option markets the quotation of trading volatility was seen to follow similar patterns. Implied trading volatility increased directly with the strong rise in yields reaching a peak of 17.50 % on 27/06/94. The point to make about this quote was that, instead of forecasting the level of future volatility, it was merely reacting to history. The price action following this period was much more subdued which suggested that there was backward looking volatility pricing behaviour. What was significant about this behaviour was that it suggested, in contrast to Jorion's (1995) analysis of the foreign exchange markets, that traded volatility responded to changes in yields rather than providing reliable forecasts of future yield volatility.

The new research in this case would test the hypothesis that SFE bond futures option implied volatility was an accurate forecaster of realised market volatility. The results of this study, as per the findings in Chapter 7, would be critical for anyone attempting to hedge a fixed interest portfolio. If implied volatility could be shown to be a leading volatility indicator than hedgers would be assured of paying the correct price for their interest rate insurance.

The ten year bond futures "price action" also reflected this type of wait and see behaviour. The futures price change distribution was found to be leptokurtic in nature. The return distribution had a higher peak and thicker tails than would be present under a normal distribution. This result suggested that the market's behaviour can be broken into two parts. The first, where the futures prices trade in tight ranges as the market searches for new signals and a second where the market, on the back of powerful new information, attempts to move to a completely new price level. These findings provide the impetus for future research into the potential profitability of a trading strategy that buys and sell "breaks" of identified trading ranges.

Fourthly, the option pricing analysis suggested that the Yield Error Margin based model had some limitations. There existed the positive probability of economically significant errors. The model in a practical sense definitely had room for improvement. The Yield Error Margin distributions that were constructed from this paper's modelling approach could be assumed to have represented the last or end step in an equivalent binomial or lattice based option pricing structure. In the case of the 2 week "Composite" Yield Error Margin database, see Figure 8. in Chapter 5, this "final step" distribution was 240 basis points wide and contained 25 pricing nodes. In contrast the binomial based option pricing model presented by Amin and Morton (1994) had ten steps in its structure and 2^{10} or 1024 pricing nodes in its final step. This was seen to give a good approximation of the true underlying continuous Eurodollar futures distribution.

Over the sample period the market consistently adjusted its volatility view. This factor in combination with the changing time to maturity of options, meant that a new underlying distribution was being assumed by the market for the ten year bond futures price, everyday over the 01/01/94 to 31/12/94 sample. This implied that the discrete nature of the Yield Error Margin databases may have constrained the model's ability to "fine tune" the underlying futures price distribution. A new version of the current model would potentially involve incorporating the 2 week Yield Error Margin distribution into a binomial structure. To price a two month option there would be four steps with sixteen distinct Yield Error Margin distributions related to the final step. A more precise bond futures distribution was likely to result. The development and testing of this version of the current model was a definite area of future research.

The final area of new research is focused on the still developing Australian Superannuation / Funds Management Industry. The results in Chapter 7 suggested that naively hedged bond portfolios outperformed unhedged bond portfolios over 1994. The attraction of hedging, given these type of findings, was that it may directly affect the survival and ultimately the prosperity of an individual fund. A potential hypothesis would be one that tests the proposition that there existed a significant correlation between hedging policy and the survival of individual funds.

8.7 Summary of Model Performance

The 1994 sample period was a unique time in the financial markets. The end of the early 1990's world wide recession had been potentially signalled by the decision of the US Federal Reserve to tighten monetary policy in February 1994. This was the first move of its type in over five years. This led to a new period of uncertainty about the medium term direction of interest rates. As documented in this paper the Australian bond market performed extremely poorly over 1994. It appeared that the risk premium required to hold longer dated Australian securities was significantly increased over the period. It can be suggested, that the market justified this higher risk premium by looking to past experience when Australia was seen as a high inflation country.

In response ten year bond yields increased by 425 basis points while the average change in yields over 1990 to 1995 was only 231 basis points. Despite this acute level of volatility, the term structure based model was utilised reliably in a variety of roles such as pricing futures, options and hedging a fixed interest portfolio.

The interest rate risk pricing assumptions in this paper attempted to provide an alternative way of assessing the dynamics of the term structure. At its centre was the derivation of an Australian market arbitrage free implied forward term structure. To approximate the stochastic behaviour of the term structure this implied forward was modified by the distribution of errors in the market's interest rate forecasts. The resultant probability weighted distributions of the term structure allowed interest rate risk to be estimated and hence contingent claims could be priced. The model's contribution to the literature can be summarised by the four main results associated with the research.

The first, was that the derived Australian market term structures were found to be economically significant. They were found to price SFE ten year bond futures accurately with a sample average error of only 0.2 of 1 basis point. The model term structure was able to evaluate a market traded security of a similar risk free nature with a high degree of reliability. This result also suggested that the SFE ten year bond futures were, in a theoretical sense, equivalent to Ho and Lee's (1986) arbitrage free certainty forward. To ensure the absence of riskless arbitrage these securities must reflect the implied forwards of today's yield curve.

Secondly, the errors in the market's term structure based interest rate forecasts were found to be generally statistically significant. The market consistently underestimated the capacity for yields to rise over the sample period. There was persistence in market expectations. The market delayed changing its behaviour until it was clear that the interest rate shocks were permanent. This type of lagging behaviour was also present in the way market makers quoted volatility and in the underlying price action of the SFE ten year bond futures contract. This suggests that there may exist trading strategies that can be used to exploit this type of market behaviour.

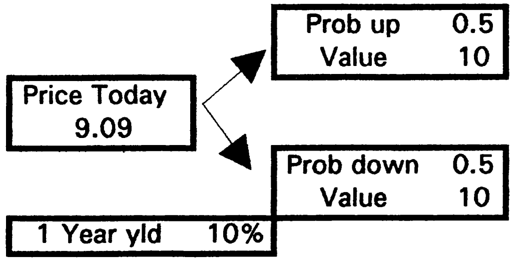
Thirdly, the distribution of these forecasting errors formed the basis of the so called Yield Error Margin databases. These constructions attempted to provide information about the likely distribution of the term structure over a given forward period. These databases in conjunction with the implied forward term structures were applied to the pricing of SFE ten year bond futures options. This methodology, on average, was able to replicate the option prices observed in the market. There was however a bias with In - The - Money put options priced more reliably than the other option series included in the analysis. This bias, combined with the fact that the model had the propensity to produce errors that were economically significant, suggests that further work was needed on the underlying distributional assumptions. To be more successful the Yield Error Margin assumptions need to be incorporated into a framework that makes more formal the relationship of the pricing distribution with changes in volatility and time to maturity.

The final finding was that the hedging simulation reaffirmed the rewards from utilising interest rate insurance. The "naive" strategy of purchasing put options to protect the value of a fixed interest portfolio was shown to reduce the probability of large negative returns. The fixed interest portfolio manager implementing this more risk averse strategy over the sample period would have been rewarded by attaining lower losses at an overall reduced level of risk. However, it was clear that the effectiveness of this strategy was constrained in a high volatility environment.

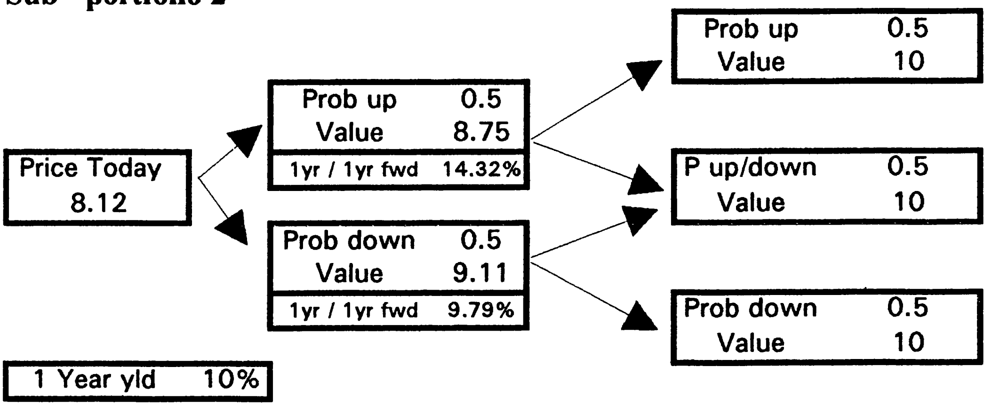
Overall, the research associated with this paper provided some important insights into the stochastic behaviour of the Australian market term structure that supported the hypothesis that the market's implied *N*-period forward interest rate forecasts have a central role in risk management. The distributional methodology that was presented, although not a first best set of assumptions at this stage of its development, does provide a meaningful starting point from which to establish more sophisticated empirical interest rate risk pricing models.

APPENDIX A : Development of the BDT forward rate lattice

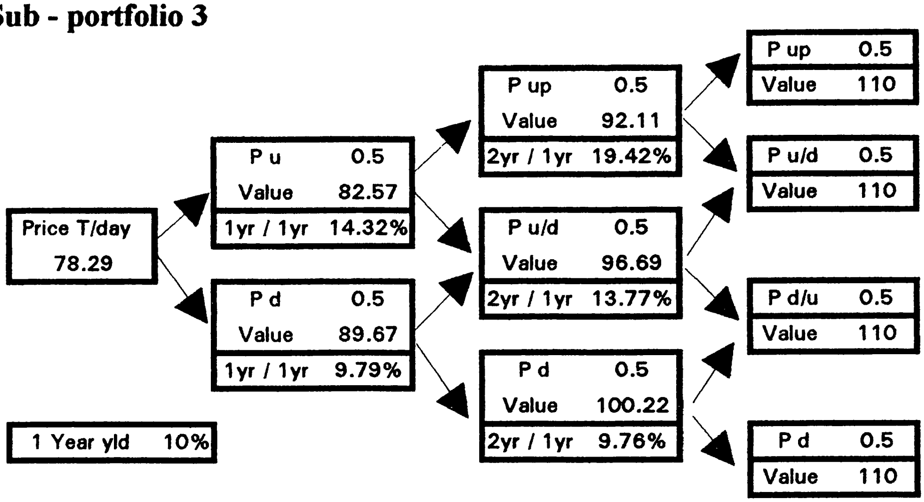
Sub - portfolio 1



Sub - portfolio 2

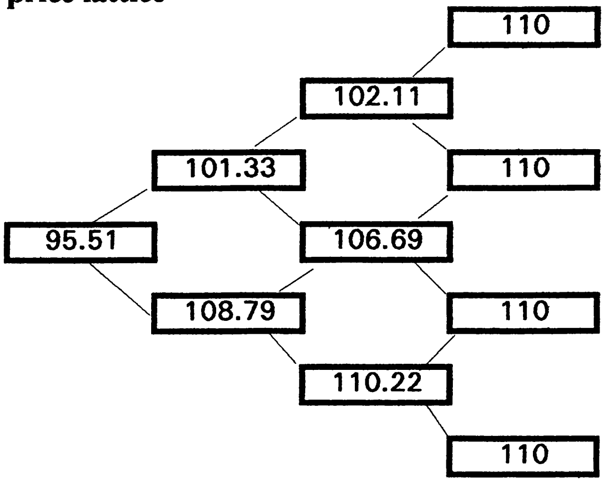


Sub - portfolio 3

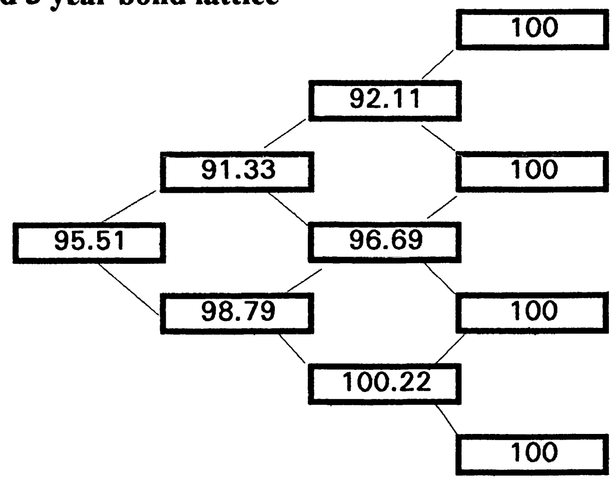


Appendix A continued:

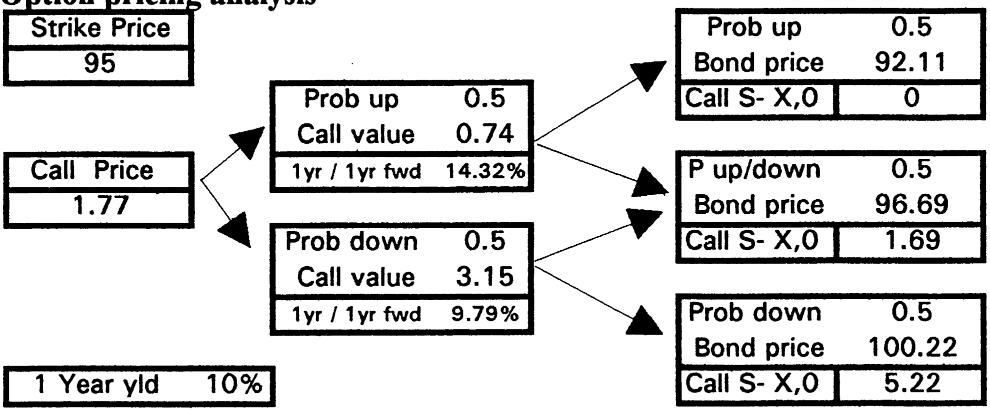
Full 3 year bond price lattice



Coupon adjusted 3 year bond lattice



Option pricing analysis



APPENDIX B: Market data used for term structure derivation 04/01/94

Table A. Short "Anchor" Rates

Term	Mkt.	Input
O/N	4.75	4.75
1mth. [(a)]	4.82	4.8
2mth.	4.84	4.82
3mth.	4.84	4.82
4mth.	4.85	4.83
5mth.	4.85	4.83
6mth.	4.85	4.83
1yr.[(b)]	5.14	5.04

(a) 0.02 % was subtracted from 1 - 6 months (b) 0.10 % was subtracted from 1 year Swap

Table B. Commonwealth Government "Hot Stock" Bonds

Maturity	Coupon	Yield
15/02/95	13	5.2
15/04/95	12.5	5.15
15/09/95	10.5	5.35
15/07/96	13	5.71
15/03/97	12.5	5.92
15/09/97	12.5	6.06
15/01/98	12.5	6.14
15/08/98	7	6.26
15/03/99	6.25	6.4
15/07/99	12	6.41
15/04/00	7	6.53
15/07/00	13	6.53
15/11/01	12	6.65
15/10/02	10	6.75
15/08/03	9.5	6.78
15/09/04	9	6.84
15/07/05	7.5	6.88
15/11/06	6.75	6.91

APPENDIX C : Deriving zero coupon discount factors for the 15/02/95 bond

(Trade Date 04/01/94 Settlement Date 07/01/94)

Step 3 : Linearly interpolating zero coupon discount factors

Days	Date	Yield	Discount	Interp. Date	Interp. Disc.
1	7/01/94	4.75%	1		
31	7/02/94	4.80%	0.996026		
				15/02/94	*0.9949956*
59	7/03/94	4.82%	0.99242		
90	7/04/94	4.82%	0.98846		
120	7/05/94	4.83%	0.984612		
151	7/06/94	4.83%	0.980675		
181	7/07/94	4.83%	0.97688		
				15/08/94	*0.9716107*
365	7/01/95	5.04%	0.952018		

Step 4: Finding the zero coupon discount factor for maturity date 15/02/95

Days	Date	Cash Flow	Discount	Ann. Yield [b]	PV
39	15/02/94	6.5	0.994996	4.80733	6.46747
220	15/08/94	6.5	0.971611	4.89418	6.31546
404	15/02/95	106.5	*0.944639* [a]	*5.2801276*	*100.60406*

Bond Price	**113.387**
Mkt Price	***113.387***

(a) Patterned area = The market price of the bond used to calculate the remaining zero coupon discount factor. (b)Annual yield = $((1/((\text{Discount factor})^{(365/\text{days to maturity})}))-1 \times 100$

APPENDIX D : Cash and Carry SFE 10 year bond futures price 04/01/94

(i) March 1994 SFE bond pricing basket as at 04/01/94

Maturity	Coupon	Yield	Price	NextCoupon	Amount
15/10/02	10	6.75	123.535	15/04/94	5
*15/08/03	9.5	6.78	122.704	15/02/94	4.75
15/09/04	9	6.84	119.003	15/03/94	4.5
15/07/05	7.5	6.88	108.462	15/01/94	3.75

(ii) Calculating the Cash and Carry price of the 15/08/03 Bond

B_t = \$122.704. Y_{ft-t+3} = 4.84 %. $ft - t{+}3$ = 68 days.
The bond paid a coupon on 15/02/94 so c = \$4.75. $I_t[Y_{ft-ct}]$ = 4.834 %. $ft{-}ct$ = 29.
Substitute these values into $B_{(CAC)ft}$.

$$B_{(CAC)ft} = \$122.704 \times (1+((0.0484\,\%)\times(68/365))) - \$4.75\,(1+((0.04834\,\%)\times(29/365))).$$

Forward price = \$119.042176. Forward equivalent yield 16/03/94 = 6.835 %.

This calculation was performed for every bond in the pricing basket.

(iii) Implied forward bond yields as at 16/03/94 (Cash Settlement date)

Maturity	Forw. Price	Equiv. Yld.
15/10/02	124.6489	6.80889151
*15/08/03	119.042176	6.83454206
15/09/04	115.575457	6.8939599
15/07/05	105.660137	6.92912396
	Av. yield	6.86662936

(iv) Closing SFE futures price vs calculated Cash and Carry price 04/01/94

The spread = Market yield - Cash and Carry yield. This has been converted to basis points. The spread in Table (iv) should therefore be read as 4/10 of 1 basis point.

Futures Price	93.13
Yld. :100-Price	6.87
CAC Fwd yield	6.866
Spread	-0.4

(i) The cashflow structure of the 15/08/03 bond as at 04/01/94

Days	Date	Cash Flow	Discount	Ann. Yield	PV
39	15/02/94 [(a)]	4.75	0.994996	4.807	4.726
220	*15/08/94	4.75	0.976107	4.894	4.615
404	15/02/95	4.75	0.944663	5.28	4.487
.	.	.	.	.	.
.	.	.	.	.	.
3142	15/08/02	4.75	0.557677	7.019	2.645
3326	15/02/03	4.75	0.537725	7.046	2.554
3507	15/08/03	104.75	0.51906	7.063	54.372

Bond Price	122.704

(a) Note that Zd_{m-ft} the implied forward zero coupon factor could not be calculated in this case. The 15/02/94 cashflow falls before the ten year bond futures cash settlement date so that $m < ft$.

(ii) Calculating the implied forward discount factor for the 15/08/94 date

$$Zd_{m-ft} = \frac{Zd_{m-t+3}}{Zd_{ft-t+3}} \qquad (3.7)$$

Where ;

Zd_{m-t+3} = The zero coupon discount factor for the 220 day period, m-t+3, between 07/01/94 to 15/08/94, with a zero coupon yield of 4.894 % = (1 / 1 +((0.04894)(220/365))) = 0.976107.

Zd_{ft-t+3} = The zero coupon discount factor for the 68 day period, ft - t+3, between the bond settlement date 07/01/94 and the ten year bond futures cash settlement date 16/03/94. The zero coupon yield was 4.84% = (1 / 1 + ((0.0484)(68/365))) = 0.99123309.

Zd_{m-ft} = 0.976107 / 0.99123301 = 0.98020407. This represented the implied forward zero coupon discount factor in 68 days time for the 152 day period, m-ft, between 16/03/94 and 15/08/94.

(iii) Substitute the forward discount factors into the cashflow structure

Days	Date	Cash Flow	Discount	Ann. Yield	PV
-29	15/02/94	0	0	0	0
152	*15/08/94	4.75	0.980204	4.918	4.655
336	15/02/95	4.75	0.952994	5.369	4.526
.	.	.	.	.	.
.	.	.	.	.	.
3074	15/08/02	4.75	0.56261	7.068	2.672
3258	15/02/03	4.75	0.542481	7.092	2.576
3439	15/08/03	104.75	0.523651	7.107	54.852

Bond Price	119.021

This calculation was performed for every bond in the pricing basket.

(iv) The model derived SFE ten year bond futures equivalent yield.

Futures Price	93.13
Yld. :100-Price	6.87
Model yield	6.869
Spread Zero	-0.1

(i) 04/01/94 Zero coupon discount factors

Days	Date	Discount	Yield
8	15/01/94	0.998974	4.79275
39	15/02/94*	0.994996	4.807331
67	15/03/94	0.991398	4.819126
98	15/04/94	0.987434	4.822684
128	15/05/94	0.983596	4.829541
159	15/06/94	0.979663	4.829647
189	15/07/94	0.975799	4.84484
220	15/08/94	0.971611	4.894187
251	15/09/94	0.967422	4.934196
281	15/10/94	0.963368	4.966954
312	15/11/94	0.95918	4.996478
342	15/12/94	0.955126	5.021994
.	.	.	.
.	.	.	.
1500	15/02/98	0.776501	6.34869
1528	15/03/98	0.772488	6.36035
.	.	.	.
.	.	.	.
4695	15/11/06	0.410425	7.168725

To construct the two week Yield Error Margin statistic for the 04/01/94 trading day, whose bond settlement date t+3 was 07/01/94, the assumptions presented in Equation (3.14) and Equation (3.15) were activated. The trading date 18/01/94 was two weeks from 04/01/94. Its bond settlement date t+3 was 21/01/94.

$$Zd_{m-N} = \frac{Zd_{m-t+3}}{Zd_{N-t+3}} \tag{3.14}$$

Zd_{m-t+3} = The zero coupon discount factor for the 39 day period, m-t+3, between 07/01/94 and 15/02/94, with a zero coupon yield of 4.807% = (1 / 1 +((0.04807)(39/365))) = 0.994996.

Zd_{N-t+3} = The zero coupon discount factor for the 14 day period, $N - t+3$, between 07/01/94 and 21/01/94 was linearly interpolated from the 15/01/94 and 15/02/94 dates. It was found to equal 0.9982. Today's fourteen day zero coupon discount factor was used as the basis for all the two week forward zero coupon discount factors.

Zd_{m-N} = 0.994996 / 0.9982 = 0.996785. This represented the implied forward zero coupon discount factor in 14 days time for the 25 day period between, m-N, 15/02/94 and 21/01/94. This discount factor was recorded in Table (ii) below. The methodology associated with Equation (3.14) was applied to all those zero coupon discount factors associated with the 07/01/94 term structure whose dates $m > N$.

(ii) Implied 2 Week forward zero coupon discount curve 18/01/94

Days	Date	Discount	Yield
25	15/02/94*	0.996785	4.814438
53	15/03/94	0.99318	4.825595
84	15/04/94	0.989209	4.827358
114	15/05/94	0.985364	4.833827
145	15/06/94	0.981425	4.833028
175	15/07/94	0.977554	4.848857
206	15/08/94	0.973358	4.900956
237	15/09/94	0.969161	4.942446
267	15/10/94	0.9651	4.975997
298	15/11/94	0.960904	5.005969
328	15/12/94	0.956843	5.031709
•	•	•	•
1486	15/02/98	0.777897	6.36344
1514	15/03/98	0.773877	6.374937
•	•	•	•
4681	15/11/06	0.411163	7.175905

Appendix F continued:

(iii) The actual observed term structure as at 18/01/94

Days	Date	Discount	Yield
25	15/02/94	0.996821	4.758134
53	15/03/94	0.993259	4.767967
84	15/04/94	0.989316	4.778076
114	15/05/94	0.985474	4.796434
145	15/06/94	0.981517	4.808064
175	15/07/94	0.977729	4.809739
206	15/08/94	0.973706	4.834549
237	15/09/94	0.969654	4.86033
267	15/10/94	0.965733	4.881965
298	15/11/94	0.961682	4.901975
328	15/12/94	0.957761	4.919713
•	•	•	•
1486	15/02/98	0.783432	6.178346
1514	15/03/98	0.779417	6.192168
•	•	•	•
4681	15/11/06	0.426196	6.876231

The difference between today's two week implied forward average ten year bond futures basket yield for the 18/01/94 trading date and the actual average ten year bond futures basket yield observed on 18/01/94 represented the two week Yield Error Margin statistic for the 04/01/94 trading date. Consistent with Appendix E the nearby March 1994 ten year bond futures contract was used to provide the underlying Commonwealth coupon bond pricing basket.

(iv) Implied forward average basket yield vs Actual average basket yield

Bond	Implied Yield	Actual Yield	Pts. Diff.
15/10/02	6.7627	6.51	25.27
15/08/03	6.7917	6.54	25.17
15/09/04	6.85175	6.58	27.175
15/07/05	6.8906	6.62	27.06
Average	6.8241875	6.5625	26.16875

Appendix F continued:

$$\text{YEM}_t = \left(\left(\sum_{i=1}^{n} YB_{IFNt}\right) / \, n\right)_t \; - \; \left(\left(\sum_{i=1}^{n} YB_{ANt}\right) / \, n\right)_{Nt} \tag{3.15}$$

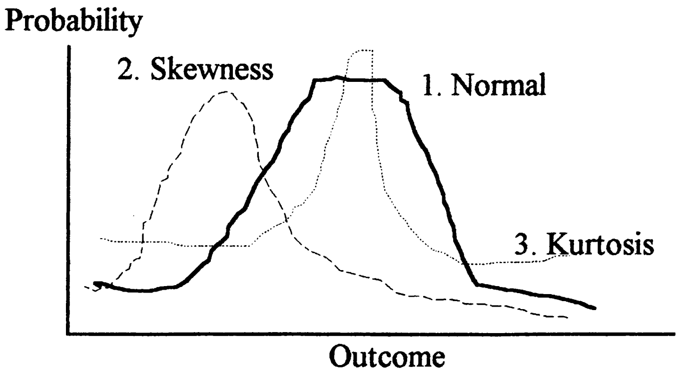
YB_{IFNt} = The two week implied forward average basket yield for the 18/01/94 trading date = 6.824 %

YB_{ANt} = The actual average basket yield observed on the two week forward date 18/01/94 = 6.5625 %

YEM_t = The forecasting error two weeks forward from today's 04/01/94 trading date = 6.824 % - 6.5625 % = 26.17 yield basis points. The error was around 1/4 of 1.00 %.

Today's implied two week forward average basket yield was 26.2 basis points higher than the actual average basket yield that was observed on 18/01/94. The market had expected yields to rise slightly over the two week interval when in fact they actually fell. This result represented the first input into the two week Yield Error Margin database for the sample period 01/01/94 to 31/12/94.

APPENDIX G : The characteristics of the normal distribution



The benchmark normal distribution has a density function with the following characteristics;

1. It was symmetrical and bell shaped.

2. The distribution was fully described by its mean and variance. The higher moments of skewness and kurtosis can be ignored. The mean, mode and median were all equal.

3. The probability that a single observation of a normally distributed variable x will lie within two standard deviations of its mean was approximately 95 %. This implies that 95 % of the outcomes of a normal distribution were within two standard deviations of the mean.

APPENDIX H: Calculating implied volatility from SFE 10 year bond option prices

The Scenario:

On 21/08/95 the ten year bond futures contract was trading at a price of 90.91. The Sept. 1995 90.75 call strike was trading on a bid / offer basis of 28 / 31 basis points. Attachment five incorporates these prices. The time to expiry of the call was 25 days (21/08/95 to 15/09/95). The floor quote for volatility was given as 11.20 %. The objective of the research was to determine if the floor quoted volatility was consistent with a mid - point 90.75 call strike option price of 29.5 basis points.

1. The first step in the solution process involved converting the ten year bond futures price and strike price into SFE By - Laws bond prices. For example the ten year bond futures price 90.91 was converted to a semi - annual yield (100 - 90.91) / 200 = 0.04545.

	Futures	Exercise
Y	0.04545	0.04625
V	0.9565259	0.9557945
V^{20}	0.4110879	0.4048467
SFE Price	118852.96	117693.75

2. The next step required calculating d_1 and d_2 using the initial 11.2 % volatility quote.

Calculating d1 and d2	
LN F/X	0.0098012311
$0.5 \times \text{Vol.}^2 \times T\text{-}t$	0.00042958904110
Top of d_1	0.0102308201
Top of d_2	0.0093716421
$\text{Vol.} \times \sqrt{T\text{-}t}$	0.02931174
d_1	0.349034901
d_2	0.319723161

3. The third step transformed $d1$ and $d2$ into $N(d1)$ and $N(d2)$ respectively. The normal distribution was approximated by a polynomial function. Hull, (1993) presents a 5th order polynomial that was accurate to six decimal places.

$$N(x) = 1 - N'(x)\,(a_1K + a_2K^2 + a_3K^3 + a_4K^4 + a_5K^5)$$

If $d1, d2 > 0$ than $N(d1, d2) = N(x)$

If $d1, d2 < 0$ than $N(d1, d2) = 1 - N(x)$

Where ;

$N'(x) = 1 / \sqrt{2\pi} \times e^{-(x^{\wedge}2)/2}$

$K = 1 / 1 + \gamma x$

$\gamma = 0.2316419$

$a_1 = 0.319381530$ $a_2 = -0.356563782$ $a_3 = 1.781477937$

$a_4 = -1.821255978$ $a_5 = 1.330274429$

$N'(d1)$=	$0.3989423e\text{-}(d1^{\wedge}2)/2$	**0.375366944**
K	$1/1+0.236419 \times d1$	**0.925196813**
Polynomial	**Brackets fn(a_i)**	**0.968469976**
$N(d1)$	$1\text{-}N'(d1) \times$brackets	**0.636468385**

$N'(d2)$=	$0.3989423e\text{-}(d2^{\wedge}2)/2$	**0.379064091**
K	$1/1+0.236419 \times d2$	**0.931045573**
Polynomial	**Brackets fn(a_i)**	**0.988194804**
$N(d2)$	$1\text{-}N'(d2) \times$brackets	**0.625410835**

Appendix H continued:

4. Step four substituted the SFE By - Law bond prices from step one and $N(d1, d2)$ from step three into Equation (3.19) to calculate the dollar value of the 90.75 strike call option.

$$C = FN(d1) - XN(d2) \tag{3.19}$$

$$C = \$118{,}852.96 \times (0.636468385) - \$117{,}693.75 \times (0.625410835)$$

$$C = \$75{,}646.15 - \$73{,}606.95 = \$2039.20$$

5. Step five converted the dollar price to a market consistent quote in yield basis points terms. The dollar price was divided by the price sensitivity of the SFE By - Laws bond price to a one basis point change in yield at the strike bond price. For the 90.75 strike a one basis point change was worth $72.00.

In market terms the September 1995 90.75 call would be quoted as $2039.20 / $72.00 = 28.3 points with an assumed implied volatility of 11.2 %.

6. Step six repeated steps one to five because the 11.2 % volatility quote did not generate a mid - point option price quote.

A 11.6 % volatility quote was found to be consistent with the 29.5 mid-point option price.

APPENDIX I: Calculating 30 day historical volatility July to August 95

Date	Futures Close	Futures Yield	PRICE	Ln
10/07/95	91.19	8.81	120919.51	
11/07/95	91.15	8.85	120621.30	-0.002469
12/07/95	91.14	8.86	120546.90	-0.000617
13/07/95	91.145	8.855	120584.09	0.0003085
14/07/95	91.15	8.85	120621.30	0.0003085
17/07/95	90.97	9.03	119291.70	-0.011084
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
11/08/95	90.955	9.045	119181.81	0.0052167
14/08/95	90.915	9.085	118889.44	-0.002456
15/08/95	90.865	9.135	118525.36	-0.003067
16/08/95	90.885	9.115	118670.81	0.0012264
17/08/95	90.86	9.14	118489.03	-0.001533
18/08/95	90.87	9.13	118561.70	0.0006131
21/08/95	90.9	9.1	118780.05	0.00184

Av. Daily Vol.	0.005527612
Annual Vol.	8.7 %

APPENDIX J : ATM put option price using simulated YEM database in Table 3.2

YEM Pts.	Bond Yld. Distr.	Bond price Distribution	Payoff [*X-B*, 0]	YEM weight payoff	Frequency Outcome	Relative Frequency
30	7.169	134069.05	3664.02	219.84	6	0.06
20	7.069	134931.55	2801.52	252.14	9	0.09
10	6.969	135801.49	1931.58	424.95	22	0.22
0	6.869	136678.97	1054.10	284.61	27	0.27
-10	6.769	137564.04	169.03	35.50	21	0.21
-20	6.669	138456.80	0.00	0.00	12	0.12
-30	6.569	139357.30	0.00	0.00	3	0.03
					100	1.0

X = 93.25	137733.07	*$ / PRICE*	*1217.04*
		Pts. / $89.07	13.7

$$\left(\left(\sum_{i=1}^{n} Y_{B(distn.)ot}\right)/n\right)_{jt} = \left(\left(\sum_{i=1}^{n} Y_{B(model)ot}\right)/n\right)_{t} \pm YEM_{jt} \qquad (3.22)$$

1. The first step in pricing the SFE ten year bond futures ATM put option was to calculate the model ten year bond futures average basket yield associated with the option expiration date *ot*. The shadowed area in the main body of the table shows this equal to 6.869 %.

$$\left(\left(\sum_{i=1}^{n} Y_{B(model)ot}\right)/n\right)_{t} = 6.869\ \%$$

2. The distribution of the bond futures average basket yields was generated by modifying 6.869 % by $\pm$ the recorded Yield Error Margins. For example in the case where Yield Error Margin equalled +10 basis points the *j*th ten year bond futures average basket yield would be equal to 6.969 % .

$$\left(\left(\sum_{i=1}^{n} Y_{B(model)ot}\right)/n\right)_{t} = 6.869\ \% + YEM_{jt} = +10 \text{ basis points or } 0.10\ \%$$

$$\left(\left(\sum_{i=1}^{n} Y_{B(distn.)ot}\right)/n\right)_{jt} = 6.969\ \%$$

Appendix J continued:

3.The simulated Yield Error Margin database in Table 3.2 had a range of only ± 30 basis points. Thus k = 7 in this example. This generated a distribution of seven average basket yields.

4.These yields were converted to SFE By - Laws bond prices and compared to the bond price associated with the ATM option strike 93.25 / 6.75 %. Its SFE By - Law price was \$137,733.07. The payoff [X - B , 0] was determined by subtracting the individual *j*th bond price from the option strike bond price. Where $X < B$ corresponded with the put option finishing out of the money at expiration *ot*.

$$P_{(model)t} = \sum_{j=1}^{k} p_j \times (X - B_j)_{ot} \qquad (3.23)$$

$$\left(\left(\sum_{i=1}^{n} YB_{(distn.)ot} \right) / n \right)_{jt} = 6.969\ \% \quad \text{Bond price equivalent } B_j = \$135{,}801.49$$

$$(X - B_j)_{ot} = \$137{,}733.07 - \$135{,}801.49 = \$1931.58$$

5.The bond price payoffs were then weighted by their individual probabilities so that the expected payoff of the ATM put at the expiration date *ot* could be determined.

$$(X - B_j)_{ot} = \$1931.58 \qquad p_j = 0.22$$

$$p_j \times (X - B_j)_{ot} = \$424.95$$

6.Summing these ITM probability weighted payoffs produced the dollar option price \$1217.04. Consistent with Appendix H the dollar price was divided by the dollar value of a 1 basis point change in yield at the option strike price. The model 93.25 put option settlement price was calculated as \$1217.04 / \$89.07 = 13.7 yield basis points per \$100,000 bond futures contract unit.

$$P_{(model)t} = \$1217.04 / \$89.07 = 13.7 \text{ basis points.}$$

APPENDIX K: Market data used and additional assumptions

Section 3.2 : Derivation of the Australian Market Term Structure

Data $n = 248$	Source	Purpose	Assumptions
Bank Bill 1 - 6 months.	Reuters Page: BBSW available 10.15 a.m.	To provide short rate "anchor" data for term structure.	Bank Bill - 0.02% (2 points) equivalent to Treasury note.
1 year Swap Rate	Telerate Tactician Closing Data 4.30 p.m.	" As above "	Swap - 0.10 % (10 points) equivalent to govt. bond.
Commonwealth Government bond data for so called "Hot stocks".	Telerate Tactician closing data 4.30 p.m. and Reuters Page:GLTT.	To provide coupon bond data for the derivation of the term structure.	" Hot stocks" were seen as the most actively traded Commonwealth bonds.
SFE ten year bond futures	SFE via Knightridder Financial Services. Closing Data 4.30 p.m.	For testing Hypotheses 1 - 3. Term structure model vs observed futures prices.	Convert Futures price to an equivalent yield. (100 - Closing futures price)

N.B. The full data set was actually from 01/01/94 to 31/01/95. This was used in the forecasting work in Chapter 4.

Section 3.3: Calculation of the Yield Error Margin database

Data $n = 248$ to 208	Source	Purpose	Assumptions
The Commonwealth Government bonds that made up the pricing basket of the "nearby" 10 year Bond futures contract.	Telerate Tactician closing data 4.30 p.m. and Reuters Page:GLTT	To compare actual N- period forward yields with term structure based N- period forecasts. Testing hypothesis 4.	Only incorporated a subset of the term structure for testing.
SFE ten year bond futures	SFE via Knightridder Financial Services. Closing Data 4.30 p.m.	To test Hypotheses 5 and 6 whether the ten year bond futures distribution was normal.	Convert Futures price to an equivalent yield. (100 - Closing futures price)
Daily SFE ten year bond futures option settlement prices.	SFE Statistics department	To allow calculation of an implied option volatility series and to test Hypothesis 7.	These option settlement prices reflected closing futures prices.
SFE ten year bond futures	SFE via Knightridder Financial Services. Closing Data 4.30 p.m.	To allow calculation of historical bond futures volatility series For testing Hypothesis 8.	Convert Futures price to an equivalent yield. (100 - Closing futures price) Convert this yield to SFE By- Law bond prices.

Section 3.4: Pricing Fixed Interest Contingent Claims

Data $n = 217$	Source	Purpose	Assumptions
Daily SFE ten year bond futures option settlement prices. Calls and Puts : ATM , ITM and OTM.	SFE Statistics department.	To allow testing of model approximations of market prices and Hypotheses 9 and 10.	The definition of ATM as that strike $\pm$ 12.5 basis points from closing futures price.

Section 3.5 Rewards from Hedging Fixed Interest Portfolios

Data $n = 163$	Source	Purpose	Assumptions
Daily SFE ten year bond futures put option settlement prices.	SFE Statistics department.	To allow testing of hedged vs unhedged fixed interest portfolio via hypothesis 11.	The YEM distribution was used to determine the appropriate put option strike.

APPENDIX L : Daily results model futures prices vs market

January 1994 SFE 10 year bond futures pricing

Date	Fut. Price[a]	Fut. Yield [b]	CAC [c]	Spread [d]	Model	Spread
4/01/94	93.13	6.87	6.8666303	0.3369739	6.8692522	0.074777
5/01/94	93.245	6.755	6.7488177	0.6182255	6.7513541	0.3645907
6/01/94	93.235	6.765	6.7565588	0.8441152	6.7583733	0.662675
7/01/94	93.21	6.79	6.7884674	0.1532619	6.7909272	-0.09272
10/01/94	93.235	6.675	6.6747865	0.0213477	6.6772248	-0.22248
11/01/94	93.34	6.66	6.6638616	-0.386164	6.6660883	-0.608825
12/01/94	93.395	6.605	6.6011811	0.3818907	6.6034345	0.15655
13/01/94	93.39	6.61	6.6156354	-0.5635404	6.618122	-0.8122
14/01/94	93.38	6.62	6.6229239	-0.2923883	6.625115	-0.5115
17/01/94	93.375	6.625	6.6220081	0.2991863	6.624191	0.0809
18/01/94	93.405	6.595	6.600945	-0.5945006	6.603075	-0.8075
19/01/94	93.445	6.555	6.555547	-0.0547006	6.5575398	-0.253975
20/01/94	93.415	6.585	6.5960169	-1.1016913	6.597993	-1.2993
21/01/94	93.445	6.555	6.564001	-0.9001044	6.5659145	-1.09145
24/01/94	93.45	6.55	6.5528908	-0.2890787	6.55483	-0.483
25/01/94	93.58	6.42	6.4285853	-0.8585324	6.430459	-1.0459
27/01/94	93.55	6.45	6.463722	-1.3722037	6.4654618	-1.546175
28/01/94	93.545	6.455	6.4757639	-2.0763858	6.477463	-2.2463
31/01/94	93.6	6.4	6.4166422	-1.6642239	6.4183158	-1.831575

Av. Spr.	-0.3946586	Av. Spr.	-0.6059688
Std. dev.	0.78970328	Std. dev.	0.77087228
Correl. Cx.	0.99925179	Correl. Cx.	0.99925638

(a) Fut. Price = 4.30 p.m. closing 10 year bond futures price (b) Fut. Yield = 100 - Price. (c) CAC = The cash and carry futures implied yield. Model = Model derived futures equivalent yield. (d) Spread = basis point spread: Observed futures yield - model yield. The results in the table should be read in yield point terms i.e. 0.3 = 3/10 of 1 basis point. The basic summary statistics were defined in Table 4.2..

Appendix L continued:

February 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/02/94	93.565	6.435	6.4481422	-1.3142248	6.4479715	-1.29715
2/02/94	93.53	6.47	6.4851065	-1.5106536	6.486626	-1.6626
3/02/94	93.505	6.495	6.5148389	-1.9838934	6.516325	-2.1325
4/02/94	93.48	6.52	6.531862	-1.1862007	6.533246	-1.3246
7/02/94	93.375	6.625	6.6425663	-1.7566255	6.6439721	-1.8972075
8/02/94	93.395	6.605	6.6213859	-1.6385879	6.6236382	-1.863815
9/02/94	93.42	6.58	6.5892809	-0.9280916	6.5905188	-1.051875
10/02/94	93.475	6.525	6.5277991	-0.2799148	6.529015	-0.4015
11/02/94	93.45	6.55	6.5574412	-0.7441199	6.5586175	-0.86175
14/02/94	93.495	6.505	6.5160554	-1.1055388	6.5172153	-1.221525
15/02/94	93.435	6.565	6.5758841	-1.0884128	6.5760253	-1.1025325
16/02/94	93.435	6.565	6.5687899	-0.3789917	6.569771	-0.4771
17/02/94	93.325	6.675	6.6841009	-0.9100926	6.685062	-1.0062
18/02/94	93.1	6.9	6.9027342	-0.2734172	6.9036425	-0.36425
21/02/94	93.035	6.965	6.972464	-0.7463969	6.9733615	-0.83615
22/02/94	93.08	6.92	6.9263882	-0.6388167	6.9271358	-0.713575
23/02/94	93.085	6.915	6.9212603	-0.6260305	6.9219683	-0.696825
24/02/94	92.925	7.075	7.0788978	-0.3897808	7.0795493	-0.454925
25/02/94	92.805	7.195	7.1986579	-0.365789	7.1992685	-0.42685
28/02/94	92.915	7.085	7.0820541	0.2945894	7.082637	0.2363

Av. Spr.	-0.8785495	Av. Spr.	-0.9778315
Std. dev.	0.5786758	Std. dev.	0.60387639
Correl. Cx.	0.99982733	Correl. Cx.	0.99981216

Appendix L continued:

March 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/03/94	92.815	7.185	7.1790361	0.5963873	7.1795975	0.54025
2/03/94	92.71	7.29	7.2941674	-0.4167443	7.294587	-0.4587
3/03/94	92.845	7.155	7.1551055	-0.0105545	7.155475	-0.0475
4/03/94	92.56	7.44	7.4374798	0.2520191	7.437806	0.2194
7/03/94	92.69	7.31	7.3136127	-0.361267	7.3139045	-0.39045
8/03/94	92.79	7.21	7.2123415	-0.2341522	7.2125703	-0.257025
9/03/94	92.595	7.405	7.4096316	-0.4631635	7.4097113	-0.471125
10/03/94	92.5	7.5	7.4985044	0.1495636	7.49857	0.143
11/03/94	92.44	7.56	7.534768	2.5231997	7.5382838	2.171622
14/03/94	92.565	7.435	7.4200721	1.4927921	7.4232428	1.17572
15/03/94	92.395	7.605	7.6006868	0.4313188	7.6038916	0.11084
16/03/94	92.59	7.41	7.40668	0.3319989	7.409794	0.0206
17/03/94	92.515	7.485	7.4867008	-0.1700835	7.4896806	-0.46806
18/03/94	92.47	7.53	7.51641	1.3589989	7.5195512	1.0448755
21/03/94	92.29	7.71	7.716725	-0.6724964	7.7198238	-0.98238
22/03/94	92.32	7.68	7.6846482	-0.4648241	7.6877306	-0.77306
23/03/94	92.5	7.5	7.4980951	0.1904933	7.501034	-0.1034
24/03/94	92.315	7.685	7.6810272	0.3972839	7.684005	0.0995
25/03/94	92.235	7.765	7.7703051	-0.5305126	7.7732642	-0.82642
28/03/94	92.005	7.995	8.0006174	-0.5617373	8.0035411	-0.854114
29/03/94	92.035	7.965	7.9657858	-0.0785765	7.9685797	-0.357972
30/03/94	91.995	8.005	7.9969467	0.8053299	7.9998628	0.51372
31/03/94	91.955	8.045	8.0472564	-0.2256431	8.05003	-0.503

Av. Spr.	0.18867959	Av. Spr.	-0.0197252
Std. dev.	0.77249634	Std.dev.	0.74061647
Correl. Cx.	0.99957846	Correl. Cx.	0.99962882

Appendix L continued:

April 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
5/04/94	91.675	8.325	8.3357656	-1.0765638	8.3385268	-1.35268
6/04/94	91.845	8.155	8.1596319	-0.4631877	8.1621385	-0.7138455
7/04/94	91.84	8.16	8.1543229	0.5677069	8.1568964	0.31036
8/04/94	91.845	8.155	8.157483	-0.2482986	8.1600226	-0.50226
11/04/94	91.76	8.24	8.2424759	-0.2475948	8.2450382	-0.50382
12/04/94	91.815	8.185	8.1904421	-0.5442058	8.1928742	-0.78742
13/04/94	91.92	8.08	8.0740002	0.5999771	8.076332	0.3668
14/04/94	91.775	8.225	8.2270073	-0.2007285	8.2292968	-0.42968
15/04/94	91.675	8.325	8.3242122	0.0787845	8.326446	-0.1446
18/04/94	91.8	8.2	8.1890983	1.090171	8.1913694	0.86306
19/04/94	91.44	8.56	8.5625199	-0.2519889	8.5636888	-0.3688762
20/04/94	91.405	8.595	8.6037503	-0.875028	8.6048566	-0.98566
21/04/94	91.61	8.39	8.3858306	0.4169409	8.3878968	0.21032
22/04/94	91.7	8.3	8.3028003	-0.2800268	8.3047752	-0.47752
26/04/94	91.775	8.225	8.2197149	0.5285138	8.2217219	0.327812
27/04/94	91.745	8.255	8.2565507	-0.155073	8.2583938	-0.33938
28/04/94	91.745	8.255	8.2595854	-0.4585391	8.2613234	-0.63234
29/04/94	91.485	8.515	8.5167539	-0.1753898	8.5185524	-0.35524

Av. Spr.	-0.0941406	Av. Spr.	-0.3063872
Std. dev.	0.55179815	Std. dev.	0.54694599
Correl. Cx.	0.99939731	Correl. Cx.	0.99937566

Appendix L continued:

May 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
2/05/94	91.39	8.61	8.6047356	0.5264411	8.6064916	0.35084
3/05/94	91.44	8.56	8.554516	0.548397	8.5562476	0.37524
4/05/94	91.14	8.86	8.8488528	1.1147166	8.8505644	0.94356
5/05/94	91.13	8.87	8.8740848	-0.4084793	8.8758066	-0.58066
6/05/94	91.1	8.89	8.87971	1.0289979	8.880654	0.9346
9/05/94	90.97	9.03	9.0193984	1.0601563	9.0204061	0.959394
10/05/94	91.225	8.775	8.774051	0.0949019	8.7755072	-0.05072
11/05/94	91.14	8.86	8.8525135	0.7486502	8.8538376	0.61624
12/05/94	91.015	8.985	8.9784618	0.6538174	8.9797326	0.52674
13/05/94	91.085	8.915	8.9161504	-0.1150389	8.9173904	-0.23904
16/05/94	91.305	8.695	8.6915768	0.3423199	8.692788	0.2212
17/05/94	91.225	8.775	8.7749518	0.0048159	8.7761228	-0.11228
18/05/94	91.545	8.455	8.455018	-0.0017982	8.456047	-0.1047
19/05/94	91.48	8.52	8.5141755	0.5824486	8.515171	0.4829
20/05/94	91.525	8.475	8.4723243	0.2675723	8.4732794	0.17206
23/05/94	91.39	8.61	8.6099416	0.0058374	8.6108868	-0.08868
24/05/94	91.25	8.75	8.7517257	-0.1725736	8.7526134	-0.26134
25/05/94	91.185	8.815	8.817542	-0.2541984	8.818294	-0.329396
26/05/94	91.325	8.675	8.680886	-0.5885964	8.6811702	-0.61702
27/05/94	91.225	8.775	8.7800447	-0.5044682	8.780248	-0.5248
30/05/94	91.3	8.7	8.6999744	0.0025642	8.7006158	-0.06158
31/05/94	91.175	8.825	8.8290996	-0.4099605	8.8297292	-0.47292

Av. Spr.	0.20575105	Av. Spr.	0.09725627
Std. dev.	0.51825922	Std. dev.	0.49750039
Correl. Cx.	0.99948352	Correl. Cx.	0.99952473

Appendix L continued:

June 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/06/94	91.085	8.915	8.916653	-0.1652951	8.9171396	-0.21396
2/06/94	91.13	8.87	8.8669105	0.3089522	8.867461	0.2539
3/06/94	91.2	8.8	8.8080324	-0.8032415	8.8054192	-0.54192
6/06/94	91.41	8.59	8.5826663	0.7333666	8.5829964	0.70036
7/06/94	91.33	8.67	8.6653465	0.4653511	8.665618	0.4382
8/06/94	91.33	8.67	8.6591646	1.0835393	8.6592944	1.07056
9/06/94	91.13	8.87	8.8673659	0.2634146	8.8677032	0.22968
10/06/94	91.08	8.92	8.9165714	0.3428614	8.9199615	0.003852
14/06/94	90.885	9.115	9.1054618	0.9538213	9.108926	0.6074
15/06/94	90.99	9.01	8.9940863	1.5913706	8.9974668	1.25332
16/06/94	90.69	9.31	9.3002992	0.9700844	9.3037682	0.62318
17/06/94	90.605	9.395	9.3929489	0.205109	9.3946938	0.03062
20/06/94	90.215	9.785	9.7962358	-1.1235759	9.7997688	-1.47688
21/06/94	90.025	9.975	9.9918115	-1.6811499	9.9954404	-2.04404
22/06/94	90.08	9.92	9.9185385	0.1461491	9.9230502	-0.30502
23/06/94	90.225	9.775	9.7683151	0.668487	9.7719342	0.30658
24/06/94	90.215	9.785	9.7763306	0.8669415	9.7835912	0.14088
27/06/94	89.85	10.15	10.146006	0.399408	10.149922	0.0078
28/06/94	90.06	9.94	9.9279651	1.2034864	9.9317452	0.82548
29/06/94	90.175	9.825	9.8007867	2.4213304	9.8045988	2.04012
30/06/94	90.195	9.805	9.7817461	2.3253915	9.786282	1.8718

Av. Spr.	0.53218105	Av. Spr.	0.2772339
Std. dev.	0.98863211	Std. dev.	0.94640555
Correl. Cx.	0.99982249	Correl. Cx.	0.99984201

Appendix L continued:

July 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/07/94	90.19	9.81	9.7803998	2.9600156	9.7842344	2.57656
4/07/94	90.13	9.87	9.852316	1.768403	9.8559754	1.40246
5/07/94	90.14	9.86	9.8455133	1.4486738	9.846021	1.397898
6/07/94	90.245	9.755	9.7378932	1.7106835	9.7410432	1.39568
7/07/94	90.265	9.735	9.7253511	0.9648946	9.7275092	0.74908
8/07/94	90.275	9.725	9.7139155	1.1084547	9.7168734	0.81266
11/07/94	90.12	9.88	9.8647635	1.5236515	9.8678224	1.21776
12/07/94	90.16	9.84	9.8251761	1.4823875	9.8281822	1.18178
13/07/94	90.225	9.775	9.7596499	1.535009	9.7633118	1.16882
14/07/94	90.255	9.745	9.7292281	1.5771868	9.7319916	1.30084
15/07/94	90.475	9.525	9.5136376	1.1362389	9.5162448	0.87552
18/07/94	90.545	9.455	9.4445676	1.0432369	9.4471796	0.78204
19/07/94	90.485	9.515	9.5056558	0.934425	9.5082406	0.67594
20/07/94	90.465	9.535	9.5320751	0.2924934	9.5345052	0.04948
21/07/94	90.285	9.715	9.7177765	-0.2776518	9.7201718	-0.51718
22/07/94	90.34	9.66	9.6533682	0.6631824	9.6557704	0.42296
25/07/94	90.295	9.705	9.7012069	0.3793143	9.7036158	0.13842
26/07/94	90.355	9.645	9.6427214	0.2278578	9.6451692	-0.01692
27/07/94	90.49	9.51	9.5034098	0.6590179	9.5056572	0.43428
28/07/94	90.285	9.715	9.7222276	-0.7227576	9.7233464	-0.83464
29/07/94	90.34	9.66	9.6658173	-0.5817259	9.6679418	-0.79418

Av. Spr.	0.94442816	Av. Spr.	0.68663133
Std. dev.	0.86751148	Std. dev.	0.82418186
Correl. Cx.	0.997977	Correl. Cx.	0.998188

Appendix L continued:

August 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
2/08/94	90.52	9.48	9.4749756	0.5024386	9.4769866	0.30134
3/08/94	90.395	9.605	9.6020441	0.2955891	9.6038428	0.11572
4/08/94	90.69	9.31	9.2969938	1.3006151	9.2988108	1.11892
5/08/94	90.62	9.38	9.3727949	0.7205056	9.374576	0.5424
8/08/94	90.43	9.57	9.5652713	0.4728667	9.5670472	0.29528
9/08/94	90.54	9.46	9.449337	1.0663011	9.4511162	0.88838
10/08/94	90.52	9.48	9.4667383	1.3261732	9.4683662	1.16338
11/08/94	90.455	9.545	9.5463863	-0.1386307	9.5479592	-0.29592
12/08/94	90.375	9.625	9.6193094	0.5690646	9.620911	0.4089
15/08/94	90.45	9.55	9.5365033	1.3496716	9.5380408	1.19592
16/08/94	90.425	9.575	9.5703868	0.4613219	9.571959	0.3041
17/08/94	90.745	9.255	9.2572681	-0.2268119	9.2587764	-0.37764
18/08/94	90.705	9.295	9.2963894	-0.1389387	9.2977728	-0.27728
19/08/94	90.59	9.41	9.4098157	0.0184299	9.4111796	-0.11796
22/08/94	90.655	9.345	9.3478365	-0.2836456	9.3491558	-0.41558
23/08/94	90.585	9.415	9.4088257	0.6174329	9.410105	0.4895
24/08/94	90.625	9.375	9.3719729	0.3027135	9.3730492	0.19508
25/08/94	90.695	9.305	9.314006	-0.9005985	9.3150414	-1.00414
26/08/94	90.61	9.39	9.3929945	-0.2994469	9.3934214	-0.34214
29/08/94	90.67	9.33	9.3371117	-0.7111721	9.3380432	-0.80432
30/08/94	90.62	9.38	9.3798221	0.0177896	9.3805721	-0.057208
31/08/94	90.615	9.385	9.3912034	-0.6203388	9.3919006	-0.69006

Av. Spr.	0.25915137	Av. Spr.	0.11984873
Std. dev.	0.65414095	Std. dev.	0.62781684
Correl. Cx.	0.99847207	Correl. Cx.	0.99858157

Appendix L continued:

September 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/09/94	90.555	9.445	9.4399574	0.5042589	9.4405824	0.44176
2/09/94	90.545	9.455	9.4584364	-0.3436443	9.458998	-0.3998
5/09/94	90.455	9.545	9.542967	0.2033033	9.5435552	0.14448
6/09/94	90.48	9.52	9.5133538	0.6646211	9.514183	0.5817
7/09/94	90.345	9.655	9.6487432	0.6256767	9.648972	0.6028
8/09/94	90.3	9.7	9.69322	0.6779951	9.6934002	0.65998
9/09/94	90.32	9.68	9.6714643	0.8535673	9.6715592	0.84408
12/09/94	89.875	10.125	10.121749	0.325084	10.121832	0.3168
13/09/94	89.74	10.26	10.246834	1.316552	10.25171	0.829
14/09/94	89.595	10.405	10.403602	0.139844	10.408433	-0.34325
15/09/94	89.695	10.305	10.298079	0.692056	10.302843	0.21575
16/09/94	89.875	10.125	10.103908	2.10925	10.10764	1.736
19/09/94	89.715	10.285	10.266878	1.812198	10.27034	1.466
20/09/94	89.635	10.365	10.353638	1.136191	10.358348	0.66525
21/09/94	89.69	10.31	10.285567	2.443308	10.290163	1.98375
22/09/94	89.675	10.325	10.307674	1.732554	10.312148	1.28525
23/09/94	89.66	10.34	10.317026	2.297367	10.321618	1.83825
26/09/94	89.585	10.415	10.401499	1.350133	10.406008	0.89925
27/09/94	89.625	10.375	10.367318	0.76818	10.371865	0.3135
28/09/94	89.52	10.48	10.466519	1.348142	10.470875	0.9125
29/09/94	89.585	10.415	10.405541	0.945898	10.409815	0.5185
30/09/94	89.505	10.495	10.487376	0.762442	10.4918	0.32

Av. Spr.	1.01658987	Av. Spr.	0.71961591
Std. dev.	0.72428724	Std. dev.	0.63499254
Correl. Cx.	0.99986278	Correl. Cx.	0.99987094

Appendix L continued:

October 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
4/10/94	89.53	10.47	10.461102	0.889782	10.46305	0.69500
5/10/94	89.575	10.425	10.402267	2.273313	10.40665	1.83500
6/10/94	89.68	10.32	10.296142	2.385825	10.300425	1.95750
7/10/94	89.645	10.355	10.344671	1.032863	10.348663	0.63375
10/10/94	89.765	10.235	10.222186	1.281408	10.226368	0.86325
11/10/94	89.745	10.255	10.251527	0.347297	10.255598	-0.05975
12/10/94	89.645	10.355	10.358112	-0.31121	10.361865	-0.68652
13/10/94	89.505	10.495	10.49465	0.035003	10.498473	-0.34725
14/10/94	89.645	10.355	10.345266	0.973375	10.3491	0.59000
17/10/94	89.65	10.35	10.331003	1.89967	10.33428	1.57200
18/10/94	89.67	10.33	10.313749	1.625068	10.317435	1.25650
19/10/94	89.66	10.34	10.324154	1.584589	10.327673	1.23275
20/10/94	89.75	10.25	10.234642	1.535831	10.238105	1.18950
21/10/94	89.71	10.29	10.272597	1.740339	10.276093	1.39075
24/10/94	89.74	10.26	10.252619	0.738144	10.259408	0.05925
25/10/94	89.625	10.375	10.360905	1.409539	10.36482	1.01800
26/10/94	89.455	10.545	10.540246	0.475362	10.543915	0.10850
27/10/94	89.45	10.55	10.539053	1.094659	10.542663	0.73375
28/10/94	89.435	10.565	10.542369	2.263117	10.545855	1.91450
31/10/94	89.375	10.625	10.622227	0.27732	10.625695	-0.06950

Av. Spr.	1.1775647	Av. Spr.	0.79434875
Std. dev.	0.76420568	Std. dev.	0.77226759
Correl. Cx.	0.99798087	Correl. Cx.	0.99789796

Appendix L continued:

November 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/11/94	89.335	10.665	10.653551	1.144864	10.656945	0.805500
2/11/94	89.32	10.68	10.671249	0.875125	10.674438	0.556250
3/11/94	89.335	10.665	10.64692	1.80796	10.650013	1.498750
4/11/94	89.32	10.68	10.66599	1.401022	10.669013	1.098750
7/11/94	89.19	10.81	10.796205	1.379533	10.79913	1.087000
8/11/94	89.18	10.82	10.784032	3.596809	10.78705	3.295000
9/11/94	89.215	10.785	10.769675	1.53246	10.772301	1.269907
10/11/94	89.325	10.675	10.666515	0.848511	10.66905	0.595000
11/11/94	89.27	10.73	10.715727	1.427347	10.71819	1.181000
14/11/94	89.305	10.695	10.680996	1.400398	10.683374	1.162600
15/11/94	89.38	10.62	10.598055	2.194534	10.600527	1.947325
16/11/94	89.28	10.72	10.700583	1.94169	10.702778	1.722250
17/11/94	89.23	10.77	10.759394	1.060638	10.76156	0.844000
18/11/94	89.235	10.765	10.750527	1.447276	10.74923	1.577000
21/11/94	89.195	10.805	10.791631	1.336946	10.793568	1.143250
22/11/94	89.25	10.75	10.73947	1.053047	10.74133	0.867000
23/11/94	89.49	10.51	10.494699	1.530128	10.497145	1.285500
24/11/94	89.525	10.475	10.466549	0.845092	10.468028	0.697250
25/11/94	89.595	10.405	10.394669	1.033128	10.396033	0.896750
28/11/94	89.545	10.455	10.433379	2.162143	10.434673	2.032750
29/11/94	89.58	10.42	10.405971	1.402932	10.407555	1.244475
30/11/94	89.465	10.535	10.527228	0.77723	10.528215	0.678500

Av. Spr.	1.46358241	Av. Spr.	1.249355
Std. dev.	0.62341355	Std. dev.	0.6142777
Correl. Cx.	0.99891555	Correl. Cx.	0.99869817

December 1994 SFE 10 year bond futures pricing

Date	Fut. Price	Fut. Yield	CAC	Spread	Model	Spread
1/12/94	89.59	10.41	10.402042	0.795836	10.402968	0.70325
2/12/94	89.555	10.445	10.438446	0.655436	10.438449	0.65515
5/12/94	89.905	10.095	10.085735	0.926532	10.086463	0.85375
6/12/94	89.825	10.175	10.179667	-0.466731	10.180308	-0.53075
7/12/94	89.655	10.345	10.3459	-0.08998	10.34638	-0.13800
8/12/94	89.685	10.315	10.328239	-1.323867	10.327393	-1.23925
9/12/94	89.765	10.235	10.249509	-1.4509	10.245745	-1.07448
12/12/94	89.79	10.21	10.211552	-0.155227	10.21165	-0.16500
13/12/94	89.475	10.525	10.511406	1.359449	10.519435	0.55650
14/12/94	89.605	10.395	10.389488	0.551244	10.397585	-0.25850
15/12/94	89.735	10.265	10.250359	1.464149	10.258923	0.60775
16/12/94	89.68	10.32	10.306763	1.323705	10.315353	0.46475
19/12/94	89.7	10.3	10.294865	0.513513	10.30339	-0.33900
20/12/94	89.77	10.23	10.208059	2.194121	10.216515	1.34850
21/12/94	89.805	10.195	10.182198	1.280247	10.189793	0.52075
22/12/94	89.73	10.27	10.254023	1.59771	10.261835	0.81650
28/12/94	89.95	10.05	10.039116	1.088398	10.04643	0.35700
29/12/94	89.94	10.06	10.046643	1.33575	10.055083	0.49175

Av. Spr.	0.64441028	Av. Spr.	0.201704
Std. dev.	0.99530437	Std. dev.	0.69200947
Correl. Cx.	0.99709601	Correl. Cx.	0.99832511

Figure(i) Correlation between time and pricing error

Basis point pricing error: Market - Model

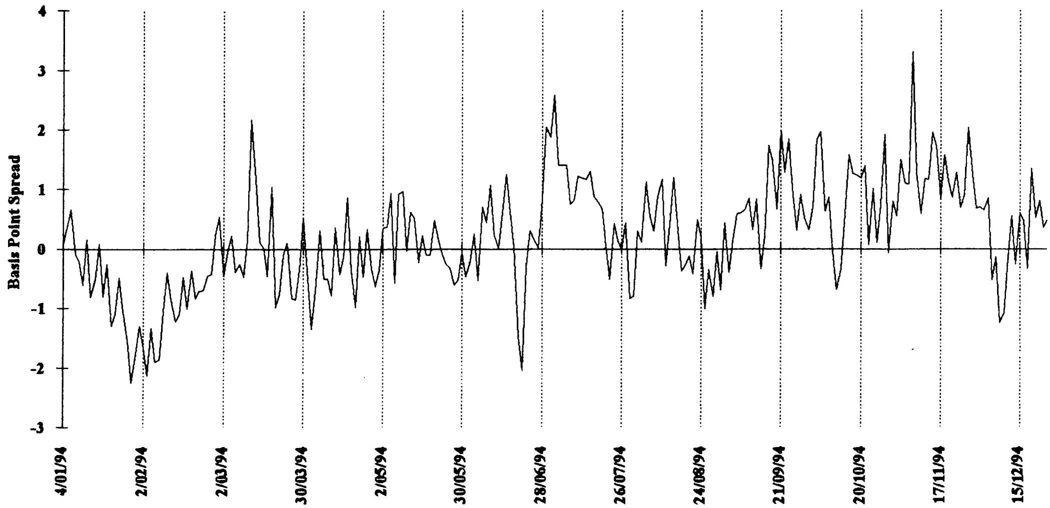
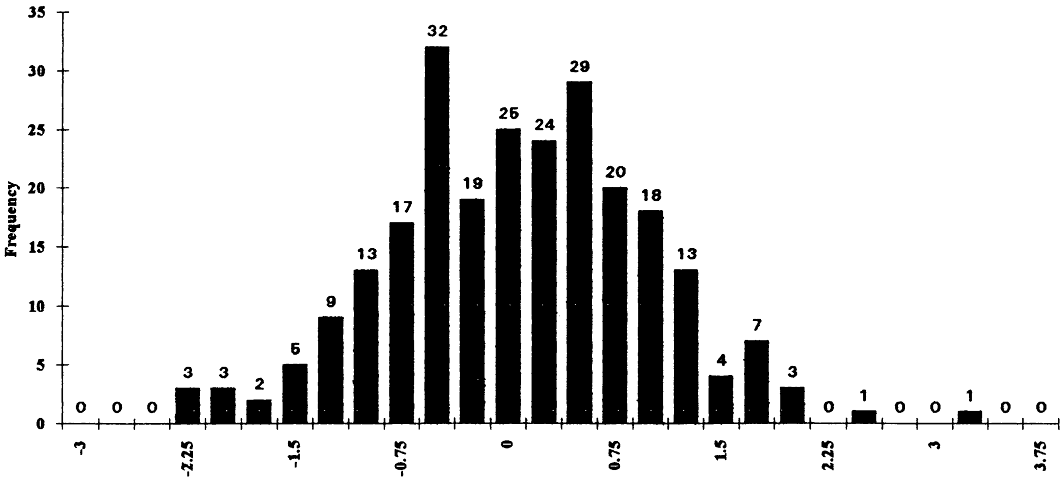


Figure (ii) Relative frequency distribution of futures pricing errors

Numbers on top of bars denoted relative frequency of error n =248



Figure(iii) Correlation between time and pricing error

Basis point pricing error: Market - Model

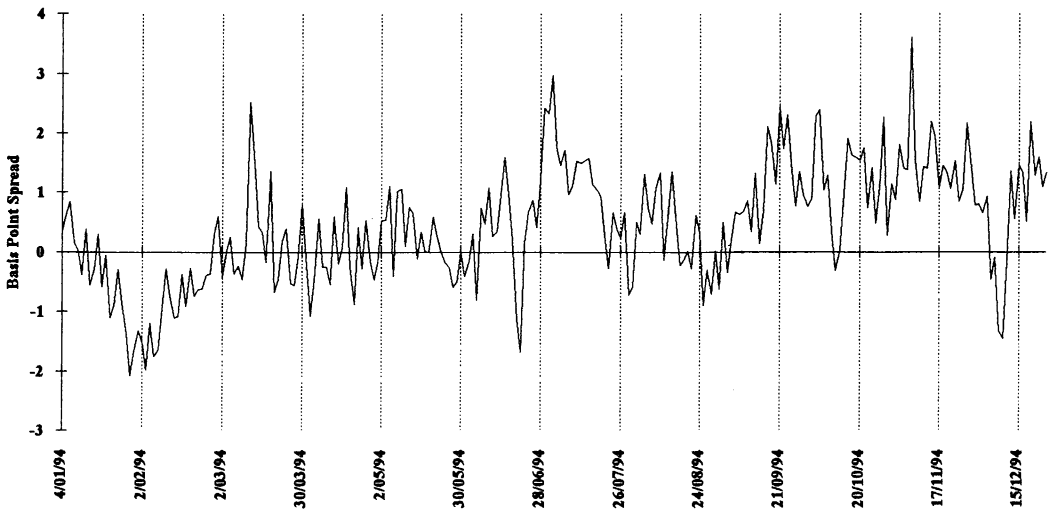
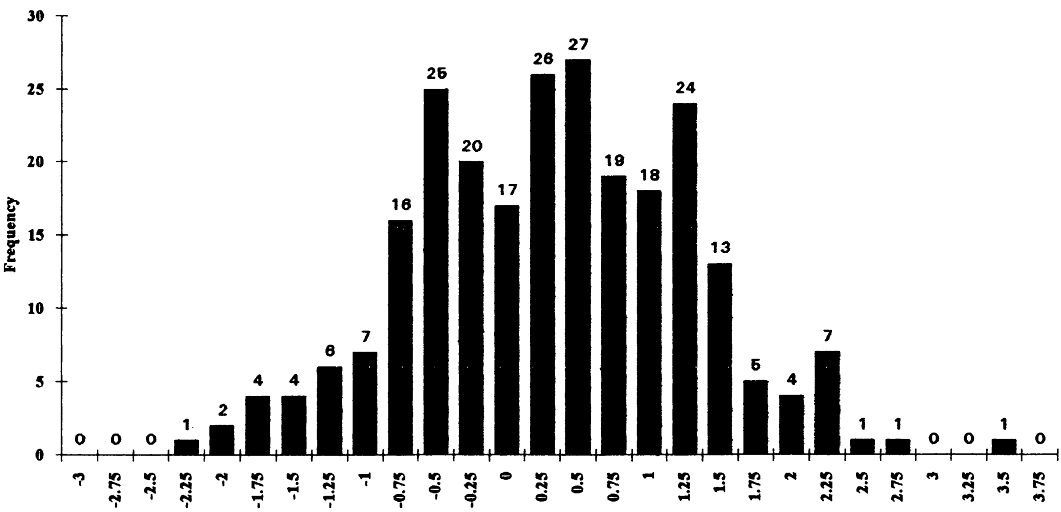


Figure (iv) Relative frequency distribution of futures pricing errors

Numbers on top of bars denoted relative frequency of error n =248



APPENDIX O : Daily results Yield Error Margin databases

January 1994 Yield Error Margin databases

Date	2 week	1 month	2 month	3 month
4/01/94	26.168258	33.2680186	-56.613278	-133.53331
5/01/94	19.097134	10.700374	-56.045414	-145.46445
6/01/94	15.85487	11.383947	-55.353737	-127.48611
7/01/94	22.20283	14.592535	-52.102854	-123.64825
10/01/94	12.148874	14.79187	-82.012159	-143.73773
11/01/94	23.304196	10.79353	-76.009767	-144.78094
12/01/94	13.624616	8.587115	-71.037681	-183.12363
13/01/94	15.066476	10.045585	-69.548926	-133.37667
14/01/94	14.57543	10.732664	-68.873858	-147.62969
17/01/94	20.312131	-5.984438	-75.33385	-144.02235
18/01/94	15.29825	-29.762523	-80.625162	-146.17589
19/01/94	7.03425	-41.199532	-104.62709	-187.19633
20/01/94	8.068127	-37.211592	-100.55716	-187.56779
21/01/94	1.975758	-40.392466	-103.78355	-169.51304
24/01/94	-8.782964	-52.110866	-101.53093	-154.47496
25/01/94	-18.987123	-76.410107	-122.19715	-167.17886
27/01/94	-6.258724	-61.361453	-141.35881	-167.3344
28/01/94	-8.001108	-60.167527	-140.15136	-166.53756
31/01/94	-9.784357	-66.056645	-151.09831	-197.72016
Av. Spr.	8.57457	-18.19797	-89.94006	-156.34222
Std. dev.	13.15359	34.82890	30.96739	21.81576

Appendix O continued:

February 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
1/02/94	-12.774609	-72.784496	-176.03632	-198.54005
2/02/94	-8.241209	-80.470869	-172.1441	-194.60208
3/02/94	-16.987473	-63.705499	-169.17313	-191.63647
4/02/94	-36.721203	-90.173382	-167.43027	-217.38773
7/02/94	-32.655965	-66.798813	-138.44386	-224.34405
8/02/94	-30.177821	-58.838531	-141.02346	-226.50391
9/02/94	-32.914366	-81.821216	-152.4748	-229.80937
10/02/94	-54.707491	-96.911158	-158.69616	-211.24585
11/02/94	-63.699965	-86.892805	-155.69783	-217.02027
14/02/94	-56.219831	-79.685107	-156.30057	-203.52027
15/02/94	-59.943615	-91.382807	-161.38111	-201.43939
16/02/94	-72.193094	-73.389099	-149.54468	-202.18608
17/02/94	-46.871617	-69.256316	-137.84089	-196.72949
18/02/94	-53.247981	-50.501036	-115.66713	-142.80679
21/02/94	-33.957475	-62.928899	-128.08881	-165.32911
22/02/94	-28.484804	-64.483434	-124.68434	-170.06588
23/02/94	-48.732667	-46.984445	-118.61847	-170.5936
24/02/94	-41.890459	-48.803936	-102.60184	-154.43684
25/02/94	-22.834948	-44.929253	-90.407055	-148.78871
28/02/94	-23.146086	-79.305795	-104.9074	-149.29047
Av. Spr.	-38.82013	-70.50234	-141.05811	-190.81382
Std. dev.	17.75348	15.49348	25.13324	27.95635

Appendix O continued:

March 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
1/03/94	-31.094129	-102.247965	-128.85241	-161.18293
2/03/94	-0.778689	-90.634379	-112.24843	-144.40587
3/03/94	-22.120929	-104.68103	-126.41708	-152.60496
4/03/94	3.053242	-76.15284	-128.85913	-109.31974
7/03/94	-28.775517	-70.75821	-155.84735	-122.20919
8/03/94	-35.833542	-81.397262	-166.17933	-132.23761
9/03/94	2.032196	-69.589092	-146.00438	-132.61932
10/03/94	-6.665502	-60.584226	-112.77468	-113.28818
11/03/94	-22.308537	-68.694939	-127.45482	-141.22132
14/03/94	-56.365675	-78.60704	-125.3099	-152.70088
15/03/94	-35.421703	-70.366734	-107.38103	-124.1062
16/03/94	-57.363495	-76.223903	-126.58828	-173.23887
17/03/94	-54.510908	-68.428642	-126.97343	-174.13429
18/03/94	-51.605061	-65.510591	-92.511153	-210.34381
21/03/94	-60.072279	-65.108907	-102.17122	-209.46463
22/03/94	-63.21116	-60.16396	-105.36291	-206.76655
23/03/94	-64.304815	-70.396814	-123.89359	-212.00222
24/03/94	-45.998293	-52.466571	-105.75223	-193.61306
25/03/94	-37.560924	-43.69633	-103.52985	-220.63087
28/03/94	-23.424891	-25.029416	-69.104909	-176.34851
29/03/94	-21.747895	-53.775116	-72.594314	-168.13279
30/03/94	-7.286793	-50.665994	-69.4815	-163.51127
31/03/94	-17.267524	-45.752002	-77.361755	-158.43863
Av. Spr.	-32.11447	-67.43182	-113.59364	-163.15312
Std. dev.	21.74183	18.18931	25.91956	33.61934

Appendix O continued:

April 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
5/04/94	-21.841649	-52.51025	-32.5014	-134.74483
6/04/94	-43.011501	-70.600751	-50.094681	-141.94063
7/04/94	-22.412987	-84.80513	-50.635344	-141.15568
8/04/94	-14.004831	-84.522426	-70.081619	-154.54072
11/04/94	2.449995	-59.811956	-70.526058	-146.00139
12/04/94	-2.716518	-71.145322	-75.740374	-147.52532
13/04/94	-17.773328	-82.865759	-87.392436	-152.98073
14/04/94	-3.077889	-45.667515	-72.087601	-134.37639
15/04/94	-18.816483	-36.076332	-51.732632	-103.75769
18/04/94	-35.760953	-26.046611	-143.08274	-110.54894
19/04/94	0.918597	5.046592	-82.872464	-72.587259
20/04/94	-23.996107	13.267739	-101.52405	-76.906979
21/04/94	-47.912766	-36.054468	-142.41859	-117.35227
22/04/94	-56.757381	-44.298664	-144.73427	-119.66758
26/04/94	-54.459663	-45.575963	-175.41655	-127.15858
27/04/94	-58.636596	-51.798588	-171.70527	-109.79099
28/04/94	-70.833499	-43.591493	-150.3136	-131.19099
29/04/94	-39.287933	-18.026284	-111.02473	-99.292061
Av. Spr.	-29.32953	-46.39351	-99.10469	-123.41772
Std. dev.	22.45508	28.02923	45.06452	24.29240

Appendix O continued:

May 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
2/05/94	-8.446959	-25.910152	-108.65751	-71.65569
3/05/94	-21.66672	-24.768933	-113.76238	-89.35732
4/05/94	39.098213	26.719238	-84.030824	-29.230784
5/05/94	35.734778	29.227772	-80.297978	-34.004421
6/05/94	40.546024	29.88231	-68.88683	-53.204421
9/05/94	40.632803	15.33589	-67.182779	-26.748004
10/05/94	2.280249	0.776173	-91.973887	-53.483901
11/05/94	3.5321	-9.638849	-84.043065	-53.326224
12/05/94	29.611309	2.97568	-67.510189	-47.727807
13/05/94	13.579918	-3.271335	-67.519059	-45.935882
16/05/94	-0.761548	-44.692757	-59.422956	-72.569335
17/05/94	-5.313333	-45.178013	-50.948299	-33.212991
18/05/94	-45.897861	-116.518475	-83.441407	-70.072365
19/05/94	-35.059344	-110.589084	-83.281509	-75.222777
20/05/94	-33.085508	-114.796034	-90.080264	-73.38971
23/05/94	2.791775	-100.396311	-93.018096	-65.259326
24/05/94	8.683075	-86.14846	-78.573919	-46.903309
25/05/94	15.889919	-115.423631	-73.080619	-34.369392
26/05/94	-18.597728	-129.169688	-80.258697	-56.329691
27/05/94	1.283578	-119.220689	-56.955487	-40.472582
30/05/94	-24.976921	-92.635189	-77.483113	-53.000175
31/05/94	-12.06073	-79.621139	-67.308569	-41.186014
Av. Spr.	1.26350	-50.59371	-78.53261	-53.03010
Std. dev.	25.44554	56.67207	15.42869	17.15562

Appendix O continued:

June 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
1/06/94	7.1931	-68.042	-40.2223	-31.4242
2/06/94	-27.2303	-82.2925	-44.7718	-43.5778
3/06/94	-42.266729	-88.540486	-63.482424	-58.343334
6/06/94	-103.802533	-99.140414	-82.85042	-78.500478
7/06/94	-114.553421	-89.426407	-74.376935	-83.45844
8/06/94	-109.100718	-88.8872	-75.009824	-88.500959
9/06/94	-73.17914	-82.616296	-42.256081	-64.679997
10/06/94	-84.43235	-92.702614	-54.191259	-120.09128
14/06/94	-80.957774	-60.995838	-42.572527	-110.73338
15/06/94	-79.816935	-50.677956	-53.538258	-111.72434
16/06/94	-48.440647	-14.09919	-26.610965	-62.095932
17/06/94	-39.580832	-5.16195	13.205567	-68.909949
20/06/94	-6.553919	26.313568	44.486164	-36.445327
21/06/94	13.781822	27.390496	64.02738	-10.33588
22/06/94	18.268786	27.134288	57.054598	-20.088032
23/06/94	4.600465	7.189631	35.817149	-35.788171
24/06/94	7.003485	8.395602	40.701403	-42.607227
27/06/94	28.3811	64.20493	81.01823	-2.76525
28/06/94	10.854196	21.499355	59.323383	-34.304901
29/06/94	5.133858	14.658605	46.726944	-40.226764
30/06/94	6.66057	12.911185	40.615744	-51.686003
Av. Spr.	-33.71609	-29.18501	-5.56696	-56.96608
Std. dev.	47.67640	53.04787	55.45142	32.76103

Appendix O continued:

July 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
1/07/94	27.883237	27.16177	34.469284	-49.413544
4/07/94	40.95656	55.918126	31.367283	-42.317815
5/07/94	33.986898	47.365413	30.329187	-37.751415
6/07/94	20.79384	17.62969	22.758388	-37.663593
7/07/94	1.201593	16.240504	7.997728	-43.20092
8/07/94	6.2122	15.0705	2.417	-32.3682
11/07/94	16.2687	32.0022	-25.3396	-19.3127
12/07/94	18.0652	20.797	-29.3162	-34.5412
13/07/94	25.4373	22.5138	-30.4447	-54.3473
14/07/94	1.0287	19.2913	-48.4371	-43.4364
15/07/94	-14.8985	-2.1803	-59.8056	-63.6297
18/07/94	-7.3539	14.6628	-63.8705	-69.2883
19/07/94	3.1064	9.3532	-57.7142	-63.5126
20/07/94	-6.8526	18.054928	-63.0855	-51.8037
21/07/94	41.4518	36.5052	-37.9221	-37.1601
22/07/94	27.7801	30.1392	-46.6538	-42.6929
25/07/94	13.5924	38.5159	-50.7729	-48.7513
26/07/94	19.3251	24.941	-56.7267	-72.5549
27/07/94	3.8436	16.6345	-67.3509	-86.5266
28/07/94	17.414	38.2157	-54.6511	-64.062
29/07/94	4.537859	32.659454	-54.662735	-77.796194
Av. Spr.	13.98955	25.30914	-29.40070	-51.05388
Std. dev.	15.70221	13.37417	35.48945	16.90381

Appendix O continued:

August 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
2/08/94	-9.5813	1.7369	-79.9369	-105.8247
3/08/94	33.6837	5.9794	-67.1348	-86.6256
4/08/94	-0.3073	-24.4252	-98.0148	-119.8871
5/08/94	-4.0579	-16.9023	-84.8191	-125.0837
8/08/94	21.2244	-12.5765	-47.2344	-106.4861
9/08/94	3.7382	-22.0152	-59.2303	-114.702
10/08/94	9.151	-65.3203	-57.1881	-102.8185
11/08/94	22.6223	-57.3729	-52.0966	-99.5173
12/08/94	22.3513	-50.0955	-55.2677	-88.7361
15/08/94	19.689	-57.7182	-61.2089	-88.9355
16/08/94	18.8568	-35.3875	-59.6876	-96.2311
17/08/94	-13.2347	-82.7701	-89.8907	-133.7064
18/08/94	-14.2028	-78.863	-83.5693	-128.3997
19/08/94	-4.7316	-67.4786	-73.4217	-117.3721
22/08/94	-19.3593	-77.4983	-126.8665	-123.7474
23/08/94	-10.3259	-72.2321	-67.7103	-91.7404
24/08/94	-27.5423	-83.9831	-71.4203	-92.7529
25/08/94	-37.7769	-89.8191	-88.5663	-91.5828
26/08/94	-25.8712	-81.8858	-97.8434	-87.9159
29/08/94	-78.365464	-87.956244	-103.11391	-91.015966
30/08/94	-68.742436	-91.900488	-98.776491	-98.810697
31/08/94	-82.53595	-90.763655	-97.419758	-84.985779
Av. Spr.	-11.15083	-56.32944	-78.20081	-103.49444
Std. dev.	32.53184	31.99881	20.44781	15.35657

Appendix O continued:

September 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
1/09/94	-67.502214	-83.689867	-103.95439	-81.045949
2/09/94	-46.682778	-81.802428	-103.71022	-82.632362
5/09/94	-54.247456	-67.768966	-107.58122	-39.67935
6/09/94	-65.256643	-60.194546	-110.60327	-50.991233
7/09/94	-45.152546	-50.908047	-96.645364	-53.173577
8/09/94	-42.914972	-34.442978	-92.892068	-46.772091
9/09/94	-45.926525	-36.63608	-91.384911	-41.051808
12/09/94	-11.28576	-4.30708	-36.75061	8.67602
13/09/94	-11.65943	-24.05542	-42.81038	-14.76843
14/09/94	-6.10673	5.56842	-27.39382	11.41416
15/09/94	-10.42043	-3.31227	-29.65655	14.0043
16/09/94	-37.793662	-22.49431	-65.30449	-10.12123
19/09/94	-19.46422	-5.54364	-52.06753	6.31887
20/09/94	-11.08945	11.59787	-43.43201	26.62285
21/09/94	-11.87701	1.152	-50.18847	19.80271
22/09/94	0.86407	4.63606	-42.71759	15.06912
23/09/94	-2.86947	5.47685	-17.4486	15.62295
26/09/94	17.18958	-14.33965	-2.74802	45.70316
27/09/94	11.16449	-17.39737	-6.46287	42.16515
28/09/94	10.78844	-8.05561	3.57639	52.13758
29/09/94	-8.74914	-21.89082	0.13114	45.22486
30/09/94	13.97792	-13.64166	-3.65463	48.67826
Av. Spr.	-20.22791	-23.72952	-51.07725	-3.12709
Std. dev.	26.02463	28.65687	39.45645	42.19591

Appendix O continued:

October 1994 Yield Error Margin database

Date	2 week	1 month	2 month	3 month
4/10/94	14.865	-20.098	37.819	34.536
5/10/94	8.032	-38.854	31.914	13.21
6/10/94	6.222	-49.421	-4.544	-8.1
7/10/94	6.794	-44.702	0.269	-2.801
10/10/94	-3.779	-43.914	5.989	-1.801
11/10/94	-11.757	-45.543	4.434	-6.093
12/10/94	-18.688	-32.045	15.065	11.806
13/10/94	-5.1	-18.664	20.414	14.852
14/10/94	-19.971	-33.233	5.451	11.921
17/10/94	-29.231	-42.557	12.757	11.644
18/10/94	-33.988	-42.329	11.002	9.523
19/10/94	-34.744	-46.422	12.026	6.114
20/10/94	-41.063	-55.267	14.499	-11.104
21/10/94	-39.09	-51.506	18.308	-21.721
24/10/94	-53.357	-20.942	30.64	-17.297
25/10/94	-43.527	-3.258	41.487	5.177
26/10/94	-22.422	10.934	59.554	38.557
27/10/94	-12.396	10.81	59.409	38.415
28/10/94	-16.915	10.869	59.75	44.746
31/10/94	-5.614	9.723	62.109	30.806
Av. Spr.	-17.78645	-27.32095	24.91760	10.11950
Std. dev.	19.35321	23.25065	21.66514	19.23998

Appendix O continued:

November 1994 Yield Error Margin database

Date	2 week	1 month	2 month
1/11/94	5.652	25.254	53.994
2/11/94	-2.846	23.43	55.763
3/11/94	-11.108	56.159	53.324
4/11/94	-7.828	58.048	39.878
7/11/94	0.5	45.241	42.357
8/11/94	6.514	48.015	42.814
9/11/94	27.44	52.65	39.856
10/11/94	19.941	45.77	30.554
11/11/94	31.976	50.652	40.279
14/11/94	24.45013	38.8552	45.67726
15/11/94	18.64055	43.7209	37.09954
16/11/94	17.38603	48.38117	46.95878
17/11/94	35.64922	54.91329	53.63279
18/11/94	30.88689	54.21582	52.10037
21/11/94	70.41216	70.02874	29.6531
22/11/94	55.87289	57.92492	24.29462
23/11/94	15.01823	33.14661	-0.33983
24/11/94	14.01533	51.68494	3.23343
25/11/94	14.99156	44.55041	7.98906
28/11/94	22.25545	48.4596	33.23061
29/11/94	0.16687	44.72687	30.05561
30/11/94	23.19611	52.02214	42.39241
Av. Spr.	18.78102	47.62953	36.58167
Std. dev.	19.41116	10.62859	16.03101

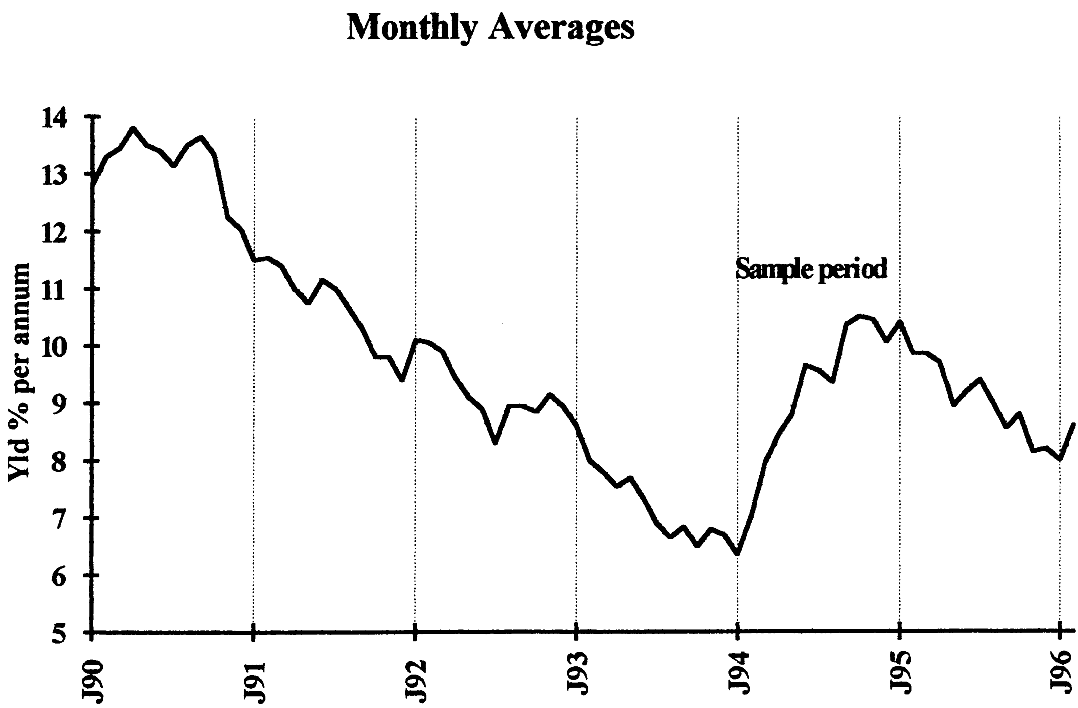
N.B. There were no 3 month Yield Error Margin results due to the data ending on 31/01/95.

December 1994 Yield Error Margin database

Date	2 week	1 month
1/12/94	23.60214	37.11937
2/12/94	21.63783	40.06304
5/12/94	-12.61451	-19.89164
6/12/94	5.12627	-21.05154
7/12/94	24.82377	-3.9662
8/12/94	16.18571	-5.63482
9/12/94	7.46507	-13.71977
12/12/94	25.91826	-3.71031
13/12/94	45.50192	5.17184
14/12/94	33.92453	5.23745
15/12/94	19.8627	-7.98475
16/12/94	20.63226	-2.30428
19/12/94	17.7704779	-7.3743749
20/12/94	9.362869	-23.969
21/12/94	6.889	-26.524
22/12/94	-11.567	-34.292
28/12/94	-37.221302	-16.39877
29/12/94	-29.21266	-15.48324
Av. Spr.	10.44930	-6.37294

N.B. There were no 2 month Yield Error Margin results due to the data ending on 31/01/95.

APPENDIX P : Australian 10 year bond yields 1990 - 1996



Average yearly yield ranges 1990 -1995

	Average Range
1990	1.75%
1991	2.15%
1992	1.80%
1993	2%
1994	4.15%
1995	1.90%
All Sample	2.31%

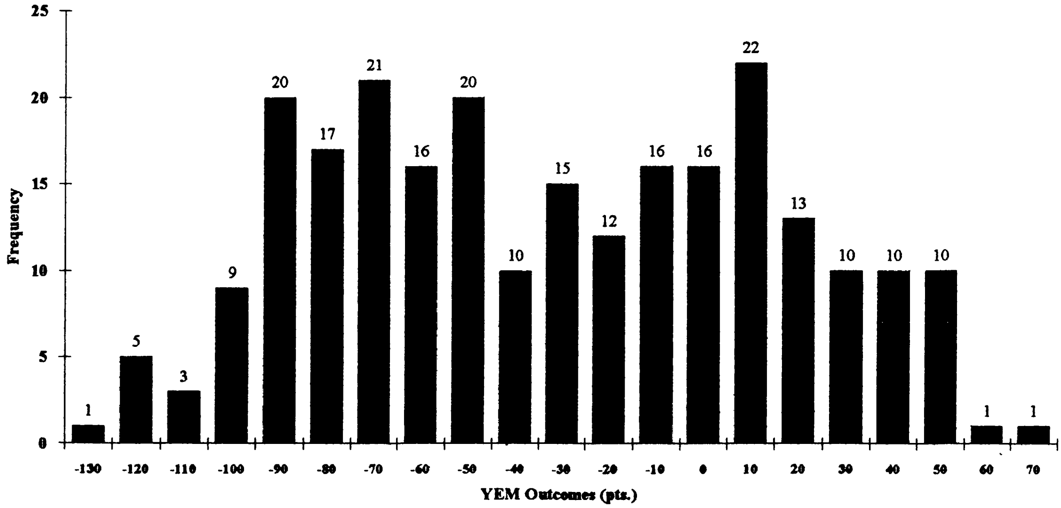
APPENDIX Q : Relative frequency distributions 1 month to 3 months

1 month Yield Error Margin database: Relative frequency distribution

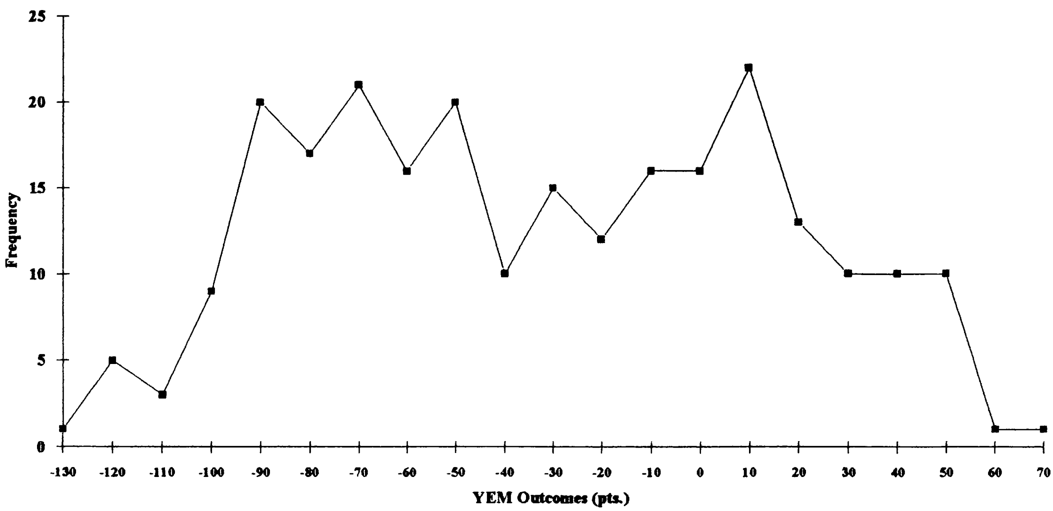
YEM Pts.	Frequency Outcome	Relative Frequency
-130	1	0.4
-120	5	2.0
-110	3	1.2
-100	9	3.6
-90	20	8.1
-80	17	6.9
-70	21	8.5
-60	16	6.5
-50	20	8.1
-40	10	4.0
-30	15	6.0
-20	12	4.8
-10	16	6.5
0	16	6.5
10	22	8.9
20	13	5.2
30	10	4.0
40	10	4.0
50	10	4.0
60	1	0.4
70	1	0.4
Total	248	100.0

1 month Yield Error Margin : Relative frequency distribution histogram

Numbers on top of the bars denoted relative frequency of error



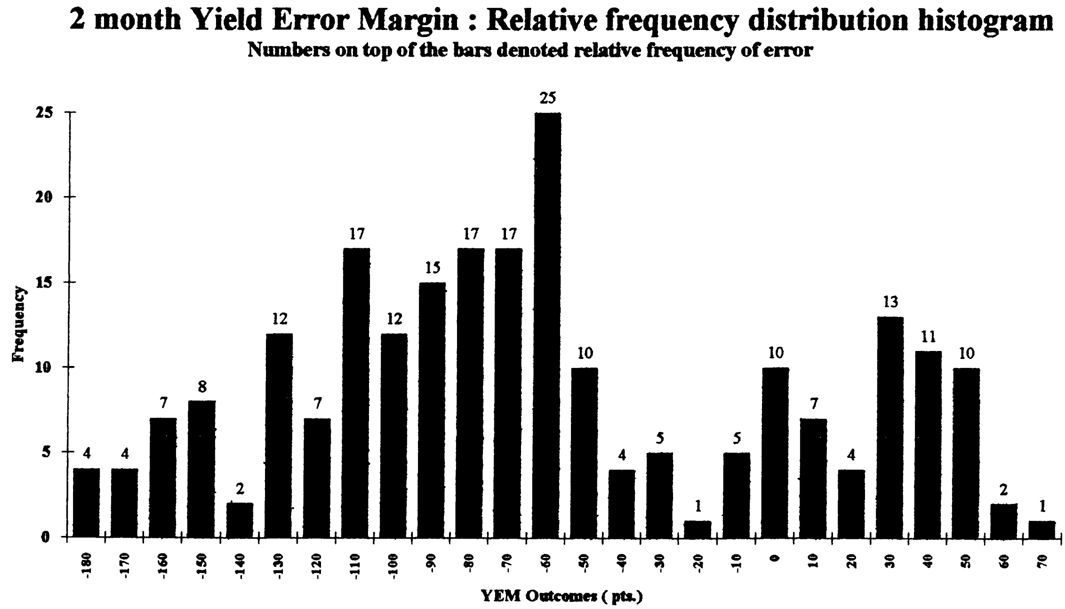
1 month Yield Error Margin : Approximate density function

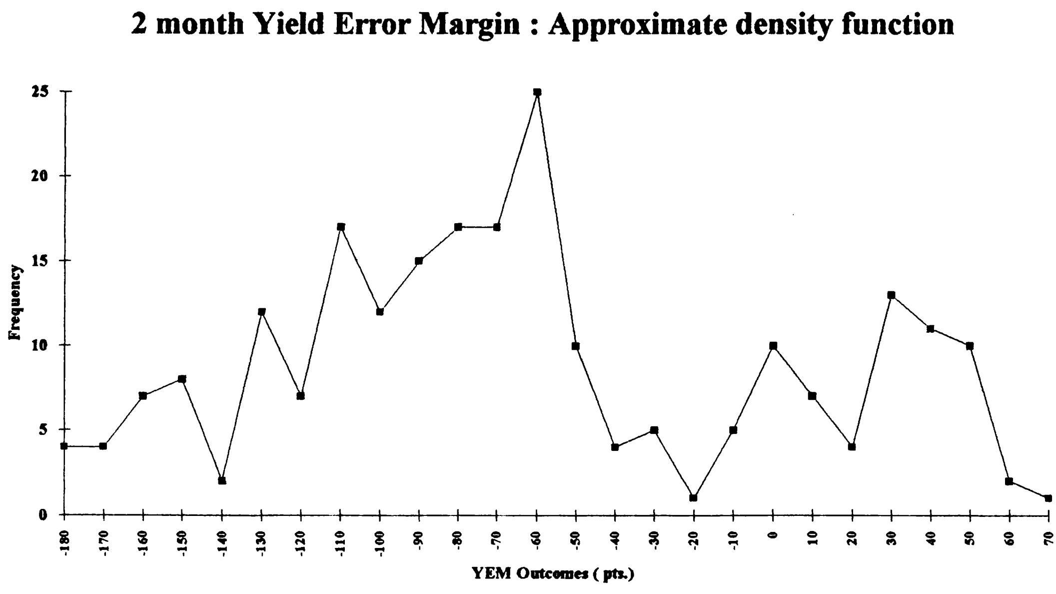


2 month Yield Error Margin database: Relative frequency distribution

YEM pts.	Frequency Outcome	Relative Frequency
-180	4	1.7
-170	4	1.7
-160	7	3.0
-150	8	3.5
-140	2	0.9
-130	12	5.2
-120	7	3.0
-110	17	7.4
-100	12	5.2
-90	15	6.5
-80	17	7.4
-70	17	7.4
-60	25	10.9
-50	10	4.3
-40	4	1.7
-30	5	2.2
-20	1	0.4
-10	5	2.2
0	10	4.3
10	7	3.0
20	4	1.7
30	13	5.7
40	11	4.8
50	10	4.3
60	2	0.9
70	1	0.4
Total	230	100.0

Appendix Q continued:





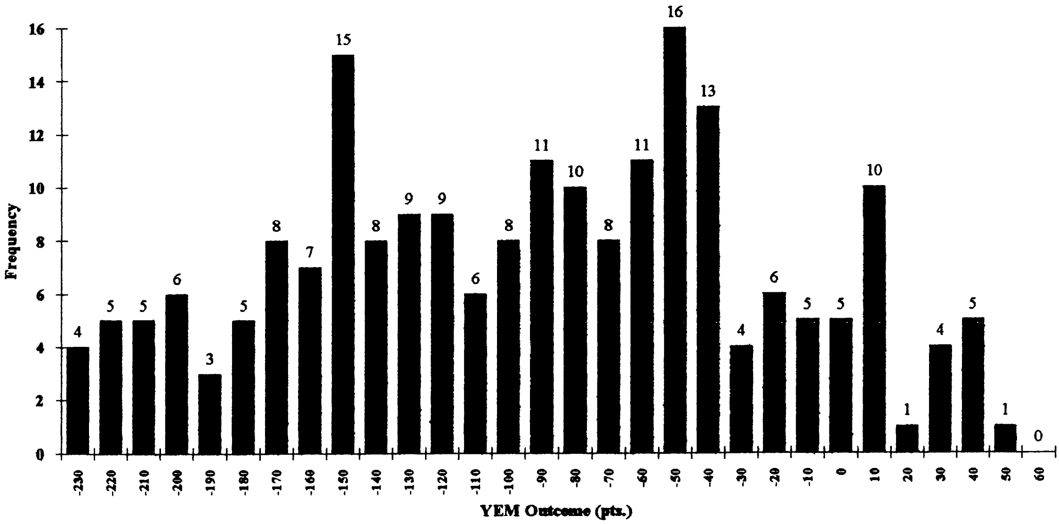
3 month Yield Error Margin database: Relative frequency distribution

YEM pts.	Frequency Outcome	Relative Frequency
-230	4	1.9
-220	5	2.4
-210	5	2.4
-200	6	2.9
-190	3	1.4
-180	5	2.4
-170	8	3.8
-160	7	3.4
-150	15	7.2
-140	8	3.8
-130	9	4.3
-120	9	4.3
-110	6	2.9
-100	8	3.8
-90	11	5.3
-80	10	4.8
-70	8	3.8
-60	11	5.3
-50	16	7.7
-40	13	6.3
-30	4	1.9
-20	6	2.9
-10	5	2.4
0	5	2.4
10	10	4.8
20	1	0.5
30	4	1.9
40	5	2.4
50	1	0.5
60	0	0.0
Total	208	100.0

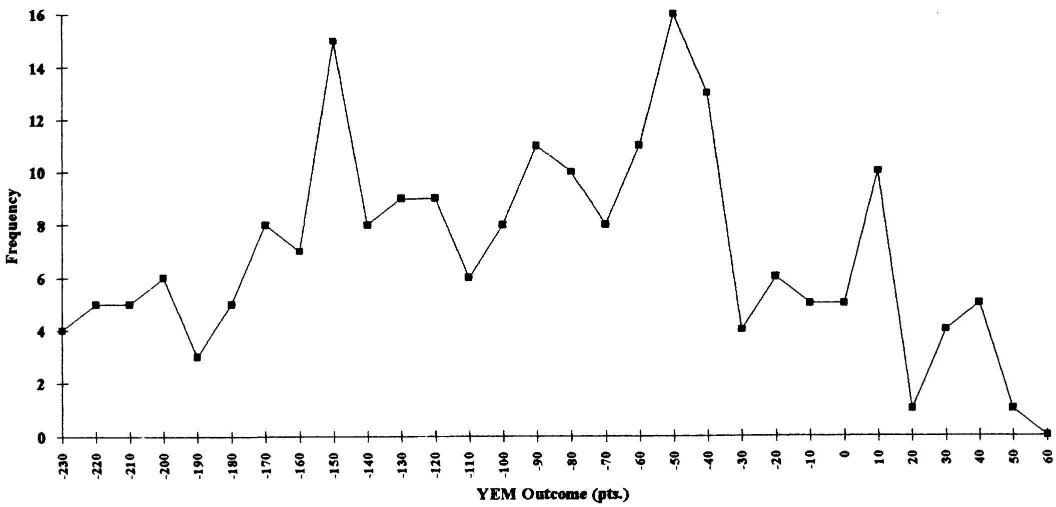
Appendix Q continued:

3 month Yield Error Margin : Relative frequency distribution histogram

Numbers on top of the bars denoted relative frequency of error



3 month Yield Error Margin : Approximate density function



APPENDIX R: SFE 10 year bond futures: relative frequency distribution

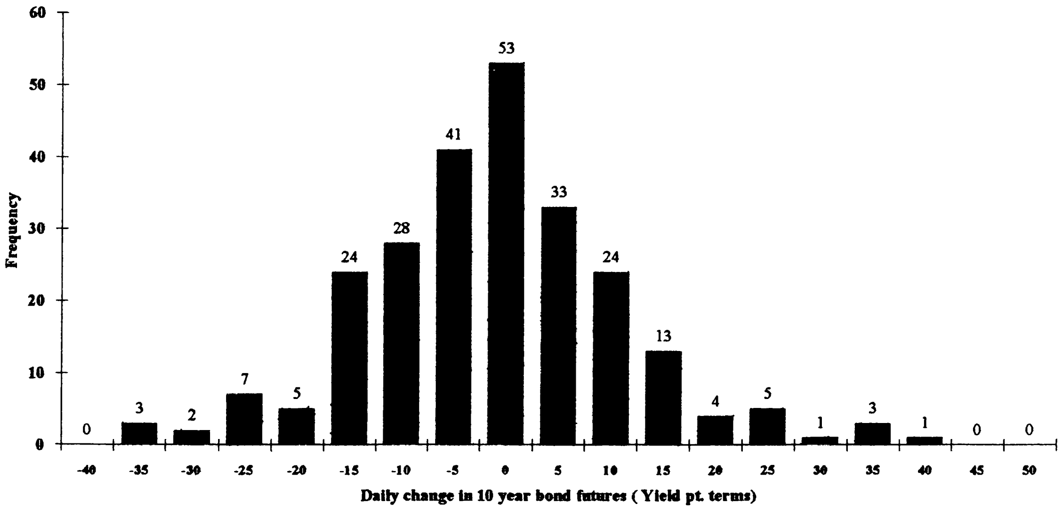
Daily [a] Chg.	Frequency Outcome	Relative Frequency
-40	0	0.0
-35	3	1.2
-30	2	0.8
-25	7	2.8
-20	5	2.0
-15	24	9.7
-10	28	11.3
-5	41	16.6
0	53	21.5
5	33	13.4
10	24	9.7
15	13	5.3
20	4	1.6
25	5	2.0
30	1	0.4
35	3	1.2
40	1	0.4
45	0	0.0
50	0	0.0
Total	**247**	**100.0**

(a) Daily Chg. = Equals change in yield from one close of the market to the next. The change was measured in yield basis point terms. Closing yield = 100 - futures closing price. Intervals in this case were 5 basis points wide.

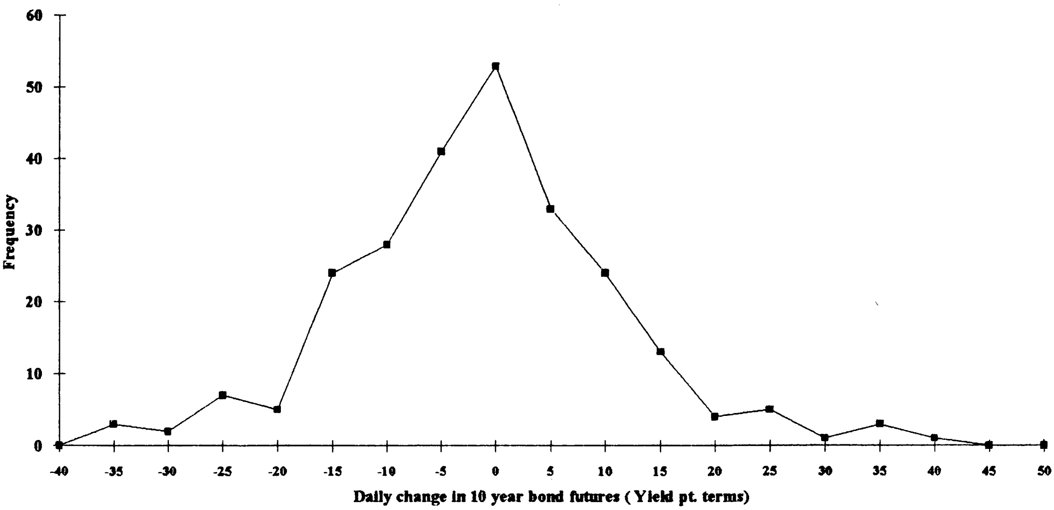
Appendix R continued:

10 year bond futures : Relative frequency distribution histogram

Numbers on top of the bars denoted relative frequency of daily change



10 year bond futures: Approximate density function



APPENDIX S: 2 week YEM volatility converted to historical volatility

Month	Av. YEM. Vol.	Max. Vol.	Min. Vol.	Range
Feb-94	9.02	9.60	8.07	1.53
Mar-94	6.69	7.54	6.38	1.16
Apr-94	6.72	7.05	6.30	0.75
May-94	9.39	10.14	6.96	3.18
Jun-94	12.70	14.56	10.01	4.54
Jul-94	12.11	14.83	4.38	10.44
Aug-94	5.31	9.58	4.37	5.21
Sep-94	9.48	10.46	8.19	2.27
Oct-94	7.17	9.48	5.55	3.93
Nov-94	8.70	9.24	5.75	3.49
Dec-94	5.44	8.25	4.78	3.47
All Sample	8.35	14.83	4.37	10.46

2 week Yield Error Margin converted volatility 16/02/94 to 31/12/94
% per Annum

APPENDIX T : "Mirror" & "Composite" 2 Week YEM distributions

"Mirror" 2 week Yield Error Margin distribution

YEM Pts.	Frequency Outcome	Relative Frequency
120	1	0.4
110	2	0.8
100	0	0.0
90	3	1.2
80	5	2.0
70	7	2.8
60	13	5.2
50	15	6.0
40	20	8.1
30	19	7.7
20	28	11.3
10	28	11.3
0	37	14.9
-10	33	13.3
-20	22	8.9
-30	8	3.2
-40	5	2.0
-50	1	0.4
-60	0	0.0
-70	1	0.4
Total	248	100

Appendix T continued:

"Composite" 2 week Yield Error Margin distribution

YEM Pts.	Frequency Outcome	Relative Frequency
-120	1	0.2
-110	2	0.4
-100	0	0.0
-90	3	0.6
-80	5	1.0
-70	8	1.6
-60	13	2.6
-50	16	3.2
-40	25	5.0
-30	27	5.4
-20	50	10.1
-10	61	12.3
0	74	14.9
10	61	12.3
20	50	10.1
30	27	5.4
40	25	5.0
50	16	3.2
60	13	2.6
70	8	1.6
80	5	1.0
90	3	0.6
100	0	0.0
110	2	0.4
120	1	0.2
Total	496	100.0

APPENDIX U : ATM put option price 04/01/94

YEM Pts. (b)	Bond Yld. Distr. (c)	Bond price Distribution	Payoff [X-B, 0](d)	YEM weight payoff (e)	Frequency Outcome (f)	Relative Frequency
-130	5.569	148805.73	0.00	0.00	1	0.2
-120	5.669	147823.41	0.00	0.00	5	1.0
-110	5.769	146849.63	0.00	0.00	3	0.6
-100	5.869	145884.32	0.00	0.00	9	1.8
-90	5.969	144927.38	0.00	0.00	20	4.0
-80	6.069	143978.75	0.00	0.00	17	3.4
-70	6.169	143038.34	0.00	0.00	22	4.4
-60	6.269	142106.07	0.00	0.00	17	3.4
-50	6.369	141181.86	0.00	0.00	30	6.0
-40	6.469	140265.63	0.00	0.00	20	4.0
-30	6.569	139357.30	0.00	0.00	25	5.0
-20	6.669	138456.80	0.00	0.00	25	5.0
-10	6.769	137564.04	169.03	12.95	38	7.7
0	6.869 (a)	136678.97	1054.10	68.01	32	6.5
10	6.969	135801.49	1931.58	147.98	38	7.7
20	7.069	134931.55	2801.52	141.21	25	5.0
30	7.169	134069.05	3664.02	184.68	25	5.0
40	7.269	133213.94	4519.13	182.22	20	4.0
50	7.369	132366.14	5366.93	324.61	30	6.0
60	7.469	131525.57	6207.50	212.76	17	3.4
70	7.569	130692.18	7040.89	312.30	22	4.4
80	7.669	129865.89	7867.18	269.64	17	3.4
90	7.769	129046.63	8686.44	350.26	20	4.0
100	7.869	128234.33	9498.74	172.36	9	1.8
110	7.969	127428.94	10304.13	62.32	3	0.6
120	8.069	126630.37	11102.70	111.92	5	1.0
130	8.169	125838.57	11894.50	23.98	1	0.2
					496	100.0

Strike 93.25	137733.07(g)	*\$ PRICE*	*2577.20*
		Pts. / \$89.07	28.9(h)

(a) The model derived futures equivalent yield for 04/01/94.(b) "Composite" Yield Error Margin distribution constructed in Chapter 5.(c) The Yield Error Margin distribution was applied to the model derived futures equivalent yield to produce a distribution of futures equivalent yields. These yields were converted by the SFE By - Laws bond pricing formula to a bond price distribution.(d) The payoff associated with the distribution of bond prices. If X < B the payoff = 0.(e) The probability weighted payoff of X-B. The sum of these payoffs equalled the outright \$ price of a option on a \$100,000 futures contract.(f) As defined in Chapter 5 this equals the number of times a particular error outcome occurred.(g) The SFE By - Laws bond price associated with the strike price X.(h) The final yield point model option price. It was set by the overall \$ price / \$ value of a ± 1 point move in yield at the strike By - Laws bond price.

APPENDIX V : Daily results model option prices vs market

January 1994 10 year bond futures options : Calls

	OTM [a]			ATM			ITM		
Date	Model	Mkt.	Spr. [b]	Model	Mkt.	Spr.	Model	Mkt.	Spr.
4/01/94	9.6	6	3.6	18.5	13.5	5	31.6	27	4.6
5/01/94	13.3	8.5	4.8	24	18.5	5.5	39.4	34	5.4
6/01/94	13.1	8	5.1	23.7	18	5.7	38.9	33	5.9
7/01/94	12	7.5	4.5	22.1	17	5.1	36.6	32.5	4.1
10/01/94	6.8	11	-4.2	17.8	22.7	-4.9	36.9	39.8	-2.9
11/01/94	7.1	10.5	-3.4	18.5	23	-4.5	37.8	41	-3.2
12/01/94	3.2	5	-1.8	9.3	13	-3.7	22.8	26.5	-3.7
13/01/94	3	5	-2	8.6	12.5	-3.9	21.7	26	-4.3
14/01/94	2.9	4.5	-1.6	8.4	12	-3.6	21.2	25.5	-4.3
17/01/94	3	4	-1	8.4	11	-2.6	21.2	24.5	-3.3
18/01/94	3.2	4.5	-1.3	9.1	12	-2.9	22.8	26	-3.2
19/01/94	3.9	5	-1.1	11	13.5	-2.5	25.4	29	-3.6
20/01/94	3.3	4.5	-1.2	9.3	12	-2.7	23.2	25.5	-2.3
21/01/94	3.8	5.5	-1.7	10.6	13.5	-2.9	25.4	29	-3.6
24/01/94	4	4.5	-0.5	11.3	13	-1.7	26.5	28	-1.5
25/01/94	7.1	8	-0.9	17.6	19.5	-1.9	36.6	38.5	-1.9
27/01/94	5.7	7	-1.3	15.6	17.5	-1.9	33.6	35.5	-1.9
28/01/94	5.5	6.5	-1	14.9	17	-2.1	32.6	34	-1.4
31/01/94	7.4	8	-0.6	18.3	20	-1.7	37.7	39.5	-1.8

OTM		ATM		ITM	
Av. Spr.	-0.295	Av. Spr.	-1.17	Av. Spr.	-1.205
St.dev.	2.707	St.dev.	3.563	St.dev.	3.421
Cr. Cx.	0.636	Cr. Cx.	0.757	Cr. Cx.	0.874

(a) The daily pricing results for OTM, ATM and ITM call options. (b) The spread in this case measured the difference in yield basis points between model and market prices. This measured the errors in model prices.

Appendix V continued:

January 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
4/01/94	17	14	3	28.9	25.5	3.4	44.6	42	2.6
5/01/94	12.8	9	3.8	22.8	19	3.8	36.8	34	2.8
6/01/94	13.1	9.5	3.6	23.1	19.5	3.6	37.2	34.5	2.7
7/01/94	14.2	11	3.2	24.8	20.5	4.3	39.4	35.5	3.9
10/01/94	10.5	7.5	3	19.4	15.2	4.2	32.3	28.4	3.9
11/01/94	3.4	6.5	-3.1	9.5	14	-4.5	23	26.5	-3.5
12/01/94	7.4	12	-4.6	18.8	23.5	-4.7	37.4	40	-2.6
13/01/94	7.9	12	-4.1	19.8	23.5	-3.7	38.7	40.5	-1.8
14/01/94	8.1	12.5	-4.4	20.3	24	-3.7	39.2	41	-1.8
17/01/94	8	12	-4	20.2	23	-2.8	39.1	41	-1.9
18/01/94	7.4	10.5	-3.1	18.8	21.5	-2.7	37.4	38.5	-1.1
19/01/94	6.1	9.5	-3.4	16.2	19	-2.8	33.7	35	-1.3
20/01/94	7.3	10	-2.7	18.4	20.5	-2.1	37	37.5	-0.5
21/01/94	6.3	9.5	-3.2	16.6	19	-2.4	34.4	35.5	-1.1
24/01/94	5.9	8	-2.1	15.8	18	-2.2	33.1	34.5	-1.4
25/01/94	3.5	5	-1.5	10.1	11.5	-1.4	24	25	-1
27/01/94	4.2	5	-0.8	11.5	12.5	-1	26.5	26.5	0
28/01/94	4.4	5	-0.6	12	12.5	-0.5	27.4	27	0.4
31/01/94	3.4	4	-0.6	9.6	10	-0.4	23.2	23.5	-0.3

	OTM		ATM		ITM
Av. Spr.	-1.137	Av. Spr.	-0.82	Av. Spr.	-0.105
St.dev.	2.988	St.dev.	3.103	St.dev.	2.221
Cr. Cx.	0.654	Cr. Cx.	0.819	Cr. Cx.	0.936

Appendix V continued:

February 1994 10 year bond futures options : Calls

Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/02/94	6.1	7	-0.9	16.5	18	-1.5	34.9	36.5	-1.6
2/02/94	5.2	6	-0.8	14.3	16	-1.7	31.8	33.5	-1.7
3/02/94	4.7	4	0.7	12.8	14	-1.2	29.4	31	-1.6
4/02/94	4.4	4	0.4	12	12.5	-0.5	27.9	29	-1.1
7/02/94	2.6	2	0.6	7.8	8	-0.2	19.8	21	-1.2
8/02/94	3	2.5	0.5	8.4	8.5	-0.1	21.3	22	-0.7
9/02/94	3.4	2.5	0.9	9.6	8.5	1.1	23.6	23	0.6
10/02/94	4.4	3	1.4	12.25	11	1.25	28.3	27	1.3
11/02/94	3.9	2.5	1.4	10.99	10	0.99	26	25.5	0.5
14/02/94	4.7	3	1.7	12.8	11	1.8	29.3	28.5	0.8
15/02/94	3.9	2.5	1.4	10.2	8	2.2	24.7	23	1.7
16/02/94	3.6	1.5	2.1	10.46	8	2.46	25.1	22.5	2.6
17/02/94	6.5	4.5	2	17.3	16	1.3	36.1	35.5	0.6
18/02/94	7.5	6.5	1	19.2	18	1.2	38.8	38	0.8
21/02/94	5.6	4.5	1.1	15.1	14.5	0.6	32.9	32.5	0.4
22/02/94	6.8	5.5	1.3	17.8	17.5	0.3	36.8	35	1.8
23/02/94	6.9	5.5	1.4	18.1	17.5	0.6	37.3	37	0.3
24/02/94	3.5	3	0.5	10	10	0	24.4	25.5	-1.1
25/02/94	6.1	7	-0.9	16.5	18	-1.5	34.9	36.5	-1.6
28/02/94	3.8	2	1.8	9.9	8	1.9	23.7	23	0.7

	OTM		ATM		ITM
Av. Spr.	0.88	Av. Spr.	0.45	Av. Spr.	0.075
St.dev.	0.899	St.dev.	1.267	St.dev.	1.305
Cr. Cx.	0.863	Cr. Cx.	0.95	Cr. Cx.	0.975

Appendix V continued:

February 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/02/94	3.85	4.5	-0.65	10.8	11.5	-0.7	25.2	25.5	-0.3
2/02/94	4.6	5	-0.4	12.4	13	-0.6	28.2	27.5	0.7
3/02/94	5.2	5	0.2	13.9	13.5	0.4	30.5	28.5	2
4/02/94	5.5	6	-0.5	14.8	14.5	0.3	31.8	31	0.8
7/02/94	8.6	8.5	0.1	21.6	20.5	1.1	40.8	39	1.8
8/02/94	8	7.5	0.5	20.2	19	1.2	39.2	37.5	1.7
9/02/94	7.2	6	1.2	18	17	1	36.4	35	1.4
10/02/94	5.5	6	-0.5	14.6	13.5	1.1	31.5	30.5	1
11/02/94	6.1	5	1.1	16.2	14.5	1.7	33.7	32	1.7
14/02/94	5.2	3.5	1.7	13.9	11.5	2.4	30.5	28	2.5
15/02/94	6.6	4.5	2.1	17.2	14.5	2.7	35.2	33	2.2
16/02/94	6.45	4	2.45	16.9	14.5	2.4	34.7	32.5	2.2
17/02/94	3.7	2.5	1.2	10.3	8.5	1.8	24.3	22	2.3
18/02/94	3.2	2.5	0.7	9	8	1	22.2	21.5	0.7
21/02/94	4.3	3.5	0.8	11.9	11	0.9	27.15	26	1.15
22/02/94	3.6	3.5	0.1	9.9	9.5	0.4	23.8	23	0.8
23/02/94	3.5	3	0.5	9.7	9	0.7	23.5	22	1.5
24/02/94	6.8	8	-1.2	17.5	17.5	0	35.6	35	0.6
25/02/94	3.7	5.5	-1.8	10.9	12.5	-1.6	25.3	26.5	-1.2
28/02/94	6.8	6.5	0.3	17.6	16.5	1.1	35.8	35	0.8

OTM		ATM		ITM	
Av. Spr.	0.395	Av. Spr.	0.865	Av. Spr.	1.218
St.dev.	1.072	St.dev.	1.08	St.dev.	0.915
Cr. Cx.	0.798	Cr. Cx.	0.957	Cr. Cx.	0.987

Appendix V continued:

March 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	**Model**	**Mkt.**	**Spr.**	**Model**	**Mkt.**	**Spr.**	**Model**	**Mkt.**	**Spr.**
11/03/94	25.5	21.5	4	37.6	32.5	5.1	51.8	46	5.8
14/03/94	30.9	26	4.9	43.8	38	5.8	58.8	53.5	5.3
15/03/94	22.7	19	3.7	34.2	29.5	4.7	47.8	42.5	5.3
16/03/94	31.6	24	7.6	44.6	38	6.6	59.7	54.5	5.2
17/03/94	27.8	22	5.8	40.1	33.5	6.6	54.7	48.5	6.2
18/03/94	26.4	20	6.4	38.5	31	7.5	52.8	45.5	7.3
21/03/94	28.7	23	5.7	41.2	35	6.2	55.8	50	5.8
22/03/94	30.2	24	6.2	43	36.5	6.5	57.8	52	5.8
23/03/94	27.4	19	8.4	39.5	30.5	9	54	45.5	8.5
24/03/94	30.6	22	8.6	43.4	34	9.4	58.3	49.5	8.8
25/03/94	26.4	21	5.4	38.55	32	6.55	52.8	46.5	6.3
28/03/94	27.4	23	4.4	39.6	34.5	5.1	54	49	5
29/03/94	28.8	24	4.8	41.3	36	5.3	55.9	50	5.9
30/03/94	27.4	25	2.4	39.6	37	2.6	54	51.5	2.5
31/03/94	25	23.5	1.5	37	35	2	51	48.5	2.5

OTM		ATM		ITM	
Av. Spr.	5.32	Av. Spr.	5.93	Av. Spr.	5.747
St.dev.	2.01	St.dev.	1.99	St.dev.	1.736
Cr. Cx.	0.615	Cr. Cx.	0.741	Cr. Cx.	0.857

Appendix V continued:

March 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	**Model**	**Mkt.**	**Spr.**	**Model**	**Mkt.**	**Spr.**	**Model**	**Mkt.**	**Spr.**
11/03/94	27	27	0	38.1	38.5	-0.4	50.8	52.5	-1.7
14/03/94	22.3	21.5	0.8	32.8	31.5	1.3	44.8	44	0.8
15/03/94	29.8	28	1.8	41.3	40	1.3	54.4	54	0.4
16/03/94	21.7	20	1.7	32.2	29	3.2	44.2	41.5	2.7
17/03/94	24.9	21.5	3.4	35.8	32	3.8	46.3	45.5	0.8
18/03/94	26.2	23.5	2.7	37.2	34	3.2	49.8	48	1.8
21/03/94	23.6	21	2.6	34.2	32.5	1.7	46.3	44	2.3
22/03/94	22.4	19.5	2.9	32.8	29.5	3.3	44.6	42	2.6
23/03/94	25.4	20	5.4	36.2	30.5	5.7	48.8	43.5	5.3
24/03/94	22.5	18	4.5	33.1	27.5	5.6	45.2	41	4.2
25/03/94	26.2	22	4.2	37.2	33.5	3.7	49.8	47	2.8
28/03/94	25.4	23.5	1.9	36.2	34	2.2	48.8	47.5	1.3
29/03/94	24.1	22	2.1	34.9	32.5	2.4	47.2	45.5	1.7
30/03/94	25.4	26.5	-1.1	36.2	37.5	-1.3	48.8	50.6	-1.8
31/03/94	27.5	28	-0.5	38.7	39.5	-0.8	51.4	53	-1.6

	OTM		ATM		ITM
Av. Spr.	2.16	Av. Spr.	2.327	Av. Spr.	1.44
St.dev.	1.839	St.dev.	2.096	St.dev.	2.066
Cr. Cx.	0.829	Cr. Cx.	0.866	Cr. Cx.	0.9

Appendix V continued:

April 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
5/04/94	23.4	23	0.4	35	33.5	1.5	48.2	47	1.2
6/04/94	31.5	29	2.5	44.4	41.5	2.9	58.9	57.5	1.4
7/04/94	31.7	28	3.7	44.8	39	5.8	59.3	55	4.3
8/04/94	31.5	26	5.5	44.5	38.5	6	59	54.5	4.5
11/04/94	27.5	21	6.5	39.8	32.5	7.3	53.7	47.5	6.2
12/04/94	30	23	7	43.8	35	8.8	56.9	52	4.9
13/04/94	10.9	16	-5.1	20.4	26.5	-6.1	34.35	40	-5.65
14/04/94	28.3	21	7.3	40.7	32.5	8.2	54.75	47.5	7.25
15/04/94	23.9	17	6.9	35.6	27.5	8.1	48.9	42	6.9
18/04/94	15.3	19.5	-4.2	27	31.5	-4.5	42.5	47	-4.5
19/04/94	24.4	20	4.4	35.8	30.5	5.3	50.2	45	5.2
20/04/94	22.7	19.5	3.2	33.7	29.5	4.2	47.7	43	4.7
21/04/94	32.6	27	5.6	45.3	39.5	5.8	61	55.5	5.5
22/04/94	24.8	19.8	5	36.75	30.5	6.25	50.2	44.5	5.7
26/04/94	28.6	21.5	7.1	40.25	33	7.25	55.2	48	7.2
27/04/94	26.9	20	6.9	39.2	31	8.2	52.9	45.5	7.4
28/04/94	26.8	20.5	6.3	38.9	31.5	7.4	52.8	46	6.8
29/04/94	26.4	22	4.4	38.1	33	5.1	52.8	47.5	5.3

OTM		ATM		ITM	
Av. Spr.	4.078	Av. Spr.	4.861	Av. Spr.	4.128
St.dev.	3.677	St.dev.	4.166	St.dev.	3.781
Cr. Cx.	0.767	Cr. Cx.	0.774	Cr. Cx.	0.819

Appendix V continued:

April 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
5/04/94	29.6	29.5	0.1	40.6	41	-0.4	53.6	55	-1.4
6/04/94	22.2	22.5	-0.3	32.3	32	0.3	44.3	44.5	-0.2
7/04/94	22	20.5	1.5	32	30	2	44	42.5	1.5
8/04/94	22.1	20.5	1.6	32.2	29	3.2	44.2	41.5	2.7
11/04/94	22.1	19.5	2.6	36	31.5	4.5	48.6	45	3.6
12/04/94	25.6	21	4.6	33.6	28.5	5.1	45.8	41.5	4.3
13/04/94	23.4	20	3.4	40.8	34.5	6.3	52.75	48.5	4.25
14/04/94	28.5	23	5.5	35.3	30	5.3	47.7	43	4.7
15/04/94	24.9	19.5	5.4	39.9	35	4.9	52.8	49	3.8
18/04/94	29	24.5	4.5	33.6	26.5	7.1	45.8	39.5	6.3
19/04/94	23.3	16.5	6.8	39.8	36.5	3.3	52.2	50.5	1.7
20/04/94	28	26	2	41.9	39	2.9	54.4	53.5	0.9
21/04/94	29.8	27	2.8	31.7	28.5	3.2	43.1	41	2.1
22/04/94	20.9	19	1.9	38.9	35.5	3.4	51.7	49	2.7
26/04/94	28.2	24.5	3.7	35	30.5	4.5	47.4	43.5	3.9
27/04/94	26.1	21	5.1	36.6	31.5	5.1	49.3	45	4.3
28/04/94	26.3	21.5	4.8	36.8	32	4.8	49.4	45.5	3.9
29/04/94	25.6	24	1.6	36.9	34.5	2.4	48.8	48	0.8

OTM		ATM		ITM	
Av. Spr.	3.2	Av. Spr.	3.772	Av. Spr.	2.769
St.dev.	1.981	St.dev.	1.925	St.dev.	1.943
Cr. Cx.	0.798	Cr. Cx.	0.867	Cr. Cx.	0.899

Appendix V continued:

May 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
2/05/94	22.6	18.5	4.1	33.7	28.5	5.2	47.7	41.5	6.2
3/05/94	24.7	18.5	6.2	36.2	29.5	6.7	50.6	43.5	7.1
4/05/94	22.5	19	3.5	34.4	29	5.4	48	42	6
5/05/94	21.4	16.5	4.9	33.2	27.5	5.7	46.5	40.5	6
6/05/94	33	25.5	7.5	46.2	38.5	7.7	61.5	54.5	7
9/05/94	26.4	19	7.4	38.5	30	8.5	52.8	44.5	8.3
10/05/94	25.7	17	8.7	38.2	30	8.2	52.5	43.5	9
11/05/94	9.75	12	-2.25	19	23	-4	32.5	36.5	-4
12/05/94	14.1	16.5	-2.4	25.2	27.5	-2.3	40.8	42.5	-1.7
13/05/94	17.1	17.5	-0.4	28.5	29.5	-1	45.2	46	-0.8
16/05/94	15.1	14.5	0.6	27.3	26	1.3	43.4	42	1.4
17/05/94	12.1	11.5	0.6	22.8	21.5	1.3	37.6	37.5	0.1
18/05/94	14.9	12.5	2.4	26	24	2	42.5	40	2.5
19/05/94	12.9	8.5	4.4	22.8	19	3.8	38.4	34	4.4
20/05/94	5.9	8.5	-2.6	14.9	19	-4.1	33.1	35	-1.9
23/05/94	3.1	4.5	-1.4	8.5	12	-3.5	22.3	25	-2.7
24/05/94	4.8	8	-3.2	13.4	18	-4.6	30.5	33.5	-3
25/05/94	3.6	6.5	-2.9	10.68	15	-4.32	25.1	29	-3.9
26/05/94	6.4	10	-3.6	17.6	21.5	-3.9	36.5	39	-2.5
27/05/94	4.2	6	-1.8	12.2	15	-2.8	28.2	30.5	-2.3
30/05/94	5.9	7	-1.1	16.4	17.5	-1.1	34.8	35.5	-0.7
31/05/94	3.4	4.5	-1.1	10	11.5	-1.5	24.4	26	-1.6

OTM		ATM		ITM	
Av. Spr.	1.252	Av. Spr.	1.031	Av. Spr.	1.495
St.dev.	3.943	St.dev.	4.621	St.dev.	4.404
Cr. Cx.	0.952	Cr. Cx.	0.957	Cr. Cx.	0.947

Appendix V continued:

May 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
2/05/94	29.3	27.5	1.8	41.1	39.5	1.6	53.3	54	-0.7
3/05/94	26.1	24	2.1	38.7	35.5	3.2	50.6	49	1.6
4/05/94	29.6	28	1.6	41.1	40	1.1	54.8	54.5	0.3
5/05/94	30.7	27.5	3.2	42.5	39.5	3	56.2	53	3.2
6/05/94	20.6	18	2.6	30.9	27.5	3.4	42.7	39.5	3.2
9/05/94	26.3	22.5	3.8	37.2	33	4.2	49.9	46.5	3.4
10/05/94	26.4	20.5	5.9	37.5	31.5	6	50.7	44	6.7
11/05/94	16.5	22.5	-6	28	34	-6	44.2	47.5	-3.3
12/05/94	11.9	16	-4.1	21.4	26	-4.6	34.8	39.5	-4.7
13/05/94	10.1	12	-1.9	18.7	21	-2.3	31.3	33.5	-2.2
16/05/94	10.9	11	-0.1	20.2	20.5	-0.3	33.9	34	-0.1
17/05/94	13.6	13	0.6	24	24	0	39.1	38.5	0.6
18/05/94	11.3	10	1.3	21.2	19.5	1.7	34	33	1
19/05/94	13.3	11	2.3	24	21	3	37.7	35.5	2.2
20/05/94	4.3	7.5	-3.2	12.2	16.5	-4.3	26.9	31	-4.1
23/05/94	7.6	11	-3.4	19.9	23	-3.1	38	40	-2
24/05/94	4.9	8.5	-3.6	13	18	-5	28.75	33	-4.25
25/05/94	6.5	10.5	-4	16.9	21.5	-4.6	35.6	37.5	-1.9
26/05/94	3.6	6	-2.4	10.1	14	-3.9	24.7	27.5	-2.8
27/05/94	5.5	8	-2.5	14.7	17.5	-2.8	32.6	33	-0.4
30/05/94	3.9	5	-1.1	10.9	12.5	-1.6	26.7	26.5	0.2
31/05/94	6.8	8.5	-1.7	17.5	19	-1.5	36.4	36	0.4

OTM		ATM		ITM	
Av. Spr.	-0.4	Av. Spr.	-0.58	Av. Spr.	-0.166
St.dev.	3.114	St.dev.	3.5	St.dev.	2.88
Cr. Cx.	0.959	Cr. Cx.	0.96	Cr. Cx.	0.961

Appendix V continued:

June 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
10/06/94	31	30.5	0.5	44	43	1	58.9	58.5	0.4
14/06/94	22.5	22.5	0	34	33	1	47.5	46	1.5
15/06/94	27.4	26.5	0.9	39.7	38	1.7	54.1	52	2.1
16/06/94	25.9	26.5	-0.6	36.8	38	-1.2	50.8	51.5	-0.7
17/06/94	32.2	34.5	-2.3	45.4	47.5	-2.1	60.6	64	-3.4
20/06/94	37.9	34.5	3.4	50.1	46	4.1	64.5	60	4.5
21/06/94	39.7	40	-0.3	52.3	52	0.3	66.9	66.5	0.4
22/06/94	45.2	44	1.2	56.5	57	-0.5	71.7	72	-0.3
23/06/94	38.4	40	-1.6	50.8	50.5	0.3	65.3	64.5	0.8
24/06/94	37.9	36.5	1.4	50.2	48.5	1.7	64.6	62.5	2.1
27/06/94	44.3	47	-2.7	57.6	60	-2.4	73	75.5	-2.5
28/06/94	42.7	43	-0.3	55.8	55.5	0.3	70.9	74.5	-3.6
29/06/94	36.9	35.5	1.4	49.1	47	2.1	63.2	60.5	2.7
30/06/94	39.4	36.5	2.9	50.1	47.5	2.6	64.4	61.5	2.9

OTM		ATM		ITM	
Av. Spr.	0.279	Av. Spr.	0.636	Av. Spr.	0.493
St.dev.	1.776	St.dev.	1.802	St.dev.	2.417
Cr. Cx.	0.969	Cr. Cx.	0.972	Cr. Cx.	0.962

Appendix V continued:

June 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
10/06/94	22.2	25	-2.8	32.7	35	-2.3	44.75	47.5	-2.75
14/06/94	30	32.5	-2.5	41.6	44.5	-2.9	55.9	58.5	-2.6
15/06/94	25.3	28	-2.7	36.2	39	-2.8	48.7	52	-3.3
16/06/94	26	32.5	-6.5	38.1	43.5	-5.4	50.6	57.5	-6.9
17/06/94	21.2	28	-6.8	31.5	37	-5.5	43.5	49	-5.5
20/06/94	37	38.5	-1.5	47.9	49.5	-1.6	60.5	62.5	-2
21/06/94	35.4	38.5	-3.1	46.1	49.5	-3.4	58.4	62	-3.6
22/06/94	32.4	38.5	-6.1	42.7	49	-6.3	54.6	61	-6.4
23/06/94	36.5	42	-5.5	47.4	53	-5.6	59.8	66	-6.2
24/06/94	37	41	-4	47.9	52	-4.1	60.4	65	-4.6
27/06/94	31.7	40	-8.3	41.8	50	-8.2	53.6	62	-8.4
28/06/94	32.9	39.5	-6.6	43.3	49.5	-6.2	55.2	61.5	-6.3
29/06/94	37.9	43	-5.1	48.9	54.5	-5.6	61.5	67.5	-6
30/06/94	37.1	42	-4.9	48	53	-5	60.5	66.5	-6

OTM		ATM		ITM	
Av. Spr.	-4.743	Av. Spr.	-4.64	Av. Spr.	-5.039
St.dev.	2.014	St.dev.	1.842	St.dev.	1.908
Cr. Cx.	0.942	Cr. Cx.	0.956	Cr. Cx.	0.954

Appendix V continued:

July 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/07/94	39.5	35.5	4	50.2	47	3.2	64.6	61	3.6
4/07/94	34.6	32	2.6	46.4	43	3.4	60.2	56.5	3.7
5/07/94	35.1	31.5	3.6	46.9	42.5	4.4	60.8	56	4.8
6/07/94	39.9	34	5.9	52.5	46	6.5	67.3	60.5	6.8
7/07/94	27.8	33	-5.2	40.75	45	-4.25	54.2	60.5	-6.3
8/07/94	28.3	33.5	-5.2	41.4	45.5	-4.1	54.8	61	-6.2
11/07/94	34.3	37	-2.7	46.9	50	-3.1	62.3	65.5	-3.2
12/07/94	23.8	26.5	-2.7	35.6	38	-2.4	49.3	51.5	-2.2
13/07/94	27.8	28.5	-0.7	38.9	40	-1.1	53.2	54	-0.8
14/07/94	28.1	27.5	0.6	40.5	39	1.5	55.1	54	1.1
15/07/94	26.5	25	1.5	38.7	36.5	2.2	53	50.5	2.5
18/07/94	29.8	26	3.8	42.4	38	4.4	57.2	53	4.2
19/07/94	26.9	23.5	3.4	39.1	34.5	4.6	53.5	49	4.5
20/07/94	25.75	21	4.75	37.8	32.5	5.3	51.9	47.5	4.4
21/07/94	28.7	26	2.7	41.2	38	3.2	55.8	54	1.8
22/07/94	33.1	28	5.1	44.7	40.5	4.2	57.4	56.5	0.9
25/07/94	29.4	25.5	3.9	42.1	37.5	4.6	56.8	53	3.8
26/07/94	32.2	27	5.2	45.4	40	5.4	60.6	56	4.6
27/07/94	13.2	17.5	-4.3	23.8	28.5	-4.7	39	43	-4
28/07/94	14.3	21	-6.7	25.5	32.5	-7	41.9	49	-7.1
29/07/94	16.5	21.5	-5	28.6	35	-6.4	45.3	51	-5.7

OTM		ATM		ITM	
Av. Spr.	0.693	Av. Spr.	0.945	Av. Spr.	0.533
St.dev.	4.152	St.dev.	4.371	St.dev.	4.399
Cr. Cx.	0.818	Cr. Cx.	0.809	Cr. Cx.	0.777

Appendix V continued:

July 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/07/94	36.9	41.5	-4.6	47.9	53	-5.1	60.4	66	-5.6
4/07/94	39.2	43.5	-4.3	51.4	55	-3.6	64.2	68.5	-4.3
5/07/94	39.6	41.5	-1.9	50.9	53.5	-2.6	63.7	67	-3.3
6/07/94	35.2	35.5	-0.3	45.9	46.5	-0.6	58.1	59.5	-1.4
7/07/94	34.7	33.5	1.2	45.3	43.5	1.8	57.5	56.5	1
8/07/94	34.2	33	1.2	44.8	43	1.8	56.9	55.5	1.4
11/07/94	30.4	28	2.4	40.5	38	2.5	51.9	50	1.9
12/07/94	38	35.5	2.5	50	47	3	62.7	60	2.7
13/07/94	36.2	31.5	4.7	46.9	42.5	4.4	59.3	55.5	3.8
14/07/94	34.8	28	6.8	45.5	38.5	7	57.7	51.5	6.2
15/07/94	26.1	28	-1.9	37.1	39	-1.9	49.7	52.5	-2.8
18/07/94	23.2	23	0.2	33.9	33.5	0.4	46.1	46.5	-0.4
19/07/94	25.7	25	0.7	36.7	36	0.7	49.2	49.5	-0.3
20/07/94	26.8	26	0.8	37.9	36	1.9	50.6	49	1.6
21/07/94	24.2	25	-0.8	34.9	34.5	0.4	47.3	47.5	-0.2
22/07/94	21.6	22	-0.4	32.1	31.5	0.6	44.1	44	0.1
25/07/94	23.5	23	0.5	34.2	33	1.2	46.5	46	0.5
26/07/94	21.2	20	1.2	31.6	29.5	2.1	44.4	41.5	2.9
27/07/94	12.7	19	-6.3	23	29.5	-6.5	38	43.5	-5.5
28/07/94	24.2	20	4.2	35	29	6	47.4	42	5.4
29/07/94	10.3	16.5	-6.2	19	26	-7	31.8	37.5	-5.7

OTM		ATM		ITM	
Av. Spr.	-0.014	Av. Spr.	0.31	Av. Spr.	-0.095
St.dev.	3.388	St.dev.	3.713	St.dev.	3.468
Cr. Cx.	0.915	Cr. Cx.	0.909	Cr. Cx.	0.92

Appendix V continued:

August 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
2/08/94	14.2	15	-0.8	25.3	26.5	-1.2	41	42	-1
3/08/94	10	11	-1	19.1	20	-0.9	32.5	34	-1.5
4/08/94	11.7	11.5	0.2	21.7	22	-0.3	36.1	36	0.1
5/08/94	18.2	20.5	-2.3	31.2	33.5	-2.3	48.5	50	-1.5
8/08/94	11.1	13	-1.9	20.8	23	-2.2	34.8	37	-2.2
9/08/94	15.1	16.5	-1.4	26.8	28	-1.2	42.8	44.5	-1.7
10/08/94	14.5	16	-1.5	25.9	27	-1.1	41.7	42	-0.3
11/08/94	11.7	13.5	-1.8	21.8	23.5	-1.7	36.1	37.5	-1.4
12/08/94	9.5	10.5	-1	18.4	19.5	-1.1	31.5	32.5	-1
15/08/94	12.1	11	1.1	22.2	21	1.2	36.8	35.5	1.3
16/08/94	11	9.5	1.5	21	19	2	34.6	33	1.6
17/08/94	13.1	10.5	2.6	23.6	20.5	3.1	38.8	35.5	3.3
18/08/94	11.7	8.5	3.2	21.7	17.5	4.2	36.1	32	4.1
19/08/94	16.7	12	4.7	29	24	5	43	41.5	1.5
22/08/94	3.3	5	-1.7	9.2	13	-3.8	23	26.5	-3.5
23/08/94	7.3	9.5	-2.2	18.8	22	-3.2	38.3	40	-1.7
24/08/94	8.4	10	-1.6	21.3	23	-1.7	41.6	42.5	-0.9
25/08/94	3.8	5	-1.2	10.7	13	-2.3	25.4	27.5	-2.1
26/08/94	7.8	9	-1.2	19.9	22	-2.1	39.8	41	-1.2
29/08/94	3.6	3.5	0.1	9.7	11	-1.3	22.8	25.5	-2.7
30/08/94	8.2	9	-0.8	20.7	21.5	-0.8	40.9	42	-1.1
31/08/94	7.9	8	-0.1	20.4	20.7	-0.3	39.9	40	-0.1

OTM		ATM		ITM	
Av. Spr.	-0.323	Av. Spr.	-0.55	Av. Spr.	-0.545
St.dev.	1.866	St.dev.	2.289	St.dev.	1.88
Cr. Cx.	0.891	Cr. Cx.	0.911	Cr. Cx.	0.957

Appendix V continued:

August 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
2/08/94	12.1	14.5	-2.4	21.7	24.5	-2.8	35.4	38	-2.6
3/08/94	16.5	19.5	-3	28.1	30.5	-2.4	43.6	46	-2.4
4/08/94	14.4	16.5	-2.1	25.2	28	-2.8	39.9	42	-2.1
5/08/94	9.1	12.5	-3.4	17.3	21.5	-4.2	29.3	33.5	-4.2
8/08/94	15.1	19	-3.9	26.2	30	-3.8	41.2	45	-3.8
9/08/94	11.2	15	-3.8	20.5	24	-3.5	33.7	37.5	-3.8
10/08/94	11.8	15	-3.2	21.3	25	-3.7	34.8	38.5	-3.7
11/08/94	14.4	17	-2.6	25.2	28	-2.8	39.8	42.5	-2.7
12/08/94	17.1	19.5	-2.4	29.1	32	-2.9	44.7	47.5	-2.8
15/08/94	14.1	15.5	-1.4	24.2	26	-1.8	39.2	41	-1.8
16/08/94	15.3	15.5	-0.2	26.4	26.5	-0.1	41.4	41.5	-0.1
17/08/94	13.1	11	2.1	23.2	21	2.2	37.4	35.5	1.9
18/08/94	14.4	11.5	2.9	25.1	22	3.1	39.8	37.5	2.3
19/08/94	10.1	7	3.1	18.7	15	3.7	31.4	28	3.4
22/08/94	7.3	11	-3.7	18.5	22.5	-4	37.2	39	-1.8
23/08/94	3.3	6	-2.7	9.4	13	-3.6	22.7	25.5	-2.8
24/08/94	2.8	4.5	-1.7	8	10.5	-2.5	20.2	19	1.2
25/08/94	6.3	8	-1.7	16.6	18.5	-1.9	34.4	35	-0.6
26/08/94	3.1	4.5	-1.4	8.8	11	-2.2	21.5	23	-1.5
29/08/94	7	8.5	-1.5	17.9	19	-1.1	36.2	36	0.2
30/08/94	2.9	4.5	-1.6	8.3	9.5	-1.2	20.7	21	-0.3
31/08/94	3.1	3	0.1	8.6	9	-0.4	21.5	21.5	0

OTM		ATM		ITM	
Av. Spr.	-1.568	Av. Spr.	-1.76	Av. Spr.	-1.273
St.dev.	2.035	St.dev.	2.241	St.dev.	2.134
Cr. Cx.	0.924	Cr. Cx.	0.95	Cr. Cx.	0.969

Appendix V continued:

September 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
13/09/94	27.2	32.5	-5.3	39.4	44.5	-5.1	53.8	59	-5.2
14/09/94	31.7	37.5	-5.8	44.6	50	-5.4	59.7	65.5	-5.8
15/09/94	25.4	30.5	-5.1	36.8	42	-5.2	50.8	55.5	-4.7
16/09/94	22.5	27.5	-5	34	38	-4	47.6	51	-3.4
19/09/94	38.5	34	4.5	50.9	46	4.9	65.4	60	5.4
20/09/94	34.5	31.5	3	46.3	42.5	3.8	60	55.5	4.5
21/09/94	37.6	32.5	5.1	49.9	44.5	5.4	64.1	59	5.1
22/09/94	24.5	31	-6.5	36.4	42.5	-6.1	50.3	56.5	-6.2
23/09/94	24.1	29.5	-5.4	35.9	41	-5.1	49.7	55.5	-5.8
26/09/94	31.7	36.5	-4.8	44.7	49.5	-4.8	59.8	64.5	-4.7
27/09/94	34.8	38.5	-3.7	46.7	51.5	-4.8	62	67.5	-5.5
28/09/94	28.7	33.5	-4.8	41.1	45.5	-4.4	55.7	60.5	-4.8
29/09/94	31.6	34.5	-2.9	44.5	47	-2.5	59.6	63	-3.4
30/09/94	27.7	29.5	-1.8	40	41.5	-1.5	54.4	56	-1.6

OTM		ATM		ITM	
Av. Spr.	-2.75	Av. Spr.	-2.49	Av. Spr.	-2.579
St.dev.	3.971	St.dev.	4.081	St.dev.	4.281
Cr. Cx.	0.629	Cr. Cx.	0.651	Cr. Cx.	0.672

Appendix V continued:

September 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
13/09/94	35.7	35	0.7	46.4	45.5	0.9	58.8	58.5	0.3
14/09/94	32	30	2	41.3	40.5	0.8	54	52.5	1.5
15/09/94	37.7	36	1.7	48.8	47.5	1.3	61.4	60.5	0.9
16/09/94	39.3	38.5	0.8	51.4	50.5	0.9	64.3	64	0.3
19/09/94	36.4	38.5	-2.1	47.3	49.5	-2.2	59.7	62	-2.3
20/09/94	40	42	-2	51.4	53.5	-2.1	64.3	67.5	-3.2
21/09/94	37.2	40	-2.8	48.2	50.5	-2.3	60.7	63	-2.3
22/09/94	38.1	38.5	-0.4	49.3	50	-0.7	61.9	63	-1.1
23/09/94	38.4	39.5	-1.1	49.3	50	-0.7	61.9	63	-1.1
26/09/94	31.2	30.5	0.7	41.3	41	0.3	52.6	53	-0.4
27/09/94	29.9	29.5	0.4	40.7	39	1.7	51.1	50.5	0.6
28/09/94	32.4	33	-0.6	44.1	43.5	0.6	56	56	0
29/09/94	31.4	29	2.4	41.4	38.5	2.9	52.9	50.5	2.4
30/09/94	34.5	30	4.5	45.1	41	4.1	57.1	54	3.1

	OTM		ATM		ITM
Av. Spr.	0.3	Av. Spr.	0.393	Av. Spr.	-0.093
St.dev.	1.988	St.dev.	1.878	St.dev.	1.803
Cr. Cx.	0.919	Cr. Cx.	0.948	Cr. Cx.	0.963

Appendix V continued:

October 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
4/10/94	29	29.5	-0.5	41.6	41.5	0.1	56.1	57	-0.9
5/10/94	31.7	31	0.7	44.7	43.5	1.2	59.8	59	0.8
6/10/94	25	23.5	1.5	36.9	35	1.9	51.4	48.5	2.9
7/10/94	24.1	21.5	2.6	34.5	32	2.5	48.5	45	3.5
10/10/94	28.4	23	5.4	40.8	35	5.8	55.8	51	4.8
11/10/94	27	22.5	4.5	39.2	33.5	5.7	54	48	6
12/10/94	22.4	18.5	3.9	33.8	28.5	5.3	47.6	43	4.6
13/10/94	27.4	24	3.4	39.6	35.5	4.1	54.1	51	3.1
14/10/94	22.9	19	3.9	34.5	29.5	5	48.1	42.5	5.6
17/10/94	10.7	16	-5.3	20	26	-6	33.8	40	-6.2
18/10/94	11.2	15.5	-4.3	20.8	26	-5.2	34.9	40	-5.1
19/10/94	10.8	15	-4.2	20.3	25.5	-5.2	34.2	39	-4.8
20/10/94	13.8	17.5	-3.7	24.7	29	-4.3	40.2	44.5	-4.3
21/10/94	12.5	16.5	-4	23.3	27.5	-4.2	37.6	41.5	-3.9
24/10/94	13.1	17	-3.9	23.6	28	-4.4	38.8	43.5	-4.7
25/10/94	18.8	22.5	-3.7	32.2	36	-3.8	49.6	52.5	-2.9
26/10/94	11.9	16.5	-4.6	21.9	27.5	-5.6	36.4	41.5	-5.1
27/10/94	11.9	16	-4.1	21.9	27	-5.1	36.4	41	-4.6
28/10/94	11.8	15.5	-3.7	21.8	25.5	-3.7	36.3	39.5	-3.2
31/10/94	9.4	12	-2.6	18.2	21.5	-3.3	31.1	34.5	-3.4

OTM		ATM		ITM	
Av. Spr.	-0.935	Av. Spr.	-0.96	Av. Spr.	-0.89
St.dev.	3.734	St.dev.	4.418	St.dev.	4.28
Cr. Cx.	0.914	Cr. Cx.	0.904	Cr. Cx.	0.907

Appendix V continued:

October 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
4/10/94	23.9	28.5	-4.6	34.6	38.5	-3.9	47	51.5	-4.5
5/10/94	21.7	26	-4.3	32.1	35.5	-3.4	44.1	48.5	-4.4
6/10/94	27.5	30.5	-3	38.8	42	-3.2	51.5	55	-3.5
7/10/94	29.6	30.5	-0.9	41.1	42.5	-1.4	54.2	56.5	-2.3
10/10/94	24.4	24	0.4	35.2	33.5	1.7	47.7	45.5	2.2
11/10/94	25.2	23.5	1.7	36.6	34	2.6	49.2	47.5	1.7
12/10/94	30.2	28.5	1.7	41.8	39	2.8	54.9	53.5	1.4
13/10/94	25.4	25	0.4	36.2	35	1.2	48.8	48.5	0.3
14/10/94	29.6	28	1.6	41.1	39.5	1.6	54.2	54	0.2
17/10/94	28.9	25	3.9	39.6	36	3.6	53.4	50.5	2.9
18/10/94	28.2	23	5.2	39.5	34	5.5	52.4	48	4.4
19/10/94	27.5	23	4.5	40	34.5	5.5	52.9	48.5	4.4
20/10/94	24.9	19	5.9	35.7	29	6.7	48.2	42.5	5.7
21/10/94	13.7	20.5	-6.8	24.1	31.5	-7.4	38.5	45	-6.5
24/10/94	13.1	19	-5.9	23.2	29	-5.8	37.4	42.5	-5.1
25/10/94	8.6	14.5	-5.9	16.7	23.5	-6.8	28.4	35	-6.6
26/10/94	14.3	21	-6.7	25	32	-7	39.6	45.5	-5.9
27/10/94	14.3	21	-6.7	25	32	-7	39.6	45.5	-5.9
28/10/94	14.3	21	-6.7	25.1	32	-6.9	39.7	45.6	-5.9
31/10/94	17.3	22	-4.7	29.4	34	-4.6	45.1	49	-3.9

OTM		ATM		ITM	
Av. Spr.	-1.545	Av. Spr.	-1.31	Av. Spr.	-1.565
St.dev.	4.474	St.dev.	4.844	St.dev.	4.162
Cr. Cx.	0.778	Cr. Cx.	0.78	Cr. Cx.	0.825

November 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/11/94	18.2	20	-1.8	31.1	32	-0.9	48.4	48	0.4
2/11/94	16.2	19	-2.8	28.3	31	-2.7	44.8	47	-2.2
3/11/94	17.5	18	-0.5	29.6	30	-0.4	46.6	46.5	0.1
4/11/94	16.5	17	-0.5	28.7	29	-0.3	45.4	45.5	-0.1
7/11/94	11.7	11.5	0.2	21.7	21.5	0.2	36	35.5	0.5
8/11/94	12.1	10	2.1	22.2	19.5	2.7	36.9	33.5	3.4
9/11/94	12.6	11	1.6	23	21	2	37.9	35.5	2.4
10/11/94	16.4	14	2.4	28.6	26	2.6	45.2	42.5	2.7
11/11/94	14.5	10.5	4	25.8	21	4.8	41.6	37	4.6
14/11/94	15.8	11	4.8	27.7	22	5.7	44.1	38.5	5.6
15/11/94	10.1	10.7	-0.6	18.8	14.5	4.3	32.7	27.5	5.2
16/11/94	15	10	5	26.7	21	5.7	42.7	37	5.7
17/11/94	4.8	7.5	-2.7	13.1	17	-3.9	29.8	32	-2.2
18/11/94	5.1	6.5	-1.4	13.6	16	-2.4	30.7	31.5	-0.8
21/11/94	4.1	4.5	-0.4	11.6	12.5	-0.9	27	27.5	-0.5
22/11/94	5.2	5.5	-0.3	14	15	-1	31.9	32	-0.1
23/11/94	5.1	7.5	-2.4	13.7	17	-3.3	30.9	33	-2.1
24/11/94	5.7	7.5	-1.8	15.4	18	-2.6	33.3	35	-1.7
25/11/94	7.7	9.5	-1.8	19.7	21.5	-1.8	39.6	40	-0.4
28/11/94	6.5	7.5	-1	17.3	18	-0.7	36.1	35.5	0.6
29/11/94	7.4	9	-1.6	18.9	20.5	-1.6	38.5	38.5	0
30/11/94	4.5	5	-0.5	12.3	14	-1.7	28.4	29	-0.6

OTM		ATM		ITM	
Av. Spr.	0.00	Av. Spr.	0.17	Av. Spr.	0.93
St.dev.	2.333	St.dev.	2.943	St.dev.	2.552
Cr. Cx.	0.887	Cr. Cx.	0.893	Cr. Cx.	0.92

Appendix V continued:

November 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
1/11/94	9.9	14	-4.1	18.5	23.5	-5	31.1	36	-4.9
2/11/94	10.4	14.5	-4.1	19.3	24	-4.7	32.1	37	-4.9
3/11/94	9.8	12.5	-2.7	18.2	21.5	-3.3	30.8	34	-3.2
4/11/94	10.2	13	-2.8	19	22	-3	31.7	35	-3.3
7/11/94	14.4	16	-1.6	25.3	27.5	-2.2	39.9	43	-3.1
8/11/94	14	15	-1	24.6	26.5	-1.9	39.2	42	-2.8
9/11/94	13.5	13.5	0	23.9	24.5	-0.6	38.2	39	-0.8
10/11/94	10.3	9.5	0.8	19.1	18.5	0.6	33.2	31.5	1.7
11/11/94	11.7	9.5	2.2	21.3	19	2.3	35.5	33.5	2
14/11/94	10.6	8	2.6	19.7	16.5	3.2	32.7	30	2.7
15/11/94	16.7	14.5	2.2	27.9	26.5	1.4	43.3	43.5	-0.2
16/11/94	11.3	8.5	2.8	20.2	18	2.2	33.2	32	1.2
17/11/94	13.2	9	4.2	23.3	19	4.3	37.5	34	3.5
18/11/94	12.8	7.5	5.3	22.7	27.5	-4.8	37.5	33	4.5
21/11/94	5.7	8	-2.3	15.5	18	-2.5	32.6	34.5	-1.9
22/11/94	4.7	6.5	-1.8	12.6	15	-2.4	28.6	30	-1.4
23/11/94	4.8	9	-4.2	12.9	18	-5.1	29	33	-4
24/11/94	4.2	7	-2.8	11.7	15.5	-3.8	26.7	30	-3.3
25/11/94	3.1	5	-1.9	8.7	12	-3.3	21.7	25	-3.3
28/11/94	3.7	5	-1.3	10.3	13	-2.7	24.4	27	-2.6
29/11/94	3.3	5	-1.7	9.2	12.5	-3.3	22.5	26	-3.5
30/11/94	5.4	7.5	-2.1	14.6	17.5	-2.9	31.4	33.5	-2.1

	OTM		ATM		ITM
Av. Spr.	-0.65	Av. Spr.	-1.70	Av. Spr.	-1.35
St.dev.	2.763	St.dev.	2.812	St.dev.	2.778
Cr. Cx.	0.755	Cr. Cx.	0.861	Cr. Cx.	0.869

Appendix V continued:

December 1994 10 year bond futures options : Calls

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
13/12/94	26.4	23.5	2.9	38.5	34.5	4	52.8	49	3.8
14/12/94	32.1	28.5	3.6	45.1	41.5	3.6	60.3	57.1	3.2
15/12/94	26.9	23.5	3.4	39.1	34.5	4.6	53.4	49	4.4
16/12/94	25.4	19	6.4	36.2	29.5	6.7	50.1	43	7.1
19/12/94	12	18	-6	21.5	28.5	-7	37.3	42.5	-5.2
20/12/94	15.2	21.5	-6.3	25.9	33	-7.1	43.5	48	-4.5
21/12/94	16.2	22	-5.8	27.4	34	-6.6	45.5	49.5	-4
22/12/94	13.1	18.5	-5.4	23.6	29.5	-5.9	38.7	44	-5.3
28/12/94	12	17	-5	22.1	27.5	-5.4	36.7	41.5	-4.8
29/12/94	11.5	17.5	-6	21.4	28	-6.6	35.7	42	-6.3

OTM		ATM		ITM	
Av. Spr.	-1.82	Av. Spr.	-1.97	Av. Spr.	-1.16
St.dev.	5.167	St.dev.	5.838	St.dev.	5.111
Cr. Cx.	0.83	Cr. Cx.	0.813	Cr. Cx.	0.84

December 1994 10 year bond futures options : Puts

	OTM			ATM			ITM		
Date	Model	Mkt.	Spr.	Model	Mkt.	Spr.	Model	Mkt.	Spr.
13/12/94	26.2	26	0.2	37.2	37	0.2	49.9	50.5	-0.6
14/12/94	21.3	21	0.3	31.7	31	0.7	43.7	43	0.7
15/12/94	25.8	25	0.8	36.7	36	0.7	50.4	49.5	0.9
16/12/94	28.2	25	3.2	39.5	36	3.5	53.3	50.5	2.8
19/12/94	27.1	22.5	4.6	38.9	33.5	5.4	51.7	47.5	4.2
20/12/94	24	20.5	3.5	34.8	31	3.8	47.2	44.5	2.7
21/12/94	22.9	18.5	4.4	33.6	28.5	5.1	45.8	41.5	4.3
22/12/94	25.9	20.5	5.4	36.7	31.5	5.2	49.4	45.5	3.9
28/12/94	27.1	21.5	5.6	38.3	32.5	5.8	50.9	46.5	4.4
29/12/94	27.7	22.5	5.2	39	34	5	51.8	48	3.8

OTM		ATM		ITM	
Av. Spr.	3.32	Av. Spr.	3.54	Av. Spr.	2.71
St.dev.	2.137	St.dev.	2.192	St.dev.	1.779
Cr. Cx.	0.574	Cr. Cx.	0.655	Cr. Cx.	0.829

APPENDIX W: Option model pricing bias ATM calls vs ITM puts

Figure (i) Relative frequency distribution of ATM calls pricing errors

Numbers on top of the bars denoted relative frequency of error

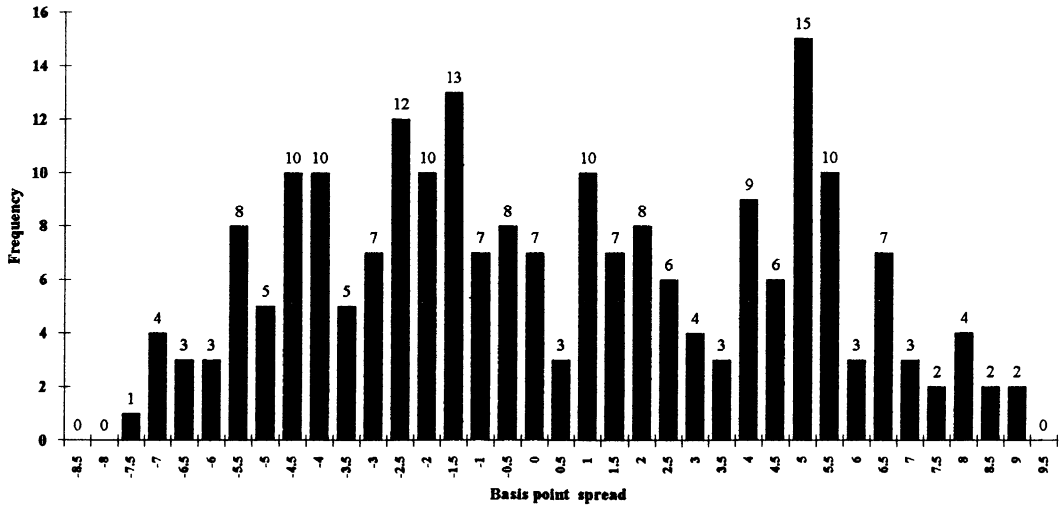
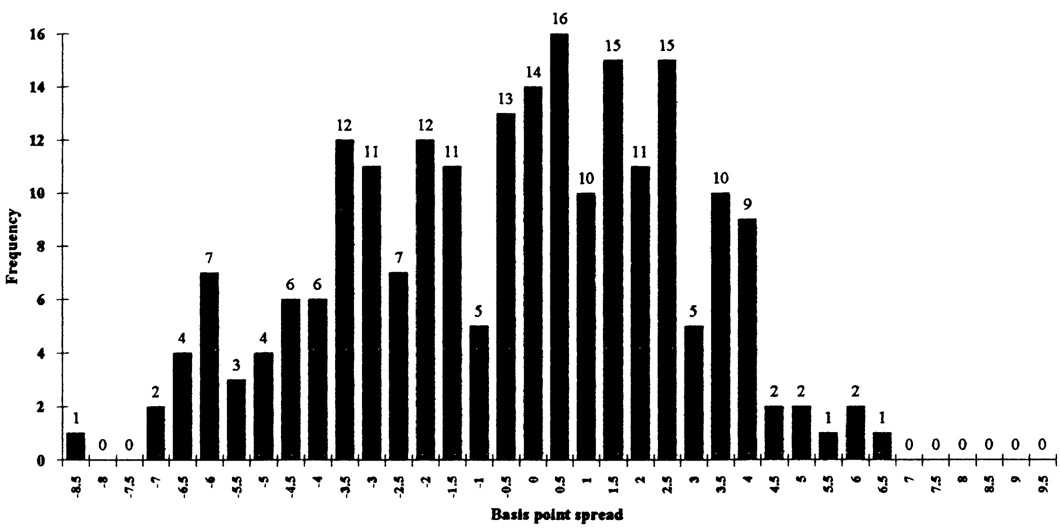


Figure (ii) Relative frequency distribution of ITM puts pricing errors

Numbers on top of the bars denoted relative frequency of error



APPENDIX X : Hedging strategy results 04/01/94

1. Today's value of the Portfolio: This was the benchmark value

Maturity	Coupon	*Bt*	*ct*	Amount	*It*[*Y*(*ft* - *ct*)]	*ft* - *ct*
15/10/02	10	123.535	15/04/95	5	0	0
15/08/03	9.5	122.704	15/02/94	4.75	4.825714286	29
15/09/04	9	119.003	15/03/94	4.5	4.84	1
15/07/05	7.5	108.462	15/01/94	3.75	4.768064516	60
	Total *Vpt*	4,737,040.00				

2. Today's implied end period portfolio value and return

Maturity	Fwd yields	*B(model) ft*	*cft*	*B+c*
15/10/02	6.812	124.628	0	124.62797
15/08/03	6.837	119.022	4.768212114	123.78988
15/09/04	6.896	115.558	4.500596712	120.05904
15/07/05	6.931	105.643	3.779392179	109.42223
		Coupon Contrib.	130482.01	
Days Run	71	Total *It*[*Vpft*]	4,778,991.10	
		Total *It*[*Ypft*]	4.553	

3. Actual Unhedged Return 16/03/94

Maturity	*Bft*	*cft*	*B+c*
15/10/02	121.489	0	121.489
15/08/03	115.736	4.768212114	120.5042121
15/09/04	112.294	4.500596712	116.7945967
15/07/05	102.471	3.779392179	106.2503922
		Total *Vpft*	4,650,382.01
		Unhed.Ret. *Ypft*	-9.40

4. Hedged Return 16/03/94

Futures Close	Current Futures	OPTION STRIKE	OPTION COST	PUT PTS	$/PT	Portfolio Units
92.595	93.13	92.75	5.9	15.5	$85.32	47

Puts Held	47
Cost	23845.69092
Revenue	62645.45918
Net Rev.	*38799.76827*
Vpft (hedge)	4,689,181.78
Ypft (hedge)	-5.19

5. Part of YEM Distribution

Return	Points chg.	Portfol. Value
4.55	0	4778991.100
1.20	10	4748133.706
-2.12	20	4717548.461
-5.41	30	4687232.659
-8.67	40	4657183.627
-11.90	50	4627398.718

APPENDIX Y: Hedged portfolio versus the unhedged portfolio

January 1994

Date	Act. R [(a)]	HR. [(b)]
4/01/94	-9.4	-5.19
5/01/94	-13.8	0.52
6/01/94	-13.8	0.49
7/01/94	-12.99	0.82
10/01/94	-17.73	-1.33
11/01/94	-19.2	-2.2
12/01/94	-21.33	5.7
13/01/94	-21.22	6.2
14/01/94	-21.36	6.28
17/01/94	-22.6	-3.1
18/01/94	-23.91	-3.8
19/01/94	-26.46	6
20/01/94	-25.34	-4.5
21/01/94	-27.35	6.36
24/01/94	-29.54	7.11
25/01/94	-36.12	3.22
27/01/94	-36.06	4.9
28/01/94	-36.33	5.5
31/01/94	-42.98	18.8
Av. Ret.	-24.08	2.725263
Std. Dev.	9.076389	5.74143

(a)Act.R = The portfolio return associated with an unhedged portfolio over the given investment horizon. (b) HR. = The portfolio return associated with a hedged portfolio over the investment horizon.

Appendix Y continued:

February 1994

Date	Act. R	HR.
1/02/94	-41.4	19.66
2/02/94	-40.7	6.2
3/02/94	-40.55	7.42
4/02/94	-40.69	7.7
7/02/94	-37.14	12.9
8/02/94	-39.66	12.7
9/02/94	-43.35	10.74
10/02/94	-48.95	9.38
11/02/94	-48.9	11.11
14/02/94	-56.67	8.9
15/02/94	-53.99	15.5
Av. Ret.	-44.7273	11.11
Std. Dev.	6.40195	3.943006

The investment horizon was the futures maturity date. Thus after the date February 15 the horizon was < 1 month and not included in the sample. This also applied to the May, August and November periods.

March 1944

Date	Act. R	HR.
11/03/94	-32.66	-12.2
14/03/94	-36.5	-7.9
15/03/94	-32.65	-11.45
16/03/94	-37.8	-8.25
17/03/94	-36.1	-13
18/03/94	-36.04	-12.8
21/03/94	-32.21	-10.4
22/03/94	-33.5	-10.9
23/03/94	-38.8	-6.9
24/03/94	-34.59	-10.9
25/03/94	-32.66	-10.5
28/03/94	-27.6	-14.1
29/03/94	-29.2	-14.2
30/03/94	-28.7	-16.1
31/03/94	-27.6	-15.6
Av. Ret.	-33.1073	-11.68
Std. Dev.	3.611635	2.720018

Appendix Y continued:

April 1994

Date	Act. R	HR.
5/04/94	-20.86	-18.34
6/04/94	-27.02	-22.09
7/04/94	-27.64	-21.44
8/04/94	-28.02	-21.33
11/04/94	-26.44	-20.08
12/04/94	-28.76	-22.51
13/04/94	-33.55	-24.8
14/04/94	-28.7	-21.3
15/04/94	-25.65	-20.5
18/04/94	-32.2	-22.9
19/04/94	-18.1	-25.2
20/04/94	-17.1	-24.8
21/04/94	-26.4	-22.7
22/04/94	-30.5	-24.6
26/04/94	-36.6	-27.8
27/04/94	-36.6	-27.6
28/04/94	-36.7	-28.4
29/04/94	-25.3	-32.9
Av. Ret.	-28.1189	-23.8494
Std. Dev.	5.769146	3.581281

May 1994

Date	Act. R	HR.
2/05/94	-22.6	-32.9
3/05/94	-25.9	-34.1
4/05/94	-11.08	-23.31
5/05/94	-10.07	-21.6
6/05/94	-10.73	-21.6
9/05/94	-2.44	-18
10/05/94	-18.14	-34.83
11/05/94	-13.97	-23.83
12/05/94	1.24	-10.8
13/05/94	-10.78	-19.96
Av. Ret.	-12.447	-24.093
Std. Dev.	8.296694	7.724485

Appendix Y continued:

June 1994

Date	Act. R	HR.
10/06/94	-17.87	-1.11
14/06/94	-14.28	-3.47
15/06/94	-17.19	-5.29
16/06/94	-10.1	-6.5
17/06/94	-8.05	-5.4
20/06/94	1.9	-6.01
21/06/94	6.97	-1.2
22/06/94	5.2	-6.05
23/06/94	1	-7.45
24/06/94	1.37	-7.98
27/06/94	11.7	0.03
28/06/94	5.6	-6.32
29/06/94	1.98	-8.4
30/06/94	1.5	-8.9
Av. Ret.	-2.16214	-5.28929
Std. Dev.	9.513731	2.829165

July 1994

Date	Act. R	HR
1/07/94	1.4	-9.1
4/07/94	3.2	-7.5
5/07/94	3.1	-7.5
6/07/94	-0.65	-8.5
7/07/94	-1.18	-8.48
8/07/94	-1.66	-8.9
11/07/94	3.3	-7.87
12/07/94	1.95	-6.65
13/07/94	-0.52	-7.7
14/07/94	-1.75	-7.7
15/07/94	-9.6	-5.77
18/07/94	-12.6	-6.5
19/07/94	-10.7	-5.3
20/07/94	-10.14	-5.15
21/07/94	-3.1	-8.5
22/07/94	-5.8	-9.97
25/07/94	-4.1	-9
26/07/94	-6.75	-10.3
27/07/94	-13.3	-4.66
28/07/94	-4	-7.7
29/07/94	-6.72	-0.8
Av. Ret.	-3.79143	-7.3119
Std. Dev.	5.265135	2.135635

Appendix Y continued:

August 1994

Date	Act. R	HR.
2/08/94	-16.98	-4.1
3/08/94	-11.5	-1.16
4/08/94	-27.7	-0.16
5/08/94	-24.5	0.35
8/08/94	-15.4	-3.45
9/08/94	-22.5	-7.8
10/08/94	-22.7	-8
11/08/94	-18.6	-4.2
12/08/94	-14.4	-1.46
15/08/94	-21.7	-4.4
Av. Ret.	-19.598	-3.438
Std. Dev.	5.057496	2.905538

September 1994

Date	Act. R	HR.
13/09/94	6.73	0.43
14/09/94	10.5	3
15/09/94	7.88	1.08
16/09/94	2.96	-4.45
19/09/94	7.3	-0.32
20/09/94	9.66	1.13
21/09/94	7.73	-0.36
22/09/94	8.4	0.09
23/09/94	8.68	0.4
26/09/94	11.34	2.6
27/09/94	10.45	1.9
28/09/94	13.2	6
29/09/94	11.45	2.83
30/09/94	14.1	5.09
Av. Ret.	9.312857	1.387143
Std. Dev.	2.860878	2.55974

Appendix Y continued:

October 1994

Date	Act. R	HR.
4/10/94	14.1	5.05
5/10/94	12.1	3.74
6/10/94	8.7	1.5
7/10/94	10.29	3
10/10/94	6.4	-2.07
11/10/94	7.44	-1
12/10/94	11.03	4.07
13/10/94	16.09	7.03
14/10/94	10.77	3.96
17/10/94	10.65	4.46
18/10/94	10.05	4.55
19/10/94	10.32	4.32
20/10/94	6.79	-1.1
21/10/94	8.4	-1.09
24/10/94	8.27	-0.22
25/10/94	13.13	6.66
26/10/94	21.27	11.62
27/10/94	21.39	12.57
28/10/94	21.85	11.91
31/10/94	27.19	20.77
Av. Ret.	12.8115	4.9865
Std. Dev.	5.812889	5.678758

November 1994

Date	Act. R	HR.
1/11/94	29.9	22.22
2/11/94	30.44	22.95
3/11/94	29.88	23.27
4/11/94	31.48	24.45
7/11/94	41.66	32.34
8/11/94	43.11	34.14
9/11/94	41.44	33.15
10/11/94	35.62	29.62
11/11/94	39.76	33.59
14/11/94	40.71	35.03
15/11/94	35.52	24.67
Av. Ret.	36.32	28.67545
Std. Dev.	5.232206	5.160501

APPENDIX Z : Hypotheses test results

Chapter 4 : Deriving the Australian Market Term Structure

Hypotheses Tests 1 to 3	Results	Comments
Ho: Av. difference between SFE 10 yr. bond futures and model prices = 0.	Accept *Ho*	*The derived term structure produced accurate SFE 10 yr. bond futures prices.*
Ho: Av. difference between SFE 10 yr. bond futures and CAC = 0.	Accept *Ho*	*The CAC methodology produced accurate SFE 10 yr. bond futures prices.*
Ho: The term structure model produced unbiased estimates of those 10 yr. bond futures prices observed in the market.	Reject *Ho*	*The model prices were not found to be unbiased. However the high R^2 and t - stats . did suggest a strong relationship.*

Chapter 5 : Characteristics of the Yield Error Margin Database

Hypotheses Tests 4 to 8	Results	Comments
Ho:Av. difference between forecasted yields and actual yields = 0.	Accept *Ho* for only the 2 wk. YEM.	*The market's implied forecasts of forward yields were generally found to have significant errors.*
Ho: The YEM and 10 yr. bond futures distributions were normally distributed.	Accept *Ho* for only the 3 mth. YEM.	*The YEM and 10 yr. bond futures distributions were not found to be normally distributed. The 3 mth. YEM distribution was the exception.*
Ho: The YEM and 10 yr. bond futures distributions did not have significant levels of skewness or kurtosis.	Skewness: Reject *Ho* for only the 2wk. YEM. Kurtosis: Accept *Ho* for only 2wk. YEM.	*The 10 year bond futures distribution was found to be leptokurtic or peaky in nature. The YEM's distributions were, except for 2 wk., platykurtic.*
Ho: Av. difference between historic volatility and implied volatility = 0	Accept *Ho*	*There was a reasonable relationship between the two volatility series.*
Ho: The converted 2 week YEM database produced unbiased estimates of historic volatility.	Reject *Ho*	*The converted 2 wk. YEM database only demonstrated a modest relationship with historic volatility.*

Chapter 6 : Pricing Fixed Interest Contingent Claims

Hypotheses Tests 9 & 10	Results	Comments
Ho: Av. difference between SFE 10 yr. bond futures option prices and model prices = 0	Accept *Ho*	*On average the term structure based model produced reasonable estimates of SFE 10 yr. bond future option prices.*
Ho: The term structure model produced unbiased estimates of those futures option prices observed in the market.	Accept *Ho* for only the ITM put series.	*The option model prices were generally found to be biased. The model ITM put series however was found to be unbiased.*

Chapter 7 : Rewards from Hedging Fixed Interest Portfolios

Hypothesis Test 11	Results	Comments
Ho: Av. difference between hedged and unhedged portfolio returns = 0	Reject *Ho*	*The naive hedging strategy significantly modified the return of the portfolio.*

ATTACHMENT 1:

No. 92-24

PRICING OF GOVERNMENT SECURITIES

The Bank is making available to interested parties a consolidated set of the formulae it uses to calculate prices and yields for Commonwealth Government securities. Apart from bringing together the various formulae in a convenient form, the release is designed to :

i) remind market participants of the change in the Bank's treatment of near-maturing bonds, where the maturity date is not a business day. This change was announced in March 1992 and will affect the 13% May 1993 stock ($1.6 billion outstanding) from 1 November, when it enters its final 6 1/2 months; it will be priced as if it were to mature on Monday, 17 May 1993.

ii) clarify the treatment of the pricing of Treasury Capital Indexed Bonds in the ex-interest period. Some confusion has existed in this area.

T-3

Reserve Bank of Australia
SYDNEY

27 October 1992

Enquiries:
Mr M.G. Bush
Head of Domestic Markets
(02) 551-8300

Mr P.J. McWilliam
Manager
Information Office
(02) 551-9720

Attachment 1 continued:

PRICE AND YIELD FORMULAE USED BY RESERVE BANK OF AUSTRALIA

This note is an update of an article published in the November 1980 Bulletin; it takes account of some changes in market practices and instruments.

Introduction

A fixed interest security consists of a series of future coupon payments, usually of equal size, and the repayment of the principal at maturity. Any formula used to put a value on these payments must rest on certain assumptions. Once these are established, the calculations required to derive a price are relatively simple.

The yield obtained by the buyer of a security (or given up by the seller) is the mathematical solution to the equation of value of all the payments involved i.e. the purchase price on one side and the present (discounted) value of the series of payments to be received on the other. However, a buyer would only obtain the calculated "yield to maturity" if all payments received before maturity were invested at this yield until maturity. In practice, this will rarely be the case but people's views about re-investment rates during the life of the security being priced can influence their desired yield as calculated by the relevant formula. The main purpose of the formula is to ensure that buyers and sellers are "talking the same language".

A yield could be expressed for any defined period but it is convenient to quote yearly rates. With bonds, it is conventional in Australia that these annual rates are obtained by doubling an **effective or true** half-yearly rate. That is to say, the usual calculations involve rates of return earned over a half year. The yearly rate is simply double this half-yearly rate and should properly be regarded as a nominal rate as it ignores the compounding (or re-investment effect) of the half-yearly rate over the year. The use of effective half-yearly rates accords with the fact that, in Australia, interest is normally paid half-yearly. Some correction may be needed for comparison of yields e.g. with those overseas, where interest periods are different from this.

There is a minor practical difficulty with the use of the half year as a unit of measurement, as the number of days in a half year varies from 181 to 184. As interest payments are made half-yearly, they will generally accumulate over alternating periods of 181 and 184 days (or 182 and 183 days). Acknowledgment of these different rates of accrual is necessary to avoid discontinuities in the progress of the price and the Bank's price and yield formulae take account of this fact. Obviously, this impairs the idea of the redemption yield as a mathematically precise concept and emphasises that it is more a matter of what is conventionally accepted.

Attachment 1 continued:

The foregoing should make plain that certain questions concerning price and yield calculations may have no single right answer. The aim should be to accord with the facts and to produce convenient, consistent and generally acceptable formulae. The formulae described below are presented with this aim in mind.

Treasury bonds

The formulae which the Bank uses are:

(a) Basic formula $$P = v^{f/d}\left(g\left(1+a_{\overline{n}|}\right) + 100v^{n}\right) \tag{1}$$

(b) Ex interest securities $$P = v^{f/d}\left(g\,a_{\overline{n}|} + 100v^{n}\right) \tag{2}$$

(c) Near-maturing bonds (specifically, those entitling a purchaser to only the final coupon payment and repayment of principal).

$$P = \frac{100+g}{1+\left(\frac{f}{365}\right)i} \tag{3}$$

In these formulae:

P = the price per $100 face value

v = $\dfrac{1}{1+i}$

where $100i$ = the half-yearly yield (per cent) to maturity in formulae (1) and (2), or the annual yield (per cent) to maturity in formula (3).

f = the number of days from the date of settlement to the next interest-payment date in formulae (1) and (2) or to the maturity date in formulae (3)*.

d = the number of days in the half year ending on the next interest-payment date.

g = the half-yearly rate of coupon payment per $100 face value.

n = the term in half years from the next interest-payment date to maturity.

$$a_{\overline{n}|} = v + v^{2} + \ldots + v^{n} = \frac{1-v^{n}}{i}$$

Attachment 1 continued:

The following notes provide further explanation of the above formulae and give some examples of their application.

(a) Basic formula

As an example of the working of the formula, the price per $100 face value on 1 November 1992 to yield 8.30% p.a. to maturity for a 12.5% 15 January 1998 bond is calculated (with $i = .0415$, $f = 75$, $d = 184$, $g = 6.25$ (i.e. half of 12.5) and $n = 10$) as $121.132.

The problem of finding the yield consistent with a given price must be approached indirectly, as equations such as (1) above cannot be solved directly. To find the required yield, an iterative process may be used successively to approximate the given price. Alternatively, straight-line interpolation using prices near the given price enables the yield to be derived accurately.

As mentioned, while the yield to maturity is expressed as an annual rate, the calculations are in terms of returns for a half year i.e. effective half-yearly rates. Also, to accord with the fact that the half year is treated as the basic accounting period, the price of a bond between interest-payment (coupon) dates is calculated by discounting back using $v^{f/d}$, f/d being the fraction of the half year to the next payment.

There are modifications required in using this formula in some situations, as discussed in the following paragraphs.

(b) Ex interest securities

With these securities, either there is no coupon payable by the issuer at the next half-yearly interest date or the next coupon payment is not available to a purchaser of the securities on the market because, for example, they have gone "ex interest" in the week or two leading up to distribution of coupon payments. In either case, calculation of an ex interest (or NX) price is effected by the removal of the "1" from the term $1+a_{\overline{n}|}$ in formula (1), thereby adjusting for the fact that the purchaser will not receive a coupon payment at the next interest payment date.

Commencing January 1993, the "ex interest" period for Treasury bonds will be reduced from 14 days to 7 days. Also, trading of these securities (i.e. Registry transfers) will be permitted until they are within 7 days (previously a month) of maturity.

ATTACHMENT 2:

10-Year Commonwealth Treasury Bond Futures

Contract Unit:	Commonwealth Government Treasury bonds with a face value of A$100,000, a coupon rate of 12 per cent per annum and a term to maturity of ten years, no tax rebate allowed.
Cash Settlement Price:	The arithmetic mean of quotations provided at 9.45 am, 10.30 am and 11.15 am on the last day of trading by 12 dealers, randomly selected for each time, at which they would buy and sell a series of bonds previously declared by the Exchange for that contract month excluding the two highest and two lowest buying quotations and the two highest and two lowest selling quotations for each bond.
Mandatory Cash Settlement:	All bought and sold contracts in existence as at the close of trading in the contract month shall be settled by the Clearing House at the cash settlement price.
Quotations:	Prices shall be quoted in yield per cent per annum in multiples of 0.005 per cent. For quotation purposes the yield shall be deducted from 100.00. (The minimum fluctuation of 0.005 per cent equals approximately $44 per contract, varying with the level of interest rates.)
Contract Months:	March, June, September and December up to two quarter months ahead.
Termination of Trading:	The fifteenth day of the cash settlement month (or the next succeeding business day where the fifteenth day is not a business day). Trading ceases at 12.00 noon.
Settlement Day:	The business day following the last permitted day for trading.
Trading Hours:	Floor - 8.30 am to 12.30 pm; 2.00 pm to 4.30 pm SYCOM - 4.40 pm to 6.00 am

Options on 10-Year Commonwealth Treasury Bond Futures

Contract Unit:	One A$100,000 face value, 12% coupon, 10-year Treasury bond futures contract for a specified contract month on the Sydney Futures Exchange.
Exercise Prices:	Set at intervals of 0.25% per annum yield. New option exercise prices created automatically as the underlying futures contract price moves.
Premium:	Quoted in yield per cent per annum.
Contract Months:	Put and call options available on futures contracts up to two quarter months ahead.
Expiry:	At 12.00 noon on the last day of trading in the underlying futures contract (the fifteenth day of the month or the next succeeding business day).
Exercise of Options:	Options may be exercised on any business day up to and including the day of expiry. In-the-money options are automatically exercised at expiry unless abandoned.

ATTACHMENT 3:

CIRCULAR TO FLOOR MEMBERS
ASSOCIATE MEMBERS
LOCAL MEMBERS
MARKET REPRESENTATIVES

NO. 309/93

TEN YEAR AND THREE YEAR BOND SERIES FOR THE MARCH 1994 CONTRACT MONTH

Further to Circular No. 288/93, this is to advise Members that the Board of the Exchange at its meeting held on 28 September 1993, resolved that the March 1994 Ten Year and Three Year Treasury Bond Futures and Options Contract be listed on Wednesday, 29 September 1993, with futures and options contracts to be listed that evening on SYCOM.

In accordance with TB.4(a) and TRB.4(a), the series of Treasury Bonds declared by the Board for cash settlement of the March 1994 contract months are:-

MARCH 1994					
THREE YEAR BOND CONTRACT			TEN YEAR BOND CONTRACT		
13.0%	July	1996	10.0%	October	2002
12.5%	March	1997	9.5%	August	2003
12.5%	September	1997	9.0%	September	2004
12.5%	January	1998	7.5%	July	2005

Please note that both series are different from the basket with prevails for the December 1993 basket.

The Nineteenth and Twentieth Schedules to the Articles have been updated and copies are attached.

Barbara Jones

BARBARA JONES
COMPANY SECRETARY

29 SEPTEMBER 1993

Attachment 3 continued:

CIRCULAR TO FLOOR MEMBERS
ASSOCIATE MEMBERS
LOCAL MEMBERS
MARKET REPRESENTATIVES

NO. 401/93

TEN YEAR AND THREE YEAR BOND SERIES FOR THE JUNE 1994 CONTRACT MONTH

Further to Circular No. 395/93 this is to advise Members that the Board of the Exchange at its meeting held on 14 December 1993, resolved that the June 1994 Ten Year and Three Year Treasury Bond Futures and Options Contract be listed on Thursday 16 December 1993, with futures and options contracts to be listed that evening on SYCOM.

In accordance with TB.4(a) and TRB.4(a), the series of Treasury Bonds declared by the Board for cash settlement of the June 1994 contract months are:-

JUNE 1994					
THREE YEAR BOND CONTRACT			TEN YEAR BOND CONTRACT		
13.0%	July	1996	10.0%	October	2002
12.5%	March	1997	9.5%	August	2003
12.5%	September	1997	9.0%	September	2004
12.5%	January	1998	7.5%	July	2005
			6.75%	November	2006

Please note that the Three Year series is but the Ten Year series is not different from the baskets which prevail for March 1994.

The Nineteenth and Twentieth Schedules to the Articles have been updated and copies are attached.

Barbara Jones

BARBARA JONES
COMPANY SECRETARY

15 DECEMBER 1993

Attachment 3 continued:

CIRCULAR TO FLOOR MEMBERS
ASSOCIATE MEMBERS
LOCAL MEMBERS
MARKET REPRESENTATIVES

NO. 101/94

TEN YEAR AND THREE YEAR BOND SERIES FOR THE SEPTEMBER 1994 CONTRACT MONTH

Further to Circular No. 87/94 this is to advise Members that the Board of the Exchange at its meeting held on 24 March 1994, resolved that the September 1994 Ten Year and Three Year Treasury Bond Futures and Options Contract is as set out below. The series will be listed on Monday, 28 March 1994, with futures and options contracts to be listed that evening on SYCOM.

In accordance with TB.4(a) and TRB.4(a), the series of Treasury Bonds declared by the Board for cash settlement of the September 1994 contract months are:-

SEPTEMBER 1994					
THREE YEAR BOND CONTRACT			TEN YEAR BOND CONTRACT		
12.5%	March	1997	10.0%	October	2002
12.5%	September	1997	9.5%	August	2003
12.5%	January	1998	9.0%	September	2004
7.0%	August	1998	7.5%	July	2005
			6.75%	November	2006

Please note that the Ten Year series is the same but the Three Year series is not the same as the baskets which prevail for June 1994.

The Nineteenth and Twentieth Schedules to the Articles have been updated and copies are attached.

Barbara Jones

BARBARA JONES
COMPANY SECRETARY

25 MARCH 1994

Attachment 3 continued:

CIRCULAR TO: FLOOR MEMBERS
ASSOCIATE MEMBERS
LOCAL MEMBERS
MARKET REPRESENTATIVES

NO. 209/94

RECOMMENDED BOND SERIES FOR THE DECEMBER 1994 CONTRACT MONTH

The Bond Committee at its meeting on 9 June 1994 considered the basket stocks to underlie the December 1994 Treasury Bond contract months. The Committee recommended that the following bond series underlie the December 1994 Three Year and Ten Year Treasury Bond Contracts:

DECEMBER 1994					
THREE YEAR BOND CONTRACT			TEN YEAR BOND CONTRACT		
12.5%	March	1997	9.5%	August	2003
12.5%	September	1997	9.0%	September	2004
12.5%	January	1998	7.5%	July	2005
7.0%	August	1998	6.75%	November	2006

Please note that the bond series listed above are a recommendation only and are subject to ratification by the Board of the Exchange at its meeting on 21 June 1994. As such, the Board is entitled to make amendments where appropriate.

GREG JARVIS
RESEARCH ANALYST

9 JUNE 1994

CIRCULAR TO FLOOR MEMBERS
ASSOCIATE MEMBERS
LOCAL MEMBERS
MARKET REPRESENTATIVES NO. 328/94

TEN YEAR AND THREE YEAR BOND SERIES FOR THE MARCH 1995 CONTRACT MONTH

Further to Circular No. 314/94 this is to advise Members that the Board of the Exchange at its meeting held on 14 September 1994, resolved that the March 1995 Ten Year and Three Year Treasury Bond Futures and Options Contract is as set out below. The series will be listed on Friday 16 September 1994, with futures and options contracts to be listed that evening on SYCOM.

In accordance with TB.4(a) and TRB.4(a), the series of Treasury Bonds declared by the Board for cash settlement of the March 1995 contract months are:-

MARCH 1995					
THREE YEAR BOND CONTRACT			TEN YEAR BOND CONTRACT		
12.5%	March	1997	9.5%	August	2003
12.5%	September	1997	9.0%	September	2004
12.5%	January	1998	7.5%	July	2005
7.0%	August	1998	6.75%	November	2006

Please note that both series are the same as the baskets which prevail for December 1994.

The Nineteenth and Twentieth Schedules to the Articles have been updated and copies are attached.

Barbara Jones.

BARBARA JONES
COMPANY SECRETARY 15 SEPTEMBER 1994

ATTACHMENT 4:

TE.8.2 Options Settlement Price Procedures

Following the establishment of the underlying futures contract Daily Settlement Prices, indicative Options Daily Settlement Prices shall be calculated by the Exchange having regard to implied volatilities supplied by traders in the pit, Futures Daily Settlement Prices, and other relevant factors and displayed on the price reporting screens in each options pit.

Option traders shall have 5 minutes to object to any of the indicative Settlement Prices. The Pit Committee shall arbitrate in the case of disagreement; however this should in no way derogate from the powers of the Exchange to make a final decision concerning Daily Settlement Prices.

After the indicative Option Daily Settlement Prices are agreed unless a different option Settlement Price is determined by the Chief Executive they (or the prices determined by the Chief Executive) shall be confirmed as the Exchange Daily Settlement Prices on the price reporting screens and transmitted to the Clearing House.

TE.8.3 Overnight Options Futures Price Procedures (Added 1/11/93)

Overnight options futures prices shall be calculated in the manner set out in the relevant market by-Laws.

Each component of an overnight options futures price shall be recorded and displayed in the Pit as it is established. If any trader disagrees with the component as so recorded and displayed, he shall immediately notify such disagreement to an Exchange official. The Pit Committee shall arbitrate any disagreement and advise the Exchange accordingly, however this does not derogate from the power of the Exchange to determine the Overnight Option Futures Price in accordance with the relevant market by-laws.

Any attempt to alter the Overnight Options futures price or any component thereof by artificial means shall be deemed behaviour prejudicial to the Exchange and its markets.

TE.9. TRADING RULES RELATING TO MULTIPLE REPRESENTATION.

TE.9.1. All aspects of the Floor Trading Etiquette will apply equally, insofar as is possible, to joint representatives as they do to single representatives on the understanding that no advantage may be claimed in the event of joint representatives, should any rule appear to grant them such an advantage. They will for such purposes be deemed as one only.

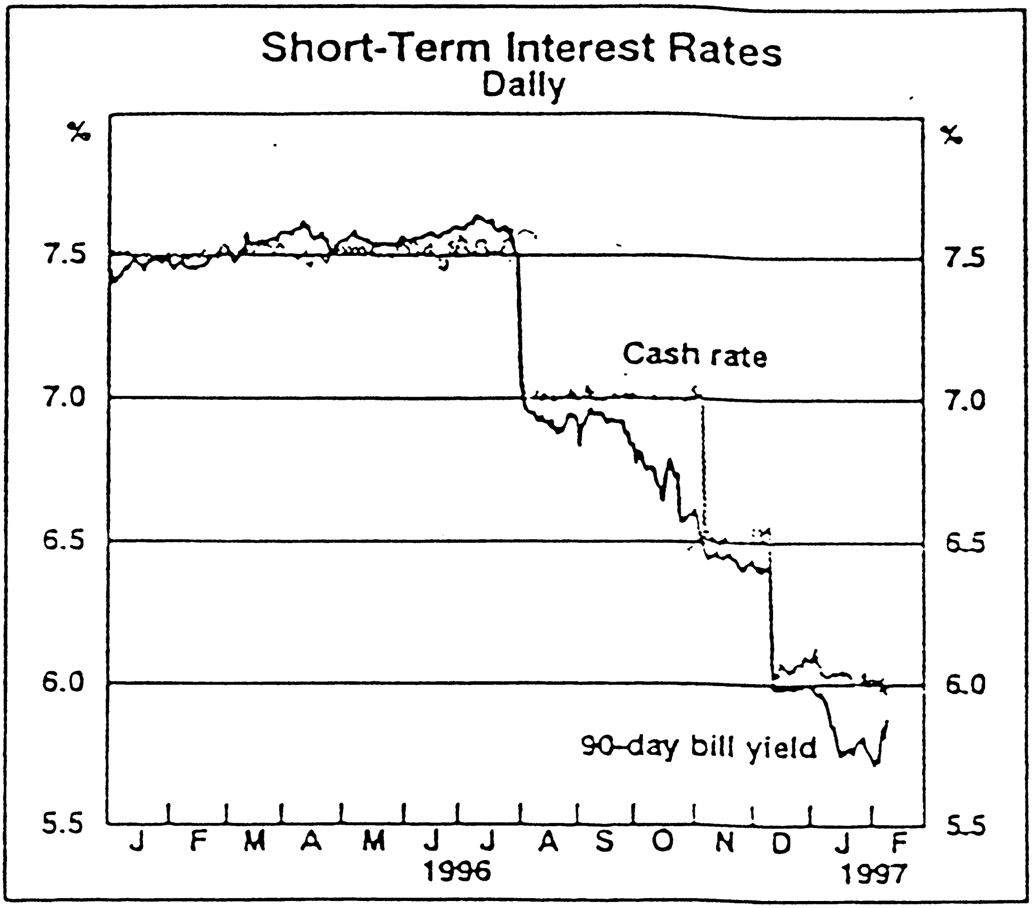
FEBRUARY '95

ATTACHMENT 5:

1#YTCU5+ 2#YTCU5- 0#YTCU5+

RIC	BID	ASK	LAST	NAME	TIME
YTC8950I5	1.495	1.525	0.000	YTC SEP5 8950 C	23:35
YTC8950U5	0.000	0.030	0.005	YTC SEP5 8950 P	23:44
YTC8975I5	1.215	1.245	0.000	YTC SEP5 8975 C	23:35
YTC8975U5	0.000	0.030	0.000	YTC SEP5 8975 P	23:35
YTC9000I5	0.950	0.980	0.000	YTC SEP5 9000 C	23:35
YTC9000U5	0.000	0.030	•0.010	YTC SEP5 9000 P	02:13
YTC9025I5	0.695	0.725	0.000	YTC SEP5 9025 C	02:14
YTC9025U5	0.005	0.035	•0.025	YTC SEP5 9025 P	02:14
YTC9050I5	0.470	0.500	0.000	YTC SEP5 9050 C	23:35
YTC9050U5	0.040	0.070	•0.055	YTC SEP5 9050 P	02:30
YTC9075I5	0.280	0.310	0.000	YTC SEP5 9075 C	01:45
YTC9075U5	0.110	0.140	0.000	YTC SEP5 9075 P	23:35

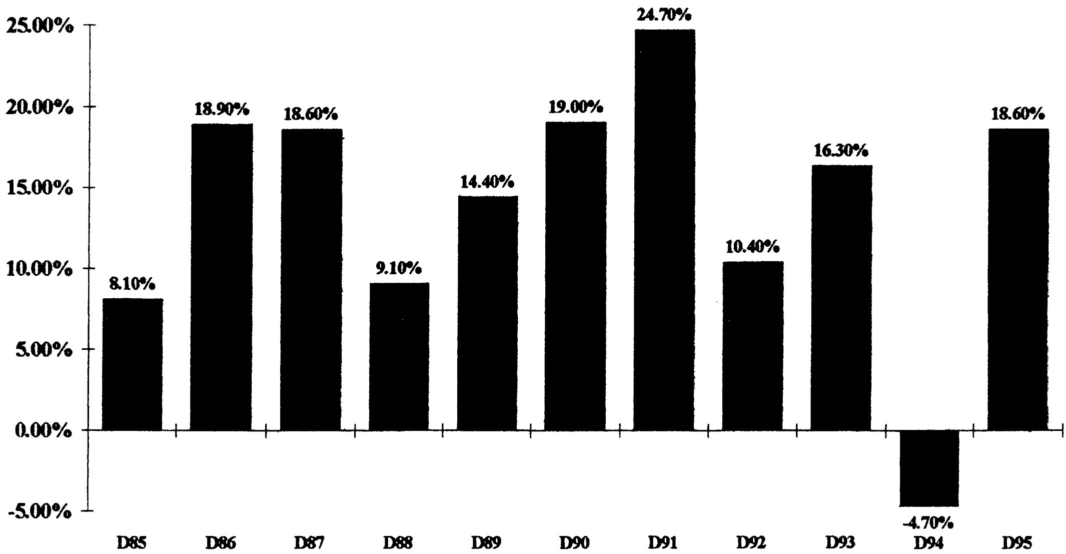
ATTACHMENT 6:



ATTACHMENT 7:

Fixed Interest Annual Return Performance 1985 - 1995

(SBC ALL BOND INDEX 3yr to 10yr)



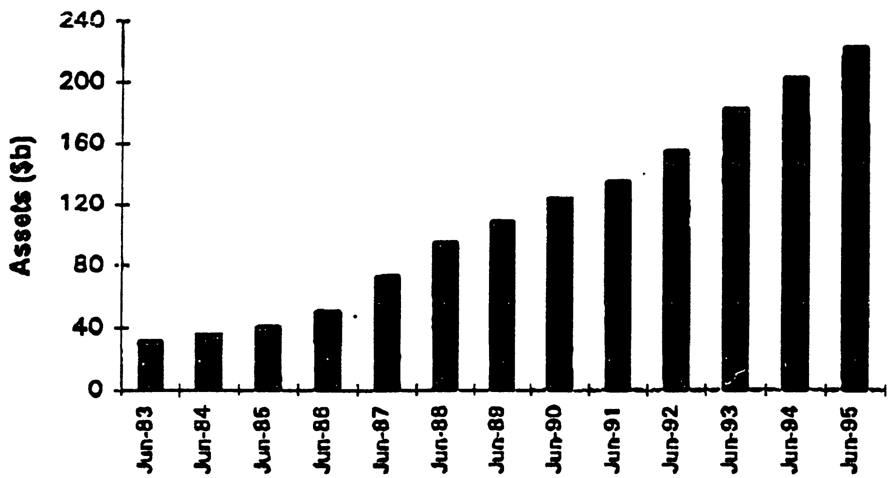
Source : William M. Mercer

ATTACHMENT 8:

Superannuation Assets - 1983 to 1995

	Assets $b
Jun-83	32
Jun-84	36
Jun-85	40
Jun-86	51
Jun-87	73
Jun-88	95
Jun-89	108
Jun-90	124
Jun-91	135
Jun-92	155
Jun-93	183
Jun-94	203
Jun-95	223

Superannuation Assets
1983-1995



Source: Insurance and Superannuation Commission

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