Managing Disturbances in Supply Chain Systems

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Managing Disturbances in Supply Chain Systems

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This thesis presents a study of disturbance management in production-inventory and supply chain systems. The study focuses on generating and analysing the recovery decision after the occurrence of a disturbance on a real-time basis. In this thesis, the developed approach was divided into several steps. At first, a plan was developed without considering any disturbance. Then a mathematical model was formulated to obtain a revised plan after the occurrence of a disturbance in the system. An efficient heuristic approach was proposed for solving the mathematical model in order to obtain the recovery plan. The mathematical model and heuristic approach were also extended to consider multiple disturbances, one after another as a series, on a real-time basis, by incorporating a modified version of those developed for a single disturbance. In case of multiple disturbances, if a new disturbance occurs during the recovery time window of another, a new revised plan which considers the effects of both disturbances was derived. Finally, the experimentation was conducted and the heuristic results were compared with other standard solution approaches to judge and validate the results. The framework was applied for managing production disruption in four different types of production-inventory and supply chain systems. The framework was also applied to two other models for managing demand fluctuation and supply disruption.

Two of the models, developed in two-stage and three-stage production-inventory systems, were also tested using real-life cases. It was found that the developed approaches were more beneficial than the company's existing practice. The results of the experimental analysis showed that the optimal recovery plan is highly dependent on the shortage cost parameters such as, back orders and lost sales costs, and to the disturbance duration. For a certain range of disturbance duration and cost values, it was found that back orders were more attractive, and in such cases, back orders cost was less than the lost sales cost. On the other hand, when back orders cost were more than the lost sales cost, the solution had lost sales in their recovery plan.

In the final work of the thesis, a simulation model was developed to analyse the effects of different types of randomly generated disturbance events. A significant number of random experiments were conducted to judge the simulation model, and to make the simulation model closer to real-world processes.

Dedication

To My Family

My Daughter Rupkotha Paul

My Father Milon Krishna Paul

My Mother Shilpi Rani Paul

My Wife Ananna Paul

My Brother Ripon Kumar Paul

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I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.

Sanjoy Kumar Paul

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Keywords

Production-Inventory System

Supply Chain System

Disruption/Disturbance Management

Risk Management

Mathematical Modelling

Quantitative Model

Recovery Model

Real-Time Disruption/Disturbance Management

Production Disruption

Demand Fluctuation

Supply Disruption

Heuristic

Simulation Model

Abstract

This thesis presents a study of disturbance management in production-inventory and supply chain systems. The study focuses on generating and analysing the recovery decision after the occurrence of a disturbance on a real-time basis. In this thesis, the developed approach was divided into several steps. At first, a plan was developed without considering any disturbance. Then a mathematical model was formulated to obtain a revised plan after the occurrence of a disturbance in the system. An efficient heuristic approach was proposed for solving the mathematical model in order to obtain the recovery plan. The mathematical model and heuristic approach were also extended to consider multiple disturbances, one after another as a series, on a real-time basis. Finally, the experimentation was conducted and the heuristic results were compared with other standard solution approaches to judge and validate the results. The framework was applied for managing production disruption: (1) in a single-stage imperfect production-inventory system, (2) in a two-stage production-inventory system, (3) in a three-stage mixed production-inventory system, and (4) in a supply chain network with multiple entities in each stage. The framework was also applied to two other models: (1) for managing demand fluctuation in a supplier-retailer coordinated system, and (2) for managing supply disruption in a three-tier supply chain system. In this thesis, three different types of disturbances were explored, namely (1) production disruption, (2) raw material supply disruption, and (3) demand fluctuation.

In real-life situations, multiple disturbances, as a series, can happen at any time and at any stage of the system. This thesis considered multiple disturbances, one after another in a series, that may or may not affect the plans revised after previous disturbances. If a new disturbance occurs during the recovery time window of another, a new revised plan which considers the effects of both disturbances must be derived. Accordingly, as this is a continuous process, an extended mathematical model and heuristic approach was

developed to deal with a series of disturbances on a real-time basis, by incorporating a modified version of those developed for a single disturbance.

The results of the experimental analysis showed that the optimal recovery plan is highly dependent on the shortage cost parameters such as, back orders and lost sales costs, and to the disturbance duration. For a certain range of disturbance duration and cost values, it was found that back orders were more attractive, and in such cases, back orders cost was less than the lost sales cost. On the other hand, when back orders cost were more than the lost sales cost, the solution had lost sales in their recovery plan.

In the final work of the thesis, a simulation model was developed to analyse the effects of different types of randomly generated disturbance events that were not known in advance. The simulation model considered all three types of disturbances, namely (1) production disruption, (2) raw material supply disruption, and (3) demand fluctuation. A good number of random experiments were conducted to judge the simulation model, and to make the simulation model closer to real-world processes.

The developed approaches were tested by solving a significant number of randomly generated test problems. The sensitivity analysis was carried out for the model parameters. Two of the models, developed in two-stage and three-stage production-inventory systems, were also tested using real-life cases from a pharmaceutical company. It was found that the developed approaches were more beneficial than the company's existing practice.

List of Publications

Journal Articles

1. Paul, S. K., Sarker, R., and Essam, D., "Managing disruption in an imperfect production-inventory system", *Computers and Industrial Engineering*, Vol. 84, pp. 101-112, 2015. (Based on Chapter 3)
2. Paul, S. K., Sarker, R., and Essam, D., "Real time disruption management for a two-stage batch production-inventory system with reliability considerations", *European Journal of Operational Research*, Vol. 237(1), pp. 113-128, 2014. (Based on Chapter 4)
3. Paul, S. K., Sarker, R., and Essam, D., "A disruption recovery plan in a three-stage production-inventory system", *Computers and Operations Research*, Vol. 57, pp. 60-72, 2015. (Based on Chapter 5)
4. Paul, S. K., Sarker, R., and Essam, D., "Managing real-time demand fluctuation under a supplier-retailer coordinated system", *International Journal of Production Economics*, Vol. 158, pp. 231-243, 2014. (Based on Chapter 7)
5. Paul, S.K., Sarker, R., and Essam, D., "Managing risk and disruption in production-inventory and supply chain systems: a review", *Journal of Industrial and Management Optimization*, Vol. 12(3), pp. 1009-1029, 2016. (Based on Chapter 2)
6. Paul, S.K., Sarker, R., and Essam, D., "A quantitative model for disruption mitigation in a supply chain", *European Journal of Operational Research*, under review, 2015. (Based on Chapter 6)
7. Paul, S.K., Sarker, R., and Essam, D., "A quantitative and simulation approach for managing supply disruption in a three-tier supply chain", *Journal of Intelligent Manufacturing,* under review, 2015. (Based on Chapter 8)

8. Paul, S.K., Sarker, R., and Essam, D., "A reactive mitigation approach for a manufacturing supply chain system using simulation", *Computers and Operations Research*, under review, 2015. (Based on Chapter 9)

Articles in Lecture Notes

1. Paul, S.K., Sarker, R., and Essam, D., "A disruption recovery model in a production-inventory system with demand uncertainty and process reliability", In *Computer Information Systems and Industrial Management*, Lecture Notes in Computer Science, Vol. 8104, pp. 511-522, Springer, 2013.

Conference Papers

1. Paul, S.K., Sarker, R., and Essam, D., "A production inventory model with disruption and reliability considerations", *Proceedings to the 43rd International Conference on Computers and Industrial Engineering*, pp. 291-304, Hong Kong, 16-18 October, 2013.

2. Paul, S.K., Sarker, R. A., and Essam, D. L., "Managing supply disruption in a three-stage supply chain with multiple suppliers and retailers", *Proceedings of The IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pp. 194-198, Malaysia, 09-12 December, 2014.

List of Acronyms

EPQ	Economic Production Quantity
EOQ	Economic Order Quantity
PS	Pattern Search
GA	Genetic Algorithm
SC	Supply Chain
SCDM	Supply Chain Disruption/Disturbance Management
DC	Distribution Centre
STD	Standard Deviation

List of Parameters and Variables

Chapter 3

Single disruption case

S_t	Set-up time for a cycle
δ_i	Idle time for cycle i
D	Demand per year (units per year)
H	Holding cost per unit per year ($ per unit per year)
r	Reliability of the production process – which is known from the historical data of the production system
Q	Economic lot size per ideal production cycle with process reliability r
A	Set up cost per cycle ($ per set-up)
P	Production rate (units per year) in 100% reliable system
u_i	Production down time for cycle i (set-up time + idle time)
M	Number of cycles in the recovery time window – given from management
T_d	Disruption duration
q	Pre-disruption production quantity
T_0	Production time for q
$X_{i,0}$	Production quantity for normal cycle i
X_i	Production quantity for cycle i of the recovery time window– which is the decision variable; $i = 1,2, \dots\dots, M$
$T_{i,0}$	Production up time for cycle i of a normal cycle
T_i	Production up time for cycle i in the recovery time window
B	Unit back order cost per unit time ($ per unit per unit time)
L	Unit lost sales cost ($ per unit)
C_P	Per unit production cost ($ per unit)

C_R Rejection cost per unit ($ per unit)

C_I Inspection cost as a percentage of production cost

m_1 Mark-up of selling price ($m_1 C_P$) of each acceptable item, it must be greater than 1

Multiple disruptions case

$\delta_{i,n}$ Idle time for cycle i before the n^{th} disruption

l New disrupted cycle number from the previous disruption

$u_{i,n}$ Production down time for cycle i before the n^{th} disruption (set-up time + idle time)

$T_{d,n}$ Disruption duration of the n^{th} disruption

q_n Pre-disruption production quantity of the n^{th} disruption

$T_{0,n}$ Production time for q_n

$X_{i,n}$ Production quantity for cycle i of the recovery time window after the n^{th} disruption– which is the decision variable; $i = 1,2, \dots\dots, M$

$T_{i,n}$ Production up time for cycle i in the recovery time window after the n^{th} disruption

Chapter 4

S_{t1} Set-up time for a cycle at the first stage

S_{t2} Set-up time for a cycle at the second stage

$\delta_{1i,n}$ Idle time for a cycle *i* before the n^{th} disruption at the first stage

$\delta_{2i,n}$ Idle time for a cycle *i* before the n^{th} disruption at the second stage

D Demand per year (units per year)

H_1 Holding cost per unit per year at the first stage ($/unit/year)

H_2 Holding cost per unit per year at the second stage ($/unit/year)

r Reliability of the production process – which is known from the historical data of the production system

Q Combined production lot size per normal cycle with reliability r

A_1 Set-up cost per cycle at the first stage ($ per set-up)

A_2 Set-up cost per cycle at the second stage ($ per set-up)

P Production rate (units per year) in 100% reliable system

l New disrupted cycle number from previous disruption

$X_{i,0}$ Production quantity for a normal cycle i at the first stage

$Y_{i,0}$ Production quantity for a normal cycle i at the second stage

$X_{i,n}$ Production quantity for cycle *i* of the recovery window at the first stage after the n^{th} disruption– which is a decision variable; *i = 1, 2, 3, ..., M*

$Y_{i,n}$ Production quantity for cycle *i* of the recovery window at the second stage after the n^{th} disruption– which is a decision variable; *i = 1, 2, 3, ..., M*

$u_{1i,n}$ Production down time for cycle *i* before the n^{th} disruption at the first stage (set-up time + idle time)

$u_{2i,n}$ Production down time for cycle *i* before the n^{th} disruption at the second stage (set-up time + idle time)

M Number of cycles to recovery the disruption – which is given by the management

$T_{d1,n}$ Disruption duration in the n^{th} disruption at the first stage

$T_{d2,n}$ Disruption duration in the n^{th} disruption at the second stage

$q_{1,n}$ Pre-disruption production quantity in the n^{th} disruption at the first stage

$q_{2,n}$ Pre-disruption production quantity in the n^{th} disruption at the second stage

$T_{01,n}$ Production time for $q_{1,n}$

$T_{02,n}$ Production time for $q_{2,n}$

$T_{1i,0}$ Production up time for a normal cycle *i* at the first stage

$T_{2i,0}$ Production up time for a normal cycle *i* at the second stage

$T_{1i,n}$ Production up time for a cycle *i* in the recovery window at the first stage after the n^{th} disruption

$T_{2i,n}$ Production up time for a cycle *i* in the recovery window at the second stage after the n^{th} disruption

$T_{idle,n}$ Idle time of production at the stage 2 due to the n^{th} disruption at the first stage

B Unit back order cost per unit time ($/unit/time)

L Unit lost sales cost ($ per unit)

C_{P1} Per unit production cost at the first stage ($ per unit)

C_{P2} Per unit production cost at the second stage ($ per unit)

C_{R1} Rejection cost per unit at the first stage

C_{I1} Inspection cost as a percentage of the production cost at the first stage

C_{I2} Inspection cost as a percentage of the production cost at the second stage

m_1 Mark-up of the unit selling price $[m_1(C_{P1} + C_{P2})]$ of the acceptable items (must be greater than 1)

Chapter 5

X_0 Batch size in stage 1 under ideal conditions (known)

T_0 Processing time of batch in first stage under ideal conditions (constant and known)

S_{t1} Set-up time in stage 1 (time per set-up)

S_{t2} Set-up time in stage 2 (time per set-up)

S_{t3} Set-up time in stage 3 (time per set-up)

A_1 Set-up cost in stage 1 ($ per set-up)

CP_1 Production cost per unit in stage 1 ($ per unit)

CM_1 Cost per unit material loss in stage 1 ($ per unit)

D Average demand per unit time

CP_2 Production cost per unit in stage 2 ($ per unit)

C_2 Capacity of machine in stage 2 (units per unit time)

H_2 Holding cost per unit per unit time in stage 2 ($ per unit per unit time)

CP_3 Production cost per unit in stage 3 ($ per unit)

C_3 Capacity of machine in stage 3 (units per unit time)

n Number of sub-lots in stages 2 and 3 for each full batch in stage 1

Y_0 Size of each sub-lot in stage 2 under ideal conditions

Z_0 Size of each sub-lot in stage 3 under ideal conditions

I_1 Idle time after processing batch in stage 1

I_2 Idle time after processing n sub-lots in stage 2

I_3 Idle time after processing n sub-lots in stage 3

T_p Pre-disruption duration

T_d Disruption duration

T_R Recovery time

M Number of batches within recovery time window

X_i Size of batch i in stage 1 after disruption; $for\ i = 1,2, \dots, M$ (decision variable)

Y_j Size of sub-lot j in stage 2 after disruption; $for\ j = 1,2, \dots, Mn$ (decision variable)

Z_j Size of sub-lot j in stage 3 after disruption; $for\ j = 1,2, \dots, Mn$ (decision variable)

B Back orders cost per unit per unit time ($ per unit per unit time)

L Lost sales cost per unit ($ per unit)

m_1 Mark-up of unit selling price [$m_1(CP_1 + CP_2 + CP_2)$] (must be greater than 1)

Chapter 6

i	Plant index
j	DC index
k	Retailer index
I	Number of plants
J	Number of DCs
K	Number of retailers
P_i	Production quantity of plant i under ideal conditions
CP_i	Maximum production capacity of plant i under ideal conditions
CD_j	Maximum handling capacity of DC j
X_{ij}	Transportation quantity from plant i to DC j under ideal conditions
Y_{jk}	Transportation quantity from DC j to retailer k under ideal conditions
D_k	Demand of retailer k
p_i	Production cost per unit at plant i
H_{1i}	Holding cost per unit per period at plant i
H_{2j}	Handling cost per unit at DC j
H_{3k}	Holding cost per unit per period at retailer k
T_{1ij}	Transportation cost per unit from plant i to DC j
T_{2jk}	Transportation cost per unit from DC j to retailer k
OC_j	Operating cost of DC j
SC_i	Spare capacity of plant i
t_n	Start time of disruption at n^{th} plant as fraction of period
T_{dn}	Disruption duration for n^{th} plant as fraction of period
P'_{im}	Production quantity after disruption at plant i in period m
X'_{ijm}	Transportation quantity from plant i to DC j after disruption in period m
Y'_{jkm}	Transportation quantity from DC j to retailer k after disruption in period m
D'_{km}	Quantity received by retailer k after disruption in period m
L	Lost sales cost per unit
B	Back-orders cost per unit per period
S	Outsourcing cost per unit
M	Number of periods in recovery window

Chapter 7

Single fluctuation case

Q_1	Production lot size for manufacturer $= X_{i,0}$
Q_2	Ordering lot size for retailer $= Y_{i,0}$
P	Annual production rate
r	Process reliability
D	Annual demand rate
S_t	Set-up time for a cycle for the manufacturer
H_1	Holding cost for the manufacturer ($ per unit per year)
H_2	Holding cost for the retailer ($ per unit per year)
A_1	Set-up cost for the manufacturer ($ per set-up)
A_2	Ordering cost for the retailer ($ per order)
C_P	Production cost per unit
C_R	Rejection cost per unit
C_I	Inspection cost as a percentage of production cost
C_L	Per unit cost due to loss of production
B	Back order cost ($ per unit per unit time)
L	Lost sales cost for retailer ($ per unit)
m_1	Mark-up of selling price ($m_1 C_P$) – must be greater than 1
$\frac{Y_{i,0}}{D}$	Cycle time for normal cycle i
M	Number of cycles in the revised planning window
X_i	Lot size for cycle i in the revised planning window, for the manufacturer after the fluctuation ($i = 1, 2, \ldots, M$)
Y_i	Lot size for cycle i in the revised planning window, for the retailer after the fluctuation ($i = 1, 2, \ldots, M$)
ΔD	Change of demand rate for the fluctuation
T_d	Fluctuation period for the fluctuation
U_d	Unfulfilled demand after the fluctuation
q	Pre-fluctuation inventory level

Multiple fluctuations case

n Fluctuation number

$X_{i,n}$ Lot size for cycle i in the revised planning window, for the manufacturer after the n^{th} fluctuation $(i = 1, 2, \dots., M)$

$Y_{i,n}$ Lot size for cycle i in the revised planning window, for the retailer after the n^{th} fluctuation $(i = 1, 2, \dots., M)$

ΔD_n Change of the demand rate for the n^{th} fluctuation

$T_{d,n}$ Fluctuation period for the n^{th} fluctuation

$U_{d,n}$ Unfulfilled demand after the n^{th} fluctuation

q_n Pre-fluctuation inventory level

Chapter 8

Single disruption case

D_j	Annual demand of the final product of retailer j
D	Annual total demand of the final product = $\sum_{j=1}^{J} D_j$
d_i	Annual demand of raw material i
B_1	Back order cost for the manufacturer ($ per unit per unit time)
B_2	Back order cost for retailer ($ per unit per unit time)
B_{qkj}	Back order quantity of retailer j during the k^{th} cycle
L_1	Lost sales cost for the manufacturer ($ per unit)
L_2	Lost sales cost for a retailer ($ per unit)
H_{1i}	Holding cost of raw material i ($ per unit per year)
H_2	Holding cost of the final product at the manufacturer ($ per unit per year)
H_{3j}	Holding cost of retailer j ($ per unit per year)
N_i	Units of raw material i required to produce one unit of the final product
K	Number of cycles in the revised plan – known from management
P	Annual production rate ($P > D$)
Q	Production lot size
q_i	Supply lot size of raw material i
Q_j	Delivery lot size of the final product for retailer j
S_{1i}	Ordering cost of raw material i ($ per order)
S_2	Set-up cost of the manufacturer ($ per order)
S_{3j}	Ordering cost of retailer j ($ per order)
s_t	Set-up time after the production of a lot
T_{idle}	Idle time after the production of a lot = $\frac{Q}{D} - \frac{Q}{P} - s_t$
T_{dm}	Supply disruption duration of the m^{th} raw material
X_{ki}	Supply lot size of raw material i in the revised plan
Y_k	Production lot size in the revised plan
Z_{kj}	Delivery lot size of the final product to retailer j in the revised plan

Multiple disruption case

l Number of cycles to the current disruption from the previous disruption

$X_{ki,n}$ Supply lot size in the revised plan after the n^{th} disruption

$Y_{k,n}$ Production lot size in the revised plan after the n^{th}disruption

$Z_{kj,n}$ Delivery lot size in the revised plan after the n^{th} disruption

$T_{dm,n}$ Actual disruption duration for the n^{th} disruption

Chapter 9

n	Number of planning periods in planning horizon
D_i	Demand of period i
P	Maximum production capacity of each period
B_i	Beginning inventory in period i
B_{n+1}	Beginning inventory which should be kept in period $(n+1)$
E_i	Ending inventory in period i
AP_i	Actual production in period i
SC_i	Spare capacity in period i
R_i	Quantity received by retailer at period i
N	Units of raw material required to produce one unit of final product
A	Set-up cost at the manufacturing plant
r	Process reliability of manufacturing plant
RM_i	Raw material supply quantity for period $i\ = N * \frac{AP_i}{r}$
C_p	Production cost per unit
C_d	Delivery cost per unit
C_r	Raw material cost per unit
H_1	Raw material holding cost per unit per period
H_2	Ending inventory holding cost per unit
C_L	Cost per unit due to decrease of demand
C_I	Inspection cost as a percentage of the production cost
C_R	Rejection cost per unit
S	Selling price per unit
B	Back orders cost per unit per period
L	Lost sales cost per unit = revenue loss per unit + cost of reputation loss per unit
X_i	Production quantity in period i in revised plan
Y_i	Delivery quantity in period i in revised plan
Z_i	Raw material quantity in period i in revised plan
b_i	Beginning inventory in revised plan
e_i	Ending inventory in revised plan

Demand fluctuation parameter

δ Demand fluctuation amount

Production disruption parameters

t_s Disruption start time as fraction of duration of period

T_{dp} Disruption duration as fraction of duration of period (≤ 1- t_s)

q Pre-disruption production quantity $= t_s * P$

Supply disruption parameter

T_{ds} Disruption duration as fraction of duration of period (≤ 1)

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Chapter 1 Introduction

1.1 Overview

The modelling of production-inventory, as well as supply chain management systems, is a challenging research topic in operations research and computer science. Production-inventory and supply chain systems exist, in many organisations, in different forms and degrees, depending upon the size and nature of the organisations, the products produced and supplied, the production facilities, the wholesalers and the retailers. A key issue for the success of any organisation in a supply chain environment is ensuring the smooth functioning of each and every entity in their supply chain, while managing the predictable and unpredictable disturbances and risks.

In reality, there are several disturbing factors that are involved in supply chain systems (Sodhi and Chopra, 2004), such as disruption in production and supply, fluctuation in demand, and uncertainty in demand and supply. An imperfect production environment is also an important factor that has a significant impact on a company’s production volume. Without a proper response to all these factors, the entire system can be imbalanced and the organisation may face huge financial loss, as well as loss of goodwill.

In a supply chain system, the disturbing sources can be either external or internal. For example, natural disaster, machine breakdown, power failure, labour strike, and political instability are all causes for supply chain disruption and delay. These types of disturbances can happen at any part of the system and at any time. But the effect of the disturbance can propagate to the whole supply chain system from the upstream to the downstream stages. A disturbance interrupts the flow of materials and the system may become inoperable for a certain period of time, resulting in loss of productivity. Without a proper response, it can take longer time for the affected system to recover (Sheffi and Rice, 2005).

In recent years, there have been many disturbances that have affected entire supply chain systems. For example, the 2011 earthquake and tsunami in Japan, which ultimately resulted in major supply chain disruptions across many sectors, caught many companies by surprise. The automotive supply chain, especially for the Japanese companies Toyota and

Honda, faced massive loss in their production and sales. In fact, Toyota lost its position as top global car producer in 2011 (Park et al., 2013). In the second half of 2011, a month long flooding in Thailand also had a significant impact on global supply capabilities in a number of high tech sectors. For example, Intel, a renowned disk drive company, lost about $1 billion in their sales in the fourth quarter of 2011, because they were unable to source the hard drives that were needed to make new machines.

In previous studies, most researchers focused on modelling different production-inventory systems under ideal conditions, although some also considered process reliability and fuzziness of variables. Only a little has been done with the consideration of any disturbance in production and supply. The reported research has so far focused only on a single disturbance, either in the production or delivery system. But in real-life, multiple disturbances, one after another as a series, may occur in a system, with or without having dependency between them. So there is a need to study multiple disturbances cases on a real-time basis. The incorporation of process reliability and uncertainty, with multiple disturbances recovery, will make the proposed study unique, and also closer to real-life problem scenarios. In the supply chain environment, previous studies were conducted mainly with single line supply with one, two or three nodes. In practice, a supply chain is a complex network. It has multiple suppliers, production facilities, warehouses and retailers. Any form of disturbance in any node in the network, will affect other parts of the network that may hence incur huge financial loss as well as reputation damage (Supply Chain Resilience 2010 Report).

In supply chain modelling, several real-life disturbing factors should be considered, such as: disruption or delay in production and supply, fluctuation in demand, uncertainty in demand, machine breakdown and natural disaster (Sodhi and Chopra, 2004). Production disruption is defined as any form of interruption that can be caused due to shortage of material, machine breakdown or unavailability, or any other form of disturbance (either accidental or man-made). Production disruption is a very familiar event in production environments. Disruptions or delay in supply is also a very common scenario in supply chain network. An entire network can be affected due to supply disruption or delay. Demand fluctuation is another important disturbing factor and a supply chain plan can be imbalanced due to this, and consequently an organisation can face financial and reputation losses. The development of an appropriate real-time disturbance recovery policy can help to minimise losses and also maintain the goodwill of a company. In practice, a production system may not be perfect, as in cases where a fraction of the produced items can be defective. However, the percentage of defective items depends on operating conditions. It is generally accepted that higher process reliability, and hence lower percentage of defective products, usually entails higher operating costs. So process reliability is a key risk factor in imperfect production environments, which hence has significant impact on costs and profits. Uncertainty of product demand is also a real-life consideration when developing a production-inventory and supply chain model.

The task involved in supply chain disturbance management modelling is not easy, especially when it is particularly sensitive to unexpected disruption (Hishamuddin, 2013). The case becomes more complex, when multiple disturbances, one after another, are considered. The most important task is to optimise the operational decisions, such as revised plan of supply, production and distribution, after the occurrence of each single or series of disturbances. So the development of appropriate methodologies, to recover from both a single and multiple disturbances under a real-life supply chain environment, will help decision makers to make accurate and prompt decisions during critical times.

1.2 Motivation and Scope of Research

This section briefly discusses the scope of the research in supply chain disturbance management, and the motivation for carrying out this thesis.

Over the last few decades, one of the popular research topics in operations research and industrial engineering has been supply chain management. It is a challenging topic, as it involves multiple entities, such as suppliers, manufacturers, wholesalers and retailers. The topic becomes more challenging when any entity faces any sudden disturbance. In general, it is very important to develop a recovery plan to minimise the effect of any disturbance.

Due to disturbance, the entire plan of an organisation can be distorted, such that shortage of goods and unfulfilled customer demand can occur. The development of an appropriate recovery policy can help to minimise losses and maintain the goodwill of a company. As of the literature, there exist limited studies that considered disturbances in production and supply chain systems and that also developed approaches to obtain a recovery plan. A very few studies have been found in the literature, which developed a recovery model after the occurrence of a sudden disturbance. Xia et al. (2004) developed a general disruption management approach for a two-stage production and inventory control system that incorporated a penalty cost for deviations of the new plan from the original plan. They introduced a disruption interval which is divided into three parts: pre-disruption, in-disruption and post-disruption, which allowed detailed analysis of disruption effects. They formulated the model as a quadratic mathematical programming problem and introduced the concept of a disruption recovery time window. A production disruption recovery model, for a single disruption within a single-stage and single item production system, has been developed by Hishamuddin et al. (2012), for obtaining a recovery plan within a user defined time window, which was an extension of the model of Xia et al. (2004). The study considered back order, as well as a lost sales option. They further extended the concept to develop a transportation disruption recovery plan in a two-stage production and inventory system with a single supplier and a single retailer (Hishamuddin et al., 2013). Recently, they have also applied the back order and lost sales concepts to develop a supply disruption recovery model in a two-echelon supply chain system with a single supplier and a single retailer (Hishamuddin et al., 2014). As, only a very few research works have been found, it is not sufficient for real-life disturbance problems.

In real-life, there are several disturbing factors, such as disruption in production and supply, fluctuation in demand, and imperfect production environment, which should be considered while developing a model for managing disturbance in a supply chain system. Any disturbances can happen suddenly, and cannot be predicted in advance. The task of optimising supply chain problems is not easy, especially when it is particularly sensitive to unexpected disturbance. Moreover, multiple types of disturbing factors may be present in a single problem. It is important for any organisation to optimise its operational decisions, after the occurrence of an unexpected disturbance, particularly for its production, supply, and distribution plan. In addition, in real-life cases, multiple disturbances can happen, one after another as a series, on a real-time basis. In this case, an appropriate strategy should be developed for managing multiple disturbances on a real-time basis. So a vital aspect of supply chain disturbance management is the development of an appropriate quantitative approach, so that decision makers can make decisions promptly and accurately.

1.3 Problem Statement

In the modern era of business, every manufacturing and/or service organisation is a part of a supply chain system. Such systems have multiple tiers, such as supplier, manufacture and retailer, and multiple entities at each tier. An example of a three-tier supply chain system with multiple entities is presented in Figure 1.1. As a supply chain system consists of multiple entities at each tier, the supply, production and distribution system form a complex network. It is a challenging optimisation problem to determine the optimal supply, production and distribution plan for such a complex network. In this network, any entity can face some uncontrollable and sudden disturbances, such as disruption in supply at the supplier end, disruption at the production system, and fluctuation of demand at the retailer end. The effects of a disturbance are not merely local; rather there is a high probability of impact on the other entities in the multiple tiers of a supply chain (Hishamuddin, 2013). For example, consider a three-tier supply chain that has multiple suppliers, manufacturers and retailers. If there is a disturbance in this supply system, then it is more likely that the supply, production and distribution plans will be affected. So the entire supply chain plan should be re-optimised, for a certain future period, to minimise the effect of any disturbance. It becomes a more challenging optimisation problem when an optimal recovery plan is developed, after the occurrence of a disturbance, so that the effect of a sudden disturbance is minimised.

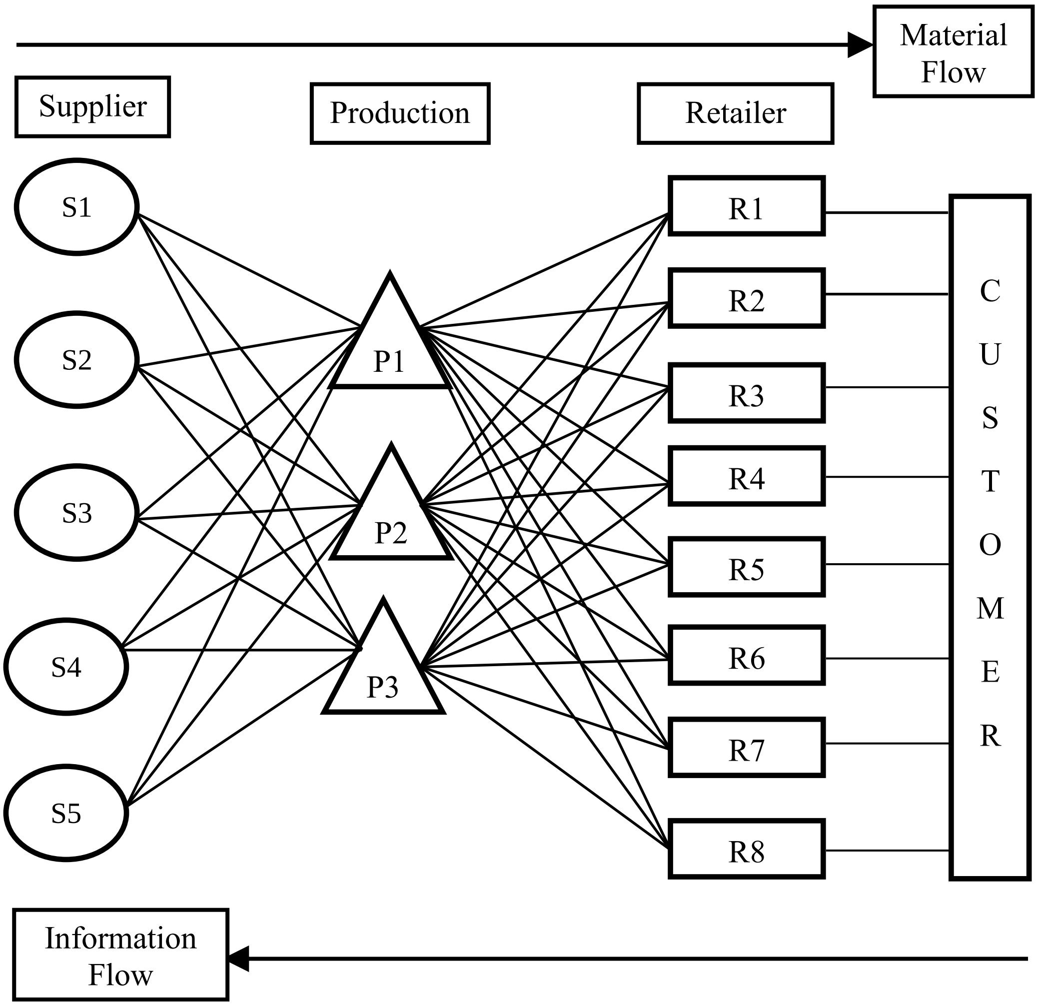

Figure 1.1: A three-tier supply chain network with multiple entities at each tier

In the literature, many studies have investigated ways to deal with disturbance in supply chains. The majority of these works consider proactive mitigation approaches, as such disturbance management strategy, in which additional inventory is held in the system for the entire period to protect against disruptions. A very few works have been found in the literature which developed a cost effective recovery strategy for managing disturbance, after the occurrence of a disturbance (Hishamuddin, 2013). In this thesis, the recovery strategies include the actions, which are taken only after the occurrence of a disturbance. These strategies include pure backorders, pure lost sales, outsourcing, or a mix of them to handle shortages in satisfying demand (Hishamuddin, 2013). In the event of a disturbance, all parties, such as suppliers, manufacturers and retailers, which are affected, must react appropriately in a timely manner in order to minimise potential losses. In this thesis, the objective is to minimise the effect of a disturbance in a supply chain system. The main decision is to determine revised supply, production and distribution schedule during the recovery-time window subject to demand, supply, production, and delivery constraints.

In real-life situations, a supply chain system can also face multiple disturbances, one after another, as a series. When a disturbance occurs, a revised plan can be generated by solving the model for a single disturbance. If a new disturbance occurs after the recovery time window of another disturbance, then the later one can be considered as an independent disturbance and the recovery plan can be made in a similar manner as the previous one. After finalizing the revised plan, if another disturbance occurs within the recovery time window, then the supply, production and distribution plans need to be revised again to consider the effect of both disturbances. This makes the case more complex for recovery planning. In practice, to minimise the effect of disturbances, they must be dealt with on a real-time basis, whether there is a single occurrence of disturbance or a series of disturbances. In this thesis, the recovery plan is also generated for a series of disturbances, as long as disturbances take place in a system.

The research problem considered in this thesis can be briefly summarised as follows.

i. A real-life supply chain system with imperfect production process.
ii. A disturbance is experienced at any point of the system and at any time. The disturbance includes disruption in supply and production, fluctuation in demand or a mix of them at both supply and demand.
iii. Generating a revised plan for supply, production and distribution, just after the occurrence of a disturbance, so the system can return to its normal schedule after a certain period of time.
iv. Generating a revised plan for a series of disturbances on a real-time basis. The plan may be revised after each disturbance, as long as disturbances take place in the system, while considering the effect of all previous disturbances.

In this research, a few general assumptions have been made as follows.

i. A single type of item is produced in the system.
ii. The production rate is greater than the demand rate.
iii. There is no inventory buffer in the system.

iv. There is no safety stock in the system.

v. The recovery-time window begins immediately after the occurrence of a disturbance.

vi. The recovery-time window is determined by the management of the organisation.

1.4 Importance of this Research

That supply chain disturbance management is an important problem, can be judged from two examples. The first, according to Sodhi and Chopra (2004), occurred because of lightning strikes at Phillips plant in New Mexico on March 17, 2000. This strike caused a massive surge in the surrounding electrical grid, and later it turned into a fire in the Royal Phillips Electronics plant and thereby damaged millions of microchips. Nokia Corporation and Ericsson were two major customers of the Phillips plant. To obtain backup supply, Nokia took proactive measures by redesigning its products and began switching its chip orders to other Phillips plant immediately after the fire disaster. In contrast, Ericsson employed a single sourcing policy. As a result, Ericsson had no other source of microchips, which consequently disrupted their production for months and cost $400 million in lost sales.

The second case is based on a Business Continuity Institute's study, entitled Supply Chain Resilience 2010, which reported that awareness of supply chain risks is increasing, but that many companies remain exposed to high levels of risk. They reported that seventy-two percent of survey respondents (310 organisations) experienced at least one disruption in their supply chain. The average number of supply chain disruptions reported by respondents was five, and for ten percent of companies the financial cost of supply chain disruptions was at least 500,000 Euros. Supply chain disruptions not only cause financial loss, but can also damage a company's brand or reputation as a result of third party failures. This fact was reported by twenty percent of companies that suffered damage to their reputation, while over fifty percent experienced a loss of productivity. In the retail sector, supply chain disruptions are almost always expected, and this sector has the worst problems with disruptions, with an average of ten per year.

From the above examples, it is clear that supply chain systems can be imbalanced because of inappropriate responses to disturbances, and organisations can consequently face huge financial loss, as well as loss of goodwill. Therefore, the development of efficient and accurate methodologies/algorithms to obtain optimal plans for inventory policies is a very important research topic. This has consequently sparked the interest of a wide range of researchers to study topics of supply chain disturbance management.

In recent years, the amount of supply chain disturbance management research is increasing significantly, which can be judged from Figure 1.2, which shows that research on this topic has increased exponentially over the last decades. This graph, presented in Figure 1.2, was plotted based on a search with the keywords "supply chain disturbance management" from Scopus. Now-a-days, disturbance management is a most important issue for supply chain systems and both academicians and practitioners are motivated to contribute in this research area, which is also the motivation behind the thesis.

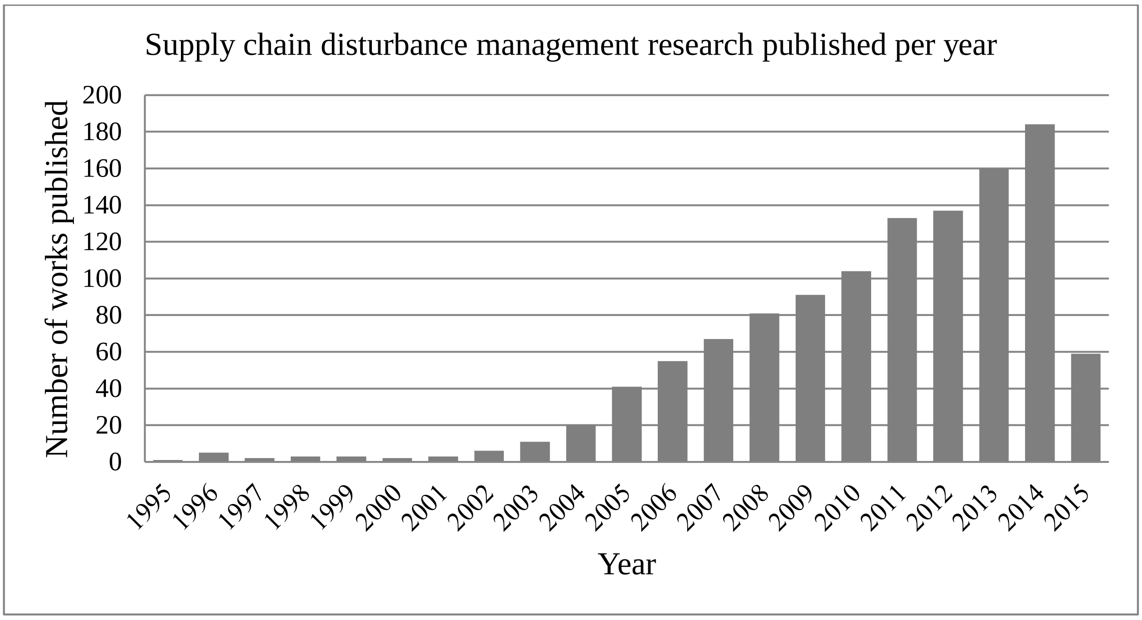

Figure 1.2: The number of research articles on supply chain disturbance management since 1995 (Source: Scopus)

1.5 Objective of the Thesis

The main objective of this thesis, is to develop quantitative decision tools to deal with different types of disturbances within a supply chain. These tools consist of mathematical modelling and heuristics developed to solve such models. Specifically, it considers real-life problems, such as production and supply disruption, demand fluctuation, and imperfect production in production-inventory and supply chain systems. The objective is to first determine the optimal inventory policies for each node as a recovery strategy, then subsequently optimising the entire supply chain as a whole and to finally determine the optimal plan if a system faces either a single or multiple disturbances on a real-time basis. The main objective has been divided into seven sub-objectives that were sequentially performed during the course of this research. The sub-objectives are described below, along with the steps taken to achieve them.

Objective 1: study an imperfect production-inventory system under production disruption.

- Develop a new mathematical model for a production-inventory system under production disruption with consideration of process reliability.
- Develop a solution approach to solve the mathematical model for managing a single disruption.
- Extend the solution approach for managing multiple disruptions, one after another as a series, on a real-time basis.
- Conduct experimental studies using different disruption scenarios.
- Perform sensitivity analysis.

Objective 2: study a two-stage production-inventory system under disruption.

- Develop a mathematical model by extending the previous single-stage production-inventory model to a two-stage system.
- Develop an efficient solution approach to solve the mathematical model for managing a single disruption at any stage.
- Extend the solution approach for managing multiple disruptions, at any stage as a series, on a real-time basis.

- Conduct experimental studies using different disruption scenarios.
- Perform sensitivity analysis.
- Implement the model to a real-world case from a pharmaceutical company.

Objective 3: study a three-stage mixed production-inventory system under disruption.

- Develop a mathematical model by extending the previous two-stage production-inventory model to a three-stage system under a mixed production environment.
- Develop an efficient heuristic to solve the mathematical model for managing a single disruption at any stage.
- Extend the heuristic for managing multiple disruptions, at any stage as a series, on a real-time basis.
- Conduct experimental studies using different disruption scenarios.
- Perform sensitivity analysis.
- Implement the model to a real-world case from a pharmaceutical company.

Objective 4: study a supply chain network under production disruption.

- Develop an ideal supply chain model and solve the model using traditional optimisation software.
- Develop a mathematical model for managing disruption with multiple manufacturing plants, multiple distribution centres, and multiple retailers.
- Develop an efficient heuristic to solve the mathematical model for managing a single disruption at any plant.
- Extend the heuristic for managing multiple disruptions, at any plant as a series, on a real-time basis.
- Conduct experimental studies using different disruption scenarios.
- Perform sensitivity analysis.

Objective 5: study demand fluctuation in a supplier-retailer coordinated system.

- Develop a mathematical model in a two-stage supplier-retailer system under demand fluctuation.

- Develop an efficient heuristic to solve the mathematical model for managing a single fluctuation at the retailer end.
- Extend the heuristic for managing multiple fluctuations, one after another as a series, on a real-time basis.
- Conduct experimental studies using different fluctuation scenarios.
- Perform sensitivity analysis.

Objective 6: study supply disruption in a three-tier supply chain system.

- Develop a mathematical model for a three-tier supply chain under raw material supply disruption.
- Develop an efficient heuristic to solve the mathematical model for managing a single disruption at any supplier.
- Extend the heuristic for managing multiple disruptions, at any supplier as a series, on a real-time basis.
- Conduct experimental studies using different disruption scenarios.
- Develop a simulation model to study real-world scenarios.
- Perform sensitivity analysis.

Objective 7: study multiple disturbances in a supply chain system and design a simulation model.

- Develop a mathematical model for a supply chain system under different types of disturbances: production disruption, demand fluctuation and supply disruption.
- Develop an efficient heuristic for managing each type of disturbance.
- Extend the heuristic for managing multiple disturbances during a period.
- Conduct experimental studies using different disturbance scenarios.
- Design a simulation model to test random disturbance instances.
- Perform sensitivity analysis.

1.6 Contributions to Scientific Knowledge

There are several scientific contributions that this thesis proposes to the literature of supply chain disruption, chiefly consisting of development of new mathematical models and their solution approaches. The specific contributions are outlined as follows.

- A real-time disruption recovery model for an imperfect production-inventory system is developed and an efficient solution approach is proposed to solve the model for both a single and multiple disruptions (Chapter 3), which is an extension of the work of Hishamuddin et al. (2012). The model determines the revised schedule and ensures recovery is achieved in a timely and cost effective manner. The model proposes a reactive strategy, after the occurrence of a disruption, on a real-time basis. The developed model also considers a more complex problem of multiple disruptions, one after another, as a series and determines a recovery plan after the occurrence of each disruption, as long as disruption takes place in the system. Both lost sales and back orders options are considered in the recovery strategy. To the best of my knowledge, this is an extended work that considers all the above characteristics in a single study.
- The research work, developed in chapter 3, is extended for managing production disruption in a two-stage (chapter 4) and three-stage mixed (chapter 5) production-inventory system and a supply chain network with multiple manufacturing plants, multiple distribution centres and multiple retailers (chapter 6). In all cases, an efficient mathematical model and heuristic approach is developed for managing both a single and multiple disruptions on a real-time basis. The models developed in chapters 4 and 5 are also implemented to real-world cases from a pharmaceutical company. A predictive mitigation approach is also developed in addition to the study in chapter 6.
- Chapter 7 develops a quantitative approach for managing demand fluctuation in a supplier-retailer coordinated system. An efficient heuristic is developed for managing both a single and multiple fluctuations on a real-time basis. The developed approach revises the schedule after the occurrence of each fluctuation as long as fluctuations take place in the system. To the best of my knowledge, there is

no such work in the literature that develops an approach for managing real-time demand fluctuation.

- A mathematical and simulation approach is developed for managing raw material supply disruptions in a three-tier supply chain (chapter 8). An efficient heuristic is developed first to obtain the revised plan after the occurrence of a disruption and after that the heuristic is extended to deal with multiple disruptions. The results are analysed and validated by carrying out experimental studies after developing a simulation approach. There is no such model in the literature that considers multiple disruptions in a three-tier supply chain.
- Finally, a mathematical and simulation model is developed for managing different types of disturbances in a supply chain system (chapter 9). Disturbances due to a production disruption, demand fluctuation, and raw material supply disruption are considered in this study. An efficient mathematical and heuristic approach is first developed for managing each type of disturbance. A simulation model is also developed that tests random disturbance occurrences to make the model closer to the real-world instances. Finally, the results are validated by carrying out experimental studies using different disturbance scenarios and by performing a sensitivity analysis. To the best of my knowledge, there is no such work in the literature that considers all the above disturbances in a single study, and develops a simulation model for managing those disturbances.

1.7 Organisation of the Thesis

This thesis has ten chapters and is organized as follows:

In chapter 1, an introduction to the thesis is presented. It first provides an overview of the research field, followed by the motivation and scope of the research. It also presents the problem statement and importance of the thesis. The objectives and a list of scientific contributions stemming from this research are also presented. The last section of the chapter presents the organisation of the thesis.

Chapter 2 provides a literature review and background study of the topics covered in this thesis. It provides a brief discussion on production-inventory and supply chain problems and their impact on real-life situations. Then the review is conducted on the basis of comparing various works published in this research domain, especially research that considered real-life risk factors, such as imperfect production process, disruption in production, supply, transportation, and fluctuation in demand while developing their models. The mathematical models and the solution approaches used in solving production-inventory and supply chain models, using both hypothetical and real-world problem scenarios, are also reviewed.

In chapter 3, a production disruption management model is developed for an imperfect single-stage production-inventory system. A significant number of numerical examples and random experimentation are presented to explain the usefulness and benefits of the developed model.

Chapter 4 extends the research work for a two-stage production-inventory system. Then a new mathematical model and heuristic approach is developed to deal with both single and multiple disruptions on a real-time basis.

In chapter 5, the research work is further extended for managing disruption in a three-stage mixed production-inventory system. The results are analysed by developing a new heuristic for managing both a single and multiple disruptions, using both random data, and also a real-world case from a pharmaceutical company.

A combined predictive and reactive mitigation approach for managing disruption in a supply chain network with multiple manufacturing plants, multiple distribution centres and multiple retailers is developed in chapter 6.

Chapter 7 presents a new mathematical and heuristic approach for managing demand fluctuation for a supplier-retailer coordinated system. The model is developed for managing both a single and multiple fluctuations, one after another as a series, on a real-time basis. A significant number of experiments are performed to analyse the results.

In chapter 8, a supply disruption management model is developed for a three-tier supply chain system. A new dynamic mathematical and heuristic approach is developed that is capable of dealing with a single and multiple supply disruptions on a real-time basis. This chapter also develops a simulation model to analyse the effect of randomly generated disruption events that are not known in advance.

A simulation model for managing multiple types of disturbances in a supply chain system is developed in chapter 9. A new and efficient heuristic is proposed for each disturbance type, to obtain a revised plan after the occurrence of a disturbance on a real-time basis. The effects of different types of randomly generated disturbance events, not known in advance, are analysed by developing a simulation model.

In chapter 10, the main findings from the thesis are summarized. The chapter concludes the thesis with a discussion of possible future research directions.

Since this thesis discusses different types of models for real-life supply chain problems, such as production disruption, supply disruption, demand fluctuations, and imperfect production, it can be presented to readers in different ways. Figure 1.3 shows the flowchart of this thesis for readers with different interests.

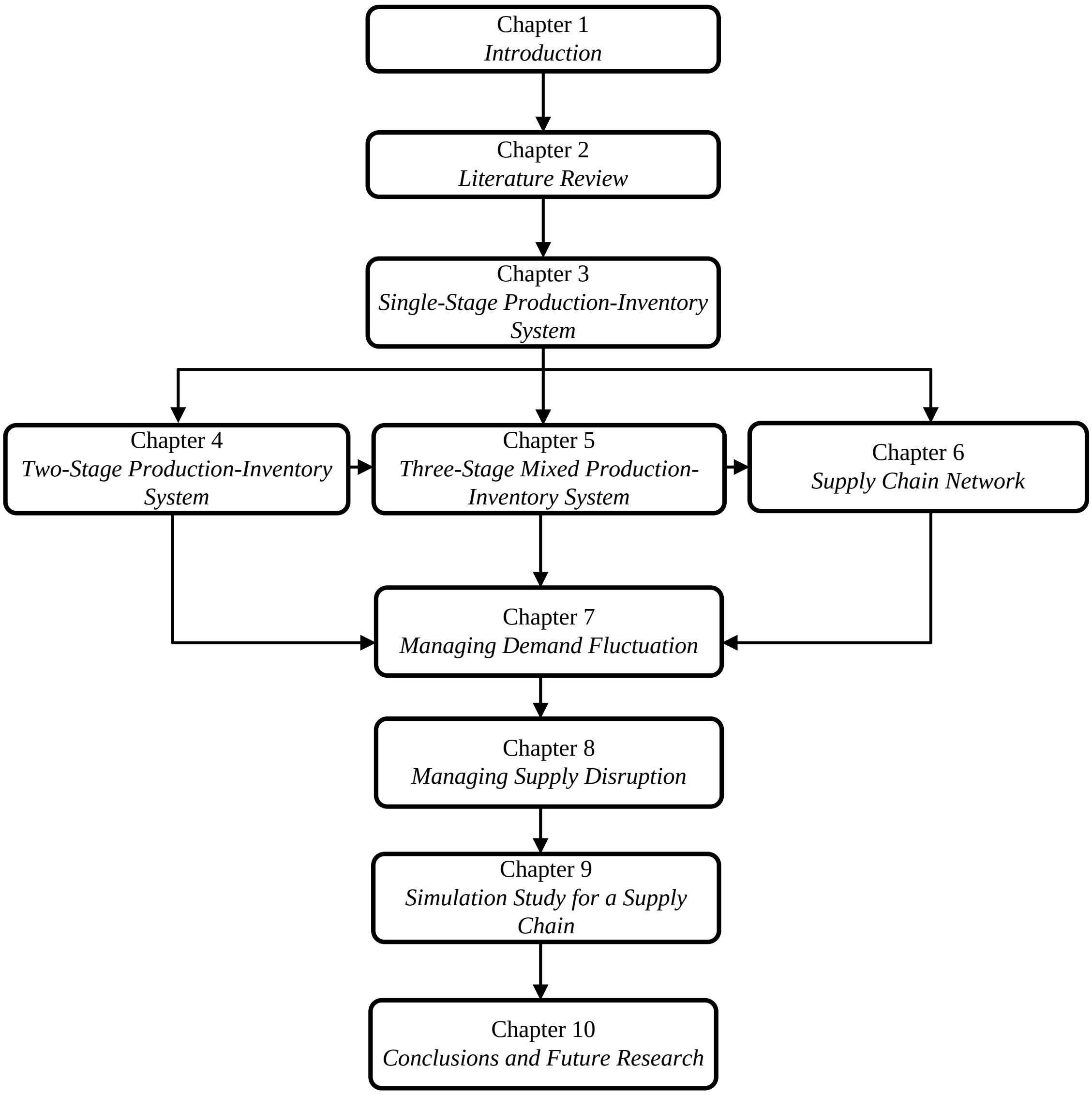

Figure 1.3: Flowchart of the thesis

Chapter 2 Literature Review

In this chapter, a literature review is presented on the topic of disturbance management in production-inventory and supply chain systems. The review is conducted on the basis of comparing various works published in this research domain, especially the papers considered real-life risk factors such as imperfect production process, disruption in production, supply, and demand while developing the models for production-inventory and supply chain systems. The emphasis is given on the assumptions and the types of problems considered in the published research. The focus is given on reviewing the mathematical models and the solution approaches in solving the models using both hypothetical and real world problem scenarios. Finally after the review, the research gaps are identified, which are the motivating factors for the research work in this thesis.

2.1 Introduction

Over the last half a century, one of the most widely studied research topics, in Operations Research and Industrial Engineering, is the production-inventory and supply chain system. Production-inventory and supply chain systems exist, in every organisation, in different forms and degrees, depending upon the size and nature of the organisation, the products produced and supplied, the production facilities, the wholesalers and the retailers. Ensuring the smooth functioning of each and every entity in a supply chain, and managing the predictable and unpredictable disturbances, is a key issue for the success of any organisation in a supply chain environment.

Recently, the disturbance management has become an important topic in this research area. In reality, there are several disturbance factors that are involved in supply chain systems (Sodhi and Chopra, 2004), such as: disruption in the production, supply, transportation, and uncertainty in demand and supply. An imperfect production environment is also an important factor that has significant impacts on company's production and supply. Without a proper response to all these factors, the entire system can be imbalanced and the organisation will face huge financial loss, as well as loss of goodwill. An organisation should have an appropriate recovery plan that minimises the impact of risk factors in supply, production and distribution.

A number of studies have been conducted in the past to develop models for facing risk in the production and supply chain systems. The literature basically consists of studies on various types of models, such as: models on imperfect production process, models production-inventory management with disruptions, models on supply chain management with disturbances, models used different solution approach, and models applied to a real-life case. In this chapter, the focus will be given on reviewing papers that incorporate the risk factors while developing models in production-inventory and supply chain systems.

2.2 Models

In the previous researches, a good number of papers developed models on production-inventory and supply chain. These works are categorized into four classes: (i) modelling for ideal system, (ii) modelling for imperfect production process, (iii) modelling with risk and disruption, and (iv) modelling for disruption recovery.

2.2.1 Modelling for the Ideal System

During the early stage, researchers focused on developing production-inventory models under ideal conditions, where the system is 100% perfect (no disruption). A few examples of such studies are, the development of the basic economic order quantity (EOQ) model ((Harris, 1990, reprint from 1913) and Wilson, 1934) and the development of the basic economic production quantity (EPQ) model (Taft, 1918), which was an extension of the EOQ model. Later, many researchers used EOQ and EPQ models in their studies. For example Cheng (1989) considered production process reliability in a single-stage imperfect production process to develop an EPQ model. Goyal and Gunasekaran (1990) also applied the basic concept of the EPQ model to determine optimal lot sizes in a multi-stage production system that minimised the sum of all costs. Other such extensions of EPQ models, in single-stage production-inventory systems, were developed in the research of periodic review stochastic inventory system (Chan and Song, 2003), a single-item single-stage inventory system with stochastic demand with periodic review where the system must order either none or at least as much as a minimum order quantity (Kiesmüller et al., 2011), the studies in steady-state average inventory and backorder levels for each product (Shiue and Altiok, 1993), an effective production ordering policy in a capacity-constrained production and inventory system (Ishii and Imori, 1996) and a direct and intuitive way of deriving the lot sizes (Hill, 2000) and a mathematical model to determine the optimal batch size under a periodic delivery policy in a single-stage production-inventory system (Sarker and Khan, 2001).

In supply chain systems, researchers also focused on developing models under ideal conditions during the early stage. A few examples of such supply chain studies include: a single product, single warehouse and multiple retailers based distribution system (Petrovic et al., 2008), a single manufacturer and single retailer model with demand and manufacturing cost as fuzzy variables (Zhou et al., 2008), a single period and two-stage supply chain coordination problem (Xu and Zhai, 2010) and a three-stage system consisting of supplier, manufacturer and retailer which produces a combination of perfect and imperfect quality items (Sana, 2011). Recently, Sana (2012) developed a model for a three stage supply chain where the systems may produce defective items. The production rate, order quantity and number of shipments are decision variables, where the objective is to maximise the expected total profit. Pal et al. (2012) developed an inventory model for multiple items produced by a manufacturer. It considered multiple suppliers, one manufacturer and multiple retailers with deterministic demand. They maximised the total integrated profit of the supply chain by determining the optimal ordering lot sizes of the raw materials. In recent years, a few more studies, on developing supply chain models under ideal conditions, can be found in Bottani and Montanari (2010), Choi et al. (2013), and Xu and Meng (2014).

For a two-stage single item supply chain system, with a lot-for-lot condition under an ideal situation, a non-disruption model is developed by Banerjee (1986). This is worth introducing here, because our two-stage demand fluctuation model have been derived based on this model. The original model is modified to incorporate the real-time demand fluctuation. The following notations were used in the model:

D Annual demand or usage of the inventory item

P Vendor's annual production rate for this item

A Purchaser's ordering cost per order

S Vendor's set-up cost per set-up

r Annual inventory carrying charge

C_v Unit production cost incurred by the vendor

C_p Unit purchase cost paid by the purchaser

Q Order or production lot size in units

The general cost function for the purchaser was derived as follows:

$$TRC_p = \frac{DA}{Q} + \frac{Q}{2} rC_p \tag{2.1}$$

The general cost function for the vendor was derived as follows:

$$TRC_v = \frac{DS}{Q} + \frac{DQ}{2P} rC_v \tag{2.2}$$

In the previous production-inventory and supply chain studies, most researchers focused on modelling different lot-sizing, coordination and optimisation systems under ideal conditions. But in real-life situations, there are several risk factors that are involved in production and supply chain systems, such as: disruption in the production, demand and supply (Sodhi and Chopra, 2004), imperfect production process and demand uncertainty. Without a proper response to all these risk factors, the entire system can be imbalanced and the organisation will face huge financial loss, as well as loss of goodwill.

2.2.2 Modelling for Imperfect Production Process

The above studies, with many others, are conducted under ideal conditions. However, imperfect production process is very common in real-life. Process reliability is used to consider imperfect production environment, in the production-inventory modelling, that has significant impact on costs and profits (Cheng, 1989). At first, process reliability was considered by Cheng (1989) in a single period inventory system and was formulated as an unconstrained geometric programming problem. Later it was extended by Bag et al. (2009) by considering product demand as a fuzzy random variable. Recently, process reliability in an imperfect production process was incorporated to determine the optimal product reliability and production rate that achieves the highest total integrated profit (Sana, 2010), to study an unreliable supplier in a single-item stochastic inventory system (Mohebbi and Hao, 2008) and to analyse an EPQ model with price and advertising demands under the effect of inflation (Sarkar and Moon, 2011). Recently, Paul et al. (2013) also extended the model of Cheng (1989), that considered product demand and inventory holding cost as a fuzzy random variable and maximised the graded mean integration value of the total profit. Some other models, which considered process reliability while modelling in production-inventory, were developed by Tripathy et al. (2003), Jaber et al. (2009), Leung (2007), Panda and Maiti (2009), Tripathy and Pattnaik (2011), Pal et al. (2013), and Masud et al. (2014).

The models, developed for imperfect production process, extended the literature significantly. These works also helped to apply the models in many real-life production processes. But in this competitive business era, the consideration of only process reliability is not sufficient to make the model realistic. Other risk factors, such as disruption in supply, production and demand should be considered while developing a realistic production-inventory model. It is worth to discuss here, because imperfect production environment is considered while developing our real-time disruption recovery models for single-stage and two-stage production-inventory system. Process reliability is also considered for demand fluctuation model in a two-stage supply chain system.

2.2.3 Modelling with Disruption

Snyder et al. (2012) provided an extensive review on supply chain disruption management models. However, in order to meaningful review for this thesis, the focus is given on the most relevant and recent topics in these works and the disruption recovery models in supply chain systems. Disruption management strategies can be categorised into three main groups (Tomlin, 2006): (i) mitigation strategies, (ii) recovery strategies, and (iii) passive acceptance. Mitigation strategies require the company to act in advance of a disruption, regardless of whether the disruptions actually occur or not (Jr and Taskin, 2008). The examples of mitigation strategies include increasing of safety stock, multiple sourcing, expanding the capacity, increasing visibility and setting up alternative transportation modes (Jr and Taskin, 2008). The recovery strategies include the action, which is taken only after the occurrence of a disruption. This strategy may include alternative sourcing, rescheduling of plan for a future period and/or rerouting the transport system (Hishamuddin, 2013). Lastly, passive acceptance, which is accepting the risks without any action, may be more appropriate in certain solutions when the costs of mitigation or recovery strategies outweigh their potential advantages. In the literature, most of the researches focused of mitigation strategies to manage the risks due to disruption. Recently, some researches have been carried out by applying recovery strategies. In case of sudden disruption, recovery strategies could be more effective than mitigation strategies. In this chapter, the focus will be given on reviewing papers that study on production-inventory and supply chain models for managing disruption risks.

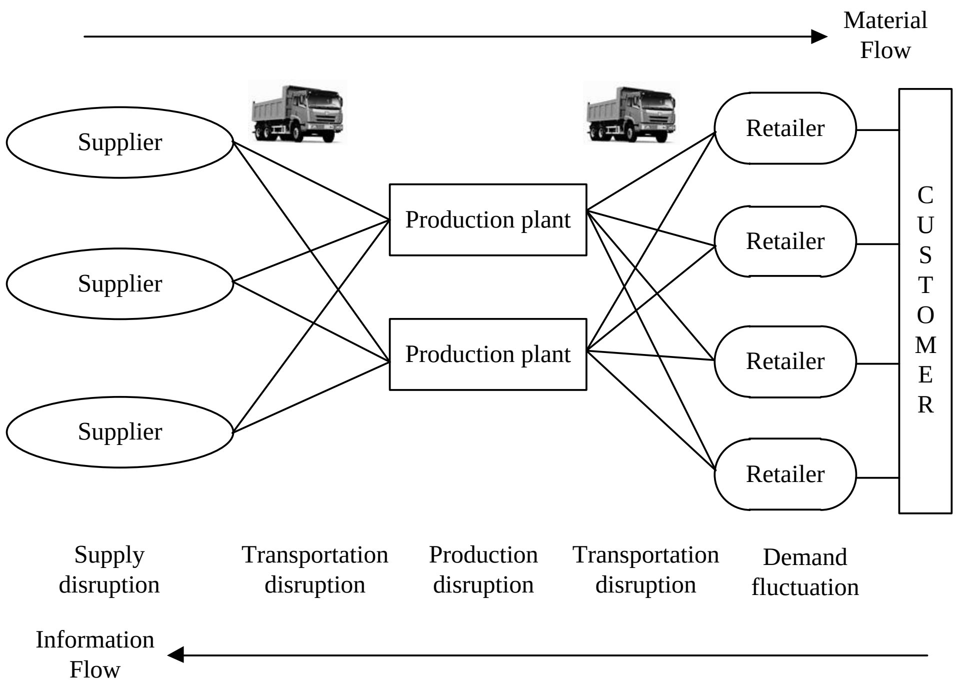

Figure 2.1: Different disruptions in a supply chain system

Figure 2.1 presents a typical supply chain system with different disruptions. The supply chain disruption risk is classified into four categories: (i) disruption in production, (ii) disruption in supply, (iii) disruption in transportation, and (iv) fluctuation in demand, which are shown in Figure 2.1. Production disruption includes any form of interruption in the production that may be caused due to shortage of material, machine breakdown and unavailability, or any other form of disturbance (either accidental or man-made). A supply disruption can be defined as any form of interruption in the material supply that may be caused due to delay, unavailability, or any other form of disturbance. The transportation disruptions include any form of interruption in the transportation system between supplier and manufacturer, and manufacturer and retailer, that may be caused due to breakdown, road work, strike, and natural disaster like flood, earthquake etc. Lastly, demand fluctuation can be defined as any kind of variation in product demand at the retailer end. Demand can be increased or decreased for a certain period of time.

2.2.3.1 Disruption in Production

Lin and Gong (2006) analysed the impact of machine breakdown on an EPQ (Economic Production Quantity) model for deteriorating items in a single-stage production system. They considered a fixed period of repair time. They minimised an expected total cost per unit time, that consisting of setup, corrective maintenance, inventory carrying, deterioration, and lost sales costs. Widyadana and Wee (2011) extended the model of Lin and Gong (2006) for deteriorating items with random machine breakdown and stochastic repair time with uniform and exponential distribution. Recently, Wee and Widyadana (2013) considered production delay which is due to random machine unavailability and shortages to develop an integrated single-vendor single-buyer inventory model. They considered lost sales option, and two kinds of machine unavailability distributions uniformly and exponentially distributed. An EPQ model with a Poisson distributed machine breakdown was considered by Chiu et al. (2007) to determine an optimal production run time. They developed a total inventory cost function, under EPQ situations, with and without breakdown in a single-stage production system. They assumed some portion of the products produced was defective, which meant that it had to be scraped or reworked. Moinzadeh and Aggarwal (1997) considered a (s, S) production-inventory policy with random disruptions and exponential time between breakdowns in an unreliable bottleneck system. A two-stage supply chain, consisting of retailer and supplier, was considered by Zeynep Sargut and Qi (2012), where random disruption may occur at both retailer and supplier, and where unfulfilled customer demand is lost. The proposed model minimises an expected annual cost in finding the order quantity of the retailer. Schmitt and Snyder (2012) developed an inventory model that considered two options: (i) an unreliable supplier and (ii) a reliable but expensive supplier. For both cases, they considered disruption and recovery probability with yield uncertainty to find the optimal order and reserve quantities.

Recently, Hishamuddin et al. (2012) developed a production disruption recovery model in a single-stage production-inventory system, which considered both back order and lost sales options. Chiu et al. (2013) considered breakdown in equipment for developing an optimal replenishment policy for an economic production quantity (EPQ) inventory model. They assumed that the machine will go immediately to under repair whenever a breakdown occurs and the production resumes immediately after the machine is fixed and restored. Taleizadeh et al. (2014) considered interruption in the manufacturing process to develop an economic production quantity (EPQ) inventory model. They studied a multi-product and single-machine EPQ model and permitted the shortage as backordered.

2.2.3.2 Disruption in Supply

Supply disruption is another important consideration in production and inventory modelling. In the inventory and supply chain disruption management, highest numbers of works have been carried out for managing supply disruptions. In the early years, Parlar and Berkin (1991) and Parlar and Perry (1996) developed inventory models that considered supplier availability with deterministic product demand under a continuous review framework. Özekici and Parlar (1999) considered back orders to analyse a production-inventory model under random supply disruptions. Weiss and Rosenthal (1992) developed an optimal inventory policy for EOQ inventory systems which may have a disruption in either supply or demand. They considered disruption is known a priori and it lasts a random length of time. Some other models of supply disruptions that considered deterministic or probabilistic product demand in their inventory models, can be found in Mohebbi (2003), Mohebbi (2004), Li et al. (2004), Tomlin (2006), Mohebbi and Hao (2008), Chopra et al. (2007), Qi et al. (2010) and Schmitt et al. (2010). There are a few studies that considered both supply and demand disruptions with deterministic product demand, such as Xiao and Yu (2006) and Ross et al. (2008).

Recently, Hou et al. (2010) studied a buy-back contract between a buyer and a backup supplier when the buyer's main supplier experiences disruptions and explored the main supplier's recurrent supply uncertainty through comparative studies and numerical examples. Pal et al. (2012b) considered two suppliers to supply the raw materials to the manufacturer, where the main supplier may face supply disruption after a random time and the secondary supplier is perfectly reliable but more expensive than the main supplier, to develop a model in a multi-echelon supply chain. Snyder (2014) introduced a simple but effective approximation for a continuous-review inventory model and considered supplier experiences "wet" and "dry" (operational and disrupted) periods whose durations are exponentially distributed. Recently, Qi (2013) considered two supplier concept; (i) supplier 1: primary supplier (cheaper) and (ii) supplier 2: backup supplier (expensive but reliable) to manage supply disruption for a single item continuous-review inventory problem. He considered two strategies to recover from a disruption; (i) If supplier 1 is available when the inventory level at the retailer reaches the reorder point, the retailer orders from supplier 1 and (ii) the retailer will reroute to the backup supplier if supplier 1 still does not recover from the disruption when the cap of waiting is reached. Hishamuddin et al. (2014) applied the back order and lost sales concept to manage supply disruption in a two-stage supply chain, which consists of single supplier and single retailer. Some other recent works on managing supply disruption can be found in Yang et al. (2009), Li et al. (2010), Qi et al. (2010), Zhang et al. (2013), Hu et al. (2013), Yan et al. (2014), and Pal et al. (2014).

2.2.3.3 Disruption in Transportation

In the literature, transportation disruption has got much less attention compare to production and supply disruptions. This type of disruption stops the flow of goods, where as other types of disruption may stop production of goods and supply of raw materials as well (Hishamuddin, 2013). Giunipero and Eltantawy (2004) discussed about transportation disruption in general in their study, but did not specify the strategies on how to fact it. Wilson (2007) investigated the effect of a transportation disruption on supply chain performance using system dynamics simulation in a 5-echelon supply chain system, which is presented in Figure 2.2. Four types of disruptions were considered in the study: (i) transportation disruption between the warehouse and the retailer, (ii) transportation disruption between the tier 1 supplier (manufacturer) and the warehouse, (iii) transportation disruption between the tier 2 supplier and the tier 1 supplier, and (iv) transportation disruption between the raw material supplier and the tier 2 supplier. It was observed that the greatest impact occurs when transportation is disrupted between the tier 1 supplier and warehouse.

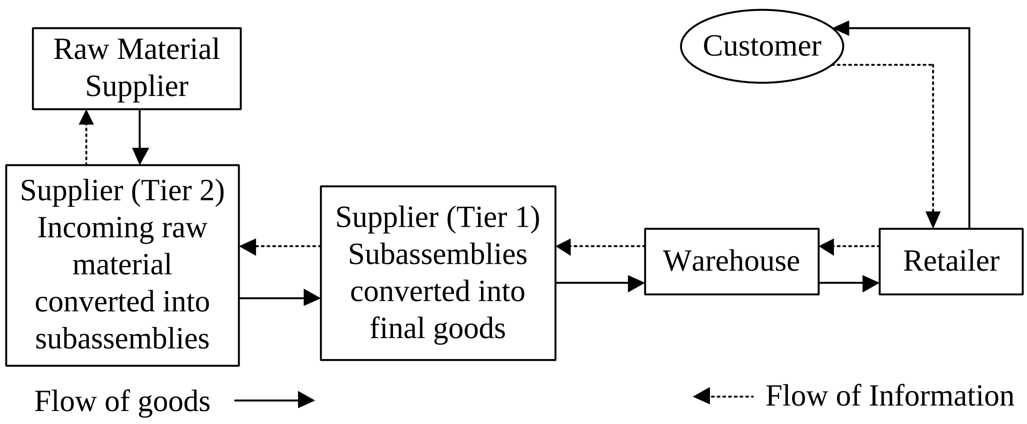

Figure 2.2: Flow of goods and information: the traditional structure (Wilson, 2007).

Zhang and Figliozzi (2010) focused on the performance of international and domestic transport and logistics systems as perceived by Chinese importers and exporters. They provided significant information regarding international freight transport chains, the impact of delays on supply chain operations and the subsequent costs, companies' delay and disruption planning, and managers' perspectives on future transport and logistics developments. Unnikrishnan and Figliozzi (2011) formulated a mathematical model for a new type of freight network assignment problem in a dynamic environment and in the presence of probable network disruptions or significant delays. Recently, Hishamuddin et al. (2013) proposed a recovery strategy for managing transportation disruption in a two-echelon supply chain system. They considered both back orders and lost sales options to recover after the occurrence of a sudden disruption. Although Hishamuddin et al. (2013) proposed a quantitative strategy for managing disruption, there is still lack of analysis for multiple disruptions, on a real-time basis.

2.2.4 Supply Chain Disruption and Risk Management

Supply chain risk management is aimed at managing risks in complex and dynamic supply and demand networks (Wieland and Wallenburg, 2012). There are some papers in the literature which focused on managing supply chain network disruption and risks. Tang (2006) presented certain "robust" strategies, for mitigating supply chain disruptions, which possess two properties. First, strategies for enabling a supply chain to manage the inherent fluctuations efficiently regardless of the occurrence of major disruptions. Second, strategies for making a supply chain become more resilient in the face of major disruptions. Craighead et al. (2007) derived six propositions relating the severity of supply chain disruptions to the supply chain design characteristics of density, complexity, and node criticality and to the supply chain mitigation capabilities of recovery and warning. Those six propositions augmented extant knowledge as to what risk factors are present within a supply chain, how vulnerable a supply chain is to these risks, how resilient a supply chain is to some given risks, and what can be done to prevent or reduce the occurrences of severe supply chain disruptions. Xiao et al. (2007) introduced a supply chain coordination model with one manufacturer and two competing retailers and studied the coordination of the supply chain with demand disruptions. They found that an appropriate contractual arrangement can fully coordinate the supply chain and the manufacturer can achieve a desired allocation of the total channel profit by varying the unit wholesale price and the subsidy rate. Manuj and Mentzer (2008) proposed a comprehensive risk management and mitigation model for global supply chains, that brought together the concepts, frameworks, and insights from several disciplines – primarily logistics, supply chain management, operations management, strategy, and international business management. Wu et al. (2007) presented a network-based modelling methodology to determine how changes or disruptions propagate in supply chains and how those changes or disruptions affect the supply chain system. The modelling approach provided insights to better manage supply chain systems that face disruptions and thus allow quicker response times, lower costs, higher levels of flexibility and agility, lower inventories, lower levels of obsolescence and reduced demand amplification throughout the chain. Recently, Atoei et al. (2013) proposed a reliable capacitated supply chain network design model by considering random disruptions in both distribution centers and suppliers and determined the optimal location of distribution centers (DC) with the highest reliability, the best plan to assign customers to opened DCs and assigns opened DCs to suitable suppliers with lowest transportation cost. Bradley (2014) analysed the differences between frequent and rare risks for supply chain disruptions, and proposed a new and improved risk measurement and prioritization method to account for the characteristics of rare risks. Some other supply chain disruption and risk management model can be found in Kleindorfer and Saad (2009), Xiao et al. (2005), Yu and Qi (2004), Finch (2004), Huang et al. (2006), Skipper and Hanna (2009), Lavastre et al. (2012), Wu et al. (2013), Salehi Sadghiani et al. (2015), Friesz et al. (2011), and Chopra and Sodhi (2014).

2.2.4.1 Multiple Sourcing

Several papers have been found in the literature, which used multiple sourcing strategies to manage supply chain disruption risks. Yu et al. (2009) evaluated the impacts of supply disruption risks on the choice between the famous single and dual sourcing methods in a two-stage supply chain with a non-stationary and price-sensitive demand. They obtained the expected profit functions of the two sourcing modes in the presence of supply chain disruption risks and then identified critical values of the key factors affecting the final choice. Xanthopoulos et al. (2012) proposed a newsvendor-type inventory models for capturing the trade-off between inventory policies and disruption risks in a dual-sourcing supply chain. They developed the models for both risk neutral and risk-averse decision-makers and obtained the closed-form analytical solutions the determination of the optimal expected total profit of the retailer/wholesaler.

Recently, Gong et al. (2014) determined the optimal ordering and pricing policies in each period over a planning horizon, and analysed the impacts of supply source diversification. They showed that, when both suppliers are unreliable, the optimal inventory policy in each period is a reorder point policy and the optimal price is decreasing in the starting inventory level of the period. They also showed that, having supply source diversification or higher supplier reliability increases the firm's optimal profit and lowers the optimal selling price. Silbermayr and Minner (2014) studied the supply interruptions mitigation and management with sourcing from multiple suppliers. They studied a supply chain with one buyer facing Poisson demand who can procure from a set of potential suppliers who are not perfectly reliable. They modelled by a Semi-Markov decision process where demands, lead times and availability of suppliers are stochastic. Some other models used multiple sourcing strategy to manage supply chain risk can be found in Blome and Henke (2009), Fang et al. (2013), Lu et al. (2011), Heese (2015), Serel (2015), Sajadieh and Thorstenson (2014), and Sawik (2014).

2.2.5 Modelling for Disruption Recovery

The disruption is a very familiar event in the production and supply chain environment. This is a concern because companies face financial, as well as reputation losses, due to disruption. Due to disruption, the entire plan of the organisation can be distorted causing shortage of goods and unfulfilled customer demand. The development of an appropriate recovery policy can help to minimise losses and maintain the goodwill of a company. As of the literature, there exist limited studies that considered disruptions in the production and supply chain system and that also develop approaches to obtain a recovery plan. If a system is disrupted for a given period of time (known as disruption duration), it is necessary to revise the production schedule (known as recovery plan) for some periods in the future (known as recovery time window) until the system returns to normal schedule (Hishamuddin et al., 2012). In some studies, it is assumed that the recovery time window must be specified by the management of the production system.

A very few studies have been found in the literature, which developed a recovery model after the occurrence of a sudden disruption. Xia et al. (2004) developed a general disruption management approach for a two-stage production and inventory control system and they incorporated a penalty cost for deviations of the new plan from the original plan. They introduced the disruption interval which is divided into three parts: pre-disruption, in-disruption and post-disruption, which allowed detailed analysis of disruption effects. They formulated the model as a quadratic programming problem and introduced the concept of a disruption recovery time window. Eisenstein (2005) introduced the flexible dynamic produce-up-to policy that is able to respond to disruption by adjusting the amount of idle time during recovery and re-established the target idle time as the schedule recover.

A production disruption recovery model, for a single disruption within a single-stage and single item production system, has been developed by Hishamuddin et al. (2012), for obtaining a recovery plan within a user defined time window, which was an extension of the model of Xia et al. (2004). The study considered back order, as well as the lost sales option. They have further extended the concept to develop a transportation disruption recovery plan in a two-stage production and inventory system with single supplier and single retailer (Hishamuddin et al., 2013). Recently, they have also applied the back order and lost sales concepts to develop a supply disruption recovery model in a two-echelon supply chain system with single supplier and single retailer (Hishamuddin et al., 2014). Some other disruption recovery models in the production-inventory and supply chain system can be found in Gallego (1994), Qi et al. (2004), Tang and Lee (2005), Yang et al. (2005), Chen et al. (2015), and Shao and Dong (2012).

The model developed by Hishamuddin et al. (2012), which was an extension of Xia et al. (2004), enhanced the disruption recovery literature significantly. The model considered the disruption in the form of schedule interruption that is not known in priori. The model considered both back order and lost sales option and developed an efficient heuristic to determine the optimal recovery plan and the recovery cycles, after the occurrence of a disruption, are presented in Figure 2.3.

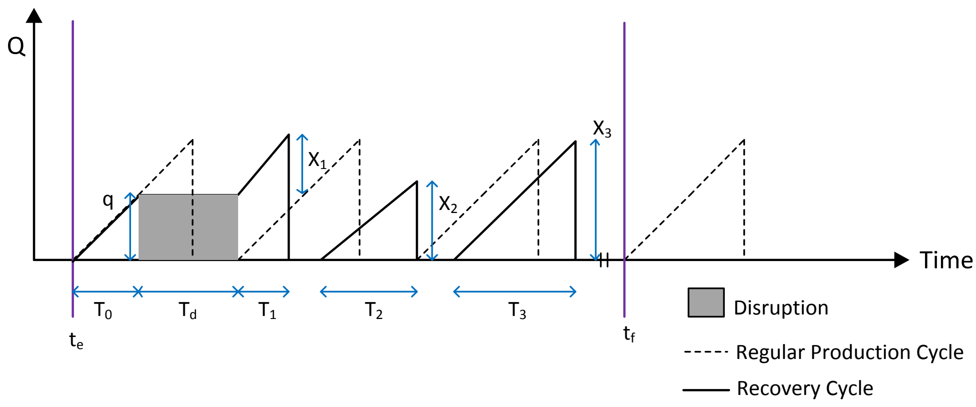

Figure 2.3: Disruption recovery plan of Hishamuddin et al. (2012)

The following notations are used to formulate the model of Hishamuddin et al. (2012):

A	Set-up cost for a cycle
D	Demand rate for a product
H	Annual inventory holding cost
P	Production rate
Q	Production lot size in the original schedule
T_d	Disruption duration
q	Pre-disruption production quantity in a cycle
T_0	Production time for q
u	Production down time for a normal cycle
t_e	Start of recovery time window
t_f	End of recovery time window
T	Production cycle time for a normal cycle
ρ	Production up time for a normal cycle
B	Unit back order cost per unit time
L	Unit lost sales cost
X_i	Production quantity for cycle i in the recovery window
T_i	Production up time for cycle i in the recovery window
S_t	Set-up time for a cycle
δ	Idle time for a cycle
n	Number of cycles in the recovery window

The cost function, which is the sum of all the cost components in the recovery window, for their model is presented below:

$$TC(X_i, n) = (A.n) + \left(\frac{H}{2P}\left(q^2 + 2qP.(T_d + S_t) + q.X_1 + \sum_{i=1}^{n} X_i^2\right)\right)$$
$$+ \left(B \left((X_1 + q)\left(T_d + S_t + \frac{X_1 + q}{P} - \frac{Q}{P}\right)\right.\right.$$
$$\left.\left. + \sum_{i=2}^{n}\left(T_d + i.S_t + \sum_{j=1}^{i}\frac{X_j}{P} + \frac{q}{P} - i.\frac{Q}{P} - u(i-1)\right)\right)\right)$$
$$+ \left(L\left(nQ - (X_1 + q) - \sum_{i=2}^{n} X_i\right)\right) \tag{2.3}$$

Subject to the following constraints, presented in (2.4) – (2.8):

$$X_1 \leq Q - q \tag{2.4}$$

$$X_i \leq Q;\ \text{for } i = 2,3,\ldots, n \tag{2.5}$$

$$\sum_{i=1}^{n} X_i \leq P(nT - nS_t - T_d) - q \tag{2.6}$$

$$X_1 + q + \sum_{i=2}^{n} X_i \geq nTD - \left(nQ - \sum_{i=1}^{n} X_i - q\right) \tag{2.7}$$

$$\sum_{j=1}^{i} X_j \geq i.Q + (i-1)P.u - P.T_d - P.iS_t - q \tag{2.8}$$

This is the most closely related research, to the current study, where a single-stage production-inventory system with only one practical issue (i.e., production disruption) is considered. The objective of their study was to minimise the total cost of recovery in a single occurrence of disruption. This study is interested to analyse the post disruption effects under imperfect production process, where the systems can face both a single and multiple disruptions, one after another as a series, on a real-time basis. To the best of knowledge, this is the first study to consider this.

2.2.6 Summary of Literature Review for Different Models

The summary of the literature review for different models is presented in Table 2.1. It is observed that, most of the studies considered single risk factor while developing the model. Most of the models considered a simple supply chain network with only a single occurrence of disruption. In real-life, multiple disruptions can happen one after another as a series. A very few models have been found in the area of disruption recovery and most of them developed a single disruption recovery model. So it can be said that there is a lack of quantitative disruption and risk management models to help the decision maker to make prompt and accurate decision.

Table 2.1: Summary of review for different models

Modelling type	Description	Author (year)	Remarks
Imperfect production	Production system is not 100% perfect and produces some defective items. The term, process reliability, is used for imperfect production system.	Cheng (1989); Bag et al. (2009); Sana (2010); Mohebbi and Hao (2008); Sarkar and Moon (2011); Paul et al. (2014); Jaber et al. (2009); Leung (2007); Masud et al. (2014); Panda and Maiti (2009); Sarkar (2012); Tripathy and Pattnaik (2011); Tripathy et al. (2003).	The models, developed for imperfect production process, extended the literature significantly. But in this competitive business era, the consideration of only process reliability is not sufficient to make the model realistic.
Disruption	**Production disruption:** Any form of interruption in the production that may be caused due to shortage of material, machine breakdown and unavailability, or any other form of disturbance (either accidental or man-made).	Lin and Gong (2006); Widyadana and Wee (2011); Wee and Widyadana (2013); Chiu et al. (2007); Moinzadeh and Aggarwal (1997); Zeynep Sargut and Qi (2012); Schmitt and Snyder (2012); Hishamuddin et al. (2012); Chiu et al. (2013); Taleizadeh et al. (2014).	Most of the developed models for managing production disruption considered a single-stage production-inventory system and only a single disruption.
	Supply disruption: Any form of interruption in the material supply that may be caused due to delay, unavailability, or any other form of disturbance.	Parlar and Berkin (1991); Parlar and Perry (1996); Özekici and Parlar (1999);Weiss and Rosenthal (1992); Mohebbi (2003); Mohebbi (2004); Li et al. (2004); Tomlin (2006); Mohebbi and Hao (2008); Chopra et al. (2007); Qi et al. (2010); Schmitt et al. (2010); Xiao and Yu (2006); Ross et al. (2008); Hou et al. (2010); Pal et al. (2012b); Snyder (2014); Qi (2013); Hishamuddin et al. (2014); Yang et al. (2009); Li et al. (2010); Qi et al. (2010); Zhang et al. (2013); Hu et al. (2013); Yan et al. (2014); Pal et al. (2014).	Developed models for managing supply disruption mainly for a two-stage supply chain system with single supplier and single retailer and also for managing only a single disruption.

Table 2.1: Summary of review for different models (Continued)

Modelling type	Description	Author (year)	Remarks
Disruption	**Transportation disruption:** Any form of interruption in the transportation system that may be caused due to breakdown, road work, strike, and natural disaster like flood, earthquake etc.	Giunipero and Eltantawy (2004); Wilson (2007); Zhang and Figliozzi (2010); Unnikrishnan and Figliozzi (2011); Hishamuddin et al. (2013).	Transportation disruption has got much less attention compare to production and supply disruptions. Most of studies developed model for a single disruption.
Supply chain risk	Risk management in complex and dynamic supply and demand networks	Atoei et al. (2013); Bradley (2014); Chopra and Sodhi (2014); Craighead et al. (2007); Finch (2004); Huang et al. (2006); Kleindorfer and Saad (2009); Lavastre et al. (2012); Manuj and Mentzer (2008); Skipper and Hanna (2009); Tang (2006); Wieland and Wallenburg (2012); Wu et al. (2007); Wu et al. (2013); Xiao et al. (2005, 2007); Yu and Qi (2004); Blome and Henke (2009); Fang et al. (2013); Sajadieh and Thorstenson (2014); Sawik (2014); Yu et al. (2009); Gong et al. (2014); Silbermayr and Minner (2014); Xanthopoulos et al. (2012)	A plenty of papers have been found in the literature. They considered different risk factors, such as risk from different disruptions, sourcing, flexibility, and reliability. No study considered all risk factors together in a single study.
Disruption recovery	Development of appropriate recovery policy, after the occurrence of a disruption, on a real-time basis.	Hishamuddin et al. (2012); Hishamuddin et al. (2014); Hishamuddin et al. (2013); Eisenstein (2005); Gallego (1994); Qi et al. (2004); Shao and Dong (2012); Tang and Lee (2005); Xia et al. (2004); Yang et al. (2005)	A very few studies have been found in the literature, which developed a recovery models after the occurrence of a sudden disruption. No study considered all disruptions together.

2.3 Solution Approach

In the literature, several solution approaches have been applied to solve the model. These approaches can be broadly classified as: traditional optimisation approach, heuristic approach, simulation approach, and search algorithm approach. In case of solving complex models, researchers focused on developing heuristic, rather than applying standard search algorithm. Many researchers used simulation techniques to solve and make the model as real world process.

2.3.1 Traditional Optimisation Approach

If the supply chain risk management problem is simple and static then it can be solved by using a traditional optimisation approach. A few examples of such researches include: use of linear programming (Kabak and Ülengin, 2011), geometric programming (Cheng, 1989; Bag et al., 2009; and Paul et al., 2013), quadratic programming (Xia et al., 2004), and branch and bound method (Baghalian et al., 2013). In real-life situations, supply chain risk management model is a dynamic and complex problem. This limits the applicability of traditional optimisation approaches to solve the risk management model.

2.3.2 Heuristic Approach

Heuristics are a subset of strategies (Gigerenzer and Gaissmaier, 2011). Heuristic rules have the advantage of being simple to understand, easy to apply and very inexpensive to use in computer programs (Talbot and Patterson, 1979). A few papers have been found in the literature, which developed a heuristic to solve their model. Usually, the heuristic was developed when the mathematical model was complex to solve. In the production-inventory and supply chain management field, several studies have been found in the literature, which developed a heuristic to solve the complex models. For example: for finding near optimal policies of a production-inventory system subject to exponential distributed disruptions (Moinzadeh and Aggarwal, 1997), for managing production disruption in a single-stage production-inventory system(Hishamuddin et al., 2012), for managing transportation disruption in a two-echelon supply chain (Hishamuddin et al., 2013), and for managing supply disruption in a two-echelon supply chain (Hishamuddin et al., 2014).

Abboud (2001) developed an efficient algorithm to obtain the optimal solution, which relaxed the constant recovery length assumptions made by Moinzadeh and Aggarwal (1997). Arreola-Risa and DeCroix (1998) presented an algorithm, to find optimal decision variables and insights behaviour of the optimal policy parameters, for a continuous review inventory system with Poisson demand, zero lead time and partial backorders. Hishamuddin et al. (2012) developed an efficient heuristic approach to determine the optimal values of production quantities and number of recovery cycles for solving their recovery model in a single-stage production-inventory system. The heuristic consists of three main strategies. *Strategy 1*: the total back order plan, *Strategy 2*: the available capacity allocation and *Strategy 3*: the minimum back order requirement. Some other recent papers, which developed a heuristic, can be found in Hill and Galbreth (2008), Perron et al. (2010), Liu and Chen (2011), Liao et al., (2009), and Al-Rifai and Rossetti (2007).

2.3.3 Simulation Approach

Simulation is defined as the imitation of the operation of a real-world process or system over time (Banks et al., 2001). Simulation enables decision makers to improve operational efficiency and performance through its ability to incorporate the inherent uncertainties in complex real system (Keskin et al., 2010). It is a very common tool in the literature, which was used to evaluate the complex models in inventory and supply chain.

In this section, a brief review, of using simulation approach in inventory and supply chain risk management, is discussed. Wu and Olson (2008) considered three types of risk evaluation models within supply chains: chance constrained programming (CCP), data envelopment analysis (DEA), and multi-objective programming (MOP) models. They modelled the various risks in the form of probability and simulation of specific probability distribution in a supply chain consisting of three levels and used simulated data with representative distributions. Longo and Mirabelli (2008) presented an advanced modelling approach and a simulation model for supporting supply chain management. They considered two objectives. The first objective was to develop a flexible, time-efficient and parametric supply chain simulator starting from a discrete event simulation package and the second objective was to provide a decision making tool for supply chain management. They analysed the effects of inventory control policies, lead times, customers' demand intensity and variability, on different supply chain performance measures. Pierreval et al. (2007) performed a dynamic analysis of the behaviour of an automotive industry supply chain through simulation, which was based on Forrester's system dynamic paradigms.

In recent years, some studies were conducted by using the simulation approach. For example: application of Monte Carlo simulation for quantifying supply chain disruption risk (Schmitt and Singh, 2009), reducing risk from both supply disruptions and demand uncertainty in a multi-echelon supply chain(Schmitt and Singh, 2012), development of second version of supply chain operations reference model (SCOR), which is a simulation based tool for dynamic supply chain analysis (Persson, 2011) and the model was also tested in a case company: Alfa Laval at Ronneby, Sweden – a manufacturer of heat exchangers and analysis the effect of supply disruptions in a single-product inventory system which involved a supplier, a retailer, and differentiated customers by considering partial backordering when a stockout occurs (Li and Chen, 2010). Some other recent simulation studies can be found in Che (2012), Carvalho et al. (2012), Mobini et al. (2013), Betts (2014), and Ramanathan (2014).

From the literature, it can be that simulation is a proven and strong tool for analysis of complex and dynamic systems. For this reason, the simulation technique is utilized for managing disruptions in production-inventory and supply chain systems.

2.3.4 Search Algorithm

Search algorithms, such as: genetic algorithm (GA), simulated annealing (SA), ant colony algorithm (ACA), and particle swarm optimisation (PSO) are also applied to solve the models developed in production-inventory and supply chain model. These are standard solution techniques to solve the model. This is worth to discuss here, because standard search algorithm is used to develop a heuristic for solving the model for managing disruptions, in single-stage and two-stage production-inventory system, on a real-time basis. Among the entire search algorithm, genetic algorithm was widely used. A few recent papers which used genetic algorithm can be found in Guchhait et al. (2013), Pasandideh et al. (2011), Gupta et al. (2009), Jawahar and Balaji (2009), and Costa et al. (2010).

Other search algorithm, such as SA, ACA and PSO were also applied to solve the model developed in production-inventory and supply chain. Simulated annealing was used in Kuik and Salomon (1990), Tang (2004), and Diabat (2014). Ant colony algorithm was used by Silva et al. (2009), Nia et al. (2013), and Liao et al. (2011). Particle swarm optimisation was used by Yang and Lin (2010), and Deleris and Erhun (2005).

The summary of the literature review for different solution approaches is presented in Table 2.2. It is observed that, most studies focused on using search algorithm to solve the models. A good number of works also have been found which developed heuristic and simulation approach to solve the complex models. In case of dynamic and complex problem, it is worth to develop a combined heuristic and simulation approach to make the model easy to implement and more closer to a real-world process.

Table 2.2: Summary of review for different solution approaches

Solution approach	Description	Author (year)	Remarks
Traditional optimisation	Includes linear programming, geometric programming, quadratic programming, branch and bound method.	Cheng (1989); Bag et al. (2009); Paul et al. (2013); Kabak and Ülengin (2011); Xia et al. (2004); Baghalian et al. (2013).	Not suitable to solve dynamic and complex problem.
Heuristic	Heuristics are a subset of strategies to find the near optimal solutions.	Moinzadeh and Aggarwal (1997); Hishamuddin et al. (2012); Hishamuddin et al. (2013); Hishamuddin et al. (2014); Abboud (2001); Arreola-Risa and DeCroix (1998); Hill and Galbreth (2008); Perron et al. (2010); Liu and Chen (2011); Liao et al., (2009); Al-Rifai and Rossetti (2007).	Simple to understand, easy to apply, computationally inexpensive. Not able to obtain optimal solution.
Simulation	The operation of a process or system over time to make it closer to a real-world process.	Wu and Olson (2008);Longo and Mirabelli (2008); Pierreval et al. (2007); Schmitt and Singh (2009); Schmitt and Singh (2012); Persson (2011); Li and Chen (2010); Che (2012); Carvalho et al. (2012); Mobini et al. (2013); Betts (2014); Ramanathan (2014).	Make the model closer to a real-world process when real-world data is not available.
Search algorithm	Existing search algorithm, such as; genetic algorithm (GA), simulated annealing (SA), ant colony algorithm (ACA), and particle swarm optimisation (PSO).	**GA:** Guchhait et al. (2013); Pasandideh et al. (2011); Gupta et al. (2009); Jawahar and Balaji (2009); Costa et al. (2010). **SA:** Kuik and Salomon (1990); Tang (2004); Diabat (2014). **ACA:** Silva et al. (2009); Nia et al. (2013); Liao et al. (2011). **PSO:** Deleris and Erhun (2005); Yang and Lin (2010).	Iterative method and higher computational time.

2.4 Application in Real-Life Case

Application of the model in a real-life case is an important way to judge the model. Several recent studies have been found in the literature, which applied the model in a real-life case. A few examples include: managing disruptions within the supply chain of a large US retailer (Oke and Gopalakrishnan, 2009), simulation study for risk assessment and management of supply chain for an industrial case (Tuncel and Alpan, 2010), development of a set of propositions about how companies manage supply risks in financial crises by using in-depth case studies conducted among eight European enterprises (Blome and Schoenherr, 2011), application in an automotive spare parts manufacturer in Iran to manage supply chain disruption(Zegordi and Davarzani, 2012), design of robust supply chain against disruption and application in a real-life case study from the agri-food industry (Baghalian et al., 2013), developing sustainable supply chain for UK construction industry (Dadhich et al., 2015), and application of ethanol supply disruption management model and methodology to Brazilian refineries system (de Barros and Szklo, 2015).

In the recent years, the researchers have started to implement their models in a real-life case. It is very important to judge the usefulness, benefits and applicability of the model.

2.5 Chapter Summary

In this chapter, a literature review has been presented in the field of managing disturbance and risk in production-inventory and supply chain systems. In this section, the review is summarized and future research directions are provided, based on the research gaps in the literature.

2.5.1 Summary of Literature Review

In the literature, most of the previous studies considered only one risk factor such as uncertainty, and a very little has been done for managing other disturbance and risk factors such as imperfect production process, and disruption in production, supply and demand, and their combination. No single study considered all disturbance and risk factors. In addition, the study on multiple disturbances on a real-time basis is very rare. However in a supply chain environment, any type of disturbance can happen, one after another, at any point in time. Furthermore, no study covered multiple disturbances, one after another as a series, whether dependent or independent, in a supply chain environment on a real-time basis. By implementing the developed approach in real-world case problems, one can judge the performance of the approach. However a very few models were found to be implemented for real-life disturbance management. Some papers developed a heuristic to solve the model, but a very little has been done to develop the combined heuristic and simulation approach to operate the model as a real-world process. So it can be concluded that more research is needed to develop a real-time disturbance and risk management system that covers all the risk factors.

2.5.2 Research Direction

From the literature review summarized above, it can be concluded that more research is needed to develop a disturbance management model that covers imperefect production process, and disruptions in production, supply and demand. Some of the future research directions include:

i. Consideration of multiple disturbance and risk factors in a single study.
ii. Development of a real-time disturbance management model for production-inventory system.
iii. Consideration of different disturbance and risk factors, such as disruption in production, demand, and supply and imperfect production process.
iv. Extension of the disturbance management model for supply chain systems.
v. Consideration of multiple disturbances, one after another as a series, either dependent or independent, on a real-time basis.
vi. Development of both the heuristic and simulation approach: (a) to make the model simple, (b) to improve operational efficiency and performance of the model and (c) to operate the model as a real-life process.
vii. Applying the developed approach in a real-life case to judge and validate the model.

In the literature, a reasonable number of works have been found in the area of supply chain disturbance and risk management. Still there is a lack of quantitative disturbance and risk management models to help the decision maker to make prompt and accurate decision. This thesis will take a step to fulfil the research gaps found in the literature by developing quantitative disturbance and risk management models in production-inventory and supply chain systems.

In the next chapter, this thesis starts to fulfil the research gaps by developing a disruption recovery model for a single-stage imperfect production-inventory system. After then, it will extend the research on developing some other quantitative models for managing different types of disturbances in production-inventory and supply chain systems.

Chapter 3 Single-Stage Imperfect Production-Inventory System

This chapter presents a new disruption recovery model for an imperfect single-stage production-inventory system. For it, the system may unexpectedly face either a single disruption or a mix of multiple dependent and/or independent disruptions. This chapter begins with a description of the problem, followed by the formulation of the mathematical model for rescheduling the production plan, after the occurrence of a single disruption, which maximises the total profit during the recovery time window. The mathematical model, developed for a single disruption, is solved by using both a pattern search and a genetic algorithm, and the results are compared using a good number of randomly generated disruption test problems. This chapter also presents an extension of single disruption case for managing multiple disruptions that occur one after another as a series, on a real-time basis and a new dynamic solution approach is developed for managing multiple disruptions. Finally, this chapter presents some numerical examples and a set of sensitivity analysis to explain the usefulness and benefits of the developed model.

3.1 Introduction

Batch production systems are very common and popular in advanced manufacturing environments. In these production systems, the products are produced and delivered in batches, because that helps to reduce costs and increase profitability. However, in real-life cases, the production process may not be perfect and they may face production interruptions, such as: machine breakdown, raw material shortage or any other type of system failure. In an imperfect production system, it is expected that a certain percentage of products will be defective. As a result, process reliability is considered as an important factor in real production environments. Hence, the consideration of production interruption with process reliability, in any production-inventory system, will make the research problem closer to those in the real-world.

In this chapter, a mathematical model is developed to deal with disruptions on a real-time basis. That means, the current plan is revised after experiencing any real disruption. Such a disruption is not known in advance and it is impossible to be predicted. It is assumed that both the disruption and the duration of disruption will follow a stochastic process. From the literature review, it is clear that the previous disruption management studies, in production-inventory systems, were mainly focused on managing a single disruption. In real-life, production processes can face multiple disruptions, one after another, as a series. These disruptions may or may not affect the recovery plans of the previous disruptions. If a new disruption occurs during the recovery time window of a previous disruption, a new recovery plan that considers the effect of both disruptions must be derived. So it could be a continuous process that must be dealt with on a real-time basis. This real-time disruption management, in an imperfect production-inventory system, is considered in this study. The most closely related research, to this study, is the one published by Hishamuddin *et al.* (2012), where they assumed that the production system produces 100% accurate items. Moreover, they developed the recovery model for managing only a single occurrence of disruption. The problem presented in this chapter is much more complex because it has developed a mathematical model and solution approach that deals with both single and multiple (mix of dependent and/or independent) disruptions on a real-time basis. The model also considers process reliability because imperfect production processes are very common in real-life. The objective of this model is to maximise the total profit as the revenue varies with production process reliability. The total profit function includes the revenues from the sale of non-defective items and the relevant costs.

Here, first, a constrained non-linear mathematical model is developed for dealing with a single occurrence of disruption and solved the mathematical model to obtain a recovery plan by using both a pattern search and a genetic algorithm. A good number of disruption test problems have generated by using a uniform random distribution and the results, obtained from both a pattern search and a genetic algorithm, are compared. The model also considered a series of disruptions that occur at different points in time. If a new disruption occurs during the recovery time of another disruption, a revised recovery plan incorporating the effect of both disruptions must be derived, which makes the algorithm more complex. In this case, a new revised plan must be derived after the occurrence that considers the effect of both disruptions. So it is a continuous process that must be dealt with on a real-time basis. In this chapter, a new mathematical model and dynamic solution approach are developed to deal with a mix of multiple dependent and/or independent disruptions, as a series, on a real-time basis. To the best knowledge, this is the first quantitative approach, which extends the model of Hishamuddin et al. (2012) and develops a new mathematical model and solution approach for managing both single and multiple disruptions in an imperfect production-inventory system. The most closely related research work, to this chapter, is the one published by Hishamuddin et al. (2012), where a single-stage production system with single occurrence of production disruption is considered. The objective of their study was to minimise the total cost of recovery period in a single occurrence of disruption. The problem presented in this chapter is much more complex because two practical issues (production disruption and process reliability) and cases of both a single and multiple occurrences of disruptions are considered. The results for both the single and multiple disruptions are discussed, and some numerical examples are presented to demonstrate the usefulness of the proposed approach. With the proposed approach, manufacturers can determine a recovery plan in real-time, whenever their production system experiences either a single independent disruption or a series of a mix of dependent and independent disruptions.

The main contributions of this chapter can be summarized as follows.

i. Modelling a production-inventory system under production disruption. Here, the disruption is not known a priori and it is impossible to predict, so that requires that the production plan be revised after experiencing the disruption on a real-time basis.

ii. Considering process reliability while developing the disruption management model, because imperfect production processes are common in real-life.
iii. Developing a new solution approach that deals with a mix of multiple dependent and/or independent disruptions, as a series, on a real-time basis.
iv. Conducting experimental studies to analyse these approaches and perform sensitivity analysis using different disruption scenarios.

3.2 Problem Description

In this section, the problem of disruption in a single-stage imperfect production-inventory system is described. The system may face either a single or multiple disruptions during the production up-time. To manage the system efficiently, it is necessary to generate a recovery plan after the occurrence of each disruption. If the production system faces a new disruption after the recovery time window of the previously occurred disruption, then it is called a single disruption that is relatively simple to solve. However, if the system faces a new disruption within the recovery time window of a previous disruption, then the case becomes more complex, as the effect of both the previous and present disruptions must be taken into consideration to develop the revised plan.

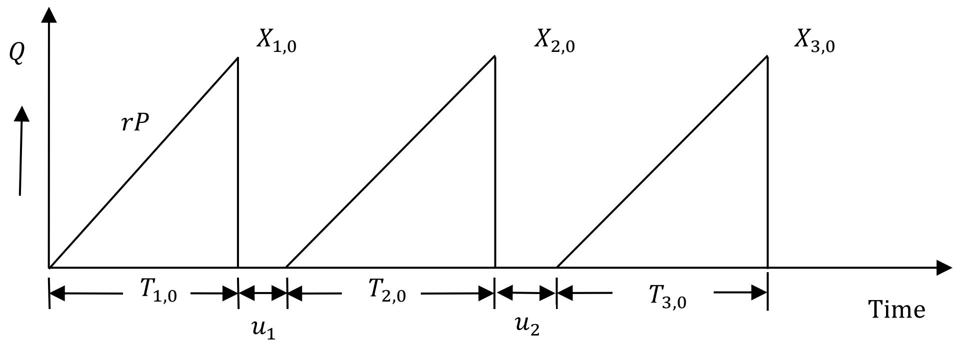

Figure 3.1: An ideal single-stage production system

Figure 3.1 presents an ideal single-stage batch production-inventory system that produces one type of product. The product is produced in batches and after completing each batch, there is a production down time which is the summation of set-up and idle time. After that, production starts again for a new batch. In an ideal production-inventory system, the production quantity for each cycle i is $X_{i,0}$ ($i = 1,2 \dots M$), and they are all equal to Q. As the system may produce some defective items, processing reliability is considered to calculate the effective production rate (rP). The **recovery plan** is a new schedule that includes the revised production quantities in each cycle, and ensuring the maximisation of the total profit in the recovery time window. The number of future cycles allocated to return to the original production schedule from the disrupted cycle, is known as the recovery time window**,** and is decided by the management of the organisation. As the model assumes that the production rate is higher than the demand rate, there is an idle timeslot between every two consecutive batches. If the production system is disrupted for a time period, known as disruption duration, the utilization of the idle timeslots, in future production cycles, may help to recover from the disruption. However, it may involve costly backorder and/or lost sales due to both a long disruption duration and delayed delivery.

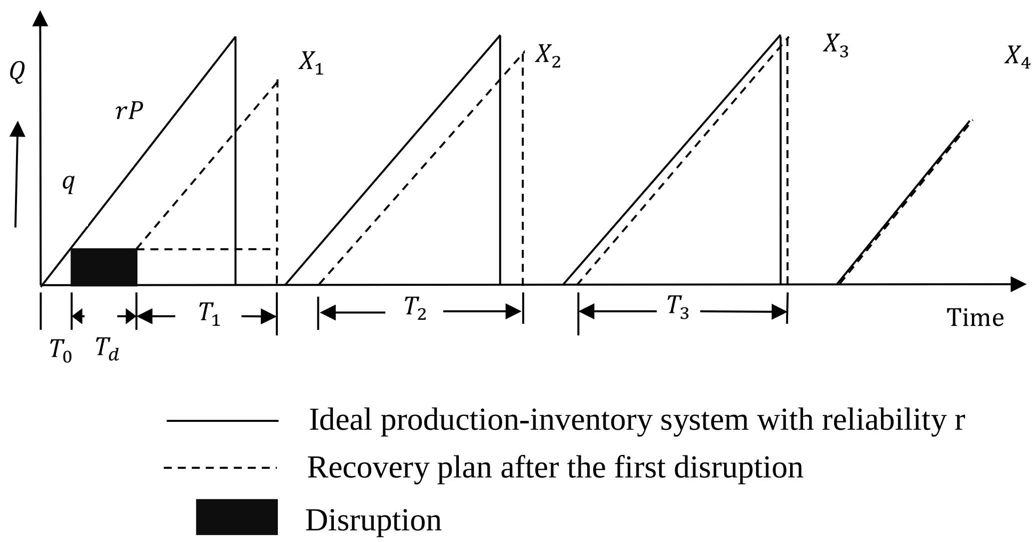

Figure 3.2: Disruption recovery plan after a single disruption

Figure 3.2 presents an example of a recovery plan after the occurrence of a single production disruption. The production system becomes inoperable for a certain period due to a disruption and after then it operates normally. The recovery plan starts just after the ending point of the disruption and continues during the recovery time window. For a single disruption case, the production quantities are revised during the recovery time window by utilizing the idle timeslots, which considers both the back orders and lost sales options. After the disruption, a recovery plan is proposed to revise the production quantities X_i; $i = 1,2, \ldots, M$, during the recovery time window, which is shown as a dashed line and the production plan is updated according the revised plan. In this case, the ideal production schedule is taken as a base plan. After the recovery time window, the production process returns to its normal schedule.

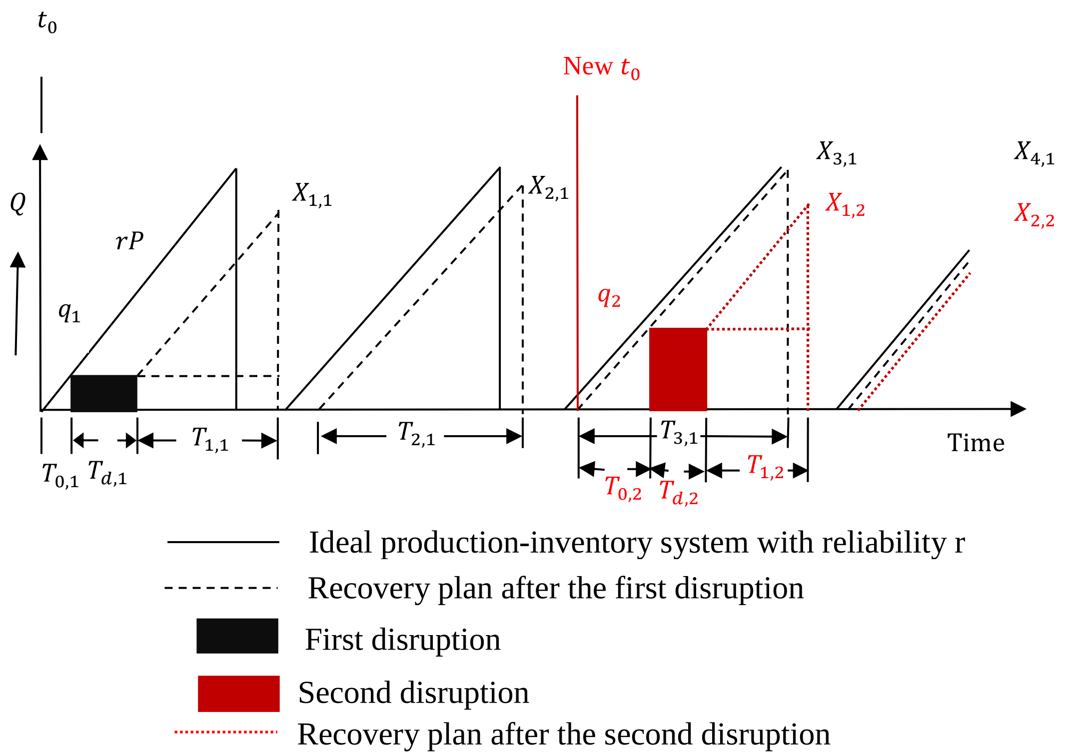

Figure 3.3: Disruption recovery plan for a series of disruptions

Figure 3.3 presents the recovery mechanism for multiple disruptions, one after another, as a series. In this case, the production quantities are revised after the occurrence of each disruption. Figure 3.3 presents the recovery plan for two dependent disruptions as a sample case. The effect of the previous disruption is taken into consideration while revising the production plan for the later disruption. After the first disruption, a recovery plan is proposed to revise the production quantities $X_{i,1}$; $i = 1,2, \ldots, M$, during the recovery time window, which is shown as a dashed line and the production plan is updated according to the revised plan. Again, after the second disruption, which is during the recovery time window of the first disruption, the production quantities in the each cycle $X_{i,2}$; $i = 1,2, \ldots, M$ are again revised while considering the effect of both disruptions, which is shown as a dotted line in Figure 3.3. The updated production plan, after the first disruption, is taken as a base environment while developing the recovery plan after the second disruption. So, the recovery plan after the second disruption is obtained by solving the problem under the updated production environment, such as updated disruption scenario, constraints and objective function. If there is any further disruption, the recovery mechanism will continue in the same way. This is a continuous process that must be dealt with on a real-time basis. The necessary changes can be dealt with by updating the decision variables, objective function and constraints of the model developed for the previous single disruption case.

3.2.1 Notations used in the Study

In this chapter, the following notations are used to formulate the mathematical model after the occurrence of a single disruption.

S_t Set-up time for a cycle

δ_i Idle time for cycle i

D Demand per year (units per year)

H Holding cost per unit per year ($ per unit per year)

r Reliability of the production process – which is known from the historical data of the production system

Q Economic lot size per ideal production cycle with process reliability r

A Set up cost per cycle ($ per set-up)

P Production rate (units per year) in 100% reliable system

u_i Production down time for cycle i (set-up time + idle time) = $S_t + \delta_i = \frac{X_{i,0}}{D} - \frac{X_{i,0}}{rP}$

M Number of cycles in the recovery time window – given from management

T_d Disruption duration

q Pre-disruption production quantity

T_0 Production time for q = $\frac{q}{rP}$

$X_{i,0}$ Production quantity for normal cycle i

X_i Production quantity for cycle i of the recovery time window– which is the decision variable; $i = 1,2, \dots\dots, M$

$T_{i,0}$ Production up time for cycle i of a normal cycle = $\frac{X_{i,0}}{rP}$

T_i Production up time for cycle i in the recovery time window = $\frac{X_i}{rP}$

B Unit back order cost per unit time ($ per unit per unit time)

L Unit lost sales cost ($ per unit)

C_P Per unit production cost ($ per unit)

C_R Rejection cost per unit ($ per unit)

C_I Inspection cost as a percentage of production cost

m_1 Mark-up of selling price ($m_1 C_P$) of each acceptable item, it must be greater than 1

For the multiple disruptions case, the model is generalized by formulating the mathematical model after the occurrence of the n^{th} disruption. The following additional notations, with subscript, are used for a series of disruptions.

$\delta_{i,n}$ Idle time for cycle i before the n^{th} disruption

l New disrupted cycle number from the previous disruption

$u_{i,n}$ Production down time for cycle i before the n^{th} disruption (set-up time + idle time)

$$= S_t + \delta_{i,n} = \frac{X_{l+i-1,n-1}}{D} - \frac{X_{l+i,n-1}}{rP}$$

$T_{d,n}$ Disruption duration of the n^{th} disruption

q_n Pre-disruption production quantity of the n^{th} disruption

$T_{0,n}$ Production time for $q_n = \frac{q_n}{rP}$

$X_{i,n}$ Production quantity for cycle i of the recovery time window after the n^{th} disruption– which is the decision variable; $i = 1,2, \ldots\ldots, M$

$T_{i,n}$ Production up time for cycle i in the recovery time window after the n^{th} disruption $= \frac{X_{i,n}}{rP}$

3.2.2 Assumptions of the Study

In this study a number of assumptions have been made, which are as follows.

i. The production rate is greater than its demand rate.
ii. A type of single item is produced in the production system.
iii. All products are inspected and defective products are rejected.
iv. The recovery cycle will start immediately after the disruption occurs.
v. Total cost of interest and depreciation per production cycle $F(A, r)$ is inversely related to set-up cost (A) and is directly related to the process reliability (r), according to the following general power function (Cheng, 1989):

$$F(A, r) = aA^{-b}r^{c}$$

where a, b and c are positive constants chosen to provide the best fit of the estimated cost function (Cheng, 1989).

To fulfill the demand on time, it is commonly assumed that the production rate is higher than its demand rate. However, for a higher demand rate, the model can easily be revised with an option for outsourcing. It was observed, that lot-for-lot production systems are common in many real-life production-inventory systems, which was discussed in Sarker and Khan (2001). From considering the customer satisfaction point of view, no defective item will be delivered to the customers, which is important for many real businesses. To make the disruption recovery meaningful in practice, the recovery plan will be generated after each disruption is experienced by the system. In other words, the recovery plan is generated on a real-time basis. Finally, as an imperfect production process is considered, there is a significant amount of interest and depreciation cost and these have been considered in our mathematical model. The equation of interest and depreciation cost is taken from the paper of Cheng (1989).

3.3 Model Formulation

In this section, a mathematical model is developed after the occurrence of a single production disruption. In the following subsections, the equations for economic lot size (Q) are derived under an ideal condition, and the relevant costs and revenue after the occurrence of disruption. Finally, the disruption recovery problem is formulated as a non-linear constrained optimisation problem, that maximises the total profit during the recovery time window, subject to capacity, demand, delivery and transportation constraints. The decision variables are the production quantities in each cycle during the recovery time window.

3.3.1 Derivation of Q

For a single item batch production system, with the lot-for-lot condition under ideal conditions (as shown in Figure 3.1), as considered by Sarker and Khan (1999), along with process reliability, then the economic lot size can be formulated as $Q = \sqrt{\frac{2ArP}{H}}$.

Here,

Annual set-up cost $= \frac{D}{Q}A$

Annual holding cost $= \frac{Q}{2} H \frac{D}{rP}$

Total set-up and holding cost $= \frac{D}{Q} A + \frac{Q}{2} H \frac{D}{rP}$

To minimise total cost, $\frac{d}{dQ}\left(\frac{D}{Q} A + \frac{Q}{2} H \frac{D}{rP}\right) = 0$

After simplifying, economic lot size, $Q = \sqrt{\frac{2ArP}{H}}$ (3.1)

3.3.2 Cost Formulation

In this section, equations for the different costs are derived that are involved in the recovery time window. These are the holding, set-up, back order, lost sales, production, rejection, inspection and interest and depreciation costs. Holding cost is determined as unit holding cost multiplied by total inventory during the recovery time window, which is equivalent to the area under the curve of Figure 3.2. Set-up cost is calculated as cost per set-up multiplied by the number of set-ups in the recovery time window. Back order is the portion of an order that cannot be delivered at the scheduled time, but that will be delivered at a later date when available, and the back order cost is determined as the unit back order cost multiplied by the number of back order units and it’s time delay (Hishamuddin et al., 2012). When there is a demand, but the item is out of stock and the customer will not wait for the stock to be replenished, lost sales cost exists. Lost sales cost is determined as unit lost sales cost multiplied by lost sales units (Hishamuddin et al., 2012). Unit production cost multiplied by the total quantity produced during the recovery time window is the total production cost. As the process reliability is r, the rejection rate is $(1 - r)$. The rejection cost is determined as the rejection cost multiplied by the total rejected quantities (Paul et al., 2014). Inspection cost is considered as a certain percentage of the production cost (Paul et al., 2014). The cost of interest and depreciation equation is considered as a general power function, which is taken from the paper of Cheng (1989).

Holding cost

$$= H\left[\frac{1}{2}qT_0 + q(T_d + S_t + T_1) + \frac{1}{2}X_1\,T_1 + \frac{1}{2}X_2\,T_2 + \cdots + \frac{1}{2}X_M\,T_M\right]$$

$$= \frac{1}{2}H\left[q\,\frac{q}{rP} + 2q\left(T_d + S_t + \frac{X_1}{rP}\right) + X_1\,\frac{X_1}{rP} + X_2\,\frac{X_2}{rP} + \cdots + X_M\,\frac{X_M}{rP}\right]$$

$$= \frac{1}{2}H\left[\frac{q^2}{rP} + 2q\left(T_d + S_t + \frac{X_1}{rP}\right) + X_1\,\frac{X_1}{rP} + X_2\,\frac{X_2}{rP} + \cdots + X_M\,\frac{X_M}{rP}\right]$$

$$= \frac{1}{2}H\left[\frac{q^2}{rP} + 2q(T_d + S_t) + \frac{2qX_1}{rP} + \sum_{i=1}^{M}\frac{(X_i)^2}{rP}\right] \tag{3.2}$$

Set-up cost

$$= AM \tag{3.3}$$

Production cost

$$= C_P P\left(\sum_{i=1}^{M} T_i + T_0\right)$$

$$= \frac{C_P}{r}\left(\sum_{i=1}^{M} X_i + q\right) \tag{3.4}$$

Rejection cost

$$= C_R(1-r)P\left(\sum_{i=1}^{M} T_i + T_0\right)$$

$$= C_R\left(\frac{1}{r} - 1\right)\left(\sum_{i=1}^{M} X_i + q\right) \tag{3.5}$$

Inspection cost

$$= \frac{C_I C_P}{r}\left(\sum_{i=1}^{M} X_i + q\right) \tag{3.6}$$

Cost of interest and depreciation

$$= Ma\ (A)^{-b}(r)^{c} \tag{3.7}$$

Back-order cost

$$= B\left[(X_1+q).delay_1+\sum_{i=2}^{M}X_i.delay_i\right]$$

$$= B\left[(X_1+q)\left[T_d+\frac{q}{rP}+\frac{X_1}{rP}-\frac{X_{1,0}}{rP}\right]\right.$$
$$\left.+\sum_{i=2}^{M}X_i.\left[T_d+(i-1)S_t+\frac{q}{rP}+\sum_{j=1}^{i}\frac{X_j}{rP}-\sum_{j=1}^{i}\frac{X_{j,0}}{rP}-\sum_{j=1}^{i-1}(u_j)\right]\right]$$

$$= B\left[(X_1+q)\left[T_d+\frac{q}{rP}+\frac{X_1}{rP}-\frac{X_{1,0}}{rP}\right]\right.$$
$$\left.+\sum_{i=2}^{M}X_i.\left[T_d+(i-1)S_t+\frac{q}{rP}+\sum_{j=1}^{i}\frac{X_j}{rP}-\sum_{j=1}^{i}\frac{X_{j,0}}{rP}-\sum_{j=1}^{i-1}\left(\frac{X_{j,0}}{D}-\frac{X_{j,0}}{rP}\right)\right]\right] \tag{3.8}$$

Lost sales cost

$$= L\sum_{i=1}^{M}X_{i,0}-LrP(T_0+T_1+T_2+\cdots+T_M)$$

$$= L\sum_{i=1}^{M}X_{i,0}-LrP\left(\frac{q}{rP}+\frac{X_1}{rP}+\frac{X_2}{rP}+\cdots+\frac{X_M}{rP}\right)$$

$$= L\left(\sum_{i=1}^{M} X_{i,0} - \sum_{i=1}^{M} X_i - q\right) \tag{3.9}$$

3.3.3 Revenues

The selling price of the acceptable items, during the recovery time window, is determined as unit selling price multiplied by the demand in the recovery time window.

Revenues

$$= m_1 C_P\, D\left[\sum_{i=1}^{M} T_i + T_0 + MS_t\right]$$

$$= m_1 C_P\, D\left[\sum_{i=1}^{M} \frac{X_i}{rP} + \frac{q}{rP} + MS_t\right] \tag{3.10}$$

3.3.4 Final Mathematical Model

The total profit during the recovery time window, which is the objective function, is determined after a particular disruption as follows:

Total profit (TP) = Total revenues – Total costs

After incorporating all the equations from (3.2) to (3.10), the objective function is obtained as follows:

$$\boldsymbol{MaxTP}(X_{i,n}) = m_1 C_P\, D\left[\sum_{i=1}^{M}\frac{X_i}{rP}+\frac{q}{rP}+MS_t\right]$$
$$-\frac{1}{2}H\left[\frac{q^2}{rP}+2q(T_d+S_t)+\frac{2qX_1}{rP}+\sum_{i=1}^{M}\frac{(X_i)^2}{rP}\right]$$
$$-AM-\left[\frac{C_P}{r}+C_R\left(\frac{1}{r}-1\right)+\frac{C_I C_P}{r}\right]\left(\sum_{i=1}^{M}X_i+q\right)$$
$$-Ma\,(A_1)^{-b}(r)^c$$
$$-B\left[(X_1+q)\left[T_d+\frac{q}{rP}+\frac{X_1}{rP}-\frac{X_{1,0}}{rP}\right]\right.$$
$$+\sum_{i=2}^{M}X_i.\left[T_d+(i-1)S_t+\frac{q}{rP}+\sum_{j=1}^{i}\frac{X_j}{rP}\right.$$
$$\left.\left.-\sum_{j=1}^{i}\frac{X_{j,0}}{rP}-\sum_{j=1}^{i-1}\left(\frac{X_{j,0}}{D}-\frac{X_{j,0}}{rP}\right)\right]\right]$$
$$-L\left(\sum_{i=1}^{M}X_{i,0}-\sum_{i=1}^{M}X_i-q\right)$$
(3.11)

Subject to the following demand, capacity, delivery and transportation constraints:

$$X_{i,0}=Q \tag{3.12}$$

$$X_1+q\le X_{1,0} \tag{3.13}$$

$$X_i\le X_{i,0}\;;\; i\;=\;2,3,\ldots.,M \tag{3.14}$$

$$rP\ge D \tag{3.15}$$

$$r\le 1 \tag{3.16}$$

$$\sum_{i=1}^{M} X_i + q \leq rP\left(\sum_{i=1}^{M} \frac{X_{i,0}}{D} - MS_t - T_d\right) \tag{3.17}$$

$$\sum_{i=1}^{M} X_i + q \geq \left(\frac{\sum_{i=1}^{M} X_i + q}{rP} + MS_t\right) D - \left(\sum_{i=1}^{M} X_{i,0} - \sum_{i=1}^{M} X_i - q\right) \tag{3.18}$$

$$\frac{X_1 + q}{D} - \frac{X_2}{rP} - S_t \geq 0 \tag{3.19}$$

$$\frac{X_i}{D} - \frac{X_{i+1}}{rP} - S_t \geq 0;\ i = 2, 3, \ldots, M \tag{3.20}$$

$$T_d + \frac{q}{rP} + \frac{X_1}{rP} - \frac{X_{1,0}}{rP} \geq 0 \tag{3.21}$$

$$T_d + (i-1)S_t + \frac{q}{rP} + \sum_{j=1}^{i} \frac{X_j}{rP} - \sum_{j=1}^{i} \frac{X_{j,0}}{rP} - \sum_{j=1}^{i-1}\left(\frac{X_{j,0}}{D} - \frac{X_{j+1,0}}{rP}\right) \geq 0;\quad i = 2, 3, \ldots, M \tag{3.22}$$

$$X_i \geq 0;\ i = 1, 2, \ldots, M \tag{3.23}$$

Equation (3.12) is the constraint for the cycle production quantity of the ideal system. Equations (3.13) and (3.14) ensure that the production quantity in each cycle of the recovery time window is less than or equal to the ideal production quantity due to delivery and transportation requirements. Equation (3.15) ensures that the production rate of the item is greater than its demand rate. The process reliability constraint is confirmed by equation (3.16). Equation (3.17) represents the production capacity constraint during the recovery time window. Demand during the recovery time window is met by equation (3.18). Non-negative idle time is ensured by (3.19) and (3.20). Equations (3.21) and (3.22) ensure non-negative delay time. The non-negativity constraint for all the decision variables is ensured by (3.23).

Similarly, for a series of disruptions, the formulation of the mathematical model can be updated after the n^{th} disruption by using the notations for multiple disruptions. The updated mathematical model for a series of disruptions is presented as follows**.**

The objective function, after the n^{th} disruption, is determined as follows:

$$\begin{aligned}
\boldsymbol{MaxTP}(X_{i,n}) = {} & m_1 C_P\, D\left[\sum_{i=1}^{M}\frac{X_{i,n}}{rP}+\frac{q_n}{rP}+MS_t\right] \\
& -\frac{1}{2}H\left[\frac{(q_n)^2}{rP}+2q_n\left(T_{d,n}+S_t\right)+\frac{2q_nX_{1,n}}{rP}\right. \\
& \left.+\sum_{i=1}^{M}\frac{(X_{i,n})^2}{rP}\right]-AM \\
& -\left[\frac{C_P}{r}+C_R\left(\frac{1}{r}-1\right)+\frac{C_IC_P}{r}\right]\left(\sum_{i=1}^{M}X_{i,n}+q_n\right) \\
& -Ma\,(A_1)^{-b}(r)^c \\
& -B\left[(X_{1,n}+q_n)\left[T_{d,n}+\frac{q_n}{rP}+\frac{X_{1,n}}{rP}-\frac{X_{l,n-1}}{rP}\right]\right. \\
& +\sum_{i=2}^{M}X_{i,n}\cdot\left[T_{d,n}+(i-1)S_t+\frac{q_n}{rP}+\sum_{j=1}^{i}\frac{X_{j,n}}{rP}\right. \\
& -\sum_{j=1}^{i}\frac{X_{l+j-1,n-1}}{rP} \\
& \left.\left.-\sum_{j=1}^{i-1}\left(\frac{X_{l+j-1,n-1}}{D}-\frac{X_{l+j,n-1}}{rP}\right)\right]\right] \\
& -L\left(\sum_{i=1}^{M}X_{l+i-1,n-1}-\sum_{i=1}^{M}\frac{X_{i,n}}{rP}-q_n\right)
\end{aligned} \tag{3.24}$$

Subject to the following demand, capacity, delivery and transportation constraints:

$$X_{i,0}=Q \tag{3.25}$$

$$X_{1,n}+q_n\le X_{l,n-1} \tag{3.26}$$

$$X_{i,n} \leq X_{l+i-1,n-1}\;;\; i \;=\; 2,3,\ldots.,M \tag{3.27}$$

$$rP \geq D \tag{3.28}$$

$$r \leq 1 \tag{3.29}$$

$$\sum_{i=1}^{M} X_{i,n} + q_n \leq rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{D} - MS_t - T_{d,n}\right) \tag{3.30}$$

$$\begin{aligned}\sum_{i=1}^{M} X_{i,n} + q_n \geq {} & \left(\frac{\sum_{i=1}^{M} X_{i,n} + q_n}{rP} + MS_t\right) D \\ & - \left(\sum_{i=1}^{M} X_{l+i-1,n-1} - \sum_{i=1}^{M} X_{i,n} - q_n\right)\end{aligned} \tag{3.31}$$

$$\frac{X_{1,n} + q_n}{D} - \frac{X_{2,n}}{rP} - S_t \geq 0 \tag{3.32}$$

$$\frac{X_{i,n}}{D} - \frac{X_{i+1,n}}{rP} - S_t \geq 0;\; i \;=\; 2,3,\ldots.,M \tag{3.33}$$

$$T_{d,n} + \frac{q_n}{rP} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP} \geq 0 \tag{3.34}$$

$$\begin{aligned}T_{d,n} + (i-1)S_t + \frac{q_n}{rP} + \sum_{j=1}^{i} \frac{X_{j,n}}{rP} - \sum_{j=1}^{i} \frac{X_{l+j-1,n-1}}{rP} \\ - \sum_{j=1}^{i-1}\left(\frac{X_{l+j-1,n-1}}{D} - \frac{X_{l+j,n-1}}{rP}\right) \geq 0;\; i \;=\; 2,3,\ldots.,M\end{aligned} \tag{3.35}$$

$$X_{i,n} \geq 0;\; i \;=\; 1,2,\ldots.,M \tag{3.36}$$

3.4 Solution Approach

In this section, firstly, the mathematical model is solved, for a single occurrence of disruption, by applying a standard search algorithm. For experimentation, the test disruption problems are generated by using a uniformly random distribution. Then a solution approach was developed for managing multiple disruptions, as a series of disruptions, on a real-time basis.

3.4.1 Solution Approach for a Single Disruption

The mathematical model, for a single disruption, can be solved by applying a standard search algorithm. As there is no standard test set available for the disruption problem considered in this study, to validate the results, two different search techniques: pattern search (PS) and genetic algorithm (GA) have been chosen. A number of test problems have been generated by using a uniformly random distribution and solved those problems using both of the two techniques. The genetic algorithm is a very popular technique to solve complex non-linear constrained optimisation problems. GAs are general purpose optimisation algorithms which apply the rules of natural genetics to explore a given search space (Homaifar, Qi, and Lai, 1994). Pattern search is also a well-accepted technique, and can be applied to solve nonlinear constrained optimisation problems (Lewis and Torczon, 2002). In this study, the pattern search technique is hybridized by the Latin hypercube search method. The results, obtained from both the GA and PS search techniques, are compared for a good number of randomly generated disruption test problems.

3.4.2 Solution Approach for a Series of Disruptions

In this section, a solution approach is proposed to generate the recovery plan for a series of disruptions on a real-time basis. When a disruption occurs, a recovery plan can be generated by solving the mathematical model for a series of disruptions, as presented in Section 3.3.4. The first disruption can be managed by using the single disruption case.

Then if a new disruption later occurs after the recovery time window of any previous disruption, it can be considered as another single disruption case that does not affect the revised plan of the previous disruption. But if a new disruption occurs within the recovery time window of any disruption, then it may affect the revised plan of the previous disruption, and the revised plan for that recovery time window must be considered as a set of additional restrictions. So after finalizing a recovery plan, if another disruption occurs within the recovery time window, then the recovery plan needs to be revised to consider the effect of both disruptions. This can be done by simply updating some of the parameters, to represent the changed scenario, in the same mathematical model. For example, the new disruption duration, the pre-disruption quantity, the cycle number from the previous disruption and the new limit on production in each cycle, must be updated to re-optimise the plan for the current disruption. The objective function and constraints are also updated for the changed situation. So a search technique can still be used, but it must be used again with the changed situation. Both the PS and GA search techniques have been used to develop the solution approaches for a series of disruptions on a real-time basis. In the proposed approach, the search technique must run every time after an occurrence of disruption, to re-optimise the recovery plan, as long as disruption takes place in the system. The main steps of the proposed solution approach for a series of disruptions, on a real-time basis, can be presented as follows.

Step 1**:** Input all information about the ideal system and determine Q by using the equation (3.1).

Step 2: Assign $X_{i,0} = Q$.

Step 3: Set n =1 for the first disruption.

Step 4: Input disruption scenario: disrupted cycle number (l), pre-disruption quantity (q_n) and disruption duration ($T_{d,n}$).

Step 5: Initialize the starting time of the recovery time window from the beginning of the disrupted cycle.

Step 6: Solve the mathematical model for a series of disruptions, as presented in Section 3.3.4, by using a search algorithm (PS and GA), under the updated disruption scenario.

Step 7: Update the value of $X_{i,n}$ as the revised lot size from ***Step 6*** and record the revised production plan.

Step 8: If there is any other disruption, go to ***Step 4*** and repeat ***Steps 4-7***.

Step 9: Stop.

The solution approaches were coded in MATLAB R2012a with the help of its optimisation toolbox, and were executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU. The parameters used for PS and GA, to design the solution approach, are presented below.

3.4.2.1 Parameters for PS

The following parameters are used in the proposed PS based solution approach.

Maximum number of iterations: 100* Number of variables

Polling order: Success

X tolerance: 1e-8

Function tolerance: 1e-8

Nonlinear constraint tolerance: 1e-8

Cache tolerance: 1e-8

Search method: Latin Hypercube

Maximum function evaluations: 1000000

Other parameters are set as default in the optimisation toolbox.

3.4.2.2 Parameters for GA

The following parameters are used in the proposed GA based solution approach.

Population Size: 50

Population type: Double vector

Crossover Fraction: 0.8

Maximum number of generations: 3,000

Function tolerance: 1e-8

Nonlinear constraint tolerance: 1e-8

Hybrid function: Pattern search

Other parameters are set as default in the optimisation toolbox

3.4.3 Range of Parameters

For experimentation purposes, the following data ranges are considered with a discrete uniform distribution for the disruption problem.

Pre-disruption quantity: $q = [0, Q]$

Disruption duration: $T_d = [0.000001, \frac{Q-q}{rP}]$

Lost sales cost: $L = [1, 100]$

Back order cost: $B = [1, 150]$

Set-up cost: $A = [5, 300]$

Holding cost: $H = [1, 5]$

3.5 Experimentations and Results Analysis

In this section, the results have been analysed for both a single and multiple disruptions on a real-time basis. In this chapter, a disruption scenario is defined as a combination of some pre-disruption situation and the timing of the duration of a disruption. In reality, these parameters follow a stochastic process, and in this chapter, uniform random variables are used for them. To test the proposed mathematical approach, seventy disruption test problems are generated by using a uniform probability distribution, by changing the parameters for the given intervals as presented in section 3.4.3, and have solved those problems using both PS and GA search techniques. The results, obtained from the two different search techniques, are also compared. As the comparison gave consistent results, the results of a few sample test problems are presented in this section.

3.5.1 Single Disruption

For the single disruption problem, the following data were used to analyse the results.

P=500000, D=450000, A=50, H=1.2, B=10, L=15, S_t=0.000057, C_P=40, C_I=0.01, C_R=8, a=1000, b=0.5, c=0.75, m_1=2.5, M=5, r = 0.95.

From the given data, $X_{i,0} = Q$ is calculated as:

$$X_{i,0} = Q = \sqrt{\frac{2ArP}{H}} \approx 6292$$

Although the study has experimented on 70 random disruption test problems, for illustrative purpose, three different sample instances were generated by arbitrarily changing the disruption data. The parameters for the three instances are shown in Table 3.1.

Table 3.1: Disruption data for three test instances

Test instance	Disruption duration (T_d)	Pre-disruption quantity (q)
1	0.0025	850
2	0.0060	1225
3	0.0090	675

Table 3.2: Results for the three test instances

Test instance	Search Technique	Production quantity in the recovery time window					Total Profit (Best, Mean, STD)
		X_1	X_2	X_3	X_4	X_5	
1	GA	544	6292	6292	6292	6292	1640541, 1640503,15.5
	PS	544	6292	6292	6292	6292	1640540, 1640502, 16.2
2	GA	438	5899	6180	6238	6292	1557556, 1557504, 14.8
	PS	446	5898	6087	6275	6272	1557539, 1557496, 13.6
3	GA	456	5486	5750	6033	6292	1462235, 1462213, 12.7
	PS	481	5598	5616	5896	6197	1462109, 1462091, 13.2

The test instances were solved using both the PS and GA search techniques. The results (Best, Mean and STD) obtained, out of 30 independent runs, for all three test instances, have been presented in Table 3.2. The results include the revised production quantities in each of five cycles of the recovery time window and the total profit. The production system returns to the original production schedule after the recovery time window, with $X_6 = 6292$, $X_7 = 6292$ and so on. From Table 3.2, it is clear that both techniques generated very similar results.

3.5.2 Comparison of Results

In order to judge the consistency of the solutions, the best results, out of the 30 independent runs, obtained from both the PS and GA search techniques have been compared. For this purpose, the results, obtained from both the PS and GA search techniques, have been compared for seventy random disruption test problems. The test problems are generated by using a uniform random distribution within the data range provide in Section 3.4.3. In this random experiment, the average percentage of deviation, between the results obtained from the two approaches, was only 0.003438%, which can be considered as negligible. Moreover, this deviation may merely be because of rounding of the values of the decision variables. The average percentage of deviation of the results was calculated by using equation (3.37). The comparison of results, between two search techniques for seventy random disruptions, is presented in Figure 3.4. It is observed that the results, obtained from the two search techniques, are very much consistent.

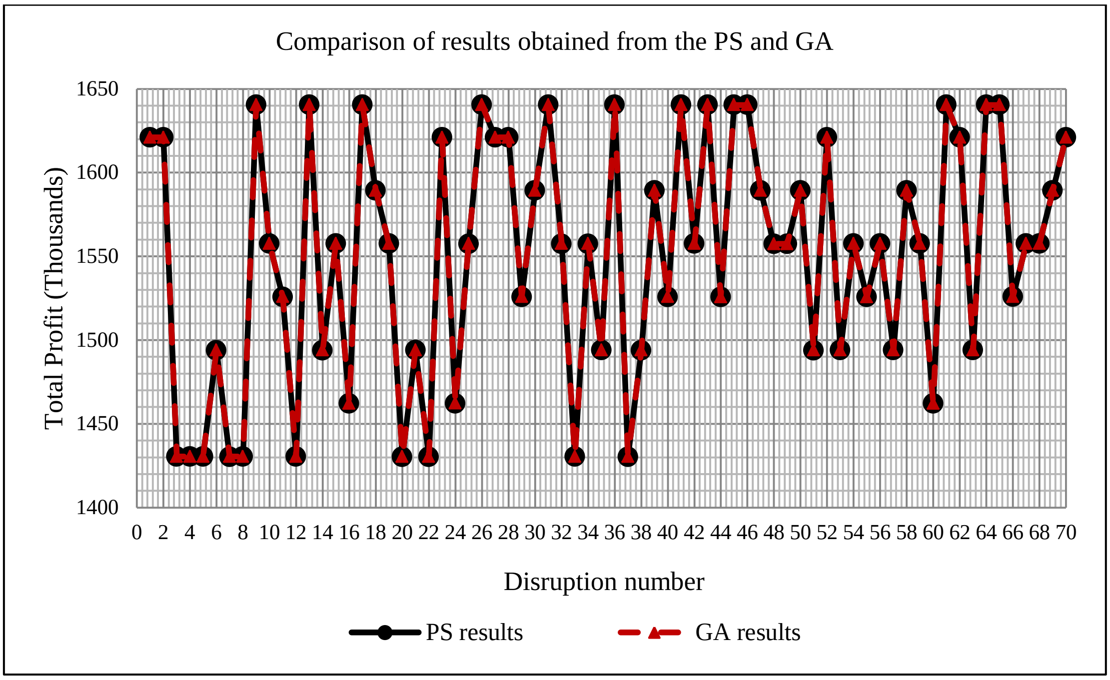

Figure 3.4**:** Comparison of results for seventy random disruption problems

Average percentage of deviation

$$= \frac{1}{N}\sum \frac{|\text{Total profit from GA} - \text{Total profit from PS}|}{\text{Total profit from PS}} \times 100\% \tag{3.37}$$

Here, N = Number of test problems.

3.5.3 Multiple Disruptions

To demonstrate the usefulness of our proposed solution approach for solving multiple disruptions, as a series of disruptions, over a period of time, the basic data of the single disruption problem is used, which is presented in Section 3.5.1. In any production-inventory system, a series of disruptions can occur, one after another, on a real-time basis.

If a disruption occurs after the recovery time window of another disruption, then the later one can be considered as an independent disruption, and a revised plan can be made similarly to the single disruption case. However, if a disruption occurs during the recovery time window of another disruption that occurred earlier, a revised plan, incorporating the effect of both disruption must be derived, which makes the case more complex for revised planning. For experimentation, the disruption scenarios for a series of disruptions, one after another, were generated randomly. Table 3.3 presents a case problem with different random combinations of disrupted cycle number, pre-disruption quantity and disruption duration. Although these disruptions can happen continuously, a case problem with only five disruptions is presented as a sample representation.

The production quantities, during the recovery time window, are revised immediately after each disruption take places in the production system. The problem was solved using the solution approach, developed in Section 3.4.2 for a series of disruptions, and then the best result was recorded, out of 30 independent runs, for each disruption. After the first disruption, the production quantities ($X_{i,1}; i = 1 - 5$) were revised for the next five cycles of the recovery time window, as shown in Tables 3.4 and 3.5. Once again, the second disruption occurred within the recovery time window of the first disruption, and so the parameters, objective function and constraints were updated for the changed situation. So the new production quantities ($X_{i,2}; i = 1 - 5$) were then revised again immediately, after the occurrence of the second disruption, under the updated situation. This process continued if the system experienced any new disruption. For each disruption, the revised production quantities and the maximum total profit (best result) were recorded. Table 3.4 presents the results obtained from the GA based approach and Table 3.5 for that of PS. It is also observed, that both techniques confirm very similar results.

Table 3.3: A case problem for series of disruptions

Disruption number (n)	Disrupted cycle number from previous disruption (l)	Pre-disruption quantity (q_n)	Disruption duration ($T_{d,n}$)
1	1	765	0.0030
2	4	875	0.0055
3	3	480	0.0100
4	5	1090	0.0045
5	3	585	0.0065
...	...	...	...

Table 3.4: The results obtained from GA based approach

Disruption number (n)	Production quantity in the recovery time window					Total Profit
	$X_{1,n}$	$X_{2,n}$	$X_{3,n}$	$X_{4,n}$	$X_{5,n}$	
1	5527	6292	6292	6292	6292	1640383
2	5417	5676	5949	6251	6292	1573316
3	4569	5302	5569	5850	6148	1409071
4	4789	6164	6170	6277	6292	1597157
5	4898	5745	6029	6292	6292	1534123
...	...	...	...	...	...	...

Table 3.5: The results obtained from PS based approach

Disruption number (n)	Production quantity in the recovery time window					Total Profit
	$X_{1,n}$	$X_{2,n}$	$X_{3,n}$	$X_{4,n}$	$X_{5,n}$	
1	5527	6292	6292	6292	6292	1640381
2	5054	5943	6213	6134	6240	1573355
3	4709	5344	5560	5824	6101	1414336
4	4812	6029	6278	6255	6269	1594574
5	5160	5889	6170	5947	6156	1537449
...	...	...	...	...	...	...

3.6 Sensitivity Analysis

The total profit changes with the different parameters. In this section, changes of the total profit, in comparison with the back order cost, lost sales cost, disruption duration, production process reliability and pre-disruption quantity are analysed. For each study, only one variable is changed, and the remainder is kept the same as in section 3.5.1. This sensitivity is analysed for the recovery plan after the second disruption of the results analysis section, and the PS search technique was used to analyse the sensitivity.

Figures 3.5 and 3.6 show the changes of the total profit with back order cost and lost sales cost respectively. In this analysis, disruption duration and pre-disruption quantity are kept constant as 0.01 and 0 respectively. The total profit decreases with the increment of both of the back order and lost sales cost. This means, that both of the back order and lost sales cost are present in the optimal solution. But the effect of the lost sales cost on the total profit is more than that of the back order cost because of compromises in the production quantities.

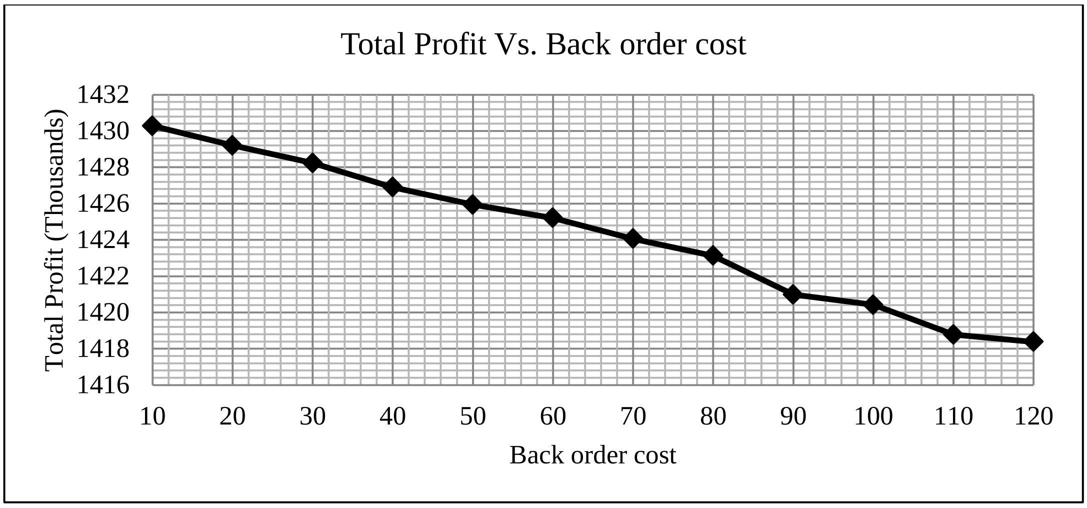

Figure 3.5**:** Changes of total profit with back order cost

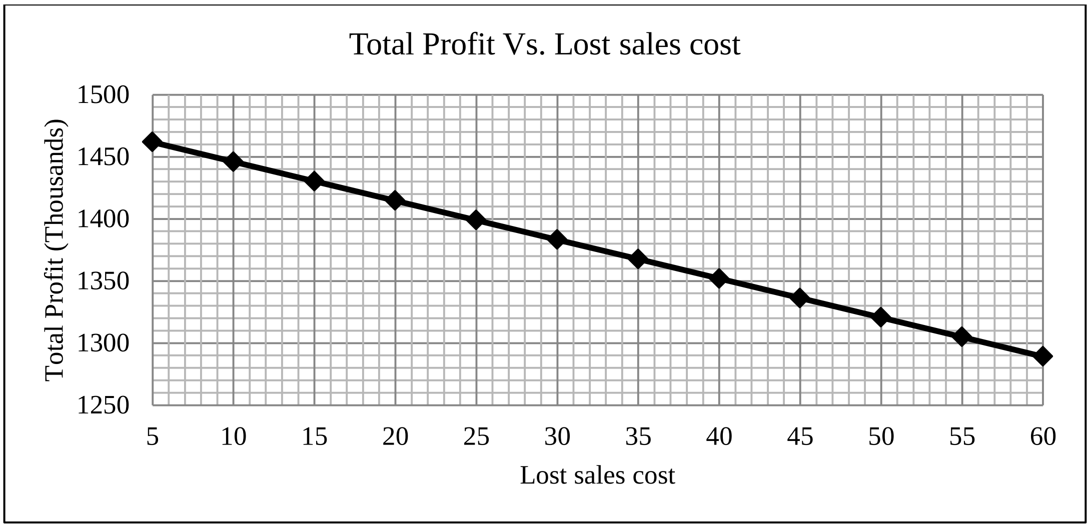

Figure 3.6**:** Changes of total profit with lost sales cost

The total profit decreases with the length of the disruption duration, as shown in Figure 3.7. In this analysis, the pre-disruption quantity is kept fixed as 0. It is observed, that when the disruption duration is between 0.001 and 0.003, the total profit does not change significantly, because only back orders are present in the solution. But after then, there is a linearly decreasing trend of the total profit. This is because of the introduction of lost sales cost in the solutions after the disruption duration becomes longer than 0.003. The total profit increases with production process reliability, which is illustrated in Figure 3.8. The holding, set-up, production, inspection and rejection costs all decrease with the increment of process reliability. In this case, the maximum total profit is obtained when the production process produces 100% good products (process reliability is equal to 1). There is no effect of the pre-disruption quantity on the total profit, which is shown in Figure 3.9. In this analysis, the disruption duration is kept constant as 0.01.

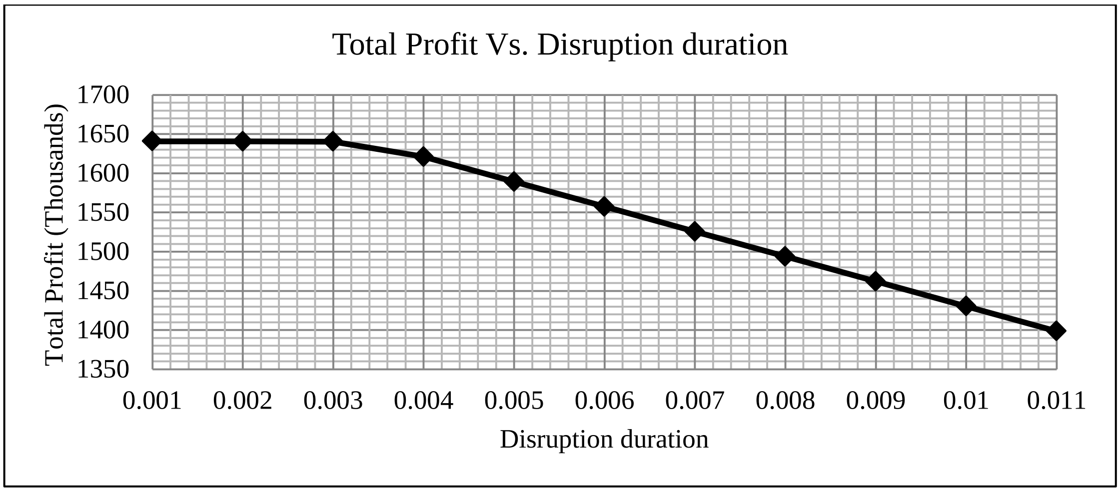

Figure 3.7: Changes of total profit with disruption duration

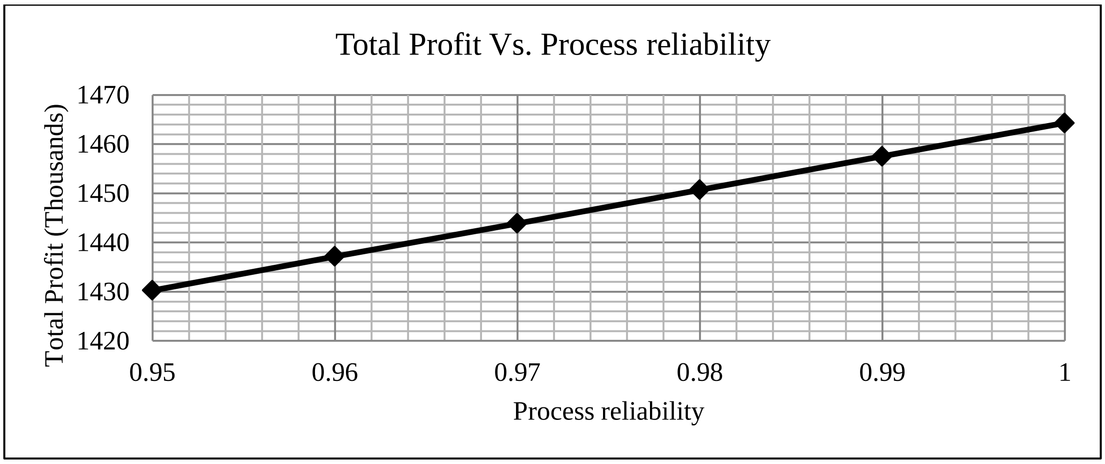

Figure 3.8: Changes of total profit with process reliability

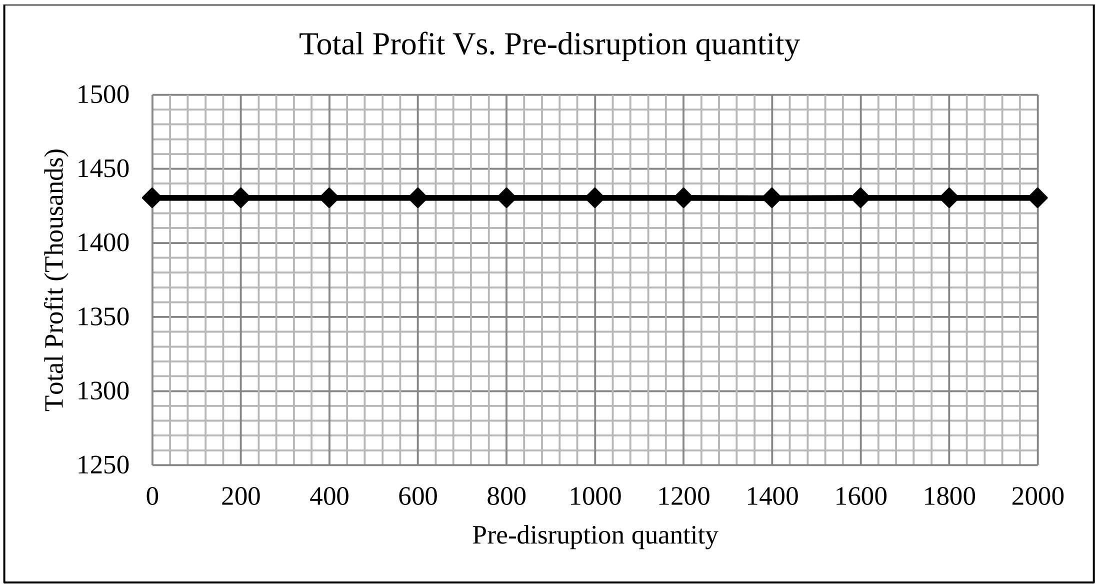

Figure 3.9**:** Changes of total profit with pre-disruption quantity

3.7 Chapter Summary

The main objective of this chapter was to develop a disruption recovery plan in an imperfect production environment. In real-life production lines, a disruption can happen at any time at any point of production. Without a proper response to the disruption recovery, the organisation can face huge financial and reputation loss due to a disruption. So it is important to develop an appropriate recovery plan after the occurrence of a sudden disruption. Moreover, imperfect production processes are very common in real-life and they have significant impacts on companies' loss and profit. So it is also important to consider an imperfect production environment while developing a production-inventory model. In real-life situations, the production systems can face both single and multiple disruptions. In the single disruption case, disruption occurs suddenly when the production system operates under a pre-assigned plan. In the multiple disruptions case, disruptions occur one after another as a series, and new disruptions may or may not affect the revised production plans of the previous disruptions. A new mathematical model and a dynamic solution approach were developed, which can handle both single and multiple disruptions, on a real-time basis. This chapter also demonstrated how the proposed methodology can be implemented for real-time disruption recovery planning with some numerical examples and randomly generated test problems.

An extension of the study of chapter 3, which considers demand uncertainty, is presented in Appendix A.

In chapter 3, the disruption recovery problem for an imperfect single-stage production-inventory system that is subject to production disruption has been addressed. The extension of this study is to explore the problem for a two-stage production-inventory system, which I have chosen as the direction of my research work in the next chapter.

Chapter 4 Two-Stage Production-Inventory System

This chapter extends the work from chapter 3, in which a new real-time disruption management model is presented for a two-stage production-inventory system. In this system, the production may be disrupted, for a given duration of time, either at one or both stages. In this chapter, firstly, a mathematical model is developed to suggest a recovery plan for a single occurrence of disruption at either stage. Secondly, multiple disruptions are considered, for which a new disruption may or may not affect the recovery plan of earlier disruptions. A new solution approach is proposed which deals with a series of disruptions over a period of time, which can be implemented for disruption recovery on a real-time basis. This chapter also presents some numerical examples and a real-world case study to explain the benefits of our proposed approach.

4.1 Introduction

The study in this chapter is an extension of the study in chapter 3. This chapter considers a two-stage batch production-inventory system that incorporates both production disruption and process reliability. Thus unlike the single-stage case considered in the previous chapter, the model in this chapter consists of two stages and the disruption can occur in either one stage or both stages at any time. The problem presented in this chapter is much more complex because a two-stage production-inventory system with process reliability is considered. In this study, methodologies have been developed for managing (i) a single occurrence of disruption at any stage, and (ii) a series of disruptions on a real-time basis.

The main contributions of this chapter can be summarized as follows.

i. Modelling a production-inventory system for a two-stage production-inventory system under production disruption.

ii. Considering process reliability, because imperfect production processes are common in real-life.

iii. Developing a dynamic solution approach that deals both a single and multiple disruptions at any stage on a real-time basis.

iv. Implementing the model to a real-world case from a pharmaceutical company.

4.2 Problem Description

In this section, a two-stage production system is discussed that produces one type of product (also known as a single product). The product is produced in batches, and once a batch is completed at the first stage, the whole batch is then transported to the second stage for final processing. A typical two-stage single-item batch production system is presented in Figure 4.1, where the inventory built up in each stage is shown for batch processing. The production of non-defective items in any batch is dependent on the process reliability. As it is assumed that the production capacity is higher than the demand, there is an idle timeslot between the consecutive production batches. If the production system is disrupted for a given period of time, the utilization of the idle timeslots for production, may help to recover from the disruption. However, it may result in costly backorder and/or lost sales due to delayed delivery.

Figure 4.2 shows a typical recovery plan where the system is recovered using the idle production time with a revision of the production quantity in each cycle. In this figure, the dashed line represents the revised production-inventory plan for the recovery of disruptions that occurred at the first and second stages. After the first disruption at the first stage, the production quantities, $X_{i,1}$ and $Y_{i,1}$ $(i = 1,2, \dots, M)$, are revised to generate the recovery plan, which is shown as a black dashed line. Again, for the disruption at the second stage, which is during the recovery time window of an earlier disruption, the production quantities $X_{i,2}$ and $Y_{i,2}$ $(i = 1,2, \dots, M)$ are revised to consider the effect of both of the disruptions, which is shown as a red dashed line. These disruptions may occur at any time, and the duration of disruption may vary from one occurrence to the next. After each disruption, the production quantity in each cycle must be revised, within the allowed recovery time window, as a plan for recovery. The shaded area in Figure 4.2 represents the disruption duration in a production cycle. The system can also limit the recovery time window to a pre-specified number of production cycles.

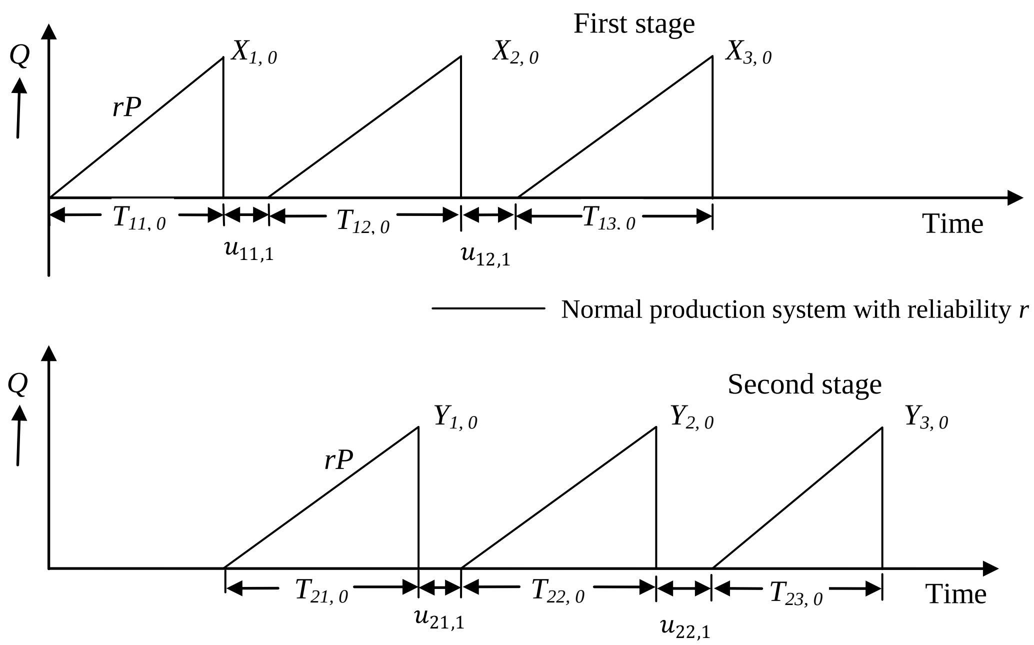

Figure 4.1: An ideal two-stage production-inventory system

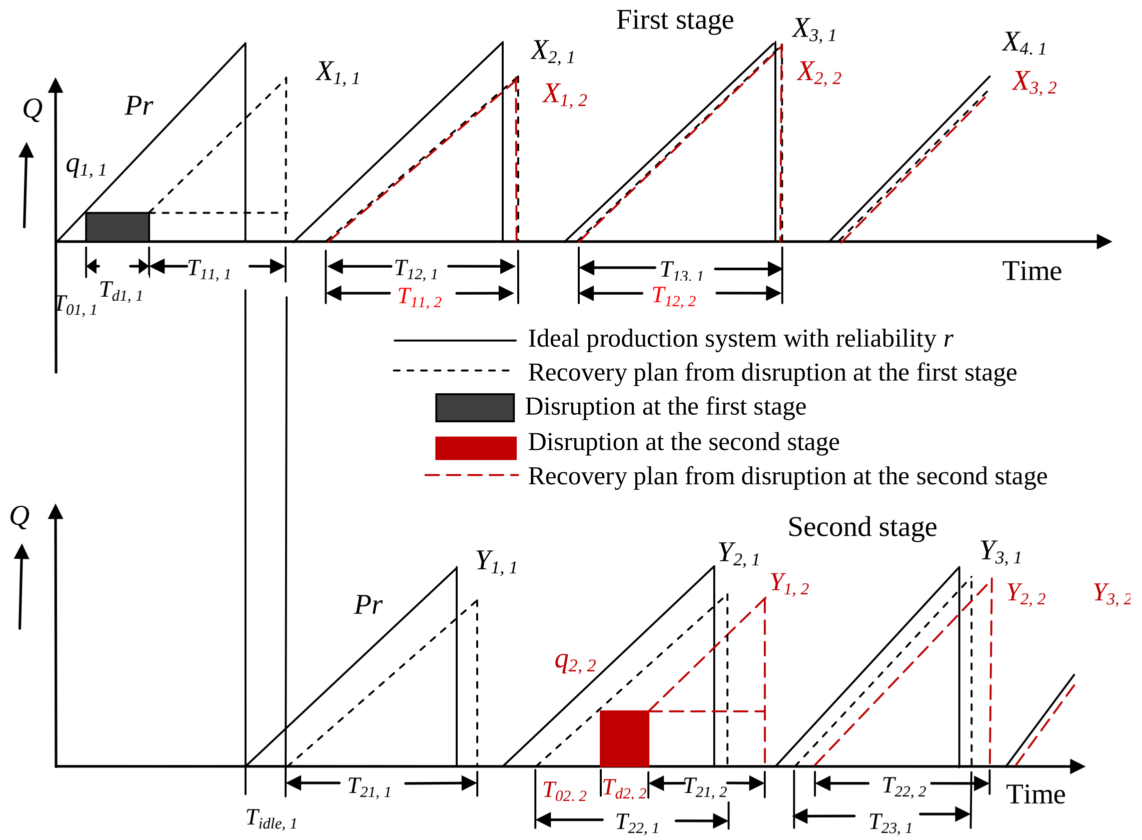

Figure 4.2: Recovery plan from the multiple disruptions

If a disruption occurs after the recovery time window of another disruption, then the later one can be considered as an independent disruption and the recovery plan can be made similar to the previous one. However, a disruption occurs within the recovery time window of another disruption occurred earlier, it makes the case more complex for recovery planning. In practice, to minimise the effect of disruptions, they must be dealt with on a real-time basis, whether this is single occurrence of disruption or a series of disruptions. In this study, the disruption recovery problems are considered as follows: (i) single occurrence of disruption, (ii) a series of independent disruptions, and (iii) a series of mix of independent and dependent disruptions.

4.2.1 Notations used in this Study

To formulate the mathematical model, the following notations are used.

S_{t1} Set-up time for a cycle at the first stage

S_{t2} Set-up time for a cycle at the second stage

$\delta_{1i,n}$	Idle time for a cycle *i* before the n^{th} disruption at the first stage
$\delta_{2i,n}$	Idle time for a cycle *i* before the n^{th} disruption at the second stage
D	Demand per year (units per year)
H_1	Holding cost per unit per year at the first stage ($/unit/year)
H_2	Holding cost per unit per year at the second stage ($/unit/year)
r	Reliability of the production process – which is known from the historical data of the production system
Q	Combined production lot size per normal cycle with reliability r
A_1	Set-up cost per cycle at the first stage ($ per set-up)
A_2	Set-up cost per cycle at the second stage ($ per set-up)
P	Production rate (units per year) in 100% reliable system
l	New disrupted cycle number from previous disruption
$X_{i,0}$	Production quantity for a normal cycle i at the first stage
$Y_{i,0}$	Production quantity for a normal cycle i at the second stage
$X_{i,n}$	Production quantity for cycle *i* of the recovery window at the first stage after the n^{th} disruption– which is a decision variable; *i = 1, 2, 3, ..., M*
$Y_{i,n}$	Production quantity for cycle *i* of the recovery window at the second stage after the n^{th} disruption– which is a decision variable; *i = 1, 2, 3, ..., M*
$u_{1i,n}$	Production down time for cycle *i* before the n^{th} disruption at the first stage (set-up time + idle time) = $S_{t1} + \delta_{1i,n} = \frac{X_{l+i-1,n-1}}{D} - \frac{X_{l+i,n-1}}{rP}$
$u_{2i,n}$	Production down time for cycle *i* before the n^{th} disruption at the second stage (set-up time + idle time) = $S_{t2} + \delta_{2i,n} = \frac{Y_{l+i-1,n-1}}{D} - \frac{Y_{l+i,n-1}}{rP}$
M	Number of cycles to recovery the disruption – which is given by the management
$T_{d1,n}$	Disruption duration in the n^{th} disruption at the first stage
$T_{d2,n}$	Disruption duration in the n^{th} disruption at the second stage
$q_{1,n}$	Pre-disruption production quantity in the n^{th} disruption at the first stage
$q_{2,n}$	Pre-disruption production quantity in the n^{th} disruption at the second stage
$T_{01,n}$	Production time for $q_{1,n} = \frac{q_{1,n}}{rP}$
$T_{02,n}$	Production time for $q_{2,n} = \frac{q_{2,n}}{rP}$

$T_{1i,0}$ Production up time for a normal cycle *i* at the first stage = $\frac{X_{i,0}}{Pr}$

$T_{2i,0}$ Production up time for a normal cycle *i* at the second stage = $\frac{Y_{i,0}}{Pr}$

$T_{1i,n}$ Production up time for a cycle *i* in the recovery window at the first stage after the n^{th} disruption = $\frac{X_{i,n}}{Pr}$

$T_{2i,n}$ Production up time for a cycle *i* in the recovery window at the second stage after the n^{th} disruption = $\frac{Y_{i,n}}{Pr}$

$T_{idle,n}$ Idle time of production at the stage 2 due to the n^{th} disruption at the first stage $= T_{01,n} + T_{d1,n} + T_{1,n} - T_{l,n-1} = \left(\frac{q_{1,n}}{rP} + T_{d1,n} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP}\right)$

B Unit back order cost per unit time ($/unit/time)

L Unit lost sales cost ($ per unit)

C_{P1} Per unit production cost at the first stage ($ per unit)

C_{P2} Per unit production cost at the second stage ($ per unit)

C_{R1} Rejection cost per unit at the first stage

C_{I1} Inspection cost as a percentage of the production cost at the first stage

C_{I2} Inspection cost as a percentage of the production cost at the second stage

m_1 Mark-up of the unit selling price $[m_1(C_{P1} + C_{P2})]$ of the acceptable items (must be greater than 1)

4.2.2 Assumptions of the Study

In this study, the following assumptions have been made:

i. The production rate of the item is greater than its demand rate.

ii. The original production line is perfectly balanced, that means the production rates for the item are equal for the both stages.

iii. The recovery cycle will start just after the disruption occurs. The disruption can occur in either single or both stages at any point in time.

iv. There are equal numbers of cycles in the disruption recovery windows of both stages.

v. All products are inspected and defective products are rejected.

vi. The total cost of interest and depreciation per production cycle $C(A,r)$ is inversely related to set-up cost (A) and is directly related to process reliability (r) according to the following general power function (Cheng, 1989):

$$C(A,r) = aA^{-b}r^{c}$$

Where a, b and c are positive constants chosen to provide the best fit of the estimated cost function. This assumption is based on the fact that to reduce the costs of production set-up and of scraping and reworking shoddy products, substantial investment in improving the reliability of the production process is necessary.

4.3 Model Formulation

In this section, a mathematical model is developed for a single occurrence of disruption at either stage of the production-inventory system. In the following few subsections, first, the equation for the economic production quantity is presented for a perfect system under an ideal situation, and the equations for relevant costs and revenue are derived for the imperfect production system under disruption. Finally, the disruption recovery problem is formulated as a non-linear constrained optimisation problem that maximises the total profit subject to capacity, demand, and stage linking constraints. The decision variables are the production quantities in each cycle during the recovery time window. Note that the total profit function is derived from the revenue from acceptable items and the relevant costs.

4.3.1 Optimal Q under Ideal Conditions

For a two-stage single item perfect production system, with lot-for-lot condition under an ideal situation as considered by Sarker and Khan (2001), the combined economic production quantity (as shown in Figure 4.1) can be formulated as $Q = \sqrt{\frac{2rP(A_1+A_2)}{H_1+H_2}}$.

Here,

Annual set-up cost at the first stage $= \frac{D}{Q}A_1$

Annual holding cost at the first stage $= \frac{Q}{2} H_1 \frac{D}{rP}$

Annual set-up cost at the second stage $= \frac{D}{Q} A_2$

Annual holding cost at the second stage $= \frac{Q}{2} H_2 \frac{D}{rP}$

Total cost $= \frac{D}{Q} A_1 + \frac{Q}{2} H_1 \frac{D}{rP} + \frac{D}{Q} A_2 + \frac{Q}{2} H_2 \frac{D}{rP}$

To minimise the total cost, $$\frac{d}{dQ}\left(\frac{D}{Q} A_1 + \frac{Q}{2} H_1 \frac{D}{rP} + \frac{D}{Q} A_2 + \frac{Q}{2} H_2 \frac{D}{rP}\right) = 0 \qquad (4.1)$$

After simplifying, $$Q = \sqrt{\frac{2rP(A_1 + A_2)}{H_1 + H_2}} \qquad (4.2)$$

4.3.2 Cost Formulation for the First Stage

In this section, the different cost equations, for the first stage of an imperfect production process, are derived with a single occurrence of disruption. In Hishamuddin et al. (2012), it is assumed that Q is the regular lot size in every cycle and that when a disruption is experienced the revised lot sizes (decision variables) are represented as X_1, X_2, ..., X_M, for the next M cycles. In their study, the cycle experiencing disruption is numbered as the first cycle in their study and the system allows M number of cycles to recover from disruption. To generalize the model under multiple disruptions, in this study, the revised lot quantities in a cycle i of the recovery window after n^{th} disruption are defined as $X_{i,n}$ and $Y_{i,n}$ and the original lot quantities in a cycle i are represented as $X_{i,0}$ and $Y_{i,0}$ in the first and second stage respectively.

The costs considered in this study are the holding, set-up, production, rejection, inspection, and depreciation costs. The total holding cost is computed as the unit holding cost multiplied by the total inventory during the recovery time window, which is equivalent to the area under the curve of recovery window of Figure 4.2. The total set-up cost is equal to the cost per set-up multiplied by the number of set-ups in the recovery time window. The total production cost is obtained by multiplying the unit production cost by the total quantity produced during the recovery time window. As the production reliability is assumed to be r, the rejection rate is (1-r). The rejection cost is determined as the unit rejection cost multiplied by the total number of rejected items (Paul et al., 2014). The inspection cost is considered as a certain percentage of the production cost. The cost for interest and depreciation is considered as a general power function as suggested by Cheng (1989).

If a disruption occurs at the first stage of the system, at the l^{th} cycle from the previous disruption, the different costs are calculated as follows.

Holding cost

$$= H_1\left[\frac{1}{2}q_{1,n}T_{01,n} + q_{1,n}\left(T_{d1,n} + S_{t1} + T_{11,n}\right) + \frac{1}{2}X_{1,n}T_{11,n} + \frac{1}{2}X_{2,n}T_{12,n} + \cdots + \frac{1}{2}X_{M,n}T_{1M,n}\right]$$

$$= \frac{1}{2}H_1\left[q_{1,n}\frac{q_{1,n}}{rP} + 2q_{1,n}\left(T_{d1,n} + S_{t1} + \frac{X_{1,n}}{rP}\right) + X_{1,n}\frac{X_{1,n}}{rP} + X_{2,n}\frac{X_{2,n}}{rP} + \cdots + X_{M,n}\frac{X_{M,n}}{rP}\right]$$

$$= \frac{1}{2}H_1\left[\frac{(q_{1,n})^2}{rP} + 2q_{1,n}\left(T_{d1,n} + S_{t1}\right) + \frac{2X_{1,n}q_{1,n}}{rP} + X_{1,n}\frac{X_{1,n}}{rP} + X_{2,n}\frac{X_{2,n}}{rP} + \cdots + X_{M,n}\frac{X_{M,n}}{rP}\right]$$

$$= \frac{1}{2}H_1\left[\frac{(q_{1,n})^2}{rP} + 2q_{1,n}\left(T_{d1,n} + S_{t1}\right) + \frac{2X_{1,n}q_{1,n}}{rP} + \sum_{i=1}^{M}\frac{(X_{i,n})^2}{rP}\right] \tag{4.3}$$

Set-up cost

$$= A_1 M \tag{4.4}$$

Production cost

$$= C_{P1}P\left(\sum_{i=1}^{M}T_{1i,n} + T_{01,n}\right)$$

$$= \frac{C_{P1}}{r}\left(\sum_{i=1}^{M}X_{i,n} + q_{1,n}\right) \tag{4.5}$$

Rejection cost

$$= C_{R1}(1-r)P\left(\sum_{i=1}^{M} T_{1i,n} + T_{01,n}\right)$$

$$= C_{R1}\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{i,n} + q_{1,n}\right) \quad (4.6)$$

Inspection cost

$$= \frac{C_{I1}C_{P1}}{r}(\sum_{i=1}^{M} X_{i,n} + q_{1,n}) \quad (4.7)$$

Cost of interest and depreciation

$$= Ma\,(A_1)^{-b}(r)^c \quad (4.8)$$

If the disruption is at the second stage, at the l^{th} cycle from the previous disruption, the different costs are calculated as follows.

Holding cost

$$= \frac{1}{2}H_1\left[\sum_{i=1}^{M} \frac{(X_{i,n})^2}{rP}\right] \quad (4.9)$$

Set-up cost

$$= A_1 M \quad (4.10)$$

Production cost

$$= C_{P1}P\left(\sum_{i=1}^{M} T_{1i,n}\right)$$

$$= \frac{C_{P1}}{r}\left(\sum_{i=1}^{M} X_{i,n}\right) \quad (4.11)$$

Rejection cost

$$= C_{R1}(1-r)P\left(\sum_{i=1}^{M} T_{1i,n}\right)$$

$$= C_{R1}\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{i,n}\right) \quad (4.12)$$

Inspection cost

$$= \frac{C_{I1} C_{P1}}{r} (\sum_{i=1}^{M} X_{i,n}) \tag{4.13}$$

Cost of interest and depreciation

$$= Ma\ (A_1)^{-b} (r)^c \tag{4.14}$$

4.3.3 Cost Formulation for the Second Stage

In this section, the different costs are determined, which are involved at the second stage due to disruption in an imperfect production process. For a single disruption, the costs considered are: holding, set-up, back order, lost sales, production, inspection and depreciation costs. As the second stage is considered as the finishing or packaging stage, it is assumed that there is no rejection cost in this stage. The holding, set-up, production, inspection and depreciation costs are determined in the same way as for the first stage. Back order is the portion of an order that cannot be delivered at the scheduled time, but will be delivered at a later date when available. This back order cost is determined as unit back order cost multiplied by back order units and it's time delay (Hishamuddin et al., 2012). When there is demand, but the item is out of stock and the customer will not wait for the item, the result is a lost sale. Lost sales cost is determined as unit lost sales cost multiplied by lost sales units (Hishamuddin et al., 2012).

If the production system has a disruption at the first stage at the l^{th} cycle from the previous disruption, the following costs are calculated:

Holding cost

$$= H_2 \left[\frac{1}{2} Y_{1,n} T_{21,n} + \frac{1}{2} Y_{2,n} T_{22,n} + \cdots + \frac{1}{2} Y_{M,n} T_{2M,n}\right]$$

$$= H_2 \left[\frac{1}{2} Y_{1,n} \frac{Y_{1,n}}{rP} + \frac{1}{2} Y_{2,n} \frac{Y_{2,n}}{rP} + \cdots + \frac{1}{2} Y_{M,n} \frac{Y_{M,n}}{rP}\right]$$

$$= \frac{1}{2} H_2 \left[\sum_{i=1}^{M} \frac{(Y_{i,n})^2}{rP}\right] \tag{4.15}$$

Set-up cost

$$= A_2 M \tag{4.16}$$

Back-order cost

$$= B\left[(Y_{1,n}).delay_1 + \sum_{i=2}^{M}(Y_{2,n}.delay_i\right]$$

$$= B\left[(Y_{1,n})\left[T_{ilde,n} + \frac{Y_{1,n}}{rP} - \frac{Y_{l,n-1}}{rP}\right]\right.$$

$$+ \sum_{i=2}^{M} Y_{2,n}.\left[T_{ilde,n} + (i-1)S_{t2} + \sum_{j=1}^{i}\frac{Y_{j,n}}{rP}\right.$$

$$\left.\left. - \sum_{j=1}^{i}\frac{Y_{l+j-1,n-1}}{rP} - \sum_{j=1}^{i-1}\left(u_{2j,n}\right)\right]\right]$$

$$= B\left[(Y_{1,n})\left[T_{ilde,n} + \frac{Y_{1,n}}{rP} - \frac{Y_{l,n-1}}{rP}\right]\right.$$

$$+ \sum_{i=2}^{M} Y_{2,n}.\left[T_{ilde,n} + (i-1)S_{t2} + \sum_{j=1}^{i}\frac{Y_{j,n}}{rP}\right.$$

$$\left.\left. - \sum_{j=1}^{i}\frac{Y_{l+j-1,n-1}}{rP} - \sum_{j=1}^{i-1}\left(\frac{Y_{l+j-1,n-1}}{D} - \frac{Y_{l+j,n-1}}{rP}\right)\right]\right] \tag{4.17}$$

Lost sales cost

$$= L\sum_{i=1}^{M} Y_{l+i-1,n-1} - LrP\left(T_{21,n} + T_{22,n} + \cdots + T_{2M,n}\right)$$

$$= L\sum_{i=1}^{M} Y_{l+i-1,n-1} - LPr\left(\frac{Y_{1,n}}{rP} + \frac{Y_{2,n}}{rP} + \cdots + \frac{Y_{M,n}}{rP}\right)$$

$$= L\left(\sum_{i=1}^{M} Y_{l+i-1,n-1} - \sum_{i=1}^{M} Y_{i,n}\right) \tag{4.18}$$

Production cost

$$= C_{P2} rP\left(\sum_{i=1}^{M} T_{2i,n}\right)$$

$$= C_{P2}\left(\sum_{i=1}^{M} Y_{i,n}\right) \tag{4.19}$$

Inspection cost

$$= C_{I2} C_{P2}(\sum_{i=1}^{M} Y_{i,n}) \tag{4.20}$$

Cost of interest and depreciation

$$= Ma\,(A_2)^{-b} \tag{4.21}$$

If the production system has a disruption at the second stage at the l^{th} cycle from the previous disruption, the following costs are calculated:

Holding cost

$$= H_2\left[\frac{1}{2} q_{2,n} T_{02,n} + q_{2,n}\left(T_{d2,n} + S_{t2} + T_{21,n}\right) + \frac{1}{2} Y_{1,n} T_{21,n} + \frac{1}{2} Y_{2,n} T_{22,n} + .. + \frac{1}{2} Y_{M,n} T_{2M,n}\right]$$

$$= H_2\left[\frac{1}{2} q_{2,n} \frac{q_{2,n}}{rP} + q_{2,n}\left(T_{d2,n} + S_{t2} + \frac{Y_{1,n}}{rP}\right) + \frac{1}{2} Y_{1,n} \frac{Y_{1,n}}{rP} + \frac{1}{2} Y_{2,n} \frac{Y_{2,n}}{rP} + \cdots + \frac{1}{2} Y_{M,n} \frac{Y_{M,n}}{rP}\right]$$

$$= \frac{1}{2} H_2\left[\frac{(q_{2,n})^2}{rP} + 2q_{2,n}\left(T_{d2,n} + S_{t2}\right) + \frac{2q_{2,n} Y_{1,n}}{rP} + \sum_{i=1}^{M} \frac{(Y_{i,n})^2}{rP}\right] \tag{4.22}$$

Set-up cost

$$= A_2 M \tag{4.23}$$

Back order cost

$$= B\left[(Y_{1,n} + q_{2,n}).\, delay_1 + \sum_{i=2}^{M} Y_{i,n}.\, delay_i\right]$$

$$= B\left[(Y_{1,n} + q_{2,n})\left[T_{d2,n} + \frac{Y_{1,n} + q_{2,n}}{rP} - \frac{Y_{l,n-1}}{rP}\right]\right.$$

$$+ \sum_{i=2}^{M} Y_{i,n}.\left[T_{02,n} + T_{d2,n} + (i-1)S_{t2} + \sum_{j=1}^{i} \frac{Y_{j,n}}{rP}\right.$$

$$\left.\left. - \sum_{j=1}^{i} \frac{Y_{l+j-1,n-1}}{rP} - \sum_{j=1}^{i-1} \left(u_{2j,n}\right)\right]\right]$$

$$= B\left[(Y_{1,n} + q_{2,n})\left[T_{d2,n} + \frac{Y_{1,n} + q_{2,n}}{rP} - \frac{Y_{l,n-1}}{rP}\right]\right.$$

$$+ \sum_{i=2}^{M} Y_{i,n}.\left[\frac{q_{2,n}}{rP} + T_{d2,n} + (i-1)S_{t2} + \sum_{j=1}^{i} \frac{Y_{j,n}}{rP}\right.$$

$$\left.\left. - \sum_{j=1}^{i} \frac{Y_{l+j-1,n-1}}{rP} - \sum_{j=1}^{i-1} \left(\frac{Y_{l+j-1,n-1}}{D} - \frac{Y_{l+j,n-1}}{rP}\right)\right]\right] \tag{4.24}$$

Lost sales cost

$$= L \sum_{i=1}^{M} Y_{l+j-1,n-1} - LPr\left(T_{02,n} + T_{21,n} + T_{22,n} + \cdots + T_{2M,n}\right)$$

$$= L \sum_{i=1}^{M} Y_{l+j-1,n-1} - LrP\left(\frac{q_{2,n}}{rP} + \frac{Y_{1,n}}{rP} + \frac{Y_{2,n}}{rP} + \cdots + \frac{Y_{M,n}}{rP}\right)$$

$$= L\left(\sum_{i=1}^{M} Y_{l+j-1,n-1} - q_{2,n} - \sum_{i=1}^{M} Y_{i,n}\right) \tag{4.25}$$

Production cost

$$= C_{P2} rP \left(\sum_{i=1}^{M} T_{2i,n} + T_{02,n} \right)$$

$$= C_{P2} \left(\sum_{i=1}^{M} Y_{i,n} + q_{2,n} \right) \tag{4.26}$$

Inspection cost

$$= C_{I2} C_{P2} (\sum_{i=1}^{M} Y_{i,n} + q_{2,n}) \tag{4.27}$$

Cost of interest and depreciation

$$= Ma\ (A_2)^{-b} \tag{4.28}$$

4.3.4 Formulation for Revenue

The selling price during the recovery time window is determined as the unit selling price multiplied by the demand during the recovery time window.

If the system faces disruption at the first stage, then the revenue from the selling price in the recovery window is determined as follows:

$$TR_1 = m_1 (C_{P1} + C_{P2})\ D \left[\sum_{i=1}^{M} T_{2i,n} + MS_{t2} \right]$$

$$= m_1 (C_{P1} + C_{P2})\ D \left[\sum_{i=1}^{M} \frac{Y_{i,n}}{rP} + MS_{t2} \right] \tag{4.29}$$

If the system faces disruption at the second stage, then the revenue from the selling price in the recovery window is determined as follows:

$$TR_2 = m_1 (C_{P1} + C_{P2})\ D \left[T_{02,n} + \sum_{i=1}^{M} T_{2i,n} + MS_{t2} \right]$$

$$= m_1 (C_{P1} + C_{P2})\ D \left[\frac{q_{2,n}}{rP} + \sum_{i=1}^{M} \frac{Y_{i,n}}{rP} + MS_{t2} \right] \tag{4.30}$$

4.3.5 Final Mathematical Model

The total profit function, that is the objective function, is determined for a particular disruption recovery time window as follows:

Total Profit (TF) = Revenues from selling price - Total costs at the first stage - Total costs at the second stage, subject to the constraints presented in (4.31- 4.79).

After a disruption in the first stage

$$X_{i,0} = Y_{i,0} = Q \tag{4.31}$$

[Normal cycle production quantity is equal to Q]

$$X_{1,n} + q_{1,n} \le X_{l,n-1} \tag{4.32}$$

[$X_{1,n} + q_{1,n}$is less than or equal to $X_{l,n-1}$ for delivery and transportation requirements]

$$X_{i,n} \le X_{l+i-1,n-1}\ ;\ i = 2,3,4, \ldots\ldots, M \tag{4.33}$$

[$X_{i,n}$is less than or equal to $X_{l+i-1,n-1}$ for delivery and transportation requirements]

$$Y_{1,n} = X_{1,n} + q_{1,n} \tag{4.34}$$

[To balance the production system]

$$Y_{i,n} = X_{i,n}\ ;\ i = 2,3,4, \ldots\ldots, M \tag{4.35}$$

[To balance the production system]

$$rP \ge D \tag{4.36}$$

[Production rate is greater than the demand rate]

$$r \le 1 \tag{4.37}$$

[Reliability must be less than or equal to 1]

$$\sum_{i=1}^{M} X_{i,n} + q_{1,n} \le rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{D} - MS_{t1} - T_{d1,n}\right) \tag{4.38}$$

[Production capacity constraint at the first stage]

$$\sum_{i=1}^{M} Y_{i,n} \le rP\left(\sum_{i=1}^{M} \frac{Y_{l+i-1,n-1}}{D} - MS_{t2}\right) \tag{4.39}$$

[Production capacity constraint at the second stage]

$$\sum_{i=1}^{M} Y_{i,n} \ge \left(\frac{\sum_{i=1}^{M} Y_{i,n}}{Pr} + MS_{t2}\right) D - \left(\sum_{i=1}^{M} Y_{l+i-1,n-1} - \sum_{i=1}^{M} Y_{i,n}\right) \tag{4.40}$$

[Demand during recovery window is accounted for]

$$\frac{X_{1,n}+q_{1,n}}{D}-\frac{X_{2,n}}{rP}-S_{t1}\geq 0 \tag{4.41}$$

[Ensure non negative idle time at the stage 1]

$$\frac{X_{i,n}}{D}-\frac{X_{i+1,n}}{rP}-S_{t1}\geq 0; i=2,3,\dots,M \tag{4.42}$$

[Ensure non negative idle time at the stage 1]

$$\frac{Y_{1,n}}{D}-\frac{Y_{2,n}}{rP}-S_{t2}\geq 0 \tag{4.43}$$

[Ensure non negative idle time at the stage 2]

$$\frac{Y_{i,n}}{D}-\frac{Y_{i+1,n}}{rP}-S_{t2}\geq 0; i=2,3,\dots,M \tag{4.44}$$

[Ensure non negative idle time at the stage 2]

$$T_{idle,n}+\frac{Y_{1,n}}{rP}-\frac{Y_{l,n-1}}{rP}\geq 0 \tag{4.45}$$

[Ensure non-negative delay]

$$T_{ilde,n}+(i-1)S_{t2}+\sum_{j=1}^{i}\frac{Y_{j,n}}{rP}-\sum_{j=1}^{i}\frac{Y_{l+j-1,n-1}}{rP} -\sum_{j=1}^{i-1}\left(\frac{Y_{l+j-1,n-1}}{D}-\frac{Y_{l+j,n-1}}{rP}\right)\geq 0 \tag{4.46}$$

[Ensure non-negative delay]

$$\frac{q_{1,n}}{rP}+T_{d1,n}+\frac{X_{1,n}}{rP}-\frac{X_{l,n-1}}{rP}\geq 0 \tag{4.47}$$

[Ensure non-negative idle time at the second stage due to disruption at the first stage]

$$\frac{q_{1,n}}{rP}+T_{d1,n}+\sum_{j=1}^{i}\frac{X_{j,n}}{rP}+(i-1)\,S_{t1} \leq \frac{Y_{l,n-1}}{rP}+T_{ilde,n}+\sum_{j=1}^{i-1}\frac{Y_{j,n}}{rP}+(i-1)\,S_{t2}; \tag{4.48}$$

$i=2,3,4,\dots\dots,M$

[To ensure second stage production has been started after completing at the first stage]

$$X_{i,n}, Y_{i,n}\geq 0;\ i=1,2,3,\dots.,M \tag{4.49}$$

[Non-negativity constraint]

<u>After a disruption in the second stage</u>

$$X_{i,0} = Y_{i,0} = Q \tag{4.50}$$

[Normal cycle production quantity is equal to Q]

$$X_{1,n} = X_{l,n-1} \tag{4.51}$$

[As $X_{l,n-1}$ is already in produced at the first stage]

$$X_{2,n} = X_{l+1,n-1} \tag{4.52}$$

[As $X_{l+1,n-1}$ is already in production at the first stage]

$$X_{i,n} \leq X_{l+i+1,n-1}; i = 3,4, \ldots\ldots, M \tag{4.53}$$

[$X_{i,n}$is less than or equal to $X_{l+i-1,n-1}$ for delivery and transportation requirements]

$$Y_{1,n} = X_{l,n-1} - q_{2,n} \tag{4.54}$$

[$Y_{1,n}$is equal to $X_{l,n-1} - q_{2,n}$ as it is already produced at the first stage]

$$Y_{2,n} = X_{l+1,n-1} \tag{4.55}$$

[As $X_{l+1,n-1}$ is already in production at the first stage]

$$Y_{i,n} \leq Y_{l+i-1,n-1};\ i = 3,4, \ldots\ldots, M \tag{4.56}$$

[$Y_{i,n}$ is less than or equal to $Y_{l+i-1,n-1}$ for delivery and transportation requirements]

$$Y_{i,n} = X_{i,n}; i = 2,3,4, \ldots\ldots, M \tag{4.57}$$

[To keep the production process balanced]

$$rP \geq D \tag{4.58}$$

[Production rate is greater than the demand rate]

$$r \leq 1 \tag{4.59}$$

[Reliability must be less than or equal to 1]

$$\sum_{i=1}^{M} X_{i,n} \leq rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{D} - MS_{t1}\right) \tag{4.60}$$

[Production capacity constraint at the first stage]

$$\sum_{i=1}^{M} Y_{i,n} + q_{2,n} \leq rP\left(\sum_{i=1}^{M} \frac{Y_{l+i-1,n-1}}{D} - MS_{t2} - T_{d2,n}\right) \tag{4.61}$$

[Production capacity constraint at the second stage]

$$\sum_{i=1}^{M} Y_{i,n} + q_{2,n} \geq \left(\frac{\sum_{i=1}^{M} Y_{i,n} + q_{2,n}}{Pr} + MS_{t2}\right) D - \left(\sum_{i=1}^{M} Y_{l+i-1,n-1} - \sum_{i=1}^{M} Y_{i,n} - q_{2,n}\right) \tag{4.62}$$

[Demand during recovery window is accounted for]

$$\frac{X_{i,n}}{D} - \frac{X_{i+1,n}}{rP} - S_{t1} \geq 0 \tag{4.63}$$

[Ensure non negative idle time at the stage 1]

$$\frac{Y_{1,n} + q_{2,n}}{D} - \frac{Y_{2,n}}{rP} - S_{t2} \geq 0 \tag{4.64}$$

[Ensure non negative idle time at the stage 2]

$$\frac{Y_{i,n}}{D} - \frac{Y_{i+1,n}}{rP} - S_{t2} \geq 0 \tag{4.65}$$

[Ensure non negative idle time at the stage 2]

$$T_{d2,n} + \frac{Y_{1,n} + q_{2,n}}{rP} - \frac{Y_{l,n-1}}{rP} \geq 0 \tag{4.66}$$

[Ensure non-negative delay]

$$\frac{q_{2,n}}{rP} + T_{d2,n} + (i-1)S_{t2} + \sum_{j=1}^{i} \frac{Y_{j,n}}{rP} - \sum_{j=1}^{i} \frac{Y_{l+j-1,n-1}}{rP} - \sum_{j=1}^{i-1}\left(\frac{Y_{l+j-1,n-1}}{D} - \frac{Y_{l+j,n-1}}{rP}\right) \geq 0 \tag{4.67}$$

[Ensure non-negative delay]

$$\sum_{j=1}^{i-1}\left(\frac{X_{l+j-1,n-1}}{D} - \frac{X_{l+j,n-1}}{rP}\right) + \sum_{j=2}^{i} \frac{X_{j,n}}{rP} \leq \frac{q_{2,n}}{rP} + T_{d2,n} + \sum_{j=1}^{i-1} \frac{Y_{j,n}}{rP} + (i-1)\, S_{t2} \tag{4.68}$$

[To ensure second stage production has been started after completing at the first stage] $i = 2,3,4, \ldots\ldots, M$

$$X_{i,n}, Y_{i,n} \geq 0;\; i = 1,2,3, \ldots\ldots, M \tag{4.69}$$

[Non-negativity constraint]

4.3.6 Relationships between the Parameters

Back order and lost sales play an important role in determining the optimal recovery plan. In fact, the optimal recovery plan is sensitive to the relative magnitude of back order and lost sales costs. A Lemma and a few propositions help to understand the relationships between these two cost parameters and the other parameters. Lemma 1 and proposition 1 are derived to prove the condition of the existence of back order and lost sales respectively in the solution. Proposition 2 proves the effect of the disruption duration and proposition 3 confirms the effect of process reliability on the solution.

Lemma 1: For a given A_1, A_2, H_1, H_2, B, L, if $X_{i,n-1} = X_{n-1}$, $X_{i,n} = X_n$ and $Y_{i,n-1} = Y_{n-1}, Y_{i,n} = Y_n$ then back orders are more attractive than lost sales, if the number of recovery cycles is equal or more than $\lceil M \rceil$. Here, $M = \frac{PrT_{d1,n}}{rP\left(\frac{X_{n-1}}{D} - S_{t1}\right) - X_{n-1}}$ for disruption at the first stage, and $M = \frac{PrT_{d1,n}}{rP\left(\frac{Y_{n-1}}{D} - S_{t2}\right) - Y_{n-1}}$ for disruption at the second stage.

Proof: From the production capacity constraints (4.38) and (4.61),

$\sum_{i=1}^{M} X_{i,n} + q_{1,n} \leq rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{D} - MS_{t1} - T_{d1,n}\right)$ [For first stage disruption]

$\sum_{i=1}^{M} Y_{i,n} + q_{2,n} \leq rP\left(\sum_{i=1}^{M} \frac{Y_{l+i-1,n-1}}{D} - MS_{t2} - T_{d2,n}\right)$ [For second stage disruption]

From the eq. (4.32) and (4.33)

$$X_{1,n} + q_{1,n} \leq X_{l,n-1}$$
$$X_{i,n} \leq X_{l+i-1,n-1}; i = 2,3,4, \ldots\ldots, M$$

From the eq. (4.54) - (4.57)

$$Y_{1,n} = X_{l,n-1} - q_{2,n}$$
$$Y_{2,n} = X_{l+1,n-1}$$
$$Y_{i,n} \leq Y_{l+i-1,n-1};\ i = 3,4, \ldots\ldots, M$$
$$Y_{i,n} = X_{i,n}; i = 2,3,4, \ldots\ldots, M$$

Using these upper bounds,

$$\sum_{i=1}^{M} X_{i,n} + q_{1,n} \leq rP\left(\frac{MX_{n-1}}{D} - MS_{t1} - T_{d1,n}\right)$$
$$\sum_{i=1}^{M} Y_{i,n} + q_{2,n} \leq rP\left(\frac{MY_{n-1}}{D} - MS_{t2} - T_{d2,n}\right)$$

After simplifying,

$$M \geq \frac{\sum_{i=1}^{M} X_{i,n} + q_{1,n} + rPT_{d1,n}}{rP\left(\frac{X_{n-1}}{D} - S_{t1}\right)} \tag{4.70}$$

$$M \geq \frac{\sum_{i=1}^{M} Y_{i,n} + q_{2,n} + rPT_{d2,n}}{rP(\frac{Y_{n-1}}{D} - S_{t2})} \tag{4.71}$$

Again from the constraints for disruption at the first stage,

$$X_{1,n} + q_{1,n} \leq X_{l,n-1}$$

$$X_{i,n} \leq X_{l+i-1,n-1}; i = 2,3,4, \dots.., M$$

From the constraints for disruption at the second stage,

$$Y_{1,n} = X_{l,n-1} - q_{2,n} = Y_{l,n-1} - q_{2,n}$$

$$Y_{2,n} = X_{l+1,n-1} = Y_{l+1,n-1}$$

$$Y_{i,n} \leq Y_{l+i-1,n-1};\ i = 3,4, \dots.., M$$

Lost sales exist in the optimal solution only if $X_{1,n} + q_{1,n} < X_{l,n-1}, X_{i,n} < X_{l+i-1,n-1}$ and $Y_{1,n}+q_{2,n} < Y_{l,n-1}, Y_{i,n} < Y_{l+i-1,n-1}$.

Using (4.70) and (4.71), back order will be more attractive compare to lost sales when,

$$M \geq \frac{MX_{n-1} +\ rPT_{d1,n}}{rP\left(\frac{X_{n-1}}{D} - S_{t1}\right)} \tag{4.72}$$

$$M \geq \frac{MY_{n-1} +\ rPT_{d2,n}}{rP(\frac{Y_{n-1}}{D} - S_{t2})} \tag{4.73}$$

From the (4.72) and (4.73)

$$\lceil M \rceil = \frac{rPT_{d1,n}}{rP(\frac{X_{n-1}}{D} - S_{t1}) - X_{n-1}} \tag{4.74}$$

[For the disruption at the first stage]

$$\lceil M \rceil = \frac{rPT_{d2,n}}{rP\left(\frac{Y_{n-1}}{D} - S_{t2}\right) - Y_{n-1}} \tag{4.75}$$

[For the disruption at the second stage]

From the (4.74) and (4.75), this implies that, back orders will be more attractive compare to lost sales if the number of the recovery cycles is $M = \frac{rPT_{d1,n}}{rP(\frac{X_{n-1}}{D} - S_{t1}) - X_{n-1}}$ for disruption at the first stage and $M = \frac{rPT_{d2,n}}{rP\left(\frac{Y_{n-1}}{D}-S_{t2}\right)-Y_{n-1}}$ for disruption at the second stage. Since M must be an integer so $\lceil M \rceil$ is used.

Proposition 1: For a given A_1, A_2, H_1, H_2, B, L, if $X_{i,n-1} = X_{n-1}$, $X_{i,n} = X_n$ and $Y_{i,n-1} = Y_{n-1}$, $Y_{i,n} = Y_n$ and $B \gg L$, lost sales will exist in the recovery plan if the number of recovery cycles is less than $\frac{rPT_{d1,n}}{rP(\frac{X_{n-1}}{D}-S_{t1})-X_{n-1}}$ for disruption at the first stage and $\frac{rPT_{d2,n}}{rP\left(\frac{Y_{n-1}}{D}-S_{t2}\right)-Y_{n-1}}$ for disruption at the second stage.

Proof: This is the opposite consequence of **Lemma 1.**

Proposition 2: The recovery time window will be longer with a longer disruption duration.

Proof: From the (4.74) and (4.75),

$$\lceil M \rceil = \frac{rPT_{d1,n}}{rP\left(\frac{X_{n-1}}{D} - S_{t1}\right) - X_{n-1}}$$

[For the disruption at the first stage]

$$\lceil M \rceil = \frac{rPT_{d2,n}}{rP\left(\frac{Y_{n-1}}{D} - S_{t2}\right) - Y_{n-1}}$$

[For the disruption at the second stage]

So, the recovery time window will be longer with a longer disruption duration.

Proposition 3: The recovery time window will be shorter with higher process reliability.

Proof: After reorganising the (4.74) and (4.75),

$$\lceil M \rceil = \frac{T_{d1,n}}{\left(\frac{X_{n-1}}{D} - S_{t1}\right) - \frac{X_{n-1}}{rP}}$$

[For the disruption at the first stage]

$$\lceil M \rceil = \frac{T_{d2,n}}{\left(\frac{Y_{n-1}}{D} - S_{t2}\right) - \frac{Y_{n-1}}{rP}}$$

[For the disruption at the second stage]

It can be said that, with higher process reliability, the recovery time window will be shorter.

4.4 Solution Approach

In this section, firstly, the model for the single occurrence of disruption at either the first or second stage is solved. Then a solution approach is developed for managing the production system with a series of multiple disruptions on a real-time basis.

4.4.1 Solution Approach for a Single Disruption

The mathematical model developed in the previous section, for a single disruption, is a constrained nonlinear programming model which can be solved using a standard search algorithm. To validate the results, two different search approaches: genetic algorithm (GA) and pattern search (PS) have been chosen, to solve the generated test problems.

4.4.2 Solution Approach for a Series of Disruptions

This section proposes a solution approach to generate a recovery plan for a series of disruptions on a real-time basis. When a disruption occurs, a recovery plan can be generated by solving the corresponding mathematical model as discussed in the preceding sub-section. After finalizing the recovery plan, if another disruption occurs within the recovery time window, the recovery plan needs to be revised. Note that the revised plan may need to be updated again, if any further disruption is experienced, and so on. Every time a disruption occurs, the optimisation model developed earlier remains the same; however, some of the parameters must be updated to represent the changed situation. For example, the length of new disrupted period, the number of cycles for recovery, the pre-disruption quantity, and the new limit on production in each cycle must be updated to re-optimise the current disruption scenario. These changes are reflected in the objective function and constraints for re-optimisation. So a search technique, such as the pattern search or genetic algorithm, can still be used, but instead repeatedly with the changed parameters and constraints. That means, the algorithm must run, every time a disruption takes place, to re-optimise the recovery plan. If there is no disruption, the system should follow the original production plan. The main steps of the proposed solution approach for a series of disruptions can be presented as follows.

Step 1*: Determine Q.*

Step 2*: Assign $X_{i,0} = Y_{i,0} = Q$.*

Step 3*: Set n=1, for the first disrupted cycle.*

Step 4*: Set the start time of the recovery window to be the beginning of the disrupted cycle.*

Step 5*: Update the parameters (such as disrupted stage and cycle number, the number of cycles for recovery window, and pre-disruption stock).*

Step 6*: Solve the mathematical model using a search algorithm.*

Step *7: Update the value of $X_{i,n}$ and $Y_{i,n}$ as the revised lot quantities from* ***Step 6.***

Step 8*: If there is any other disruption, go to* ***Step 4.***

Step 9*: Stop.*

The above mentioned solution approaches are coded in MATLAB R2012a with the help of its optimisation toolbox, and were executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU. Parameters are used for PS and GA to design the solution approach is presented below.

4.4.2.1 Parameters for PS

In the proposed PS based solution approach, following PS parameters are used to solve the model:

Maximum number of iterations: 100* Number of variables

Polling order: Success

X tolerance: 1e-6

Function tolerance: 1e-6

Non-linear constraint tolerance: 1e-6

Cache tolerance: 1e-6

Search method: Latin hypercube

Other parameters are set as the default in the optimisation toolbox.

4.4.2.2 Parameters for GA

In the proposed GA based solution approach, following GA parameters are used to solve the model:

Population Size: 100

Population type: Double vector

Hybrid function: Pattern search

Crossover Fraction: 0.8

Maximum number of generations: 5000

Function tolerance: 1e-8

Non-linear constraint tolerance: 1e-8

Other parameters are set as the default in the optimisation toolbox.

4.4.3 Parameters of the Disruption Problem

For experimentation, the following range of data is considered for the disruption problem.

Pre-disruption quantity: $q_{1,n} = [0,\ X_{l,n-1}]$, $q_{2,n} = [0,\ Y_{l,n-1}]$

Disruption duration: $T_{d1,n} = [0.00001,\ \frac{X_{l,n-1}}{rP}]$, $T_{d2,n} = [0.00001,\ \frac{Y_{l,n-1}}{rP}]$

Lost sales cost: $L = [5, 300]$

Back order cost: $B = [5, 500]$

Set-up cost: $A_1 = [5, 300]$, $A_2 = [5, 300]$

Holding cost: $H_1 = [1, 5]$, $H_2 = [1, 5]$

4.5 Experimentation and Results Analysis

In this section, the results of single disruption, as well as multiple disruptions, have been analysed at either one or both stages. To test the proposed approach, 100 test problems have been solved. The test problems are generated randomly by changing the parameters for the given intervals as presented in section 4.4.3. The solutions have been compared with their upper and lower bounds. As the comparison shows consistent results, the result of few sample test problems have been presented in this section.

4.5.1 Single Disruption

The disruption recovery plan for a single disruption either at the first or second stage, has been generated, using both GA and PS, for two problem scenarios with the following data.

S_{t1}=0.000057, S_{t2}=0.000045, A_1=50, A_2=30, H_1=1.2, H_2=1.3, D = 400000, r =0.90, P=500000, M=5, B=10, L=15, $C_{P1} = 30$, $C_{P2} = 10$, $C_{R1} = 12$, $C_{I1} = 0.01$, $C_{I2} = 0.01$, $a = 1000$, $b = 0.5$, $c = 0.75$, $m_1 = 2.5$

For the first scenario, a disruption is considered at the first stage with a pre-disruption quantity of 1200 units and disruption duration of 0.0080. This problem has been solved using both the pattern search and genetic algorithm, and the results (Best, Mean, STD) obtained, out of 30 independent runs, have been presented in Table 4.1. The results include the revised lot quantities in each of the five cycles of the recovery window and the total profit.

Table 4.1: Results for a single disruption at the first stage

Approach	Revised lot quantity										Total Profit (Best, Mean, STD)
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
PS	3792	4992	5366	5366	5366	5366	5366	5366	5366	5366	1158572, 1158406, 24.6
GA	3820	5020	5337	5337	5366	5366	5366	5366	5366	5366	1158549, 1158375, 25.8

The second scenario considered a disruption at the second stage, with a pre-disruption quantity of 600 units and disruption duration of 0.0076. The detailed results are presented in Table 4.2. From the results in Tables 4.1 and 4.2, it is clear that the solutions produced by both the techniques are very similar.

Table 4.2: Results for a single disruption at the second stage

Approach	Revised lot quantity										Total Profit (Best, Mean, STD)
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
PS	4766	5366	5366	5366	5201	5201	5364	5364	5365	5365	1170445, 1170237,26.3
GA	4733	5333	5366	5366	5198	5198	5366	5366	5366	5366	1170444, 1170226, 28.4

4.5.2 Multiple Disruptions

To demonstrate the usefulness of our proposed algorithm in solving different scenarios with multiple disruptions, over a period of time, the basic data of the single disruption cases is used. For experimental purposes, a series of random production disruptions were generated at random point in time, either at the first or second stage. Table 4.3 presents a case problem with different random combinations of disrupted stage, disrupted cycle number, pre-disruption quantity, and disruption duration. Although these disruptions happen continuously, a case problem has been presented with only seven disruptions as a sample representation. The production quantities are revised immediately after each disruption takes place in the system. For the first disruption (from the Table 4.3), the production quantities ($X_{i,1}$ and $Y_{i,1}$ for i =1 to 5) are revised for the next five cycles (recovery time window) as shown in Table 4.4. When the second disruption occurred within the recovery time window of the first disruption, the variables and constraints are updated for the changed environment. That means, the new variables $X_{i,2}$ and $Y_{i,2}$ are now revised for the next five cycles (for new recovery time window for the second disruption) from the point of disruption. This process continues as long as disruptions occur in the system. The problem is solved using the proposed algorithm where the search algorithm (either PS or GA) is run for 30 times for each disruption, and the best result is then recorded. For each disruption, the revised production quantity in each cycle and the maximum total profit (best result) are presented, in Table 4.4 for the search technique PS, and in Table 4.5 for GA.

Table 4.3: A disruption case problem

Disruption number (n)	Disrupted Stage	Disrupted cycle number from previous disruption	Pre-disruption quantity	Disruption duration
1	Second	1	750	0.0045
2	First	2	450	0.0078
3	Second	3	500	0.0030
4	First	3	1225	0.0058
5	Second	2	0	0.0068
6	First	4	500	0.0098
7	First	2	0	0.0087
...	...	...	...	...

Table 4.4: The detailed results with PS

Disruption number (n)	Revised lot quantity										Total Profit
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
1	5366	4616	5366	5366	5366	5366	5366	5366	5366	5366	1181054
2	4678	5128	5365	5365	5348	5348	5358	5358	5347	5347	1163792
3	5348	4848	5358	5358	5347	5347	5366	5366	5366	5366	1179532
4	4121	5346	5366	5366	5366	5366	5366	5366	5366	5366	1179939
5	5366	5366	5366	5366	5366	5366	5366	5366	5366	5366	1180563
6	4082	4582	5128	5128	5350	5350	5230	5230	5359	5359	1111322
7	4619	4619	5121	5121	5229	5229	5359	5359	5366	5366	1119822

Table 4.5: The detailed results with GA

Disruption number (n)	Revised lot quantity										Total Profit
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
1	5366	4616	5366	5366	5366	5366	5366	5366	5366	5366	1181053
2	4682	5312	5316	5316	5366	5366	5366	5366	5366	5366	1163798
3	5366	4866	5366	5366	5366	5366	5366	5366	5366	5366	1181523
4	4141	5366	5366	5366	5366	5366	5366	5366	5366	5366	1180814
5	5366	5366	5366	5366	5366	5366	5366	5366	5366	5366	1180560
6	4165	4665	4884	4884	5366	5366	5366	5366	5366	5366	1111303
7	4505	4505	4996	4996	5366	5366	5366	5366	5366	5366	1115328

The total profits (best result) obtained from the pattern search and genetic algorithm based solution approach have also been presented in Tables 4.4 and 4.5 respectively. It is observed that both techniques confirm very similar results, but PS based solution approach performs slightly better than GA in most cases. GA is a stochastic search process that may converge towards local optima, or even arbitrary points rather than the global optimum of the problem. On the other hand, PS is combined with the Latin hypercube search method and this helps it to provide better performance than other random search based algorithms. This may be the reason for PS' slightly better solutions over GA in this study.

4.5.3 Comparing with Lower and Upper Bounds

The problem considered in this chapter is different from the existing literature in many different ways. So there is no algorithm or benchmark problem that can be used for comparison of our proposed algorithm. However, the upper bound and lower bound of the solutions can be derived and compared. For this purpose, two bounds have been derived and an alternative model as follows.

- Upper bound that is based on the production-inventory system without any disruption (that is under ideal condition);
- Lower bound that permits only lost sales to recover from disruption; and
- An alternative model that considers recovery within the disrupted cycle.

Upper bound: The total profit in a production-inventory system, for a given number of cycles (n_c), under ideal condition (that is without any disruption) should be the upper bound of total profit that can be expressed as follows.

Total Profit = Total Revenue – Total Cost

$$= n_c\{[m_1(C_{P1}+C_{P2})Y_{i,0}] - \frac{1}{2}H_1\left[\frac{(X_{i,0})^2}{rP}\right] - A_1 - \frac{C_{P1}}{r}(X_{i,0}) - \left(\frac{1}{r}-1\right)(X_{i,n})$$
$$- \frac{C_{I1}C_{P1}}{r}(X_{i,0}) - a\,(A_1)^{-b}(r)^c - \frac{1}{2}H_2\left[\frac{(Y_{i,0})^2}{rP}\right] - A_2 - C_{P2}(Y_{i,0})$$
$$- C_{I2}C_{P2}(Y_{i,0}) - a\,(A_2)^{-b}\}$$

Lower Bound: Suppose the disruption is recovered via lost sales only. That means the shortage due to disruption will not be fulfilled. For a given number of cycles, the total profit for this system should be the lower bound, because the shortages are not managed in an efficient manner. In this system, the production quantity in the disrupted cycles will be lower than the pre-assigned production quantity. For a given number of cycles, the lower bound can be calculated from {(Upper Bound) – $(n_c \times L \times rP \times Disruption\ period)$}.

<u>An Alternative Model</u>: An alternative model can be developed where the recovery must be planned within the disrupted cycle using the idle time of the cycle and lost sales as well as back order options. If the disruption duration is less than idle time of that cycle, then there will be no lost sales. This model uses the same principle of our disruption management model, but the only difference is that the system is recovered in one cycle. The total profit of this model will provide a tighter lower bound as discussed below.

To calculate the bounds, 25 disruptions have generated randomly within 52 production cycles. The total profit, as well as the lost sales and back orders, for the upper bound, lower bound, alternative model and this model, are calculated and compared in Table 4.6.

Table 4.6: Comparison of the results from 52 production cycles

Cost or Profit (in million $)	Upper Bound	Disruption Recovery model	Alternative Model	Lower Bound
Total back order cost	0	0.08262	0.00099	0
Total lost sales cost	0	0.277	0.986	1.229
Total profit	15.294	13.922	10.700	9.565

The upper and lower bounds are the best and worst case scenarios of the production system respectively. From Table 4.6, it is clear that the proposed disruption recovery plan provides a much better solution than the lower bound as well as the tighter bound, while the alternative model shows better results than the lower bound solution.

The total profit for each production cycle, obtained from the above four alternative solutions, is presented in Figure 4.3. When a production cycle faces a disruption, the total profit of that cycle goes to its lower bound if the system is recovered with only the lost sales option. However, for a disruption free cycle, the profit is the same as the original production system. For the lower bound scenario, it is observed that the profit goes down to negative in the 16th and 28th cycle because of the higher disruption duration in those cycles. In summary, the proposed disruption model clearly shows significantly better solutions in comparison to both lower bounds.

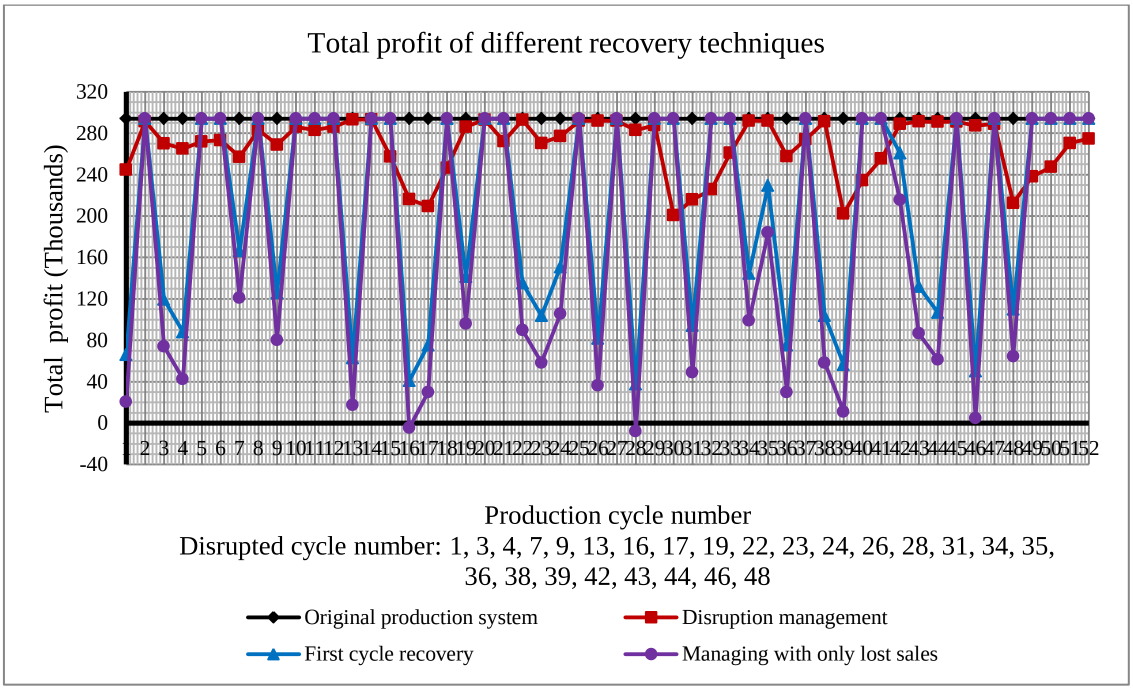

Figure 4.3: Total profit in each production cycle for different recovery techniques

4.5.4 A Real-Life Case Study

A real world production problem has been discussed in this section. A specific manufacturing line of a pharmaceutical company has been studied. The company produces several types of products such as tablet, capsule, syrup, ointment and cream, and injection and drops. Some of their products have very high demand that need special attention such as a dedicated production line for each product or few of them. A dedicated production line is studied that produces only one product (known as XPA-C tablet). This is basically a tablet of paracetamol group that is widely used as pain and fever reliever. The production process of the tablet is presented in Figure 4.4. In the process, first, the raw materials (paracetamol power, caffeine and binders) are mixed properly in the form of granulation. Then the mixture is moved to a compression machine that compresses the powder into tablets of uniform size and weight. To form a tablet, the granulated material is metered into a cavity formed by two punches and a die, and then the punches are pressed together with appropriate force to fuse the material together. To ensure quality of the products, the tablets are inspected during and after the compression. After finishing the compression process, the tablets are moved to the packaging section for blistering and packing. In blistering, the tablets are filled in a blister pack, which is known as a tablet strip, is useful for protecting the product against external factors such as humidity and contamination for extended periods of time. There are ten tablets per tablet strip. Finally, the tablet strips are filled in a carton and shipped to the store.

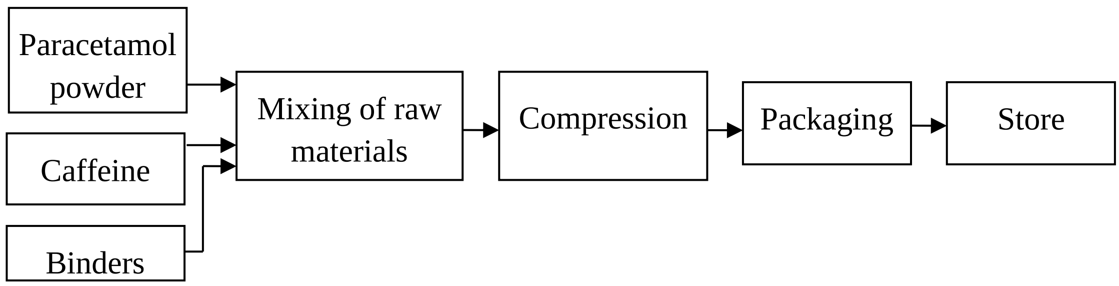

Figure 4.4: Production process of the XPA-C tablet

The tablets are produced in a batch in all processes of mixing, compression and packaging. After completing the processing of a batch in a stage, it is moved to the next stage for further processing. It was observed that the mixing stage has no record of disruption for a long period of time. For this reason, the mixing process is kept out of the disruption study. So the system is defined with compression and packaging processes, where the compression represents the first stage and the packaging as the second stage. Once a batch of tablets from compression is shipped to packaging area, the packaging is organized as a batch. Like any other manufacturing processes, the machine breakdown is a very common event in this production environment. After a breakdown, it takes some times to repair the machines and start the production again. Currently, the company is using only lost sales option, which is a substandard approach (as described in section 4.5.3), to recover from the disruptions. It is also observed that, breakdown can occur at any stage at any time and compression stage has more machine breakdown than packaging stage. It is also common that, a new breakdown can occur within the recovery time window of the previous breakdown.

This disruption case problem can be solved by using this proposed approach as outlined in section 4.5.3. To solve the disruption case problem, some data were collected directly from the production line and the historical record of the company. Some others are approximated by consulting with the plant manager.

The annual forecasted demand is determined based on the historical record which is 4928400 strips and the production capacity of the line is approximately 5916000 strips per year. Some defective items are produced in the systems. The process reliability is calculated as an average of non-defective items over the last five years which is 92%.

After processing of each batch, it is a standard process to clean and inspect the machines thoroughly before starting a new batch. The time involved in checking and cleaning is considered set-up time. The set-up time is calculated as 0.525 hours for the compression stage and 0.438 hours for the packaging stage with the help of time study. The set-up cost is considered as the costs for labour in preparing raw materials, and cleaning and inspecting machines. As suggested by the company, the set-up cost is 1725 Taka (21.5625 USD) for compression stage and 1075 Taka (13.4375 USD) for packaging stage. The annual holding cost per unit per year is approximated as 21 Taka (0.2625 USD) and 45 Taka (0.5625 USD) for the compression and packaging stage respectively. The backorder and lost sales costs are also approximated as 1600 Taka/strip/year (20 USD) and 40 Taka/strip (0.5 USD) respectively.

Production, rejection and inspection costs are collected as follows.

Production cost at compression stage = 15.5 Taka/strip (0.19375 USD)
Production cost at packaging stage = 5.2 Taka/ strip (0.065 USD)
Cost for rejection = 8 Taka/ strip (0.1 USD)
Inspection cost for both stages = 1% of production cost

Cost of interest and depreciation is approximated as based on theory (Cheng, 1989) with the parameters, $a = 1000$, $b = 0.5$, and $c = 0.75$.

Table 4.7: Data for machine breakdown scenario

Disruption number (n)	Disrupted Stage	Disrupted cycle number from previous disruption	Pre-disruption quantity	Disruption duration (hours)
1	Compression	1	3550	11.2
2	Packaging	11	0	6.3
3	Compression	4	2040	8.1
4	Compression	12	950	18.6
5	Compression	7	1220	16.5
6	Packaging	2	2425	14.2
7	Compression	14	1160	22.8
8	Compression	10	1840	5.8
9	Packaging	14	210	15.4
10	Compression	3	150	23.6

The breakdown scenarios are collected from the company register. The proposed model has been implemented for a series of mix of ten dependent and independent disruptions for 73 production cycles. Seven of those disruptions were occurred in the compression stage and the rest three are at packaging stage. The recovery plan after each disruption is implemented immediately. The breakdown scenarios are presented in Table 4.7.

This disruption case problem is solved by our proposed approach. The batch sizes are revised immediately after each breakdown takes place in the system. The problem is solved using the PS based proposed algorithm and the best results are recorded. For each disruption, the revised batch size in each cycle and the maximum total profit are presented in Table 4.8.

Table 4.8: The detailed results using PS

Disruption number (n)	Revised batch size after the n^{th} disruption										Total Profit
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
1	16606	20156	21490	21490	21490	21490	21490	21490	21490	21490	28987
2	21490	21490	21490	21490	21490	21490	21490	21490	21490	21490	30177
3	18593	20633	21490	21490	21490	21490	21490	21490	21490	21490	29740
4	18688	19638	21366	21366	21489	21489	21490	21490	21490	21490	27144
5	18530	19750	21490	21490	21490	21490	21490	21490	21490	21490	27707
6	19065	16640	21490	21490	21490	21490	21490	21490	21490	21490	28236
7	17238	18398	19996	19996	21489	21489	21490	21490	21490	21490	25509
8	19107	20987	21490	21490	21490	21490	21490	21490	21490	21490	30301
9	21280	21070	21490	21490	21490	21490	21490	21490	21490	21490	27944
10	18016	18166	19741	19741	21480	21480	21490	21490	21490	21490	25594

Table 4.9: Comparison of the results from 73 production cycles

	Ideal System	Proposed model	Current Policy
Total back order cost	0	11490.80	0
Total lost sales cost	0	9358.48	44268.70
Total profit	524691.50	497165.20	448782.98
Percentage of improvement	10.78%		

Currently, the company is using only lost sales option to recover from production disruptions which is discussed in Section 4.5.3. The benefits of implementing of our disruption recovery model are summarized in Table 4.9. It is observed that, the proposed disruption technique clearly shows significantly better results in comparison to company's current policy. Under an ideal condition, the company will have a total profit of 524691.50USD from 73 production cycles. Under the disrupted environment, this profit reduces to 448782.98USD, if the company uses the lost sales option to recover from the disruptions. In this case, the total lost sales is 44268.70USD. If the proposed approach is applied, the total profit will be increased to 497165.20USD with both back order and lost sales cost 11490.80 and 9358.48 USD respectively. In this case, the total profit is 10.78% more than the existing practice.

4.6 Sensitivity Analysis

There are some important variables that have a significant impact on the total profit. These variables are: process reliability, disruption duration, pre-disruption quantity, back order cost and lost sales cost. In this section, the relationship of these variables with the total profit is analysed. The pattern search based algorithm was applied to perform the sensitivity analysis.

This section presents a number of studies, in each of them one variable is changed. The reminder always have default values of 0.9 for process reliability, 0.008 for first stage disruption duration, 0.0085 for second stage disruption duration, 0 for pre-disruption quantity for both stages, 10 for back order and 15 for lost sales.

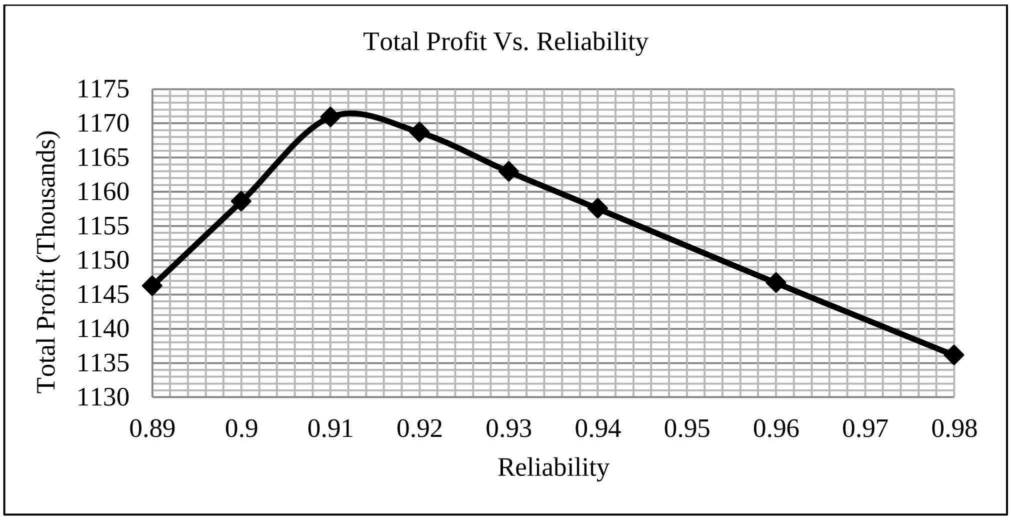

Figure 4.5: Changes of total profit with reliability when disruption is in the first stage

The relationship between process reliability and the total profit is shown in Figure 4.5. It is observed from Figure 4.5, that the total profit is highest when process reliability is 0.91.

Figure 4.6 presents the changes to total profit as pre-disruption quantity changes when the process is disrupted at the first stage. It shows that, the total profit decreases very little with changes to pre-disruption quantity.

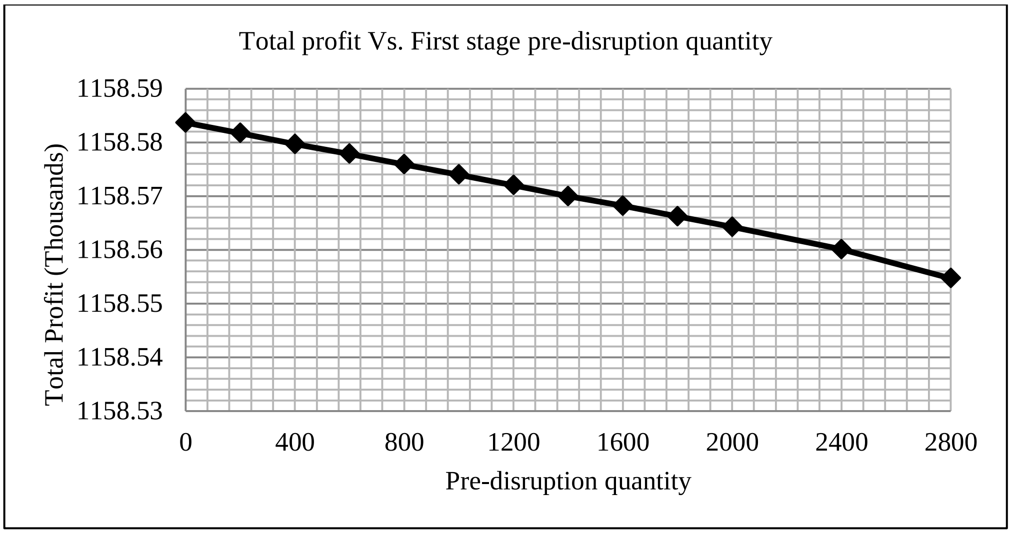

Figure 4.6: Changes of total profit with the pre-disruption quantity when disruption is in the first stage

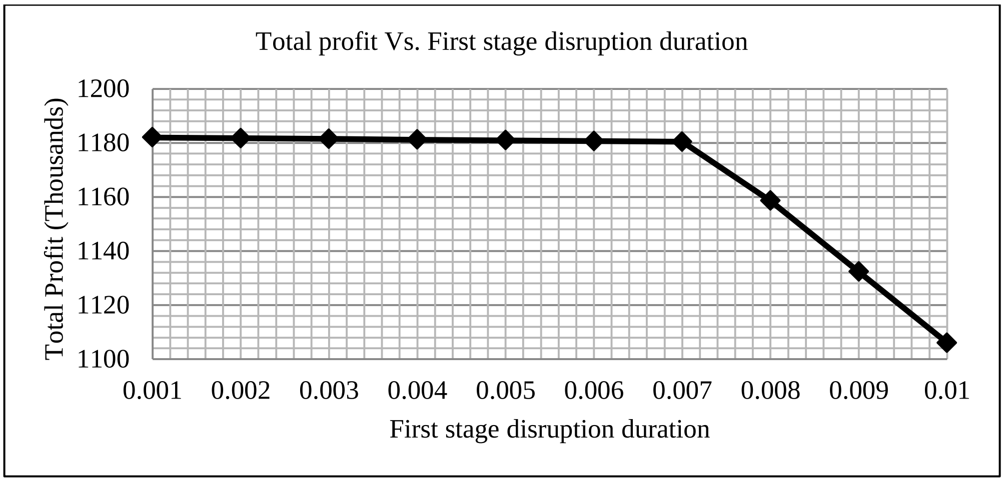

Figure 4.7: Changes of total profit with first stage disruption duration

The variation of total profit with disruption duration for when a process is disrupted at the first stage is presented in Figure 4.7. It is observed, that the total profit decreases very slowly when the disruption duration is less than 0.007. However, after then the rate of decrement is very high. This is because of the existence of the lost sales cost. This is because when the disruption duration is high, then lost sales exists in the optimal solution.

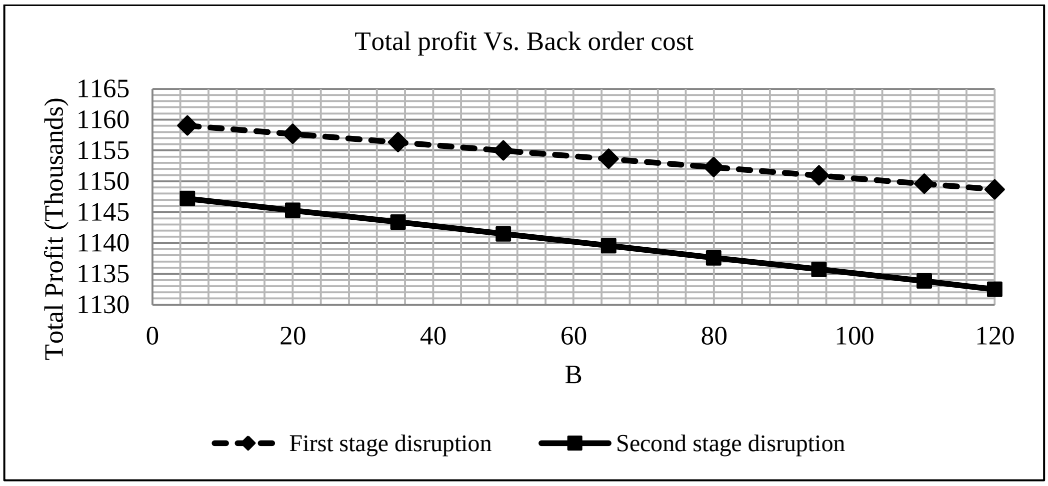

Figure 4.8: Changes of total profit with back order cost

Total profit decreases along with back order and lost sales cost for disruptions at either the first or second stage. Figures 4.8 and 4.9 show the changes of the total profit with the back order and lost sales cost respectively. From Figures 4.8 and 4.9, it is observed, that both back order and lost sales exist in the optimal solution.

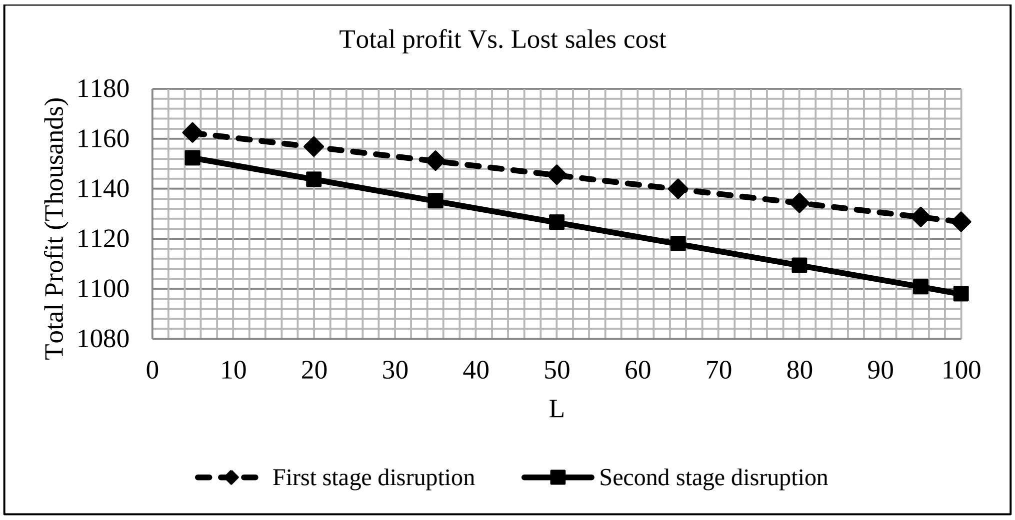

Figure 4.9: Changes of total profit with lost sales cost

Figure 4.10 presents the changes of total profit with pre-disruption quantity when the process is disrupted at the second stage. It is observed that the total profit decreases very little with increased pre-disruption quantity.

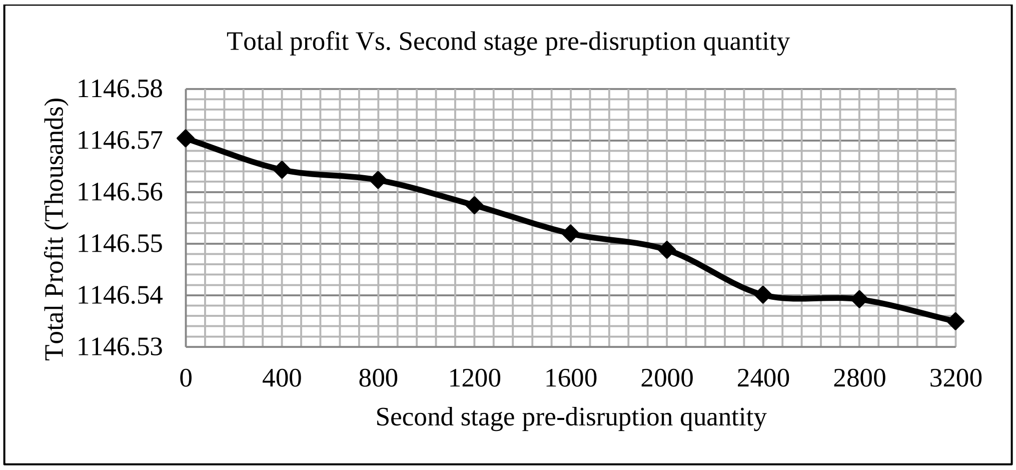

Figure 4.10: Changes of total profit with pre-disruption quantity when disruption is in the second stage

Total profit decreases with disruption duration when the process is disrupted at the second stage, as presented in Figure 4.11. After the disruption duration of 0.007, the rate of decrement is very high. This is because of existence of the lost sales in the optimal solution.

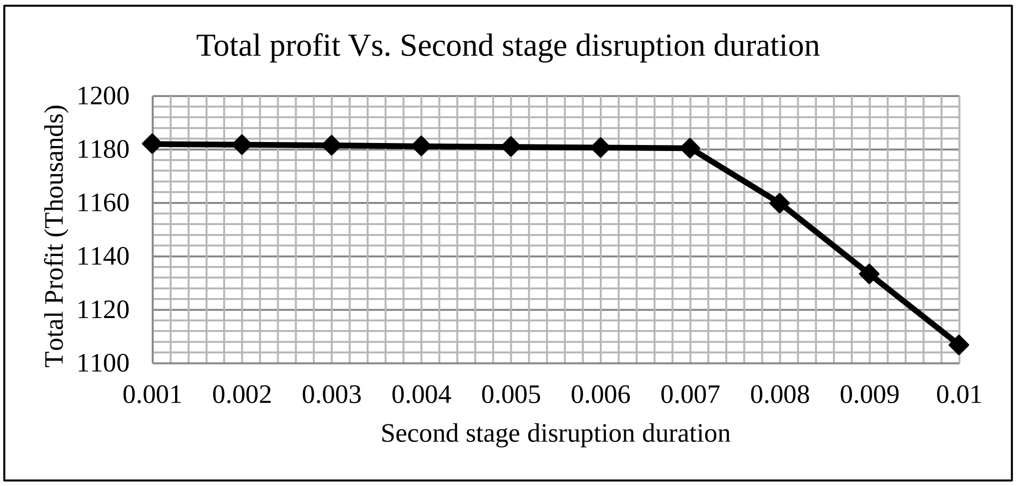

Figure 4.11: Changes of total profit with second stage disruption duration

4.7 Chapter Summary

The objective of this research was to develop a real-time recovery plan from production disruptions for a two-stage and single item batch production-inventory system. The model is developed to recover from either single disruption or a series of production disruptions on a real-time basis, while considering both back order and lost sales options. Process reliability was also considered because imperfect production processes are common and the total profit function was maximised to obtain optimal recovery plans. A pattern search and genetic algorithm based solution approaches were proposed to obtain the recovery plan either for a single or a series of disruptions. Both methods show similar results, but pattern search with the Latin hypercube search based solution approach performed better than the genetic algorithm based approach.

It was assumed that the lot size is fixed throughout the planning period under ideal conditions. But in real-life situations, the lot is needed to be split to meet the transportation and warehouse capacity. In some situations, there may have more than two stages production systems but our model is applicable only for two-stage production system. The demand rate is assumed to be known and constant but practically, the demand can be fluctuated. There may have special orders outside of regular demand trend and sometimes a priority can be given to produce for special orders. For all these variations, the management must decide how to deal with them when the proposed approach is applied.

In chapter 4, the disruption recovery problem for a two-stage production-inventory system with process reliability that is subject to production disruption has been addressed. The extension of this study is to explore the problem for a three-stage mixed production-inventory system, which I have chosen as the direction of my research work in the next chapter.

Chapter 5 Three-Stage Mixed Production-Inventory System

This chapter proposes a recovery plan for managing disruptions in a three-stage production-inventory system under a mixed production environment. First, a mathematical model is developed to deal with a disruption at any stage while maximising total profit during the recovery time window. A new and efficient heuristic is proposed for solving the developed mathematical model. Second, multiple disruptions are considered, where a new disruption may or may not affect the recovery plans of earlier disruptions. The heuristic, developed for a single disruption, is extended to deal with a series of disruptions so that it can be implemented for disruption recovery on a real-time basis. The heuristic solutions with are compared with those obtained by a standard search algorithm for a set of randomly generated disruption test problems. Finally, this chapter presents some numerical examples and a real-world case study to demonstrate the benefits and usefulness of the proposed approach.

5.1 Introduction

The study in this chapter is an extension of the study in chapter 4. This chapter considers a three-stage production-inventory system under mixed production environment. Thus unlike the two-stage case considered in the previous chapter, the model in this chapter consists of three stages and the disruption can occur at any stage at any time. The problem presented in this chapter is much more complex because a three-stage production-inventory system under mixed production environment is considered.

Batch production is a very common and popular technique in manufacturing systems in which products are produced in batches to minimise the overall production cost while maximising utilization of the available capacity. Sometimes, the batch size and processing time can be constant, depending on the nature of the process, as well as on the capacity of the equipment. In some cases, the processing time can be either dependent or independent of the batch sizes. For example, the mixing time of raw materials is independent of the batch size because a quantity of them which does not use the full capacity of the equipment can be mixed. In real-life production lines, it is very common to process materials or products in a series of stages, one after another, to obtain the final products. There are numerous industries, such as pharmaceutical, textile and manufacturing that produce products using multiple stages, during which the production environment can be either similar or different, and processes such as batch or continuous production or a combination of both. Even if the production environment is continuous, a product may be produced in batches due to there being higher production capacity than demand.

This study has been motivated by the disruption scenarios observed in a real-life pharmaceutical production line. That production line consists of three sequential processes (known as mixing, compression, and packaging) which can easily be defined as three stages of the production process. The production process starts with a discrete batch production in the mixing stage and is followed by two continuous production processes in the compression and packaging stages. The production line is sometime disrupted, mainly due to machine breakdowns that occur at any stage of the line without having any prior knowledge. Although management repairs the machines as soon as possible, it is not easy to reschedule the production line to minimise the overall loss with a minimum effect on customer goodwill. This is a common problem in many industrial units, and hence it requires a new real-time problem solving approach, such as the disruption recovery method proposed in this chapter.

In this chapter, a new and efficient heuristic is proposed for solving the developed mathematical model, with its results compared with the solutions obtained from a pattern search using a set of randomly generated disruption test problems. This study also considers multiple disruptions, one after another in a series, that occur in any stage at any point in time and may or may not affect the plans amended after previous disruptions. If a new disruption occurs during the recovery time window of another, a new revised plan which considers the effects of both disruptions must be derived. Accordingly, as this is a continuous process, the heuristic is extended to deal with a series of disruptions on a real-time basis by incorporating a modified version of that developed for a single disruption.

This is the first quantitative model that develops a disruption recovery model for both a single and multiple disruptions, on a real-time basis, in a three-stage mixed production-inventory system. Finally, this study shows how the proposed methodology can be applied to real-time disruption recovery planning, with randomly generated test problems, as well as a real-world case problem from the aforementioned pharmaceutical company.

The main contributions of this chapter can be summarized as follows.

i. Development of a mathematical model for disruption recovery in a three-stage mixed production-inventory system. As a disruption scenario is not known in advance and not possible to predict, the recovery plan is revised for periods after the disruption occurs on a real-time basis.
ii. Development of a new efficient heuristic for generating a revised production plan after a disruption.
iii. Extension of this heuristic to deal with multiple disruptions on a real-time basis. As any new disruptions may or may not affect the plans revised after previous ones, their scenarios may be considered dependent and independent, both of which the extended heuristic can handle.
iv. Application of the developed methodology to a real-world case problem from a pharmaceutical company.

5.2 Problem Description

The ideal three-stage production system is shown in Figure 5.1. In it, stage 1 processes the raw materials as a batch and the production procedures in stages 2 and 3 are continuous. As the system requires different processing techniques, it is recognized as a mixed-production environment. In Figure 5.1, X_0 is the batch size in the first stage and, as it is less than or equal to the capacity of the equipment, the processing time is independent of it and, therefore, fixed. After the first stage, the whole batch is transferred to the second stage for further processing into smaller lots (size Y_0), whereby Y_0 is limited by the capacity of the transfer bucket and is equal to $\frac{X_0}{n}$, where n is a positive integer. Then, each batch is transferred to the third stage for further processing with a lot size of Z_0 and, finally, the finished products are transferred to storage.

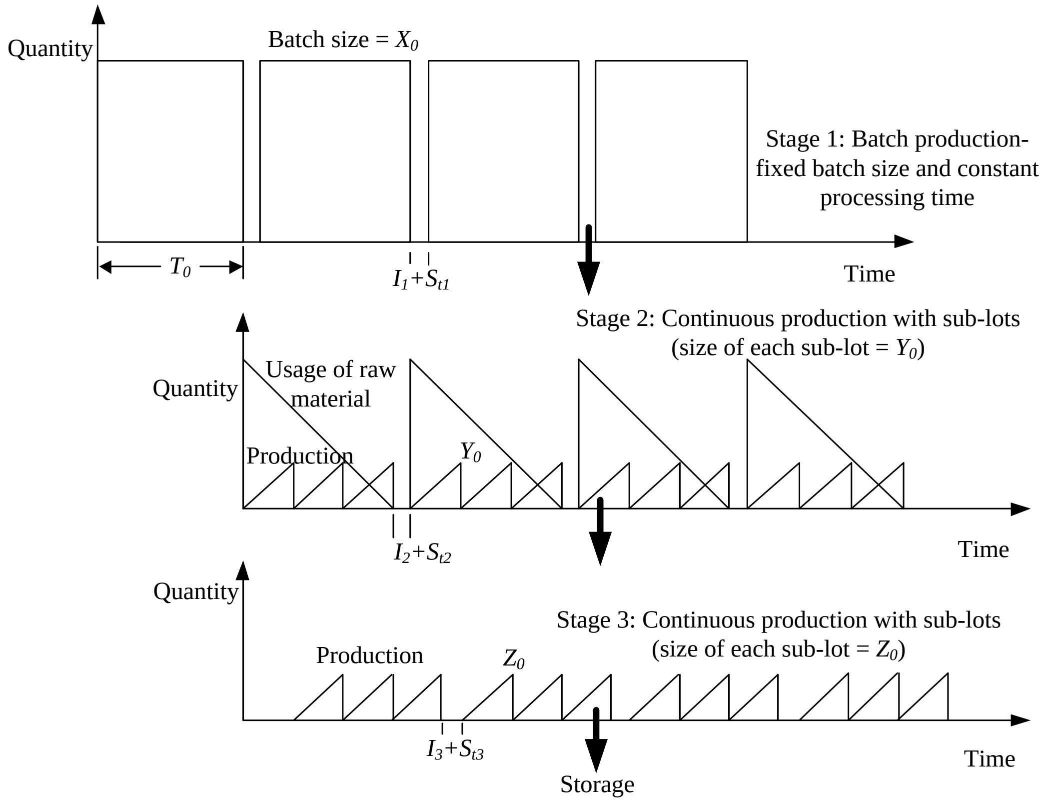

Figure 5.1: Ideal three-stage production system

A disruption is any kind of interruption in a production system, for example, a machine breakdown, raw material shortage, power cut, labour strike, etc., which can happen in any stage and at any time in the process. Once the system is disrupted, it is assumed that it will be inoperable for a period of time, known as *disruption duration*, with the strategy taken to recover from the disruption known as a *recovery strategy*. In this chapter, to recover from a disruption, the following two cost factors are considered.

i. **Back orders:** the portion of demand that cannot be fulfilled at the scheduled time but will be delivered at a later date, with a penalty, if the production system is capable.

ii. **Lost sales:** the portion of demand lost if customers will not wait for the required stock to be replenished following the production process not being capable of fulfilling demand.

After the occurrence of a disruption in a system, its planned production quantities for all stages are revised for some periods in the future (known as *recovery periods* or *recovery time windows*) until the system returns to its normal schedule, which is known as a *recovery plan* (Paul et al., 2014, in press). As, if a disruption occurs in any stage, it has significant impacts on other stages because they all operate in a coordinated fashion, all their production plans must be revised after the occurrence of a disruption to minimise the overall loss incurred. Although, in this study, the recovery periods is assumed to be specified by the management of the system, it can be considered a decision variable in the model.

In any industrial production environment, the system can face multiple disruptions, one after another, on a real-time basis. In this case, one disruption can occur within the recovery periods of another which is known as a dependent disruption and, as this is a complex situation, the combined effect of dependent disruptions should be considered in the development of a recovery plan. This is achieved by re-optimising the production schedule within the new recovery time window under the changed production environment. The proposed heuristic (discussed earlier) for dealing with a single disruption is later extended to consider multiple disruptions on a real-time basis and is capable of handling dependent, independent and mixes of dependent and independent disruptions on a real-time basis.

5.2.1 Notation used in this Study

To formulate the mathematical model, the following notations are used.

X_0 Batch size in stage 1 under ideal conditions (known)

T_0 Processing time of batch in first stage under ideal conditions (constant and known)

S_{t1} Set-up time in stage 1 (time per set-up)

S_{t2} Set-up time in stage 2 (time per set-up)

S_{t3} Set-up time in stage 3 (time per set-up)

A_1 Set-up cost in stage 1 ($ per set-up)

CP_1 Production cost per unit in stage 1 ($ per unit)

CM_1 Cost per unit material loss in stage 1 ($ per unit)

D	Average demand per unit time
CP_2	Production cost per unit in stage 2 ($ per unit)
C_2	Capacity of machine in stage 2 (units per unit time)
H_2	Holding cost per unit per unit time in stage 2 ($ per unit per unit time)
CP_3	Production cost per unit in stage 3 ($ per unit)
C_3	Capacity of machine in stage 3 (units per unit time)
n	Number of sub-lots in stages 2 and 3 for each full batch in stage 1 $= \left\lceil \frac{X_0}{\text{Capacity of transfer bucket between stages 2 and 3}} \right\rceil$
Y_0	Size of each sub-lot in stage 2 under ideal conditions $= \frac{X_0}{n}$
Z_0	Size of each sub-lot in stage 3 under ideal conditions $= \frac{X_0}{n}$
I_1	Idle time after processing batch in stage 1 $= \frac{X_0}{D} - T_0 - S_{t1}$
I_2	Idle time after processing n sub-lots in stage 2 $= \frac{X_0}{D} - \frac{X_0}{C_2} - S_{t2}$
I_3	Idle time after processing n sub-lots in stage 3 $= \frac{X_0}{D} - \frac{X_0}{C_3} - S_{t3}$
T_p	Pre-disruption duration
T_d	Disruption duration
T_R	Recovery time
M	Number of batches within recovery time window
X_i	Size of batch i in stage 1 after disruption; $for\ i = 1,2,\dots,M$ (decision variable)
Y_j	Size of sub-lot j in stage 2 after disruption; $for\ j = 1,2,\dots,Mn$ (decision variable)
Z_j	Size of sub-lot j in stage 3 after disruption; $for\ j = 1,2,\dots,Mn$ (decision variable)
B	Back orders cost per unit per unit time ($ per unit per unit time)
L	Lost sales cost per unit ($ per unit)
m_1	Mark-up of unit selling price $[m_1(CP_1 + CP_2 + CP_2)]$ (must be greater than 1)

5.2.2 Assumptions of the Study

In this study, the following assumptions are made.

i. The production rate in any stage is greater than the average demand rate.

ii. A single item is produced in the system.

iii. The sizes of the sub-lots in stages 2 and 3 are determined based on the capacity of the transfer bucket.

iv. The recovery time window begins immediately after the occurrence of a disruption which can occur in any stage at any point in time.

The model is developed for a single type of item and, as is common, assumes that, to fulfil demand on time, the production rate is higher than the demand rate. However, for a higher demand rate, it can easily be revised by using an option for outsourcing. In a multi-stage production environment, it is common to use a transfer bucket to transport semi-processed materials between stages. In this study, the sub-lot sizes are determined based on the capacity of the transfer bucket to balance the production system. To make the recovery process meaningful in practice, the revised plan is generated after the disruption is experienced by the system, that is, on a real-time basis.

5.3 Model Formulation

In this section, a mathematical model is developed for a single disruption in any stage in a three-stage production system. Firstly, the equations are derived for different costs and revenue, and then the disruption recovery problem is formulated as a constrained mathematical model in which the objective is to maximise total profit during the recovery periods, with the total profit function derived by subtracting all the relevant costs from the revenue. Finally, relevant capacity, delivery, demand and stage-balancing constraints are developed to make the model realistic. The decision variables are revised production quantities in each cycle, and both the total back orders and lost sales during the recovery periods. In modelling the recovery planning problem, different batch sizes for different cycles are considered, for example, X_i for cycle i in stage 1 and, similarly, Y_i and Z_i in stages 2 and 3 respectively. Although one lot size can be considered for all cycles in a stage, allowing different sizes makes the model more flexible (or more general) for optimisation. As the model uses both of back orders and lost sales costs, its solution may suggest different quantities for different production cycles in order to maximise total profit. To establish balanced coordination among the stages, the lot sizes in stages 2 and 3 are considered as equal. However, one can easily modify the model to have a different relationship, such as one lot size in stage 3 being equal to multiple lot sizes in stage 2 or vice versa.

5.3.1 Cost and Revenue Calculations

In this section, the different cost and revenue equations are derived for a single disruption's recovery time window considering the set-up, holding, production back orders and lost sales costs. The total set-up cost is determined as the cost per set-up multiplied by the number of set-ups, and the production cost the per unit production cost multiplied by the quantity produced, both during the recovery periods. The average holding cost is calculated as the unit holding cost multiplied by the total inventory during the recovery periods, the back orders cost as the unit back order cost multiplied by the number of back order units and its time delay, and the lost sales cost as unit lost sales cost multiplied by the number of lost sales units. The revenue during the recovery periods is determined as the unit selling price multiplied by the quantity produced during the recovery periods, and total profit, which is the objective function, calculated by subtracting all the costs from the revenue.

5.3.2 Mathematical Model for Disruption at First Stage

In some production environments, a batch of products (or semi-products) may be unacceptable due to the effects of a disruption during processing. In others, an entire batch of products may be either unaffected or recovered by applying corrective actions. For this reason, two scenarios are considered for disruption recovery in this mathematical model: (i) no loss of materials; and (ii) 100% loss of materials.

5.3.2.1 Scenario 1: No Loss of Materials

In this scenario, as there is no loss of materials due to a disruption, pre-disruption processed materials can be used during the recovery periods.

Number of batches in recovery periods, $M = \left\lceil \frac{T_R}{T_0 + I_1} \right\rceil$

Cost formulation

Set-up cost in first stage

$$= A_1 M \tag{5.1}$$

Production cost in first stage

$$= CP_1 * (X_1 + X_2 + \cdots + X_M)$$

$$= CP_1 * \sum_{i=1}^{M} X_i \tag{5.2}$$

Production cost in the second stage

$$= CP_2 * (Y_1 + Y_2 + \cdots + Y_{Mn})$$

$$= CP_2 * \sum_{j=1}^{Mn} Y_j \tag{5.3}$$

Average raw material holding cost in the second stage

$$= \frac{H_2}{2} * \left(X_1 \frac{X_1}{C_2} + X_2 \frac{X_2}{C_2} + \cdots + X_M \frac{X_M}{C_2} \right) = \frac{H_2}{2C_2} * \sum_{i=1}^{M} {X_i}^2 \tag{5.4}$$

Production cost in the third stage

$$= CP_3 * (Z_1 + Z_2 + \cdots + Z_{Mn}) = CP_3 * \sum_{j=1}^{Mn} Z_j \tag{5.5}$$

Back orders cost

$$= B * \sum_{i=1}^{k} X_i \left(T_d + (i-1)S_{t1} - iI_1\right) \tag{5.6}$$

Lost sales

$$= L * (MX_0 - \sum_{i=1}^{M} X_i) \tag{5.7}$$

Revenue formulation

Revenue

$$= m_1 (CP_1 + CP_2 + CP_3) * \sum_{i=1}^{M} X_i \tag{5.8}$$

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised and obtained by using equations (5.1) to (5.8) and subject to constraints (5.9) to (5.17).

$$X_i \leq X_0;\ \forall i \tag{5.9}$$

[to ensure delivery and transportation constraints]

$$Y_j \leq Y_0;\ \forall j \tag{5.10}$$

[to ensure delivery and transportation constraints]

$$Z_j \leq Z_0;\ \forall j \tag{5.11}$$

[to ensure delivery and transportation constraints]

$$MX_0 \geq \sum_{i=1}^{M} X_i \tag{5.12}$$

[to ensure delivery and transportation constraints]

$$I_1, I_2, I_3 \geq 0 \tag{5.13}$$

[to ensure non-negative idle time]

$$Y_{(i-1)n+1} + Y_{(i-1)n+2} + \cdots + Y_{in} = X_i;\ \forall i \tag{5.14}$$

[to balance batches and sub-lots]

$$Y_j = Z_j;\ \forall j \tag{5.15}$$

[to balance production system]

$$T_d + (k-1)S_{t1} - kI_1 \geq 0;\ For\ k = 1,2,\ldots,\left\lceil \frac{T_d}{I_1} \right\rceil \tag{5.16}$$

[to ensure non-negative delay]

$$X_i, Y_j \text{ and } Z_j \geq 0;\ \forall i,j \tag{5.17}$$

5.3.2.2 Scenario 2: 100% Loss of Materials

In this scenario, as materials are completely lost due to a disruption, the pre-disruption processed materials cannot be used during the recovery periods. Therefore, a recovery policy which is proposed, as the disruption duration is considered a summation of the pre-disruption and actual disruption durations, it is $(T_d + T_p)$.

Cost formulation

Set-up cost in first stage

$$= A_1 M \tag{5.18}$$

Production cost in first stage

$$= CP_1 * (X_1 + X_2 + \cdots + X_M)$$

$$= CP_1 * \sum_{i=1}^{M} X_i \tag{5.19}$$

Cost due to material loss

$$= CM_1 * X_0 \tag{5.20}$$

Production cost in the second stage

$$= CP_2 * (Y_1 + Y_2 + \cdots + Y_{Mn}) = CP_2 * \sum_{j=1}^{Mn} Y_j \tag{5.21}$$

Average raw material holding cost in the second stage

$$= \frac{H_2}{2} * \left(X_1 \frac{X_1}{C_2} + X_2 \frac{X_2}{C_2} + \cdots + X_M \frac{X_M}{C_2}\right) = \frac{H_2}{2C_2} * \sum_{i=1}^{M} {X_i}^2 \tag{5.22}$$

Production cost in the third stage

$$= CP_3 * (Z_1 + Z_2 + \cdots + Z_{Mn}) = CP_3 * \sum_{j=1}^{Mn} Z_j \tag{5.23}$$

Back orders cost

$$= B * \sum_{i=1}^{k} X_i \,(T_d + T_p + (i-1)S_{t1} - iI_1) \tag{5.24}$$

Lost sales

$$= L * (MX_0 - \sum_{i=1}^{M} X_i) \tag{5.25}$$

Revenue formulation

Revenue

$$= m_1(CP_1 + CP_2 + CP_3) * \sum_{i=1}^{M} X_i \tag{5.26}$$

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised and subject to constraints (5.27) to (5.35).

$$X_i \leq X_0;\ \forall i \tag{5.27}$$
[to ensure delivery and transportation constraints]

$$Y_j \leq Y_0;\ \forall j \tag{5.28}$$
[to ensure delivery and transportation constraints]

$$Z_j \leq Z_0;\ \forall j \tag{5.29}$$
[to ensure delivery and transportation constraints]

$$MX_0 \geq \sum_{i=1}^{M} X_i \tag{5.30}$$
[to ensure delivery and transportation constraints]

$$I_1, I_2, I_3 \geq 0 \tag{5.31}$$
[to ensure non-negative idle time]

$$Y_{(i-1)n+1} + Y_{(i-1)n+2} + \cdots + Y_{in} = X_i;\ \forall i \tag{5.32}$$
[to balance batches and sub-lots]

$$Y_j = Z_j;\ \forall j \tag{5.33}$$
[to balance production system]

$$T_d + T_p + (k-1)S_{t1} - kI_1 \geq 0;\ For\ k = 1,2,\dots,\left\lceil \frac{T_d + T_p}{I_1} \right\rceil \tag{5.34}$$
[to ensure non-negative delay]

$$X_i, Y_j \text{ and } Z_j \geq 0;\ \forall i, j \tag{5.35}$$

Proposition 1: the production system will be optimally recovered using only the back orders cost if (i) $T_d \leq MI_1$ for no loss of materials and (ii) $(T_d + T_p) \leq MI_1$ for 100% loss of materials.

Proof: the idle time after producing a batch in stage 1 is I_1. As there are M cycles in the recovery time window (T_R), the total idle time in it is MI_1.

If the disruption duration is less than MI_1, the production system is capable of managing the disruption duration within T_R and there will only be a delay in product delivery. Therefore, it can be said that, if $T_d \leq MI_1$ for no loss of materials and $(T_d + T_p) \leq MI_1$ for

100% loss of materials, the production process will be optimally recovered using the back orders cost.

Proposition 2: sales will be lost in the recovery policy if (i) $T_d > MI_1$ for no loss of materials and (ii) $(T_d + T_p) > MI_1$ for 100% loss of materials.

Proof: this is the opposite consequence to that in **Proposition 1.**

Proposition 3: the minimum recovery periods, without incurring lost sales in the solution, is $(T_0 + I_1) * \left\lceil \frac{T_d}{I_1} \right\rceil$ for no loss of materials and $(T_0 + I_1) * \left\lceil \frac{T_d + T_p}{I_1} \right\rceil$ for 100% loss of materials.

Proof: from the condition of the existence of only back orders for no loss of materials,

$$T_d \leq MI_1$$

$$\Rightarrow \; M \geq \frac{T_d}{I_1}$$

As M is considered as an integer, it can be written as:

$$\Rightarrow \; M \geq \left\lceil \frac{T_d}{I_1} \right\rceil$$

$$\Rightarrow \; \frac{T_R}{T_0 + I_1} \geq \left\lceil \frac{T_d}{I_1} \right\rceil \text{ [inserting value of } M\text{]}$$

After simplifying,

$$T_R \geq (T_0 + I_1) * \left\lceil \frac{T_d}{I_1} \right\rceil \tag{5.36}$$

Similarly, for 100% loss of materials,

$$T_R \geq (T_0 + I_1) * \left\lceil \frac{T_d + T_p}{I_1} \right\rceil \tag{5.37}$$

5.3.3 Mathematical Model for Disruption at Second Stage

The formulation of the mathematical model for a single occurrence of disruption in stage 2 is presented as follows.

Cost formulation

Number of batches in recovery periods, $M = \left\lceil \frac{T_R}{\frac{X_0}{C_2}+I_2} \right\rceil$

$l =$ disrupted sub-lot number (any single number between 1 and n)

If $T_p \geq \frac{iY_0}{C_2}$, then $l = i + 1$; $(i = 0,1,2,..,n-1)$

Pre-disruption raw material level, $X_p = X_1 - T_p C_2$ (5.38)

Set-up cost at first stage

$$= A_1 M \tag{5.39}$$

Production cost at first stage

$$= CP_1 * (X_1 + X_2 + \cdots + X_M)$$

$$= CP_1 * \sum_{i=1}^{M} X_i \tag{5.40}$$

Production cost at second stage

$$= CP_2 * (Y_1 + Y_2 + \cdots + Y_{Mn})$$

$$= CP_2 * \sum_{j=1}^{Mn} Y_j \tag{5.41}$$

Average raw material holding cost at second stage

$$= \frac{H_2}{2C_2} * \sum_{i=1}^{M} {X_i}^2 + H_2 * \left\{ X_p T_d + \sum_{i=2}^{k} X_i \left(T_d - (i-1) I_2\right) \right\} \tag{5.42}$$

Production cost at third stage

$$= CP_3 * (Z_1 + Z_2 + \cdots + Z_{Mn})$$

$$= CP_3 * \sum_{j=1}^{Mn} Z_j \tag{5.43}$$

Back orders cost

$$= B\{(Y_l + Y_{l+1} + \cdots + Y_n) * T_d + (Y_{n+1} + Y_{n+2} + \cdots + Y_{2n}) * (T_d - I_2) + \cdots + \left(Y_{(k-1)n+1} + Y_{(k-1)n+2} + \cdots + Y_{kn}\right) * (T_d - (k-1)I_2)\} \tag{5.44}$$

Lost sales

$$= L * (MnY_0 - \sum_{j=1}^{Mn} Y_j) \tag{5.45}$$

Revenue formulation

Revenue

$$= m_1(CP_1 + CP_2 + CP_3) * \sum_{i=1}^{Mn} Y_i \tag{5.46}$$

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised and subject to constraints (5.47) to (5.55).

$X_i \leq X_0; \forall i$
[to ensure delivery and transportation constraints] (5.47)

$Y_j \leq Y_0; \forall j$
[to ensure delivery and transportation constraints] (5.48)

$Z_j \leq Z_0; \forall j$
[to ensure delivery and transportation constraints] (5.49)

$Y_j = Y_0; j = 1,2, \dots, l-1$
[to ensure pre-disruption production constraint] (5.50)

$I_1, I_2 \text{ and } I_3 \geq 0$
[to ensure non-negative idle time] (5.51)

$Y_{(i-1)n+1} + Y_{(i-1)n+2} + \cdots + Y_{in} = X_i; \forall i$
[to balance batches and sub-lots] (5.52)

$Y_j = Z_j; \forall j$
[to balance production system] (5.53)

$T_d - (k-1)I_2 \geq 0; For\ \ k = 1,2, \dots, \left\lceil \frac{T_d}{I_2} \right\rceil$
[to ensure non-negative delay] (5.54)

X_i, Y_j and $Z_j \geq 0; \forall i, j$ (5.55)

Proposition 4: the production system will be optimally recovered using only the back orders cost if $T_d \leq MI_2$ and using both the back orders and lost sales costs if $T_d > MI_2$.

Proof: as there are M cycles in the recovery time window (T_R), the total idle time in it is MI_2. If the disruption duration is less than MI_2, the production system is capable of producing all the lost items due to the disruption within T_R. So there will be a delay in product delivery, and in this case, having only back orders will be the most appropriate for generating the optimal solution. Therefore, it can be said that the production system will be optimally recovered using the back orders cost if $T_d \leq MI_2$.

If the disruption duration is greater than the idle time, the system is not capable of producing all the lost items during the recovery periods. In this case, the system will utilize its idle time to produce some of the lost items. So the system will experience both lost sales and back orders (delay delivery) in generating solutions if $T_d > MI_2$.

Proposition 5: the minimum recovery periods, without incurring lost sales in the solution, is $\left(\frac{X_0}{C_2} + I_2\right) * \left\lceil\frac{T_d}{I_2}\right\rceil$

Proof: from the condition of the existence of only back orders,

$$T_d \leq MI_2$$

As M is considered as an integer, it can be written as:

⇨ $M \geq \left\lceil\frac{T_d}{I_2}\right\rceil$

⇨ $\frac{T_R}{\frac{X_0}{C_2}+I_2} \geq \left\lceil\frac{T_d}{I_2}\right\rceil$ [inserting value of M]

After simplifying,

$$T_R \geq \left(\frac{X_0}{C_2} + I_2\right) * \left\lceil\frac{T_d}{I_2}\right\rceil \qquad \textbf{(proven)}$$

5.3.4 Mathematical Model for Disruption at Third Stage

The formulation of the mathematical model for a single occurrence of disruption in stage 3 is presented as follows.

Cost formulation

Number of batches in recovery periods, $M = \left\lceil \frac{T_R}{\frac{X_0}{C_3}+I_3} \right\rceil$

$l =$ disrupted sub-lot number (any single number between 1 and n)

Set-up cost at first stage

$$= A_1 M \tag{5.56}$$

Production cost at first stage

$$= CP_1 * (X_1 + X_2 + \cdots + X_M)$$

$$= CP_1 * \sum_{i=1}^{M} X_i \tag{5.57}$$

Production cost at second stage

$$= CP_2 * (Y_1 + Y_2 + \cdots + Y_{Mn})$$

$$= CP_2 * \sum_{j=1}^{Mn} Y_j \tag{5.58}$$

Average raw material holding cost at second stage

$$= \frac{H_2}{2C_2} * \sum_{i=1}^{M} {X_i}^2 \tag{5.59}$$

Production cost at third stage

$$= CP_3 * (Z_1 + Z_2 + \cdots + Z_{Mn})$$

$$= CP_3 * \sum_{j=1}^{Mn} Z_j \tag{5.60}$$

Back orders cost

$$= B\{(Z_l + Z_{l+1} + \cdots + Z_n) * T_d + (Z_{n+1} + Z_{n+2} + \cdots + Z_{2n}) * (T_d - I_3) + \cdots + \left(Z_{(k-1)n+1} + Z_{(k-1)n+2} + \cdots + Z_{kn}\right) * (T_d - (k-1)I_3)\} \tag{5.61}$$

Lost sales

$$= L * \left(MnZ_0 - \sum_{j=1}^{Mn} Z_j\right) \tag{5.62}$$

Revenue formulation

Revenue

$$= m_1(CP_1 + CP_2 + CP_3) * \sum_{i=1}^{Mn} Z_i \tag{5.63}$$

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised and subject to constraints (5.64) to (5.72).

$$X_i \leq X_0;\ \forall i \tag{5.64}$$
[to ensure delivery and transportation constraints]

$$Y_j \leq Y_0;\ \forall j \tag{5.65}$$
[to ensure delivery and transportation constraints]

$$Z_j \leq Z_0;\ \forall j \tag{5.66}$$
[to ensure delivery and transportation constraints]

$$Z_j = Z_0;\ j = 1,2,\ldots,l-1 \tag{5.67}$$
[to ensure pre-disruption production constraint]

$$I_1, I_2\ and\ I_3 \geq 0 \tag{5.68}$$
[to ensure non-negative idle time]

$$Z_{(i-1)n+1} + Z_{(i-1)n+2} + \cdots + Z_{in} = X_i;\ \forall i \tag{5.69}$$
[to balance batches and sub-lots]

$$Y_j = Z_j;\ \forall j \tag{5.70}$$
[to balance production system]

$$T_d - (k-1)I_3 \geq 0;\ For\ \ k = 1,2,\ldots,\left\lceil\frac{T_d}{I_3}\right\rceil \tag{5.71}$$
[to ensure non-negative delay]

$$X_i, Y_j \text{ and } Z_j \geq 0; \forall i, j \tag{5.72}$$

Proposition 6: the production system will be optimally recovered using only the back orders cost if $T_d \leq MI_3$ and using both the back orders and lost sales costs if $T_d > MI_3$.

Proof: this is the same as that for **Proposition 4**.

Proposition 7: the minimum recovery periods, without incurring lost sales in the solution, is $\left(\frac{X_0}{C_3} + I_3\right) * \left\lceil\frac{T_d}{I_3}\right\rceil$

Proof: this is the same as that for **Proposition 5.**

5.4 Solution Approach

With the help of the developed propositions, a heuristic is first developed to solve the model for a single occurrence of disruption. Then, it is extended to manage multiple disruptions, as a series, on a real-time basis. A disruption scenario can be defined as the combination of a disrupted stage, and pre-disruption and disruption durations. As, in reality, these parameters follow stochastic processes, in this study, uniform random variables are used to generate them. However, one can use different probability distributions. To judge the quality of the heuristic solutions, another standard search technique is also used to solve the model.

5.4.1 Heuristic for Single Disruption

The proposed heuristic for a single occurrence of disruption in any stage is described. Different parameters for an ideal system are input in Step 1 and disruption scenarios generated using uniformly random variables input in Step 2. If there is any disruption in stage 1, in Step 3, the model determines a recovery plan which is generated in Steps 4 and 5 after occurrences of disruptions in stages 2 and 3 respectively. In steps 4 and 5, the production system utilizes the idle time for possible recovery from a disruption. If the idle time is greater than the disruption duration, then the production system is capable of producing the lost items due to the disruption, but there will be a delay in product delivery. For this reason, back orders will be the only appropriate means to generate the solution. If the idle time is less than the disruption duration, then the production system is capable of producing only a part of the lost items during the recovery periods, and so the system will generate partly lost sales and partly delayed delivery (for back orders). When lost sales are appropriate for generating the solution, the minimum recovery periods, without incurring lost sales, is determined by using propositions 5 and 7. The steps in the proposed heuristic are as follows.

Step 1: input scenarios of three stages in ideal production system.
Step 2: input disrupted stage with pre-disruption (T_p) and actual disruption (T_d) period.
Step 3: for disruption in stage 1, determine disruption duration. If materials completely lost, disruption duration equal to $\left(T_p + T_d\right)$, otherwise, T_d.

3.1: input recovery periods (T_R).
3.2: determine number of batches and total idle time in T_R.
3.3: if total idle time ≥ disruption duration, obtain recovery plan using equations (5.73) to (5.75).

$$X_i = X_0;\ \forall i \tag{5.73}$$
$$X_i = X_0;\ \forall i \tag{5.74}$$
$$Z_j = Z_0;\ \forall j \tag{5.75}$$

Determine all costs, revenue and total profit.

3.4: if total idle time < disruption duration, obtain recovery plan using equations (5.76) to (5.81).

$$X_1 = 0 \tag{5.76}$$
$$X_i = X_0;\ \text{For } i = 2,3, \dots, M \tag{5.77}$$
$$Y_j = 0;\ \text{For } j = 1,2, \dots, n \tag{5.78}$$
$$Y_j = Y_0;\ \text{For } j = n+1, n+2, \dots, Mn \tag{5.79}$$
$$Z_j = 0;\ \text{For } j = 1,2, \dots, n \tag{5.80}$$
$$Z_j = Z_0;\ \text{For } j = n+1, n+2, \dots, Mn \tag{5.81}$$

Determine all costs, revenue and total profit.

Minimum recovery periods, without including lost sales in solution, $(T_0 + I_1) * \left\lceil \frac{\text{Disruption duration}}{I_1} \right\rceil$.

Step 4: for disruption in stage 2

4.1: input disrupted sub-lot number (l), disruption duration (T_d) and recovery periods (T_R).
4.2: determine total idle time in T_R.
4.3: if total idle time ≥ disruption duration, obtain recovery plan using equations (5.82) to (5.84).

$$X_i = X_0;\ \forall i \tag{5.82}$$
$$X_i = X_0;\ \forall i \tag{5.83}$$
$$Z_j = Z_0;\ \forall j \tag{5.84}$$

Determine all costs, revenue and total profit.

4.4: if total idle time < disruption duration,
Lost time (T_L) = T_d − total idle time,
Production capacity in lost time = $C_2 T_L$ and
Number of sub-lots in lost time (m) = $\left\lfloor \frac{C_2 T_L}{Y_0} \right\rfloor$
Then, obtain recovery plan using equations (5.85) to (5.92).

$$Y_j = Y_0;\ \text{For } j = 1,2, \dots, 2n \tag{5.85}$$
$$Y_j = 0;\ \text{For } j = 2n+1, 2n+2, \dots, 2n+m \tag{5.86}$$

$Y_{2n+m+1} = Y_0 - C_2T_L + mY_0$ (5.87)

$Y_j = Y_0$; For $j = 2n + m + 2, 2n + m + 3, .., Mn$ (5.88)

$X_1 = X_0$ (5.89)

$X_2 = X_0$ (5.90)

$X_i = Y_{(i-1)n+1} + Y_{(i-1)n+2} + \cdots + Y_{in}$; For $i = 3,4, .., M$ (5.91)

$Z_j = Y_j; \forall j$ (5.92)

Determine all costs, revenue and total profit.

Minimum recovery periods, without including lost sales in solution, $\left(\frac{X_0}{C_2} + I_2\right) * \left\lceil\frac{T_d}{I_2}\right\rceil$.

Step 5: for disruption in stage 3

5.1: input disrupted sub-lot number (l), disruption duration (T_d) and recovery periods (T_R).

5.2: determine total idle time in T_R.

5.3: if total idle time ≥ disruption duration, obtain recovery plan from equations (5.93) to (5.95).

$X_i = X_0; \forall i$ (5.93)

$X_i = X_0; \forall i$ (5.94)

$Z_j = Z_0; \forall j$ (5.95)

Determine all costs, revenue and total profit.

5.4: if total idle time $<$ disruption duration,

Lost time (T_L) $= T_d -$ total idle time,

Production capacity in lost time $= C_3T_L$

Number of sub-lots in lost time (m) = $\left\lfloor\frac{C_3T_L}{Z_0}\right\rfloor$

Then, obtain recovery plan from equations (5.96) to (5.103).

$Z_j = Z_0$; For $j = 1,2, .., 2n$ (5.96)

$Z_j = 0$; For $j = 2n + 1, 2n + 2, \dots, 2n + m$ (5.97)

$Z_{2n+m+1} = Z_0 - C_3T_L + mZ_0$ (5.98)

$Z_j = Z_0$; For $j = 2n + m + 2, 2n + m + 3, .., Mn$ (5.99)

$Y_j = Z_j; \forall j$ (5.100)

$X_1 = X_0$ (5.101)

$X_2 = X_0$ (5.102)

$X_i = Y_{(i-1)n+1} + Y_{(i-1)n+2} + \cdots + Y_{in}$; For $i = 3,4, .., M$ (5.103)

Determine all costs, revenue and total profit.

Minimum recovery periods, without including lost sales in solution, $\left(\frac{X_0}{C_3} + I_3\right) * \left\lceil\frac{T_d}{I_3}\right\rceil$.

Step 6: record the results.

Step 7: stop.

5.4.2 Extended Heuristic for Multiple Disruptions

In this section, the heuristic developed for recovering from a single occurrence of disruption, is extended to manage multiple disruptions on a real-time basis. To do this, a recovery plan is obtained from the heuristic after each disruption, with the revised production scenarios saved and then used as a foundation for recovering from the next disruption. The steps in the extended heuristic for managing multiple disruptions are described below.

Step A: determine and input ideal conditions (fixed batch size, sub-lot size, machine capacity, difference cost data etc.).

Step B: input disrupted stage, disruption duration and time since previous disruption.

Step C: solve model with proposed heuristic developed in Section 5.4.1 using updated parameters, such as disruption scenario, objective function and constraints.

Step D: update decision variables from ***Step C*** and record revised production plan after disruption.

Step E: if another disruption, go to ***Step B*** and repeat ***Steps B-D***.

Step F: stop.

5.4.3 An Alternative Approach

The mathematical model developed in Section 3 for a single disruption is a constrained nonlinear one which can be solved using a standard search algorithm. As there is no standard test set available for the problem considered in this research, to validate and judge the quality of the results obtained from the heuristic, another approach, namely a pattern search (PS)-based technique, is chosen to solve it. Both methods were coded in MATLAB R2012a and executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU, with their best results from 10 independent runs compared. In the PS-based technique, the following parameters were used.

Maximum number of iterations: 100* Number of decision variables

Maximum function evaluation: 1000000

Polling order: Random

X tolerance: 1e-10

Function tolerance: 1e-10

Nonlinear constraint tolerance: 1e-10

Cache tolerance: 1e-10

Search method: Latin hypercube

Other parameters were set as the defaults in the optimisation toolbox of MATLAB R2012a.

5.4.4 Range of parameters

For experimentation, the following data range was considered with a discrete uniform distribution for the disruption problem.

Disruption duration in the first stage: [0.0001, T_0]

Disruption duration in the second stage: [0.0001, $\frac{X_0}{C_2}$]

Disruption duration in the third stage: [0.0001, $\frac{X_0}{C_3}$]

Lost sales cost: $L =$ [2, 50]

Back orders cost: $B =$ [0.1, 10]

Set-up cost: $A_1 =$ [5, 300]

Holding cost: $H_2 =$ [0.005, 2]

Production cost: CP_1,CP_2, CP_3 =[0.5, 10]

5.5 Analysis of Experimentation and Results

In this section, the solutions for both the single and multiple disruption cases are analysed. To judge their quality those obtained from the proposed heuristic, this study has experimented using 90 disruption test problems randomly generated using a uniform random distribution by changing the parameters for the given intervals (presented in Section 5.4.4) which were solved using both the proposed heuristic and PS-based approaches. As their results were consistent, only those for a few sample test problems are discussed in this section. Then, the heuristic was modified for multiple disruptions on a real-time basis, as described in Section 5.4.2.

5.5.1 Single Disruption

In this section, the solutions for a single disruption are analysed. Although it was experimented on 90 random disruption test problems, for illustrative purposes, five different sample instances were used by arbitrarily changing the disruption data, with their parameters shown in Table 5.1. For the disruption in stage 1, the two scenarios, that is, with no loss of materials and 100% loss of materials, were considered test problems 1 and

2 respectively, with three other problems used for disruptions in stages 2 and 3, and the following data for analysing the results. Although, as production in stages 2 and 3 is continuous, it was considered that no set-up was required for them, it could be considered if appropriate.

D = 8000 units per day, X_0 = 3000 units, T_0 = 0.30 days, S_{t1} = 1 hour, S_{t2} = 0, S_{t3} = 0, A_1 = 50, C_2 = 10000 units per day, C_3 = 9000 units per day, CP_1 = 5, CP_2 = 3, CP_3 = 1, CM_1 = 5, H_2 = 0.01 per unit per day, bucket capacity = 1000 units, B = 0.5, L = 20, m_1 = 2.5, T_R = 2 days.

Table 5.1: Five test problems with single disruption

Disruption problem	Disrupted stage	Disruption duration (days)	Pre-disruption duration (days)
1	1 (no loss of materials)	0.15	0.00
2	1 (100% loss of materials)	0.23	0.035
3	2	0.20	0.060
4	3	0.10	0.085
5	3	0.28	0.008

The test problems shown in Table 5.1 were solved using the proposed heuristic and PS-based techniques, with the results presented in Table 5.2. Both approaches provided similar solutions to all test problems. The heuristic showed insignificant changes in results for different runs and for PS technique, the best results recorded out of 10 independent runs. To recover from disruptions, only the back orders cost was used for problems 1, 3 and 4, only the lost sales cost for problem 2 and both for problem 5. If any recovery plan used the lost sales cost, the minimum recovery periods without lost sales were also determined. For problems 2 and 5, the minimum recovery periods were 2.667 and 2.625 days respectively (when using only the back orders cost). If management does not specify the recovery periods, the system may use this minimum recovery periods to avoid the lost sales cost.

Table 5.2: Results obtained from heuristic and PS

Disruption problem	Total profit		Deviation (%)	Computational time (seconds)		Recovery strategy	Minimum recovery periods without lost sales (days)
	Heuristic	PS (best result)		Heuristic	PS		
1	241898.0	241898.0	0.0	1.60	4.72	Only back orders	--
2	127177.5	127177.5	0.0	1.91	6.53	Only lost sales	2.667
3	242409.5	242409.4	0.0	0.75	28.74	Only back orders	--
4	242560.5	242560.5	0.0	0.74	28.60	Only back orders	--
5	232442.2	232442.2	0.0	0.73	33.49	Both back orders and lost sales	2.625

To judge the quality of the solutions obtained from the proposed heuristic, 90 test problems with disruption were randomly generated using a uniform distribution. They were solved through both approaches, with the best results (out of 10 independent runs) obtained by the PS technique. Although the two approaches produced similar solution quality (with insignificant differences, as shown in Table 5.3), there was a significant difference in their computational times. In terms of quality, the average percentage of deviation, calculated using equation (5.104), between the results from the two approaches was only 0.00034% which could be considered negligible. Indeed, it may merely have been due to errors in rounding the values of the decision variables. Apart from its capability to produce quality solutions, the heuristic took significantly less average computational time than the PS technique (see the second column in Table 5.3).

$$\text{Percentage of deviation} = \frac{(\text{Total profit from heuristc} - \text{Total profit from PS})}{\text{Total profit from PS}} \times 100\% \qquad (5.104)$$

Table 5.3: Comparison of results for 90 different disruption test problems

Approach	Average computational time (seconds)	Average percentage of deviation (%)
Heuristic	1.024	0.00034
PS	20.36	

5.5.2 Multiple Disruptions

To demonstrate the usefulness of the proposed heuristic for solving different scenarios with a series of disruptions over a period of time, this study used the basic data of the single disruption problem presented in Section 5.5.1 which could manage the first disruption. Then, if another occurred after the recovery periods of a previous one, it could be considered another single disruption that would not affect the revised plan based on the previous disruption. However, as a new disruption within the recovery periods of any previous one may affect this revised plan, its revised plan for its recovery periods must be considered a set of additional restrictions. For experimental purposes, a series of ten dependent disruptions, one after the other, with different conditions, is generated randomly as shown in Table 5.4. Although they could happen continuously, this study presents only ten as a sample representation.

To maximise total profit in the system, the batch and sub-lot sizes were revised immediately after each disruption took place. The problem was then solved using the proposed heuristic for multiple disruptions, as presented in Section 5.4.2, with the results recorded after each disruption and shown in Table 5.5 for lost sales, back orders and total profit. It is observed that, in the recovery plans, there were only back orders for disruption numbers 1, 7, 8, 9 and 10, and to maximise total profit, only lost sales for disruption number 6 while both lost sales and back orders were present for disruption numbers 2, 3, 4 and 5.

Table 5.4: Disruption scenarios for series of disruptions

Disruption number	Disrupted stage	Disruption duration (days)	Time since previous disruption (days)
1	1	0.10	--
2	3	0.28	0.50
3	2	0.16	0.67
4	2	0.24	1.50
5	3	0.18	0.67
6	1	0.27	1.20
7	1	0.20	1.60
8	3	0.08	0.80
9	2	0.22	0.33
10	3	0.14	1.75
....	....	....	....

Table 5.5: Results obtained from heuristic for series of disruptions

Disruption number	Total lost sales	Total back orders	Total profit
1	0	212.50	242460
2	5400	1186.60	232440
3	5400	675.23	232960
4	5400	705.23	232910
5	5400	588.83	233040
6	60000	0	142180
7	0	1420.00	241253
8	0	1227.50	241450
9	0	1230.00	241440
10	0	887.50	241790
....	....	....	....

5.6 A Real-Life Case Study

The developed model was used to solve a real-world case problem involving a pharmaceutical company's paracetamol tablet production line which had three stages, mixing, compression and packaging, and was dedicated to only one product. A flow diagram of the production line is presented in Figure 5.2. In this process, the raw materials were initially blended for a fixed time and then the blended material moved to the compression stage to shape each tablet. The shaped tablets were then blistered and packed in the third stage and, finally, the finished items were transferred to storage.

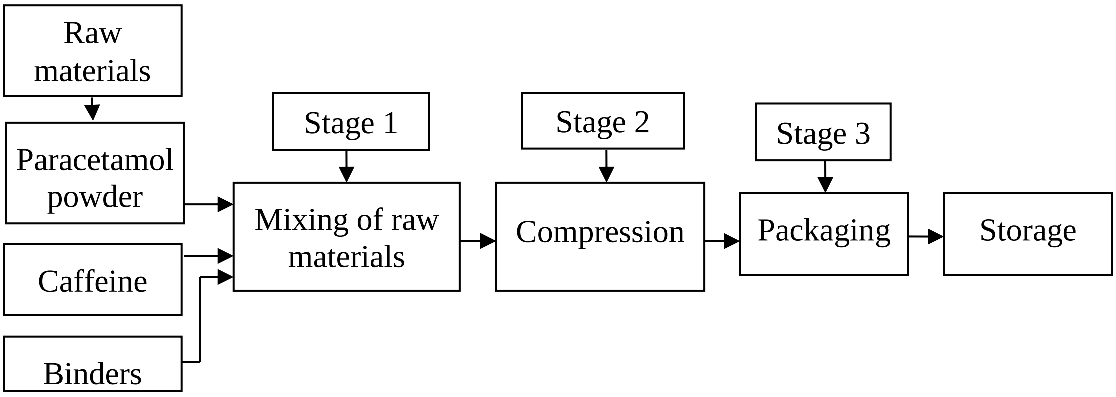

Figure 5.2: Three-stage production process for pharmaceutical product

The relevant data were collected from historical records, including a number of disruption scenarios, for all three stages in the case study. It was observed that machine breakdown was a common disruption, whereby the system became inoperable for a certain period of time. It was also observed that the compression and packaging stages had more machine breakdowns than the mixing stage. However, although the mixing stage suffered fewer disruptions, as the materials could be completely lost, scenarios of both loss and no loss of materials are considered for a disruption in the first stage. Moreover, it was observed that machine breakdown could occur in any stage at any time. Currently, the company uses only the lost sales cost to recover from disruptions which means that its production capacity during these periods always leads to shortages and materials are completely lost.

This disruption problem could be solved by applying our developed model and the proposed heuristic. To demonstrate this, some data was collected directly from the production line and historical records of the company while some was approximated by consulting with the plant manager.

The average daily demand of 9,260 strips was determined from historical records, and the capacity of the mixing cylinder was approximately 168.3 kg (equivalent to 3,366 strips per batch and 10 tablets per strip) for the mixing stage. The processing time for a batch, which was independent of the batch size, was 0.3 days. The capacities of the compression and packaging stages were 10,960 strips per day and 10,230 strips per day respectively, and that of the transfer bucket used to transfer materials from the second to third stage 12,000 tables (equivalent to 1,200 strips).

After processing a batch in the mixing stage, it was the company's standard procedure to clean the cylinder which involved some time and cost. The time was considered the set-up time and calculated as 1.10 hours following observations from a time study, and the set-up cost required for labour to prepare the raw materials and clean the cylinder was taken as 2,250 Taka (28.125 USD), as the company suggested. As it was a dedicated continuous production line, the cleaning times for the compression and packaging stages could be considered negligible in comparison with the production time. Therefore, it was assumed their set-up times to be 0 in this case study. The back orders and lost sales costs were also approximated as 10 Taka/strip/day (0.125 USD) and 50 Taka/strip (0.625 USD) respectively while the daily holding cost per unit for materials used in compression was approximated as 0.4 Taka (0.005 USD).

The following costs of production and material loss were collected.

Production cost in mixing stage = 11.2 Taka/strip (0.14 USD)

Production cost in compression stage = 7.6 Taka/strip (0.095 USD)

Production cost in packaging stage = 6.5 Taka/strip (0.08125 USD)

Cost per unit of material loss = 11.2 Taka/strip (0.14 USD)

Based on the above, seven breakdown scenarios were observed within the observation period of 59 days and relevant data collected. Of them, four were in compression, two in packaging and one in mixing, as presented in Table 5.6.

Table 5.6: Data for disruption scenarios

Disruption number (n)	Disrupted stage	Time since previous disruption (days)	Disruption duration (hours)
1	Compression	--	6.30
2	Compression	10.50	4.27
3	Packaging	1.20	6.94
4	Compression	21.40	5.76
5	Mixing	7.20	4.69
6	Packaging	15.10	3.10
7	Compression	1.60	5.38

After consulting with the plant manager, the recovery periods was set as 2 days. Our proposed heuristic was implemented to solve the disruption problem. The total lost sales, back orders and profits in the revised plan for each disruption are presented in Table 5.7.

Table 5.7: Detailed results using heuristic

Disruption number	Total lost sales (USD)	Total back orders (USD)	Total profit (USD)
1	0	211.67	10738.89
2	0	86.51	10874.02
3	306.26	434.24	9989.91
4	0	173.81	10779.41
5	2103.80	0	6793.021
6	0	76.03	10886.13
7	0	147.16	10807.92

The benefits of implementing the proposed model are presented in Table 5.8 which shows that it achieved significantly better results than the company’s current practice. Under ideal conditions, the company would have a profit of 253914.41 USD within the observation period of 59 days whereas, under the disrupted environment, this would reduce to 234138.63 USD if the company used the lost sales cost to recover from disruptions, with a lost sales cost of 10974.87 USD. However, if our proposed model was applied, total profit would increase to 248063.04 USD, with the back orders and lost sales costs 1129.35 and 2410.00 USD respectively.

Table 5.8: Comparison of results during observation period

Cost/Profit	Ideal system (no disruption)	Proposed model	Current practice
Total cost of back orders (USD)	0	1129.35	0
Total cost of lost sales (USD)	0	2410.00	10974.87
Total profit (USD)	253914.41	248063.04	234138.63

A comparison of the daily total profits obtained has been presented graphically in Figure 5.3. It confirms that the proposed approach obtains better results than the company's current practice. It is observed that the total profit reduces significantly in the case of current practice when a disruption occurs. This is because the current practice uses only lost sales cost to recover from a disruption. However, our proposed model ensures minimum reduction of the total profit for the disruption scenarios. This is because the proposed model uses both back orders and lost sales costs to recover from a disruption. The only exceptional scenario is disruption number 5 in Table 5.6, where the total profit both from the current practice and the proposed model is same (Figure 5.3).

The variation of total profit depends on the disruption duration, lost sales and back order costs. The change of total profit with duration of disruption, at all three stages, is presented in Figure 5.4, which shows that total profit decreases significantly with larger disruption durations. For disruption at the mixing stage, total profit decreases suddenly at the disruption duration of 3 hours, because of the commencement of lost sales due to product unavailability. Before that, only back orders were used to generate the solution. For disruption at the packaging stage, total profit decreases from the disruption duration of 6 hours because only back orders are optimal for less than 6 hours, but both back orders and lost sales are required after 6 hours to generate the best solutions. The rate of decrease of total profit with disruption duration is much lower for disruption at the compression stage.

This is because of the presence of only back orders in the best solution when the disruption duration is less than 9 hours, but after that, both back order and lost sales are required to generate the solutions.

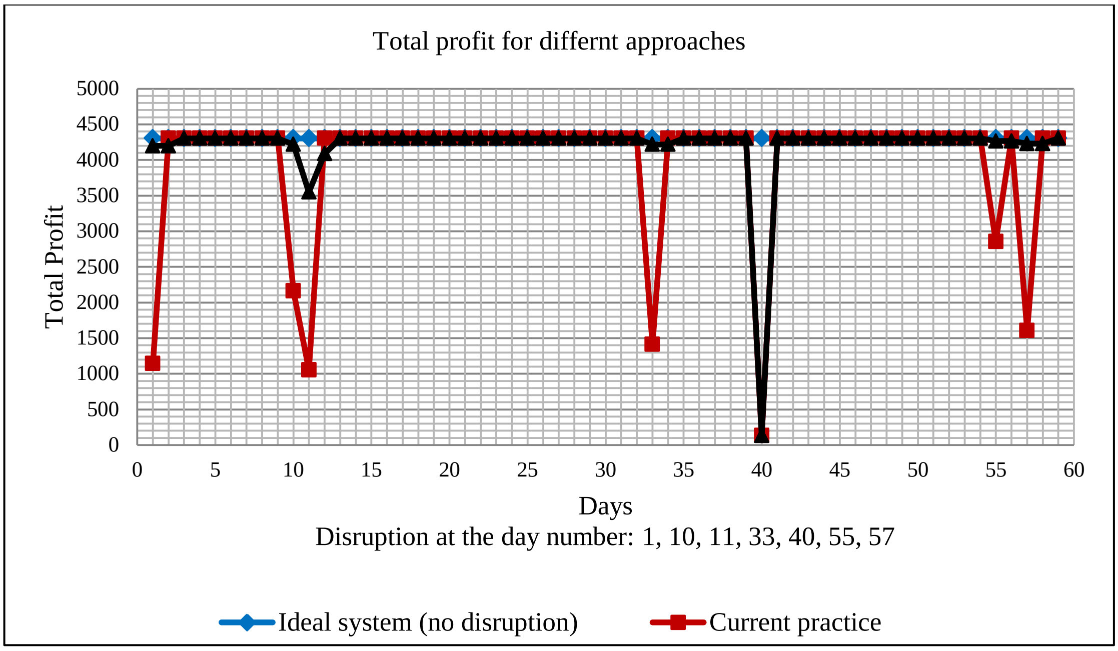

Figure 5.3: Graphical presentation of comparison of results from different approaches

The changes of total profit with lost sales cost is presented in Figure 5.5. The disruption and pre-disruption durations were taken as 7 and 0 hours respectively for this analysis. The remainder of the parameters have the default values of the case study. It was observed, that for disruptions at the mixing and packaging stage, total profit decreases with lost sales cost. But the rate of decrement was much higher for disruption at the mixing stage. This is because of the presence of only lost sales in the solution for disruption at the mixing stage, and the presence of both lost sales and back order for disruption at the packaging stage. On the other hand, at the compression stage, total profit did not change with lost sales cost. This is because of the presence of only back orders in the solution.

The relationship between total profit and back order cost is presented in Figure 5.6. The disruption and the pre-disruption durations were taken as 2 and 0 hours respectively for this analysis. Total profit decreases with back order cost for any disruption at the mixing, compression or packaging stage. This is because back order costs are always present in the solution for all cases.

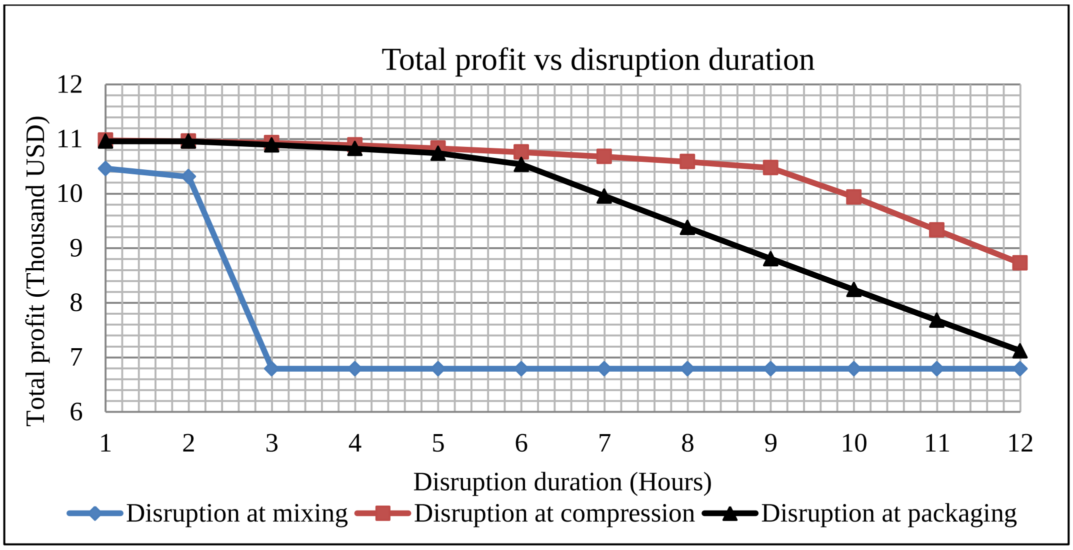

Figure 5.4: Changes of total profit with disruption duration for the case study

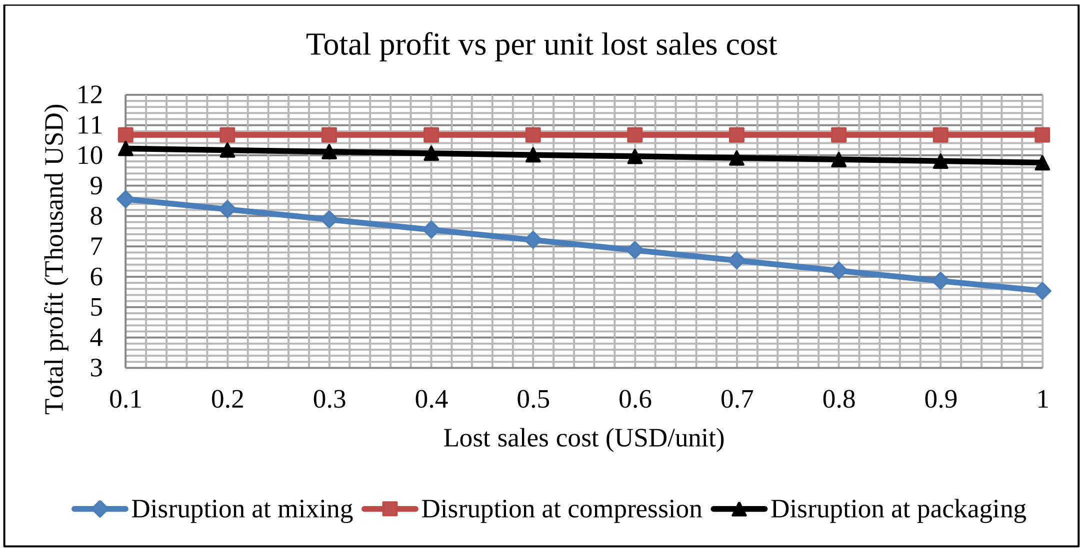

Figure 5.5: Changes of total profit with lost sales cost

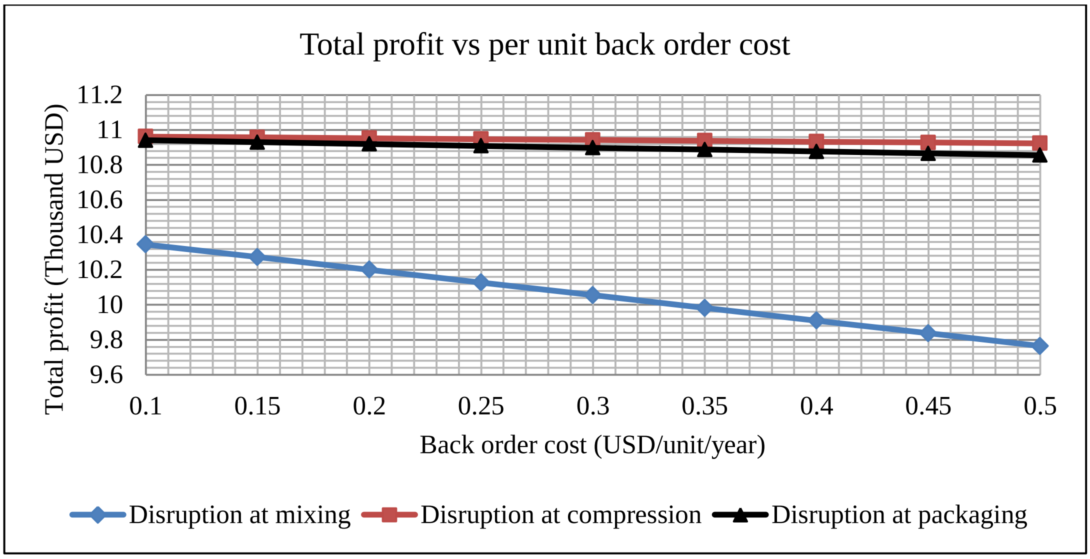

Figure 5.6: Changes of total profit with back order cost

In the above study, the proposed approach shows mostly better, but no worse performance, as compared to current practice. However, in some scenarios the proposed approach provides the same solution as current practice where the disruption is at the mixing stage and the disruption duration is more than 3 hours. This is because the proposed approach requires only lost sales cost to generate solutions, and current practice also uses lost sales in this scenario.

This study has proposed the approach based on a real-life problem from a pharmaceutical company. However this disruption recovery model can be applied to other similar production systems that countenance single or multiple production disruptions at any stages. The study considered fixed lot sizes at different stages throughout the planning horizon, which was based on ideal production conditions. The considered production system also has a mix of discrete and continuous batch production. However, in some production systems, the lot may be split, or may not be fixed, to meet transportation and warehouse capacities. Although this approach is applicable to three-stage production systems, the concept can be used for any number of stages. The annual demand rate is assumed to be known and constant, but practically demand can fluctuate. There may also be special orders outside of the regular demand trend, and sometimes a priority should be given to produce such special orders. For all these variations, management must decide how to deal with them when the proposed approach is applied.

5.7 Chapter Summary

The main objective of this study was to develop an appropriate recovery policy for managing disruptions in a three-stage mixed-production environment. A mathematical model was developed, and then a new heuristic for managing both single and multiple disruptions on a real-time basis proposed. The results from the heuristic were compared with those from another search algorithm for a set of randomly generated disruption test problems. Both approaches produced very similar results, with the average percentage of deviation only 0.00034% which can be considered negligible. The proposed approach was also implemented to solve a real-world disruption problem of a pharmaceutical company. It was proven that the developed mathematical model and proposed heuristic can be easily applied to manage both single and multiple disruptions in a three-stage mixed-production system. With the help of this model, an organisation could increase its profit margin and, thus, decrease its loss due to a disruption. Finally, as customer satisfaction could also be greatly increased, a company’s reputation could be enhanced. This proposed mathematical and heuristic technique offers a potentially very useful quantitative approach for helping decision-makers make prompt and accurate decisions regarding revising production plans whenever a sudden (or series of) disruption occurs in a mixed-production environment.

In chapter 5, the disruption recovery problem for managing production disruption in a three-stage mixed production-inventory system has been addressed. The extension of this study is to explore the problem for a supply chain network, which I have chosen as the direction of my research work in the next chapter.

Chapter 6 Mitigation Approaches for a Supply Chain Network

In this chapter, a three-stage supply chain network, with multiple manufacturing plants, distribution centres and retailers, is considered. Under ideal conditions, an ideal plan is generated in an infinite planning horizon and updated in a finite planning horizon if there are any changes in the data. Then, a predictive mitigation planning approach is developed for obtaining a better supply chain plan. In it, as the production process in any manufacturing plant may face an unexpected disruption at any time, a quantitative model is formulated for determining production and distribution plans under disrupted conditions while minimising the total supply chain cost and revising these plans over a finite future planning period. Multiple disruptions are also considered as a series, where a new disruption may or may not affect the recovery plans of earlier disruptions and requiring the revision of some plans after the occurrence of each disruption on a real-time basis. An efficient heuristic capable of dealing with both single and multiple disruptions on a real-time basis is developed. Finally, this chapter presents some numerical examples, and results comparison to explain both the usefulness and the advantages of the proposed approaches.

6.1 Introduction

A supply chain is a network that receives inputs or raw materials from suppliers, produces final products at its manufacturing facilities and delivers those products to customers through a distribution network. Every manufacturing and service industry is part of a supply chain network which can have multiple manufacturing plants, multiple distribution centres (DCs) and multiple retailers. There are numerous industries, such as the pharmaceutical, textile and manufacturing, that supply, produce and distribute products using a supply chain network. Depending on the number of entities in each tier of a network, it can be very complex and, in a real-life one, any information can be changed at any time. Therefore, an ideal plan should be updated to incorporate changes in order to generate a better plan. Although some changes in data may be known well in advance, others may not but can be detected using appropriate prediction models. Such predictions will help to generate a better supply chain plan than the one designed for ideal conditions.

Supply chain entities can also face many sudden uncontrollable problems, such as a production disruption in a manufacturing plant which can be defined as any form of interruption in the manufacturing system, including a material shortage, machine breakdown, natural disaster or any other form of accidental or man-made disturbance. Supply chain disruption management is an important research topic, as is obvious in the examples provided in Chapter 1.

From the literature review, it is clear that most previous research focused on supply chain coordination and optimisation problems under ideal conditions, although a reasonable number of studies developed recovery models after the occurrence of a disruption on a real-time basis. Previous disruption recovery models in production-inventory and supply chain systems were focused mainly on developing models for a single disruption. Moreover, most previous supply chain models considered a single supplier and single retailer which limited their applicability in real-life situations.

In this chapter, after generating the ideal supply chain plan, if there is any variation in the data in any period, this plan is updated for a finite period on a rolling horizon basis when the new information is known. In real-life situations, some changes may not be known in advance but can be predicted using appropriate prediction models. Therefore, in this chapter, a predictive mitigation planning approach is developed, with the predicted data used to generate a revised plan on a rolling planning horizon basis. Finally, a new attempt is made to develop a quantitative disruption recovery (reactive mitigation) model for a supply chain network consisting of multiple manufacturing plants, DCs and retailers. Disruptions due to technical and internal problems are considered, which take place more frequently (repetitive type) and for short durations. In real-life, a system can face a series of production disruptions (known as multiple disruptions), one after another, at any plant. If a new disruption occurs at any plant during the revised planning window of a previous production disruption, the production and distribution plan must be revised again considering the effects of both disruptions. Therefore, this can be a continuous process that must be dealt with on a real-time basis. A real-time disruption management scenario in a three-tier supply chain network, where the disruptions are not known a priori, is considered in this study. This means that the current plan is revised immediately after a disruption occurs as this disruption is impossible to predict. As it is assumed that any disruption event is random, disruption scenarios are generated by using a uniformly random distribution to determine characteristics such as the disruptions' start times and durations. A new mathematical and heuristic approach is developed for obtaining a revised plan after the occurrence of a single disruption or series of disruptions on a real-time basis.

The main contributions of this chapter can be summarized as follows.

i. Developing an updated supply chain plan for a finite period on a rolling horizon basis to incorporate any changes in the data.
ii. Developing a predictive mitigation planning approach for obtaining a better supply chain plan.
iii. Developing a new quantitative approach for managing production disruptions which are not known in advance. After a disruption, the production and distribution plan is revised for a finite period in the future on a real-time basis.
iv. Developing a new heuristic for generating a revised plan after a production disruption. The heuristic results are compared with those from another established solution technique for a good number of randomly generated test problems.
v. Extending the heuristic to deal with multiple disruptions on a real-time basis. This heuristic is capable of determining a revised plan after each disruption occurs for as long as disruptions take place in the system.

6.2 Problem Description

Firstly, this chapter develops a supply chain model under ideal conditions for an infinite planning horizon. Basically, this ideal plan is used to determine the cycle length which is required for planning and analysis. A three-tier supply chain network with multiple entities in each tier (such as manufacturing plants, DCs and retailers) is considered, as presented in Figure 6.1. In the ideal system, products are produced in the manufacturing plants and then moved to DCs and, finally, distributed to retailers from the DCs according to retailers' demands. The total supply chain cost is minimised to obtain the ideal production and distribution plan. The ideal system is formulated mathematically as a constrained programming problem, where the objective is to minimise the total supply chain cost subject to capacity, distribution and demand constraints. The decision variables are the production quantity in each plant i, and the quantities transported from plant i to DC j, and from DC j to retailer k.

In a real-life situation, a supply chain's input data may vary at any time due to, for example, changes in demand, cost, production capacity and amount of raw materials. If there is any variation in any period, the ideal plan must be updated with the new known information. In fact, the model is run on a rolling horizon basis to incorporate changes in the data and is known as an **updated plan**.

Some changes in the data may be known well in advance, as discussed above. However, others may not be but can be detected using appropriate prediction models. Such predictions will help to generate a better plan than the one developed under ideal conditions. In this chapter, the predicted data are used with the rolling horizon planning model to generate the revised plan which is known as **predictive mitigation**, and its methodology illustrated in Figure 6.2. Historical data are used to calculate the base forecast which is updated by predicting some future event, such as a demand fluctuation, unexpected incident and natural incident, with examples presented in Table 6.1. A rule and logic-based fuzzy inference system (FIS) is used to quantify the value of a qualitatively predicted event, with its working principle illustrated in Figure 6.3 (Paul, 2015). If there is no future event, the base forecast is used for a prediction on which revision of the supply chain plan is based on a rolling horizon.

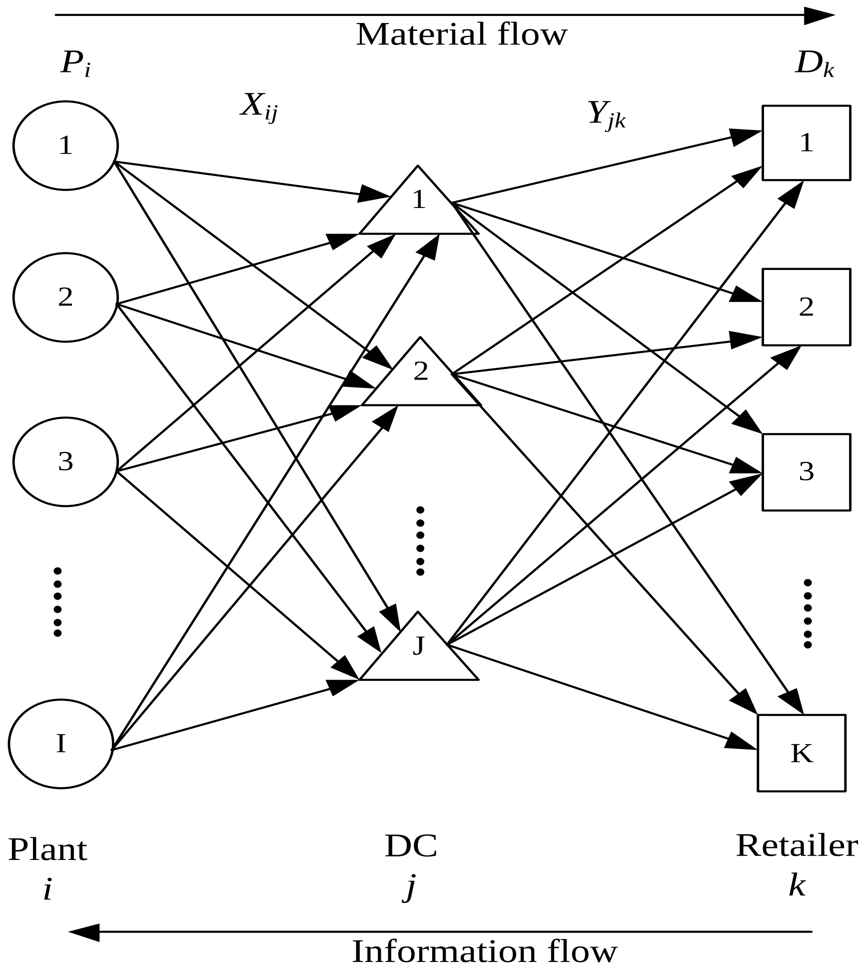

Figure 6.1: Ideal supply chain network for infinite planning horizon

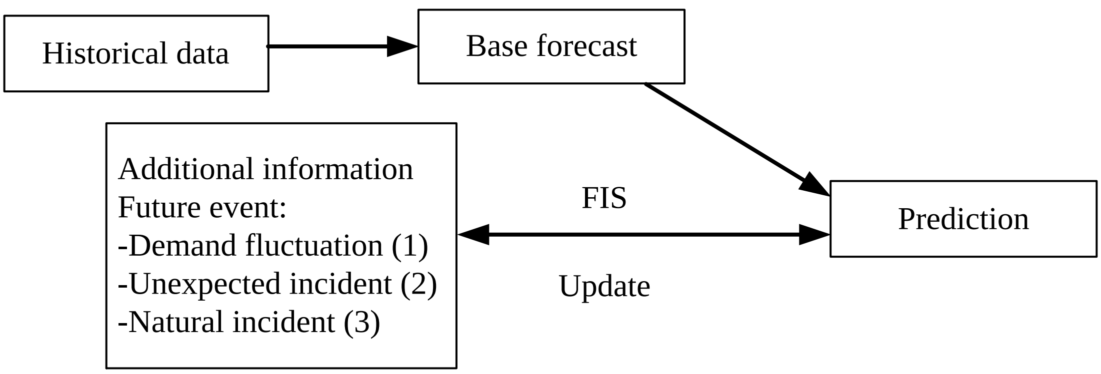

Figure 6.2: Process of prediction for predictive mitigation

Table 6.1: Examples of future events (Y - yes and N - no)

Event	Predicted period			
	1	2	…..	F
Demand fluctuation	N	N	…..	Y
Unexpected incident	N	Y	…..	N
Natural incident	Y	N	…..	N

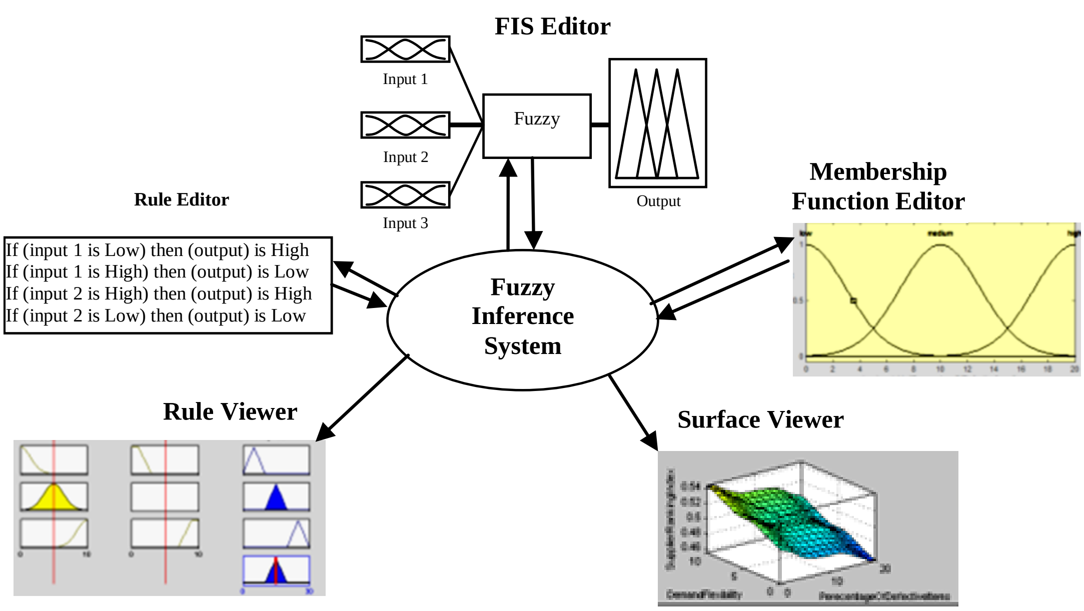

Figure 6.3: Rule and logic-based FIS (Paul, 2015)

Finally, this chapter develops a **recovery plan** which is actually a **reactive mitigation**. In real-life situations, any supply chain can face a sudden disruption at any time. After such an occurrence, the production and distribution plan must be revised for a finite period in the future so that losses can be minimised and the system returns to its ideal plan as quickly as possible.

A production disruption is a familiar event in any manufacturing environment. If there is a sudden disruption at any plant, as that plant will be inoperable for a certain period of time, there will be a loss of production quantity. Afterwards, the main objective is to minimise that loss by revising the production and distribution plan for a finite period in the future, with the revision mechanism presented in Figure 6.4. After a disruption with a duration of T_{dn}, the plan is revised for a future finite planning period (e.g., for the next M periods) which is known as a recovery window.

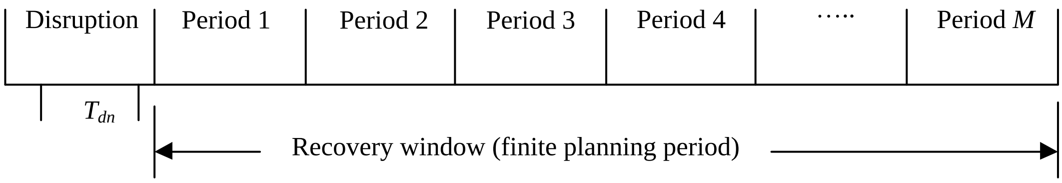

Figure 6.4: Mechanism of recovery plan for managing disruption

The considered system may face multiple disruptions, one after the other in a series, within its recovery window. When a disruption occurs, a revised plan can be generated by solving the mathematical model for a single disruption. After finalizing the revised plan, if another disruption occurs within the recovery window, this plan needs to be revised again to consider the effects of both disruptions. This can be done by simply updating some of the parameters in the same mathematical model, for example, the newly disrupted plant, the start time of the disruption, the disruption duration, the quantity produced before starting the revised plan and the demand to be filled, to represent the changed scenario in order to re-optimise the plan for the current disruption. After every disruption, the plan is revised for a finite period in the future as long as disruptions occur in the system.

In this study, the following strategies for managing a disruption are considered.

i. **Back orders:** if a production quantity is lost, the portion of demand that cannot be filled at the scheduled time but will be delivered at a later date when available is known as the back-orders quantity.

ii. **Lost sales**: if the system is not capable of filling demand after a disruption and customers will not wait for stock to be replenished, demand is lost.

iii. **Outsourcing**: if the production system is not capable of filling the demand on time, management may want to purchase some items from another company at a higher cost.

6.2.1 Notations used in this Study

The following notations are used in this study to formulate the mathematical model.

i Plant index

j DC index

k Retailer index

I Number of plants

J Number of DCs

K Number of retailers

P_i Production quantity of plant i under ideal conditions

CP_i Maximum production capacity of plant i under ideal conditions

CD_j Maximum handling capacity of DC j

X_{ij} Transportation quantity from plant i to DC j under ideal conditions

Y_{jk} Transportation quantity from DC j to retailer k under ideal conditions

D_k Demand of retailer k

p_i Production cost per unit at plant i

H_{1i} Holding cost per unit per period at plant i

H_{2j} Handling cost per unit at DC j

H_{3k} Holding cost per unit per period at retailer k

T_{1ij} Transportation cost per unit from plant i to DC j

T_{2jk} Transportation cost per unit from DC j to retailer k

OC_j Operating cost of DC j

SC_i Spare capacity of plant i

t_n Start time of disruption at n^{th} plant as fraction of period

T_{dn} Disruption duration for n^{th} plant as fraction of period

P'_{im} Production quantity after disruption at plant i in period m

X'_{ijm} Transportation quantity from plant i to DC j after disruption in period m

Y'_{jkm} Transportation quantity from DC j to retailer k after disruption in period m

D'_{km} Quantity received by retailer k after disruption in period m

L Lost sales cost per unit

B Back-orders cost per unit per period

S Outsourcing cost per unit

M Number of periods in recovery window

6.2.2 Assumptions of the Study

In this study, the following assumptions are made.

i. A single type of item is produced in the system.
ii. No inventory buffers are present in the system.
iii. The ideal plan is updated if any information changes.
iv. The ideal plan is revised according to the predicted data.
v. The recovery plan is developed after the occurrence of a disruption.
vi. The revised plan considers back orders, lost sales and outsourcing options.

6.3 Model Formulation

In this section, the mathematical model is formulated for both an ideal and disrupted supply chain system. The ideal plan is updated if there are any changes in the data and also revised according to any prediction of future changes for a finite planning period. In the case of managing a disruption, the model is re-formulated to incorporate the effect of a disruption and the production and distribution plan revised for a finite planning period. After the recovery window, the production and distribution plan reverts to its ideal plan.

6.3.1 Formulation of Ideal Plan

In this section, the different costs are calculated to formulate the mathematical model for the ideal system. The production cost is determined as the per unit production cost multiplied by the production quantity, the average holding cost as the unit holding cost multiplied by the total inventory, the transportation cost as the unit transportation cost multiplied by the transportation quantity, the total operating cost as the sum of the operating cost of each DC, and the handling cost of distribution the unit handling cost multiplied by the total handling quantity. Finally, the different costs are summed to obtain the objective function to be minimised subject to capacity, distribution and demand constraints, where P_i, X_{ij} and Y_{jk} are decision variables. The final mathematical model is considered a constrained programming problem.

Costs at plant

Production cost

$$= \sum_{i=1}^{I} p_i P_i \tag{6.1}$$

Average holding cost

$$= \sum_{i=1}^{I} \frac{1}{2} H_{1i} P_i \tag{6.2}$$

Transportation cost

$$= \sum_{j=1}^{J} \sum_{i=1}^{I} T_{1ij} X_{ij} \tag{6.3}$$

Costs at DCs

Operating cost

$$= \sum_{j=1}^{J} OC_j \tag{6.4}$$

Handling cost

$$= \sum_{j=1}^{J} \sum_{i=1}^{I} H_{2j}\, X_{ij} \tag{6.5}$$

Transportation cost

$$= \sum_{k=1}^{K} \sum_{j=1}^{J} T_{2jk} Y_{jk} \tag{6.6}$$

Costs at retailer

Average holding cost

$$= \sum_{k=1}^{K} \frac{1}{2} H_{3k} D_k \tag{6.7}$$

Objective function

The total supply chain cost (TC), which is the objective function, is derived using equations (6.1) to (6.7) and equals the total plant cost + total DC cost + total retailer cost,

$$TC = \sum_{i=1}^{I} p_i P_i + \sum_{i=1}^{I} \frac{1}{2} H_{1i} P_i + \sum_{J=1}^{J} \sum_{i=1}^{I} T_{1ij} X_{ij} + \sum_{j=1}^{J} OC_j$$
$$+ \sum_{j=1}^{J} \sum_{i=1}^{I} H_{2j}\, X_{ij} + \sum_{k=1}^{K} \sum_{j=1}^{J} T_{2jk} Y_{jk} + \sum_{k=1}^{K} \frac{1}{2} H_{3k} D_k \tag{6.8}$$

Here, P_i, X_{ij} and Y_{jk} are decision variables, subject to the following constraints.

$$P_i \leq CP_i;\ \forall i \tag{6.9}$$

$$P_i = \sum_{j=1}^{J} X_{ij}\ ;\ \forall i \tag{6.10}$$

$$\sum_{i=1}^{I} X_{ij} = \sum_{k=1}^{K} Y_{jk};\ \forall j \tag{6.11}$$

$$\sum_{i=1}^{I} X_{ij} \leq\ CD_j;\ \forall j \tag{6.12}$$

$$\sum_{j=1}^{J} Y_{jk} = D_k;\ \forall k \tag{6.13}$$

$$\sum_{i=1}^{I} P_i = \sum_{k=1}^{K} D_k \tag{6.14}$$

$$P_i, X_{ij} \text{ and } Y_{jk} \geq 0;\ \forall i,j,k \tag{6.15}$$

The production quantity of each plant is less than or equal to the maximum capacity of that plant (equation (6.9)), the constraints for distribution from the plant to DCs and DCs to retailers equations (6.10) and (6.11) respectively, the capacity constraints of the DCs equation (6.12), the demand of the retailers equation (6.13) and total production is equal to total demand (equation (6.14)) while equation (6.15) is a non-negativity constraint.

The formulations for the updated and predictive mitigation planning approaches are presented in Appendix B.

6.3.2 Formulation for Reactive Mitigation Approach

In this section, a mathematical model for revising the production and distribution plan for a finite planning period after the occurrence of a production disruption, with the objective of minimising the total supply chain cost, is developed. As the recovery strategy involves back orders, outsourcing and lost sales options, there are additional cost equations for them. The back orders cost is determined as the unit back orders cost multiplied by the number of back-orders units and the time delay (Paul et al, 2015, in press), the lost sales cost as the unit lost sales cost multiplied by the number of lost sales units (Paul et al., 2015) and the outsourcing cost by the quantity outsourced per unit purchase cost.

If there is a disruption at the n^{th} plant for a duration of T_{dn} with a start time of t_n, the production quantity loss after a single disruption can be determined using equations (6.16) and (6.17).

If $t_n + T_{dn} < \frac{P_n}{CP_n}$

$$D' = CP_n * T_{dn} - min\left\{\sum_{i=1}^{I} SC_i, \sum_{i=1}^{I} CP_i * (1 - t_n - T_{dn})\right\} \tag{6.16}$$

If $t_n + T_{dn} > \frac{P_n}{CP_n}$

$$D' = CP_n * \left(\frac{P_n}{CP_n} - t_n\right) - min\left\{\sum_{i=1}^{I} SC_i, \sum_{i=1}^{I} CP_i * (1 - t_n - T_{dn})\right\} \tag{6.17}$$

As this quantity needs to be filled during the recovery window, back orders, lost sales and outsourcing options are considered so that the total supply chain cost during this time can be minimised.

Costs at plant

Production cost

$$= \sum_{m=1}^{M} \sum_{i=1}^{I} p_i P'_{im} \tag{6.18}$$

Average holding cost

$$= \sum_{m=1}^{M} \sum_{i=1}^{I} \frac{1}{2} H_{1i} P'_{im} \tag{6.19}$$

Transportation cost

$$= \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{i=1}^{I} T_{1ij} X'_{ijm} \tag{6.20}$$

Cost at DCs

Operating cost

$$= M \sum_{j=1}^{J} OC_j \tag{6.21}$$

Handling cost

$$= \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{i=1}^{I} H_{2j} X'_{ijm} \tag{6.22}$$

Transportation cost

$$= \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{J} T_{2jk} Y'_{jkm} \tag{6.23}$$

Costs at retailer

Average holding cost

$$= \sum_{m=1}^{M} \sum_{k=1}^{K} \frac{1}{2} H_{3k} D'_{km} \tag{6.24}$$

Back orders cost

$$= B \sum_{m=1}^{M} Units\ delay\ at\ period\ m * delay_m$$

$$= B \left[\sum_{m=1}^{M} m \left(\sum_{i=1}^{I} P'_{im} - \sum_{i=1}^{I} P_i \right) \right] \tag{6.25}$$

Outsourcing cost

$$= S \left(M \sum_{i=1}^{I} P_i + D' - \sum_{m=1}^{M} \sum_{i=1}^{I} P'_{im} \right) \tag{6.26}$$

Lost sales cost

$$= L\left(M\sum_{i=1}^{I} P_i + D' - \sum_{m=1}^{M}\sum_{i=1}^{I} P'_{im}\right) \tag{6.27}$$

If $S \leq L$, then the lost sales cost = 0, otherwise the outsourcing cost =0.

The total supply chain cost (TC), which is the objective function, is derived using equations (6.18) to (6.27) and equals the total plant cost + total DC cost + total retailer cost + back orders cost + outsourcing cost + lost sales cost, where P'_{im}, X'_{ijm}, Y'_{jkm} and D'_{km} are decision variables.

$$P'_{im} \leq CP_i;\ \forall i, m \tag{6.28}$$

$$P'_{im} = \sum_{j=1}^{J} X'_{ijm}\ ;\ \forall i, m \tag{6.29}$$

$$\sum_{i=1}^{I} X'_{ijm} = \sum_{k=1}^{K} Y'_{jkm}\ ;\ \forall j, m \tag{6.30}$$

$$\sum_{i=1}^{I} X'_{ijm} \leq\ CD_{j,m};\ \forall j, m \tag{6.31}$$

$$\sum_{j=1}^{J} Y'_{jkm} = D'_{km};\ \forall k, m \tag{6.32}$$

$$M\sum_{i=1}^{I} P_i + D' \geq \sum_{m=1}^{M}\sum_{i=1}^{I} P'_{im} \tag{6.33}$$

$$P_i, X_{ij} \text{ and } Y_{jk} \geq 0;\ \forall i, j, k \tag{6.34}$$

The production quantity of each plant in the revised plan is less than or equal to its maximum capacity in the revised plan (equation (6.28)), the constraints for distribution from the plant to DCs and from DCs to retailers equations (6.29) and (6.30) respectively, the capacity constraint of the DCs equation (6.31), the constraint for the quantity received by each retailer equation (6.32), the total lost sales quantity, which should be non-negative, equation (6.33) and the non-negativity of the decision variables equation (6.34).

Proposition 1: for a given P_i, CP_i, CD_j and D_k, and the n^{th} disrupted plant, if $B \ll L, S$, the recovery plan will use only the back orders option if $D' \leq M \sum_{i=1}^{I} SC_i$.

Proof: let a production disruption occur at the n^{th} plant starting at period t_n with a disruption duration of T_{dn}.

For a disruption, if the spare capacity is greater than or equal to the demand to be filled during the recovery window, as the production system is capable of producing and meeting that demand, the revised plan will utilize only the back-orders option as condition of existence of only back orders:

$$M \sum_{i=1}^{I} P_i + D' = \sum_{m=1}^{M} \sum_{i=1}^{I} P'_{im}$$

$$\Rightarrow \quad D' = \sum_{m=1}^{M} \sum_{i=1}^{I} P'_{im} - M \sum_{i=1}^{I} P_i$$

$$\Rightarrow \quad D' \leq M \sum_{i=1}^{I} CP_i - M \sum_{i=1}^{I} P_i \qquad \text{(using the equation (6.28))}$$

$$\Rightarrow \quad D' \leq M \left(\sum_{i=1}^{I} CP_i - \sum_{i=1}^{I} P_i \right)$$

$$\Rightarrow \quad D' \leq M \sum_{i=1}^{I} SC_i \tag{6.35}$$

Therefore, it can be said that, if $D' \leq M \sum_{i=1}^{I} SC_i$, the system will utilize only the back orders option.

Proposition 2: for a given P_i, CP_i, CD_j and D_k, and the n^{th} disrupted plant, the revised plan will use both the lost sales/outsourcing and back orders options if $D' > M \sum_{i=1}^{I} SC_i$.

Proof: this is the opposite consequence to that of **Proposition 1.**

6.4 Solution Approaches

In this section, solution approaches for both ideal and disrupted systems are developed. A standard solution technique for solving the ideal supply chain system is proposed and applied to obtain updated and predictive mitigation plans for changes in the data and future predictions respectively. An efficient heuristic for managing a single disruption in the system is developed and then extended to be implemented for managing multiple disruptions on a real-time basis.

6.4.1 Solution Approach for Generating Supply Chain Plan

The ideal production and distribution plan is obtained using the branch and bound algorithm of the LINGO optimisation software to solve the model for the system which is a constrained programming problem, and is also applied to obtain the updated and predictive mitigation plans.

6.4.2 Heuristic for Managing Single Disruption

A heuristic is designed to obtain the revised plan after an occurrence of a single disruption at any plant. Firstly, both the ideal and disrupted systems are solved using the LINGO optimisation software and then the heuristic efficiently solves the disruption management model through the following steps.

Step 1: Input all the information about production and distribution under ideal conditions.

Step 2: Obtain an ideal production and distribution plan by solving the mathematical model for ideal situations and also determine the spare capacity in each plant.

Step 3: Input a production disruption scenario involving a disrupted plant, disruption start time (t_n) and disruption duration (T_{dn}).

Step 4: Determine the production plan.

4.1. If $B \leq L, S$:

4.1.1 if $D' \leq M \sum_{i=1}^{I} SC_i$, use the spare capacity to revise the plan until the unfilled demand is met;

4.1.2 if $D' > M \sum_{i=1}^{I} SC_i$, use both the spare capacity and lost sales/outsourcing options;

4.1.2.1 if $L > S$, use the outsourcing option and

4.1.2.2 if $L \leq S$, use the lost sales option.

4.2. If $B > L, S$:

4.2.1 if $L > S$, use the outsourcing option to revise the plan or

4.2.2 if $L \leq S$, use the lost sales option to revise the plan.

Step 5: Determine the distribution plan.

5.1. If $D' \leq M \sum_{i=1}^{I} SC_i$, determine the distribution plan by varying only the transportation quantity while using the same path as the ideal plan.

5.2. If $D' > M \sum_{i=1}^{I} SC_i$, determine the distribution plan by varying only the transportation quantity while using the same path as that obtained from LINGO for $D' > M \sum_{i=1}^{I} SC_i$.

Step 6: Record the results and determine the different costs.

Step 7: Stop.

6.4.3 Proposed Heuristic for Multiple Disruptions

In this section, the heuristic first developed to manage a single disruption is extended to manage multiple production disruptions on a real-time basis. When a disruption occurs, a revised plan can be generated by solving the mathematical model using the proposed heuristic for a single disruption. Then, if another disruption occurs, the plan should be revised again to consider the effects of both disruptions. This can be done by simply updating some of the parameters in the same mathematical model to represent the changed scenario; for example, the newly disrupted plant, start time of the disruption, disruption duration, quantity produced before starting the revised plan and demand to be filled in the revised plan. The objective function and constraints are also updated for the changed situation. Therefore, the heuristic for a single disruption can still be used but must be slightly modified for the changed situation to be capable of dealing with a series of disruptions on a real-time basis. In the proposed approach, the heuristic must be run every time a disruption occurs to re-optimise the revised plan while there are disruptions in the system.

For a series of disruptions, the production quantity loss after the s^{th} disruption can be determined using equations (6.36) and (6.37).

If $t_n + T_{dn} < \frac{P_n}{CP_n}$

$$D'_s = D'_{s-1} - \left(\sum_{m=1}^{l} \sum_{i=1}^{I} P'_{im} - l * \sum_{i=1}^{I} P_i \right) + CP_n * T_{dn} - min \left\{ \sum_{i=1}^{I} SC_i , \sum_{i=1}^{I} CP_i * (1 - t_n - T_{dn}) \right\} \quad (6.36)$$

If $t_n + T_{dn} > \frac{P_n}{CP_n}$

$$D'_s = D'_{s-1} - \left(\sum_{m=1}^{l} \sum_{i=1}^{I} P'_{im} - l * \sum_{i=1}^{I} P_i \right) + CP_n * \left(\frac{P_n}{CP_n} - t_n \right) - min \left\{ \sum_{i=1}^{I} SC_i , \sum_{i=1}^{I} CP_i * (1 - t_n - T_{dn}) \right\} \quad (6.37)$$

Here, l is the new disrupted period since the previous disruption.

The main steps in the proposed heuristic for a series of disruptions on a real-time basis can be presented as follows.

Step 1: Input all the information about production and distribution under ideal conditions.

Step 2: Determine the optimal plan under ideal conditions.

Step 3: Input the disruption scenario (disrupted plant, disrupted period since the previous disruption, disruption start time (t_n) and disruption duration (T_{dn})).

Step 4: Update the loss of production quantity using equations (6.36) and (6.37).

Step 5: Revise the production plan for the corresponding disruption using the proposed heuristic developed in Section 6.4.2.

Step 6: Record and update the optimal production and distribution plan from **Step 5** after the disruption occurs.

Step 7: If there is any other disruption, go to **Step 3**.

Step 8: Stop.

The heuristic for managing both a single disruption and multiple disruptions is coded in MATLAB R2012a and executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU.

6.5 Experimentations and Analysis of Results

In this section, the experiments and results are discussed for both the ideal and disrupted systems, and the updated and predictive mitigation plans for a good number of randomly generated test problems. For the disrupted system, the results for both a single disruption and multiple disruptions are analysed. The test problems are solved using both the heuristic, and branch and bound algorithm of the LINGO optimisation software. To judge the quality of the heuristic solutions, the results, obtained from the two different techniques, are compared.

6.5.1 Experimentation for Ideal System

The following data were considered for an ideal supply chain network.

$$I = 2; J = 3; K = 6;\ CP_i = [2000, 2700]; D_k = [450, 500, 650, 725, 800, 1000];$$

$$CD_j = [2500, 2000, 1500];\ p_i = [19, 22]; H_{1i} = [1.2, 1]; H_{2j} = [1.5, 1.2, 0.8];$$

$$H_{3k} = [1.5, 1.2, 0.8, 1.75, 1, 0.9];\ \ OC_j = [10000, 15000, 8000]$$

$$T_{1ij} = \begin{bmatrix} 3.0 & 4.0 & 5.0 \\ 5.0 & 6.0 & 3.0 \end{bmatrix};\ T_{2jk} = \begin{bmatrix} 6.0 & 2.0 & 4.0 & 3.0 & 6.0 & 4.0 \\ 8.0 & 5.0 & 3.0 & 2.0 & 6.0 & 4.0 \\ 6.0 & 2.0 & 8.0 & 6.0 & 6.0 & 7.0 \end{bmatrix}$$

$$T_{1ij} = \begin{bmatrix} 3.0 & 4.0 & 5.0 \\ 5.0 & 6.0 & 3.0 \end{bmatrix};\ T_{2jk} = \begin{bmatrix} 6.0 & 2.0 & 4.0 & 3.0 & 6.0 & 4.0 \\ 8.0 & 5.0 & 3.0 & 2.0 & 6.0 & 4.0 \\ 6.0 & 2.0 & 8.0 & 6.0 & 6.0 & 7.0 \end{bmatrix}$$

The ideal system was solved using the branch and bound algorithm of the LINGO optimisation software, with the optimal plan for minimising the total supply chain cost obtained and presented in Table 6.2. The total minimum cost for the ideal system was 156456.9.

Table 6.2: Optimal production and distribution plan for ideal system

Production plan		Distribution plan										
			DC				Retailer					
P_1	P_2	Plant	1	2	3	DC	1	2	3	4	5	6
2000	2125	1	0	2000	0	1	0	0	0	0	0	625
		2	625	0	1500	2	0	0	650	725	250	375
		–	–	–	–	3	450	500	0	0	550	0

6.5.2 Experimentation for Updated and Predictive Mitigation Plans

In this section, the results for both the updated and predictive mitigation plans are discussed using random data.

6.5.2.1 Updated Plan

The results for 50 random test problems were analysed and the test problems were generated by varying the demand and cost data. The problems were solved using LINGO to obtain an updated plan in a finite planning horizon of three periods, with their total cost patterns presented in Figure 6.5.

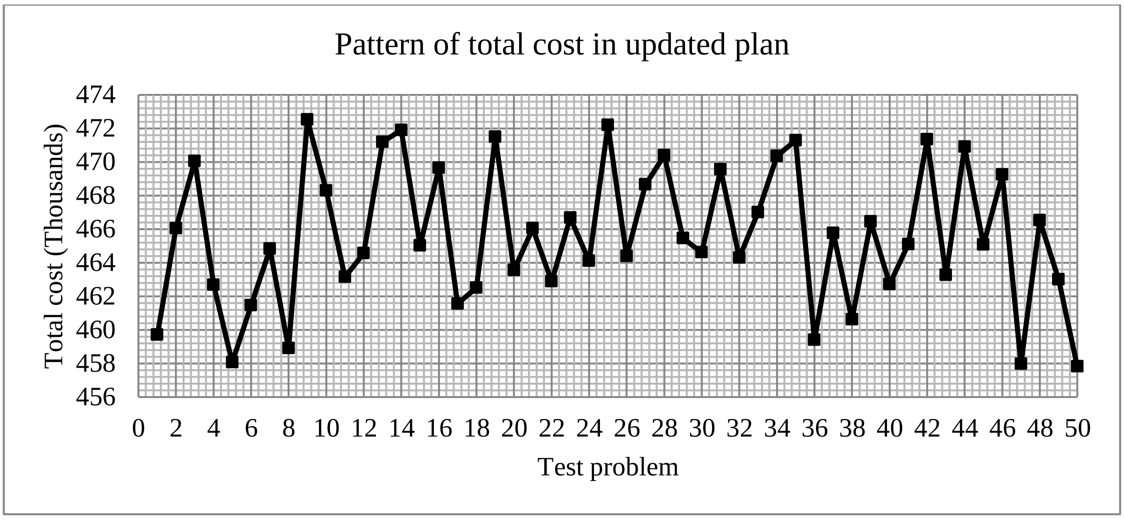

Figure 6.5: Total cost of random experimentation for updated plan

6.5.2.2 Predictive Mitigation Plan

In this section, the results were analysed for the predictive mitigation approach. The demand data was predicted by applying the rule and logic-based FIS developed using the fuzzy toolbox of MATLAB R2012a. Then, the plan was revised by solving the formulation for the predicted demand. For this analysis, the demand was predicted in the 3rd period and revised the plan accordingly in a finite planning horizon of three periods using LINGO. Table 6.3 presents the range of data considered for different predicted events with the formula for obtaining the predicted value:

$$\text{predicted value} = \text{base forecast} \pm \text{value from FIS} \tag{6.38}$$

A number of rules was generated for relating all the inputs to their predicted values and then determine these values for 50 random test problems using the rule viewer of the FIS. For each test problem, the supply chain plan was revised according to LINGO's predictions. The pattern of the total cost obtained from this experiment is presented in Figure 6.6.

Table 6.3: Range of data for predictions

Event	NH	NM	NL	PL	PM	PH
Demand fluctuation	[-1 -0.8 -0.6]	[-0.7 -0.5 -0.3]	[-0.4 -0.2 0]	[0 0.2 0.4]	[0.3 0.5 0.7]	[0.6 0.8 1]
Unexpected incident	---	---	---	[0 0.2 0.4]	[0.3 0.5 0.7]	[0.6 0.8 1]
Natural incident	[-1 -0.8 -0.6]	[-0.7 -0.5 -0.3]	[-0.4 -0.2 0]	[0 0.2 0.4]	[0.3 0.5 0.7]	[0.6 0.8 1]
NH: negative high, NM: negative medium, NL: negative low PL: positive low, PM: positive medium, PH: positive high						

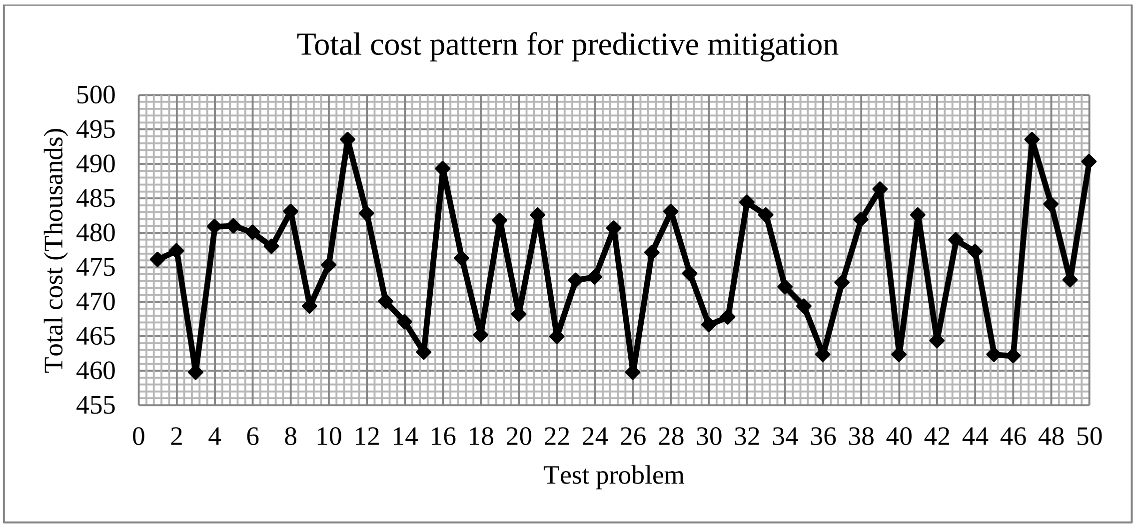

Figure 6.6: Total cost pattern of random experimentation for predictive mitigation

6.5.3 Single Disruption

For the single disruption problem, the same basic data was used as for the ideal system. For illustrative purposes, sixteen different disruption scenarios were generated and their parameters are presented in Table 6.4. The disruption start times were classified as early, middle and late, and disruption durations as low, medium and high. As, when the disruption start time was in the late range, it was not possible to have a high disruption duration because the latter was dependent on the former, two scenarios (1-L-H and 2-L-H) were absent from the design of this experiment. Then, 10 random test problems were generated for each scenario by varying the cost data using the following additional data.

$$B = 10, L = 50, S = 60, \text{ and } M = 2$$

Table 6.4: Design of experiment with total of sixteen scenarios

Disrupted plant	Disruption start time	Disruption duration	Scenario name
1	0.60 (late)	–	–
		0.35 (medium)	1-L-M
		0.20 (low)	1-L-L
1	0.30 (middle)	0.55 (high)	1-M-H
		0.33 (medium)	1-M-M
		0.20 (low)	1-M-L
1	0.10 (early)	0.70 (high)	1-E-H
		0.50 (medium)	1-E-M
		0.18 (low)	1-E-L
2	0.55 (late)	–	–
		0.30 (medium)	2-L-M
		0.15 (low)	2-L-L
2	0.35 (middle)	0.60 (high)	2-M-H
		0.40 (medium)	2-M-M
		0.10 (low)	2-M-L
2	0.05 (early)	0.75 (high)	2-E-H
		0.30 (medium)	2-E-M
		0.12 (low)	2-E-L

The sixteen scenarios in Table 6.4, each with ten random test problems, were solved using both the heuristic and LINGO optimisation software. The results for all 160 random disruption test problems were compared and the average percentage of deviation between those obtained from the two approaches, calculated by equation (6.39), was only 0.0007% which, again, was negligible. Moreover, this may merely have been due to the rounding of the values of the decision variables. It can be said that the results obtained from the two approaches were very consistent.

Average percentage of deviation

$$= \frac{1}{N} \sum \frac{|\text{Total profit from heuristic} - \text{Total profit from LINGO}|}{\text{Total profit from LINGO}} \times 100\% \qquad (6.39)$$

Here, $N =$ total number of test problems.

6.5.4 Series of Disruptions

To demonstrate the usefulness of the proposed heuristic for solving different scenarios with multiple production disruptions, the same basic data was used as for the ideal and disrupted systems presented in Sections 6.5.1 and 6.5.3 respectively. In a supply chain system, a series of production disruptions can occur at any plant, one after another, on a real-time basis. The first disruption can be managed using the single disruption approach discussed in the previous section. If another disruption occurs at any plant during the recovery window of the previous one, as this may affect the revised plan of previous disruptions, this plan must be considered a set of additional restrictions. For experimental purposes, several random disruptions were generated to occur one after another. Table 6.5 presents cases with different random combinations of a disrupted plant, disruption start times and disruption durations. Although disruptions can happen continuously within a production cycle, Table 6.5 presents only five disruptions as a sample representation. The production and distribution plan were revised immediately after each disruption occurred in the system. The problem was solved using the proposed heuristic, and the results after each disruption for the total supply chain and total lost sales cost in the revised plan are presented in Table 6.6. As it was observed that, the system utilized both the spare capacity and lost sales options for the first three disruptions for revising the plan and, capable of revising plan using only back-orders options for the 4th disruptions.

Table 6.5: A case problem for multiple disruptions

Disruption number	Disrupted plant	Disrupted period since previous disruption	Disruption start time	Disruption duration
1	1	--	0.05	0.85
2	2	3	0.22	0.70
3	2	1	0.55	0.25
4	1	4	0.35	0.30
5	2	2	0.70	0.10
…	…	…	…	…

Table 6.6: Summary of results for series of disruptions

Disruption number	Total cost	Total back-orders cost	Total lost sales cost	Total outsourcing cost
1	379990	17250	4000	0
2	376240	17250	250	0
3	378990	17250	3000	0
4	317912	250	0	0
5	316830	0	0	0
…	…	…	…	…

6.5.5 Effect of Disruption Duration

As the disruption duration has a significant impact on the total supply chain, back orders and lost sales costs, its relationships with these factors are analysed. This section presents a number of studies, in each of which only one variable is changed while the other parameters have the default values of the ideal system presented in Section 6.5.1.

Figures 6.7, 6.8 and 6.9 respectively show the changes in the total supply chain, back orders and lost sales costs for varying disruption durations. In this analysis, the disruption start time is kept constant at 0 and it is observed that the total supply chain cost increases with increasing durations of disruptions at both plants 1 and 2.

In Figure 6.7, it is observed that the revised plan uses no additional cost when the disruption durations are less than or equal to 0.20 and 0.30 for disruptions at plant 1 and 2 respectively. This is because the system utilizes the spare capacity of the disrupted period to fill the quantity loss. However, then there is an increasing trend in the total supply chain cost because of the introduction of back orders and lost sales costs in the solutions.

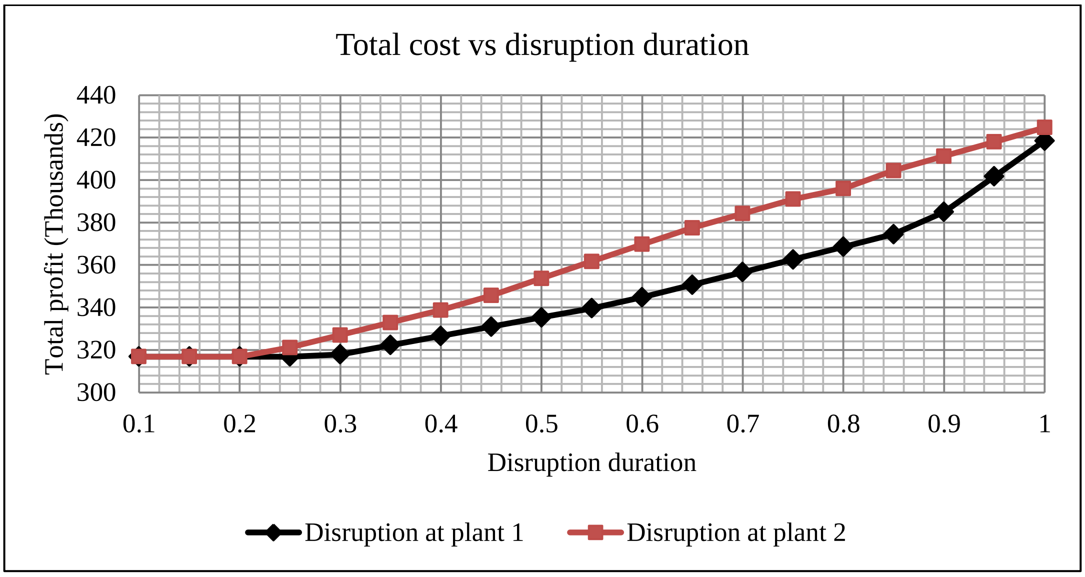

Figure 6.7: Changes in total supply chain cost for varying disruption durations

Figure 6.8 presents changes in the back orders cost with different disruption durations. It is observed that they occur in the system after disruption durations of 0.2 and 0.3 at plants 1 and 2 respectively and increase up to those of 0.65 and 0.85 respectively. Then, the lost sales cost appears in the system and the back orders cost becomes a fixed amount.

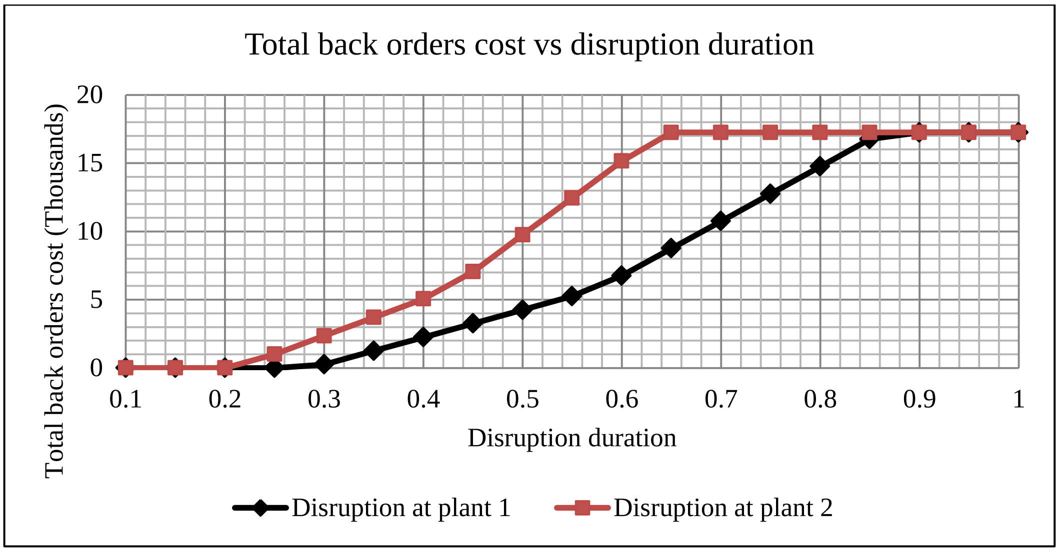

Figure 6.8: Changes in total back orders cost for varying disruption durations

Figure 6.9 presents changes in the lost sales cost for different disruption durations. It is observed that they occur in the system after disruption durations of 0.6 and 0.85 at plants 1 and 2 respectively and then increases. Previously, the system was capable of recovering using only the back orders option.

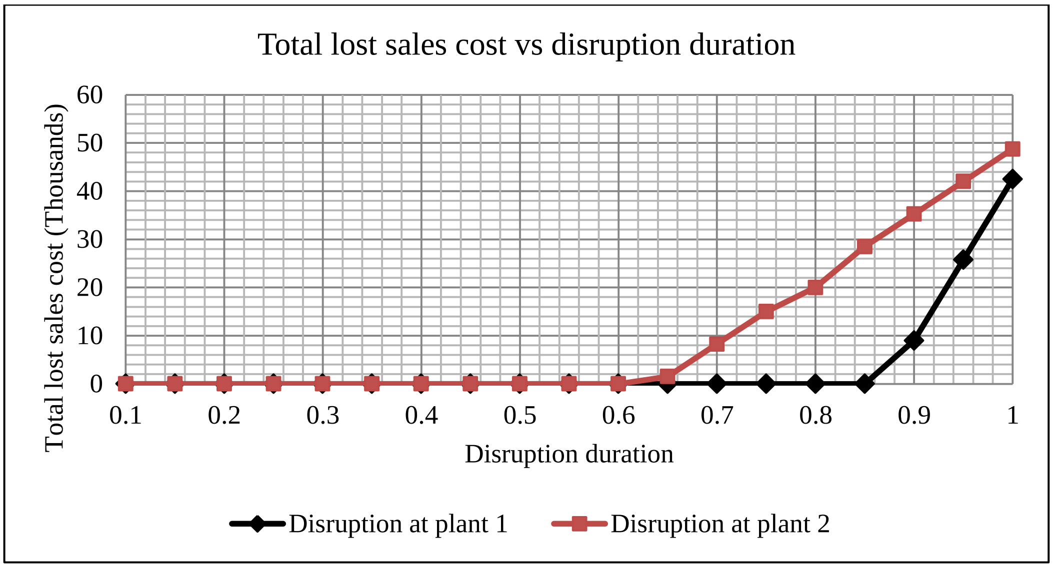

Figure 6.9: Changes in total lost sales cost for varying disruption durations

6.6 Chapter Summary

The main objective of this chapter was to develop both predictive and reactive mitigation planning approaches for a supply chain, and to revise the plans based on any future prediction and after the occurrence of a production disruption on a real-time basis. This chapter introduced a three-stage supply chain network model with multiple numbers of manufacturing plants, DCs and retailers. It was formulated as a constrained programming problem in which the objective was to minimise the total supply chain cost. The ideal supply chain system worked in an infinite rolling planning horizon. The plans were revised if there were any changes in data and predictions using the developed prediction methodology. The production and distribution plan was revised again after a disruption in the system for a finite planning period in the future so that the system could return to its ideal plan as quickly as possible. An efficient heuristic for obtaining a revised plan for either a single disruption or series of disruptions on a real-time basis was proposed. Sixteen different scenarios were developed, each with ten randomly generated disruption test problems, and compared the performances of the heuristic and LINGO for them. It was shown that the average percentage of deviation in the results was only a negligible 0.0007%. Therefore, it can be said that the proposed mathematical and heuristic approach offers a potentially very useful quantitative means of helping decision makers arrive at prompt and accurate decisions regarding both predictive and reactive mitigation plans.

The disruption recovery problems, for managing production disruption in different production-inventory and supply chain systems, have been addressed in this and previous chapters. As fluctuation in demand is one of the major sources of risk and it can imbalance the total supply chain plan, it would be interesting to investigate this specific type of disruption. Furthermore, considering the lack of research concerning demand fluctuation, I have considered it as the direction of my next research work, as discussed in the next chapter.

Chapter 7 Managing Demand Fluctuation

This chapter considers a supplier-retailer system, that operates under an agreed coordinated policy, with an imperfect production process and a possibility of having demand fluctuation. In this chapter, a dynamic planning process is proposed to deal with short-term demand fluctuations. To do this, a mathematical model is first developed for a single fluctuation, either for increasing or decreasing demand rate. The model generates a revised plan, after the occurrence of the fluctuation event. A new and efficient heuristic is proposed to solve the developed model. Secondly, multiple fluctuations are considered, for which a new occurrence may or may not affect the revised plan of earlier occurrences and the heuristic is extended so that is capable of dealing with multiple demand fluctuations on a real-time basis. A good number of random test problems is generated and also solved using a genetic algorithm, in order to compare the solutions with our heuristic. Finally, this chapter presents some numerical examples and sensitivity analysis to explain the usefulness of the developed model.

7.1 Introduction

This chapter deals with demand fluctuations on a real-time basis. That means, the current plan is revised after experiencing any real demand fluctuation. Such a fluctuation is not known in advance and it is impossible to be predicted. It is assumed that both the demand fluctuation and the duration of fluctuation will follow a stochastic process. To do this, a mathematical model is developed for dealing with a single occurrence of demand fluctuation. Heuristics are proposed to solve the models for managing both a single and multiple fluctuations. The objective in the model is to maximise the total profit as the revenue varies with production process reliability. The main contributions of this chapter can be summarized as follows:

i. Modelling a supplier-retailer coordinated system under demand fluctuation. Here, the demand fluctuation is not known in advance and it follows a stochastic process.

So the plan is revised, for a future period, after experiencing the fluctuation, on a real-time basis.

ii. Developing a new heuristic to generate the revised plan after the occurrence of a demand fluctuation. The heuristic is able to produce quality solutions with little computational time.

iii. Extending the heuristic to deal with multiple fluctuations on a real-time basis. In the multiple fluctuations case, any new fluctuations may or may not affect the revised plans of the previous fluctuations. These two cases may be introduced as dependent and independent scenarios. The extended heuristic is capable of dealing with both scenarios.

For better understanding of the demand fluctuation problem, a definition of the different terms used in this chapter is provided below.

***Demand fluctuation*:** Any kind of variation in product demand. Demand can be increased or decreased for a certain period of time, which is known as the fluctuation period.

***Process reliability*:** Percentage of non-defective products produced in the production system (Cheng, 1989).

***Revised plan*:** If the demand variation occurs for a given period of time, it is necessary to revise the schedule for some periods in the future (known as a revised planning window) until the system returns to normal schedule. It is known as a revised plan.

***Backorder*:** If the demand rate is increased for a certain period of time, then the portion of demand that cannot be fulfilled at the scheduled time, but that will be delivered at a later date when available, is known as the backorder quantity.

***Lost sales*:** If the demand rate is increased for a certain period of time and the production process is not capable of fulfilling that demand, then customers will sometime not wait for the stock to be replenished, and so that demand is lost.

***Loss of production*:** If the demand rate is decreased for a certain period of time, then the production process has to reduce the lot size because of the decreasing demand, this will reduce profit. Note that, in this case, there are no backorder or lost sales.

7.2 Problem Description

In this section, a two-stage supply-retailer coordinated system is discussed, where a manufacturer produces a single product and it is supplied to a retailer. The product is produced in batches, and once a batch is completed at the manufacturer, the whole batch is then transported to the retailer for sale to customers (Sarker and Khan, 1999). The system is presented in Figure 7.1, where the inventory built up in each stage is batch processed. The production of non-defective items in any batch is dependent on production process reliability. As it is assumed that the production capacity is higher than the demand, there is an idle time-slot between the consecutive production batches. The lot sizes of the manufacturer and retailer are assumed to be the same as this is a perfectly balanced system.

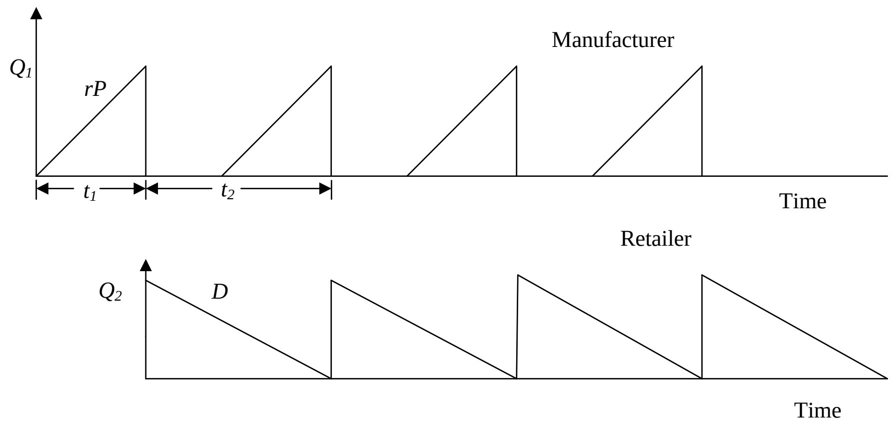

Figure 7.1: Original supplier-retailer coordinated system

The product demand can fluctuate, such as an increase or decrease in the demand rate, for a certain period of time. If the demand rate is increased for a given period of time, the utilization of idle time-slots in the production system may help to meet the additional demand. However, as the retailer receives the products as a lot, the additional demand must be met from the future supplies (that is from the revised lot). This may result in costly backorder and/or lost sales due to delayed delivery. If the demand is decreased for a given period of time, then the production process has to reduce the future lot size in some cycles to avoid excessive inventory.

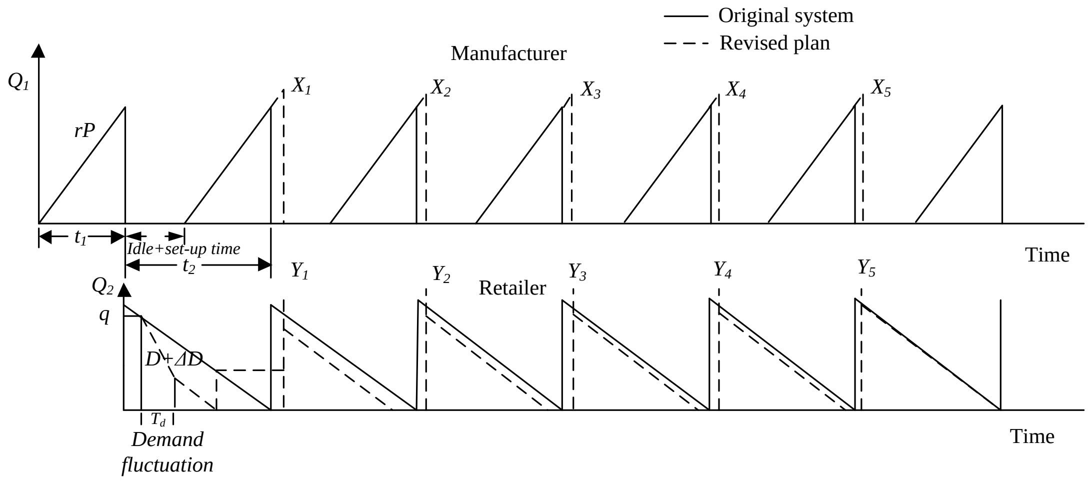

Figure 7.2: Managing a single demand fluctuation

Figure 7.2 shows a typical revised plan caused by an increasing demand rate, where the system uses the idle production time to update the production quantity for a given number of cycles. The revised production plan will start immediately after the cycle experiences the demand fluctuation. As shown in the figure, the demand fluctuation occurred with an increasing rate of ΔD for a period T_d. After the fluctuation period, the demand rate returned to its normal rate. As the system experienced an increased demand fluctuation for a period, there is an unfulfilled demand after the fluctuation. Our approach attempts to satisfy this unfulfilled demand by using the idle production time during the revised planning window. In the figure, the dashed line represents the revised plan after the demand fluctuation. After the occurrence of the demand fluctuation, the future lot sizes for the immediately next cycle of the fluctuated cycle, X_i and Y_i $(i = 1,2, \dots, M)$, will be determined to maximise the profit for the changed scenario, which is shown as a dashed line. Figure 7.3 shows an alternate case, namely for when the demand rate is decreased for a period. In this case the system, unless changed, will have some excess inventory at the end of the cycle. So the system has to reduce the future production lot size to match with the demand as soon as possible to avoid excessive inventory, as shown in Figure 7.3.

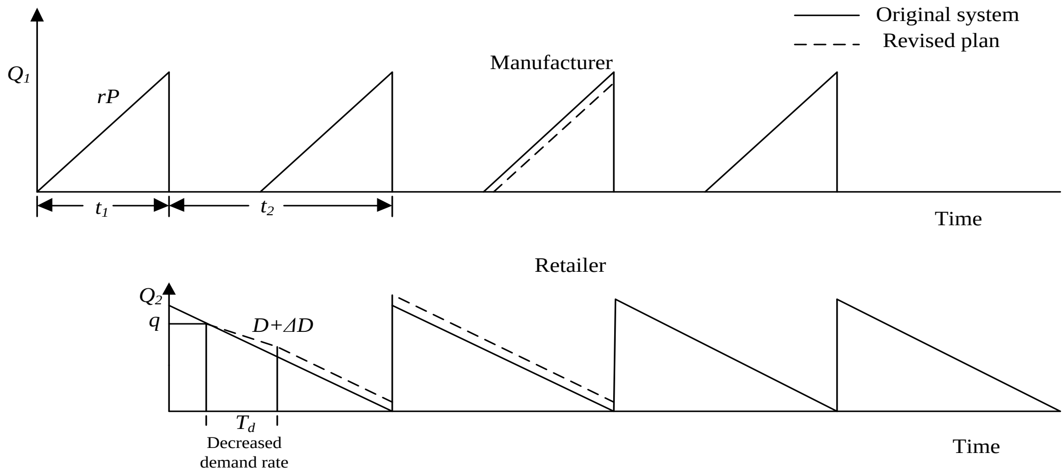

Figure 7.3: Revised plan for $U_d < 0$

7.2.1 Notations used in this Study

In this chapter, the following notations have been used to formulate the model that is applied after the occurrence of a single fluctuation.

Q_1	Production lot size for manufacturer $= X_{i,0}$
Q_2	Ordering lot size for retailer $= Y_{i,0}$
P	Annual production rate
r	Process reliability
D	Annual demand rate
S_t	Set-up time for a cycle for the manufacturer
H_1	Holding cost for the manufacturer ($ per unit per year)
H_2	Holding cost for the retailer ($ per unit per year)
A_1	Set-up cost for the manufacturer ($ per set-up)
A_2	Ordering cost for the retailer ($ per order)
C_P	Production cost per unit
C_R	Rejection cost per unit
C_I	Inspection cost as a percentage of production cost
C_L	Per unit cost due to loss of production
B	Back order cost ($ per unit per unit time)
L	Lost sales cost for retailer ($ per unit)

m_1	Mark-up of selling price $(m_1 C_P)$ – must be greater than 1
$\frac{Y_{i,0}}{D}$	Cycle time for normal cycle i
M	Number of cycles in the revised planning window
X_i	Lot size for cycle i in the revised planning window, for the manufacturer after the fluctuation $(i = 1, 2, \ldots, M)$
Y_i	Lot size for cycle i in the revised planning window, for the retailer after the fluctuation $(i = 1, 2, \ldots, M)$
ΔD	Change of demand rate for the fluctuation
T_d	Fluctuation period for the fluctuation
U_d	Unfulfilled demand after the fluctuation
q	Pre-fluctuation inventory level

7.2.2 Assumptions of the Study

In this study, the following assumptions have been made.

i. The actual production rate is greater than its demand rate.
ii. The products are produced in a lot, and after the production of each lot, it is delivered to the retailer (a lot for lot system).
iii. The recovery cycle will start immediately after the cycle that is experiencing the demand fluctuation.
iv. There are equal numbers of cycles in the revised planning window of both stages.
v. All products are inspected and defective items are rejected.

To fulfill the demand on time, it is commonly assumed that the production rate is higher than its demand rate. However, for a higher demand rate, the model can easily be revised with an option for outsourcing. It was observed that, a lot-for-lot production and delivery system is common in many real-life production-inventory systems, which was discussed in Sarker and Khan (2001). Demand fluctuation is not new in many practical production-delivery systems. To make the recovery process meaningful in practice, the revised plan will be generated after the demand fluctuation is experienced by the system. In other words, the revised plan is generated on a real-time basis. To have a balance supplier-retailer coordinated system, it is assumed that both stages have an equal number of cycles in the revised planning window. Such an assumption will help to better manage the flow of materials within the system. Finally, from the customer satisfaction point of view, no defective item will be delivered to the customers, which is important for many real businesses.

7.3 Model Formulation

In this section, a mathematical model is developed for a single occurrence of demand fluctuation, of either increasing or decreasing rate. In the following few subsections, an equation for the joint economic lot size is first presented under ideal conditions, and then the equations for relevant costs and revenue are derived, by considering imperfect production environment with demand fluctuation. Finally, the revised plan is formulated as a non-linear constrained optimisation problem that maximises the total profit, which is derived from the revenue from acceptable items and the relevant costs, subject to production capacity and product demand constraints. The decision variables are: the revised lot size, backorder quantity, lost sales quantity, and loss of production quantity, in each cycle during the revised planning window.

7.3.1 Optimal Lot Size under Ideal Conditions

For a two stage single item supply chain system, with a lot-for-lot condition under an ideal situation, as considered by Banerjee (1986), the joint economic lot size (as shown in the Figure 7.1) can be formulated as $Q_1 = Q_2 = \sqrt{\frac{2D(A_1+A_2)}{\frac{H_1 D}{rP}+H_2}}$.

Here,

Annual holding cost for manufacturer $= \frac{Q_1}{2} H_1 \frac{D}{rP}$

Annual set-up costs for manufacturer = $\frac{D}{Q_1} A_1$

Annual holding costs for retailer $= \frac{Q_2}{2} H_2$

Annual set-up cost for retailer $=\frac{D}{Q_2}A_2$

So, annual total cost $(TC) = \frac{Q_1}{2}H_1\frac{D}{rP} + \frac{D}{Q_1}A_1 + \frac{Q_2}{2}H_2 + \frac{D}{Q_2}A_2$

Now, $Q_1 = Q_2 = Q$

So, $TC = \frac{Q}{2}H_1\frac{D}{rP} + \frac{D}{Q}A_1 + \frac{Q}{2}H_2 + \frac{D}{Q}A_2$

To minimise total cost, $\frac{d(TC)}{DQ} = 0$

⇨ $\frac{H_1 D}{2rP} - \frac{DA_1}{Q^2} + \frac{H_2}{2} - \frac{DA_2}{Q^2} = 0$

⇨ $Q^2 = \frac{2D(A_1+A_2)}{\frac{H_1 D}{rP}+H_2}$

⇨ $Q^* = \sqrt{\frac{2D(A_1+A_2)}{\frac{H_1 D}{rP}+H_2}}$

Joint economic lot size, $Q_1 = Q_2 = \sqrt{\frac{2D(A_1+A_2)}{\frac{H_1 D}{rP}+H_2}}$ (7.1)

7.3.2 Calculation of U_d after a Fluctuation

The unfulfilled demand, which results from an increased demand rate, is calculated after the occurrence of a fluctuation. It is equal to the multiplication of the fluctuation period and the change of demand rate.

$$U_d = \Delta D \times T_d \tag{7.2}$$

i. If $U_d > 0$, then the demand will be unfulfilled. It is hence required to revise the production lot sizes for a future period and use idle timeslots to fulfill the demand if possible. Lost sales and/or backorder will be present in the revised plan.

ii. If $U_d < 0$, then the production process has to reduce the future lot size because of decreasing demand, which will hence decrease the overall profit. There will be no lost sales or backorder.

iii. $\Delta D = 0$ for the original supplier-retailer coordinated system

7.3.3 Cost Formulation for Manufacturer

In this section, the different cost equations are derived, for the manufacturer, after a single demand fluctuation at the retailer end. The holding, set-up, production, rejection, inspection and interest and depreciation costs are considered in this study. The total holding cost is computed as the unit holding cost multiplied by the total inventory during the revised planning window. The total set-up cost is equal to the cost per set-up multiplied by the number of set-ups in the revised planning window. The total production cost is obtained by multiplying the unit production cost by the total quantity produced during the revised planning window. As the production reliability is assumed to be r, the rejection rate is (1-r). The rejection cost is determined as the unit rejection cost multiplied by the total number of rejected items. The inspection cost is considered as a certain percentage of the production cost (Paul et al., 2014). The cost for interest and depreciation is considered as a general power function as suggested by Cheng (1989).

Holding cost

$$= H_1 \times \frac{X_1}{2} \times \frac{X_1}{rP} + H_1 \times \frac{X_2}{2} \times \frac{X_2}{rP} + \cdots + H_1 \times \frac{X_M}{2} \times \frac{X_M}{rP}$$

$$= \frac{H_1}{2rP} \times \left[{X_1}^2 + {X_2}^2 + \cdots + {X_M}^2\right]$$

$$= \frac{H_1}{2rP} \times \left[\sum_{i=1}^{M} {X_i}^2\right] \qquad (7.3)$$

Set-up cost

$$= A_1 \times M \qquad (7.4)$$

Production cost

$$= C_P P \times \left[\sum_{i=1}^{M} \frac{X_i}{rP}\right] = \frac{C_P}{r} \times \left[\sum_{i=1}^{M} X_i\right] \qquad (7.5)$$

Rejection cost

$$= C_R(1-r)P \times \left[\sum_{i=1}^{M} \frac{X_i}{rP}\right]$$

$$= C_R \left(\frac{1}{r} - 1\right) \times \left[\sum_{i=1}^{M} X_i\right] \tag{7.6}$$

Inspection cost

$$= \frac{C_P C_I}{r} \times \left[\sum_{i=1}^{M} X_i\right] \tag{7.7}$$

Cost of interest and depreciation

$$= Ma\,(A_1)^{-b} r^c \tag{7.8}$$

7.3.4 Cost Formulation for Retailer

In this section, the different costs for the retailer are derived. The holding, ordering, lost sales, backorder cost and cost due to loss of production are considered for the cost formulation for the retailer. If $U_d \geq 0$, lost sales and backorder costs will be present in the formualtion. If $U_d < 0$, costs due to loss of production will be present in the formulation. The total holding cost is computed as the unit holding cost multiplied by the total inventory during the revised planning window. The total ordering cost is equal to the cost per order multiplied by the number of orders in the revised planning window. Back order cost is determined as unit back order cost multiplied by back order units and it's time delay (Hishamuddin et al., 2012). The lost sales cost is determined as unit lost sales cost multiplied by lost sales units (Hishamuddin et al., 2012). The cost due to loss of production is determined as units lost in production multiplied by lost production units.

For $\boldsymbol{U_d \geq 0}$

Holding cost

$$= \frac{H_2}{2D}\left[\left\{Y_1 - \frac{(Y_1 - Y_{1,0})D}{rP} - U_d\right\}^2 + \left\{Y_2 - \frac{(Y_2 - Y_{2,0})D}{rP} - \left(Y_{1,0} - Y_1 + U_d\right)\right\}^2 + \cdots + \left\{Y_M - \frac{(Y_M - Y_{M,0})D}{rP} - \left(\sum_{i=1}^{M-1} Y_{i,0} - \sum_{i=1}^{M-1} Y_i + U_d\right)\right\}^2\right] \tag{7.9}$$

Ordering cost

$$= A_2 \times M \tag{7.10}$$

Lost sales cost

$$= L\left[U_d + \sum_{i=1}^{M} Y_{i,0} - \sum_{i=1}^{M} Y_i\right] \tag{7.11}$$

Back order cost

$= B \times \sum_{i=1}^{M}[\text{units delayed in cycle } i \times \text{delay time of cycle } i]$

$$= B \times U_d\left[\frac{U_d}{D} + \frac{\left(Y_1 - Y_{1,0}\right)}{rP}\right] + B \times \left[\frac{(Y_1 - Y_{1,0})D}{rP} \times \frac{\left(Y_1 - Y_{1,0}\right)}{rP}\right] + B \times \left(Y_{1,0} - Y_1 + U_d\right)\left[\frac{Y_{1,0} - Y_1 + U_d}{D} + \frac{\left(Y_2 - Y_{2,0}\right)}{rP}\right] + B \times \left[\frac{(Y_2 - Y_{2,0})D}{rP} \times \frac{\left(Y_2 - Y_{2,0}\right)}{rP}\right] + \cdots + B \times \left(\sum_{i=1}^{M-1} Y_{i,0} - \sum_{i=1}^{M-1} Y_i + U_d\right)\left[\frac{\sum_{i=1}^{M-1} Y_{i,0} - \sum_{i=1}^{M-1} Y_i + U_d}{D} + \frac{\left(Y_M - Y_{M,0}\right)}{rP}\right] + B \times \left[\frac{(Y_M - Y_{M,0})D}{rP} \times \frac{\left(Y_M - Y_{M,0}\right)}{rP}\right] \tag{7.12}$$

For $\boldsymbol{U_d < 0}$

Holding cost

$$= |U_d| \times \frac{Y_{1,0}}{D} \times H_2 + \frac{H_2}{2D}\left[\sum_{i=1}^{M} {Y_i}^2\right] \tag{7.13}$$

Ordering cost

$$= A_2 \times M \tag{7.14}$$

Cost due to loss of production

$$= C_L|U_d| \tag{7.15}$$

7.3.5 Formulation for Revenue

The total revenue during the revised planning window is determined as the unit selling price multiplied by the products produced during that period as follows:

Total revenue

$$= m_1 C_P \times \left[\sum_{i=1}^{M} Y_i\right] \tag{7.16}$$

7.3.6 Final Mathematical Model

The total profit during the revised planning window, that is the objective function, is determined for a particular fluctuation as follows:

Total profit (TP) = Total revenue - total costs for manufacturer – total costs for retailer.

Subject to the following constraints:

For $\boldsymbol{U_d \geq 0}$

$$X_{i,0} = Q_1;\ \forall i \tag{7.17}$$

$$Y_{i,0} = Q_2;\ \forall i \tag{7.18}$$

$$X_i \geq X_{i,0};\ \forall i \tag{7.19}$$

$$Y_i \geq Y_{i,0};\ \forall i \tag{7.20}$$

$$X_i \leq \left(\frac{X_{i,0}}{D} - S_t\right) rP;\ \forall i \tag{7.21}$$

$$Y_i \leq Y_{i,0} + \left(\sum_{j=1}^{i-1} Y_{j,0} - \sum_{j=1}^{i-1} Y_j + U_d\right);\ \forall i \tag{7.22}$$

$$\sum_{i=1}^{M-1} Y_{i,0} - \sum_{i=1}^{M-1} Y_i + U_d \geq 0 \tag{7.23}$$

$$\sum_{i=1}^{M} Y_i \leq \sum_{i=1}^{M} Y_{i,0} + U_d \tag{7.24}$$

$$r \leq 1 \tag{7.25}$$

$$rP > D \tag{7.26}$$

$$Y_i = X_i;\ \forall i \tag{7.27}$$

$$X_i, Y_i\ \geq 0; \forall i \tag{7.28}$$

For $\boldsymbol{U_d \geq 0}$

$$X_{i,0} = Q_1;\ \forall i \tag{7.29}$$

$$Y_{i,0} = Q_2;\ \forall i \tag{7.30}$$

$$X_i \leq X_{i,0};\ \forall i \tag{7.31}$$

$$Y_i \leq Y_{i,0};\ \forall i \tag{7.32}$$

$$\sum_{i=1}^{M} Y_i = \sum_{i=1}^{M} Y_{i,0} - |U_d| \tag{7.33}$$

$$r \leq 1 \tag{7.34}$$

$$rP > D \tag{7.35}$$

$$X_i = Y_i;\ \forall i \tag{7.36}$$

$$X_i, Y_i \geq 0; \forall i \tag{7.37}$$

For $U_d \geq 0$, equations (7.17) and (7.18) ensure normal cycle lot size is equal to Q_1 and Q_2. Equations (7.19) and (7.20) ensure the revised lot size will be greater than the normal lot size because of the need to satisfy the unfulfilled demand. Production capacity is met by equations (7.21) and (7.22). The unfulfilled demand cannot be negative in the revised planning window, which is ensured by equations (7.23) and (7.24). Equation (7.25) ensures that process reliability must be less than or equal to 1. The production rate is greater than the demand rate, which is met by equation (7.26). Equation (7.27) ensures the balanced coordinated system. The non-negativity constraint is ensured by equation (7.28).

For $U_d < 0$, equations (7.29) and (7.30) ensure normal cycle lot size is equal to Q_1 and Q_2. Equations (7.31) and (7.32) ensure the revised lot size will be equal or smaller than the normal lot size. Equation (7.33) ensures the reduction of lot size by $|U_d|$ to return to the original system. Equation (7.34) ensures that process reliability must be less than or equal to 1. The production rate must be greater than the demand rate, which is ensured by equation (7.35). Equation (7.36) ensures the balanced coordinated system. Another non-negativity constraint is ensured by equation (7.37).

7.3.7 Conditions of Existence of Different Costs

The back order and lost sales costs play an important role in determining the revised plan for an increasing demand rate. In fact, the revised plan is sensitive to the relative magnitude of the back order and lost sales costs. The reduction of production is necessary if the demand rate is decreased. A few propositions help to understand the conditions of the existence of these three cost parameters in the solution. Proposition 1 and 2 are derived to prove the condition of the presence of back order and lost sales respectively in the solution. Proposition 3 proves the condition of the existence of loss of production and proposition 4 confirms the condition of the presence of only lost sales in the solution.

Proposition 1: For a given A_1, A_2, H_1, H_2, B, L, M and if $B\frac{U_d}{D} \leq L$ and $U_d > 0$, then only back orders will be present in the solution when $U_d \leq rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$.

Proof: After a fluctuation, unfulfilled demand, $U_d = \Delta D \times T_d$

Now, idle time in the production cycle, $i = \left(\frac{X_{i,0}}{D} - \frac{X_{i,0}}{rP} - S_t\right)$

As there are M number of cycles in the revised plan, so total idle times in the revised panning window

$$= \sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t$$

Maximum production capacity in the idle times of the revised panning window

$$= rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$$

When $U_d \leq rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$, then the production process is only capable to produce that U_d by using only idle timeslots. So only back orders will then be present in the solution.

Proposition 2 : For a given A_1, A_2, H_1, H_2, B, L, M and if $B\frac{U_d}{D} \leq L$ and $U_d > 0$, then lost sales will exist in the solution when $U_d > rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$.

Proof: It is the opposite consequence of **Proposition 1.**

Proposition 3: For a given A_1, A_2, H_1, H_2, B, L and M, loss of production quantity will be present in the solution when $U_d < 0$.

Proof: When demand rate decreases for a certain period of time, then U_d becomes negative, which is shown in Figure 7.3. This means that some products $(|\Delta D| \times T_d)$ will remain at the end of that cycle at the retailer stage. The production process has to reduce the quantity (U_d), to avoid excessive inventory within the revised planning period, to return to the original system. So when $U_d < 0$, loss of production quantity will be present because of reducing the production quantity by $|U_d|$, as shown in Figure 7.3.

Proposition 4: For a given A_1, A_2, H_1, H_2, B, L, M and if $U_d > 0$, then only lost sales will be present in the solution when $B\frac{U_d}{D} > L$.

Proof: After a fluctuation, unfulfilled demand $U_d = \Delta D \times T_d$

The minimum time required to get the next order is $\frac{U_d}{D}$, that is the delay time.

Now, $B =$ Back order cost per unit per unit time

So, back order cost per unit in the delay time is $B\frac{U_d}{D}$.

Again, $L =$ Lost sales cost ($ per unit)

So, if $B\frac{U_d}{D} > L$, then the back order costs will be greater than the lost sales costs. That's why only lost sales will be present in the solution when $B\frac{U_d}{D} > L$ and total profit is maximised.

7.4 Solution Approach

In this section, a heuristic is firstly developed, with the support of the propositions introduced in the last section, to solve the model for a single occurrence of fluctuation, either for increasing or decreasing demand rate. Then, the heuristic is further developed for managing multiple demand fluctuations on a real-time basis. A fluctuation scenario is defined as the combination of both the quantum and the duration of a demand variation. In reality, these parameters follow a stochastic process, and in this study, uniform random variables are assumed for them. However, one can consider Poisson or any other stochastic process.

7.4.1 Proposed Heuristic for a Single Fluctuation

The heuristic for a single occurrence of fluctuation is briefly described here. For a single demand fluctuation, different parameters are initialized and joint economic lot sizes for the coordinated system are calculated by using equation (7.1), $X_{i,0}$ and $Y_{i,0}$ are also assigned in Step 1. Demand fluctuation scenarios for an independent fluctuation are given as input through Steps 2 and 3. Unfulfilled demand is calculated in Step 4. Step 5 describes the solution technique for $U_d > 0$. In Step 5(a), if backorder cost is less than lost sales cost, then the system uses idle timeslots of the revised planning window to obtain the revised plan. If the production system is capable of producing the excess demand in the revised planning window, then there will be no lost sales. Otherwise both backorder and lost sales options will be present in the solution. But if backorder cost is greater than lost sales cost, then the system will use only the lost sales option to obtain the results, which is described in Step 5(b). For $U_d \leq 0$, because of decreasing demand the production process has to reduce the lot size to obtain the solution, which is described in Step 6. The lot sizes for both supplier and retailer are recorded and different costs and profit are calculated and recorded in Step 7.

Step 1: *Initialize the parameters and calculate Q_1 and Q_2 using equation (7.1). Assign $X_{i,0} = Q_1$ and $Y_{i,0} = Q_2$.*

Step 2: *Input the number of recovery cycles (M) in the revised planning window.*

Step 3: *Input the change of demand rate (ΔD) and the period (T_d) of the fluctuation.*

Step 4: *Determine the unfulfilled demand after the fluctuation by using the equation (7.2).*

Step 5: *For $U_d > 0$*

a) *For $\left(B\frac{U_d}{D}\right) \leq L$*

i. *If $rP\left(\frac{X_{1,0}}{D} - \frac{X_{1,0}}{rP} - S_t\right) \geq U_d$*
Then, calculate the revised lot size using equations (7.38) and (7.39).

$$X_1 = X_{1,0} + U_d \tag{7.38}$$

$$X_i = X_{i,0};\ i = 2,3,\dots,M \tag{7.39}$$

ii. *Else If $rP\left(\frac{X_{1,0}}{D} - \frac{X_{1,0}}{rP} - S_t\right) < U_d \leq rP\left(\sum_{i=1}^{2}\frac{X_{i,0}}{D} - \sum_{i=1}^{2}\frac{X_{i,0}}{rP} - 2S_t\right)$*
Then, calculate the revised lot size using equations (7.40), (7.41) and (7.42).

$$X_1 = X_{1,0} + rP\left(\frac{X_{1,0}}{D} - \frac{X_{1,0}}{rP} - S_t\right) \tag{7.40}$$

$$X_2 = X_{2,0} + U_d - rP\left(\frac{X_{1,0}}{D} - \frac{X_{1,0}}{rP} - S_t\right) \tag{7.41}$$

$$X_i = X_{i,0};\ i = 3,4, \dots, M \tag{7.42}$$

....

iii. *Else If* $rP\left(\sum_{i=1}^{M-1}\frac{X_{i,0}}{D} - \sum_{i=1}^{M-1}\frac{X_{i,0}}{rP} - (M-1)S_t\right) < U_d \le rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$
Then, calculate the revised lot size using equations (7.43), (7.44), (7.45) and (7.46).

$$X_1 = X_{1,0} + rP\left(\frac{X_{1,0}}{D} - \frac{X_{1,0}}{rP} - S_t\right) \tag{7.43}$$

$$X_2 = X_{2,0} + rP\left(\frac{X_{2,0}}{D} - \frac{X_{2,0}}{rP} - S_t\right) \tag{7.44}$$

....

$$X_{M-1} = X_{M-1,0} + rP\left(\frac{X_{M-1,0}}{D} - \frac{X_{M-1,0}}{rP} - S_t\right) \tag{7.45}$$

$$X_M = X_{M,0} + U_d - rP\left[\sum_{i=1}^{M-1}\frac{X_{i,0}}{D} - \sum_{i=1}^{M-1}\frac{X_{i,0}}{rP} - (M-1)S_t\right] \tag{7.46}$$

iv. *Else If* $U_d > rP\left(\sum_{i=1}^{M}\frac{X_{i,0}}{D} - \sum_{i=1}^{M}\frac{X_{i,0}}{rP} - MS_t\right)$
Then, calculate the revised lot size using equation (7.47).

$$X_i = X_{i,0} + rP\left(\frac{X_{i,0}}{D} - \frac{X_{i,0}}{rP} - S_t\right);\ i = 1,2, \dots, M \tag{7.47}$$

b) For $\left(B\frac{U_d}{D}\right) > L$
Calculate the revised lot size using equation (7.48).

$$X_i = X_{i,0};\ i = 1,2, \dots, M \tag{7.48}$$

Step 6*: For* $U_d \le 0$

Calculate the revised lot size using equations (7.49), (7.50) and (7.51).

$$X_1 = X_{1,0} \tag{7.49}$$

$$X_2 = X_{2,0} - |U_d| \tag{7.50}$$

$$X_i = X_{i,0};\ i = 3,4, \dots, M \tag{7.51}$$

Step 7*: Record the revised lot size* (X_i) *and update the revised lot size for retailer* (Y_i)*. Also calculate total lost sale costs, backorder costs, costs due to loss of production and total profit and record the revised plan after the fluctuation.*

Step 8: *Stop.*

7.4.2 Extended Heuristic for Multiple Fluctuations

The following notations are considered for the multiple fluctuations case.

n Fluctuation number

$X_{i,n}$ Lot size for cycle i in the revised planning window, for the manufacturer after the n^{th} fluctuation ($i = 1, 2, \dots, M$)

$Y_{i,n}$ Lot size for cycle i in the revised planning window, for the retailer after the n^{th} fluctuation $(i = 1, 2, \ldots., M)$

ΔD_n Change of the demand rate for the n^{th} fluctuation

$T_{d,n}$ Fluctuation period for the n^{th} fluctuation

$U_{d,n}$ Unfulfilled demand after the n^{th} fluctuation

q_n Pre-fluctuation inventory level

Figure 7.4 shows the management mechanism for multiple demand fluctuations on a real-time basis. As shown in the figure, the first demand fluctuation occurred with an increasing rate of ΔD_1 for a period $T_{d,1}$. After the occurrence of the first demand fluctuation, the future lot sizes for the immediately next cycle of the fluctuated cycle, $X_{i,1}$ and $Y_{i,1}$ $(i = 1,2, \ldots, M)$, will be determined to maximise the profit for the changed scenario, which is shown as a dashed line. If a fluctuation occurs after the revised planning window of another fluctuation, then the later one can be considered as an independent fluctuation, and a revised plan can be made similarly to the previous one. However, if a fluctuation occurs during the revised planning window of another fluctuation that occurred earlier, a revised plan, incorporating the effect of both fluctuations must be derived, which makes the case more complex for revised planning.

In Figure 7.4, the second fluctuation occurs with an increasing rate of ΔD_2 and for a period $T_{d,2}$, which is during the revised planning window of the first fluctuation. After the second fluctuation, another revised plan must be derived, which incorporates the effect of both the first and second fluctuations. In Figure 7.4, after the second fluctuation, the production quantities $X_{i,2}$ and $Y_{i,2}$ $(i = 1,2, \ldots, M)$ are revised again within the allowed revised planning window, to consider the effect of both fluctuations, which is shown with a dotted line. The plan is revised in the same way if there is any further demand fluctuation. These fluctuations may occur at any point in time, and the duration of fluctuation may vary from one occurrence to the next. After each fluctuation, the lot size in each future cycle must be revised, within the allowed revised planning window, as a plan for improvement.

For multiple fluctuations cases, the unfulfilled demand $(U_{d,n})$ is calculated using equation (7.52). The portion of unfulfilled demand from any previous recovery is taken into consideration to derive the equation of $U_{d,n}$.

$$U_{d,n} = U_{d,n-1} + \Delta D_n \times T_{d,n} - \sum_{i=1}^{l} X_{i,n-1} + \sum_{i=1}^{l} X_{i,0} \tag{7.52}$$

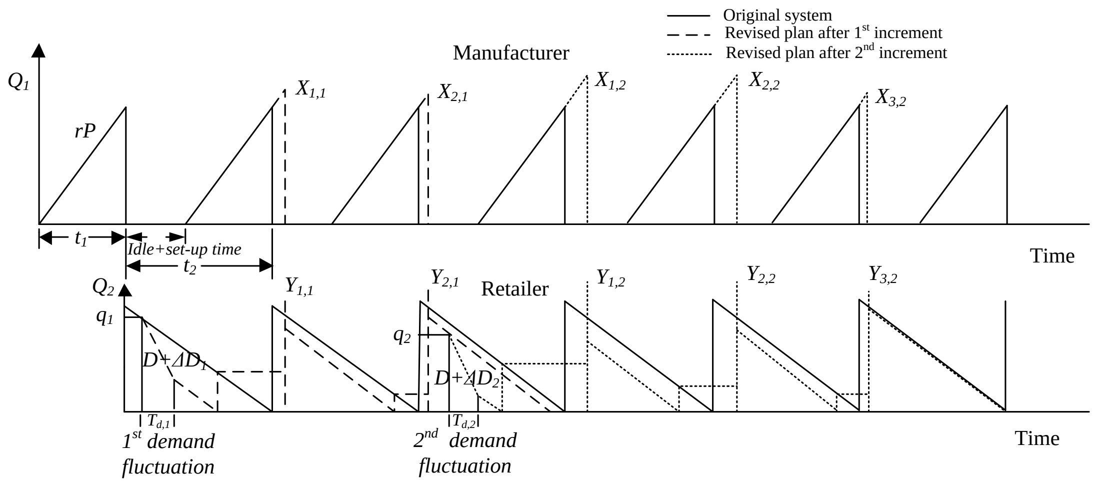

Figure 7.4: Managing multiple demand fluctuations

To manage these situations the heuristic for a single fluctuation, is further developed for managing multiple fluctuations as a series of demand fluctuations on a real-time basis, which as described in the following. After the first fluctuation, the demand fluctuation scenarios are given as input in **Step A** and after then, the unfulfilled demand is calculated. Then the model is solved by using the heuristic developed for a single fluctuation and the revised plan is recorded. If there is any new fluctuation, the new fluctuation scenarios are given as input in **Step F** and the unfulfilled demand is calculated by using equation (52). Once again the model is solved by using the heuristic for the updated parameters. If there is any other fluctuation, this loop will be repeated as long as any more demand fluctuations take place in the system, during which the revised plan after each fluctuation is recorded. The steps of the extended heuristic are as follows.

Step A: *Input the change of the demand rate and the fluctuation period for the first fluctuation.*
Step B: *Determine unfulfilled demand,* $U_{d,n} = \Delta D_n \times T_{d,n}$.
Step C: *Update the parameter* $U_d = U_{d,n}$ *and also update the decision variables as* $X_i = X_{i,n}$ *and* $Y_i = Y_{i,n}$.
Step D*: Solve the model by using steps* ***5 to 7*** *of the heuristic developed in Section 4.1 under the updated parameters.*
Step E: *Record the revised plan after the fluctuation.*
Step F: *If there is any other fluctuation,*

F.1 *Later when known, input the fluctuated cycle number (l), change of demand rate (ΔD_n) and fluctuation period ($T_{d,n}$) for the fluctuation.*
F.2 *Calculate $U_{d,n}$ by using equation (52).Go to* ***Step C.***
Step G: *Stop.*

7.4.3 Parameters for the Demand Fluctuation Problem

For experimentation, the following data range is considered with a discrete uniform (DU) distribution for the demand fluctuation problem.

Pre- fluctuation inventory level: $q =$ DU $\left[0,\ Y_{1,0}\right]$

Fluctuation period:

For $\Delta D > 0$; $T_d =$ DU $\left[0.0001,\ \frac{q}{D+\Delta D}\right]$

For $\Delta D < 0$; $T_d =$ DU $\left[0.0001,\ \frac{q}{D}\right]$

Lost sales cost: $L =$ DU [5, 100]

Back order cost: $B =$ DU [5, 1000]

Set-up cost: $A_1 =$ DU [20, 500]

Ordering cost: $A_2 =$ DU [20, 500]

Holding cost: $H_1 =$ DU [1, 10], $H_2 =$ DU [1, 10]

Process reliability: $r =$ DU [$\frac{D}{P}$, 1]

7.5 Experimental Results and Analysis

In this section, the results have been analysed for a single fluctuation, as well as multiple fluctuations on a real-time basis. For the experimentation, the fluctuation scenarios are generated using a uniform probability distribution, within the given intervals of the parameters, as presented in section 5.4.3. For a single demand fluctuation, the proposed heuristic is used, as developed in Section 4.1, for single fluctuation to analyse the results. Then the heuristic was modified for multiple fluctuations on a real-time basis, as indicated in Section 5.4.2.

7.5.1 Single Fluctuation

The revised plan for a single fluctuation either for increasing or decreasing demand rate has been generated, using the proposed heuristic, for three problem scenarios with the following data.

$$P = 500000,\ r = 0.95,\ D = 450000,\ S_t = 0.000114,\ H_1 = 2.5,\ H_2 = 3.0,\ A_1 = 120,$$
$$A_2 = 150,\ C_P = 40,\ C_R = 40,\ C_I = 0.02,\ C_L = 15,\ B = 20,\ L = 15,\ a = 1000,\ b = 0.5,$$
$$c = 0.75,\ m_1 = 2,\ M = 5$$

7.5.1.1 Increasing demand rate

Two different scenarios are considered to analyse the results for increasing demand rate. In scenario 1, it is observed that both backorders and lost sales are present in the revised plan and only backorders are present in the revised plan of scenario 2. As it is considered there are five production cycles in the revised planning window, the system will return to normal production plan at the sixth cycle after a fluctuation.

i. Scenario 1

The first fluctuation problem is given as:

Pre- fluctuation inventory level: 6000

Change of demand rate: 250000

Fluctuation period: 0.007

Next the fluctuation case problem was solved using the proposed heuristic for the single fluctuation. The results were obtained and are presented below:

Lost sales costs = 2250
Backorder costs = 414.5
Costs due to loss of production = 0
Total profit = 1225731
Revised lot size:
$X_{i,1}$ = 7047, 7047, 7047, 7047, 7047
$Y_{i,1}$ = 7047, 7047, 7047, 7047, 7047

It can be observed that both backorders and lost sales are present in the revised plan.

ii. Scenario 2

The second fluctuation problem is given as:

Pre- fluctuation inventory level: 5000

Change of demand rate: 200000

Fluctuation period: 0.004

The results were obtained and are:

Lost sales costs = 0

Backorder costs = 67.3

Costs due to loss of production = 0

Total profit = 1200237

Revised lot size:

$X_{i,1}$ = 7047, 7047, 6887, 6727, 6727

$Y_{i,1}$ = 7047, 7047, 6887, 6727, 6727

It can be observed that only backorders are present in the revised plan.

7.5.1.2 Decreasing demand rate

The third fluctuation problem, for a decreasing demand rate, is given as:

Pre- fluctuation inventory level: 5500

Change of demand rate: -150000

Fluctuation period: 0.006

The results were obtained and are:

Lost sales costs = 0

Backorder costs = 0

Costs due to loss of production = 13500

Total profit = 1127471

Revised lot size:

$X_{i,1}$ = 6727, 5827, 6727, 6727, 6727

$Y_{i,1}$ = 6727, 5827, 6727, 6727, 6727

It can be observed that the revised plan in obtained by using only loss of production quantity.

7.5.2 Results Comparison

In order to judge the quality of the solutions of heuristic, the above three problems were also solved using a genetic algorithm (GA). GA is a very popular technique to solve complex nonlinear constrained optimisation problems. GAs are general purpose optimisation algorithms which apply the rules of natural genetics to explore a given search space (Homaifar et al., 1994). Both approaches were coded in MATLAB R2012a, and were executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU. The best results obtained from the GA, out of 10 independent runs, were compared with the heuristic results.

Table 7.1: Results comparison between the heuristic and genetic algorithm

Test case	Fluctuation scenario	Total Profit		Deviation (%)	Running time (Seconds)	
		Heuristic	GA (best result)		Heuristic	GA
1	$q = 6000$ $\Delta D = 250000$ $T_d = 0.007$	1225731	1225610	0.0098	0.90	88.74
2	$q = 5000$ $\Delta D = 200000$ $T_d = 0.004$	1200237	1200263	0.0021	0.75	26.94
3	$q = 5500$ $\Delta D = -150000$ $T_d = 0.006$	1127471	1127352	0.0105	0.82	1038.60

The heuristic showed insignificant changes in results for different runs and for GA technique, the best results recorded out of 10 independent runs. It is observed that, the results, obtained from the heuristic and the genetic algorithm, are almost identical. Table

7.1 shows the comparison of the maximum total profit obtained and the computational time required, for three fluctuation scenarios, by the two approaches. For our experimentation, 68 scenarios have been developed with uniformly random fluctuations within the range of data presented in section 7.4.3. The scenarios have then divided in two classes. Among all of the scenarios, 32 are longer range fluctuations, which have fluctuation periods greater than 0.005, and the other are shorter range fluctuations, which have fluctuation periods less than 0.005. The summary of the results, obtained from the heuristic and the GA, is presented in Table 7.2. It is observed that the average percentage of deviation between the results from the two approaches is very small. The percentage of deviation of the results was calculated by using equation (7.53). This deviation may merely be because of rounding the values of the decision variables. Apart from the ability to produce quality solutions, the heuristic has a significantly lower average computational time in comparison to the genetic algorithm approach.

$$\text{Percentage of deviation} = \frac{(\text{Total profit from the heuristc} - \text{Total profit from GA})}{\text{Total profit from GA}} \times 100\% \tag{7.53}$$

Table 7.2: Results comparison for 68 fluctuations cases

Fluctuation class	Average deviation (%)	Average computational time (Seconds)	
		Heuristic	GA
Longer range fluctuation	0.0089	0.7421	72.93
Shorter range fluctuation	0.0023	0.7473	18.5

7.5.3 Multiple Fluctuations

To demonstrate the usefulness of our proposed heuristic in solving different scenarios with multiple fluctuations, over a period of time, the basic data is used from the single fluctuation cases.

Table 7.3: A demand fluctuation case problem

Fluctuation number (n)	Disrupted cycle number from previous fluctuation (l)	Pre-fluctuation inventory level (q_n)	Change of demand rate (ΔD_n)	Fluctuation period ($T_{d,n}$)
1	--	6500	300000	0.0065
2	3	4500	-120000	0.0100
3	5	5700	220000	0.0080
4	2	3000	100000	0.0040
5	3	4000	-200000	0.0060
6	1	6000	250000	0.0030
7	4	4200	100000	0.0050
8	2	5500	180000	0.0070
9	5	2000	-250000	0.0025
10	4	3500	200000	0.0035
......	.....	.....	.....	.....

In any supply chain system, a series of demand fluctuations can occur, one after another, on a real-time basis. The first fluctuation can be managed by using the single fluctuation approach discussed in the previous section. Then if a new fluctuation later occurs after the revised planning period of any previous fluctuation, it can be considered as another single fluctuation case that does not affect the revised plan of the previous fluctuation. But if a new fluctuation occurs within the revised planning window of any fluctuation, then it may affect the revised plan of the previous fluctuation, and the revised plan for that planning window must be considered as a set of additional restrictions. For experimental purposes, a series of random fluctuations was generated to occur one after another, either for increasing or decreasing demand rate. Table 7.3 presents a case problem with different random combinations of fluctuated cycle number, pre-fluctuation inventory level, and fluctuation period. Although the fluctuations happen continuously, a case problem is presented with only ten fluctuations as a sample representation. The lot sizes were revised immediately after each fluctuation takes place in the system. The problem was solved by using the proposed heuristic for multiple fluctuations, which is presented in Section 7.4.2, and the result was recorded. Table 7.4 presents, for each fluctuation, the set of revised lot sizes in each cycle and also the maximum total profit.

Table 7.4: The results for the demand fluctuation case problem

Fluctuation number (n)	Revised lot size										Total profit
	$X_{1,n}$	$Y_{1,n}$	$X_{2,n}$	$Y_{2,n}$	$X_{3,n}$	$Y_{3,n}$	$X_{4,n}$	$Y_{4,n}$	$X_{5,n}$	$Y_{5,n}$	
1	7047	7047	7047	7047	7047	7047	7047	7047	7047	7047	1222647
2	6727	6727	6517	6517	6727	6727	6727	6727	6727	6727	1161853
3	7047	7047	7047	7047	7047	7047	7047	7047	7047	7047	1225577
4	7047	7047	7047	7047	7047	7047	7047	7047	6967	6967	1225264
5	6727	6727	6087	6087	6727	6727	6727	6727	6727	6727	1140427
6	6837	6837	6727	6727	6727	6727	6727	6727	6727	6727	1176159
7	7047	7047	6907	6907	6727	6727	6727	6727	6727	6727	1189774
8	7047	7047	7047	7047	7047	7047	7027	7027	6727	6727	1216243
9	6727	6727	6102	6102	6727	6727	6727	6727	6727	6727	1141175
10	7047	7047	7047	7047	6787	6787	6727	6727	6727	6727	1196750
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

7.5.4 Comparing with Two Alternative Approaches

To compare the results of the proposed approach with two other alternative approaches, eight fluctuations have been generated randomly, for increasing demand rate, within 32 cycles. The total profit, as well as the lost sales and back orders, for the two other alternative approaches and our approach, are calculated and compared in Table 7.5. The two alternative approaches are:

i. *Managing with only lost sales*: In this approach, the unfulfilled demand due to demand fluctuation is considered as lost sales. This means, from the next cycle of fluctuation, it will return to the normal schedule.

ii. *First cycle revision*: This approach is the same as our model, except that the fluctuation will be recovered in one cycle. If the system is capable to produce the unfulfilled demand by using idle time of one cycle, then there will be no lost sales. Otherwise, the fluctuation will be recovered with both backorders and lost sales options.

Table 7.5: Comparison of the results from 32 cycles

Cost or Profit	Original system	Demand fluctuation management (Proposed approach)	First cycle revision	Managing with only lost sales
Total backorder costs	0	578.1	188.3	0
Total lost sales costs	0	0	89400	127800
Total profit	7502822	7799669	7502861	7375362

From Table 7.5, it is clear that the proposed demand fluctuation management plan provides a much better solution than the other two alternative approaches, while the first cycle revision plan shows better results than the managing with only lost sales.

The total profit for each cycle, obtained from the above three alternative solutions and the original system, is presented in Figure 7.5. When a cycle faces a fluctuation, the total profit of that cycle goes to its *lower bound* if the plan is revised with only the lost sales option. However, for a fluctuation free cycle, the profit is the same as the original production system. The first cycle revision approach shows better results than the lower bound. In summary, the proposed demand fluctuation management model clearly shows significantly better solutions in comparison to other alternative approaches.

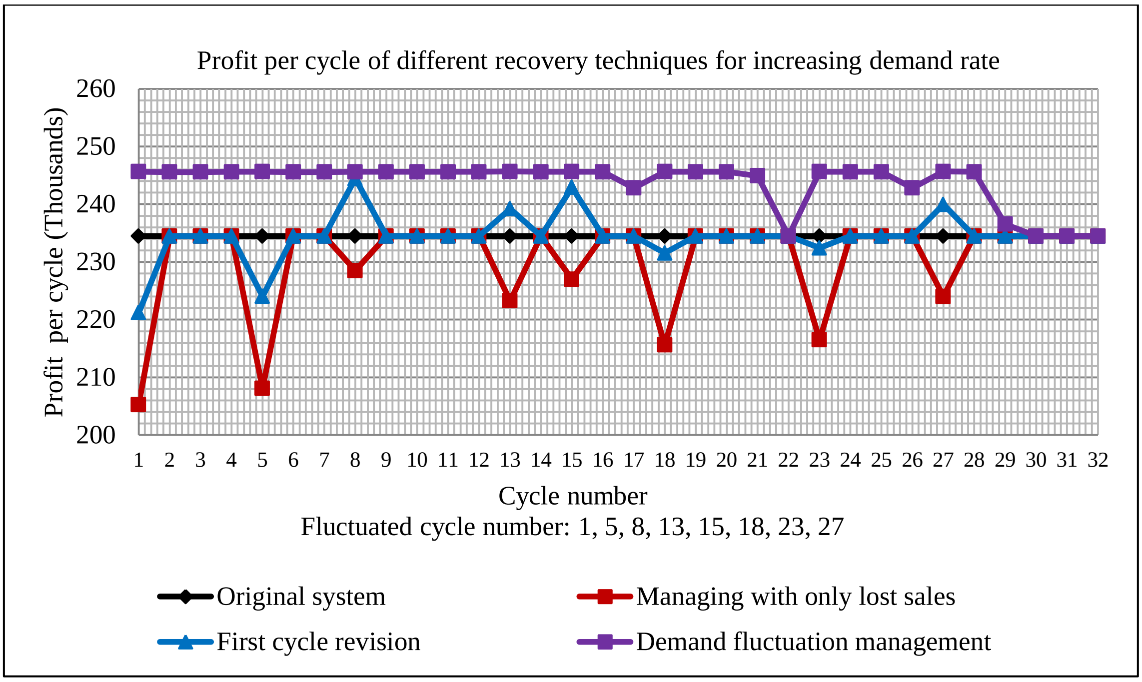

Figure 7.5: Total profit in each cycle for different recovery techniques

7.6 Sensitivity Analysis

There are some important parameters that have a significant impact on the total profit. These parameters are: fluctuation period, back order cost, lost sales cost, process reliability, set-up cost, ordering cost and holding cost. In this section, the relationship of these variables with the total profit is analysed.

This section presents a number of studies, where in each of them one parameter is considered. Other parameters will have the default values of 200000 for increasing demand rate, -100000 for decreasing demand rate, 0.009 for increasing demand fluctuation period, 6000 for pre-fluctuation inventory level and the remainder are kept the same as in section 7.5.1.

The relationship between total profit and fluctuation period for increasing demand rate is presented in Figure 7.6. The total profit increases with the fluctuation period, because an increasing lot size is necessary to satisfy the unfulfilled demand. But after 0.008 of fluctuation period, the total profit decreases because of the commencement of lost sales costs. Backorder costs increase with fluctuation period. Figure 7.7 presents the changes of total profit with fluctuation period for a decreasing demand rate. Total profit decreases with fluctuation period because of the increasing costs due to loss of production.

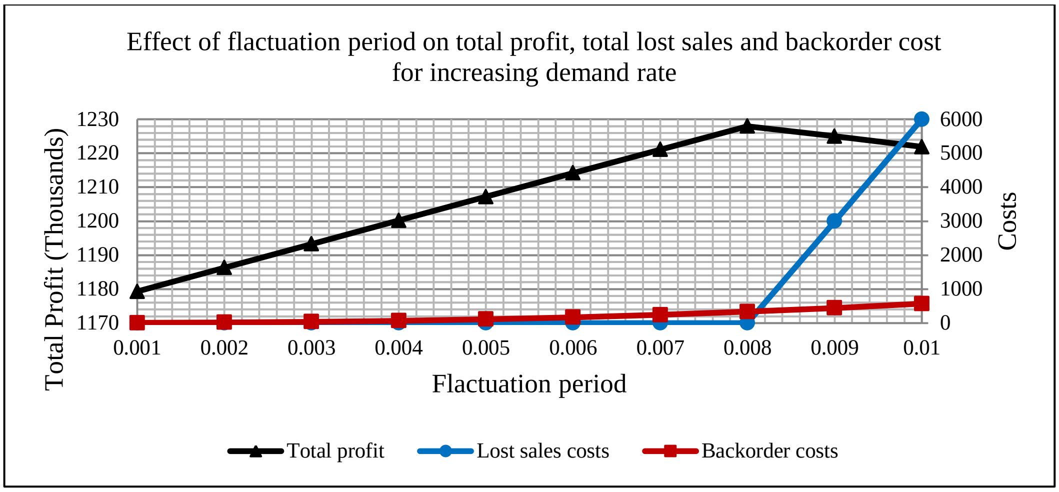

Figure 7.6: Changes of total profit, lost sales and backorder costs with a fluctuation period for an increasing demand rate

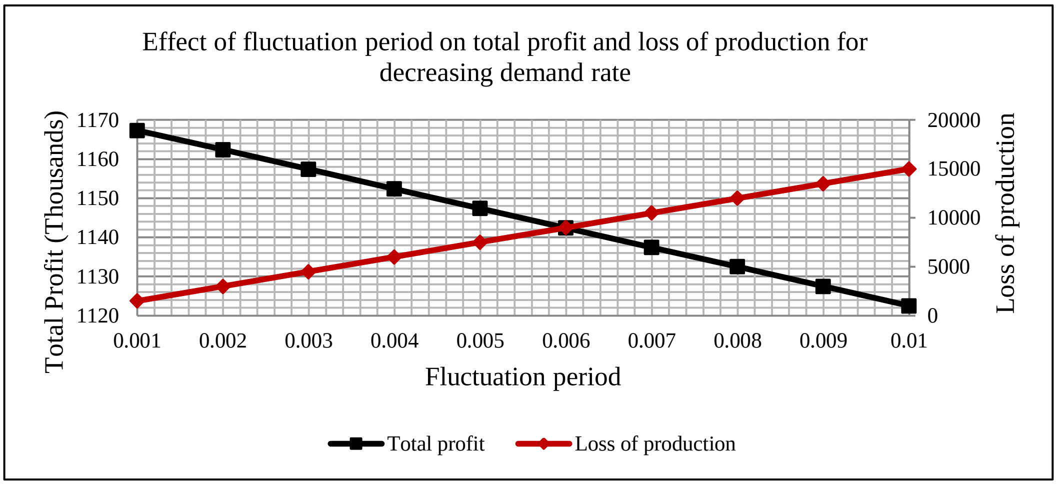

Figure 7.7: Changes of total profit and costs due to loss of production with a fluctuation period for a decreasing demand rate

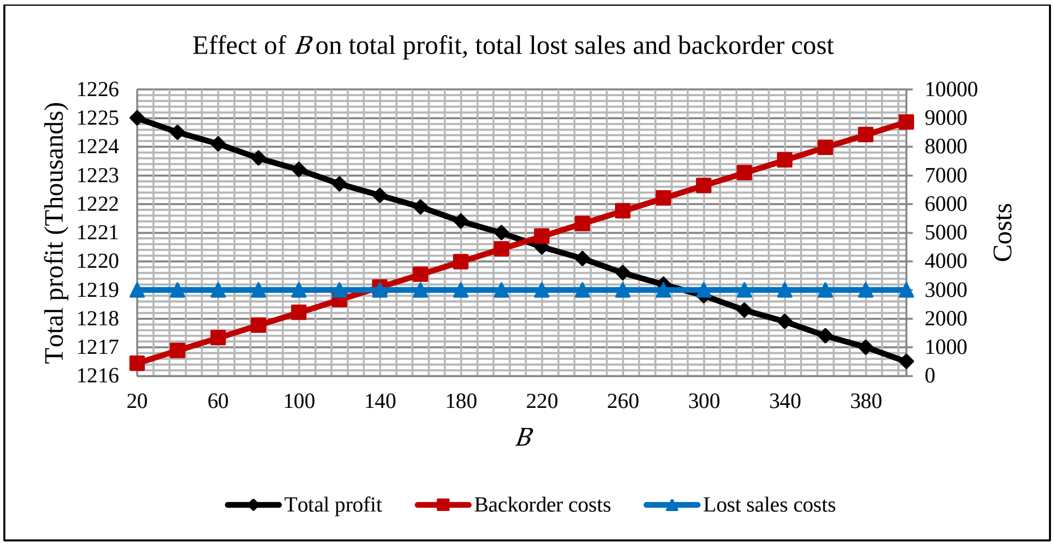

Figure 7.8: Changes of total profit, lost sales and backorder costs with B

The effect of backorder cost on the total profit is illustrated in Figure 7.8. Total profit decreases with *B*, due to the increase of total backorder costs. Total lost sales costs remain unchanged. The total profit also decreases with *L*, due to the increase of total lost sales costs, which is presented in Figure 7.9. Total backorder costs remain unchanged.

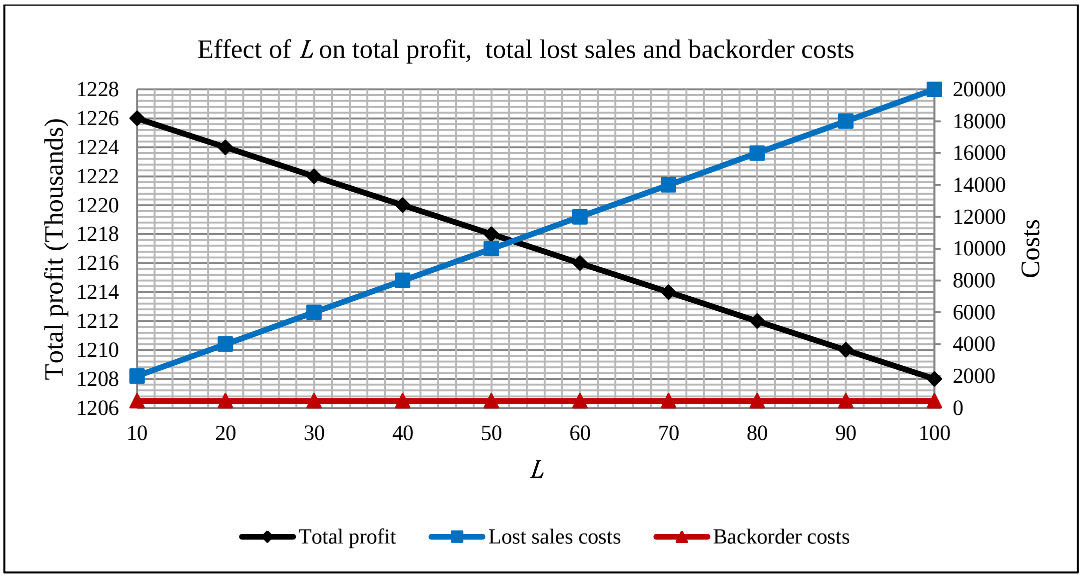

Figure 7.9: Changes of total profit, lost sales and backorder costs with *L*

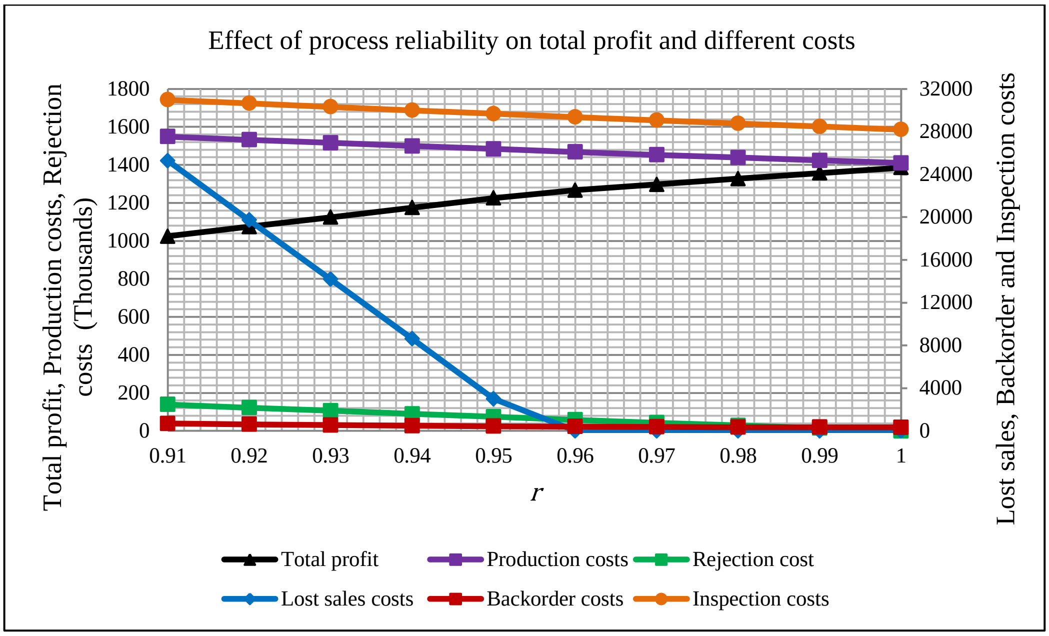

Figure 7.10: Changes of total profit and other cost with process reliability

Figure 7.10 demonstrates the effect of production process reliability on total profit and other relevant costs. Production, rejection, lost sales, backorder and inspection costs decrease, but the cost of interest and depreciation increase, with process reliability. Thus, total profit increases with process reliability.

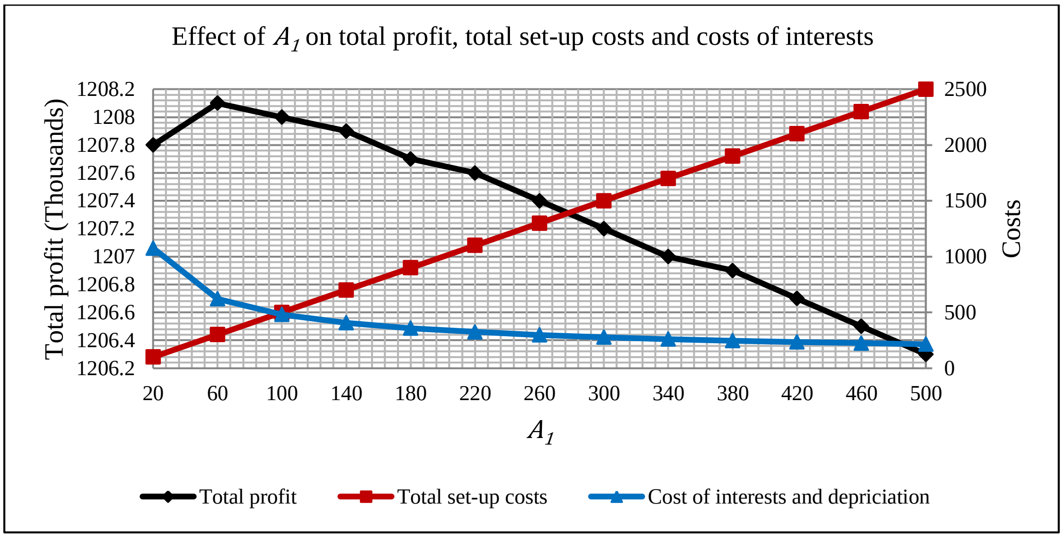

Figure 7.11: Changes of total profit and other costs with A_1

The relationship between total profit and set-up cost is shown in Figure 7.11. Cost of interest and depreciation decreases and total set-up cost increases with the cost per set-up. Ultimately, total profit decreases with the increment of cost per set-up. Total profit decreases with the cost of per order, which is shown in Figure 7.12, because of the increasing total ordering cost.

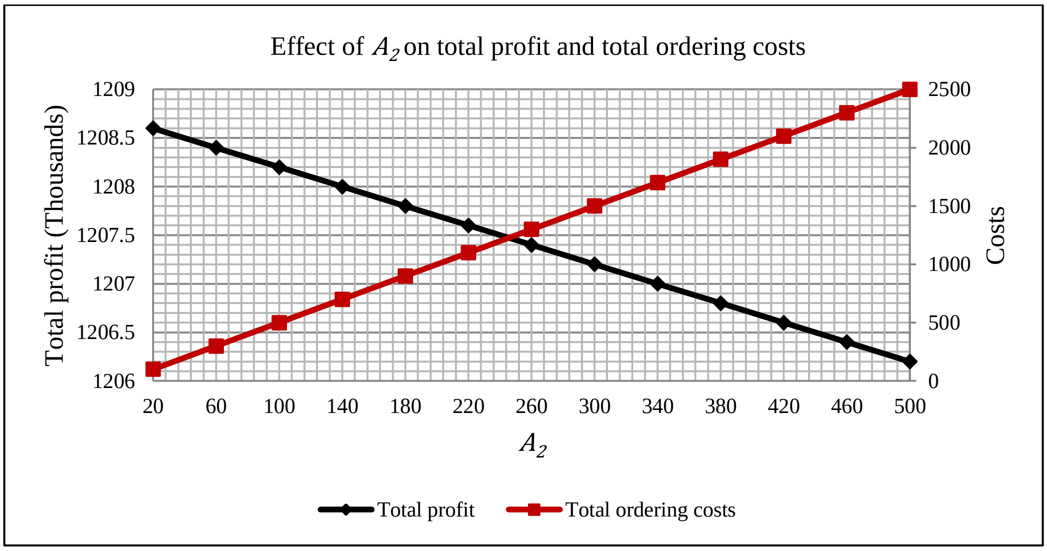

Figure 7.12: Changes of total profit and total ordering costs with A_2

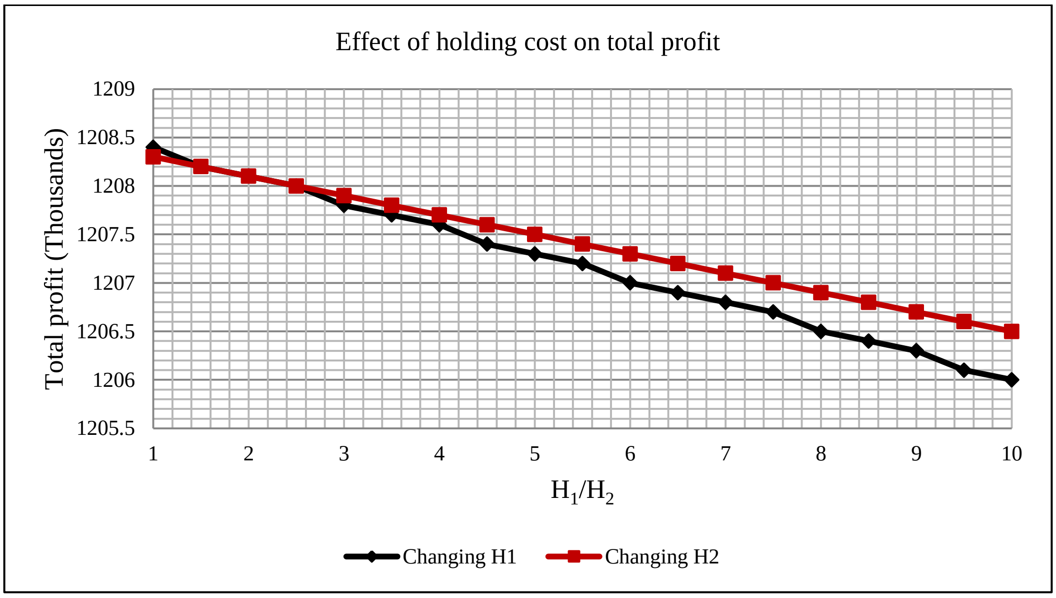

Figure 7.13: Changes of total profit with H_1 and H_2

The relationship between total profit and per unit holding cost is presented in Figure 7.13. It is observed that the total profit decreases with per unit holding cost for both supplier and retailer.

7.7 Chapter Summary

The main objective of this study was to develop a disturbance management model in a two-stage supplier-retailer coordinated system due to demand fluctuations, and with an imperfect production process. The model was formulated as a non-linear constrained optimisation problem to maximise the total profit in the revised planning window, and a heuristic was proposed to obtain the revised plan, after the occurrence of a fluctuation. The model was also solved by using a genetic algorithm based search technique, with a uniformly distributed random fluctuation scenarios. It was observed that our proposed heuristic was capable of producing quality solutions with significantly less computational time, as compared to the genetic algorithm.

In real life situations, multiple demand fluctuations can happen, whether dependent or independent, one after another as a series on a real-time basis. The extended heuristic was also capable to solve the model that deals with multiple demand fluctuations on a real-time basis. In this case, the solutions have been generated after each fluctuation with changed parameters. The proposed model offers a potentially very useful quantitative study to help decision makers to make prompt and accurate decisions on the revised plan, whenever a sudden or a series of demand fluctuations occur in a supply chain system.

The disruption management problem, for managing demand fluctuation in a supplier-retailer coordinated system, has been addressed in chapter 7. As supply disruption is also a major source of risk and it can also imbalance the supply chain plan, it would be interesting to investigate this specific type of disruption. Furthermore, considering the lack of research for managing supply disruption in three-tier supply chain, I have chosen it as my research work in the next chapter.

Chapter 8 Managing Supply Disruption

In this chapter, a quantitative approach is developed for managing supply disruption for a supply chain. A three-tier supply chain system is considered, with multiple raw material suppliers, a single manufacturer and multiple retailers, where the system may face sudden disruption in its raw material supply. First, a mathematical model is developed that generates a recovery plan after the occurrence of a single disruption. Here, the objective is to minimise the total cost during the recovery time window while being subject to supply, capacity, demand, and delivery constraints. An efficient heuristic is developed to solve the model for a single disruption. Second, multiple disruptions are also considered, where a new disruption may or may not affect the recovery plans of earlier disruptions. A new dynamic mathematical and heuristic approach is also developed that is capable of dealing with multiple disruptions, one after another as a series, on a real-time basis. The heuristic solutions are compared with those obtained by a standard search algorithm for a set of randomly generated disruption test problems. Finally, this chapter develops a simulation model to analyse the effect of randomly generated disruption events that are not known in advance.

8.1 Introduction

In the modern business era, supply chains are an important part of many businesses. A manufacturing supply chain is a network which receives raw materials from suppliers as input, which it processes in manufacturing plants, to obtain final products for delivery to customers through a distribution network. A standard three-tier supply chain network consists of suppliers, manufacturers and retailers. Suppliers supply raw materials to manufacturers, and after processing in a manufacturing plant, final products are delivered to retailers according to their demand. In reality, a supply chain can face many uncontrollable problems, such as production and supply disruption (Sodhi and Chopra, 2004). Without a proper response to those problems, a supply chain system can be imbalanced, and an organisation can consequently face huge financial loss, as well as loss of customer goodwill.

From the literature review, it is clear that most of the research developed supply chain models under ideal conditions. Although a few of them developed disruption recovery models, most of them considered a single supplier and a single retailer, which limits the applicability of such studies. To overcome this limitation, this chapter develops a quantitative and simulation approach to recover from a supply disruption, for a three-tier supply chain with multiple suppliers and multiple retailers. Existing studies developed disruption recovery policies for only a single supply disruption. In this chapter, a three-tier supply chain system is considered, that deal with both single, as well as multiple disruptions, on a real-time basis. Disruption events that are not known and cannot be predicted in advance are considered. A mathematical model is first developed for coping with a single supply disruption. Then a new efficient heuristic is proposed for generating a revised plan after a disruption. In the experimental study, a random probability distribution is used to generate disruption parameters, such as disrupted raw material, and disruption durations. Then, the mathematical model is solved to obtain the revised plan after the occurrence of a disruption. Multiple disruptions are also considered, one after another in a series, that can occur at any time at any supplier and that may or may not affect the plans revised after previous disruptions. If a new disruption occurs during the recovery time window of another, a new revised plan which considers the effects of both disruptions must be derived. Accordingly, as this is a continuous process, the mathematical model and the heuristic are extended to deal with a series of disruptions on a real-time basis, by incorporating a modified version of those developed for a single disruption. The heuristic solutions have compared with those obtained by a standard search algorithm for a good number of randomly generated disruption test problems to validate the performance of the proposed heuristic. Finally, a simulation model is developed to analyse the effect of randomly generated disruption events that are not known in advance. Many random experiments and their numerical results have been performed to explain the usefulness of the developed models and methodologies.

The main contributions of this chapter can be summarized as follows.

i. Development of a new mathematical model for managing raw material supply disruption in a three-tier supply chain system with multiple suppliers, a single manufacturer and multiple retailers. As a disruption scenario is not known in advance and is not possible to predict, the recovery plan is revised for some future periods after the disruption occurs on a real-time basis.
ii. Development of a new efficient heuristic for generating a revised plan after a disruption.
iii. Extension of this heuristic to deal with multiple disruptions on a real-time basis. As any new disruptions may or may not affect the plans revised after the previous ones, their scenarios may be dependent or independent, both of which the extended heuristic can handle.
iv. The conduct of many random experiments to validate the heuristics and develop a simulation model which closely emulates real-world processes.

8.2 Problem Description

A three-tier manufacturing supply chain system with multiple raw material suppliers and retailers is considered. It is assumed that each supplier supplies one type of raw material. That means the number of suppliers is equal to the number of different types of raw materials required in the production process. The products are produced in batches in a single manufacturing plant. After production, the products are delivered to the retailers according to their demand. In an ideal plan, the optimal supply, production and delivery quantities are q_i, Q and Q_j respectively, which is shown as a solid line in Figure 8.1. However the system can face a sudden supply disruption at any time. To manage the system efficiently, it is necessary to generate a recovery plan after the occurrence of a disruption. In Figure 8.1, after a supply disruption, a recovery plan is generated to revise the supply (X_{ki}), production (Y_k) and delivery (Z_{kj}) quantities during the recovery time window, which is shown as a dashed line. The objective is to minimise the total cost during the recovery time window, while being subject to supply, production capacity, demand, and delivery constraints.

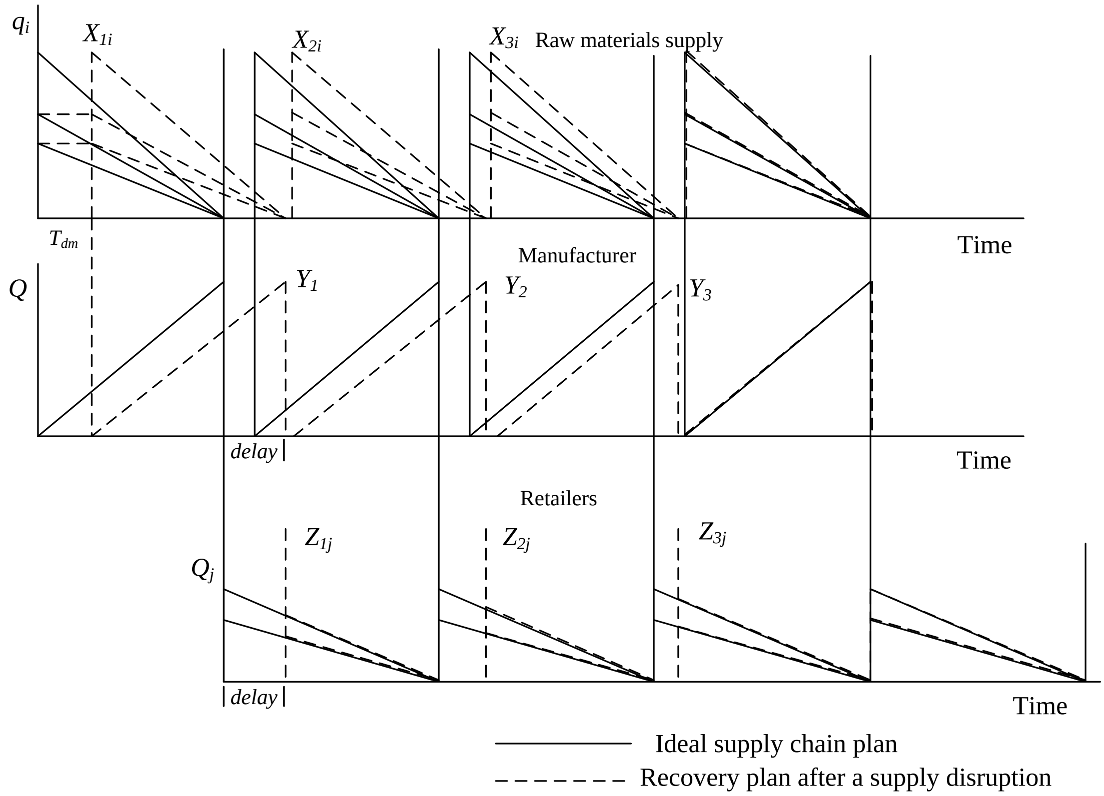

Figure 8.1: Recovery plan after the occurrence of a disruption

In real-life situations, the supply chain system can face multiple supply disruptions, one after another, as a series. When a disruption occurs, a revised plan can be generated by solving the mathematical model for a single disruption. If a new disruption occurs after the recovery time window of another disruption, then the later one can be considered as an independent disruption and the recovery plan can be made similar to the previous one. After finalizing the revised plan, if another disruption occurs within the recovery time window, then the supply, production and delivery plan need to be revised again to consider the effect of both disruptions. This makes the case more complex for recovery planning. In practice, to minimise the effect of disruptions, they must be dealt with on a real-time basis, whether this is a single occurrence of disruption or a series of disruptions. For a series of disruptions, the plan is revised every time, after each occurrence of a disruption, as long as disruptions take place in the system.

8.2.1 Disruption Recovery Strategy

A **supply disruption** can be defined as any form of interruption in the raw material supply. It may be caused due to delay, unavailability, or any other form of disturbance. The **recovery/revised plan** is a new schedule that includes the revised supply, production and delivery quantities in each cycle, for future periods, while ensuring the minimisation of the total cost in the recovery time window. The number of future cycles allocated to return to the original schedule from the disrupted cycle, defines the **recovery time window**, and is decided by the management of the organisation. As it is assumed that the production rate is higher than the demand rate, there is an idle timeslot between any two consecutive production cycles. If the raw material supply is interrupted for a time period, known as **disruption duration**, the utilization of the idle timeslots, in future production cycles, may help to recover from the disruption. However, it may also involve costly back order and/or lost sales due to a long disruption duration and delayed production and delivery. In this chapter, to recover from a disruption, the following two options are considered.

i. **Back orders:** the portion of demand that cannot be fulfilled at the scheduled time, but that will be delivered at a later date, with a penalty, if the system is capable.
ii. **Lost sales:** the portion of demand lost if customers will not wait for the required stock to be replenished as a consequence of the system not being capable of fulfilling demand.

8.2.2 Real-Time Disruption Recovery

A disruption recovery plan is basically a rescheduling of supply, production and delivery plans for some future periods, after the occurrence of a disruption, in order to return to its normal plan. A disruption event that is not known and cannot be predicted in advance is considered. In this chapter, random disruption scenarios are considered which can be defined as combinations of disrupted raw material, and disruption durations. In any supply chain environment, the system can face multiple disruptions, one after another, on a real-time basis. In this case, one disruption can occur within the recovery window of another, which is known as a dependent disruption, and as this is a complex situation, the combined effect of dependent disruptions should be considered in the development of a recovery plan. This is achieved by re-optimising the supply, production and delivery plans within the new recovery window under the changed supply, production and delivery environment. The proposed mathematical model and heuristic (discussed earlier) for dealing with a single disruption are later extended to consider multiple disruptions on a real-time basis, and are capable of handling dependent, independent and mixtures of dependent and independent disruptions on a real-time basis.

8.2.3 Notations used for a Single Disruption Case

In this study, the following notations have been used for a single disruption case.

D_j	Annual demand of the final product of retailer j
D	Annual total demand of the final product = $\sum_{j=1}^{J} D_j$
d_i	Annual demand of raw material i
B_1	Back order cost for the manufacturer ($ per unit per unit time)
B_2	Back order cost for retailer ($ per unit per unit time)
B_{qkj}	Back order quantity of retailer j during the k^{th} cycle
L_1	Lost sales cost for the manufacturer ($ per unit)
L_2	Lost sales cost for a retailer ($ per unit)
H_{1i}	Holding cost of raw material i ($ per unit per year)
H_2	Holding cost of the final product at the manufacturer ($ per unit per year)
H_{3j}	Holding cost of retailer j ($ per unit per year)
N_i	Units of raw material i required to produce one unit of the final product
K	Number of cycles in the revised plan – known from management
P	Annual production rate ($P > D$)
Q	Production lot size
q_i	Supply lot size of raw material i
Q_j	Delivery lot size of the final product for retailer j
S_{1i}	Ordering cost of raw material i ($ per order)
S_2	Set-up cost of the manufacturer ($ per order)
S_{3j}	Ordering cost of retailer j ($ per order)

s_t Set-up time after the production of a lot

T_{idle} Idle time after the production of a lot $= \frac{Q}{D} - \frac{Q}{P} - s_t$

T_{dm} Supply disruption duration of the m^{th} raw material

X_{ki} Supply lot size of raw material i in the revised plan

Y_k Production lot size in the revised plan

Z_{kj} Delivery lot size of the final product to retailer j in the revised plan

8.2.4 Assumptions of the Study

The following assumptions have been made in this study.

i. The production rate is greater than the demand rate.
ii. A single item is produced in the system.
iii. The recovery plan starts just after the occurrence of a disruption.
iv. The recovery plan considers both lost sales and back order options.
v. No inventory buffers are present in the system.

8.3 Mathematical Modelling

In this section, a mathematical model is developed for managing a single occurrence of a disruption caused by a supply disruption, by firstly presenting a mathematical model for an ideal supply chain plan. Then, a revised plan is formulated as a constrained optimisation problem that minimises total cost, which is derived from the relevant costs, subject to production capacity, supply, delivery, and product demand constraints. The decision variables are the revised quantities of production, delivery, supply, back orders and lost sales during the recovery time window. Some propositions are also developed to analyse the properties of some important parameters.

8.3.1 Mathematical Model for the Ideal Plan

The economic supply, production and delivery sizes under ideal conditions are derived in this section. The optimal ideal plan is obtained by minimising the total annual holding, ordering and set-up cost.

Annual raw material holding cost $= \frac{QD}{2P}\sum_{i=1}^{I} N_i H_{1i}$ (8.1)

Annual raw material ordering cost $= \frac{D}{Q}\sum_{i=1}^{I} S_{1i}$ (8.2)

Manufacturer annual holding cost $= \frac{Q}{2} H_2 \frac{D}{P}$ (8.3)

Manufacturer annual set-up cost $= \frac{D}{Q} S_2$ (8.4)

Retailer annual holding cost $= \frac{Q}{2D}\sum_{j=1}^{J} D_j H_{3j}$ (8.5)

Retailer annual ordering cost $= \frac{D}{Q}\sum_{j=1}^{J} S_{3j}$ (8.6)

Total cost,

$$TC = \frac{QD}{2P}\sum_{i=1}^{I} N_i H_{1i} + \frac{D}{Q}\sum_{i=1}^{I} S_{1i} + \frac{Q}{2} H_2 \frac{D}{P} + \frac{D}{Q} S_2 + \frac{Q}{2D}\sum_{j=1}^{J} D_j H_{3j} + \frac{D}{Q}\sum_{j=1}^{J} S_{3j} \quad (8.7)$$

Now, to minimise the total cost, $\frac{d}{dQ}(TC) = 0$

After simplifying, the optimal ideal plan is obtained from (8.8) – (8.10).

$$Q = \sqrt{\frac{2D\left(\sum_{i=1}^{I} S_{1i} + S_2 + \sum_{j=1}^{J} S_{3j}\right)}{\frac{D}{P}\sum_{i=1}^{I} N_i H_{1i} + \frac{H_2 D}{P} + \frac{1}{D}\sum_{j=1}^{J} D_j H_{3j}}} \quad (8.8)$$

$$q_i = N_i Q \quad (8.9)$$

$$Q_j = \frac{QD_j}{D} \quad (8.10)$$

8.3.2 Mathematical Model for a Single Disruption

In this section, a mathematical model is developed for managing a supply disruption. To formulate the mathematical model for determining the revised plan after a supply disruption, the costs of holding, ordering, set-up, back orders and lost sales are considered. Finally, a mathematical model is obtained in which the total cost is to be minimised subject to capacity, delivery, supply, and product demand constraints.

8.3.2.1 Different Costs

Raw material holding cost

$$= \sum_{i=1}^{I} \frac{X_{1i}}{2} H_{1i} \frac{Y_1}{P} + \sum_{\forall i \neq n} X_{1i} T_{dm} H_{1i} + \sum_{k=1}^{K} \sum_{i=2}^{I} \frac{X_{ki}}{2} H_{1i} \frac{Y_k}{P} \quad (8.11)$$

Raw material ordering cost

$$= K \sum_{i=1}^{I} S_{1i} \quad (8.12)$$

Manufacturer holding cost

$$= \sum_{k=1}^{K} \frac{Y_k^2}{2P} H_2 \quad (8.13)$$

Manufacturer set-up cost

$$= KS_2 \quad (8.14)$$

Manufacturer back order cost

$$= B_1 \sum_{k=1}^{K} Y_k . delay_k \quad (8.15)$$

Here,

$$delay_k = T_{dm} + \sum_{i=1}^{k} \frac{Y_i}{P} + (k-1)s_t - \frac{(k-1)Q}{D} - \frac{Q}{P}$$

Manufacturer lost sales cost

$$= L_1 (KQ - \sum_{k=1}^{K} Y_k) \quad (8.16)$$

Retailer holding cost

$$= \sum_{k=1}^{K} \sum_{j=1}^{J} \frac{(Z_{kj} - B_{qkj})^2}{2D_j} H_{3j} \tag{8.17}$$

Retailer ordering cost

$$= B_2 \sum_{k=1}^{K} \sum_{j=1}^{J} \frac{delay_k}{2} B_{qkj} \tag{8.18}$$

Retailer back order cost

$$= B_2 \sum_{k=1}^{K} \sum_{j=1}^{J} \frac{delay_k}{2} B_{qkj} \tag{8.19}$$

Retailer lost sales cost

$$= L_2 \left(K \sum_{j=1}^{J} Q_j - \sum_{k=1}^{K} \sum_{j=1}^{J} Z_{kj} \right) \tag{8.20}$$

8.3.2.2 Final Mathematical Model for a Single Disruption

The total cost function, which is the objective function, is obtained by adding all the costs presented in (8.11) – (8.20) and subject to the following constraints presented in (8.21) – (8.28).

$Y_k \leq Q;\ \forall k$ [To meet the delivery requirements] (8.21)

$X_{ki} \leq q_i;\ \forall i, k$ [Raw material supply constraint] (8.22)

$Z_{kj} \leq Q_j;\ \forall j, k$ [Final product delivery constraint] (8.23)

$delay_k \geq 0;\ \forall k$ [Non-negative delay time] (8.24)

$\frac{Q}{D} - \frac{Q}{P} - s_t \geq 0$ [Non-negative idle time] (8.25)

$KQ - \sum_{k=1}^{K} Y_k \geq 0$ [Lost sales quantity constraint] (8.26)

$\sum_{k=1}^{K} Y_k \leq P\left(K * \frac{Q}{D} - (K-1) * s_t - T_{dm}\right)$ [Production capacity constraints] (8.27)

$X_{ki}, Y_k, Z_{kj} \geq 0;\ \forall i, j, k$ [Non-negative constraint] (8.28)

Proposition 1: For a given H_{1i}, H_2, H_{3j}, S_{1i}, S_2, S_{3j}, D_j, P, B_1, B_2, L_1 and L_2, the revised plan will only use the back order option if $T_{dm} \leq KT_{idle}$.

Proof: Idle time per cycle, $T_{idle} = \frac{Q}{D} - \frac{Q}{P} - s_t$. As there are K cycles in the recovery plan, so the total idle time during the revised plan is KT_{idle}. The quantity to be produced during the idle time is PKT_{idle}.

Now, quantity loss during the duration of the disruption is $T_{dm}P$. The system will thus be able to recover by using only back order options, if the quantity to be produced during the idle time is greater than the quantity loss during the disruption duration.

So, $PKT_{idle} \geq T_{dm}P$, hence $T_{dm} \leq KT_{idle}$.

Proposition 2: For a given H_{1i}, H_2, H_{3j}, S_{1i}, S_2, S_{3j}, D_j, P, B_1, B_2, L_1 and L_2, both back order and lost sales will exist in the revised plan if $T_{dm} > KT_{idle}$.

Proof: This is the opposite consequence of **Proposition 1**.

Proposition 3: For a given H_{1i}, H_2, H_{3j}, S_{1i}, S_2, S_{3j}, D_j, P, B_1, B_2, L_1 and L_2, the revised plan will only use the lost sales option if $(B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) > (L_1 + L_2)$.

Proof: Idle time per cycle $= \frac{Q}{D} - \frac{Q}{P} - s_t$ and total back order cost per unit per unit time $= B_1 + B_2$. So, back order cost per unit $= (B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right)$

Now, lost sales cost per unit $= (L_1 + L_2)$. Hence if $(B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) > (L_1 + L_2)$, then the back order cost will be higher than the lost sales cost, so it is favorable that the revised plan will only use the lost sales option.

Proposition 4: For a given H_{1i}, H_2, H_{3j}, S_{1i}, S_2, S_{3j}, D_j, P, B_1, B_2, L_1 and L_2, the back order quantity of retailer to its customers is $B_{qkj} = Z_{kj} - D_j\left(\frac{Q}{D} - delay_k\right)$ if $Z_{kj} \geq D_j\left(\frac{Q}{D} - delay_k\right)$, and is $B_{qkj} = 0$ if $Z_{kj} < D_j\left(\frac{Q}{D} - delay_k\right)$.

Proof: After a disruption, the delay time for delivering the final product of the k^{th} cycle to a retailer is $delay_k$. So, the remaining period of the demand cycle is $\left(\frac{Q}{D} - delay_k\right)$. The demand during the remaining period for retailer j is $D_j\left(\frac{Q}{D} - delay_k\right)$. Now the quantity received by retailer j in the k^{th} cycle is Z_{kj}. If $Z_{kj} \geq D_j\left(\frac{Q}{D} - delay_k\right)$, then the excess quantity than $D_j\left(\frac{Q}{D} - delay_k\right)$ should be back ordered. So, the retailer back order quantity, $B_{qkj} = Z_{kj} - D_j\left(\frac{Q}{D} - delay_k\right)$ and if $Z_{kj} < D_j\left(\frac{Q}{D} - delay_k\right)$, then the retailer back order quantity, $B_{qkj} = 0$, because back orders are no longer needed in this condition.

8.3.3 Dynamic Mathematical Model for a Series of Disruptions

Based on the formulation of the mathematical model for a single disruption, developed a dynamic mathematical model have also developed for a series of disruptions. Here the mathematical model is presented after the n^{th} disruption. The following additional notations have been used for the mathematical formulation.

l	Number of cycles to the current disruption from the previous disruption
$X_{ki,n}$	Supply lot size in the revised plan after the n^{th} disruption
$Y_{k,n}$	Production lot size in the revised plan after the n^{th}disruption
$Z_{kj,n}$	Delivery lot size in the revised plan after the n^{th} disruption
$T_{dm,n}$	Actual disruption duration for the n^{th} disruption

The term $T^*_{dm,n}$ is used as the disruption duration to determine the new revised plan, which considers the effect of both the previous and the current disruption. It is calculated by using the equations (8.29) and (8.30).

For the first disruption:

$$T^*_{dm,1} = T_{dm,1} \tag{8.29}$$

From the second disruption:

$$T^*_{dm,n} = \begin{cases} T_{dm,n} + T^*_{dm,n-1} - l\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right); \text{ if } T^*_{dm,n-1} > l\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) \\ T_{dm,n}; \text{ if } T^*_{dm,n-1} \leq l\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) \text{ or } l > K \end{cases} \tag{8.30}$$

8.3.3.1 Different Costs in the Recovery Plan after the n^{th} Disruption

Raw material holding cost

$$= \sum_{i=1}^{I} \frac{X_{1i,n}}{2} H_{1i} \frac{Y_{1,n}}{P} + \sum_{\forall i \neq n} X_{1i,n} T_{dm,n} H_{1i} + \sum_{k=1}^{K} \sum_{i=2}^{I} \frac{X_{ki,n}}{2} H_{1i} \frac{Y_{k,n}}{P} \tag{8.31}$$

Raw material ordering cost

$$= K \sum_{i=1}^{I} S_{1i} \tag{8.32}$$

Manufacturer holding cost

$$= \sum_{k=1}^{K} \frac{Y_{k,n}^2}{2P} H_2 \tag{8.33}$$

Manufacturer ordering cost

$$= KS_2 \tag{8.34}$$

Manufacturer back order cost

$$= B_1 \sum_{k=1}^{K} Y_{k,n} * delay_{k,n} \tag{8.35}$$

Here,

$$delay_{k,n} = T_{dm,n} + \sum_{i=1}^{k} \frac{Y_{i,n}}{P} + (k-1)s_t - \frac{(k-1)Q}{D} - \frac{Q}{P}$$

Manufacturer lost sales cost

$$= \sum_{k=1}^{K} \sum_{j=1}^{J} \frac{(Z_{kj,n} - B_{qkj,n})^2}{2D_j} H_{3j} \tag{8.36}$$

Retailer holding cost

$$= \sum_{k=1}^{K}\sum_{j=1}^{J}\frac{(Z_{kj,n} - B_{qkj,n})^2}{2D_j}H_{3j} \tag{8.37}$$

Retailer ordering cost

$$= K\sum_{j=1}^{J} S_{3j} \tag{8.38}$$

Retailer back order cost

$$= B_2\sum_{k=1}^{K}\sum_{j=1}^{J}\frac{delay_{k,n}}{2}B_{qkj,n} \tag{8.39}$$

Retailer lost sales

$$= L_2\left(K\sum_{j=1}^{J} Q_j - \sum_{k=1}^{K}\sum_{j=1}^{J} Z_{kj,n}\right) \tag{8.40}$$

8.3.3.2 Final Mathematical Model for a Series of Disruptions

The total cost function after the n^{th} disruption, which is the objective function, is obtained by adding all the costs in (8.31) – (8.40) and subject to the following constraints presented in (8.41) – (8.47).

$Y_{k,n} \leq Q;\ \forall k$	[To meet the delivery requirements]	(8.41)
$X_{ki,n} \leq q_i;\ \forall i, k$	[Raw material supply constraint]	(8.42)
$Z_{kj,n} \leq Q_j;\ \forall j, k$	[Final product delivery constraint]	(8.43)
$delay_{k,n} \geq 0;\ \forall k$	[Non-negative delay time]	(8.44)
$KQ - \sum_{k=1}^{K} Y_{k,n} \geq 0$	[Lost sales quantity constraint]	(8.45)
$\sum_{k=1}^{K} Y_{k,n} \leq P(K * \frac{Q}{D} - (K-1) * s_t - T_{dm,n})$	[Production capacity constraints]	(8.46)
$X_{ki,n}, Y_{k,n}, Z_{kj,n} \geq 0; \forall i, j, k$	[Non-negative constraint]	(8.47)

8.4 Solution Approaches

In this section, a heuristic is developed to solve the developed model for a single disruption. To judge the quality of the heuristic solutions, the model is also solved by applying a pattern search (PS) technique, which is a standard search algorithm for solving constrained optimisation problems. A simulation model is also developed to make the disruption problem closer to a real-world process. Finally, the heuristic is extended for managing multiple disruptions, one after another as a series, on a real-time basis. Both the heuristics and the PS technique were coded in MATLAB R2012a, and were executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU. The parameters used in the PS technique are presented as follows.

Maximum number of iterations: 100* Number of variables
Polling order: Random
X tolerance: 1e-8
Function tolerance: 1e-8
Non-linear constraint tolerance: 1e-8
Cache tolerance: 1e-8
Search method: Latin hypercube
Maximum function evaluations: 10^6
Other parameters are set as the default in the optimisation toolbox of MATLAB R2012a.

8.4.1 Proposed Heuristic for a Single Disruption

In this section, a heuristic is developed for managing a single disruption. The steps of the heuristic are as follows.

Step 1: Input all information about the ideal system.

Step 2: Determine Q, q_i and Q_j for the optimal ideal plan by using (8.8) – (8.10) and also determine production time, cycle time and idle time.

Step 3: Input disruption information, such as: disrupted raw material, disruption duration and recovery period.

Step 4: If $(B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) \leq (L_1 + L_2)$ and $T_{dm} \leq K * T_{idle}$, then

$Y_k = Q; \forall k$

$X_{ki} = N_i * Y_k;\ \forall i, k$

$Z_{kj} = \frac{Y_k * D_j}{D};\ \forall j, k$

If $T_{dm} \leq T_{idle}$, then

$delay_1 = T_{dm} + \frac{Y_1}{P} - \frac{Q}{P}$

$delay_k = 0$; For $k = 2, 3, \ldots, K$

If $T_{idle} < T_{dm} \leq 2T_{idle}$, then

$delay_1 = T_{dm} + \frac{Y_1}{P} - \frac{Q}{P}$

$delay_2 = T_{dm} + \frac{Y_1 + Y_2}{P} + s_t - \frac{Q}{D} - \frac{Q}{P}$

$delay_k = 0$; for $k = 3, 4, \ldots, K$

……

If $(K-1)T_{idle} < T_{dm} \leq KT_{idle}$, then

$delay_k = T_{dm} + \sum_{i=1}^{k} \frac{Y_i}{P} + (k-1)s_t - \frac{(k-1)Q}{D} - \frac{Q}{P};\ \forall k$

Step 5: If $(B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) \leq (L_1 + L_2)$ and $T_{dm} > KT_{idle}$, then

$Y_1 = Q$

$Y_2 = Q - P\left[T_{dm} - K * \left(\frac{Q}{D} - \frac{Q}{P} - s_t\right)\right]$

$Y_k = Q$; For $k = 3, 4, \ldots, K$

$X_{ki} = N_i * Y_k;\ \forall i, k$

$Z_{kj} = \frac{Y_k * D_j}{D};\ \forall j, k$

$delay_k = T_{dm} + \sum_{i=1}^{k} \frac{Y_i}{P} + (k-1)s_t - \frac{(k-1)Q}{D} - \frac{Q}{P};\ \forall k$

Step 6: If $(B_1 + B_2)\left(\frac{Q}{D} - \frac{Q}{P} - s_t\right) > (L_1 + L_2)$ then

$Y_1 = Q - T_{dm} * P$

$Y_k = Q$; *For* $k = 2, 3, \ldots, K$

$X_{ki} = N_i * Y_k;\ \forall i, k$

$Z_{kj} = \frac{Y_k * D_j}{D};\ \forall j, k$

$delay_k = 0;\ \forall k$

Step 7: Determine the lost sales and back order quantities.

Step 8: Determine the different costs and record the results.

Step 9: Stop.

8.4.2 Simulation Model

A simulation model is developed to make the disruption model closer to a real-world problem by using the following steps.

Step A: Generate a random number for choosing a disrupted raw material by using a uniform distribution.

Step B: Generate a random number for the disruption duration by using an exponential distribution.

Step C: Solve the disruption management problem by using the heuristic for a single disruption.

Step D: Repeat steps **A** to **C** 2000 times.

Step E: Record the results.

Step F: Stop.

8.4.3 Extended Heuristic for Multiple Disruptions

The heuristic, developed for a single disruption, is extended for managing multiple disruptions, one after another as a series, on a real-time basis. To do this, a recovery plan is obtained from the heuristic after each disruption, with the revised production, supply and delivery plans saved and then used as a foundation for recovering from the next disruption. The steps in the extended heuristic for managing multiple disruptions are described below.

Step 1: Input the disrupted raw material and $T_{dm,1}$ for the first disruption.

Step 2: Update the parameter $T_{dm} = T^{*}_{dm,1}$ and also update the decision variables as $X_{ki} = X_{ki,1}$ and $Y_k = Y_{k,1}$ and $X_{kj} = X_{kj,1}$

Step 3: Solve the model by using the heuristic for the single disruption under the updated parameters.

Step 4: Record the revised plan and calculate the different costs.

Step 5: If there is any other disruption,

5.1 Later when known, input the disrupted raw material, disrupted cycle number from the previous disruption (l), and disruption duration ($T_{dm,n}$) for the next disruption.

5.2 Calculate $T^*_{dm,n}$ by using equation (8.30).

5.3 Update the disruption duration as $T_{dm} = T^*_{dm,n}$ and also update the decision variables as $X_{ki} = X_{ki,n}$, $Y_k = Y_{k,n}$ and $X_{kj} = X_{kj,n}$.

5.4 Go to **step 3.**

Step 6: Record the results.

Step 7: Stop.

8.5 Experimentations and Results Analysis

In this section, the results have been analysed for both the ideal and revised plans by performing random experimentations. The results for both a single and multiple disruptions have also analysed.

8.5.1 Ideal Plan

The following data are considered for the ideal supply chain plan with three raw material suppliers and four retailers.

$$I = 3;\ J = 4;\ D_j = [15{,}000, 25{,}000, 20{,}000, 30{,}000];\ P = 100{,}000;\ N_i = [1, 3, 2];$$

$$H_{1i} = [2, 2.5, 2.2];\ S_{1i} = [100, 80, 120];\ H_2 = 3;\ S_2 = 150;\ H_{3j} = [1.2, 1.5, 1.7, 1.4];$$
$$S_{3j} = [50, 60, 60, 50];\ s_t = 0.000228$$

The equations (8.8) – (8.10) were used to determine the ideal plan which is obtained as follows.

$Q = 2{,}689.6$;
$q_i = [2689.6, 8068.8, 5379.2]$; and
$Q_j = [448.3, 747.1, 597.7, 896.5]$

8.5.2 Results Analysis for a Single Disruption

In this section, the solutions for a single disruption are analysed. Although it was experimented on many random disruption test problems, for illustrative purposes, six different sample instances were used by arbitrarily changing the disruption data, with their parameters shown in Table 8.1. The same data of the ideal plan was used with the following additional data to obtain the revised plan.

$$B_1 = 20, B_2 = 10, L_1 = 25, L_2 = 15 \text{ and } K = 5$$

Table 8.1: Disruption instances for a single disruption

Instance number	Disrupted raw material	Disruption duration
1	1	0.005
2	1	0.020
3	2	0.010
4	2	0.025
5	3	0.008
6	3	0.022

The results for the disruption instances are presented in Table 8.2, which includes back orders, lost sales and total cost. It is observed that, the heuristic showed insignificant changes in results for different runs. For a sample representation, the revised plans for disruption instances 1 and 2 are presented in Table 8.3.

Table 8.2: Results for disruption instances

Instance number	Total Back Order Cost	Total Lost Sales Cost	Total Cost
1	402.94	0	7236.50
2	2,672.56	24790.60	34408.30
3	1339.69	0	8131.25
4	2904.75	44790.60	54272.06
5	889.46	0	7738.52
6	2762.74	32790.60	42351.55

8.5.3 Results Analysis for a Single Disruption

To judge the quality of the solutions obtained from the proposed heuristic, the solutions of 150 test problems were compared with the best result (out of 30 independent runs) obtained from the pattern search. Those random test problems have been generated by using a uniform distribution and by varying the data of the disruption parameters. The comparison showed that the proposed heuristic is capable of producing high quality solutions. In terms of the quality of the solutions, the average deviation of results between the two approaches is only 0.000283%, which can be considered as negligible. The percentage of deviation was calculated by using the equation (8.48).

$$\text{Percentage of deviation} = \frac{|\text{Total profit from heuristc} - \text{Total profit from PS}|}{\text{Total profit from PS}} \times 100\% \qquad (8.48)$$

Table 8.3: Revised plans for disruption instances 1 and 2

<table>
<tr><th>Disruption instance</th><th colspan="6">Revised plan</th></tr>
<tr><td rowspan="15">1</td><td colspan="6">Revised raw material supply plan</td></tr>
<tr><td>Supplier</td><td>X_{1i}</td><td>X_{2i}</td><td>X_{3i}</td><td>X_{4i}</td><td>X_{5i}</td></tr>
<tr><td>1</td><td>2689.6</td><td>2689.6</td><td>2689.6</td><td>2689.6</td><td>2689.6</td></tr>
<tr><td>2</td><td>8068.8</td><td>8068.8</td><td>8068.8</td><td>8068.8</td><td>8068.8</td></tr>
<tr><td>3</td><td>5379.2</td><td>5379.2</td><td>5379.2</td><td>5379.2</td><td>5379.2</td></tr>
<tr><td colspan="6">Revised production plan</td></tr>
<tr><td></td><td>Y_1</td><td>Y_2</td><td>Y_3</td><td>Y_4</td><td>Y_5</td></tr>
<tr><td></td><td>2689.6</td><td>2689.6</td><td>2689.6</td><td>2689.6</td><td>2689.6</td></tr>
<tr><td colspan="6">Revised delivery plan</td></tr>
<tr><td>Retailer</td><td>Z_{1j}</td><td>Z_{2j}</td><td>Z_{3j}</td><td>Z_{4j}</td><td>Z_{5j}</td></tr>
<tr><td>1</td><td>448.3</td><td>448.3</td><td>448.3</td><td>448.3</td><td>448.3</td></tr>
<tr><td>2</td><td>747.1</td><td>747.1</td><td>747.1</td><td>747.1</td><td>747.1</td></tr>
<tr><td>3</td><td>597.7</td><td>597.7</td><td>597.7</td><td>597.7</td><td>597.7</td></tr>
<tr><td>4</td><td>896.5</td><td>896.5</td><td>896.5</td><td>896.5</td><td>896.5</td></tr>
<tr><td colspan="6"></td></tr>
<tr><td rowspan="14">2</td><td colspan="6">Revised raw material supply plan</td></tr>
<tr><td>Supplier</td><td>X_{1i}</td><td>X_{2i}</td><td>X_{3i}</td><td>X_{4i}</td><td>X_{5i}</td></tr>
<tr><td>1</td><td>2689.6</td><td>2069.9</td><td>2689.6</td><td>2689.6</td><td>2689.6</td></tr>
<tr><td>2</td><td>8068.8</td><td>6209.6</td><td>8068.8</td><td>8068.8</td><td>8068.8</td></tr>
<tr><td>3</td><td>5379.2</td><td>4139.7</td><td>5379.2</td><td>5379.2</td><td>5379.2</td></tr>
<tr><td colspan="6">Revised production plan</td></tr>
<tr><td></td><td>Y_1</td><td>Y_2</td><td>Y_3</td><td>Y_4</td><td>Y_5</td></tr>
<tr><td></td><td>2689.6</td><td>2069.9</td><td>2689.6</td><td>2689.6</td><td>2689.6</td></tr>
<tr><td colspan="6">Revised delivery plan</td></tr>
<tr><td>Retailer</td><td>Z_{1j}</td><td>Z_{2j}</td><td>Z_{3j}</td><td>Z_{4j}</td><td>Z_{5j}</td></tr>
<tr><td>1</td><td>448.3</td><td>345.0</td><td>448.3</td><td>448.3</td><td>448.3</td></tr>
<tr><td>2</td><td>747.1</td><td>575.0</td><td>747.1</td><td>747.1</td><td>747.1</td></tr>
<tr><td>3</td><td>597.7</td><td>460.0</td><td>597.7</td><td>597.7</td><td>597.7</td></tr>
<tr><td>4</td><td>896.5</td><td>690.0</td><td>896.5</td><td>896.5</td><td>896.5</td></tr>
</table>

8.5.4 Random Experimentation

Many disruption test problems were generated randomly for each raw material supply, and solved them using the heuristic. The total cost pattern have been analysed for random occurrences of disruption over 500 random scenarios, and variations in the different costs according to the disruption duration.

Five hundred random scenarios were generated for the duration of a supply disruption using an exponential distribution within the range of [0.0001, 1], and the total cost pattern, for disruption of raw material 1, is presented in Figure 8.2. It was determined that the mean and standard deviation values of the total cost were 13.2000 and 11.8620 thousand respectively, and the maximum and minimum values were 54.6130 and 6.7060 thousand respectively.

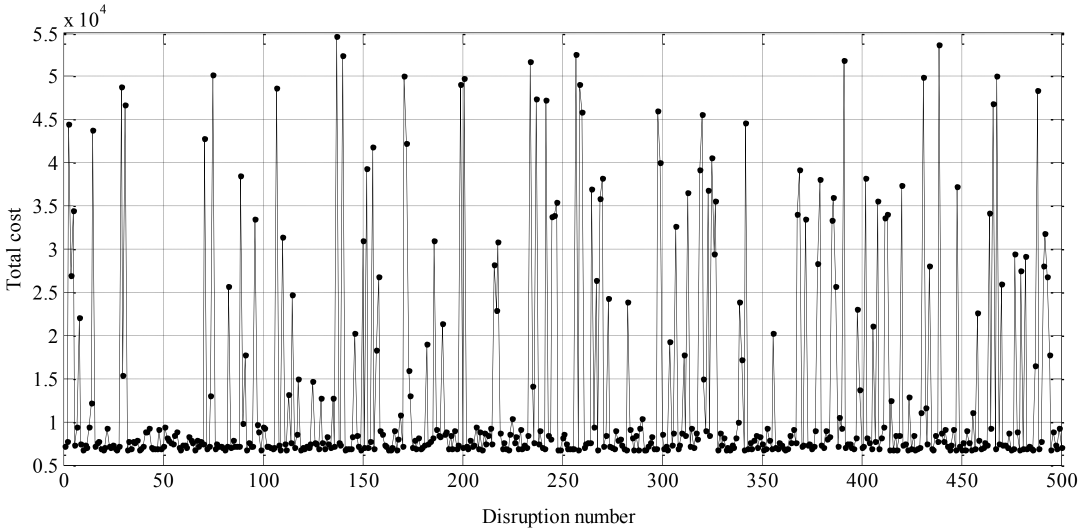

Figure 8.2: Total cost vs. disruption number for disruption at raw material 1

The variations in the different costs in relation to the duration of a supply disruption, for disruption of raw material 1, are presented in Figure 8.3. The total cost increases slowly when the duration is less than 0.014 because only back orders are present in the the revised plan. Then, the total cost increases at a higher rate with disruption durations because of the lost sales cost being included in the plan and both back orders and lost sales are present in the revised plan. Figure 8.4 presents the variations of back orders and lost sales quantities in relation to the disruption duration for disruption of raw material 1. The back orders quantity increases with the disruption duatrion when the duration is less than 0.014 and no lost sales are then present. After then, the lost sales quantity enters in the revised plan and both back orders and lost sales quantities are present. Similar properties have also been found for disruption of raw materials 2 and 3.

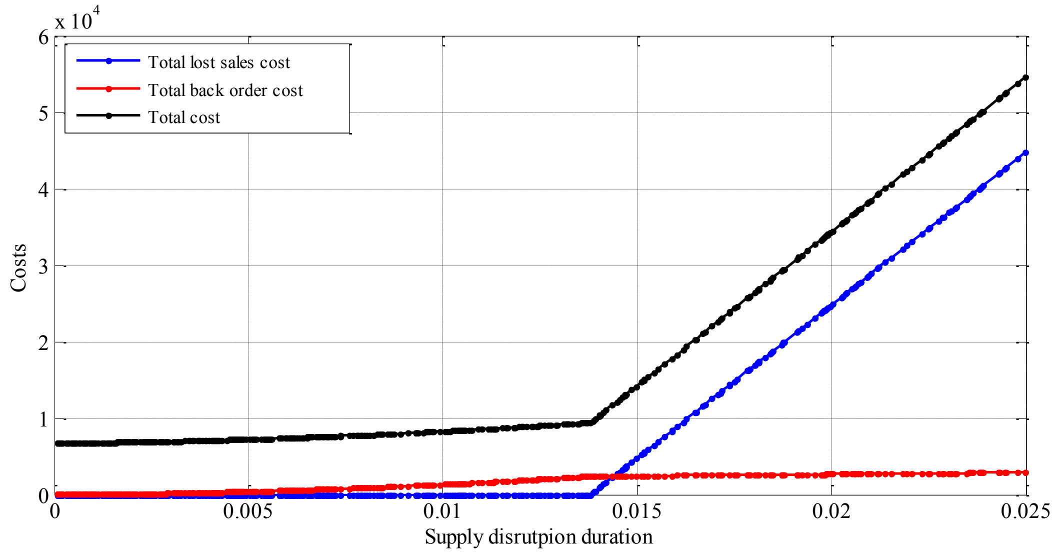

Figure 8.3: Different costs vs. disruption duration for disruption of raw material 1

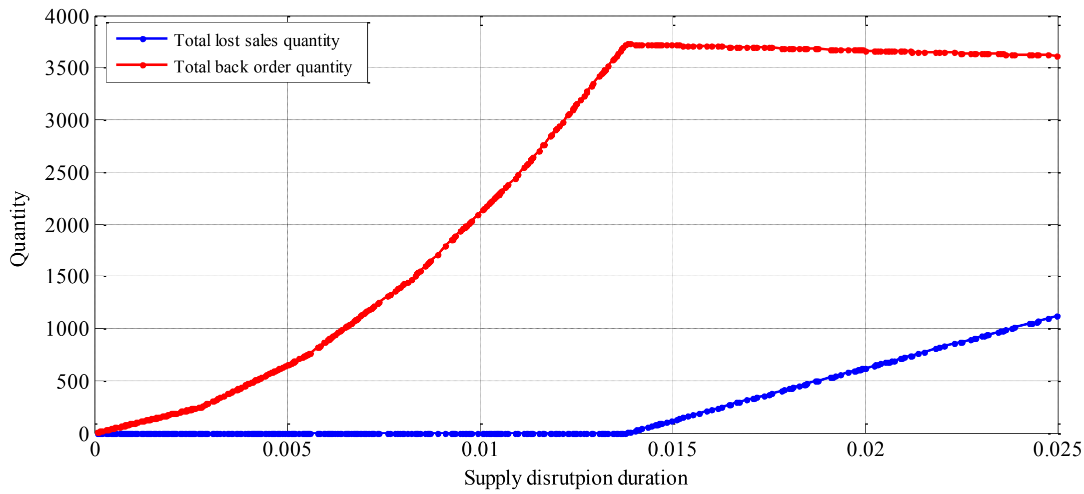

Figure 8.4: Lost sales and back order quantity vs. disruption duration for disruption of raw material 1

It was also generated 500 random scenarios for the duration of a supply disruption, for disruption of raw material 2 and 3, using an exponential distribution within the range of

[0.0001, 1], and the total cost patterns are presented in Figures 8.5 and 8.6 respectively. It was determined that the mean and standard deviation values of the total cost were 13.0800 and 11.4840 thousand respectively, and the maximum and minimum values were 53.8530 and 6.7044 thousand respectively for disruption of raw material 2 and those values were 12.0260, 10.4250, 54.4580 and 6.7009 thousands respectively for disruption of raw material 3.

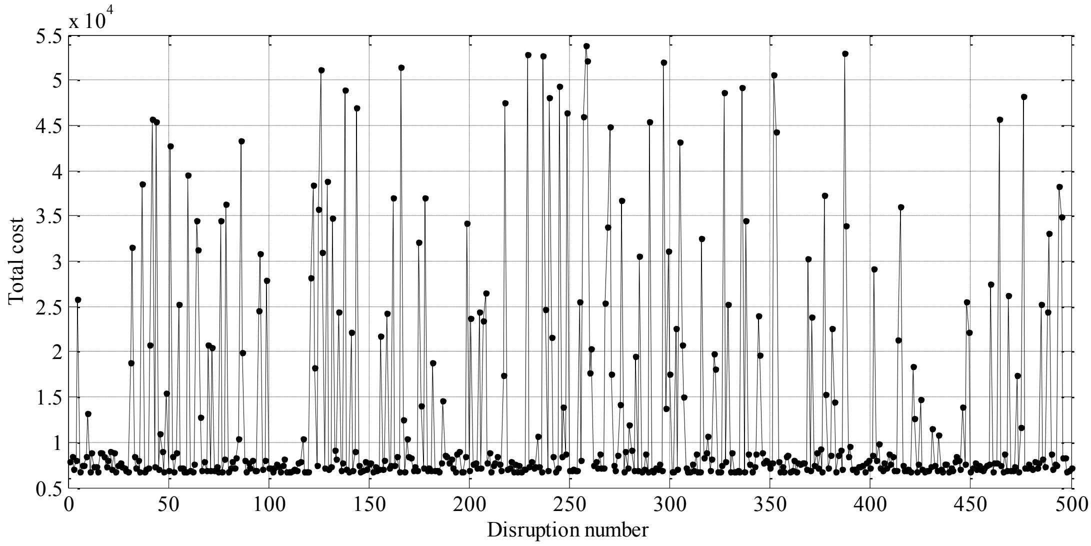

Figure 8.5: Total cost vs. disruption number for disruption of raw material 2

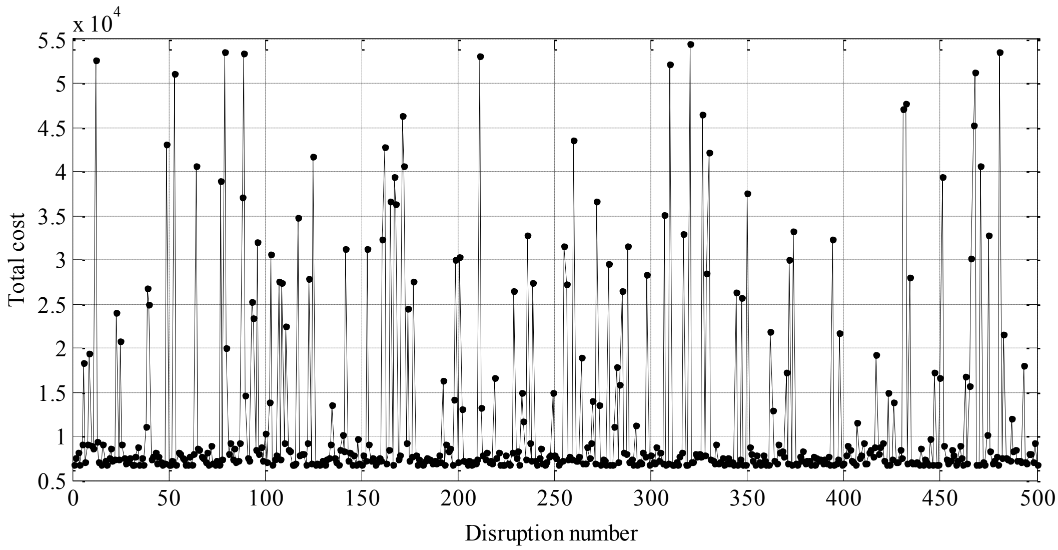

Figure 8.6: Total cost vs. disruption number for disruption of raw material 3

8.5.5 Simulation Results

The simulation model presented in Section 8.4.2 was ran to make the supply chain disruption problem close to a real-world process. Random supply disruption durations were generated by using an exponential distribution, and the histogram of the disruption duration is presented in Figure 8.7. The different costs patterns for random disruption occurrences over the 2000 random scenarios are presented in Figure 8.8. The mean, standard deviation, maximum and minimum values of different costs were calculated, and they are presented in Table 8.4.

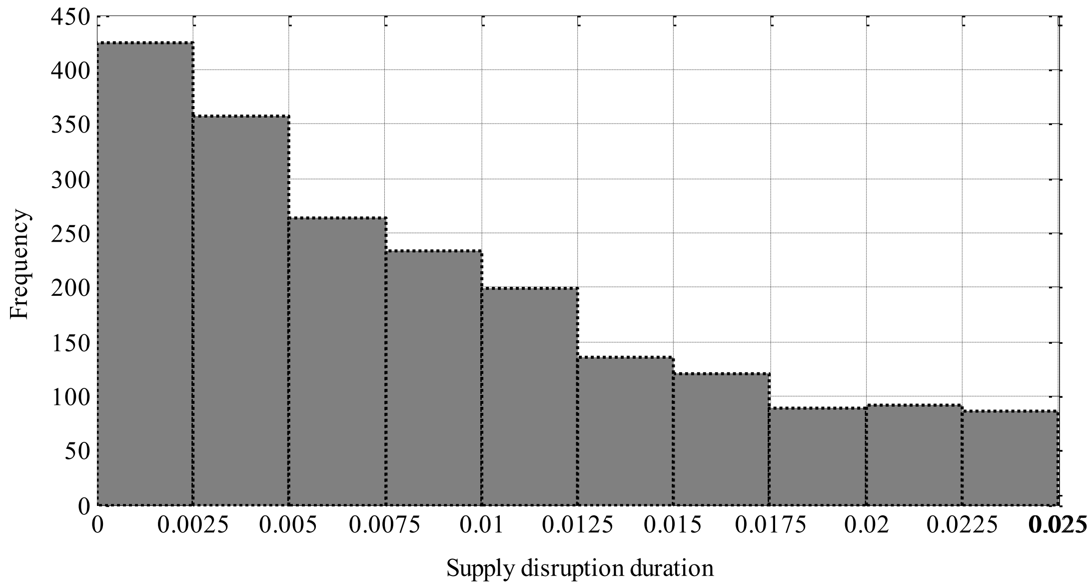

Figure 8.7: Histogram of disruption duration for the simulation

Table 8.4: Statistic of different costs for the simulation run

Cost type	Mean (Thousands)	Standard deviation (Thousands)	Maximum (Thousands)	Minimum (Thousands)
Total back orders cost	1.0936	1.0005	2.9037	0
Total lost sales cost	4.5365	10.4750	44.7070	0
Total cost	12.4370	11.2020	54.3960	6.7012

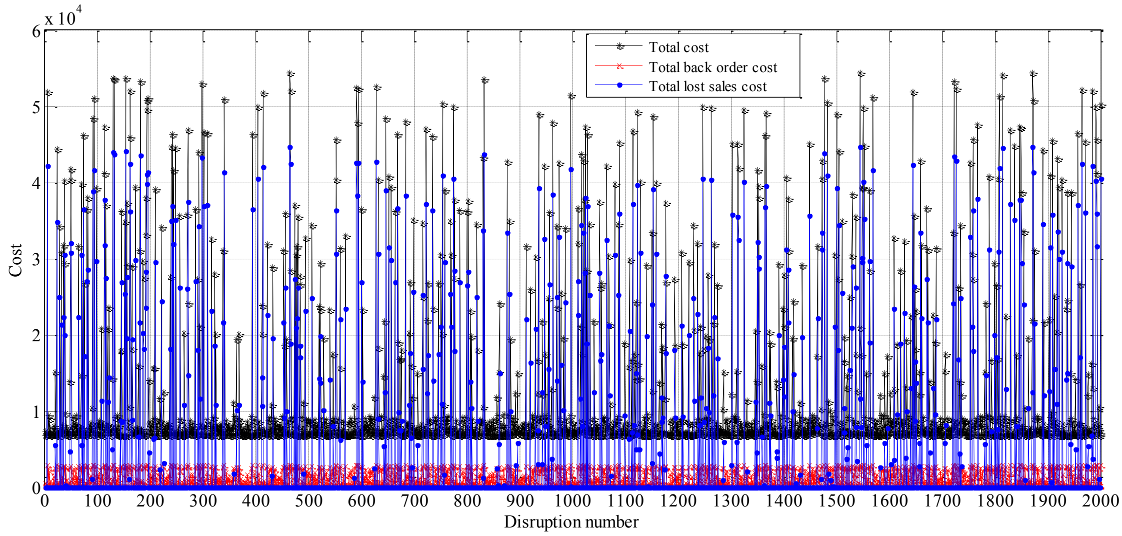

Figure 8.8: Simulation results of different costs for 2000 runs

8.5.6 Results Analysis for a Series of Disruptions

To demonstrate the usefulness of the proposed heuristic for solving different scenarios with a series of disruptions, one after another, over a period of time, the basic data of the single disruption problem presented in Sections 8.5.1 and 8.5.2 was used. The first disruption can be solved by using the heuristic developed for a single disruption. Then, if another disruption occurs after the recovery window of a previous one, it could be considered another single disruption that would not affect the revised plan based on the previous disruption. However, as a new disruption within the recovery window of any previous one may affect the previous revised plan, the revised plan for its recovery window must be considered as a set of additional restrictions. For experimental purposes, a series of ten dependent disruptions was generated randomly, one after another, as shown in Table 8.5. Although they could happen continuously, only ten is presented as a sample representation.

To minimise the total cost in the system, the supply, production and delivery quantities were revised immediately, after each disruption took place, for the next five cycles. The problem was then solved using the proposed heuristic for multiple disruptions, as presented in Section 8.4.3, with the results recorded after each disruption, the total lost sales cost, total back orders cost and total cost are shown in Table 8.6.

Table 8.5: A random case for a series of disruptions

Disruption number	Disrupted raw material	Disrupted cycle number	Disruption duration
1	2	--	0.009
2	3	4	0.016
3	1	6	0.012
4	3	8	0.007
5	3	2	0.014
6	2	8	0.020
7	1	3	0.006
8	1	5	0.022
9	2	7	0.013
10	3	4	0.018
---	---	---	---

Table 8.6: Results for the series of disruptions

Disruption number	Total back orders	Total lost sales	Total cost
1	1,105.22	0	7,892.47
2	2,503.01	8,790.60	18,167.13
3	1,871.36	0	8,846.95
4	715.29	0	7,548.71
5	2,481.99	6,706.85	16,071.98
6	2,672.56	24,790.60	34,112.44
7	2,574.09	15,664.97	25,198.68
8	2,949.74	48,455.57	58,357.03
9	2,171.73	0	8,971.95
10	2,670.71	24,623.09	34,110.28
---	---	---	---

To compare and judge the heuristic solutions of the multiple disruptions, another solution approach have also developed for multiple disruptions by using the PS technique. Then the solutions of 30 randomly generated test problems were compared. The comparison showed that the average percentage of deviation was only the negligible amount of 0.000008%. So the proposed heuristic is also capable of handling multiple disruptions on a real-time basis.

8.6 Chapter Summary

The main objective of this chapter was to develop a quantitative approach to recover from supply disruptions in a three-tier supply chain system. A new mathematical and heuristic approach was developed for managing a single supply disruption. Then the mathematical model and the heuristic were extended to develop a dynamic approach for managing multiple supply disruptions on a real-time basis. These heuristics were validated by comparing the results from another standard solution technique which showed that the average percentage of deviation was a negligible amount for a good number of randomly generated test problems. A large set of random experiments was performed to analyse the characteristics of the developed models, and finally, a simulation model was developed to enable solving the supply disruption problem as a real-world process.

The proposed approach offers a potentially very useful quantitative approach to help decision makers to make prompt and accurate decisions on a real-time recovery plan, whenever a sudden, or a series of supply disruptions, takes place in a three-tier supply chain system. The supply chain system can return to its normal supply, production and delivery plan as quickly as possible after a supply disruption with the help of this approach, and thereby minimise its total costs and enhance its reputation.

The disruption management problem, for managing supply disruption in a three-tier supply chain, has been addressed in chapter 8. In this and previous chapters, different types disruptions such as disruption production and supply and demand fluctuation have been considered. Each chapter focused on a distinctive disruption type. In real-world case, however, multiple types of disruption can happen in a single setting of supply chain. The next chapter will combine all the disruption types under a single supply chain system. The effect of each and multiple types of disruption, on a three-tier supply chain, will be analysed and discussed by developing a quantitative and simulation approach.

Chapter 9 A Simulation Model for a Supply Chain System

This chapter presents a reactive mitigation approach for a three-tier manufacturing supply chain, which has a single supplier, single manufacturer and single retailer under imperfect production environment, in which three types of sudden disturbances are considered: demand fluctuation, and production and raw material supply disruptions. Firstly, a mathematical model is developed for generating an ideal plan under imperfect production, for a finite planning horizon, while maximising total profit and then re-formulated it by incorporating each type of disturbance. Then a new and efficient heuristic is proposed for each disturbance type to obtain the revised plan after the occurrence of a disturbance on a real-time basis. The heuristic solutions are compared with those obtained by a standard solution technique for a set of randomly generated test problems which demonstrates the consistent performance of the developed heuristics. Another heuristic is also developed for managing the combined effects of multiple disturbances in a period. The effects of different types of randomly generated disturbance events, not known in advance, are analysing by developing a simulation model. Finally, this chapter presents some numerical results and significant number of random experimentations to explain the usefulness of the developed models and methodologies.

9.1 Introduction

In real-life, every manufacturing and service organisation is part of a supply chain; for example, pharmaceutical, textile and manufacturing enterprises that supply, produce and distribute products use supply chain networks. In real-life situations, the production system of a manufacturing plant can be imperfect. This means that the manufacturing system can produce some defective items. Moreover, every supply chain has multiple entities, such as a supplier, manufacturing plant and retailer, which face many uncontrollable sudden disturbances, such as a demand fluctuation at the retailer's end, a production disruption in the manufacturing plant and a supply disruption at the supplier's end (Paul et al., 2016). Every organisation should have appropriate strategies for managing these types of real-life factors in their supply chain system.

Most previous studies in the literature considered ideal supply chain systems and a reasonable number developed models for after the occurrence of a disturbance on a real-time basis. They were focused mainly on developing models for a single type of disturbance, with no one study considering multiple types of disturbances. Although some papers developed a heuristic to solve their models, very little has been done to develop a combined heuristic and simulation approach to make the model closer to a real-world process. Therefore, it can be said that more research is needed to develop a real-time disturbance management system that covers all types of disturbances. In this chapter, manufacturing process reliability is considered to develop the ideal supply chain plan as imperfect production processes are common in real-life. Mathematical and heuristic approaches are also developed for managing three types of disturbances, a demand fluctuation, and production and supply disruptions, on a real-time basis. In a real-life supply chain system, multiple types of disturbances can happen together during a single period. Therefore, another heuristic is also developed that considers the combined effects of multiple disturbances in a certain period. Finally, a simulation approach is developed for making the disturbance management problem closer to a real-world process and performs a great deal of random experimentation to validate the heuristics and analyse the results.

The main contributions of this chapter can be summarized as follows.

i. Consideration of manufacturing process reliability while developing the mathematical model for the ideal system and the ideal plan will be updated immediately if there are any changes in data.
ii. Development of a new disturbance management model for a supply chain system which considers a demand fluctuation, and production and raw material supply disruptions. As such a disturbance scenario is not known in advance and follows a stochastic process, after it occurs, the original plan is revised for a future period on a real-time basis.

iii. Development of a new and efficient heuristic for each type of disturbance to generate the revised plan after the occurrence of a disturbance.

iv. Development of a new heuristic capable of dealing with multiple disturbances in a period by considering their combined effects in order to develop the revised plan.

v. The conduct of random experimentations to validate the heuristics and develop a simulation model which closely emulates real-world processes.

For a better understanding of the disturbance management problem, definitions of the different terms used in this chapter are provided as follows.

Process reliability: percentage of non-defective products produced in the production system (Cheng, 1989).

Demand fluctuation: any kind of variation in product demand. Demand can be increased or decreased for a certain period of time which is known as its fluctuation period (Paul et al., 2014a).

Production disruption: any kind of interruption in the production system (Paul et al., 2014b); for example, machine breakdown, power cut, raw material shortage, etc.

Supply disruption: any form of interruption to the raw material supply that may be caused by a delay, unavailability or any other form of disturbance (Hishamuddin et al., 2014).

Ideal plan: a supply, production and delivery plan developed under ideal conditions (no disruption).

Revised plan: if a disturbance occurs in the system, it is necessary to revise the plan for some periods in the future until the system returns to its normal schedule.

Back orders: after the occurrence of a disturbance, a portion of demand that cannot be fulfilled at the scheduled time but will be delivered at a later date when available.

Lost sales: if, after the occurrence of a disturbance, the production process is not capable of fulfilling that demand, as customers will sometimes not wait for stock to be replenished, demand is lost.

9.2 Problem description

In this section, the different disturbance problems that occur in a real-life supply chain system are described and presented in Fig. 9.1. It is considered that a demand fluctuation can happen at the retailer end, a production disruption at the manufacturing plant and a supply disruption at the supplier end. After a disruption occurs in a system, the production, supply and delivery plan has to be revised so that the effect of the disruption is minimised; in other words, total profit is maximised.

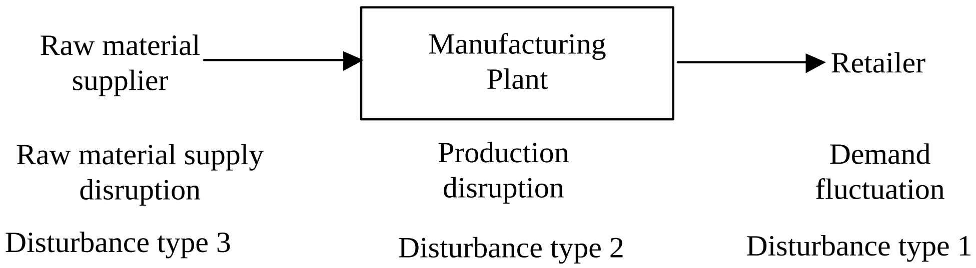

Figure 9.1: Different disturbances in a supply chain system

In this system, first, the ideal production, supply and delivery plan is developed for n periods under imperfect production system which is updated after each period for the next n periods on a rolling horizon basis. The term process reliability (r) is used to express imperfect production environment. The ideal plan is presented in Table 9.1, where the decision variables are AP_i, R_i, RM_i, B_i and E_i, and the total profit is maximised.

Table 9.1: Ideal plan for n periods

Variable	Period 1	Period 2	Period 3	...	Period n
Demand (D_i)	D_1	D_2	D_3	...	D_n
Production capacity (P)	rP	rP	rP	...	rP
Actual production (AP_i)	AP_1	AP_2	AP_3	...	AP_n
Beginning inventory (B_i)	B_1	$AP_1 + B_1 - R_1$	$\sum_{j=1}^{2} AP_j - \sum_{j=1}^{2} R_j + B_1$	...	$\sum_{j=1}^{n-1} AP_j - \sum_{j=1}^{n-1} R_j + B_1$
Ending inventory (E_i)	$AP_1 + B_1 - R_1$	$\sum_{j=1}^{2} AP_j - \sum_{j=1}^{2} R_j + B_1$	$\sum_{j=1}^{3} AP_j - \sum_{j=1}^{3} R_j + B_1$	...	$\sum_{j=1}^{n} AP_j - \sum_{j=1}^{n} R_j + B_1$
Received by retailer (R_i)	$B_1 + AP_1 - E_1$	$B_2 + AP_2 - E_2$	$B_3 + AP_3 - E_3$	...	$B_n + AP_n - E_n$
Raw material quantity (RM_i)	RM_1	RM_2	RM_3	...	RM_n

Table 9.2: Revised plan for managing disturbance

Variable	Period 1	Period 2	Period 3	...	Period n
Production (X_i)	X_1	X_2	X_3	...	X_n
Received by retailer (Y_i)	Y_1	Y_2	Y_3	...	Y_n
Beginning inventory (b_i)	b_1	$X_1 + b_1 - Y_1$	$\sum_{j=1}^{2} X_j - \sum_{j=1}^{2} Y_j + b_1$	...	$\sum_{j=1}^{n-1} X_j - \sum_{j=1}^{n-1} Y_j + b_1$
Ending Inventory (e_i)	$X_1 + b_1 - Y_1$	$\sum_{j=1}^{2} X_j - \sum_{j=1}^{2} Y_j + b_1$	$\sum_{j=1}^{3} X_j - \sum_{j=1}^{3} Y_j + b_1$	...	$\sum_{j=1}^{n} X_j - \sum_{j=1}^{n} Y_j + b_1$
Raw material quantity (Z_i)	Z_1	Z_2	Z_3	...	Z_n

Finally, this chapter develops a **recovery plan** which is actually a **reactive mitigation**. In real-life situations, any supply chain can face a sudden disturbance at any time. After such an occurrence, the plan must be revised for a finite period in the future so that losses can be minimised and the system returns to its ideal plan as quickly as possible. After the occurrence of a disturbance, X_i, Y_i, Z_i, b_i and e_i are changed to obtain the revised plan presented in Table 9.2 while the objective is still to maximise total profit.

9.2.1 Notations used in the Study

The following notations are used in this study.

n	Number of planning periods in planning horizon
D_i	Demand of period i
P	Maximum production capacity of each period
B_i	Beginning inventory in period i
B_{n+1}	Beginning inventory which should be kept in period $(n+1)$
E_i	Ending inventory in period i
AP_i	Actual production in period i
SC_i	Spare capacity in period i
R_i	Quantity received by retailer at period i
N	Units of raw material required to produce one unit of final product
A	Set-up cost at the manufacturing plant
r	Process reliability of manufacturing plant
RM_i	Raw material supply quantity for period i $= N * \frac{AP_i}{r}$
C_p	Production cost per unit
C_d	Delivery cost per unit
C_r	Raw material cost per unit
H_1	Raw material holding cost per unit per period
H_2	Ending inventory holding cost per unit
C_L	Cost per unit due to decrease of demand
C_I	Inspection cost as a percentage of the production cost
C_R	Rejection cost per unit
S	Selling price per unit
B	Back orders cost per unit per period

L Lost sales cost per unit = revenue loss per unit + cost of reputation loss per unit

X_i Production quantity in period i in revised plan

Y_i Delivery quantity in period i in revised plan

Z_i Raw material quantity in period i in revised plan

b_i Beginning inventory in revised plan

e_i Ending inventory in revised plan

Demand fluctuation parameter

δ Demand fluctuation amount

Production disruption parameters

t_s Disruption start time as fraction of duration of period

T_{dp} Disruption duration as fraction of duration of period (≤ 1- t_s)

q Pre-disruption production quantity $= t_s * P$

Supply disruption parameter

T_{ds} Disruption duration as fraction of duration of period (≤ 1)

9.2.2 Assumptions of the Study

In this chapter, the following assumptions are made:

i. The total production capacity is greater than its demand rate ($rP * n > \sum_{i=1}^{n} D_i$);
ii. A single item is produced in the system;
iii. A supplier can supply any amount of raw material;
iv. The total cost of interest and depreciation per production cycle $C(A, r)$ is inversely related to set-up cost (A) and is directly related to process reliability (r) according to the following general power function(Cheng, 1989):

$$C(A, r) = aA^{-b}r^{c}$$

where a, b and c are positive constants chosen to provide the best fit of the estimated cost function;
v. The ideal plan is updated after each period for the next n periods (rolling planning horizon);
vi. The beginning inventory in the first period is known; and
vii. The plan is revised after the occurrence of a disturbance on a real-time basis.

9.3 Mathematical Modelling

In this section, a mathematical model is developed for managing a single occurrence of a disturbance caused by a demand fluctuation, production disruption or supply disruption by firstly presenting a mathematical model for an ideal supply chain plan. Then, a revised plan is formulated as a constrained optimisation problem that maximises total profit, which is derived from the revenue, obtained from acceptable items, minus relevant costs, subject to production capacity, delivery and product demand constraints. The decision variables are the revised quantities of production, delivery, supply, back orders and lost sales in each period in this plan.

9.3.1 Mathematical Model for Ideal Plan

In the ideal plan, the costs for production, rejection, inspection, depreciation, holding, delivery and raw material purchases as well as the revenue from acceptable items are calculated. Then, a mathematical model is developed as a constrained optimisation problem in which the objective is to maximise total profit subject to capacity, delivery, inventory and product demand constraints.

Calculations of Different Costs and Revenue

Total production cost

$$= \frac{C_p}{r}\sum_{i=1}^{n} AP_i \tag{9.1}$$

Total rejection cost

$$= C_R\left(\frac{1}{r}-1\right)\sum_{i=1}^{n} AP_i \tag{9.2}$$

Total inspection cost

$$= \frac{C_I C_p}{r}\sum_{i=1}^{n} AP_i \tag{9.3}$$

Cost of interest and depreciation

$$= na\, A^{-b} r^{c} \tag{9.4}$$

Raw material holding cost

$$= \frac{1}{2} H_1 \sum_{i=1}^{n} RM_i$$

$$= \frac{1}{2r} H_1 N \sum_{i=1}^{n} AP_i \tag{9.5}$$

Total raw material cost

$$= \sum_{i=1}^{n} C_r RM_i$$

$$= \frac{NC_r}{r} \sum_{i=1}^{n} AP_i \tag{9.6}$$

Total delivery cost

$$= \sum_{i=1}^{n} C_d R_i \tag{9.7}$$

Total ending inventory holding cost

$$= H_2 \sum_{i=1}^{n} E_i \tag{9.8}$$

Total revenue

$$= S \sum_{i=1}^{n} AP_i \tag{9.9}$$

Final Mathematical Model

The objective function, total profit (TP) = total revenue – total costs, is obtained as:

$$\begin{aligned} TP = S \sum_{i=1}^{n} AP_i & \\ & - \left[\frac{C_p}{r} \sum_{i=1}^{n} AP_i + C_R \left(\frac{1}{r} - 1 \right) \sum_{i=1}^{n} AP_i + \frac{C_I C_p}{r} \sum_{i=1}^{n} AP_i \right. \\ & + na\, A^{-b} r^{c} + \frac{1}{2r} H_1 N \sum_{i=1}^{n} AP_i + \frac{NC_r}{r} \sum_{i=1}^{n} AP_i \\ & \left. + \sum_{i=1}^{n} C_d R_i + H_2 \sum_{i=1}^{n} E_i \right] \end{aligned} \tag{9.10}$$

Here,

$R_i = B_i + AP_i - E_i;\ \forall i$

$$B_i = \sum_{j=1}^{i-1} AP_j - \sum_{j=1}^{i-1} R_j + B_1;\ \forall i \neq 1$$

$$E_i = \sum_{j=1}^{i} AP_j - \sum_{j=1}^{i} R_j + B_1;\ \forall i$$

$$RM_i = N * \frac{AP_i}{r}$$

subject to constraints (9.11) – (9.17).

$E_i \geq 0; \forall i$	[Ending inventory cannot be negative]	(9.11)
$B_i \geq 0; \forall i$	[Beginning inventory cannot be negative]	(9.12)
$E_n = B_{n+1}$	[Beginning inventory for $(n+1)^{th}$ period]	(9.13)
$\sum_{i=1}^{n} AP_i = \sum_{i=1}^{n} D_i - B_1 + B_{n+1}$	[Total production must be equal to total demand]	(9.14)
$AP_i \leq rP$	[Actual production must be less than maximum production capacity]	(9.15)
$R_i = D_i$	[Delivery quantity must be equal to demand]	(9.16)
AP_i, R_i and $RM_i \geq 0$	[Non-negativity constraint	(9.17)

9.3.2 Disturbance Management Model

In this section, a mathematical model is developed for managing a demand fluctuation, with those for production and supply disruptions presented in Appendix C and Appendix D respectively.

9.3.2.1 Managing Demand Fluctuation

To formulate the mathematical model for determining the revised plan after a demand fluctuation, the costs of production, rejection, inspection, depreciation, delivery, holding and raw material purchases are considered. For $\delta > 0$, both back orders and lost sales costs are considered and, for $\delta < 0$, the cost due to a decrease in demand, and determine revenue from the selling price. Finally, a mathematical model is developed in which total profit is to be maximised subject to capacity, delivery, product demand, and inventory constraints.

(a) For $\boldsymbol{\delta > 0}$

Total production cost

$$= \frac{C_p}{r} \sum_{i=1}^{n} X_i \tag{9.18}$$

Total rejection cost

$$= C_R \left(\frac{1}{r} - 1\right) \sum_{i=1}^{n} X_i \tag{9.19}$$

Total inspection cost

$$= \frac{C_I C_p}{r} \sum_{i=1}^{n} X_i \tag{9.20}$$

Cost of interest and depreciation

$$= na\, A^{-b} r^{c} \tag{9.21}$$

Total raw material cost

$$= \sum_{i=1}^{n} C_r Z_i$$

$$= \frac{NC_r}{r} \sum_{i=1}^{n} X_i \tag{9.22}$$

Raw material holding cost

$$= \frac{1}{2r} H_1 N \sum_{i=1}^{n} X_i \tag{9.23}$$

Total delivery cost

$$= \sum_{i=1}^{n} C_d\, Y_i \tag{9.24}$$

Total ending inventory holding cost

$$= H_2 \sum_{i=1}^{n} e_i \tag{9.25}$$

Back orders cost

$$= B \sum_{i=1}^{n} i(X_i - AP_i) \tag{9.26}$$

Lost sales cost

$$= L\left(\sum_{i=1}^{n} AP_i + \delta - \sum_{i=1}^{n} X_i\right) \tag{9.27}$$

Total revenue

$$= S \sum_{i=1}^{n} X_i \tag{9.28}$$

Final mathematical model for $\boldsymbol{\delta > 0}$

The objective function, total profit = total revenue – total costs, which is to be maximised subject to constraints (9.29) – (9.36).

$e_i \geq E_i; \forall i$	[Constraint of ending inventory]	(9.29)
$b_i \geq B_i; \forall i$	[Constraint of beginning inventory]	(9.30)
$X_i \leq rP;\ \forall i$	[Production quantity must be less than or equal to maximum capacity]	(9.31)
$\sum_{i=1}^{n} X_i \leq \sum_{i=1}^{n} D_i + b_{n+1} - b_1 + \delta$	[Limitation of total production quantity]	(9.32)
$\sum_{i=1}^{n} X_i \geq \sum_{i=1}^{n} AP_i$	[Limitation of total production]	(9.33)
$\sum_{i=1}^{n} Y_i \geq \sum_{i=1}^{n} R_i$	[Limitation of total delivery]	(9.34)

$$\sum_{i=1}^{n} Y_i \leq \sum_{i=1}^{n} D_i + \delta \quad \text{[Limitation of total delivery]} \tag{9.35}$$

$$X_i, Y_i, Z_i \geq 0; \forall i \quad \text{[Non-negativity constraint]} \tag{9.36}$$

(b) For $\boldsymbol{\delta < 0}$

Total production cost

$$= \frac{C_p}{r} \sum_{i=1}^{n} X_i \tag{9.37}$$

Total rejection cost

$$= C_R \left(\frac{1}{r} - 1\right) \sum_{i=1}^{n} X_i \tag{9.38}$$

Total inspection cost

$$= \frac{C_I C_p}{r} \sum_{i=1}^{n} X_i \tag{9.39}$$

Cost of interest and depreciation

$$= na\, A^{-b} r^{c} \tag{9.40}$$

Total raw material cost

$$= \sum_{i=1}^{n} C_r Z_i$$

$$= \frac{N C_r}{r} \sum_{i=1}^{n} X_i \tag{9.41}$$

Raw material holding cost

$$= \frac{1}{2r} H_1 N \sum_{i=1}^{n} X_i \tag{9.42}$$

Total delivery cost

$$= \sum_{i=1}^{n} C_d\, Y_i \tag{9.43}$$

Total ending inventory holding cost

$$= H_2 \sum_{i=1}^{n} e_i \tag{9.44}$$

Cost due to decrease in demand

$$= C_L\left(\sum_{i=1}^{n} AP_i - \sum_{i=1}^{n} X_i\right) \tag{9.45}$$

Total revenue

$$= S\sum_{i=1}^{n} X_i \tag{9.46}$$

Final mathematical model for $\delta < 0$

The objective function, total profit = total revenue – total costs, which is to be maximised subject to constraints (9.47) – (9.52).

$e_i \geq E_i; \forall i$	[Constraint of ending inventory]	(9.47)
$b_i \geq B_i; \forall i$	[Constraint of beginning inventory]	(9.48)
$X_i \leq rP;\ \forall i$	[Limitation of production quantity of each period]	(9.49)
$\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} D_i + b_{n+1} - b_1 - \delta$	[Limitation of total production quantity]	(9.50)
$Y_i = \sum_{i=1}^{n} R_i - \delta$	[Limitation of total delivery quantity]	(9.51)
$X_i, Y_i, Z_i \geq 0; \forall i$	[Non-negativity constraint]	(9.52)

9.4 Solution Approaches

In this section, solution approaches are developed for both the ideal and revised plans, and some heuristics are proposed for solving the mathematical models developed to manage disturbances.

9.4.1 Solution Approach for Ideal Plan

As the mathematical model developed for the ideal plan belongs to a constrained program, it is solved using the SIMPLEX method.

9.4.2 Proposed Heuristic for Managing Disturbance

A heuristic is developed for managing each disturbance type, i.e., a demand fluctuation, and production and supply disruptions, as well as another for handling multiple disturbances in a period.

9.4.2.1 Heuristic 1 for Managing Demand Fluctuation

The steps in heuristic 1 for managing a demand fluctuation are as follows.

Step 1: Input data for ideal plan.
Step 2: Determine and record ideal plan.
Step 3: Determine spare capacity (SC_i) of each period.
Step 4: Input demand fluctuation amount (δ).
Step 5: For $\delta > 0$

5.1 For $0 \leq \delta \leq SC_1$

If $B \leq L$, Then

$X_1 = AP_1 + \delta$

$X_i = AP_i$; $i = 2, 3\ldots n$

If $B > L$, Then

$X_i = AP_i$; $i = 1, 2, 3\ldots n$

5.2 For $k = 2$ to n

For $\sum_{j=1}^{k-1} SC_j < \delta \leq \sum_{j=1}^{k} SC_j$

If $B \leq \frac{L}{k}$, Then

$X_i = AP_i + SC_i$; $i = 1, 2, 3\ldots k-1$

$X_k = AP_k + \delta - \sum_{j=1}^{k-1} SC_j$

$X_i = AP_i$; $i = k+1, k+2\ldots n$

If $\frac{L}{k} < B \leq \frac{L}{k-1}$, Then

$X_i = AP_i + SC_i$; $i = 1, 2, 3\ldots k-1$

$X_i = AP_i$; $i = k, k+1, k+2\ldots n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1$

$X_i = AP_i$; $i = 2, 3\ldots n$

If $B > L$, Then

$X_i = AP_i$; $i = 1, 2, 3\ldots n$

5.3 For $\delta > \sum_{j=1}^{n} SC_j$

If $B \leq \frac{L}{n}$, Then

$X_i = AP_i + SC_i$; $i = 1, 2, 3\ldots n$

If $\frac{L}{n} < B \leq \frac{L}{n-1}$, Then

$X_i = AP_i + SC_i$; $i = 1, 2, 3\ldots n-1$

$X_n = AP_n$

……….

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1$

$X_i = AP_i$; $i = 2, 3 \ldots n$

If $B > L$, Then

$X_i = AP_i$; i =1, 2, 3…n

Step 6: For $\delta < 0$

$X_1 = AP_1 - |\delta|$

$X_i = AP_i$; $i = 2, 3 \ldots n$

Step 7: Determine raw material required and final product delivery quantity.

$Y_i = B_i + X_i - E_i$; i =1, 2, 3…n

$Z_i = N * \frac{X_i}{r}$; i =1, 2, 3…n

Step 8: Determine different costs, total profit, and back orders and lost sales quantities.

Step 9: Stop.

9.4.2.2 Heuristic 2 for Managing Production Disruption

The steps in heuristic 2 for managing a production disruption are presented in Appendix E.

9.4.2.3 Heuristic 3 for Managing Raw Material Supply Disruption

The steps in heuristic 3 for managing a raw material supply disruption are presented in Appendix F.

9.4.3 Validation of Heuristics

To validate the heuristics, 100 random test problems are generated for each disturbance type and solved them using both the respective heuristic and SIMPLEX method, and compared the results.

9.4.4 Combined Effects of Multiple Disturbances

A demand fluctuation happens at the retailer end, a supply disruption at the supplier end and a production disruption at the manufacturing plant. Multiple disturbances can happen together in a period, in which case, their effects must be considered when formulating a revised plan. A heuristic is proposed to deal with multiple disturbances and use random data to develop multiple disturbance scenarios.

9.4.4.1 Heuristic 4 for Managing Multiple Disturbances

The steps in heuristic 4 for managing multiple disturbances in a period are presented in Appendix G.

9.5 Simulation Model

A simulation model is developed to make the supply chain disturbance problem closer to a real-world problem using the following steps.

Step 1: Generate a random number for disturbance type (1-4).

Step 2:

2.1 if disturbance type = 1, generate a random number for amount of demand fluctuation (δ) using normal distribution with mean 500 and standard deviation 250.

2.2 if disturbance type =2, generate random number for disruption start time (t_s) using uniform distribution and disruption duration (T_{dp}) through exponential distribution between 0 and 1 and 0.00001 and 1– t_s respectively.

2.3 if disturbance type =3, generate a random number for supply disruption duration (T_{ds}) using Poisson distribution between 0.00001 and 1.

2.4 if disturbance type = 4, generate random number for disturbance scenario of each disturbance type for multiple disturbances in a period.

Step 3:

3.1 if disturbance type =1, then run heuristic 1.

3.2 if disturbance type =2, then run heuristic 2.

3.3 if disturbance type =3, then run heuristic 3.

3.4 if disturbance type = 4, then run heuristic 4.

Step 4: Repeat steps 1-3 for 4000 times.

Step 5: Record and analysis of results.

Step 6: Stop.

The heuristics and simulation model are coded in MATLAB R2012a and executed on an Intel core i7 processor with 8.00 GB RAM and a 3.40 GHz CPU.

9.6 Experimentations and Analysis of Results

In this section, the results have been analysed for both the ideal and revised plans by performing random experimentations.

9.6.1 Ideal Plan

The following data are considered for the ideal supply chain plan.

$n = 12$, P =1200, $B_1 = 300$, $B_{n+1} = 200$, $N = 2$, $A = 50$, $C_p = 2$, $C_d = 0.5$, $C_r = 1.5$,

$H_1 = 0.5$, $H_2 = 0.5$, $S = 20$, $r = 0.98$, $C_I = 0.02$, $C_R = 4$, $a = 1000$, $b = 0.5$, $c = 0.75$,

$D_i =$ [1000 1200 1500 1100 1000 800 900 1200 1300 1200 1500 1000]

The SIMPLEX method is used to solve the mathematical model developed in Section 9.3.1 to obtain the ideal plan for the next 12 periods which is presented in Table 9.3 and has a total profit of 184.05 thousand.

Table 9.3: Ideal plan

Parameter	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12
D_i	1000	1200	1500	1100	1000	800	900	1200	1300	1200	1500	1000
AP_i	1048	1176	1176	1100	1000	1044	1176	1176	1176	1176	1176	1176
B_i	300	348	324	0	0	0	244	520	496	372	348	24
E_i	348	324	0	0	0	244	520	496	372	348	24	200
R_i	1000	1200	1500	1100	1000	800	900	1200	1300	1200	1500	1000
RM_i	2139	2400	2400	2245	2041	2131	2400	2400	2400	2400	2400	2400

9.6.2 Revised Plan

The following additional cost data are considered to obtain the revised plan.

$B = 3$, $L = 15$ and C_L =10

9.6.2.1 Revised Plan after a Demand Fluctuation

In the case of a demand fluctuation, the revised plan is generated using its proposed heuristic. Although it was experimented using random data from both uniform and normal distributions, only a sample result, for $\delta = 500$, is presented in the revised plan in Table 9.4.

Table 9.4: Revised plan after demand fluctuation

Parameter	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12
X_i	1176	1176	1176	1176	1176	1164	1176	1176	1176	1176	1176	1176
Y_i	1128	1200	1500	1176	1176	920	900	1200	1300	1200	1500	1000
Z_i	2400	2400	2400	2400	2400	2376	2400	2400	2400	2400	2400	2400
Total profit = 184.84 thousand Total back orders cost = 6.096 thousand Total lost sales cost = 0												

9.6.2.2 Revised Plan after a Production Disruption

In the case of a production disruption, the revised plan is generated using its proposed heuristic. Although it was experimented using random data from both uniform and exponential distributions, only a sample result, for t_s =0.1 and T_{dp} =0.5, is presented in the revised plan in Table 9.5.

Table 9.5: Revised plan after production disruption

Parameter	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12
X_i	588	1176	1176	1176	1176	1176	1176	1176	1176	1176	1176	1176
Y_i	540	1200	1500	1176	1176	932	900	1200	1300	1200	1500	1000
Z_i	2139	1461	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400
Total profit = 177.09 thousand Total back orders cost = 4.776 thousand Total lost sales cost = 1.14 thousand												

9.6.2.3 Revised Plan after a Supply Disruption

In the case of a supply disruption, the revised plan is generated using its proposed heuristic. Although it was experimented using random data from both uniform and Poisson distributions, only a sample result, for T_{ds} =0.6, is presented in the revised plan in Table 9.6.

Table 9.6: Revised plan after supply disruption

Parameter	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10	Period 11	Period 12
X_i	470	1176	1176	1176	1176	1176	1176	1176	1176	1176	1176	1176
Y_i	422	1200	1500	1176	1176	932	900	1200	1300	1200	1500	1000
Z_i	2139	1221	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400
Total profit = 173.70 thousand Total back orders cost = 4.776 thousand Total lost sales cost = 2.904 thousand												

9.6.3 Comparison of Results and Validation of Heuristics

To validate the heuristics developed for managing demand fluctuation, and production and supply disruptions, 300 random test problems (100 for each disruption type) were generated by varying the back orders and lost sales cost data and disturbance parameters. Then, the test problems were solved using both the heuristic and SIMPLEX method. As the mean percentages of deviation between the respective heuristic and SIMPLEX results, calculated using equation (9.53), are almost 0.00% for all three disruption types, it can be said that the heuristics are capable of producing accurate solutions.

The average percentage of deviation

$$= \frac{1}{M} \sum \frac{|\text{Total profit from heuristc} - \text{Total profit from SIMPLEX}|}{\text{Total profit from SIMPLEX}} \times 100\% \qquad (9.53)$$

Here, M = the number of test problems.

9.6.4 Which Disturbance Type is More Severe?

To compare the severity of each disturbance type, 500 more test problems are generated for each disturbance using a uniform probability distribution and solved them using the proposed corresponding heuristic. The means and standard deviations of total profit from the results are determined, as presented in Table 9.7. The following data range of disturbance parameters is considered.

(a) Demand fluctuation amount = [0, $\sum_{\forall i} SC_i$]

(b) Supply disruption duration = [0.0001, 1]

(c) Production disruption duration = [0.0001, 1-t_s]

Table 9.7: Total profit for each disruption type

Disturbance type	Total profit (thousands)	
	Mean	Standard deviation
Demand fluctuation	185.19	0.3533
Production disruption	178.08	5.3127
Supply disruption	175.37	7.2898

As can be seen, the mean total profit reduces significantly in the case of a supply disruption because the effect of this disruption starts at the beginning of, and may continue until the end of, a period. Therefore, it can be said that its effect is more severe than those of the other two.

9.6.5 Experimentation using Random Data

Five hundred random test problems are generated for each type of disturbance and solved them using the appropriate heuristic. The total profit pattern is analysed for random occurrences of disturbance over the 500 random scenarios, and variations in the different costs and total profit according to the amount of disturbance.

9.6.5.1 Experimentation for Demand Fluctuation

Five hundred random data scenarios are generated for demand fluctuations using a normal distribution with mean = 500 and standard deviation = 250, and present the total profit pattern in Figure 9.2. It was determined that the mean and standard deviation values of total profit are 183.77 and 2.1175 thousand respectively, and the maximum and minimum values 185.56 and 172.60 thousand respectively.

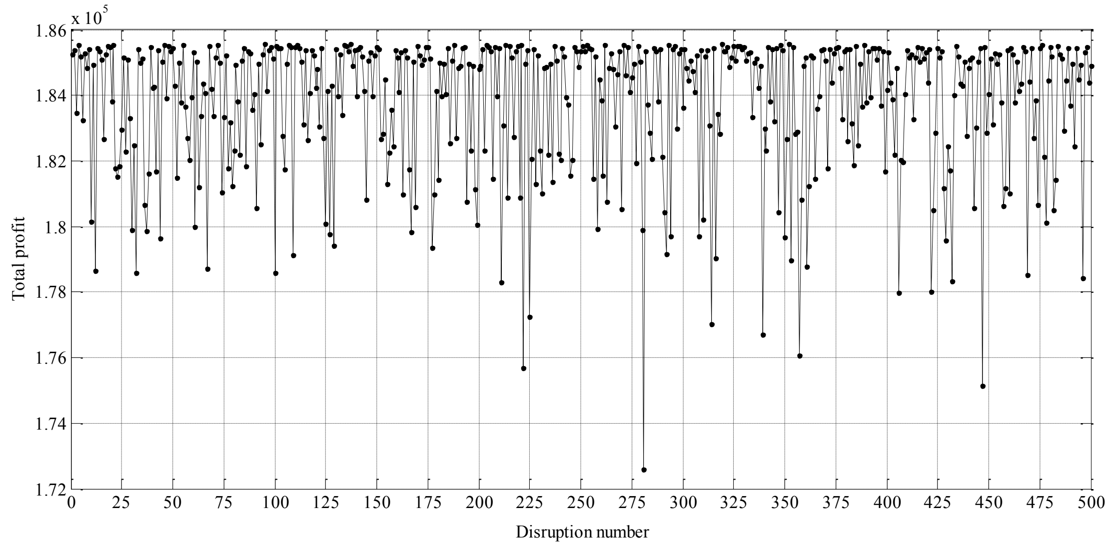

Figure 9.2: Total profit vs random demand fluctuation

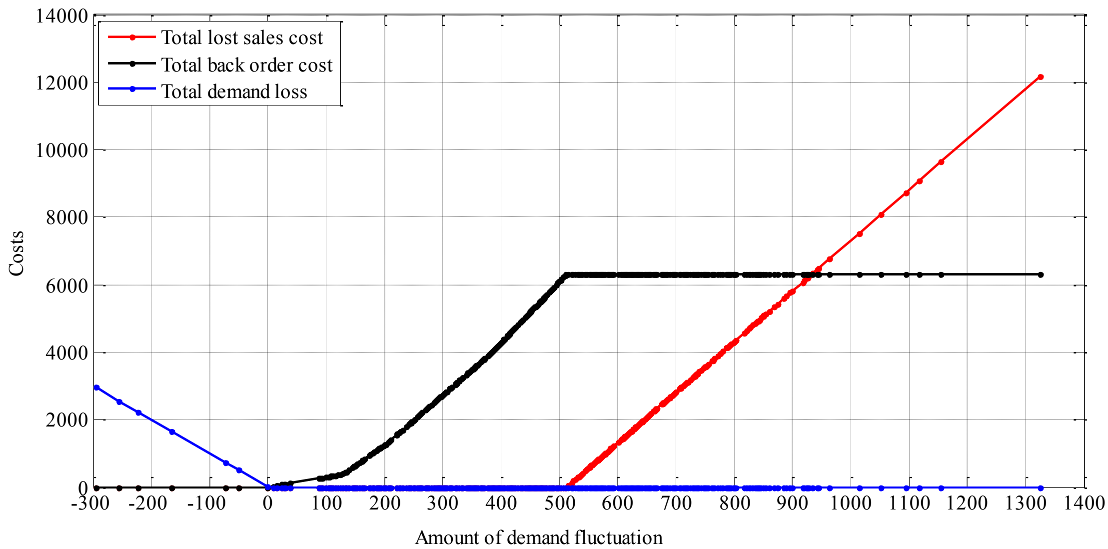

Figure 9.3: Different costs vs amount of demand fluctuation

Figure 3 presents the variations in different costs with the amount of demand fluctuation. It was obeserved that the cost due to loss of demand exists only when the fluctuation amount is negative but there are no back orders or lost sales. However, when the fluctuation amonut is positive, both back orders and lost sales are present in the revised plan. The back orders cost increases with fluctuation amounts up to 512 when there are no lost sales because the revised plan is capable of fulfilling the demand using only back orders. Then, lost sales is introduced into the plan and the back orders cost becomes a fixed amount so that both back orders and lost sales are present.

The variations in total profit with demand fluctuation amounts are presented in Figure 9.4. For a negative fluctuation, the total profit decreases with the fluctuation amount but, for a positive one, it is greater than that in the ideal plan when the fluctuation amount is up to 512 because the revenue earned is greater than the cost incurred due to the increase in demand. Then, the total profit decreases with the fluctuation amount because of the lost sales cost being introduced into the system.

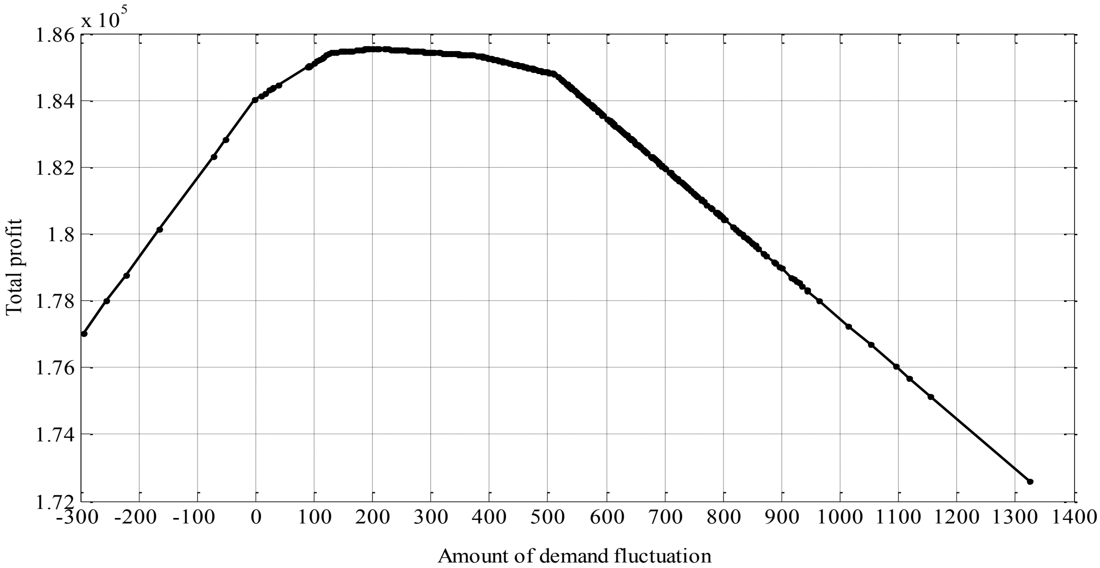

Figure 9.4: Total profit vs amount of demand fluctuation

9.6.5.2 Experimentation for Production Disruption

Five hundred random scenarios are generated for a production disruption using a uniform distribution within the range of (0, 1) for t_s and an exponential distribution within the range of (0.0001, $1-t_s$) for T_{dp}. The total profit pattern for these random production disruption occurrences is presented in Figure 9.5. It was determined that the mean and standard deviation values of total profit are 181.26 and 4.2136 thousand respectively, and the maximum and minimum values 184.05 and 161.83 thousand respectively.

Figure 9.6 presents the variations in different costs according to the duration of the production disruption. It was observed that there are no back orders or lost sales costs when the disruption duration is less than 0.11. Then, the back orders cost is introduced into the system and increases with disruption durations up to 0.43 because the revised plan is capable of satisfying the production loss using only back orders. After a disruption duration of 0.43, the lost sales cost is included in the revised plan and the back orders cost becomes a fixed amount so that both back orders and lost sales costs are present.

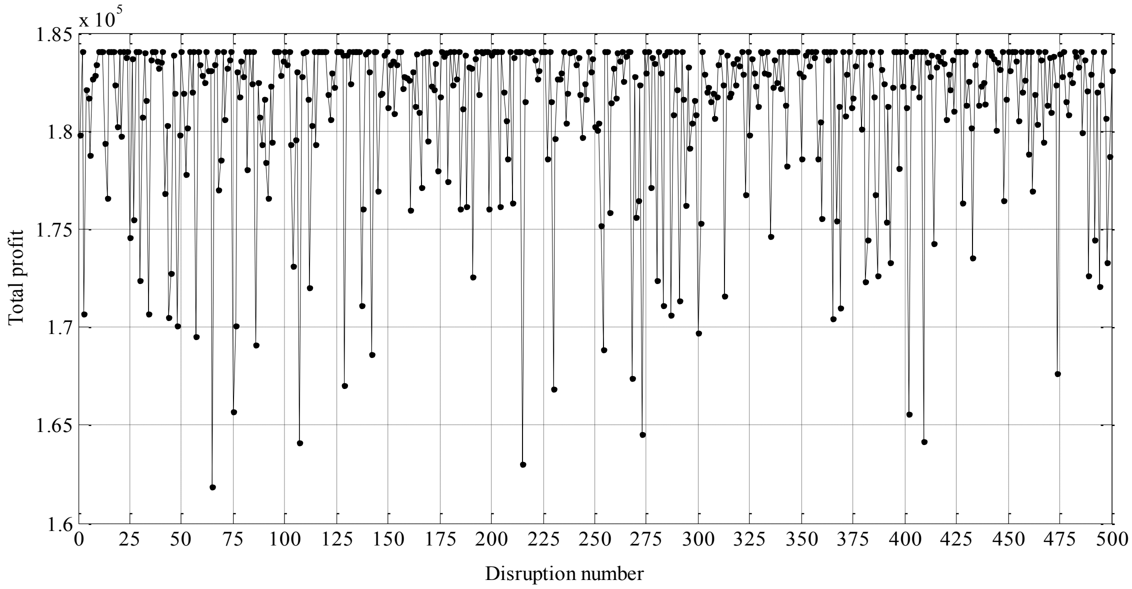

Figure 9.5: Total profit vs random production disruption

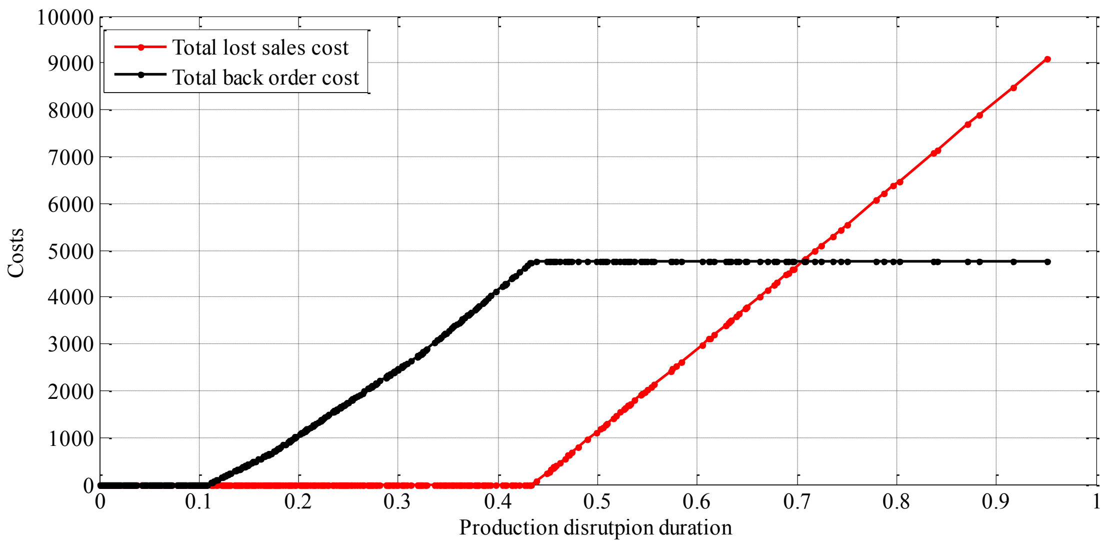

Figure 9.6: Different costs vs duration of production disruption

The variations in total profit in relation to the duration of a production disruption are presented in Figure 9.7. The total profit does not change when the duration is less than 0.11 because no back orders or lost sales costs are present and the revised plan is capable of compensating for the production loss in its first period. Then, the total profit decreases slowly with disruption durations up to 0.43 because only back orders are present. Following a disruption duration of 0.43, total profit decreases at a higher rate because of the lost sales cost being included in the plan.

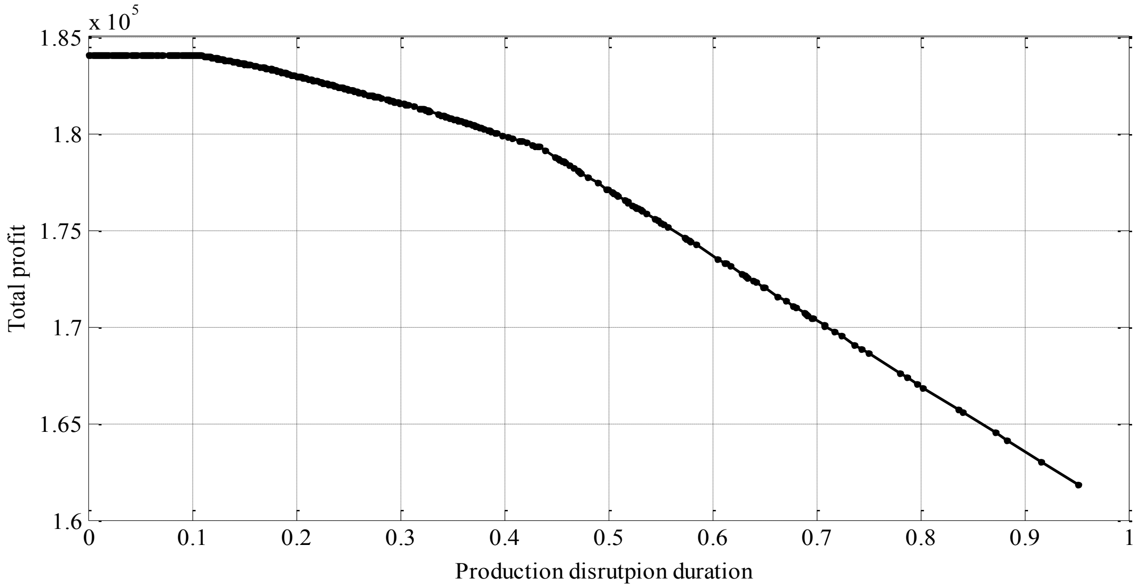

Figure 9.7: Total profit vs duration of production disruption

9.6.5.3 Experimentation for Raw Material Supply Disruption

Five hundred random scenarios are generated for the duration of a supply disruption using a Poisson distribution within the range of (0.0001, 1), and the total profit pattern is presented in Figure 9.8. It was determined that the mean and standard deviation values of total profit are 177.16 and 7.1467 thousand respectively, and the maximum and minimum values 184.05 and 160.48 thousand respectively. Figures 9.9 and 9.10 present the variations in different costs and total profit respectively for different supply disruption durations which are similar to those in Figure 9.6 and 9.7.

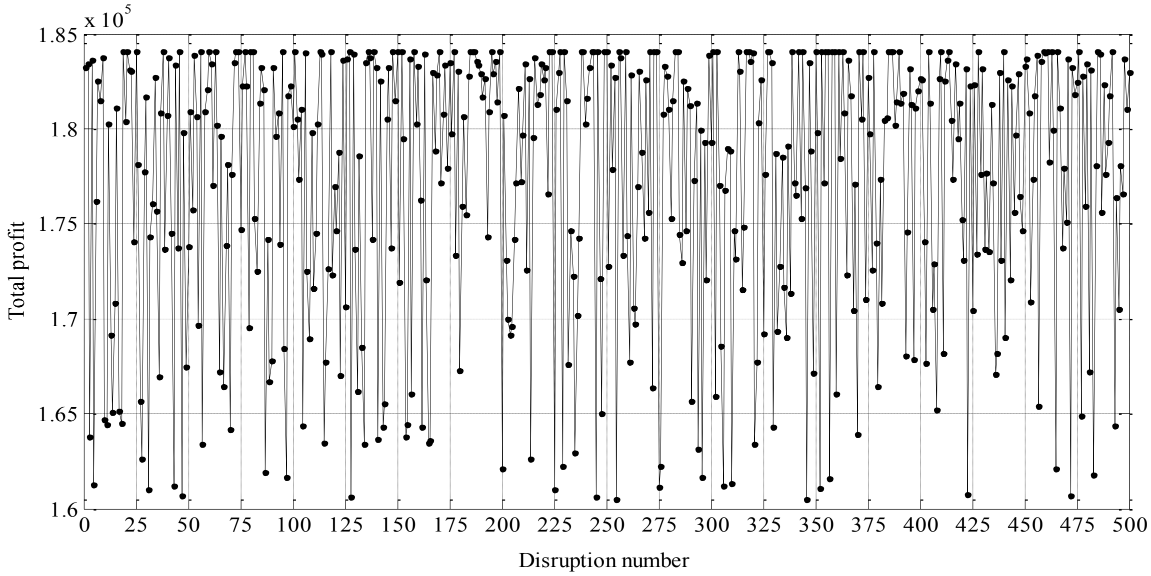

Figure 9.8: Total profit vs random supply disruption

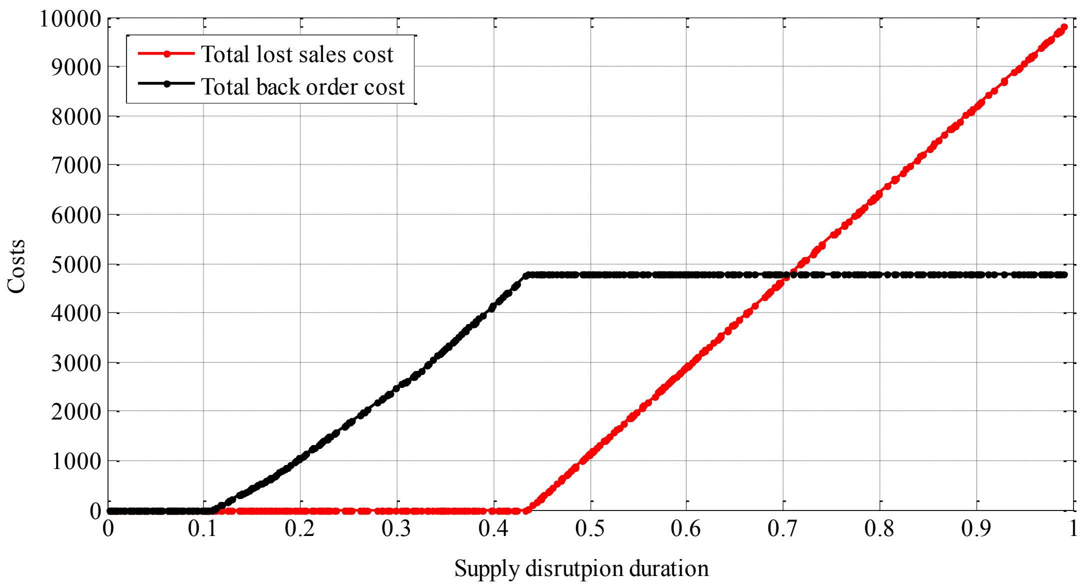

Figure 9.9: Different costs vs duration of supply disruption

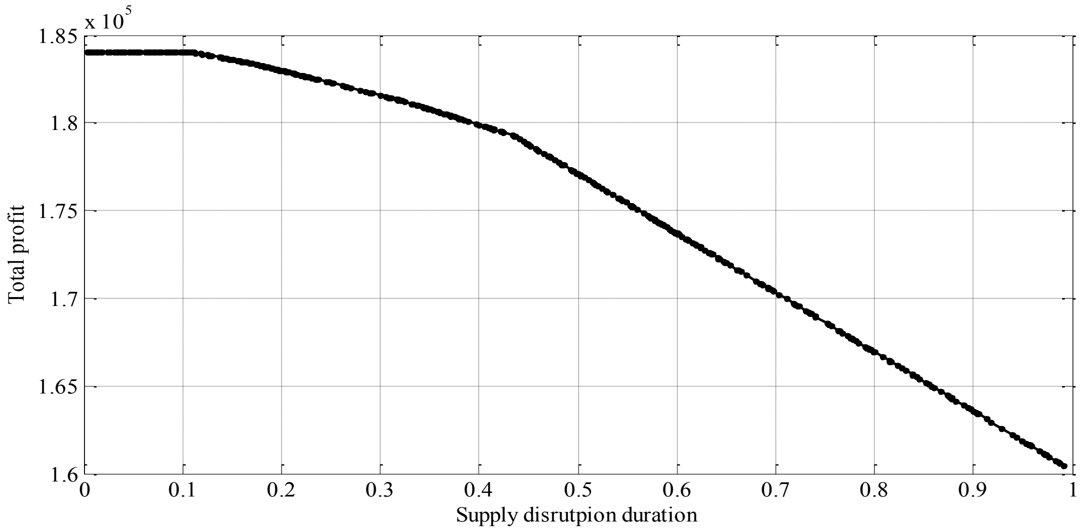

Figure 9.10: Total profit vs duration of supply disruption

9.6.5.4 Experimentation for Multiple Disturbances

For this experimentation, 500 random scenarios are generated for multiple disturbances in a period and solve them using the proposed heuristic which considers the combined effects of multiple disturbances. The results are presented in Figure 9.11 in which it can be observed that total profit varies significantly and that the mean total profit reduces greatly with mean and standard deviation values of 165.70 and 9.4252 thousand respectively.

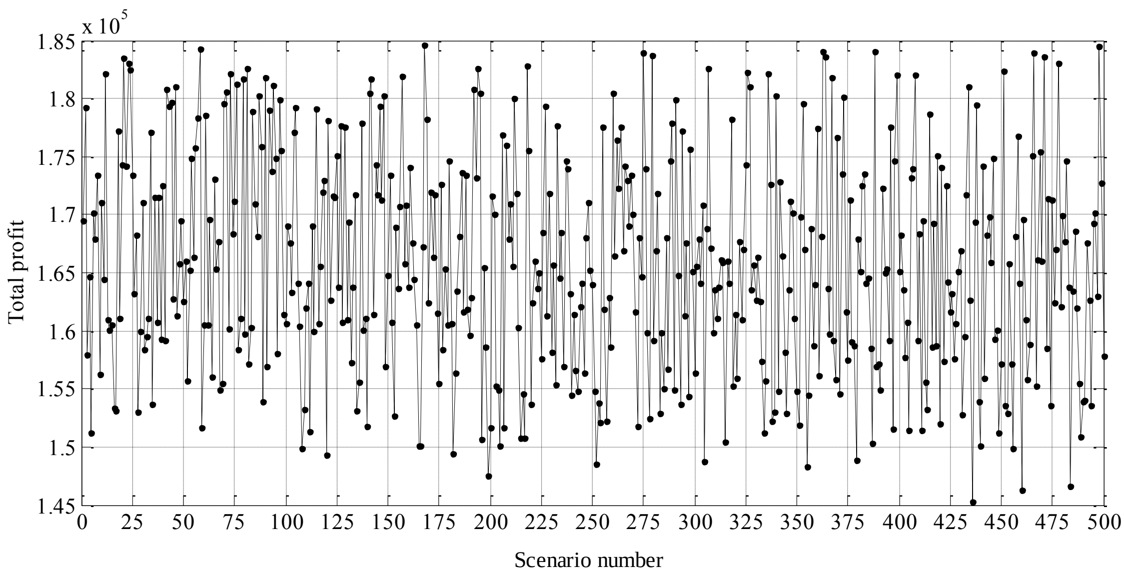

Figure 9.11: Total profit vs random multiple disturbances

9.6.6 Simulation Results

The simulation model, presented in Section 9.5, was run to make the supply chain disturbance problem close to a real-world process. The total profit pattern for random disturbance occurrences over the 4000 random scenarios is presented in Figure 9.12. The mean and standard deviation values of total profit were calculated as 176.85 and 9.2721 thousand respectively, and the maximum and minimum values as 185.56 and 142.48 thousand respectively.

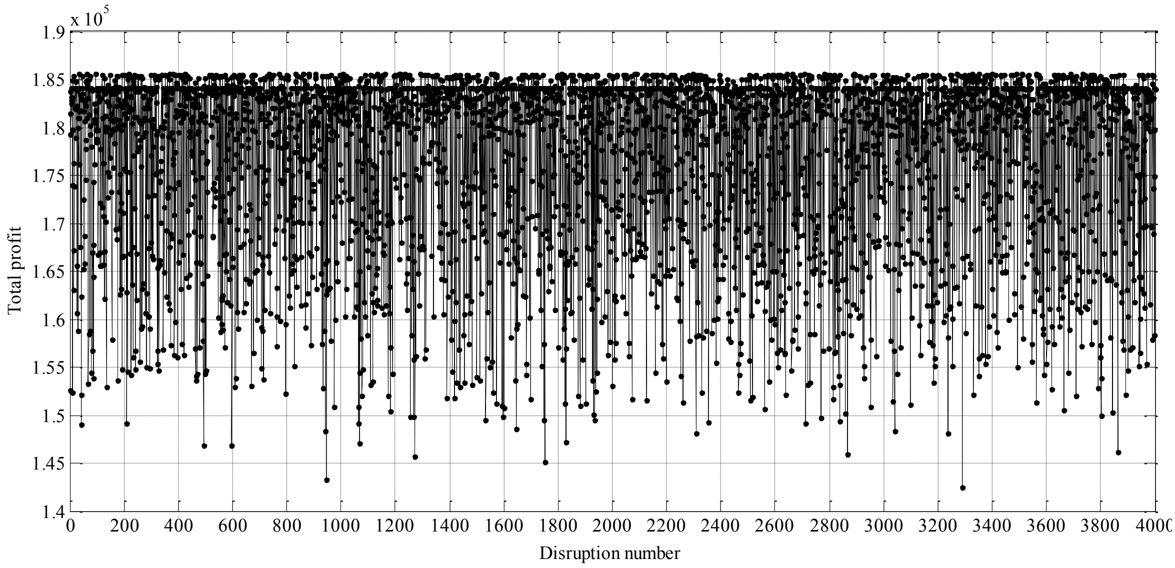

Figure 9.12: Total profit vs occurrences of random disruption from simulation run

9.7 Chapter Summary

The main objective of this study was to develop a disturbance management model for a manufacturing supply chain system which considers demand fluctuations, and production and raw material supply disruptions under an imperfect production environment. A new mathematical model and efficient heuristic were developed for each disruption type to manage disturbances on a real-time basis. These heuristics were validated by comparing the results from another standard solution technique which showed that the average percentage of deviation was 0.00% for a set of randomly generated test problems. Also, a new heuristic was developed to consider multiple disturbances in a period. A random experimentation was performed to analyse the characteristics of the developed models and, finally, a simulation model was developed to make solving a disturbance problem a real-world process.

Chapter 10 Conclusions and Future Research

This chapter briefly summarizes the research that has been conducted in this thesis, as well as its findings and conclusions. Possible directions for future research are also presented.

10.1 Summary of Research and Conclusions

In this thesis, disturbance management models for production-inventory and supply chain systems are studied, developed, and analysed. In production-inventory systems, the models are developed for managing production disruption in single, two, and three-stage systems. In supply chain systems, the models are developed for managing production disruption, demand fluctuation, and supply disruption on a real-time basis.

The framework of the developed models was divided into several steps. At first, an ideal plan was developed without any disturbance. Then a mathematical model was developed, as a constrained mathematical programming problem, to obtain a revised plan after the occurrence of a disturbance in the system. A heuristic approach was developed to solve the mathematical model to obtain the recovery plan. The mathematical model and heuristic approach were also extended to consider multiple disturbances, one after another as a series, on a real-time basis. Finally, random experimentation was conducted and the heuristic results were compared with other standard solution approaches to judge and validate the results.

The framework was applied to six different problems from production-inventory and supply chain systems. The first four models were for managing production disruption for a (i) single-stage production-inventory, (ii) a two-stage production-inventory, (iii) a three-stage mixed production-inventory, and (iv) a three-tier supply chain system. The remaining two models were for managing (v) demand fluctuation in a suppler-retailer system, and (vi) raw material supply disruption in a supply chain system. The replenishment decisions for the production, ordering, supply, and distribution quantities were determined, as well as the quantity of back-orders and/or lost sales was also decided for the revised plan. Some managerial insights about how a decision maker should respond to all types of disturbances during the operations have been provided for all six models. In the final work, a simulation model was also developed to combine all the disturbances under a supply chain environment to investigate the effect of various disruption scenarios. The simulation model was also experimented with a good number of random test problems from different probability distribution. Several insights were also proposed from the simulation results.

The experimental results and findings of each of the above contributions are summarised below.

10.1.1 A Single-Stage Production-Inventory System with Disruption

The main objective of this study was to develop a disruption recovery plan in an imperfect production environment. In real-life production lines, a disruption can happen at any time at any point of production. Moreover, imperfect production processes are very common in real life and they have significant impacts on companies' loss and profit. So it is also important to consider an imperfect production environment while developing a production-inventory model. In real-life situations, the production systems can face both single and multiple disruptions. In the single disruption case, disruption occurs suddenly when the production system operates under a pre-assigned plan. In the multiple disruptions case, disruptions occur one after another as a series, and new disruptions may or may not affect the revised production plans of the previous disruptions. A new mathematical model and a dynamic solution approach was developed, which can handle both single and multiple disruptions, on a real-time basis.

The model was categorised as a non-linear constrained optimisation program, which was solved using both GA and PS based heuristic approaches. The results obtained from both approaches were compared, for a good number of randomly generated disruption test problems, to judge the consistency of the results. The multiple disruptions, one after another as a series, were also considered, as multiple disruptions are very common in real-life situations.

From analysis of the experimental results, it was observed that the recovery plan was dependent on the shortage cost parameters, such as back orders and lost sales costs, and to the disturbance duration. For a certain range of disturbance duration and cost values, it was found that back orders were more attractive, and in such cases, back orders cost was less than the lost sales cost. On the other hand, when back orders cost is more than the lost sales cost, the solution will have lost sales in the recovery plan. It was also demonstrated how the proposed methodology can be implemented for real-time disruption recovery planning with some numerical examples and randomly generated test problems. Additionally, a sensitivity analysis was performed to show the effects of various important parameters on the total profit of the system.

10.1.2 A Two-Stage Production-Inventory System

Chapter 4 extended the works in Chapter 3, to develop a disruption recovery model in a two-stage production-inventory system. The model was developed to recover from either single disruption or a series of production disruptions on a real-time basis, while considering both back order and lost sales options. Pattern search and genetic algorithm based solution approaches were proposed to obtain the recovery plan for either a single or a series of disruptions. The results were compared for a good number of randomly generated test problems. Both methods showed similar results, but pattern search with the Latin hypercube search based solution approach performed better than the genetic algorithm based approach. The proposed approach was also compared with two other existing practices: (i) only lost sales option and (ii) first cycle recovery. It was observed that the proposed disruption recovery plan provided a much better solution than the existing practices. The model was also implemented to a real-world case from a pharmaceutical company to validate the model, and found that the total profit was 10.78% more than the existing practice. Furthermore, the results and findings were similar to that of the models developed in Chapters 3 and 4, which also supports the validity of the model.

10.1.3 A Three-Stage Mixed Production-Inventory System

In Chapter 5, the research extended the works in Chapter 4, to develop a recovery plan for a three-stage mixed production-inventory system. A mathematical model was developed, and then a new heuristic for managing both single and multiple disruptions on a real-time basis was proposed. The results from the heuristic were compared with those from another search algorithm for a set of randomly generated disruption test problems. Both approaches produced very similar results, with the average percentage of deviation being only 0.00034%, which can be considered negligible. The proposed approach was also implemented to solve a real-world disruption problem of a pharmaceutical company. It was proven that the developed mathematical model and proposed heuristic can be easily applied to manage both single and multiple disruptions in a three-stage mixed-production system.

10.1.4 A Supply Chain Network with Disruption

The main objective of this chapter was to develop both predictive and reactive mitigation planning approaches for a supply chain, and to revise the plans based on any future prediction and after the occurrence of a production disruption on a real-time basis. Chapter 6 introduced a three-stage supply chain network model with multiple numbers of manufacturing plants, DCs and retailers. It was formulated as a constrained programming problem in which the objective was to minimise the total supply chain cost. The ideal supply chain system worked in an infinite rolling planning horizon. The plans were revised if there were any changes in data and predictions were generated using the developed prediction methodology. The production and distribution plan was revised again after a disruption in the system for a finite planning period in the future, so that the system could return to its ideal plan as quickly as possible. An efficient heuristic for obtaining a revised plan for either a single disruption or series of disruptions, on a real-time basis, was proposed. An experiment was designed with sixteen different scenarios, each with ten randomly generated disruption test problems, and the performances of the heuristic and LINGO for them were compared. It was shown that the average percentage of deviation in the results was only a negligible 0.0007%. Therefore, it can be said that the proposed mathematical and heuristic approaches offer a potentially very useful quantitative means of helping decision makers arrive at prompt and accurate decisions, regarding both predictive and reactive mitigation plans.

10.1.5 Managing Demand Fluctuation

Chapter 7 developed a risk management model in a two-stage supplier-retailer coordinated system with demand fluctuations, and with an imperfect production process. The model was formulated as a non-linear constrained optimisation problem to maximise the total profit in the revised planning window, and a heuristic was proposed to obtain the revised plan, after the occurrence of a fluctuation. The model was also solved by using a genetic algorithm based search technique, with uniformly distributed random fluctuation scenarios. It was observed that the proposed heuristic was capable of producing quality solutions with average deviations of 0.0089% and 0.0023% for longer and shorter range fluctuations respectively, and with significantly less computational time, as compared to the genetic algorithm. The proposed approach was also compared with two other alternative approaches, (i) managing with only lost sales and (ii) first cycle revision. It was observed that the proposed demand fluctuation management model clearly showed significantly better solutions in comparison to other alternative approaches. Additionally, a sensitivity analysis was also performed to show the effects of various important parameters on the total profit of the system.

10.1.6 Managing Raw Material Supply Disruption

The main objective of Chapter 8 was to develop a quantitative approach to recover from supply disruptions in a three-tier supply chain system. A new mathematical and heuristic approach was developed for managing a single supply disruption. Then the mathematical model and the heuristic were extended to develop a dynamic approach for managing multiple supply disruptions, one after another as a series, on a real-time basis. These heuristics were validated by comparing with the results from the PS technique, which showed that the average percentage of deviation was a negligible amount for a good number of randomly generated test problems. In terms of the quality of the solutions, the average deviation of results between the two approaches was only a negligible amount of 0.000283%. A large set of random experiments was performed to analyse the characteristics of the developed models. Finally, a simulation model was also developed to enable solving the supply disruption problem as a real-world process and the simulation model was analysed for 2000 random scenarios, which were generated by using an exponential distribution

10.1.7 A Simulation Model

Chapter 9 developed a disturbance management approach for a manufacturing supply chain system which considered demand fluctuations, and production and raw material supply disruptions under an imperfect production environment. A new mathematical model and an efficient heuristic were developed for each disturbance type to manage disturbances on a real-time basis. These heuristics were validated by comparing the results from another standard solution technique which showed that the average percentage of deviation was 0.00% for a set of randomly generated test problems. Also, a new heuristic was developed to consider multiple disturbances in a period. Random experimentation was performed to analyse the characteristics of the developed models, and finally, a simulation model was developed to make solving a disturbance problem a real-world process, and the results were analysed for a large set of random test problems.

10.1.8 Summary of Researches

In summary, this thesis has contributed to the supply chain disturbance management literature in a number of ways. The models have been developed to manage real-world disturbances faced by many firms across their supply chain. In real-life cases, multiple disturbances can happen, one after another as a series. This is the most complex scenario, as the effect of both of the previous and current disturbances must be taken into consideration while developing the plan. This complex scenario has been considered in the developed models of this thesis. An appropriate solution approach, for managing both a single and multiple disturbances, has been developed for each of the problem considered in this thesis. The development of these new solution approaches is considered as another novel contribution of this thesis. The models can be run immediately after the occurrence of each disturbance and then the customised output will provide decisions without further processing of outputs and interpretations. The solution approaches can be used easily by any decision maker and it eliminates the cost of acquiring costly software to solve such models, while it facilitates firms to achieve a recovery schedule on a real-time basis.

The proposed approaches offer a potentially very useful quantitative method for helping decision-makers arrive at prompt and accurate decisions regarding a real-time recovery plan, whenever a single and/or multiple disturbances takes place in a supply chain system. Using the developed models, an organisation can return to its normal supply, production and delivery schedule as quickly as possible after the occurrence of a disturbance, and thereby, increase its profit margin and enhance its reputation.

10.2 Future Research Directions

Various avenues of further research stem from the work carried out in this thesis. The current research can be extended in a number of different ways. Several potential works have been identified and are described below.

In the presented research, the models were developed for managing disturbances, after the occurrence, on a real-time basis. Extending the models to develop a new approach for combining the predictive mitigation approach with real-time disturbance management techniques would be a worthwhile extension. Additionally, it would be worthwhile to incorporate environmental aspects, such as lowering supply chain costs by reducing travel distances, carbon emission, production costs, product waste, and unplanned activities. Another interesting extension would be to relax the assumption of a single type of item, so as to consider multiple types of items, as well as to analyse the impacts of disturbances on different types of items in a multi-tier supply chain system.

In addition, several aspects could be introduced into the developed models, and some of them are listed as following.

i. Considering safety-stock level and analysing the effect of disturbance on safety stock, and determining the optimum level to minimise the effect of a disruption.
ii. Considering lead time factors and analysing the effect of disturbance on lead time and disruption recovery.
iii. Considering different shipment policies, such as the multiple-lot for lot, including equal-sized shipment policy, geometric shipment policy and mixtures of them.
iv. Implementing the developed models to different types of real-life supply chain systems, such as food and coal supply chain systems.

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Appendix A A Disruption Recovery Model with Demand Uncertainty

This appendix presents an extension of the study performed in chapter 3. The model is extended by considering demand as uncertain variable. In this study, demand is taken as a triangular fuzzy number (TFN) to tackle uncertainty and the main objective is to maximise the graded mean integration value (GMIV) of it.

A.1 Notations

The notations are same as considered in chapter 3. Only change is in demand per year as demand is a fuzzy variable as following.

$\widetilde{D}$ Fuzzy demand per year

A.2 Costs and Revenues Formulation

The formulation for different costs and revenues are similar as presented in chapter 3. The mathematical model is formulated as follows.

Holding cost

$$= \frac{1}{2} H \left[\frac{(q_n)^2}{rP} + 2q_n\left(T_{d,n} + S_t\right) + \frac{2X_{1,n}q_n}{rP} + \sum_{i=1}^{M} \frac{(X_{i,n})^2}{rP} \right] \tag{A.1}$$

Set-up cost

$$= AM \tag{A.2}$$

Production cost

$$= \frac{C_P}{r} \left(\sum_{i=1}^{M} X_{i,n} + q_n \right) \tag{A.3}$$

Rejection cost

$$= C_R\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{i,n}+q_n\right) \tag{A.4}$$

Inspection cost

$$= \frac{C_I C_P}{r}(\sum_{i=1}^{M} X_{i,n}+q_n) \tag{A.5}$$

Cost of interest and depreciation

$$= Ma\,(A_1)^{-b}(r)^c \tag{A.6}$$

Back-order cost

$$\begin{aligned} = B\Bigg[&(X_{1,n}+q_n)\left[T_{d,n}+\frac{q_n}{rP}+\frac{X_{1,n}}{rP}-\frac{X_{l,n-1}}{rP}\right] \\ &+\sum_{i=2}^{M} X_{i,n}.\left[T_{d,n}+(i-1)S_t+\frac{q_n}{rP}+\sum_{j=1}^{i}\frac{X_{j,n}}{rP}\right. \\ &\left.-\sum_{j=1}^{i}\frac{X_{l+j-1,n-1}}{rP}-(i-1)u\right]\Bigg] \end{aligned} \tag{A.7}$$

Lost sales cost

$$= L\left(\sum_{i=1}^{M} X_{l+i-1,n-1}-\sum_{i=1}^{M} X_{i,n}-q_n\right) \tag{A.8}$$

Revenues

$$= m_1 C_P\,\widetilde{D}\left[\sum_{i=1}^{M}\frac{X_{i,n}}{rP}+\frac{q_n}{rP}+MS_t\right] \tag{A.9}$$

Total profit, the objective function, is derived by subtracting all costs from the total revenues. Considering all the equations from (A.1) to (A.9), the objective function is obtained as follows.

$$\text{Max } \widetilde{Z} = \text{Total Revenues- Total Costs} \tag{A.10}$$

A.3 Fuzzy Parameter

In this study, product demand is considered as a triangular fuzzy number (TFN) to tackle uncertainty. A TFN $\widetilde{D}$ is specified by a triplet (d_1, d_2, d_3) and is defined by its continuous membership function $\mu_{\widetilde{D}}(x): x \rightarrow [0,1]$ as follows:

$$\mu_{\widetilde{D}}(x) = \begin{cases} L(x) = \left(\dfrac{x - d_1}{d_2 - d_1}\right) & if\ d_1 \leq x \leq d_2 \\ R(x) = \left(\dfrac{d_3 - x}{d_3 - d_2}\right) & if\ d_2 \leq x \leq d_3 \\ \quad 0 & otherwise \end{cases} \tag{A.11}$$

$L(x)$ and $R(x)$ indicate the left and right branch of the TFN $\widetilde{D}$ respectively. An α-cut of $\widetilde{D}$ can be expressed by the following interval (Lee and Yao, 1998):

$D(\alpha) = [d_1 + (d_2 - d_1)\alpha,\ d_3 - (d_3 - d_2)\alpha],\ \ \alpha \in [0,1]$

The graded mean integration value (GMIV) of a LR-fuzzy number is introduced by Chen and Hsieh (1999). The graded mean integration representation method is based on the integral value of the graded mean α-level of the LR-fuzzy number for defuzzifing LR-fuzzy numbers. By considering $\widetilde{D}$ is a LR-fuzzy number and according to Chen and Hsieh (1999), the GMIV of $\widetilde{D}$ is defined as:

$$G(\widetilde{D}) = \frac{\int_0^1 \left(\frac{\alpha}{2}\right)\{L^{-1}(\alpha) + R^{-1}(\alpha)\}\,d\alpha}{\int_0^1 \alpha d\alpha} = \int_0^1 \alpha\{L^{-1}(\alpha) + R^{-1}(\alpha)\}\,d\alpha \tag{A.12}$$

A.4 Disruption Recovery with Fuzzy Demand

In this section, fuzziness of demand is incorporated to the final mathematical model. The GMIV of the expected total profit function is evaluated. Relevant constraints are also developed with the GMIV of expected fuzzy demand. After simplifying the equation (A.10), the following equation of the total profit is obtained:

$$\widetilde{Z} = C\,\widetilde{D} + Y \tag{A.13}$$

Here,

$$C = m_1 C_P \left[\sum_{i=1}^{M} \frac{X_{i,n}}{rP} + \frac{q_n}{rP} + MS_t \right]$$

$$\begin{aligned} Y = -\frac{1}{2} H \left[\frac{(q_n)^2}{rP} + 2q_n (T_{d,n} + S_t) + \frac{2X_{1,n} q_n}{rP} + \sum_{i=1}^{M} \frac{(X_{i,n})^2}{rP} \right] - AM \\ - \left[\frac{C_P}{r} + C_R \left(\frac{1}{r} - 1 \right) + \frac{C_I C_P}{r} \right] \left(\sum_{i=1}^{M} X_{i,n} + q_n \right) - Ma\,(A_1)^{-b} (r)^c \\ - B \left[(X_{1,n} + q_n) \left[T_{d,n} + \frac{q_n}{rP} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP} \right] \right. \\ + \sum_{i=2}^{M} X_{i,n} \cdot \left[T_{d,n} + (i-1) S_t + \frac{q_n}{rP} + \sum_{j=1}^{i} \frac{X_{j,n}}{rP} - \sum_{j=1}^{i} \frac{X_{l+j-1,n-1}}{rP} \right. \\ \left. \left. - (i-1)u \right] \right] - L \left(\sum_{i=1}^{M} X_{l+i-1,n-1} - \sum_{i=1}^{M} X_{i,n} - q_n \right) \end{aligned}$$

Now, considering the fuzzy random demand $\widetilde{\mathrm{D}}$ with the given set of data $(\tilde{d}_1, \tilde{p}_1), (\tilde{d}_2, \tilde{p}_2), (\tilde{d}_3, \tilde{p}_3), \ldots\ldots, (\tilde{d}_v, \tilde{p}_v)$, the profit ($\tilde{Z}$) is also a fuzzy random variable and its expectation is a unique fuzzy number (Bag et al., 2009) which is,

$$E\widetilde{Z} = C \sum_{k=1}^{v} \tilde{d}_k \tilde{p}_k + Y$$

In this study, demand data are considered as a triangular fuzzy number (TFN). Demand TFN and associated probabilities are taken as a triplet $\left(\underline{d_k}, d_k, \overline{d_k}\right)$ and $\left(\underline{p_k}, p_k, \overline{p_k}\right)$ respectively. Here, *k= 1, 2, 3...., v*. Then the fuzzy expected profit function will also be a TFN, $E\widetilde{Z} = \left(\underline{\mathrm{EZ}}, \mathrm{EZ}, \overline{\mathrm{EZ}}\right)$ which is determined as follows:

$$EZ = E[Z(\alpha = 1)] = C \sum_{k=1}^{v} d_k\, p_k + Y$$

$$\underline{EZ} = E[Z_L(\alpha = 0)\,] = C\sum_{k=1}^{v} \underline{d_k}\,\underline{p_k} + Y$$

$$\overline{EZ} = E[Z_R(\alpha = 0)] = C\sum_{k=1}^{v} \overline{d_k}\,\overline{p_k} + Y$$

Here the α-level set of the fuzzy number $E\widetilde{Z}$ are considered as $EZ(\alpha) = E[Z(\alpha)] = \left[E\big(Z_L(\alpha)\big), E\big(Z_R(\alpha)\big)\right]$; $0 \leq \alpha \leq 1$ and different α -cut intervals for the fuzzy number $E\widetilde{Z}$ are obtained for different α between 0 and 1.Taking, α-cut on both sides of equation of $E\widetilde{Z}$.

$$E\tilde{Z}_\alpha = C\sum_{k=1}^{v} \tilde{d}_{k\alpha}\tilde{p}_{k\alpha} + Y$$

The arithmetic interval of fuzzy demand and associated probabilities using an α-cut is determined as follows.

$$\tilde{d}_{k\alpha} = \left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right), \qquad \overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]$$

$$\tilde{p}_{k\alpha} = \left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right), \qquad \overline{p_k} - \alpha(\overline{p_k} - p_k)\right]$$

By using these arithmetic intervals, $E\tilde{Z}_\alpha$ is evaluated as:

$$E\tilde{Z}_\alpha = \left[\left[C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y\right],\right.$$
$$\left.\left[C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\,[\overline{p_k} - \alpha(\overline{p_k} - p_k)]\; + Y\right]\right]$$

From the representation of graded mean integration methods based on the integral value of the graded mean α-level of the LR-fuzzy number of the total profit, $\mathrm{L}^{-1}(\alpha)$ and $\mathrm{R}^{-1}(\alpha)$ are obtained as follows.

$$\mathrm{L}^{-1}(\alpha) = C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y$$

$$\mathrm{R}^{-1}(\alpha) = C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\left[\overline{p_k} - \alpha\left(\overline{p_k} - p_k\right)\right] + Y$$

The unique fuzzy number $\mathrm{G}(E\widetilde{Z})$ is determined by substituting the value of $\mathrm{L}^{-1}(\alpha)$ and $\mathrm{R}^{-1}(\alpha)$ to the equation (A.12),

$$G(E\widetilde{Z}) = \int_0^1 \left[\alpha\left[C\sum_{k=1}^{v}\left[\underline{d_k} + \alpha\left(d_k - \underline{d_k}\right)\right]\left[\underline{p_k} + \alpha\left(p_k - \underline{p_k}\right)\right] + Y\right]\right] d\alpha$$
$$+ \int_0^1 \left[\alpha\left[C\sum_{k=1}^{v}\left[\overline{d_k} - \alpha\left(\overline{d_k} - d_k\right)\right]\left[\overline{p_k} - \alpha\left(\overline{p_k} - p_k\right)\right] + Y\right]\right] d\alpha$$

After integrating and simplifying the above equation of $G(E\widetilde{Z})$, the GMIV of the total profit function, which is to be maximised, and obtained as:

$$\text{Max } \mathrm{G}(E\widetilde{Z}) = C(Z_1 + Z_2) + Y \qquad \text{(A.14)}$$

Here,

$$Z_1 = \sum_{k=1}^{v}\left\{\frac{1}{2}\underline{d_k}\,\underline{p_k} + \frac{1}{3}\underline{d_k}\left(p_k - \underline{p_k}\right) + \frac{1}{3}\underline{p_k}\left(d_k - \underline{d_k}\right) + \frac{1}{4}\left(d_k - \underline{d_k}\right)\left(p_k - \underline{p_k}\right)\right\}$$

$$Z_2 = \sum_{k=1}^{v}\left\{\frac{1}{2}\overline{d_k}\,\overline{p_k} - \frac{1}{3}\overline{d_k}\left(\overline{p_k} - p_k\right) - \frac{1}{3}\overline{p_k}\left(\overline{d_k} - d_k\right) + \frac{1}{4}\left(\overline{d_k} - d_k\right)\left(\overline{p_k} - p_k\right)\right\}$$

The GMIV of the expected fuzzy demand, $\mathrm{G}(E\widetilde{D}) = Z_1 + Z_2$

Subject to the following constraints presented in (A.15) – (A.24):

$$X_{i,0} = Q \tag{A.15}$$

$$X_{1,n} + q_n \leq X_{l,n-1} \tag{A.16}$$

$$X_{i,n} \leq X_{l+i-1,n-1} \text{ ; } i = 2,3,4, \ldots\ldots, M \tag{A.17}$$

$$rP \geq G\big(E\widetilde{D}\big) \tag{A.18}$$

$$\sum_{i=1}^{M} X_{i,n} + q_n \leq rP\left(\sum_{i=1}^{M} \frac{X_{l+i-1,n-1}}{G(E\widetilde{D})} - MS_t - T_{d,n}\right) \tag{A.19}$$

$$\sum_{i=1}^{M} X_{i,n} + q_n \geq \left(\frac{\sum_{i=1}^{M} X_{i,n} + q_n}{rP} + MS_t\right) G(E\widetilde{D}) - \left(\sum_{i=1}^{M} X_{l+i-1,n-1} - \sum_{i=1}^{M} X_{i,n} - q_n\right) \tag{A.20}$$

$$\frac{X_{1,n} + q_n}{G(E\widetilde{D})} - \frac{X_{2,n}}{rP} - S_t \geq 0 \tag{A.21}$$

$$\frac{X_{i,n}}{G(E\widetilde{D})} - \frac{X_{i+1,n}}{rP} - S_t \geq 0; \; i \; = \; 2, 3, \ldots., M \tag{A.22}$$

$$T_{d,n} + \frac{q_n}{rP} + \frac{X_{1,n}}{rP} - \frac{X_{l,n-1}}{rP} \geq 0 \tag{A.23}$$

$$T_{d,n} + (i-1)S_t + \frac{q_n}{rP} + \sum_{j=1}^{i} \frac{X_{j,n}}{rP} - \sum_{j=1}^{i} \frac{X_{l+j-1,n-1}}{rP} - (i-1)u \geq 0; \quad i = 2,3,4, \ldots\ldots, M \tag{A.24}$$

A.5 Solution Approach

A genetic algorithm based heuristic is proposed to solve the model. Genetic algorithm is very popular technique to solve complex non-linear constrained optimisation problem. GAs are general purpose optimisation algorithms which apply the rules of natural genetics to explore a given search space (Homaifar et al., 1994). The heuristic is designed to make a recovery plan from a single or a series of production disruptions. The proposed heuristic revises the production lot size of each cycle as long as disruptions take place in the system.

For a series of disruptions, the heuristic revises the lot size of each cycle by considering the effect of all previous dependent disruptions. The proposed genetic algorithm based heuristic is presented in the Figure A.1. The above mentioned heuristic is coded in MATLAB R2012a with the help of its optimisation toolbox.

In the proposed heuristic, following GA parameters are used to solve the model. Population size: 100; Population type: Double vector; Crossover fraction: 0.8; Maximum number of generations: 3000; Function tolerance: 1e-6; Nonlinear constraint tolerance: 1e-6, and other parameters are set as default of the optimisation toolbox.

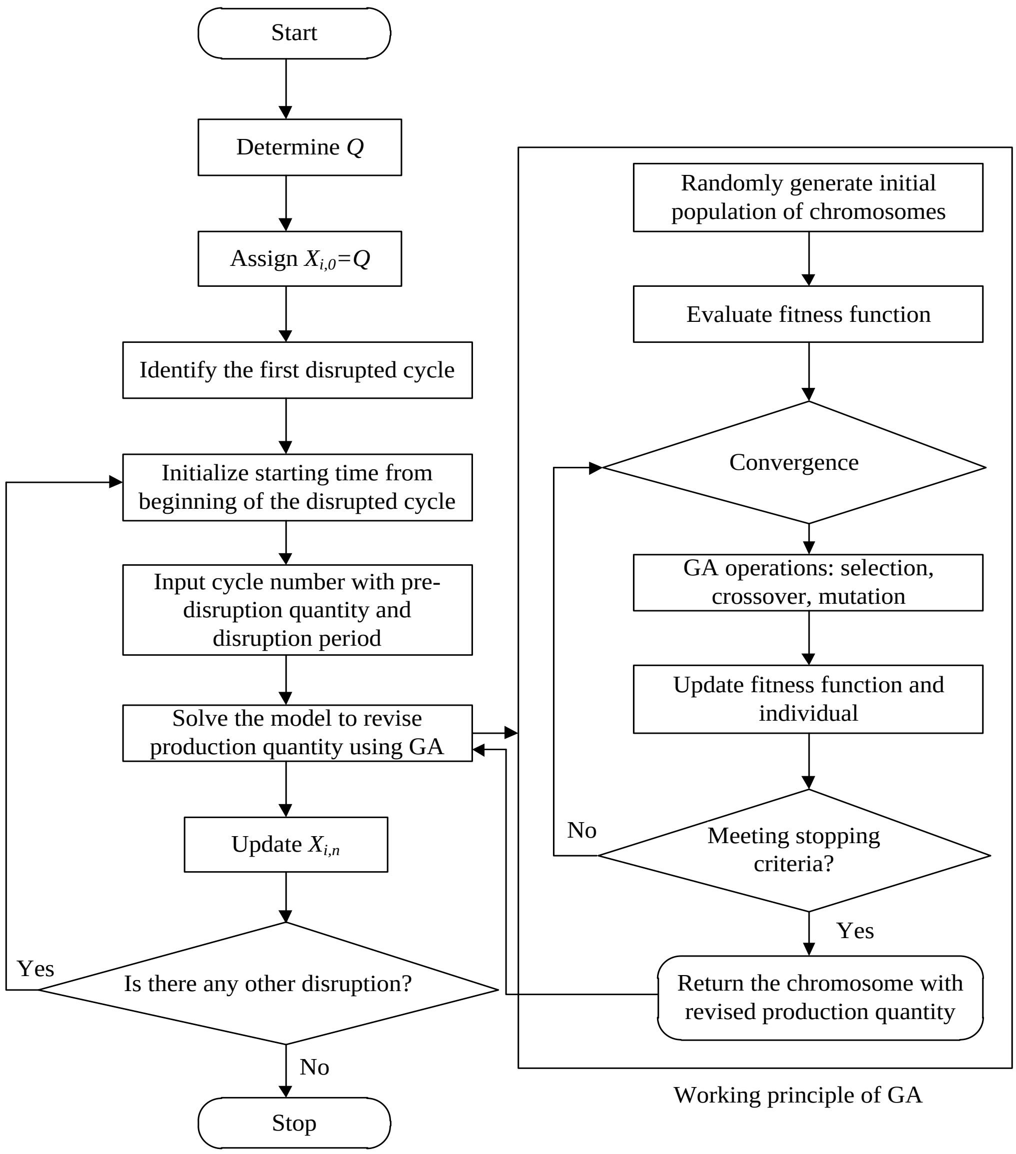

Figure A.1: Flowchart of proposed GA based heuristic

A.6 Result Analysis

Results have been analysed for both single and multiple disruptions on a real-time basis. For single disruption, there is only one random disruption in the system and there is no more disruption within the recovery period. For multiple disruptions, there is a series of disruptions, one after another, and results have been analysed on a real-time basis.

A.6.1 Results Analysis for Single Disruption

Following data are considered to analyse the results for a single disruption.

$S_t = 0.000057, u = 0.000077,\ A = 60,\ H = 1.4,\ r = 0.92,\ P = 550000,\ M = 5,$ $B = 20,\ L = 25,\ T_{d1} = 0.01,\ q_1 = 500, C_P = 40,\ C_R = 15,\ C_I = 0.01, a = 1000,\ b = 0.5, c = 0.75, m_1 = 2.5$, and demand data are considered as TFN which is shown in the Table A.1.

Table A.1: Demand data as TFN and associated probabilities

Demand rate	Probability
(300000, 320000, 340000)	(0.050, 0.055, 0.060)
(350000, 370000, 390000)	(0.144, 0.150, 0.156)
(400000, 420000, 440000)	(0.293, 0.300, 0.307)
(450000, 470000, 490000)	(0.194, 0.202, 0.210)
(500000, 520000, 540000)	(0.104, 0.110, 0.117)
(550000, 570000, 590000)	(0.094, 0.100, 0.106)
(600000, 620000, 640000)	(0.088, 0.093, 0.098)

The problem is solved using the proposed GA based heuristic. The results are obtained from 30 different runs. The best recovery plan after single disruption is shown in Table A.2. The production system returns to original schedule from the sixth cycle after the disruption with $X_{6,1} = 6586,\ X_{7,1} = 6586$ and so on. The maximum total profit in the recovery period is obtained as 1381112.5.

Table A.2: Best results obtained for the single disruption

Revised production quantity					Total profit
$X_{1,1}$	$X_{2,1}$	$X_{3,1}$	$X_{4,1}$	$X_{5,1}$	
5305	5638	6066	6469	6563	1381112.5

A.6.2 Results Analysis for a Series of Disruptions

In this case, a series of disruptions is considered, which is shown in Table A.3. In this series of disruptions, seven disruptions are considered and each one occurs within the recovery period of the previous disruption. Other data remain same as in section A.6.1.

Table A.3: Data for the series of disruptions

Disruption number (n)	Disrupted cycle number from previous disruption	Pre-disruption quantity	Disruption duration
1	1	1000	0.0045
2	2	650	0.0092
3	3	500	0.0025
4	5	1500	0.0065
5	2	0	0.0110
6	4	800	0.0098
7	3	0	0.0078
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The production system with multiple disruptions is also solved using the GA based heuristic on a real-time basis. The results are obtained from 30 different runs. Production quantity in each cycle is revised after each disruption considering the effect of entire dependent disruptions to maximise the total profit in the recovery period. The best recovery plan obtained from the heuristic for the series of disruptions is shown in Table A.4. The production system returns to original schedule from the sixth cycle after each disruption with $X_{6,n} = 6586$, $X_{7,n} = 6586$ and so on.

Table A.4: Best results obtained for the series of disruptions

Disruption number (n)	Revised production quantity					Total profit
	$X_{1,n}$	$X_{2,n}$	$X_{3,n}$	$X_{4,n}$	$X_{5,n}$	
1	5585	6561	6545	6567	6563	1545915.7
2	5272	5671	6111	6553	6572	1404616.5
3	5611	6548	6570	6541	6550	1524257.8
4	4804	6539	6433	6476	6522	1506722.6
5	5271	5647	5994	6336	6381	1324912.7
6	4941	5668	5936	6271	6532	1364154.2
7	5657	5914	6049	6486	6443	1407321.9
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A.7 Summary

The objective of this research was to incorporate demand uncertainty in developing a disruption recovery model in a production-inventory system, which was an extension of the study presented in chapter 3. A single or a series of disruptions on a real-time basis was considered to make the model applicable in practical problems. A genetic algorithm based heuristic was proposed to solve the model with single or multiple disruptions on a real-time basis. This model can be applied in an imperfect production process where the process countenances a single or multiple production disruptions and product demand is uncertain.

Appendix B Formulation of Updated and Predictive Mitigation Plans

B.1 Mathematical Formulation

F	Number of periods in finite planning horizon
P_{if}	Production quantity of plant i at period f
CP_{if}	Maximum production capacity of plant i at period f
CD_{jf}	Maximum handling capacity of DC j at period f
X_{ijf}	Transportation quantity from plant i to DC j at period f
Y_{jkf}	Transportation quantity from DC j to retailer k at period f
D_{kf}	Demand of retailer k at period f
p_{if}	Production cost per unit at plant i at period f
H_{1if}	Holding cost per unit per period at plant i at period f
H_{2jf}	Handling cost per unit at DC j at period f
H_{3kf}	Holding cost per unit per period at retailer k at period f
T_{1ijf}	Transportation cost per unit from plant i to DC j at period f
T_{2jkf}	Transportation cost per unit from DC j to retailer k at period f
OC_{jf}	Operating cost of DC j at period f

Costs at plant

$$\text{Production cost } = \textstyle\sum_{f=1}^{F}\sum_{i=1}^{I} p_{if}P_{if} \qquad \text{(B.1)}$$

$$\text{Average holding cost} = \textstyle\sum_{f=1}^{F}\sum_{i=1}^{I}\frac{1}{2}H_{1if}P_{if} \qquad \text{(B.2)}$$

$$\text{Transportation cost} = \textstyle\sum_{f=1}^{F}\sum_{j=1}^{J}\sum_{i=1}^{I} T_{1ijf}X_{ijf} \qquad \text{(B.3)}$$

Costs at DCs

$$\text{Operating cost} = \textstyle\sum_{f=1}^{F}\sum_{j=1}^{J} OC_{jf} \qquad \text{(B.4)}$$

$$\text{Handling cost} = \textstyle\sum_{f=1}^{F}\sum_{j=1}^{J}\sum_{i=1}^{I} H_{2jf}\,X_{ijf} \qquad \text{(B.5)}$$

Transportation cost $= \sum_{f=1}^{F} \sum_{k=1}^{K} \sum_{j=1}^{J} T_{2jkf} Y_{jkf}$ (B.6)

Cost at retailer

Average holding cost $= \sum_{f=1}^{F} \sum_{k=1}^{K} \frac{1}{2} H_{3kf} D_{kf}$ (B.7)

Final Mathematical Model

The total supply chain cost (*TC*), which is the objective function, is derived using equations (B.1) – (B.7), and equals the total plant cost + total DC cost + total retailer cost, with P_{if}, X_{ijf} and Y_{jkf} decision variables, subject to the following constraints presented in (B.8) – (B.13).

$P_{if} \leq CP_{if}; \forall i, f$ (B.8)

$P_{if} = \sum_{j=1}^{J} X_{ijf}; \forall i, f$ (B.9)

$\sum_{i=1}^{I} X_{ijf} = \sum_{k=1}^{K} Y_{jkf}\ ; \forall j, f$ (B.10)

$\sum_{i=1}^{I} X_{ijf} \leq CD_{jf}; \forall j, f$ (B.11)

$\sum_{f=1}^{F} \sum_{i=1}^{I} P_{if} = \sum_{f=1}^{F} \sum_{k=1}^{K} D_{kf}$ (B.12)

P_{if}, X_{ijf} and $Y_{jkf} \geq 0; \forall i, j, k, f$ (B.13)

Appendix C Formulation for Managing Production Disruption

In this appendix, the mathematical model for determining the revised plan after a production disruption is formulated. The costs of production, delivery, holding, raw materials, back orders and lost sales, and the revenue from the selling price are calculated. Finally, a mathematical model is developed in which the total profit to be maximised is subject to capacity, demand, delivery and inventory constraints.

Total production cost $= \frac{C_p}{r}\sum_{i=1}^{n} X_i$ (C.1)

Total rejection cost $= C_R\left(\frac{1}{r}-1\right)\sum_{i=1}^{n} X_i$ (C.2)

Total inspection cost $= \frac{C_I C_p}{r}\sum_{i=1}^{n} X_i$ (C.3)

Cost of interest and depreciation $= na\ A^{-b}r^{c}$ (C.4)

Total raw material cost $= \sum_{i=1}^{n} C_r Z_i = \frac{NC_r}{r}\sum_{i=1}^{n} X_i$ (C.5)

Raw material holding cost $= \frac{1}{2r}H_1 N\sum_{i=1}^{n} X_i$ (C.6)

Total delivery cost$= \sum_{i=1}^{n} C_d\, Y_i$ (C.7)

Total ending inventory holding cost $= H_2\sum_{i=1}^{n} e_i$ (C.8)

Back orders cost $= B\sum_{i=2}^{n}(i-1)(X_i - AP_i)$ (C.9)

Lost sales cost $= L(\sum_{i=1}^{n} AP_i - \sum_{i=1}^{n} X_i)$ (C.10)

Total revenue $= S\sum_{i=1}^{n} X_i$ (C.11)

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised and subject to constraints (C.12) – (C.20).

$e_i \geq E_i; \forall i$ [Constraint of ending inventory] (C.12)

$b_i \geq B_i; \forall i$[Constraint of beginning inventory] (C.13)

$X_1 \leq r(P - T_{dp} * P)$ [Limitation of production quantity in first period] (C.14)

$X_i \leq rP; \forall i \neq 1$ [Limitation of production quantity in each period] (C.15)

$X_i \geq AP_i; \forall i \neq 1$ [Constraint for production in revised plan] (C.16)

$\sum_{i=1}^{n} X_i \leq \sum_{i=1}^{n} D_i + b_{n+1} - b_1$ [Limitation of total production quantity] (C.17)

$\sum_{i=1}^{n} AP_i - \sum_{i=1}^{n} X_i \geq 0$ [Constraint of lost sales quantity] (C.18)

$\sum_{i=1}^{n} Y_i \leq \sum_{i=1}^{n} D_i$ [Limitation of total delivery] (C.19)

$X_i, Y_i, Z_i \geq 0; \forall i$ [Non-negativity constraint] (C.20)

Appendix D Formulation for Managing Raw Material Supply Disruption

In this appendix, the mathematical model is formulated for determining the revised plan after a supply disruption in which the total profit is maximised subject to the following capacity, demand, delivery and inventory constraints.

Total production cost $= \frac{C_p}{r}\sum_{i=1}^{n} X_i$ (D.1)

Total rejection cost $= C_R\left(\frac{1}{r}-1\right)\sum_{i=1}^{n} X_i$ (D.2)

Total inspection cost $= \frac{C_I C_p}{r}\sum_{i=1}^{n} X_i$ (D.3)

Cost of interest and depreciation $= na\ A^{-b} r^{c}$ (D.4)

Total raw material cost $= \sum_{i=1}^{n} C_r Z_i = \frac{NC_r}{r}\sum_{i=1}^{n} X_i$ (D.5)

Raw material holding cost $= \frac{1}{2r} H_1 N \sum_{i=1}^{n} X_i$ (D.6)

Total delivery cost$= \sum_{i=1}^{n} C_d\, Y_i$ (D.7)

Total ending inventory holding cost $= H_2 \sum_{i=1}^{n} e_i$ (D.8)

Back orders cost $= B\sum_{i=2}^{n}(i-1)(X_i - AP_i)$ (D.9)

Lost sales cost $= L(\sum_{i=1}^{n} AP_i - \sum_{i=1}^{n} X_i)$ (D.10)

Total revenue $= S\sum_{i=1}^{n} X_i$ (D.11)

Final mathematical model

The objective function, total profit = total revenue – total costs, which is to be maximised subject to constraints (D.12) – (D.20).

$e_i \geq E_i; \forall i$ [Constraint of ending inventory] (D.12)

$b_i \geq B_i; \forall i$[Constraint of beginning inventory] (D.13)

$X_1 \leq r(P - T_{ds} * P)$ [Limitation of production quantity in first period] (D.14)

$X_i \leq rP; \forall i \neq 1$ [Limitation of production quantity in each period] (D.15)

$X_i \geq AP_i; \forall i \neq 1$[Constraint for production in revised plan] (D.16)

$\sum_{i=1}^{n} X_i \leq \sum_{i=1}^{n} D_i + b_{n+1} - b_1$ [Limitation of total production quantity] (D.17)

$\sum_{i=1}^{n} AP_i - \sum_{i=1}^{n} X_i \geq 0$ [Constraint of lost sales quantity] (D.18)

$\sum_{i=1}^{n} Y_i \leq \sum_{i=1}^{n} D_i$ [Limitation of total delivery] (D.19)

$X_i, Y_i, Z_i \geq 0; \forall i$ [Non-negativity constraint] (D.20)

Appendix E Heuristic 2 for Managing Production Disruption

Step 1: Input data for ideal plan.

Step 2: Determine and record ideal plan.

Step 3: Determine spare capacity (SC_i) of each period.

Step 4: Input production disruption start time (t_s) and duration (T_{dp}) and determine loss of production = $T_{dp} * rP$.

Step 5: If 0≤ $T_{dp} * rP \le SC_1$, Then

$X_i = AP_i$; $i = 1, 2, 3\ldots n$

Step 6: For k =2 to n

For $\sum_{j=1}^{k-1} SC_j < T_{dp} * rP \le \sum_{j=1}^{k} SC_j$

If $B \le \frac{L}{k-1}$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_i = AP_i + SC_i$; $i = 2, 3\ldots k-1$

$X_k = AP_k + T_{dp} * rP - \sum_{j=1}^{k-1} SC_j$

$X_i = AP_i$; $i = k$+1, k+2... n

If $\frac{L}{k-1} < B \le \frac{L}{k-2}$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_i = AP_i + SC_i$; $i = 2, 3\ldots k-1$

$X_i = AP_i$; $i = k$, k+1, k+2... n

………..

If $\frac{L}{2} < B \le L$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_2 = AP_2 + SC_2$

$X_i = AP_i$; $i = 3, 4\ldots n$

If $B > L$, Then

$X_1 = AP_1 + SC_1 - T_d * rP$

$X_i = AP_i$; i =2, 3…n

Step 7: For $T_d * rP > \sum_{j=1}^{n} SC_j$

If $B \leq \frac{L}{n-1}$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_i = AP_i + SC_i$; i =2, 3…n

If $\frac{L}{n-1} < B \leq \frac{L}{n-2}$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_i = AP_i + SC_i$; $i =$ 2, 3…$n - 1$

$X_n = AP_n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_2 = AP_2 + SC_2$

$X_i = AP_i$; $i =$ 3, 4…n

If $B > L$, Then

$X_1 = AP_1 + SC_1 - T_{dp} * rP$

$X_i = AP_i$; i =2, 3…n

Step 8: Determine raw material required and final product delivery quantity.

$Y_i = B_i + X_i - E_i$; i =1, 2, 3…n

$Z_1 = N * \frac{AP_1}{r}$

$Z_2 = \frac{1}{r}[N * X_2 - N * (AP_1 - X_1)]$

$Z_i = N * \frac{X_i}{r}$; $i =$ 3, 4…n

Step 9: Determine different costs, total profit and quantities of back orders and lost sales.

Step 10: Stop.

Appendix F Heuristic 3 for Managing Raw Material Supply Disruption

Step 1: Input data for ideal plan.

Step 2: Determine and record ideal plan.

Step 3: Determine spare capacity (SC_i) of each period.

Step 4: Input supply disruption duration (T_{ds}) and determine loss of production (LP).

Step 5: If $0 \leq LP \leq SC_1$, Then

$X_i = AP_i$; $i = 1, 2, 3 \ldots n$

Step 6: For k =2 to n

For $\sum_{j=1}^{k-1} SC_j < LP \leq \sum_{j=1}^{k} SC_j$

If $B \leq \frac{L}{k-1}$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i + SC_i$; $i = 2, 3 \ldots k - 1$

$X_k = AP_k + LP - \sum_{j=1}^{k-1} SC_j$

$X_i = AP_i$; $i = k+1, k+2 \ldots n$

If $\frac{L}{k-1} < B \leq \frac{L}{k-2}$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i + SC_i$; $i = 2, 3 \ldots k - 1$

$X_i = AP_i$; $i = k, k+1, k+2 \ldots n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_2 = AP_2 + SC_2$

$X_i = AP_i$; $i = 3, 4 \ldots n$

If $B > L$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i$; i =2, $3 \ldots n$

Step 7: For $LP > \sum_{j=1}^{n} SC_j$

If $B \leq \frac{L}{n-1}$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i + SC_i$; i =2, 3…n

If $\frac{L}{n-1} < B \leq \frac{L}{n-2}$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i + SC_i$; $i =$ 2, 3…$n - 1$

$X_n = AP_n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_2 = AP_2 + SC_2$

$X_i = AP_i$; $i = 3, 4…n$

If $B > L$, Then

$X_1 = AP_1 + SC_1 - LP$

$X_i = AP_i$; i =2, 3…n

Step 8: Determine raw material required and final product delivery quantity.

$Y_i = B_i + X_i - E_i$; i =1, 2, 3…n

$Z_1 = N * \frac{AP_1}{r}$

$Z_2 = \frac{1}{r}[N * X_2 - N * (AP_1 - X_1)]$

$Z_i = N * \frac{X_i}{r}$; $i = 3, 4…n$

Step 9: Determine different costs, total profit and quantities of back orders and lost sales.

Step 10: Stop.

Appendix G Heuristic 4 for Managing Multiple Disturbances

Step 1: Input data for ideal plan.

Step 2: Determine and record ideal plan.

Step 3: Determine spare capacity (SC_i) of each period.

Step 4: Input demand fluctuation, supply disruption and/or production disruption scenario.

Step 5: Determine unfulfilled demand, $D_u = \delta + \left(T_{ds} + T_{dp}\right) * rP$

Step 6: For $D_u > 0$

6.1 For $0 \leq D_u \leq SC_1$

If $B \leq L$, Then

$X_1 = AP_1 + D_u - \left(T_{ds} + T_{dp}\right) * rP$

$X_i = AP_i$; $i = 2, 3 \ldots n$

If $B > L$, Then

$X_i = AP_i$; $i = 1, 2, 3 \ldots n$

6.2 For $k = 2$ to n

For $\sum_{j=1}^{k-1} SC_j < D_u \leq \sum_{j=1}^{k} SC_j$

If $B \leq \frac{L}{k}$, Then

$X_1 = AP_1 + SC_1 - \left(T_{ds} + T_{dp}\right) * rP$

$X_i = AP_i + SC_i$; $i = 2, 3 \ldots k - 1$

$X_k = AP_k + D_u - \sum_{j=1}^{k-1} SC_j$

$X_i = AP_i$; $i = k+1, k+2 \ldots n$

If $\frac{L}{k} < B \leq \frac{L}{k-1}$, Then

$X_1 = AP_1 + SC_1 - \left(T_{ds} + T_{dp}\right) * rP$

$X_i = AP_i + SC_i$; $i = 2, 3 \ldots k - 1$

$X_i = AP_i$; $i = k, k+1, k+2 \ldots n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1 - (T_{ds} + T_{dp}) * rP$

$X_i = AP_i$; $i = 2, 3\dots n$

If $B > L$, Then

$X_i = AP_i$; $i =1, 2, 3\dots n$

6.3 For $D_u > \sum_{j=1}^{n} SC_j$

If $B \leq \frac{L}{n}$, Then

$X_1 = AP_1 + SC_1 - (T_{ds} + T_{dp}) * rP$

$X_i = AP_i + SC_i$; $i = 2, 3\dots n$

If $\frac{L}{n} < B \leq \frac{L}{n-1}$, Then

$X_1 = AP_1 + SC_1 - (T_{ds} + T_{dp}) * rP$

$X_i = AP_i + SC_i$; $i = 2, 3\dots n - 1$

$X_n = AP_n$

………..

If $\frac{L}{2} < B \leq L$, Then

$X_1 = AP_1 + SC_1$

$X_i = AP_i$; $i = 2, 3\dots n$

If $B > L$, Then

$X_i = AP_i$; $i =1, 2, 3\dots n$

Step 7: For $D_u < 0$

$X_1 = AP_1 - |D_u|$

$X_i = AP_i$; $i = 2, 3\dots n$

Step 8: Determine raw material required and final product delivery quantity.

$Y_i = B_i + X_i - E_i$; $i =1, 2, 3\dots n$

$Z_i = N * \frac{X_i}{r}$; $i =1, 2, 3\dots n$

Step 9: Determine different costs, total profit and quantities of back orders and lost sales.

Step 10: Stop.