THE UNIVERSITY OF NEW SOUTH WALES

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SCHOOL OF ECONOMICS

EXTERNAL ECONOMIES

AND

ECONOMIC DEVELOPMENT

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FOREWORD

I am greatly indebted to Professor M.C. Kemp and Dr. K. Rivett for their guidance and general assistance throughout. Professor Kemp's suggestions as to the mathematical formulation of certain problems and the handling of some of the more difficult manipulations have been particularly valuable. Dr. Rivett's devoted interest in the topic, his constructive comments and criticisms at various stages have been essential. Without their kind help, this thesis could not be written. However, I alone am responsible for the mistakes that remain.

P.N.V.T.

CONTENTS

EXTERNAL ECONOMIES AND ECONOMIC DEVELOPMENT.

CHAPTER I

Introduction

External Economies (henceforth abbreviated to E.E.) are not new in economic theory. A.Marshall first defined them as "those dependent on the general development of the industry". These economies take the form of "improved organisation, improved methods or machinery which are accessible to the whole industry", "development of mechanical appliances of division of labour and of the means of transport and improved organisation of all kinds", "advances made by subsidiary industries", "growth of correlated branches of industries which mutually assist one another, perhaps being concentrated in the same localities", "the growth of knowledge and the progress of the arts", "newspapers and trade and technical publications".(1)

Marshall used the concept of E.E. to explain the falling industry supply curve in terms consistent with perfectly competitive equilibrium, in which each firm experiences rising marginal cost. Marshall's explanation salvages competition, but implies the

1. A.Marshall [13] pp. 266, 318, 615, 808, 615, 317, 266.

inefficiency of the price mechanism: taxes and subsidies are required to harmonise private with social products.

With the cooling down of the cost controversy kindled by this conclusion in the 1920's, E.E. faded into the background; but recently, with the resurgence of interest in economic development, it has returned to the stage in a new and versatile role.

Modern concepts of E.E. are still Marshallian: J.Viner defines them as "those which accrue to particular concerns as the result of the expansion of output by their industry as a whole, and which are independent of their own individual outputs".(2)

J.Robinson defines External Economics in marginal terms: "When a new firm enters the industry, it may enable all the firms to produce more cheaply, so that, while each produces at its minimum average cost, the cost of the minimum is reduced".(3) External Economies are thus attributed to new entry, which, by adding output to that of the industry, causes the marginal costs of component firms to shift downward.

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2. J.Vines [29] p.217.
3. J.Robinson [20] p.340

M.Fleming defines External Economies as any "increment in direct net product, in net factor supply, in tax quantum, in government services, in net psychic income, in terms of trade ... which is brought about by the actions of a particular firm, other than a change in its own direct net product".(4)

In this thesis, I shall retain the Marshallian definition of External Economies as "those economics (in the forms of greater output, lower costs, better profit) which depend on the development of the industry (or industries), and the exploitation of which no one firm alone could monopolize".

Various types of E.E. will be classified (ch.II) according to their "modus operandi", and put to work in several branches of economic theory:

E.E. and the Marginal Theory of Distribution (Chapter III)

E.E. and the allocation of resources (Chapter IV)

E.E. and Investment Techniques (Chapter V)

E.E. and Balanced Growth (Chapter VI).

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4. M.Fleming (6) p.255-256

Chapter II

CLASSIFICATION OF EXTERNAL ECONOMIES

"No man is a law unto himself." Similarly, the economic activities (consumption and production) of a particular decision-making unit (household, firm, government) may change the economic environment in the light of which a second unit takes decisions. If the change is "beneficial" to the second unit, the activity of the first unit is said to generate an economy; if it is "detrimental", the activity is said to generate a diseconomy. Whether the change is considered beneficial or detrimental depends on the values of the beholder, the economist. And he will normally choose the values of the second unit.

EXTERNAL ECONOMIES OF CONSUMPTION.

The interdependence of the Consumption and consumption plans of individuals and groups has long been known. The consumption of a particular commodity by an individual or a group may increase, decrease, or leave unchanged, the satisfaction of some others. A noisy party, at which everyone enjoys himself, could be a nuisance to occupants of the flat next door. Well kept gardens, beautiful lawns, modern buildings, on the other hand, beside the enjoyment and comfort they provide their owners, are sometimes sources of satisfaction, available free of charge, to passers-by and tourists. Note, however, that the same activities may generate E.E. to some people and external diseconomies to others: a beautiful residence confers E.E. of consumption on passers-by, but may cause envy and disquietude to a neighbour who wants to "keep up with the Joneses", and who feels uncomfortably inferior. The high income and consumption of some people may give a person pain or pleasure, and so may his knowledge of people's misery. All those cases are referred to as E.E. of Consumption. In what follows, however, I shall concentrate on the production side only.

EXTERNAL ECONOMIES OF PRODUCTION.

Production activities also generate E.E. These may be the effects of producers on consumers (E.g. Pigou's example of factory smoke and laundry bills) and vice versa, but I shall confine my attention to the interdependence of producers only.

E.E. may be defined as those economic gains (in the form of greater output, lower costs, or better prices accruing externally to the firms causing them), which depend on the general development of the industry (industries) and which are not subject to exploitation by any one firm alone.

A distinction may be made between total, average and marginal E.E. Total E.E. are the total difference made to the output or revenue or profit of a firm I by the total level of activity of some other firm II. This difference could be measured by subtracting the output which firm I would produce or the revenue which it would obtain if firm II did not exist, from the output it actually produces or the revenue it actually obtains when firm II operates. For example, if as a result of firm II achieving the level of production x_2, firm I, using the same amount of factors l_1^*, s_1^* as before, experiences an increase in its output x_1 from say $x_1^*(l_1^*, s_1^*)$ to $x_1^{**}(l_1^*, s_1^*, x_2)$, then the difference $x_1^{**}-x_1^*$ is a measure of total E.E. If the level of production x_2 results in a downward shift of firm I's total cost curve from say $C_1^*(x_1)$ to $C_1^{**}(x_1, x_2)$ and in a consequent adjustment of output x_1 say from x_1^* to x_1^{**}, so as to equate marginal cost and revenue again, then total E.E. may be measured by the vertical distance (AB in fig.I or ab times x_1 in fig.4) between the two total cost curves, corresponding to the new level x_1^{**}.

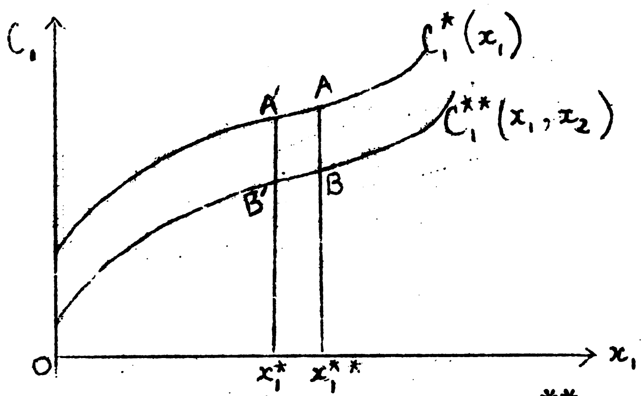

fig I

Whether AB (corresponding to x_1^{**}) or A^1B^1 (corresponding to x_1^*) in fig. 1 is chosen as a measure of total E.E. is a matter of definition. The difference between AB and A^1B^1 may not be very great anyhow, especially when x_1^* and x_1^{**} are close to each other.

Average E.E. are obtained by dividing total E.E. by the number of units of x_2, i.e.

$$\frac{x_1^{**}(l_1^*, s_1^*, x_2) - x_1^*(l_1^*, s_1^*)}{x_2} \quad \text{or} \quad \frac{AB}{x_2} \quad \text{(in fig.1)}$$

in each case. Average E.E. are rather difficult to calculate, as it is not easy to single out the exact contribution to output or revenue which is made by each factor of production. It is easier to work at the margin: When all the factors of production of a firm are held constant, and the output produced, or factors used, by another firm are allowed to vary slightly, any difference made to the output or revenue of the first firm, as a result of this infinitesimal change, may be taken to be a measure of marginal E.E. Of course, this difference may be negative, positive or zero, depending on whether there exist external diseconomies, economies or no economies at all, i.e. no interdependence between the two producers in question.

Zero Marginal E.E. may imply complete absence of E.E., but may also indicate the point where all E.E. have been fully exploited, i.e. stationary values of the total E.E. curve. Second order conditions are normally needed to determine whether total E.E. are at a maximum or minimum at that point. Where average E.E. are independent of the scale of firm II's operation, i.e., total economies are proportional to this scale of operation, the distinction between the average and marginal E.E. is futile, since they are the same. But when average E.E. vary with firm II's scale of operation, the two diverge: Marginal E.E. are greater than average E.E. when the latter are an increasing function of firm II's scale of operation, and vice versa. Average E.E. need not be a straight line: they may be scalloped, fluctuating or discontinuous; in which case, marginal E.E. would follow the same pattern.

E.E. may be technological or pecuniary, factor- or output-generated, and static or dynamic. They may also be examined separately for the cases of perfect and imperfect competition in factor and product markets.

TECHNOLOGICAL AND PECUNIARY EXTERNAL ECONOMIES

Technological E.E.[1] are the differences made to output x_1 of a firm by the presence of output x_2 produced by, or factors l_2, s_2 used by, some other firm II, within or outside the industry. This interaction is direct and external to the market mechanism: products and factors are all measured in appropriate physical units, and both factor costs and product prices are left out of account. If firm II could appropriate all his product x_2 or charge firm I for this favourable interaction, there would be no E.E.: the notion of E.E. implies in-appropriability.

Economies of this type are called, after Viner,[2] Technological External Economies, because firm II affects the technological conditions of firm I's production directly, without the intermedium of market prices. This interaction helps reduce the

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(1) Where the effects of one producer on another are detrimental or neutral, there exist External Diseconomies, and neutral economies respectively. These may be considered as the special cases where E.E. are negative and zero respectively. I use E.E. in this general meaning.

(2) J. Viner, [29] p.213

technological coefficients: the same level of output could be produced by using less of each factor l_1, s_1; or alternatively, the use of the same amount of each factor now enables the production of a larger output. This reduction in production coefficient may be neutral in the sense that each factor is saved in the same proportion; but it may well be biased towards labour, or capital saving. Marshall's examples of "improved organisation, improved methods, the growth of knowledge, trade and technical publications"(1) consequent "on the general development of the industry" may be illustrations of technological E.E.

The picture may be sharpened by introducing marginal E.E. The development of the industry (industries) is only possible if at least one firm expands its scale of production, or at least one new firm is established. If this "marginal" firm enables the existing firms to produce more efficiently, it will create marginal technological E.E. In the case where this reduction in technological coefficient is due to innovation, it may be called innovational E.E. Pure research may be considered as an invisible commodity, costly to produce. But once it has been completed and its results have been published, it

(1) A.Marshall, [13] p.615 and 808

becomes more or less a free good susceptible of various applications. The adoption of new methods of production by application of these results will save factors and actualize innovational E.E. Professor Meade's[1] unpaid factor case is another illustration of technological E.E.: in producing apples, the apple farmer also provides free food for bees. The increase in honey output without a corresponding increase in factors used, is due to the increase in apple blossoms.

Assume two firms producing x_1 and x_2 respectively, using the same factors l and s. If the scale of operation of x_2 influences the production of x_1, say if part of x_2 or its by-product, now becomes a free factor of production of x_1, the two production functions may be written as:

$$x_1 = f(l_1, s_1, x_2)$$
$$x_2 = g(l_2, s_2) \qquad \ldots(1)$$

and their isoquants shown in the following diagrams:

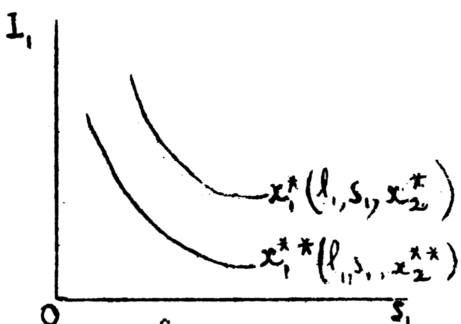


fig. 2

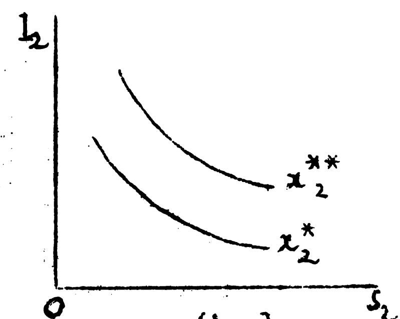


fig. 3

(1) J.E.Meade [14] pp. 54 - 67.

In fig. 2, each family of x_1 isoquants (only one of which, x_1^*, is shown) corresponds to a given level of production x_2^* in fig. 3. If x_2 confers technological E.E. on x_1, an increase in x_2 (from isoquant x_2^* to x_2^{**}, with $x_2^{**} > x_2^*$) may be represented as reducing the production coefficient of x_1, i.e., the x_1^* level could be produced with less of each factor, as indicated, in fig. 2, by a downward shift of $x_1^*(l_1, s_1, x_2^*)$ to $x_1^{**}(l_1, s_1, x_2^{**})$, with $x_1^* \equiv x_1^{**}$. On the other hand, isoquant $x_1^{**}(l_1, s_1, x_s^{**})$, belonging to a different family to x_1^*, does not have to lie entirely below x_1^*: it may cut it from above or below, i.e., technological E.E. need not be neutral: they could be factor-saving or -using.

We have been concerned so far with direct or technological E.E. When the effects of a level of production x_2 are felt by the revenue or profit of some other firm I through market mechanism, on the other hand, there is a case of pecuniary external economies. These take the forms of better factor and/or product prices.

Changes in factor prices are often identified with E.E. As the industry expands, factor-supplying firms may benefit, in their expansion, from lower production costs, if they are producing under conditions of increasing returns. These would be partly or totally passed on to their customers in the forms of lower factor prices.

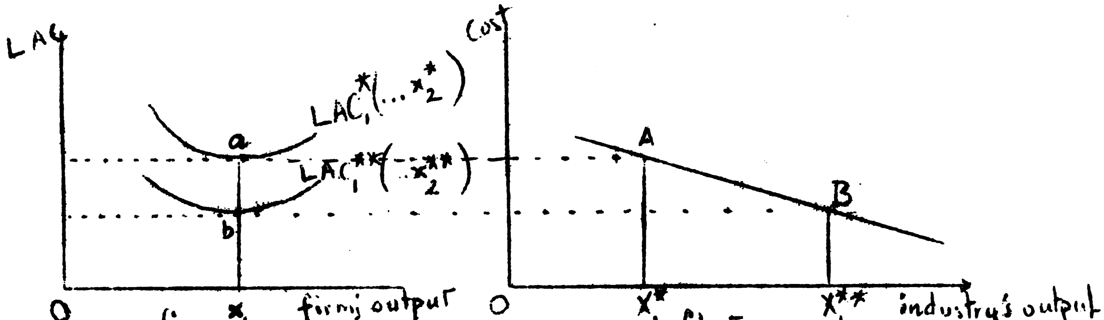


fig 4 fig 5

If x_2 confers pecuniary E.E. on x_1, then a higher level of x_2, say x_2^{**} (with $x_2^{**} > x_2^*$), would cause the long run average cost LAC_1^* (fig.4) to shift downward to LAC_1^{**}. There exists one LAC_1 corresponding to each level of industry output, changes in which are brought about by the entry of new firms or by the expansion of existing firms. (Here, that marginal firm causing the change and E.E., is referred to as firm II, producing x_2). The vertical difference between LAC_1^* and LAC_1^{**} is a measure of pecuniary E.E. in terms of cost savings, and the difference made to the industry supply is the measurement of E.E. in terms of output. Industry costs would be lower as can be seen from fig. 5 (where A and B correspond to a and b (fig. 4) respectively). It is easy to see that rising firms' marginal costs (at a and b in fig. 4) both before and after the shift is compatible with falling industry marginal cost, i.e., constant returns to scale at firms' level (at a,b) under perfect competition with external economies, are quite compatible with increasing returns to scale at industry level. This explains why a firm benefiting from E.E. cannot expand to the point of becoming a monopolist.

If a and b (fig.4) are the points corresponding to actual levels of production of the firm, AB is the supply curve of the industry, the negative slope of which reflects E.E. Note that A_1B_1 ... is the locus of all the relevant points on the rising supply curves of the industry (not shown), corresponding to each number of component firms and each level of their production. Where price is determined depends on the industry demand curve. So long as the latter cuts AB from above, stability obtains. If it cuts AB from below, the system is condemned to eternal instability; and if it coincides with AB, the system breaks down under the weight of indeterminacy. Note that AB may eventually reach a minimum and start rising, which occurs when E.E. have been fully exploited. Note also that LAC_1^{**}(fig.4) need not be equidistant from LAC_1^{*} at every point: LAC_1^{**}'s minimum point may well lie to the right of the vertical going through LAC_1^{*} 's minimum, which implies that x_1 not only benefits from costs savings, but also from increasing returns: at the actual level of production, LAC_1^{**}may be increasing and LAC_1 constant or falling.

The removal of the assumption of perfect competition would not invalidate the analysis: when firms are free to fix their prices, and with unchanged demand conditions, lower cost conditions due to E.E. would generally mean larger output, lower prices and better profit, depending on the price elasticity of demand.

Beside the form of lowering production costs, pecuniary E.E. may also take the form of raising of, and increasing the demand for, the product of the beneficiary firms. The expansion of x_2 is normally associated with a fall in its price p_2. A lower p_2 would cause the demand (d_1) for x_1 to increase (decrease) if x_1 and x_2 are complements (substitutes). In the complementary case, p_1 would normally rise unless the supply of x_1 is infinitely elastic. This means a better revenue (R_1) for firm I, and, under unchanged cost conditions, a better profit (P_1). Even if a higher price p_1 in money terms does not obtain, a fall in p_2 may still be a stimulus to the expansion of x_1, as p_1 in terms of x_2 (i.e. $\frac{p_1}{p_2}$) is better. All these are forms of pecuniary E.E.

The income effect of the fall in p_2 need not be confined to x_1 and x_2, but could be spread on the demand for other commodities which will expand in response. This expansion pattern may be effected at the cost of contraction of the industry producing the substitutes of x_2. This expansion-contraction pattern normally has effects on relative factor prices, except in the unlikely case where expansion just offsets contraction, and factor proportions are the same in both the contracting and expanding industries.

The dissection of E.E. into technological and pecuniary is rather arbitrary: overall E.E. may be zero and yet there may exist either technological or pecuniary E.E. or both, but they may be offset by external diseconomies; or technological economies may be offset by pecuniary diseconomies, and vice versa. For example, as output x_1 increases due to an expansion of output x_2, the price p_1 of x_1 may fall, leaving revenue (R_1) and profit (P_1) of firm I unaffected, i.e., there are zero overall E.E. in spite of the presence of technological E.E. Similarly, costs savings realised in the production of x_1 due to x_2's expansion may be matched by a fall in p_1, leaving R_1 and P_1 unaffected. Better prices (p_1) may be difficult to detect: as p_2 falls, an unchanged price p_1 really implies a better $\frac{p_1}{p_2}$ and is a stimulus to the expansion of x_1: there exist E.E. in real terms, not in money terms. Moreover, lower production costs, usually identified with pecuniary E.E., could be the result of either technological or pecuniary E.E. or both. Lower costs could be a form either of technological or of pecuniary E.E. Lower factor prices may be the result of technological improvements realised in x_2 production,ie., pecuniary E.E. may be the results of technological E.E.

The best way to express overall E.E. is perhaps to refer to the profit function (P_1) of the firm benefiting from E.E.:

$$P_1 = P_1(l_1, s_1, x_2) \quad .... \ (2)$$

Where P_1 = Profit of firm I

l_1, s_1 = factors used in firm I

x_i = output of each firm ($i = 1,2$)

More explicitly, profit (P_1) may be written as the difference between total revenue ($R_1 = p_1 x_1$) and total cost ($C_1(l_1,s_1,x_2)$), i.e.

$$P_1 = p_1 x_1(l_1,s_1,x_2) - C_1(l_1,s_1,x_2) \quad ... \ (2^a)$$

The effects of x_2 on P_1, measuring output-generated E.E., could be expressed as:

$$\frac{dP_1}{dx_2} = p_1 \frac{\partial x_1}{\partial x_2} + x_1\left(\frac{\partial p_1}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial p_1}{\partial x_2}\right) - \frac{\partial C_1}{\partial x_2} \quad . . . \ (2b)$$

where only the first term $p_1 \frac{\partial x_1}{\partial x_2}$ is pure technological E.E.; $\frac{\partial C_1}{\partial x_2}$ and $x_1 \frac{\partial p_1}{\partial x_1} \frac{\partial x_1}{\partial x_2}$ are mixed and $x_1 \frac{\partial p_1}{\partial x_2}$ is pure pecuniary E.E.

Now $\frac{\partial p_1}{\partial x_1} < 0$ as a rule; $\frac{\partial x_1}{\partial x_2} > 0$ and $\frac{\partial C_1}{\partial x_2} < 0$ for the case of E.E. and vice versa for External diseconomies; $\frac{\partial p_1}{\partial x_2}$ is positive if x_1 and x_2 are complementary and negative if they are competitive goods.

Under perfect competition, any single firm is too small to affect product price, i.e., p_1 is a constant, and (2^b) becomes

$$\frac{dP_1}{dx_2} = p_1 \frac{\partial x_1}{\partial x_2} - \frac{\partial C_1}{\partial x_2} \quad \ldots . \; (2c)$$

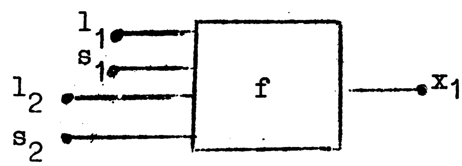
a marginalist concept which includes both technological and pecuniary E.E. The profit function is thus an indicator of overall E.E.

It is easy to see that profit (P_1) may be unaffected if technological E.E. are neutralised by pecuniary External Diseconomies, and vice versa. Thus, although a good indicator of overall E.E., ~~although a good indicator of~~ profit function, as a means of detecting various kinds of External Economies, is a poor discriminator.

FACTOR AND OUTPUT-GENERATED EXTERNAL ECONOMIES.

Technological and pecuniary E.E. may be either factor, or output-generated. When the level of output produced in firm II favourably affects firm I's profit (P_1), E.E. are output-generated. The examples given so far in the above section are all of this kind.

When the factors used in the production of X_2 directly influence the production (X_1) or revenue (R_1) of the beneficiary firm I, there are factor-generated E.E., i.e.



or $X_1 = f(l_1, s_1, l_2, s_2)$

or more generally, $P_1 = P_1\,(l_1,\ s_1,\ l_2\ s_2)$ where P_1 is the profit function, which takes into account both technological and pecuniary E.E.

Factor-generated technological E.E. may be represented in figure 6. The favourable effect of a change in the level of factor (l_2 or s_2) used in firm 11, say from l_2^* to l_2^{**} (with $l_2^{**} > l_2^*$),

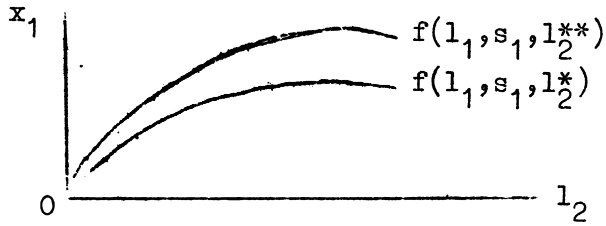


Fig. 6.

may be shown as causing the production function to shift upward, thus increasing the productivity of the factors of production of the beneficiary firm. It is left to the reader to visualise the case of factor generated pecuniary E.E., and the more general case of factor generated overall E.E. as represented by $P_1\ (l_1, s_1, l_2, s_2)$. Where there are factor-generated E.E., X_2 is to be mentally replaced by l_2 and/or s_2 in the previous section (pp. 10-19). For example (2a) on page 17 is to be read:

$$P_1 = P_1 X_1\ (l_1,\ s_1, l_2, s_2\ ) - C_1(l_1, S_1, l_2, S_2) \ldots (2d)$$

and the results (2c) on page 18 are to be read as:

$$\frac{dP_1}{dl_2} = p_1 \frac{\partial x_1}{\partial l_2} - \frac{\partial C_1}{\partial l_2}\ ; \quad \frac{dP_1}{ds_2} = p_1 \frac{\partial x_1}{\partial s_2} - \frac{\partial C_1}{\partial s_2} \quad \ldots \ldots (2e)$$

The lowering of cost (C_1) due to the level of factors (l_2, s_2) used by firm 11, may be shown in figure 7.

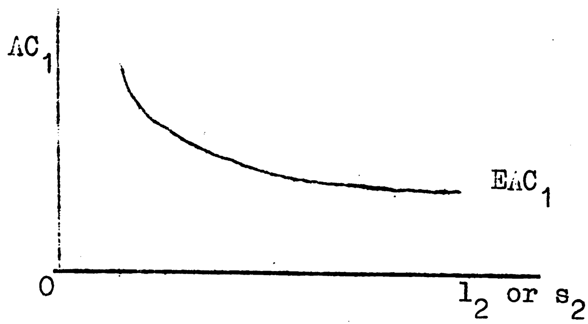


Fig. 7

EAC_1 represents the locus of the various average costs curves of firm 1 corresponding to the actual level of x_1, given each level of l_2 or s_2 used. EAC_1 is downward sloping by the assumption of E.E. The rate at which EAC_1 falls depends on whether X_1 is produced under constant or decreasing or increasing costs in the absence of E.E.

The horizontal axis could in fact be split up into l_2 or s_2 and x_1 axes: EAC_1 is a function of both the scale of production of x_1 and the level of factors l_2, s_2 used by firm 11.

If the horizontal axis of fig. 7 represented X_1 only, the effects of E.E. could be shown, as before, by a downward shift of the EAC_1 : there exists a different EAC_1 curve corresponding to each level of l_2 or s_2.

Note that the distinction between factor and output-generated E.E. is meaningful only when factors are partly substitutable: when the production function admits of a unique factor combination, output-generated E.E. are also factor-generated E.E.

STATIC AND DYNAMIC EXTERNAL ECONOMIES

External Economies may also be classified into static and dynamic, which, in turn, may be technological or pecuniary, factor or output-generated.

Static E.E. are those which are considered as accruing instantly: the process of adjustment, the time path of these adjustments as well as the effects of x_2's past history on x_1 or revenue (R_1) or profit (P_1) of the beneficiary firm (I) are all telescoped. The E.E. considered so far are all static. For example, the production or profit functions of the beneficiary firm

$$x_1 = f(l_1, s_1, x_2)$$

$$P_1 = P_1(l_1, s_1, x_2, l_2, s_2)$$

imply that as x_2 or l_2, s_2 appear, x_1 or P_1 are affected without delay. Allthese variables belong to the same period of time, or are rather timeless, undated, and the past behaviour of x_2, $l_2 s_2$ exerts no influence on current x_1 or P_1. In fact this is an oversimplification in many cases. The concept of dynamic E.E. takes the adjustment process into account.

There exist dynamic E.E. whenever the product, revenue, costs, or more generally profit of a firm is dependent on, among other factors, the output produced, or factors used, by some other firm at some previous time. For example $x_{1_t} = f(l_{1_t}, s_{1_t}, x_{2_{t-1}}, x_{2_{t-2}} \ldots)$

or using the profit function:

$$P_{1_t} = P_1(l_{1_t}, s_{1_t}, \sum_{t=-\infty}^{T} x_2\ t)$$

where x_i = outputs $(i = 1,2)$

P_1 = Profit of the beneficiary firm

l, s = factors of production

If these time periods are short enough, we may consider the rate of change of time as continuous, i.e.

$$x_1(t) = f\left[l_1(t), s_1(t), \int_{-\infty}^{T} x_2 dt\right]$$

$$\text{or } P_{1(t)} = P_1\left[l_1(t)_1 s_1(t), \int_{-\infty}^{T} w_t x_2 dt\right]$$

Where w is weighted average

Pure research is an example of dynamic E.E. Research is an output, costly to produce, but under present institutions, it is not always marketable. There usually is a time lag between invention and innovations, which gives E.E. a dynamic character.

A firm which enters the industry in an established area would benefit, free of charge, from the labour force already trained in previous periods by pioneering firms. That is an illustration of factor-generated dynamic technological E.E.

Suppose that in the absence of E.E., output x_1 grows along AB (fig.8). With E.E., it will grow say along AC.

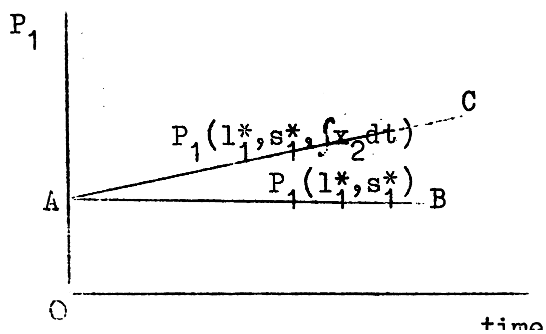


Figure 8.

There is a different AC line corresponding to each level of production and rate of growth of x_2 over time. AB need not be a straight and horizontal line: it may grow at an exponential rate, i.e. upward sloping, or decline at a certain rate, i.e. downward sloping, in which case AC would have to be adjusted upward or downward accordingly.

Dynamic pecuniary E.E. exhibit much the same features: there is a time lag between the expansion of the industry (due to the increase in the output of at least one firm, old or new), and the time its effects on the profit function of the beneficiary firms are felt; a time lag between the fall in production costs and product price revision. Those pecuniary E.E. which take the forms of cost savings may be pictured as giving the beneficiary firm's long run average cost curve a falling time shape. In Fig. 9, AC_1 in the absence of E.E. is shown by the top line $C_1(l_1^*, s_1^*)$ and in the presence of E.E., by the bottom line.

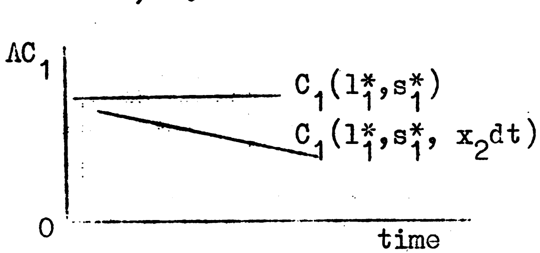


fig. 9

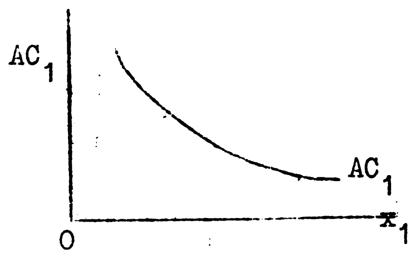


fig. 10

The top line need not be a horizontal straight line, of course, it may be upward or downward sloping, in which case the bottom line would have to be adjusted accordingly. For a horizontal top line, the bottom line may be scalloped, fluctuating, or may reach a minimum and start rising, which implies that E.E. may eventually be fully exploited. Newspapers, technical publications etc., going hand in hand with the general development of industries, partly account for this fall in costs over time.

One of the characteristics of dynamic E.E. is cost irreversibility. In the literature, fig.10 is usually drawn to show irreversibility: As x_1 contracts, average cost (AC_1) would not follow the path it has taken in falling down when output expands. Elsewhere, it is said that the falling supply schedule must be interpreted as showing price as a function of the quantity supplied, not vice versa, as it makes no sense to say that at lower prices firms would be willing and prepared to supply more.

I think a good deal of confusion could be avoided, if we split the horizontal axis of fig. 10 into x_1 and time axis. When time is kept fixed, and AC_1 is a function of output alone, the cost curve is perfectly reversible, provided plants and machineries are meccano sets which could be assembled, dismantled and reassembled at no costs and in no time. Contracting output would then mean higher unit costs and prices. That is the case of static E.E. with cost reversibility. Where both output and time are allowed to vary, a fall in costs could mean a time long enough for tastes to be changed, technical know how to be acquired both from the firm's own experience and from that of the industry, overhead investments to be undertaken, population to grow. Contracting output to a smaller scale need not, and usually does not, mean higher costs: costs behave as if they have forgotten the level which they emerge from and are quite willing to land back on some other spot on a lower level. That is dynamic E.E. with cost irreversibility.

Like static E.E., dynamic pecuniary E.E. could be brought about by change in product price. As p_2 falls, x_1 would be demanded in greater quantity if x_1 and x_2 are complementary. It usually takes time for this complementarity to be developed or discovered. That is Hirschman's "entailed wants".(1) These time lags make the dynamic E.E.

The above classification of E.E. adopted here, imperfect as it is, has the merit of separating the direct and market interdependence of producers, and are useful in the applications which are to follow.

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(1) A.O.Hirschman [8] p.68.

CHAPTER III

EXTERNAL ECONOMIES AND THE MARGINAL THEORY OF DISTRIBUTION

The Marginal Theory of Distribution rests on the proof that if all factors of production are rewarded according to the values of their marginal product, then the total product will be exactly disposed of. The "adding up" problem goes back to the classical "residual" theory which considers rent and profit as residues left over when the other factors have received their marginal product.(1) Subsequently an attempt was made to prove that the residual rent or profit are also rent or profit as marginal product.(2)
The next development is to premise and assume product exhaustion, based on fixed coefficients of production.(3) Euler's theorem has been used to prove that if the production function is homogeneous of first degree, i.e., there is constant returns to scale and if each factor is paid its marginal product, total product is completely exhausted. Discussions are generally confined to perfect competition, as under Monopoly, constant returns to scale

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(1) A.Berry (1) pp. 923-924.

(2) J.Chapman (3) pp. 523-28.

(3) L.Walras (30)

K.Wicksell (31)

do not normally obtain.[1][2]

In what follows, an attempt will be made to show in a simplified model of perfect competition in both factor and product markets, the implications of E.E. for the Marginal Productivity theory of Distribution.

Imagine two firms producing identical or different products (e.g. milk, or milk and wheat). The scale of operation of these firms is assumed to be too small to affect factors and products' prices. In equilibrium, only normal profit is earned, and the scale of production is indicated by the point of tangency between AC and the horizontal Demand curve. Constant returns to scale prevail at that point. We know that if each factor is paid its marginal product (which happens to be equal to average product at that point of tangency), total product is exactly disposed of. Euler's theorem is very adequate for the purpose of explaining this.

Let x_i = products ($i = 1,2$)

l_i, s_i, c_i = factors

A = constant

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(1) For a review of the theories of distribution see J.Robinson [19] pp.398-414. Stigler [27]

(2) The only case where constant returns to scale obtain under monopoly is that in which the monopolist's marginal cost and marginal revenue intersect vertically underneath the minimum point of his average cost curve.

If the two production functions

$$x_i = x_i \, (l_i, s_i, c_i) \quad \ldots .(1)$$

are both homogeneous of degree one in all l_i, s_i, c_i, we have, from Euler's theorem:

$$x_i = \frac{\partial x_i}{\partial l_i} l_i + \frac{\partial x_i}{\partial s_i} s_i + \frac{\partial x_i}{\partial c_i} c_i \quad \ldots\ldots (1a)$$

i.e. if factors are rewarded accordingly to their marginal product, output will be exhausted. For a verification, x_i may be given a form of, say, Cobb-Douglas function $x_i = A_i l_i^{\alpha_i} s_i^{\beta_i} c_i^{\gamma_i}$ $(i = 1,2)$ complete exhaustion of products will follow from the assumption of homogeneity i.e. of $\alpha_i + \beta_i + \gamma_i = 1$.

If x_i confers E.E. on x_1 and there is only one factor (l_i) used in each firm (the generalisation to the many-factor case will be made later), each producing under constant returns to scale, the two production functions may be written as

$$x_1 = f(l_1, x_2)$$
$$x_2 = g(l_2) \quad \ldots \quad (2)$$

where f and g are homogeneous of degree one in l_1, x_2 and in l_2 respectively.

Competition will ensure the equalisation of factor's reward (w) in both firms, i.e.

$$w = f_{l_1} = pg_{l_2}$$

where $f_{l_1} = \frac{\partial f}{\partial l_1}$; $g_{l_2} = \frac{\partial g}{\partial l_2}$

$p = \frac{p_2}{p_1}$ i.e., the first commodity is chosen as unit.

In fact, l_2 does contribute to the production of x_1 as an argument of x_2 in (2), and is not rewarded for it under laisser faire competition (3). To do justice to the factor generating E.E. socially optimal factor's reward should equate not $f_{l_1} = pg_{l_2}$ in (3) but

$$f_{l_1} = pg_{l_2} \left(1 + \frac{1}{p} f_{x_2}\right) \quad \ldots \quad (4)$$

To bring these optimum rewards about in a free enterprise economy, a subsidy rate of 100 $(\frac{1}{p} f_{x_2})$% would have to be paid to l_2, if l_2's reward (w_2) is to be brought into equality with l_1's reward (w_1); or a tax rate of 100 $\left(\frac{\frac{1}{p} f_{x_2}}{1+\frac{1}{p} f_{x_2}}\right)$% would have to be imposed on firm I, if w_1 is to be reduced to equality with w_2. The tax revenue, or the subsidy cost, is exactly equal to the abnormal profit $f_{x_2}x_2$ realised by firm I as a result of these output-generated E.E. This "abnormal profit" is what is left over after l_1 has been paid according to its private marginal product f_{l_1}. This governmental interference has the effect of preventing firm I from "reaping where it has not sown" and redistributing to each factor according to its real contribution to social output.

The generalisation of this result to the many-factor case does not challenge the validity of the above conclusions: (2) is extended to

$$x_1 = f\,(l_1, s_1, c_1, x_2)$$
$$x_2 = g\,(l_2, s_2, c_2) \quad \ldots \quad (5)$$

where x_i = outputs

l_i, s_i, c_i = factors $(i = 1,2)$

f,g are assumed homogenous in all their arguments. Under individual profit maximization, the private Marginal Distribution Theory leads to

$$f_{l_1} = pg_{l_2}$$
$$f_{s_1} = pg_{s_2}$$
$$f_{c_1} = pg_{c_2}$$

Under Pareto optimality, the social marginal distribution theory gives:

$$f_{l_1} = g_{l_2} \, (p + f_{x_2})$$
$$f_{s_1} = g_{s_2} \, (p + f_{x_2}) \qquad \ldots (6)$$
$$f_{c_1} = g_{c_2} \, (p + f_{x_2})$$

The community could afford to pay factors this, under conditions of constant returns to scale, as social product is completely exhausted as can be seen from:

$$l_1 f_{l_1} + s_1 f_{s_1} + c_1 f_{c_1} + (l_2 g_{l_2} + s_2 g_{s_2} + c_2 g_{c_2})(p + f_{x_2})$$
$$= x_1 - f_{x_2} x_2 + x_2(p + f_{x_2}) = x_1 + px_2 \qquad \ldots (7)$$

This total social factor payment $x_1 + px_2$ is exactly equal to total social revenue $x_1 + px_2$. Thus, under perfect competition in both factor and product markets, if one firm confers E.E. on another and there are constant returns to scale, the payment to factors according to their social marginal product will result in a complete exhaustion of products. The payment according to their private marginal product will under-exhaust social output. Market rewards fail to do justice to factors of production and government interference is needed to bring the Pareto optimum about.

Each firm confers E.E. on the other.

There may be cases when the two products benefit each other. Each firm, in this case, confers E.E. on the other, and benefits from E.E. caused by the other firm. Again, only the one-factor case of output-generated E.E. will be examined, the many-factor case as well as the factor-generated E.E. are left to the reader. The homogeneity assumption will be lifted to give the problem a more general nature. The production functions may be written as:

$$x_1 = f(l_1, x_2)$$
$$x_2 = g(l_2, x_1)$$

or given the definite forms of, say:

$$x_1 - l_1^{\alpha_1} x_2^{\beta_1} = 0$$
$$x_2 - l_2^{\alpha_2} x_1^{\beta_2} = 0 \quad \ldots\ldots\ldots \quad (8)$$

where x_i = outputs
l_i = factors $(i = 1,2)$

Differentiating with respect to l_1

$$\frac{dx_1}{dl_1} - \beta_1 \frac{x_1}{x_2}\frac{dx_2}{dl_1} = \frac{\alpha_1}{l_1} x_1$$
$$-\beta_2 \frac{x_2}{x_1}\frac{dx_1}{dl_1} + \frac{dx_2}{dl_1} = 0$$

Solving

$$\frac{dx_1}{dl_1} = \frac{\begin{vmatrix} \frac{\alpha_1}{l_1}x_1 & -\beta_1\frac{x_1}{x_2}\overset{\frac{dx_2}{dl_1}}{} \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\beta_1\frac{x_1}{x_2} \\ -\beta_2\frac{x_2}{x_1} & 1 \end{vmatrix}} = \frac{\frac{x_1}{l_1}\alpha_1}{1-\beta_1\beta_2}$$

Similarly,

$$\frac{dx_2}{dl_1} = \frac{\alpha_1\beta_2\frac{x_2}{l_1}}{1-\beta_1\beta_2}$$

Similarly, differentiating (8) with respect to l_2 and solving:

$$\frac{dx_1}{dl_2} = \frac{\alpha_2\beta_1\frac{x_1}{l_2}}{1-\beta_1\beta_2}$$

and

$$\frac{dx_2}{dl_2} = \frac{\frac{\alpha_2}{l_2}x_2}{1-\beta_1\beta_2}$$

Social factorial rewards are:

$$w_1^* = \frac{dx_1}{dl_1} + p\frac{dx_2}{dl_1} = \frac{\alpha_1 x_1}{l_1}\,\frac{1+\beta_2 p\frac{x_2}{x_1}}{1-\beta_1\beta_2}$$

$$w_2^* = p\frac{dx_2}{dl_2} + \frac{dx_1}{dl_2} = \frac{\frac{\alpha_2}{l_2}(px_2+\beta_1 x_1)}{1-\beta_1\beta_2} \quad \ldots (9)$$

These are to be compared with private factors' rewards of::

$$w_1 = f_{l_1} = \frac{\alpha_1 x_1}{l_1}$$

and $$w_2 = pg_{l_2} = \frac{\alpha_2}{l_2}px_2 \quad \ldots\ldots\ldots\ldots (10)$$

This implies that under laisser faire conditions, a subsidy rate of

$$\beta_2\frac{x_2}{x_1}\,\frac{p + \frac{\beta_1 x_1}{x_2}}{1 - \beta_1\beta_2} \quad \text{would have to be paid to } l_1$$

and of

$$\frac{\beta_1\frac{x_1}{x_2}\left(1 + p\,\frac{\beta_2 x_2}{x_1}\right)}{p\,(1 - \beta_1\beta_2)} \quad \ldots\ldots\ (11)$$

to be paid to l_2 to bring this Pareto optimality about.

If the subsidy is paid, total factor rewards (W^*) in this simple model would be:

$$W^* = w_1^* l_1 + w_2^* l_2 = \frac{\alpha_1(x_1 + \beta_2 p x_2) + \beta_1(p x_2 + \beta_1 x_1)}{1 - \beta_1\beta_2}$$

$$= \frac{1}{1 - \beta_1\beta_2}\left[x_1(\alpha_1 + \alpha_2\beta_1) + p x_2(\alpha_1\beta_2 + \alpha_2)\right] \quad . \ . \ (12)$$

Whether this payment W^* would exhaust social product or not, i.e., whether $W^* \gtrless x_1 + p x_2$, depends on α_i, β_i $(i = 1,2)$. As x_1 and $p x_2$ in (12) are all positive economically, only $\alpha_1 + \alpha_2\beta_1$, $\alpha_1\beta_2 + \beta_1$ and $1 - \beta_1\beta_2$ call for examination.

Put $\alpha_1 + \beta_1 = c$

$\alpha_2 + \beta_2 = d$,

the two expressions $\alpha_1 + \alpha_2\beta_1$ and $\alpha_1\beta_2 + \beta_1$ may be written as:

$$\begin{aligned}\alpha_1 + \alpha_2\beta_1 &= c - \beta_1 + \beta_1(d - \beta_2)\\ &= (1 - \beta_1\beta_2) + \beta_1(d - 1) + (c - 1)\end{aligned}$$

Similarly,

$$\alpha_2 + \alpha_1\beta_2 = (1 - \beta_1\beta_2) + \beta_2(c - 1) + (d - 1)$$

Three possible cases suggest themselves:

Case (1) $\dfrac{\alpha_1 + \alpha_2\beta_1}{1 - \beta_1\beta_2} > 1$; $\dfrac{\alpha_1\beta_2 + \alpha_2}{1 - \beta_1\beta_2} > 1$

$W^* > x_1 + px_2$, i.e., there is overexhaustion of social product. This implies that

(a) $\beta_1\beta_2 < 1$ or $\beta_1(d - 1) + (c - 1) > 0$
$\beta_2(c - 1) + (d - 1) > 0$

$c > 1$ $d > 1$ for $\beta_1, \beta_2 \lesseqgtr 0$
$c > 1$ $d < 1$ for $\beta_1 \lesseqgtr 0, \beta_2 > 0$
$c < 1$ $d > 1$ for $\beta_1 > 0, \beta_2 \gtrless 0$
$c > 1$ $d < 1$ for $\beta_1, \beta_2 < 0$

(b) $\beta_1\beta_2 > 0$ or $\beta_1(d - 1) + (c - 1) < 0$
$\beta_2(c - 1) + (d - 1) < 0$

$c > 1$ $d > 1$ for $\beta_1, \beta_2 < 0$
$c > 1$ $d < 1$ for $\beta_1 > 0, \beta_2 \gtrless 0$
$c < 1$ $d > 1$ for $\beta_1 \gtrless 0, \beta_2 > 0$
$c < 1$ $d < 1$ for $\beta_1 , \beta_2 \gtrless 0$

Case (2) $\dfrac{\alpha_1 + \alpha_2\beta_1}{1 - \beta_1\beta_2} < 1$; $\dfrac{\alpha_1\beta_2 + \beta_1}{1 - \beta_1\beta_2} < 1$

that means product under-exhaustion, i.e., $W^* < x_1 + px_2$.

The sub-cases (a) and (b) are the same as those under case (1) except that (a) only holds if

$$\frac{\alpha_1 + \alpha_2\beta_1}{1 - \beta_1\beta_2} > 0 \quad \text{and} \quad \frac{\alpha_1\beta_1 + \alpha_2}{1 - \beta_1\beta_2} > 0$$

Case (3) $$\frac{\alpha_1 + \alpha_2\beta_1}{1 - \beta_1\beta_2} = \frac{\alpha_1\beta_2 + \beta_1}{1 - \beta_1\beta_2} = 1$$

$W^* = x_1 + px_2$, i.e., there is complete exhaustion of social output.[1] This implies that $c = d = 1$.

These three cases give a fairly general and complete picture of the implications of factor's payment according to their social marginal product, for the cases of external economies (in other words, $\beta_1, \beta_2 > 0$), external diseconomies ($\beta_1, \beta_2 < 0$) and no externalities ($\beta_1 = \beta_2 = 0$), and all that in both firms at once or in one of them, i.e., $\beta_1 \neq 0$, $\beta_2 = 0$; $\beta_1 \lesseqgtr 0$, $\beta_2 = 0$), the cases of increasing decreasing or constant returns to scale, i.e., $\alpha_i + \beta_i$ are greater, less, than or equal to, zero ($i = 1,2$). Thus only under conditions of constant returns to scale, would the Marginal Theory of Distribution lead to complete exhaustion of social output. That is the case of decreasing returns to each factor l_i taken separately in each firm, but constant returns to those taken together; i.e., the increase in l_i by λ would lead to an increase in each output x_i by less than λ, but to an increase in social output by λ: the private marginal product of factors falls short of their social marginal product. Sub-cases (a) are more realistic economically than (b) as normally $0 < \beta_1, \beta_2 < 1$.

Finally, when only one firm confers EE. on the other, i.e., $\beta_1\beta_2 = 0$, which means either:

(i) $\beta_1 = 0$; $\beta_2 > 0$, in which case, $W^* = \alpha_1 x_1 + (\alpha_1\beta_2 + \alpha_2)px_2$.

As $\beta_1 = 0$ implies $\alpha_1 = 1$, i.e, if x_2 in the production function $x_1 = l_1^{\alpha_1} x_2^{\beta_1}$ (8) contributes nothing to output x_1, then l_1 alone

1. Professor's J.E. Meade's unpaid factor (system 5, [14], p. 58) is this special case (3) extended to two factors.

produces the entire output x_1. Thus social factor payment becomes $W^* = x_1 + px_2(\alpha_2 + \beta_2)$, which implies complete, over-, or under-exhaustion of social output, depending on whether $\alpha_2 + \beta_2 \gtreqless 1$.

Similarly, $\beta_1\beta_2 = 0$ may also mean:

(ii) $\beta_2=0$; $\beta_1 > 0$ and $W^* = (\alpha_1 + \alpha_2\beta_1)x_1 + \alpha_2 px_2$

which, as above, implies $\alpha_2 = 1$ and $W^* = (\alpha_1 + \beta_1)x_1 + px_2$ [1]

This again, means over-, or under-exhaustion of product depending on whether $\alpha_1 + \beta_1 \gtrless 1$, i.e., on whether there are increasing or decreasing returns to scale in l_1 and x_2 in the function $x_1 = l_1^{\alpha_1} x_2^{\beta_1}$ in (8). In the particular case of constant returns to scale in l_1 and x_2 in (8), complete exhaustion of product will follow again: Thus, system (2) above becomes a particular case of this overall picture.

Finally, in the particular case where $\beta_1\beta_2 = 1$, the whole system breaks down.

If subsidies rates (11) are to be paid to salvage competition and bring about Pareto optimum, the total subsidy costs would be:

$$\frac{1}{1 - \beta_1\beta_2}\,[\,\beta_1x_1(\alpha_1\beta_2 + \alpha_2) + \beta_2px_2(\alpha_1 + \alpha_2\beta_1)] \quad . \; . \; . \; . \quad (13)$$

In the particular case of constant returns to scale, replacing α_i by $1 - \beta_i$ $(i = 1,2)$ (Euler's theorem), in the above gives: $\beta_1x_1 + \beta_2px_2$, which is precisely the sum of abnormal profit realised in both firms together, after paying l_i the value of their marginal product to each firm. Thus, these subsidy costs may be met by a 100% profit tax. This tax on prodit would have its impact on the allocation of resources, which will be examined in the next chapter.

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1. Professor Meade's unpaid factor case (system 4), [14] page 57 is this special case (ii) where $\beta_2=0$, $\beta_1=1-\alpha_1$ both $\alpha_1,\beta_1>0$ extended to two factors.

CHAPTER IV

EXTERNAL ECONOMIES

AND THE ALLOCATION OF RESOURCES

The effectiveness of competitive resources allocation in bringing about Pareto optimality had been questioned long ago by A.Marshall and A.C.Pigou.(1) Government interference is therefore necessary.

This conclusion has been challenged.(2)

This chapter is an application of the conclusions arrived at earlier, to the problem of resource allocation, and an attempt to show the validity of the Marshall-Pigou argument in a model where E.E. exist.

The allocation problem where E.E. are present will be examined under perfect competition in both factor and product markets. I shall start with the case of perfect competition with no E.E., then introduce E.E. to examine the difference they make.

(a) Perfect Compeition with no E.E.

The best way to study the allocation of resources under perfect competition is, perhaps, to examine the profit functions (P_1 and P_2) of two firms. Profit is defined in the usual way as the difference between total revenue and total cost. The scale of operation of each of the two firms in question is assumed to be

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(1) A.Marshall [13] pp.467-470

A.C.Pigou [17] pp.172-179

(2) See Ch.I above, pp.1-4

too small to affect product and factor prices.

Profits are:

$$P_1 = f(l_1,s_1) - C_1(l_1,s_1)$$

$$P_2 = pg(l_2,s_2) - C_2(l_2,s_2) \quad \ldots (1)$$

where f and g are the two production functions,

$C_i = wl_i + rs_i$ $(i = 1,2)$ = cost functions

l_i, s_i = factors of production used by two firms $(i = 1,2)$

w, r = factor prices which are equal for two firms, i.e. $w_1 = w_2 = w$
$r_1 = r_2 = r$

$p = \frac{P_2}{P_1}$ i.e. commodity one is taken as unit of measurement.

First order profit maximization conditions imply the following allocation of resources:

$$\frac{\partial P_1}{\partial l_1} = f_{l_1} - w = \frac{\partial P_2}{\partial l_2} = pg_{l_2} - w = 0$$

$$\frac{\partial P_1}{\partial s_1} = f_{s_1} - r = \frac{\partial P_2}{\partial s_2} = pg_{s_2} - r = 0$$

$$\text{or } w = f_{l_1} = pg_{l_2}$$
$$r = f_{s_1} = pg_{s_2} \quad \ldots\ldots\ldots (2)$$

i.e., factors are used up to the point where an additional unit of any one factor would make no difference to profit P_i; in other words, up to the point where the value of factors' marginal product is equal to factor price.

Product prices will be

$$P = \frac{f_{s_1}}{g_{s_2}} = \frac{f_{l_1}}{g_{l_2}}$$

$$\text{or } P = \frac{f_{s_1}}{f_{l_1}} = \frac{g_{s_2}}{g_{l_2}} \quad \ldots (3)$$

i.e. factors are allocated in such a way that the ratio of their marginal product is the same in each firm.

Thus factors are paid according to the value of their private marginal product as seen by the profit maximizing firms working separately, and product prices are set at the competitive level. This business situation also reflects a social optimum, as can be seen by joining the two firms together:

$$P = f(l_1,s_1) + pg(l_2,s_2) - w(l_1+l_2) - r(s_1+s_2)$$

First order conditions also give $\frac{\partial P}{\partial l_i} = \frac{\partial P}{\partial s_i} = 0$ (i=1,2)

or $w = f_{l_1} = pg_{l_2}$

$$r = f_{s_1} = pg_{s_2} \text{ and } p = \frac{f_{s_1}}{f_{l_1}} = \frac{g_{s_2}}{g_{l_2}} \quad \ldots (4)$$

Thus, there is no divergence between the private and social values of w, r and p.

(b) E.E. and Individual Profit maximization.

Let us now introduce technological E.E. into the above simplified model and retain all the other assumptions made.

Suppose part of x_2 (or its sub-product, or factors used by x_2) now slips out of producer II's hands and favourably affects I's production. Then x_2 could be considered as a free factor, i.e.

$$P_1 = f(l_1,s_1,x_2) - wl_1 - rs_1$$
$$P_2 = pg(l_2,s_2) - wl_2 - rs_2 \quad \ldots (5)$$

Under perfect competition

$$w = f_{l_1} = p g_{l_2}$$

$$r = f_{s_1} = p g_{s_2}$$

$$p = \frac{f_{s_1}}{g_{s_2}} = \frac{f_{l_1}}{g_{l_2}}$$

$$\text{or } g_{l_2} = \frac{w}{p}\ ;\ g_{s_2} = \frac{r}{p} \qquad \ldots (6)$$

i.e. factors are used up to the point where the contribution made by the last units to revenue is equal to their real cost at the margin. Thus, competitive allocation is the same as if no E.E. existed at all.

(c) <u>Technological External Economies and Pareto efficiency</u>

The socially optimal resources allocation may be shown by joining the two firms together under a single management. First order joint profit maximization conditions imply:

$$P = f\left\{l_1, s_1, x_2(l_2, s_2)\right\} + pg(l_2, s_2) - w(l_1 + l_2) - r(s_1 + s_2) \qquad \ldots\ldots\ldots(7)$$

First order conditions are:

$$\frac{\partial P}{\partial l_1} = f_{l_1} - w = 0$$

$$\frac{\partial P}{\partial l_2} = f_{x_2} g_{l_2} + p g_{l_2} - w = 0$$

or ~~similarly,~~ $w = f_{l_1} = f_{x_2} g_{l_2} + p g_{l_2}$ (3)

similarly, $r = f_{s_1} = f_{x_2} g_{s_2} + p g_{s_2}$ ... (8)

i.e. factors are used up to the point where

---

(3) As a numerical example, we may put ($p = 1$, $f_{l_1} = £44$, $g_{l_2} = 40$, $f_{x_2} = 0.1$, $f_{s1} = £88$, $g_{s2} = £80$)

and (8) becomes $w = f_{l_1} = f_{x_2} g_{l_2} + p g_{l_2}$

$= £44 = 0.1 \times 40 + 40$

$r = f_{s_1} = £_2 g_{s_2} + p g_{s_2}$

$= £88 = (0.1)\ 80 + 80$

$$g_{l_2} = \frac{w}{p + f_{x_2}} = \frac{f_{l_1}}{p + f_{x_2}}$$

$$g_{s_2} = \frac{r}{p + f_{x_2}} = \frac{f_{s_1}}{p + f_{x_2}} \quad \ldots (9)$$

$$p = \frac{f_{l_1} - f_{x_2} g_{l_2}}{g_{l_2}} = \frac{f_{s_1} - f_{x_2} g_{s_2}}{g_{s_2}} \quad \ldots (10)$$

$$= \frac{f_{l_1}}{g_{l_2}} - f_{x_2} = \frac{f_{s_1}}{g_{s_2}} - f_{x_2}$$

A comparison of (9) and (10) with (6) shows that, so long as there are E.E., i.e. $f_{x_2} > 0$ ($g_{l_2} > 0$ of course):

(i) The use of factors in firm II will be pushed further in (9) than in (6), under the usual assumption of diminishing returns, i.e., pushed to the point (9) where their marginal product declines further than at (6).

(ii) Alternatively, if could be said that where E.E. prevail, the competitive product price (p) corresponding to the same output level (i.e. at the output level where g_{l_2} in (10) has the same value as g_{l_2} in (6)) is higher than socially optimal price, as could be seen from (10). Only when there are no E.E., or when these have been fully exploited (i.e. $f_{x_2} = 0$) would Pareto efficiency give the same results as competitive allocation.

Thus, it seems clear that where E.E. are present, competitive allocation fails to bring about Pareto optimality. The Marshall-Pigou conclusion is therefore perfectly valid in this case. Thus to save competition and bring about Paretian efficiency at once, a subsidy rate of $\frac{1}{p} f_{x_2}$ is to be given to firm II; or a tax rate of $\dfrac{\frac{1}{p}f_{x_2}}{1+\frac{1}{p}f_{x_2}}$ is to be levied on factors used in firm I.

Under the subsidization scheme, allocation would be determined by the two conditions:

$$g_{1_2}\left(1+\frac{1}{p}f_{x_2}\right)=\frac{f_{1_1}}{p} = \frac{w}{p}$$

or $$g_{1_2} = \frac{w}{p+f_{x_2}}$$

similarly, $$g_{s_2} = \frac{r}{p+f_{x_2}} \qquad \text{....(11)}$$

Under the tax scheme,

$$g_{1_2} = \frac{f_{1_1}}{p}\left(1 - \frac{\frac{1}{p}f_{x_2}}{1+\frac{1}{p}f_{x_2}}\right)$$

$$= \frac{f_{1_1}}{p+f_{x_2}} = \frac{w}{p+f_{x_2}}$$

Similarly, $$g_{s_2} = \frac{f_{s_1}}{p+f_{x_2}} = \frac{r}{p+f_{x_2}} \qquad \text{...(11)}$$

i.e., subsidization or taxation would lead to the same resource allocation.(4)

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(4) In the numerical example (footnote 3, p.), it can be seen that the tax rate $t = \dfrac{\frac{1}{p}f_{x_2}}{1+\frac{1}{p}f_{x_2}} = \dfrac{.1}{1+.1} = \dfrac{1}{11}$

Under tax (at tax rate t)	Under subsidy (at rate $\frac{1}{p}f_{x_2}$)
$w_1\,(1-t) = w_2$	$w_2(1+0.1) = w_1$
or £44 $(1-\frac{1}{11})$ = £40	or £40 (1+ 0.1 = £44
$r_1\,(1-t) = r_2$	$r_2\,(1+0.1) = r_2$
or £88 $(1-\frac{1}{11}$ = £80	or £80 (1 + 0.1) = £88

From the above analysis, it is clear that:

(i) Only when no E.E. exist, or when they have been fully exploited, would competitive allocation reflect Pareto efficiency. In other words, only when $x_2(l_2, s_2)$ is completely absent from function f in (7), or when $f_{x_2} = 0$, would w, r, g_{l_2}, g_{s_2}, p under (a) be the same as those under (a). In which case, no interference is warranted: competitive allocation is efficient and ensures Pareto optimality. But as soon as E.E. appear, they differ; taxes and subsidies prove necessary in order to bring them into equality with one another, i.e. to harmonise the private and social values of factor and product prices.

(ii) Since taxation and subsidization would bring about the same allocation (11) it may be tentatively suggested that where economic growthis to be maximized it would be best to resort to taxation as a source of funds for productive re-investment. This raises the conflict between welfare and growth, or in other words, between the present and future generation's welfare. There are also political difficulties involved: the taxation course implies sacrificing the welfare of the present generation for that of the future, but I do not propose to go into these here. Suffice to say that taxation would be the appropriate course if the aim is not maximizing living standard but economic growth.

(iii) As x_2 expands in (11), the E.E. it generates will continue to benefit x_1, and to raise tax revenues in future years. Such a tax is likely to be a disincentive for firm I, but it appears to have its incidence on factors used in firm I, leaving untouched that part of abnormal profit P_1. We may examine these two cases separately.

If the tax incidence falls on entrepreneur I, producing under constant returns to scale

$$P_1 = f_{l_1}l_1 + f_{s_1}s_1 + f_{x_2}x_2 - (f_{l_1}l_1 + f_{s_1}s_1)t - f_{l_1}l_1 - f_{s_1}s_1$$

$$= f_{x_2}x_2 - tC_1$$

where t is tax rate $\dfrac{\frac{1}{p}f_{x_2}}{1+\frac{1}{p}f_{x_2}}$

$$C_1 = \text{costs of firm I} = wl_1 + rs_1$$

$$= f_{l_1}l_1 + f_{s_1}s_1$$

Abnormal profit of firm I ($= f_{x_2}x_2$) is thus reduced by tC_1. So long as $f_{x_2}x_2 > tC_1$, profit tax need not have disincentive effect, except at the margins, on the production of x_1. If $f_{x_2}x_2 = tC_1$, firm I still earns normal profit. Only when $f_{x_2}x_2 < tC_1$ will tax have disincentive effect.

If tax can be shifted entirely to factors, there would be no changes in the cost figures, although factors $l_1 s_1$ are now left with less money, i.e. $C_1(1-t)$, after paying the tax. This is equivalent to a wage-and-rent cut, and abnormal profit $P_1 = f_{x_2}x_2$ remains intact. In underdeveloped countries, wage earners have high marginal propensity to consume, landowners have conspicuous and lavish expenditures patterns (sumptuous weddings, wasteful funerals, extravagant entertainment and other demonstration items), profit earners remain the main savers. National income could be written as

$$
\begin{aligned}
Y &= R + W + P \\
&= Cw + Cr + S \\
&= C + S \\
&= C + I \qquad \ldots (12)
\end{aligned}
$$

where R,W = Rent and Wage bills

P = Profit

C = Aggregate consumption

Cw, Cr = Cohsumption of wage and rent earners

I = Investment

S = Savings

This is very much the "widow's cruse" case where wage and rent-earners spend all their income and entrepeneurs save all theirs. Savings are then invested, S = I. Taxing factors of production in this model is equivalent to forced savings, which is sorely needed for capital formation. This course of action, provided it is feasible, does not interfere with the scale of firm II's production and raises reinvestible tax revenue at once without discouraging production x_1 (by leaving its abnormal profit intact).

Subsidization, while bringing about Pareto efficiency, would not have the same effects: with a subsidy rate of $\frac{1}{p} f_{x_2}$, factors used in x_2 would get better rewards, enjoy a higher living standard, x_2 will expand thus benefiting x_1, but the cost of subsidy is to be incurred, which would be made available for capital formation under the taxation alternative. This course of action is equivalent to a rise in wage and rent, and is surely recommendable if the maximand is to be the welfare of the present generation at the expense of the future.

Even if the "widow's cruse" assumption is removed, so long as the marginal propensity to save out of profit (Sp) exceeds the marginal propensity to save out of factors' earning (Sf), the share of profit still determines capital formation,(5) i.e.

$$\frac{I}{Y} = (Sp - Sf)\frac{P}{Y} + Sf \qquad \dots (13)$$

This means that for given Sp and Sf, an increase in the share or profit ($\frac{P}{Y}$) will lead to an increase in investment ($\frac{I}{Y}$). This corroborates our belief in the taxation course as the most recommendable, on growth maximization grounds, as it redistributes income in favour of re-investible profit; economic growth depending on $\frac{I}{Y}$ and $\frac{I}{Y}$ depending on profit share.

(d) Pecuniary E.E. and Pareto efficiency

The same conclusion about the allocation of resources holds for the case of pecuniary E.E., namely, the market mechanism fails to bring about Pareto efficiency, and interference is justified.

Assume that two firms produce the same commodity under perfect competition, and that there are output

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(5) See N.Kaldor [10] pp.83-100

generated pecuniary E.E. in both industries:

$$C_1 = C_1(x_1,x_2)$$

$$C_2 = C_2(x_2,x_1)$$

$$P_1 = px_1 - C_1(x_1,x_2)$$

$$P_2 = px_2 - C_2(x_2,x_1) \qquad \ldots \quad (14)$$

where C_i = costs of production of each firm ($i = 1,2$)

P_i = profit

x_i = outputs

x_i refer to the points on the respective isoquants of x_i tangent to given isocosts; i.e., points of profit maximization. This enables us to consider C_i as functions of x_i directly, not of different factor combinations (i.e. other points on the same isoquants), as only one such combination satisfies the tangency requirements (i.e. efficiency conditions).

In fact, $C_i(x_1,x_2)$ may not sufficiently distinguish pecuniary E.E., as they may still contain technological E.E. in them. For example, x_2 may favourably affect C_1 by cheapening factor prices(pecuniary) but also by increasing x_1 without any corresponding increase in costs (technological), thus resulting in cheapening the cost per unit. To obviate this difficulty, the above equations may be made more explicit

$$P_1 = px_1(l_1,s_1) - C_1\{x_1(l_1,s_1),\ x_2(l_2,s_2)\}$$

$$P_2 = px_2(l_2,s_2) - C_2\{x_2(l_2,s_2),\ x_1(l_1,s_1)\} \ldots (14a)$$

i.e. there is complete absence of technological E.E.

x_i being produced by their own paid factors l_i, s_i.[6]

Individual profit maximization under perfect competition will allocate resources as to have:

$$\frac{\partial P_i}{\partial x_i} = 0, \quad \text{i.e.,} \quad p_i = \frac{\partial C_i}{\partial x_i} \quad \text{and} \quad \frac{\partial^2 P_i}{\partial x_i^2} < 0 \text{ or } \frac{\partial^2 C_i}{\partial x_i^2} > 0 \quad \ldots (14d)$$

i.e. price equal to marginal cost (MC) and MC rising.

The socially optimal allocation could be shown by joining the two firms together, i.e.

$$P = p(x_1+x_2) - C_1(x_1,x_2) - C_2(x_1,x_2) \quad \ldots (15)$$

At stationary values, stability conditions require

$$p = \frac{\partial C_1}{\partial x_1} + \frac{\partial C_2}{\partial x_1} = \frac{\partial C_2}{\partial x_2} + \frac{\partial C_1}{\partial x_2} \quad \ldots (15a)$$

and

$$\begin{vmatrix} -c_{11}^{(1)} - c_{11}^{(2)} & -c_{12}^{(1)} - c_{12}^{(2)} \\ -c_{12}^{(1)} - c_{12}^{(2)} & -c_{22}^{(1)} - c_{22}^{(2)} \end{vmatrix} > 0 \quad \ldots (15b)$$

(where $c_{11}^{(1)} = \frac{\partial^2 C_1}{\partial x_1^2}$, $c_{12}^{(2)} = \frac{\partial^2 C_2}{\partial x_1 \partial x_2}$ etc...

and $-c_{11}^{(1)} - c_{11}^{(2)} < 0$ ... (15b)

---

(6) To spell out the cost cheapening aspect of pecuniary E.E., the above may be written more explicitly as:

$$P_1 = px_1 - s_1 r(x_2) - l_1 w(x_2)$$

$$P_2 = px2 - s_2 r(x_1) - l_2 w(x_1) \quad \ldots (14b)$$

or $P_1 = pf(l_1,s_1) - s_1 r(s_2) - l_1 w(l_2)$

$$P_2 = pg(l_2,s_2) - s_2 r(s_1) - l_2 w(l_1) \quad \ldots (14c)$$

As (14b) and(14c) will arrive at the same conclusions, as will be shown later, in what follows, I shall retain the simpler form (14a) i.e. the profit functions (14a) are free from all technological E.E. elements.

i.e. The sufficient conditions for (15a) to be maxima require the principal minors of the Hessian (15b) to alternate in sign (provided the $C^{(i)}$ functions are twice differentiable and Young's theorem holds). This implies that for true maximum,

$C^{(1)}_{11} + C^{(2)}_{22} > 0$ and $C^{(1)}_{22} + C^{(2)}_{11} > 0$, i.e.[7] price equal to marginal social cost (MSC) (in 15a) and (SMC) rising.

It could be seen at once that when E.E. are present in both firms, $\frac{\partial C_2}{\partial x_1} < 0$ and $\frac{\partial C_1}{\partial x_2} < 0$ and the equation of price with SMC would result in greater output in both firms, SMC being lower than private MC. Alternatively, it could be shown that the socially optimal price (in 15a) is lower than private competitive price(in 14).[8]

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(7) This follows from the alternation of signs of the Hessian. $\begin{vmatrix} -C^{(1)}_{11} - C^{(2)}_{11} & -C^{(1)}_{12} - C^{(2)}_{12} \\ -C^{(1)}_{12} - C^{(2)}_{12} & -C^{(1)}_{22} - C^{(2)}_{22} \end{vmatrix} > 0$ and $-C^{(1)}_{11} - C^{(2)}_{11} < 0$ jointly imply that $-C^{(1)}_{22} - C^{(2)}_{22} < 0$, or $C^{(1)}_{22} + C^{(2)}_{22} > 0$

(8) The reader could easily verify that the conclusions also hold for the more explicit forms (14b) and (14c). e.g. Under separate profit maximization

$$\frac{\partial P_i}{\partial l_i} = \frac{\partial P_i}{\partial s_i} = 0 \quad (i = 1,2) \qquad \text{or}$$

$$pf_{l_1} = pg_{l_2} = w$$

$$pf_{s_1} = pg_{s_2} = r$$

$$\text{or } p = \frac{w}{f_{l_1}} = \frac{r}{f_{s_1}} = \frac{w}{g_{l_2}} = \frac{r}{g_{s_2}}$$

The benefit to the consumers could be shown by a lower final equilibrium price: As a result of joint production, greater output is produced at the same price, thus shifting the industry supply curve to the right,

---

8. (Continued) Under joint profit maximization:

$$P = p\left[f(l_1,s_1) + g(l_2,s_2)\right] - (s_1+s_2)r(s_1,s_2)-(l_1+l_2)w(l_1l_2)$$

The same marginal conditions $\frac{\partial P}{\partial l_i} = \frac{\partial P}{\partial s_i} = 0$ lead to

$$pf_{l_1} - (l_1+l_2)\frac{\partial w}{\partial l_1} = pg_{l_2} - (l_1 + l_2)\frac{\partial w}{\partial l_2} = w$$

$$pf_{s_1} - (s_1+s_2)\frac{\partial r}{\partial s_1} = pg_{s_2} - (s_1 + s_2)\frac{\partial r}{\partial s_2} = r$$

$$\text{or}\quad p = \frac{1}{f_{l_1}}\left[w + (l_1+l_2)\frac{\partial w}{\partial l_1}\right] = \frac{1}{g_{s_2}}\left[r + (s_1+s_2)\frac{\partial r}{\partial s_2}\right] \ldots \quad (15d)$$

etc...

The conclusions are still the same: with the payment of the same wage rate as under private maximization Pareto efficiency will ensure the employment of factors up to the point where the value of their marginal productivity declines further than under individual maximization, so long as there exist pecuniary E.E., i.e., $\frac{\partial w}{\partial l_i} < 0$, $\frac{\partial r}{\partial s_i} < 0$. This implies a greater employment of resources, under the usual assumptions of diminishing returns. Alternatively, it could be said that, corresponding to the same levels of output produced and factors used, and with the same payment to factors, the socially optimal price, as could be seen from (15d) is lower than individually optimal price. Second orders conditions are also fulfilled, and are left to the reader.

which would mean a lower new price provided the demand curve of the industry has less than infinite price elasticity. Lower price, greater output are the outcome. Better profits are also secured, which are a measurement of E.E.(9)

If the assumption of perfect competition is to be removed, the above becomes a comparison between the individual profit maximization of (15a) corrected:

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(9) As a numerical example (15) may be given a form: (conventional U-shaped cost curves):

$$C_1 = 0.1x_1^2 + 5x_1 + 200 - 0.1x_2^2$$
$$C_2 = 0.3x_2^2 + 3x_2 + 100 - 0.02x_1^2$$
$$p = 15$$

Under perfect competition, individual profit maximization:

$$P_i = 15x_i - C_i$$
$$\frac{\partial P_1}{\partial x_1} = 15 - .2x_1 - 5 = 0 \quad \text{or } x_1 = 50, \quad P_1 = 90$$
$$\frac{\partial P_2}{\partial x_2} = 15 - 0.6x_2 - 3 = 0 \quad \text{or } x_2 = 20, \quad P_2 = 70$$

Total outputs of both firms $= x_1 + x_2 = 70$

Total profits $= P_1 + P_2 = 160$

Under Pareto efficiency, first order conditions require

$$P^* = 15(x_1 + x_2) - (C_1 + C_2)$$
$$\frac{\partial P^*}{\partial x_1} = 0 \quad \text{or } x_1^* = 62$$
$$\frac{\partial P^*}{\partial x_2} = 0 \quad \text{or } x_2^* = 30$$

Total output $= x_1^* + x_2^* = 92$ which is better than 70

Total profit $= P_1^* + P_2^* = 192.48$ --- 160

Note however that taxes and subsidies are normally

$$p_i = \frac{\frac{\partial C_i}{\partial x_i}}{1 + \frac{1}{\eta_{x_i p_i}}} \qquad \text{where } \eta_{x_i p_i} = \frac{dx_i}{dp_i}\,\frac{p_i}{x_i} \quad \text{or}$$

Price elasticity of demand
($\eta < 0$ generally)

$$\text{and } p_1^* = \frac{\frac{\partial C_1}{\partial x_1} + \frac{\partial C_2}{\partial x_1}}{1 + \frac{1}{\eta_{x_1 p_1}}}$$

$$\text{and } p_2^* = \frac{\frac{\partial C_2}{\partial x_2} + \frac{\partial C_1}{\partial x_2}}{1 + \frac{1}{\eta_{x_2 p_2}}} \qquad \text{.... (15c)}$$

(where x_i^*, p_i^* = socially optimal product and their prices) (i = 1,2)

It is clear that so long as there are pecuniary E.E., i.e. $\frac{\partial C_2}{\partial x_1} < 0$ and $\frac{\partial C_1}{\partial x_2} < 0$, $\cancel{P_1^* < P_1}$ $p_i^* < p_i$ (i = 1,2) the firm's downward sloping demand curve implies that if $\cancel{P_1^* < P_1}$ $p_i^* < p_i$, then $x_i^* > x_i$ (fig. 12 and 13). (P_i^* = price (capital P_i is typing mistake))

---

9. (continued) necessary devices to bring Pareto efficiency conditions about. Thus to save competition and have Pareto efficiency at once, a tax is to be imposed on P_1^* and a subsidy is to be given to P_2^* as $P_1^* = 125.6$ or $P_1^* > P_1$; but $P_2^* = 66.8 < P_2$;otherwise the lure of profit is insufficient for firm II to prduce x_2^* . Tax-subsidy is all the more necessary if one sector suffers from external diseconomies. This could be seen by changing the signs of either one of the last term of the cost functions, i.e.

$$C_1 = .1\,x_1^2 + 5x_1 + 200 + .1\,x_2^2$$

$$\text{or} \quad C_2 = .3\,x_2^2 + 3x_2 + 100 + .02x_1^2$$

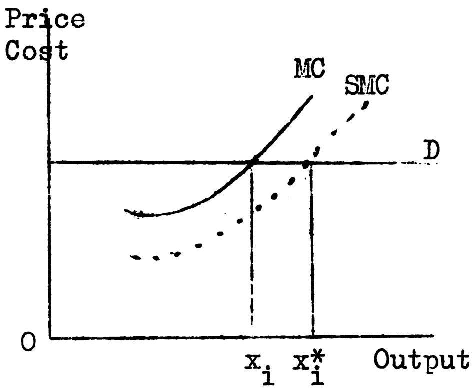

Fig. 12
EE under Perfect competition

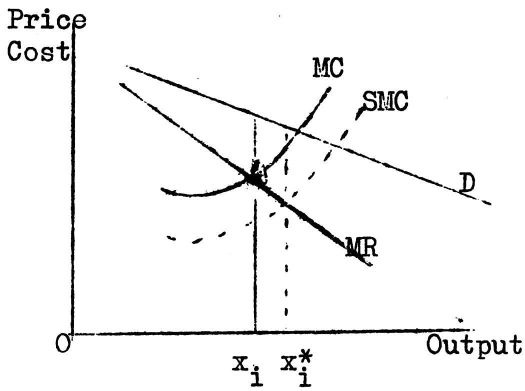

Fig. 13
EE under imperfect competition

From the above, it could be seen that Pareto efficiency conditions imply that under perfect competition, price is to be equated with social MC, not private MC. This equality ensures that the consumers rate of substitution will equal, not the individual rate of transformation but to the society's. For this to eventuate, the alternative to socialism is the well known tax-subsidy scheme, the magnitude of which can be calculated from demand and supply functions. Let these functions under private profit maximization be D(p) and S (p), and under Pareto optimality $S_i^*(p)$. The equality S(p) = D(p) determines competitive price p, and the equality $S_i^*(p)$ = D(p) determines optimal price p^* and quantity $S_i^*(p^*)$. A unit tax t_i is thus needed, such that $S_i(p^*-t_i) = S_i^*(p^*)$, solving, $t_i = f_i(p^*)$. (Wide numerical example, footnote 8, pp. 49-50)

Thus we come to the Marshall-Pigou conclusion that when E.E. are present, the firms causing E.E. produce short of, and the firms causing External Diseconomies produce beyond, the socially optimum level of output. Taxes and subsidies are necessary corrective measures to bring this social optimum about and salvage individual profit maximization. This seems to be against the popular belief that pecuniary E.E. do not call for Government interference.

The same conclusions apply to both Technological and Pecuniary E.E.

CHAPTER V

EXTERNAL ECONOMIES AND INVESTMENT TECHNIQUES

The problem of Investment Techniques has received some attention lately.(1) This chapter is neither a review nor a re-examination of Investment Techniques. It simply purports to point out the implications of E.E. for the choice of techniques, with special reference to the latest work done on the subject.(2)

The Investment Criteria put forth so far may be classified as output and growth maximizing. Maximizing current output, for a given capital stock, in a country where capital is scarce and labour superabundant, amounts to the use of relatively more labour per unit of capital. The adoption of labour-intensive techniques in this context, serves the short run policy of maximizing national income and employment, but it implies a heavier wage bill to be paid, and a heavier unit cost to incur, as the increase in output will eventually be less than the increase in labour cost. Wage earners having a high

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(1) J.J.Polak [18] pp.208-40; N.S.Buchanan [2] ch.6, A.E.Kahn [9] pp.38-61; H.B.Chenery [4] pp.76-96; W.Galenson & H.Leibenstein [7] pp.343-70; A.K.Sen [22] pp.561-84, [23] pp.466-84 and [24]. For a complete ibiliography on Investment Criteria, see United Nations [28] pp.30-45.

(2) See A.K.Sen [24] .

marginal propensity to consume, near enough to unity, savings available for investment would be small, and the rate of growth of income would be slow. Labour intensive methods would fail to pass the test if the maximand is not output but the surplus of output over current consumption, available for re-investment, i.e. if the maximand is the rate of economic growth. This growth maximizing criterion, applied in an economy where labour is abundant relative to capital, i.e., in Underdeveloped Countries, would lead to the employment of less labour per unit of capital, i.e., to the adoption of capital intensive techniques. These two criteria normally lead to conflicting practical conclusions. This could be shown in fig. 14.

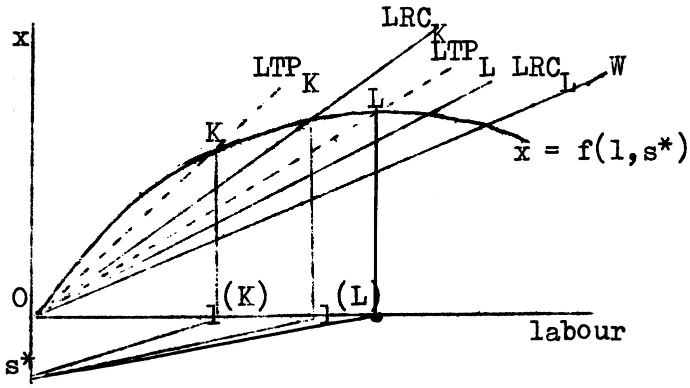

Fig. 14

Assume, for simplicity, that we have a short-run production function $x = f(l,s^*)$ homogeneous of degree one (say $x = As^{\alpha}l^{1-\alpha}$) where l is labour, s^* is a fixed capital stock somehow made available in the economy to start the ball rolling. OW shows total labour cost at a constant wage rate. The capital cost of s^*, being a fixed cost, could be represented by adding a constant to OW, i.e., the total cost schedule lies above OW and parallel to it. This would not alter the nature of the choice as the points of output maximization and surplus maximization are still L and K (fig.14) respectively.(1) Consider the Revenue function $R = As^{\alpha} l^{1-\alpha}$ and the Cost function $C = rs + wl$ where A is a constant, price is unity, or money price could be dispensed with by assuming that all costs are to be part in physical units of x. R is shown net of depreciation.

The Labour intensive technique will produce output $Ll^{(L)}$ (fig.14) employing $Ol^{(L)}$ labour, thus maximizing output x, i.e.,

$$\frac{\partial x}{\partial l} = 0 \quad \text{and} \quad \frac{\partial^2 x}{\partial l^2} < 0 \qquad \ldots (1)$$

The capital intensive technique will choose point K on the production curve, employing $l^{(K)}$ and maximizing the surplus of output over cost (x - C), i.e.

$$\frac{\partial}{\partial l}(x - C) = 0$$
$$\frac{\partial^2}{\partial l^2}(x - C) < 0 \qquad \ldots (2)$$

---

(1) Except when this total cost curve lies above L, but below K on the production function $x = f(l_1 s)$. In that case, the labour intensive technique (L) would result in a negative re-investible saving. and thus a negative rate of growth. (L) thus drops out leaving (K) alone for consideration.

If the Cost function is reduced, for simplicity, to labour cost alone as shown in fig. 14, and if wage earners are assumed to spend what they earn, capitalists to save what they get, the adoption of the capital intensive technique, in this context, leads to maximizing re-investible savings, i.e.

$$\frac{\partial}{\partial l}(x - wl) = 0 \quad \text{and} \quad \frac{\partial^2}{\partial l^2}(x - wl) < 0$$

Marginal conditions (1) and (2) determine labour employment in each case.

If the production function $f(l_1 s)$ is homogeneous of degree one, if the state of the arts and wage rate remain unchanged over time, and if we keep re-investing the surplus $(x - wl)$ realised each year, we shall have:

$$\frac{s_1}{s_2} = \frac{l_1}{l_2} = \frac{x_1}{x_2} \quad \text{or} \quad x_2 = \frac{s_2}{s_1}x_1$$

and $$\frac{s_1}{s_n} = \frac{l_1}{l_n} = \frac{x_1}{x_n} \quad \text{or} \quad x_n = \frac{s_n}{s_1}x_1$$

where s_i, l_i, x_i $(i = 1,2,\ldots,n)$ represent factors used and output produced in each year from 1 to n.

The sum total of output thus produced from 1 to n is

$$x_1 + x_2 + x_5 + \ldots\ldots\ldots + x_n$$

$$= x_1 + \frac{s_2}{s_1}x_1 + \frac{s_3}{s_2}\frac{s_2}{s_1}x_1 + \ldots\ldots\ldots + \frac{s_n}{s_1}x_1$$

$$= x_1\left(1 + \frac{s_2}{s_1} + \frac{s_3}{s_1} + \ldots\ldots\ldots + \frac{s_n}{s_1}\right)$$

or $$\sum_{i=1}^{n} x_i = \frac{x_1}{s_1}\sum_{i=1}^{n} s_i \qquad \ldots (3)$$

Similarly, $$\sum_{i=1}^{n} x_i = \frac{x_1}{l_1}\sum_{i=1}^{n} l_i$$

So, each of the factors used and output produced will grow at the same rate, if this re-investment process continues and production is under constant returns to scale, i.e.

$$\frac{\Delta s}{s} = \frac{\Delta l}{l} = \frac{\Delta x}{x}$$

$$\text{or} \quad \frac{\frac{\Delta s}{\Delta t}}{s} = \frac{\frac{\Delta l}{\Delta t}}{l} = \frac{\frac{\Delta x}{\Delta t}}{x}$$

$$\text{or when } \Delta t \longrightarrow 0, \quad \frac{\dot{s}}{s} = \frac{\dot{l}}{l} = \frac{\dot{x}}{x}$$

In other words, if the surplus $(x - wl)$ is re-invested, the capital stock s, will become s_2 in year 2, s_3 in year 3 and so on. With $s_2 > s_1$ and $s_3 > s_2$, the partial production function of fig.14 will shift upward overtime, and the locus of all the relevant points satisfying (1) and (2) will lie on the prolonged OL and OK respectively. I shall call them the two long run product curves LTP_K and LTP_L. The same degree of capital intensity (i.e. $\frac{Os_i^{(K)}}{Ol_i^{(K)}} = a$ and $\frac{Os_i^{(L)}}{Ol_i^{(L)}} = b$, a and b being two constants) will be adhered to over time. In other words, all the triangles $Ol_i^{(K)}s_i^{(K)}$ and $Ol_i^{(L)}s_i^{(L)}$ are similar respectively (K and L refer to capital-, and labour-intensive techniques respectively; $i = 1,2,\ldots..$nth year). If a fixed cost of capital (rs) is to be added each time, the short run fixed cost will be parallel to OW, but the locus of all the relevant points corresponding to each technique will havo a greater slope than does OW. I shall call them long run cost curves LRC_K and LRC_L respectively with LRC_K having a greater slope than LRC_L. It is also interesting to note that income distribution is different at different points on the Production Function (Ox) (fig. 14). The choice of point K for example, implies a maximization of the capitalists' share. The choice of point L means an improvement of Labour's share. However, once any point on the production function Ox has been chosen, in other words, when given income-distribution has been decided upon once for all at the beginning, the relative shares of capitalists and wage earners, will remain unchanged over time. So once the model starts working, the "widow's cruse" assumption is no longer needed. The assumption of a constant marginal propensity to save out of wage and profit respectively will be sufficient for the model to work, as income distribution does not change over time.

---

1. This should be obvious. If proof should be needed fig. 14 could be reproduced. In fig. 14a, from similar triangles OK_1W_1 and OK_2W_2, we get: $\frac{OK_1}{OK_2} = \frac{OW_1}{OW_2} = \frac{K_1W_1}{K_2W_2}$. From similar triangles $OK_1L_1{}^{(K)}$ and $OK_2l_2{}^{(K_2)}$ we get $\frac{OK_1}{OK_2} = \frac{K_1 l_1{}^{(K_1)}}{K_2 l_2 \cancel{(K_2)}} = \frac{Ol_1{}^{(K_1)}}{Ol_2{}^{(K_2)}}$

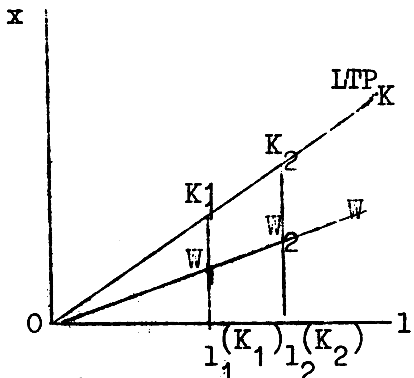

Fig. 14a

$\therefore \frac{K_1W_1}{K_2W_2} = \frac{K_1 l_1{}^{(K_1)}}{K_2 l_2{}^{(K_2)}} \therefore \frac{K_1W_1}{K_1 l_1}(K_1) = \frac{K_2W_2}{K_2 l_2}(K_2)$

i.e., capitalists' share $\frac{K_iW_i}{K_i l_i}(K)$ $(i = 1,2,...n)$ does not change over time. Similarly, for the labour intensive technique (K_i are to be replaced by L_i in fig. 14a and) the same results hold, i.e. $\frac{L_iW_i}{L_i l_i}(L)$ remains constant over time.

The question now is that of comparing

$$\sum_{i=1}^{U} x_i^{(K)} \quad \text{with} \quad \sum_{i=1}^{U} x_i^{(L)} \quad \ldots\ldots\ldots (5)$$

at any time U $(1 < U < n)$ to see which technique gives a greater time series of output up to U.

In the light of our above analysis of External Economies, however, this comparison (5) leaves much to be desired. When output x or factors $(1_x, S_x)$ used in sector x confer E.E. on some other sectors, y,z of the Economy, the acceptance or rejection of a technique on the basis of comparison (5) would not do justice to the project in question. Suppose sectors y or z benefit from E.E. generated by x or 1_x or S_x. Then if we assume, for simplicity, that y and z are also produced under conditions of constant returns to scale, we can write

$$y = y(1_y, S_y, x) = y_1 1 \pm y_s s + y_x x$$

Similarly $z = z_1 1 \pm z_s s \pm z_x x$

where $y_1 1 = \dfrac{\partial y}{\partial 1_y} 1_y$ etc...

x, y, z = outputs x,y and z

$1_x, s_x$ = factors used in sector x.

Where there are factor-generated technological E.E. write $Y(1_y, s_y, 1_x, s_x)$ for example, instead of $y(1_y, s_y, x)$.

If y and z are the yearly outputs, the total external effects (E_x) conferred on y and z by x from year 1 to n are:

$$E_{x_i} = \sum_{1}^{n} Y_x x \pm \sum_{1}^{n} z_x x \ldots\ldots\ldots (6)$$

The assessment of the social value of the two techniques should take E_{x_i} into account, i.e., comparison (5) should become that of

$$\sum_{1}^{U} \bar{x}_i^{(K)} \quad \text{with} \quad \sum_{1}^{U} \bar{x}_i^{(L)} \quad \ldots\ldots\ldots\ldots 7$$

Where $\sum_{1}^{U} \bar{x}_i^{(K)} = \sum_{1}^{U} \bar{x}_i^{(K)} \pm E_{xi}^{(K)}$

and $\sum_1^U \bar{x}_i(L) = \sum_1^U x_i(L) + E_{x_i}^{(L)}$
i.e., the social value $\bar{x}_i^{(k)}$ and $\bar{x}_i^{(L)}$ of the two techniques K and L respectively includes both the time series of outputs produced by them and the time series of External Economies $E_x^{(K)}$ and $E_x^{(L)}$ the two techniques would generate, from year 1 to U, U being the year interested in. Comparison should be made between these social values, and which technique is to be chosen depends on whether

$$\sum_1^U \bar{x}_i^{(K)} \gtrless \sum_1^U \bar{x}_i^{(L)}$$

Since all these output figures are expected future values some discount has to be made. The introduction of U is one way of discounting them: any output produced beyond U is discounted to zero, and any output up to U has its full undiscounted value.

Similarly, when E.E. accrue in the forms of cost cheapening, the procedure remains the same. If as x expands, factor price W becomes cheaper in y-sector, then $\frac{1}{y}\frac{\partial w}{\partial x}$ is a measure of this cost saving in any one year. Output y will usually expand to fulfill marginal conditions. If y- surplus is reinvested, these annual cost savings will have a time series themselves. This is due to x, and therefore, in the evaluation of x, this cost savings time series will have to be deducted from the cost of x-production. Thus the social costs of x are lower than its private costs. This case is left to the reader to visualise in more detail.

---

1. An alternative method of discounting future output is to apply an arbitrary discount rate $\rho = \frac{1}{1+r}$ and compare $\sum_1^n \rho^i \bar{x}_i^{(K)}$ with $\sum_1^n \rho^i \bar{x}_i^{(L)}$ (n is the life of a project; if a project is assumed to last for ever, n= ∞

(Footnote 3 continued)
If a rate of technological progress $\theta = e^{mt}$ is allowed for, the above become: $\sum_{1}^{n} \rho^{i} \theta_i x_i^{-(K)}$ and $\sum_{1}^{n} \rho^{i} \theta_i x_i^{-(L)}$ where $\theta_i = e^{mt} = e^{mi}$ where t takes all values i from year 1 to n.

The E.E. generated by the two techniques need not be the same. The capital intensive technique for example, may give rise to technological or pecuniary E.E. or both, while the labour intensive technique may not do so to the same extent. Looked at in this light, the capital, or labour-intensive techniques may best be applied in one country rather than in another, in the same country at one time rather than at another, depending, beside factor endowment, on the economic conditions, the stage of economic development, the atmosphere of growth.
(E.G., Spirit of enterprise, of enthusiasm following an independence movement, and so on). These are at least partly accounted for by E_{x_i}. Capital intensive techniques, for example, would postulate as prerequisite conditions the existence of some skilled labour, a certain state of the arts, a certain atmosphere in which it can survive and prosper. Without a suitable atmosphere, it would be abortive and doomed to premature extinction. An often cited example is the case of Pakistan where mechanisation of agriculture took place at a time when skilled labour was not available. Tractors broke down and could not be fixed, due to the absence of mechanics, garage and service stations etc...

In such cases, capital intensive techniques are to die of suffocation and inanition, not having the E.E. needed; and would not generate any E.E. In these situations, labour intensive methods may generate more E.E. It would be advisable to use labour intensive techniques in earlier stages of economic development - except where machines are necessary substitutes of of skilled labour needed and not available - to build up conditions favourable to economic growth, e.g., dams, roads, etc..

When these conditions are mature, when awakening has taken place, the adherence to the traditional labour intensive method may have the effects of perpetuating low productivity and slowing down economic growth. These are necessary conditions for a technique to generate E.E. This is important, especially in underdeveloped countries where capital stock is small, marginal efficiency of capital high, and the effects of E.E. are all the more pronounced than they would be in developed countries.

The introduction of E.E. might reverse the choice made without them. If, for example, the excess of $E_{x_i}^{(L)}$ over $E_{x_i}^{(K)}$ is large enough, then, adding them up to the total products, streams may make the surplus of L greater than that of K, thus swinging our decision in favour of L on the very grounds of surplus maximization (2). Labour intensive technique in such a case would maximize both current and future output, i.e. both current output and the rate of growth of that output. The same thing could be said about K technique if $E_{x_i}^{(K)}$ is sufficiently larger than $E_{x_i}^{(L)}$. Thus the consideration of E.E. may cause a project or technique[3] rejected to be accepted and vice versa.

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3. Project and technique could be used interchangeably here as only the best technique would be chosen for each project, and thus for each project, only one technique is relevant.

Thus, in terms of fig. 14, if the total product curve $f(1,s^*)$ lies entirely below the cost curve (OW), the project would be rejected. With the introduction of E_x, it might well be accepted. That is the case of most overhead

Investments. Railways, for example, involve heavy initial expenditures and losses perhaps, but their contribution to national income and welfare will justify their being undertaken. As External Economies E_x do not accrue to x-producers, a subsidy would have to be paid for x to be undertaken by private entrepeneurs. These subsidies may be financed by taxes raised on beneficiary sectors. If x is a public sector no subsidy would be needed: this public business may afford to run at a loss, which loss could be made up by taxes raised in the Economy as a whole.

Thus, in the assessment of the social merit of an Investment technique, comparison (9) should have to be used instead of (5). In other words, the choice of Investment techniques made in the light of this overall picture may be entirely different from that made by using other criteria. It would be different from "Polak - Buchanan-Kahn-Chenery[2], from Galenson-Leibenstein[3], from Sen's[4] conclusions.

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1. This problem has been discussed in earlier chapters (ch. 111. & 1V).
2. J. J. Polak (18) pp 208-40; N. S. Buchanan (2)ch.6, A.E.Kahn (9) pp 38-61; H.B. Chenery (4) pp.76-96.
3. W. Galenson & H. Leibenstein (7) pp. 343-70;
4. A.K. Sen (22) 561-84; (23)pp. 466-84;(24)

Fig. 14 could be adjusted to reflect E.E. very roughly. In fig. 15, factor-generated E.E. are shown. In y-sector the short run production function $y(s_y^*, l_y)$ is shown in quadrant III. With a given capital stock S_y^* (not shown),

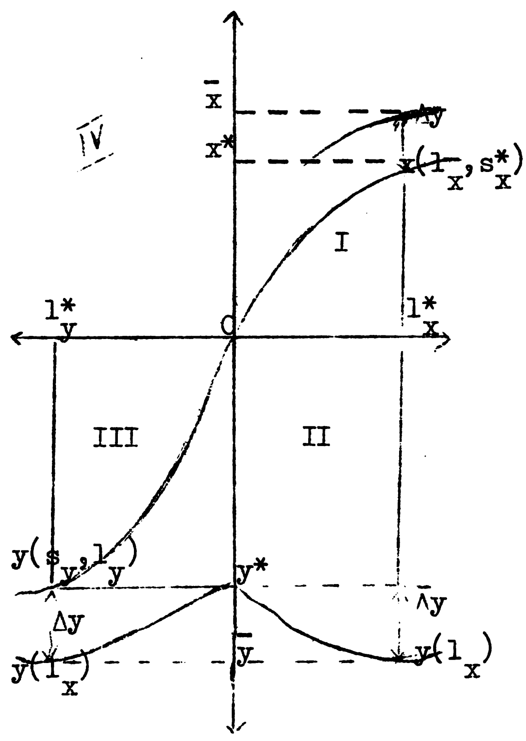

Figure 15

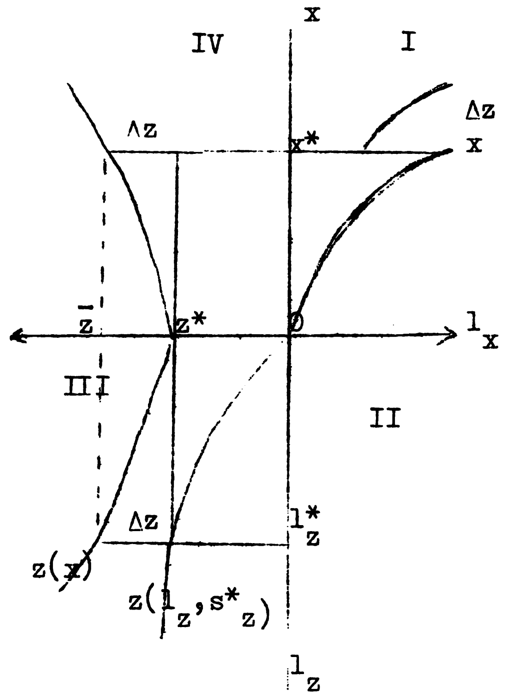

Figure 16

the employment of l^*_y will produce y. If l_x generates E.E. which benefit y, then l^*_x will raise y by $\Delta y = \bar{y} - y^*$ (quadrant 11 where only the contribution of l_x to y is shown). That Δy is usually added to y in quadrant 111, but is really attributable to l_x. Thus in the assessment of the social merit of x, Δy should be added, not to y (in quadrant 111) but to x (in quadrant 1), after being converted into appropriate units. This would be equivalent to the partial production function x (l_x, S^*_x) being shifted upward, which would push both point K (surplus maximizing 2) and L (output maximizing 1) to the right, provided factor prices do not change. This implies a greater labour absorption for both techniques, and a larger re-Investment surplus.

In figure 16, output-generated E.E. are shown. A given of x produced (quadrant 1) is shown to increase the production of Z by $\Delta Z = \bar{Z} - Z^*$ (quadrant 1V). This ΔZ is usually added up to Z^* (l_z, S_z) in quadrant 111, produced with a given capital stock S_z (not shown), and l_z^*. In fact, this ΔZ is attributable to the presence of output x, and therefore should be added to x in the assessment of the latter's social merit. The shape of E.E. is convex to origin in fig. 16, concave in fig. 15. This reflects the fact that, at certain stages, E.E. may exhibit increasing returns before eventually reaching the diminishing returns phase. Thus, considered in isolation, only x^* is imputed to x in both cases. But with E.E., $\bar{x} = x^* \pm \Delta y \pm \Delta z$, or more precisely

$$\bar{x} = x^* \pm \Delta y \, P_1 \pm \Delta Z P_2 \quad \text{where } P_1 = \frac{P_y}{P_x}$$

$$P_2 = \frac{P_z}{P_x}$$

i.e., x is taken as unit of measurement. This roughly corresponds to our $\bar{x}$ in (7) above, in a particular year. In terms of fig. 14, the long run social total product curve thus corrected would lie above LTP_k or LTP_L respectively, as the case may be, when E.E. are present and below them when external Diseconomics prevail.

If the surplus of output over cost realised in each sector is re-invested, x,y,z, will grow at an exponential rate. In figure 17, K and L represent the time series

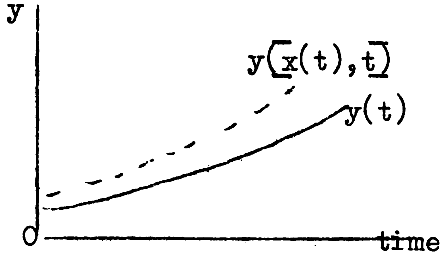

Fig. 18

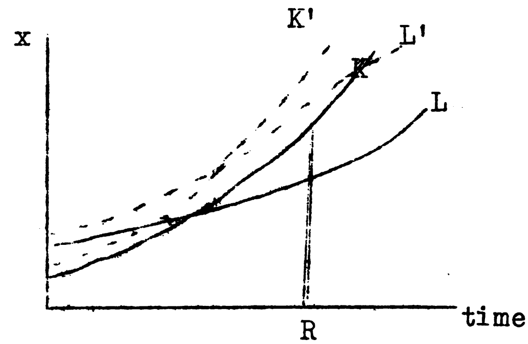

Fig. 17

of output x produced by adopting the capital -, and labour-intensive techniques respectively. The full lines K & L show their rate of growth in the absence of E.E. Labour intensive technique (1) gives a greater initial output but less surplus available for re-Investment and consequently a slower rate of growth than the capital intensive technique (K) would give. Provided t is large enough, there would eventually come a period R^1 where the cumulative output produced by K-technique will catch up with that produced by the L-technique, i.e. $\sum_{i=1}^{R} x_i^{(K)} = \sum_{i=1}^{R} x_i^{(L)}$

1. R is called "Period of Recovery" by A. K. Sen (24) p. 32 & 33.

For any period of time to the left of R., L-technique is to be preferred; and to the right of R, K-technique is to be chosen.

Now if x confers E.E. on some other sector y, the time series of y(t) (fig. 18) will shift upward, and becomes the broken line y(t,x(t)) in fig. 18. The producers of y reap where they have not sown: this net increase in y due to output x is really attributable to x, and should be added to L and K in fig. 17. This would be equivalent to an upward shift of L and K, and normally displace R to the right.

When technological E.E. are generated by factor l_x, L (fig.17) would shift upward more than would K, as it employs relatively more labour. This will move R to R^1 to the right of R, in favour of L-technique. In the absence of E.E., any period of time longer than R will speak in favour of K-technique. Now the period R R^1 would be in favour of L: The arrival of R has been delayed by E.E..

On the other hand, if E.E. are generated by S_x, K would move upward more than would do L, thus shifting R to R" (R" being to the left of R), thus tilting the choice in favour of K. technique. Any output during period R"R would be produced by L-Technique, in the absence of E.E., and by K. Technique, when E.E. prevail. E.E. thus bring period R about sooner.

The effects of pecuniary E.E. may be analysed in a similar way: When y-sector benefits from cheaper factor price W, as a result of x-expansion, or l_x used in x-sector, y-producers will employ more labour ($l^{(K^1)}$ instead of $l^{(K)}$ in fig. 19) and expand output y from $Y^{(K)}$ to $Y^{(K^1)}$ if they are to equate Marginal productivity of labour to real wage; i.e. if they use capital intensive technique. Re-investible surplus may be greater,

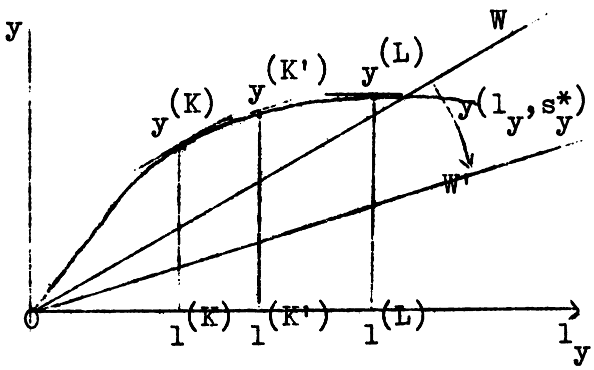

Fig. 19

depending on the extent to which W falls. In any case, in terms of fig. 18, the time series Y(x(t),t) will be higher than y(t) and the intercept of Y(t,x(t) with the y-axis would be higher than that of Y(t). If y-producers have chosen to maximize output y ($Y^{(L)}$ in fig. 19) before, they would not employ any more labour and produce any more output, as output has been maximised. But the re-investible surplus of output over labour cost as a result of x-generated pecuniary E.E. in the form of w-cheapening will be larger, and the rate of growth of y will be faster. In terms of fig. 18 Y (x(t),t) will have the same origin as Y(t) at t=o, but will then diverge from y(t) in the upward direction as time goes on.

Similarly, if x-generated E.E. accrue in the form of r-cheapening, with the same initial surplus, a greater capital stock would be obtainable, which implies an upward shift of y(1,s) and a greater reinvestible surplus of y over labour cost if W remains unchanged. This means both a higher origin for y(t) at t=o and a greater rate of growth of y over time.

All these benefits of E.E. are, of course, due to x, and should be taken into account in the assessment of the social value of Investment techniques used in x. This could be done by adding these streams of E.E. to the K & L lines of fig. 18. The result would be a displacement of R, which will alter the choice made without any consideration paid to E.E. Again, taxes and subsidies may prove necessary for private entrepeneurs to undertake the production of X[1].

The reverse of this all could be said about External Diseconomies.

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1. This point has been discussed earlier. ch. 111 & 1V and footnote 1 p.69.

The appraisal of Investment Techniques is thus to be made in the light of this dynamic overall picture, where E.E. play an important part, rather than in isolation. Where no E.E. exist or all E.E. are exploited, the private and social values of Investment Techniques are the same, but where E.E. are present, they diverge. This problem is very important, as a well chosen Investment Technique is the core of the Investment problem, and capital formation directly determines the rate of economic growth.

Chapter VI

EXTERNAL ECONOMIES AND BALANCED GROWTH

The idea of correlated growth linked with EE goes as far back as A. Marshall: "The growth of correlated branches of industries which mutually assist one another..." 1. Next, A. Young and Rosenstein-Rodan[2] could be considered the heralds of the Balanced Growth doctrine. R. Nurkse[3] developed their idea along the same line. Like A. Young, he interpreted A. Smith's market size in terms of demand: "Where any single enterprise might appear quite inauspicious and impracticable, a wide range of projects in different industries may succeed because they will all support each other..." [4]. A. Lewis[5] joined in with a new emphasis put on the balance of Agriculture and Industry, Export and home consumption.

The balanced growth doctrine has been challenged by M. Fleming, J. Sheahan, R. Findlay, A.O. Hirschman and P. Streeten[6].

The concept of balanced growth seems to have a plastic meaning, susceptible of various interpretations. It may mean a simultaneous development of all industries, or, it may mean concentrated growth[7]; it may mean the rate of growth of outputs as determined by the community's marginal propensity to consume agricultural products as compared with

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1. A. Marshall [13], p. 441.
2. A. Young [31], pp. 527-42; P.N. Rosentein-Rodan [21], pp. 202-11.
3. & 4. R. Nurkse [15], chapter I, especially p. 13 & 19.
5. W.A. Lewis [12], pp. 275-83.
6. M. Fleming [6], pp. 241-56; J. Sheahan [25], pp. 183-97; R. Findlay [5], pp. 339-46; A.O. Hirschman [8]; P. Streeten [26], pp. 167-90.
7. J. Sheahan, for example, writes: "If balanced growth includes the case of concentrated development of one or a few industries, then the point is acceptable; if it is meant to support the policy of matching production to demand changes by expanding most or all industries simultaneously, the position is untenable". [25], p. 184.

manufactures[8], or the rate of outputs growth as determined by the "pattern of consumers' demand for each others' products" [9].

In this paper, I shall define Balanced growth as accruing when all variables grow at the same rate. For example, a balanced growth of x, y, z exists when $\frac{\dot{x}}{x} = \frac{\dot{y}}{y} = \frac{\dot{z}}{z}$ (where $\dot{x} = \frac{dx}{dt}$ etc...). I shall examine the problem of balanced growth where EE are present, or, more precisely, I shall examine the conditions of feasibility and desirability of balanced growth, under static and dynamic EE.

FEASIBILITY OF BALANCED GROWTH

I. STATIC EXTERNAL ECONOMIES

When EE are present, there are technically necessary conditions for balanced growth. When these conditions are not fulfilled, insisting on balanced growth is asking the impossible. The one-factor case will be examined here, and the results will then be generalised to the many-factor case.

Consider the two production functions:

$$\begin{aligned} x_1 &= x_1[l_1(t),\ x_2(t)] \\ x_2 &= x_2[l_2(t),\ x_1(t)] \end{aligned} \qquad \text{.......... (1)}$$

where l_i = factor of production $(i=1,2)$

x_i = output $(i=1,2)$

t = time

The proportional rates of change of outputs are:

$$\begin{aligned} \frac{\dot{x}_1}{x_1} &= \frac{\partial x_1}{\partial l_1}\frac{\dot{l}_1}{x_1} + \frac{\partial x_1}{\partial x_2}\frac{\dot{x}_2}{x_1} \\ \frac{\dot{x}_2}{x_2} &= \frac{\partial x_2}{\partial l_2}\frac{\dot{l}_2}{x_2} + \frac{\partial x_2}{\partial x_1}\frac{\dot{x}_1}{x_2} \end{aligned} \qquad \text{. . . . . . . (2)}$$

where $\dot{x}_i = \frac{dx_i}{dt}$; $\dot{l}_i = \frac{dl_i}{dt}$

8. W.A. Lewis [12], p? 278 and 283.

9. P. Streeten [26], p. 170.

Or, in elasticity forms, putting $\lambda_i = \frac{\dot{l}_i}{l_i}$; $\mu_i = \frac{\dot{x}_i}{x_i}$

and $\eta x_i l_i = \frac{\partial x_i}{\partial l_i} \frac{l_i}{x_i} = \frac{\partial(\log x_i)}{\partial(\log l_i)}$ = elasticity of x_i with respect to l_i etc...

$$\begin{aligned} \mu_1 &= \eta x_1 l_1 \lambda_1 + \eta x_1 x_2 \mu_2 \\ \mu_2 &= \eta x_2 l_2 \lambda_2 + \eta x_2 x_1 \mu_1 \end{aligned} \quad \ldots\ldots\ldots (3)$$

(1) <u>One-factor case</u>

(a) Balanced growth of outputs, i.e., $\mu_1 = \mu_2 = \mu (= \text{constant})$.

Equation (3) becomes

$$\begin{aligned} \mu(1 - \eta x_1 x_2) &= \eta x_1 l_1 \lambda_1 \\ \mu(1 - \eta x_2 x_1) &= \eta x_2 l_2 \lambda_2 \end{aligned}$$

Solving, $\lambda_1 = \frac{\mu(1 - \eta x_1 x_2)}{\eta x_1 l_1}$

$$\text{and} \quad \lambda_2 = \frac{\mu(1 - \eta x_2 x_1)}{\eta x_2 l_2} \quad \ldots.(4)$$

λ_1 and λ_2 are the required rates of growth of inputs in this one-factor case with output-generated static EE, if balanced growth of outputs is to be achieved. In the particular case where (1) are homogeneous of degree one in all their arguments respectively, in other words, where there are constant returns to scale, $\eta x_1 l_1 + \eta x_1 x_2 = \eta x_2 l_2 + \eta x_2 x_1 = 1$, and (4) becomes:

$$\begin{aligned} \lambda_i &= \mu \\ \lambda_1 &= \lambda_2 \end{aligned} \quad \ldots\ldots (4a)$$

i.e., the balanced growth of outputs, when production is under constant returns to scale, requires all inputs to grow at the same rate, which is equal to the rate of growth of outputs themselves. This result is obvious enough: if doubling inputs would double outputs, then the conditions for doublind outputs is doubling inputs.

(b) Balanced growth of inputs, i.e., $\lambda_1 = \lambda_2 = \lambda$

Equation (4) could be written

$$\begin{aligned} \mu_1 &= \eta_{x_1 l_1}\lambda + \eta_{x_1 x_2}\mu_2 \\ \mu_2 &= \eta_{x_2 l_2}\lambda + \eta_{x_2 x_1}\mu_1 \end{aligned} \qquad \ldots\ldots (5)$$

Solving

$$\mu_1 = \frac{\begin{vmatrix} \eta_{x_1 l_1}\lambda & -\eta_{x_1 x_2} \\ \eta_{x_2 l_2}\lambda & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\eta_{x_1 x_2} \\ -\eta_{x_2 x_1} & 1 \end{vmatrix}}$$

$$= \frac{\lambda(\eta_{x_1 l_1} + \eta_{x_1 x_2}\eta_{x_2 l_2})}{1 - \eta_{x_1 x_2}\eta_{x_2 x_1}}$$

Similarly, $$\mu_2 = \frac{\lambda(\eta_{x_2 l_2} + \eta_{x_2 x_1}\eta_{x_1 l_1})}{1 - \eta_{x_1 x_2}\eta_{x_2 x_1}} \qquad \ldots\ldots (5a)$$

In the particular case of constant returns to scale in both sectors, this becomes:

$$\mu_1 = \frac{\lambda[1 - \eta_{x_1 x_2} + \eta_{x_1 x_2}(1 - \eta_{x_2 x_1})]}{1 - \eta_{x_1 x_2}\eta_{x_2 x_1}} = \lambda$$

Similarly, $\mu_2 = \lambda$, or $\mu_1 = \mu_2 = \lambda$ $\quad \ldots\ldots (5b)$

Thus μ_i (i=1,2) are the required rates of growth of outputs in two sectors if the balanced growth of inputs is to be maintained. μ_i depend on the values of different elasticities. In the particular case of constant returns to scale, the two warranted rates of growth of outputs are equal to each other and equal each to the rate of inputs' growth. In the general case, these rates of growth of outputs are unequal to each other and to the rate of growth of inputs, i.e., $\mu_1 \neq \mu_2 \neq \lambda$ (5a).

(2) <u>Many-factor case</u>

Let us now consider the case of many factors, l_i and s_i (i=1,2) in the two sectors, when there exist static EE. The production functions are:

$$x_1 = x_1[\,l_1(t),\ s_1(t),\ x_2(t)]$$
$$x_2 = x_2[\,l_2(t),\ s_2(t),\ x_1(t)] \qquad \ldots\ldots\ (6)$$

The proportional rates of outputs growth are:

$$\frac{\dot{x}_1}{x_1} = \frac{\partial x_1}{\partial l_1}\frac{\dot{l}_1}{x_1} + \frac{\partial x_1}{\partial s_1}\frac{\dot{s}_1}{x_1} + \frac{\partial x_1}{\partial x_2}\frac{\dot{x}_2}{x_1}$$

$$\frac{\dot{x}_2}{x_2} = \frac{\partial x_2}{\partial l_2}\frac{\dot{l}_2}{x_2} + \frac{\partial x_2}{\partial s_2}\frac{\dot{s}_2}{x_2} + \frac{\partial x_2}{\partial x_1}\frac{\dot{x}_1}{x_2}$$

or, in elasticity forms,

$$\mu_1 = \eta x_1 l_1 \lambda_1 + \eta x_1 s_1 \gamma_1 + \eta x_1 x_2 \mu_2$$
$$\mu_2 = \eta x_2 l_2 \lambda_2 + \eta x_2 s_2 \gamma_2 + \eta x_2 x_1 \mu_1 \qquad \ldots\ (6a)$$

where $\mu_i = \frac{\dot{x}_i}{x_i}$; $\lambda_i = \frac{\dot{l}_i}{l_i}$; $\gamma_i = \frac{\dot{s}_i}{s_i}$ (i=1,2)

(a) Balanced growth of all inputs, i.e., $\lambda_i = \gamma_i = \lambda$ (= constant)

If we want all inputs to grow at a balanced rate, which, although actually a function of time, is assumed ,for simplicity, to be a constant (λ) over time. Equation (6a) now becomes:

$$\mu_1 - \eta x_1 x_2 \mu_2 = \lambda(\eta x_1 s_1 + \eta x_1 l_1)$$
$$\mu_2 - \eta x_2 x_1 \mu_1 = \lambda(\eta x_2 s_2 + \eta x_2 l_2) \qquad \ldots\ldots\ (7)$$

Solving, $$\mu_1 = \frac{\lambda[\eta x_1 s_1 + \eta x_1 l_1 + \eta x_1 x_2(\eta x_2 s_2 + \eta x_2 l_2)]}{1 - \eta x_1 x_2 \eta x_2 x_1}$$

and $$\mu_2 = \frac{\lambda[\eta x_2 s_2 + \eta x_2 l_2 + \eta x_2 x_1(\eta x_1 s_1 + \eta x_1 l_1)]}{1 - \eta x_1 x_2 \eta x_2 x_1} \qquad \ldots\ (7a)$$

In the particular case of constant returns to scale in both sectors, i.e., (6) are homogeneous of degree one in all their arguments, replacing $\eta x_1 s_1 + \eta x_1 l_1$ by $1 - \eta x_1 x_2$ and $\eta x_1 s_2 + \eta x_2 l_2$ by $1 - \eta x_2 x_1$ in (7a) gives:

$$\mu_1 = \lambda$$
$$\mu_2 = \lambda$$
$$\text{or} \quad \mu_1 = \mu_2 = \lambda \quad \ldots\ldots \quad (7b)$$

So if we want to maintain balanced growth of all inputs, i.e., $\lambda_i = \gamma_i$ where EE are present, the required rates of growth of outputs are given by μ_1 and μ_2 in (7a). Put in another way, if factors' supply in a country grow at a balanced rate, then the rates of growth of outputs we could obtain are μ_1 and μ_2 in (7a). These μ_i depend on the value of different elasticities, and are usually unequal to each other, but proportional to the rate of growth of inputs. Only in the particular case of constant returns to scale are the two outputs required to grow at the same rate which is equal to that of inputs, i.e., $\mu_1 = \mu_2 = \lambda$ in (7b). These results could be verified by using the production functions of the forms:

$$x_1 = l_1^{\alpha_1} s_1^{\alpha_2} x_2^{\alpha_3}$$
$$x_2 = l_2^{\beta_1} s_2^{\beta_2} x_1^{\beta_3}$$

with $\alpha_1 + \alpha_2 + \alpha_3 \neq 1$ and

$\beta_1 + \beta_2 + \beta_3 \neq 1$ in the general case,

and $\alpha_1 + \alpha_2 + \alpha_3 = \beta_1 + \beta_2 + \beta_3 = 1$ in the case of constant returns to scale. The results could be obtained by simply replacing

$\eta x_1 l_1$ by α_1 ; $\eta x_1 s_1$ by α_2 ; $\eta x_1 x_2$ by α_3

$\eta x_2 l_2$ by β_1 ; $\eta x_2 s_2$ by β_2 ; $\eta x_2 x_1$ by β_3 , in (7a):

$$\mu_1 = \frac{\lambda[\alpha_1 + \alpha_2 + \alpha_3(\beta_2 + \beta_1)]}{1 - \alpha_3 \beta_3}$$

$$\mu_2 = \frac{\lambda[\beta_2 + \beta_1 + \beta_3(\alpha_2 + \alpha_1)]}{1 - \alpha_3 \beta_3}$$

fo the general case, and remembering that $\alpha_1 + \alpha_2 + \alpha_3 = \beta_1 + \beta_2 + \beta_3 = 1$

where x_1 and x_2 are homogeneous of degree one,

$$\mu_1 = \frac{\lambda[1 - \alpha_3 + \alpha_3(1-\beta_3)]}{1 - \alpha_3\beta_3} = \frac{\lambda(1 - \alpha_3\beta_3)}{1 - \alpha_3\beta_3} = \lambda$$

and similarly, $\mu_2 = \lambda$

for the case of constant returns to scale.

(b) Balanced growth of each input in both sectors,

$$\text{i.e.,} \quad \frac{\dot{l}_i}{l_i} = \lambda \quad ; \quad \frac{\dot{s}_i}{s_i} = \gamma \; ; \quad \text{and} \quad \lambda \neq \gamma \quad (i=1,2)$$

Equation (6a) now becomes

$$\begin{aligned} \mu_1 - \eta x_1x_2\mu_2 &= \eta x_1l_1\lambda + \eta x_1s_1\gamma \\ -\eta x_2x_1\mu_1 + \mu_2 &= \eta x_2l_2\lambda + \eta x_2s_2\gamma \end{aligned} \quad . \; . \; . \; (8)$$

$$\mu_1 = \frac{\begin{vmatrix} \eta x_1l_1\lambda + \eta x_1s_1\gamma & -\eta x_1x_2 \\ \eta x_2l_2\lambda + \eta x_2s_2\gamma & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\eta x_1x_2 \\ -\eta x_2x_1 & 1 \end{vmatrix}} = \frac{\lambda(\eta x_1l_1 + \eta x_1x_2}{}$$

$$= \frac{\lambda(\eta x_1l_1 + \eta x_1x_2\eta x_2l_2) + \gamma(\eta x_1s_1 + \eta x_1x_2)}{1 - \eta x_1x_2\eta x_2x_1}$$

Similarly,

$$\mu_2 = \frac{\lambda(\eta x_2l_2 + \eta x_2x_1\eta x_1l_1) + \gamma(\eta x_2s_2 + \eta x_2x_1\eta x_1s_1)}{1 - \eta x_1x_2\eta x_2x_1} \quad . \; . \; . \; (8a)$$

Thus, when inputs grow at different rates, but each input grows at the same rate in each of the two sectors in which EE appear, outputs must grow at different rates, (i.e., $\mu_1 \neq \mu_2 \neq \lambda \neq \gamma$); insisting on balanced growth is advocating the impossible.

(c) Balanced growth of outputs (i.e., $\mu_1 = \mu_2 = \mu$ constant)

If inputs grow at different rates, but the same input used in one sector grows at the same rate as the same input in the other sector, i.e., $\frac{\dot{l}_i}{l_i} = \lambda$; $\frac{\dot{s}_i}{s_i} = \gamma$, equation (6a) becomes:

$$\begin{aligned} \eta x_1 l_1 \lambda + \eta x_1 s_1 \gamma &= \mu(1 - \eta x_1 x_2) \\ \eta x_2 l_2 \lambda + \eta x_2 s_2 \gamma &= \mu(1 - \eta x_2 x_1) \quad \ldots (9) \end{aligned}$$

Solving,

$$\lambda = \frac{\mu[\eta x_2 s_2(1 - \eta x_1 x_2) - \eta x_1 s_1(1 - \eta x_2 x_1)]}{\eta x_1 l_1 \eta x_2 s_2 - \eta x_2 l_2 \eta x_1 s_1}$$

$$\gamma = \frac{\mu[\eta x_1 l_1(1 - \eta x_2 x_1) - \eta x_2 l_2(1 - \eta x_1 x_2)]}{\eta x_1 l_1 \eta x_2 s_2 - \eta x_2 l_2 \eta x_1 s_1} \quad \ldots (9a)$$

If (6a) is homogeneous of degree one with respect to all their arguments, i.e., constant returns to scale prevail in both sectors, replacing $1 - \eta x_1 x_2$ by $\eta x_1 l_1 + \eta x_1 s_1$; and $1 - \eta x_2 x_1$ by $\eta x_2 l_2 + \eta x_2 s_2$ into (9) gives:

$$\lambda = \mu$$

$$\gamma = \mu$$

$$\text{or,} \quad \lambda = \gamma = \mu \quad \ldots\ldots (9b)$$

i.e., if outputs are to grow at a balanced rate, inputs do not have to grow at the same rate, nor does either one have to grow at the same rate as outputs' growth rate, except in the special case of constant returns to scale (9b). The rate of input growth are proportional to the balanced rate of growth of outputs, and their magnitude depends on various elasticities.

II. DYNAMIC EXTERNAL ECONOMIES

We now turn to the problem of balanced growth when dynamic EE exist. Output in one sector is now dependent, among other factors, on the rate of change of output produced, or factors used, in the other sector. Only the case of one factor of production under the conditions of output-generated dynamic EE will be examined here. The generalisation of the results to the many-factor case is left to the reader.

Consider the two production functions

$$x_1 = x_1(l_1 , \dot{x}_2)$$
$$x_2 = x_2(l_2 , \dot{x}_1) \qquad \ldots\ldots\ldots (10)$$

where x_i, l_i are outputs and factors,

$\dot{x}_i = \dfrac{dx_i}{dt}$ etc..., t=time, i = 1,2

The proportional rates of growth of outputs are:

$$\frac{\dot{x}_1}{x_1} = \frac{\partial x_1}{\partial l_1}\frac{\dot{l}_1}{x_1} + \frac{\partial x_1}{\partial \dot{x}_2}\frac{\ddot{x}_2}{x_1}$$

$$\frac{\dot{x}_2}{x_2} = \frac{\partial x_2}{\partial l_2}\frac{\dot{l}_2}{x_2} + \frac{\partial x_2}{\partial \dot{x}_1}\frac{\ddot{x}_1}{x_2} \qquad \ldots\ldots (10a)$$

If the rates of growth of outputs are a constant over time, and we assume they are, for simplicity, i.e., $\dfrac{\dot{x}_i}{x_i} = \mu$ (constant), i = 1,2, then

$$\frac{d}{dt}\left(\frac{\dot{x}_i}{x_i}\right) = \frac{x_i\ddot{x}_i - \dot{x}_i^2}{x_i^2} = 0$$

$$\text{i.e.,} \quad \frac{\ddot{x}_i}{\dot{x}_i} = \frac{\dot{x}_i}{x_i} = \mu.$$

Substituting this into (10a), putting $\dfrac{\dot{x}_i}{x_i} = \mu$ and $\dfrac{\dot{l}_i}{l_i} = \lambda_i$ gives, in elasticity forms:

$$\mu_1 = \eta x_1 l_1 \lambda_1 + \eta x_1 \dot{x}_2 \mu_2$$
$$\mu_2 = \eta x_2 l_2 \lambda_2 + \eta x_2 \dot{x}_1 \mu_1 \qquad \ldots (10b)$$

(a) Balanced Growth of Outputs, (i.e., $\frac{\dot{x}_i}{x_i} = \mu$, i=1,2)

Equations 10b now become:

$$\mu = \eta x_1 l_1 \lambda_1 + \eta x_1 \dot{x}_2 \mu$$
$$\mu = \eta x_2 l_2 \lambda_2 + \eta x_2 x_1 \mu \quad \ldots \text{(10c)}$$

Soving,

$$\lambda_1 = \frac{\mu(1 - \eta x_1 \dot{x}_2)}{\eta x_1 l_1}$$

$$\lambda_2 = \frac{\mu(1 - \eta x_2 \dot{x}_1)}{\eta x_2 l_2} \quad \ldots\ldots \text{(10d)}$$

where η stands for elasticity, e.g., $\eta x_1 \dot{x}_2 = \frac{\partial x_1 \dot{x}_2}{\partial \dot{x}_2 x_1}$

Thus, λ_1 and λ_2 in (10d) are the required rates of growth of inputs to ensure the balanced growth of outputs, when dynamic EE prevail in both sectors. If $\eta x_1 \dot{x}_2 = \eta x_2 \dot{x}_1 = 1$, balanced growth breaks down: outputs would have to grow at an infinite rate ($\mu =$) and inputs would not have to grow at all (i.e., $\lambda_i = 0$).

This could be verified by giving (10) a form of, say,

$$x_1 = l_1^{\alpha_1} (\dot{x}_2)^{\beta_1} \qquad \text{or,} \quad \frac{\dot{x}_1}{x_1} = \alpha_1 \frac{\dot{l}_1}{l_1} + \beta_1 \frac{\ddot{x}_2}{\dot{x}_2}$$

$$x_2 = l_2^{\alpha_2} (\dot{x}_1)^{\beta_2} \qquad \text{or,} \quad \frac{\dot{x}_2}{x_2} = \alpha_2 \frac{\dot{l}_2}{l_2} + \beta_2 \frac{\ddot{x}_1}{x_1}$$

For $\frac{\dot{x}_i}{x_i} = \mu$ constant, (i = 1,2), $\frac{\ddot{x}_i}{\dot{x}_i} = \frac{\dot{x}_i}{x_i} = \mu$, putting $\frac{\dot{l}_i}{l_i} = \lambda_i$

gives: $$\mu = \frac{\alpha_1}{1 - \beta_1} \lambda_\phi = \frac{\alpha_2}{1 - \beta_2} \lambda_2$$

When $\beta_i = 1$, only zero balanced growth is possible.

(b) Balanced Growth of inputs (i.e., $\frac{\dot{l}_i}{l_i} = \lambda, \quad i = 1,2$)

If inputs grow at the same rate λ in both sectors, (10b) would become:

$$\mu_1 - \eta x_1\dot{x}_2\mu_2 = \eta x_1 l_1 \lambda$$
$$\mu_2 - \eta x_2 x_1 \mu_1 = \eta x_2 l_2 \lambda \quad \ldots\ldots (11)$$

Solving,

$$\mu_1 = \frac{\lambda(\eta x_1 l_1 + \eta x_1\dot{x}_2\ \eta x_2 l_2)}{1 - \eta x_1\dot{x}_2\ \eta x_2\dot{x}_1}$$

$$\mu_2 = \frac{\lambda(\eta x_1 l_2 + \eta x_1 l_1 \eta x_2\dot{x}_1)}{1 - \eta x_1\dot{x}_2\ \eta x_2\dot{x}_1} \quad \ldots\ldots (11a)$$

This could be verified by giving to (10) a form of, say,

$$x_1 = l_1^{\alpha_1} (\dot{x}_2)^{\beta_1}$$
$$x_2 = l_2^{\alpha_2} (\dot{x}_1)^{\beta_2}$$

for $\frac{\dot{l}_i}{l_i} = \lambda$,

$$\mu_1 = \alpha_1\lambda + \beta_1\mu_2$$
$$\mu_2 = \alpha_2\lambda + \beta_2\mu_1$$

Solving,

$$\mu_1 = \frac{\lambda(\alpha_1 + \alpha_2\beta_1)}{1 - \beta_1\beta_2}$$
$$\mu_2 = \frac{\lambda(\alpha_2 + \alpha_1\beta_2}{1 - \beta_1\beta_2}$$

Thus, if inputs grow at a balanced rate, the required rates of output growth are given in (11a). They are not equal to each other, and are both proportional to the rate of inputs' growth.

In the special cases where $\eta x_1 l_1 = -\eta x_1\dot{x}_2\eta x_2 l_2$, $\mu_1 = 0$; and $\eta x_2 l_2 = -\eta x_1 l_1 \eta x_2\dot{x}_1$, $\mu_2 = 0$, that is, only zero balanced growth is possible.

If $\eta_{x_1\dot{x}_2}\,\eta_{x_2\dot{x}_1} = 1$, $\mu_i =$ and $\lambda = 0$, i.e., outputs would have to grow at an infinite rate and inputs do not have to grow at all: Balanced growth is doomed to break down.

DESIRABILITY OF BALANCED GROWTH

So far, we have examined the technical possibility of Balanced Growth. We have seen the rates of growth of inputs required to ensure balanced growth of outputs in different cases. Similarly, we have seen the different rates of growth of outputs compatible with the given balanced growth of inputs. We have also examined the cases where balanced growth is bound to break down.

But the technical possibility, where it exists, of balanced growth is no guarantee that balanced growth will necessarily occur. In other words, even where balanced growth is technically feasible, it may not be desired by enterpreneurs who may find it unprofitable in the absence of subsidies. We now propose to examine the conditions of desirability of balanced growth.

Common sense tells us that for investment to be desirable, entrepreneurs must be able to dispose of their products profitably, i.e., there must be demand for their products. This seems to suggest that for balanced growth of outputs to be willingly undertaken, when it is technically possible, there must be balanced growth of demand, i.e., if $\frac{\dot{x}_i}{x_i} = \mu$, we must have $\frac{\dot{d}_i}{d_i} = \mu$ . . . (23)

(where d_i = demand $(i = 1,2)$ for outputs x_1 and x_2)

for investment in x_i to be undertaken by entrepreneurs having accurate foresight. We shall examine the conditions required to bring the rates of growth of d_i into line with μ, in the case consumers do not want to take up all x_1 and x_2 produced.

Thus, we require demand to have a balanced rate of growth to keep in line with production, i.e., $\frac{d_1}{d_2} = \theta =$ constant over time, and we study the conditions required to bring this about.

The demand functions may be written in the usual way as dependent on price and income, e.g.,

$$d_2 = (p + \theta)^{\alpha} y^{\beta} \quad \ldots\ldots (24)$$

(where y = income, i.e., $y = d_1 + pd_2$

$$= (p + \theta)d_2$$

and price p is in terms of the first commodity, i.e., $p = \frac{p_2}{p_1}$)

Thus $(p + \theta)$ is price in terms of p_1, with a new origin we find convenient to introduce. Demand for x_1 is written by way of residual

$$d_1 = y - pd_2 \quad \ldots\ldots\ldots\ldots (25)$$

$$\begin{aligned} d_2 &= (p + \theta)^{\alpha} y^{\beta} \quad \text{, replacing y by } (p + \theta)d_2 \text{ ,} \\ &= (p + \theta)^{\alpha} d_2^{\beta} (p + \theta)^{\beta} \\ &= (p + \theta)^{\alpha + \beta} d_2^{\beta} \\ &= (p + \theta)^{\frac{\alpha + \beta}{1 - \beta}} \end{aligned}$$

The rate of change of d_2 is:

$$\dot{d}_2 = \frac{\alpha + \beta}{1 - \beta} (p + \theta)^{\frac{\alpha + \beta}{1 - \beta} - 1} \cdot (\dot{p} + \dot{\theta})$$

And the proportional rate of change of d_2 over time, (remembering that θ is a constant and $\dot{\theta} = 0$), is:

$$\frac{\dot{d}_2}{d_2} = \frac{\dot{p}\,\frac{\alpha + \beta}{1 - \beta} (p + \theta)^{\frac{\alpha + \beta}{1 - \beta} - 1}}{(p + \theta)^{\frac{\alpha + \beta}{1 - \beta}}}$$

$$= \frac{\alpha + \beta}{1 - \beta} \frac{\dot{p}}{p + \theta} \qquad \ldots\ldots (26)$$

or
$$\frac{\dot{p}}{p + \theta} = \frac{\dot{d}_2}{d_2} \frac{1 - \beta}{\alpha + \beta} \qquad \ldots\ldots\ldots (27)$$

$$= \mu_2 \frac{1 - \beta}{\alpha + \beta} \qquad \text{where } \mu_i = \frac{\dot{d}_i}{d_i}, \quad (i=1,2)$$

Similarly,

$$\begin{aligned} d_1 &= y - pd_2 \\ &= \theta d_2 \\ &= \theta(p + \theta)^{\frac{\alpha + \beta}{1 - \beta}} \end{aligned}$$

The proportional rate of growth of demand for x_1 thus is:

$$\frac{\dot{d}_1}{d_1} = \frac{\dot{p} \frac{\alpha + \beta}{1 - \beta} \theta(p + \theta)^{\frac{\alpha + \beta}{1 - \beta} - 1}}{\theta(p + \theta)^{\frac{\alpha + \beta}{1 - \beta}}}$$

$$= \frac{\alpha + \beta}{1 - \beta} \frac{\dot{p}}{p + \theta} \qquad \ldots\ldots (28)$$

or
$$\frac{\dot{p}}{p + \theta} = \mu_1 \frac{1 - \beta}{\alpha + \beta} \qquad \ldots\ldots\ldots\ldots\ldots (29)$$

$$\therefore \mu_1 = \mu_2 = \mu$$

i.e., demand for the two products must grow at the same rate as outputs.

For this to eventuate when people are reluctant to allocate all additional income on the expenditure of the commodities, in the proportion in which they are produced, i.e., when the income elasticity of demand is different from unity, in other words, when the marginal propensity to consume is different from the average propensity, a change in relative prices is a condition "sine qua non" to ensure balanced growth of outputs and of demands. The rate of change of relative prices required is given in (27) and (29).

I now examine the implications of assuming different values for price- and income-elasticities, i.e., for α and β in (27) and (29).

Case (a) $\alpha = -1$

$$\frac{\dot{p}}{p^*} = \mu \frac{1-\beta}{\beta - 1} = -\mu$$

where $p^* = p + \theta$, i.e., price with a new origin.

Case (b) $\alpha = 0$

$$\frac{\dot{p}}{p^*} = \frac{1-\beta}{\beta}\mu$$

Case (c) $\alpha = \infty$, $\frac{\dot{p}}{p^*} = 0$

Case (d) $-\infty < \alpha < -1$

Put $\alpha = -1 - \epsilon$ with $\epsilon > 0$

$$\frac{\dot{p}}{p^*} = \frac{1}{\mu}\left(\frac{-1+\beta-\epsilon}{1-\beta}\right)$$

$$= \frac{1}{\mu}\left(-1 - \frac{\epsilon}{1-\beta}\right)$$

So $\frac{\dot{p}}{p^*} < 0$ if $\beta < 0$ and $\mu > 0$ and $\frac{\dot{p}}{p^*} => 0$ if $\beta < -\alpha$ and $\mu > 0$.

Case (e) $\quad \beta = 1, \qquad \frac{\dot{p}}{p^*} = 0$

Case (f) $\quad \beta > 1$ and $-\infty \leq \alpha \leq 0$

$$0 \leq \beta \leq \infty$$

(i) If $/\alpha/ < /\beta/$, $\frac{\dot{p}}{p^*} < 0$

(ii) If $/\alpha/ > /\beta/$, $\frac{\dot{p}}{p^*} > 0$

Case (g) $\quad \beta < 1$

$$\frac{\dot{p}}{p^*} > 0 \quad \text{for } /\alpha/ < /\beta/$$

$$\frac{\dot{p}}{p^*} < 0 \quad \text{for } /\alpha/ > /\beta/$$

Case (h) $\quad \beta = 0$

$$\frac{\dot{p}}{p^*} = \frac{\mu}{\alpha} \gtrless 0 \text{ depending on whether } \alpha \gtrless 0$$

Case (i) $\quad /\alpha/ = /\beta/$

$$\frac{\dot{p}}{p^*} = +\infty \quad \text{if } \beta < 1$$

$$\frac{\dot{p}}{p^*} = -\infty \quad \text{if } \beta > 1$$

$$\frac{\dot{p}}{p^*} = \text{indeterminate if } \beta = 1$$

Limits of $\frac{\dot{p}}{p^*}$ as t tends to infinity and zero

$\gamma = \frac{\dot{p} + \theta}{p + \theta}$ where θ = constant, $\dot{\theta} = 0$, could be written as:

$\dot{p} - \gamma p = \gamma\theta$, whose solution is

$p = Ae^{\gamma t} - \theta$ where A is an arbitrary constant.

With the initial condition at $t = 0$,

$p_o = A - \theta$, or $A = p_o + \theta$

The general solution of this differential equation is

$$p(t) = (p_o + \theta)e^{\gamma t} - \theta \quad \ldots\ldots \quad (30)$$

The proportional rate of change of relative price is:

$$\frac{\dot{p}}{p} = \frac{(p_o + \theta)\gamma e^{\gamma t}}{(p_o + \theta)e^{\gamma t} - \theta}$$ which is, by division of fraction,

$$= \frac{\gamma\theta}{(p_o + \theta)e^{\gamma t} - \theta} + \gamma$$

Thus, if (1) $\gamma < 0$ and t tends to ∞, $(p_o + \theta)e^{\gamma t}$ tends to 0

$$\frac{\dot{p}}{p} \longrightarrow \gamma + \frac{\gamma\theta}{0 - \theta} \longrightarrow 0$$

If $\gamma < 0$ and $t \longrightarrow 0$

$$\frac{\dot{p}}{p} \longrightarrow \gamma + \frac{\gamma\theta}{p_o + \theta - \theta} \longrightarrow \gamma + \frac{\gamma\theta}{p_o} \longrightarrow \gamma(1 + \frac{\theta}{p_o})$$

if (2) $\gamma > 0$ and $t \longrightarrow \infty$

$$\frac{\dot{p}}{p} \longrightarrow \gamma + \frac{\gamma\theta}{\infty - \theta} \longrightarrow \gamma$$

If $\gamma > 0$ and $t \longrightarrow 0$

$$\frac{\dot{p}}{p} \longrightarrow \gamma + \frac{\gamma\theta}{-\theta} \longrightarrow 0.$$

This result could be shown in figures 17 (for $\gamma > 0$) and 18 (for $\gamma < 0$) or, alternatively, for $\gamma > 0$ and $t \longrightarrow \infty$, we may write $\frac{\dot{p}}{p} \longrightarrow \gamma$ as $\dot{p} \longrightarrow \gamma p$ and consider γ as a slope of that linearised function; similarly, $\dot{p} = \gamma(1 + \frac{\theta}{p_o})p$

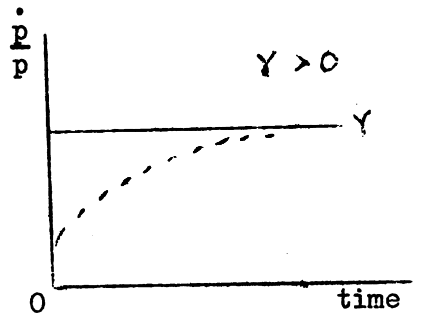

fig. 17

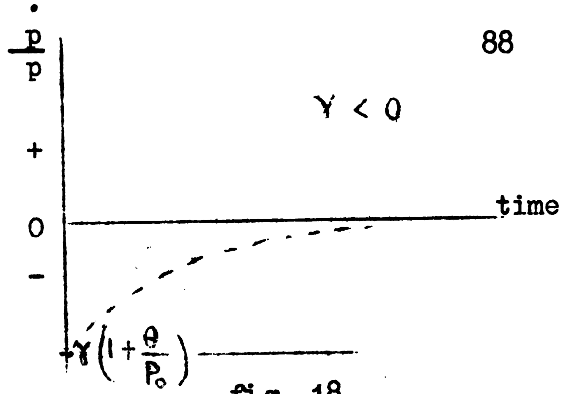

fig. 18

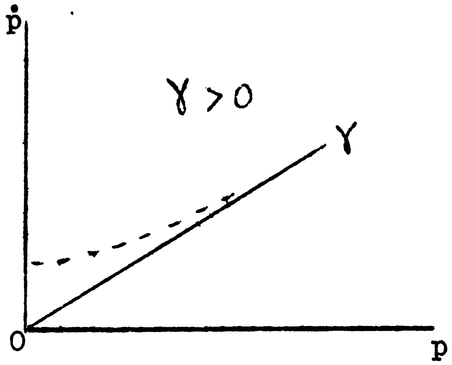

fig. 19

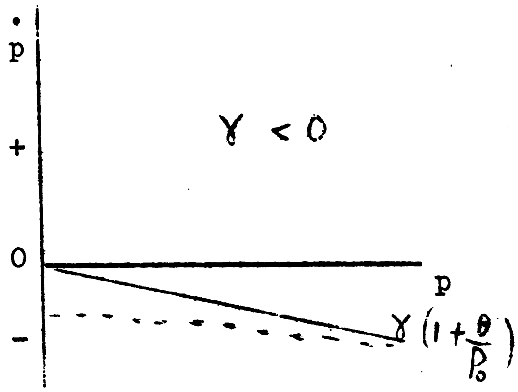

fig. 20

Some conclusions emerge from the above analysis: For balanced growth of outputs to be desirable, even when technical conditions of production are all fulfilled, demand must grow at the same rate as outputs. However, in a free society, there is no guarantee that consumers would gear their consumption pattern to the rate of growth output. Demand is a matter of taste, determined by consumers' sovereignty in their utility maximization. This taste changes over time at different rates. As the community grows richer, the consumption of food and necessaries seem to grow at a slower rate than that of other items; in other words, income elasticity of demand for food is less than unity. Similarly, consumers respond differently to changes in relative prices, depending on whether the commodity the price of which falls, is a superior or inferior good.

The different cases above show the rate at which relative prices must change, in order to bring demand into line with outputs all the time. If the commodity is not prepared to take up everything that has been produced, then the price of the unwanted commodity must fall relatively to other prices, to an extent sufficient for it to be cleared. Case (c) for example, shows that if demand is infinitely price elastic, relative prices do not have to change in order to bring balanced growth of outputs and demand into line with each other. Case (e) means that if the income elasticity of demand is unity, i.e. marginal propensity to consume equals average propensity to consume, no change in relative prices is needed to bring about the demand conditions of balanced growth. This could be seen in fig.21 where A

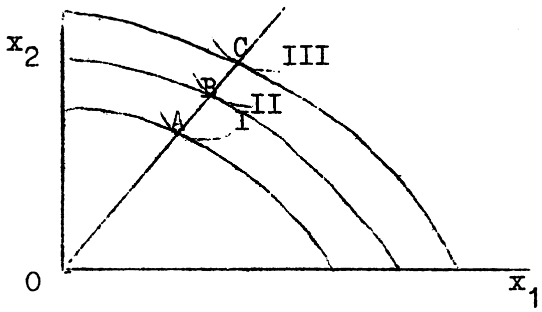

fig. 21

is the initial point of equilibrium between the supply of, and demand for, the two commodities. Balanced growth of output means that the locus of all production points A, B, C.... is a straight line going through origin O, and the tangents to the relevant transformation curves at A, B, C are all parallel, i.e. the marginal rate of transformation remains unchanged as outputs grow. Balanced growth of demand implies that the locus of all relevant points of consumers' preferred position A, B, C on the indifference curves I, II, II (fig.21) is also a straight line going through origin O, and the tangents to these indifference curves at A B, C etc... are all parallel to one another. Balanced growth of output and demand therefore implies that these two loci in fact coincide with each other, and the tangents to the various production possibilities curves (measuring the marginal rate of transformation) also coincide with the tangents to the various indifference curves (measuring the marginal rate of substitution). These tangents also measure the equilibrium relative price p which, as can be seen, does not have to change. That is the case where $\frac{\dot{p}}{p} = 0$.

In case (a) where price elasticity of demand is -1, the proportional rate of change of relative prices and demand vary inversely in exactly the same proportions, when relative price falls, relative demand rises in the same proportions.

Similarly, the remaining cases show, for different values of income and price elasticities, the required rate of price changes to make balanced growth of desirable outputs.

It is hoped that the above analysis sheds some light on the problem of balanced growth and helps remove some doubts about the problem. Thus when E.S. are present, balanced growth is possible in some cases, impossible in some others. The conditions necessary to bring about balanced growth have been spelled out, and it is easy to see that, unless these conditions are fulfilled, balanced growth is just a chimera. The conditions of technical possibility of balanced growth might shed some light on both cases of "complementary" (Nurkse) and "competitive" (Fleming)(10) relationships between industries. In the absence of technological progress, insisting on balanced growth of outputs where inputs do not grow at the required rate, is asking the impossible: a "competitive" relationship develops. Like all necessary conditions, the technically necessary conditions of balanced growth are no guarantee that it must occur: an appropriate rate of demand growth is also required, if subsidies are not to be used. That is partly Lewis' case in reverse.(11) Lewis let demand growth

(10) See R.Nurkse 15 Ch.1 also R Nurkse 16 p.365 seq. J.Fleming 6 pp 241-56.

(11) W.A.Lewis 12 pp.275-283

set the pace and gears the rate of output growth to it. I let outputs grow at a balanced rate and gear the growth rate of demand to it, by appropriate changes in relative prices. But there are limits within which this change can "do the tick": if foodstuff is to grow at a given rate, there is little the change in relative prices can do, to make people eat it all up: price -, and income - elasticities are both important considerations. These cases of elasticities have been studied in detail above.

Chapter VII

SUMMARY AND CONCLUSION

This thesis is an exercise in the classification of EE with some applications to economic theory. It is hoped that the classification has served to spell out more fully a concept which is so often heard of, amd which nevertheless, has remained rather vague. The few applications are intended to clarify such controversial issues as the Marginal Theory of Distribution, the allocation of resources, the choice of Investment Techniques and the problem of Balanced Growth, in a framework where EE are present.

Thus, the complete exhaustion of product, which is the core of the Marginal Theory of Distribution, has been shown to eventuate only in some particular cases. Where outputs are not produced under conditions of constant returns to scale, and there are EE, the payment to factors according to their social marginal product cannot exhaust social output (see chapter III).

The study of the problem of resources allocation has shown that EE cause the divergence between social and private values. Competitive allocation guided by the criterion of individual profit maximization and based on private values, falls short of (exceeds) the social optimun figures,when EE (diseconomies) are present. This proves the validity of the Marshall-Pigou argument that Government interference is needed to bring about Pareto optimality and save free competition at once (chapter IV).

As far as the problem of Investment Techniques is concerned, it has been shown that, EE causing divergence between private and social values, adherence to private marginal conditions (i.e., equating the value of labour's marginal product with real wages, leading to the choice of a capital intensive technique, or taking on labour till the value of its marginal product is zero, leading to the choice of labour intensive technique) may both result in a wrong choice of technique. Where there are no externalities, social and private values coincide, but when an Investment Technique generates EE, its private values understate its soicial merit. The choice is thus to be made according to these social values, in the light of an overall picture. It is easy to see that the choice thus made would be different from the one actually made in the light of private values. Again, as these EE are not appropriable, taxes and subsidies are needed for the right choice to be made, by private investors.

The chapter on Balanced growth is intended to help clarify a controversial issue. It has shown the necessary and sufficient conditions of balanced growth where there are EE. These conditions cut right through the condusion which has led R. Nurkse[1] to advocate the simultaneous development of all sectors as a means of providing market for one anothe's product; and Fleming[2], P. Streeten[3] to conclude that the reverse is recommendable. In fact, it is easy to see that R. Nurkse[1] after A. Young and Rosenstein-Rodan[5]; overconcentrating attention on the demand side, only saw "complementary relationship" in balanced growth, while M. Fleming[2], absorbed with the supply side, only saw "competitive relationship". In fact, in Marshall's terminology,[6] the cutting is effected by both the upper and lower blades of a pair of scissors: Our necessary conditions state the feasibility or otherwise of balanced growth, and our "desirability" condition (chapter VI) examine Nurkse's complementary relations in detail, and in addition, the conditions necessary (changes in relative price) to turn "non-complementarity" into complementarity.

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1. R. Nurkse [15] especially chapter I, pp. 4-31.
2. M. Fleming [6] pp. 241-56.
3. P. Streeten [26] pp. 167-90.
4. A. Young [32] pp. 527-42.
5. Rosentein-Rodan [21] pp. 201-11.
6. A. Marshall [13] p. 348.

In the study of the "feasibility" conditions, a sub-problem emerges for future investigation: Where EE are present, if inputs do not grow at the rate required to have balanced output growth, what rate of factor substitution would be required in order to ensure balanced output growth. The problem may be formulated as follows:

Condition (a) : Unbalanced input growth, i.e.,

$$\frac{\dot{l}}{l} = \lambda \quad \text{and} \quad \frac{\dot{s}}{s} = \gamma \quad . \; . \; . \; . \; (a)$$

where λ and γ are two constants different from each other.

Condition (b) : Efficiency (i.e., full employment) growth, obtained by maximization of output $x_2 = l_2^{\alpha_2} s_2^{\beta_2} x_1^{\sigma_2}$ subject to a constant level of production $x_1 = l_1^{\alpha_1} s_1^{\beta_1} x_2^{\sigma_1}$ . . . (1)

This could be dealt with by using Lagrangean multiplier λ^*, and forming equation V (remembering that full employment is assumed, i.e., $l = l_1 + l_2$ and $s = s_1 + s_2$):

$$V = (l-l_1)^{\alpha_2}(s-s_1)^{\beta_2}x_1^{\sigma_2} + \lambda^*[x_1 - l_1^{\alpha_1}s_1^{\beta_1}\{(l-l_1)^{\alpha_2}(s-s_1)^{\beta_2}x_1^{\sigma_2}\}^{\sigma_1}]$$

First order conditions are obtained by differentiating V with respect to s_1, l_1 and λ^* and setting them equal to zero; which gives efficient growth conditions:

$$\frac{\beta_1 l_1}{\alpha_1 s_1} = \frac{\beta_2(l-l_1)}{\alpha_2(s-s_1)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (b)$$

Condition (c) : Balanced output growth, i.e., $\frac{\dot{x}_i}{x_i} = \mu$ $(i = 1,2)$ or

$$\alpha_1 \frac{\dot{l}_1}{l_1} + \beta_1 \frac{\dot{s}_1}{s_1} + \sigma_1\mu = \alpha_2 \frac{\dot{l}_2}{l_2} + \beta_2 \frac{\dot{s}_2}{s_2} + \sigma_2\mu = \mu \; . \; . \; . \; . \; (c)$$

If factors grow at the constant rates λ and γ, i.e.,

$$l = l_o e^{\lambda t} \qquad \text{and} \qquad s = s_o e^{\gamma t}$$

then $\dot{l}_1 + \dot{l}_2 = \lambda l$ and $\dot{s}_1 + \dot{s}_2 = \gamma s$

$$= \lambda(l_1 + l_2) \qquad\qquad = \gamma(s_1 + s_2)$$

Put $\dfrac{\dot{l}_1}{l_1} = \lambda_1$ and $\dfrac{\dot{s}_1}{s_1} = \gamma_1$

$$\frac{\dot{l}_2}{l_2} = \lambda_2 \qquad\qquad \frac{\dot{s}_2}{s_2} = \gamma_2$$

$$\frac{l_2}{l_1} = \rho$$

Then $\dot{l}_1 + \dot{l}_2 = \lambda(l_1 + l_2)$

or, $\lambda_1 + \lambda_2\rho = \lambda(1+\rho)$ . . . . (2)

Similarly, $\dot{s}_1 + \dot{s}_2 = \gamma(s_1 + s_2)$

or $\gamma_1 + \gamma_2\dfrac{s_2}{s_1} = \gamma(1 + \dfrac{s_2}{s_1})$

As $s_2 = s - s_1$ and $l_2 = l - l_1$, replacing $\dfrac{s_2}{s_1}$ by

$\dfrac{\alpha_1\beta_2 l_2}{\alpha_2\beta_1 l_1} = \dfrac{\alpha_1\beta_2}{\alpha_2\beta_1}\rho$ from condition (b), gives

$$\gamma_1 + \gamma_2\frac{\alpha_1\beta_2}{\alpha_2\beta_1} = \gamma(1 + \frac{\alpha_1\beta_2}{\alpha_2\beta_1}\rho) \quad . . . . (3)$$

We now have to satisfy the three equations (c), (2) and (3), in 5 unknowns λ_1, λ_2, γ_1, γ_2 and μ. If the constant μ happens to be the value μ determined by the technical production conditions compatible with the rates of input growth and efficiency conditions, we are left with 3 equations in 4 unknowns. The usual way to solve them is to assign to one of the unknowns an arbitrary value and solve the system in terms of it. Another way is to assume $\frac{l_2}{l_1}$ constant here, and have $\dot{l}_2 = \rho \dot{l}_1$ or $\frac{\dot{l}_2}{l_2} = \frac{\dot{l}_1}{l_1}$

$$\text{or} \quad \lambda_2 = \lambda_1 \quad \ldots\ldots (4)$$

And (2) gives $\lambda_1(1 + \rho) = \lambda(1 + \rho)$

$$\text{or} \quad \lambda_1 = \lambda \quad \ldots\ldots (5)$$

Thus we have 2 equations in two unknowns γ_1, γ_2 i.e.,

$$\beta_1\gamma_1 - \beta_2\gamma_2 = (\alpha_2 - \alpha_1)\lambda + (\sigma_2 - \sigma_1)\mu$$

$$\gamma_1 + \frac{\alpha_1\beta_2\rho}{\alpha_2\beta_1}\gamma_2 = \gamma\left(1 + \frac{\alpha_1\beta_2}{\alpha_2\beta_1}\rho\right)$$

which could be solved for γ_1 and γ_2. However, this is true only for the particular case where μ as determined by the system happens to be the constant rate μ at which we want outputs to grow, and $\lambda_1 = \lambda_2 = \lambda$ happens to satisfy efficiency conditions (b). What is needed is a general answer to the question.

Finally, it is important to bear in mind the difficulty of empirical measurement of EE, as various factors are so entangled that it is rather impossible to isolate and identify them with absolute certainty. For example, when a firm experiences a better profit, it is difficult to say which part of that profit is due to technological progress, to windfall elements, to internal economies and finally to EE. The blurred line of demarcation between technological and pecuniary, factor and output-generated, EE also constitutes another difficulty. All these practical difficulties which surround us should warn us against the danger of inconsiderate applications of the results of this analysis for any policy recommendation, without specific and careful consideration.

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<u>Abbreviations for Journals</u>

A.E.R.	American Economic Review
B.O.U.I.S.	Bulletin of the Oxford University Institute of Statistics
C.J.E.	Canadian Journal of Economics and Political Science
E.D.C.C.	Economic Development and Cultural Change
E.J.	Economic Journal
E.R.	Economic Record
I.E.J., I.E.R.,	Indian Economic Journal, Review.
I.E.P.	International Economic Papers
J.P.E.	Journal of Political Economy
M.S.	Manchester School of Economics and Social Studies
O.E.P., N.S.	Oxford Economic Papers, New Series
Q.J.E.	Quaterly Journal of Economics
R.E.Stat.	Review of Economics and Statistics
R.E.Stud.	Review of Economic Studies

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