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by

T.R. Fietz

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Key Words

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Preface

The work reported herein formed part of the programme of unsponsored research carried out in the Water Engineering Department of the School of Civil Engineering by Mr. T.R.Fietz, Senior Lecturer, as a contribution to an evaluation of methods which might be applied to pipe networks particularly with small time sharing computers.

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Associate Professor of Civil Engineering,
Officer-in-Charge.

Abstract

The simple loop method is applied to the analysis of steady water flow in networks of pipes, pumps and reservoirs. Improved head-discharge relations are presented for pipe and pump lines. The effects of loop set selection, convergence criteria, node head calculation, and convergence acceleration techniques are investigated. A computer program, based on the conclusions reached, and suitable for use on a small time-sharing computer, is presented.

Notation

a, b, c	Coefficients in friction factor approximation for a pipe
A, B, C	Empirical coefficients in head-discharge relation for a pipe or pump
d	Pipe diameter
e	Nikuradse pipe roughness
e/d	Relative roughness of a pipe
f	Darcy friction factor for a pipe
f_f	Head loss-discharge function for a pipe
f_f'	Slope of tangent to above at a particular discharge
f_p	Head rise-discharge function for a pump
f_p'	Slope of tangent to above at a particular discharge
g	Gravitational acceleration
h_f	Head loss in a pipe
h_{f_j}	Head loss contribution to a loop by pipe j
h_p	Head rise through a pump
h_{p_j}	Head rise contribution to a loop by pump j
l	Pipe length
m	Iteration number
n	Exponent in exponential head loss expression for a pipe
n_{p_i}	Number of pipes in loop i
n_{u_i}	Number of pumps in loop i
N_l	Number of loops in network
N_n	Number of nodes in network
N_p	Number of lines in network
N_r	Number of reservoir nodes in network
q	Flow correction for a loop
q_1	Flow correction for loop 1
Q	Flow in a line
Q_j	Flow in line j
Q_j^m	Flow in line j after iteration m

r	Resistance coefficient in exponential head loss expression for a pipe
R_e	Reynolds number
T_j	Direction of loop through line j
V	Mean velocity in a pipe
W_q	Maximum absolute value of q for any loop for convergence
W_h	Maximum absolute value of out of balance head for any loop for convergence
X_i	Head at node at beginning of loop i
Y_i	Head at node at end of loop I
ΔHa	Maximum absolute value of change in any node head for convergence
ΔQa	Maximum absolute value of change in any line flow for convergence
ΔQr	Maximum absolute value of relative variation in any line flow for convergence
ν	Kinematic viscosity

Table of Contents

1. Introduction

1.1 Contents

This report describes an investigation of some aspects of the simple loop method for analysing steady flow in water supply networks. The factors looked at are head-discharge relations for pipe and pump lines; loop set selection; convergence criteria; node head calculations; and improving the speed of convergence.

Most of the investigation has been carried out by applying a computer program to analyse a small test network. The final version of the program is included as an appendix to the report.

1.2 Network Definition

The network consists of a series of lines which join at the nodes. For any line one end node is arbitrarily defined as the upstream node and the flow as positive when from the upstream to the downstream node. Three empirical constants are sufficient to define the head-discharge relation for any line.

Nodes are either reservoirs, junctions with no external flow, or junctions with outflow or inflow.

A closed loop is a non-intersecting path through the network which returns to its starting point. An open loop is a non-intersecting path joining two reservoirs.

1.3 Methods of Steady Flow Analysis of Networks

The common methods of mathematical analysis of pipe networks are based on either satisfaction of continuity at the nodes, or conservation of energy around any closed loop in the network. Both methods were stated by Hardy Cross (Ref. 1). Satisfying continuity at the nodes gives rise to the "method of balancing heads", also called the "node method". Considering head losses around closed loops gives rise to the "loop method", also called the "mesh" or "grid" method. In either case there results a set of simultaneous equations which are non-linear in the pertinent variables (node heads for the node method, line discharges for the loop method).

The conventional method of solution of the simultaneous equations is to solve them one at a time. For example, in the simple loop method of Hardy Cross (Ref. 1) the same correction q is applied to all line discharges in a loop to correct any head loss imbalance. The process is repeated for the next loop, and so on. A more recent method, described by Epp and Fowler (Ref. 2), involves simultaneous solution of the equations with the flow corrections q_1, q_2 for loops 1 and 2 etc. as the unknowns. Similarly in the simple node method the head at each node is corrected in turn to restore continuity, for example see McCormick and Bellamy (Ref. 3). The modern counterpart, using simultaneous solution of the non-linear equations, was originally used by Martin and Peters (Ref. 4).

The simple loop method has been used for the present investigation, for two main reasons. Firstly because of the small storage of the small time-sharing computer used for the investigation - the modern method requires a much larger computer. Secondly so that the progress of the solution could be readily interpreted physically - one of the attractions of the simple loop or node methods. Many of the conclusions reached by using the simple loop method can be applied directly to the modern method.

2. The Simple Loop Method

2.1 Basis of the Method

The solution is by iteration or successive correction to an initial guess for the line discharges. The procedure is repeated until some specified convergence criterion is satisfied, or until the specified maximum number of iterations is reached.

The initial values for line flows must obey continuity at the nodes. For each node a linear equation relating the flows in lines at the node is thus obtained. There will be more unknown line flows than equations available, however, and the additional equations are obtained from closed or open loops. For a closed loop the sum of the head changes around the loop is zero for a balanced network. For an open loop the sum of the head changes should equal the known head difference between the two reservoirs at the ends of the loop. Care must be taken to use the correct number N_1 of independent loops, given by Barlow and Markland (Ref. 5) as:-

$$N_1 = N_p + N_r - N_n \qquad (1)$$

where N_p is the number of lines, N_r is the number of reservoirs, and N_n is the total number of nodes, including reservoirs.

2.2 Finding the Flow Correction q for a Loop

We require the flow correction q to apply to the line flows in loop i so that:-

$$\left(X_i - Y_i\right) - \sum_{j=1}^{n_{p_i}} h_{f_j} + \sum_{j=1}^{n_{u_i}} h_{p_j} = 0 \qquad (2)$$

that is, to balance the loop. Here X_i and Y_i are the heads at the beginning and the end of loop i respectively. $X_i - Y_i$ is zero for a closed loop. n_{p_i} and n_{u_i} are the number of pipe lines and pump lines in loop i respectively, and h_{f_j} and h_{p_j} are the head changes through a pipe line and a pump line respectively.

The flow correction q may be estimated by using a Taylor's series expansion about the existing flow Q_j in any line j:-

$$(X_i - Y_i) - \sum_{j=1}^{n_{p_i}} (f_f(Q_j) + f_f'(Q_j)\, q)$$

$$+ \sum_{j=1}^{n_{u_i}} (f_p(Q_j) + f_p'(Q_j)\, q) = 0 \qquad (3)$$

where second and higher order terms are neglected. Here f_f is the head loss-discharge function for a pipe; f_p is the head rise - discharge function for a pump; f_f' and f_p' are the slopes of tangents to the functions f_f and f_p respectively, where the slopes are determined for the existing flow Q_j.

Rearranging equation (3) gives the first order estimate for the flow correction q:-

$$q = \frac{\left(X_i - Y_i\right) - \sum_{j=1}^{n_{p_i}} f_f(Q_j) + \sum_{j=1}^{n_{u_i}} f_p(Q_j)}{\sum_{j=1}^{n_{p_i}} f_f'(Q_j) - \sum_{j=1}^{n_{u_i}} f_p'(Q_j)} \qquad (4)$$

In the numerator of equation (4) due account must be taken of the sign of a head change term relative to the direction of the loop through the line concerned. The numerator is the out of balance head resulting from using the current estimate of line flows.

The denominator of equation (4) is made up of the sums of the slopes of tangents to the head-discharge curves for the pipe and pump lines in the loop. A divergent solution may result if the denominator becomes small relative to the numerator.

2.3 Line Characteristics

2.3.1 Long Pipes

An exponential form of the head loss expression for a pipe is commonly used for the loop method:-

$$h_f = rQ^n \tag{5}$$

where r is the resistance coefficient and n the exponent. The values of r and n for a pipe may be found for a specified range of discharge (see Streeter, Ref. 6). Alternatively the flow may be assumed to be wholly rough wall turbulent so that $n = 2$, and differentiation of equation (5) to obtain the f_f' function is simplified. The Hazen-Williams and Manning formulae, rearranged into the exponential form of equation (5), are also used, for example by Epp and Fowler (Ref. 2). The inaccuracies of using these formulae have been discussed by Vallentine (Ref. 7).

The preferred head loss expression for a pipe is the universal Darcy-Weisbach formula:-

$$h_f = f \; \frac{l}{d} \; \frac{V^2}{2g} \tag{6}$$

where f is the friction factor; l is the pipe length; d is the pipe diameter; and V the mean velocity. The friction factor is given by the Colebrook-White equation:-

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{e}{3.7d} + \frac{2.51}{R_e\sqrt{f}} \right) \tag{7}$$

where e is the Nikuradse wall roughness, and R_e is the Reynolds number. Equation (7) is implicit in f and cannot be combined directly with equation (6). As an explicit relation between h_f and Q is required for differentiation (to give an f_f' function) we use the explicit form of equation (7) derived by Wood (Ref. 8).

$$f = a + b \; R_e^{-c} \tag{8a}$$

where a, b and c are functions of the relative roughness e/d:-

$$a = .094 \left(\frac{e}{d}\right)^{.225} + .53 \left(\frac{e}{d}\right) \tag{8b}$$

$$b = 88 \left(\frac{e}{d}\right)^{.44} \tag{8c}$$

$$c = 1.62 \left(\frac{e}{d}\right)^{.134} \tag{8d}$$

Equation (8) gives f values within four per cent or less of those from equation (7) and is far more convenient to use. Combining equations (6) and (8) gives

$$h_f = AQ^2 + BQ^{2-C} \tag{9a}$$

where
$$A = \frac{8\ al}{g\pi^2 d^5} \tag{9b}$$

$$B = \frac{8bl}{g\pi^2 d^5} \left(\frac{4}{\pi d \nu}\right)^{-c} \tag{9c}$$

$$C = c \tag{9d}$$

A, B and C are the three empirical head loss coefficients necessary to relate head loss to discharge for a pipe. Also equation 9(a) may be readily differentiated with respect to Q. Note that the head loss coefficient A for a pipe is identical with the resistance coefficient r of equation (5) when the flow is wholly rough wall turbulent.

Equation 9(a) is shown plotted in Figure 1(A).

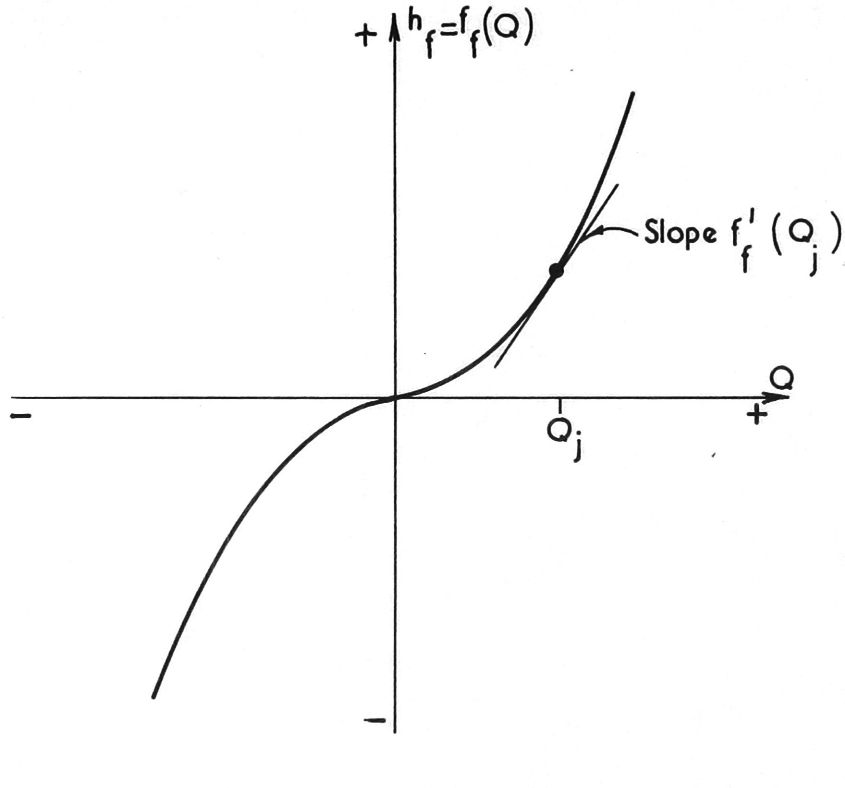
For a pipe j in a loop the term $f_f(Q_j)$ in equation (4) becomes:-

$$f_f(Q_j) = \text{Sign}\,(Q_j)\; T_j \left\{ A_j \left(|Q_j|\right)^{2} + B_j \left(|Q_j|\right)^{2-C_j} \right\} \tag{10}$$

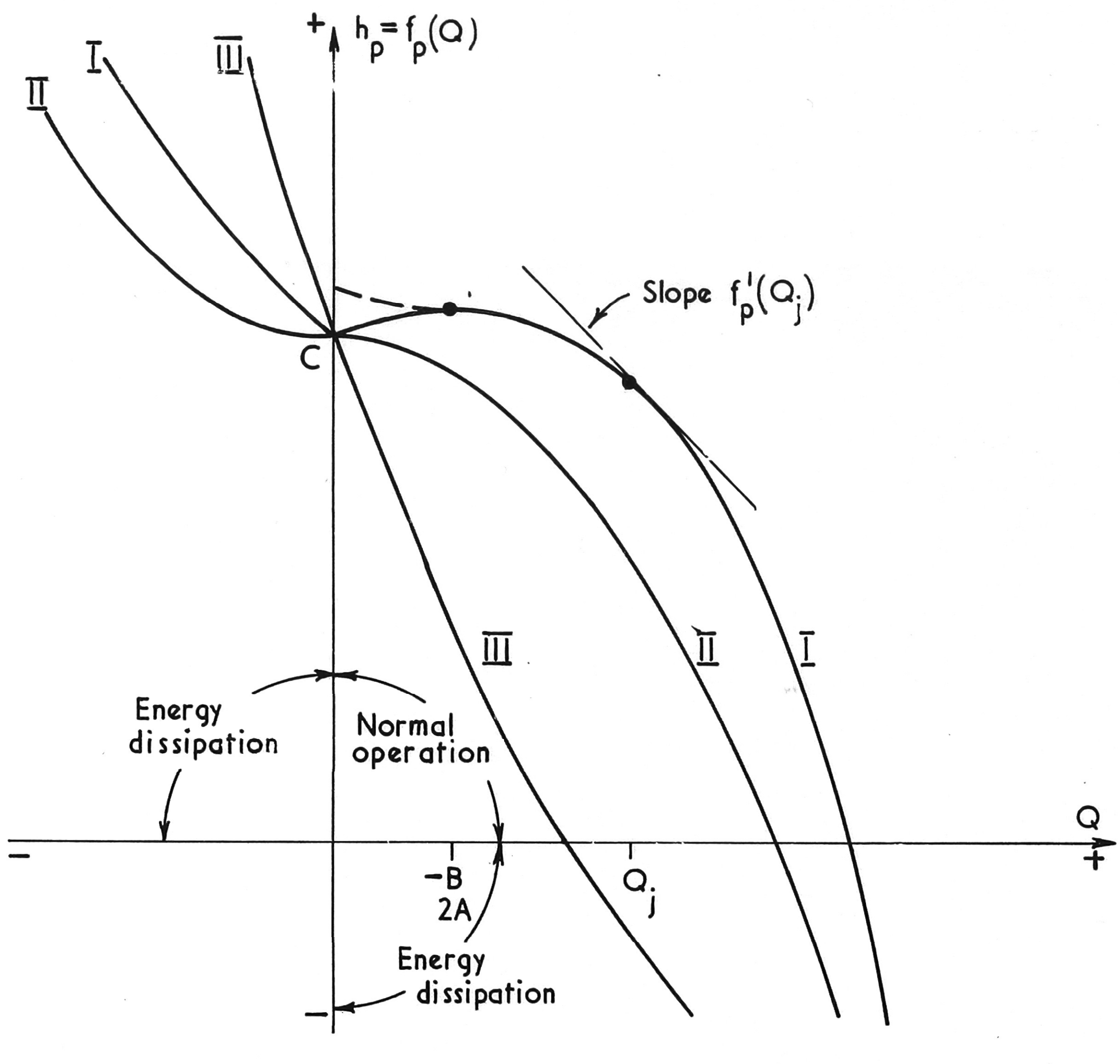
where T_j indicates the direction of the loop through the pipe j, being set equal to +1 when passing from the upstream to the downstream node, and -1 for the converse.

The slope of the tangent to the head-discharge curve for a pipe is always positive so that the term $f_f'(Q_j)$ for a pipe j in equation (4) is given by:-

$$f_f'(Q_j) = 2A_j\left(|Q_j|\right) + (2-C_j)\, B_j \left(|Q_j|\right)^{1-C_j} \tag{11}$$



(A) HEAD DISCHARGE CURVE FOR A PIPE



(B) HEAD DISCHARGE CURVES FOR PUMPS

FIGURE I : HEAD DISCHARGE CURVES FOR PIPES AND PUMPS

2.3.2 Pumps

The head rise-discharge curve for a rotodynamic pump is normally presented by a graph or tabular data. Although computer algorithms are available for interpolating head rise for a given discharge from tabular data, and for finding the slope of the tangent to the empirical curve, it is much more convenient to use an explicit relation between head rise and discharge. The explicit relation normally used is a polynomial in Q. Epp and Fowler (Ref. 2) and Daniel (Ref. 9) have used a fourth order polynomial, while McCormick and Bellamy (Ref. 3) used a second order polynomial. The latter has been adopted here as it is convenient to have the same number (3) of empirical coefficients applying to a pump line as to a pipe line.

The pump head rise-discharge relations used are shown in Figure 1(B). For Case I, $A < 0$, $B > 0$ and $C > 0$; for Case II, $A < 0$, $B < 0$, and $C > 0$; and for Case III, $A > 0$, $B < 0$, $C > 0$, and $\left(C - \frac{B^2}{2A}\right) < 0$.

For normal operation in the first quadrant for all three cases:-

$$h_p = AQ^2 + BQ + C \tag{12a}$$

This equation is also assumed to apply to the zone of energy dissipation in the fourth quadrant.

For operation in the zone of energy dissipation in the second quadrant (where the flow is reversed but the pump is still being driven in the forward direction), some allowance is made for the head being greater than the shut-off head C by using:-

$$h_p = |AQ^2| + |BQ| + C \tag{12b}$$

For a pump j in a loop the term $f_p(Q_j)$ in equation (4) then becomes:-

$$f_p(Q_j) = T_j\,(A_jQ_j^2 + B_jQ_j + C_j) \text{ for } Q_j \geq 0 \tag{13a}$$

$$f_p(Q_j) = T_j\,(|A_jQ_j^2| + |B_jQ_j| + C_j) \text{ for } Q_j < 0 \tag{13b}$$

The slope $f_p'(Q_j)$ of the tangent to the head-discharge curve is always negative for cases II and III. For Case I the slope is negative except for the discharge range $0 < Q_j < \left(\frac{-B}{2A}\right)$ in the first quadrant, where the small positive slope may cause the denominator of equation (4) to become small relative to the numerator. To avoid this $f_p'(Q_j)$ is taken as negative (that is as the slope of the dotted curve in Figure 1(B)) so that q will be underestimated if the Case I pump is operating in this region during the course of the solution. The term $f_p'(Q_j)$ in equation (4) is then given by:-

$$f_p'(Q_j) = -|2A_jQ_j + B_j| \text{ for } Q_j \geq 0 \text{ for Cases I, II and III} \tag{14a}$$

$$f_p'(Q_j) = 2A_j|Q_j| - B_j \text{ for } Q_j < 0, \text{ Case I} \tag{14b}$$

$$f_p'(Q_j) = 2A_j|Q_j| + B_j \text{ for } Q_j < 0, \text{ Case II} \tag{14c}$$

$$f_p'(Q_j) = 2A_j Q_j + B_j \text{ for } Q_j < 0, \text{ Case III} \tag{14d}$$

2.4 Loop Definition

For any network several alternative sets of independent loops are possible. A desirable set is one which produces a well conditioned set of simultaneous equations so that convergence will be rapid and without long term drift.

For the modern method of solving all loop flow corrections simultaneously a desirable loop set is one consisting of loops of small extent, that is containing as few lines as possible. These "natural" loops are required for minimising the computer storage requirement (Epp and Fowler, Ref. 2).

For the simple loop method the computer storage requirement is already small and the objective is to minimise the amount of computation required to converge on a solution. The amount of computation depends on the number of iterations required to converge on a solution, and, to a lesser extent, on the number of lines appearing in the individual loops. Voyles and Wilke (Ref. 10) have shown that the number of iterations required is minimised when loops are selected to minimise the resistance of lines common to two or more loops. The resistance of a pipe line may be taken as the coefficient A of equation 9(b). For a pump line it is suggested that the resistance be taken as the absolute value of the tangent slope at the design point on the head-discharge curve for the pump. Minimising the resistance of common lines is equivalent to selecting loops so that the sum of the loop resistances is minimised. Computer algorithms for finding minimum resistance loops have been described by Travers (Ref. 11), and Hyman and Jones (Ref. 12).

Barlow and Markland (Ref. 5) argue that loop selection by computer algorithm may lead to zones of loop concentration and that the computer time required may be excessive. They have used manual loop selection with principal loops of large extent and comprising low resistance lines, and subsidiary loops of small extent and comprising high resistance lines.

2.5 Convergence Criteria

2.5.1 Absolute Criteria

Criteria based on changes in line flows or node heads from one iteration to the next are:-

(i) the absolute value of the flow change in any line is less than ΔQ_a;

(ii) the absolute value of the head change at any node is less than ΔH_a.

Criteria using loop flow correction properties are:-

(iii) the absolute value of the loop flow correction $|q|$ for any loop is less than W_q;

(iv) the absolute value of the numerator of equation (4) is less than W_h for any loop.

Criterion (i) has been used by Epp and Fowler (Ref. 2); (iii) by Travers (Ref. 11); and (iv) by Hoag and Weinberg (Ref. 13). The main disadvantage of criteria (i) and (ii) is that additional computer storage is required to save values from the previous iteration. Criteria (iii) or (iv) have a much smaller storage requirement.

2.5.2 Relative Criteria

A relative criterion corresponding to criterion (i) above is:-

(v) Any $$\left|\frac{Q_j^{m} - Q_j^{m-1}}{Q_j^{m}}\right| \leq Q_r \qquad (15)$$

where the superscript refers to the iteration number.

There is no point in defining a relative criterion for node heads as these are measured relative to an arbitrary datum.

2.6 Calculation of Node Heads

When the solution has converged sufficiently the line flows are used for calculating the node heads. Starting with a line with one of its nodes a reservoir, the head at the second node is found by applying a head-discharge relation, such as equation 9(a). A line connected to the second node is then used, and so on through the network. The way in which the lines are selected may be either arbitrary, for example by following the initially assigned line numbers, or by following a minimum resistance tree, as suggested by Travers (Ref. 11). The line flows from the converged solution are all of about the same accuracy, that is about W_q if convergence criterion (iii) is used. The percentage error in line flow, and hence in head change, is therefore minimised in high flow, low resistance lines. When there is a choice in selecting the next line then the lowest resistance line should be used. Prim (Ref. 14) has described a simple procedure for constructing a network tree built up of the low resistance lines.

2.7 Improving the Speed of Convergence

Assuming that a good set of loops has been selected for the network, then further ways of improving convergence have been suggested.

Barlow and Markland (Ref. 5) suggest using an over-relaxation factor applied to the loop flow correction q of equation (4). They have used values up to 1.4.

Hoag and Weinberg (Ref. 13) have suggested a selective loop technique as a method of accelerating convergence. They propose doing only the large out of balance loops on alternate (say even-numbered) iterations.

2.8 Difficulties with Pumps and Non-Return Valves

During the course of the solution a pump may be required to operate outside the zone of normal operation, i.e. outside the first quadrant of Figure 1(B). This is allowed for by the approximate head-discharge relations for the second and fourth quadrants which are given in section 2.3.2 above. In this way continuity of flow may be maintained in the network. If the converged solution indicates pump operation outside the first quadrant (or even away from the optimum operating point) then the network components and demands are mismatched and design changes are indicated.

Similarly the presence of a non-return valve in a line may complicate the solution. If a non-return valve is assumed to shut when the line flow is reversed during the course of the solution then continuity is affected. To preserve continuity a redistribution of line flows is required along the shortest path connecting the end nodes of the line. An easier alternative is to ignore the presence of non-return valves (except that they may add an equivalent length to a pipe line) during the course of the solution. If the final solution indicates a reverse flow in a line containing a non-return valve then either the system components and the demands are mismatched, or that line should be removed from the network.

3. The Computer Program

A computer program, incorporating the conclusions of this investigation, is shown in flow chart form in Appendix A. The program is split into three sub-programs which communicate via common variables.

The first sub-program inputs line and node data and computes head loss coefficients for pipe lines. The coefficient A for a pipe line is equal to the exponential resistance coefficient r so that the output from this program is useful for loop selection. The second sub-program inputs the loop data.

The third sub-program performs the loop method analysis. Any consistent unit system may be used e.g. English Engineering units or S.I. units. Flow rates of imperial gallons per minute may be substituted for cusecs when the former system is used.

4. Use of Program on a Test Network

4.1 The Test Network

The test network, taken from McCormick and Bellamy (Ref. 3) is shown in Figure 2. There are 22 lines, one of which is a pump, and 18 nodes, including three reservoirs.

The results of using various loop sets etc. have been compared with those from a node method analysis of the same network. The node method analysis was a simple node by node technique, similar to that of McCormick and Bellamy (Ref. 3), but incorporating the pipe line head loss-discharge relation given by equation (9), as well as some other improvements.

The conclusions reached are strictly only applicable to the test network, but probably apply to similar small town water supply networks.

4.2 Effect of Pipe Line Head-Discharge Relation

Using loop set M (see Table 2), the network was analysed using an exponential head loss expression (corresponding to equation (5)) for the pipe lines. This expression was obtained by putting $B_j = 0$ in equations (10) and (11). The results shown in Table 1 indicate a significant difference in both line flows and node heads compared to the node method results. The most important difference is the over-estimation of node heads when

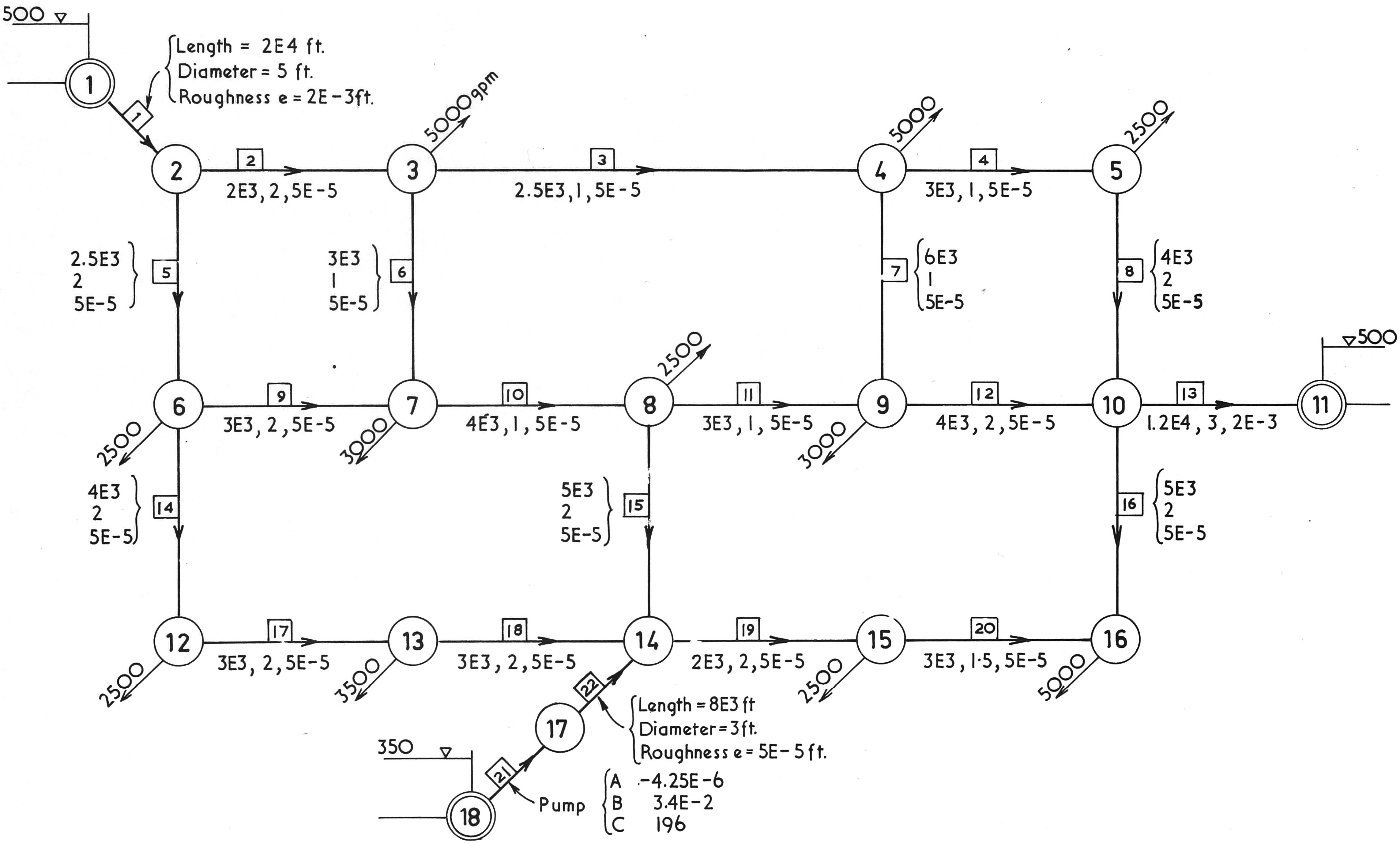


FIGURE 2 : TEST NETWORK

the exponential head loss expression is used. This could be important when minimum heads are required, for example for fire insurance purposes.

4.3 Effect of Loop Set Selection

From equation (1), 7 loops are required for the test network. The various loop sets tried and the number of iterations to converge are shown in Table 1. The results using these loop sets are compared with the node method results in Table 1.

From Tables 1 and 2, loop set M, with minimum resistance loops arranged in ascending order, appears to be the most efficient in both speed and accuracy. Loop set A, selected according to Barlow and Markland (Ref. 5), converges slightly more rapidly than loop set M, but the results show considerable discrepancies from the node method results.

4.4 Tests Using Various Convergence Criteria

Criterion (i) of Section 2.5.1 stopped the procedure after the same number of iterations as criterion (iii) when the respective limiting values were equal, that is when $\Delta Q_a = W_q$. As the latter criterion (iii) requires far less computer storage, and less program steps, it has been used in the program in the Appendix. As the loop method converges very rapidly when minimum resistance loops are used, then the value of W_q for criterion (iii) may be made quite small with little additional computation. For example, using loop set M, putting W_q = 100 gpm required 5 iterations, while putting W_q = 2.5 gpm required 9 iterations.

Criterion (iv) has been tried but appeared to be insensitive compared to criterion (iii). Changing W_h for criterion (iv) from 1.0 to 0.1 ft. required 2 additional iterations while changing W_q for criterion (iii) from 100 gpm to 10 gpm required 3 additional iterations.

4.5 Effect of Node Head Calculation Method

If the node heads are found arbitrarily from the converged line flows then serious error may result. In Table 1 node heads for loop set A are compared with the node method results for the case where the node heads were calculated by using the arbitrarily assigned line numbers shown in Figure 2. Serious under-estimation of some node heads results, for example node 9 where the head is found relative to node 4 by using the high resistance line 7.

Table 1: Results from Loop Method Compared with Node Method

Line No.	Node Method Flow, gpm	% Variation in Flow for Loop Set				
		M	N	H	A	M with $h_f = AQ^2$
1	16500				+1.1	+2.4
2	8027			-0.1	+8.6	+1.3
3	2254			+0.1	+3.9	+0.8
4	-1606	+0.1	+0.1		+10.8	-1.9
5	8473				-6.0	+3.6
6	773			-1.0	+78.1	+11.0
7	-1140	-0.1	-0.1	-0.2	-23.0	+ 1.0
8	-4106				+4.2	-0.7
9	2690			+0.3	-20.3	+0.9
10	463	+0.2	+0.2	+0.2	+12.6	+23.3
11	537	-0.4	-0.4	-2.0	+ 0.6	+28.5
12	-3603			+0.2	- 7.4	- 3.9
13	-11029			+0.1	- 1.5	- 3.6
14	3283			-0.2	+ 1.2	+ 8.7
15	-2574	-0.1	-0.1	-0.5	- 2.1	+ 1.7
16	3320				- 2.0	- 6.8
17	783	+0.1	+0.1	-1.0	+ 5.1	+36.4
18	-2715			+0.4	- 1.4	-10.4
19	4180				+ 1.6	+ 5.4
20	1680	+0.1	+0.1	-0.1	+ 4.0	+13.4
21	9467				- 0.2	- 0.1
22	0467				- 0.2	- 0.1

Table 1 (cont'd.) Results from Loop Method Compared with Node Method

Node No.	Node Method Head, ft.	Head Change in Feet for Loop Set					
		M	N	H	A	M with $h_f = AQ^2$	A with asc. line nos.
1	500						
2	494.5				-0.1	+0.1	-0.1
3	485.6				-1.6	+2.5	-1.6
4	454.1			-0.1	-4.1	+10.4	-4.1
5	474.1				+0.2	+4.1	+0.2
6	482.2				+1.2	+3.0	+1.2
7	480.5			-0.1	+1.8	+3.6	-11.5
8	477.8				+1.2	+3.9	-12.2
9	475.2	-0.1	-0.1		+1.1	+3.9	-12.2
10	479.1			+0.1	+0.6	+2.3	+0.6
11	500						
12	478.9				+1.1	+3.8	+1.1
13	478.7				+1.1	+3.8	+1.1
14	480.5	-0.1	-0.1		+1.0	+2.9	-12.2
15	477.8			+0.1	+1.0	+3.7	-12.3
16	474.9				+0.7	+4.2	+ 0.7
17	487			-0.1	+1.0	+0.6	+ 1.0
18	350						

Notes: (1) Over-relaxation factor = 1.4 for all runs

(2) Convergence criterion (iii), W_q = 5 gpm

(3) % Flow variation in a line = $\frac{|\text{Flow this set}| - |\text{Flow node method}|}{|\text{Flow node method}|} \times \frac{100}{1}$

% flow variation taken to nearest 0.1%

(4) Head change at node = Head this set - Head node method

Head change $<$ 0.1 ft. not shown

Table 2: Loop Set Details and Number of Iterations to Converge

Loop Set	Selection Method	Lines in Loop Number 1	2	3	4	5	6	7	No. Itns. to converge
M	Minimum resistance loops in ascending order	21, 22, 18, 17, 14, 5, 1	21, 22, 19, 20, 16, 13	2, 6, 9, 5	11, 12, 16, 20, 19, 15	9, 10, 15, 18, 17, 14	1, 2, 3, 4, 8, 13	4, 8, 12, 7	8
N	Natural loops	2, 6, 9, 5	3, 7, 11 10, 6	4, 8, 12, 7	9, 10, 15, 18, 17, 14	11, 12, 16, 20, 19, 15	21, 22, 18, 17, 14, 5, 1	21, 22, 19, 20, 16, 13	9
H	Maximum resistance loops in descending order	1, 2, 6, 10, 11, 7, 4, 8, 13	3, 7, 11, 10, 6	11, 7, 4, 8, 16, 20 19, 15	4, 8, 12, 7	9, 10, 11, 12, 16, 20, 19, 18, 17, 14	2, 6, 10, 15, 18, 17, 14, 5	21, 22, 15, 10, 6, 2, 1	25
A	Using Barlow and Markland (Ref. 5) rules	21, 22, 19, 20, 16, 13	21, 22, 18, 17, 14, 5, 1	1, 2, 3 4, 8, 13	2, 3, 4 8, 16, 20, 19, 18, 17, 14, 5	3, 7, 11 10, 6	9, 10, 15, 18, 17, 14	11, 12, 16, 20, 19, 15	7

Notes (i) Over-relaxation factor = 1.4

(ii) Convergence criterion (iii), W_q = 5 gpm

The error in node heads for loop set A is reduced to tolerable values for most nodes by selecting the lines to form a minimum resistance tree. In the case of "good" loop sets (M and N) this procedure gives results which are practically identical with those from the node method. The same shape of tree will result irrespective of the starting point in the network. The order of adding lines to the tree is the important feature for node head calculations.

4.6 Attempts to Improve the Speed of Convergence

The effect of applying an over-relaxation factor to the loop flow correction is shown in Table 3. For the test network a value of 1.4 was optimal, and this about halved the number of iterations required to reach convergence.

The selective loop technique of Hoag and Weinberg (Ref. 13) was tried on the test network without any net saving in execution time. Unless the criterion for accepting a loop for processing is set close to the final convergence criterion then all loops are processed after a few iterations. The saving in execution time by rejecting a few loops for the first few iterations would appear to be more than outweighed by the time spent examining loop out of balance and switching on alternative iterations.

5. Conclusions

1. By using an explicit approximation, the Colebrook-White equation for friction factor may be applied to the loop method of pipe network analysis.

2. The operation of a pump, both within and outside the range of normal operation, may be approximated by a second order polynomial in discharge.

3. A loop set with minimum resistance loops arranged in ascending order is the most satisfactory for speed of convergence and accuracy of the solution.

4. A convergence criterion based on examination of the value of the loop flow corrections after each iteration is an efficient and sufficiently sensitive way of terminating the iterative procedure.

5. When the line flows have converged the node heads should be calculated by using lines in the order in which they are added to a minimum resistance tree, starting with a line connected to a reservoir.

6. Over-relaxation factors up to about 1.4 can improve the speed of convergence of the procedure.

Table 3: Effect of Over-Relaxation Factors

Over-relaxation Factor	No. Iterations to Converge
1.0	17
1.1	15
1.2	13
1.3	11
1.4	8
1.6	13

7. When factors 1 to 6 above are used in the simple loop method then the results obtained are virtually identical with those from a node method analysis of the same network.

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APPENDIX A: COMPUTER PROGRAM

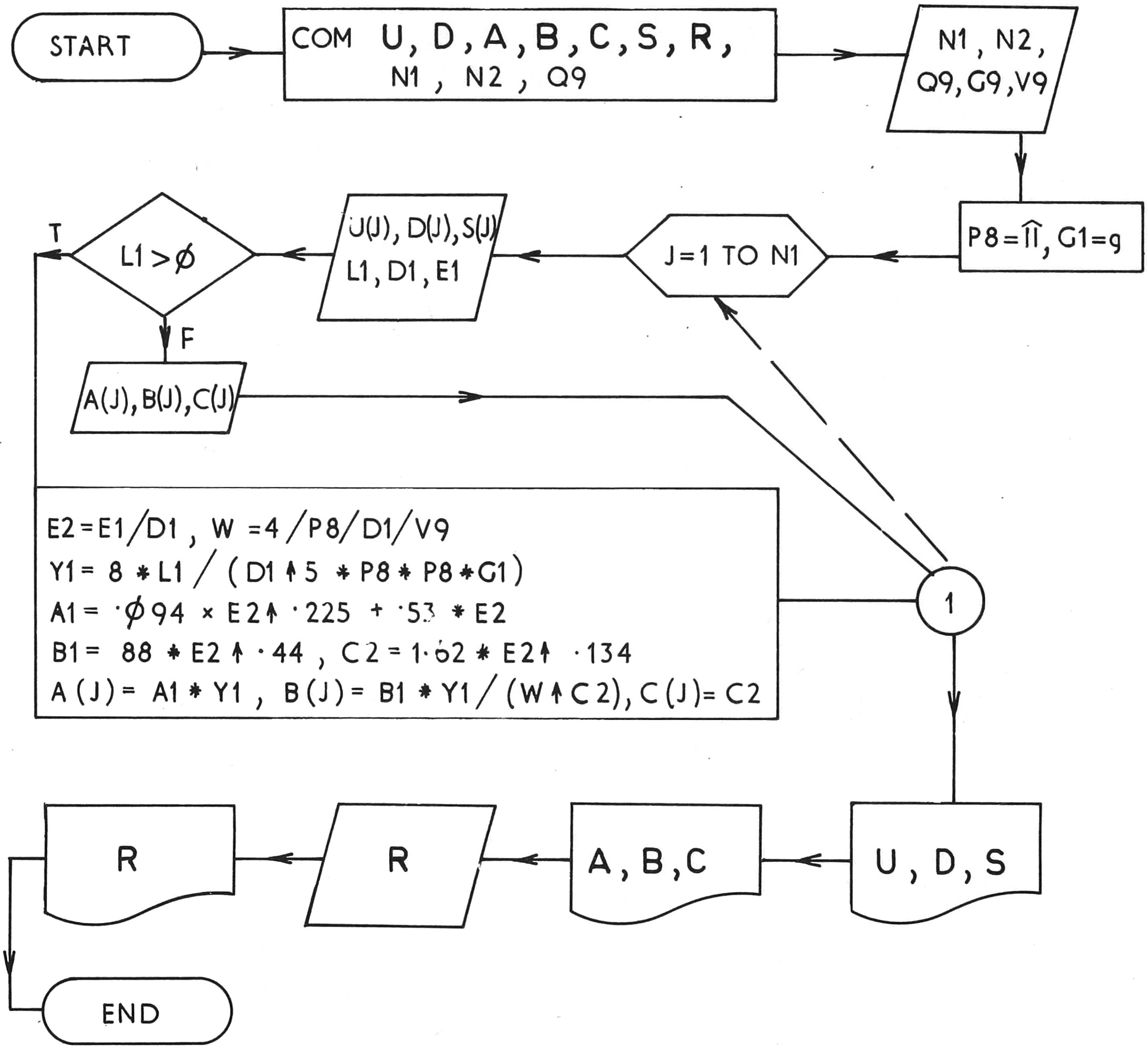
Notation

Variable in text	Variable in program	Appears in sub-program	Definition
A, B, C	A(J), B(J), C(J)	Com. 1, 2, 3	Empirical coefficients in head-discharge relation for a pipe or pump.
	D(J)	Com. 1, 2, 3	Node at upstream end of line j
d	D1	1	Pipe diameter
e	E1	1	Nikuradse pipe roughness
e/d	E2	1	Relative roughness of a pipe
g	G1	1	Gravitational acceleration
	G9	1	Unit weight of water
	H(I)	3	Current head at node i
	I1	3	Maximum number of iterations
	L(I)	Com. 2, 3	Number of lines in loop i
l	L1	1	Pipe length
m	M	3	Iteration number
	M4	3	Number of iterations to converge
N_p	N1	Com. 1, 2, 3	Number of lines
N_n	N2	Com. 1, 2, 3	Number of nodes
N_l	N3	Com. 2, 3	Number of loops
	N4	Com. 2, 3	Number of lines in minimum resistance tree
	01	3	Over-relaxation factor
	P(I, K)	Com. 2, 3	Identification number of k'th line in loop i
	P2	3	Results print trigger, = 0 print only converged iteration, = 1 print all iterations
	P3	3	Loop print trigger, = 0 print none, = 1 print all loop variables for each iteration

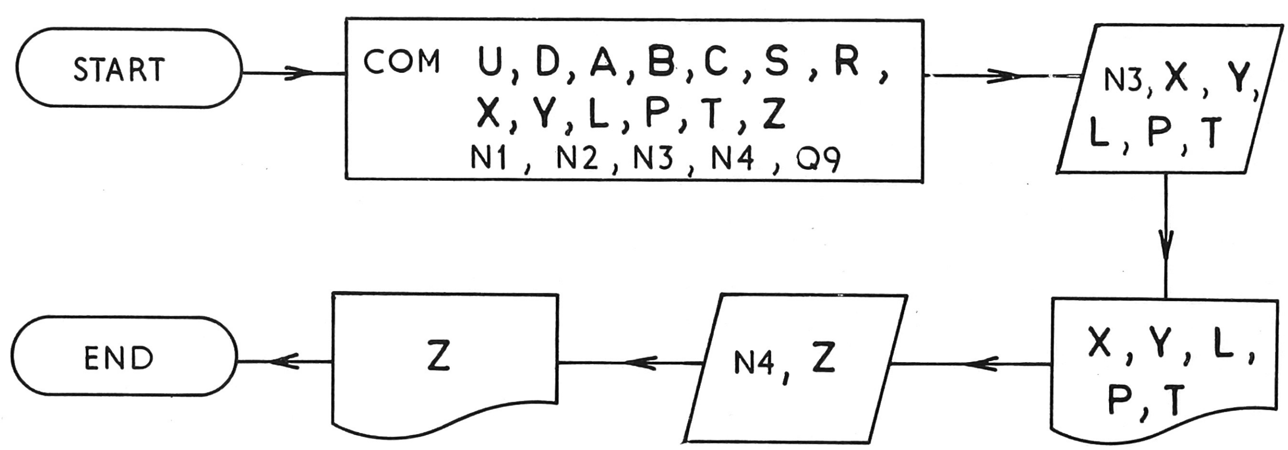
APPENDIX A (cont'd.): COMPUTER PROGRAM

Notation

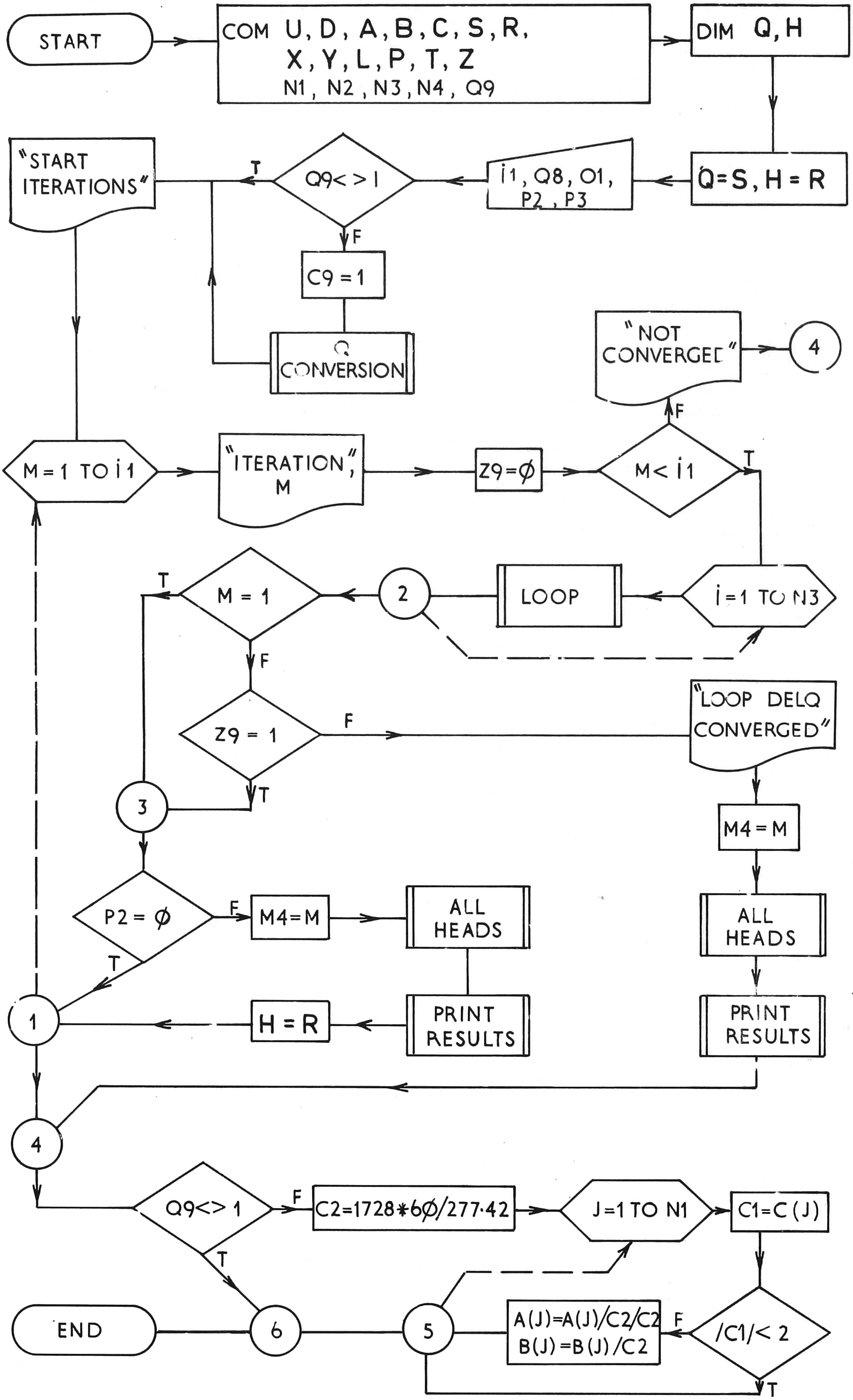
Variable in text	Variable in program	Appears in sub-program	Definition
	P8	1	⇑
Q_j	Q(J)	3	Current flow in line j
q	Q5	3	Flow correction for a loop
W_q	Q8	3	Maximum absolute value of Q5 for any loop for convergence
	Q9	Com. 1, 2, 3	Flow unit, = 1 for gpm
	R(I)	Com. 1, 2, 3	Initial head at node i, = 0 for a junction
	S(J)	Com. 1, 2, 3	Initial guess of flow in line j
$\sum f_f(Q_j)$	S1	3	Sum of head loss contributions to a loop by pipe lines
$\sum f_p(Q_j)$	S2	3	Sum of head rise contributions to a loop by pump lines
$\sum f_f'(Q_j)$	S3	3	Sum of tangent slopes for pipe lines in a loop
$-\sum f_p'(Q_j)$	S4	3	-1 x Sum of tangent slopes for pump lines in a loop
$X_i - Y_i$	S5	3	Known head difference between ends of a loop
T_j	T(I, K)	Com. 2, 3	Direction of loop i through k'th line in loop, = +1 from up-stream to downstream node, -1 for converse
	U(J)	Com. 1, 2, 3	Node at downstream end of line j
ν	V9	1	Kinematic viscosity of water
	X(I)	Com. 2, 3	Node at start of loop i
	Y(I)	Com. 2, 3	Node at end of loop i
	Z(I)	Com. 2, 3	Identification number of i'th line in minimum resistance tree
	Z9	3	Convergence indicator, = 0 converged, = 1 not converged



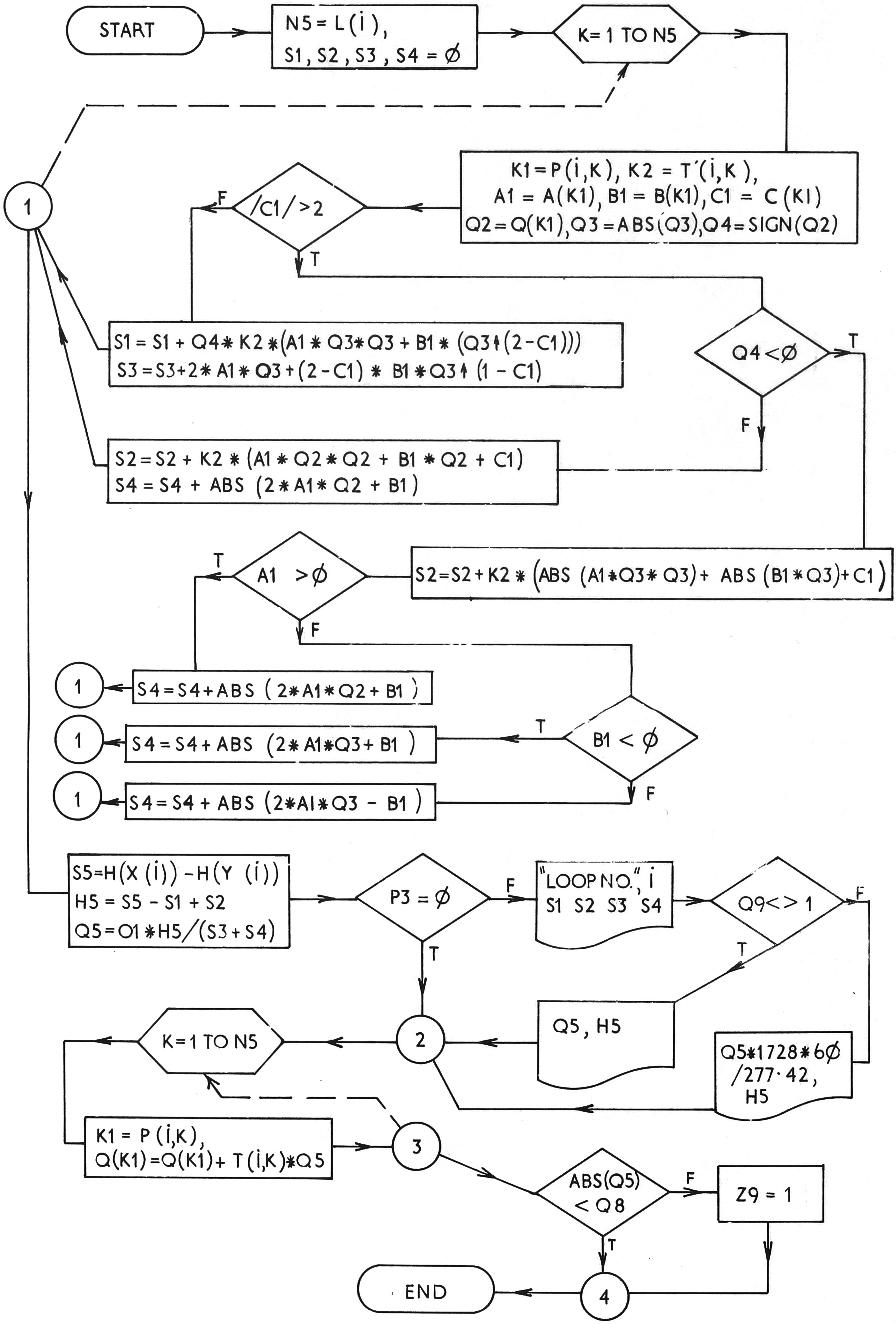
SUB PROGRAM 1: LINE AND NODE DATA INPUT



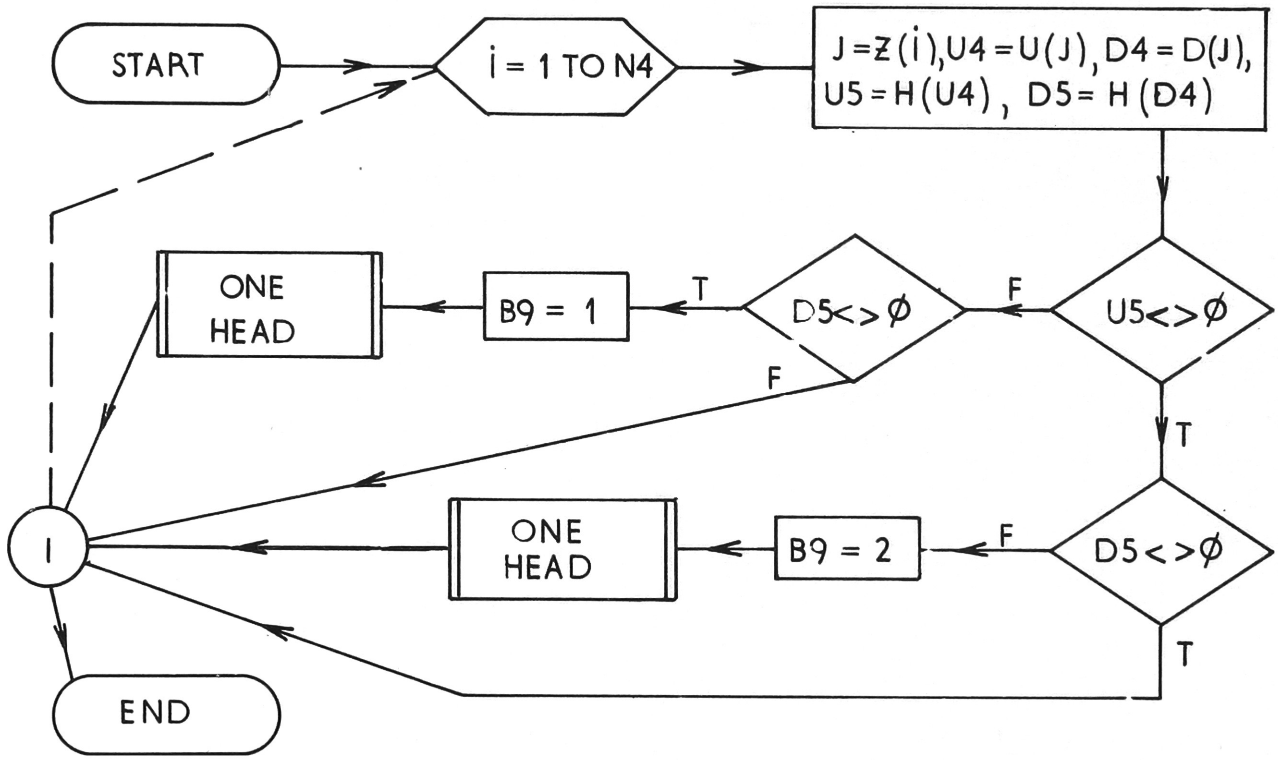
SUB-PROGRAM 2: LOOP DATA INPUT



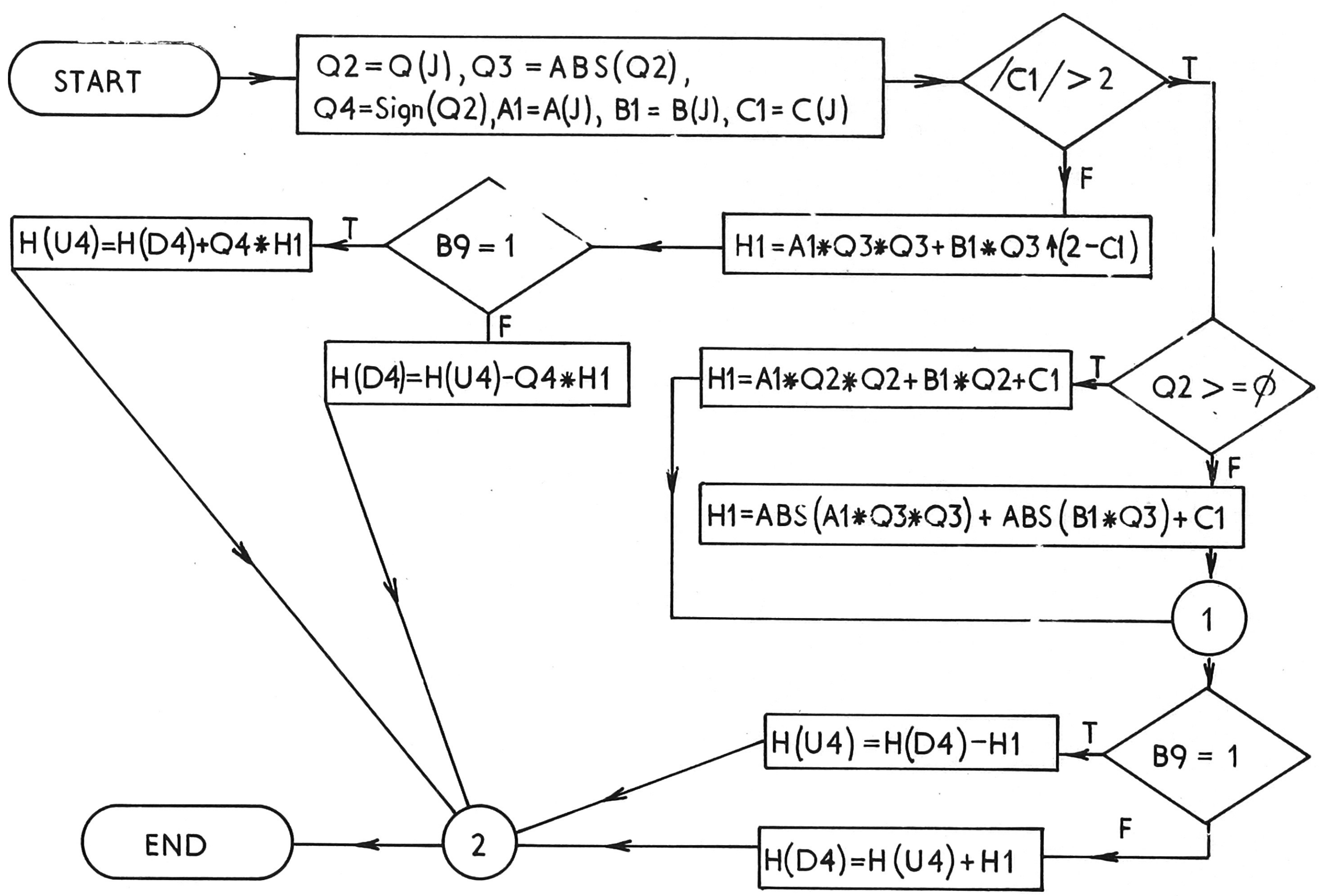
SUB-PROGRAM 3: LOOP METHOD ANALYSIS



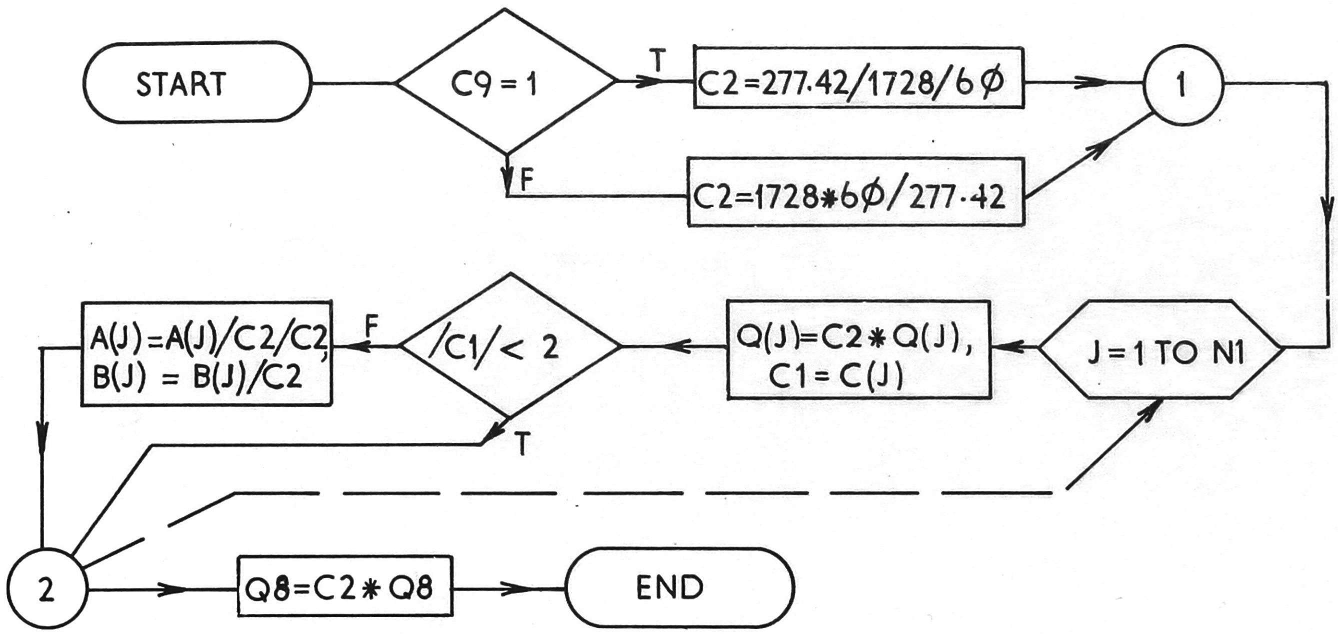
SUBROUTINE "LOOP"



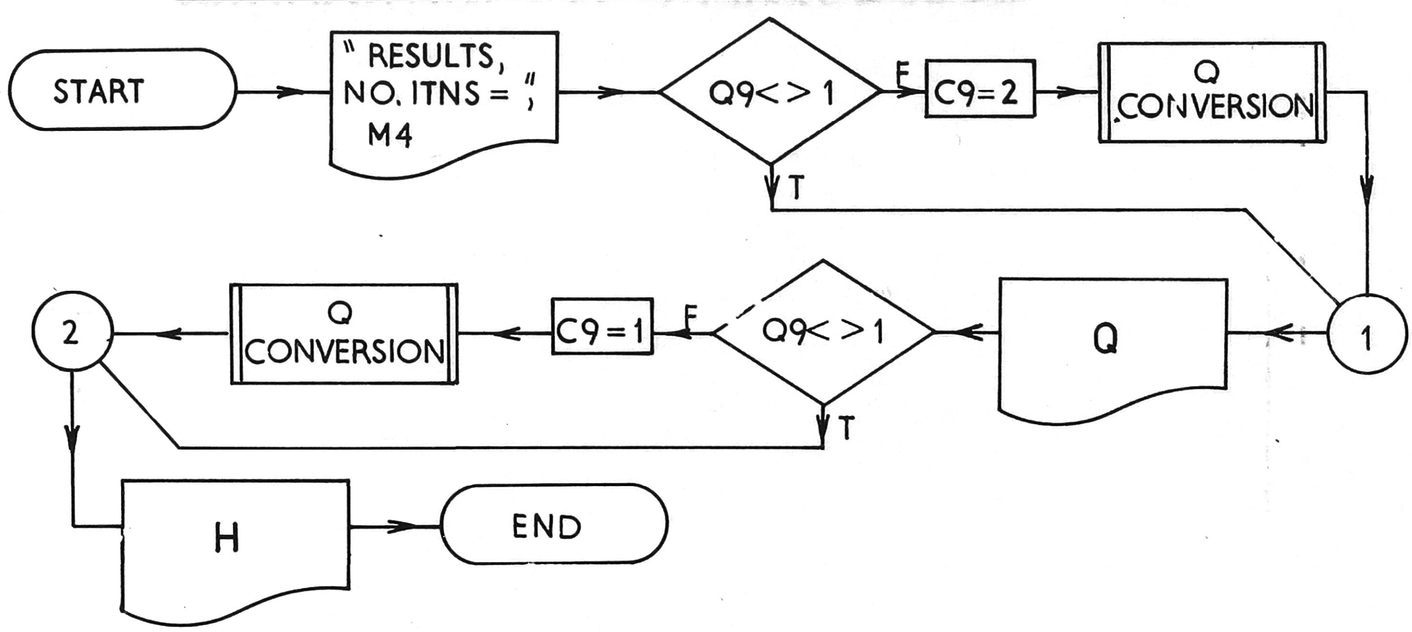
SUBROUTINE "ALL HEADS"



SUBROUTINE "ONE HEAD"



SUBROUTINE "Q CONVERSION"



SUBROUTINE "PRINT RESULTS"