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THE STEROSCOPIC LOCALISATION
OF MOVING OBJECTS

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The original work described in this thesis has been carried out by the undersigned, and no part of the work has been submitted for a higher degree at any other University or institution.

S U M M A R Y

The literature on the stereoscopic localisation of moving objects is reviewed and discussed. Two main issues emerge from the review. The first is the failure of the latency hypothesis to account for some of the results of previous investigations into the Pulfrich phenomenon; the second is the apparent displacement (localisation error) of an object moving in a frontal plane and viewed binocularly and with equal retinal luminance.

The Pulfrich phenomenon is examined theoretically. Some reasons for the facts not accounted for by the latency hypothesis are presented, and a mathematical description of the apparent path of the Pulfrich pendulum is developed.

A theory for the localisation error is also presented. The theory is named the "L-F-S" theory, and it involves the variation of visual latent period with retinal location, fixation disparity, and a path sampling hypothesis. The first two factors have been established in various contexts by other investigators, although their magnitudes in a localisation error experiment need to be studied. The path sampling hypothesis is concerned with the way in which an observer organises his sensory input during an experiment. It states that the observer bases his decision on the apparent path of movement while a moving stimulus object is approaching a stationary reference point. The hypothesis can be made to account for variations of the localisation error with velocity, as well as some of the discrepancies in the latency hypothesis of the Pulfrich phenomenon.

An experiment devised to test the path sampling hypothesis is described. In the experiment, an observer fixates a stationary rod in his median plane, while another rod follows a circular horizontal path, concave towards the observer. A psychophysical method is used to determine the apparent distance at which the moving rod intersects the median plane. The prediction of the path sampling hypothesis is that the apparent distance should increase as the curvature of the path is flattened. The experimental design makes use of recent developments in up-and-down (staircase) techniques; these are fully reviewed and discussed in an appendix. A method for applying feedback during the experiment is devised and used. The results support the notion that path shape influences the localisation error, but a systematic relationship cannot be demonstrated. The reason for this is that for a given set of conditions, the localisation error varies significantly both from day to day, and over periods as short as 25 minutes. While this variability makes comparisons between conditions difficult, it is in itself an interesting result, not previously reported. Its significance for the L-F-S theory is discussed, as are other factors such as the selection of psychophysical method, eye movements, and adaptation.

An experimental design which partially overcomes the difficulties imposed by the variability of localisation error is used in demonstrating a consistent relationship between the error and background luminance level. As luminance is increased, the apparent distance of the moving target also increases. This is not in complete agreement with the results of an earlier investigation, and it is tentatively concluded that the effect of luminance level is a characteristic function of the observer.

Finally, the status of the L-F-S theory is re-assessed. It is suggested that the theory is a useful model on which to base future research.

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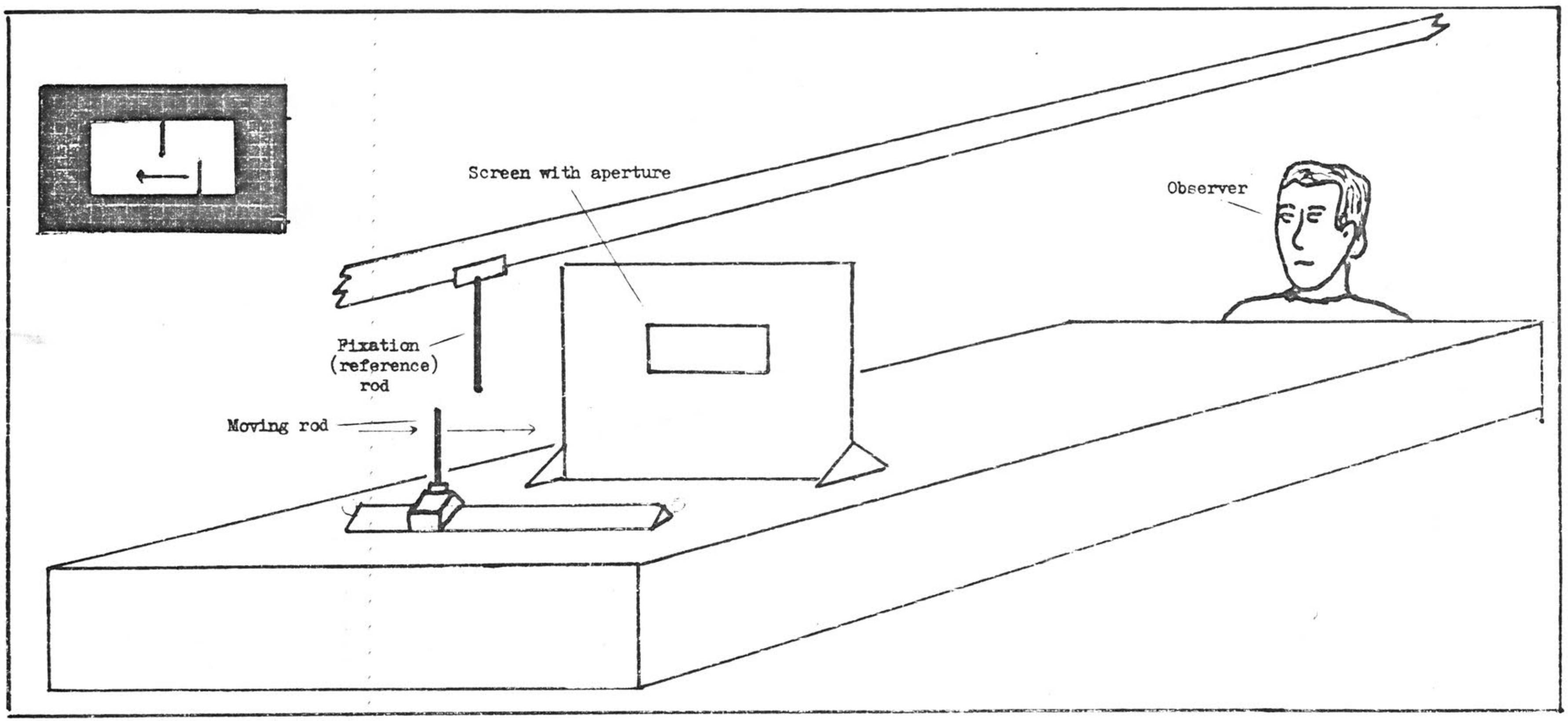

Figure I.1. The basic experimental situation. The observer's task is to judge whether the moving rod passes behind or in front of the stationary rod. His view of the stimulus configuration is shown at top left.

CHAPTER I

STEREOPSIS, MOTION, AND THE PULFRICH PHENOMENON

I.1 INTRODUCTION

(a) The Basic Problem

An observer views, through an aperture in a screen, a stationary object in his median plane (see Fig. I.1). A second object is made to traverse the field along a path parallel to the horizontal plane. The observer's task is to judge whether the moving object passes in front of or behind the stationary one.

This experimental paradigm is the basis of the study presented here. It is one in which the so-called monocular cues for space perception (relative size, interposition, perspective, light and shade) are absent or minimised, leaving stereopsis as the only means by which the observer can make accurate judgments.

A priori, one could expect that the sensitivity with which these judgments can be made will decrease as the moving object's velocity increases, as is the case with dynamic visual acuity (Miller & Ludvigh, 1962). There is no immediate reason to expect a change in the relative localisation of the two objects. Nevertheless, such a change can occur, as has been demonstrated by the experiments of Lit (1960c, 1964, 1966) and those to be described here.

Lee (1970a) has pointed out that the *binocular-kinetic* mode of visual space perception, the mode with which we are concerned here, "has been virtually neglected, despite the fact that it is the most common mode of visual space perception". Most previous investigations have been in the context of the Pulfrich stereoscopic phenomenon (Pulfrich, 1922), which occurs when an attenuator (filter) is placed before one eye during the binocular viewing of a moving object. Even Lit's experiments arose from Pulfrich-type experiments.

Ordinarily, stereoscopic perceptions are not made with an attenuator in front of one eye. However, the Pulfrich phenomenon is measurable, and its extent can be predicted by a widely-held theory: the latency hypothesis. Discrepancies between these predictions and empirical findings need not be accepted as evidence against the latency hypothesis; rather, they suggest that the binocular-kinetic mode is not a simple extension of the binocular-static mode. The same "laws" of retinal disparity and stereoscopic perception apply to both modes, but in the kinetic mode there are temporal factors to be taken into account.

A consideration of the Pulfrich phenomenon is important in understanding the stereoscopic localisation of moving objects in general. For this reason, and also because the literature is sparse in non-Pulfrich experiments, the phenomenon is discussed in detail in subsequent sections.

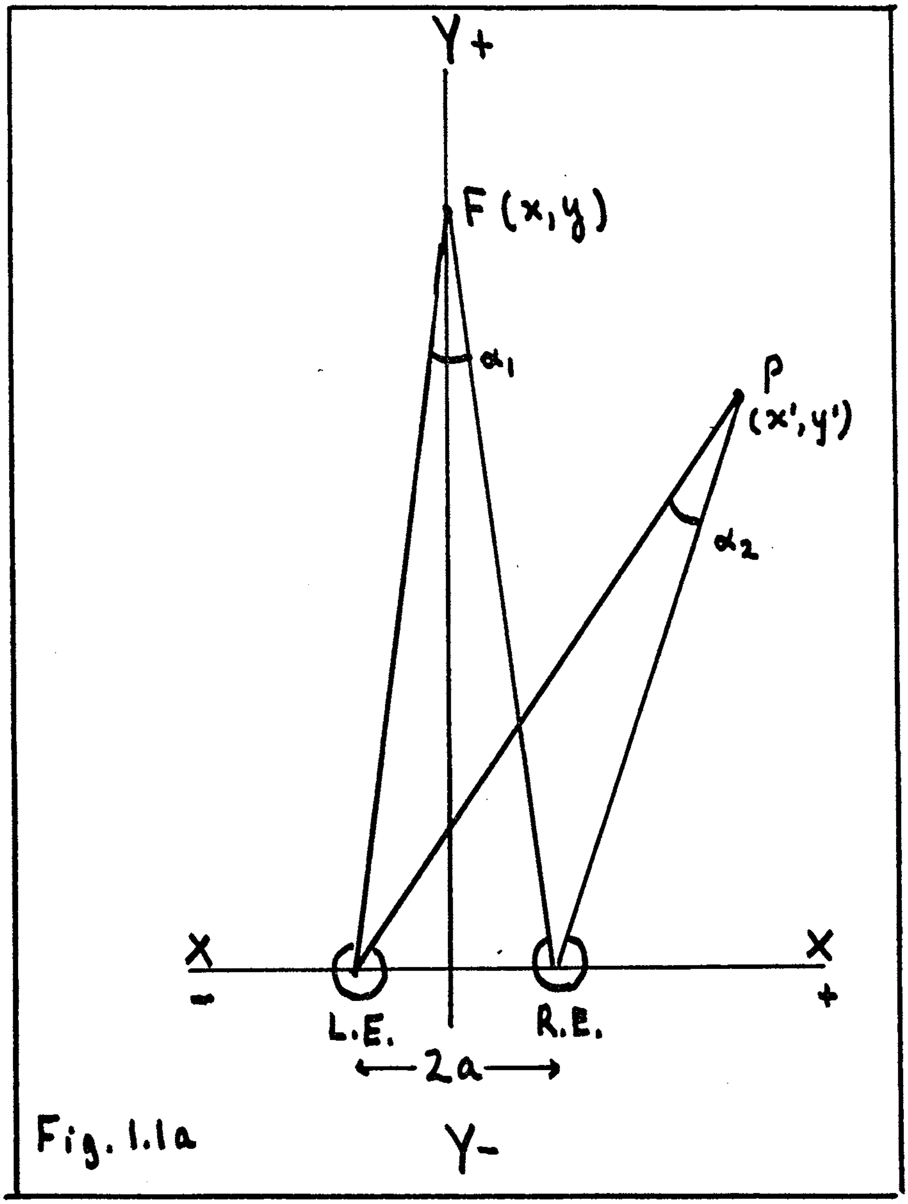

Figure I.1a. Illustrating conventions and co-ordinate system used in the text.

I.1 (b) Definitions and conventions.

(i) Co-ordinate system.

In most of the diagrams presented in the first three chapters, a plan view of the subject/stimulus system is depicted (see Fig. I.1a). A Cartesian co-ordinate system is used, with its origin at the centre of the inter-ocular base-line of the subject, whose eyes are on the x-axis of the co-ordinate system.

The subject's inter-ocular distance (distance between nodal points) is designated as 2a in order to simplify equations.

(ii) Localisation errors.

If y is the true distance of a point, and y' its apparent distance, then the localisation error is specified by the quantity $(y' - y)$. Thus a positive localisation error means that the apparent distance is greater than the real distance.

(iii) Angular magnitudes.

In order to facilitate comparisons, localisation errors are generally reported in terms of their angular magnitude, derived in the same way as the angular disparities describing, for example, stereoscopic thresholds (Ogle, 1962, p.284). Thus, in Fig. I.1a, the angular disparity corresponding to the localisation error $(y' - y)$ is given by $(x_1 - x_2)$, which to sufficient accuracy is equal to:

$$2a\ (y' - y)\ /\ (y.y').$$

If 2a, y, and y' are all in the same units, the expression gives the angle in radians. Multiplication by 206265 converts to seconds of arc.

(iv) Angular velocities.

Unless otherwise specified, angular velocities refer to the velocity of an object relative to the observer. Thus, if V cms/sec is the linear velocity of an object moving in a plane y cms from the observer, its angular velocity is $\tan^{-1}(V/y)$. Strictly speaking, angular velocity varies with lateral angle, but this expression is convenient, and sufficiently accurate for comparisons.

I.2 Some aspects of stereoscopic localisation.

The purpose of this section is not to present a complete discussion of theories of stereoscopic space perception, but to highlight and clarify some of the more relevant aspects.

(a) Stereoscopic localisation as a relative judgment.

Stereopsis is usually defined (e.g. Graham, 1965, p.524) as the discrimination "of difference in distance brought about by ... retinal disparity".

By definition, then, stereopsis is not concerned with judgments of absolute distance, but with the perception of space between objects, or between points on a solid object.

This implies that there must be at least two objects in the visual field for stereopsis can occur; a single point cannot be "seen stereoscopically".

The objects giving rise to disparate images need not be viewed simultaneously. This is indicated by experiments with after-images (Ogle & Reiher, 1962). Also, in the classical demonstration of the Pulfrich phenomenon (see Section I.3 for a description), the apparent elliptical path of a moving object can be perceived in the absence of any other object.

Thus, in the situation illustrated in Fig. I.1, stereopsis gives no information as to how far away either of the rods is from the observer. However, if the moving rod was following, say, a circular path, the non-linearity could be perceived stereoscopically, even if the stationary rod was not present.

(b) Stereopsis and the apparent fronto-parallel plane.

The apparent fronto-parallel plane (AFPP) is obtained by having a subject fixate a vertical rod at some distance in his median plane. Other vertical rods at various lateral positions are adjusted so that all the rods appear to be in the same vertical plane, parallel to the subject's frontal plane. (Ogle, 1962, Chapter 16).

The empirical AFPP is invariably a curved surface, usually concave towards the observer. The curvature is related to the curvature of the horopter, which is defined by Ogle (1962, p.326) as "that surface in space, for a constant fixation point of the eyes, any point of which would have images in the two eyes that would fall on

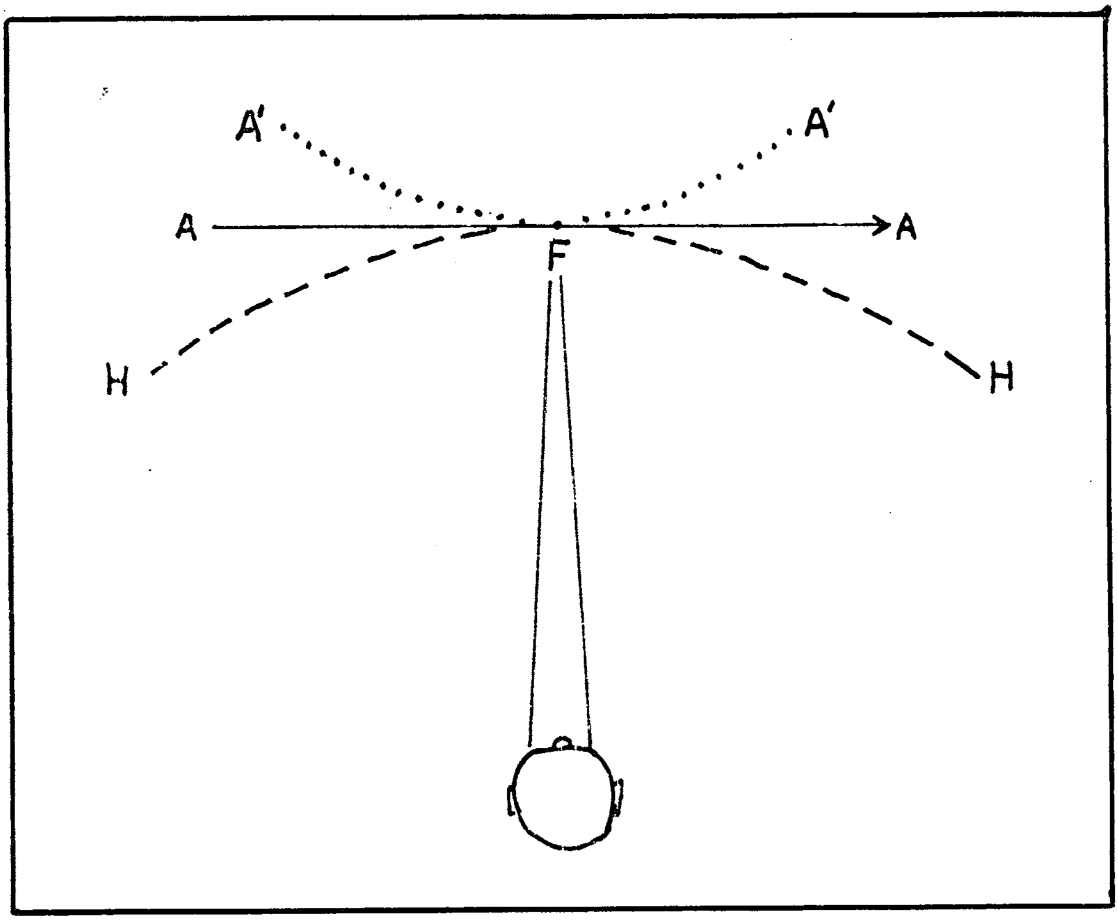

Fig. I.2. The observer fixates at F while an object moves along the linear path A – A. Stereoscopically, distances relative to F are referred to the apparent fronto-parallel plane, which may be curved as shown by HH. As a result, the moving object may appear to follow a curved path such as A' – A'.

corresponding retinal elements".

The horopter curvature must always influence stereoscopic judgments. The inverse of the empirical AFPP experiment is that objects which actually do lie in the objective fronto-parallel plane appear to be arranged along a curved surface.

Thus, in the experiment shown in Fig. I.1, if the lower rod moves along a straight line parallel to the observer's frontal plane, stereoscopically it should appear to follow a curved path. If, as seems to be the rule, the horopter is concave towards the observer, then the path of movement should be a curve convex towards the observer. At the same time, if the true path passes directly beneath the vertical rod, the apparent path should also appear to do so (see Fig. I.2).

(c) <u>The stability of retinal correspondence</u>.

Numerous horopter studies (summarised by Ogle, 1962, Chapter 16) indicate that there is an asymmetry in the spatial distribution of corresponding points on the retinae of the two eyes. The horopter is not a circle, as would be predicted if the distribution was symmetrical, but part of a somewhat flatter curve. Ogle (1950) has shown that this curve belongs to the family of conic sections, and with Ogle's analysis the stability of the organisation of retinal correspondence can be investigated.

From his own and Helmholtz's data on the AFPP at different viewing distances (Ogle, 1962, Chapter 16.V), Ogle concluded that this organisation was not stable with change in fixation distance. Amigo (1965, 1967) made similar findings with respect to asymmetric convergence; he concluded that there is a tendency on the part of observers to maintain a constant individual shape of the horopter curve.

Other experiments, particularly those with afocal magnifying lenses (Ogle, 1950) show that for a given observer, and for given conditions of viewing distance and convergence, the organisation of retinal correspondence _is_ stable. This is supported by neurophysiological experiments which demonstrate retinal correspondence at the cortex of the cat (summarised in Bishop, 1970).

Thus whatever the shape of the horopter may be, it would be expected that a subject's performance with a stereoscopic task would be similar from one day to the next, provided the experimental conditions were kept constant.

(d) _The stereoscopic threshold_.

The stereoscopic threshold seems to be at a minimum at a slightly extrafoveal angle of 15 to 21 min arc (Hirsch & Weymouth, Fabre & Lapouille, cited in Ogle, 1962, Chapter 15.II), and rises as peripheral angle increases. Typically, the minimum stereoscopic threshold is of the order of 10 sec arc or less at or near the fovea, rising to about 50 sec arc at a 10° peripheral angle.

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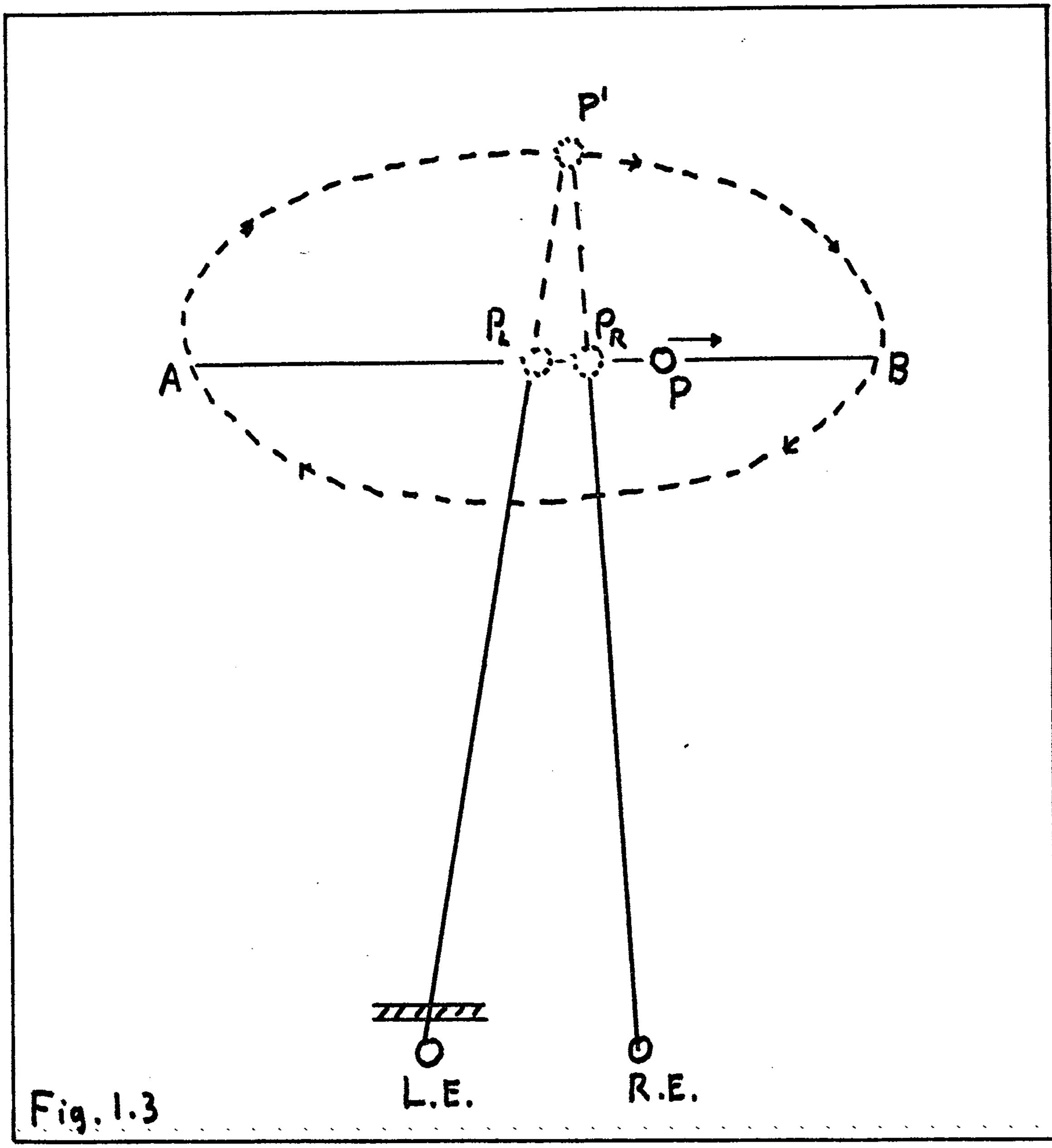

Figure I.3. The Pulfrich phenomenon. When viewed with a filter in front of the left eye, a pendulum bob moving along A-B appears to follow a curved path. According to the latency hypothesis, when the bob is at P, the apparent positions of the uniocular images are P_R and P_L, giving rise to a single fused image at P'.

Thus stereopsis provides a cue for the perception of relative distance much more sensitive than other cues such as convergence and apparent size. In any stereoscopic experiment where thresholds of 40 sec arc or less are obtained, it would be difficult to conclude that something other than stereopsis was used for the judgments.

I.3 The Pulfrich Phenomenon.

An observer with normal binocular vision watches a pendulum, swinging in a plane parallel to his frontal plane, with a filter in front of his left eye. The pendulum bob appears to follow an elliptical path, approaching the observer during its right-to-left swing, and receding during the left-to-right phase. If the filter is placed before the right eye, the direction of the apparent path is reversed.

The phenomenon was first reported by Pulfrich (1922). (It is interesting to note that Pulfrich was blind in one eye (Enright, 1970), and so never observed the effect which bears his name). Fig. I.3 illustrates the phenomenon, and also indicates a widely accepted theory of its causation. In the figure, the pendulum is moving from left to right along its path AB. The observer, whose eyes are indicated at L.E. and R.E., has a filter in front of his left eye. The apparent path of the pendulum is shown by the dotted ellipse.

According to the latency hypothesis, first put forward by Pulfrich (1922), the effect is due to the latent period of vision, that is, the time between the appearance of an image on a retinal element and the arrival of excitations due to that stimulus at cortical (or post-cortical) cells. The latent period increases as the intensity of the stimulus decreases; direct and indirect support for this relationship can be found in Gotch (1904), Granit (1947), Bernhard (1940), Arden & Weale (1954), Roufs (1963), May (1964).

Thus in Fig. I.3, when the pendulum bob, moving from left to right, is at P, it is "seen" by the right visual system at a slightly earlier position, P_R. The left eye has a longer latent period, because the filter has reduced the stimulus intensity, and its image is at P_L. If the fixation point (not shown in the diagram) is, say, somewhere along AB, then the two uniocular images P_L and P_R in effect arise from non-corresponding retinal points. A "fused" image of the bob is seen at P', which is the intersection of the projections of these two points through the nodal points of the eyes.

The amount of disparity depends on the difference between the latencies and the velocity of the object. The pendulum bob, moving in approximate simple haronic motion, has its maximum velocity at the centre of its path, so here the disparity will be greatest. The disparity decreases towards the ends of the swing, accounting for the elliptical apparent path.

A full analysis of the path of the Pulfrich pendulum is given in Chapter II.

As Ogle points out, the latency hypothesis "has generally accounted for the facts" (Ogle, 1962, p.301). It does not account for all of the facts; however, in view of the striking evidence supporting the inverse relationship between visual latency and stimulus intensity, the hypothesis cannot be rejected without seriously questioning the classical concepts of stereopsis and retinal correspondence.

I.4 Previous studies.

(a) Monocular adaptation.

Pulfrich presented the phenomenon as a method for heterochromatic photometry: the visual transmittance of a coloured filter used for eliciting the phenomenon could be found by placing appropriate neutral filters of known transmittance in front of the other eye until the effect was no longer seen. Engelking & Poos (1924, cited by Lederer, 1957) discussed the limitations of the method, and also reported that the phenomenon occurred when one eye was dark-adapted and the other eye light adapted. The image seen by the dark-adapted eye appeared brighter, but the direction of the apparent path indicated that this eye had the longer latency. They accounted for this by assuming that the latent period is greater for rods than for cones. Lythgoe (1938) could not confirm the observations of Engelking & Poos, but found that a light shining into one eye produced the same effect as a filter in front of the other eye.

Rock & Fox (1949), using six subjects, found that the Pulfrich phenomenon did occur when one eye was dark-adapted. Their results " ... would seem to indicate that the retinal elements, more particularly the dark-adapted cones, decreased their temporal latency of response as they become light-adapted" (Rock & Fox, 1949, p. 284). In other words, the dark-adapted eye has the longer latency, which is precisely the same as the conclusion reached by Engelking & Poos (according to Lederer, 1957, and Kronenberger, 1926). For some reason, Rock & Fox go on to say that their result contradicts that of Engelking & Poos; one can only assume that there was an error in their translation.

Diamond (1958) reported an experiment in which the spokes of a wheel rotating in a frontal plane appeared to be displaced when a peripheral inducing field was in the view of one eye. Diamond's experiment was essentially the same as that of Lythgoe, and similar results were obtained: the inducing field had the same effect as a filter in front of the other eye. (Diamond was apparently unaware of Lythgoe's 1938 report). According to Diamond, his finding "supports the notion that a different physiological mechanism subserves contrast brightness reduction than that which subserves filter brightness reduction", which is too obvious to require further comment. He goes on to suggest that the eye with the inducing field has a reduced latency, brought about by entoptic light scatter in that eye. This is a reasonable explanation, and is in accordance with Engelking & Poos' observations with light- and dark-adapted eyes.

(b) Monocular observations.

Kronenberger (1926) reported that for large differences in adaptation between the two eyes the Pulfrich phenomenon was either absent or reversed. He did not make any quantitative measurements of the apparent displacement, but his anecdotal report strongly suggests that binocular vision was inoperative during the observations. Apparently, the difference in adaptation was so great that binocular vision could not be maintained, one eye taking up its phoria position. The direction of apparent rotation could be changed by an effort of will.

Katz & Schwartz (1955) placed Polaroid sheets in front of a light traversing a horizontal track in simple harmonic motion. With Polaroid filters at appropriate axes in front of his eyes, an observer twelve feet from the stimulus could be made to see the whole track with both eyes, or each half of the track with either the homolateral eye or the contralateral eye. Their subjects reported non-linear paths under all conditions, with or without a uniocular filter.

It seems, then, that a pseudo-Pulfrich effect can occur under conditions of monocular viewing. Curthoys (1964) had naive observers view luminous pendulum bobs in a dark room. They consistently reported that the bobs followed curved paths under conditions of monocular viewing and equiluminance binocular viewing, as well as with a filter in front of one eye. Curthoys did not make measurements of the effects, depending only on the observers' reports.

Curthoys+ (1965) pointed out that his results could be interpreted in terms of the ambiguity of retinal image motion. Consider a point moving with constant angular velocity around a circular path in a horizontal plane. An eye viewing this point some distance away in the same horizontal plane will have an image which is moving across the retina with simple harmonic motion: the same kind of motion as the image of the swinging pendulum. Thus there is nothing in the retinal image which can differentiate an object moving with varying velocity along a straight line from one which is following a curved path with constant angular velocity.

Curthoys' observations are therefore in the same class as other kinetic illusions such as the kinetic depth effect (Dember, 1963, p.183) and the rotating trapezoid (Ames, 1951). The issue is complicated by the fact that the misinterpretation of retinal movement can occur under binocular conditions, where stereopsis should provide an unequivocal cue for correct localisation. One can suggest that with a

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\+ Some of the ideas in this paragraph arose during informal conversations. The author accepts responsibility for any mis-statements of Curthoys' concepts.

kinetic array the stereoscopic perception of distance is weakened (cf. increases in the kinetic stereoscopic threshold reported by Lit, 1966), thereby enabling conflicting monocular cues to manifest themselves. Whether or not monocular cues could actually modify stereoscopic localisation remains to be seen.

It may be pointed out that the monocular pseudo-Pulfrich effect does not affect the validity of the latency hypothesis, although it could alter the compellingness of the true Pulfrich phenomenon. Perhaps it is imparsimonious to have two explanations for phenomena which give rise to the same subjective reports, but on the other hand there is no reason to expect that a binocular phenomenon should have the same mechanism as a monocular one.

(c) Complex stimulus arrays.

Gerard (1935) described manifestations of the Pulfrich phenomenon with arrays more complex than the oscillating pendulum. In one experiment, an object moving in a horizontal circular path was observed by a binocular subject with a filter before one eye. With the appropriate filter, the object appears to move in a flat plane; a more dense filter makes the object appear to follow an elliptical path in a direction opposite to the real one. (This is explained logically by Gerard in terms of the latency hypothesis).
If several objects are made to move along concentric paths with different radii, they appear to follow different paths

when viewed with a filter in front of one eye, giving effects which are more striking if the illumination is arranged so that objects are brighter when they appear further away. Gerard states (in translation): "In this experiment, the visual latent period explanation of the stereo-effect is of minor importance because differences in the visual latent period only give the first 'push' to develop the phenomenon - the whole phenomenon seems to depend on 'sensory comprehension' ". That is, retinal disparity may be the stimulus for the perception of depth differences, but the way in which the perception of the entire array is organised depends on non-stereoscopic factors.

(d) Eye movements.

Fischer & Mex (1950) found that the Pulfrich phenomenon occurred when the eyes followed the moving pendulum, as well as when fixation was directed at a stationary object near the centre of the path, although the effect was greater with static fixation. Rosemann & Buchmann (1953) reported that the apparent depth of the excursion of the pendulum bob was not affected by fixating the bob itself or by using a stationary fixation point. The discrepancy between these two reports may be due to the fact that Fischer & Mex were discussing subjective reports, whereas Rosemann & Buchmann made measurements of the displacement. Nevertheless, the influence of eye movements on the Pulfrich phenomenon is of great importance, particularly if one notes that if each eye has a different latent period, all eye movements are in effect disjunctive, and spatio-temporal relationships may be altered.

Rosemann & Buchmann (cited in Lederer, 1957) did record eye-movements during observations of the Pulfrich effect. Using electro-oculography, they found that with a fixation point no eye movements greater than one or two degrees occurred, and there were no regular optokinetic movements. What remain to be done are similar experiments with apparatus capable of better resolution than the electro-oculograph, and capable of recording movements of each eye separately.

(e) Stimuli moving along inclined paths.

Fischer & Kaiser (1950) examined the Pulfrich effect with a stimulus moving back and forth along a line with sinusoidally varying velocity. The path could be varied from the horizontal to the vertical meridian. They expected that the amount of apparent displacement would decrease with the cosine of the angle of inclination, since the horizontal disparity would decrease in a similar fashion. They found that the decrease differed from the cosine function, and use this result to attack the latency hypothesis. What they did not take into account was that as the path of movement was changed from horizontal to vertical, an increasing amount of vertical disparity was introduced, in addition to the horizontal disparity. Fusion may have become difficult, or cyclo-torsional movements may have occurred to counteract the oblique disparity; in either case, the magnitude of the Pulfrich phenomenon could not be predicted from the latency hypothesis alone.

(f) Changes in phenomenal size.

Observers often report (e.g. Engel & Fischer, 1950) that the pendulum bob appears to increase in size as it approaches, and decreases as it recedes. This is a manifestation of size constancy; the retinal image size is the same for all apparent distances, and hence there is a phenomenal change in size.

(g) Shape of the path of the Pulfrich Pendulum.

Engel & Fischer (1950) also reported that the apparent path of the Pulfrich pendulum was not elliptical, but pear-shaped. A mathematical analysis of the apparent path is presented in Chapter II, where it is shown that although geometrically the path is always symmetrical, there are conditions in which large and rapid changes of disparity make it unlikely that stereopsis can operate accurately. Furthermore, one must take into account the curvature of the horopter (see Chapter I.2b), and the transformation of stereoscopic space into a non-Euclidean space. These could account for reports of non-elliptical paths. One might well recall Gerard's description of latency differences only giving the first "push" to develop the observed phenomena.

(h) Practical implications.

Some authors have been concerned with manifestations of the Pulfrich phenomenon during automobile driving. Lederer (1957) showed that displacements could occur with objects moving in directions other than frontally. He suggested that in night driving, one eye may be exposed more than the other to the headlights of an oncoming vehicle, leaving that eye relatively light-adapted. With a latency difference of as little as 4 msec., and at ordinary rates of motor car travel, quite large stereoscopic displacements could occur. These could account for otherwise inexplicable collisions at night.

Gramberg-Danielsen (1963) reported a case in which a patient left the rooms of an ophthalmologist with the pupil of one eye dilated by a mydriatic. The patient drove off in his motor car, and collided with a cyclist at an intersection. Because of the greater intensity of the retinal image in the eye with the dilated pupil, Gramberg-Danielsen concluded that the Pulfrich phenomenon was responsible for the accident.

Strickland, Ward, & Allen (1966) had five subjects make observations with a filter in front of one eye while sitting in a moving motor car. As well as changes in apparent distances, the subjects reported distortions in size and depth. All of the observations were attributable to latency differences and the Pulfrich effect.

Enright (1970) also commented on the distortions observed from a motor car while wearing a monocular filter.

He also suggested that the Pulfrich phenomenon could be used to create three-dimensional movies and television. The limitations of such systems are obvious, since a pseudo-stereoscopic effect would occur if the camera or the objects depicted moved in the wrong direction.

I.5 Lit's experiments.

(a) Unequal retinal illuminance.

The most reliable and extensive quantitative data on the Pulfrich phenomenon have been those reported by Lit (1949, 1951, 1960(a), 1960(b)).

Initially, Lit (1949) used a stimulus rod oscillating with simple harmonic motion in a frontal plane, duplicating the conditions of the original Pulfrich demonstration. The subject, with unequal neutral density filters before his eyes, adjusted a stationary rod until it was equidistant with either the furthest or nearest point of the apparent path. With mathematical equations similar to those developed in Chapter Ib, the latency difference corresponding to each apparent displacement could be calculated.

In these experiments, retinal illumination was the only variable studied. The results were generally in agreement with the latency hypothesis. The latency difference was found to increase systematically with increasing differences between retinal illumination, and Lit was able to fit his results into a model in which the absolute visual latent period varies with the inverse of the logarithm of stimulus intensity.

With one subject, Lit found, as predicted by the latency hypothesis, that the latency difference was the same for both directions of movement. With his other subject, however, the displacement for one direction of movement was not the same as that calculated by using the latency difference for the other direction. Part of this discrepancy was due to the fact that even with equal retinal illumination, this subject had a localisation error, the fixation rod being adjusted about 5 mm (approx. 94.50 sec arc) in front of the 'true' plane of oscillation. That is, the oscillating target appeared to be moving in a plane nearer than its true plane. The displacement was the same for both directions of movement.

Lit noted that Wolfflin, in 1925, had reported a similar localisation error, but in the opposite direction.

In the 1949 paper, Lit suggested that the error was related to fixation disparity (see Chapter III.2), since the subject was esophoric for the fixation distance of the apparatus. Exactly how fixation disparity can affect stereoscopic localisation was not made clear by Lit; the problem will be further discussed in Chapter III.

Lit & Hyman (1951) reported experiments in which the stimulus moved with constant linear velocity rather than simple harmonic motion. Constant velocity simplifies the

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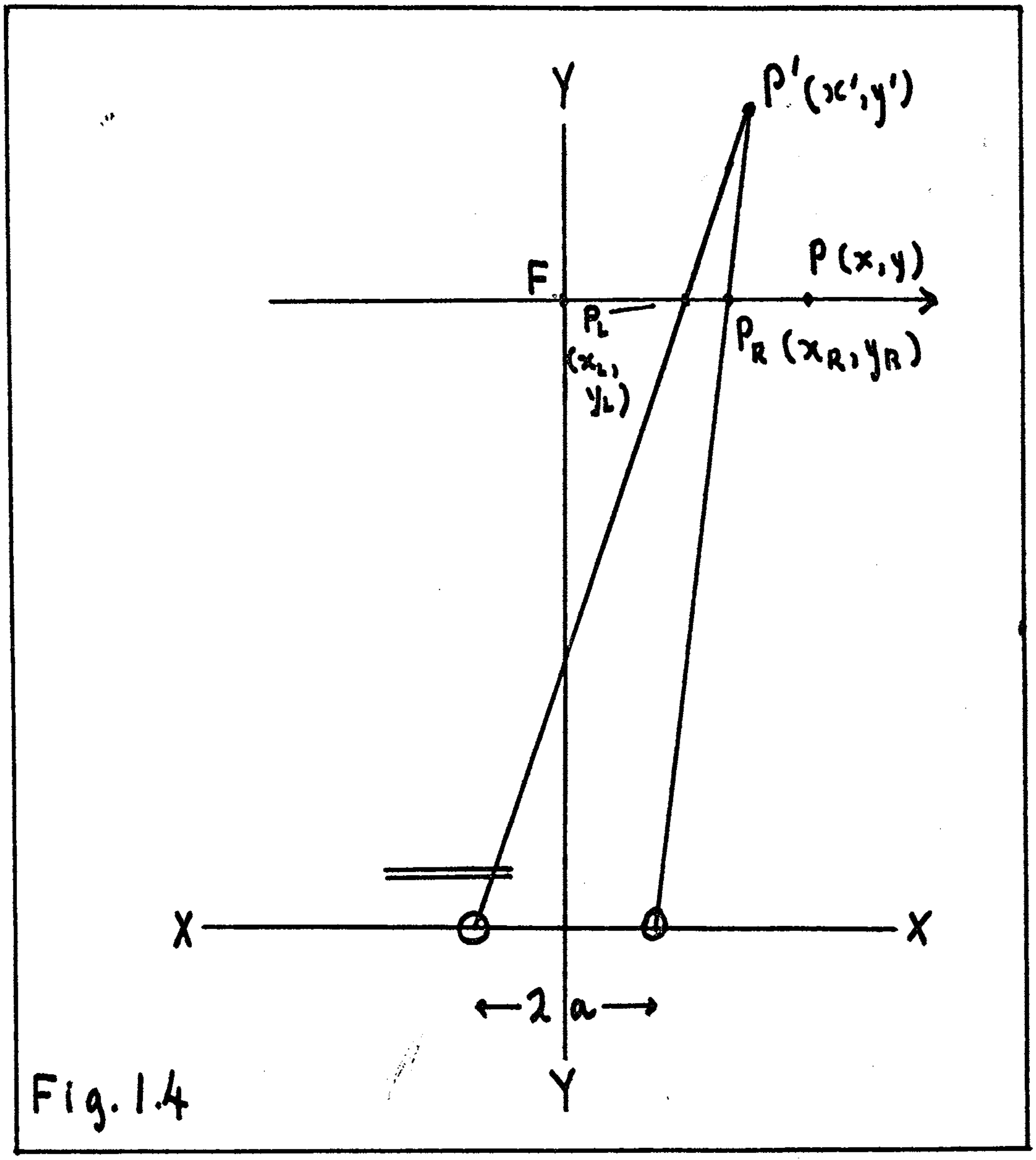

Figure I.4. Derivation of latency difference from apparent displacement (see text).

calculations, and furthermore reduces the possibility of sinusoidally varying velocity being misinterpreted as movement in depth (see Section I.4b).

(b) Calculation of latency differences;

The calculation of latency difference from the apparent displacement is derived from the geometry of Fig. I.4. Here, a co-ordinate system is used with its origin ($x = 0$, $y = 0$) at the centre of the interocular base-line. The conventions of the Cartesian co-ordinate system are used, with velocities in the left-to-right direction being considered as positive vectors.

In the figure, a point, whose momentary position P has co-ordinates (x, y), is moving from left to right with constant velocity V. A filter is placed before the left eye, so that the latency t_L for this eye is greater than the latency t_R of the right eye. Thus, when the object is actually at P, the uniocular images are at P_L and P_R, giving rise to a "fused" image at P'.

The co-ordinates of P_L and P_R are

$$\left.\begin{aligned} x_L &= x - V.t_L \\ x_R &= x - V.t_R \\ y_L &= y_R = y \end{aligned}\right\} \quad \ldots \quad \ldots \quad (I.1)$$

From simple geometry

$$y'/y = (x' + a)/(x_L + a)$$

and

$$y'/y = (x' - a)/(x_R - a)$$

which can be solved to give x' and y' independent of each other.

$$x' = a(x_L + x_R)/(x_L - x_R + 2a)$$
$$y' = 2ay/(x_L - x_R + 2a)$$

Substituting the relationships of equations I.1 gives

$$x' = a\left[2x - V(t_L + t_R)\right] / \left[2a - V(t_L - t_R)\right] \quad \ldots \quad \text{I.2}$$

and $$y' = 2ay/ \left[2a - V(t_L - t_R)\right] \quad \ldots \quad \ldots \quad \text{I.3}$$

Equation I.3 is the more important. The interocular distance 2a, the observation distance y, and the velocity V are experimental constants, and the apparent distance y' is found by the experiment, enabling the latency difference $(t_L - t_R)$ to be computed:

$$(t_L - t_R) = 2a(y' - y)/Vy' \quad \ldots \quad \ldots \quad \text{I.3a}$$

Equations I.3 and I.3a are equivalent to the equations given by Lit & Hyman (1951) but give a better idea of the predictions made by the latency hypothesis.

(c) <u>Effects of observation distance, target thickness, and velocity</u>.

In the experiments described by Lit & Hyman, observation distance (y in equation I.3) was the variable studied. However, the linear velocity V was also varied, in order to keep angular velocity constant at 18.91°/second. The results were in good agreement with the latency hypothesis. For a given set of illumination conditions, the solution of equation I.3a was the same for all values of y and V.

Lit (1960a) used the same apparatus to see if target thickness had any effect on the magnitude of the Pulfrich phenomenon. For an observation distance of 100 cms, he

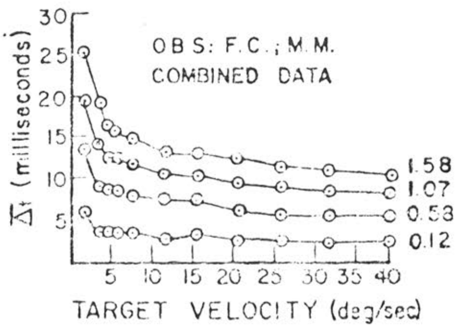

Fig. I.5. From Lit (1960(b), Fig. 4. Each data point is the latency difference obtained at the angular velocity indicated on the abscissa. The figures at the end of each line are log (E_R/E_L), E_R and E_L being the retinal luminances for the right and left eyes. Latency differences were calculated from apparent displacements, as in Equation I.3a.

found no differences for target thicknesses from 0.031 inch to 0.460 inch. However, in these experiments he found that the computed latency difference decreased as the angular velocity of the target increased. That is, apparent displacement was not related to angular velocity in the way predicted by the latency hypothesis.

The relationship between target velocity and apparent displacement was considered further in Lit's next paper (Lit, 1960b). Figure 1.5 illustrates the average results, giving computed latency difference as a function of target velocity. For velocities less than about 15°/sec the apparent displacements do not agree with the predictions of the latency hypothesis: they increase as velocity decreases, giving an increase in the computed latency difference. For higher velocities, the latency difference is constant, in accordance with the hypothesis. The discrepancy at lower velocities could not be accounted for in terms of any localisation errors obtained with equal retinal luminance.

(d) Equal retinal illuminance: the localisation error.

In subsequent experiments, Lit studied the localisation error obtained under conditions of equal retinal illuminance. Two subjects demonstrated opposite localisation errors (Lit, 1960c). One subject was slightly esophoric, and had a positive localisation error, the oscillating target appearing consistently beyond its true plane. The other

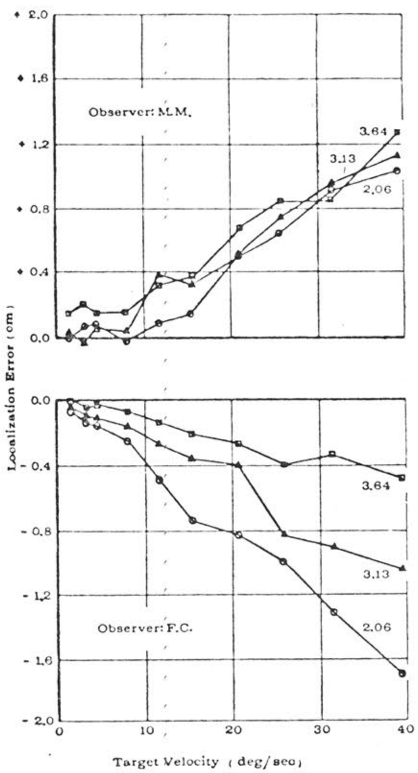

Fig. I.6a. From Lit (1960(c), Fig. 2). Localisation error as a function of target velocity at 3 luminance levels. Data from two observers.

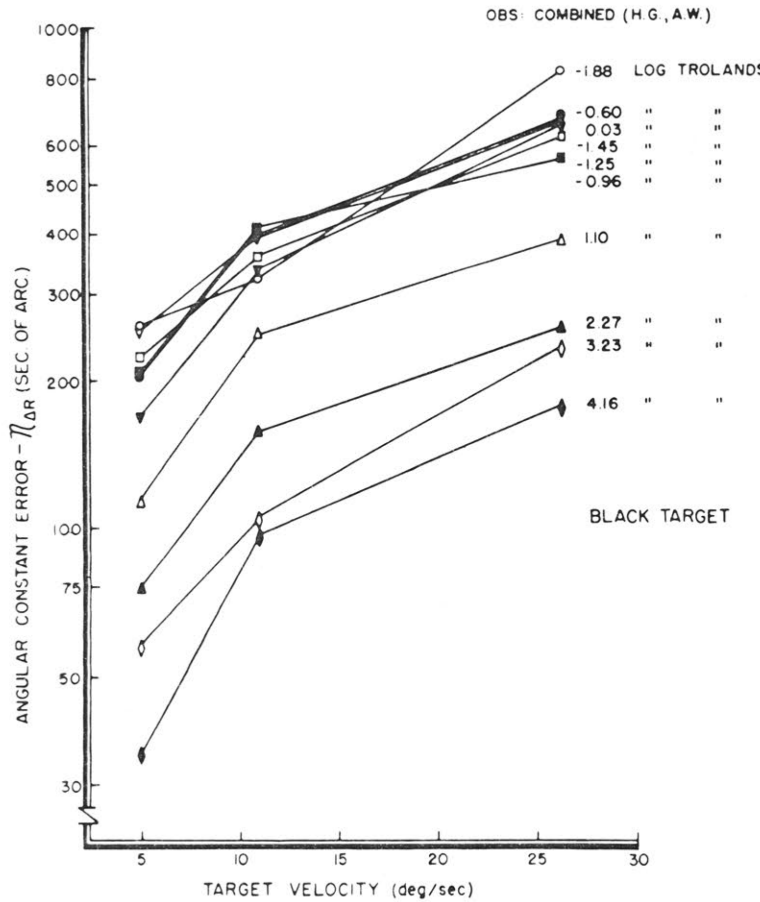

Fig. I.6b. From Lit, 1967 (Fig. 7). Localisation error as a function of velocity at 10 luminance levels. Combined data for two observers.

subject, who was slightly exophoric, had a negative localisation error.

The error varied both with illumination and with target velocity (see Fig. 1.6a). The variation with illumination was more marked for one observer (F.C.) than the other (M.M.). In both cases, the trend was for the oscillating target to appear further away as retinal illuminance increased; for example, at the highest target velocity F.C.'s localisation error was about -1.7 cms for the lowest luminance, and only -0.5 cm. for the highest, whereas the corresponding results for subject M.M. were + 1.0 cm. and + 1.3 cms.

In both cases, the localisation error increased with target velocity.

The thresholds (Lit, 1964) in terms of the average deviations of the experimental data for each set of stimulus conditions, increased more or less linearly from about 12 sec arc at a velocity of 1.49°/sec to 50 sec arc at 39.05°/sec. The retinal illuminance levels used were all in the photopic range, and did not have any marked effect on the threshold.

Additional experiments (Lit, 1966) showed that the threshold did decrease as luminance was changed from the scotopic to the photopic range. The variation had the typical scotopic-photopic discontinuity found with other visual functions when illumination is changed from low to high levels (e.g., visual acuity, as demonstrated by Fletcher _et al_, 1966); in the same paper, Lit described a similar variation in stereoscopic acuity for stationary targets. Further data in this paper confirmed that the localisation error was a function of both target velocity and retinal illuminance level (Fig. I.6b). The variation with illuminance level was not stated as in Lit's 1960c paper; with three subjects and over a wider range of luminance levels, Lit found that positive as well as negative errors decreased with increased illuminance.

I.6 Harker's saccadic suppression theory.

Harker (1967) presented an alternative explanation for the Pulfrich effect. He based his theory on evidence by Dodge (1900), Holt (1903), Ditchburn (1955) and Zuber & Stark (1966) that vision is suppressed during saccadic eye movements. The period of suppression is thought to increase as target intensity decreases; vision is recovered as soon as the saccade is completed.

Harker postulated that during observation of the Pulfrich pendulum, the observer makes small involuntary predictive eye movements (Westheimer, 1954) despite his efforts to keep fixation steady. The eye movements are conjunctive, but because of the difference in target intensity induced by the filter, the period of suppression in one eye is longer than that of the other. At the onset of suppression, the moving stimulus would be at a different position for each eye. According to Harker, these disparities are consistent with the displacements seen with the Pulfrich effect.

Furthermore, irregularities in the repetition pattern of the saccades can be made to account for the often reported asymmetries of the apparent path of the pendulum (see Section I.4g).

Harker presented some experimental support for his theory. He viewed a pendulum through an episcotister, arranged so that the order of exposure was either right eye/binocular/left eye or left eye/binocular/right eye. The pendulum appeared to follow a curved path, anticlockwise for the first sequence and clockwise for the second.

The episcotister certainly provided conditions of intermittent vision similar to those postulated by the saccadic suppression theory, and thus far the theory is supported. The theory is so open-ended that it can not be properly tested until very small eye movements can be measured during the course of an experiment with the Pulfrich pendulum. Any result can be accounted for simply by assuming that the appropriate eye movements occurred.

The discrepancy between the empirical and theoretical displacements found by Lit to occur with low target velocities (Lit, 1960b; see previous section) could be interpreted in terms of the saccadic suppression theory by assuming that fewer movements occur as velocity is increased. That this may be the case is indicated by Westheimer's results (Westheimer, 1954). At the same time, Lit's results were fairly repeatable, and one must make the further assumption that eye movements patterns for a given stimulus velocity are essentially constant from one experimental session to the next. Whether or not this is so remains to be seen.

On comparing the saccadic suppression theory with the latency hypothesis, it may be seen that both are based on the concept of retinal disparity as the stimulus for the stereoscopic perception of space. The evidence for a visual latent period which varies with intensity has been well-established (see Section I.3), while that in support of saccadic suppression is still inconclusive. Thus while the saccadic suppression theory of the Pulfrich phenomenon can be said to be the more parsimonious, application of Occam's Razor tends to favour the latency hypothesis.

It may be that both theories are correct, latency differences being responsible for most of the Pulfrich phenomenon, and saccadic suppression accounting for such discrepancies as asymmetries in the apparent path of the pendulum, and Lit's results with varying target velocity.

I.7 Spatio-temporal integration: Lee's experiments.

Lee (1970a) reported experiments similar to that of Harker (1967), but with more refined apparatus. In Lee's experiments, each eye received a regular sequence of exposures of a rod oscillating with pendulum-like motion in a frontoparallel plane. The sequence of exposures is

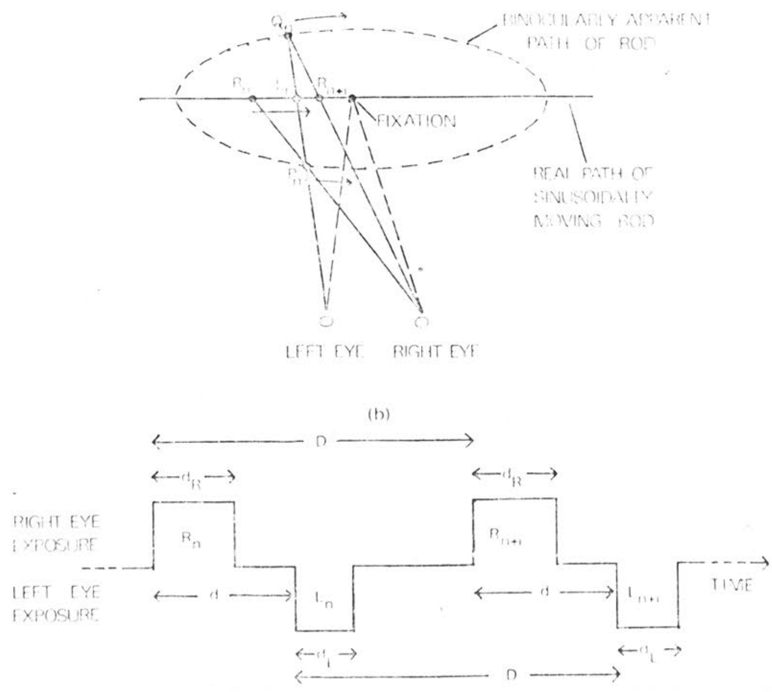

Fig. I.7. From Lee (1970a), Fig. 1. A rod moving along the path shown in (a) is illuminated according to the temporal sequence shown at (b). In (a), R_n and R_{n+1} are successive right-eye images of the rod; L_n is a left eye image which could be paired with either R_n or R_{n+1} to give a stereoscopic image at P_n or Q_n.

illustrated in Fig. I.7. The onset-onset inter-exposure interval D was the same for both eyes, while the exposure times d_R and d_L could be set independently. Also, the sequence of exposures could be put out of phase by a delay factor, d.

When viewed in this way, the oscillating rod followed an apparent path similar to that of the Pulfrich phenomenon. This demonstrates that non-simultaneous monocular information can be temporally integrated to give a binocular depth percept.

Lee describes the temporal integration process in terms of binocular pairing: " ... the angular-position information in the exposure to one eye is paired with the information in a non-simultaneous exposure to the other eye, to give rise to binocular-kinetic depth perception." He assumes, logically, that pairing will occur between temporally adjacent inputs; the question is whether a given input to the right eye is paired with the preceding input to the left eye, or with the succeeding input. For a given set of conditions, one pairing would result in a clockwise apparent path, the other in a anti-clockwise path.

In one set of experiments, Lee used an inter-exposure interval D of 50 msec and equal exposure times, $d_R = d_L =$ 10 msec. The delay was varied from 0 to 50 msec. The subjects had to report whether the rod appeared to follow a clockwise or anticlockwise path.

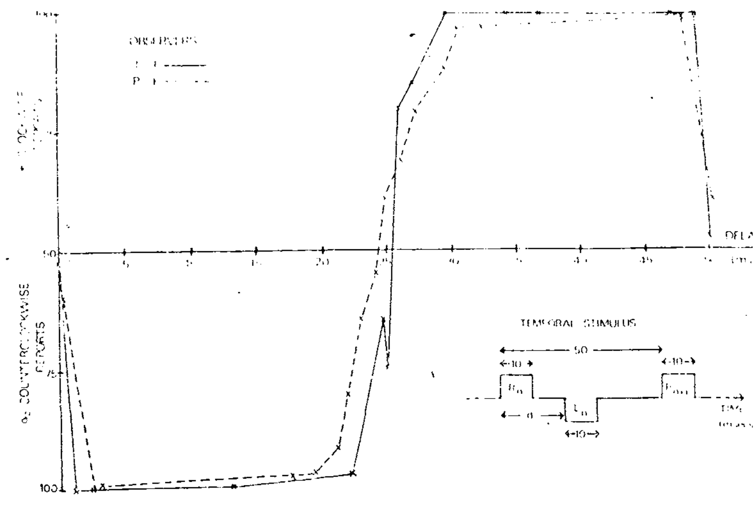

Fig. I.8. From Lee (1970a), Fig. 3.
Results of some of Lee's experiments
(see text).

For delays of from (approximately) 1 to 22 msec, the apparent path was anticlockwise in almost 100% of the reports. At a delay of 25 msec, the reports were equally distributed between clockwise and anticlockwise, and for delays greater than 28 msec and up to about 49 msec, the reports were almost exclusively clockwise (see Fig. I.8). With delays of 0 and 50 msec, the exposures were simultaneous, and a flat path was reported.

These results indicated that binocular pairing tended to occur between temporally proximal adjacent exposures.

In these experiments, the inter-exposure interval was such that flicker was perceptible in the stimulus. Subsequently, Lee found that the phenomenon occurred even for a flicker rate of 50 Hz ($D = 20$ msec, $d_R = d_L = 4$ msec). For smaller values of D, the effect declined and could not be elicited when D was less than 10 msec.

Thus the phenomenon can occur under flicker-fusion conditions, which is an interesting result, for as Lee points out "if we make the reasonable assumption that the existence of the phenomenon is dependent upon the perceptual analysis of temporally discontinuous input, it follows that information about such temporal discontinuity may still be available for perceptual processing, even though the illumination is seen as temporally continuous".

The reports of direction of path became more heterogeneous as the inter-exposure interval D was increased. The results indicated that if there was a delay greater than about 100 msec between right- and left- eye inputs, binocular pairing could not occur.

Lee next considered the fact that the spatial disparities were essentially proportional to the temporal disparities. For example, in the sequence of exposures in Fig I.7 (b), the angular disparity between R_n and L_n is less than that between L_n and R_{n+1}. Lee asked whether the relevant factor in binocular pairing was temporal or angular disparity. He found that if the intensity to one eye was reduced with a neutral density filter, there was a change in the turnabout delay (that value of d for which reports of clockwise and anti-clockwise were equally distributed). The filter did not alter the angular disparities, but changed the effective neural temporal disparities, and the variation in turnabout delay was in accordance with binocular pairing occurring between that pair of neural signals between which there is the smaller temporal disparity.

Since Lee used the latency hypothesis of the Pulfrich phenomenon to arrive at the latter conclusion, it would be circular reasoning to claim that the results support the latency hypothesis. Nevertheless, at this stage if one is to accept Lee's model of binocular pairing, which is a reasonable concept, the latency hypothesis must also be accepted.

Lee went on to ask whether the temporal disparity which determined binocular pairing was the onset-onset, the offset-offset, or the onset-offset disparity, or some other disparity. With experiments in which the exposure times to the two eyes, d_R and d_L were unequal and varied, he concluded that the relevant factor was either the offset-onset disparity, or the relative "mean" disparities between neural signals. Subsequent experiments, in which pairs of exposures to the right eye were alternated with single exposures to the left eye, indicated that the offset-onset dealy was the relevant variable.

It may be recalled that Harker (1967) (see Section I.6) assumed that the disparities due to the offset-offset relationship in saccadic suppression were responsible for the Pulfrich phenomenon. Lee's results somewhat weaken the saccadic suppression theory, but at the same time it must be remembered that in saccadic suppression, the exposures to each eye always overlap, and both have simultaneous onset. These specific conditions were not duplicated in Lee's work.

In another paper, Lee (1970b) reported an experiment in which the rod, oscillating as before with a pendulum-type motion, was illuminated by a stroboscope flashing at 20 Hz. The display was viewed with a filter in front of one eye, and the rod appeared to follow a path qualitatively similar to that of the Pulfrich pendulum.

Despite the similarity, this phenomenon is not the same as the Pulfrich phenomenon. The following extract from Lee (1970b) shows why this is so:

"... suppose that the 0.3 log unit filter is over the left eye, thereby increasing its latency by T msec. Since the illumination is stroboscopic, with a 50 msec interval between flashes, the moving rod is visible only at discrete positions along its motion path. Imagine such a sequence of successive positions of the rod labelled P_n, P_{n+1}, P_{n+2}... The corresponding sequence of neural signals from the right eye are r_n, r_{n+1}, r_{n+2}, ... and occur at times t msec, (t + 50) msec, (t + 100) msec, ... the corresponding left-eye signals are l_n, l_{n+1}, l_{n+2}, ... and occur at times (t + T) msec, (t + 50 + T) msec, (t + 100 + T) msec ... (the value of t is, of course, arbitrary). Thus the signals from the two eyes alternate temporally, and it is known that under such conditions binocular pairing, affording disparity information, takes place between those left and right eye signals which are closer together in time (Lee, 1970a). Now it may be assumed that the latency increase T produced by the 0.3 log unit filter is appreciably less than 25 msec (i.e., half the interval between strobe flashes). It follows that binocular pairings (r_n, l_n), (r_{n+1}, l_{n+1}), (r_{n+2}, l_{n+2}) ... between <u>simultaneous</u> exposures will be formed. (...) But since each signal of a binocular pair corresponds to the <u>same</u> spatial position of the rod in the frontoparallel plane (e.g. signals r_n and l_n each correspond to position P_n), it might be expected that the disparity information in each binocular pair would give rise to veridical perception of the rod moving in a frontoparallel plane. That the rod is seen to be moving around an elliptical path in depth is therefore puzzling. Apparently, reducing the luminance to one eye changes the disparity information in the binocular pairs."

The phenomenon is not easy to explain. Lee suggested, tentatively, that a lateral inhibition mechanism might be responsible. An exposure to one eye could inhibit the nearer side of the image of the target in the next exposure to the same eye. If this inhibition was dependent on luminance level, it would be different for each eye in Lee's experimental situation, resulting in an effective disparity which could account for the observed effect.

One must ask whether such lateral inhibition could occur with continuous illumination of a moving rod. If so, then a further explanation of the Pulfrich effect presents itself. However, Lit (1960a) showed that target thickness had no effect on the extent of the Pulfrich phenomenon. A lateral inhibition theory would suggest that the amount of inhibition would be related to retinal image thickness, so that Lit's evidence does not support such a theory.

I.8 Spatio-temporal integration: Julesz's demonstration.

Julesz & White (1969) prepared a film loop from pairs of random dot stereograms (Julesz, 1964). Each pair

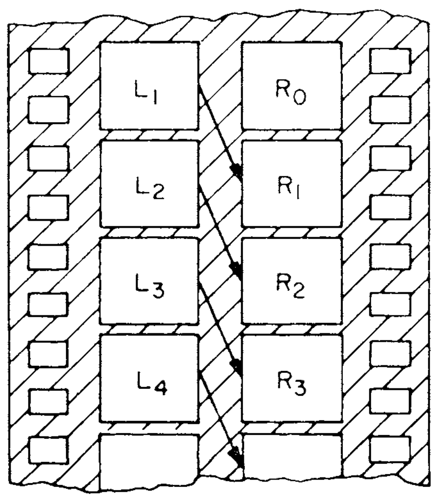

Fig. I.9. (From Julesz and White, 1969). Section of film loop used to demonstrate the effect of a filter on visual latency (see text). (L_1, R_1), (L_2, R_2) ... are pairs of random dot stereograms, each pair different from the others.

consisted of two matrices of randomly placed dots, identical except for a central square which was shifted laterally in one matrix relative to the other. When viewed in a stereoscope, such a pair gives a strong stereoscopic impression of a square in front of or behind a surrounding matrix.

If a succession of such stereograms are presented cinematically, binocular viewing still results in the stereoscopic percept, even though the pattern on each frame is different from that of any other (providing, of course, that the same disparity relations were present throughout).

In the film loop under consideration, each frame did not have a corresponding pair of stereograms; instead, the left-eye half in the second frame was paired with the right-eye half belonging to the first frame, and so on (see Fig. I.9). When viewed binocularly (using appropriate prisms and polaroid filters to facilitate fusion) the display consisted only of visual noise, since the uniocular images from each frame could not be combined into a single percept. However, a neutral density wedge filter in front of the eye with the "leading" matrix could be adjusted so that fusion and stereopsis did occur.

Julesz & White concluded that their experiment demonstrated unambiguously that filtering of one eye delays the arrival of neural signals from that eye to higher centres, thus lending considerable support to the latency hypothesis of the Pulfrich phenomenon.

I.9 Summary

The literature on stereoscopic localisation of moving objects has been reviewed and discussed. Most of the literature has been concerned with the Pulfrich phenomenon, and it is shown that the latency hypothesis accounts for most aspects of the phenomenon.

What have yet to be explained, with respect to the Pulfrich phenomenon, are the asymmetries in the apparent path of the pendulum which have often been reported, and the discrepancy between the theoretical and empirical displacements found for low target velocities (Section I.5).

Harker's saccadic suppression theory (Section I.6) has been proposed to account for asymmetries in the reported path. It is suggested here that the saccadic suppression theory need not replace the latency hypothesis, but that both mechanisms might be operative. (Julesz & White (1969) make a similar comment.) Furthermore, the exact shape of the path of the Pulfrich pendulum is difficult to determine quantitatively, much reliance being placed on subjective reports. Of interest here is the statement of Gerard (1935), to the effect that while retinal disparity is the stimulus for the perception of depth differences, the way in which the perception of the entire array is organised depends on non-stereoscopic factors (Section I.4c). These other factors include the misinterpretation of retinal image movement (Section I.4b). In Chapter II, the role of "perceptually impossible" stimuli in the Pulfrich phenomenon will be considered.

Some experiments by Lee (Section I.7) dealihg with spatio-temporal integration in binocular space perception have been discussed. In general, these experiments lend only indirect support to the latency hypothesis.

The localisation error demonstrated by Lit under conditions of equal retinal illuminance is of basic importance to this thesis. Some theories as to its causation will be presented in Chapter III, while the experiments to be reported confirm that such errors do occur.

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CHAPTER II

MATHEMATICAL ANALYSIS OF THE PATH OF THE PULFRICH PENDULUM

II.1 Introduction.

If it is assumed that:-

(a) stereoscopically perceived space is Euclidean and congruent with physical ("objective") space,

(b) the apparent and reported path of the Pulfrich pendulum is determined solely on the basis of retinal disparity,

(c) visual latency for a given level of stimulus luminance is constant over all regions of the retina, and

(d) the pendulum bob is moving with simple harmonic motion (SHM) along a horizontal line in a frontal plane,

then a mathematical description of the apparent path can be made.

Objections to each of these assumptions are made in Chapters I and III, so that such a mathematical description can only be a rough approximation of the stimulus presented to the cortex. It is presented here to stress that the various non-symmetrical shapes described by many observers (e.g., Engel & Fischer, 1950, Miles, 1953) cannot be fully explained geometrically on the basis of the above assumptions. However, the analysis given below does indicate that under some conditions, the theoretical path is perceptually undefinable, thereby accounting for at least some of the subjective reports.

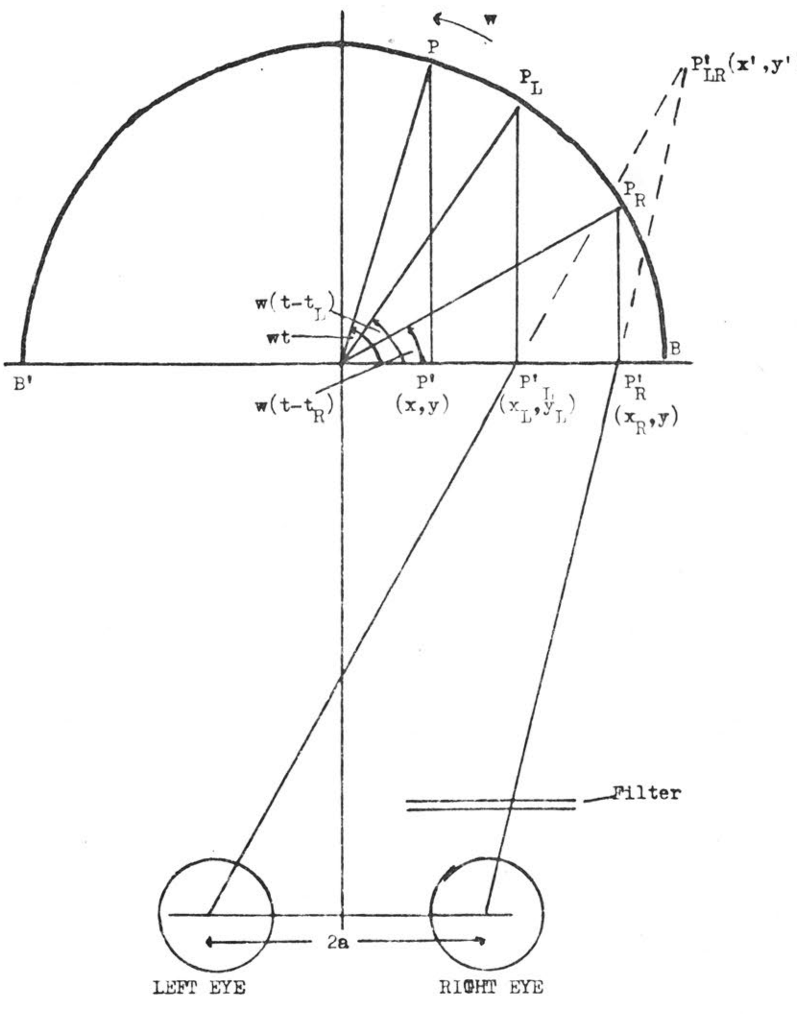

Fig. II.1. Illustrating the generation of simple harmonic motion by the projection of a point moving with constant angular velocity, and the derivation of the fused image P_{LR} from the uniocular images P'_L and P'_R.

Lederer (1957) presented a mathematical analysis which suggested that the apparent path of the pendulum is "an ellipse-like curve whose major axis is rotated with respect to the objective path ...". Such a rotation could account for some of the reported shapes, particularly if it is remembered that the observer is trying to interpret the path of an object moving in depth. Unfortunately, there is a flaw in Lederer's analysis. It is implicit in his argument that at a given moment, the velocities of the two uniocular images are the same. This is not so; for example, when the image of the non-filtered eye has reached its maximum velocity, at the centre of its path, the other image is still approaching this point, and still accelerating. At each end of the path, when the non-filtered image is momentarily at rest, the other image is still decelerating. The two images cannot co-incide at the end of the path, as indicated by Lederer's diagrams.

II.2 The Mathematical Description.

Simple harmonic motion can be considered as the projection on to a diameter of a circle of a particle moving with constant angular velocity around that circle (Ference, Lemon, & Stephenson, 1964). In Figure II.1, P is the momentary position of the particle generating SHM along the path BB'. P', the projection of P, is therefore a position of an object moving in SHM along BB'.

If w (in degrees per second) is the angular velocity of P and r the radius of the circle, then the frequency is w/360 oscillations per second, the period is 360/w seconds, and the amplitude is r.

At a time t seconds after the start of a cycle (i.e., after P has passed through B), the displacement of P' is given by:

$$x = r.\cos(wt) \qquad \dots \quad \dots \quad \text{II.1}$$

Now consider an observer watching the bob moving along BB', with a filter in front of his right eye, as illustrated in Fig. II.1. A co-ordinate system is superimposed so that the centres of projection of the left and right eyes are respectively (-a, 0) and (a, 0), and the oscillating object is in a frontal plane at a distance y from the observer. (At this stage, the actual positions of the fixation axes are irrelevant, since the Euclidean concept of stereoscopic space is concerned with differences between angles, not their individual magnitudes).

t_L and t_R are the response latencies for the left and right eyes, and $\Delta.t = (t_L - t_R)$.

When, as in Figure II.1, the pendulum bob is at P' (x,y), the left eye image is at P'_L (x_L, y), and the right image at P'_R (x_R, y).

From equation II.1:

$$x_L = r.\cos\left[w(t - t_L)\right] \qquad \dots \quad \dots \quad \text{II.2}$$

and

$$x_R = r.\cos\left[w(t - t_R)\right] \qquad \dots \quad \dots \quad \text{II.3}$$

The "fused" image is at P'_{LR}, whose co-ordinates are, from simple geometry,

$$x' = \frac{a\ (x_L + x_R)}{x_L - x_R + 2a} \qquad \ldots \quad \ldots \quad \text{II.4}$$

$$y' = \frac{2ay}{x_L - x_R + 2a} \qquad \ldots \quad \ldots \quad \text{II.5}$$

What follows now is the derivation of an equation relating x' and y' in terms of the angular velocity w, the amplitude of oscillation r, the inter-ocular separation 2a, and the observation distance y. (Note that the relation between angular velocity and frequency of oscillation n is $n = w/2\pi$).

By trigonometric transformation, equations .2 and .3 become:

$$x_R = r\ \left[(\cos wt).\ (\cos wt_R) + (\sin wt).\ (\sin wt_R)\right]$$

$$x_L = r\ \left[(\cos wt).\ (\cos wt_L) + (\sin wt).\ (\sin wt_L)\right]$$

and thus:

$$x_L + x_R = r\left[\cos wt.(\cos wt_L + \cos wt_R) + \sin wt.(\sin wt_L + \sin wt_R)\right] \qquad \ldots \quad \ldots \quad \text{II.6}$$

$$x_L - x_R = r\left[\cos wt.(\cos wt_L - \cos wt_R) + \sin wt.(\sin wt_L - \sin wt_R)\right] \qquad \ldots \quad \ldots \quad \text{II.7}$$

The immediate aim is to remove the time variable t.

Let $\cos wt_L + \cos wt_R = B \qquad \sin wt_L + \sin wt_R = C$

$\cos wt_L - \cos wt_R = D \qquad \sin wt_L - \sin wt_R = E$

so that:

$$x_L + x_R = r\ (B.\ \cos wt. + C.\ \sin wt)$$

and $x_L - x_R = r\,(D.\ \cos wt + E.\ \sin wt)$

From equations II.4 and .5,

$$x' = \frac{\underline{ar}\,(B.\ \cos wt + C.\ \sin wt)}{r\,(D.\ \cos wt + E.\ \sin wt) + 2a}$$

$$y' = \frac{2ay}{r\,(D.\ \cos wt + E.\ \sin wt) + 2a}$$

Rearranging terms,

$$(\cos wt).(aB - x'D) + (\sin wt).(aC - x'E) = x'(2a/r) \quad \ldots \quad \ldots \quad \text{II.8}$$

$$(\cos wt).D + (\sin wt).E = (y - y')\,(2a/ry') \quad \ldots \quad \text{II.9}$$

For further simplification, make the substitutions

$J = x'(2a/r)$ $\qquad K = (y-y')\,(2a/ry')$

$M = aB - x'D$ $\qquad N = aC - x'E$

so that equations II .8 and II.9 become

$$J = M.\cos wt + N.\sin wt \quad \ldots \quad \ldots \quad \text{II.10}$$

$$K = D.\cos wt + E.\sin wt \quad \ldots \quad \ldots \quad \text{II.11}$$

Equations II.10 and II.11 can be solved as a pair of simultaneous equations, giving:

$\sin wt = (MK - JD)/(ME - DN)$

and $\cos wt = (EJ - NK)/(ME - DN)$

But

$\sin^2(wt) + \cos^2(wt) = 1,$

therefore

$$(MK - JD)^2 + (EJ - NK)^2 = (ME - DN)^2 \quad \ldots \quad \ldots \quad \text{II.12}$$

Equation II.12 is independent of the time variable t.

It remains only to expand equation II.12 and make the appropriate re-substitutions.

$$MK = (aB - x'D)(y - y')(2a/ry')$$
$$= (2a/ry')(ayB - ay'B - yx'D + x'y'D)$$
$$JD = x'D(2a/r)$$
$$= (2a/ry')(x'y'D)$$
$$(MK - JD) = (2a/ry')(ayB - ay'B - yx'D)$$
$$(MK - JD)^2 = (2a/ry')^2.(a^2y^2B^2 + a^2(y')^2B^2 + y^2\ (x')^2D^2 - 2a^2yy'B^2 - 2ay^2x'BD + 2ayx'y'BD)$$
$$EJ = (x'E)(2a/r)$$
$$= (2a/ry')(x'y'E)$$
$$NK = (aC - x'E)(y - y')\ (2a/ry')$$
$$= (2a/ry')(ayC - ay'C - yx'E + x'y'E)$$
$$(EJ - NK) = (2a/ry')(-ayC + ay'C + yx'E)$$
$$(EJ - NK)^2 = (2a/ry')^2.(a^2y^2C^2 + a^2(y')^2C^2 + y^2\ (x')^2E^2 - 2a^2yy'C^2 - 2ay^2x'CE + 2ayx'y'CE)$$
$$(MK-JD)^2 + (EJ-NK)^2 = (2a/ry')^2.\ \big[a^2y^2(B^2 + C^2) + a^2(y')^2(B^2+C^2) + y^2(x')^2(D^2+E^2) - 2a^2yy'(B^2+C^2) - 2ay^2x'(BD+CE) + 2ayx'y'(BD + CE)\big]$$
$$= \text{L.H.S. of equation II.12}$$

Substituting the full values of B, C, D and E, we get:

$$(B^2 + C^2) = \cos^2 wt_L + \cos^2 wt_R + \sin^2 wt_L + \sin^2 wt_R + 2\cos wt_L.\cos wt_R + 2\sin wt_L.\sin wt_R$$
$$= 2\ \big[1 + \cos\ (wt_L - wt_R)\big]$$
$$= 2\ \big[1 + \cos\ (w.\ \Delta t)\big]$$

where $\Delta t = t_L - t_R$

Similarly,

$$(D^2 + E^2) = \cos^2 wt_L + \cos^2 wt_R + \sin^2 wt_L + \sin^2 wt_R$$
$$- 2 \cos wt_L . \cos wt_R - 2 \sin wt_L . \sin wt_R$$
$$= 2 \left[1 - \cos (w. \Delta t) \right]$$
$$(BD + CE) = \cos^2 wt_L - \cos^2 wt_R + \sin^2 wt_L - \sin^2 wt_R$$
$$= 0 \quad \text{(zero)}$$

The left hand side of equation II.12 is therefore:

$$(2a/ry')^2 \left[2a^2y^2(1 + \cos w.\Delta t) + 2a^2(1 + \cos w.\Delta t)(y')^2 + 2y^2(1 - \cos w. \Delta t)\ (x')^2 - 4a^2y\ (1 + \cos w.\Delta t)(y') \right]$$

The right hand side of equation II.12 is:

$$(ME - DN)^2 = (aBE - x'DE - aCD + x'DE)^2$$
$$= a^2(BE - CD)^2$$

Here it is convenient to substitute

$\cos wt_L = e$, $\cos wt_R = f$, $\sin wt_L = g$, $\sin wt_R = h$

so that

$$(BE - CD) = (e + f)\ (g - h) - (g + h)\ (e - f)$$
$$= eg - eh + fg - fh - eg + fg - eh + fh$$
$$= 2fg - 2eh$$
$$= 2(\cos wt_R .\ \sin wt_L - \cos wt_L .\ \sin wt_R)$$
$$= 2 \sin (w. \Delta t)$$

and the right hand side of equation II.12 is:

$$4a^2 . \sin^2 (w. \Delta t)$$

After rearranging terms and removing the common factor $4a^2$, the expansion of equation II.12 gives the following expression for the path of the Pulfrich Pendulum:

$$2y^2(1 - \cos w. \Delta t)(x')^2 + \left[2a^2(1 + \cos w. \Delta t) - r^2 . \sin^2 w.\Delta t \right] (y')^2 - 4a^2y(1 + \cos w.\Delta t)(y') + 2a^2y^2(1 + \cos w. \Delta t) = 0 \quad \ldots \text{II.13}$$

II.3 Discussion

Equation II.13 is of the second degree, therefore the path is a curve of the family of conic sections.

There is no term in $x'y'$: the path is not rotated about the co-ordinate system.

Finally, there is no term in x', so that the path is symmetrical about the y axis (the observer's median plane).

Thus, within the framework of the assumptions stated earlier, there is no mathematical basis for apparent paths which are rotated or asymmetrical. However, equation II.13 facilitates the study of the theoretical path, and from such a study some interesting points emerge.

The shape of the path can be determined by examining the coefficients of $(x')^2$, $(y')^2$, and (y'). Let these be designated K_1, K_2, and K_3 respectively.

If K_1 and K_2 are of like sign, the curve is an ellipse. If either is zero, it is a parabola, and if they are of opposite sign, the curve is an hyperbola (Keane & Senior, 1961).

For example, let $a = 3$ cms., $r = 30$ cms., $y = 100$ cms., and $\Delta t = .02$ sec. (Some of the data reported by Lit (1949) indicate that such a high difference in latencies could occur with a retinal illuminance difference of 2 log units).

A parabola would be obtained if $K_1 = 0$, that is, when $\cos w. \Delta t = 1.0$, or $w. \Delta t = 360.0^{\circ}$. For $\Delta t = .02$, w would be $18{,}000^{\circ}$/sec: the pendulum would be oscillating

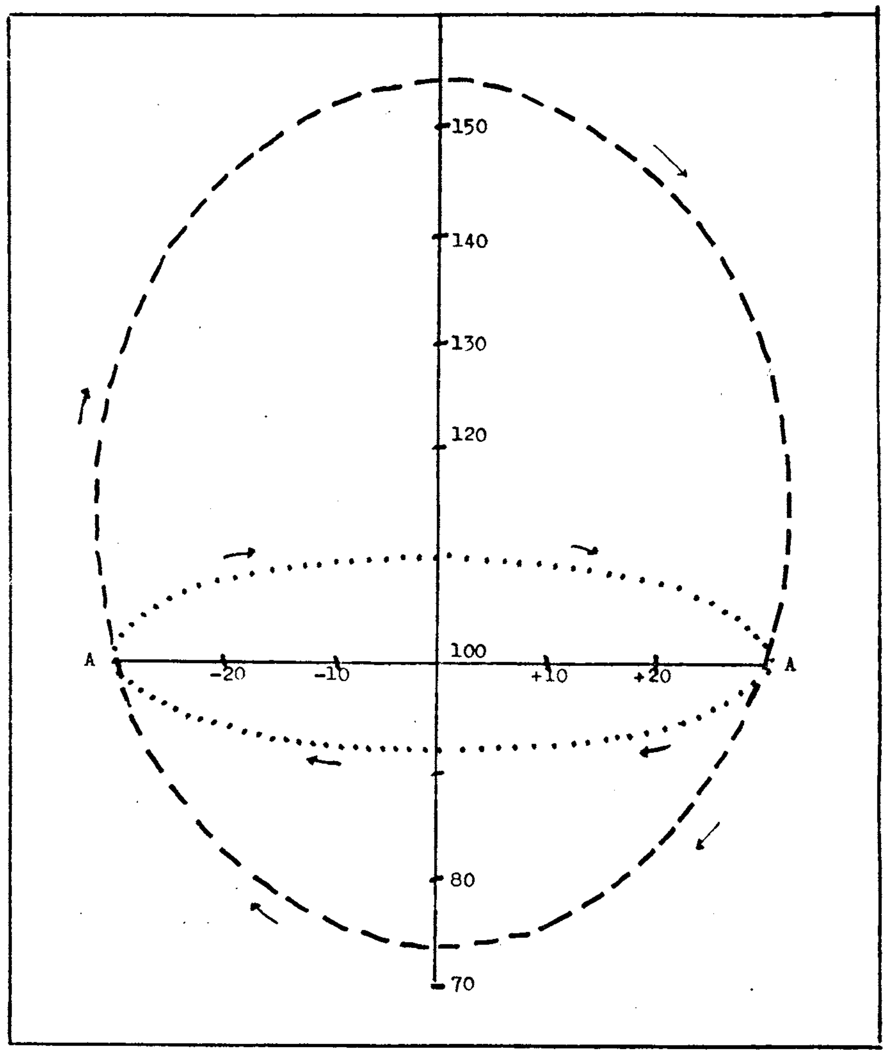

Fig. II.2. Theoretical paths of the Pulfrich Pendulum. $2a = 6$ cms. $r = 30$ cms. $y = 100$ cms. Latency difference $(t_L - t_R) = 20$ msec. Smaller ellipse obtained at .1388 oscillations ~~per~~ second; larger ellipse at .555 oscillations per second.

50 times each second. This is a somewhat unrealistic requirement. In the subsequent discussion, other unrealistic cases will be ignored.

For the other parabolic condition ($K_2 = 0$), the equation can be solved for cos w. Δt:

$$2a^2(1 + \cos w.\Delta t) - r^2.\sin^2 w.\Delta t = 0$$

from which

$$\cos w.\Delta t = (r^2 - 2a^2)/r^2$$

In our numerical example, the curve is a parabola when w = 573.817°/sec., requiring a high but not unrealistic frequency of 1.594 oscillations per second.

For lower frequencies, the curve is an ellipse, and for higher frequencies it is an hyperbola.

Figure II.2 shows two ellipses. The smaller ellipse occurs for a frequency of .1388 Hz., and resembles the figures usually drawn to resemble the apparent path (e.g., Ogle, 1962, Lit 1949). The other ellipse has its major axis at right angles to the plane of oscillation, and is obtained at .555 Hz. Such a path, viewed in depth by an observer whose eyes are in the same plane, could conceivably be described as a pear shaped figure.

Fig. II.3 (a) shows the elongated ellipse obtained at a frequency of 1.388 Hz, while Fig. II.3 (b) illustrates the parabolic case. In the latter, the letters in the figure indicate successive stages in the cycle. During the first half cycle, while the pendulum bob is moving from right to

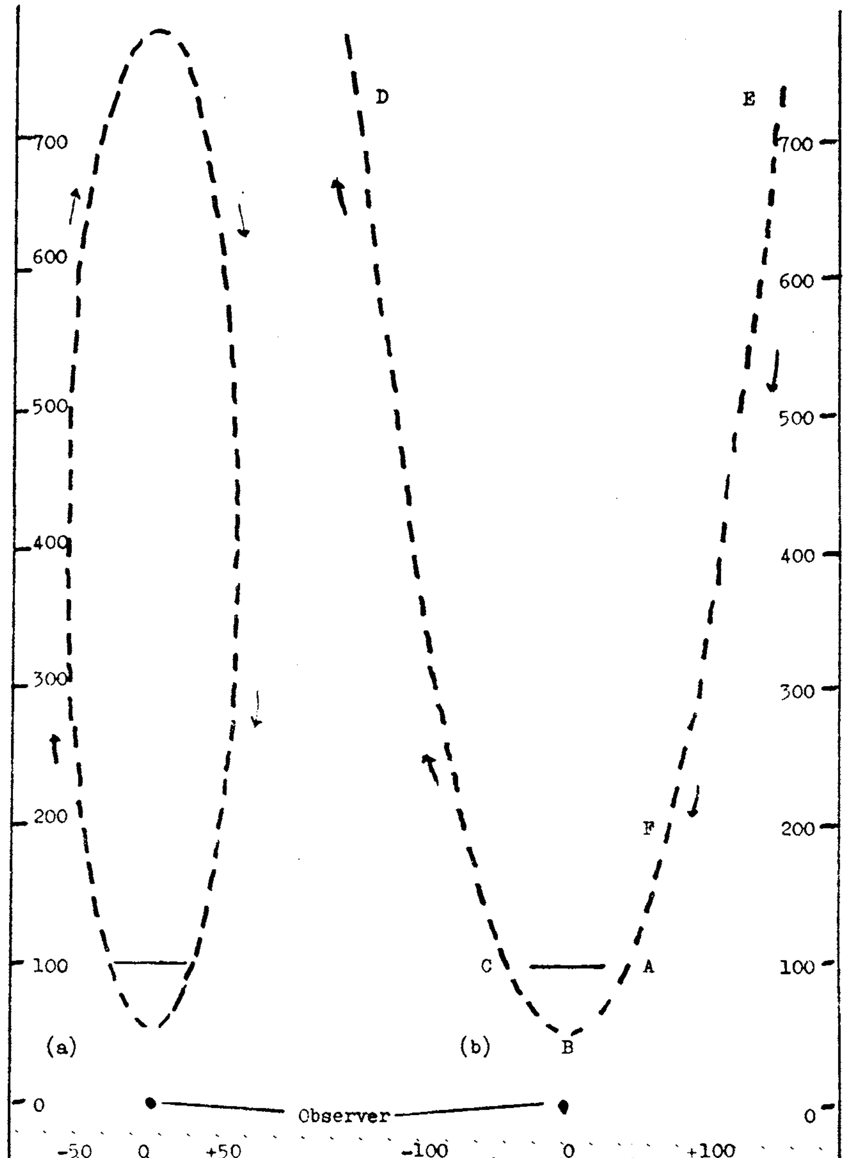

Fig. II.3. Theoretical paths of the Pulfrich pendulum. Same viewing conditions as Fig. II.2. (a) Elongated ellipse obtained at 1.388 oscillations per second. (b) Parabola obtained at 1.59 oscillations persecond. Letters A, B, C, etc. indicate successive stages of the cycle referred to in the text.

left, the apparent path is A $\rightarrow$ B $\rightarrow$ C. During the third quarter cycle, the bob apparently moves very rapidly along C $\rightarrow$ D, being momentarily at infinity when $x_L - x_R + 2a = 0$ (see equation II.5). In the last quarter cycle, the bob completes the parabola, along E $\rightarrow$ F $\rightarrow$ A.

It is worth noting that during the second half of the cycle, the apparent velocity of the pendulum bob exceeds the velocity of light. It is improbable that such a path is perceivable, let alone describable. A likely report would be that, while the pendulum is moving from left to right, it appears at an indefinite distance.

Fig. II.4 shows the hyperbola obtained when the frequency is 1.618 Hz. In the first half-cycle, the pendulum apparently moves in the usual manner along A $\rightarrow$ B $\rightarrow$ C, first approaching and then receding. During the third quarter cycle, it continues to recede, until a stage is reached when the denominator of equation II.5 is zero. After this, the denominator is negative, and the path is along E $\rightarrow$ F $\rightarrow$ G: "behind" the observer. Finally, the denominator becomes zero again and then positive, and the path H $\rightarrow$ I $\rightarrow$ A is completed.

In stereoscopic vision, a "negative" distance is not paradoxical. It can be obtained, for example, with a stereoscopic pair, if the fixation points have the same separation as the observer's interocular separation, and

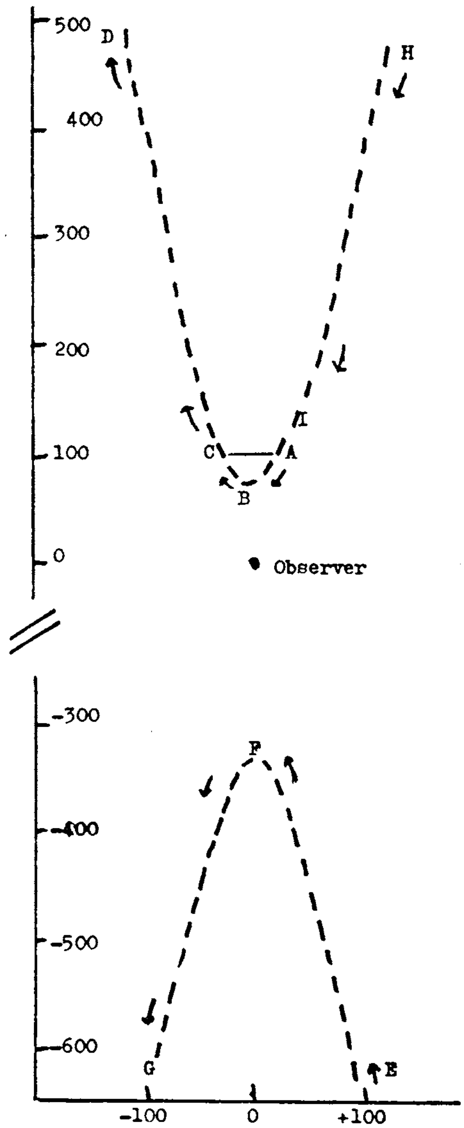

Figure II.4. Theoretical (hyperbolic) path of the Pulfrich pendulum at 1.618 oscillations per second (same conditions as for Figs. II.2 and II.3).

another pair of points is arranged to give crossed retinal disparity (within Panum's fusional areas). In Fig. II.4, the "negative" part of the path could be perceived at some very great distance; more likely, in view of the rapid changes of apparent position and the large disparities, the distance may be indefinite, or the bob could be seen in diplopia.

Once again, a likely report would be of a closed, asymmetrical figure. One might well recall the comment of Gerard (1935), to the effect that disparities brought about by latency differences "only give the first 'push' to develop the phenomenon".

II.4 Summary

1. A mathematical description of the apparent path of the Pulfrich pendulum is developed.
2. The apparent path is symmetrical and not rotated.
3. Under some conditions, the apparent path may be elliptical, with the major axis at right angles to the true plane of oscillation, or it may be parabolic or hyperbolic.
4. It is suggested that a path which is an elongated ellipse, a parabola, or an hyperbola, could be described by an observer as a pear-shaped figure, not necessarily symmetrical.

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CHAPTER III

ORIGINS OF THE LOCALISATION ERROR.

III.1 Introduction.

Lit has found that a rod oscillating in a frontal plane may be localised in nearer to or further from the observer than its true plane (Lit, 1960c, 1964, 1966). Lit's findings have been discussed in Chapter I.5, and are summarised briefly as follows:

(a) The localisation error varies in extent and direction from one observer to another.

(b) It is independent of direction of movement, and cannot be attributed to an inherent difference between the losses of light by absorption or scattering in each eye.

(c) The localisation error increases with target velocity.

(d) It decreases with increased luminance level.

(e) The angular extent of the localisation error is in the range from zero (for low velocities and high luminance) to 800 sec arc (for high velocities and scotopic luminance levels).

(f) The stereoscopic threshold angle for the equidistance settings increases as velocity increases, and decreases as luminance is increased, over the range of from 10 sec arc to 200 sec arc.

(g) The direction of the localisation error may be related to the phoria of the observer. Esophoric observers tend to have a positive error (moving rod localised further away than its true plane), while exophoric observers have a positive error.

In this chapter, some theories as to the causation of the localisation error are presented and discussed.

III.2 <u>The nature of fixation disparity</u>.

Lit (1949, p. 180; 1960c, p. 973) suggested that the localisation error may be related to fixation disparity, on the basis of the difference in direction of the localisation error for esophoric and exophoric observers.

Before further examining this proposition, a discussion of some aspects of fixation disparity is appropriate.

When a person fixates an object under conditions of normal binocular viewing, his fixation axes may not intersect exactly at the point of regard, but at some distance in front of or behind that point (Ogle, 1964, Chapter 8). Although the images of the fixation point are not located on corresponding retinal elements, single vision is maintained because the disparity is (normally) never greater than the extent of Panum's fusional areas at the foveal region.

The amount of fixation disparity varies with the degree to which there are fusion stimuli in the stimulus array. It is likely to be less for complex arrays with many binocularly seen details near the fixation point than for simple arrays with fewer binocular details. (Ogle, Martens, & Dyer, 1967).

Thus, a typical stereoscopic localisation experiment in which fusion details are restricted to two rods and perhaps a peripheral frame presents conditions ideal for the manifestation of fixation disparity.

The extent of the disparity also depends on the characteristics of the oculo-motor system of the observer, and can be altered by the application of spherical lenses and/or prisms. More specifically, it is related to any oculo-motor imbalance that the observer might have.

It would be expected that there is a correlation between fixation disparity and phoria. Phoria is the angular position taken up by the eyes in the absence of binocular fusion stimuli. If under these conditions the eyes converge, the subject is said to have an esophoria; if they diverge, it is an exophoria.

However, for low phorias (less than about 5 prism dioptres), there is no correlation with fixation disparity. An esophoria is just as likely to be associated with an exo-disparity (relative divergence) as with an eso-disparity (Ogle et al, 1967, p. 108). It is not possible, therefore, to predict the direction of fixation disparity from a phoria measurement.

Indeed, because of the dependence of fixation disparity on the stimulus array, it is not valid to assume that fixation disparity measured with some clinical technique is the same as that occurring under experimental conditions. Unfortunately, fixation disparity cannot be determined without introducing additional details into the array, so that the procedure of measurement may itself influence the disparity.

One is forced to the conclusion that, with presently available techniques, fixation disparity in a given set of stimulus conditions is indeterminable. It can only be assumed that it may be present in situations where binocular fusion stimuli are sparse.

Ogle and his co-writers (Ogle et al, 1968) state categorically that fixation disparity has no effect on stereoscopic depth perception (Ogle et al, p. 366). This statement is presumably based on the fact that the angular disparities which are the stimuli for stereoscopic perception are not affected by small changes in fixation position. This is not quite true, since the horopter changes shape for different observation distances, thereby altering the stereoscopic frame of reference, but such variations as may be induced by fixation disparity are so small that Ogle's dictum is acceptable.

Thus even if fixation disparity was present in Lit's experiments, it alone cannot account for the localisation errors. Additional factors, related to the kinetic nature of the experimental array, must be sought. One possible factor is the variation of latent period over different

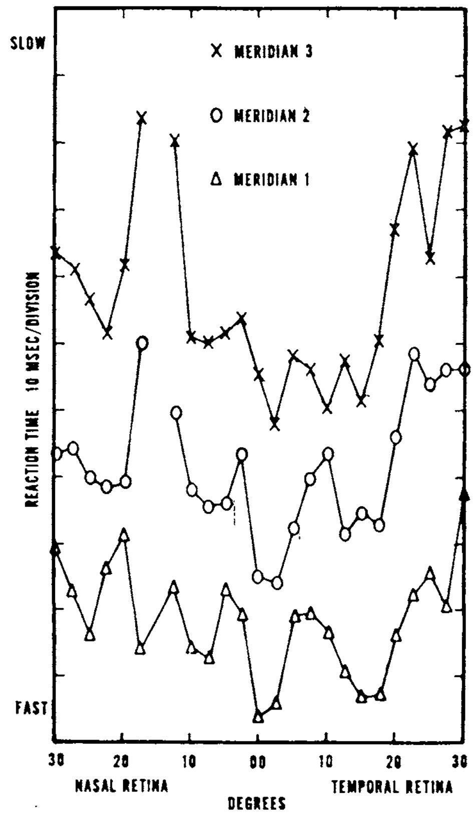

Fig. III.1. (From Payne, 1966, Fig.2). Reaction time as a function of retinal location. Meridian 2 is the horizontal meridian; meridians 1 and 3 are at $\pm 20^{\circ}$, and are not of direct releveance here. The curves are offset for clarity. The foveal RT for meridian 2 is 185 msec. Each data point is based on 120 RTs. A white background illuminated by 1.61 ml and averaged data for 3 stimulus intensities were used in obtaining each plot.

parts of the retina.

III.3 Visual latent period and retinal location.

Poffenberger (1912) found that reaction time (RT) increased as the distance from the fovea increases along the horizontal meridian. Payne (1966) confirmed Poffenberger's results, measuring his own reaction time for 33 locations along the horizontal position; in these experiments, RT was measured by having the subject lift a fingernail from a plate in response to a 1° stimulus light.

Payne's results are shown in Fig. III.1. (Data from non-horizontal meridians are not relevant here). The first item of interest is that there is a very rapid increase in RT from the fovea to points about 5 degrees on either side. Secondly, the variation in RT is not the same for both halves of the retina; for example, the difference between RT at 5 degrees nasally and 5 degrees temporally is about 5 msec.

Similar nasal-temporal differences were found by Rutschmann (1966), who used the perception of temporal* order as a measure of relative latency between the fovea and points 30° into the periphery.

Electrophysiological support for these nasal-temporal

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* The ambiguity of the word "temporal" is unfortunate, but the meaning is generally clear from the context.

differences has been presented by Auerbach _et al_ (1961), who found that in the cat, the latency of the temporal hemiretina in one eye was about 3 msec greater than that of the nasal hemiretina of the other eye.

Thus even under conditions of constant stimulus intensity there may be differences in the time taken for neural signals to reach cortical or higher centres from each eye. The origin of the delay time differences may be in the relative concentrations of rods to cones in each retinal location: the rod-cone ratio increases non-linearly from fovea to periphery (see Graham, 1965, Fig. 2.11), and there is evidence that rod delay is greater than cone delay (Guinn _et al_, 1968). There are also differences in the neural networks of peripheral as compared with foveal retihal locations (Duke-Elder, 1961, pp 246 ff), which could account for variations in latency.

Furthermore, Lang (1970) has shown that from purely optical considerations, the intensity of the retinal image of a stimulus of fixed intensity varies with retinal location because of obstruction of the entrance pupil by eyelids and lashes, changes in the shape of the entrance pupil with peripheral angle, and reflection losses at the various refracting surfaces of the eye. According to Lang (1970, fig. 22), the reduction of retinal illuminance is greater for the temporal than for the nasal _visual field_, leading one to expect an increase in latency for the nasal retina.

However, Lang's calculated losses for corresponding retinal areas differ only slightly for the central 10° area, probably not enough to cause significant latency differences.

Whatever their origin, the evidence for differences between the latent periods of different parts of the retina is quite impressive. The implications of these differences for binocular-kinetic space perception are considered in the next section.

III.4 Inherent latency differences and a path sampling hypothesis.

(a) Inherent latency differences.

The variations in visual latency described in the previous section may be said to result in _inherent_ latency differences, as opposed to the _induced_ latency differences obtained by reducing the light input to one eye.

An inherent latency difference may come about because of the differences between nasal and temporal retinal latencies, or because of differences between the latencies of adjacent retinal areas. The first of these possibilities is discussed here, while the second will be examined in Section III.5.

Consider a subject in the basic experimental paradigm discussed in Chapter I.1. He fixates the stationary rod, while the stimulus rod moves with a constant linear velocity of 20 cms per second in a frontal plane equidistant with the stationary rod. The subject's interocular distance is 6 cms, and the fixation distance is 100 cms.

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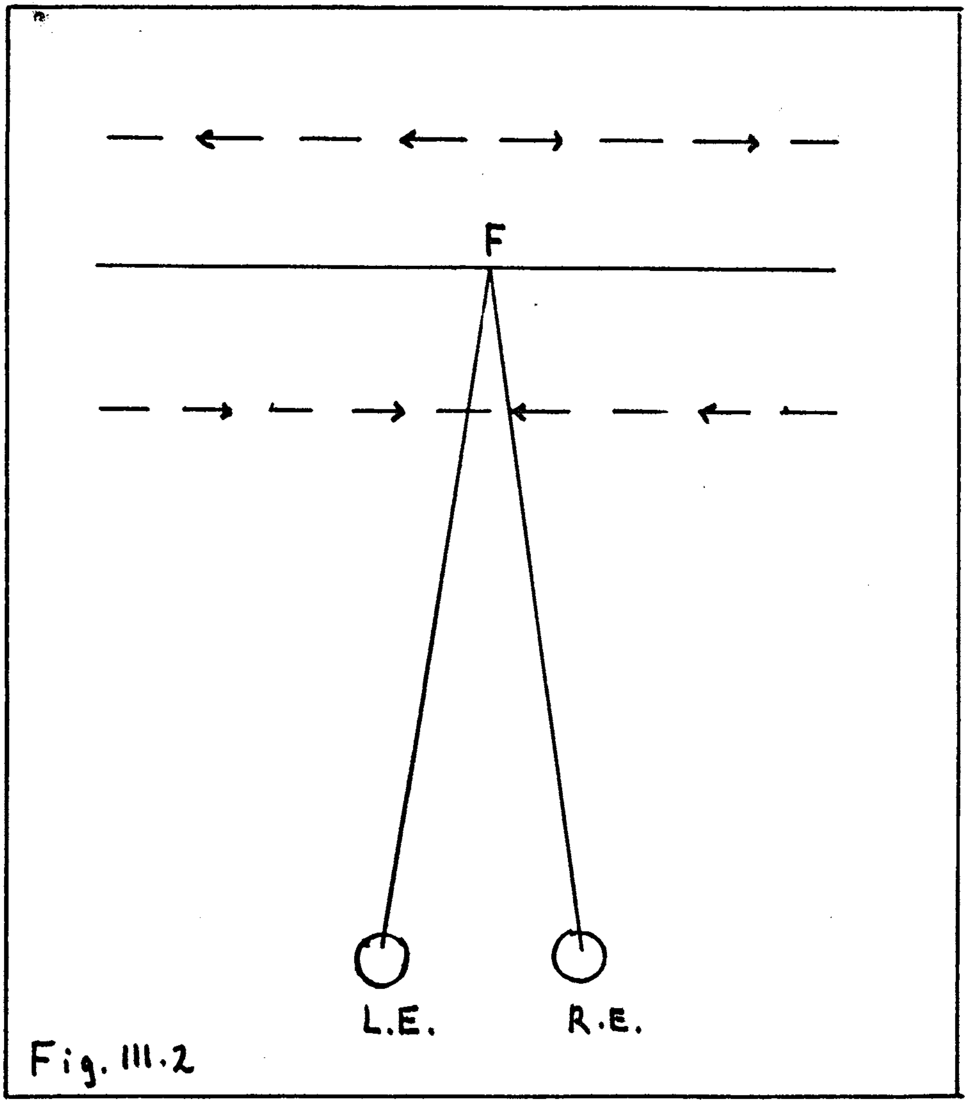

Figure III.2

Hypothetical apparent path of an object moving with constant linear velocity in a frontal plane (not to scale). Observer fixates at F, and it is assumed that the latency of the temporal hemiretinae is greater than that of the nasal by a constant amount.

Now assume that the latency difference between the nasal and temporal hemiretinas is 3 msec (as suggested in Section III.3 above). When the stimulus is moving from left to right (velocity V = + 20 cms/sec), it is initially imaged on the nasal retina of the left eye, and the temporal retina of the right eye. In terms of equation I.3, the latency difference (t_L - t_R) is - 0.003 sec (temporal latencies assumed greater than nasal latencies). According to equation I.3, the apparent position of the rod is given by:

$$
\begin{aligned}
y' &= 2ay/2a - V\,(t_L - t_R) \\
&= (6 \times 100)/(6 + 20 \times 0.003) \\
&= 99.0 \text{ cms.}
\end{aligned}
$$

Thus there should be a localisation error of - 1.0 cm, equivalent to 123.8 sec arc.

When the moving rod crosses the median plane, its image is now on the temporal retina of the left eye and the nasal retina of the right eye. The latency difference is now - 0.003 sec, and equation I.3 gives y' = 101.0 cms, or a localisation error of + 1.0 cm.

For movement in the right to left direction, the velocity is -20 cms per second, and the localisation errors are reversed: - 1.0 cm when the target is to the right of the median plane, and + 1.0 cm when it crosses over to the left. According to this development, then, the moving rod should appear to follow the path illustrated in Fig. III.2.

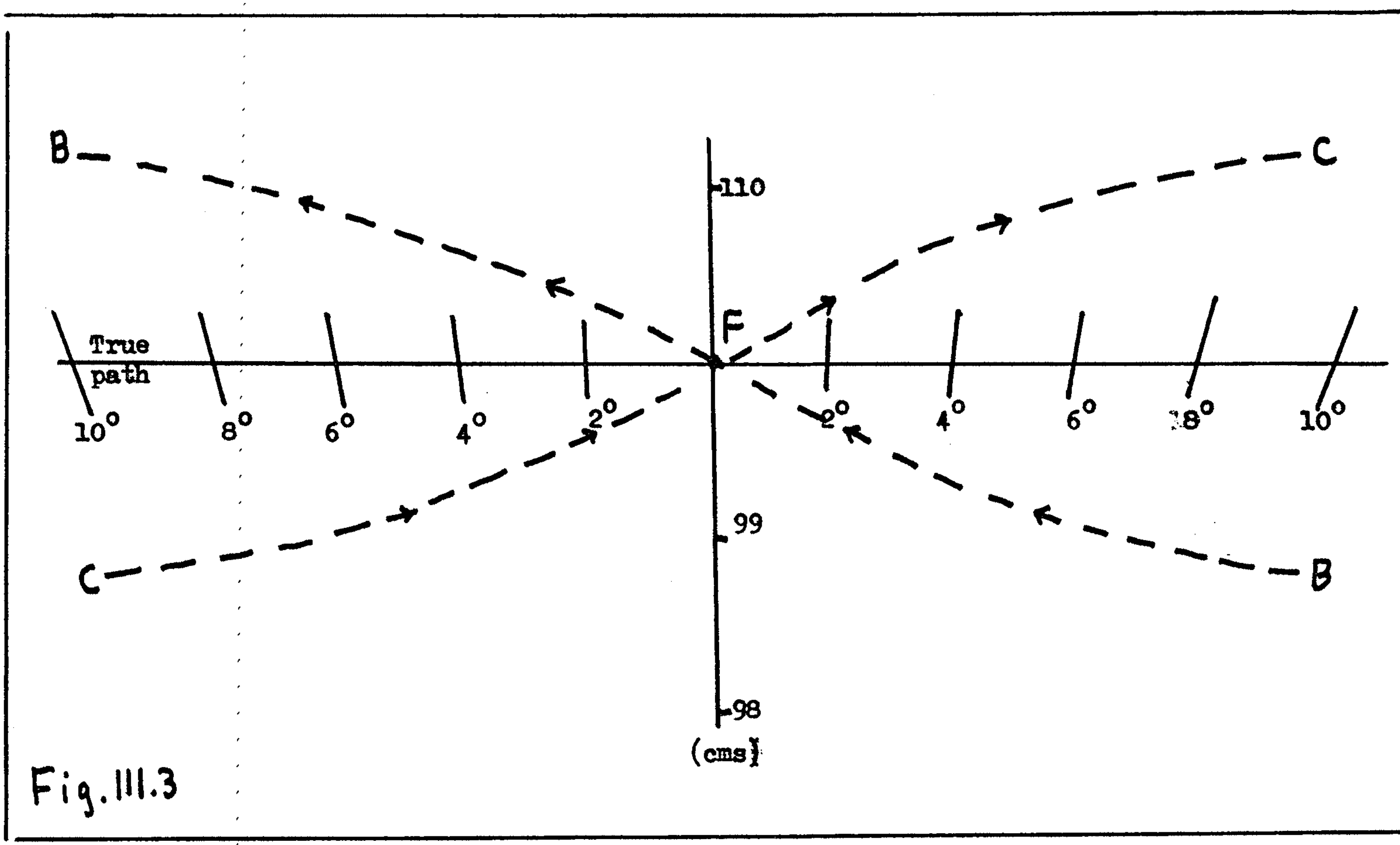

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Figure III.3. Apparent path of an object moving with constant linear velocity of ± 20 cms/sec along a frontal path. It is assumed that the temporal-nasal latency difference is 4 msec at a 10° peripheral angle, decreasing linearly to zero at the fovea. F is the fixation point. BB = apparent path for right-to-left movement; CC = apparent path for left-to-right.

The figure is hypothetical, serving only to illustrate the point. What is more likely is that the nasal-temporal latency difference is zero at the fovea, and increases towards the periphery. In Fig. III.3, it is assumed that the increase is linear, rising to 4 msec at 10^{o} into the periphery on each side. Again, $(t_L - t_R)$ is positive when the stimulus is in the right half of the visual field, negative when it is to the left. With these assumptions, the moving rod should appear to follow the curved paths shown in Fig. III.3. If the velocity of the rod is increased, similar paths are obtained, but with greater displacements.

(b) The path-sampling hypothesis.

In an experiment, the subject is asked, in effect, to judge whether the path crosses the median plane in front of or behind the fixation point F. We may ask, how does he go about making this judgment? It is not likely that he can make the judgment at the precise moment when the apparent path crosses the median plane. What is more likely is that he will begin to make his judgment while the rod is approaching the median plane, for example when it is 4^{o} to the right when moving from right to left. On this basis, he should judge the apparent path as being closer to him than the fixation object, thus demonstrating a negative localisation error. The same would occur for movement in the opposite direction.

The assumption that the subject develops his judgment criteria by "sampling" the path of the object as it approaches the fixation point accounts for some of Lit's findings on the localisation error. ("Sampling" here refers to an integration process at some post-cortical cerebral level, not to an eye-movement mechanism). It explains why the localisation error increases with velocity. Furthermore, Lit's demonstration of an inverse relationship between latency differences and illumination level (Lit, 1949) suggests a reason why the error should decrease as illumination increases (Lit, 1966).

On the other hand, the model presented here implies that all localisation errors are negative, while four of the five subjects reported by Lit (1966) had a positive error. For a positive localisation error, either the latency of the temporal error is less than that of the nasal, or the subject makes his judgments on the basis of input obtained after the target has passed the median plane.

The first alternative is not supported by the evidence cited in the preceding section. As far as the second alternative is concerned, the premise that judgments are based on only a part of the apparent path is just as much an assumption one way or the other. The theory is weakened by the fact that both assumptions must be made.

Before leaving the subject of nasal-temporal latency differences, it may be pointed out that they do provide an explanation of the asymmetries seen in the path of the Pulfrich pendulum.

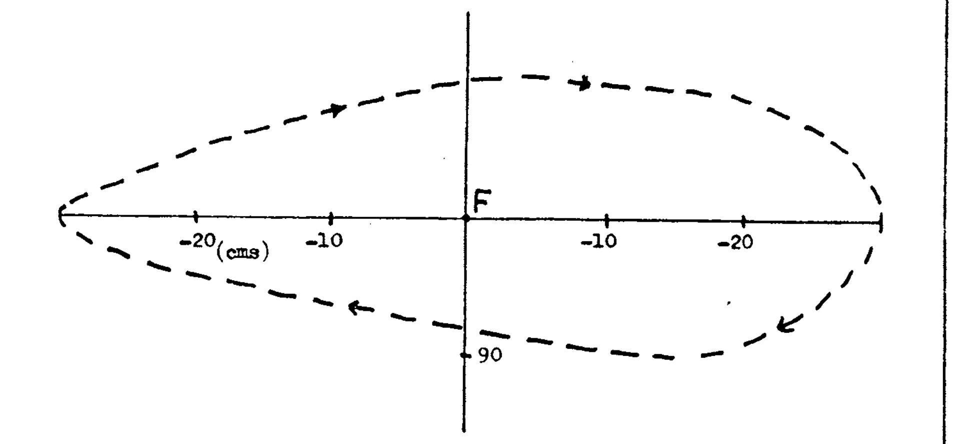

Fig. III.4

Fig. III.4. Theoretical path of a Pulfrich pendulum (filter before left eye). Other constants are the same as for the smaller ellipse in Fig. II.2, but here it is assumed that there is an inherent temporal-nasal latency difference of 5 msec (t_L-t_R) for objects at 30 cms into the right visual field (fixation at 100 cms), decreasing linearly to zero at F, and increasing negatively to -5 msec at 30 cms into the left visual field.

If the inherent latency difference varies as suggested for Fig. III.3, then a filter in front of the left eye would result in a latency difference which is maximum when the pendulum is at the extreme right of its swing, and which decreases as the pendulum moves to the left. The apparent path would then be a distorted ellipse as indicated in Fig. III.4. This figure was derived by solving equation II.13, using the same constants as for the small ellipse in Fig. II.2. However, instead of keeping the latency difference ($t_L - t_R$) constant at .02 sec, it was varied linearly from .025 sec at the extreme right of the field to .015 sec. at the left.

III.5. <u>The role of fixation disparity: the L-F-S theory</u>.

As reported by Payne (1966) and illustrated in Fig. III.1, reaction time has a minimum value of about 200 msec near the fovea, and rises rapidly for the first five degrees in both directions along the horizontal meridian. It is not possible to say whether this change in RT is due to peripheral neurophysiological factors or to the cerebral integrative processes associated with the task, but it seems reasonable to assume that peripheral factors do contribute to the variation; that is, increases because neural messages from the retinal periphery take longer to reach the cortex than from the central retina, and not only because it takes longer to process peripheral information at a cognitive level.

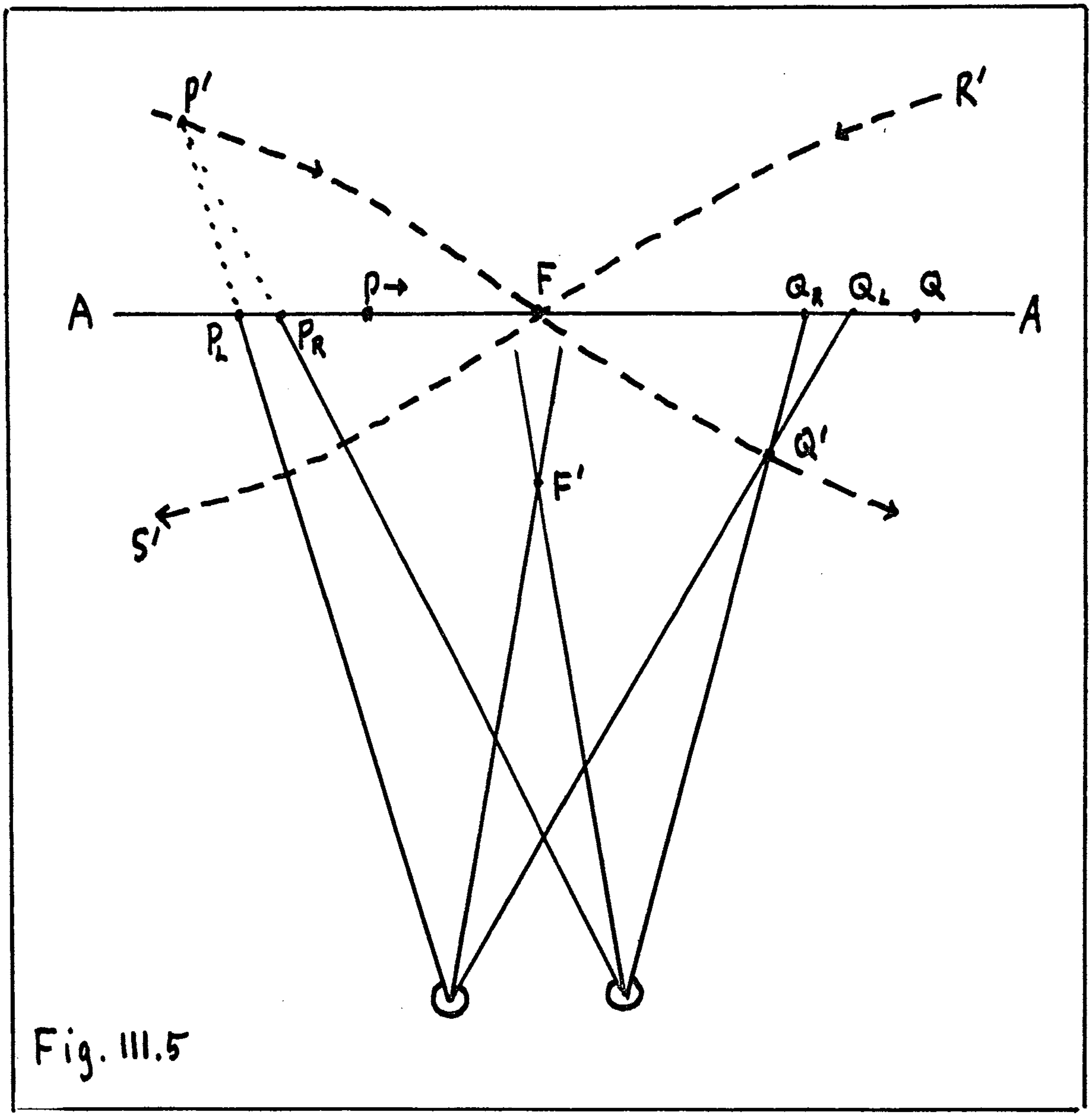

Fig. III.5. Hypothetical apparent path of a rod moving in a frontal plane along the path A-A, assuming that (a) visual latency increases from fovea to periphery in all directions in both eyes; (b) fixation disparity is present, the visual axes crossing at F' instead of at the fixation point F.

Now consider what happens when fixation disparity is present in the basic stereoscopic localisation experiment. In Fig. III.5, AA is the true path of the object, and F is a fixation point in the same frontal path as AA. The subject has an eso fixation disparity; his fixation axes cross at F', some distance nearer to him than F.

While the object is travelling from left to right in the left half of the field, its retinal image is closer to the fovea in the right eye than in the left eye. According to the above, there is effectively a latency difference. When the object is actually at P, the uniocular images are at P_R and P_L, giving rise to the fused percept at P'.

In the right half of the field, the object still travelling from left to right, the left eye image is now closer to the fovea, the uniocular images Q_R and Q_L resulting in an apparent image at Q'. The apparent path of the object is thus a curve such as that indicated by P'FQ'.

Similar reasoning gives the path R'FS' for left-to-right movement.

Fig. III.5 is similar to Fig. III.3, and by assuming again that the subject bases his judgment on input received while the object is approaching the median plane, the basis for a positive localisation error is demonstrated. As in the nasal-temporal model, the displacement of the apparent path will increase with velocity, and a variation of effective latency differences with illumination can be expected.

A negative localisation error would occur in the presence of exo-disparity.

The direction of the localisation error is in agreement with the results reported by Lit (1960c and 1964): negative for exo-disparity, positive for eso-disparity.

Lit (1964) calculated the latency differences corresponding to the linear localisation errors. From this information, and from Payne's data on reaction time (Fig. III.1), the required amount of fixation disparity can be estimated.

For Lit's subject M.M., the average latency difference was 1.01 msec. According to the theory presented here, this would represent a mean of the latency differences occurring at those parts of the retinae where the moving target is imaged while approaching the median plane. For example, the latency difference may have been 2 msecs when the target was five degrees from the median plane, decreasing to zero when the target was imaged the same distance away from each fovea.

From Payne's data (Fig. III.1), it can be estimated that the absolute latency changes by about 20 msecs per five degrees near the fovea. For a latency difference of 2 msec, a fixation disparity of .5 degree would have to occur. This value is high, being close to the reported limits of Panum's fusional areas (Ogle, 1964, p.67), and one must question whether or not such a large fixation disparity could have occurred.

For Lit's other subject, F.C., the average latency difference was -0.8 msec. (F.C. had a negative localisation error, except for some very low positive findings with small target velocities. These low findings were possibly not significantly different from zero, and were omitted in arriving at the average latency difference of -0.8). Following the reasoning of the previous paragraph, and using -1.6 msec as the highest latency difference that must have occurred, the required fixation disparity for F.C. is .4 degrees. A fixation disparity of 24 min arc is still high, but not unrealistic, in view of the reduced binocular fusion stimuli.

While the theory explains why the apparent displacement should increase with velocity, Lit's data indicates that the mean latency difference increases with velocity even though illumination conditions are held constant. For example, for the highest level of illumination, Lit's subject M.M. had a calculated latency difference of 0.47 msec at 8.16 cms/sec, and 1.40 at 68.17 cms/sec. The theory as stated would require a different amount of fixation disparity for the two stimulus conditions.

However, it may be that the subject does not begin to gather data for his judgment when the target is at some constant distance away from the median plane, but rather

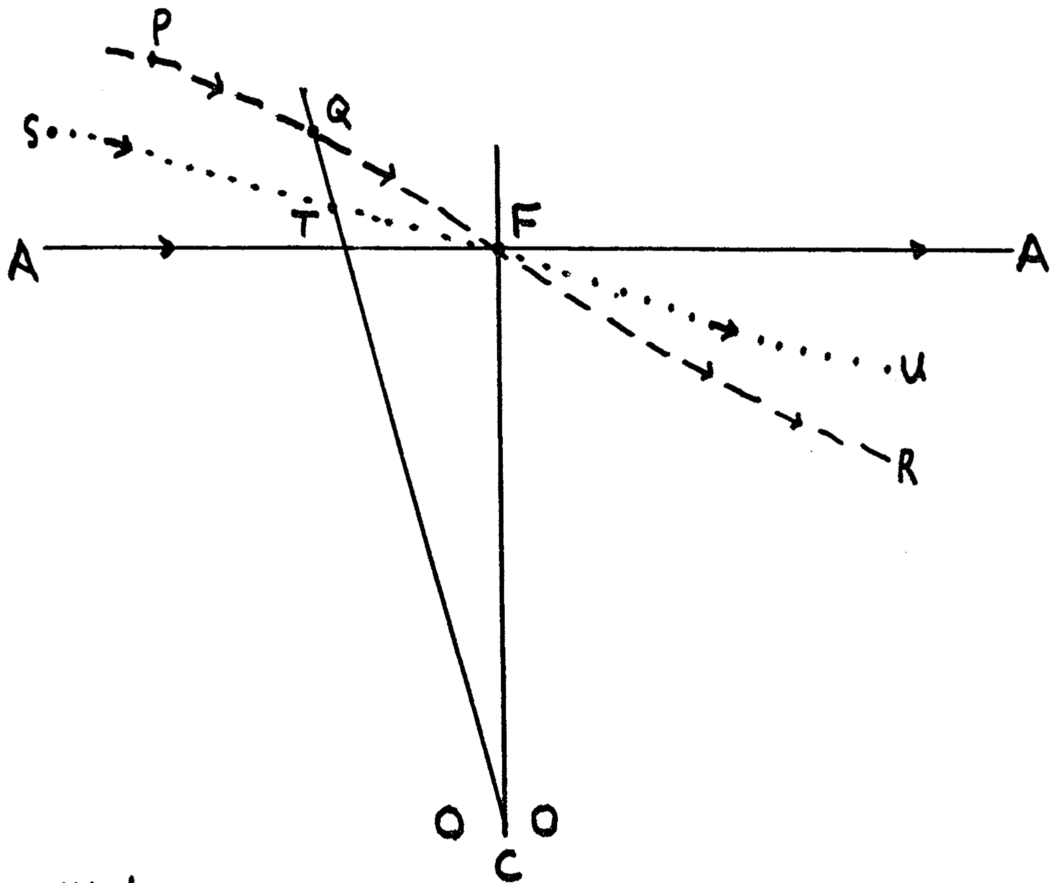

Fig. III.6

Figure III.6. PQFR and STFU are apparent paths of an object moving along AA, and viewed by an observer at C. PQFR occurs at twice that velocity which gives rise to STFU (cf. Fig. III.3). The interpretation of this figure in terms of the path sampling hypothesis is given in the text.

uses information obtained over a fixed interval of time. In Fig. III.6, two apparent paths are shown, both obtained (hypothetically) under the same luminance conditions. The path PQFR occurs at twice the target velocity which results in path STFU. If the subject always judges the distance of the apparent path on the integration of a segment of fixed angular extent, input for the integration process would be the segment QF for the high velocity, and TF for the low velocity. If on the other hand the input is ordered by time rather than distance, two segments he might use are PF and TF. The second alternative would give a larger localisation error than the first; the computed latency difference for the higher velocity would accordingly be greater than for the lower.

For convenience, the theory presented in this section will be referred to as the L-F-S theory (Latency - Fixation disparity - Sampling), the initials serving as a reminder of its three basic premises.

III.6 <u>Horopter curvature</u>.

In the theories given in the preceding two sections to account for localisation errors in the stereoscopic localisation of moving objects, a recurring assumption is that the subject bases his judgments of "further" or "nearer" on the apparent path of the object as it approaches the reference point. Thus, even if the apparent path passes directly below the reference point, as in Fig.III.6, it

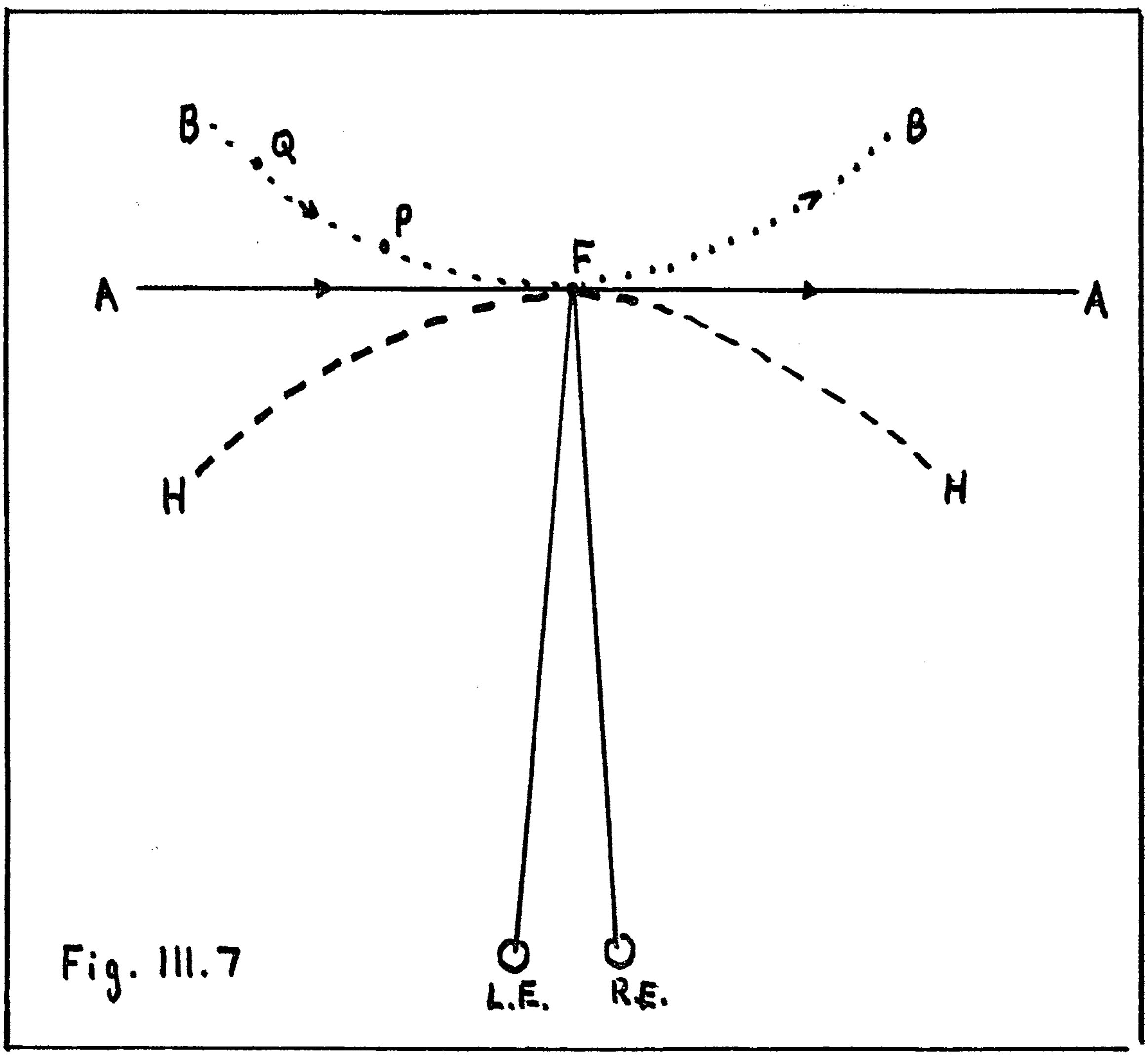

Figure III.7. HH is the apparent fronto-parallel plane for an observer fixating at F. Since judgments of stereoscopic depth differences are made relative to the AFPP, an object moving along a linear path such as AA should appear to follow a curve similar to BB.

may be judged "behind F" on the basis of a segment of path such as TF.

In the absence of latency-induced distortions of the perceived path, curvature of the stereoscopic frame of reference (the horopter) may have a similar influence on the judgment task. (See Chapter I.2b).

In Fig. III.7, HH is the apparent fronto-parallel plane (AFPP) of an observer fixating at F. According to the general theory of stereopsis, an object would have to move along HH in order to appear to follow a subjectively linear path.

An object actually moving along the linear path AA should appear to follow the curve BB.

If the path sampling hypothesis is correct, the observer should judge the object to be moving along a path behind F.

Furthermore, if the length of path which is sampled is determined by a fixed time interval, the faster the object is moving the greater will be its reported displacement. An object moving with a velocity V cms/sec may be judged according to the segment PF in Fig. III.7, whereas an object moving at 2V cms/sec may be judged according to the segment QF.

Curvature of the apparent fronto-parallel plane can therefore account for the localisation error and its increase with velocity, but not for the variation with luminance.

The example in Fig. III.7 results in a positive localisation error; for a negative localisation error, the AFPP would have to be convex towards the observer. AFPPs and horopters reported in the literature have been invariably concave (e.g. Ogle, 1964, Ch. 4; Amigo, 1967), although a convex horopter is not an impossibility (Ogle, 1964, Fig. 16). Indeed, most of the localisation errors reported by Lit (1960c, 1964, 1966) have been positive, and furthermore, the exact shape of the AFPP may vary with stimulus conditions (Chapter I.2c). The AFPP of an observer in a moving stimulus experiment might not be the same as that elicited in a multiple-rod AFPP experiment.

The deviation of the AFPP (or the horopter) from the objective plane is rarely greater than 10 mm even for lateral angles greater than 10° according to Ogle's examples, most of which are for observation distances of 40 cms. In Fig. 16 of Ogle (1964), the AFPP is only about 6 mm from the objective plane at a lateral angle of 12° and for a 76 cm observation distance.

The horopter curvature therefore can account only for some and not all of the localisation errors and their characteristics.

III.7 <u>Other Possibilities</u>.

(a) <u>Eye movements</u>.

Harker's saccadic suppression theory of the Pulfrich phenomenon has been discussed in Chapter I.6. Harker's theory (Harker, 1967) depended on:

(i) small eye movements during observation of the saccades,

(ii) suppression of vision during saccades; and

(iii) inequality of suppression intervals because of reduced stimulation to one eye.

In the kinetic stereoscopic localisation experiments with equal luminance conditions, similar eye movements as in the Pulfrich situation should occur. These would have their origins principally in the opto-kinetic reflex (Adler, 1959, p. 422), the involuntary fixation reflex making it difficult to maintain exact fixation as the moving object passes near the stationary target.

If binocular opto-kinetic movements are disjunctive, then one eye could have a different onset or offset of suppression than the other, leading to stereoscopic displacements as in the Pulfrich phenomenon.

Very little can be learned from the literature concerning the disjunction or otherwise of opto-kinetic movements, except that grosser movements are more or less conjunctive (Westheimer, 1954a).

A saccadic suppression explanation of the localisation error would require that the pattern of eye-movements was similar from one observation to the next, otherwise greatly varying results would be obtained.

If voluntary fixation movements are made to track

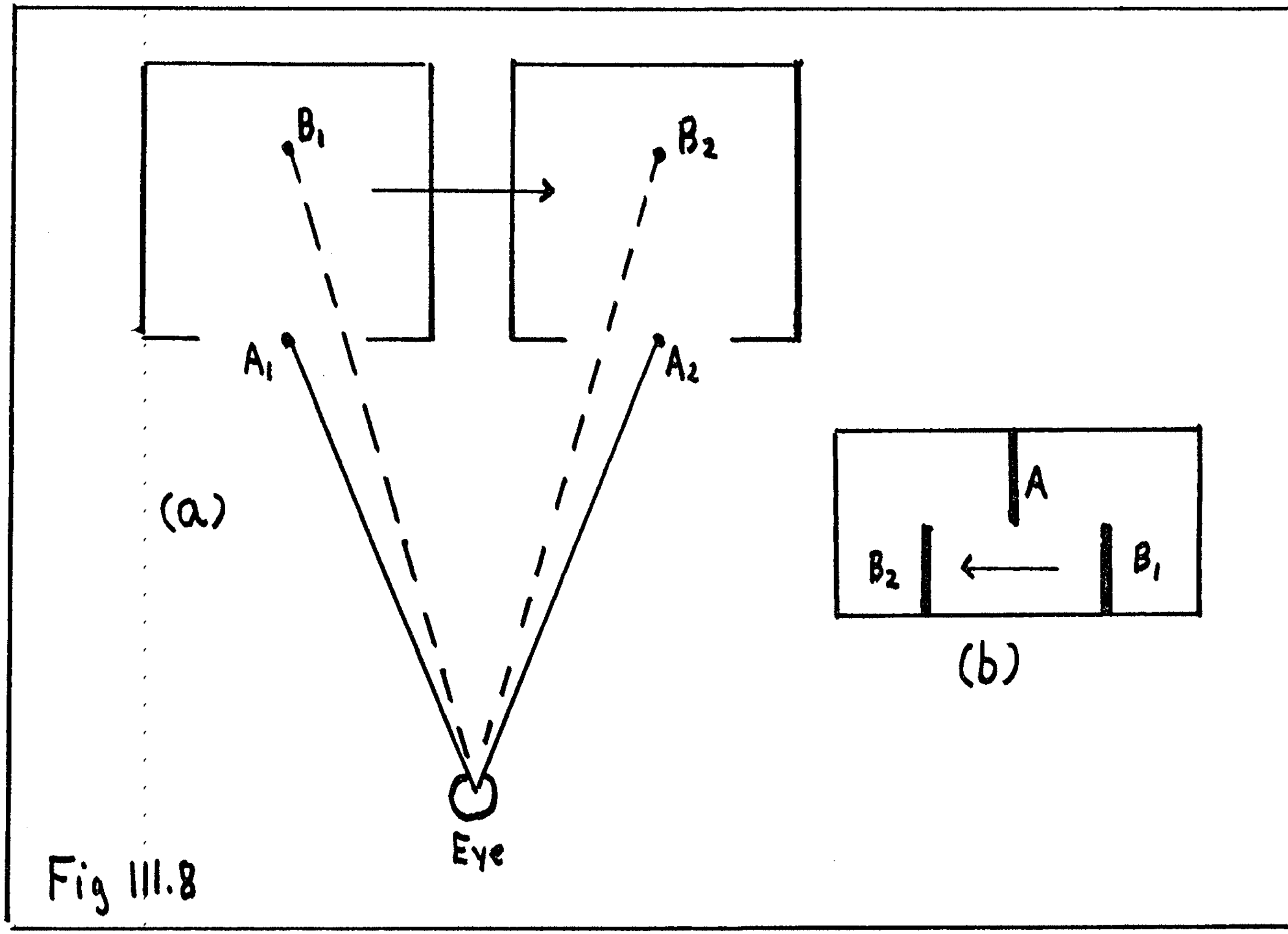

Figure III.8. In (a), A and B are the initial positions of two rods. While the observer maintains fixation on rod A, the whole array is moved to the right. The relative movement of the rods as seen by the observer is shown at (b).

the moving target, Hering's law (that in all voluntary movements an equal and simultaneous innervation goes to the appropriate muscles of both eyes; Adler, 1959, p.344) suggests that the eye movements will be conjunctive, again excluding a saccadic suppression explanation.

Gross eye movements, as might be made by an inattentive subject, could result in temporary differences in adaptation, particularly if the stimulus configuration is surrounded by a black area. These differences would contribute to a rise in threshold, but not to the magnitude of a localisation error.

In general, it must be concluded that the role of eye movements in the stereoscopic localisation of moving objects is uncertain.

(b) Motion parallax.

Fig. III.8 (a) is the plan view of a stimulus array consisting of two rods, initially at the positions A_1 and B_1. A monocular observer fixates rod A. The whole array is moved laterally to positions A_2 and B_2, the observer maintaining fixation on rod A. The appearance of the array during the movement is indicated in Fig. III.8 (b): the two rods move relative to each other, the further rod moving against the direction of movement of the whole array. This is similar to the motion parallax which occurs when there is a translatory movement of the head (although with a moving head and stationary objects, distant objects move with the movement of the head). As pointed out by Ogle

(1962, Chapter 14.3E), motion parallax is a sensitive cue for depth distances, enabling discriminations of the order of 7 to 28 sec arc to be made.

The appearance depicted in Fig. III.8 (b) is essentially the same as that in the stereoscopic localisation experiment (see Fig. I.1): one object remains in the same position in the visual field while a second moves from one side of the field to the other. Therefore the possibility should be considered that in the localisation experiment, the stimulus array is misinterpreted as relative movement caused by motion parallax.

The possibility is somewhat remote. For rod A in Fig. III.8 to remain stationary in the visual field, there must be either an eye movement or a head movement, neither of which occur in the stereoscopic localisation experiment.

Furthermore, the appearance depicted in Fig. III.8 (b) is ambiguous. If it is the result of a head movement, rod B would be interpreted as being nearer than A, whereas if the whole array moves with the head stationary, rod A would appear nearer.

Finally, in the stereoscopic localisation experiment, there is of course never an occasion when the "motion parallax" is zero, so that if localisation errors were due to this kind of misinterpretation, apparent equality of distance should never be reported.

III.8 Summary and discussion.

(a) Recapitulation.

Several explanations for the existence of localisation errors in kinetic stereoscopic perception have been presented in this chapter. That a number of theories are possible indicates that the problem is too complex to be solved by a simple and single set of factors.

The main theory, called the "L-F-S" theory, is developed in Section III.5. Its premises may be summarised as follows:

(i) Visual latency varies with retinal location, being minimal at or near the fovea and increasing into the periphery.

(ii) In the presence of fixation disparity, a stimulus array which otherwise gives rise to zero retinal disparity may be imaged on retinal areas for which there is a latency difference between the two eyes.

(iii) With a moving stimulus, these differences in latency result in a distorted apparent path, as in the latency hypothesis of the Pulfrich phenomenon.

(iv) The depth difference between the intersection of a distorted path with the median plane and a reference object in that plane is judged by sampling the path as it nears the plane.

Variations in visual latency with retinal location have been demonstrated independently (Section III.3), as

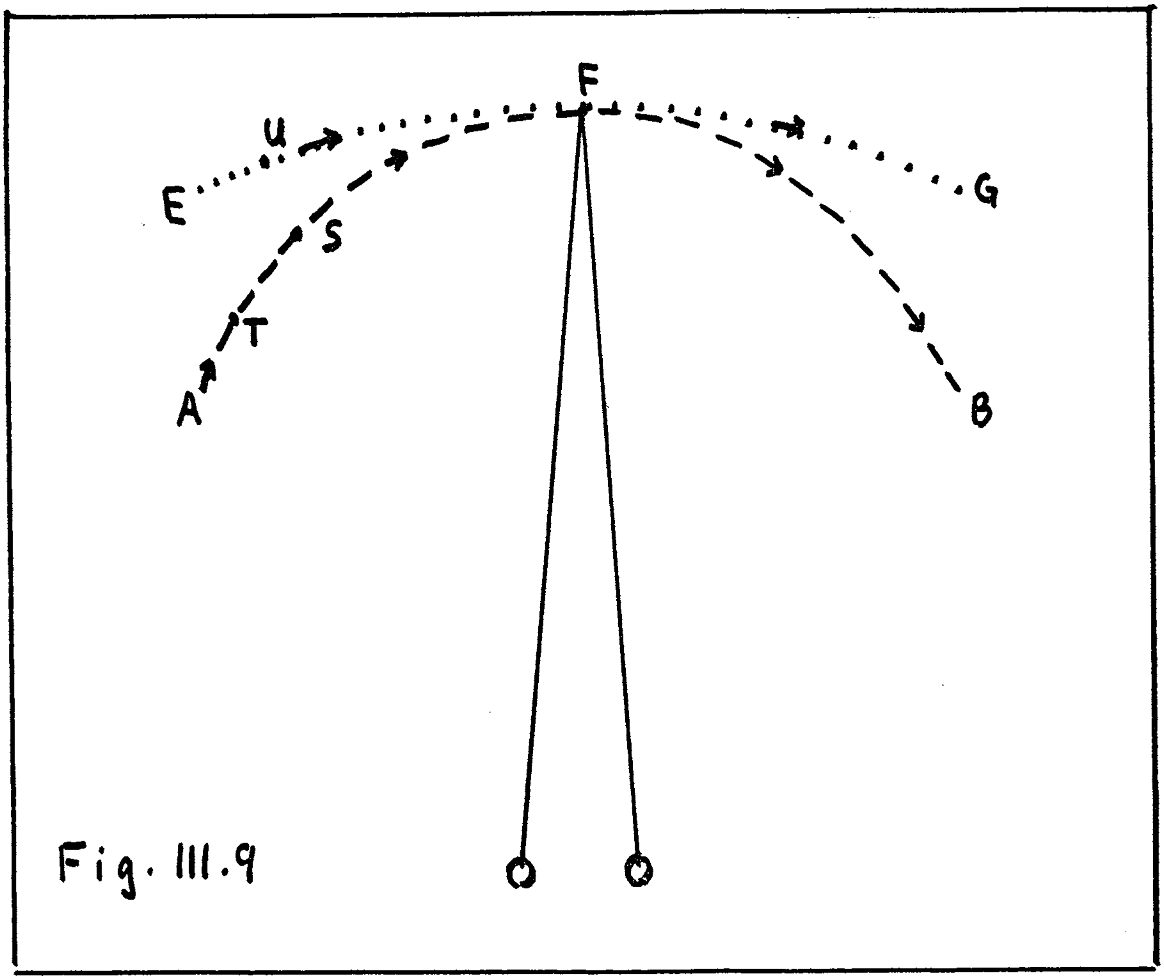

Figure III.9. An object moves along the circular path A-B while an observer fixates at F. According to the path-sampling hypothesis (see text), the object appears to intersect the median plane at some distance which is an integrated function of a segment such as S - F. For higher velocities, a longer segment (e.g. T - F) is used, increasing the localisation error. For a flatter path such as E - G, and with the same velocity as in the first case, the localisation error is reduced, judgments being based on a segment such as U - F.

has fixation disparity. The question is whether or not the magnitudes of these factors are sufficient to provide the effective disparities postulated by (iii).

As yet, there is no evidence confirming or denying the path sampling hypothesis in the fourth premise.

Other possibilities considered in this chapter are the curvature of the apparent fronto-parallel plane, the effects of eye-movements, and the misinterpretation of stimulus movement as motion parallax.

(b) Testing the L - F - S theory.

As explained in Section III.2, fixation disparity must be measured under precisely the same conditions as in the localisation experiment. Similarly, so should the variation of visual latency with retinal location. In the investigations cited in III.3, the stimulus was invariably a photic signal presented against a background of lower luminance. In the stereoscopic localisation experiments described by Lit (Chapter I.5), the stimulus was a black rod moving against an illuminated background. What is required then is a study of the latency of response to an "off" signal, rather than to an "on" signal.

A test of the path sampling hypothesis may be carried out along the lines illustrated in Fig. III.9. In the figure, an object moves initially along the circular path A-B, from left to right. An observer fixates a reference object at F, and his task is to judge whether the moving object passes nearer to him or further than F.

According to the hypothesis, the observer samples a segment of the path, for example SF, while the object is approaching the median plane. His judgment then is that the object crosses the median plane at some distance between S and F. He would display a localisation error no greater than the perpendicular distance between S and F.

Next the object is made to move along the same path, but at twice the velocity. The hypothesis holds that the sampling is time-ordered, that is, that the length of path sampled is that segment traversed during a fixed time period. With the higher velocity, the sample may be the segment T-F in the diagram, so that the localisation error is increased.

The object is now made to move along the flatter path E-G, with the same angular velocity (with reference to the subject) as in the first example with the steeper path. (Because of the curved path, angular velocities subtended at the subject will vary; for the present purpose, it may be assumed that the angular velocity is constant over the relatively small segments considered). The length of path sampled is U-F, and a localisation error not greater than the perpendicular distance between U and F should be elicited.

So far, no mention has been made of localisation errors caused by latency differences, fixation disparity, horopter curvature, or other unknown factors. Whatever their origins may be, the magnitude of these errors should be altered by varying the shape of the curved path. Thus, a positive localisation error should decrease as path curvature is increased, while negative errors should increase. Stated another way, objects moving along steeply curved paths should be localised nearer than those following flatter paths.

Experiments along these lines have been carried out, and are described in the remaining chapters of this thesis.

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Facing p. IV.1

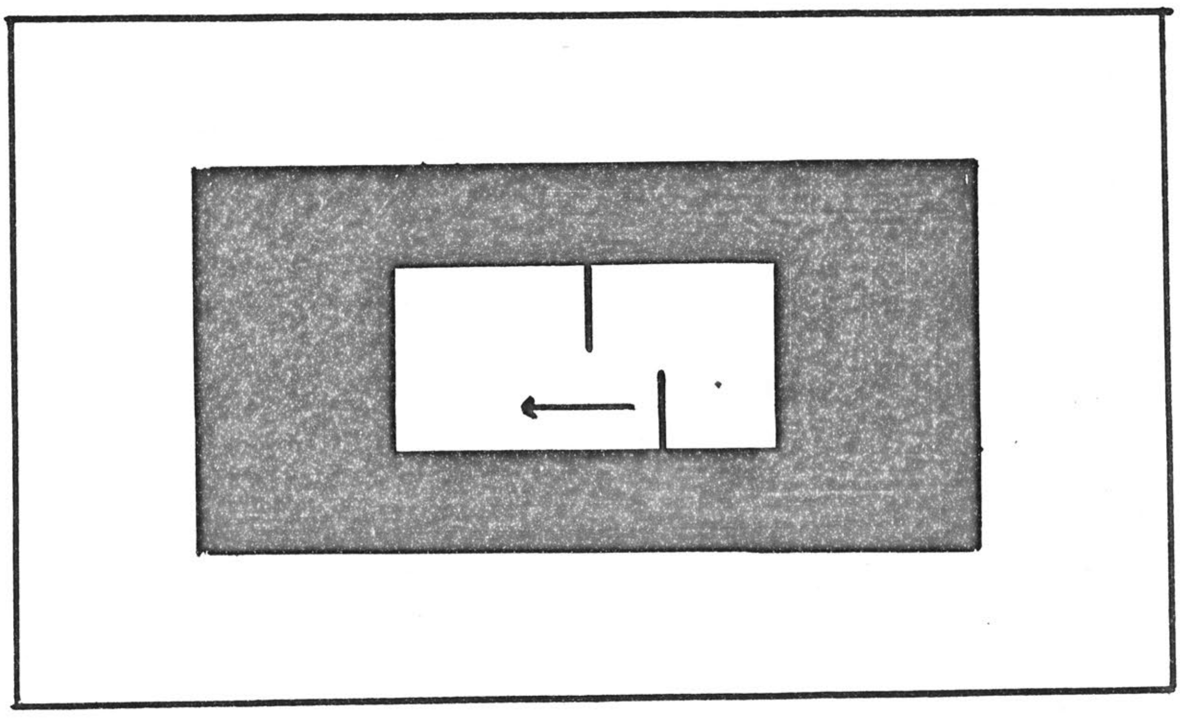

Fig. IV.1. Subject's view of the stimulus configuration. The illuminated area (field of view) was 20° long and 10° high. The tip of the upper rod (the reference object) was the fixation point, while the lower rod moved horizontally along a circular path.

CHAPTER IV

A P P A R A T U S

IV.1 Brief description.

Basically, the requirements of the apparatus were that the subject should see two vertical rods against an evenly illuminated background (see Fig. IV.1).

The upper rod (the fixation target), in the subject's median plane, was stationary during each observation, but could be adjusted to different distances from the subject. The lower rod moved with constant velocity along a circular path whose centre of curvature was in the subject's median plane.

Each trial began with the moving rod out of the field of view. The subject fixated the lower end of the stationary rod, which was set at a distance according to the rules of the method used (see below). The experimenter then set the lower rod in motion, and the subject judged whether the rod moved in front of or behind the plane containing the fixation object. He used a bell-buzzer system to signal his decision to the experimenter, who recorded the result, returned the lower rod to its starting position, and adjusted the fixation rod for the next observation.

IV.2 The Apparatus.

In order to control the illumination conditions around the subject, the apparatus was housed in a double

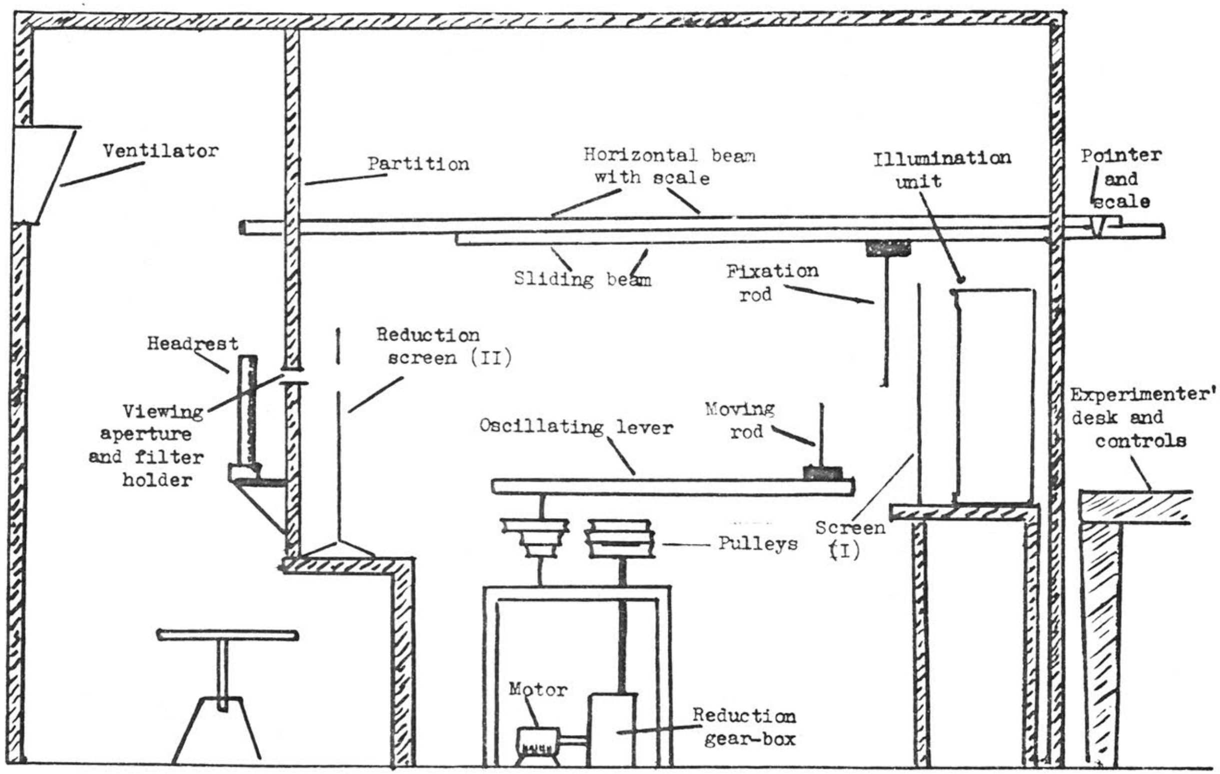

Fig. IV.2. Cross-section through the double dark-room (not to scale). Details are given in the text.

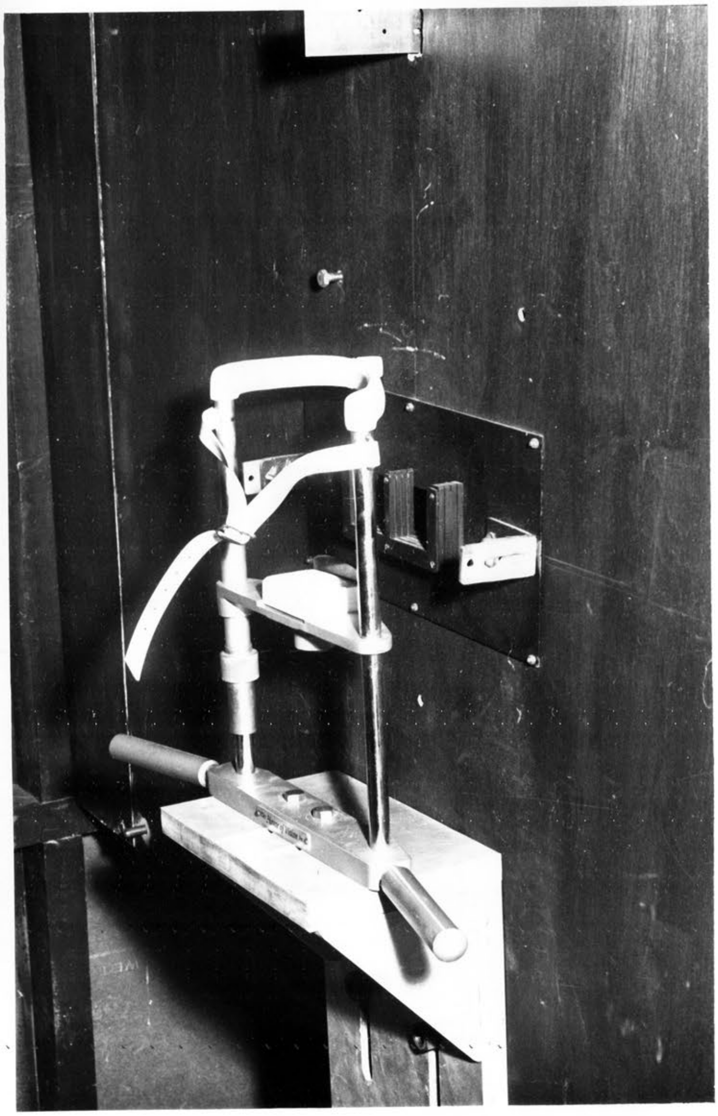

Figure IV.3. The head-and-chin rest used for positioning the subject. Immediately in front of the head rest is the viewing aperture and filter holders; on each side are the sight holes for aligning the corneas with the end of the scale protruding from the partition at the top of the picture.

dark-room (see Fig. IV.2). The two sections were separated by a partition which was light proof except for the viewing aperture.

(a) The subject's cubicle.

The subject sat on a stool, with his head in position in a head-and-chin rest which could be adjusted vertically and fore-and-aft. Immediately in front of the head rest was the viewing aperture, 10 cms long by 5 cms high, which had provision for up to three filters in front of each eye. (See Fig. IV.3).

Projecting through the partition, above the head-rest, was the end of the metal strip which carried the scale on which the distance of the fixation rod was measured. A plumb-line could be suspended from the metal strip, and by sighting through appropriate holes the projecting tabs on either side of the head-rest were adjusted until they were in line with the plumb-line. These holes were than used as reference points for positioning the subject's head before each experiment.

Two push-buttons, one actuating a bell and the other a buzzer, were held by the subject and used for signalling his decisions.

The cubicle was ventilated by a light-proof exhaust fan. Subjects could stay in position for up to an hour without reporting discomfort.

Fig. IV.4. General view of the stimulus mechanism. One wall of the dark-room has been removed, to show the motor and reduction gear (on the floor). A shaft from the gear box drives a series of stepped pulleys, which in turn move the long lever carrying the moving stimulus. Above can be seen the reference rod, with the illuminated panel at the back.

(b) The moving stimulus.

Initially, a cam mechanism was used to make a long arm oscillate with constant angular velocity. This was eventually abandoned because of difficulties in manufacturing a cam to the accuracy needed for smooth movement. A less sophisticated but more reliable device was eventually used, in which a synchronous motor drove the arm through a series of pulleys (see Figure IV.4).

The synchronous motor was specially manufactured by G.M.F Electric Motors Pty. Limited (Sydney). It provided a torque of .25 H.P. at 1500 r.p.m. The direction of rotation was controlled by a switch at the experimenter's desk.

The r.p.m. were reduced first by a 100:1 worm gear mechanism, and then by a series of pulleys. The last set of pulleys could be interchanged, for final speeds of from .5 r.p.m. to 3 r.p.m. Care was taken to ensure that the final motion was smooth. At all times, the motor was started with the rod well out of the subject's field of view, to take up any slack in the belts driving the pulleys.

As a result, an arm attached to the axle of the last pulley moved smoothly in a horizontal plane. The arm used was made of T-section aluminium, 110 cms. long, counter-balanced so that it was always horizontal. The centre of rotation could be moved to any position on the arm, although in the experiments only two positions were

Fig. IV.5. General view of the stimulus arrangement. The upper (fixation) rod and lower (moving) rod are clamped in adjustable holders. The shaft supporting the upper rod slides along the broad beam, while the lower rod is moved by the mechanism seen in Fig. IV.4. The screens used for restricting the field of view are not shown.

used, giving radii of 50 cms and 100 cms.

The entire driving mechanism could be moved to place the centre of rotation of the arm at any distance from the subject.

The rod was attached to the arm by a device which provided micrometer screw movement in all three directions, as well as rotation in two planes at right angles to each other (see Fig. IV.5). Thus, the rod could be adjusted for position and verticality. This was done as described below with a series of plumb-lines. Initially, it was intended that the arm oscillate back and forth automatically. Apart from the mechanical difficulties involved, it was found to be convenient for the experimenter to control the movement directly at all times.

Two micro-switches were arranged so that at each end of the swing a light at the experimenter's desk was activated. The light operated at two brightnesses, one for each position of the arm, and after a little practice the experimenter could monitor the arm position with his peripheral vision, enabling him to operate the apparatus rapidly.

(c) The fixation rod.

The fixation rod was suspended from a holder similar to that used for the moving rod.

An H-section aluminium beam ran through the length of the stimulus part of the darkroom. This beam was constructed as accurately as possible so as to be in the median plane of a correctly positioned subject. It carried a channel, which in turn held a T-section shaft which could slide along that channel. The sliding shaft held the holder for the fixation rod, which could thus be moved to any position in the subject's median plane.

A 200 cm scale was fixed to the H-section beam. The end of the scale projected into the subject's cubicle, providing a reference point as described in Section IV.2 (a).

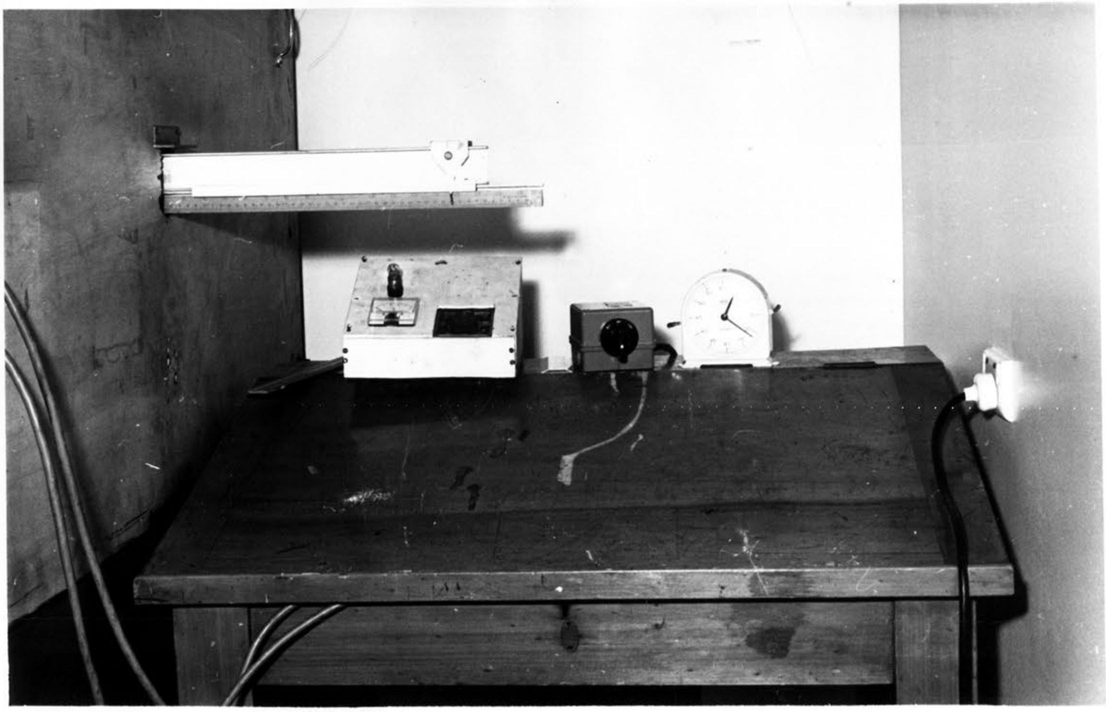

Fig. IV.6. The experimenter's desk, showing the ends of the beam and shaft carrying the fixation rod, with the scale and pointer. On the desk are (left to right) the device for monitoring the position of the moving rod, the switch controlling the motor, and a timer.

The other end of the beam and the sliding shaft both projected out of the dark-room over the experimenter's desk (see Fig. IV.6). At this end, an arbitrary millimetre scale was fixed to the sliding shaft. When the fixation rod was adjusted to its "zero" position (200 cms from the subject in the experiments described here), the pointer attached to the stationary beam could be set at the scale zero, and clamped into position. The fixation rod's position could be read to an accuracy of at least .25 mm.

Built into the fixation target was a small amount of lateral movement, which occurred when the target was adjusted from one position to another. This made it impossible for the subject to detect the direction of movement, even when, in pilot experiments, he was specifically asked to do so.

Both the fixation rod and the moving rod were made of .125 inch steel, painted with matt black paint. The angle subtended at 200 cms was 0.1°.

When the rods were aligned, the gap between their ends also subtended 0.1° at 200 cms. In some early experiments, a larger gap (0.3°) was used; this is noted in the report of the experimental results. The larger gap was dictated by the early form of the apparatus. After modifications, it was reduced in order to keep the stimuli near to the horizontal visual plane.

During construction, the position of the end of the

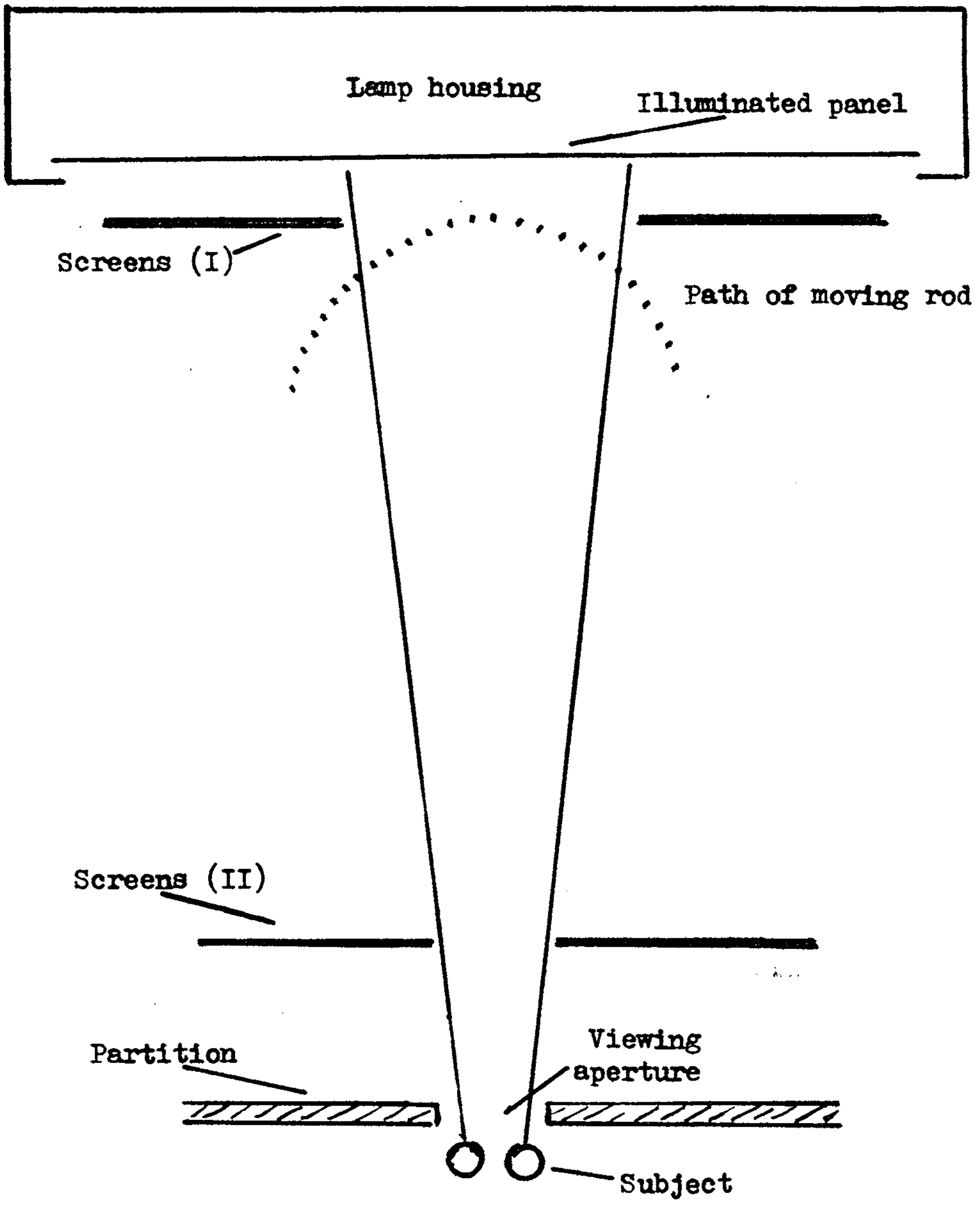

Fig. IV.7. Plan view of the apparatus (not to scale), showing arrangement of the reduction screens.

fixation rod was aligned with the sight-holes on either side of the head-rest. Thus, when the subject was in position and fixating the end of the rod, his visual axes were horizontal, while the tip of the moving rod was seen 0.1° below.

(d) Illumination Unit.

The background illumination consisted of a sheet of white translucent perspex, 100 cms long by 25 cms high, set in a box containing two thirty-inch, 60 watt tungsten strip lamps. The lamps were powered directly from the 240 volt mains. Long periods of monitoring with an S.E.I. Photometer showed that a voltage stabiliser was not necessary.

The luminance of the panel was 384 cd/m^2. It was measured before and after each experimental session, and proved to be remarkably constant, even when checked by other observers.

There were no variations in luminance over the area seen through the reducing screens.

(e) Reducing screens.

Two sets of reducing screens were used. One set was 200 cms. from the subject, with an aperture of 35.3 cms, so that the seen part of the illuminated background subtended 10° at the subject's interocular midpoint (see Fig. IV.7).

The other set was at 50 cms, and adjusted as shown in the figure so as to limit the binocular field of view to the extent of the illuminated panel.

With this arrangement, the binocularly overlapping field of view was slightly greater than 10° (10.05° for an interocular separation of 6.0 cms) while there was a small area (0.3°) seen temporally by each eye alone. In the monocularly seen area, the background illumination was occluded, and ideally, the matt black rod should not have been visible. However, the rod did reflect some light, being seen briefly as a dim object in a black surround. It was not felt that this small amount of monocular viewing would affect the experiments.

A smaller monocular area could have been achieved by having the near screens further away from the subject; however, this was undesirable for two reasons. First, there would have been more stray light reaching the subject from the walls of the darkroom with the possibility of unequal adaptation between the two eyes. Second, pilot experiments with the near screens at 150 cms indicated that these presented fusion stimuli which competed strongly with the fixation object, subjects reporting asthenopia and occasional diplopia. These reports did not occur when the screens were moved to 50 cms.

(This experience suggests a way of introducing fixation disparity without the use of prisms, as done for example by Ogle (1964).)

The near screen also reduced the vertical field of view to 5^{o}.

IV.3 Summary.

In summary, the apparatus was basically a two-rod Howard-Dolman stereoscopic threshold apparatus, with the introduction of lateral movement of the lower rod. The differences between this apparatus and that of Lit (1949, etc.) were that:

(i) the path of movement was circular, not linear;

(ii) the subject did not have control of the reference rod;

(iii) the moving rod could not be seen to decelerate, change direction, and accelerate at the start of each cycle;

(iv) the lower rod moved, not the upper rod as in Lit's apparatus.

The last difference is probably of no importance; the first is basic to the theory behind the experiments, the second precludes the method of adjustment, and the significance of the third remains to be seen.

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CHAPTER V

EXPERIMENTAL METHODS

V.1 Introduction.

Lit (1949, 1951, 1959, 1960, 1964, 1966) used the method of adjustment (Guilford, 1954) in his experiments. It will be recalled that the full extent of the target's oscillation was visible to the subject (Lit & Hyman, 1951, p. 572), including the ends of the path where the velocity was not linear. It has been already mentioned (Chapter I.4 b) that this non-linearity may introduce an illusion of movement in depth similar to the kinetic depth effect; for this reason, the apparatus described here was screened so that only the smooth part of the target's movement could be seen.

Also, in his equal illuminance experiments, Lit apparently made no distinction between left-to-right and right-to-left movement: " ...the observer adjusts this rod (the fixation rod) in the median plane until it appears to lie directly below the frontal path of the oscillating target" (Lit, 1964, p. 84). This statement appears in most of Lit's papers; it is not clear what he means by the "<u>frontal</u> path of the oscillating target", but only in relation to Pulfrich type experiments does he distinguish between left-to-right and right-to-left movement. It would seem that under equal illuminance conditions, there was no difference between directions.

There is always the possibility of a "built in" Pulfrich effect, due perhaps to unequal transmission in the subject's eyes, unequal or eccentric pupils, or unequal adaptation from stray light. In the present experiments, it was thought better to study the two directions independently; thus, at each session, only one direction of movement was used.

The method of adjustment was not appropriate for these experiments, since the stimulus appeared for discrete time intervals, followed by another interval during which it travelled in the opposite direction. Such a set-up suggests the use of a constant stimulus method, and the apparatus was designed with such a method in mind.

Furthermore, the method of adjustment does not permit analysis in terms of decision theory (Swets et al., 1961). One of the aims of the study was to explore the possibility of such an analysis (see section V.5).

A traditional constant stimulus method was used in the early experiments. Later, an "up-and-down" ("staircase") method was used, for reasons discussed later and in Appendix A.

V.2 General procedure.

Before each session, the apparatus was checked and adjusted so that all components were positioned correctly. Pulleys giving the desired r.p.m. were selected, and checked by measuring the time taken for 20 revs.

A plumb bob was suspended from the fixed beam (section IV.2 (c)) at the distance where the centre of curvature of the path was required to be. The moving rod mechanism was placed so that the pivot of the oscillating arm was in this position; this was checked by operating the motor and seeing that the centre of the pivot always remained beneath the bob of the plumb line. This adjustment could never be exact because of the small but incessant pendular motion of the plumb-line. However, the error was always less than 1 mm., negligible in relation to the other distances involved.

When the mechanism was in position, the fixation rod was set at exactly 200 cm. It was adjusted for position and verticality with two plumb lines, one of which was aligned with the 200 cm. mark on the scale of the fixed beam.

Finally, the moving rod was put into place, with the oscillating arm parallel to the fixed beam. Much care was taken to ensure that the two rods were exactly aligned, since their relative distance was the object of the experiment.

After the adjustments were completed, the motor was operated several times, and the alignment of the two rods again checked to ensure that all components were properly clamped.

Next, the sight-holes for checking head position were adjusted as described in Section IV.2 (a). Then the luminance of the illuminated background panel was measured with an S.E.I. photometer. A Variac was available to adjust the voltage of the lamps, but this never proved necessary.

When the experimenter was satisfied that everything was in order, the darkroom was sealed, and the subject positioned so that his corneal apices could be seen through the sight holes on each side of the head-rest. The chin-rest was kept at the same position throughout, and the subject told to keep both head and chin always in contact with the restraints.

The subject was given the push-button for the bell to hold in his right hand, and for the buzzer in his left hand. The following instructions were repeated before each session:

"Always fixate the lower end of the stationary rod. When the moving rod moves from left-to-right (or right-to-left, depending on the experiment), I want you to judge whether it passes in front of or behind the stationary one. If it passes in front, ring the bell which is in your right hand. If it passes behind, ring the buzzer. If you are not sure, make a guess. If you make a mistake, ring three times, then repeat your response when I tell you. If the moving rod appears to jerk, or if anything else goes wrong, tell me immediately."

At the first session, the instructions were explained with diagrams, to ensure that the subjects understood what they were to do. A card summarising the instructions was kept in the subject's cubicle.

With the subject in position, the door was closed and the experiment begun. The first ten minutes were used for finding approximately the position of apparent equal distance; these results were not used in analysis, to allow for practice and adaptation effects. During the up-and-down experiments, it was found that the responses were just as stable after five minutes as after ten, and the pre-test time was reduced accordingly.

V.3 Constant stimulus experiments.

The approximate position of subjective equal distance (PSE) was found by a modified method of limits, in which the fixation rod was moved in 3 mm steps towards or away from the observer until the response was reversed. For example, the fixation rod was set at 209 mm, with the moving rod out of the field of view to the right or to the left, depending on the particular conditions to be used in the experiment. The lower rod was then made to traverse the field, and stopped either when it reached the other end of its path, or as soon as the subject made his response. If the response was a "bell", i.e., "moving rod in front of the stationary rod", the fixation rod was moved 3 mm closer. The new setting was never reached directly, but approached with a series of oscillations, so that the subject could not be aware of the magnitude and direction of the change.

EXPT. NO. 30AFP4 SUBJ. C.J. DATE: 24-2-70 ADAPT: 10 mins.

MOVING STIMULUS CENTRE: 150 cms RADIUS: 50 cms CAM: Pulley 7 Y = 2000 mms.

R.P.M. 1.5 SPEED = °/SEC = 7.85 CMS/SEC SPEED AT S = 2.25 °/SEC.

STATIONARY (REFERENCE) ROD: Y_0' = 2000 mms SCALE ZERO = 40.0 STIM. DIFF. ($\Delta Y'$) = vary

I_R = 384 cd/m² I_L = 384 cd/m² PRESENTATION: 85 +

BAFFLES: H = 20 ° = 35 cms. AT 200 cms V = 5 ° = cms. AT 50 cms

TIME: STARTED: 11:10 A.M. ENDED: 11.32

SCALE SETTING		RESPONSES	G GUESS EQUAL	f/n	P
39.5 Y' = 2005	0	L ~~||||~~ || Z ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ |||		7/30	.23
39.4 Y' = 2006	1	L ~~||||~~ |||| Z ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ |		9/30	.30
39.25 Y' = 2007.5	2	L ~~||||~~ ~~||||~~ ~~||||~~ ||| Z ~~||||~~ ~~||||~~ |||		18/31	.58
39.1 Y' = 2009	3	L ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ | Z ||||		26/30	.87
39.0 Y' = 2010	4	L ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ | Z ||||		26/30	.87
Y' =	5	L Z			
Y' =	6	L Z			

CODE: L = BELL = Moving rod closer than stationary rod. Z = BUZZER =

Fig. V.1. Record of a constant stimulus experiment (7/10 actual size). Details are explained in the text.

While the setting was being made, the lower rod was brought back to its starting position in readiness for the next observation.

At the end of the pre-test period, the approximate PSE was calculated, and five appropriate settings were selected. The step sizes were of the order of 2 or 3 mm., based on the results of the first several experiments.

The order of presentation for the main part of the experiment was determined according to a random sequence of the integers 1 to 5. This sequence was on 100 cards, turned over one at a time for each trial.

Each observation was made as described in the first paragraph of this section. The subject was told when the main experiment began. The results were recorded on a form as shown in Fig. V.1.

At the top of the form were entered details of the experimental conditions. "Adapt" refers to the pre-test period during which no results were recorded. "Moving stimulus centre" and "radius" are respectively the distance from the subject to the centre of curvature of the stimulus path, and the radius of the path.

"Cam" refers to an early version of the apparatus, and is not relevant.

"y" is the distance at which the moving target intersects the median plane, i.e., the distance to be compared with the fixation target position.

"y_o'" is the "zero" setting of the fixation rod. Invariably this was equal to y. "Scale zero" refers to the setting on the arbitrary scale corresponding to y'_o.

"R.P.M." and "Speed" describe the movement of the target. "Speed at S" is the angular velocity of the target referred to the observation distance. Strictly speaking, this is correct only for the infinitesimally small segment of path at the median plane, since the distance between subject and moving target continuously varied along the path. However, the measure is a convenient index of the velocity conditions.

"Stim. diff." is the step size used in the constant method. As in the example shown in Fig. V.1, step sizes were not always constant. It was found that more homogeneous results were obtained with smaller step sizes between the extreme stimuli than between the central ones.

"I_R" and "I_L" are the background luminances for the right and left eyes respectively. "Presentation" denotes the start of the sequence of 100 random integers, recorded in case there was any need for the analysis of sequential effects.

The remaining details are self-explanatory.

Provision was made on the form for up to seven stimulus values. In the first column was entered the scale setting corresponding to the required stimulus value y'. The "guess/equal" column was not used, being intended for

```
PROBIT ANALYSIS OF DATA SET NO.    9
   EXPERIMENT:        CMJ/30AFP6

LINEAR ANALYSIS

     STIMULUS  EXPOSURES  RESPONSES  EMPIRICAL  EXPECTED  EXPECTED
                                       PPTN       PPTN     PROBIT

       4.000      25.0        2.0      0.080      0.082     3.606
       5.000      25.0        4.0      0.160      0.191     4.124
       6.500      24.0       12.0      0.500      0.461     4.902
       8.000      25.0       20.0      0.800      0.752     5.680
       9.000      25.0       21.0      0.840      0.885     6.198


NUMBER OF ITERATIONS IN THE PROBIT ANALYSIS:      4

EQUATION OF THE PROBIT REGRESSION LINE:
                                           Y =     1.5321 +     0.5184X


**********    MEDIAN =         6.6893

>>>>>>>>>>    95% CONFIDENCE LIMITS:      7.1896 TO     6.1898

**********    STANDARD DEVIATION =      1.9289

              CHI  SQUARE =    1.0991 FOR  3.0 DEGREES OF FREEDOM.
```

Figure V.2. Print-out from a FORTRAN program used for probit analysis of experimental data. See Appendix B for details of the program.

three-alternative experimental designs. "f/n" is the ratio of "bell" responses to the number of trials, and "p" the resulting empirical probability. Thus, "p" is the empirical probability that the moving stimulus be judged nearer than the stationary rod.

A FORTRAN IV program was written to analyse the results by probit analysis (Finney, 1962). This program is described in detail in Appendix B. Briefly, it made use of a routine provided by IBM (IBM, 1966), modified so as to yield confidence intervals (at the 95% level) as well as the median, standard deviation, the probit equation, and chi squared. A typical output page is shown in Fig. V.2.

The median yielded by the probit analysis is the result of greatest interest, being the distance at which the fixation target has to be set for the moving target to pass apparently directly beneath it. In terms of the experimental design, the median is that position at which each of the two possible responses occur with equal frequency.

In the print-out, the result is in terms of the difference between the apparent distance, y', and the true distance, y. (See Figure V.5). This quantity, (y' - y), is the localisation error. It is positive when the apparent distance is greater than the real distance.

The 95% confidence interval is based on the t-ratio, and provides a means of testing the significance of

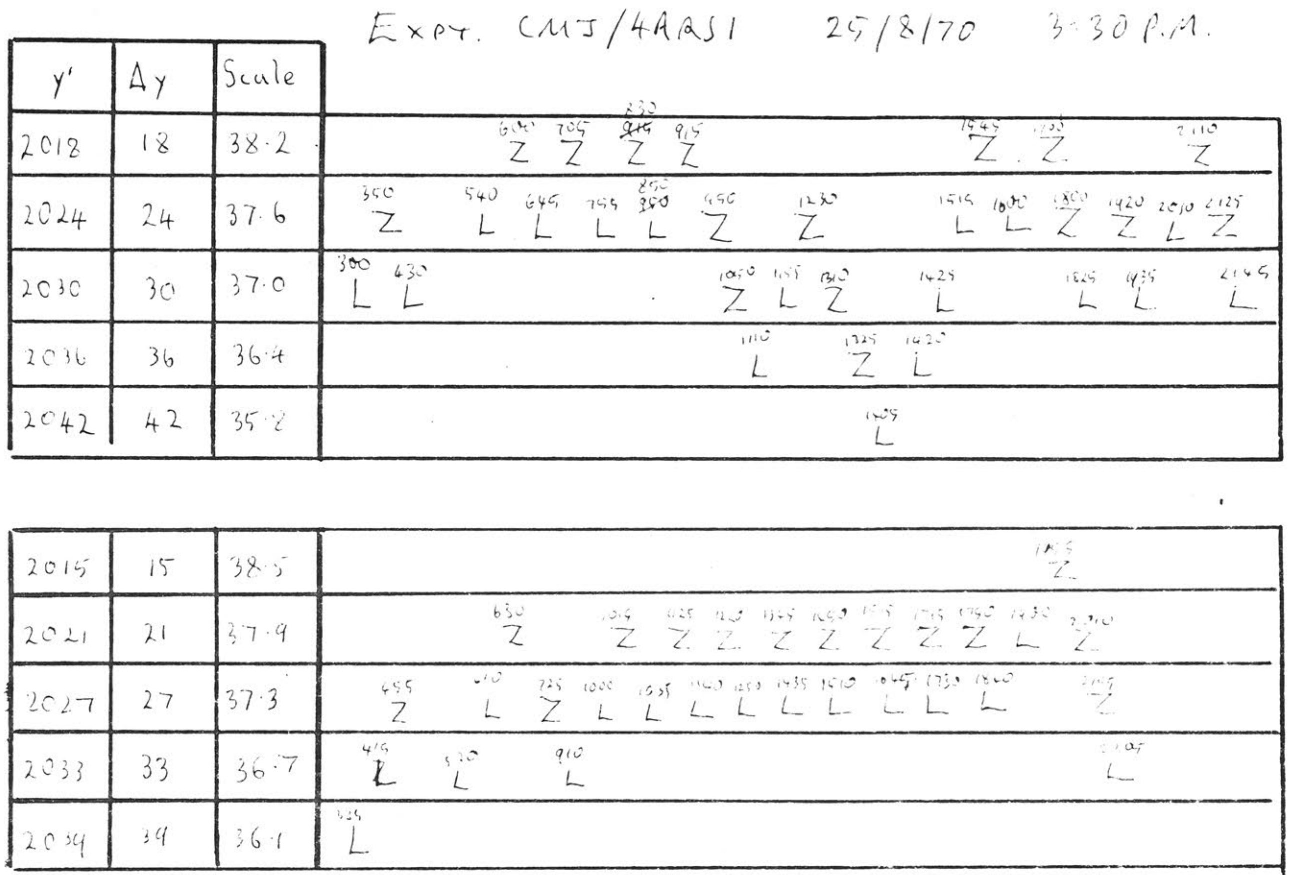


Fig. V.3. Protocol from a complementary concurrent series experiment. 'L' and 'Z' represent reports that the moving rod appeared in front of or behind the stationary rod. The numbers above each report are the time elapsed, in minutes and seconds, since the subject took up his position.

differences between medians. The chi squared value indicates the goodness of fit of the probit regression line.

V.4. <u>Up-and-down methods</u>.

The general procedure (section V.2) was the same as for the constant stimulus experiments. Identical instructions were given, and in fact the subjects were at no time aware that there had been a change of method.

The reasons for changing the method are given in the following chapter, while up-and-down methods are reviewed in Appendix A.

Initially, Cornsweet's double random staircase was used (Appendix A, Section A.7). The up-and-down transformed response rule (UDTR) of Wetherill & Levitt (1965) (Appendix A.10) was also tried, but finally Kappauf's concurrent complementary series method (Kappauf, 1967, 1969a) was found to be the most satisfactory.

Kappauf's method is a variant of Cornsweet's double staircase. Two series are used, but the stimulus levels in one series differ by half a step size from the levels in the other. The theory of the method is described in Appendix A.8.

A typical protocol from a CCS experiment is shown in Fig. V.3.

In the protocol of Fig. V.3, y', Δy, and "scale" have the same significance as in Section V.3. Once again, 'L' and 'Z' represent respectively reports that the moving

```
DATA SET NO. 411     CDN/20ARS2     4 JUNE 1970

INTERPUPILLARY DISTANCE:      62.50    MM.          TEST DISTANCE:      2000.00   MM.

SERIES 1:   36 OBSERVATIONS.      5 STIMULUS LEVELS AT STEPS OF     4.000 MM, BEGINNING AT    8.000.
          NUMBER OF SAMPLES ANALYZED:    1

DATA WERE CODED AS FOLLOWS:
   4   1   3   5   7  10   8   6   3   5   8   6   4   1   3   6   3   6   3   5   8   6   3   5   8
   6   3   6   3   5   8   6   3   6   3   6

    8.00: X            X
    4.00:O X     X    O X X X   X   X X   X X
    0.00:   X   O X O    O O X O X O O X O O O
   -4.00:    X O    O         O   O     O
   -8.00:     O

SERIES 2:   35 OBSERVATIONS.      5 STIMULUS LEVELS AT STEPS OF     4.000 MM, BEGINNING AT   10.000.
          NUMBER OF SAMPLES ANALYZED:    1

DATA WERE CODED AS FOLLOWS:
   8   5   7  10   8   5   8   5   8   6   3   6   4   1   3   5   7  10   8   5   8   6   3   5   8
   6   3   6   3   5   7  10   8   5   8

   10.00:             X
    6.00:          X O X        X   X X
    2.00: X   X X O O    X   X O X O O X   X
   -2.00:O X O O O       X O O    O     X O O
   -6.00:                  O             O

RESULTS FOR OBSERVATIONS      1 TO   36 IN SERIES 1;
                              1 TO   35 IN SERIES 2.
              MEAN   STANDARD        + OR -   ST.ERROR            ANGULAR VALUES
                      DEVIATION   95% LIMITS   OF MEAN              (SEC. ARC)

   SERIES 1   0.839     3.904                                     MEAN =   2.95 + -   4.56

   SERIES 2   0.941     4.830                                     ST. DEVN. =   14.253
 CONCURRENT
   SERIES     0.915     4.432        1.416      0.723

95% CONFIDENCE INTERVAL:  -0.501 TO   2.331
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

          **********
          END OF SET
```

Fig. V.4. Print out from a FORTRAN program for analysing data from concurrent complementary series experiments. See Appendix B for details of the program.

stimulus was in front of or behind the stationary one. ('L' and 'Z' were used for their mnemonic value: L = bell and Z = buzz).

The numbers over each response are the time (in minutes and seconds) elapsed since the subject took up his position and the dark-room door closed. In the example, the first five minutes constitute the pre-test (adaptation and practice) period. Only the results after this period were analysed.

Analysis was carried out with another FORTRAN IV computer program. Print-out from this program is shown in Fig. V.4; the program itself is discussed in Appendix B.2. The program yielded the mean of each series, and their standard deviations according to Kappauf's formula (Appendix A.5, equation A.3), as well as the mean and standard deviation for the combined series. It also gave a 95% confidence interval, based on the t-ratio and the standard error of the mean (Equation A.7 in appendix A.9). The localisation error (mean), 95% limits, and standard deviation were also given in seconds of arc, using the formula:

$$\eta = 206264.(2a).(y - y_o)/(y.y_o) \ldots \quad \ldots \quad \text{V.1}$$

in which 2a is the subject's interocular separation, y_o is the reference distance, and y is the second distance whose angular relationship with y_o is given by η. 206264 is a constant for converting from radians to seconds of arc (Chapter I.1 (b)).

The program could be made to analyse segments of the experiment as well as the whole series. In this way, variations of the localisation error with time could be studied and tested for significance.

V.5. Feedback in PSE experiments.

In psychophysical experiments, it is often desirable that the subject receive feedback during the experiment, that is, he is made aware of the accuracy of his performance after each response. This is particularly so if the results are to be interpreted in terms of decision theory (Swets, 1964).

The purpose of feedback is, basically, to ensure that the subject's judgment criteria remain the same during an experiment; sometimes, the criteria may be manipulated by changing the values associated with different decision outcomes. Such experiments yield information about the subject's "receiver operating characteristic" (ROC), the subject being considered as a signal detecting device.

In straightforward signal detection experiments, the introduction of feedback presents no problems; the signal is either there or not there, and the response can only be right or wrong. In PSE experiments, however, the question of whether a response is right or wrong is not easily resolved. For example, in the experiments described here, the subjects almost invariably localised the moving rod away from its true position. Physically, their responses

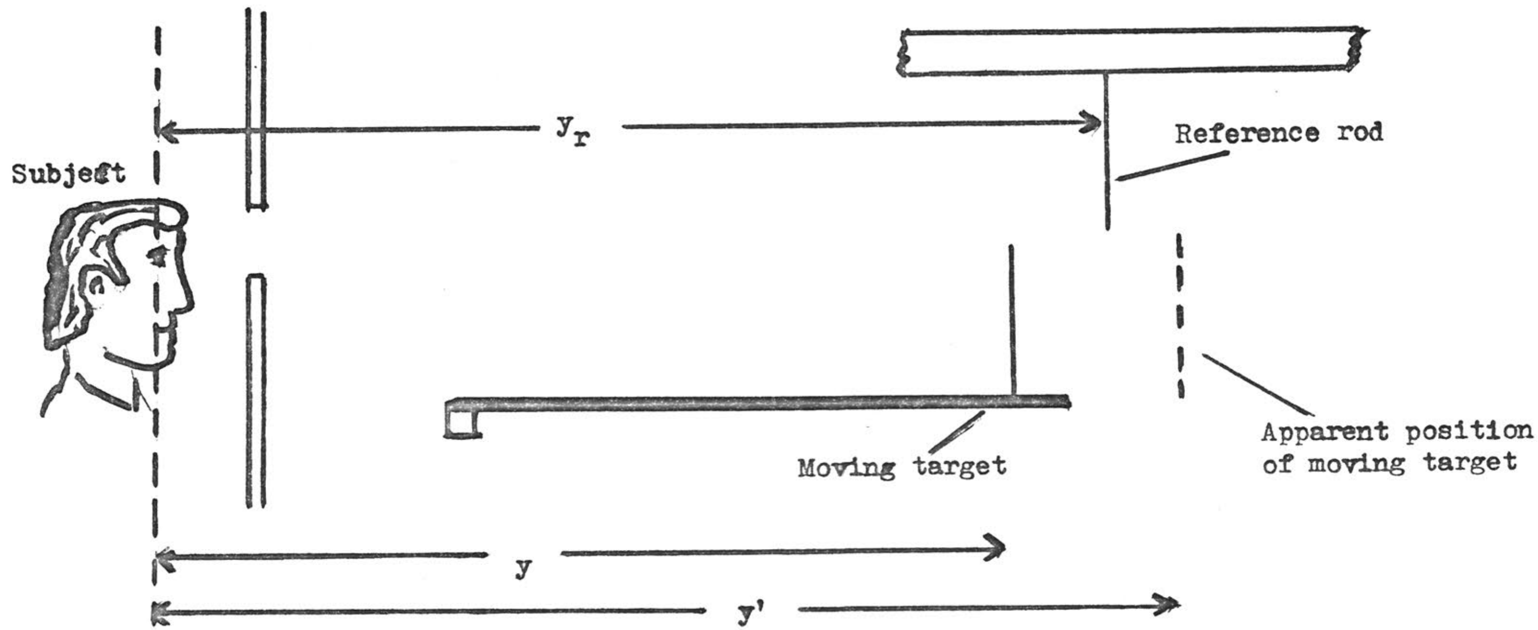


Fig. V.5. Illustrating a signal detection model of the localisation error experiments. y is the actual distance of the distal point of the path of movement, and y' its apparent distance. y_r is the setting of the reference rod for a given trial. The "signal" is $(y' - y_r)$, not $(y_r - y)$.

were nearly always wrong, yet in terms of their subjective localisation some proportion of responses at each stimulus level were correct. That is, the "signal" in these experiments is the distance between the current setting of the reference rod and the apparent position of the moving target, regardless of the latter's true position (see Fig. V.5).

In Fig. V.5, the localisation error is $(y' - y)$, the quantity sought by the experiment. The "signal" is not $(y_r - y)$, but $(y_r - y')$; thus, the correct response in Fig. V.5 would be "moving target behind stationary rod". Since the determination of y' is the purpose of the experiment, there seems to be no valid way of introducing feedback. However, in up-and-down type experiments, it is possible to make approximate estimates during the course of an experiment of the quantity being investigated. On this basis, an arbitrary system for determining the accuracy of a response was devised.

In the system, no feed-back is given until five reversals have occurred, when an approximate estimate of y' is made. The next several responses are declared "right" or "wrong" on the basis of this estimation. There are two possible difficulties here. The initial estimate may be erroneous, or there may be an actual change in y' during the experiments. In both cases, the subject may be forced to change his criteria or even modify his responses in

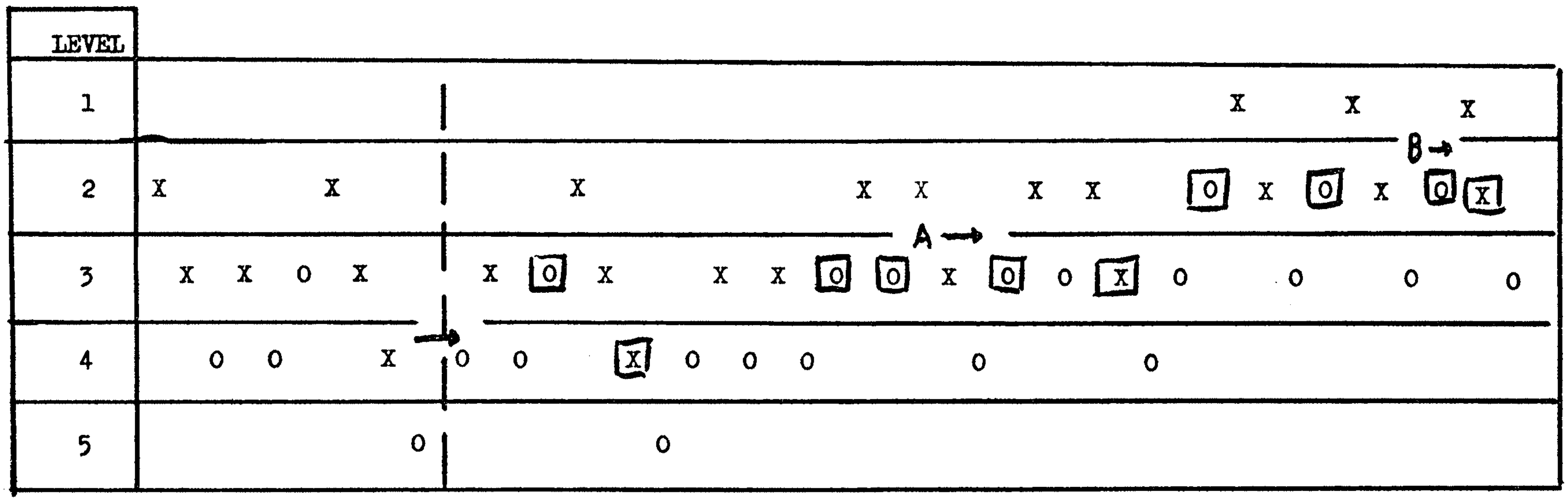


Fig. V.6. Feedback in an up-and-down PSE experiment. For the first five reversals (before the dashed line), no feedback was given. Then the PSE was estimated as being between levels 3 and 4. Subsequent responses were declared "right" or "wrong" according to this estimate. "Wrong" responses are signified by boxes. Eventually, three out of five successive responses at stimulus 3 were "wrong", and the estimate of the PSE was changed, as indicated by the arrow at 'A'. Another change was made at 'B'.

order to improve his success rate. To alleviate these problems, the last five responses at each level are taken into account.

The method is illustrated in Fig. V.6. After the first five runs (ten trials), the PSE was estimated as being between stimulus levels 3 and 4. An '0' response at levels 1, 2 or 3 was declared "wrong", as was an 'X' response at levels 4 or 5. However, of the ninth to thirteenth responses at level 3, three out of the five were '0', indicating that the PSE should be estimated as being between levels 2 and 3. Subsequent '0' responses at level 3 were therefore declared correct.

In applying the method to some of the experiments described here, no values were assigned to the different decision outcomes. The aim was simply to assist the subject in maintaining as high a success rate as possible. As will be discussed later, the effectiveness of the method was equivocal, although the subject with whom it was used reported she felt more reliable when feedback was applied than without it.

The possibilities of the method in a signal detection context have yet to be explored. Some pilot studies have indicated that it may provide a means of obtaining ROC curves in PSE type experiments.

V.6. Subjects.

Three optometry undergraduates were used as subjects for the experiments. In the second year of the optometry course, students gain some experience as subjects in psychophysical type experiments; one of the present subjects, CDN, had already completed this stage of the course, while the other two were in second year during the study, which lasted from February to October, 1970. Initially, all subjects were unaware of the theoretical background, but they gained some knowledge during the year.

Subjects were told that they may have a localisation error, but its direction was not revealed until after the experiments were completed.

No payment was made for experiments prior to July, 1970, after which subjects were paid $1.00 per experiment. This had no apparent effect on the results, but served to encourage the subjects to attend frequently and punctually.

<u>Subject CDN</u>: Male, aged 21. Visual acutiy: 6/4.5 in each eye. No significant refractive errors. Interpupillary distance = 62.5 mm. Phorias: 3 EXO at 500 cms; 8 EXO at 40 cms. No vertical phorias. Other clinical findings indicated that this subject had a low degree of <u>convergence weakness</u>, which could be manifested as an EXO fixation disparity (Ogle <u>et al</u>, 1966).

<u>Subject CMJ</u>: Female, aged 20 years. Visual acuity: 6/4.5 in each eye. No significant refractive errors.

Interpupillary distance = 61.0 mm.

Phorias: ½ EXO at 500 cms; 1 EXO at 40 cms.

0.25 left hyperphoria.

No indications of oculomotor imbalance.

Subject BK: Female, aged 19 years.

Visual acutiy (February 1970): Right eye: 6/12.

Left eye: 6/15.

Binocular: 6/9.

Interpupillary distance = 59.5 mm.

Refraction: + 0.75/-0.25 x 120 + 0.75/-0.25 x 100

Phorias: 2 ESO at 500 cms; 9 ESO at 40 cms.

No vertical phorias.

Other findings indicated the presence of convergence excess, which could result in an ESO fixation disparity.

The low visual acuity was of unknown origin, there being no apparent fundus defects. Fixation was observed directly with an ophthalmoscope and found to be central and steady. Visual acuity could not be improved by lenses. Initially, it was thought that this subject should be rejected, but early experiments showed that stereoscopic acuity was better than 9 seconds of arc, as measured on the apparatus used as a static two-rod test.

During the year, visual acuity improved, and by October it was 6/4.5 in each eye.

A correction for the refractive error given above was worn throughout.

V.7. Experimental sessions.

Experimental sessions lasted for not more than one hour. On days when more than on experiment was run on a subject, there was a rest period of at least fifteen minutes between sessions.

In the later up-and-down experiments, when background luminance was varied by introducing filters in the viewing aperture, each session consisted of two 25 minute runs, with a brief break in between when the filters were being changed.

Because of the subjects' other commitments, experiments could not always be done at the same time of day. Generally, most of CDN's sessions began at 8.30 a.m., BK's at 9.30 a.m., and CMJ's at 3.30 p.m.

-o-O-o-

CODE	GAP (degrees)	RADIUS OF PATH (cms)	CENTRE OF CURVATURE (cms)	ANGULAR VELOCITY AT SUBJECT (deg. per second)
20	0.3	50	150	4.05
2	0.1	50	150	4.05
3	0.1	50	150	2.25
4	0.1	100	100	4.50
5	0.1	100	100	2.50

LUMINANCE

A: 384 cd/m^2

B: 35.9 $cd.m^2$

DIRECTION OF MOVEMENT

R: left-to-right

F: right-to-left

EXPERIMENTAL METHOD

P: probit analysis

S: up-and-down

TABLE VI.1. Codes used for identifying experimental conditions.

Facing p. VI.1

CHAPTER VI

THE EXPERIMENTS AND THEIR RESULTS

VI.1 Identification of experimental conditions.

Each experiment is identified by the subject's initials, followed by a code which gives the experimental conditions, the method, and the number for the particular set of conditions.

For example, a typical experiment name is CDN/3ARS3.

CDN are the subject's initials.

The first digit in the code refers to the experimental conditions as set out in Table VI.1.

The second character, 'A' or 'B', identifies the background luminance. 'A' is 384 cd/m^2; 'B' is 35.9 cd/m^2.

The third character gives the direction of movement. 'R' is from left to right; 'F' is from right to left. ('R' and 'F' were settings on the controls for the moving target; the letters were retained to avoid errors in recording.)

The fourth character, 'P' or 'S', refers to the experimental method. 'P' is a constant stimulus (probit analysis) experiment, 'S' is an up-and-down (staircase or sequential) experiment.

Thus, experiment CDN/3ARS3 is one in which the subject was CDN, the experimental conditions were those of conditions 3 in Table VI.1, the background luminance was 384 cd/m^2, and an up-and-down method was used. It was

EXPT.	DATE	LOCALISATION ERROR (sec arc)	95% LIMITS (sec arc)	THRESHOLD (sec arc)	CHI SQUARED /df	p
30ARP1 30ARP2	11/2 12/2	21.51	1.12	7.62	.83/5	.95-.98
30ARP3	16/2	13.99	1.72	8.25	2.43/3	.30-.50
30ARP4	17/2	11.70	1.57	7.64	.71/3	.80-.90
30ARP5	20/3	16.96	2.20	9.27	2.54/3	.30-.50
30AFP1	18/2	Discarded - inappropriate choice of steps				
30AFP2	20/2	27.00	4.01	7.78	7.21/3	. .50
30AFP3	23/2	20.44	3.25	8.37	3.54/3	. .30
30AFP4	24/2	21.79	1.61	7.50	1.50/3	.50-.70
30AFP5	25/2	21.40	1.56	5.73	1.65/3	.50-.70
30AFP6	26/2	20.97	1.51	6.06	1.09/3	.70-.80
30AFP7	27/2	19.28	2.72	5.95	3.73/3	.20-.30
20ARP1	17/4	16.12	2.10	8.76	2.43/3	.30-.50
20AFP1	24/4	16.67	1.75	7.23	0.78/3	.80-.90

TABLE VI.2. Results of constant stimulus experiments, subject CMJ. See caption to Table VI.3.

the third experiment under all these conditions.

VI.2 <u>Constant stimulus experiments</u>.

The data of constant stimulus experiments were analysed by probit analysis (Finney, 1962), using the computer program described in Appendix B. The results are given in Tables VI.2, VI.3, and VI.4.

The results in the tables are given in seconds of arc, the angular measure being used to facilitate comparison with Lit's results and with other binocular functions such as static stereoscopic acuity and fixation disparity. The relationship between angular and linear measures is given by:

$$\eta = 206265.2a.\ (y' - y)/(y.y')$$

where 2a is the subjects interocular separation, and y and y' are the two distances under consideration. 206265 is a factor for converting radians to seconds of arc (see Chapter I.1).

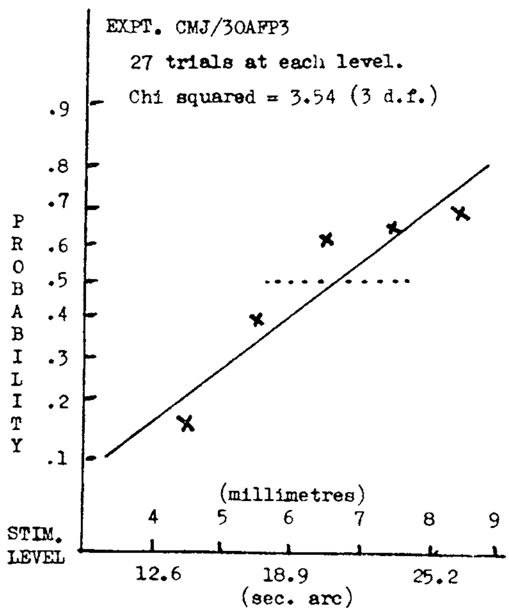
As a rule of thumb, for y = 200 cms. and 2a = 6 cms, a change of 1 mm in (y'- y) corresponds to a change of approximately 3 mm in η. Thus an angular localisation error of +21 sec arc means that the moving rod was localised about 3 mm beyond its true position.

Most of the constant stimulus experiments were done with subject CMJ, the method being changed shortly after subjects BK and CDN came into the study.

In Table VI.2, the results obtained on the first two days, experiments 30ARP1 and 30ARP2, were pooled to give a well-fitting straight line, the probability that deviations were due to chance being better than .95, as indicated in the last column. These data gave a localisation error of 21.51 sec arc, definitely demonstrating the presence of an error under the experimental conditions. However, the next two sessions gave results significantly different (at the 95% level) from the first, suggesting the possibility of variations between days. This was confirmed by expt. 30ARP5, which a result different from each of the previous results was obtained. Because of the day-to-day variation, the practice of pooling data was discontinued.

A similar variation was found in the results for the 30AFP experiments, although here there was more stability.

The day-to-day variation makes comparisons difficult, if not impossible. For example, in conditions 30ARP, the stimulus was moving from left to right, and the mean of the four localisation errors is 16.04 sec arc. For 30AFP, the conditions were the same except that the rod was moving from right to left, and the mean of the six results is 21.81 sec arc. The _t_-test shows that the difference between the two means is significant at the 5% level, although the use of a _t_-test for such small samples is highly questionable.



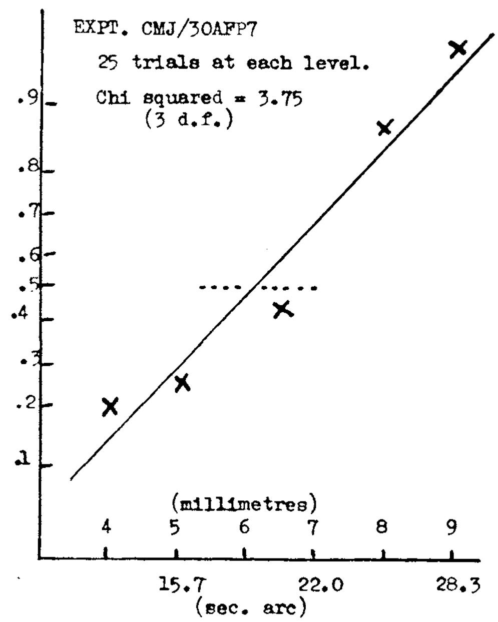


Fig. VI.1. Maximum likelihood regression line fitted to data of expt. CMJ/30AFP3.

Fig. VI.2. Maximum likelihood regression line fitted to data of expt. CMJ/30AFP7.

In both figures, empirical points are denoted by 'X'.
The dotted horizontal lines (:...) indicate the mean and the 95% confidence interval.
The ordinate is the probability of the moving target being reported in front of the stationary rod when the latter is at the position given on the abscissa.

As well as a day-to-day variation, there seemed to be changes in the localisation error during an experiment. This is reflected in the high chi squared values obtained on many occasions. Transformations of the stimulus levels from a linear scale to angular, tangent, log linear, log angular, log tangent, or log reciprocal scales did not improve the fit of the regression line.

As examples, the probit regression line for experiments CMJ/30AFP3 and CMJ/30AFP7 are shown in Figures VI.1 and VI.2.

The inconsistency of the deviations can be seen from the two examples. Any transformation leading to a better fit for the data of CMJ/30AFP3 would not be appropriate for those of CMJ/30AFP7.

Also as indicated in Fig. VI.1 and VI.2, different step sizes were tried, without improving the homogeneity of the results.

The other two subjects gave equally heterogeneous data (see Tables VI.3 and VI.4). The results could have been due to poor observation and reporting on the part of the subjects, but it was felt that there was the possibility of some change in the localisation function during the experiments. With this in mind, an up-and-down ("staircase") type of experimental design was introduced.

EXPT.	DATE	LOCALISATION ERROR (sec arc)	95% LIMITS (sec arc)	THRESHOLD (sec arc)	CHI-SQUARED /df	p
30ARP1	19/3	14.83	2.44	10.63	5.04/5	.30-.50
30ARP2	2/4	14.21	2.14	8.54	0.57/3	.90-.95
30AFP1	23/4	33.03	1.98	7.21	1.75/3	.50-.70
20ARP1	7/5	19.12	5.06	8.09	6.56/3	.05-.10

Table VI.3. Constant stimulus results for subject BK.

95% limits were computed according to Finney's method, as were the chi squared values (Finney, 1962, p.63 and p. 53). Given with each chi squared value is the number of degrees of freedom used for testing the goodness of fit of the probit regression line. 'p' in the last column is the probability that deviations of empirical data from the line were due to chance.
'THRESHOLD' is the reciprocal of the probit regression coefficient, i.e., the standard deviation of the hypothesized distribution of observations.

EXPT.	DATE	LOCALISATION ERROR (sec arc)	95% LIMITS (sec arc)	THRESHOLD (sec arc)	CHI-SQUARED /df	p
30ARP1	2/4	Discarded - inappropriate choice of steps				
30ARP2	23/4	8.72	2.5	11.09	2.77/3	.30-.50
20ARP1	7/5	1.38	3.2	18.25	3.07/4	.30-.50

TABLE VI.4. Constant stimulus results for subject CDN.

See caption to Table VI.3.

VI.3 Up-and-down experiments: Presentation of Results.

Up-and-down methods are discussed fully in Appendix A. For the bulk of the experiments, Kappauf's method of complementary concurrent series (CCS) (see Appendix A.8) was used, but the first few experiments were done with Cornsweet's double random staircase method (Appendix A.7).

A computer program was written to analyse the up-and-down data (see Appendix B.2). This program gives the mean and standard deviation of the hypothetical distribution from which the data are drawn; ideally, these results should be the same as those of a probit analysis method. The program can also be made to examine segments of the data, enabling variations with time to be examined. Fiducial limits are computed to give 95% confidence intervals.

The results are presented at the end of the chapter in three forms. In Tables VI.5, VI.10, and VI.15 are given all the results for the three subjects CMJ, BK and CDN respectively, arranged in chronological order. The first two columns give the experiment identification (see Table VI.1) and the date. The "mean" is the localisation error obtained by the experiment, and the "95% LIMIT" and "THRESHOLD" are quantities calculated as explained in the Appendix, A.5 and A.9. "THRESHOLD" is actually the standard deviation of the hypothesised distribution of responses sampled by the experiment. All results are in seconds of arc.

In the "REMARKS" column, changes of method and step-size are indicated, as well as other comments from the experimenter's diary. For some experiments, more than one set of entries are presented. On these occasions, there was an apparent change in the localisation error during the experiment. Changes were accepted as significant only if the confidence intervals derived from the 95% limits did not overlap. The time intervals within the session which yielded the different results are indicated in the "REMARKS" column.

Following each of the main Tables are sets of four tables in which the results are grouped according to experimental conditions. Experiments under conditions 20 have been omitted; these were done with a gap of $.3^{\circ}$ between the two rods (see Section IV.2 (c)), while for most of the experiments the gap was 0.1°.

Finally, Tables VI.20 to VI.22 give the means of the results for each set of conditions. Where there was a change in localisation error, the two results have been used to give the mean, rather than using the result for the whole session. For example, for conditions CMJ/2AR (see Table VI.6), there were four experiments, and at two of these experiments a change in the localisation error occurred. Thus the corresponding entries in Table VI.20 are the means of six results obtained from four sessions, indicated by "6/4" in column "N" of Table VI.20.

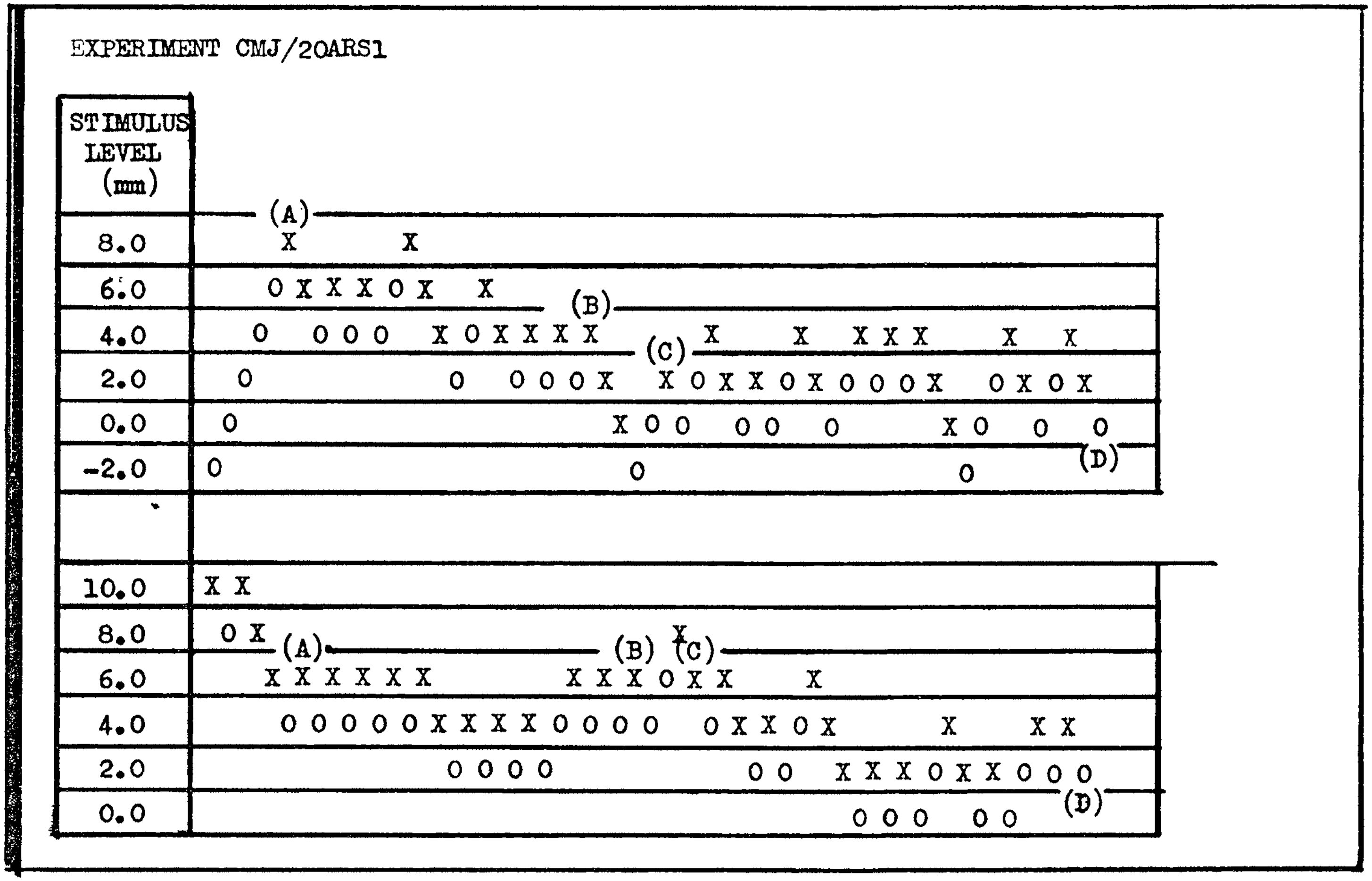


Fig. VI.3. Data from an up-and-down (homogeneous series) experiment. 'X' indicates that at the stimulus level indicated, the moving rod was reported "in front" of the stationary rod. The two series were interleaved at random during the experiment.

VI.4 Changes of localisation error within sessions.

The first up-and-down experiment with subject CMJ showed that the localisation error could in fact change during a forty minute experimental session. The data from this experiment, CMJ/20ARS1, are shown in Fig. VI.3.

Observations made during the first five minutes of the session were omitted from the analysis, which began with the observations marked 'A' in each series. At 'B', the time elapsed was 18 minutes, at 'C' it was 21 minutes, and at 'D' it was 40 minutes.

The data were analysed over the whole session (A to D, 35 minutes) and over the two periods A-B (13 minutes) and C-D (19 minutes). The three minute interval when the observations began to change was omitted from the last two analyses.

As shown in Table VI.5, (Expt. 20ARS1), the two periods gave significantly different results. For the whole 35 minute period, the localisation error was 9.54 sec arc, with a threshold (standard deviation) of 8.71". For the two periods 5 to 18 minutes and 21 to 40 minutes, the error changed from 13.51" ± 1.99 to 6.82" ± 1.95.

In an up-and-down experiment, such a change could be an artefact, perhaps the result of using too small a step size. However, in the present experiment, the step size was in the optimum range of 0.5 to 2.0 times the standard distribution (see Appendix A.3). Furthermore, one of the advantages of using two concurrent up-and-down series is that artefacts of this kind can be detected, depending on whether or not the two series changed in the same way and at the same time. In experiment CMJ/20ARS1, for the first period, the two series analysed separately gave means of 13.0" and 12.8"; for the second period, the means were 5.4" and 7.5". It is not likely that such close agreement could have occurred by chance, and it seems safe to accept the significance of the difference between the two localisation errors as indicated by the 95% limits.

All within-session variations were studied in this way, and no examples were found of series drifting in opposite directions.

The following points arise from an examination of the results:

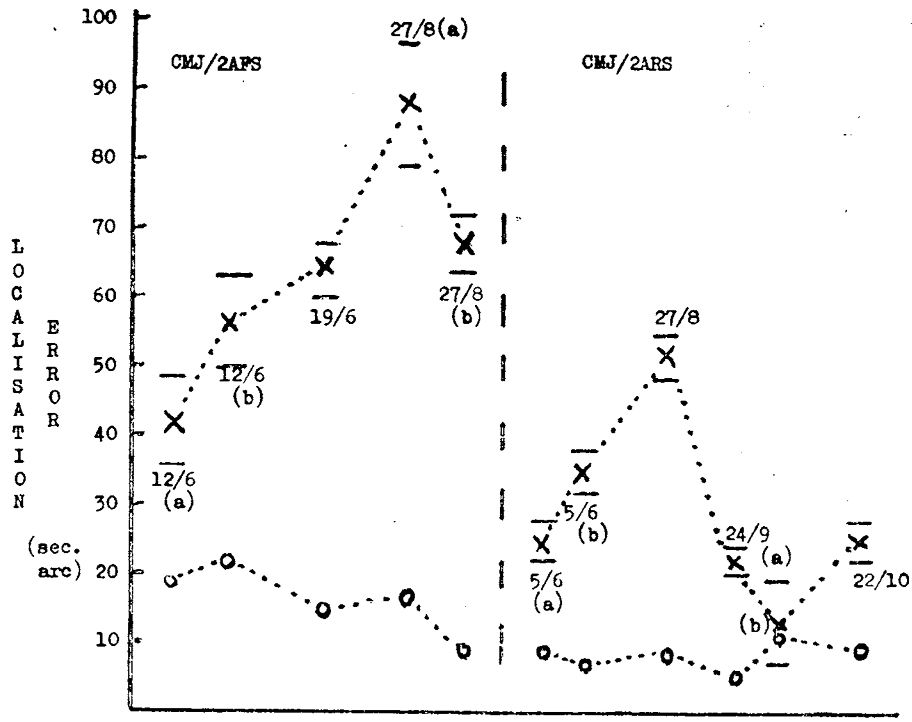
(a) Of the 43 experiments recorded for CMJ, the localisation error changed during 10 sessions. For subject BK, in 40 sessions a change was observed 7 times. For subject CDN, a change was found only twice out of 43 sessions. The subsequent remarks apply mainly to subjects CMJ and BK, although it may be noted that on many occasions CDN showed changes which were almost but not quite significant at the 95% level.

(b) Changes occurred during 20 minute sessions as well as in longer sessions. As a rule, however, the change occurred 15 to 20 minutes after the subject entered the apparatus.

(c) The localisation error did not always change in the same direction for CMJ. For example, in experiment CMJ/2ARS1, the error changed from 25.02" to 35.61". Under the same conditions 10 weeks later (CMJ/2ARS3), the error changed from 21.79" to 12.87". An increase was observed in 6 of the 10 changes. For subject BK, the localisation error, when it changed, always increased from the first to the second half of the experiment. On the two occasions when a change was observed with subject CDN, it was in opposite directions.

(d) Changes occurred more frequently during months 5 to 7 of the studies, than during the later months. It should be noted however that from 27/8, each experiment lasted only 25 minutes, so that changes over longer time intervals may have been undetected.

(e) There was no systematic change in the threshold with a change in localisation error. On 5 occasions, CMJ's threshold changed markedly. The greatest change was in experiment CMJ/3ARS3, when an increase in localisation error from 84.82" to 103.91" was associated with a <u>decrease</u> of the threshold from 40.05" to 15.48". BK's threshold tended to increase slightly with the increase in localisation error.



Radius = 50 cms   Velocity = 4.05°/sec

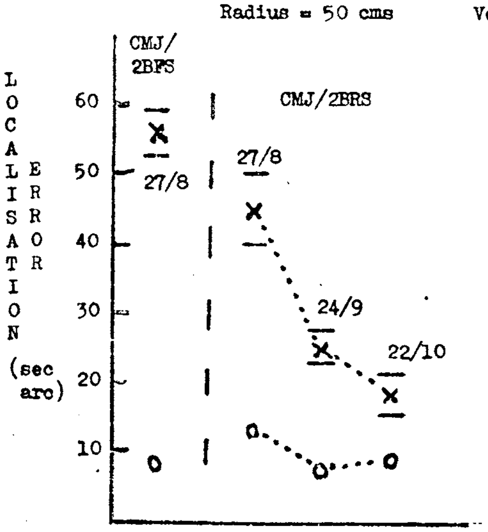


Fig. VI.4. Results for CMJ, conditions 2A (above) and 2B (left). See Table VI.1 for explanation of condition codes. Data from Table VI.6.
Each 'X' is the localisation error obtained on the date shown. The small horizontal lines are the 95% confidence limits; the circles are the thresholds.
(a) and (b) after a date indicate a significant within-session change in localisation error.

VI.5 Day to day variations in localisation error.

For all three subjects, the localisation error for a given set of conditions changed markedly from one experiment to another. Once again CMJ's results were the most variable.

In Fig. VI.4, VI.5, VI.6, and VI.7, the results for CMJ are presented graphically. These figures are plots of the data in Tables VI.6 to VI.9. Each 'X' is the localisation error obtained on the date indicated in the figure. The small horizontal lines show the 95% limits, and the circles represent the thresholds. Where two results were obtained from one session, they are indicated by (a) and (b).

The discussion now is concerned mainly with those conditions for which data were obtained on at least two separate days.

In the earlier experiments (constant stimulus method, Table VI.2) CMJ's localisation errors were of the order of 10 to 30 sec arc. It is not likely that the change to up-and-down methods affected the results; for example, the last constant stimulus experiment with conditions 20AF gave a result very similar to that of the first up-and-down experiment (experiment 20AFS1 in Table VI.5).

Nor is it likely that the alteration to the gap between the two rods had any serious effect. Experiment 20ARS3, with the 0.3° gap, gave results quite comparable

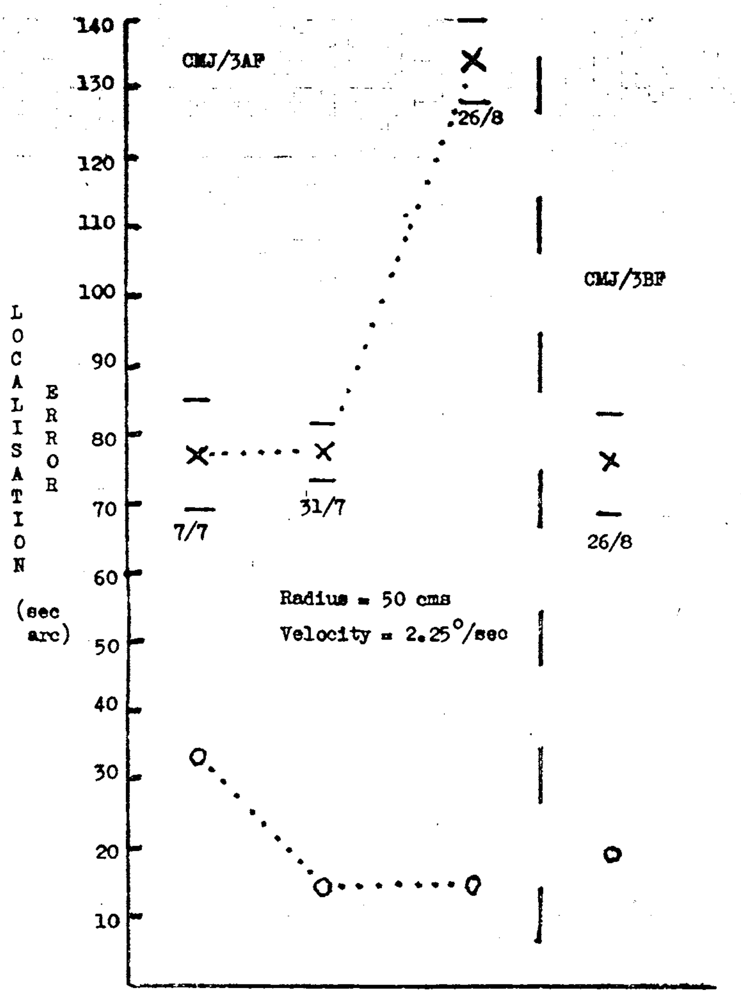


Fig. VI.5 (part A). Results for CMJ, conditions 3AF and 3BF. See caption to Figure VI.4 for explanation. Data from Table VI.7.

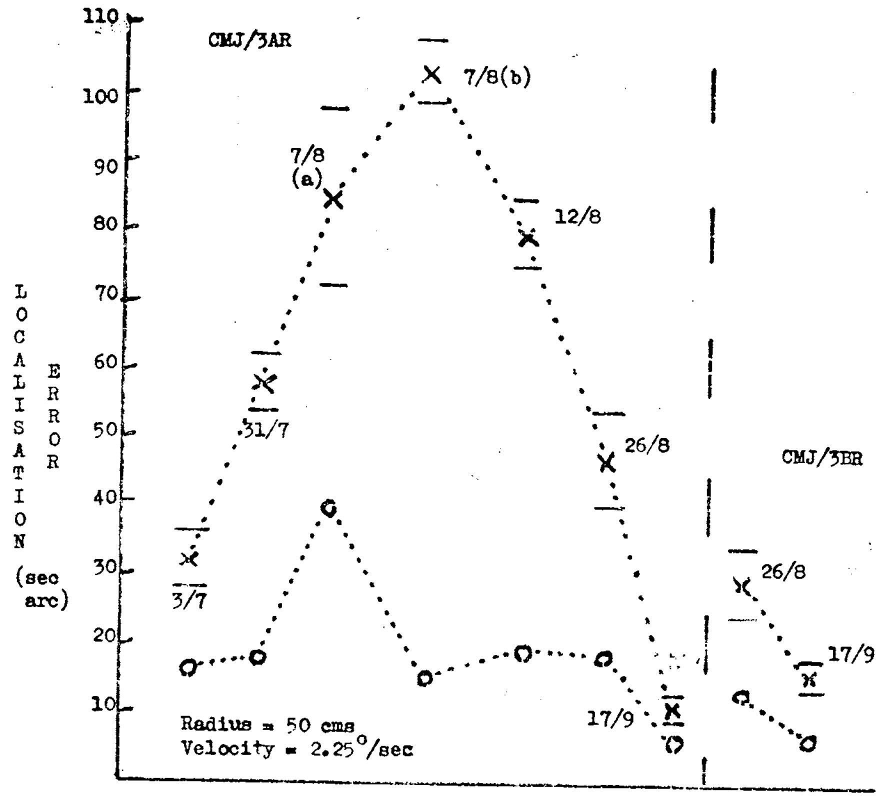


Fig. VI.5 (part B). Results for CMJ, conditions 3AR and 3BR. See caption to Fig. VI.4 for explanation. Data from Table VI.7.

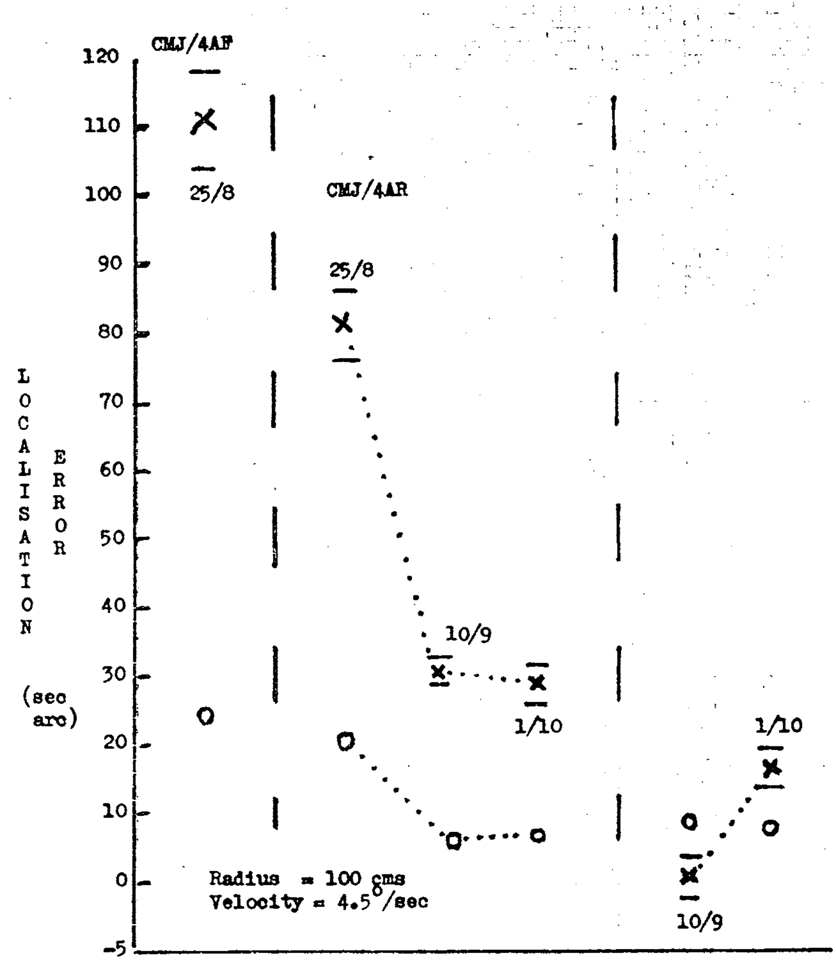


Fig. VI.6. Results for CMJ, conditions 4AF, 4AR, and 4BR. Taken from Table VI.8. See caption to Fig. VI.4 for explanation.

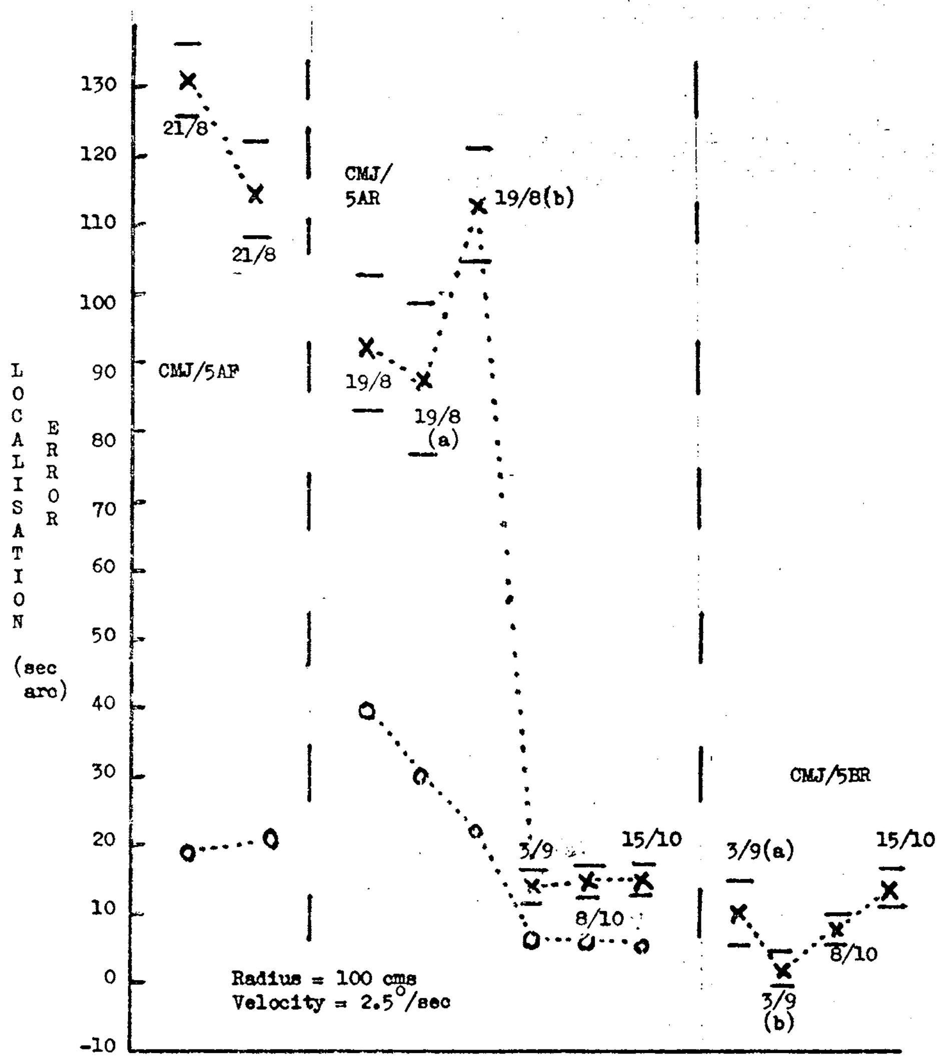


Fig. VI.7. Results for CMJ, conditions 5AF, 5AR, and 5BR. Taken from Table VI.9. See caption to Fig. VI.4 for explanation. For clarity, thresholds for the 5BR results are not shown. They were respectively 10.27, 4.55, 6.49, and 7.38 sec arc.

with those of experiment 2ARS1, where conditions were the same except for the smaller 0.1° gap.

The trend towards larger localisation errors began with experiment CMJ/20ARS2 on 18/5, when the error was, overall, about three times that of 20ARS1 ten days earlier. (20ARS2 was an experiment in which head position was purposely varied; this will be discussed later, but the results were consistent with the following 20ARS3). At the same time, there was an increase in the threshold, but not to the extent one would expect if the changes were due to extraneous factors such as inattention.

The localisation errors continued to increase, regardless of stimulus conditions, up to and including the experiments on 27/8. During this time, the thresholds did not show a related trend. For example, for conditions 2AF (Fig. VI.4), there was a trend for the threshold to decrease while the localisation error increased; in 2AR (Fig. VI.4) the threshold was fairly constant throughout.

In an attempt to stabilise CMJ's performance, the feedback procedure discussed in Section V.5 was introduced on 19/8. There was no immediate effect. However, after 27/8, the localisation errors began to decrease, and by the end of the investigation CMJ was giving results comparable to those at the beginning.

A possible reason for the gross changes in CMJ's localisation error was discovered during an interview with the subject at the end of the study. She admitted that she did not always fixate steadily the end of the stationary rod, but often watched the moving rod as it traversed the field. For the last few weeks, she again heeded her instructions and fixated as requested. This point will be discussed more fully in Chapter VII.2 (d).

The results for subject BK are plotted in Figs. VI.8, VI.9, VI.10, and VI.11. Here too significant changes can be seen in the localisation error from session to session, although not over as wide a range as in the case of subject CMJ.

The tendency was for BK's error to increase throughout the study. In conditions 2AR and 2BR, the lowest thresholds were obtained when the localisation error was highest (Fig. VI.8 (c) and (d)); under the other conditions, occasionally the threshold increased with the error, but in general thresholds remained fairly constant.

Subject CDN (Figs. VI.12, VI.13, VI.14. VI.15) was the only subject of the three to demonstrate a negative localisation error, i.e., he localised the moving rod closer to him than its actual distance. His results were also the least variable; nevertheless, significant differences from one day to another were obtained. Again, there is no clear relationship between variations in threshold and variations in localisation error.

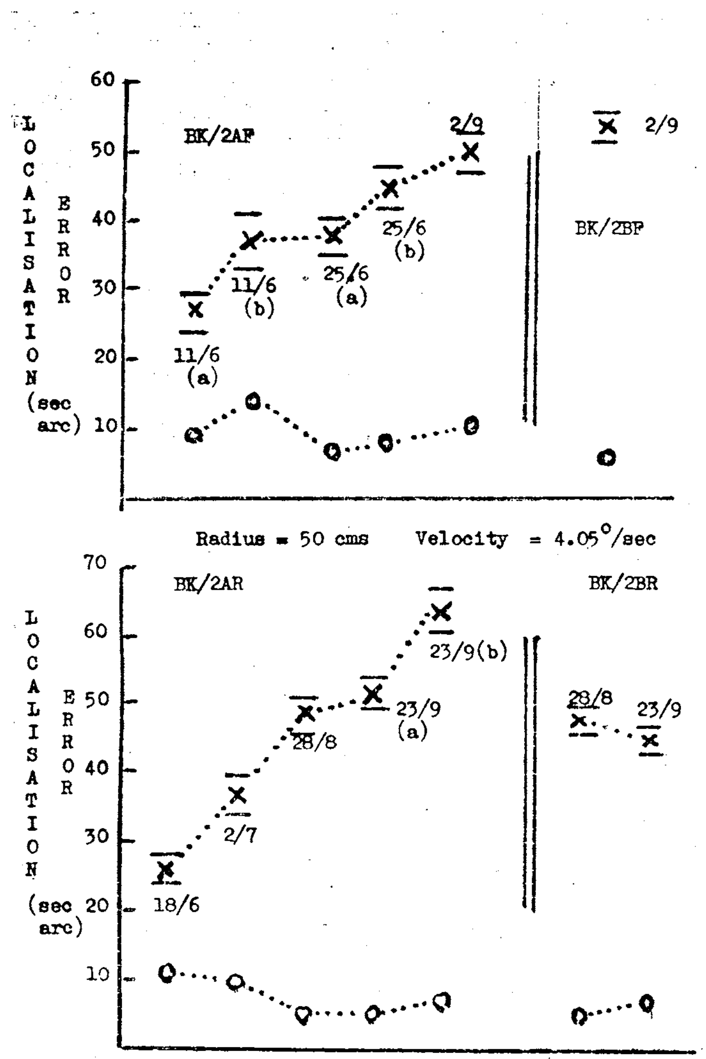


Fig. VI.8. Results for subject BK, conditions 2AF and 2BF (top graph), and 2AR and 2BR (lower graph. From Table VI.11.
See caption to Fig. VI.4 for explanation.

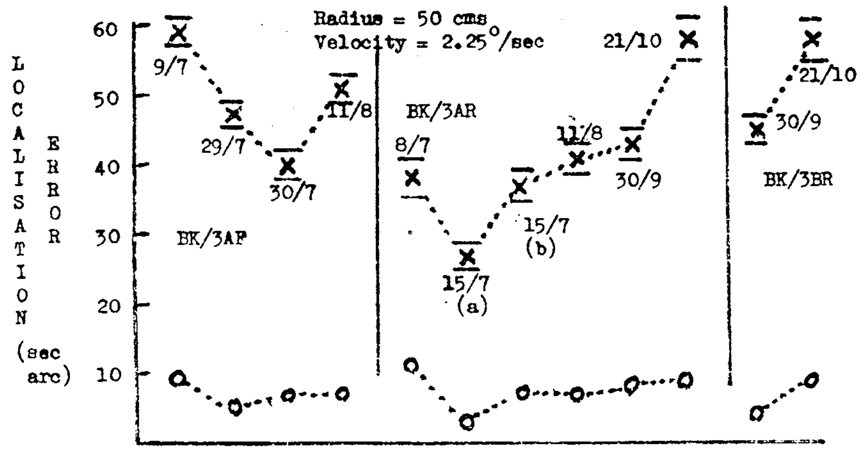


Fig. VI.9. Results for subject BK, conditions 3AF, 3AR, 3BR. From Table (VI.12).

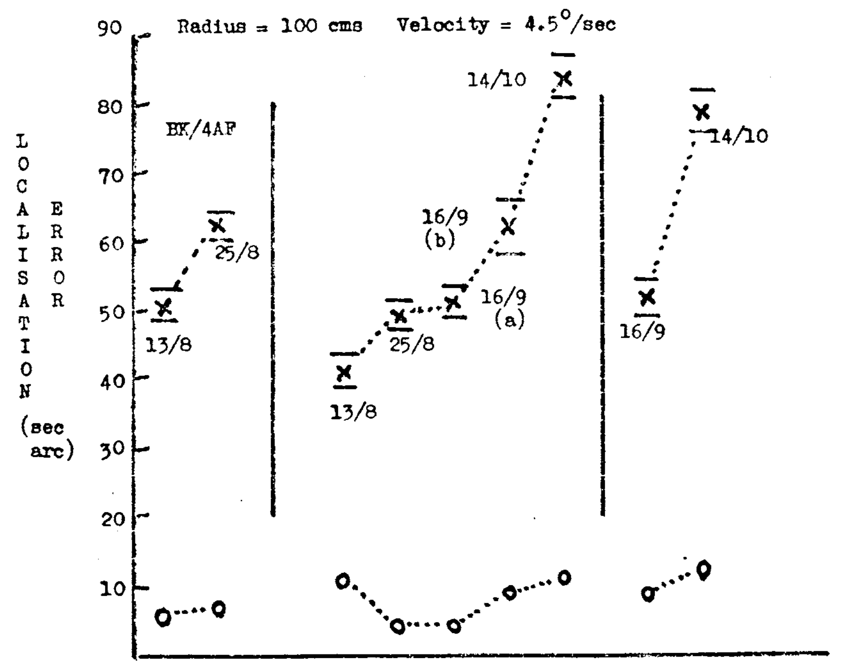


Fig. VI.10. Results for subject BK, conditions 3AF, 3AR, 3BR. From Table VI.13. See caption to Fig. VI.4.

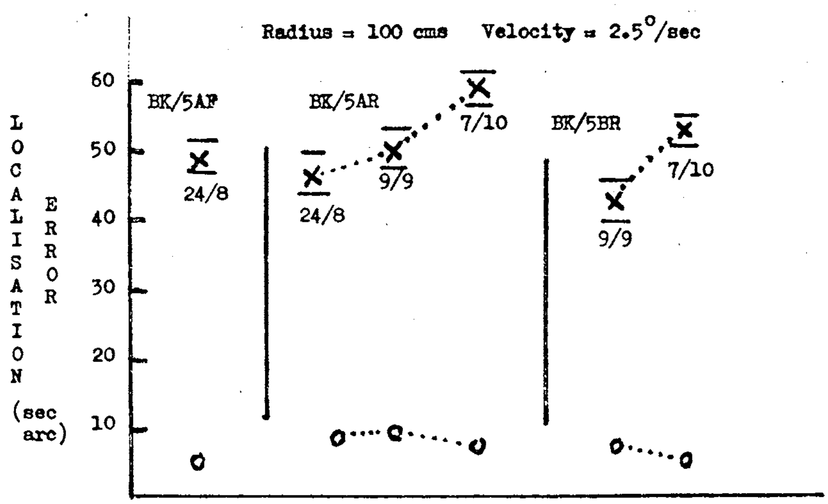


Figure VI.11. Results for subject BK, conditions 5AF, 5AR, and 5BR. From Table VI.14.

See caption to Fig. VI.4 for explanation.

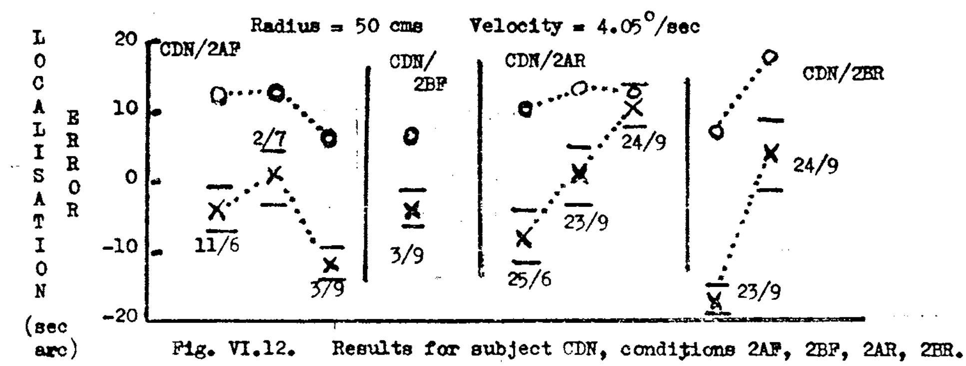


Fig. VI.12. Results for subject CDN, conditions 2AF, 2BF, 2AR, 2BR. From Table VI. 16. See caption to Fig. VI. 4 for explanation.

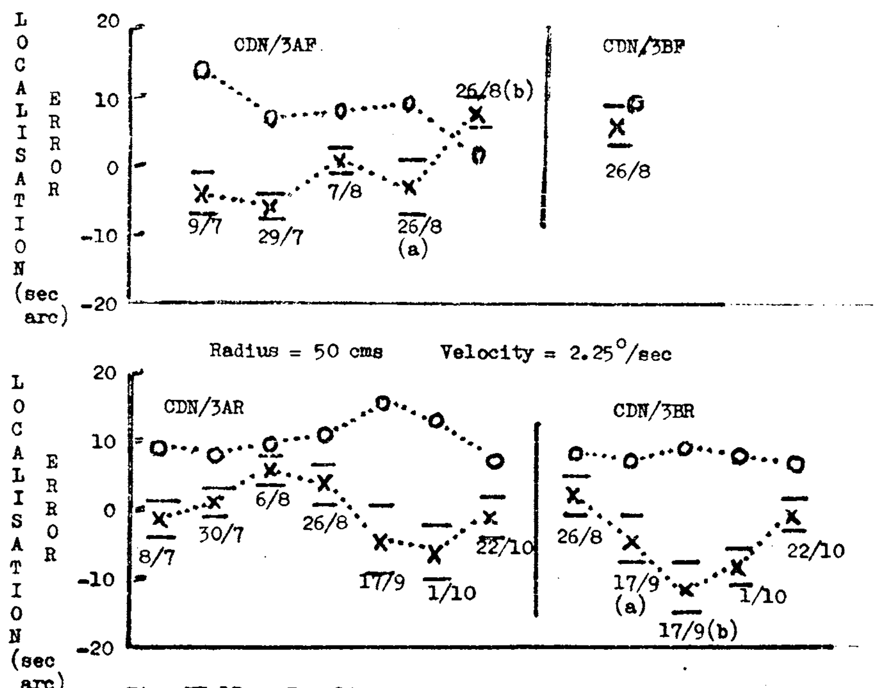


Fig. VI.13. Results for subject CDN, conditions 3AF and 3BF (upper graph), and 3AR and 3BR (lower graph). From Table VI.17. See caption to Fig. VI.4 for explanation.

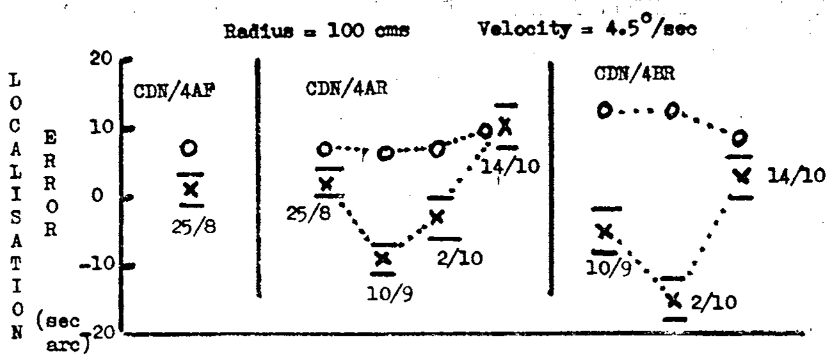


Fig. VI.14 Results for CDN, conditions 4AF, 4AR, and 4BR. Taken from Table VI.18.

See caption to Fig. VI.4 for explanation.

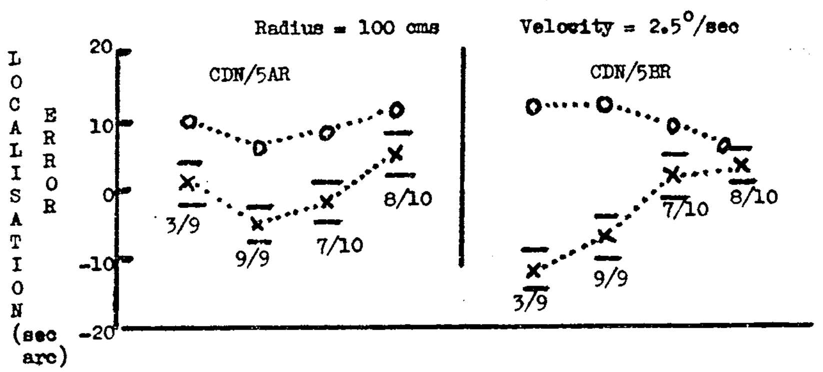


Fig. VI.15 Results for CDN, conditions 5AR and 5BR. From Table VI.19.

See caption to Fig. VI.4 for explanation.

VI.6 Influence of head position.

Following a suggestion by Amigo (personal communication), two experiments were run to see if changes in the subject's head position could lead to variations in localisation error.

The head and chin rest of the apparatus made it virtually impossible for the subject to rotate his head about a vertical axis, so that assymetric convergence and its associated changes in the longitudinal horopter (Amigo, 1967) were not likely to occur. However, it was possible for the subject to incline his head, although he was instructed to keep it pressed firmly against the restraints. Such an inclination effectively changes the plane of regard, which in turn could vary the shape of the horopter surface. This has been indicated by unpublished data obtained by Amigo (1969).

To test this idea, use was made of the fact that the subject could change the position of the chin rest while still in the experimental position. The exact amount of head inclination depended on the subject's facial anatomy. The difference in inclination between the "chin forward" and "chin back" positions was approximately 10°, much greater than differences introduced by head movements during an experiment.

In experiment CMJ/20ARS2, the chin rest was moved forward for the first twenty minutes; this is, the head was inclined backwards. The localisation error was found to be 27.08 sec arc (see Table VI.5). Then the chin rest was moved back, so that the subject's head was inclined forwards, and under these conditions, the error was 29.11 sec arc. The 95% confidence limits indicated that the change in localisation error was not significant.

Towards the end of the "chin back" position, the localisation error changed. For the first ten minutes of forward inclination, the error was 24.64 sec arc; for the last ten minutes it was 32.92 sec arc. In the last case, the threshold also increased markedly, and the two results were not quite significantly different.

A similar procedure was followed in experiment BK/20ARS2. Here, there was a significant difference between the "chin forward" and "chin back" positions: the localisation errors were 25.02 sec arc and 34.83 sec arc respectively. However, at the end of the "chin back" period, the chin rest was again moved forward. The localisation error became 38.11 sec arc, not significantly different from the "chin back" result, but certainly different from the first "chin forward" period.

The conclusion to be drawn from these experiments is that variations of localisation error during an experimental session were not likely to be due to changes of head inclination.

VI.7 Variation of localisation error with background luminance

The presence of day to day variations in localisation errors make it difficult to compare results obtained under different conditions. The ideal solution is to "interleave" conditions, that is, to run two or more concurrent series, each with a different set of conditions, and going from one series to another in the usual "double random staircase" procedure.

With the present apparatus, this was impossible, since changing conditions meant dismantling part of the apparatus, replacing pulleys, and re-calibrating. The only stimulus condition that could be easily changed was background luminance, which was varied by introducing filters in the viewing aperture.

The use of concurrent series with a different luminance for each series was out of the question, because each change would require an adaptation period, resulting in experimental sessions lasting for several hours.

The closest that one could come to the ideal solution was to run each condition for consecutive twenty-five minute periods. This did not negate the within-session variations, but these could be partly allowed for by changing the order of presentation from one session to another. Thus, if at one session the high luminance condition was studied first, then at another session the low luminance condition would be run first.

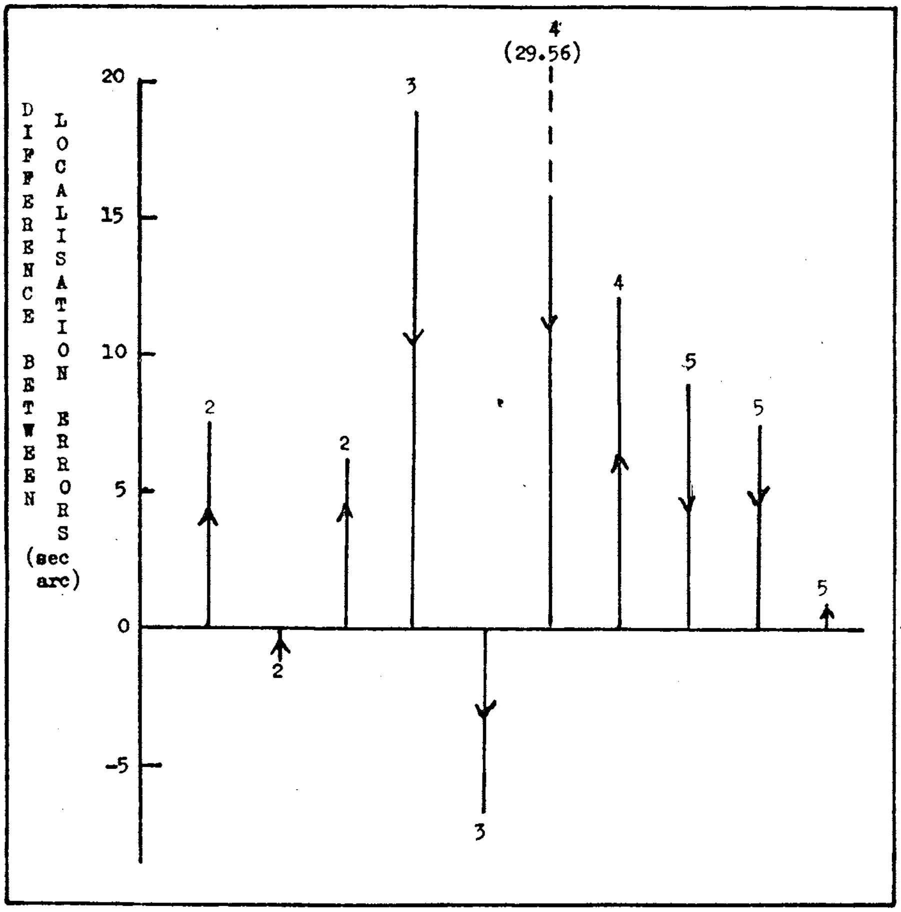


Fig. VI.16. Paired luminance experiments: subject CMJ.

Result obtained for the lower luminance at each session is plotted on the 'zero' line; the vertical line indicates the extent and sign of the change with higher luminance. The arrow on each line points to the luminance level used in the second half of the session. Numbers indicate stimulus conditions (see Table VI.1). Only left-to-right movement was used in these experiments. See text for further explanation.

Such a procedure was followed for experiments on and after 26/8. The overall design was that each condition of path radius and velocity would be run at at least two separate sessions. At each session, two levels of background luminance were introduced, both for twenty-five minute periods. The order of presentation of luminances was changed from one session to the other.

The first five minutes of each luminance condition were omitted from the analyses, to allow for adaptation.

Luminance was varied by introducing two Wratten neutral density filters in the viewing apertures. Their nominal density was 1.0; measurements with an SEI photometer were made at each session. The resulting luminance with the filters was 35.9 cd/m^2; without the filters, it was 384 cd/m^2.

In Table VI.5 (results for CMJ), experiments from 3ARS5 on 26/8 onwards were done as described here. First 3ARS5 was run for twenty-five minutes, then the filters were introduced, and 3BRS1 was run. All subsequent experiments were paired in this way.

For subject BK, the paired experiments begin with 2ARS3 and 2BRS1 (Table VI.10); for subject CDN (Table VI.15), the pairs begin with 3ARS4 and 3BRS1.

Only one direction of movement, from left to right, was used in this study.

These results are shown graphically in Figs. VI.16, VI.17 and VI.18. In these diagrams, the localisation error for the lower stimulus level has been set at zero. The vertical lines indicate the extent and sign of the difference between localisation errors for the two luminances at each session. A 'positive' line means that the moving rod was localised further away with the higher luminance than with the lower. The arrow on each line points towards the luminance level presented during the second half of the session, while the number at the end of each line gives the stimulus conditions (see Table VI.1). For example, the first vertical line in Fig. VI.16 signifies that conditions 2B (lower luminance) were presented first, followed by conditions 2A. The localisation error for 2A was 7.5 sec arc greater than for 2B.

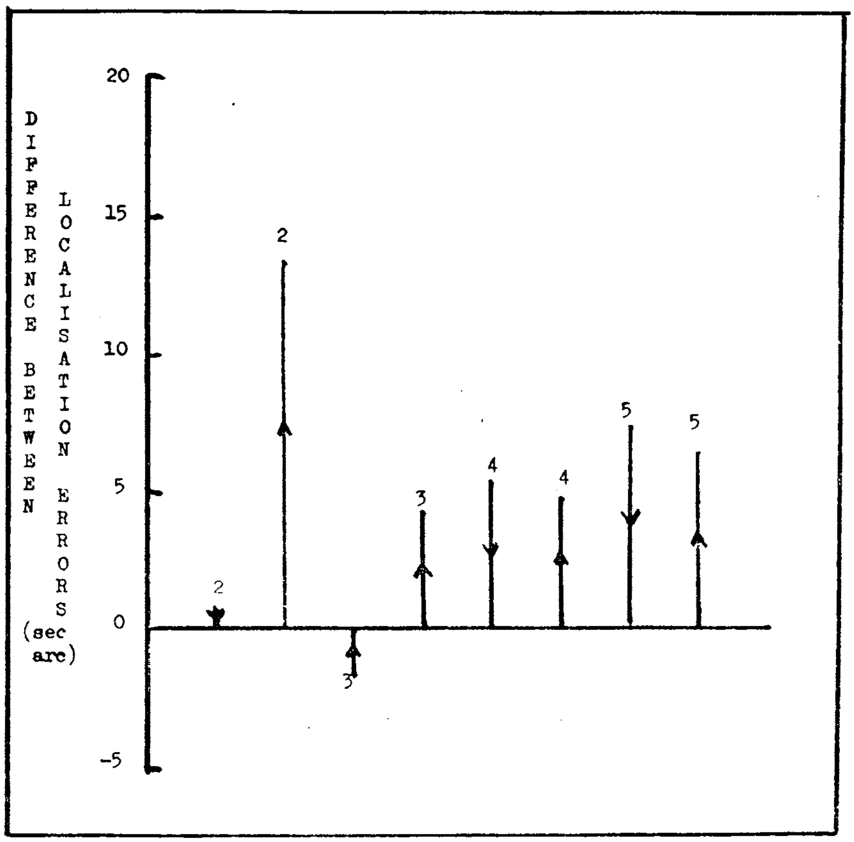


Figure. VI.17. Paired luminance experiments: subject BK.
See caption to Fig. VI.16 for explanation.

Thresholds are not shown on these figures. There was no systematic relationship between threshold and luminance level.

With subject CMJ (Fig. VI.16), 10 "paired luminance" experiments were carried out. In eight of these, the localisation error increased for the higher luminance. In six of the eight, the difference was significant according to the 95% confidence limits.

On one occasion, there was a significant decrease in localisation error with increased luminance (conditions 3BR and 3AR).

The higher luminance level was presented before the lower in exactly half of the experiments where the error increased, indicating that the changes were not due solely to the within-session drifts discussed in Section VI.4.

8 paired luminance experiments were done with subject BK (Fig. VI.17). The localisation error increased with the higher luminance in 7 of these. The difference was significant in 3 out of the 7. The reverse change was not significant.

Again, there was no indication that within-session changes were fully responsible for these results.

With subject CDN (Fig. VI.18), the localisation error increased <u>positively</u> with increased luminance; that is, negative localisation errors under the lower luminance became <u>less negative</u> when luminance was increased. This occurred in ten out of thirteen paired experiments.

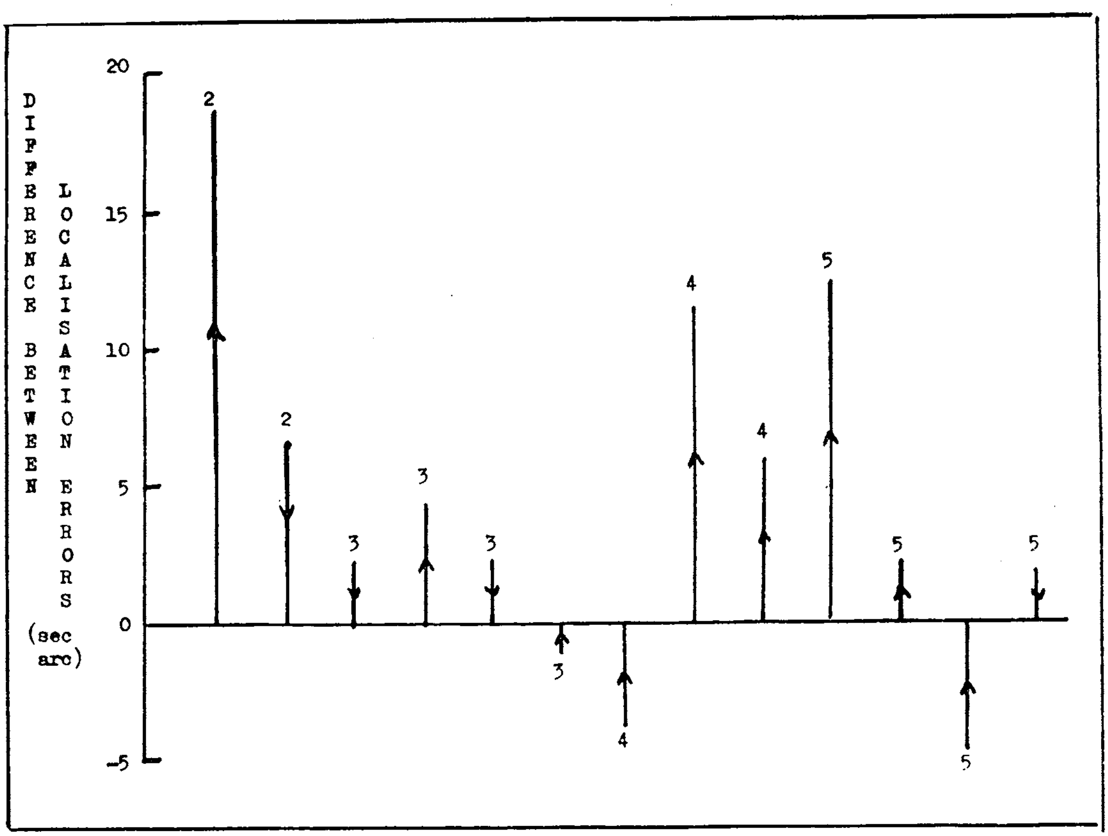


Fig. VI.18. Paired luminance experiments: subject CDN.
See caption to Fig. VI.16.

The change was significant in four of the ten experiments. None of the reverse changes were significant.

Once again, the change occurred independently of the order in which luminance levels were presented.

It may now be concluded that moving objects tend to be localised further away for high luminances than for low luminance, contrary to most of the findings of Lit (1966).

VI.8 Variation of localisation error with radius of path.

One of the aims of the study was to investigate the effect of path shape on the localisation of moving objects. As discussed in the previous section, day-to-day variations make comparisons difficult. Data sufficiently numerous for an analysis of variance are required; such an ambitious experiment was not within the scope of the present study.

However, such comparisons as can be made with the available data, while inconclusive, indicate that a large scale experiment would be fruitful.

In Tables VI.20, VI.21 and VI.22 are given the means of all the results for conditions 2, 3, 4 and 5 (see Table VI.1). These means are shown graphically in Figs. VI.19 a, VI.20, and VI.21.

Only data obtained with the stimulus moving from left to right are shown in the figures; results for the right to left movement are too sparse for even a rough comparison.

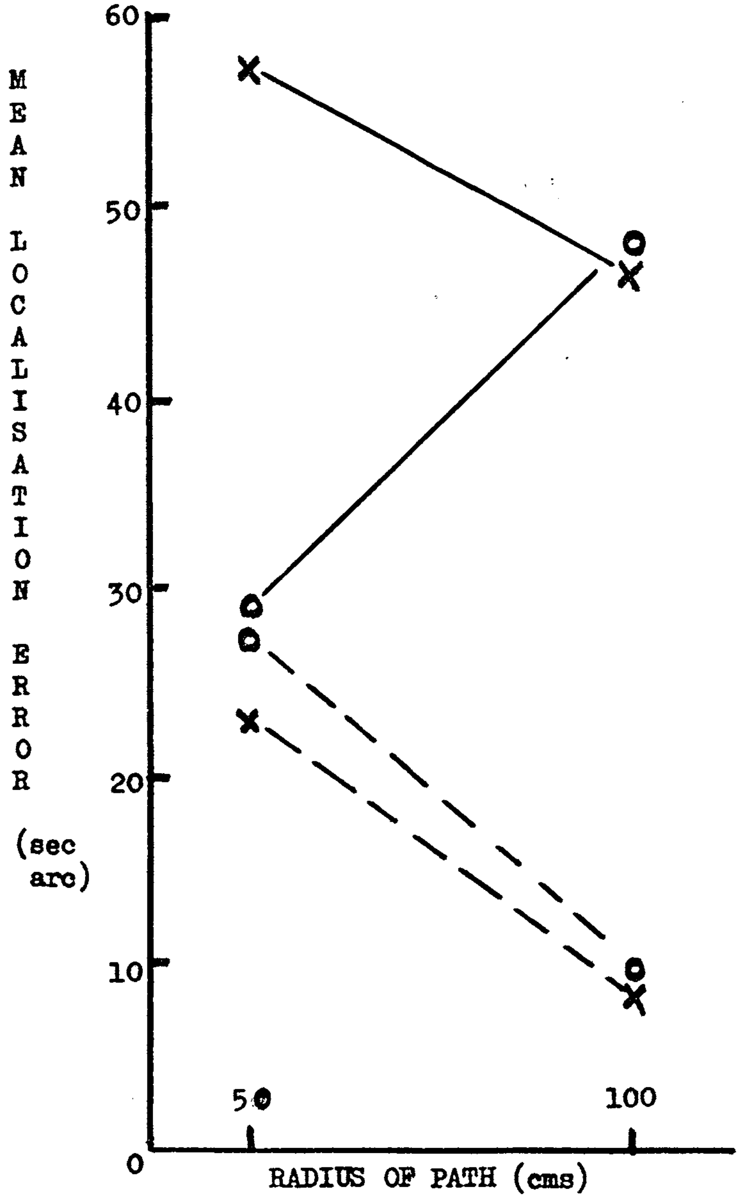


Fig. VI.19(a). Mean results for CMJ, from Table VI.20. Only left-to-right results are given.

KEY: X: 2.3^{o}/sec O: 4.2^{o}/sec
Solid lines: 384 cd/m^2
Broken lines: 35.9 cd/m^2

Mechanical limitations made it difficult to obtain identical velocities for both radii of path. The velocities used (2.25°/sec and 2.5°/sec; 4.05°/sec and 4.5°/sec) were sufficiently similar for purposes of comparison, the higher velocity being approximately twice the lower. The diagrams give the approximate mean velocities, 2.3°/sec and 4.2°/sec.

Looking first at CMJ's results (Fig. VI.19 a), it can be seen that the localisation error decreased for the flatter path for both luminance levels when the velocity was 2.3°/sec, and for the lower luminance at the higher velocity. With the higher luminance and higher velocity, on the other hand, the error increased when the path was flattened.

It will be remembered (Section VI.5) that subject CMJ was not always fixating the stationary rod as instructed, and that data obtained after 27/8 was more reliable than earlier data. The means of the more reliable data are plotted in Figure VI.19 b; the localisation error decreased for both velocities when the path radius was changed from 50 cms to 100 cms under the low luminance conditions, and increased with the higher luminance.

BK's localisation error (Fig. VI.20) increased for all conditions when the radius was increased from 50 cms to 100 cms, except for the case of low luminance and low

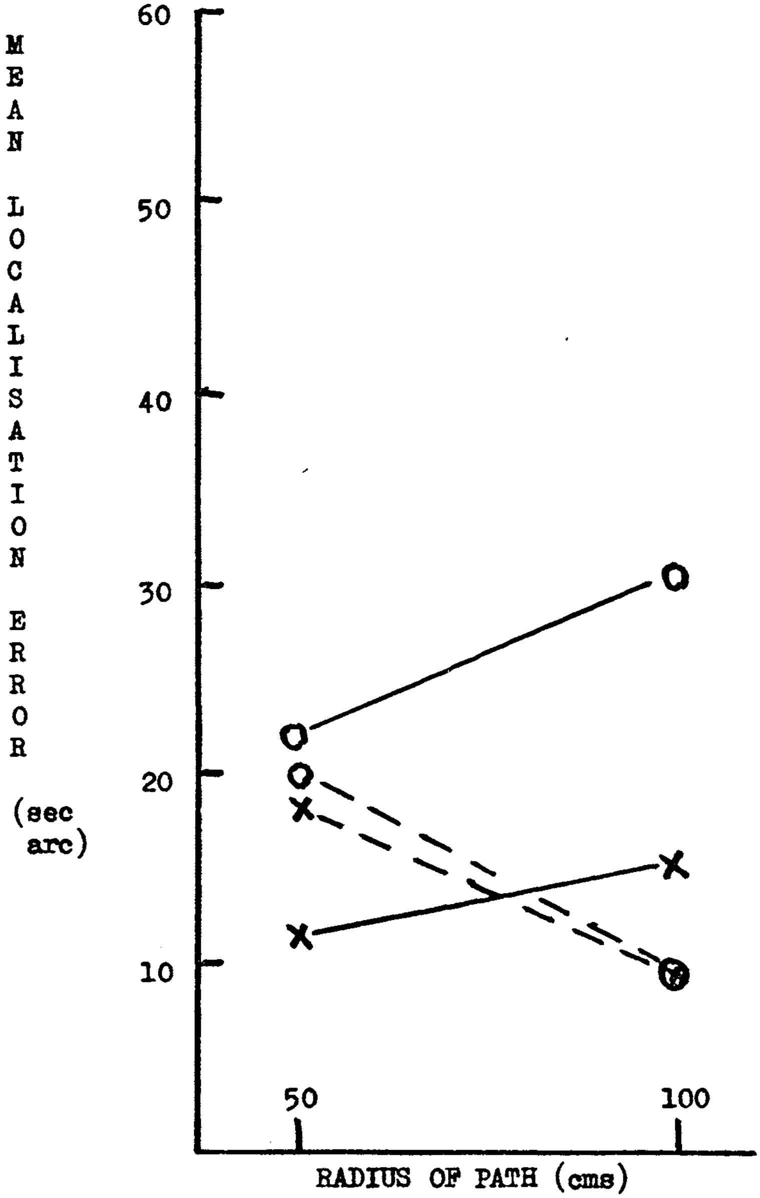


Fig. VI.19(b). Mean results for CMJ. Only data obtained after 27/8 were used to obtain these means.

KEY: X: 2.3^{o}/sec O: 4.2^{o}/sec
Solid lines: 384 cd/m^2
Broken lines: 35.9 cd/m^2

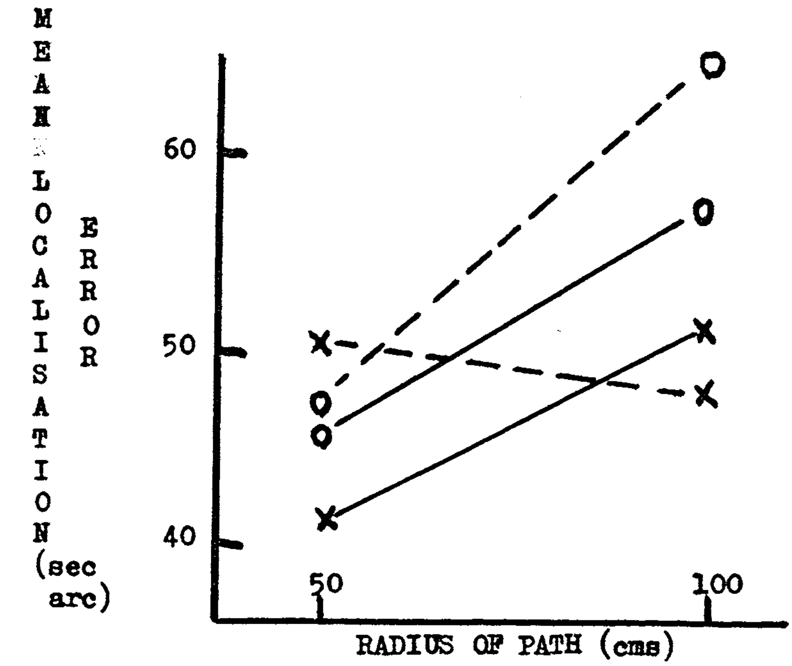


Fig. VI.20. Mean results for subject BK, from Table VI.21.

KEY

X: 2.3°/sec   O: 4.2°/sec

Solid lines: 384 cd/m²

Broken lines: 35.9 cd/m²

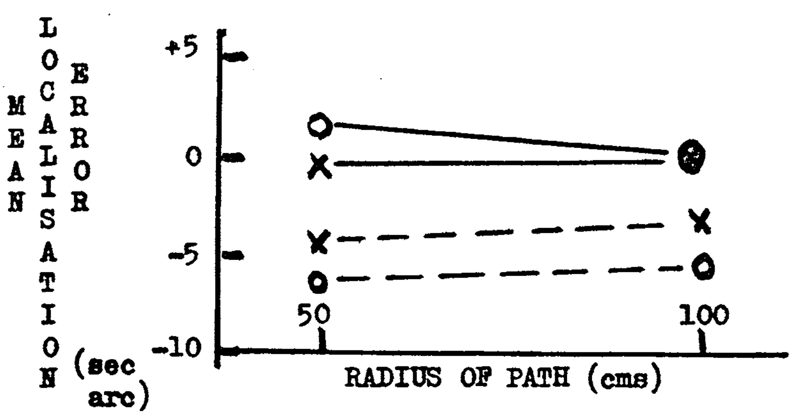


Fig. VI.21. Mean results for subject CDN, from Table VI.22.

velocity. There, the error was 51.43 sec arc for the 50 cms radius, and 47.69 sec arc for 100 cms. Both results were derived from only two experiments each, so that the significance of this difference is very dubious.

From the results of CMJ and BK one can only make the tentative conclusion that the localisation error does vary with path radius, but the kind of variation is somehow connected with luminance level.

CDN's results (Fig. VI.21) show no variation with stimulus conditions, except for the change associated with luminance level which has already been discussed.

VI.9 Summary.

In this chapter, experiments are described in which a rod was made to move along paths of different curvature. The subject had to judge whether or not the moving rod passed in front of or behind a stationary fixation rod in the median plane. A psychophysical method was used to determine the position of apparent equal distance.

As well as path curvature, velocity and luminance level were used as variables in the experiments. The results are summarised as follows:

(a) The localisation error for a given set of conditions varied significantly from session to session for all three subjects.

(b) Two of the subjects showed significant variations in localisation error over periods as short as twenty-five minutes.

(c) Head inclination was found to have no predictable or systematic effect on localisation error.

(d) For the two luminance levels used, localisation error increased positively for the higher level; that is, positive localisation errors increased with luminance levels, while negative errors decreased.

(e) The localisation error varied with changes in path curvature, but no systematic variation could be demonstrated.

(f) No systematic variation with velocity was found.

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VI.10. TABLES OF RESULTS

Tables VI.5 to VI.22

TABLE VI.5 (i)
Results for subject CMJ.
Arranged chronologically.
See Table VI.1 for explanation of experiment identifications.

All results in seconds of arc.

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
20ARS1	8/5	9.54	2.22	8.71	Step size = 6.3". Homogeneous series.
(a)		13.51	1.99	4.31	5 to 18 minutes.
(b)		6.82	1.95	5.18	21 to 40 minutes.
20AFS1	15/5	15.13	1.69	7.26	Step size 7.9"
20ARS2	18/5	28.11	2.55	11.05	Head tilt experiment.
chin f'd		27.08	3.75	11.44	
chin b'k		29.11	3.75	11.66	
b'k (a)		24.64	2.35	3.93	20 to 30 minutes
b'k (b)		32.92	7.41	15.69	30 to 40 minutes
20ARS3	20/5	27.68	2.61	11.16	
I		24.22	2.64	7.91	5 to 30 minutes
II		32.18	3.10	8.30	30 to 47 minutes
I(a)		21.29	2.15	4.59	5 to 21 minutes
I(b)		29.35	3.69	6.08	21 to 30 minutes
2ARS1	5/6	29.37	3.10	12.96	Step size: 9.4". CCS method introduced.
(a)		25.02	3.09	9.36	5 to 25 minutes.
(b)		35.61	3.04	7.24	25 to 40 minutes.
2AFS1	12/6	49.16	7.14	30.69	
(a)		42.18	6.56	19.32	5 to 22 minutes
(b)		56.65	7.44	21.73	22 to 40 minutes

TABLE VI.5 (ii) Subject: CMJ

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
2AFS2	19/6	64.52	4.25	14.86	
3ARS1	3/7	32.43	3.96	16.42	
3AFS1	7/7	76.89	8.45	33.31	Dubious threshold - S was coughing.
3AFS2	31/7	77.43	3.68	14.04	Step size = 12.6"
3ARS2	31/7	58.11	4.32	18.60	
3ARS3	7/8	96.07	8.39	37.13	
(a)		84.82	14.30	40.05	5 to 22 minutes.
(b)		103.91	5.02	15.48	22 to 40 minutes.
3ARS4	12/8	79.56	5.45	19.49	
5ARS1	19/8	93.05	9.91	39.63	
5ARS2	19/8	102.22	12.00	47.90	Feedback introduced.
(a)		88.13	11.64	29.86	5 to 24 minutes
(b)		113.61	8.10	22.69	24 to 43 minutes
5AFS1	21/8	130.72	5.83	19.56	
5AFS2	21/8	114.78	6.88	21.60	Step size = 18.8"
4AFS1	25/8	111.04	6.68	24.44	

TABLE VI.5 (iii) Subject: CMJ

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
4ARS1	25/8	81.59	5.33	20.65	
3ARS3	26/8	134.19	6.11	14.53	
3BRS1	26/8	76.65	6.86	18.87	First lower luminance experiment.
3ARS5	26/8	47.84	6.84	17.90	'Paired luminance' expts. begin
3BRS1	26/8	29.14	5.13	13.09	
2BRS1	27/8	44.66	4.82	12.72	
2ARS2	27/8	52.14	3.94	9.30	
2ARS3	27/8	76.15	6.79	21.61	
(a)		83.39	8.77	16.72	5 to 13 minutes
(b)		68.14	4.41	9.71	13 to 25 minutes
2BRS1	27/8	56.20	3.47	8.15	
5ARS3	3/9	14.29	2.54	6.35	Step size = 12.6"
5BRS1	3/9	5.44	3.47	10.49	
(a)		10.32	5.11	10.27	5 to 15 minutes
(b)		1.89	2.52	4.55	15 to 29 minutes
4ARS2	10/9	31.13	1.93	5.23	

TABLE VI.5 (iv) Subject: CMJ

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
4BRS1	10/9	1.57	3.01	9.34	
3BRS2	17/9	18.14	2.21	6.53	Step size = 9.4"
3ARS6	17/9	11.54	2.07	6.00	
2ARS3	24/9	19.33	2.75	9.24	
(a)		21.79	2.13	5.41	5 to 20 minutes
(b)		12.87	6.43	11.41	20 to 25 minutes
2BRS2	24/9	20.46	2.42	7.81	
4BRS2	1/10	17.58	2.71	8.27	
4ARS3	1/10	29.55	2.33	6.99	
5ARS4	8/10	15.42	2.12	5.72	
5BRS2	8/10	8.18	2.30	6.49	
5BRS3	15/10	14.31	2.51	7.38	
5ARS3	15/10	15.03	1.89	5.30	
2BRS3	22/10	18.57	2.86	8.69	
2ARS4	22/10	24.75	3.07	9.93	

TABLE VI.6

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
2AFS1	12/6	49.16	7.14	30.69	Right to Left
(a) *		42.18	6.56	19.32	384 cd/m^2
(b) *		56.65	7.44	21.73	
2AFS2*	19/6	64.52	4.25	14.86	
2AFS3	27/8	76.15	6.79	21.61	
(a)*		88.39	8.77	16.72	
(b)*		68.14	4.41	9.17	
2BFS1	27/8	56.20	3.47	8.15	Right to Left 35.9 cd/m^2
2ARS1	5/6	29.37	3.10	12.96	Left to right
(a)*		25.02	3.09	9.36	384 cd/m^2
(b)*		35.61	3.04	7.24	
2ARS2*	27/8	52.14	3.94	9.30	
2ARS3	24/9	19.35	2.75	9.24	
(a)*		21.79	2.13	5.41	
(b)*		12.87	6.43	11.41	
2ARS4*	22/10	24.75	3.07	9.93	
2BRS1*	27/8	44.66	4.82	12.72	Left to right
2BRS2*	24/9	20.46	2.42	7.81	35.9 cd/m^2
2BRS3	22/10	18.57	2.86	8.69	

RADIUS OF PATH = 50 cms VELOCITY = 4.05°/sec

TABLE VI.6. Results for subject CMJ, arranged according to conditions.

* See Table VI.20 for means of asterisked results.

TABLE VI.7

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
3AFS1*	7/7	76.89	8.45	33.31	Right to left
3AFS2*	31/7	77.43	3.68	14.04	384 cd/m^2
3AFS3*	26/8	134.19	6.11	14.53	
3BFS1*	26/8	76.65	6.86	18.87	Right to left 35.9 cd/m^2
3ARS1*	3/7	32.43	3.96	16.42	Left to right
3ARS2*	31/7	58.11	4.32	18.60	384 cd/m^2
3ARS3	7/8	96.07	8.39	37.13	
(a) *		84.82	14.30	40.05	
(b) *		103.91	5.02	15.48	
3ARS4*	12/8	79.56	5.45	19.49	
3ARS5*	26/8	47.84	6.84	17.90	
3ARS6*	17/9	11.54	2.07	6.00	
3BRS1*	26/8	29.14	5.13	13.09	Left to right
3BRS2*	17/9	18.14	2.21	6.53	35.9 cd/m^2

RADIUS OF PATH = 50 cms VELOCITY = 2.25°/sec

TABLE VI.7. Results for subject CMJ, arranged according to conditions.

* See Table VI.20 for means of asterisked results.

TABLE VI.8

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
4AFS1	25/8	111.04	6.58	24.44	Right to left 384 cd/m^2
4ARS1*	25/8	81.59	5.33	20.65	Left to right 384 cd/m^2
4ARS2*	10/9	31.13	1.98	5.23	
4ARS3*	1/10	29.55	2.33	6.99	
4ERS1*	10/9	1.57	3.01	9.34	Left to right 35.9 cd/m^2
4ERS2*	1/10	17.58	2.71	8.27	

RADIUS OF PATH = 100 cms VELOCITY = 4.5°/sec

TABLE VI.8. Results for subject CMJ, arranged according to conditions.

* See Table VI.20 for means of asterisked results.

TABLE VI.9

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
5AFS1*	21/8	130.72	5.83	19.56	Right to left 384 cd/m^2
5AFS2*	21/8	114.78	6.88	21.60	
5ARS1*	19/8	93.05	9.91	39.63	Left to right 384 cd/m^2
5ARS2	19/8	102.22	12.00	47.90	
(a)*		83.13	11.64	29.86	
(b)*		113.61	8.10	22.69	
5ARS3*	3/9	14.29	2.54	6.35	
5ARS4	8/10	15.42	2.12	5.72	
5ARS5	15/10	15.03	1.89	5.30	
5ERS1	3/9	5.44	3.47	10.49	Left to right 35.9 cd/m^2
(a)*		10.32	5.11	10.27	
(b)*		1.89	2.52	4.55	
5ERS2*	8/10	8.18	2.30	6.49	
5ERS3*	15/10	14.31	2.51	7.38	

RADIUS OF PATH = 100 cms VELOCITY = 2.5°/sec

TABLE VI.9. Results for subject CMJ, arranged according to conditions.

* See Table VI.20 for means of asterisked results.

TABLE VI.10 (i)

All results in sec arc.

Results for subject BK
Arranged chronologically.
See Table VI.1 for explanation of experiment identifications.

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
20ARS1	14/5	26.17	2.53	10.56	Homogeneous series. Step size 7.9"
20ARS2	19/5	31.52	3.36	17.11	Head tilt experiment. Step size 9.4"
chin f'd(a)		25.02	3.34	10.31	
chin b'k		34.83	4.05	12.43	
chin f'd(b)		38.11	5.87	12.80	
20ARS3	21/5	34.06	2.86	12.10	
(a)		29.24	2.83	7.35	5 to 21 minutes
(b)		37.94	3.24	9.96	21 to 41 minutes
20ARS4	28/5	41.62	2.24	10.08	Complementary series
20AFS2	4/6	42.39	1.88	7.61	
2AFS1	11/6	32.40	3.43	15.26	
(a)		27.64	2.99	9.35	5 to 25 minutes
(b)		36.67	4.62	14.08	25 to 42 minutes
2ARS1	18/6	26.03	2.45	10.96	
2AFS2	25/6	41.62	2.19	9.76	
(a)		38.08	2.36	7.02	5 to 23 minutes
(b)		45.19	2.77	8.41	23 to 42 minutes
2ARS2	2/7	37.24	2.82	9.63	
3ARS1	8/7	38.11	2.74	11.52	
3AFS1	9/7	59.23	2.16	9.27	
3ARS2	15/7	33.92	1.99	7.85	
(a)		26.69	1.50	3.06	5 to 22 minutes
(b)		37.17	2.51	7.17	22 to 40 minutes
3AFS2	29/7	47.49	1.48	5.49	

TABLE VI.10 (ii) Subject: BK

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
3AFS3	30/7	39.96	1.84	7.57	
3ARS3	11/8	40.84	1.79	6.85	
3AFS4	11/8	51.08	1.74	6.78	
4ARS1	13/8	41.29	2.61	10.77	
4AFS1	13/8	50.31	1.72	6.14	
5AFS1	24/8	49.21	2.18	5.45	
5ARS1	24/8	46.49	3.10	8.60	
4AFS2	25/8	61.67	2.02	7.50	
4ARS2	25/8	48.60	1.58	3.76	
2ARS3	28/8	49.00	1.96	5.19	'Paired luminance' expts. begin.
2BRS1	28/8	48.51	2.03	5.35	
2BFS1	2/9	54.38	2.10	6.24	
2AFS3	2/9	50.56	3.34	10.84	
5ARS2	9/9	49.80	3.03	8.70	
5BRS1	9/9	42.66	2.58	7.42	
4ARS3	16/9	56.99	3.89	13.88	
(a)		51.08	2.25	4.63	5 to 15 minutes
(b)		62.33	3.76	9.17	15 to 25 minutes
4BRS1	16/9	51.78	2.64	8.96	
2BRS2	23/9	44.86	2.24	7.53	
2ARS4	23/9	58.20	3.89	14.07	
(a)		52.06	2.27	4.82	5 to 15 minutes
(b)		64.62	3.41	7.93	15 to 25 minutes

TABLE VI.10 (iii)   Subject: EI

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
3ARS4	30/9	43.15	2.51	8.21	
3BRS1	30/9	44.90	1.53	4.22	
5BRS2	7/10	52.72	2.07	5.60	
5ARS3	7/10	59.38	2.62	7.70	
4BRS2	14/10	79.07	3.44	12.03	
4ARS4	14/10	83.90	3.16	11.03	
3BRS2	21/10	53.89	2.62	8.10	
3ARS5	21/10	57.95	2.75	8.75	

TABLE VI.11

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
2AFS1	11/6	32.40	3.43	15.26	
(a)*		27.64	2.99	9.35	
(b)*		36.67	4.62	14.08	Right to left
2AFS2	25/6	41.62	2.19	9.76	384 cd/m^2
(a)*		38.08	2.36	7.02	
(b)*		45.19	2.77	8.41	
2AFS3*	2/9	50.56	3.34	10.84	
2BFS1	2/9	54.38	2.10	6.24	Right to left 35.9 cd/m^2
2ARS1*	18/6	26.03	2.45	10.96	
2ARS2*	2/7	37.24	2.82	9.63	Left to right
2ARS3*	28/8	49.00	1.96	5.19	384 cd/m^2
2ARS4	23/9	58.20	3.89	14.07	
(a)*		52.06	2.27	4.82	
(b)*		64.62	3.41	7.93	
2ERS1*	23/8	48.51	2.05	5.35	Left to right
2ERS2*	23/9	44.86	2.24	7.53	35.9 cd/m^2

RADIUS OF PATH = 50 cms VELOCITY = 4.05°/sec

TABLE VI.11. Results for subject BK, arranged according to conditions.

* See Table VI.21 for means of the asterisked results.

TABLE VI.12

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
3AFS1*	9/7	59.23	2.16	9.27	Right to left
3AFS2*	29/7	47.49	1.48	5.49	384 cd/m^2
3AFS3*	30/7	39.96	1.84	7.57	
3AFS4*	11/8	51.08	1.74	6.78	
3ARS1*	8/7	38.11	2.74	11.52	
3ARS2	15/7	33.92	1.99	7.85	
(a)*		26.69	1.50	3.08	
(b)*		37.17	2.51	7.17	Left to right
3ARS3*	11/8	40.84	1.79	6.85	384 cd/m^2
3ARS4*	30/9	43.15	2.51	8.21	
3ARS5*	21/10	57.95	2.75	8.75	
3ERS1*	30/9	44.90	1.53	4.22	Left to right
3ERS2*	21/10	53.09	2.62	8.10	35.9 cd/m^2

RADIUS OF PATH = 50 cms VELOCITY = 2.25°/sec

TABLE VI.12. Results for subject BK, arranged according to conditions.

*See Table VI.21 for means of the asterisked results.

TABLE VI.13

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
4AFS1*	13/8	50.31	1.72	6.14	Right to left
4AFS2*	25/8	61.67	2.02	7.50	384 cd/m^2
4ARS1*	13/8	41.29	2.61	10.77	Left to right
4ARS2*	25/8	48.80	1.58	3.76	384 cd/m^2
4ARS3	16/9	56.99	3.89	13.88	
(a)*		51.08	2.25	4.63	
(b)*		62.33	3.76	9.17	
4ARS4*	14/10	83.90	3.16	11.08	
4ERS1*	16/9	51.78	2.64	8.96	Left to roght
4ERS2*	14/10	79.07	3.44	12.08	35.9 cd/m^2

RADIUS OF PATH = 100 cms VELOCITY = 4.5°/sec

Table VI.13. Results for subject BK, arranged according to conditions.

*See Table VI.21 for means of the asterisked results.

TABLE VI.14

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
5AFS1	24/8	49.21	2.18	5.45	Right to left 384 cd/m^2
5ARS1*	24/8	46.49	3.10	8.60	Left to right
5ARS2*	9/9	49.80	3.03	8.70	384 cd/m^2
5ARS3*	7/10	59.38	2.62	7.70	
5ERS1*	9/9	42.66	2.58	7.42	Left to right
5ERS2*	7/10	52.72	2.07	5.60	35.9 cd/m^2

RADIUS OF PATH = 100 cms VELOCITY = 2.5°/sec

TABLE VI.14. Results for subject B.K., arranged according to conditions.

* See Table VI.21 for means of the asterisked results.

TABLE VI.15

Results (in seconds of arc) for subject CDN, arranged chronologically.

See Table VI.1 for explanation of experiment identifications.

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	REMARKS
20ARS1	11/5	-6.77	3.69	12.69	Stop size = 6.3" Homogeneous series
20ARS2	4/6	+2.95	4.56	14.25	Stop size = 12.6" Complementary series
2AFS1	11/6	-4.01	2.90	11.79	
2ARS1	25/6	-8.01	3.74	10.44	
2AFS2	2/7	+0.56	3.93	12.35	
3ARS1	8/7	-1.62	2.61	9.17	
3AFS1	9/7	-3.85	3.54	14.09	
3AFS2	29/7	-5.89	2.03	7.09	
3ARS2	30/7	+0.88	2.31	8.08	
3ARS3	6/8	+6.40	2.40	9.45	
3AFS3	7/8	+0.65	2.08	7.88	
4AFS1	25/8	+1.26	2.04	7.08	
4ARS1	25/8	+1.81	2.00	6.71	
3BFS1	26/8	+6.31	2.95	9.28	
3AFS4	26/8	+1.73	4.03	12.06	
(a)		-3.36	4.15	8.75	5 to 15 minutes
(b)		+8.05	1.59	1.53	15 to 22 minutes
3ARS4	26/8	+4.54	3.28	10.67	'Paired luminance' experiments begin.
3BRS1	26/8	+2.42	3.44	8.36	
2AFS3	3/9	-12.52	2.33	5.90	
2BFS1	3/9	-4.38	2.51	6.52	
5BRS1	3/9	-11.82	3.31	11.59	
5ARS1	3/9	+0.73	3.38	9.62	
5BRS2	9/9	-7.09	3.64	11.86	
5ARS2	9/9	-4.70	2.37	6.32	
4ARS2	10/9	-8.79	2.12	6.26	
4BRS1	10/9	-5.0	3.62	12.00	
3BRS2	17/9	-8.52	3.19	10.18	
(a)		-5.11	3.68	7.83	5 to 15 minutes
(b)		-11.57	4.24	9.29	15 to 25 minutes
3ARS5	17/9	-4.30	4.71	15.69	
2BRS1	23/9	-17.45	2.23	6.87	
2ARS2	23/9	+1.17	4.07	13.89	
2ARS3	24/9	+10.57	3.51	12.25	
2BRS2	24/9	+4.02	4.78	18.02	
3ARS6	1/10	-6.18	4.07	13.28	
3BRS3	1/10	-8.39	2.65	8.27	
4BRS2	2/10	-14.75	3.71	12.15	
4ARS3	2/10	-3.18	2.29	6.90	
5ARS3	7/10	-1.85	2.85	8.46	
5BRS3	7/10	+2.51	3.26	9.59	
5ARS4	8/10	+5.63	3.62	11.60	
5BRS4	8/10	+3.82	2.21	6.02	
4BRS3	14/10	+2.83	2.59	8.42	
4ARS4	14/10	+9.77	2.80	9.55	
3ARS7	22/10	-1.44	2.64	7.67	
3BRS4	22/10	-0.64	2.38	6.75	

TABLE VI.16 All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
2AFS1*	11/6	-4.01	2.90	11.79	Right to left 384 cd/m²
2AFS2*	2/7	+0.56	3.93	12.35	
2AFS3*	3/9	-12.52	2.33	5.90	
2BFS1	3/9	-4.38	2.51	6.52	Right to left 35.9 cd/m²
2ARS1*	25/6	-8.01	3.74	10.44	Left to right 384 cd/m²
2ARS2*	23/9	+1.17	4.07	13.89	
2ARS3*	24/9	+10.57	3.51	12.25	
2BRS1*	23/9	-17.45	2.23	6.87	Left to right 35.9 cd/m²
2BRS2*	24/9	+4.02	4.78	18.02	

RADIUS OF PATH = 50 cms VELOCITY = 4.05°/sec

TABLE VI.16. Results for subject CDN, arranged according to conditions. (From Table VI. 15).

* For means of asterisked results, see Table VI.22.

TABLE VI.17 All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
3AFS1*	9/7	-5.85	3.54	14.09	Right to left 384 cd/m²
3AFS2*	29/7	-5.89	2.03	7.02	
3AFS3*	7/8	+0.65	2.08	7.83	
3AFS4	26/8	+1.73	4.05	12.06	
(a)*		-3.36	4.15	8.75	
(b)*		+8.05	1.59	1.53	
3BSF1	26/8	+6.31	2.95	9.28	Right to left 35.9 cd/m²
3ARS1*	8/7	-1.62	2.61	9.17	Left to right 384 cd/m²
3ARS2*	30/7	+0.88	2.31	8.08	
3ARS3*	6/8	+6.40	2.40	9.45	
3ARS4*	26/8	+4.54	3.28	10.67	
3ARS5*	17/9	-4.30	4.71	15.69	
3ARS6*	1/10	-6.18	4.07	13.28	
3ARS7*	22/10	-1.44	2.64	7.67	
3BRS1*	26/8	+2.42	3.44	8.36	Left to right 35.9 cd/m²
3BRS2	17/9	-8.52	3.19	10.18	
(a)*		-5.11	3.68	7.83	
(b)*		-11.57	4.24	9.29	
3BRS3*	1/10	-8.39	2.65	8.27	
3BRS4*	22/10	-0.64	2.38	6.75	

RADIUS = 50 cms VELOCITY = 2.25°/sec

TABLE VI.17. Results for CDN (from Table VI.15), arranged according to conditions. * See Table VI.22 for means of asterisked results.

TABLE VI.18 All results in sec. arc.

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
4ARS1	25/8	+1.26	2.04	7.08	Right to left 384 cd/m^2
4ARS1*	25/8	+1.81	2.00	6.71	Left to right 384 cd/m^2
4ARS2*	10/9	-8.79	2.12	6.26	
4ARS3*	2/10	+3.18	2.29	6.90	
4ARS4*	14/10	+9.77	2.80	9.55	
4BRS1*	10/9	-5.00	3.62	12.00	Left to right 35.9 cd/m^2
4BRS2*	2/10	-14.75	3.71	12.15	
4BRS3*	14/10	+2.83	2.59	8.42	

RADIUS = 100 cms VELOCITY = 4.5°/sec

TABLE VI.18. Results for subject CDN (from Table VI.15, but arranged according to conditions.)

See Table VI.22 for means of asterisked results.

TABLE VI.19

All results in seconds of arc

EXPT.	DATE	MEAN	95% LIMIT	THRESHOLD	CONDITIONS
5ARS1*	3/9	+0.75	3.38	9.62	Left to right 384 cd/m^2
5ARS2*	9/9	-4.70	2.37	6.32	
5ARS3*	7/10	-1.85	2.85	8.46	
5ARS4*	8/10	+5.63	3.62	11.60	
5BRS1*	3/9	-11.82	3.31	11.59	Left to right 35.9 cd/m^2
5BRS2*	9/9	-7.09	3.64	11.86	
5BRS3*	7/10	+2.51	3.26	9.59	
5BRS4*	8/10	+3.82	2.21	6.02	

RADIUS OF PATH = 100 cms VELOCITY = 2.5°/sec

TABLE VI.19. Results for subject CDN (from Table VI.15), arranged according to conditions.

* See Table VI.22 for means of asterisked results.

SUBJECT: CMJ			RIGHT - - - TO - - - LEFT			LEFT - - - TO - - - RIGHT		
Radius (cms)	Velocity (°/sec)	Luminance (cd/m²)	N	Mean Localisation Error (sec arc)	Mean Threshold (sec arc)	N	Mean Localisation Error	Mean Threshold (sec arc)
50	2.25	35.9	1	76.65	18.87	2	23.64	9.81
		384.0	3	96.17	20.63	7/6	59.74	19.13
	4.05	35.9	1	56.20	8.15	3	27.90	9.74
		384.0	5/3	63.98	16.36	6/4	28.70	8.77
100	2.50	35.9	0	-	-	4/3	8.67	7.17
		384.0	2	122.75	20.58	6/5	46.45	14.68
	4.50	35.9	0	-	-	2	9.57	8.81
		384.0	1	111.04	24.44	2	47.42	10.96

TABLE VI.20. Results for subject CMJ.

Each entry under "Mean Localisation Error" and "Mean Threshold" is the mean of the corresponding group of results from Tables VI.6 to VI.9 (asterisks in previous tables indicate which results were used here). 'N' is the number of results from the earlier tables contributing to the means. E.g., N = 4/3 means that there were 4 results from 3 sessions, there having been a significant change in localisation error during one of the sessions.

SUBJECT: BK			RIGHT - - - TO - - - LEFT			LEFT - - - TO - - - RIGHT		
Radius (cms)	Velocity (o/sec)	Luminance (cd/m^2)	N	Mean Localisation Error (sec arc)	Mean Threshold (sec arc)	N	Mean Localisation Error	Mean Threshold (sec arc)
50	2.25	35.9	0	-	-	2	51.43	6.49
		384.0	4	49.44	7.28	6/5	40.65	7.59
	4.05	35.9	1	54.38	6.24	2	46.69	6.44
		384.0	5/3	39.63	9.94	5/4	45.79	7.71
100	2.50	35.9	0	-	-	2	47.69	6.51
		384.0	1	49.21	5.45	3	51.89	8.33
	4.50	35.9	0	-	-	2	65.43	10.52
		384.0	2	55.99	6.82	5/4	57.48	7.88

TABLE VI. 21. Averaged results for subject BK.
See caption to Table VI.20 for explanation.
Means are derived from the asterisked data in Tables
VI.11 to VI.14.

SUBJECT: CDN			RIGHT - - - TO - - - LEFT			LEFT - - - TO - - - RIGHT		
Radius (cms)	Velocity (°/sec)	Luminance (cd/m²)	N	Mean Localisation Error (sec arc)	Mean Threshold (sec arc)	N	Mean Localisation Error (sec arc)	Mean Threshold (sec arc)
50	2.25	35.9	1	+6.31	9.28	5/4	-4.66	8.10
		384.0	5/4	-0.88	7.87	7	-0.25	10.57
	4.05	35.9	1	-4.38	6.52	2	-6.72	12.45
		384.0	3	-5.32	10.01	3	+1.24	12.19
100	2.50	35.9	0	-	-	4	-3.14	9.76
		384.0	0	-	-	4	-0.05	9.00
	4.50	35.9	0	-	-	3	-5.64	10.86
		384.0	1	+1.26	7.08	4	-0.10	7.35

TABLE VI.22. Averaged results for subject CDN.
Means are derived from the asterisked data in
Tables VI.16 to VI.19.
See caption to Table VI.20 for explanation.

CHAPTER VII

DISCUSSION AND CONCLUSIONS

VII.1 Variability of the localisation error.

The most significant (and surprising) result of the experimental investigation was the variation in localisation error which occurred from session to session with all three subjects, and within sessions for two subjects. Two groups of factors must be considered: those concerned with the experimental method and the subject's behaviour, and those which have a physiological basis.

(a) Experimental methods and judgment criteria.

It might be asked why variations in localisation error were not noted and reported by Lit, who conducted numerous experiments. An answer might be found in examining the differences, already noted in Chapter IV.3, between Lit's experiments and those reported here.

Lit used a method of adjustment, as compared with the constant stimulus presentation employed here. His subjects observed the moving rod for an unspecified number of oscillations before deciding that the two rods were equidistant, whereas in the constant stimulus presentation the subjects had to make a report after each exposure.

Furthermore, the whole extent of travel was visible to Lit's subjects, whereas in the present apparatus the moving rod appeared from behind a screen.

It may be, then, that the subjects in our experiments were less able to establish consistent judgment criteria than those of Lit. This does not imply that their reports were inconsistent, for thresholds better than 10 sec arc were obtained more often than not (see Tables VI.20 to 22). Inconsistent reports should have led to higher thresholds, and only subject CMJ showed a tendency towards increased thresholds with increased localisation errors.

Relevant to this point are the localisation error and threshold obtained for CMJ in experiments 3AFS1 and 3AFS2. The first of these, on 3/7, showed an error of 76.89 sec arc, with a threshold of 33.31 sec arc. During this experiment, the subject coughed frequently, and there was doubt as to the validity of the results. However, the next experiment under the same conditions, on 31/7, gave essentially the same localisation error (77.43 sec arc), but a lower threshold of 14.04 sec arc. Thus the subject's coughing in the first experiment can be seen as a distraction which influenced her sensitivity but not her localisation.

In terms of the path-sampling hypothesis (Chapter III.4), variations in judgment criteria could indicate that subjects did not always use the same length of path in making a decision. Reasons for this can only be speculative: subjects were told neither the velocity nor the path shape at each session, and perhaps this uncertainty, instead of stabilising their criteria as hoped, led them to establish new criteria at each session.

(b) Variations of fixation disparity.

According to Ogle *et al* (1967, pp. 47-53), fixation disparity does vary slightly from day to day. They cite some data of Mitchell & Ellerbrock: for one subject, over eight consecutive days, the disparity varied over a range of 1.86 minutes of arc; for another, the range was 0.75 minutes of arc.

In the experiments reported here, the greatest variation in localisation error for a given set of conditions was 98 sec arc, or 1.6 min arc (Experiments CMJ/5ARS2b and 5ARS5, Table VI.9). In general, the variations were over a range less than 1 min arc, not inconsistent with possible variations in fixation disparity, particularly in view of the reduced binocular fusion stimuli (see Chapter III.2).

The exact influence of changes in fixation disparity on path shape and distance judgments, according to the L - F - S theory developed in Chapter III.5, depends on the way in which latency varies over the retina. Until both these factors have been measured under experimental conditions, it can only be assumed that variations of localisation error reflect variations in fixation disparity.

(c) Adaptation

Since the latency of the visual response decreases with increased light adaptation (Chapter I.4a), variations in localisation error could be attributed to changes in the state of adaptation during an experimental session.

In the experiments reported here, luminance levels ($35.9\ cd/m^2$ or $384\ cd/m^2$) were always well into the photopic range. Subjects generally came to the experiments after being in photopic environments such as lecture rooms and common rooms, and no observations were recorded until they had been exposed to the stimulus for at least five minutes.

Thus it seemed safe to assume that the state of adaptation throughout the experiments was fairly stable.

Furthermore, Rock & Fox (1949) measured the extent of the Pulfrich phenomenon after dark adapting one eye for fifteen minutes. They found that the phenomenon disappeared after 60 seconds, supporting the statement made in the previous paragraph.

However, one experiment in the present series indicated that extreme states of pre-adaptation could have some effect. In experiment CDN/20ARS2, during the first fifteen minutes of the forty minute session the subject made observations which seemed to be randomly ordered. For the remainder of the session, his observations were more stable. The resulting localisation error was + 2.95 sec arc, with a threshold of 14.25 sec arc. The previous experiment under the same conditions gave an error of -6.77 sec arc and a threshold of 12.69 sec arc. The two localisation errors were statistically different according to the 95% limits.

On interviewing the subject after the experiment, it was found that before the session, he had run for a considerable distance in very bright sunshine. His observations were made difficult by strong after images, which occasionally caused the stimulus rods to disappear.

Subsequently, care was taken to ensure that the subjects had not been in unusual situations before the experiments.

The experience with CDN indicates that the possibility of adaptation influencing localisation error is a real one.

In general, however, there was sufficient control during the experiments to assure that the possibility was unlikely.

(d) Eye movements.

In section VI.5, it was reported that subject CMJ did not always fixate the fixation rod as instructed, but tracked the moving rod in some experiments. Since eye movement was not a controlled variable, it was not possible to discriminate between those experiments in which tracking occurred and those in which it did not. However, there were indications that localisation error increased and was more variable when the moving rod was followed.

The role of eye movements had already been discussed in Chapter III.7 (a). There, it was concluded that gross eye movements could contribute to a rise in threshold, but not to the magnitude of a localisation error. Hering's Law was invoked to exclude a saccadic suppression theory.

The L - F - S theory, as set out previously, is not strictly applicable to a situation in which there are tracking eye movements. Just how the subject would organise his responses is not at all obvious.

It could be that when tracking, the subject attends to the depth difference between the moving and stationary rod for a greater period of time, that is, sampling the path over a greater segment than when fixation is static. However, this means that the localisation error should be more negative, since the actual path was concave towards the subject. CMJ's results, during the (assumed) tracking phase, increased in the positive sense, the moving rod being localised further away than its true distance.

Alternatively, binocular fixation might not have been accurate during the tracking eye movements, altering the fixation disparity relationships.

Before any firm conclusions can be drawn, experiments with controlled eye movements are necessary. In the meantime, most of the foregoing is purely speculative.

VII.2 Variations of localisation error with path shape.

The path sampling hypothesis predicts that localisation error should increase negatively as the radius of the path is decreased (Chapter III.8b).

From Figs. VI.19 to VI.22, the following observations can be made:

(i) For all of CMJ's results (Fig. VI.19), the path sampling hypothesis is supported by the results of only one set of conditions (4.2^{o}/sec velocity and 384 cd/m^2). For the other three sets of conditions, localisation error increased positively when path radius was decreased, contrary to the predictions of the hypothesis.

(ii) For CMJ's later, more consistent, results, the hypothesis was supported for both velocities at the higher luminance level, but negated for the lower level (Fig. VI.22).

(iii) BK's results (VI.22) supported the hypothesis for all conditions except that of lower luminance and lower velocity.

(iv) CDN's results (Fig. VI.21) showed no variation whatsoever with path radius.

Because of the variability of the localisation errors, the comparisons are dubious. The path sampling hypothesis is neither confirmed nor denied, although it must be admitted that for two subjects the shape of the path did have an effect on localisation error.

VII.3 Variation of localisation error with luminance.

As pointed out in Chapter VI.7, the technique of pairing stimulus conditions, in this case luminance levels, proved to be a satisfactory experimental method.

The results showed that the localisation error increased positively with luminance level. This was in agreement with the results reported by Lit in 1960c, but not with his subsequent results (Lit, 1966), in which localisation error decreased positively with luminance.

Lit's data was based on four subjects (two reported in 1960c and two in 1966), and the present data on three subjects, so that little can be drawn from the conflicting results. Indeed, all that can be said at this stage is that the variation with luminance level seems to be a characteristic function of the subject.

VII.4 Variation of localisation error with velocity.

No systematic variation with stimulus velocity was found (see Figs. VI.19 to 22). This is not at variance with Lit's results (Lit, 1960c, 1966), which showed an increase of localisation error with velocity. The two velocities used in the present experiment were in the lower range of those used by Lit (1960c), where his graphs show little variation. The additional data presented by Lit in 1966 began at 5^{o}/sec, so that results comparable to those presented here are not available.

VII.5 Conclusions.

The most significant result of the experiments reported here has been that the localisation error of kinetic stereoscopic depth perception varies, both from day to day and for shorter (within session) periods.

This variability makes firm conclusions difficult, and poses more questions than have been answered. It certainly deserves further investigation.

It is interesting to speculate whether similar short term variations occur with other stereoscopic functions. In experimental designs using other than up-and-down methods, such variations could be disguised as increased thresholds.

The existence of variations means that results under different stimulus conditions cannot be properly compared unless the conditions are presented more-or-less concurrently Thus, in order to compare the effects of changing path shape, observations with different paths have to be interleaved during each experimental session. This presents no procedural problems, but the mechanical difficulties are somewhat forbidding.

The theory involving latency, fixation disparity, and path sampling, developed in Chapter III, continues to serve as a useful model on which to base further research. It has been fairly well established that latency varies with retinal location, and the existence of fixation disparity is indisputable (see Chapter III); what needs to be done is for these factors to be studied under the same conditions as the localisation experiments.

The oncept of path sampling remains hypothetical. Further experiments need to be made along the lines of those described here, with convex as well as concave path shapes.

Another way of testing the hypothesis is to examine the effect of reducing the field of view. The prediction would be that, by forcing the subject to use shorter path segments for his judgments, there would be a decrease in the localisation error, and perhaps an increase in the threshold. Experiments along these lines were projected by Lit & Hyman (1951), but have yet to be carried out.

The role of eye movements in kinetic stereoscopic localisation is still very uncertain. Experiments in which eye movements are monitored during a localisation task are an obvious next step.

Finally, it may be pointed out that the stereoscopic localisation of moving objects has im-portant implications for theories of space perception generally. More often than not, we make our perceptions when there is movement of the head, of the eyes, of the stimulus, or of all three, not under the static conditions which characterise most experiments in stereoscopic perception.

-o-O-o-

APPENDIX A

A REVIEW OF SEQUENTIAL METHODS FOR PSYCHOPHYSICS

A.1 Introduction.

"Up-and-down" or "staircase" methods for estimating points on a response curve have found many applications since their development during the 1940's for the testing of explosives (Dixon & Mood, 1948). Such estimates are frequently the aim of psychophysical experiments, and during the past decade there have appeared several refinements which provide reliable and useful tools for psychophysical research. However, their application to vision research is still sparse, if one is to judge from the recent literature, where several reported studies could have used "up-and-down" methods to advantage.

Admittedly, many of the developments have been in fields remote from vision and perception. As already mentioned, the original work reported by Dixon & Moon in 1948 was related to explosives, while an important analysis by Wetherill (1963) was in the context of some properties of plastic pipes. Most the psychophysical refinements have arisen from work in audition (e.g., Wetherill & Levitt, 1965, Campbell, 1969, Taylor & Creelman, 1967), but credit must be given to Cornsweet (1962) for showing how the simple up-and-down method can be made to rival the method of constant stimuli as the method of choice.

The popularity of the constant methods is another reason why up-and-down methods are not widely accepted. Indeed, according to some authors (e.g. Sidowski, 1966), it is almost heretical to claim that there may be something better than the method of constant stimuli. It will be argued in a later section that, in comparison with up-and-down methods, the constant method is inefficient, in the sense that it requires more work to be done than is really necessary.

The purpose of this Appendix is to collate and review the various refinements to the up-and-down method, and to justify their use in the experiments described elsewhere in this thesis.

A.2 The sequential nature of up-and-down experiments.

As well as "up-and-down" or "staircase" (Cornsweet, 1962), the adjectives "adaptive" (Taylor & Creelman, 1967) and "sequential" (Wetherill & Levitt, 1965) are often applied to the methods under discussion. "Up-and-Down" (UD) seems to be the most popular term, perhaps because it is aptly descriptive, and this usage will be followed here. The term "adaptive" is not particularly suitable in psychophysical applications, because of possible confusions with words such as "adaptation" which have other connotations. "Sequential" methods refers to the class of statistical techniques (Wetherill, 1966) to which the UD method belongs.

A sequential experiment may be defined as "one in which the course of the experiment depends in some way upon the results obtained" (Wetherill, 1966). More specifically, the sequential analysis is aimed at making one or both of two decisions: what factor level should be used in successive stages of the experiment, and when should the experiment be terminated. In psychophysical terms: what stimulus level should be presented for each observation, and how many observations should be made.

Sequential rules for stopping an experiment can be applied to most experimental methods. For example, with the method of adjustment one could calculate the standard error of the mean (SE_m) after each observation, stopping when a satisfactorily low value is obtained.

UD experiments are characterised by the application of sequential rules to the first decision, i.e., which stimulus to present next. Rules for stopping might or might not be sequential.

The sequential nature of UD methods may be contrasted with the classical constant method (Guilford, 1954), in which the stimulus levels and the number of observations are decided upon before the experiment begins. Apart from this distinction, the two types of methods are similar, since in both types the stimulus is varied in discrete steps, and presented in an order which is not known to the subject. In some UD designs, curve fitting techniques similar to those of the constant method can be applied.

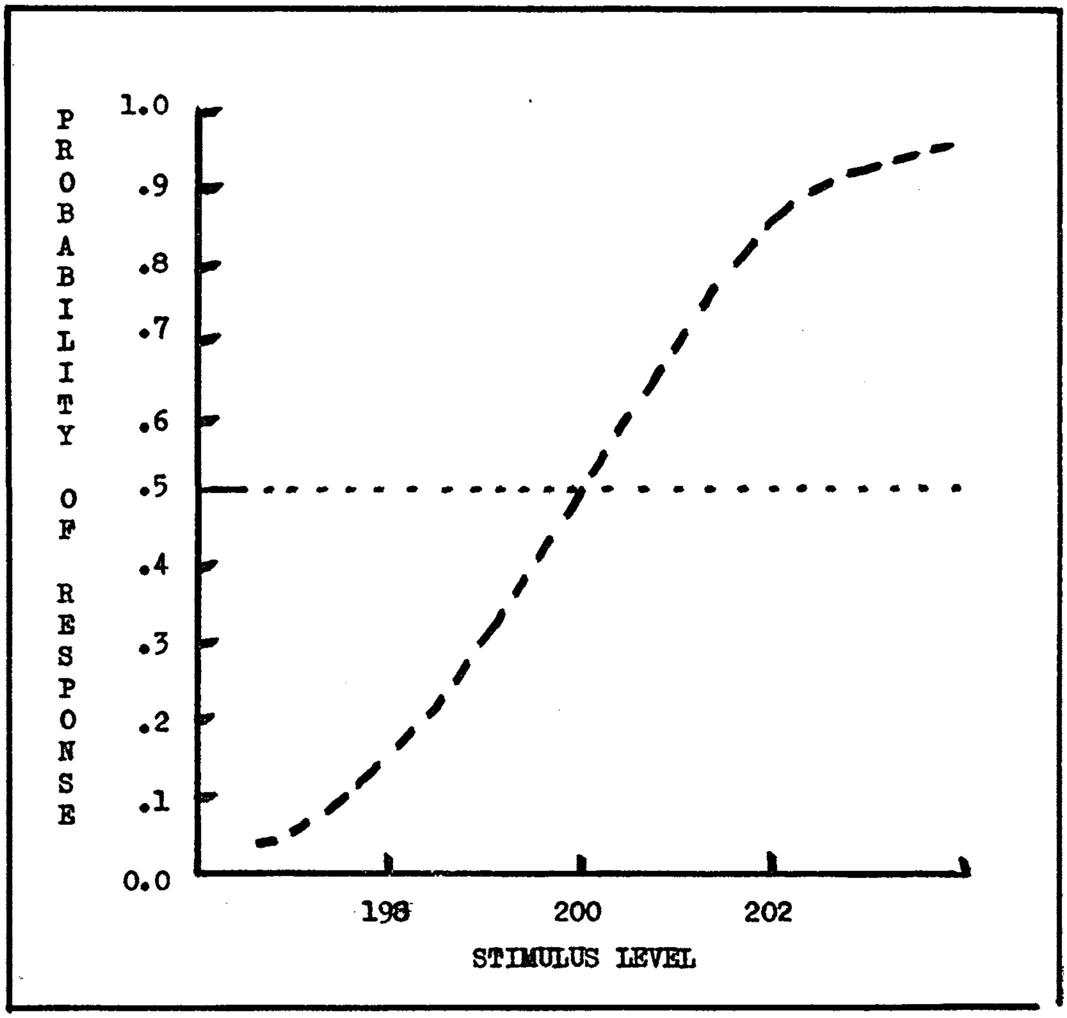

Fig. A.1. A typical psychophysical response curve.

A.3 The up-and-down method of Dixon & Mood.

For the purpose of discussion, imagine an experiment in which a horizontal line, 200 mm long, is presented somewhere in the subject's visual field. We wish to find the length of line which, when presented elsewhere in the field, is judged equal to the first. The first, fixed, length is the reference stimulus L_r; the second, variable, length is the test stimulus L_v. It is assumed that the probability of the response "L_v is greater than L_r" follows a symmetrical function as shown in Figure A.1.

As in a constant stimulus design, the point of subjective equality (PSE) is defined as the stimulus value $L_{.5}$ which elicits the required response in 50% of the observations. As a measure of the subject's sensitivity or differential threshold, we can use the standard deviation of the assumed distribution, or some other convenient percentage points such as 75% or 25%.

In the simple UD method of Dixon & Mood (1948), a guess is made at a suitable size for the steps between stimulus levels, and an initial stimulus is selected at a value close to the expected $L_{.5}$. Neither of these guesses need be accurate; for a response curve resembling the normal distribution ogive a step size within the range 0.5σ to 2σ is suitable, and a poor guess can be rectified during the course of the experiment. A poor initial guess at $L_{.5}$ means only that a few observations will be wasted.

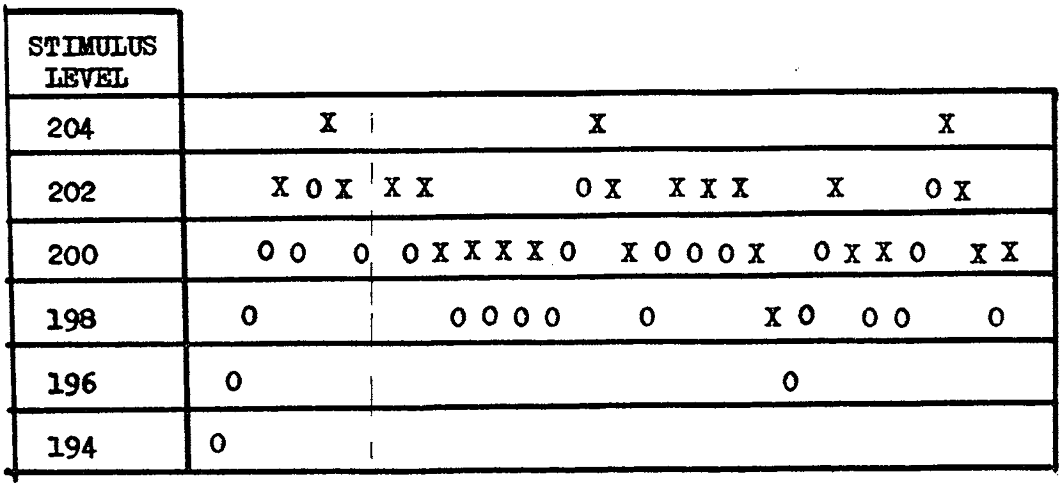

Fig. A.2. Record of an up and down experiment. At each 'X', the variable stimulus, at the indicated level, was reported as "greater than the reference stimulus", while theO's represent reports of "less than...". The ten trials before the dotted line are not used in the analysis, for reasons given in the text.
The experiment was simulated from a computer generated set of randomly selected normal deviates; mean = 200.0, standard deviation = 2.0.

In psychophysics, it may be desirable to select an initial stimulus far from $L_{.5}$, giving the subject several easy judgments with which to familiarise himself with the experiment.

For our imaginary experiment, a reasonable guess at $L_{.5}$ would be 200 mm. A step size of 2 mm will be used, with the initial presentation at 194 mm.

The rule for selecting subsequent stimuli is as follows. If the response to the previous stimulus was "L_v greater than L_r", reduce the stimulus by one step for the next observation. If the previous response was "L_v less than L_r", increase by one step.

Thus, in the experiment, the response to the first stimulus, 194 mm, might be "less", in which case the second stimulus will be 196 mm. If the second response is "greater", then the third stimulus will be 194 mm., and so on. A typical record is shown in Fig. A.2, where 'X' and 'O' represent "greater" and "less" responses respectively. In a more general terminology, they represent "hits" and "misses".

Groups of observations leading to a change in response are often referred to as "runs" (Wetherill & Levitt, 1965). In Fig. A.2, the first five observations constitute a run, the fifth and sixth are another run, and the sixth, seventh, and eighth make up the third run. There are 29 runs altogether.

Other descriptive terms such as "peaks", "valleys", and "reversals", are self explanatory.

It is desirable to have some fixed rule for excluding a number of initial observations, in order to minimise the effects of a poor initial guess at $L_{.5}$, and to make allowance for a period of familiarisation. Typical rules are based on excluding the first _k_ runs, rather than the first _m_ observations, _k_ being the same for different experiments in the study. This ensures that the first "real" observation will be close to $L_{.5}$, and that the experiment does not bias the results by beginning the series with runs close to some expected or desirable value.

If there is a question of temporal adaptation, one could decide upon some standard time period with which to define the excluded early observations.

In the simulated experiment of Fig. A.2, the first five runs (ten observations) will be excluded, leaving forty observations for the estimates.

A number of estimators for $L_{.5}$ have been proposed. Dixon & Mood (1948) suggest counting the X's and O's and using the event with the lesser frequency. The justification for this is that the number of O's at any level cannot differ by more than one from the number of X's at the next higher level. Thus the distribution of either the X's or the O's contains all of the useful information from the experiment, and the use of the less frequent response eliminates the effects of starting at too high or too low a level.

In Fig. A.2, there are twenty-one X's and nineteen O's, so that the distribution of O's will be used for the estimates. Had it been decided to include the runs before the dotted line, the low starting level would have been reflected in the greater number of O's, and the X's would have been used.

Dixon & Mood use a maximum likelihood method for obtaining estimators of $L_{.5}$ and the standard deviation. If there is some assurance that the response curve is similar to the normal cumulative ogive, and that the step sizes are constant, the following approximate formulae may be used:

$$m = L_o + d\ (A/N \pm 0.5) \qquad \dots \quad \dots \quad A1.$$

$$\underline{s} = 1.62d\ (V + .029) \qquad \dots \quad \dots \quad A2$$

In equation A.1, $\underline{d}$ is the step size, and A is the product of the coded stimulus values (see Table I) and their respective frequencies. L_o is the stimulus level coded as zero, the coding being done merely to simplify the calculations.

$\underline{m}$ is the estimate of $L_{.5}$, and can be seen to be simply the mean of the distribution, with an adjustment ($\pm$ .5d) depending on whether the analysis is based on the O's (plus) or X's (minus).

In equation A2, V is the variance of the distribution, computed in the usual way as shown in Table I, and s is the estimate of the standard deviation. As Dixon & Mood point out, it is curious that the estimate of the standard deviation is based on a linear function of the variance; the explanation is to be found in their maximum likelihood analysis.

L_V	X's	O's	L'	n.L'	n. $(L')^2$
204	2	0	3	-	-
202	8	2	2	4	8
200	10	7	1	7	7
198	1	9	0	0	0
196	0	1	-1	-1	1
Totals	21	19		10	16
		=N		=A	=B

$$\underline{m} = 200 + 2\ (10/19 + 0.5) = 200.052$$

$$V = (NB - A^2)/N^2 = .564$$

$$s = 3.24\ (.564 + .029) = 1.924$$

TABLE I. Analysis of the results shown in Fig. 2, using the estimators of Dixon & Mood (1948).

Figure A2 was based on a random sample from a normally distributed population generated by a computer. (The program generating the distribution is described in a Note towards the end of this Appendix). The population had a mean of 200.00 and a standard deviation of 2.00. Both estimates calculated in Table I are well within the range of the standard errors for small samples of the population.

The maximum estimate solution also provides estimates of the standard errors of the mean and standard deviation, SE_m and SE_s. SE_m is particularly useful, since it enables the differences between means to be tested for significance. These estimates are discussed in Section A.9, with reference to modifications suggested by Kappauf (1967).

A.4 Other Estimators of L_5.

A related estimator was suggested by Brownlee, Hodges & Rosenblatt (1953). This is the arithmetic mean of all the stimulus values presented during the experiment, omitting the first observation and adding another observation at the stimulus level which would have been presented at the (n + 1)st observation, n being the number of observations actually presented. The first observation, having been selected beforehand, contains no information about m, whereas the (n + 1)st is informative and known, even though not presented.

For the data of Figure A2, (after the dotted line), only nine observations are counted for 202, and another is added to the count for 198. The resulting mean is, fortuitously, exactly 200.00. (Since the first observation in the series was preceded by ten initial observations, it _is_ informative, and should not be discarded.)

The two estimators of _m_ so far discussed are asymptotically similar, i.e., they will tend to agree for large samples. However, Brownlee _et al_ claim that their estimator has better properties for small samples. They do not discuss the estimate _s_.

Wetherill (1966) gives an estimator for $L_{.5}$ which illustrates the similarity between the UD method and the method of limits. This estimate is the average of the midpoints of the last interval in each run. In figure 2, for the first, descending, run (after the dotted line) the midpoint is 201, as it is for the second, ascending, run. For the third run, observations 3 to 5, the midpoint of the end interval is 199. Proceeding in this way, the average of the end-interval midpoints is 200.5: Wetherill's estimate _w_ (see Table II, last column).

A Midpoint	B Ascending runs	C Descending runs	D Totals
203	2	0	2
201	5	3	8
199	5	8	13
197	0	1	1
Totals	12	12	24

Means:
a. For ascending runs (column B): 200.5
b. For descending runs (column C): 199.33
c. For all runs (column D): 199.92

TABLE II. Analysis of the data of Fig. A2. The means a and b could be interpreted as ascending and descending limits. The third mean is Wetherill's estimate _w_ (Wetherill 1966).

Wetherill's procedure is basically the same as the analysis of an ascending and descending limits experiment in which the stimulus is changed in discrete steps. It seems valid to take the average of the peak end interval midpoints as the ascending limit, and likewise use the valleys for the descending limit. The results for Fig. 2A are, from Table II, 200.5 and 199.33. Half the interval between the two limits, 0.58, could be used as an index of the differential threshold. This index cannot, of course, be compared with the maximum likelihood estimation of the standard deviation.

For large samples and with well selected step sizes, there is little to choose between the various estimators. Even with small samples (less than twenty observations), the differences between the methods is not likely to be greater than any sampling errors.

A.5 An alternative estimate of the standard deviation.

In his studies of the UD method, Kappauf (1967, 1969) used computer simulated experiments, and found that the Dixon-Mood estimate s is biased, the bias being partly a function of the number of trials. Kappauf (1967) proposes the following empirically derived estimator:

$$s_K = (1.71\ d.V.N)/(N - 1) \qquad \ldots \quad \ldots \quad \text{A3.}$$

in which s_K is Kappauf's estimate of σ, and d, V, and N have the same meanings as in equation A2.

For the example of Fig. A2, s_K is 1.02, a slightly better estimate than the Dixon-Mood result.

In many applications, one is not concerned so much with the actual value of σ, as with the way in which it varies over a range of experimental conditions. For such comparisons, Dixon & Mood's s may be preferable, if only because it is easier to calculate. For large N, the two estimates approach equality.

If confidence intervals are to be based on the estimate of σ, it may be advisable to use s_K, for reasons given in section A8.

A.6 Step sizes, and rules for stopping.

What step size to use is decided partly by the sequential nature of the UD method, as well as any prior knowledge that the experimenter might have. Ideally, a step size equal to the standard deviation should be used (Dixon & Mood, 1948). With too large a step size, the observations will oscillate between two stimulus levels, giving no information other than that $L_{.5}$ is somewhere between these two values. Too small a step size leads to inefficiency, requiring many observations for good estimates of m and s.

A step size within the range 0.5σ to 2σ is generally satisfactory. A useful rule is that most runs should be made up of three or four trials. If after a few runs it appears that this is not the case, then the step size should be

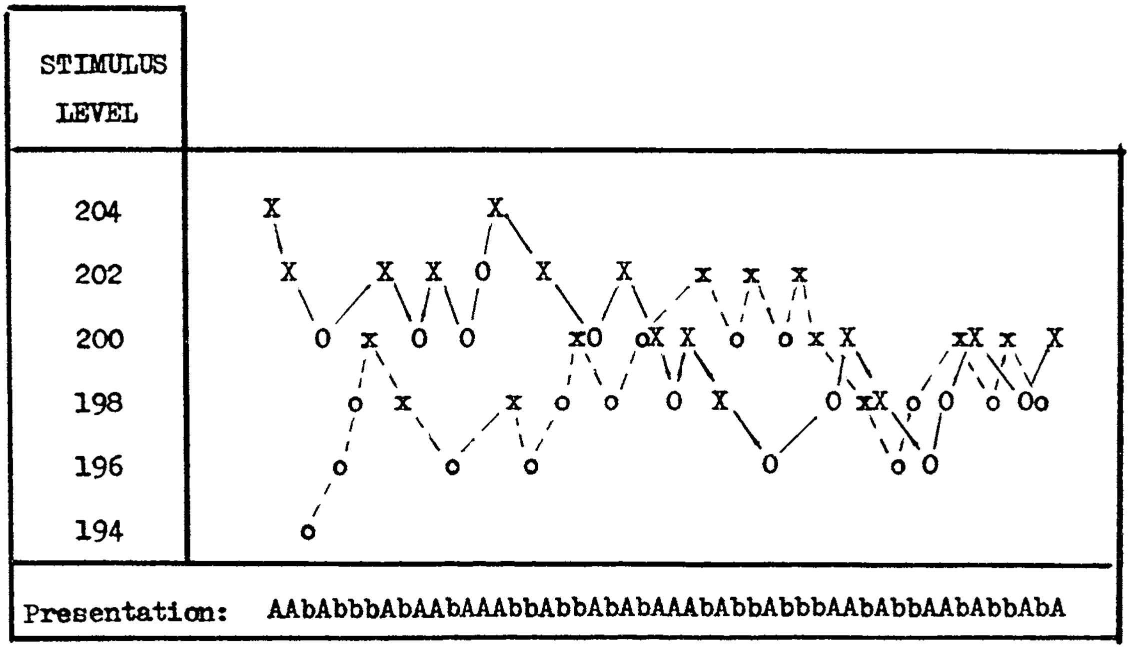

Fig. A.3. Random double staircase experiment (homogeneous complementary series). Series A (X,O) was begun at 204 stimulus units, Series B (x,o) at 194 units. The order of presentation was the randomly determined sequence shown along the bottom of the record. Drawn from the same population as Fig. A.2 (mean = 200, standard deviation = 2).

changed accordingly.

A sequential rule for stopping in UD experiments is to stop when some pre-selected value of the standard error of the mean (SE_m) is obtained. Since SE_m is dependent on <u>s</u> (see Section A9), a number of calculations have to be made after each trial, a procedure which is practical only if on-line computing facilities are available. If there is some prior knowledge of , the approximate number of trials for a given SE_m can be calculated beforehand (Section A9).

A.7 Randomly interleaved series.

An obvious objection to the UD method is that in psychophysics, the subject could detect a pattern in the stimulus presentations, and in some way adjust his responses. Also, it may not be safe to assume that successive responses are independent.

The classical constant method meets this objection by presenting the stimuli in a random order. A similar design for UD experiments was devised by Cornsweet (1962).

In Cornsweet's "random double staircase" method, two series, with different starting points, are presented. The order in which each series is visited is determined by a random sequence, as illustrated in Fig. A.3. The figure is derived from the same population as that of Fig.A.2.

In a useful terminology proposed by Kappauf (1967) Cornsweet's method, which has gained wide popularity in psychophysics, is described as <u>homogeneous concurrent series</u> (HCS).

The results may be analysed as two separate series and averaged; alternatively, the two sets may be pooled and analysed together. The second procedure may give an over-estimate of the standard deviation, particularly if both series begin three or more steps from the mean.

Making a separate analysis for the two series in Fig. A.3 gives a mean of 299.73 for Series A, and 201.0 for series B. $L_{.5}$ is the average of these two figures, 500.12, illustrating how sampling errors in one series are often counteracted by errors in the other.

Likewise, the Dixon & Mood estimates *s* are 2.6 and 1.86, giving a final estimate of 2.23 for the standard deviation.

Since one has virtually carried out two experiments, double staircase methods add greater certainty to the results.

As far as the subject is concerned, his experiences during a random double staircase experiment are little different from those of a classical constant stimulus design.

A.8 Complementary concurrent series (CCS)

According to Kappauf (1967, 1969a), the reliability of the estimates of $L_{.5}$ (*m*) and *s* (or s_K) is affected by the phase relationship between the test levels and the true value of $L_{.5}$.

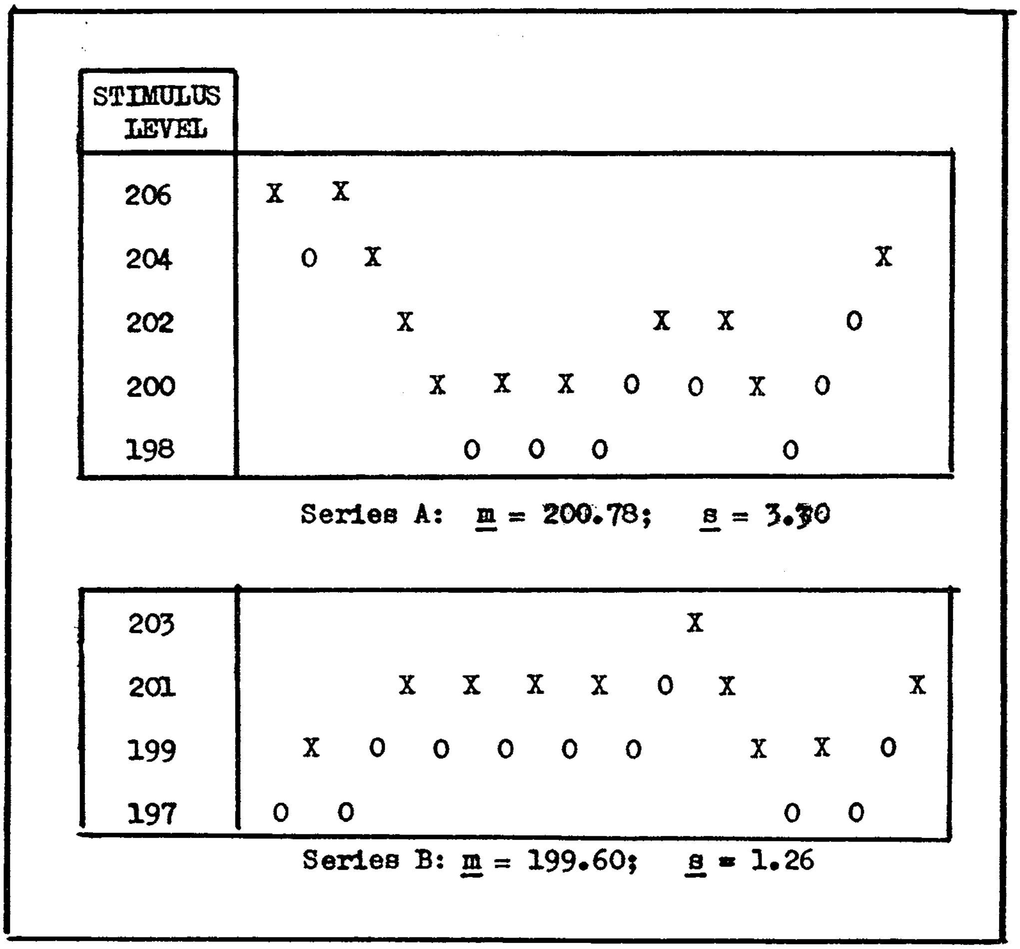

Fig. A.4. Complementary concurrent series experiment. Population mean: 200.0; standard deviation 2.0.

Phase is defined by:

$$\phi = (L_{vo} - L_{.5})/d \qquad \ldots \quad \ldots \quad A.4$$

in which L_{vo} is the test level nearest to $L_{.5}$, and _d_ is, as before, the step size.

Poor estimates are more likely to occur if _d_ is too large.

In his computer simulations, Kappauf found that _s_ and _sK_ are positively biased when the phase is ;00, and negatively biased when it is .50. The bias in _m_ is positive for ϕ = -.25, and negative for ϕ = +.25.

The biases can be eliminated by running two concurrent series which have the same step size, but in which the test levels in one series differ by half a step from the levels in the other. Kappauf calls these _complementary concurrent series_ (CCS).

In our example, a CCS design would consist of using the test levels 196, 198, 200 ... for one series, and 197, 199, 201 ... for the other. Whatever the value of $L_{.5}$ may be, the difference between the two phases will be .5, satisfying the conditions for minimum bias.

The method is illustrated in Fig. A.4, again drawn from a population with mean = 200.0, standard deviation = 2.0. In practice, the series are visited in a random order, as in Cornsweet's method.

The first series has a phase of 0.0, and as predicted by Kappauf, $\underline{s}$ = 3.30 is an over-estimate. The second series has a phase of 0.5, giving an underestimate of $\underline{s}$ = 1.26. The mean of the two estimates is 2.28, more accurate than either alone.

Since the two phases are both ideal for estimating $\underline{m}$, errors in the two results are due to other factors. Again, however, their mean, 200.19, is better than either of the individual results, 200.78 and 199.60.

The use of complementary series adds little to the complexity of double staircase designs, while removing some of the uncertainties associated with inappropriate phasing. Kappauf is justified in recommending that the method be used in psychophysics.

A.9 <u>Standard errors of the mean and standard deviation; confidence intervals</u>.

In their maximum likelihood analysis, Dixon & Mood (1948) showed that the usual estimates for the standard error of the mean (SE_m) and standard deviation (SE_s), $\sigma/\sqrt{N}$ and $\sigma/\sqrt{2N}$ respectively (Guilford, 1965), do not apply to the up-and-down solutions. They presented the formulae:

$$SE_m = G\underline{s}/\sqrt{N} \quad \ldots \quad \ldots \quad (5)$$

and

$$SE_s = H\underline{s}/\sqrt{N} \quad \ldots \quad \ldots \quad (6)$$

where $\underline{s}$ is the estimate of the standard deviation (Eq. A2), and G and H are functions of $\underline{d}/\underline{s}$. Values of G and S could be obtained from a graph published by Dixon & Mood.

The function for G is fairly linear for $\underline{d}/\underline{s}$ in the range .5 to 2.5, and a good approximation for equation A5 is given by:

$$SE_m = (0.1\underline{d} + 0.9\underline{s})/\sqrt{N} \quad \ldots \quad \ldots \quad (A.5A)$$

It must be remembered that SE_m and SE_s are estimated from estimates, and should be used with some caution. Nevertheless, Dixon & Mood indicate that SE_m can be used for calculating confidence intervals, using the $\underline{t}$ distribution for (N - 1) degrees of freedom. Thus if $\underline{r}$ is the appropriate $\underline{t}$ ratio for a given level of significance, the corresponding confidence interval is $\pm\, r.SE_m$.

For complementary concurrent series, Kappauf (1967) found that a good estimate of SE_m was given by:

$$SE_{mc} = (.82\underline{s} + .16\underline{d})/\sqrt{N_1 + N_2} \ldots \quad \ldots \quad (A.7)$$

where N_1 and N_2 refer to each series.

The number of observations necessary for a given SE_m can be predicted if there is some knowledge of the standard deviation, by solving equations (A.5) or (A.7) for N or $(N_1 + N_2)$, keeping in mind that N represents half the number of trials in a series. As Kappauf (1967) points out, the choice of $\underline{d}$ contributes only slightly to the step length, the important factor being the ratio of SE_m to $\underline{s}$.

For example, suppose we require $SE_m = .25\underline{s}$. Then from equation A.7,

$$\sqrt{N_1 + N_2} = 3.28 + .6\ (d/s)$$

For $\underline{d} = \underline{s}$, $N_1 + N_2$ is 15.05, i.e., about thirty-one observations will be required. Similarly, for $\underline{d} = 2\underline{s}$, about forty-one observations are necessary. On the other hand, $SE_m = .2\underline{s}$ requires about fifty observations for the first case, and about seventy-five for the second. Thus for a given SE_m, large step sizes can be used without a drastic increase in the number of observations; from a psychophysical viewpoint large step sizes are desirable in that the subject will have a greater proportion of easier judgments than otherwise. Kappauf's estimates (equations 3 and 7) allow for the biases due to short test lengths and large step sizes.

Kappauf (1969b) describes a computer controlled experiment in which SE_m is periodically calculated during the tests, the experiment terminating when a satisfactory SE_m is reached. This is hardly practical if there are no on-line computing facilities available, but if there are occasional rest periods an experimenter with an efficient calculating machine could use this time to compute the estimates.

The estimates of $\underline{s}$, SE_m, and SE_s are meaningful only if there is some assurance that the response curve is similar to the normal cumulative ogive. If the curve departs from normality at the tails, the estimates may still be of value, since the UD method ensures that most of the stimuli will be clustered about the mean.

The estimate of $L_{.5}$ ($\underline{m}$) requires only that the response curve be symmetrical.

Many psychophysical functions are symmetrical and/or normally distributed only if the stimulus units are on some non-linear scale, such as a logarithmic scale. The need for a transformation and its type are better indicated by a standard constant stimulus design rather than by a UD experiment, but variants of the UD method have been developed which give reliable and efficient estimates of points on the response curve other than $L_{.5}$. These methods are discussed in the following sections.

A.10 Transformed response curves.

The up-and-down transformed response rule (UDTR) was first proposed by Wetherill (1963, 1966), and discussed in a psychophysical context by Wetherill & Levitt (1965).

In a typical UDTR experiment, the stimulus level is stepped down only after two 'X' responses have occurred in succession. This has the effect of transforming the response curve defined by $p_L = F(L)$, where p_L is the probability of a response at the stimulus level L, into a function in which $p'_L = T(p_L)$. More specifically,

$$p'_L = (p_L)^n \qquad \dots \quad \dots \quad (8)$$

n here is the number of successive responses required before the stimulus level is changed. For example, if $n = 2$, the level is changed after 2 successive X's, and $p'_L = (p_L)^2$. The UD method tracks the median level of the transformed curve, hence:

```
STIMULUS
  LEVEL

  203        XX                                                   XX

  202     XO    XX  XX          XX                            XO     XO

  201                  O   XX  XO   XX  XX          XO

  200                          O          O   XX  XO

  199                                          O
```

Fig. A.5. UDTR experiment designed to find $L_{.707}$. The stimulus level was increased after every 'O' response, and decreased only after two successive X's. Analysis according to Wetherill's method gives $L_{.707} = 201.11$ (expected value 201.06). Population mean = 200.0; standard deviation = 2.0.

$$p_L = (0.5)^{\frac{1}{2}} = .707$$

That is, the UDTR strategy with n = 2 tracks $L_{.707}$, the stimulus level at which a positive response is obtained in 70.7% of trials. Such an experimental design is illustrated in Fig. A5.

Wetherill (1966) recommends using the estimate of L_p based on the mid-points of the last interval in each run (section A.4), while for samples of 100 or more the simpler average of valleys and peaks (Wetherill & Levitt, 1965) is as good as a maximum likelihood solution. In practice, there is little to choose between the two estimators. For the data of Fig. A5, the first estimate is 201.11, while the peak-valley average is 201.10; both are very close to the expected result of 201.06.

In another UDTR strategy, the rule specifies when the level should be increased, i.e., after n successive '0' responses. Here, $(1-p_L) = (1-p)^n$. For example, n = 2 would give an estimate of $L_{.293}$.

Wetherill & Levitt (1965) suggest using two randomly interleaved series, one series to track p and the other to track (1 - p). This ensures, amongst other things, that the number of 'X' responses in the whole experiment will be about the same as the number of 0's.

Other UDTR strategies are shown in Table III, which is adapted from Wetherill & Levitt (1965). The method becomes impractical for extreme points, because of the large number of observations required.

Entry No.	Response type D	U	Percentage point estimated
1	XX	0,X0	.707
2	XXX	0,X0,XX0	.794
3	XXX,XX0X	0,X0,XX00	.734
4	XXXX	0,X0, etc.	.841
5	X,0X	00	.293
6	X,0X,00X	000	.206
7	X,0X,00XX	000,00X0	.266
8	X,0X, etc.	0000	.159

TABLE III: Some possible UDTR rule patterns (adapted from Wetherill & Levitt, 1965). The stimulus level is increased after a pattern of type U, decreased after type D.

Cornsweet & Pinsker (1965), apparently working independently, used the first strategy in Table VI in a study of Weber's Law, and suggested that the point estimated was $L_{.75}$. Wales & Blake (1970) in a note on Cornsweet's method, reported computer simulations of the strategy which indicated that the level tracked was close to the 65% point. This is at variance with Wetherill's simulations (Wetherill, 1966), in which the same strategy with small step sizes gave results in agreement with the expected result, while larger step sizes appeared to track higher percentage points.

Wetherill used a logit model for his response curve, while Wales & Blake used a linear model; theoretically, this should make little difference.

It seems that studies of the influence of step size, phasing, and test length are necessary before the data of UDTR experiments can be properly analysed. Nevertheless, Wetherill's (1966) theoretical basis is sufficiently sound to justify the use of UDTR rules, particularly if one takes the precaution of using small step sizes. Small step sizes in an UDTR experiment do not necessarily increase the difficulty of the subject's task, since most of the stimuli will be away from the 50% point.

UDTR rules for estimating $L_{.5}$ are of value when the response curve is markedly assymetrical at one or both extremes. In such cases, it is desirable to limit the stimulus levels to the symmetrical part of the function. To achieve this, Levitt & Rabiner (1967) used UDTR rules which converge on the 50% point. The simplest of these is a "best-of-three" strategy, in which the stimulus level is reduced for 'XX', 'OXX', or 'XOX', and increased for 'OO', 'XOO', or 'OXO'.

Following the development of equation A8, the best-of-three strategy is defined by:

$$p_L' = p_L^2 (3 - 2p_L) \qquad \dots \quad \dots \quad A9.$$

Once again, the UD method tracks $p_L' = 0.5$, and the solution of equation A9 is $p_L = 0.5$.

STIMULUS LEVEL												NUMBER OF BLOCKS
202.4											(?)	(1)
201.8	10			11						9		3
201.2		9	8		11		9	9	7			6
200.6						8						1

Fig. A.6. A BUDTIF experiment, sampled from a population with mean = 200.0 and standard deviation = 2.0. The target level is the 75% point, the expected result being $L_{.75}$ = 201.34. Stimuli were presented in blocks of 12, the numbers in the graph showing the number of correct responses in each block. (?) is the level at which the next block would have been presented.

Evidence that the standard errors of the estimate of $L_{.5}$ are less for the "best-of-three" method than for the simple UD rule is given by Wetherill (1963) and Levitt & Rabiner (1967).

A.11 Block up-and-down methods - BUDTIF

Campbell (1963) described an experimental design with the acronym BUDTIF - Block Up-and-Down, Two Interval Forced-choice. The context was an auditory signal detection experiment, the subject having to decide which of two time intervals contained the signal, but the method could be applied to most other two alternative situations.

In BUDTIF, the stimuli are presented in blocks of 8 to 12. The stimulus level is changed after each block, the direction of the change depending on the percentage point being tracked and the number of positive (or correct) responses in the block. For example, to track the 75% point, if there are more than nine 'hits' in a block of twelve, the level is decreased for the next block, while it is increased if there are less than nine. For exactly nine 'hits', the stimulus level is not altered.

An example of a BUDTIF experiment is given in Fig.A6, again drawn from a normally distributed population of mean = 200.0, standard deviation = 2.0.

L_p is calculated as the median of all the stimulus levels at which trials were made, i.e., the median of all blocks, regardless of the proportions of correct responses.

Levels at which there were only one block are excluded, presumably as a precaution in case the experiment was started at too high or too low a level. In thedata of Fig. A6, it would seem unwise to omit the entry at 200.6, since this appears to be a valid result.

Since the analysis is based on the number of times each level was visited, regardless of the result, it would be logical to follow the rule of Brownlee _et al_ (1953) (see Section A4), omitting the first arbitrarily decided block and adding another at the level which would have followed the last block.

Following Campbell's instructions exactly gives $L_{.75} = 201.40$. If the block at 200.6 is included, $L_{.75} = 201.32$. Using Brownlee's rule, $L_{.75} = 201.38$. Each of these is sufficiently close to the expected 201.34 to suggest that any differences would be less than sampling errors.

Since there are many observations at each level, BUDTIF is amenable to analysis by curve fitting with the method of least squares. In fact, BUDTIF could be used as a means of ensuring that efficient stimulus levels are being used in an otherwise standard constant stimulus experiment.

As a measure of variability, Campbell (1963) suggests that the semi-interquartile range $\underline{Q}$ be used. He says that $\underline{Q}$ is readily calculated, but how it is calculated is not explicit in his paper. In his example, he gives Q = 0.6, but the standard deviation of the distribution of blocks is 0.665 when calculated in the usual way (Guilford, 1965), giving a Q of 0.45.

The step size should be "not too small or too large". The optimum size is such that Q's of about 0.7 step are obtained, the standard deviation is 1.0 step, and a difference of about 75% of correct judgments is obtained near the 75% level. (One might point out that the preceding statement and the first sentence of this paragraph are not very informative).

In his 1963 paper, Campbell indicated that blocks of at least eight should be used. Campbell & Lasky (1968) found that the most efficient block size is the smallest which permits tracking of the required percentage point. Thus, to track the 75% point, a block size of four is optimal.

For greater precision, Campbell & Lasky suggest that it is preferable to run several short series rather than a few long ones.

A suitable stopping rule for BUDTIF is to stop after a given stimulus level has been revisited a given number of times, four to eight revisitations being suggested as parsimonious choices.

BUDTIF methods as described by Campbell are designed explicitly for signal detection experiments, in which the signal is present during one of two time intervals, or in one of two locations (as in visual threshold experiments).

For PSE applications, interleaved series should be run, as a precaution against the subject recognising the pattern of presentations. As with UDTR rules, two interleaved series could be aimed at finding different percentage points.

A.12 PEST.

Somewhat more complex sequential rules have been devised by Taylor & Creelman (1967), in a method called Parameter Estimation by Sequential Testing (PEST).

In PEST, if p is the target probability, and T trials have been made at a stimulus level L_v, then the number of correct responses should be greater than T.p if L_v is greater than L_p, and less than T.p if L_v is less than L_p. A constant W is selected, and the stimulus level is changed when the number of correct responses is beyond the range T.p $\pm$ W. The "power" of the decision made by this test increases with W. but so does the necessary number of trials. Taylor & Creelman recommend values of W of one or two.

The step size is varied according to rules developed "partly from intuition and partly ... over many hours of computer simulations" (Taylor & Creelman, 1967). The experiment is terminated when the rules call for a step size of some pre-selected small size. No trials are made at this next level, which is itself the estimate of L_p. Thus there are no calculations to be made at the end of a PEST experiment, and the precision of the estimate is

TABLE A.4

Rules for Parameter Estimation by Sequential Testing (PEST), and ranges of the sequential test bounds for three values of the deviation limit W.

RULES FOR 'PEST'

1. Stimulus level is increased or decreased if the number of correct responses is below or above the range of N_b after T trials.

2. Step size is halved on every reversal of step direction.

3. Second step in a given direction is the same size as the first.

4. Fourth and subsequent steps in a given direction are each double their predecessor.

5. The third successive step in a given direction is doubled only if the step immediately preceding the most recent reversal did not result from a doubling.

6. The experiment is terminated when the rules call for a step of some pre-determined size.

7. The level called for by the last small step is the estimate of L_t.

T Number of trials	N_e Expected number of correct responses in T trials for $L_{.75}$ (N_e = .75T)	N_b Range of sequential test bounds (to appropriate nearest integer) $N_b = N_e \pm W$ W = 1	 W = 1.5	 W = 2
2	1.5	1 - 2	0 - 3	
3	2.25	1 - 3	1 - 3	0 - 4
4	3	2 - 4	2 - 4	1 - 5
5	3.75	3 - 4	3 - 5	2 - 5
6	4.5	4 - 5	3 - 6	3 - 6
7	5.25	5 - 6	4 - 6	4 - 7
8	6	5 - 7	5 - 7	4 - 8
9	6.75	6 - 7	6 - 8	5 - 8
10	7.5	7 - 8	6 - 9	6 - 9
11	8.25	8 - 9	7 - 9	7 - 10
12	9	8 - 10	8 - 10	7 - 11
13	9.75	9 - 10	9 - 11	8 - 11
14	10.5	10 - 11	9 - 12	9 - 12
15	11.25	11 - 12	10 - 12	10 - 13
16	12	11 - 13	11 - 13	10 - 14
17	12.75	12 - 14	11 - 14	11 - 14
18	13.5	13 - 14	12 - 15	12 - 15
19	14.25	14 - 15	13 - 15	13 - 16
20	15	14 - 16	14 - 16	13 - 17

determined by the terminating step size.

The rules for PEST are summarised in Table IV. With practice, PEST is simple to operate, particularly if one prepares a table of T.p $\pm$ W, as presented in Table IV. For a stopping step of $\frac{1}{4}$ logit units (about 0.15 S.D. on the normal ogive), Taylor & Creelman found that on the average about 60 trials were necessary for estimates of $L_{.6}$ to $L_{.75}$. This is close to the same number of trials for estimates of $L_{.75}$ with the UDTR rule, and a little less than the number required by BUDTIF.

Taylor & Creelman infer that there may be more precise estimates to be obtained by some sort of averaging procedure after a PEST experiment, but there is something to be said for a method in which "the experimenter's life is made very much easier because he does not have to record the history of the run and because he can make immediate decisions while the subject is still in the experimental situation" (Taylor & Creelman, 1967).

A.13 Sequential Effects.

One assumption of most psychophysical methods is that each response is independent from its predecessors. At the same time, it is generally recognised that a subject tends to avoid repeating the same response (Guilford, 1954). If he knows the rules governing stimulus presentation the subject might, consciously or otherwise, adjust his responses in some way. In UD experiments, biases due to these "response habits" (Cornsweet, 1962) are confounded by interleaving two (or more) series at random.

Other types of sequential effects may be due to adaptation, after-effects, or hysteresis. For example, in determining the absolute threshold for light, a supra-threshold stimulus could alter the level of retinal adaptation and hence decrease the probability of detection on the next trial.

Levitt (1968) discusses the possibility that hysteresis may be present in auditory signal detection experiments, that is, a just-audible stimulus on one trial increases auditory acuity for the next trial.

In most experiments, one wishes to avoid sequential effects; however, there may be occasions when information about sequential dependencies is required. When there are many data available, statistical analysis of the response pattern could give this information (e.g., Campbell, 1969), but a method developed by Levitt has the advantages of elegance and simplicity.

In Levitt's method (Levitt, 1968), two similar series A and B are run concurrently. However, instead of interleaving at random, series A is entered only after there has been an 'X' response in series B, which in turn is entered only after there has been an 'O' response in series A. If there are sequential dependencies, the two series should give consistently different results.

A.14 <u>UD methods compared with the constant method</u>.

(a) The efficiency of the simple UD method as compared with the method of constant stimuli (probit analysis) is discussed by Brownlee <u>et al</u> (1953). Even with accurate initial guesses at the median and standard deviation, the constant method requires at least 50% more observations than the UD method.

An example of inefficiency is to be found in a report by Williams (1970) of an experiment in depth perception. Nine stimuli were presented ten times each in a constant stimulus design. Typical results were that six of the stimuli resulted in empirical probabilities of either 1.0 or 0.0, that is, more than half of the stimulus levels were so remote from $L_{.5}$ that they gave no information about the response function. Ninety trials in a sequential design would have yielded results of much greater precision.

(b) As in the UD method, the results of a constant stimulus experiment can be affected by the phase relationship between $L_{.5}$ and the stimulus levels. The estimate of $L_{.5}$ tends towards the mean of the stimulus levels (Guilford, 1954). The effects of phase can be reduced in UD experiments with the method described in Section A8.

(c) Estimates of points away from $L_{.5}$ are imprecise with the constant method, unless the precise shape of the response curve is known. Sequential methods such as UDTR, BUDTIF, and PEST offer means of determining such points with no knowledge of the response curve.

(d) Sequential dependencies can be investigated fairly easily with an UD design (Section A.13), whereas a complicated analysis of the response pattern is necessary in a constant stimulus design.

(e) The method of constant stimuli fails if $L_{.5}$ drifts during the experimental session. Such drifts can be detected during an up-and-down experiment (Wetherill & Levitt, 1965).

(f) A series might drift during an experiment if the subject's attention lapses and he begins to respond at random. With two concurrent series, the drift is not likely to be the same in each case, enabling the random drift to be differentiated from real changes in the level being tracked. The experimenter can then take the appropriate action, such as alerting the subject or abandoning the experiment. In a constant stimulus experiment, lapses of attention are not detectable until after many trials.

It should be noted that, operationally, a drift due to inattentiveness is the same as an apparent drift due to too small a step size. The interpretation of "random drifts" therefore must be based on the extent of the drift and the experimenter's knowledge of the function being tested.

Against the advantages of the up-and-down methods must be placed the extensive theoretical and empirical background of the constant stimulus methods. If points (c), (d), (e), and (f) of the foregoing are not relevant to the experimental situation, and particularly if the response curve is known to be similar to the normal cumulative ogive, the power and rigour of the constant method cannot be denied. The choice between this and the comparative simplicity of UD methods becomes a question of expediency and personal opinion.

A.15 Some notes on response curves and computer simulations.

A convenient assumption in many psychophysical experiments is that the response curve in a two alternative situation is the same as the normal cumulative frequency ogive:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \qquad \ldots \text{(A.10)}$$

in which p (x) is the probability that a stimulus of value x will evoke the required response.   is the median (and mean) of the distribution, corresponding to $L_{.5}$, and   is the standard deviation. The function is the basis of the phi-gamma hypothesis, which is too well known to require further elaboration here (Guilford, 1954).

Other sigmoid functions have been used to describe response curves. The logistic curve (Wetherill, 1966, Finney, 1962) has the advantage that it is exactly defined by:

$$p(x) = 1/\left[1 + e^{-(\alpha + \beta x)}\right] \qquad \text{... (A.11)}$$

and is more easily applied to computer simulations than the normal ogive.

In computer simulations, one begins with a universe of numbers with an appropriate mean and dispersive properties. Thus the probability that one of these numbers, selected at random, is less than some given value will follow some function such as equations A.10 or A.11. The "given value" represents the stimulus value, and each random selection is a "trial" at that level. If the stimulus level is greater than the randomly selected number, then the response is "positive" or a "hit".

For the examples presented in this Appendix, a normally distributed set of numbers was generated with the IBM 360 computer (see Appendix B).

-o-O-o-

APPENDIX B

COMPUTER PROGRAMS

Several computer programs were written for various phases of the investigation. They were run on the IBM 360 machine at the University of New South Wales Computing Centre.

The programs were written in FORTRAN IV. The WATFOR version of FORTRAN (Blatt, 1967) was preferred because of the simplifications offered by its free input facility.

B.1. Probit Analysis.

IBM (1969) supply a prepared subroutine for probit analysis. This subroutine, PROBT, was modified in several ways:

(i) The original version was in single precision. It was found that if step sizes were small in comparison with stimulus levels, the sum of squares about the mean ($Snw(x - \bar{x})^2$) vanished, the small differences being lost to the seven significant figures allowed by four byte arithmetic. Since this sum is used as a denominator (see Finney, 1962, p. 53), the program often failed. To extend the utility of the program, it was re-written in double precision (eight byte) language.

(ii) The 6th executable statement in the original version was an arithmetic IF which had one branch jumping into the range of a DO loop. Such a jump is not permitted in most versions of FORTRAN, including that implemented at the Computing Centre, and a correction was made to ensure the compatability of the program with all systems.

B.2

(iii) Output from the original PROBT subroutine consisted of the intercept constant A and regression coefficient B, the chi squared value and associated degrees of freedom, the empirical probabilities for the various stimulus levels and the corresponding expected probits, and an error signal. The modified program also yielded the expected proportion for each level (useful for drawing graphs), 95% confidence limits, and the number of iterations, the latter being useful for studies of the efficiency of the method.

(iv) The 95% confidence limits referred to in (iii) were calculated exactly as described by Finney (1962, p. 61), allowances being made for any heterogeneity indicated by chi-squared. A subroutine, FLIM, was specially written for these computations.

The main program, PROBANAL, was written in a form suitable for any kind of constant stimulus data. The program listings which follow are self explanatory.

```
C
C
C
C
C              ***************************
C              *                         *
C              *                         *
C              *    SUBROUTINE PROBT     *
C              *                         *
C              *                         *
C              *                         *
C              ***************************
C
C     PURPOSE
C        TO OBTAIN MAXIMUM LIKELIHOOD ESTIMATES OF THE PARAMETERS
C        A AND B IN THE PROBIT EQUATION Y = A + BX.
C
C        AN ITERATIVE SCHEME IS USED.   THE INPUT TO THE SUBROUTINE
C        CONSISTS OF K DIFFERENT STIMULUS LEVELS APPLIED S TIMES EACH
C        AND THE NUMBER OF RESPONSES, R, AT EACH LEVEL.
C
C     SOURCE
C        IBM SCIENTIFIC SUBROUTINE PACKAGE IBM H20-0205 -3.
C            THE IBM ROUTINE HAS BEEN MODIFIED TO YIELD
C            ADDITIONAL DATA.
C
C     REFERENCE
C        FINNEY, 'PROBIT ANALYSIS', CAMBRIDGE UNIV. PRESS, 1962.
C
C     USAGE
C        CALL PROBT(K,X,S,R,LOG,ANS,W1,W2,PX,FID,TER,ITER)
C
C     INPUT PARAMETERS
C        K     - NUMBER OF STIMULUS LEVELS.   SHOULD BE GREATER
C                THAN 2.
C        X     - INPUT VECTOR, LENGTH K, CONTAINING STIMULUS VALUES.
C        S     - VECTOR, LENGTH K, CONTAINING THE NUMBER OF
C                 OF TRIALS AT EACH STIMULUS LEVEL.
C        R     - VECTOR, LENGTH K, CONTAINING NUMBER OF RESPONSES
C                TO EACH STIMULUS LEVEL.
C        LOG   - IMPUT OPTION CODE.
C                   K = 1 IF IT IS DESIRED TO CONVERT THE STIMULI
C                          TO COMMON LOGS.   THE STIMULI SHOULD BE
C                          GREATER THAN ZERO IN THIS CASE.
C                   K = 0 IF NO CONVERSION IS REQUIRED
```

```
C
C     OUTPUT PARAMETERS
C          ANS: VECTOR, LENGTH 4, CONTAINING
C                ANS(1) - ESTIMATE OF INTERCEPT CONSTANT A.
C                ANS(2) - ESTIMATE OF PROBIY
C                ANS(2) - ESTIMATE OF PROBIY REGRESSION COEFFICIENT B.
C                ANS(3) - CHI SQUARED VALUE, FOR TESTING GOODNESS OF
C                      FIT OF FINAL PROBIT EQUATION.
C                ANS(4) - DEGREES OF FREEDOM FOR CHI SQUARED TEST.
C          W1 - VECTOR, LENGTH K, CONTAINING THE PROPORTIONS OF
C                RESPONSES TO EACH STIMULUS LEVEL (EMPIRICAL PROBABILITY).
C          W2 - VECTOR, LENGTH K, CONTAINING VALUE OF THE EXPECTED
C                      PROBIT AT EACH LEVEL.
C          PX - VECTOR, LENGTH K, CONTAINING THE EXPECTED PROPORTION
C                      FOR EACH LEVEL.
C          FID - VECTOR, LENGTH 2, CONTAINING THE 95% CONFIDENCE LIMITS.
C          IER - ERROR SIGNAL.
C          IER = 1 IF K IS NOT GREATER THAN 2.
C          IER = 2 IF A STIMULUS IS NEGATIVE, OR IF LOG CONVERSION
C                      IS REQUIRED AND SOME STIMULUS IS ZERO.C
C          IER = 3 IF SOME ELEMENT OF S IS NOT POSITIVE.
C          IER = 4 IF NUMBER OF RESPONSES AT A LEVEL IS GREATER
C                THAN THE NUMBER OF TRIALS.
C          ***   ANALYSIS IS DONE ONLY IF IER IS ZERO.
C          ITER - NUMBER OF ITERATIONS OF THE MAXIMUM LIKELIHOOD ROUTINE.
C
C     REMARKS
C          THE PROGRAM WILL ITERATE ON THE PROBIT EQUATION UNTIL TWO
C          SUCCESSIVE SOLUTIONS PRODUCE CHANGES OF LESS THAN 10**(-7)
C
C     ***** NOTE
C          THIS SUBROUTINE IS IS DOUBLE PRECISION.   IF SINGLE
C          PRECISION IS REQUIRED, CALLS ON INBUILT FUNCTIONS MUST
C          BE MODIFIED.
C
C     ADDITIONAL SUBROUTINES REQUIRED
C          NDTR - IBM ROUTINE FOR THE NORMAL DISTRIBUTION FUNCTION.
C          NDTRI - IBM SUBROUTINE FOR THE INVERSE OF THE NORMAL
C                DISTRIBUTION FUNCTION.
C          FLIM - SUBROUTINE FOR CALCULATING FIDUCIAL LIMITS.
C
      SUBROUTINE PROBT (K,X,S,R,LOG,ANS,W1,W2,PX,FID,IER,ITER)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(K),S(K),R(K),ANS(4),W1(K),W2(K),PX(K),FID(2)
      IER=0
```

```
C     SEE IF THERE ARE ENOUGH STIMULUS LEVELS.
      IF(K-2)5,5,7
    5 IER = 1
      GO TO 90
C
C     CHECK THAT THERE ARE NO NEGATIVE STIMULUS LEVELS.
    7 DO 8 I=1,K
      IF(X(I).GE.0)GO TO 8
      IER = 2
      GO TO 90
    8 CONTINUE
C
C     IF LOG CONVERSION REQUIRED, CHECK THAT NO X IS ZERO.
      IF(LOG-1) 16,10,16
   10 DO 15 I=1,K
      IF(X(I))12,12,14
   12 IER=2
      GO TO 90
C     CONVERT X(2) TO COMMON LOGS.
   14 X(I) = DLOG10(X(I))
   15 CONTINUE
C
C     MAKE SURE THAT THE NUMBER OF RESPONSES IS NEVER GREATER THAN
C         THE NUMBER OF TRIALS.
   16 DO 18 I=1,K
      IF(S(I)-R(I)) 17,18,18
   17 IER=4
      GO TO 90
   18 CONTINUE
C
C     CHECK THAT NO X IS NEGATIVE.   IF O.K., CALCULATE THE EMPIRICAL
C         PROPORTION.
   20 DO 23 I=1,K
      IF(S(I))21,21,22
   21 IER=3
      GO TO 90
   22 W1(I)=R(I)/S(I)
   23 CONTINUE
C
C     WE GET THIS FAR ONLY IF ALL INPUT PARAMETERS ARE VALID.
C     FIRST CLEAR VARIOUS LOCATIONS FOR ACCUMULATING SUMS.
      WN=0.0
      XBAR=0.0
      SNWY=0.0
      SXX=0.0
```

```
      SXY=0.0                                                           PRBT1360
C                                                                       PRBT1370
C     MAKE A FIRST ESTIMATE OF THE REGRESSION LINE.   THIS STEP REPLACESPRBT1380
C     FINNEY'S PROCEDURE OF FITTING A PROVISIONAL LINE BY EYE.          PRBT1390
      DO 30 I=1,K                                                       PRBT1400
C     P IS THE EMPIRICAL PROPORTION FOR X(I).                           PRBT1410
      P=W1(I)                                                           PRBT1420
C     IF P IS ONE OR ZERO, IT IS NOT USED IN THE CALCULATION.           PRBT1430
      IF(P) 30, 30, 24                                                  PRBT1440
   24 IF(P-1.0) 25, 30, 30                                              PRBT1450
C     WN COUNTS THE NUMBER OF LEVELS USED IN THE CALCULATION.           PRBT1460
   25 WN=WN+1.0                                                         PRBT1470
C     GO TO THE SUBROUTINE NDTRI TO FIND THE NORMAL DEVIATE 'Z'         PRBT1480
C         CORRESPONDING TO THE PROBABILITY 'P'.                         PRBT1490
      CALL NDTRI (P,Z,D,IER)                                            PRBT1500
C     CONVERT 'Z' TO A PROBIT, BY ADDING 5.                             PRBT1510
      Z=Z+5.0                                                           PRBT1520
C     ACCUMULATE THE VARIOUS SUMS OF SQUARES AND PRODUCTS (UNWEIGHTED)  PRBT1530
C     FOR THE LEAST SQUARES COMPUTATION.                                PRBT1540
      XBAR=XBAR+X(I)                                                    PRBT1550
      SNWY=SNWY+Z                                                       PRBT1560
      SXX=SXX+X(I)**2                                                   PRBT1570
      SXY=SXY+X(I)*Z                                                    PRBT1580
   30 CONTINUE                                                          PRBT1590
C                                                                       PRBT1600
C     SOLVE THE UNWEIGHTED LEAST SQUARES EQUATIONS TO GIVE THE          PRBT1610
C     PROVISIONAL REGRESSION EQUATION.                                  PRBT1620
      B=(SXY-(XBAR*SNWY)/WN)/(SXX-(XBAR*XBAR)/WN)                       PRBT1630
      XBAR=XBAR/WN                                                      PRBT1640
      SNWY=SNWY/WN                                                      PRBT1650
      A=SNWY-B*XBAR                                                     PRBT1660
C                                                                       PRBT1670
C     CLEAR A LOCATION DD.   CALCULATE THE EXPECTED PROBITS ACCORDING   PRBT1680
C     TO THE PROVISIONAL REGRESSION EQUATION.                           PRBT1690
      DD=0.0                                                            PRBT1700
      DO 31 I=1,K                                                       PRBT1710
   31 W2(I)=A+B*X(I)                                                    PRBT1720
C                                                                       PRBT1730
C     THE ITERATIONS BEGIN AT STATEMENT 33.   FIRST CLEAR ITER          PRBT1740
C         AS A LOCATION FOR COUNTING ITERATIONS.                        PRBT1750
      ITER = 0                                                          PRBT1760
   33 ITER = ITER + 1                                                   PRBT1770
C     BEFORE PROCEEDING WITH THIS ITERATION, CLEAR VARIOUS              PRBT1780
C     LOCATIONS FOR ACCUMULATING FRESH SUMS.                            PRBT1790
      SNW = 0.0                                                         PRBT1800
```

```
      SNWX=0.0
      SNWY=0.0
      SNWXX=0.0
      SNWXY=0.0
C
      DO 50 I=1,K
C     Y IS THE LAST VALUE OF THE EXPECTED PROBIT FOR THIS LEVEL.
C     CONVERT IT TO A NORMAL DEVIATE 'D', AND GO TO SUBROUTINE NDTR
C     TO FIND THE CORRESPONDING PROBABILITY AND ORDINATE.
      Y=W2(I)
      D=Y-5.0
      CALL NDTR (D,P,Z)
C     CALCULATE THE WEIGHT ('W').
      Q=1.0-P
      W=(Z*Z)/(P*Q)
C
C     COMPUTE A WORKING PROBIT.   SELECT AN ALGORITHM DEPENDING ON
C     WHETHER Y IS .GT. OR .LT. 5.
      IF(Y-5.0) 35, 35, 40
   35 WP=(Y-P/Z)+W1(I)/Z
      GO TO 45
   40 WP=(Y+Q/Z)-(1.0-W1(I))/Z
C
C     ACCUMULATE THE VARIOUS SUMS, USING THE WEIGHT 'W'.
   45 WN=W*S(I)
      SNW=SNW+WN
      SNWX=SNWX+WN*X(I)
      SNWY=SNWY+WN*WP
      SNWXX=SNWXX+WN*X(I)**2
   50 SNWXY=SNWXY+WN*X(I)*WP
C
C     COMPUTE THE NEW ESTIMATES.
      XBAR=SNWX/SNW
      SXX=SNWXX-(SNWX)*(SNWX)/SNW
      SXY=SNWXY-(SNWX)*(SNWY)/SNW
      B=SXY/SXX
      A=SNWY/SNW-B*XBAR
C
C     COMPUTE THE NEW EXPECTED PROBIT FOR EACH LEVEL, AND COMPARE
C     IT WITH THE LAST ESTIMATE BY EXAMINING THE SUM OF THE SQUARES
C     OF THE DIFFERENCES.
      SXX=0.0
      DO 60 I=1,K
      Y=A+B*X(I)
      D=W2(I)-Y
```

```
C     STORE THE NEW ESTIMATE IN W(I).                                  PRBT2260
      SXX=SXX+D*D                                                      PRBT2270
   60 W2(I)=Y                                                          PRBT2280
C     IF THE DIFFERENCE BETWEEN THE NEW SUM OF SQUARES, SXX, AND THE   PRBT2290
C     OLD SUM, DD, IS TOO GREAT, RE-ITERATE, MATE.                     PRBT2300
      IF((DABS(DD-SXX))-(1.0D-7)) 65, 65, 63                           PRBT2310
   63 DD=SXX                                                           PRBT2320
      GO TO 33                                                         PRBT2330
C                                                                      PRBT2340
C     WE REACH HERE WHEN THE DESIRED ACCURACY HAS BEEN OBTAINED.       PRBT2350
   65 ANS(1)=A                                                         PRBT2360
      ANS(2)=B                                                         PRBT2370
C                                                                      PRBT2380
C     COMPUTE CHI SQUARED.                                             PRBT2390
      ANS(3)=0.0                                                       PRBT2400
      DO 70 I=1,K                                                      PRBT2410
      Y=W2(I)-5.0                                                      PRBT2420
C     GO TO NDTR TO FIND THE PROBABILITY ASSOCIATED WITH EACH PROBIT.  PRBT2430
      CALL NDTR (Y,P,D)                                                PRBT2440
      PX(I) = P                                                        PRBT2450
      AA=R(I)-S(I)*P                                                   PRBT2460
      DD=S(I)*P*(1.0-P)                                                PRBT2470
   70 ANS(3)=ANS(3)+AA*AA/DD                                           PRBT2480
C     CALCULATE AND STORE THE DEGREES OF FREEDOM.                      PRBT2490
      ANS(4)=K-2                                                       PRBT2500
C                                                                      PRBT2510
C     THE MEDIAN, 'X5', IS REQUIRED FOR COMPUTING FIDUCIAL LIMITS.     PRBT2520
C     ALSO NEEDED IS THE NUMBER OF DEGREES OF FREEDOM IN INTEGER FORM. PRBT2530
      X5 = (5.0 - A)/B                                                 PRBT2540
      NDFRDM = K - 2                                                   PRBT2550
      CALL FLIM(X5,XBAR,B,SNW,SNWXX,NDFRDM,ANS(3),FID)                 PRBT2560
C                                                                      PRBT2570
C     ALL CALCULATIONS ARE COMPLETED, SO BACK TO THE MAIN PROGRAM.     PRBT2580
   80 RETURN                                                           PRBT2590
C                                                                      PRBT2600
C     THE FOLLOWING ARE CONE IF THERE HAS BEEN AN ERROR, AND IER       PRBT2610
C     IS NOT ZERO.                                                     PRBT2620
   90 DO 100 I=1,K                                                     PRBT2630
      W1(I)=0.0                                                        PRBT2640
  100 W2(I)=0.0                                                        PRBT2650
      DO 110 I=1,4                                                     PRBT2660
  110 ANS(I)=0.0                                                       PRBT2670
      GO TO 80                                                         PRBT2680
      END                                                              PRBT2690
C                                                                      PRBT2700
```

```
C
      SUBROUTINE NDTR(X,P,D)
      IMPLICIT REAL*8(A-H,O-Z)
      AX = DABS(X)
      T=1.0/(1.0+.2316419*AX)
      D=0.3989423*DEXP(-X*X/2.0)
      P = 1.0 - D*T*((((1.330274*T - 1.821256)*T + 1.781478)*T -
     1  0.3565638)*T + 0.3193815)
      IF(X)1,2,2
    1 P=1.0-P
    2 RETURN
      END
C
C
      SUBROUTINE NDTRI(P,X,D,IE)
      IMPLICIT REAL*8(A-H,O-Z)
      IE=0
      IF(P)1,4,2
    1 IE=-1
      GO TO 12
    2 IF(P-1.0)7,6,1
    4 X=-.999999999999999D+74
    5 D=0.0
      GO TO 12
  6   X = .999999999999999D+74
      GO TO 5
    7 D=P
      IF(D-0.5)9,9,8
    8 D=1.0-D
    9 T2=DLOG(1.0/(D*D))
      T = DSQRT(T2)
      X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*T+0.189269*T2
     1  +0.001308*T*T2)
      IF(P-0.5)10,10,11
   10 X=-X
   11 D=0.3989423*DEXP(-X*X/2.0)
   12 RETURN
      END
C
C
C
      SUBROUTINE FLIM(XP,XM,BS,SUMNW,SUMXX,NDF,CHI,FID)
C     CALCULATES 5 PERCENT CONFIDENCE LIMITS FROM A PROBIT
C                    REGRESSION LINE.
C     INPUT
```

```
C           XP: THE TEST VALUE FOR WHICH FIDUCIAL LIMITS REQUIRED.
C           XM: WEIGHTED AVERAGE OF TEST VALUES.
C           BS: PROBIT REGRESSION COEFFICIENT
C           SUMNW: WEIGHTED SUM OF OBSERVATIONS
C           SUMXX: WEIGHTED SUM OF SQUARES OF DEVIATIONS OF X
C           NDF: NUMBER OF DEGREES OD FREEDOM.
C           CHI: VALUE OF CHI SQUARE FOR THE PROBIT LINE.
C
C     OUTPUT:
C           FID: VECTOR ARRAY, LENGTH 2, CONTAINING THE LIMITS FOR
C                5 % EITHER SIDE OF XP.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TTEST(31),FID(2)
      DATA TTEST/12.71D0, 4.3D0, 3.18D0, 2.78D0, 2.57D0, 2.45D0,
     U2.37D0, 2.31D0, 2.26D0, 2.23D0, 2.2D0, 2.18D0, 2.16D0, 2.15D0,
     U2.13D0, 2.12D0, 2.11D0, 2.10D0, 2*2.09D0, 2.08D0, 2*2.07D0,
     U3*2.06D0, 3*2.05D0, 2.04D0, 1.96D0/
C     SEE IF HETEROGENEIYY FACTOR IS NECESSARY.
C     IF SO, COMPUTE IT.  OTHERWISE, USE INFINITY D.F.
      NCHI = IDINT(CHI + 5.D-1)
      IF(NCHI.LE.NDF) GO TO 4000
      HF = CHI/NDF
      N = NDF
      GO TO 5000
 4000 HF = 1.0
      N = 31
 5000 T = TTEST(N)
      G = (HF*T*T)/(BS*BS*SUMXX)
      GA = 1.0 - G
      RES = XP - XM
      RSQ = RES*RES
      Q = (GA/SUMNW + RSQ/SUMXX)*HF
      ROOT = DSQRT(Q)
      FAC = (T/(BS*GA))*ROOT
      TERM = XP + (G/GA)*RES
      FID(1) = TERM + FAC
      FID(2) = TERM - FAC
      RETURN
      END
```

B.2. Analysis of up-and-down experiments.

Program UPDOWN was written to calculate the Dixon-Mood estimate of the mean (Appendix A.3), and Kappauf's estimates of the standard deviation (App. A.5) and the standard error of the mean (App. A.9). The t-ratio for the .95 significance level and infinite degrees of freedom was used to find fiducial limits from the standard error of the mean. In most cases, it would have been correct to use the t-ratio for fewer degrees of freedom, but since the algorithm for the standard error is only an approximation, little would have been gained by this refinement.

The raw data were converted to a form more easily manipulated by the computer by an initial program, CODING. It was advantageous to separate the coding routine from UPDOWN, because in this way erroneously punched data could be corrected while the remainder were being processed, saving considerable computer time.

CODING made use of the fact that in an up-and-down series, only the starting level, the sequence of responses, and the step size need be specified. For example, if the starting level was 10 and the step size 2, the sequence 'X O X X ... ' meant that the stimulus levels were '10, 8, 10, 12 ,...'. The final stimulus level was also supplied to CODING, so that errors could be detected. The output from CODING was a set of punched cards, containing descriptive information, step size, number of trials, and sampling requirements, as well as the sequence of responses. These cards were then used as input for UPDOWN.

As well as analysing the whole of a pair of concurrent series, UPDOWN examined sequences of specified length beginning at any required pair of responses. This enabled changes in the function studied to be detected and tested for significance.

The subroutine ANGLE converted linear results into seconds of arc, using the expression given in Chapter I.

The program listings which follow are self descriptive.

```
C     PROGRAM UPDOWN
C     FOR CALCULATING STATISTICS FROM PSYCHOPHYSICAL DATA OBTAINED
C     BY THE UP AND DPWN METHOD WITH TWO CONCURRENT COMPLEMENTARY
C     (OR HOMOGENOUS) SERIES.
C     ESTIMATORS BASED ON KAPPAUF METHOD, RATHER THAN DIXON AND MOOD.
C
C     PROVISION IS MADE FOR EXAMINING GROUPS OF OBSERVATIONS, IN ORDER
C     TO ESTABLISH TIME TRENDS, SEQUENTIAL EFFECTS, ETC.
C
C     DESCRIPTION OF INPUT (DATA) DECK:
C          CARD 1: KSET - AN INTEGER IDENTIFYING THE DATA.   PROGRAM
C                 TERMINATES ON KSET = 0
C          CARD 2: DESCRIBES THE DATA, AND IS READ INTO FORMAT AT
C                 STATEMENT 1200.   UP TO 55 CHARACTERS, INCLUDING
C                 '1' IN FIRST COLUMN.
C          CARD 3 CONTAINS SEVEN NUMBERS:
C               NOBS(1) AND NOBS(2): 2 INTEGERS, GIVING THE NUMBER
C                 OF OBSERVATIONS IN EACH SERIES.
C             STIM(1,1): TWO REAL NUMBERS, GIVING THE HIGHEST
C             STIM(2,1): STIMULUS VALUES PRESENTED IN EACH SERIES.
C             STEP: THE STEP SIZE.
C             NLEV(1): THE NUMBER OF STIMULUS LEVELS PRESENTED
C             NLEV(2): IN EACH SERIES.
C             NSAM: THE NUMBER OF SAMPLES TO BE CONSIDERED.
C
C          TWO SETS OF CARDS: CONTAINING THE RESULTS FOR EACH SERIES,
C             READ INTO ARRAYS AT NR(1,NOBS(1)) AND NR(2,(3 >(2)).
C             DATA ARE CODED AS FOLLOWS: EACH STIMULUS LEVEL IS ASSIGNE
C             TWO NUMBERS, BEGINNING WITH 1,2 FOR THE HIGHEST LEVEL,
C             3,4 FOR THE SECOND LEVEL, ETC.   A 'HIT' AT A LEVEL
C             IS CODED WITH THE APPROPRIATE ODD NUMBER, A MISS BY
C             THE APPROPRIATE EVEN NUMBER.
C
C          REMAINING CARDS ARE NSAM IN NUMBER, GIVING THE FIRST AND LAST
C             OBSERVATIONS IN EACH SERIES FOR EACH SAMPLE.
C             JSAM(1,1) AND JSAM(1,2) REFER TO THE FIRST SERIES.
C                 JSAM(2,1) AND JSAM(2,2) REFER TO THE SECOND SEROES.
C          EXAMPLE:
C          5   24   6   24
C                 OBSERVATIONS 5 TO 24 IN SERIES 1, AND 6 TO 24 IN
C                 SERIES 2, ARE TO BE TREATED AS CONCURRENT SERIES.
C
      DIMENSION NOBS(2),STIM(2,20),NLEV(2),GRAPH(124),SYM(3),TOT(2),
     1AV(2),SD(2),JSAM(2,2),NR(2,500),$NAME(80)
C
```

```
      DATA SYM/'X','O',' '/
C
C     THIS SECTION READS IN THE BASIC DATA.
  500 READ,PD,Y
 1000 READ,KSET
      IF(KSET.LT.0)GO TO 500
      IF(KSET.EQ.0)GO TO 9000
      WRITE(3,1050)KSET
 1050 FORMAT('1','DATA SET NO.',I4)
C     IER IS THE MONITOR FOR ERRORS IN SEQUENCES.   0 FOR O.K.,
C          BECOMES 1 FOR ERRORS.
      IER = 0
      READ(1,1200)$NAME
 1200 FORMAT(80A1)
      WRITE(3,1250)$NAME
 1250 FORMAT('0'/' ',80A1/)
      READ,NOBS(1),NOBS(2),STIM(1,1),STIM(2,1),STEP,NLEV(1),NLEV(2),NSAM
C
C     READ IN THE EXPERIMENTAL DATA. PRINT IT OUT IN BOTH CODED AND
C     GRAPHIC FORMS.
      DO 4600 J = 1,2
      NJ = NOBS(J)
      READ,(NR(J,K),K=1,NJ)
      WRITE(3,2000)J,NOBS(J),NLEV(J),STEP,STIM(J,1),NSAM
 2000 FORMAT('OSERIES',I2,':',I4,' OBSERVATIONS.',I6,' STIMULUS ',
     1'LEVELS AT STEPS OF',F7.3,' MM, BEGINNING AT',F10.4,'. NUMBER',
     I' OF SAMPLES ANALYZED:',I3)
      WRITE(3,2005)PD,Y
 2005 FORMAT('OINTERPUPILLARY DISTANCE:',F5.1/' TEST DISTANCE:',F10.2)
      WRITE(3,2100)(NR(J,K),K=1,NJ)
 2100 FORMAT('ODATA WERE CODED AS FOLLOWS:'/(' ',8(' ',5I3)))
      WRITE(3,4650)
C
C     THIS PART DOES THE GRAPHICAL REPRESENTATION.
      K2 = NLEV(J)
      DO 4500 K = 1,K2
      STIM(J,K) = STIM(J,1) - (K - 1)*STEP
C     SEE WHICH OBSERVATIONS OCCURRED AT THE KTH STIMULUS.
      L2 = NOBS(J)
      DO 4000 L = 1,L2
C     PUT A BLANK IN CURRENT ELEMENT OF GRAPH
 2500 GRAPH(L) = SYM(3)
C     IDENTIFY THE STIMULUS
      NSUB = (NR(J,L) + 1)/2
C     IF THIS OBSERVATION NOT AT STIM(J,K), GO TO END OF INNER LOOP.
```

```
      IF(NSUB.NE.K)GO TO 4000
C     HERE IF OBSERVATICN IS AT STIM(J,K).  FIND OUT IF IT WAS A HIT
C     OR A MISS, AND ENTER THE APPROPROATE SYMBOL.
      GRAPH(L) = SYM(1)
      IF(MOD(NR(J,L),2).EQ.0)GRAPH(L)=SYM(2)
 4000 CONTINUE
C
C     WE GET HERE WHEN ONE STIMULUS LEVEL HAS BEEN GRAPHED. PRINT IT.
      WRITE(3,4200)STIM(J,K),(GRAPH(N),N=1,NJ)
 4200 FORMAT(' ',F6.2,':',125A1)
C
C     GO BACK FOR THE NEXT STIMULUS LEVEL.
 4500 CONTINUE
C     THIS SECTION CHECKS THE DATA, MAKING SURE THAT EACH EVEN
C     NUMBER (I.E., A MISS) IS FOLLOWED BY A LESSER NUMBER, SIGNIFYING
C     AN INCREASE IN TEST LEVEL, AND VICE VERSA FOR ODD NUMBERS.
 4510 DO 4590 L = 2,L2
      LZ = L - 1
      IF(MOD(NR(J,LZ),2).EQ.0)GO TO 4540
C     LAST RESPONSE A HIT. SEE IF PRESENT STIMULUS IS LESS.
 4520 IF(NR(J,L).GT.NR(J,LZ))GO TO 4590
C     AN ERROR IF WE GET HERE, SO PRINT A MESSAGE.
      GO TO 4550
C     LAST RESPONSE A MISS. SEE IF PRESENT STIM IS GREATER.
 4540 JEST = NR(J,L) - NR(J,LZ)
      IF(JEST.LT.0)GO TO 4590
C     ERROR IF WE GET TO HERE.
 4550 IER = 1
      WRITE(3,4560)J,L
 4560 FORMAT('0 ******    ERROR    ******'/' ',20X,'IN SERIES',I2,' RES',
     1'PONSE',I3,' IS WRONG.')
 4590 CONTINUE
C     BACK FOR THE NEXT SERIES.
 4600 CONTINUE
C
C     NEW PAGE FOR RESULTS
      WRITE(3,1050)KSET
 4610 FORMAT('1')
C     NEXT LOOP TRAVERSED AS MANY TIMES AS THERE ARE SAMPLE SETS.
      DO 7000 N = 1,NSAM
      WRITE(3,4650)
 4650 FORMAT('0')
      IF(MOD(N,4).EQ.0)WRITE(3,1050)KSET
C     READ IN AND PRINT OUT DETAILS FOR A SAMPLE.
      READ(1,4700)$NAME
```

```
 4700 FORMAT(80A1)
      WRITE(3,4750)$NAME
 4750 FORMAT(' ',80A1)
      READ,JSAM(1,1),JSAM(1,2),JSAM(2,1),JSAM(2,2)
      WRITE(3,4800)JSAM(1,1),JSAM(1,2),JSAM(2,1),JSAM(2,2)
 4800 FORMAT('ORESULTS FOR OBSERVATIONS',I4,' TO',I4,' IN ',
     1'SERIES 1;'/' ',24X,I4,' TO',I4,' IN SERIES 2.')
C     IF THERE HAS BEEN AN ERROR (IER = 1), DON'T BOTHER CALCULATING.
      IF(IER.EQ.0)GO TO 4900
 4950 WRITE(3,4960)
 4960 FORMAT('0***  ANALYSIS NOT DONE - ERROR IN SEQUENCE  ***')
      GO TO 7000
C     LOOP ON J DESIGNATES A SERIES.
 4900 DO 6500 J = 1,2
C     CLEAR LOCATIONS FOR TOTALS.
      NHITS = 0
      NMISS = 0
      AH = 0
      AM = 0
      BH = 0
      BM = 0
C     NOW GO THROUGH THE OBSERVATIONS.
      K1 = JSAM(J,1)
      K2 = JSAM(J,2)
      DO 6100 K = K1,K2
      NSUB = (NR(J,K) + 1)/2
      S = STIM(J,NSUB)
      IF(MOD(NR(J,K),2).EQ.0)GO TO 6000
C     DO THIS FOR ODD NR, I.E., A HIT.
 5000 NHITS = NHITS + 1
      AH = AH + S
      BH = BH + S*S
      GO TO 6100
C     DO THIS FOR NR EVEN, I.E., A MISS.
 6000 NMISS = NMISS + 1
      AM = AM + S
      BM = BM + S*S
 6100 CONTINUE
C
C     SEE IF HITS OR MISSES SHOULD BE USED, DEPENDING ON WHICH IS LESS.
      IF(NHITS.LT.NMISS)GO TO 6300
C     THIS IS DONE IF MISSES ARE FEWER.
 6200 TOT(J) = NMISS
      A = AM
      B = BM
```

```
      AV(J) = A/TOT(J) + (STEP*0.5)
      GO TO 6400
C     THIS IS DONE IF HITS ARE FEWER.
 6300 TOT(J) = NHITS
      A = AH
      B = BH
      AV(J) = A/TOT(J) - (STEP*0.5)
 6400 VAR = (TOT(J)*B - A*A)/(TOT(J)*TOT(J)*STEP*STEP)
      SD(J) = (1.71*STEP*VAR*TOT(J))/(TOT(J) - 1.5)
C
C
 6500 CONTINUE
C     CALCULATE STATISTICS FOR THE CONCURRENT SERIES.
      AVCC = (AV(1) + AV(2))/2.0
      AVSEC = ANGLE(AVCC,Y,PD)
      SDCC = (SD(1) + SD(2))/2.0
      SDSEC = ANGLE(SDCC,Y,PD)
      SEMCC = (.82*SDCC + .16*STEP)/(SQRT(TOT(1) + TOT(2)))
      FIDCC = 1.96*SEMCC
      FIDSEC = ANGLE(FIDCC,Y,PD)
      RANGE1 = AVCC - FIDCC
      RANGE2 = AVCC + FIDCC
C     PRINT THE RESULTS.
      WRITE(3,6700)
 6700 FORMAT(' ',13X,'MEAN    STANDARD        + OR -    ST.ERROR',8X,
     1'ANGULAR VALUES'/' ',21X,'DEVIATION  95% LIMITS  OF MEAN',11X,
     2'(SEC. ARC)')
      WRITE(3,6710)AV(1),SD(1),AVSEC,FIDSEC,AV(2),SD(2),SDSEC,AVCC,SDCC,
     1FIDCC,SEMCC,RANGE1,RANGE2
 6710 FORMAT('0    SERIES 1',F8.3,F10.3,31X,'MEAN =',F8.2,' + ',F6.2/
     1'0    SERIES 2',F8.3,F10.3,31X,'ST. DEVN. =',F8.3/
     2'0 CONCURRENT'/'     SERIES',F10.3,F10.3,F12.3,F11.3/'095% ',
     3'CONFIDENCE INTERVAL:',F8.3,' TO',F8.3/' ',80('-'))
C     GO BACK FOR DETAILS OF NEXT SAMPLE, IF ANY.
 7000 CONTINUE
      WRITE(3,8000)
 8000 FORMAT('0'//'0',10X,10('*')/'0',10X,'END OF SET')
C     GO BACK FOR MORE DATA.
      GO TO 1000
 9000 WRITE(3,9500)
 9500 FORMAT('1'///'0**********   ALL STAIRCASES HAVE BEEN NEGOTIATED ',
     V'WITH SUCCESS   **********')
      STOP
      END
      FUNCTION ANGLE(DELTA,Y,PD)
```

```
YDASH = Y + DELTA
A = (PD * 15.0)/Y
B = (DELTA * 13751.0)/YDASH
ANGLE = A * B
RETURN
END
```

B.3. Random normal deviate generator.

For the simulation of psychophysical experiments (see App. A.1), a randomly ordered sample from a normally distributed population is required. IBM (1969) supply a subroutine, GAUSS, which does this by using the formulae

$$Y = \sum_{i=1}^{6} X_i - 6.0$$

$$Y' = Y * S + AM.$$

X_i is a uniformly distributed random number, obtained by another IBM subroutine, RANDU. AM and S are the required mean and standard deviation, and Y' is the resulting normally distributed random number.

A brief study was carried out to test the small sample properties of GAUSS. 500 sets of 10 random normal deviates were generated, with mean = 0 and standard deviation = 1.0. For sample sizes of 10, the theoretical standard error of the mean is .316 (= / N; Gilford, 1965). The standard deviation of the mean of the means for the 500 sets was found to be .561, somewhat high.

Another program, GROUSE, was devised for generating random normal deviates. In this routine, two random numbers were generated by RANDU. The first was in the range ± 4, and represented a possible normal deviate. The ordinate on the normal distribution curve corresponding to this deviate was then computed, and compared with the second random number, which was in the range 0.0 to .398942. If the first number was less than the second, it was returned as an appropriate normal deviate; if not, it was discarded, and another two random numbers were generated and tested.

GROUSE was found to have good small sample properties. The empirical standard error of the mean, obtained again by generating 500 samples of 10 each, was 0.327; the standard error of the standard deviations was .221 (theoretical result = / 2N = .223).

For psychophysical simulations, GROUSE has the advantage that skewed, distorted, or truncated distributions can be easily generated, by varying the ranges of the two random numbers.

```
      SUBROUTINE GROUSE(IX,AM,S,XG)
C     FOR GENERATING A RANDOM NORMAL DEVIATE FROM A
C          POPULATION WITH MEAN = AM,
C          STANDARD DEVIATION = S
C     IX IS AN ODD INTEGER, SUPPLIED FROM THE MAIN
C          PROGRAM, FOR INITIATING THE RANDOM NUMBER
C          ROUTINE 'RANDU'.
C     XG IS THE REQUIRED NORMAL DEVIATE.
C
C     FIRST GET A RANDOM NUMBER
 1000 CALL RANDU(IX,IY,YFL)
C     PUT YFL INTO THE RANGE OF THE ORDINATE OF THE
C          NORMAL DISTRIBUTION CURVE.
      Y = YFL * 0.398942
C     GET ANOTHER RANDOM NUMBER.
      IX = IY
      CALL RANDU(IX,IY,YFL)
      IX = IY
C     PUT YFL INTO THE RANGE + OR - 4.0
      X = (YFL * 8.0) - 4.0
```

B.7

```
C       COMPUTE THE ORDINATE FOR THIS DEVIATE
        POW = -(X*X)/2.0
        YDEV = .398942 * (2.71828**POW)
C       IF Y IS GREATER THAN YDEV, GO BACK AND TRY AGAIN.
C       OTHERWISE, ADJUST X TO THE PARAMETERS AND RETURN.
        IF(Y.GT.YDEV)GO TO 1000
        XG = X * S + AM
        RETURN
        END


        SUBROUTINE RANDU(IX,IY,YFL)
C       FOR COMPUTING UNIFORMLY DISTRIBUTED RANDOM NUMBERS.
C       *** SEE IBM SCIENTIFIC SUBROUTINE PACKAGE, PAGE 77,
C            1969 EDITION.
C       IX IS AN ODD INTEGER, INITIALLY SUPPLIED TO THE
C            MAIN PROGRAM.   SUBSEQUENTLY, IX IS THE
C            PREVIOUSLY OBTAINED VALUE OF IY.
C       IY  IS A RESULTANT RANDOM INTEGER, IN THE RANGE
C            ZERO TO 2**31
C       YFL IS A UNIFORMLY DISTRIBUTED RANDOM FLOATING POINT
C            NUMBER, IN THE RANGE 0 TO 1.0
        IY = IX * 65539
        IF(IY)5,6,6
      5 IY = IY + 2147483647 + 1
      6 YFL = IY
        YFL = YFL * .4656613E-9
        RETURN
        END
```

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