A Study of parameter optimisation for a rainfall-runoff model. February 1973.

Author:
Pilgrim, D. H.; Johnston, P. R.

Publication details:
Commissioning Body: Australian Water Resources Council. Research Project 68/1
Report No. UNSW Water Research Laboratory Report No. 131
0858240890 (ISBN)

Publication Date:
1973

DOI:
https://doi.org/10.4225/53/579fcdf9b5828

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Report No. 131

Australian Water Resources Council Research Project 68/1

— Analysis Component (d)

A STUDY OF PARAMETER OPTIMISATION FOR A RAINFALL—RUNOFF MODEL

by

P. R. JOHNSTON and D. H. PILGRIM

February, 1973

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Key Words

Rainfall
Runoff
Boughton Model
Optimization
Numerical analysis
Mathematical models

0/85824/089/0

PREFACE

The work described in this report was undertaken in the Hydrology Section of the School of Civil Engineering, The University of New South Wales, between November 1968 and August 1972. Funds were provided by the Australian Water Resources Council under a grant from the A.W.R.C. Water Research Fund, and this generous support is gratefully acknowledged. This study was one of the four analysis components of A.W.R.C. Research Project 68/1, "Hydrology of Small Rural Catchments". It also formed a continuation of the research on mathematical models of the rainfall-runoff process in which the School of Civil Engineering has been active over a number of years.

From the commencement of the project to February 1970, the work was carried out by Mr. F.C. Bell under the supervision of Professor E.M. Laurenson. From March 1970, the work was carried out by Mr. P.R. Johnston under the supervision of Associate Professor D.H. Pilgrim. The work described in this report relates primarily to the latter period, as the earlier work was of a preliminary nature.

H.R. VALLENTINE,
Professor and Head,
School of Civil Engineering.

SUMMARY

A detailed study on the optimisation of the parameter values of the Boughton daily rainfall-runoff model has been carried out for a number of small catchments. The optimum values of the parameters were sought using the Steepest Descent, Simplex and Davidon optimising methods. It had been intended to correlate these optimum parameter values with measurable catchment characteristics.

Rapid initial reductions in the values of the objective function were readily achieved and the solutions approached apparent optimum points on the response surface. However, several of these points were found for each catchment and there were large differences in the parameter values between the points. This type of problem has also been encountered in previously reported optimisation studies for rainfall-runoff models. It was found that further improvements in the objective function could usually be achieved by using another of the search techniques or by numerical trials, and in this way, downhill paths on the response surface were found from the apparently optimum points. This work was pursued for one of the catchments until the paths appeared to be converging, but coincidence at a true optimum could not be achieved. A number of somewhat different sets of parameter values which appeared to lie in a flat "valley" area of the response surface were obtained, and these sets gave equally good fits to the observed runoff data.

An algebraic analysis of the operation of the model and of the effect on the objective function of changes in some individual parameter values led to important findings on some of the problems encountered.

It is probable that the findings from the numerical and algebraic analyses would be applicable to all rainfall-runoff models.

ACKNOWLEDGEMENTS

Initial work in the study was carried out by Mr. F.C. Bell under the supervision of Professor E.M. Laurenson. This work included the selection of the Boughton model for use in the project, the selection of the objective function to be used for optimisation, and some preliminary optimisation work using the steepest descent method.

Mr. M.K. Smith, Research Forester, Forestry Commission of N.S.W., assisted the study by providing rainfall, runoff, evaporation and soil moisture data for the Lidsdale No. 2 catchment, and also made several helpful suggestions.

Mr. D. Doran, Professional Officer, The University of New South Wales, assisted in integrating the modified infiltration function and in extracting some runoff data for the Lidsdale No. 2 catchment.

Thanks are also due to the Reference Panel of A.W.R.C. Research Project 68/1 for helpful suggestions and advice during the course of the project. Mr. D.T. Walsh was chairman of the Panel. Professor T.G. Chapman provided helpful suggestions relating to optimising methods. The project leader, Mr. J.A.H. Brown of the Snowy Mountains Engineering Corporation, and Mr. J.A. Shaw of the A.W.R.C. Secretariat also assisted the study.

The financial assistance of the Australian Water Resources Council is gratefully acknowledged.

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1. INTRODUCTION

The study described in this report is part of the Australian Water Resources Council Research Project 68/1, "Hydrology of Small Rural Catchments", which was commenced in November, 1968. The aims of this project were to catalogue all gauged rural catchments in Australia with an area of less than 10 square miles, to process the streamflow, rainfall and other records from these catchments into computer compatible form, and to commence analysis of the records. The project is described in a four volume report (Australian Water Resources Council, 1971). The analysis phase of the project consisted of four parts, two relating to flood estimation and the other two relating to methods of yield estimation. The study reported herein was Part (d), originally titled "Yield Estimation Using a Rainfall-Runoff Process Model Approach". In the remainder of this report, the word "project" will refer to Part (d) of A.W.R.C. Research Project 68/1.

This project, as well as the other three analysis projects, was envisaged as only a commencement of analysis of the compiled data, and as having an important function in testing and providing feed-back for the data compilation and transfer procedures.

The original proposal for this project envisaged four parts. The first two were of a preliminary nature and comprised graphical summaries of data and low-flow frequency analyses. Little work was carried out on these aspects. The third and major aspect comprised selection of an appropriate model of the rainfall-runoff process, determination of optimum model parameters for about 80% of the catchments for which records were available, correlation of the parameter values with catchment characteristics, and testing of these relationships with the remaining catchments. The fourth part was a tentative proposal and was dependent on the successful completion of the third part. It involved investigating the possibility of developing direct relationships between runoff characteristics and physical and climatic characteristics of catchments, thus eliminating the use of a model in a design procedure for the estimation of catchment yield. Data availability and the need for practical usefulness of results restricted the choice of a model to one using rainfall-runoff data at time intervals of not less than one day. The Boughton model was selected for use in the project.

As the project progressed beyond its initial phase, it became obvious that the proposed objectives could not be met. Many difficulties were encountered in the searches for optimum values of the model parameters. It was recognised that any attempts to relate model parameters to catchment conditions, or to use a model for synthesising data, would be pointless if the derived parameter values were not truly optimum. Further funds were provided by the Australian Water Resources Council for extension of this project, and in this extra time, the work was concentrated on the problem of obtaining optimum parameter values.

While the original aims of the project were not achieved, many of the practical difficulties which can be expected in implementing the optimising procedures and in searching for the optimum parameter values have been identified. Also, insight into such questions as the effect of the chosen objective function and the nature of the response surface has been gained. Considerable interest has been shown in deterministic models of the rainfall-runoff process over the last decade and the development and application of several types of models have been reported. While these models vary in the degree of their sophistication in attempting to simulate physical processes, they all depend on the derivation of optimum parameter values for usefulness in application. The findings of this project indicate that such values have not been found for many of these models and that more attention should be given to the problem of optimisation in the future.

2. MATHEMATICAL MODELS OF THE RAINFALL-RUNOFF PROCESS

Mathematical models of the rainfall-runoff process usually consist of a number of stores which, conceptually, represent the moisture-holding capacity of the vegetation and soil of the catchment. The movement of water onto, within, and out of the catchment is reproduced in these models by making transfers of water into, between, and out of the stores according to the rainfall and evaporation data and known or assumed functions to represent such physical processes as infiltration and evapotranspiration. Overflow from the system of stores is regarded as modelled runoff and should, theoretically, correspond to the observed runoff from the catchment. Spatial variation of the processes and moisture storage on the catchment is not generally taken into account. Average or "lumped" values are usually used for these components in most models. Examples of rainfall-runoff models are the Stanford Watershed Model (Crawford and Linsley, 1966), the Boughton Model (Boughton, 1965, 1966), and a model being used in the Representative Basins Project of the Australian Water Resources Council (A.W.R.C., 1969). These models are potentially useful for synthesising runoff data from observed rainfall and evaporation data, for generating very long sequences of flow using synthetic rainfall and evaporation data, and for estimating catchment wetness in flood forecasting schemes.

Some constants in the functions used to represent the physical processes, and the capacities of the stores, are parameters of the model and must be assigned fixed numerical values before the model may be used to estimate the runoff for any particular catchment. The numerical values vary for different catchments because of different vegetation, soil types and soil depths, and for models which truly represent the physical process, these values would ideally be estimated from measurements of the appropriate physical variables.

For catchments where there is a period of concurrent rainfall, evaporation and runoff records, the usual method of finding the appropriate parameter values is to operate the model with estimates of these values, compare the modelled and observed runoff, and make changes to the parameter values so as to obtain the best agreement between modelled and observed runoff records. The values which give this agreement are defined as the optimum parameter values.

The term "best agreement" must be defined in a quantitative way. It is necessary to choose some feature of the observed runoff record which is to be reproduced as closely as possible by the model. Features such as the runoff volumes in certain time periods or the peak flows are often selected. A numerical measure of the fit of the modelled runoff to the observed runoff may then be formed using some function of the differences between the modelled and observed values for the selected feature. An example is the sum of squares of the differences between the modelled and observed monthly runoff volumes. The measure of fit is known as an objective function, and the optimum parameter values are those which give a minimum value of this function, i.e., those which give the best fit in terms of the chosen objective.

For a given catchment, the value of the objective function is dependent only on the values assigned to the parameters. If there are n parameters and these are represented by n of the co-ordinates of an m-dimensional co-ordinate system (where $m = n + 1$), and the remaining co-ordinate represents the objective function, then this function forms a surface in the m-dimensional space known as a response surface. The co-ordinates of a point on this surface are n parameter values and the value of the objective function obtained when the model is operated with these values. The lowest point on the surface is where the objective function is a minimum and the corresponding parameter values are regarded as the optimum parameter values. The lowest point is known as the global minimum. There may be other points on the surface which are lower than all others in their immediate vicinity (but not lower than the global minimum). Such points are known as secondary minima.

A typical response surface where $n = 2$ is illustrated in Fig. 2.1. The parameters are x_1 and x_2 and the surface is represented by a contour map in the $x_1 - x_2$ plane. Where n is greater than 2 the concept of a response surface is retained even though the surface cannot be represented visually.

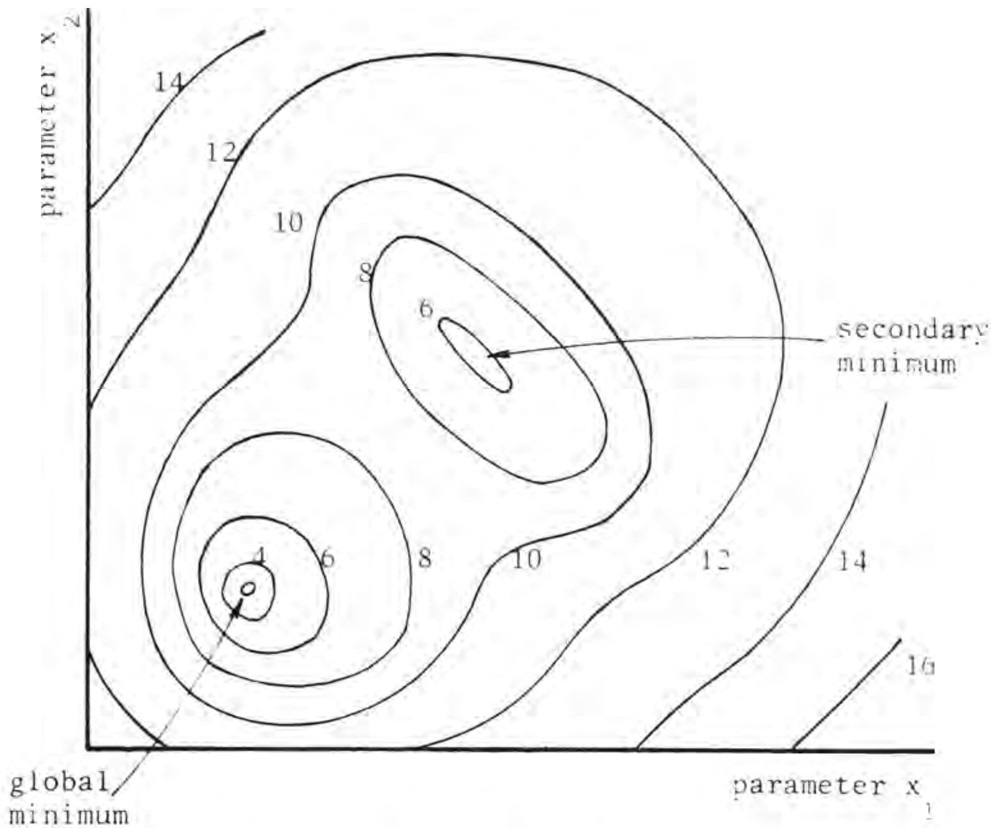


FIGURE 2.1
CONTOUR MAP OF A RESPONSE SURFACE
FOR A FUNCTION OF TWO PARAMETERS

The search for the set of optimum parameter values may be regarded as a search on the response surface for its lowest point. Techniques have been developed to conduct the search in a systematic way and these are known either as optimisation methods or simply as search techniques.

An exact fit of the modelled runoff to the observed runoff (i.e., objective function = 0) cannot be achieved, as

(i) all models are simplifications of the complex processes which actually occur on the catchment and are therefore inadequate to reproduce these processes exactly,

(ii) variations in catchment characteristics and moisture conditions are averaged both over the area of the catchment and within the time increments used for the model calculations, and

(iii) errors, some of which are unavoidable, are always present in the data.

Models may be refined to represent the physical process more closely, but this will normally involve

(i) greater complexity,

(ii) an increased number of parameters,

(iii) an increase in the computing time required to operate the model and search for the optimum parameter values, and

(iv) more stringent requirements on the data, e.g., readings at more frequent time intervals and on a more dense areal network.

If, for a given model, a set of optimum parameter values can be found for a particular catchment, it is desirable to verify these values and the validity of the model for the catchment by checking the model's ability to reproduce a period of observed runoff which was not used in the optimisation.

One of the main aims of this project was to obtain the optimum parameter values for the selected model on about 80% of the catchments included in the A.W.R.C. Research Project 68/1. The chosen objective function to be minimised for each catchment was the sum of squares of the differences between monthly observed and modelled runoff volumes. This function has been used frequently by other workers. Some comparative optimisation runs using other functions were made by Mr. Bell in the early work of this project but they did not indicate that any of these functions would be preferable.

3. THE MODEL USED IN THIS PROJECT

The Boughton Model (Boughton, 1965, 1966) was selected for use in this project. For practical usefulness, a model which uses daily data was required and the Boughton model is the most developed of these. It was thought desirable to continue previous work done with this model at The University of New South Wales. The model is illustrated in Fig. 3.1.

The Interception Store allows for water held on the surfaces of vegetation and also probably on surface litter. The topsoil layer of the catchment is assumed to have an unrestricted infiltration rate, and is represented by two moisture stores, the Upper Soil Store for the amount of water held in the topsoil between moisture levels corresponding to the simplified concepts of wilting point and field capacity, and the Drainage Store for the water temporarily held in the topsoil between field capacity and saturation. The subsoil is assumed to be much denser than the topsoil and to have a much lower infiltration rate which governs the amount of infiltration loss during a storm. It is represented by one moisture store, the Lower Soil Store. The daily infiltration rate is given by a function which is greatest when the Lower Soil Store is empty and declines exponentially as the quantity in the store increases. Evaporation occurs at the potential rate from the Interception Store until this store is emptied, after which evapotranspiration depletes the contents of both the Upper and Lower Soil Stores.

The model is operated as follows:-

(i) Rainfall first enters the Interception Store. If the capacity of this store is exceeded, it overflows into the Upper Soil Store, which in turn overflows into the Drainage Store. (The amount of water required to fill these stores on any day is the potential initial loss from that day's rainfall.)

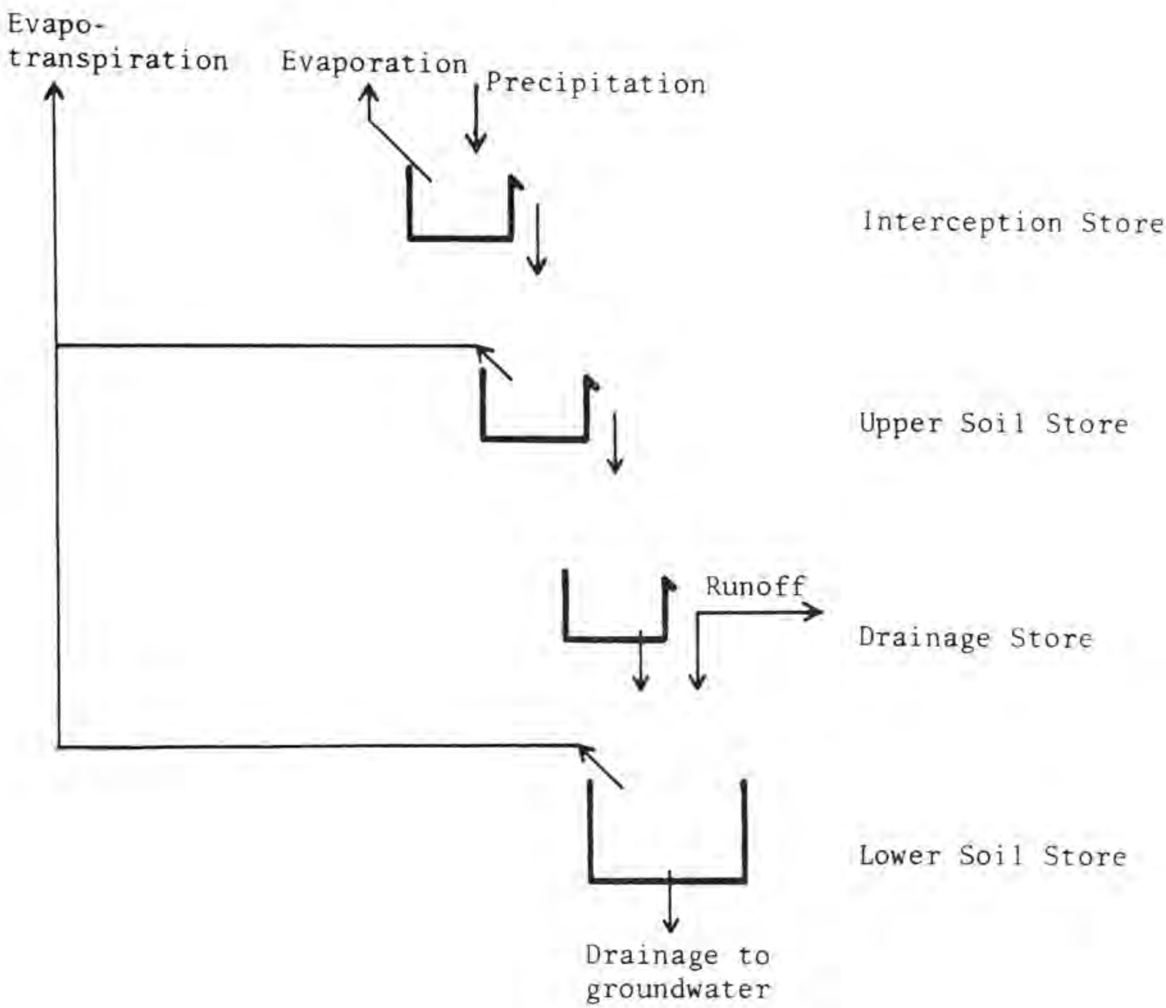


FIGURE 3.1
STRUCTURE OF THE BOUGHTON MODEL

(ii) When the Drainage Store contains water, infiltration into the Lower Soil Store occurs. The infiltration rate is determined from the equation

$$F = FC + (FO - FC)\, e^{-KF.SS}. \tag{3.1}$$

where F = daily infiltration rate

FC = daily infiltration rate when the Lower Soil Store is full

FO = daily infiltration rate when the Lower Soil Store is empty

KF = empirical constant

SS = amount of water currently held in the Lower Soil Store.

(iii) If the rainfall is sufficient to cause overflow from the Drainage Store, surface runoff commences. The amount of runoff is found from the following empirical relation:-

$$Q = P - F \tanh \left(\frac{P}{F}\right) \tag{3.2}$$

where Q = amount of runoff

P = that amount of the rainfall which overflows from the Drainage Store, i.e., rainfall less initial loss

F = daily infiltration rate.

(iv) Evaporation occurs at the potential rate from the Interception Store during and after rainfall until this store is empty. Evapotranspiration then takes place from both the Upper and Lower Soil Stores. The quantity taken from the Upper Soil Store is the lower of either

(a) PV.E

or (b) PV.EVPMAX.US/USMAX

while that taken from the Lower Soil Store is the lower of either

(a) $(1 - PV) . E$

or (b) $(1 - PV) . EVPMAX.SS/SSMAX$

where

PV = the fraction of the evapotranspiration taken from the Upper Soil Store

E = the potential evapotranspiration rate

EVPMAX = the maximum evapotranspiration rate when the soil moisture level is at field capacity

US = the amount of water currently held in the Upper Soil Store

USMAX = the capacity of the Upper Soil Store

SS = the amount of water currently held in the Lower Soil Store

SSMAX = the capacity of the Lower Soil Store.

The functions above are explained in more detail in sub-section 6.1.

(v) Depletion occurs from the Lower Soil Store to groundwater. Boughton allowed for this by applying a factor of 0.999 to the amount of water in the Lower Soil Store at the end of each day. The daily depletion quantity of water passes out of the system, as groundwater fluctuations or contributions to runoff are not included in the model.

Calculations are made on a daily basis to determine the amounts of water transferred into and out of the various stores, the amounts of water currently held in the stores, and any runoff produced, according to the above procedures.

The model has nine parameters and the notation and units given below for these parameters are used throughout this report.

Moisture Stores

VSMAX	Capacity of the Interception Store	(points)
USMAX	Capacity of the Upper Soil Store	(points)
DSMAX	Capacity of the Drainage Store	(points)
SSMAX	Capacity of the Lower Soil Store	(points)

Evapotranspiration function

PV	The fraction of evapotranspiration drawn from the Upper Soil Store	(no units)
EVPMAX	The maximum possible evapotranspiration rate when the relevant soil store is full. This is a property of the vegetation	(points/day)

Infiltration function

FC	The daily infiltration rate when the Lower Soil Store is full	(points/day)
FO	The daily infiltration rate when the Lower Soil Store is empty	(points/day)
KF	Empirical constant	(no units)

Listed below are some of the variables of the model which are referred to frequently.

VS	Contents of the Interception Store	(points)
US	Contents of the Upper Soil Store	(points)
DS	Contents of the Drainage Store	(points)
SS	Contents of the Lower Soil Store	(points)
E	Potential evapotranspiration	(points/day)

(One point equals 1/100th inch depth of water.)

Daily rainfall (averaged over the catchment area) and daily evaporation data are required to operate the model. Daily evaporation data are not available for many catchments, however, and it is often necessary to use average figures. The runoff quantities calculated by the model occur on days of rainfall only; no routing procedure is applied to the runoff, therefore recession curves do not appear in the calculated hydrograph.

The model attempts to reproduce the physical processes which occur in the catchment, but because of the use of daily data and a daily time period for calculations, complete representation of these processes is not possible. Variations in rainfall intensity during each day can not be taken into account. Only one infiltration rate is used on each day, and this rate is applied over the whole area of the catchment. The parameter values are average or "lumped" values for the whole of the catchment, and variations of catchment and moisture conditions over the area cannot be reproduced. No contribution to streamflow from groundwater depletion is allowed for and this would induce errors when using the model to compute the runoff from all but ephemeral streams.

4. FINDING THE OPTIMUM PARAMETER VALUES

The use of optimising methods to search for the optimum parameter values of rainfall-runoff models has attracted increasing interest in recent years. Some of this work is summarised below:-

Authors	Year of Publication	Model	Optimising Methods	Catchments
Lichty, Dawdy & Bergmann	1968	U.S.G.S.	Rosenbrock	One in North Carolina
Boughton	1968	Boughton	Steepest Descent	Five in New Zealand
Dawdy & Bergmann	1969	U.S.G.S.	Rosenbrock	One in southern California
Wood & Sutherland	1970	Stanford	Steepest Descent	Five in New Zealand
Murray	1970	Modified Boughton	Rosenbrock	Brenig, Wales
Chapman	1970	A.W.R.C. Representative Basins Project	Simplex	One in Central Australia
Porter & McMahon	1970	Porter & McMahon	Steepest Descent Univariate	Two near Melbourne, Australia
Ibbitt & O'Donnell	1971b	Dawdy & O'Donnell	9 methods (for comparison)	Synthetic Record
Ibbitt	1972	Dawdy & O'Donnell	Rosenbrock	Synthetic Record with known errors

In all of the studies listed above, the optimising methods adjusted the initial estimates of the parameter values so that substantial reductions were made to the objective functions. In most cases, however, it would not be possible to state that the global minimum of the response surface had been reached. Some of the authors state that it is possible to obtain similar low values of the objective function with quite different sets of parameter values (although some of the parameters may maintain a fairly constant value). Two reasons for this are usually proposed:-

(i) Inter-dependence between parameters. This is present when a change in one parameter can be compensated for by changes in one or more of the other parameters. For a two-parameter model, the effect on the response surface of inter-dependence between the parameters is to produce an elongated, almost flat-bottomed valley as illustrated in Fig. 4.1. This concept of a valley in the response surface is extended to problems of higher dimensionality (where the model has more parameters) even though the surface cannot be represented visually. Any combination of parameter values lying close to the bottom of such a valley will produce a near-optimum value of the objective function. The differences in individual parameter values between such combinations can be quite large.

The search techniques commonly in use will descend quite rapidly into a valley in the response surface from given starting points. However, progress along the floor of the valley to the lowest point is then very slow. It should also be noted that, for a given starting point outside the valley, different search techniques will arrive at different points on the floor of the valley.

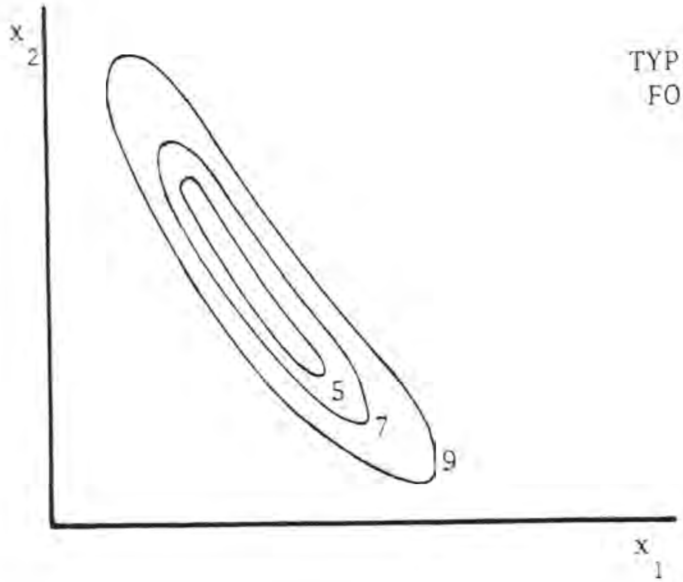


FIGURE 4.1
TYPICAL RESPONSE SURFACE
FOR TWO INTER-DEPENDENT
PARAMETERS

(ii) Indifference of the objective function to the value of parameter(s).

This is present when the objective function is not affected by large changes to the value of a parameter (without compensating changes to other parameters). The contours of the response surface are then parallel to the axis representing that particular parameter. If there is indifference to more than one parameter, the contours are not necessarily parallel to any axis, but they are very widely spaced, giving a flat area in the response surface. Search techniques are generally not able to move off such flat areas.

Indifference will occur when a parameter does not play a significant part in calculating the output from the model. This could be due to the parameter being irrelevant to the physical process being modelled, and thus a redundant part of the model. Alternatively, and of more practical concern, it is possible that the particular set of data being used with the model is such that a physically significant parameter does not become active in the model calculations. The objective function would display apparent indifference to the parameter if these data were used for optimisation, but would not be indifferent if it were possible to use a set of data which did activate the parameter in question.

There seems to be general agreement among those who have worked with rainfall-runoff models that inter-dependence between at least some of the parameters of such models exists. It seems probable that inter-dependence and indifference occur in varying degrees over the response surface for these models. Ibbitt and O'Donnell (1971a) discuss these and other features of the response surface and their effect on the search for the optimum parameter values.

An important aim of this project was to correlate the optimum parameter values of the selected model for a number of catchments with physical characteristics of the catchments (e.g., catchment area, slope, soil properties). With the likely presence of inter-dependence between parameters, the danger existed of attempting to correlate non-optimal parameter values (which, however, gave a near-optimal value of the objective function) with the catchment characteristics. The correct location of the global minimum of the response surface for each catchment was essential for the successful outcome of the project, and most of the work performed was directed towards this goal.

Optimising methods have received considerable attention from workers in the field of applied mathematics in the last ten years. Much of this activity has been reported in The Computer Journal, published by The British Computer Society. A good reference on this work is Kowalik and Osborne (1968). New methods and modifications to old methods are still being proposed, and it appears that more efficient methods will be available in the future than those in current use. The current methods may be divided into two categories; Direct Search methods and Descent methods. The general strategy behind the methods is outlined in the following two sections. The methods which were used in this project are described in greater detail, and inadequacies and problems encountered with the methods are discussed.

4.1 DIRECT SEARCH METHODS

These methods proceed from a starting point (or, for the Simplex method, a group of points) on the response surface and proceed by generating further trial points and moving towards those that progressively lower the objective function. They merely require the ability to compare the values of the objective function at different points on the response surface. They may be further subdivided into methods for finding the minimum of a function of a single variable, and methods for functions of more than one variable.

4.1.1 Methods for Functions of a Single Variable

These methods are a basic part of some of the Descent methods for functions of more than one variable, as they may be used to find the minimum along a particular direction in multi-dimensional space. Several methods are described in Kowalik and Osborne (1968) sections 2.2 and 2.3. The particular method used within the Descent methods employed in this project is illustrated in Fig. 4.2.

From the starting value of the variable x, equal sized steps are taken in the descent direction. (When moving along a direction in multi-dimensional space each step involves changing a number of variables simultaneously.) At each step, the objective function is evaluated, and the steps continue as long as the function continues to decrease. At the first step where the function begins to increase, it is assumed that the points x_1, x_2 and x_3 (Fig. 4.2) bracket the minimum. In the initial stages of the project the point x_2 was taken to be the minimum. For most of the work however, parabolic interpolation was used to predict the position of the minimum. If the function values at x_1, x_2 and x_3 are taken to be f_1, f_2 and f_3, then the minimum is predicted to be at

$$x' = x_2 - \frac{\Delta x}{2} \cdot \frac{f_3 - f_1}{f_3 - 2f_2 + f_1}$$ where Δx is the step size.

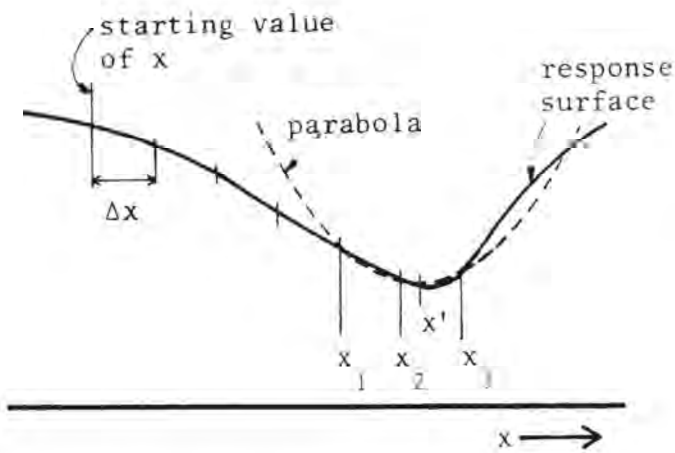


FIGURE 4.2
LOCATING THE MINIMUM OF A FUNCTION OF A SINGLE VARIABLE

The objective function is evaluated at x' and if it is lower than at x_2, is taken to be the minimum. However, sometimes the function value at x' is not lower than at x_2. This occurs when a parabola is not a good approximation of the function as in Fig. 4.3.

When this occurs, the function is evaluated at x'' equidistant from x_2 and on the opposite side from x'. The position of the minimum is then taken as either x_2 or x'', whichever has the lower function value.

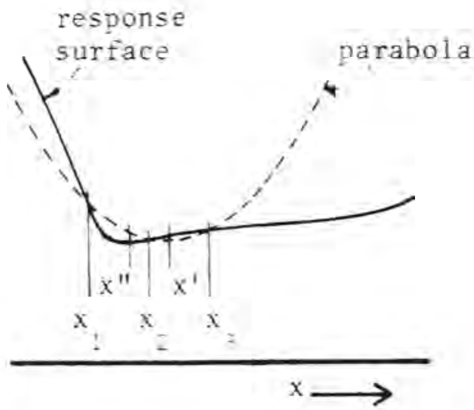


FIGURE 4.3
LOCATING THE MINIMUM WHEN
PARABOLIC INTERPOLATION
IS NOT SUCCESSFUL

4.1.2 Methods for Functions of More Than One Variable

The various Direct Search methods for functions of more than one variable differ in the way in which they generate new trial points on the response surface for evaluation. Examples of these methods are those of Hooke and Jeeves (1961), Rosenbrock (1960), various Simplex methods, e.g., Spendley et al. (1962), Nelder and Mead (1965), and a method developed from the Simplex methods by Peckham (1970).

Two versions of the Simplex technique were used in the work of this project. (The Simplex method in this context does not refer to the well-known technique for solving linear programming problems.) A simplex is a set of $n + 1$ points in n-dimensional space. If the points are equidistant, the simplex is "regular". The vertices of an equilateral triangle form a regular simplex in two dimensional space, while those of a tetrahedron do so in three dimensional space. Each point in the simplex is specified by a set of parameter values and has an associated value of the objective function. If that point with the highest function value is reflected around the centroid of the other points, it would be expected that the point so generated would be closer to the minimum. This notion is the basis of the Simplex methods.

Some work was done in this project with the method described by Spendley et al. (1962). This method uses a regular simplex and, theoretically, has advantages when trying to follow a valley in the response surface (see Beveridge and Schechter, 1970, pp. 367-383). However, the method proved disappointing in this aspect, and problems of cycling of the solution occurred, in spite of measures built into the method to prevent this.

Extensive use was made of the method of Nelder and Mead (1965). This method incorporates provisions for the simplex to change its size and shape, contracting away from high areas and expanding towards lower areas. As the minimum is approached, the simplex shrinks until, ultimately, the points become coincident at the minimum. However, the shrinking of the simplex until the points become coincident is not a sufficient condition for concluding that the minimum has been reached. Many times during the work of this project it was found that further improvement could be made from such a point by using some other method or by numerical trials. The simplex appears to shrink to a point on the floor of a valley instead of moving down the valley. It is often claimed that the method of Nelder and Mead will find the global minimum of a function provided the starting simplex is large enough to span the area containing the minimum. The results obtained in this project indicate that this is not necessarily true.

A full description of the method of Nelder and Mead is given in Appendix A1, as well as a possible reason why the method appears to be inefficient in moving along valleys in the response surface.

4.2 DESCENT METHODS

It is commonly observed that the Direct Search methods give an initially rapid reduction in the objective function, but are slow in ultimate convergence. Results obtained with the Simplex methods in this project were in accordance with this experience. The Direct Search methods use only values of the objective function and make simple comparisons of these values. Most of the Descent methods make use of additional information about the surface being searched and could thus be expected to give more rapid ultimate convergence. Many of these methods require the slope of the surface in each co-ordinate direction (i.e., the partial derivative of the objective function with respect to each parameter) at each iteration. Additionally, the conjugate direction methods assume that, close to the optimum, the surface may be approximated by a positive definite quadratic form. In general, the Descent methods search for the minimum by performing a sequence of one dimensional searches (as described in section 4.1.1). The methods differ in the means of choosing the directions over which these searches are conducted.

4.2.1 Univariate Method (Relaxation)

In this method the search directions are the co-ordinate directions, which are searched repeatedly in cyclic order until no further improvement can be made. This method does not use any more information about the response surface than the Direct Search methods use. Fig. 4.4 illustrates a case for a two parameter problem where this method would be very slow. If the sides of the valley were too steep and a large step size used, the method would stop at a non-optimum point as both +ve and -ve steps in each co-ordinate direction would lie higher up on the sides of the valley.

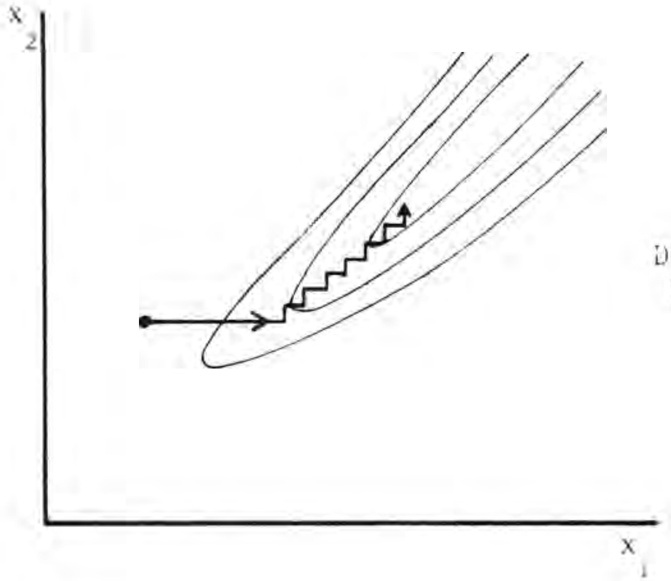


FIGURE 4.4
DIFFICULT RESPONSE SURFACE
FOR THE UNIVARIATE METHOD

4.2.2 Steepest Descent Method

Here the search direction at each iteration is the direction of steepest slope from the current point, and is defined by the vector of partial derivatives at this point. The method has been extensively used, and was used in the early work of this project. However it was found that after several iterations, little movement was made along the chosen search directions, and most of the computing effort was used in defining new search directions. Sometimes the method stopped at a point which was found to be non-optimal. These experiences suggest that the search was zig-zagging along a valley floor in a similar way to that depicted for the univariate method in Fig. 4.4. Similar experiences have apparently been reported by others. Kowalik and Osborne suggest several possible reasons for the disappointing performance of the method. The most significant reasons seem to be that the chosen search directions get into a "cage", i.e., a small number of search directions are used in cyclic fashion, and that, ultimately, the directions may become asymptotic to just two directions, so that the minimum is approached in a two dimensional sub-space.

4.2.3 Conjugate Direction Methods

Some of these methods require the partial derivatives of the function at each iteration and they all assume that in the area of the minimum, the function may be approximated by a positive definite quadratic form. The significance of these methods is that, if the function is of this form, and is a function of n variables, then the minimum will be found in n iterations. In this case, the n search directions chosen are linearly independent, i.e., no one direction may be expressed as a linear combination of the other directions. Therefore cycling through a small number of directions cannot occur. Where the function is not of the above form more iterations are required, but as the minimum is approached, the quadratic approximation improves, so the ultimate rate of convergence should be good. The quadratic form which approximates the objective function need not be known explicitly. (An example of the use of a quadratic form to approximate a function is the use of the parabola to approximate the one dimensional function in Fig. 4.2.) Methods which do not require the partial derivatives of the function are those of Powell (1964) and a modification of this method by Zangwill (1967). Two methods which do require derivatives were used in this project. The method of Fletcher and Reeves (1964) was first used, but showed little improvement on the steepest descent method in some comparative runs for one catchment. A method originally proposed by Davidon (1959) and presented definitively by Fletcher and Powell (1963) (hereafter referred to as the Davidon method) was used for much of the work in the project. This method is recommended by Kowalik and Osborne, and is described in Appendix A.

4.3 PROBLEMS IN IMPLEMENTING THE SEARCH METHODS

The problems discussed below can have a marked effect on the efficiency of the search methods. The measures taken in this project to deal with these problems are described in Appendix A2.

4.3.1 Defining the Steepest Descent Direction

The steepest descent direction at a point on the response surface is found from the slope of the surface in each of the co-ordinate directions at that point. In many optimising problems these slopes must be found by numerical methods. A simple method was first employed in this project, but it was found necessary to improve on this, and five different methods were used during the progress of the work. These methods are described in Appendix A2 and the times at which they were used are indicated at the appropriate places in this report.

4.3.2 Scaling the Parameters

Scaling may be used to change the shape of the response surface and alter such difficult features as long, flat-bottomed valleys. Scaling is achieved by transforming some of the parameters.

An idea of the important influence of scaling on the efficiency of search methods may be obtained by comparing the steepest descent searches on the two surfaces represented by the contour maps illustrated in Fig. 4.5. Where the contours are circular, steepest descent will find the minimum in one iteration, but in other cases, the number of iterations depends on the degree of "elongation" of the contours and the required accuracy in locating the minimum point.

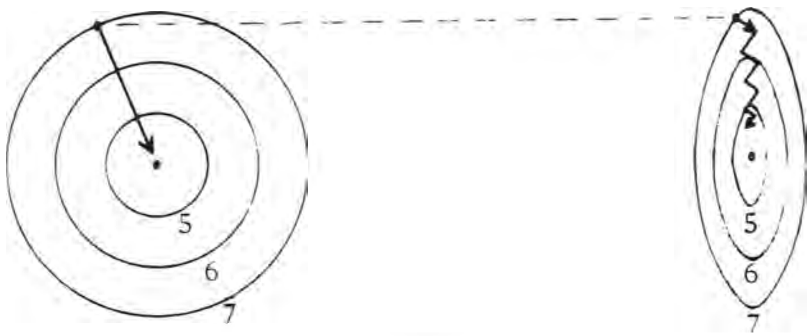


FIGURE 4.5
THE INFLUENCE OF SCALING ON
THE EFFICIENCY OF SEARCH METHODS

The parameters should therefore be transformed in such a way that the new response surface has near-circular contours, but little guidance in this matter is offered in the current literature on optimisation. The problem is discussed in more detail in Appendix A2. Some attempt at improving the shape of the response surface was made in the work with Lidsdale No. 2 catchment described in Section 7.

5. SEARCH FOR OPTIMUM PARAMETER VALUES FOR THE POKOLBIN CATCHMENTS

These four adjacent catchments are in the Pololbin area, located several miles north west of the town of Cessnock in the Hunter Valley, New South Wales. They were among the first for which concurrent rainfall and runoff records were received under the data compilation and transfer section of A.W.R.C. Research Project 68/1. The areas and periods of data used for optimisation are as follows:-

Catchment	Area (acres)	Rainfall Data	Runoff Data
First Ck (Site 1)	3500	1-1-1964 to 31-12-1964 1-4-1965 to 31-10-1968	27-10-1963 to 12-5-1969
Middle Ck. (Site 2)	1900	ditto	13-12-1963 to 13-5-1969
Deep Ck. (Site 3)	6300	ditto	13-11-1963 to 13-5-1969
Deep Ck. (Site 4)	1180	ditto	13-11-1963 to 13-5-1969

Maps and descriptions of the catchments, including soil survey information, are presented in Appendix A3.

The rainfall data available when work commenced with these catchments were daily falls at ten stations on and near the catchments. Average catchment rainfalls for the four catchments were estimated from these records by Thiessen weighting using all available records at any given time. (The individual station records were not all concurrent and the average length of record was about three years.)

Evaporation data were obtained from maps of mean monthly evaporation issued by the Commonwealth Bureau of Meteorology and converted to potential evapotranspiration by applying pan factors which were obtained from the work of Penman and used by Boughton (1965). The factors were:-

0.8	Nov., Dec., Jan., Feb.
0.7	Mar., Apr., Sept., Oct.
0.6	May, June, July, Aug.

The mean monthly figures were then divided by the number of days in the month to obtain average daily potential evapotranspiration.

Ideally, the daily potential evapotranspiration data would be estimated by an energy balance method using meteorological data or, alternatively, from daily observations of pan evaporation. Such data would rarely be available, particularly for catchments of the size included in this project. A compromise is the use of observed total monthly pan evaporation data to derive average daily figures. Records were sought from pans at Stockyard Ck., about 15 miles south west, and Tocal, about 20 miles north east of the catchments, but were only available for a few months concurrent with the rainfall and runoff data and were of poor quality.

The runoff data provided were average daily flows in cu.ft./sec. There were a few missing periods of several days in the records.

The Steepest Descent and Simplex (Nelder and Mead) methods were used to search for the optimum parameter values of the Boughton model for these catchments. Each time the methods call for a run of the data through the model (i.e., for each set of trial parameter values) it is necessary to assume starting values for the quantities of water in the various moisture stores. For the Pokolbin catchments it was assumed that the Interception Store was initially empty and that the other stores were initially half full. The squares of deviations between observed and calculated monthly runoff quantities for the first six months of model operation were not added into the objective function to allow for bias in the output of the model caused by differences between the assumed and actual (but unknown) initial quantities in the stores. The use of the above initial storage quantities and of a "warm-up" period of six months is now thought to be inappropriate, and is further discussed in Section 8.

When a period of missing record was encountered in the runoff data, operation of the model continued but any calculated runoff in that period was not added into the monthly total when calculating the objective function. A period of missing record in the rainfall data required operation of the model to cease. Operation was re-commenced with re-initialised storage contents at the end of the period of missing record and a further "warm-up" period of six months was observed before further additions were made to the objective function.

In the programme to apply the Steepest Descent method, the steepest descent directions were first defined by Method 1 described in Appendix A2. The minimum point on each descent direction was found by the procedure described in sub-section 4.1.1 without using parabolic interpolation.

In using the Simplex method, it was soon observed that after approximately fifty iterations the simplex had contracted so that there were only small differences between the points and that further reduction of the objective function was slow. However, it was found that significant further improvement could sometimes be obtained by continuing the search with the Steepest Descent method. The procedure of commencing the search with the Simplex method and continuing with the Steepest Descent method was therefore used frequently.

The general aim of this work for each catchment was to commence searches from several different sets of parameter values and, hopefully, to arrive by different routes at one set of values which could be regarded at least as a local optimum set of parameter values. Such a result would have indicated that the search techniques had been implemented correctly and that the response surfaces for the catchments were relatively easy ones on which to search for the minimum. However no two searches arrived at a common point for any of the catchments. Sometimes the parameter values for a particular catchment arrived at by the search techniques were used to start the search for optimum parameter values for another catchment. Table 5.1 lists some of the points on the response surfaces which were found by this work for the four catchments.

TABLE 5.1

POKOLBIN CATCHMENTS. POINTS ON RESPONSE SURFACES

Point No.	VSMAX (pts)	USMAX (pts)	DSMAX (pts)	SSMAX (pts)	EVPMAX (pts/day)	PV -	FC (pts/day)	FO (pts/day)	KF -	SUMSQS (pts^2)
First Creek (Site 1)										
1	3	164	24	711	60	0.8	3	674	0.005	736
2	2	192	143	805	32.5	0.51	4	419	0.010	1189
3	3.9	206	135	1264	35.3	0.44	4	736	0.011	805
Middle Creek (Site 2)										
1	3	155	24	706	60	0.9	3	667	0.005	7687
2	2	178	153	873	30.5	0.52	3.8	403	0.012	726
3	6.3	119	149	1555	25.3	0.42	1	344	0.007	1950
Deep Creek (Site 3)										
1	3	177	24	757	60.5	0.75	3	708	0.0042	187
2	2	189	193	765	33.1	0.45	4.5	408	0.009	185
3	3	104	39	828	61.3	0.69	8	695	0.0038	352
Deep Creek (Site 4)										
1	5.3	134	23.3	1190	61.4	0.62	9.74	925	0.0018	580

Some other points at which either the objective function value was significantly higher than those above or the parameter values were considered to be extreme were obtained also. Points 1 and 3 for First Creek and points 1 and 2 for Deep Creek (Site 3) illustrate the statement made earlier that different sets of parameter values may be found which give similar values of the objective function.

The steepest descent searches stopped at the points shown in Table 5.1 when no improvements were possible along the descent directions chosen at those points. This implied that the directions were being defined erroneously and prompted a reappraisal of the method used to choose the descent directions. Some inadequacies in Method 1 for defining the steepest descent directions are described in Appendix A2. Method 2, which was designed to overcome these inadequacies, is also described. The programme to apply the Steepest Descent search was modified to define the descent directions by Method 2.

The point No. 1 for Middle Ck. (Site 2) listed in Table 5.1 was then used as a starting point for this modified programme, with the following result:-

Point	VSMAX	USMAX	DSMAX	SSMAX	EVPMAX	PV	FC	FO	KF	SUMSQS
Start	3.0	155	24	706	60	0.9	3	667	0.005	7687
New Optimum	4.6	156	42	755	62.5	0.92	0	643	0.005	6383

Considerable computing effort was required to achieve this reduction in the objective function. The course of the optimisation was through 62 changes in direction. Each new definition of the descent direction by Method 2 usually required an average of 14 runs through the Boughton model. Sometimes two or three times this number of runs were required when smaller than usual parameter increments had to be used. Together with the model runs involved in stepping along each direction, the total number of runs through the Boughton model was approximately 1500, with about 4½ years of daily data per run.

A further modification was made to the programme so that, in stepping along each descent direction, the lowest point along the line was found using the parabolic fitting method described in sub-section 4.1.1. Unfortunately this did not lead to any significant improvement on the above performance, but the modification was retained as it is, theoretically, an improvement on the earlier method.

The starting point and the points at the bottom of the first nine descent directions of the search described above were then used as a starting simplex for the Simplex method. Using only 130 runs through the Boughton model, the objective function was reduced to a value of 2936. The following changes in parameter values occurred:-

	VSMAX	USMAX	DSMAX	SSMAX	EVPMAX	PV	FC	FO	KF	SUMSQS
Start	3.0	155	24	706	60	0.9	3.0	667	0.005	7687

Lowest point in Simplex after 130 model runs:-

	3.5	175	94	1093	52	0.98	3.3	606	0.0055	2936

Further alternate application of the Steepest Descent and the Simplex methods led to the following point:-

VSMAX	USMAX	DSMAX	SSMAX	EVPMAX	PV	FC	FO	KF	SUMSQS
3.8	180	96	1955	227	0.97	2.0	1578	0.0056	2757

Little further reduction in the objective function occurred, but some of the parameter values changed quite markedly. Some form of inter-action appears to be present between the parameters SSMAX, EVPMAX and FO.

Theoretically, a more satisfactory measure of the fit of the calculated runoff to the observed runoff is the sum of the squares of the deviations between the calculated and observed totals for each runoff event (see Section 9). Optimisation was therefore attempted using an objective function based on event deviations to see if the corresponding response surface was an easier one on which to locate the minimum point. This was done concurrently with some of the work described above.

The Steepest Descent programme (with Method 1 described in Appendix A2 to define the descent directions) was modified to evaluate the objective function using event deviations. The start and end of a runoff event may be defined in many ways, but for this application, the start was taken to be the first day on which a rise occurred in the observed runoff hydrograph, and this also marked the end of the previous event. Thus, many events encompassed a long period of zero runoff at the end of the recession curve.

The parameter values at points 1 and 2 for Middle Ck. (Site 2) in Table 5.1 were used to start two searches for the minimum of the event-based objective function, with the following results:-

Point	VSMAX	USMAX	DSMAX	SSMAX	EVPMAX	PV	FC	FO	KF	SUMSQS (over events)
<u>Point No. 1 (SUMSQS over months was 7687)</u>										
Start	3	155	24	706	60	0.9	3.0	667	0.005	7754
End	3	155	24	713	60	0.89	3.0	666	0.005	7640
<u>Point No. 2 (SUMSQS over months was 726)</u>										
Start	2	178	153	873	30.5	0.52	3.8	403	0.012	846
End	2	197	156	788	32	0.52	4.2	399	0.013	618

The end points of the above searches are not significantly different from the start points, which themselves were the end points of searches where the objective function was summed over monthly deviations. Also, there are only small differences in the numerical values of the two objective functions for a given set of parameter values. Inspection of the runoff record reveals that there are only several months which contain more than one runoff event. In each of these months there is no more than one event which contributes significantly to the objective functions. Under these circumstances, the two functions are virtually the same and their response surfaces would be expected to be of very similar shape.

As unique sets of optimum parameter values could not readily be found for the Pokolbin catchments and as similar difficulties were expected for other catchments it became necessary to reconsider the aims of the project. It was thought that the remaining time would best be spent in a concentrated search for the optimum parameter values for, if necessary, only one catchment. The aim was to find an efficient optimisation strategy which could then be used for other catchments. This work involved examining and introducing more efficient search methods and investigating such problems as parameter inter-dependence and the scaling of the parameters during optimisation. It has been necessary for greater clarity to describe this work in separate sections in the remainder of this report although many different aspects of the work were inter-related and were performed concurrently.

6. CHANGES INTRODUCED INTO THE BOUGHTON MODEL CALCULATIONS

Changes were made in the evapotranspiration and infiltration calculations of the Boughton model. These were found to be desirable during the algebraic analysis of the model, described later in Section 8.

The evapotranspiration and infiltration functions in the Boughton model are used to find total daily amounts for these processes given the contents of the moisture stores at the start of the day. Although only daily quantities are calculated in the changed calculations, the processes are regarded as continuous and the functions are used to relate instantaneous values of these processes to the soil store contents. As the processes influence the store contents and are at the same time functions of these contents, the daily totals used in the amended model calculations are found from expressions derived by integrating the instantaneous functions over a period of one day.

The changed calculations have the following advantages:-

(i) they prevent the occasional unrealistic result such as "over-filling" of the Lower Soil Store during infiltration and "over-emptying" of the Upper Soil Store by evapotranspiration under some combinations of parameter values, and

(ii) they allow the contents of the stores at a given time to be expressed more easily in terms of the model parameters and the data.

The changed calculations were introduced before the work with Lidsdale No. 2 catchment described in Section 7.

6.1 EVAPOTRANSPIRATION CALCULATIONS

The evapotranspiration calculations in the Boughton model are based on the simplified concepts of fixed values for the field capacity and wilting point and it is assumed that the variations of these values from real conditions do not introduce major errors into the calculations. The model assumes that when the soil moisture level is at field capacity there is a limiting rate at which evapotranspiration (abbreviated to e/t in the discussion below) will occur from the catchment. This rate is a parameter of the model, EVPMAX. As the soil dries out, the limiting e/t rate decreases linearly to zero when the soil moisture level is at wilting point. On days when the potential e/t rate, determined by meteorological conditions, is less than the limiting rate at the current soil moisture level, e/t occurs at the potential rate.

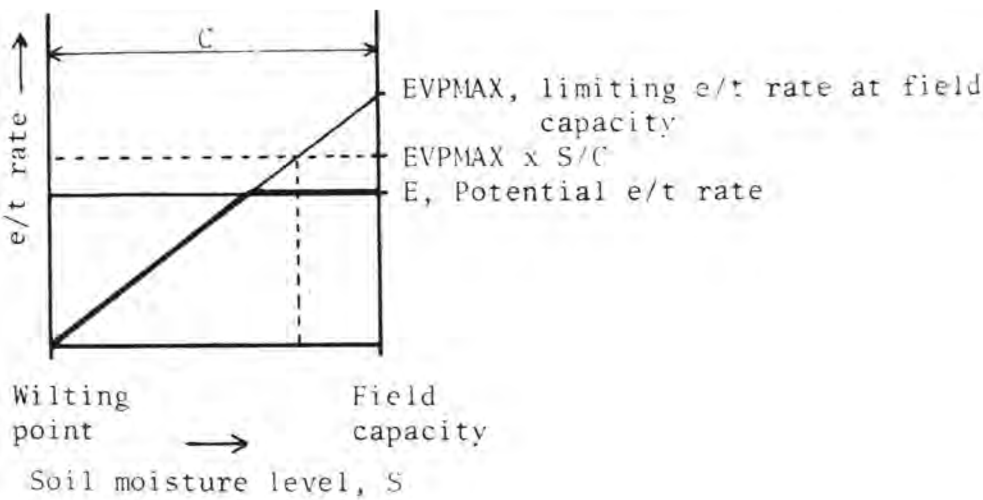


FIGURE 6.1
THE EVAPOTRANSPIRATION FUNCTION
USED IN THE BOUGHTON MODEL

Referring to Fig. 6.1 then, the actual e/t loss, on a day when the potential loss is E, is determined from the lower of the two straight lines at the current soil moisture level. The e/t function therefore consists of the two heavily drawn straight line segments. The height of the horizontal segment can change from day to day if daily potential e/t data are available.

In algebraic terms, the actual e/t loss is determined as follows:-

$$\text{if} \quad E < \frac{S}{C} \times \text{EVPMAX} \quad \text{then the e/t loss} = E \tag{6.1}$$

$$\text{if} \quad E \geq \frac{S}{C} \times \text{EVPMAX} \quad \text{then the e/t loss} = \frac{S}{C} \times \text{EVPMAX} \tag{6.2}$$

In the Boughton model the soil is represented by two stores. The e/t loss for each store is computed separately by the above method and then multiplied by PV for the Upper Soil Store and (1 - PV) for the Lower Soil Store, where the parameter PV is a fraction which would depend on the rooting of the vegetation.

In algebraic terms, the actual e/t loss from each store is determined as follows:-

(a) Upper Soil Store

$$\text{if } E < \frac{\text{US} \times \text{EVPMAX}}{\text{USMAX}} \quad \text{then the e/t loss} = \text{PV} \times E \tag{6.3}$$

$$\text{if } E \geq \frac{\text{US} \times \text{EVPMAX}}{\text{USMAX}} \quad \text{then the e/t loss} = \text{PV} \times \frac{\text{US} \times \text{EVPMAX}}{\text{USMAX}} \tag{6.4}$$

(b) Lower Soil Store

$$\text{if } E < \frac{\text{SS} \times \text{EVPMAX}}{\text{SSMAX}} \quad \text{then the e/t loss} = (1 - \text{PV}) \times E \tag{6.5}$$

$$\text{if } E \geq \frac{SS \times EVPMAX}{SSMAX} \quad \text{then the e/t loss} = (1 - PV) \times \frac{SS \times EVPMAX}{SSMAX} \tag{6.6}$$

The change which was introduced influences these calculations at the lower soil storage levels when the actual e/t is determined from the sloping segment of the e/t function. On this segment the e/t rate is a function of the current stored contents and should decrease through the day as the contents are depleted. In effect, the e/t function illustrated in Fig. 6.1 was regarded as giving an instantaneous e/t rate rather than a daily e/t quantity. The daily e/t quantity was found by integration.

This change eliminated an unrealistic calculation which sometimes occurred when equation (6.4) was applied for the Upper Soil Store. When the estimated values of PV and EVPMAX were high and the estimated value of USMAX was low, then PV x EVPMAX/USMAX became greater than 1, and the e/t loss was then greater than the current contents, US. Thus, the store could be completely emptied in one day even though the rate of loss should have been restricted by low soil moisture and zero storage approached in an exponential manner.

On the sloping segment of the e/t function, the e/t rate equals the rate of change in the soil storage level and is a function of this level. Using the symbols in Fig. 6.1, the governing equation is

$$\frac{dS}{dt} = k\,S \tag{6.7}$$

where $k = -\dfrac{EVPMAX}{C}$. (see equation 6.2)

The negative sign is included because S decreases with time. If this equation were applied for the Upper Soil Store, S would be equal to US,

k would be equal to $-\dfrac{PV \times EVPMAX}{USMAX}$.

Equation (6.7) is integrated in Appendix A4 for one day's e/t. The result is

$$S_1 = e^k \cdot S_0 . \tag{6.8}$$

The subscripts 0 and 1 denote storage contents at the start and end of the day respectively.

Applying this result to the two soil stores of the Boughton model, the amended e/t calculations are:-

(a) Upper Soil Store

$$\text{if } E < \frac{US \times EVPMAX}{USMAX} \quad \text{then } US_1 = US_0 - PV \times E, \text{ as before} \tag{6.9}$$

$$\text{if } E \geq \frac{US \times EVPMAX}{USMAX} \quad \text{then } US_1 = US_0 \times e^{-\frac{PV \times EVPMAX}{USMAX}} \tag{6.10}$$

(b) Lower Soil Store

$$\text{if } E < \frac{SS \times EVPMAX}{SSMAX} \quad \text{then } SS_1 = SS_0 - (1 - PV) \times E, \text{ as before} \tag{6.11}$$

$$\text{if } E \geq \frac{SS \times EVPMAX}{SSMAX} \quad \text{then } SS_1 = SS_0 \times e^{-\frac{(1 - PV) \times EVPMAX}{SSMAX}} \tag{6.12}$$

For the Upper Soil Store, if $E > \dfrac{US \times EVPMAX}{USMAX}$ for a period of n days, it can be shown that

$$US_n = US_0 \times e^{-\frac{n \times PV \times EVPMAX}{USMAX}} \qquad (6.13)$$

If $E < \frac{US \times EVPMAX}{USMAX}$ at the start of a day, but the discontinuity in the e/t function is crossed during the day, then

$$US_1 = USC \times e^{-\frac{EVPMAX \times (USC + PV \times E - US_0)}{USMAX \times E}} \qquad (6.14)$$

where $USC = \frac{E \times USMAX}{EVPMAX}$ and is the value of US at the discontinuity. Expressions similar to equations (6.13) and (6.14) may be found for the Lower Soil Store.

The effect of the changed calculations is to reduce the loss from the soil stores in the lower range of store contents, but the overall effect on the operation of the model appears to be small.

6.2 INFILTRATION CALCULATIONS

In the original calculations of the Boughton model, the contents of the Drainage Store infiltrate into the Lower Soil Store at a daily rate given by

$$F = FC + (FO - FC)\, e^{-KF \times SS} \qquad \text{(equation 3.1)}$$

This rate is a potential rate which is only satisfied if the contents of the Drainage Store are sufficient. In addition, part of any overflow from the Drainage Store enters the Lower Soil Store, the actual amount being determined by a function of F.

Several unsatisfactory effects are obtained when the above function for F is used. These are:-

(i) F is a function of the contents of the Lower Soil Store, but not of the capacity of the store. The contents alone do not reflect the wetness of the soil, on which the infiltration rate depends. If the capacity of the Lower Soil Store was 500 pts and the contents 400 pts, the soil would be relatively wetter, and have a lower infiltration rate, than when the capacity was 1000 pts and the contents 400 pts. However the above function will give the same value for F in both cases.

(ii) Infiltration can occur when the Lower Soil Store is full, resulting in "over-filling" of this store. When this occurs, the contents are re-set to SSMAX and the amount of "over-fill" lost from the model. It seemed preferable that all water entering the model should be accounted for and that infiltration should reduce to zero as the contents approach SSMAX.

(iii) Very large values of F are obtained when SS is low, and SS can therefore change markedly during one day. However it is assumed that infiltration goes on at the same rate during the day as the store fills. In a similar approach to that used for the evapotranspiration calculations, it was thought that the instantaneous rate should decrease through the day as the contents increase and that the daily potential infiltration amount should be found by integration.

A function for an instantaneous infiltration rate was sought where F depended on both SS and SSMAX, F approached zero as SS approached SSMAX, and which could be integrated to obtain an expression for the potential daily infiltration amount.

Equation (3.1) was first regarded as giving an instantaneous rate instead of a daily infiltration quantity. The parameter FC, which frequently tended to zero in optimisation runs, was effectively set to zero by eliminating it from the equation, and SSMAX was introduced into the exponent of e. The resulting equation was

$$F = FO \cdot e^{-KF \cdot SS/SSMAX} \tag{6.15}$$

The numerical value of KF will now be different from the KF value in equation (3.1). As SS approaches zero, this function approaches FO, but as SS approaches SSMAX, the function has a positive component, $FO \cdot e^{-KF}$. A suitable function which eliminates this is

$$F = FO \cdot e^{-KF \cdot SS/SSMAX} - \frac{SS}{SSMAX} FO \cdot e^{-KF} \tag{6.16}$$

However, this function proved too difficult to integrate and the function finally adopted was

$$F = FO \cdot e^{-KF \cdot SS/SSMAX} - FO \cdot e^{-KF} \tag{6.17}$$

This function approaches zero as SS approaches SSMAX, but approaches $FO(1 - e^{-KF})$ as SS approaches zero. However the effect of this was expected to be small. Equations 3.1 and 6.15 to 6.17 incl. are plotted for comparison in Fig. 6.2.

As infiltration proceeds, the rate of change of storage in the Lower Soil Store ie equal to the infiltration rate, i.e.,

$$\frac{d(SS)}{dt} = F = FO \cdot e^{-KF \cdot SS/SSMAX} - FO \cdot e^{-KF} \tag{6.18}$$

The change in SS over one day is the total amount of infiltration and must be found by integrating equation (6.18).

Using F now to denote the potential daily infiltration amount rather than the instantaneous rate, the result of the integration, shown in Appendix A4, is

$$F = SSMAX\left(1 + \frac{1}{KF} \cdot \ln\left(1 - D + De^{KF(SS/SSMAX - 1)}\right)\right) - SS \tag{6.19}$$

where $D = e^{-FO \cdot KF \cdot e^{-KF}/SSMAX}$

and SS = storage at the start of the day.

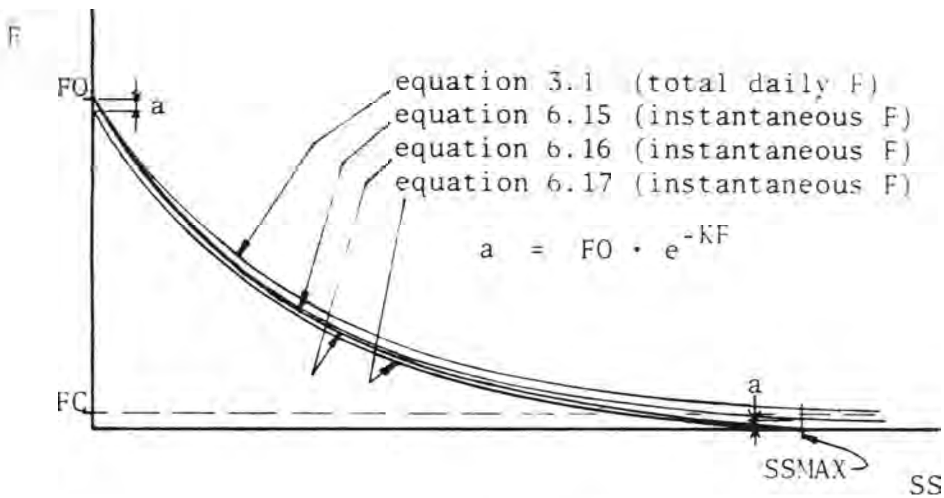


FIGURE 6.2
COMPARISON OF INFILTRATION FUNCTIONS

This F value is the potential daily infiltration amount and is only satisfied if the supply of moisture is maintained continuously throughout the day. The intermittent nature of infiltration has its effect on the model calculations when the contents of the Drainage Store are insufficient to satisfy the F value derived above and also when this value is used in the equation to calculate runoff from the amount of water which overflows the Drainage Store,

$$Q = P - F \tanh(P/F). \qquad \text{(equation 3.2)}$$

When the stored contents of the Lower Soil Store approach the capacity of the store an equilibrium state is reached where the inflow by infiltration from the Drainage Store equals the outflow to groundwater calculated by applying the subsoil depletion factor of 0.999. The store therefore never fills completely and a small flow through the store is always maintained despite the fact that the infiltration function was designed to approach zero as the store approached the full state and does not include a constant term which is independent of the soil moisture level.

The changed calculations give a lower potential daily F and higher runoff at a given level of storage. However as low infiltration and high runoff for one event tend to give low soil storage and therefore high infiltration and low runoff for the next event, the nett effect of the changed calculations over a number of events is not clear. The changed calculations were introduced into the model during preliminary work with Lidsdale No. 2 catchment and gave significant improvement in the objective function.

7. SEARCH FOR OPTIMUM PARAMETER VALUES FOR LIDSDALE NO. 2 CATCHMENT

After the work with the Pokolbin catchments described in Section 5, the Lidsdale No. 2 catchment was selected for the bulk of the optimisation work in this project for the following reasons:-

(i) pan evaporimeter records were available instead of average monthly evaporation data, enabling more accurate modelling of the drying of the catchment;

(ii) soil moisture data were also available. Comparison of the contents of the soil stores of the model with these data could provide a check on the operation of the model; and

(iv) the recording rain gauge and water level recorder charts for this catchment are held by The University of New South Wales and were readily available for scrutiny if required.

The Lidsdale No. 2 catchment is one of a group of eleven catchments in the Lidsdale State Forest, situated approximately eight miles west of Lithgow, New South Wales. The catchments are operated by The University of New South Wales with the assistance of the Forestry Commission of New South Wales. The No. 2 catchment has an area of 31.8 acres and the vegetation is planted Radiata Pine forest. Mr. M. K. Smith of the Forestry Commission has described the catchment in greater detail, (Smith, 1972), and his description is quoted here in Appendix A3.

Daily runoff figures and monthly evaporation figures were extracted from the records and made available by Mr. Smith. He also made available soil moisture data which he has been collecting and provided estimates based on physical considerations for the values of the parameters of the Boughton model for this catchment. This assistance is gratefully acknowledged.

The period of concurrent rainfall, evaporation and runoff data used for optimisation was from 17-10-1968 to 28-2-1971.

During the progress of the search for optimum parameter values for this catchment, many aspects of the optimisation problem were investigated, and sometimes this work influenced the course of the search. In the following sub-sections the course of the search is outlined, the investigations which were undertaken are described, and the resulting parameter values are discussed.

7.1 OUTLINE DESCRIPTION OF THE SEARCH

The progress of the search is shown graphically in Fig. 7.1. Sets of parameter values and their corresponding values of the objective function are presented in the same way as for the rectangular co-ordinates of a point in 9-dimensional space. The figures are always in the following order:-

VSMAX, USMAX, DSMAX, SSMAX, EVPMAX, PV, FO, KF, SUMSQS.

At the start of the search, the steepest descent direction was being found in the Davidon programme by Method 3 described in Appendix A2. Throughout the search the parameters were scaled to improve the shape of the response surface. This is also described in Appendix A2. The parameter values quoted in Fig. 7.1, however, are the equivalent untransformed values.

Two strands of the search started when the parameter values estimated from physical considerations were used as a starting point for the Simplex and Davidon methods. When run until little further change was occurring in the parameter values, these methods led to two different points. A third strand was started using the Davidon method

Strand 1 (Points No.1) — **Strand 2 (Points No.2)** — **Strand 3 (Points No.3)**

Start

- Strands 1 and 2: M.Smith's estimates [10,30,35,750,30,0.25,500,3.75,<u>38274</u>]
- Strand 3: Arbitrary point [20, 50, 60,1200, 50, 0.5,1000, 5,<u>61543</u>]

Strand 1: 167 Simplex it'ns.
Strand 2: 49 Davidon it'ns. then 150 Simplex it'ns.
Strand 3: 40 Davidon it'ns.

Stage 1

- Strand 1: [32,37,102,1138, 12, 0.58,1500,0.172,<u>5468</u>]
- Strand 2: [6,102,125,1046, 15, 0.76,864,4.224,<u>5954</u>]
- Strand 3: [22, 49, 91,1312, 41, 0.83, 993,3.344,<u>4773</u>]

Strand 1: 159 Simplex it'ns.
Strand 2: 185 Simplex it'ns.
Strand 3: 230 Simplex it'ns.

" 2

- Strand 1: [37,33,115,1173, 15,0.637,1897,0.043,<u>4868</u>]
- Strand 2: [6, 67,112,1247, 20,0.738,860,2.636,<u>4513</u>]
- Strand 3: [22, 22,208, 964, 54,0.541, 754,2.288,<u>2262</u>]

Strand 1: KF shifted, then 18 Davidon it'ns.
Strand 2: 9 Davidon it'ns.
Strand 3: 12 Davidon it'ns.

" 3

- Strand 1: [37,18,139,1286,18.7,0.603,1900, 2.93,<u>2484</u>]
- Strand 2: [7.8, 64,134,1269,18.55,0.726,886,1.052,<u>3887</u>]
- Strand 3: [23, 23,209, 960, 56,0.548, 754,2.308,<u>2188</u>]

Strand 1: 54 Simplex it'ns. only VSMAX, USMAX, PV vary
Strand 2 (first branch): 90 Simplex it'ns. only VSMAX, USMAX, PV vary
Strand 2 (second branch): 20 Davidon it'ns. only VSMAX, USMAX, PV vary
Strand 3: 8 Davidon it'ns. only VSMAX, USMAX, PV vary

" 4

- Strand 1: [53, 4,139,1286,18.7,0.573,1900, 2.93,<u>2301</u>]
- Strand 2 (after the 90 Simplex it'ns.): [61,0,134,1269,18.55,0.576,886,1.052,<u>2306</u>]
- Strand 2 (after the 20 Davidon it'ns.): [60,1,134,1269,18.55,0.58,886,1.052,<u>2317</u>]
- Strand 3: [33.6,12.6,209, 960, 56,0.511, 754,2.308,<u>2066</u>]

Strands 1 and 2: these three points combined into one point.
Strand 3: graphical opt'n of VSMAX, USMAX, EVPMAX. Davidon method made poor progress varying these parameters.

" 5

- Strands 1 and 2: [60, 1,139,1269,18.47, 0.576, 886, 1.052,<u>2318</u>]
- Strand 3: [44, 7.7,209, 960, 50,0.511, 754,2.308,<u>1977</u>]

Strands 1 and 2: 320 Simplex it'ns. FO, KF held constant
Strand 3: 110 Simplex it'ns. only VSMAX, USMAX, EVPMAX, PV vary

" 6

- Strands 1 and 2: [53.5,0.7,282,1114, 18.6, 0.513, 886, 1.052,<u>2106</u>]
- Strand 3: [44, 9.5,219, 960,48.5,0.524, 754,2.308,<u>1954</u>]

Strands 1 and 2: 314 Simplex it'ns. all params. vary.
Strand 3: 330 Simplex it'ns. all params. vary.

" 7

- Strands 1 and 2: [44.6,6.2,512, 797, 19.8,0.4595,1180,0.1275,<u>1800</u>]
- Strand 3: [35,14.7,328, 805,50.2,0.499, 485,2.174,<u>1879</u>]

Alternative KF = 3.7 gives SUMSQS = 1822

Strands 1 and 2: the points below were each generated by one Simplex run of approx. 160 it'ns. using the above point as a start but with EVPMAX progressively moved away from 19.8

Strand 3: 53 Simplex it'ns. only VSMAX, USMAX, PV vary

Stage 8 (Strand 3): [43.7, 8.9,328, 805,50.2,0.490, 485,2.174,<u>1850</u>]

Strand 3: 370 Simplex it'ns. all params. vary.

Stage 9 (Strand 3): [45.9, 8.4,299, 738, 123,0.480, 788,3.246,<u>1815</u>]

" 8 (Strands 1 and 2)

Starting value of EVPMAX	
25	48.9,6.1,567, 645, 24.9,0.4539,1195,0.0863,<u>1791</u>
30	48.7,6.3,487, 731, 27.3,0.4731, 981,0.1359,<u>1811</u>
35	48.1,6.4,515, 618, 36.6,0.4494, 983,0.1007,<u>1785</u>
40	49.1,6.2,482, 679, 33.7,0.4632,1241,0.0900,<u>1792</u>
45	49.9,6.4,466, 690, 34.9,0.4660,1166,0.1004,<u>1796</u>

Note:- Parameter values are given in the following order:-
VSMAX, USMAX, DSMAX, SSMAX, EVPMAX, PV, FO, KF, <u>SUMSQS</u>.

FIGURE 7.1

SEARCH FOR OPTIMUM PARAMETER VALUES, LIDSDALE No. 2 CATCHMENT.

from an arbitrarily selected starting point. The three points obtained at the end of these searches are listed at Stage 1 in Fig. 7.1.

Very large reductions in the objective function, from values of approximately 38,000 and 62,000 to values in the region of 5,000, were achieved by these optimising runs. As only very minor changes were occurring in the parameter values near the ends of the runs, and as the large differences in the parameter values between the points indicated that these points were widely separated on the response surface, it appeared that three distinct local minimum points had been approached. There seemed to be little possibility of making further significant improvements in the objective function. However it was felt that if interdependence existed between some of the parameters, giving long, relatively flat-bottomed valleys in the response surface, then the three points could merely be lying at different places on the floor of such a valley. If this were so the possibility existed of moving the points along the floor of the valley to its lowest point, thus bringing the three widely separated points together and locating a real minimum point. This aim was pursued in the remainder of the search. The measures undertaken to further the search at the various stages indicated in Fig. 7.1 were as follows:-

Stage 1 For the parameter values at points 1 and 3, graphs of the total contents of the Upper Soil, Drainage and Lower Soil Stores plotted against time were prepared and compared with the soil moisture data. These indicated that the value of 260 pts. being used for the initial contents of the Lower Soil Store, i.e., at 17-10-1968, was too low. A value of 600 pts. was adopted.

At about this time, Method 4 (see Appendix A2) was adopted for finding the steepest descent direction at each iteration of the Davidon method.

<u>Stage 2</u> Suspected inter-action between parameters FO and KF was studied at about this time. An alternative value for KF of 3.5 was substituted in point 1 before starting the Davidon method.

<u>Stage 3</u> Several suspected parameter inter-actions were investigated at this stage. The findings which significantly affected the course of the search were:-

(a) The objective function was relatively indifferent to a large range of combinations of FO and KF values.

(b) Bearing this in mind, and comparing the remaining parameters of points 1 and 2, only VSMAX, USMAX and PV showed significantly different values. It was found graphically that these parameters could be optimised to the same values from their values at both of the points 1 and 2, thus bringing these two points together at an improved value of the objective function. Also, improvement of point 3 was possible by optimising only the above 3 parameters.

It was therefore decided to operate the optimising methods holding all parameters except VSMAX, USMAX and PV constant.

<u>Stage 4</u> Application of the optimising methods with only VSMAX, USMAX and PV varying confirmed the conclusions above. As there were still minor differences in the parameter values

at points 1 and 2, a point was nominated as the place where strands 1 and 2 converged and this is listed at Stage 5.

It was found graphically that improvement of point 3 was possible by optimising VSMAX, USMAX and EVPMAX only. The Davidon method was operated from this point with the other parameters held constant.

Method 5 (see Appendix A2) for finding the steepest descent direction in the Davidon method was adopted at this time.

<u>Stage 5</u> On strand 3, the Davidon method encountered difficulties in optimising VSMAX, USMAX and EVPMAX and these parameters were re-set using the graphical results from Stage 4.

After the sub-optimisation work in the previous stages it was decided to allow a greater number of parameters to vary during optimisation. As difficulties had been experienced with the Davidon method, the Simplex method was used for the remaining work.

<u>Stage 6</u> At this stage optimisation was continued with all of the parameters varying once again.

<u>Stage 7</u> These points were considered to be relatively close to each other on the response surface as

(a) the differences in the VSMAX, USMAX and PV values could possibly be reduced by further optimisation varying these parameters only,

(b) the indifference of the objective function to a large range of combinations of FO and KF noted above indicated that the differences between these parameters at this stage could be neglected,
and

(c) although the values of DSMAX were significantly different, in both cases the values were tending to become unusually high.

The values of EVPMAX were still significantly different. It was suspected that a range of points could be found in which EVPMAX varied progressively between these two values.

Stages 8 & 9 All these points appear to be lying along the floor of a valley in the region of the lowest point of the valley. It is not possible to identify a distinct minimum point.

At the point where the objective function value is 1800, an alternative value of 3.7 was found for KF which gave a small rise in the value of the objective function to 1822.

The sets of parameter values at Stages 8 and 9 appear to be spread along the floor of a valley in the response surface and are probably close to the lowest point in the valley.

7.2 THE INVESTIGATIONS UNDERTAKEN DURING THE SEARCH

The investigations and actions taken at the various stages of the search are explained in greater detail in this sub-section. The aim of this work was to find downhill paths from the three widely separated points at Stage 1 with the hope that these paths would eventually converge to the one minimum point.

7.2.1 Use of the Soil Moisture Data

The contents of the soil stores of the model were compared with the soil moisture data to see if the comparison would indicate any measures which could be taken to assist in the optimisation.

The soil moisture data provided by Mr. Smith were average figures from the readings at four sites whose locations are shown in Appendix A3. Soil moisture figures for each 10 inch interval of soil down to a depth of 80 inches and an additional 5 inch interval to 85 inches were provided. Very small changes in soil moisture take place at this depth. The readings were at an average interval of eleven days, the shortest interval being 5 days and the longest, 35 days.

Total soil moisture in the 85 inches metered depth is plotted in Fig. 7.2. Some adjustments to these data were considered to be justified for the purposes of this project. There were two types of inconsistency between the changes in soil moisture in some periods and the rainfall, runoff and evaporation data for those periods:-

(i) rises in soil moisture were sometimes greater than the rainfall, and

(ii) the change in soil moisture sometimes required a loss of moisture at a significantly greater rate than the open water evaporation rate and, in a few additional cases, at a greater rate than the estimated potential evapotranspiration rate.

The periods when these inconsistencies occurred are shown in Fig. 7.2. Errors in the data for any of the four variables (rainfall, runoff, evaporation and soil moisture) could have caused these inconsistencies, or the data may have been unrepresentative of the whole catchment. Some likely reasons for the data being unrepresentative of the catchment would be:-

(i) horizontal re-distribution of soil moisture, e.g., by interflow, and

(ii) point rainfall data are not always good estimates of average catchment rainfall.

It seemed unreasonable to compare modelled soil moisture figures with observed soil moisture data which were inconsistent with the rainfall and evaporation data used in the model. Accordingly, the soil moisture figures were adjusted to comply with the other data, the changes in most cases being merely sufficient to resolve the inconsistencies. Greater adjustments could probably have been justified in many cases. The reconstituted data are also plotted in Fig. 7.2, where it is seen that the more extreme fluctuations in soil moisture have been smoothed out to some extent.

The stores of the Boughton model which represent various components of soil moisture are the Upper Soil, Drainage, and Lower Soil Stores. The calculated contents of these stores on the dates when the soil moisture measurements were taken were summed and the totals plotted for comparison with the soil moisture data. This was done using a number of different sets of parameter values, and the resulting graphs are shown in the figures listed below:-

Fig. 7.3; Graphs of modelled soil moisture with parameters values on Strands 1 and 3 at Stage 1.

Fig. 7.4; Graphs of modelled soil moisture with parameter values on Strands 1 and 3 at Stage 3.

Fig. 7.5; Graphs of modelled soil moisture with parameter values on Strands 1-2 and 3 at Stage 7.

Comparison of these graphs with the adjusted soil moisture data, also plotted on the above figures, shows that the fit of the modelled to observed soil moisture improved as optimisation of the modelled runoff proceeded. The graphs in Fig. 7.3 show that the modelled soil moisture was too low for the first few months. This prompted the change in the assumed initial contents of the Lower Soil Store from 260 pts. to 600 pts. The modelled soil moisture with near-optimum parameter values varies in the same way as the observed data, but the magnitude of the fluctuations is smaller. As the rainfall and soil moisture data are not always representative of average catchment values, the fit of the modelled to observed soil moisture appears to be reasonable.

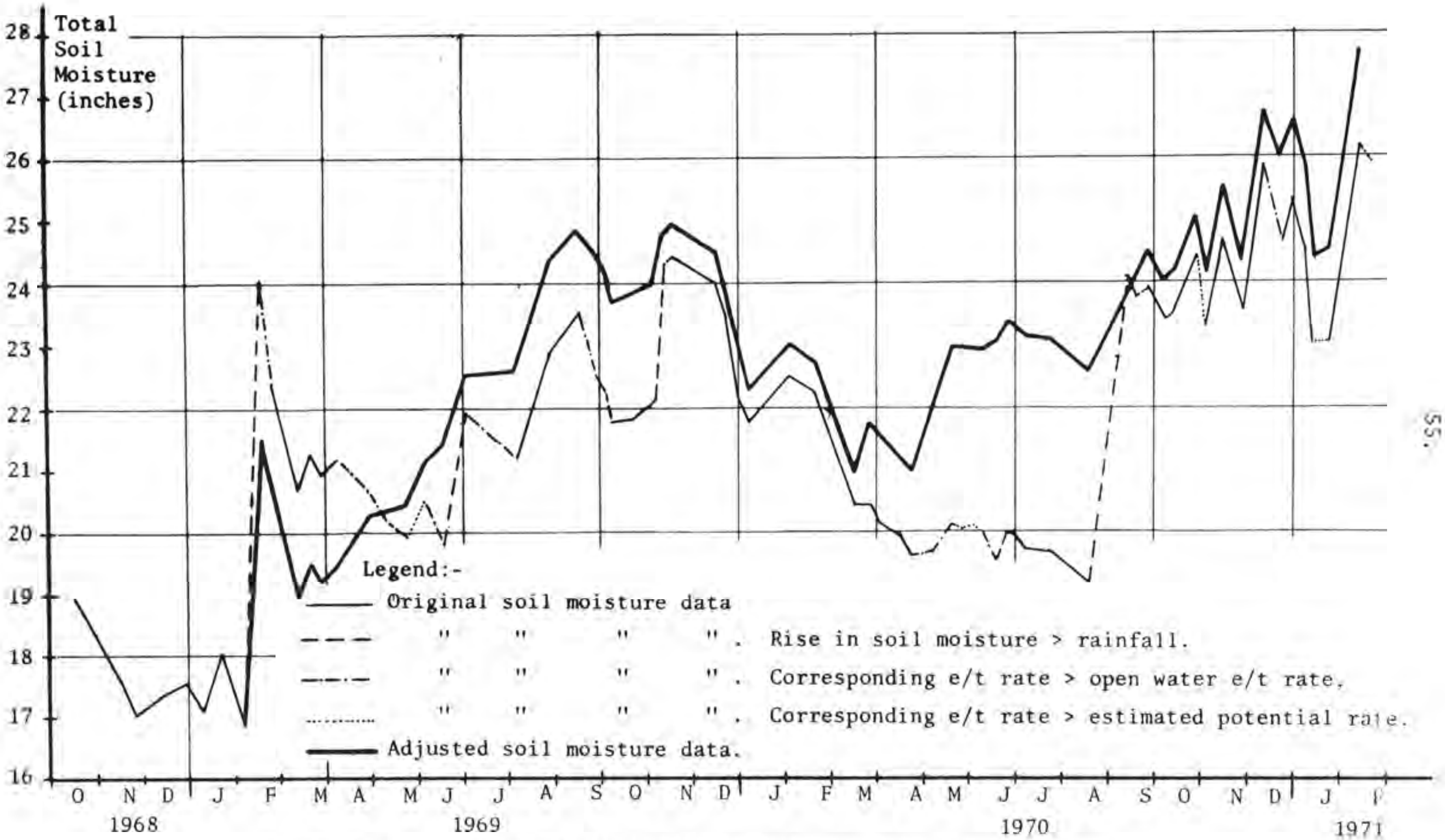


FIGURE 7.2. GRAPHS OF ORIGINAL AND ADJUSTED SOIL MOISTURE DATA

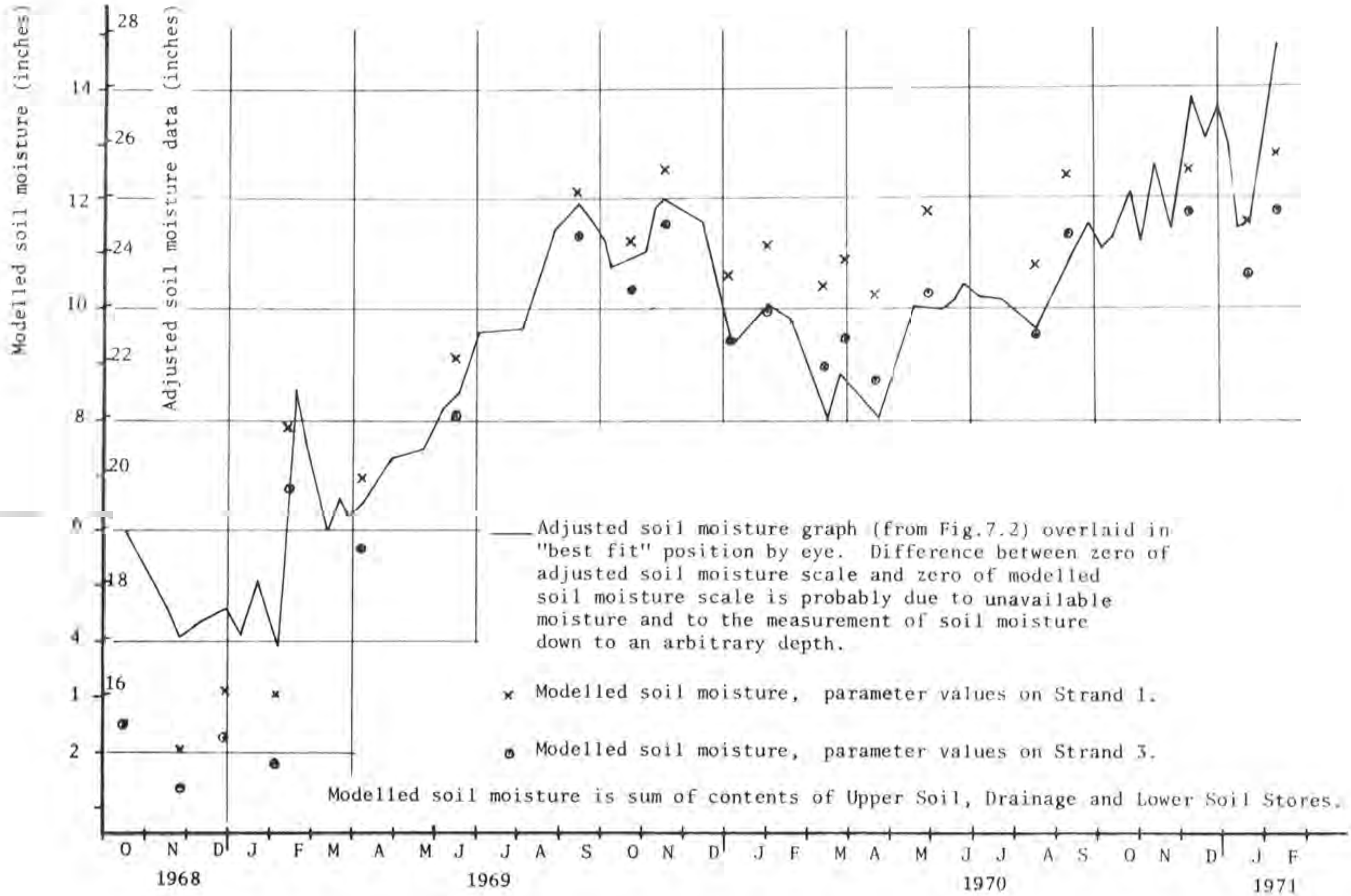


FIGURE 7.3 GRAPHS OF MODELLED SOIL MOISTURE WITH PARAMETER VALUES ON STRANDS 1 AND 3 AT STAGE 1

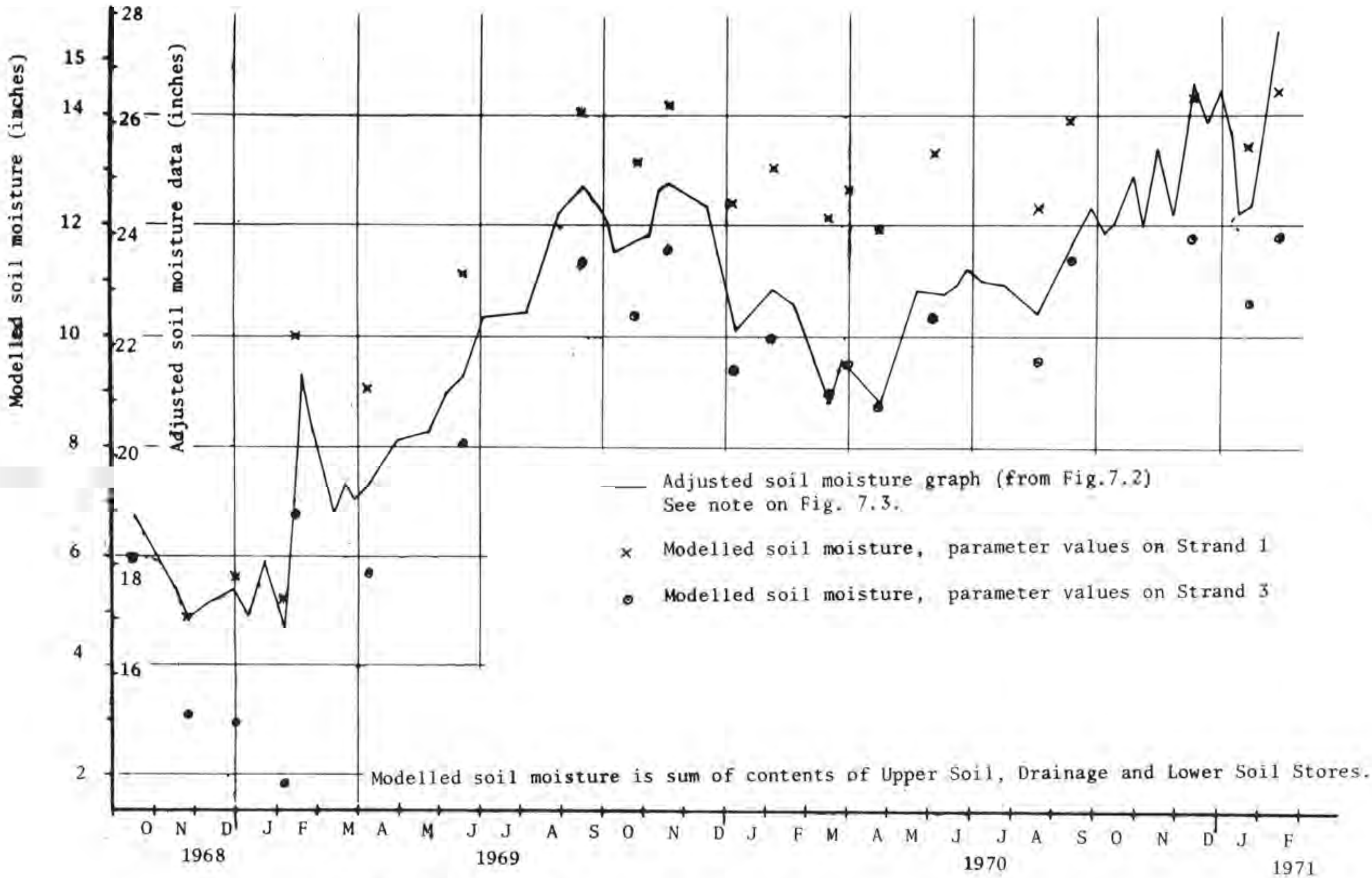


FIGURE 7.4 GRAPHS OF MODELLED SOIL MOISTURE WITH PARAMETER VALUES ON STRANDS 1 AND 3 AT STAGE 3

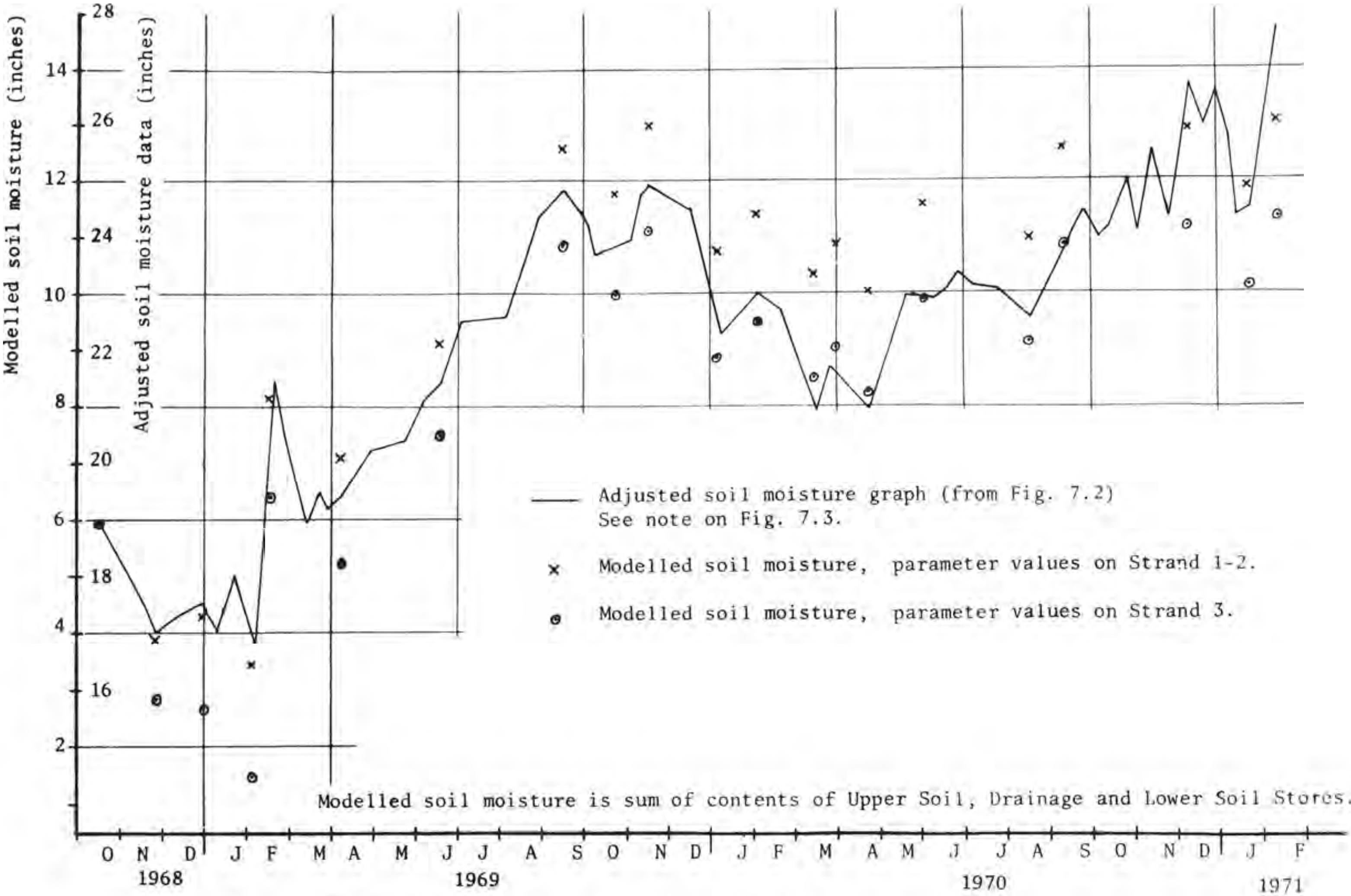


FIGURE 7.5 GRAPHS OF MODELLED SOIL MOISTURE WITH PARAMETER VALUES ON STRANDS 1-2 AND 3 AT STAGE 7.

7.2.2 Numerical Studies of Parameter Inter-Dependence

The possibility of inter-dependence was investigated numerically for a number of two-parameter and three-parameter combinations, as this could reveal opportunities to make large changes in some parameter values in circumstances where the resulting change in the objective function was small and where, consequently, progress of the optimising methods would be slow. The results of these investigations for three of the combinations, FO-KF, VSMAX-USMAX-PV, and VSMAX-USMAX-EVPMAX, influenced the search for optimum parameter values and these results are presented below. In addition, results for the DSMAX-SSMAX combination support a conclusion reached in Section 8 and these results are also described below.

(i) The Infiltration Parameters

For easy reference, the relevant equations are:-

Equation 3.1, the original equation used by Boughton to give a potential daily infiltration amount,

$$F = FC + (FO - FC)\, e^{-KF \cdot SS}$$

Equation 6.17, the equation for an instantaneous infiltration rate introduced in this project and described in sub-section 6.2,

$$\text{instantaneous } F = FO \cdot e^{-KF \cdot SS/SSMAX} - FO \cdot e^{-KF}$$

Equation 6.19, which is obtained when equation 6.17 is integrated over a time period of one day to find the potential daily infiltration amount allowing for the change in SS through the day,

$$\text{total daily } F = SSMAX\left(1 + \frac{1}{KF}\cdot \ln(1 - D + De^{KF(SS/SSMAX - 1)})\right) - SS$$

where

$$D = e^{-FO \cdot KF \cdot e^{-KF}/SSMAX}$$

Equation 3.2, to calculate runoff from the amount of water, P, which overflows from the Drainage Store,

$$Q = P - F\tanh\left(\frac{P}{F}\right), \text{ and}$$

Equation 6.11 and 6.12, to calculate evapotranspiration from the Lower Soil Store,

$$SS_1 = SS_0 - (1 - PV) \times E \quad \text{if } E < SS \times EVPMAX/SSMAX$$

$$SS_1 = SS_0 \times e^{-(1 - PV) \times EVPMAX/SSMAX}$$

$$\text{if } E \geq SS \times EVPMAX/SSMAX.$$

Interaction was suspected between the parameters FO and KF, and was first investigated using synthetic data in a single store which was similar to the Lower Soil Store of the Boughton model, and which is illustrated in Fig. 7.6.

For every inflow P there is an outflow Q given by equation 3.2, in which the F value is found from equation 6.19. The amount of water which actually enters the store is P - Q. The contents of the store are depleted by evapotranspiration according to equations 6.11 and 6.12 in which, for this case, PV = 0. Equations 3.2, 6.19, 6.11 and 6.12 therefore constitute the mathematical model of this store, and the parameters of the model are SSMAX, EVPMAX, FO and KF. A 65-day record of daily P values which contained ten rainfall events was made up for use with this model.

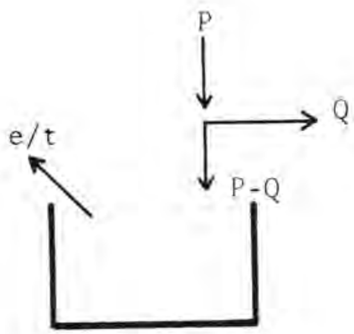


FIGURE 7.6
SINGLE STORE MODEL FOR
STUDY OF FO-KF INTER-DEPENDENCE

Arbitrary values for the parameters of the model, the initial contents of the store, and the daily potential evaporation rate may be assigned and a record of Q values generated by processing the P data through the model. If different values are then assigned to the parameters FO and KF and the P data are again processed through the model, another record of Q values is obtained and the sum of squares of the differences between the two records may be evaluated. The FO-KF response surface may be plotted by evaluating the sum of squares in this way at each node of a grid of FO and KF values. The sum of squares will of course be zero at the point where FO and KF are equal to the original arbitrary values, and these values may be regarded as the optimum values.

This procedure was carried out for two sets of arbitrary values, and the two FO - KF response surfaces are shown in Figs. 7.7 and 7.8 Strong inter-action between these parameters is evident in both cases and the surfaces are difficult ones on which to locate the minimum point using search techniques. As the infiltration rate, F, always approaches zero as SS approaches SSMAX, graphs of F versus SS (equation 6.17) for different values of FO and KF are very close together at high values of SS, but are more widely separated at lower values of SS. This probably accounts for the slightly lower degree of inter-dependence in Fig. 7.8, as the contents of the store were lower in this case.

To examine the inter-action, graphs of equation 6.17 were prepared for a number of combinations of FO and KF values which lie on the floor of the valley in Fig. 7.8. These graphs are shown in Fig. 7.9. All the curves are fairly close together at values of SS greater than about 400, but are readily distinguished from each other at lower values. As the model was operated in this case with low values of SS, the indifference to the wide range of infiltration curves was rather surprising. Graphs of equation 6.19 (the total daily infiltration amount, derived from the instantaneous rate) were then prepared for the same combinationsoof FO and KF and are shown in Fig. 7.10. These curves lie fairly close together over almost the entire range of SS values, and explain the indifference of the objective function to the corresponding range of FO and KF combinations.

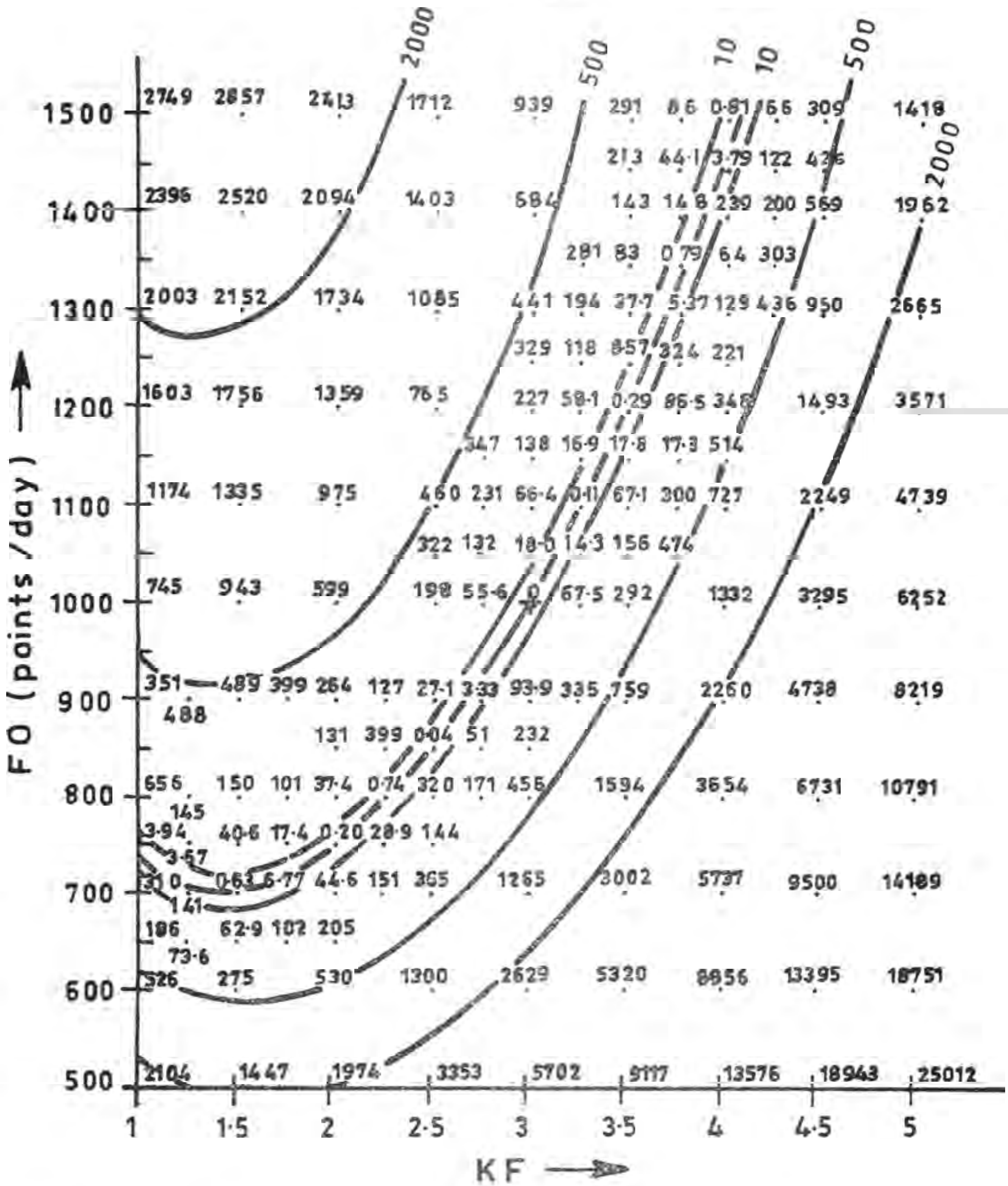
As the equation originally used by Boughton, equation 3.1, was used as a basis for equation 6.17, and as the inter-action of FO and KF could not be explained using graphs of that equation, it appeared that most of the inter-action may have been induced by the use of equation 6.19. It was thought desirable to check the amount of inter-action present when the Boughton equation was used. The one-store model was therefore operated using the original Boughton infiltration calculations and the response surfaces for FO and KF were plotted for two sets of parameter values. These are shown in Figs. 7.11 and 7.12. The inter-action, although considerably less than for the former cases, is still quite strong and the surfaces are still difficult ones on which to locate the minimum point.

Another notable difference between Figs. 7.7 and 7.8 (prepared using the modified infiltration calculations) and Figs. 7.11 and 7.12 (prepared using the original Boughton model infiltration calculations) is that, in the former, cross sections of the response surface at constant values of FO greater than about 750 have two minimum points. Thus, when using the modified infiltration calculations, there is usually an alternative value of KF for given values of the other parameters. This feature of the FO-KF response surface seems to have been introduced by the second term in equation 6.17, through which the parameter KF, which formerly only influenced the shape of the infiltration curve, now also influences the intercept of the curve with the SS = 0 axis.

After these studies with the one-store model, FO-KF response surfaces were plotted for the complete Boughton model, using both the modified infiltration calculations and Boughton's original infiltration calculations. The other model parameter values were those obtained at point 2 in Stage 3 of the search for the optimum parameter values for Lidsdale No. 2 catchment. The surfaces are plotted in Figs. 7.13 and 7.14, and they are of the same shape as those obtained with the one-store model. However, in the case of the modified infiltration calculations, the response surface is relatively flat and the valley in the surface is wide. Within the large area bounded by the 4350 contours, the objective function varies by less than 2½%. The function is therefore practically indifferent to a wide range of FO and KF combinations. The surface obtained when using Boughton's original infiltration calculations has a steep-sided but almost flat-bottomed valley. There is a much lower degree of indifference to various combinations of FO and KF. However, in moving along the floor of the valley, the value of FO changes from about 900 pts/day to about 2000 pts/day and, with a slight accompanying change in the value of KF, the corresponding change in the objective function is less than 1%. Thus, strong inter-action is still present in this case.

The above findings explain the actions taken relative to the FO and KF parameters at Stages 2, 3, 7 and 8 of the search. Thus, at each of the Stages 2 and 8 of the search, substitution of the alternative KF value was used to narrow the differences between the various points. At Stage 3 it was decided to neglect the parameters FO and KF in the following optimisation runs because of the indifference of the objective function to a wide range of combinations of these parameters. For the same reason, the differences between these parameters were ignored when comparing the points at Stage 7.

It is evident that the use of the modified infiltration calculations has induced greater inter-action between parameters which were already strongly inter-dependent. However, sound reasons were given for modifying the original infiltration calculations and as the nature of the inter-action between the parameters was now understood it was not considered necessary to revert to the original infiltration calculations. There are similarities between the modified infiltration function used in this project and the infiltration functions used in other rainfall-runoff models (including some which purport to use the Philip equation). Therefore it seems that inter-action between the infiltration parameters will be a problem for most rainfall-runoff models.

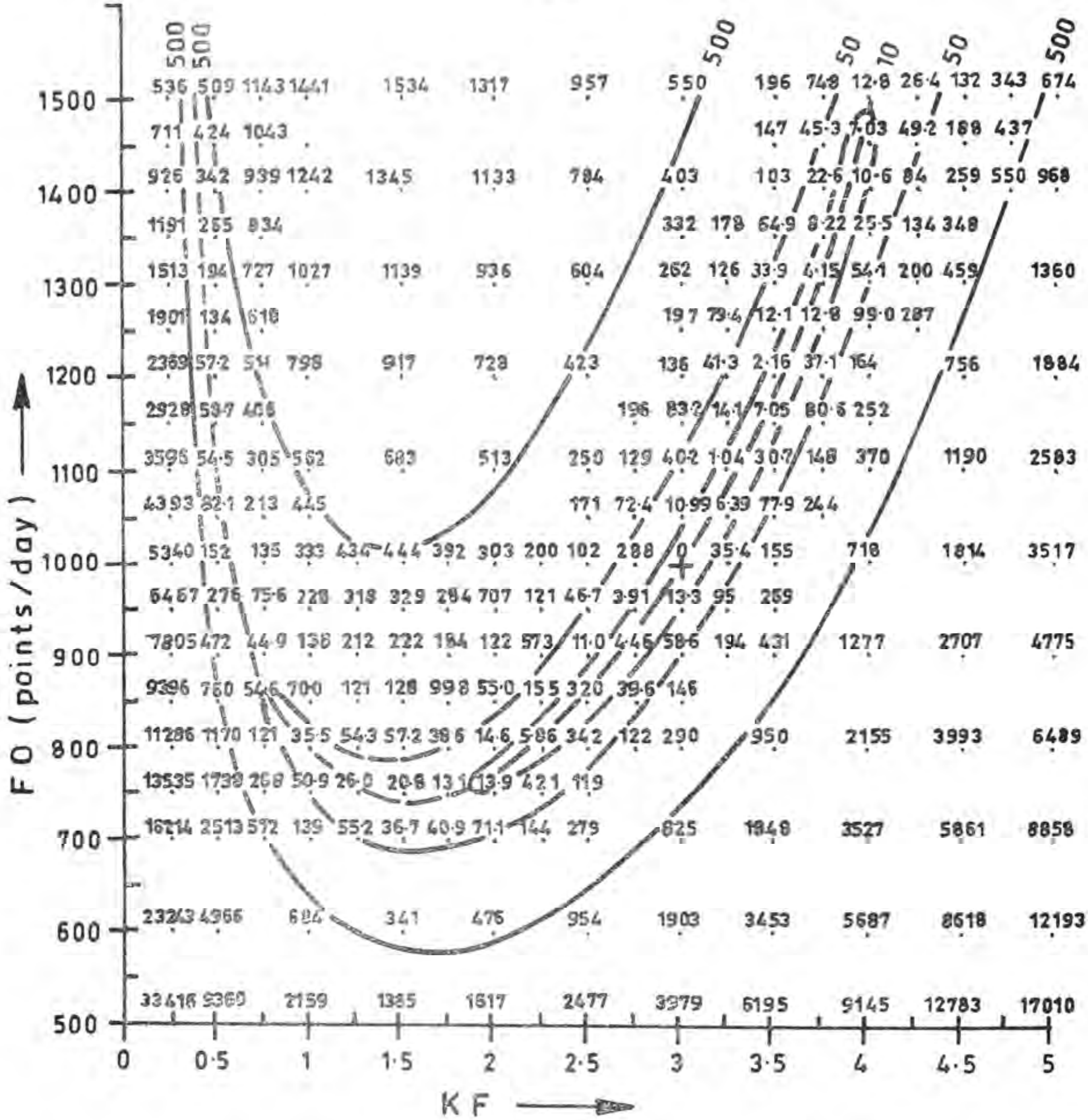


\+ Optimum parameter values - by construction.
FO = 1000 points/day, KF = 3·0

Other parameters and constants for the model were:-
EVPMAX = 35 points/day, SSMAX = 1000 points,
daily e/t = 15 points, initial store contents = 500 points.

FIG. 7·7

SINGLE STORE MODEL FO-KF RESPONSE SURFACE No1.
(MODIFIED INFILTRATION CALCULATIONS)



+ Optimum parameter values - by construction.
FO = 1000 points/day , KF = 3·0
Other parameters and constants for the model were:
EVPMAX = 40 points/day, SSMAX = 1000 points,
daily e/t = 10 points, initial store contents = 100 points.

FIG. 7·8

SINGLE STORE MODEL FO-KF RESPONSE SURFACE No. 2.
(MODIFIED INFILTRATION CALCULATIONS)

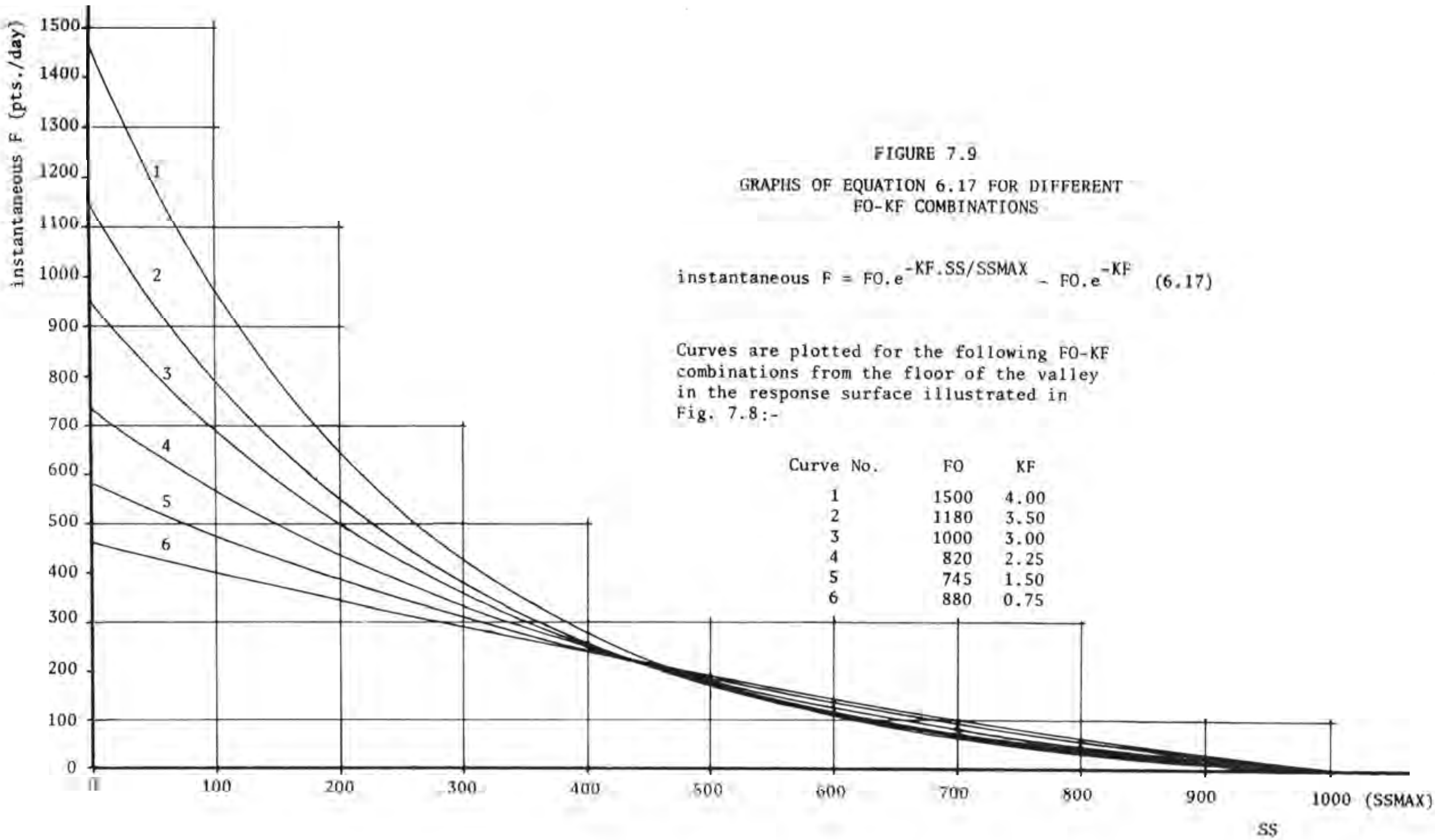
FIGURE 7.9

GRAPHS OF EQUATION 6.17 FOR DIFFERENT FO-KF COMBINATIONS

$$\text{instantaneous } F = FO.e^{-KF.SS/SSMAX} - FO.e^{-KF} \quad (6.17)$$

Curves are plotted for the following FO-KF combinations from the floor of the valley in the response surface illustrated in Fig. 7.8:-

Curve No.	FO	KF
1	1500	4.00
2	1180	3.50
3	1000	3.00
4	820	2.25
5	745	1.50
6	880	0.75



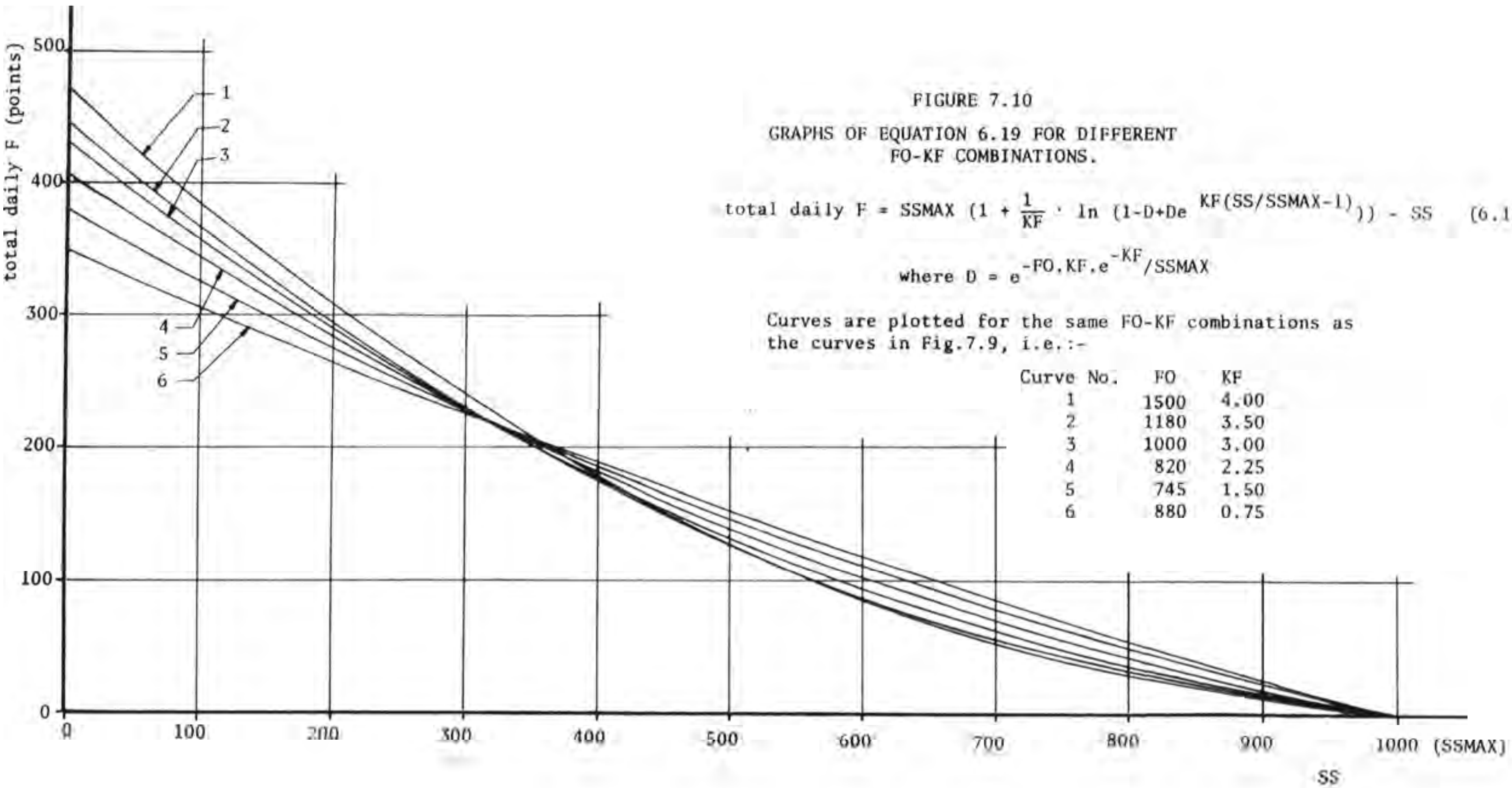


FIGURE 7.10

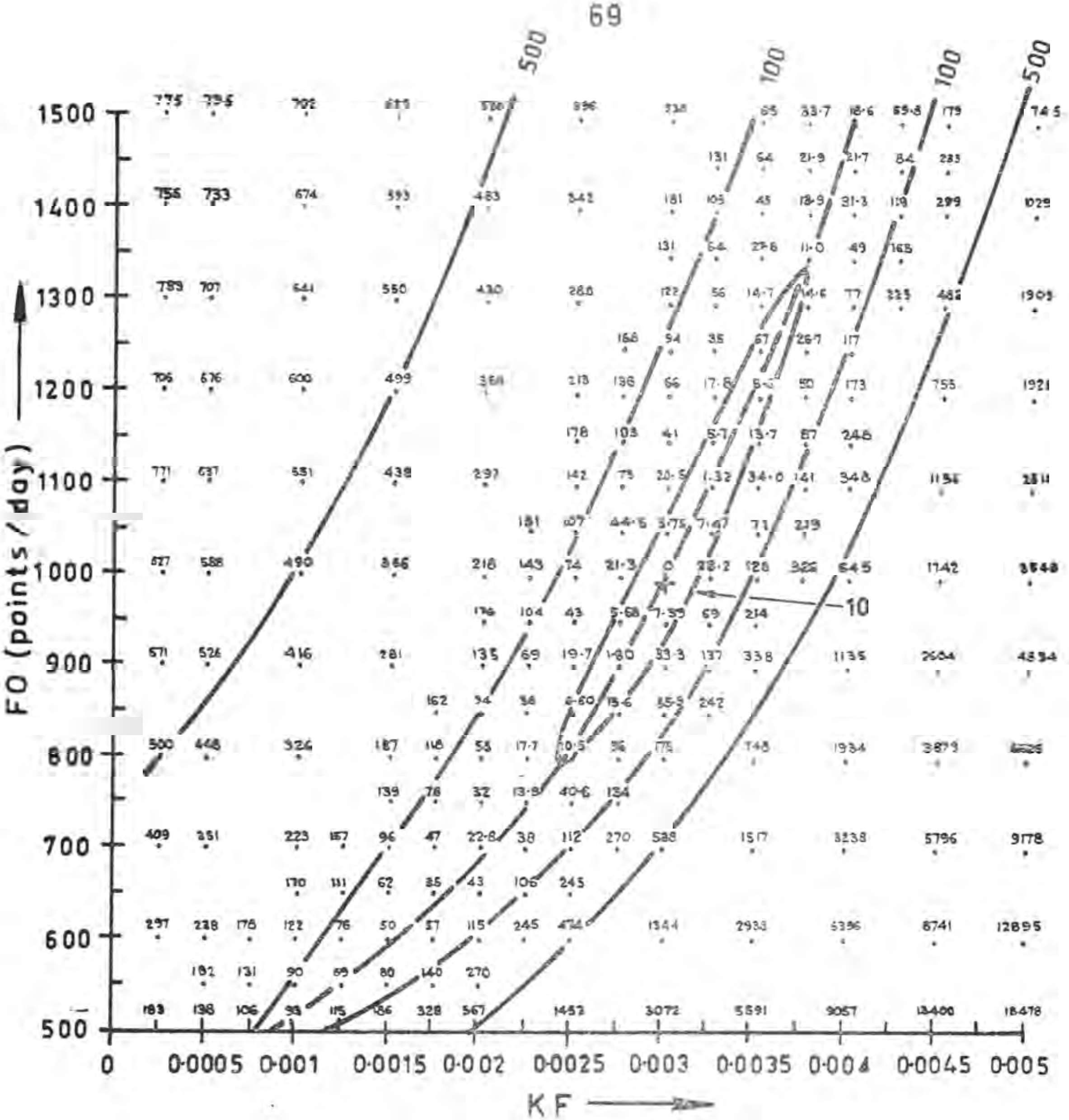
GRAPHS OF EQUATION 6.19 FOR DIFFERENT FO-KF COMBINATIONS.

$$\text{total daily F} = \text{SSMAX}\left(1 + \frac{1}{\text{KF}} \cdot \ln\left(1 - \text{D} + \text{De}^{\text{KF(SS/SSMAX}-1)}\right)\right) - \text{SS} \qquad (6.19)$$

$$\text{where D} = \text{e}^{-\text{FO.KF.e}^{-\text{KF}}/\text{SSMAX}}$$

Curves are plotted for the same FO-KF combinations as the curves in Fig.7.9, i.e.:-

Curve No.	FO	KF
1	1500	4.00
2	1180	3.50
3	1000	3.00
4	820	2.25
5	745	1.50
6	880	0.75



\+ Optimum parameter values - by construction

FO = 1000 points/day, KF = 0·003

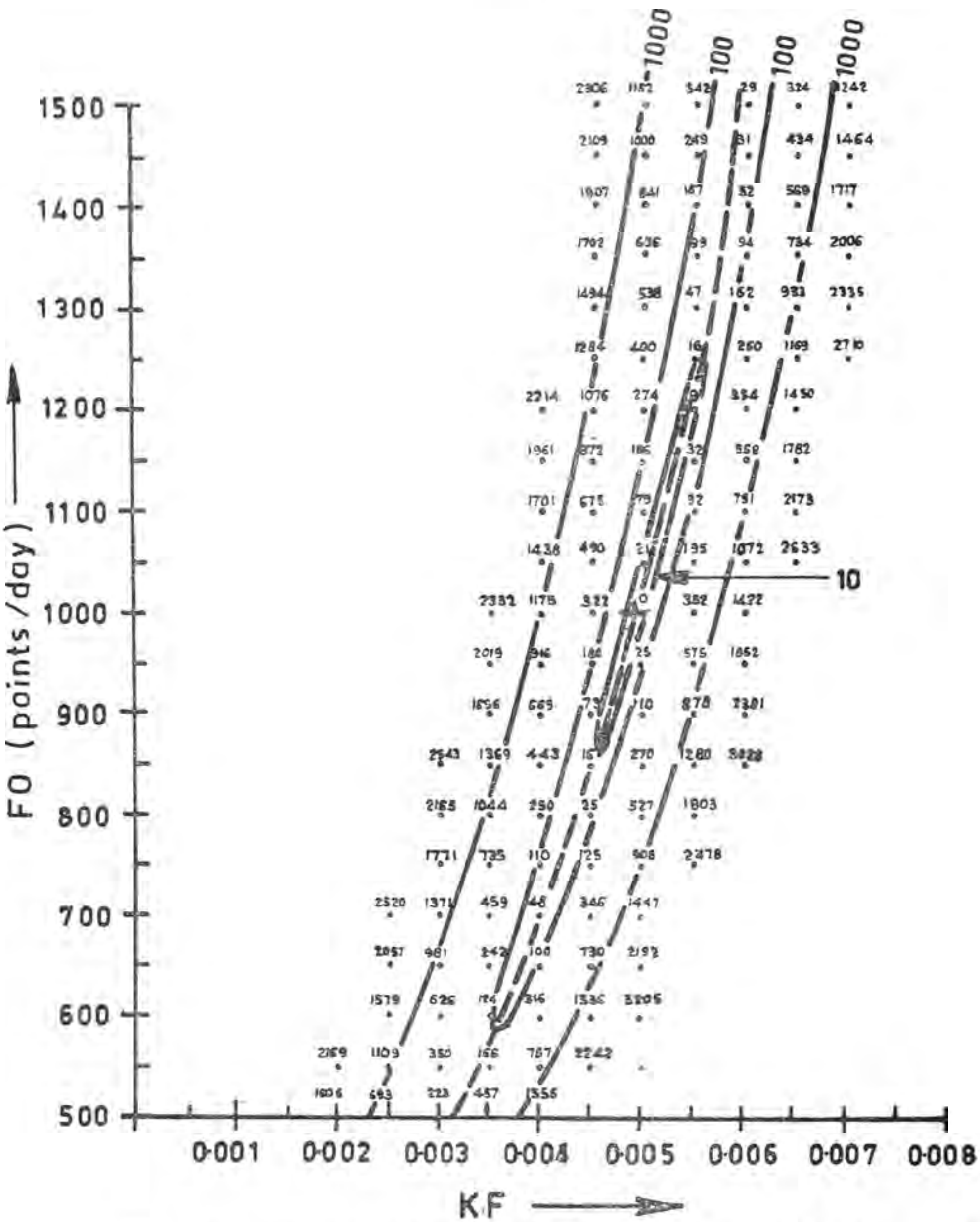
Other parameters and constants for the model were:—

EVPMAX = 40 points/day, SSMAX = 1000 points, FC = 0

Daily e/t = 10 points, initial store contents = 100 points.

FIG. 7·11

SINGLE STORE MODEL FO-KF RESPONSE SURFACE No.3.
(ORIGINAL BOUGHTON INFILTRATION CALCULATIONS).



+ Optimum parameter values - by construction.

FO = 1000 points/day, KF = 0·005

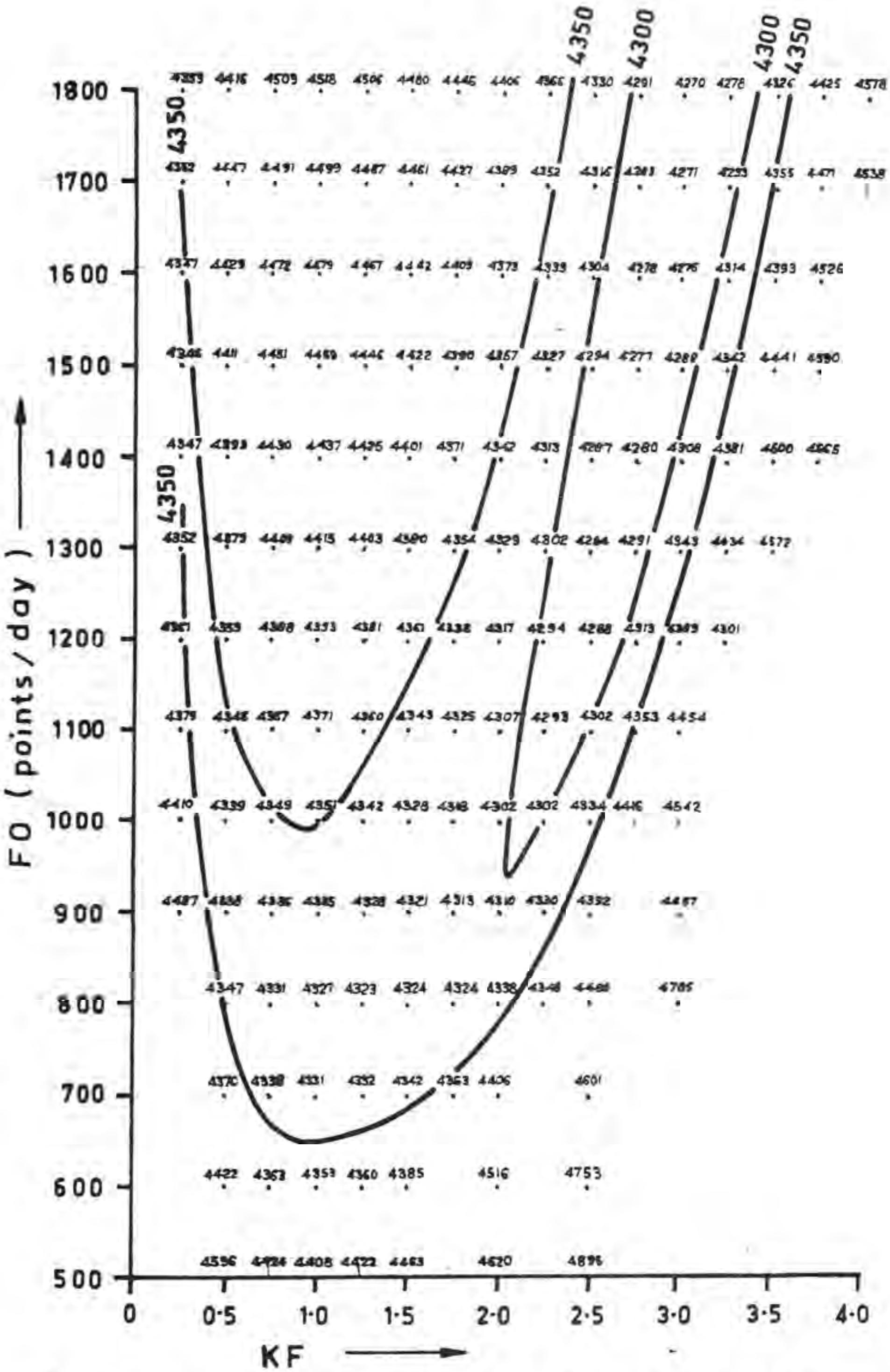
Other parameters and constants for the model were:

EVPMAX = 40 points/day, SSMAX = 1000 points, FC = 0,

Daily e/t = 10 points, initial store contents = 100 points.

FIG. 7-12

SINGLE STORE MODEL FO-KF RESPONSE SURFACE No 4. (ORIGINAL BOUGHTON INFILTRATION CALCULATIONS)

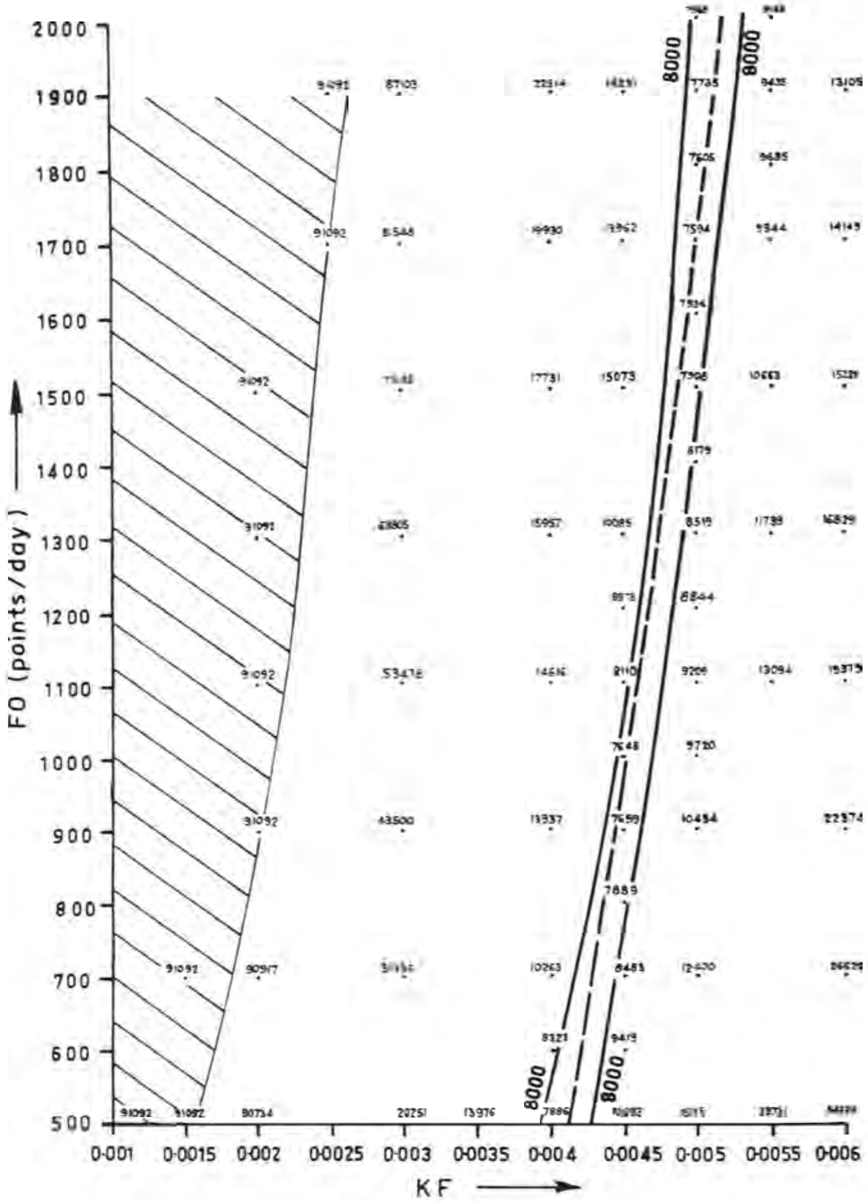


Contours are of sum of squares of monthly deviations. Values of model parameters other than FO and KF are as for point at Stage 3 (See Fig.7·1).

FIG. 7·13

BOUGHTON MODEL FO – KF RESPONSE SURFACE No.1.

(MODIFIED INFILTRATION CALCULATIONS)



Contours are of sum of squares of monthly deviations.
Values of model parameters other than FO and KF are as for point 2 at Stage 3 (See Fig.7·1)
In shaded area objective function is indifferent to changes in FO and KF. Modelled runoff is zero and objective function is sum of squares of observed monthly runoff quantities.

FIG. 7·14

BOUGHTON MODEL FO – KF RESPONSE SURFACE No 2.
(ORIGINAL BOUGHTON INFILTRATION CALCULATIONS).

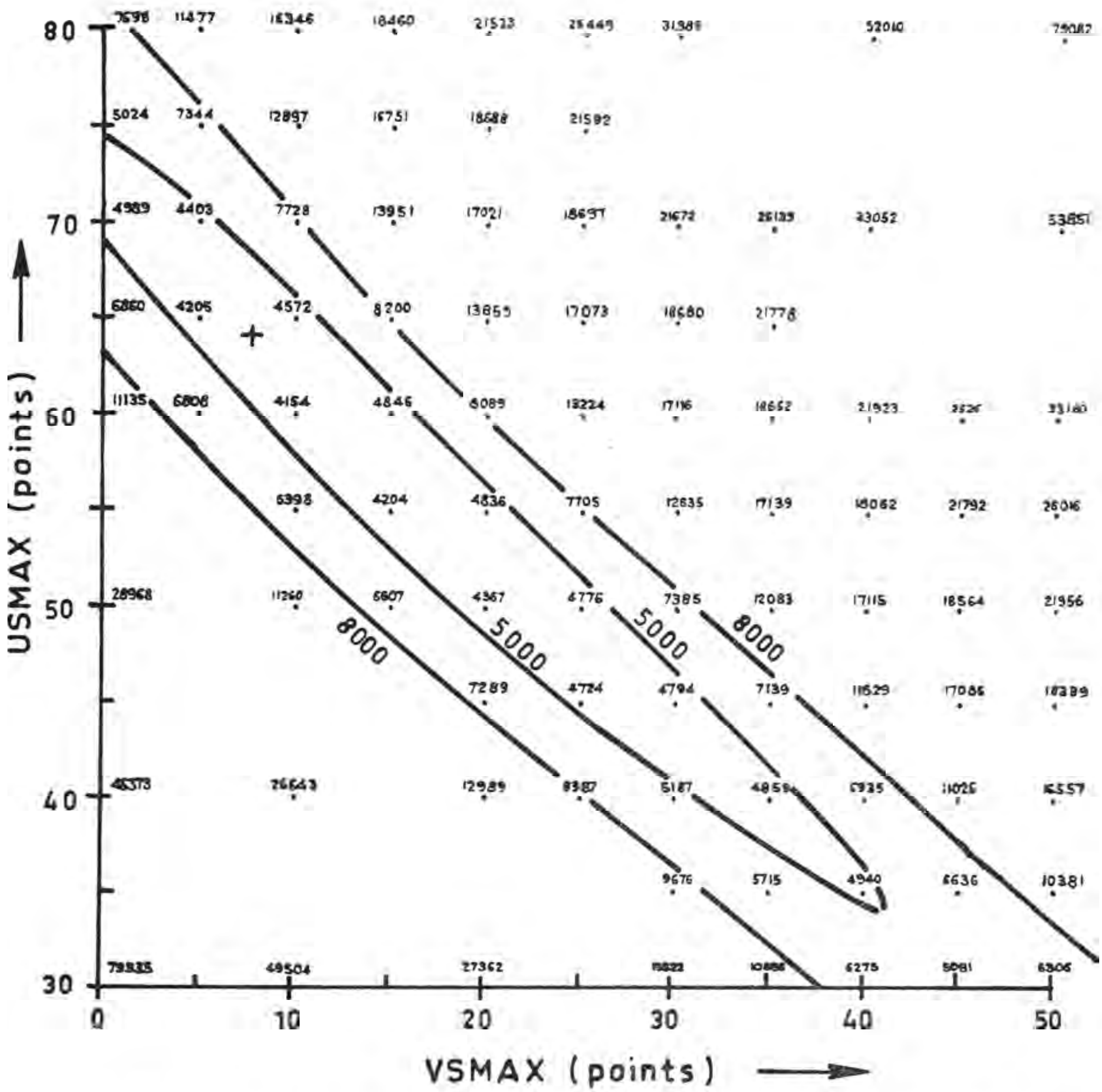
(ii) The Parameters VSMAX, USMAX and PV

At Stage 3 of the optimisation, it was noticed that the main differences between points 1 and 2 lay in the values of the above three parameters and it was felt that, if a three-way inter-action was present between these parameters, a path along the floor of a valley in the response surface could possibly be found between the two points. Inter-action between VSMAX and USMAX was indicated by the algebraic analysis described later in Section 8. The response surface for these parameters was evaluated and is plotted in Fig. 7.15. The other parameters had their values for point 2 at Stage 3 of the search. Fairly strong inter-action between VSMAX and USMAX is indicated. The response surface for USMAX and PV was also plotted, and is shown in Fig. 7.16. Inter-action between these parameters is weaker.

To investigate the possible three-way inter-action, the optimum value of VSMAX and the corresponding value of the objective function were found at each point of a grid of USMAX and PV values while the other parameters were held at their point 2 values. With these data, a surface was plotted from which the optimum values of VSMAX, USMAX and PV could be read for the fixed values of the other parameters. This surface is shown in Fig. 7.17. It is not a response surface, but a selection of points from the response surface, and is analogous to the line along the floor of a valley in a response surface for two parameters. The positions of points 1 and 2 at Stage 3 of the search are shown on this surface and it can be seen that optimisation of the three parameters from both of these points should lead to the one minimum point. The approximate path taken by the Davidon method from point 2 is indicated in the figure.

A similar surface was plotted with the remaining parameters held at the point 3 values, and is shown in Fig. 7.18. The surface is more compact in this case. The possibility of improvement in the values of VSMAX, USMAX and PV is also evident.

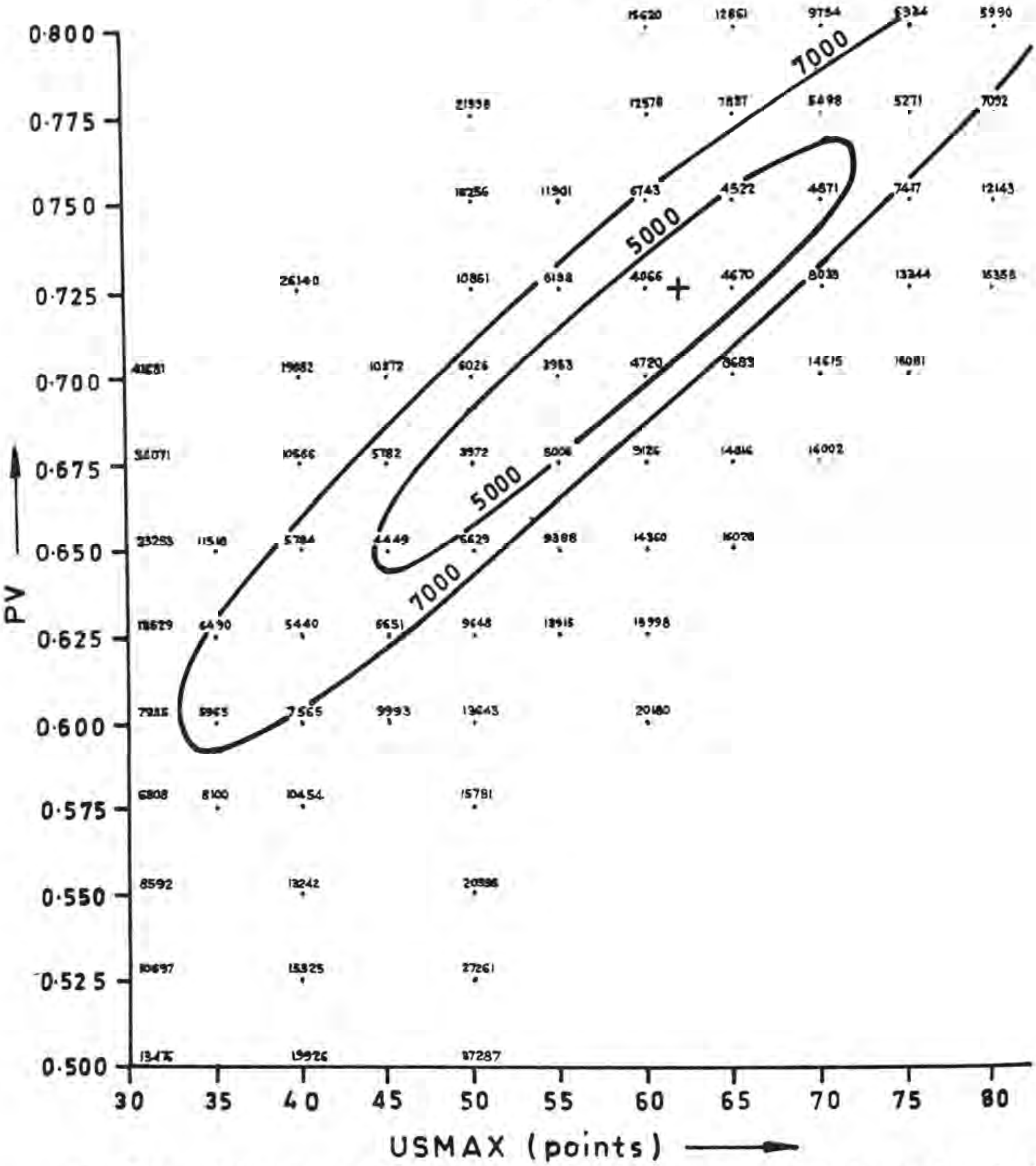
Optimising runs were made in which only VSMAX, USMAX and PV were varied to confirm that the search methods could move to the minimum points of the surfaces plotted above. The runs were successful and they resulted in the movement between Stages 3 and 4 of the search. While improvements were made in the objective function values, it is more significant that relatively large changes occurred in the values of the parameters and that these resulted in the various points moving closer together. A similar optimising run from point 3 at Stage 7 of the search gave similar results.



+ This point corresponds to point 2 at Stage 3 (See Fig. 7·1)
Contours are of Sum of Squares of monthly deviations.
Values of parameters other than VSMAX and USMAX are as for point 2 at Stage 3.

FIG. 7·15

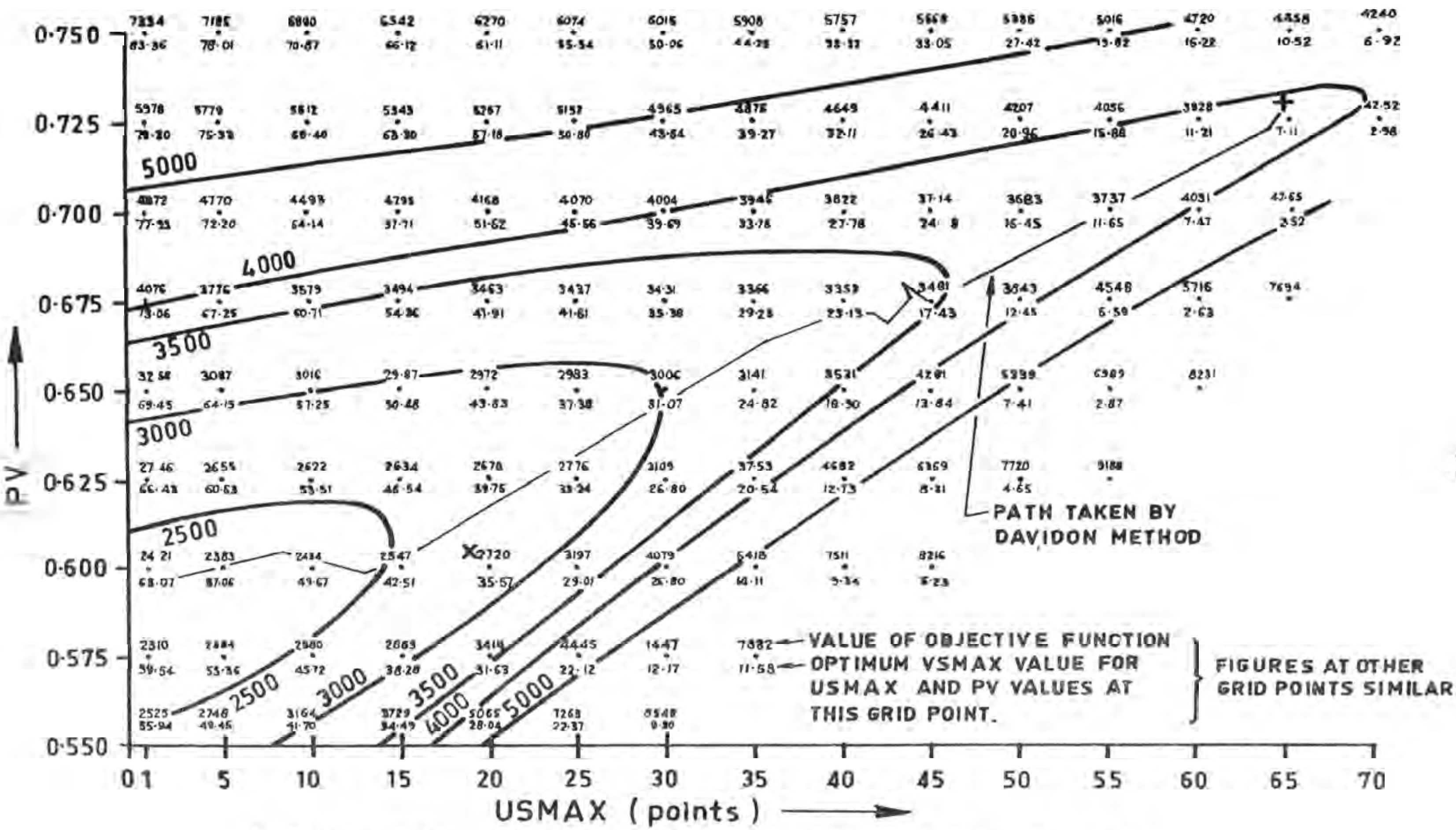
RESPONSE SURFACE FOR VSMAX AND USMAX.



+ This point corresponds to point 2 at Stage 3. (See Fig 7·1)
Contours are of Sum of Squares of monthly deviations.
Values of parameters other than USMAX and PV are as for point 2 at Stage 3.

FIG. 7·16

RESPONSE SURFACE FOR USMAX AND PV.



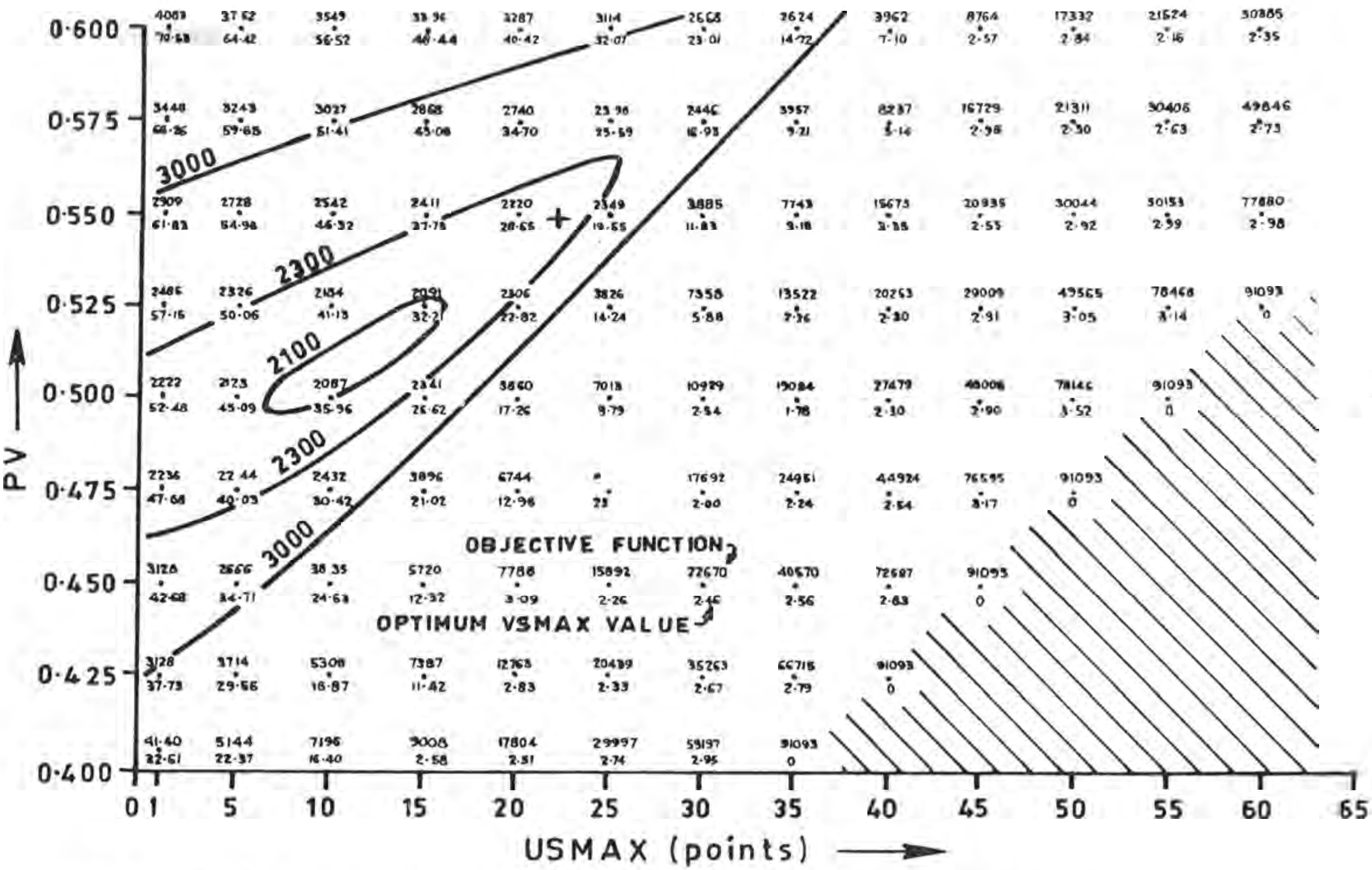
+ This point corresponds to point 2 Stage 3.(See Fig. 7·1)

x " " " " " 1 " " " "

Contours are of sum of squares of monthly deviations.

FIG. 7·17

OPTIMUM VSMAX VALUES FOR USMAX - PV COMBINATIONS, SURFACE No. 1.



+ This point correspods to point 3 at Stage 3. (See Fig. 7·1).
Contours are of Sum of Squares of monthly deviations.
In shaded area objective function is indifferent to changes in VSMAX USMAX and PV. Modelled runoff is zero and objective function is Sum of Squares of observed monthly runoff quantities.

FIG. 7·18

OPTIMUM VSMAX VALUES FOR USMAX – PV COMBINATIONS. SURFACE No. 2.

(iii) The Parameters VSMAX, USMAX and EVPMAX

A number of other three-parameter combinations were investigated after the useful results described above were obtained. Of these combinations, that of VSMAX, USMAX and EVPMAX gave results which were useful in furthering the search. By optimising the value of USMAX at each node on a grid of VSMAX and EVPMAX values, a surface similar to those described in the previous section was plotted for these three parameters. The remaining parameters were fixed at their values at point 3 of Stage 4 of the search. The surface is shown in Fig. 7.19, and indicates that some improvements in the objective function is possible by simultaneous adjustment of these three parameters.

The Davidon method, however, proved ineffective in finding the minimum point when used to search from each of two different starting points on the surface. The paths taken by the searches are shown in Fig. 7.19. At the end points of the searches, a number of iterations took place in which the parameter changes were too small to be plotted and the change in the objective function was negligible. To ensure that the search method had been implemented correctly, the optimising programme was tested on the two-dimensional problem which was used by Box (1966) as a test function and was successful in locating the minimum of that function. The objective function was then evaluated at each node of a grid of VSMAX, USMAX and EVPMAX values surrounding the end point of the second search. When the grid spacing was 0.05, 0.002, 0.05 for VSMAX, USMAX and EVPMAX respectively, no point on the grid had a lower value of the objective function than the end point of the search, but when spacings of 0.005, 0.002, 0.005 were used, about one third of the surrounding points had lower values. This grid spacing is very fine compared to the normal step size which was used in finding the steepest descent direction and in moving along the search directions, when the changes to each parameter were generally between 0.05 and 0.5. The following reasons may be advanced for the difficulties encountered with the Davidon method at this particular point of the search:-

(a) The steepest descent direction at this stage of the work was being found by Method 5, described in Appendix A2. This method assumes that the cross-sections of the response surface in the co-ordinate directions at the current point may be approximated by parabolas. The presence of a discontinuity in the response surface near this point would invalidate the assumption. (The other methods used in this project would also be invalid in this situation.)

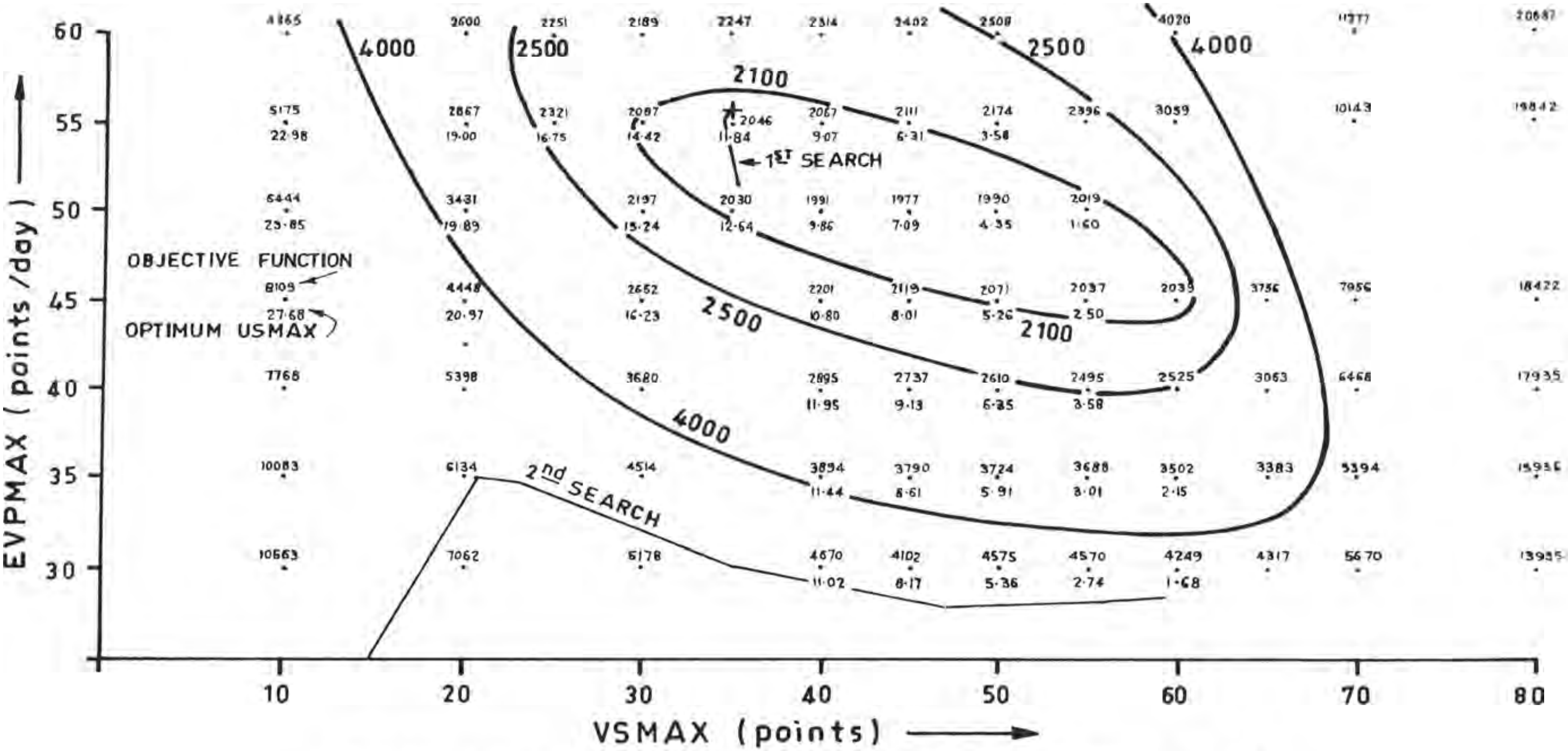
(b) The step sizes used in defining the steepest descent direction may be too large for the parabolic assumption to be valid, or the step size taken in the chosen descent direction may be too large, overstepping the minimum in this direction at the first move.

Further work would be required to identify the cause of the problem. However, if the remedy required the use of smaller step sizes than those used in this project, it may also be necessary to use "double precision" arithmetic for computing parameter changes and evaluating the objective function, and because more steps would then be required in the descent directions, the computing time would be greatly increased.

The values of VSMAX, USMAX and EVPMAX at the minimum point of the surface shown in Fig. 7.19 were substituted in point 3 resulting in the move to Stage 5 of the search for this point. Subsequently, the performance of the Simplex method on this surface was checked.

The method successfully located the minimum point of the surface after 90 iterations from a starting simplex which included the point at which the Davidon method encountered difficulty.

As it was suspected that a similar surface existed for the combined points 1 and 2 at Stage 5, a Simplex run was made with only VSMAX, USMAX and EVPMAX varying, but little change in these parameters or improvement of the objective function occurred. The parameters were probably already near the minimum of the surface in this case.



+ This point corresponds to point 3 at stage 4 (See Fig.7·1)

Contours are of sum of squares of monthly deviations.

—— Paths of searches by the Davidon method.

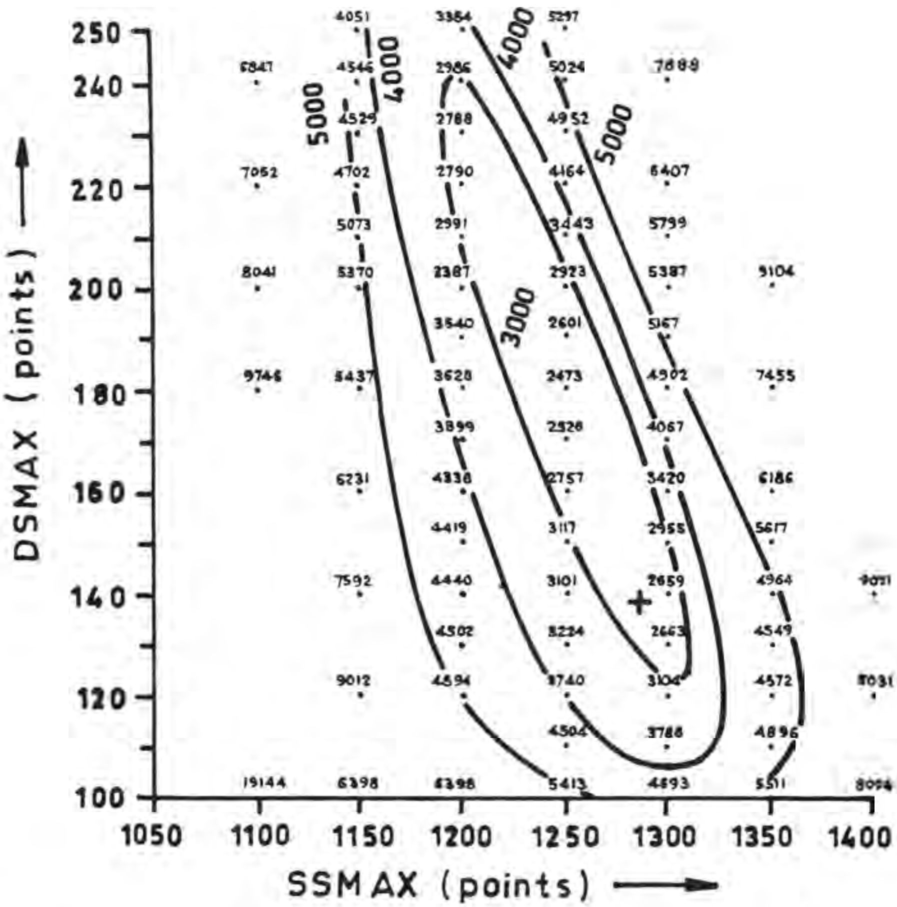
FIG. 7·19

OPTIMUM USMAX VALUES FOR VSMAX – EVPMAX COMBINATIONS.

(iv) Response Surface for DSMAX and SSMAX

Of the two-parameter combinations which were investigated, the DSMAX-SSMAX combination proved to be of interest even though it did not affect the subsequent course of the search. The response surface for these parameters was evaluated and plotted with the remaining parameter values at point 1, Stage 3 of the search, and is shown in Fig. 7.20.

Further slight improvement in the objective function may be obtained by simultaneously reducing the value of SSMAX and increasing the value of DSMAX. These movements took place during further optimisation. The degree of inter-action between the two parameters is not very strong, however, as a change in the scaling of one of the axes towards the scaling of the other by a factor of about 2 would substantially eliminate the elongation in the contours. The main interest in this surface lies in the shape of the cross-sections which are shown in Fig. 7.21. For constant values of SSMAX, graphs of the objective function vs DSMAX have discontinuities and the segments between the discontinuities appear to be parabolic. This provides numerical support for the theoretical result derived later in Section 8. Similar discontinuities are not evident in the cross-sections at constant values of DSMAX, but this could be due to the small number of points used for plotting the graphs.



+ This point corresponds to point 1 at Stage 3. (See Fig. 7·1)
Contours are of Sum of Squares of monthly deviations
Values of parameters other than DSMAX and SSMAX are as for point 1 at Stage 3.

FIG. 7·20

RESPONSE SURFACE FOR DSMAX AND SSMAX.

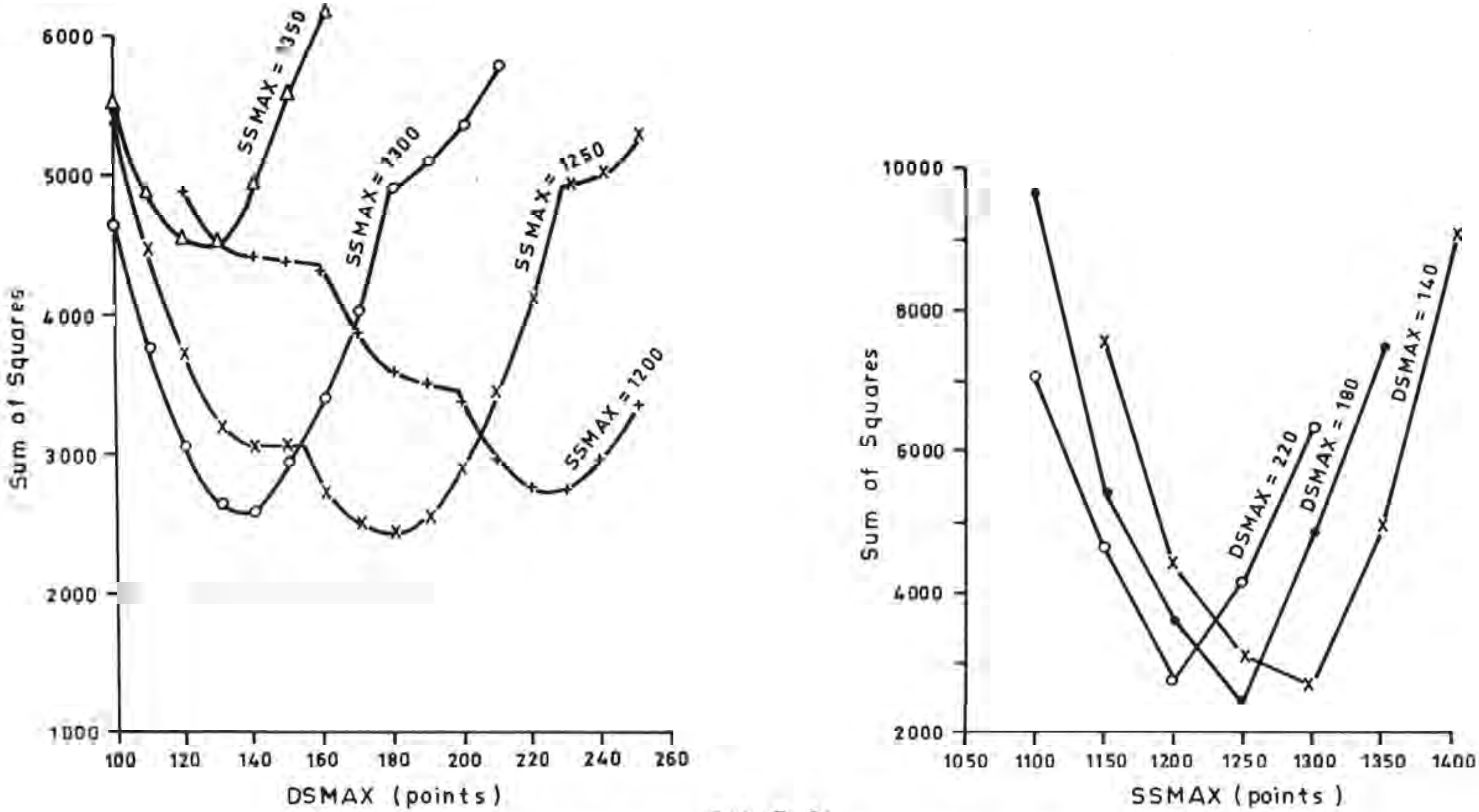


FIG. 7·21
CROSS - SECTIONS OF RESPONSE SURFACE FOR
DSMAX AND SSMAX (FIG. 7·20)

7.3 DISCUSSION OF THE PARAMETER VALUES OBTAINED FROM THE SEARCH

Although a unique set of optimum parameter values was not found for the Lidsdale No. 2 catchment, the search was effective in changing the objective function from initial values of approximately 38,000 and 62,000 to final values scattered around 1800. The points obtained at the end of the search appear to be spaced along the floor of a valley and are probably close to the lowest point in the valley. It is worthwhile considering the significance of the parameter values and the performance of the model with these values. The points are listed again in Table 7.1.

TABLE 7.1

NEAR-OPTIMUM SETS OF PARAMETER VALUES FOR LIDSDALE NO. 2 CATCHMENT

POINT NO.	VSMAX	USMAX	DSMAX	SSMAX	EVPMAX	PV	FO	KF	SUMSQS
1	44.6	6.2	512	797	19.8	0.460	1180	0.1275	1800
2	48.9	6.1	567	695	24.9	0.454	1195	0.0863	1791
3	48.7	6.3	487	731	27.3	0.473	981	0.1359	1811
4	48.1	6.4	515	618	36.6	0.449	983	0.1007	1785
5	49.1	6.2	482	679	33.7	0.463	1241	0.0900	1792
6	48.9	6.4	466	690	34.9	0.466	1166	0.1004	1796
7	43.7	8.9	328	805	50.2	0.490	485	2.174	1850
8	45.9	8.4	299	738	122.7	0.480	788	3.246	1815

It should be remembered that a value of about 3.7 may be substituted for KF in points 1 to 6 with little effect upon the value of the objective function.

7.3.1 Variation of the Parameter EVPMAX

The value of this parameter exhibits the greatest variation between the above points. However, this does not imply that the objective function is indifferent to the value of EVPMAX. For points 1 and 8 above, variation of the value of EVPMAX while the other parameters are held constant results in significant increases in the objective function as shown in Table 7.2.

TABLE 7.2

SENSITIVITY OF OBJECTIVE FUNCTION TO EVPMAX

SENSITIVITY WHEN OTHER PARAMETER VALUES ARE THOSE AT POINT NO. 1 OF TABLE 7.1		SENSITIVITY WHEN OTHER PARAMETER VALUES ARE THOSE AT POINT NO. 8 OF TABLE 7.1	
EVPMAX	OBJECTIVE FUNCTION	EVPMAX	OBJECTIVE FUNCTION
5	121,335	80	2,574
10	42,811	90	2,160
15	11,650	100	1,946
19.8	1,800	110	1,847
20	1,811	120	1,816
25	4,720	122.7	1,815
30	7,118	130	1,826
35	9,420	140	1,859
40	10,870	150	1,904
60	14,432	160	1,955
80	18,084	180	2,062

Therefore, the large changes in EVPMAX between the points listed in Table 7.1 are being accompanied by compensating changes in the other parameters and some form of inter-action exists. As the values of VSMAX, USMAX and PV show little variation, and as the objective function is known to be indifferent to a wide range of FO and KF combinations, the inter-action appears to be taking place between EVPMAX, DSMAX and SSMAX. Sub-optimisation with these three parameters may bring the points still closer together.

7.3.2 Significance of the Parameter Values

The near-optimum values for some of the parameters are rather unexpected and for a few parameters the values even appear to be inconsistent with the intended purpose of those parameters in the model.

The capacity of the Interception Store, VSMAX, at almost half an inch, is considerably higher than values normally quoted for this store. Jones (1969), after an extensive survey of the literature on interception, recommended values between 1 and 10 pts, the higher figure being for two layer vegetation such as forest with appreciable grass. The value recommended by Crawford and Linsley (1966) for heavy forest was 20 pts. It is possible that moisture other than that retained on the surfaces of vegetation evaporates at the potential rate and that, during optimisation, the capacity of the Interception Store is adjusted to include this amount of moisture. The vegetation on the Lidsdale No. 2 catchment is pine forest with a matting of dead pine needles on the ground surface, and this matting probably acts in the same way as the Interception Store. However this would probably not account fully for the higher value obtained for this catchment. A consequence of the high value is that, in winter months when the potential evaporation rate is down to about 3-5 pts/day, a period of from 10 to 15 days elapses before the Interception Store is emptied and depletion of the soil stores by evapotranspiration commences. This result is unrealistic.

It is not possible to reconcile the low value obtained for the capacity of the Upper Soil Store, USMAX, with the very high value obtained for the capacity of the Drainage Store, DSMAX. The Upper Soil Store is meant to represent the water held in the capillaries of the topsoil between wilting point and field capacity. Using the very low value of 0.6 in/ft for the Available Water Capacity of the topsoil, the value of approximately 6.5 points obtained for USMAX implies that the topsoil layer is no greater than 1¼ inches deep. As the Drainage Store is meant to represent the gravitational water in the same depth of topsoil between field capacity and saturation, the values of 3 to 5 inches obtained for DSMAX are clearly in conflict with the figures for USMAX. It is possible that these two stores are effectively operating as one combined store. If this is so then the physical interpretation of the parameter PV, the fraction of the evapotranspiration taken from the Upper Soil Store, becomes unclear. The values obtained for PV seem to be too high for the low values of USMAX.

An alternative explanation for the high values of DSMAX could be that the Drainage Store is merely acting as an artificial store to retain water which would otherwise appear as runoff and which would increase the value of the objective function.

The unexpected results indicate that, for this catchment, the parameters do not have the physical significance which they were intended to have. If similar results were obtained for other catchments, it is very unlikely that significant correlations between the parameter values and physical catchment characteristics could be found.

7.3.3 The Length of the Data Record and the Warm-Up Period

Approximately 2 years and 4 months of data were used in optimising the parameter values. If a significantly longer period of data had been available the derived parameter values may have been quite different. In forcing the model to reproduce a much longer record, "black box" effects such as the tendency of the Drainage Store to act as an artificial store would probably be reduced. The parameters would then be expected to take on values which were more meaningful from physical considerations.

A conclusion reached in the next section of this report is that the operation of the model is not independent of the assumed initial store contents until the stores have been filled during a period of heavy rainfall. For some of the sets of parameter values derived for the Lidsdale No. 2 catchment the Lower Soil Store does not approach the full state until 4-6 months from the end of the data record. The data record before this time should therefore have been contained in an extended warm-up period or, alternatively, the initial contents of the Lower Soil Store could possibly have been regarded as a parameter. More meaningful parameter values may have been derived if one of these alternatives was adopted.

7.3.4 The Lower Soil Store Depletion Factor

Depletion of the contents of the Lower Soil Store to groundwater was simulated in the Boughton model by applying a constant factor of 0.999 to the contents each day. With a typical figure of 500 pts for the contents of the store the daily depletion quantity is 0.5 pts and this would accumulate into a considerable quantity over the period of the data record. If the depletion factor were decreased to, say, 0.997 then the water extracted from the model in this way would be increased further. It is therefore possible that a small adjustment to this factor may have stopped the Drainage Store from acting as an artificial store and allowed the parameter DSMAX to take on a meaningful value. In future optimisation work the Lower Soil Store depletion factor should not be allocated a fixed value but should be regarded as another parameter to be optimised.

7.3.5 Performance of the Model with the Near-Optimum Parameter Values

Towards the end of the project, an extra year's data for the Lidsdale No. 2 catchment became available, and provided an opportunity to check the ability of the model to reproduce a period of runoff data which was not used in the search for the optimum parameter values. The model was operated with the parameter values at points nos. 1 and 8 in Table 7.1 using the extended data record, i.e., for the period 17-10-1968 to 28-2-1972. The calculated monthly runoff quantities are listed with the observed quantities and monthly rainfalls in Table 7.3 below, while similar figures on an event basis are presented in Table 7.4.

The modelling of the monthly and event runoff quantities in the period which was used for optimisation could be considered as good for engineering purposes because the large runoff amounts, which contribute most to the catchment yield, are well reproduced. This must be qualified by observing that the modelling of the flows in November 1970 is not good and that, in general, small runoff events are not reproduced by the model. In the extra one-year period the flows calculated by the model compare poorly with the observed flows. There are large errors in both under- and over-estimation of the individual events by the model. For the whole of the period of data there are only minor differences between the flows calculated by the model with the different sets of parameter values.

The total modelled runoff quantity for September 1970 is very close to the observed value. However the event data for this month show that the model badly under-estimates one event and over-estimates two events in such a way that the errors almost balance when total quantities are calculated. Similar but less obvious examples of compensating errors occur in November 1969, December 1970 and February 1971. It is unlikely that optimisation with an objective function based on event totals rather than monthly totals would achieve significantly better reproduction of individual events (see Section 9). The main reason for the poor reproduction of some events is thought to be that the model cannot simulate the effect of different rainfall intensities on the amount of runoff. Given that the runoff amount varies with the intensity of the rainfall (other factors being held constant) then the model would be expected to give reasonable reproduction for rainfall events of average intensity, to under-estimate the runoff from falls of high intensity and to over-estimate the runoff from falls of low intensity. However it is not possible to attempt a correlation between rainfall intensities and over- or under-estimation of runoff quantities. This is because the incorrect estimation by the model of the runoff for an event is accompanied by an incorrect amount of water subsequently held in soil storage, and this also influences the calculated runoff for subsequent events.

As there is apparently no way in which the effect of different rainfall intensities can be allowed for when using daily rainfall data in a daily time-period model, poor reproduction of some events by such models must always be expected.

TABLE 7.3

MODEL OUTPUT - MONTHLY FIGURES

YEAR	MONTH	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
1968	Oct.	81	0	0	0
	Nov.	111	0	0	0
	Dec.	393	0.2	0	0
1969	Jan.	235	2.2	0	0
	Feb.	750	42.0	0	0
	Mar.	261	5.3	0	0
		END OF "WARM-UP" PERIOD			
	START OF PERIOD USED FOR OPTIMISATION				
	Apr.	240	6.4	0	0
	May	217	0.0	0	0
	June	231	9.5	0	0
	July	117	0.0	0	0
	Aug.	365	11.4	0	0
	Sep.	197	6.8	0	0
	Oct.	270	0.0	0	0
	Nov.	422	49.9	50.2	49.5
	Dec.	313	50.8	46.9	47.2
1970	Jan.	584	7.6	0	0
	Feb.	266	1.0	0	0
	Mar.	251	1.5	0	0
	Apr.	198	0	0	0
	May	210	2.0	0	0
	June	146	0	0	0
	July	14	0	0	0
	Aug.	189	0	0	0

TABLE 7.3 (cont.)

YEAR	MONTH	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
1970	Sept.	433	63.0	63.3	63.0
cont.	Oct.	233	22.8	26.9	26.3
	Nov.	410	32.9	0	0
	Dec.	322	83.5	72.6	72.8
1971	Jan.	513	58.4	72.3	73.3
	Feb.	540	264.0	264.3	264.4
	END OF PERIOD USED FOR OPTIMISATION				
	START OF EXTRA PERIOD MODELLED WITH NEAR-OPTIMUM PARAMETERS				
	Mar.	135	0	0	0
	Apr.	79	0	0	0
	May	164	0	0	0
	June	51	0	0	0
	July	103	0	0	0
	Aug.	319	0.4	0	0
	Sept.	455	16.4	65.1	72.8
	Oct.	31	0	0	0
	Nov.	245	0	0	0
	Dec.	774	129.9	6.9	25.9
1972	Jan.	889	386.6	430.2	423.4
	Feb.	352	28.3	65.9	66.7

TABLE 7.4

MODEL OUTPUT - EVENT FIGURES

EVENT	START DATE	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
1	17-10-'68	0	0	0	0
2	23-10	3	0	0	0
3	25-10	78	0	0	0
4	5-11	111	0	0	0
5	4-12-'68	154	0	0	0
	16-12	55	0	0	0
	24-12	144	0.2	0	0
	27-12	38	0	0	0
	29-12	2	0	0	0
10	1- 1-'69	20	0	0	0
	14- 1	93	0	0	0
	18- 1	106	2.2	0	0
	24- 1	16	0	0	0
	5- 2	155	1.0	0	0
15	9- 2-'69	489	41.0	0	0
	18- 2	1	0	0	0
	24- 2	20	0	0	0
	26- 2	85	0	0	0
	15- 3	8	0	0	0
20	17- 3-'69	128	5.3	0	0
	22- 3	3	0	0	0
	25- 3	127	0	0	0
END OF "WARM-UP" PERIOD					
START OF PERIOD USED FOR OPTIMISATION					
	15- 4	232	6.4	0	0
	20- 4	3	0	0	0
25	2- 5-'69	9	0	0	0
	4- 5	2	0	0	0
	8- 5	2	0	0	0
	13- 5	23	0	0	0
	15- 5	71	0	0	0
30	28- 5-'69	112	0	0	0
	3- 6	34	0	0	0
	10- 6	38	0	0	0
	20- 6	157	9.5	0	0
	3- 7	4	0	0	0
35	5- 7-'69	46	0	0	0
	13- 7	6	0	0	0
	16- 7	18	0	0	0
	20- 7	43	0	0	0
	3- 8	20	0	0	0
40	13- 8-'69	179	5.7	0	0
	18- 8	16	0	0	0
	21- 8	32	0	0	0
	25- 8	93	5.7	0	0
	31- 8	85	6.8	0	0
45	6- 9-'69	29	0	0	0
	9- 9	10	0	0	0
	14- 9	55	0	0	0
	20- 9	9	0	0	0
	22- 9	27	0	0	0
50	30- 9-'69	14	0	0	0
	5-10	10	0	0	0

TABLE 7.4 cont.

EVENT	START DATE	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
	10-10	3	0	0	0
	12-10	38	0	0	0
	14-10	41	0	0	0
55	16-10-'69	40	0	0	0
	26-10	84	0	0	0
	31-10	55	0	0	0
	4-11	20	0	0	0
	6-11	217	44.3	47.0	48.1
60	12-11-'69	24	0	0	0
	14-11	98	5.6	3.2	1.5
	19-11	21	0	0	0
	22-11	3	0	0	0
	29-11	11	0	0	0
65	3-12-'69	2	0	0	0
	12-12	282	50.8	46.9	47.2
	23-12	29	0	0	0
	1- 1-'70	85	0	0	0
	7- 1	5	0	0	0
70	10- 1-'70	116	0	0	0
	17- 1	45	0	0	0
	19- 1	91	1.2	0	0
	22- 1	32	0	0	0
	26- 1	210	6.4	0	0
75	2- 2-'70	2	0	0	0
	11- 2	7	0	0	0
	13- 2	132	0	0	0
	16- 2	57	1.0	0	0
	23- 2	39	0	0	0
80	27- 2-'70	29	0	0	0
	1- 3	15	0	0	0
	6- 3	25	0	0	0
	16- 3	21	0	0	0
	20- 3	157	1.5	0	0
85	23- 3-'70	33	0	0	0
	10- 4	19	0	0	0
	12- 4	30	0	0	0
	23- 4	119	0	0	0
	28- 4	30	0	0	0
90	7- 5-'70	21	0	0	0
	15- 5	174	2.0	0	0
	21- 5	3	0	0	0
	29- 5	12	0	0	0
	1- 6	2	0	0	0
95	3- 6-'70	2	0	0	0
	6- 6	43	0	0	0
	11- 6	17	0	0	0
	16- 6	37	0	0	0
	22- 6	45	0	0	0
100	2- 7-'70	2	0	0	0
	20- 7	12	0	0	0
	2- 8	56	0	0	0
	11- 8	11	0	0	0
	18- 8	23	0	0	0
105	24- 8-'70	35	0	0	0
	27- 8	59	0	0	0
	31- 8	5	0	0	0

TABLE 7.4 cont.

EVENT	START DATE	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
	2- 9	135	52.0	0	0
	9- 9	67	1.9	0	0
110	13- 9-'70	21	0	0	0
	15- 9	52	1.5	0	0
	22- 9	40	0	0	0
	24- 9	47	0.8	21.42	20.7
	27- 9	71	6.8	41.92	42.2
115	8-10-'70	2	0	0	0
	11-10	88	0.2	0	0
	15-10	1	0	0	0
	19-10	15	0	0	0
	20-10	126	22.6	26.9	26.3
120	26-10-'70	1	0	0	0
	6-11	59	0	0	0
	8-11	51	0.3	0	0
	10-11	12	0	0	0
	12-11	83	14.9	0	0
125	16-11-'70	34	0.1	0	0
	20-11	11	0	0	0
	25-11	160	17.6	0	0
	6-12	17	0	0	0
	8-12	200	57.6	40.5	43.5
130	12-12-'70	29	8.2	0	0
	19-12	82	2.0	0.9	0.1
	22-12	2	0	0	0
	24-12	18	0	0	0
	28-12	155	15.8	31.2	29.2
135	13- 1-'71	72	0	0	0
	18- 1	108	0	0	0
	24- 1	4	0	0	0
	27- 1	6	0	0	0
	29- 1	198	5.3	0	0
140	31- 1-'71	125	53.0	72.3	73.3
	1- 2	117	76.8	86.0	86.2
	4- 2	153	83.3	94.5	94.8
	9- 2	166	101.8	83.7	83.4
	17- 2	90	2.1	0.1	0
145	24- 2-'71	12	0	0	0
	27- 2	2	0	0	0
END OF PERIOD USED FOR OPTIMISATION					
START OF EXTRA PERIOD MODELLED WITH NEAR-OPTIMUM PARAMETERS					
	3- 3	66	0	0	0
	17- 3	6	0	0	0
	21- 3	49	0	0	0
150	27- 3-'71	13	0	0	0
	1- 4	18	0	0	0
	14- 4	5	0	0	0
	18- 4	20	0	0	0
	27- 4	36	0	0	0
155	5- 5-'71	44	0	0	0
	9- 5	4	0	0	0
	13- 5	10	0	0	0
	16- 5	4	0	0	0
	20- 5	62	0	0	0

TABLE 7.4 cont.

EVENT	START DATE	RAINFALL (POINTS)	OBSERVED RUNOFF (POINTS)	MODELLED RUNOFF (POINTS) WITH THE FOLLOWING SETS OF PARAMETER VALUES FROM TABLE 7.1:-	
				POINT 1	POINT 8
160	28- 5-'71	60	0	0	0
	6- 6	4	0	0	0
	9- 6	9	0	0	0
	15- 6	12	0	0	0
	22- 6	2	0	0	0
165	27- 6-'71	4	0	0	0
	15- 7	98	0	0	0
	24- 7	2	0	0	0
	31- 7	62	0	0	0
	6- 8	136	0.4	0	0
170	11- 8-'71	2	0	0	0
	21- 8	77	0	0	0
	26- 8	3	0	0	0
	30- 8	45	0	0	0
	10- 9	172	4.0	0	0
175	15- 9-'71	39	0	0	0
	16- 9	103	7.5	18.6	26.5
	21- 9	77	3.6	35.7	35.8
	25- 9	20	0	0	0
	26- 9	41	1.3	10.8	10.8
180	8-10-'71	3	0	0	0
	15-10	10	0	0	0
	21-10	7	0	0	0
	23-10	11	0	0	0
	7-11	119	0	0	0
185	14-11-'71	59	0	0	0
	18-11	2	0	0	0
	20-11	16	0	0	0
	29-11	49	0	0	0
	3-12	11	0	0	0
190	6-12-'71	150	0	0	0
	8-12	262	18.6	0	0
	15-12	36	0	0	0
	24-12	17	0	0	0
	25-12	80	0.8	0	0
195	27-12-'71	180	97.9	0	10.8
	28-12	38	12.7	6.9	15.1
	1- 1-'72	25	0	0	0
	5- 1	116	37.9	8.8	6.4
	13- 1	112	16.2	29.3	27.2
200	14- 1-'72	183	111.1	141.5	141.8
	18- 1	42	13.7	0	0
	23- 1	29	0	0	0
	24- 1	271	140.2	177.5	174.9
	27- 1	111	67.5	73.1	73.1
205	4- 2-'72	38	0.3	0	0
	8- 2	4	0	0	0
	11- 2	32	0	0	0
	14- 2	101	0	0	0
	18- 2	99	19.9	40.6	41.4
210	20- 2-'72	53	8.1	25.3	25.3
211	26- 2-'72	25	0	0	0

Note:- Events consist of:-

(i) those rainy days associated with each peak of the observed runoff hydrograph, and

(ii) other groups of consecutive rainy days.

Each event encompasses subsequent dry days.

8. ALGEBRAIC ANALYSIS OF THE MODEL

The ultimate aim of this analysis was to develop an explicit algebraic equation for the output of the model in terms of the input data and the model parameters. Such an equation is frequently available for the optimisation problems encountered in other fields of investigation, and knowledge of the equation is helpful in solving the optimisation problem for several reasons.

(i) By combining this equation with the observed catchment output, individual equations for each of the monthly (or event) deviations between calculated and observed runoff may be written. If more equations are available than unknowns (the model parameters), as is usually the case, then the least squares techniques may be applied to solve the equations simultaneously for the parameter values. Thus, another set of methods becomes available for the solution of the optimisation problem.

(ii) A single equation for the objective function in terms of the input and output data and of the model parameters can be written by summing the individual equations written under (i) above. This equation could be used in several ways.

(a) Explicit equations for the partial derivatives of the objective function with respect to each parameter could be obtained by simple differentiation. The steepest descent direction at any point on the response surface could then be found simply by substituting the parameter values of that point into these equations. Thus the difficulties encountered in the numerical definition of the steepest descent direction in this project would be eliminated, and the overhead in computer time caused by the large number of runs through the model which are required would be considerably reduced, enabling more time to be spent in the actual search for the minimum.

(b) Inter-dependencies between parameters would possibly be revealed. For example, if the parameters x_1 and x_2 usually appeared together as $(x_1 \pm ax_2)$ in various terms of the equation, then linear inter-dependence between these parameters must be expected.

(iii) The equation would possibly indicate the likelihood of indifference to any parameter, the effect on the output of the assumed initial store contents, and the required length of the "warm-up" period.

(iv) Knowledge of the form of the equation may assist in selecting the most appropriate search technique for finding the optimum model parameter values.

In addition to the above benefits it was thought possible that the analysis could reveal a direct analytical method of finding the optimum parameter values and eliminate the use of search techniques.

8.1 ANALYSIS OF INDIVIDUAL STORES

The task of writing a comprehensive equation for the output from a mathematical model of the rainfall-runoff process is complicated by the number of conditional branches which may be taken during the operation of the model. Work was commenced by analysing a simple store, then progressed to each of the stores of the Boughton model considered in isolation. For this initial analysis it was assumed that the inflows and outflows for each of the stores were known, although this is not so when the model is considered as a whole. The analysis for each store comprised two phases:-

(i) a method was sought for finding the capacity of the store from the input and output data, and

(ii) a "model" of the store was constructed by re-arranging the equations from (i), expressing the output of the store in terms of the input data and the store capacity, considered now to be a variable. An objective function based on differences between observed and modelled output was set up and the effect on this function of changes in the store capacity was investigated.

8.1.1 A Single Input-Output Event in a Simple Store

The first physical system which was considered is as illustrated in Fig. 8.1.

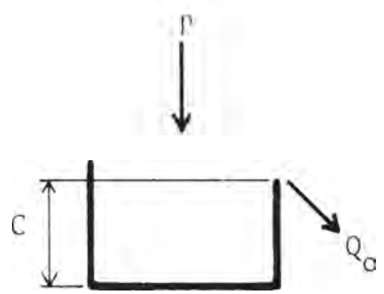


FIGURE 8.1
SINGLE INPUT-OUTPUT
EVENT IN A SIMPLE STORE

Given that the store is initially empty, that an amount of water P is poured into the store and an amount Q_o overflows, where $Q_o > 0$, then the capacity of the store is simply

$$C = P - Q_o \; .$$

Re-arranging this equation, the mathematical model of the system is:-

$$Q_c = 0 \qquad \text{if } C' \geq P \qquad (8.1a)$$

$$= P - C' \qquad \text{if } C' < P \qquad (8.1b)$$

where Q_c = output as calculated by the model
P = input (data)
C' = a parameter, the capacity of the container, whose optimum value is C.

We seek the value of C' which produces the best fit of Q_c to Q_o by minimising the objective function

$$F = (Q_o - Q_c)^2 \qquad \text{with respect to } C' .$$

Substituting from equations 8.1a and b,

$$F = Q_o^2 \qquad \text{if } C' \geq P \qquad \text{(function is constant)}$$

$$= (Q_o - P + C')^2 \qquad \text{if } C' < P \qquad \text{(function is parabolic)}$$

For the minimum of the parabolic section of the function,

$$\frac{dF}{dC'} = 2(Q_o - P + C') \cdot 1 = 0$$

$$\therefore \; C' = P - Q_o$$

In general, the graph of F vs C' is as shown in Fig. 8.2. The numbers shown are for the particular case where P = 15, Q_o = 5.

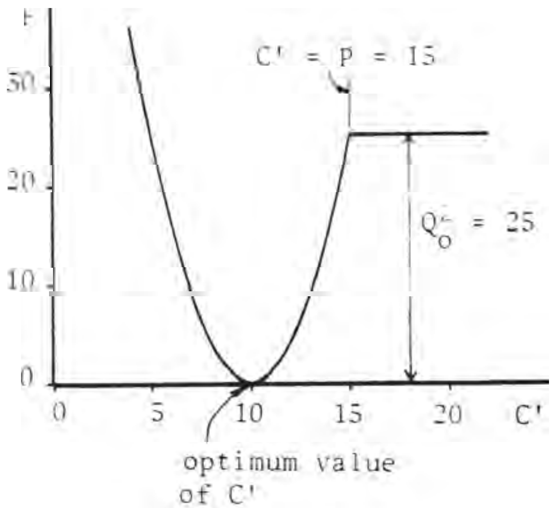


FIGURE 8.2
GRAPH OF F vs C'
FOR SIMPLE STORE

P = 15, Q_o = 5

For the special case of an event with Q_o = 0, the graph is as shown in Fig. 8.3 and the optimum value of C' is indeterminate.

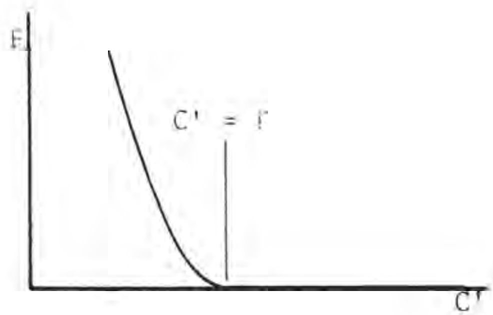


FIGURE 8.3
GRAPH OF F vs C'
FOR SIMPLE STORE
WHEN Q_o = 0

As most rainfall-runoff models are built up with components similar to the simple system shown above, it should be reasonable to expect that features of the optimisation problem for this system will also apply for the more complicated models. The most significant features appear to be:-

(i) the response surface has a discontinuity, and the objective function is indifferent to the parameter value on one side of this discontinuity. None of the optimising methods will find the optimum value of $C^{'}$ if the initial estimate of $C^{'}$ is too large to the extent that it is greater than P.

(ii) the response surface is of positive definite quadratic form when $C^{'} < P$. This is significant when using the conjugate direction descent methods of optimisation.

(iii) it is possible for the data record to be such that the optimum value of $C^{'}$ is indeterminate. This occurs for an event where $Q_o = 0$.

The above analysis may be easily generalised for an event where the contents of the container before the event were not zero. Referring to Figure 8.4, the previous analysis is used to find the empty portion of the container, C_1, and the volume is then found from $C = C_1 + S$. The graph of the objective function is shifted by the amount S, as shown in Fig. 8.5.

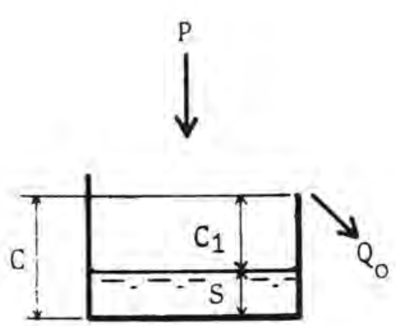


FIGURE 8.4

SINGLE INPUT-OUTPUT EVENT IN A SIMPLE STORE WITH NON-ZERO INITIAL CONTENTS.

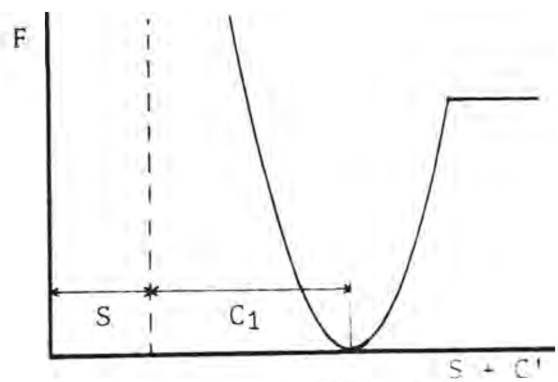


FIGURE 8.5

GRAPH OF OBJECTIVE FUNCTION FOR EVENT SHOWN IN FIG. 8.4.

Of course, if S is unknown, then C is indeterminate, and the most that can be said is that $C \geq C_1$.

8.1.2 Interception Store

The operation of this component of the Boughton model is similar to that of a simple container exposed to rainfall and evaporation. Given a record of the rainfall inputs P, the observed spills Q_o, and potential evaporation data, is it possible to find the capacity, VSMAX of the container?

From the work of the previous section, the capacity can only be found from the results of an event for which the contents immediately prior to the event are known. It can be shown that if an event occurs for which $Q_o > 0$ and $P - Q_o < ET$, where ET is the total potential evaporation since the previous rainfall (or, for the first event in the data record, since the start of the record), then the container must have been empty prior to the event and its capacity may be calculated.

Proof:- Let VS_o = contents after previous rainfall (or at start of data record)

VS_1 = contents immediately prior to event.

VS_1 is related to VS_o as follows:-

$$VS_1 = VS_o - ET \qquad \text{if} \quad ET \leq VS_o \qquad (8.2a)$$

$$= 0 \qquad \text{if} \quad ET > VS_o \qquad (8.2b)$$

Using the result of the previous section, the unfilled portion of the container, VSMAX - VS_1, is expressed in terms of the data for this event.

$$VSMAX - VS_1 = P - Q_o \tag{8.3}$$

For this event, $P - Q_o < ET$

$$\therefore \quad VSMAX - VS_1 < ET$$

Substitute for VS_1, assuming that 8.2(a) above is true.

$$VSMAX - VS_o + ET < ET$$

$$\therefore \quad VSMAX - VS_o < 0 \tag{8.4}$$

But VSMAX is positive and $VS_o \le VSMAX$, therefore equation 8.4 cannot be true and equation 8.2a cannot hold for this event. Therefore equation 8.2b is true, i.e.,

$$VS_1 = 0$$

Substituting in equation 8.3 above,

$$VSMAX = P - Q_o.$$

If there are a number of events in the data record for which $Q_o > 0$ and $P - Q_o < ET$, then they may each be solved for VSMAX. If errors are present in the P and Q_o data, the answers from the various events will be different and it is then necessary to adopt a best fit estimate of VSMAX.

Using a similar approach to that used in the previous section, the modelled output for a given event is

$$\begin{aligned} Q_c &= 0 && \text{if } C' \ge P \\ &= P - C' && \text{if } C' < P \end{aligned}$$

where the optimum value of C' is VSMAX.

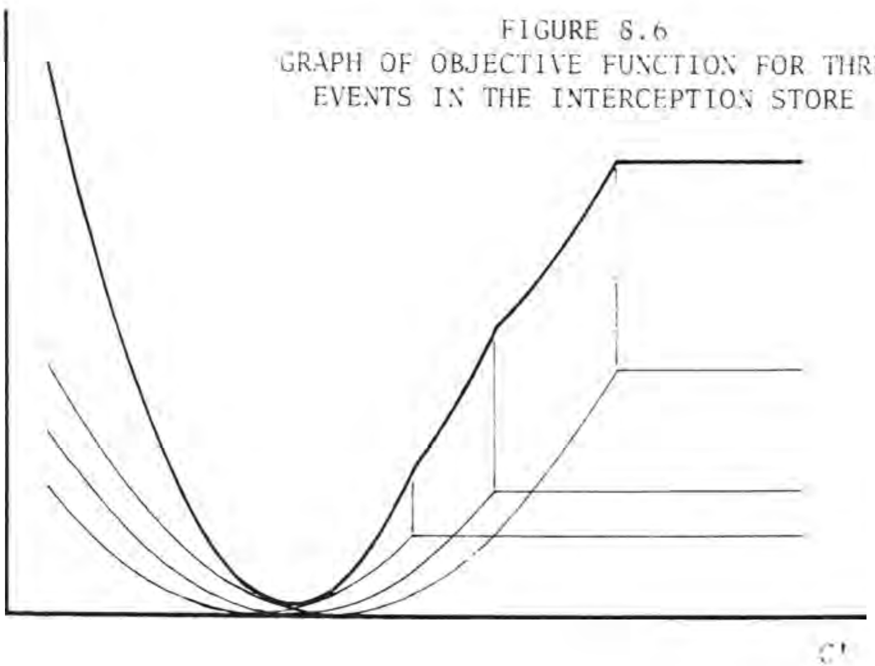
The objective is now to minimise

$$F = \sum_{i=1}^{k} (Q_{o(i)} - Q_{c(i)})^2$$

where k is the number of events being solved for VSMAX.

This function is a sum of functions which are each similar to the one shown in Fig. 8.2. However, each component function will have a different optimum point and different location of the discontinuity. The resulting response surface would appear as illustrated in Fig. 8.6.

FIGURE 8.6
GRAPH OF OBJECTIVE FUNCTION FOR THREE EVENTS IN THE INTERCEPTION STORE



In general, there will be as many discontinuities in this function as the number of events used in the solution for VSMAX. Each segment of the curve is still parabolic, however. The minimum value of the objective function is no longer zero.

If there were no gross errors in the P and Q_o data, then the minima of the component functions would lie fairly close together and the discontinuities in these functions would all lie to the right hand side of the minimum of the composite function. In these circumstances,

$$F = \sum_{i=1}^{k} (Q_{o(i)} - P_{(i)} + C')^2$$

in the area of the minimum.

For the optimum C'

$$\frac{dF}{dC'} = \sum_{i=1}^{k} 2(Q_{o(i)} - P_{(i)} + C') \cdot 1 = 0$$

$$\therefore kC' = \sum_{i=1}^{k} (P_{(i)} - Q_{o(i)})$$

$$\therefore C' = \frac{\sum_{i=1}^{k} (P_{(i)} - Q_{o(i)})}{k}$$

i.e., under the conditions stated above, the optimum value of VSMAX is the arithmetic average of the values derived from each event. This result could be useful in a direct analytical method for finding the optimum parameter values of the model.

8.1.3 Upper Soil Store

This component of the Boughton model operates in a similar way to the Interception Store except that in the lower range of storage, drying out by evapotranspiration is restricted by a function of the current stored contents in such a way that the storage depletes in an exponential manner, becoming zero only in infinite time.

Once again, the capacity is sought considering this store in isolation and given a record of inputs to the store P, overflows Q_o, and potential evapotranspiration data.

As this store never becomes empty, the data from a given event can only be used to find the empty portion of the store and the problem is similar to that shown in Fig. 8.4. The empty portion of the store is equal to P - Q_o for a given event and the capacity is

$$\text{USMAX} = P - Q_o + \text{US} \tag{8.5}$$

where US = the unknown contents of the store prior to the event.

USMAX can only be found if US can be expressed in terms of USMAX, thus eliminating one of the unknowns from equation 8.5.

It is possible to express US in terms of USMAX when it is known that the store has dried out from the full state until US is less than USC, the value of storage at the discontinuity in the evapotranspiration function (see sub-section 6.1). Furthermore, it is possible to identify those events for which the storage was less than USC immediately prior to the event and whose data may therefore be used to solve for USMAX.

(i) Expression for US in Terms of USMAX

When it is known that the store has dried out from the full state until US is less than USC, then (referring to equation 6.13)

$$\text{US} = \text{USC} \cdot e^{-m \cdot \text{PV} \cdot \text{EVPMAX}/\text{USMAX}} \tag{8.6}$$

where m = number of days since US was equal to USC.

Now $m = n - n_c$

where n = number of days since US was equal to USMAX (i.e., since the previous event where $Q_o > 0$)

and n_c = number of days to dry the store from USMAX down to USC.

Now $n_c = \dfrac{USMAX - USC}{PV \cdot E}$

where E = average daily potential evapotranspiration.

Also, $USC = \dfrac{E \cdot USMAX}{EVPMAX}$.

$$\therefore \quad n_c = \frac{USMAX - E \cdot USMAX/EVPMAX}{PV \cdot E}$$

$$= \frac{USMAX\ (EVPMAX - E)}{PV \cdot EVPMAX \cdot E}$$

Substituting for USC and m in equation 8.6,

$$US = \frac{E \cdot USMAX}{EVPMAX} \cdot e^{-(n - \frac{USMAX\ (EVPMAX - E)}{PV \cdot EVPMAX \cdot E}) \cdot \frac{PV \cdot EVPMAX}{USMAX}} \tag{8.7}$$

Thus, given the parameters PV and EVPMAX and finding n and E from the data record, it should be possible to substitute the above expression for US into equation 8.5,

$$USMAX = P - Q_o + US,$$

and solve for USMAX. This will only be valid when it is known that US < USC and, for the preceding event, $Q_o > 0$.

(ii) Events for which US < USC

In a similar approach to that which was used for the Interception Store it can be shown that, if for a given event, $Q_o > 0$ and $P - Q_o < ET$, where ET is the total potential evapotranspiration loss since the previous rainfall (or since the start of the data record), then US, the contents immediately prior to the event, must have been less than USC.

Let US_o = contents after previous rainfall

US_1 = contents immediately prior to event

US_1 is related to US_o as follows:

$$US_1 = US_o - ET \quad \text{if } US_o > USC \text{ and } ET \leq US_o - USC \qquad (8.8a)$$

$$US_1 < USC \quad \text{otherwise} \qquad (8.8b)$$

For an event satisfying the conditions that $Q_o > 0$ and $P - Q_o <$ ET, equation (8.8a) can be eliminated in the same way as was done for the Interception Store. Therefore, under these conditions, $US_1 <$ USC and the data from those events which satisfy these conditions may be used to solve for USMAX.

Summarising from (i) and (ii) above, USMAX may be found using the data from those events for which

(a) $Q_o > 0$ and $P - Q_o <$ ET and

(b) $Q_o > 0$ for the preceding event.

For each such event, an estimate of USMAX is obtained by writing down equation (8.5), substituting for US from equation (8.7), and solving for USMAX.

The task of expressing the output from this store as a function of the input and the capacity, setting up an objective function involving the observed and calculated outputs, and graphing this function against the estimated value of USMAX has not been carried out for this store.

8.1.4 Drainage Store

This component of the Boughton model operates in a similar way to the Interception Store except that its contents are depleted by infiltration into the Lower Soil Store instead of by evaporation. As the potential infiltration rate is a function of the contents of the Lower Soil Store, the operation of that store governs the operation of the Drainage Store. However, if a record of potential infiltration rates, inflow quantities P, and overflows Q_o were available, the analysis for this store would be the same as for the Interception Store and the conclusions reached for that store would all apply. In particular, the objective function would contain discontinuities and parabolic segments. The investigation of the DSMAX-SSMAX parameter combination, described in sub-section 7.2.2, part (iv), provided numerical support for this conclusion, and this was illustrated in Fig. 7.21.

8.1.5 Lower Soil Store

This store is more complex in operation than the other stores. The inflows and outflows are as shown in Fig. 8.7.

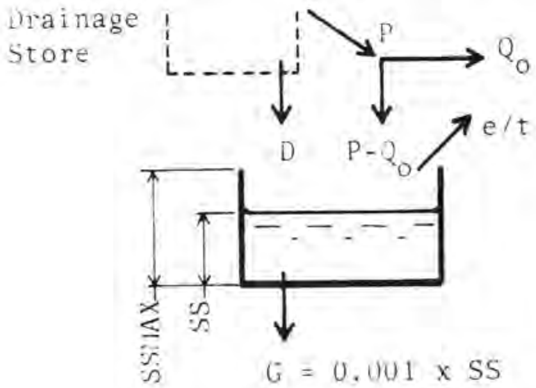


FIGURE 8.7
OPERATION OF THE
LOWER SOIL STORE

The inflows to the store are restricted by a function of the current contents in such a way that the store can never overflow. (The infiltration function was described in sub-section 6.2.) The "overflow" for this store, then, is that portion of P which cannot enter the store, and there is always a positive Q_o for every event, P, regardless of the current store contents.

Once again, the capacity of the store was sought from a record of the inflows P, outflows Q_o, and potential evapotranspiration data. The flows D and G were ignored in order to simplify the problem initially. It should be possible to modify the basic solution in order to allow for these flows.

For each event, Q_o is related to P by equation (3.2),

$$Q_o = P - F \tanh\left(\frac{P}{F}\right),$$

where $F = F(\text{SSMAX, SS, FO, KF})$ as defined by equation (6.19).

As P and Q_o are known for each event, F for each event may be found from equation (3.2). Given the parameters FO and KF, the equation (6.19) may be written for each event, the unknowns in each case being SSMAX and the contents prior to the event, SS. Thus, for any two consecutive events, two equations may be written in three unknowns, the contents prior to the first and second events, SS_1 and SS_2, and the capacity of the store, SSMAX. A third equation is

$$SS_2 = SS_1 + (P - Q_o)_1 - \text{evap. loss between the events.}$$

This equation cannot be written explicitly as the evapotranspiration calculations depend on whether soil moisture is restricting the evapotranspiration loss. However, the three equations can be solved in an iterative way as follows:-

(i) estimate a value of SSMAX.

(ii) substitute in equation (6.19) and derive corresponding values of SS_1 and SS_2.

(iii) starting from SS_1, perform the normal model evapotranspiration calculations and find SS' prior to the second event.

(iv) if the estimate of SSMAX is correct, SS' and SS_2 should be equal.

(v) adjust estimate of SSMAX and repeat from (ii).

(vi) continue iterations until the SSMAX is found which gives $SS' = SS_2$.

Allowance could be made for the D and G flows by incorporating them into the calculations at (iii) above.

Each group of two consecutive events in the data record could be used to estimate the value of SSMAX in the way outlined above. When a particular event is used as the second event in one group and also as the first event in the next group, the value of SS_2 (first group) should equal SS_1 (second group). However, due to data errors, this would probably not occur. Further work is required to find a method for obtaining the best fit estimate of SSMAX which preserves consistency between SS values.

8.2 COMBINATION OF THE INDIVIDUAL STORES

In the above analyses of the stores of the Boughton model it was assumed that the inflows and outflows for each store were known. When the stores are considered in combination as the Boughton model, only the inflow to the Interception Store and the "outflow" from the Lower Soil Store, i.e., that portion of the outflow from the Drainage Store which does not enter the Lower Soil Store, are known. It is also known that the inflow to a particular store equals the outflow from the next store above. Further work in which the top two stores are analysed in combination to express the output as a function of the two capacities, followed by consideration of the top three stores, and so on, is required.

8.3 AN ELEMENTARY EQUATION FOR THE OUTPUT FROM THE MODEL

It is possible to write an equation for the output of the Boughton model on a day when it is known from the data that sufficient rainfall has occurred to make the Drainage Store overflow and therefore to produce runoff from the model. In general the output from any of the stores except the Lower Soil Store is given by

$$Q_C = P - (C - S) \qquad \text{if} \qquad P > C - S \tag{8.9a}$$

$$= 0 \qquad \text{if} \qquad P \le C - S \tag{8.9b}$$

If the rainfall was insufficient to cause runoff, it is not known which alternative applies for any particular store. However, if runoff was produced, then all stores must have overflowed and equation (8.9a) applies in each case.

The output is given by equation (3.2),

$$Q = P_e - F \tanh\left(\frac{P_e}{F}\right),$$

where P_e is the outflow from the Drainage Store

and $F = F(\text{SSMAX, SS, FO, KF})$, as defined by equation (6.19).

For the Interception Store, outflow = P - (VSMAX - VS) and this outflow is the inflow to the Upper Soil Store.

∴ For the Upper Soil Store, outflow = P - (VSMAX - VS) - (USMAX - US).

Similarly, the outflow from the Drainage Store is

$$P_e = P - (\text{VSMAX} - \text{VS}) - (\text{USMAX} - \text{US}) - (\text{DSMAX} - \text{DS}).$$

Substituting in equation (3.2),

$$Q = P - (\text{VSMAX} - \text{VS}) - (\text{USMAX} - \text{US}) - (\text{DSMAX} - \text{DS}) - F \tanh\left(\frac{P - (\text{VSMAX} - \text{VS}) - (\text{USMAX} - \text{US}) - (\text{DSMAX} - \text{DS})}{F}\right) \tag{8.10}$$

This equation is valid for each day when the rainfall is sufficient to overflow the Drainage Store and produce runoff. If it could be evaluated for these days, the monthly modelled runoff quantities could be found simply as the sum of the runoff values on those days, thereby eliminating the present requirement to operate the model for every day in the period of record.

However, before the equation can be evaluated, it is necessary to express the store contents, VS, US, DS and SS (SS appears in the function for F) in terms of the store capacities and the other model parameters. It should be possible to do this using the evaporation data and the number of days since the previous runoff-producing rainfall, when the three higher stores would have been filled. Thus, for the Interception Store,

$$VS = VSMAX - nE \quad \text{if} \quad nE < VSMAX \tag{8.11a}$$

$$= 0 \quad \text{if} \quad nE \geq VSMAX \tag{8.11b}$$

where n = number of days since the store was filled
E = average daily evaporation.

When evaluating Q for a given day and an estimated value of VSMAX, equation (8.11a) would be adopted if nE (known from the data) was less than the estimated value of VSMAX. The second term of the equation for Q, equation (8.10), would therefore be

$$VSMAX - VS = VSMAX - (VSMAX - nE) = nE.$$

Otherwise this term would be

$$VSMAX - VS = VSMAX - 0 = VSMAX.$$

If equation (8.11a) were used for the Interception Store, then it is known that the Upper Soil Store is still full, i.e., US = USMAX.

(Evapotranspiration does not commence until the Interception Store has been emptied.) In this case, the third term in equation (8.10) would be

$$USMAX - US = USMAX - USMAX = 0.$$

Otherwise, when equation (8.11b) was used for the Interception Store, two possible alternative expressions for US are

$$US = USMAX - (n - n_i) \cdot PV \cdot E$$

$$\text{if} \quad (n - n_i) \cdot PV \cdot E < USMAX - USC$$

$$= USC \cdot e^{-(n - n_i - n_j) \cdot \frac{PV \cdot EVPMAX}{USMAX}}$$

$$\text{if} \quad (n - n_i) \cdot PV \cdot E \geq USMAX - USC$$

where n_i = time taken to empty the Interception Store

$= \dfrac{VSMAX}{E}$,

n_j = time taken to empty the Upper Soil Store down to USC

$= (USMAX - USC)/(PV \cdot E)$, and

$USC = E \cdot USMAX/EVPMAX.$

It is thus seen that there are two alternatives for the second term of equation (8.10), depending on the estimated value of VSMAX, and three alternatives for the third term, depending on the estimated values of VSMAX, USMAX, PV and EVPMAX. The task of writing down similar expressions for DS and SS would be more complex than for VS and US. However if this were accomplished it seems that the task of finding an equation for the output of the model in terms of the data and the model parameters would be substantially complete. For a given set of parameter values the equations for the daily Q values could be built up by including the appropriate terms from the available alternatives according to the simple comparative tests described above. The equations could then be evaluated and added together to give monthly total modelled outflow or event outflow as required. As only simple numerical comparisons are required when deciding which of the alternative terms will be included in the equations, both the writing and the evaluation of the equations could be performed by computer. It should also be possible to build up equations for the partial derivatives of the model output with respect to each parameter at the same time.

The use of such equations to find the output from the model is not simply an alternative to the present method of performing the daily model calculations through the entire length of the data record. The evaluation of the objective function for a given set of parameter values would be achieved simply by selecting the appropriate terms for the equations and substituting the parameter values into the equations. This should be faster than the present method.

8.4 SIGNIFICANT FINDINGS FROM THE ANALYSIS

While a general equation for the output of the Boughton model has not been derived, the procedure suggested above could enable a specific equation to be built up for a given set of data on a given catchment with a given set of parameter values. It is felt that further work along these lines would be very fruitful in the insight it might give to the solution of the optimising problem. Some reasons for this opinion were given at the start of this section. Three further points of interest arising from the above work are summarised below.

(i) As the parameter values change during optimisation, they will undoubtedly cross the values at which the terms to be included in the equations for the daily Q values change to an alternative. Discontinuities in the objective function would be expected at these values, and the descent direction should be re-defined when a discontinuity is crossed. The use of the procedures outlined above would enable this to be done.

(ii) The output from the model is a sum of equations which all have the same form as equation (8.10) above. This equation may be re-arranged as follows:-

$$Q = P - (VSMAX + USMAX + DSMAX) + (VS + US + DS) - F \tanh\left(\frac{P - (VSMAX + USMAX + DSMAX) + (VS + US + DS)}{F}\right)$$

If the store contents, VS, US and DS, were only weakly dependent on the parameters, or if they were usually near zero prior to most of the events, then strong interdependence between VSMAX, USMAX and DSMAX would be expected. This prompted some of the numerical investigations described in sub-section 7.2.2. Other inter-dependencies may be revealed when the equation is written out in full.

(iii) In writing the expressions for VS, US and DS in the equations for Q, it is necessary to make use of the fact that the relevant store was full immediately after the previous event. As the store contents are not known at the start of the period of record, the equation for Q on the first runoff day cannot be written. However the equation could be written for the next runoff day, and would be quite independent of any starting values assumed for VS, US and DS. Therefore, the length of the "warm-up" period, as far as the upper three stores of the model are concerned, need only be long enough to exclude the first runoff day from the objective function and the initial values used for the contents of these stores have no effect on the subsequent operation of the model. For the Lower Soil Store however, it appears that SS must be expressed in terms of the initial value until such time as this store approaches the full state as closely as possible. This state can be identified when the flows D and G (see Fig. 8.7) become equal. This can usually only be achieved in a period of prolonged rainfall, and such a period may not occur for some years after the start of the data record, if at all. In such cases it may be necessary to make the initial value of SS a parameter of the model and select the length of the "warm-up" period to suit the other stores. However, if a prolonged period of rainfall (sufficient to fill the Lower Soil Store until equilibrium of the D and G flows were achieved) was present near the start of the data, the subsequent operation of the Lower Soil Store would be independent of the assumed initial contents and the "warm-up" period could be selected to exclude this first wet period from the objective function.

9. COMMENTS ON CHOOSING AND EVALUATING THE OBJECTIVE FUNCTION

When attempting to model the yield of a catchment the deviations between the total observed and total calculated runoff quantities within certain time periods provide an appropriate numerical measure of the performance of the model. Choosing the objective function involves making decisions on (a) the time periods within which the deviations will be calculated and (b) the way in which the deviations will be manipulated before being summed into a single numerical index. This section contains comments related to both these decisions. In addition the effect of transforming the total calculated and observed runoff quantities before computing the deviations is discussed.

9.1 THE TIME PERIODS USED IN THE OBJECTIVE FUNCTION

The deviations may be calculated within fixed time periods such as days or months. Alternatively, variable periods which encompass the individual rainfall-runoff events may be used.

9.1.1 Fixed Time Periods

When fixed time periods are used errors may be introduced because of the time lag between the generation of runoff on the catchment and its measurement at the outlet. Most models calculate the amount of runoff at the time when it is generated and many do not reproduce the travel time to the catchment outlet.

For example, the Boughton model calculates runoff in lumped quantities on days of rainfall only, and no routing procedure is used to distribute the calculated runoff and reproduce the shape of the observed runoff hydrograph. This must be allowed for when using a fixed time period such as a month in the objective function to avoid errors caused by events such as that depicted in Fig. 9.1.

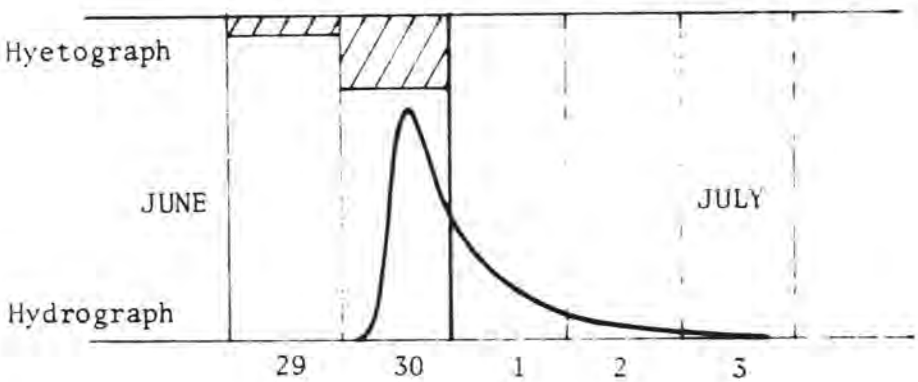


FIGURE 9.1 EVENT OVERLAPPING TWO FIXED TIME PERIODS.

As there is no modelled runoff in July from this event, the observed runoff on 1, 2 and 3 July forms a constant component in the objective function which cannot be affected in any way by changes in the parameter values. Optimisation will attempt to adjust the parameters in such a way that the model is forced to reproduce only the small amount of runoff which occurred on 30 June. This unsatisfactory result can be avoided simply by regarding the runoff in July as having occurred in June when evaluating the objective function. Thus, in the runoff data supplied to the optimising programme, the figure for 30 June should be increased by the sum of the quantities on 1, 2 and 3 July and the figure for these days given as zero.

This measure was adopted for the January-February 1971 data for Lidsdale No. 2 catchment, where an event occurred late in January followed by a second event on 1 February. The two hydrographs were separated and the February runoff from the January event was re-allocated to the data for 31 January. Mr. M. K. Smith of the Forestry Commission pointed out the need for this adjustment.

Errors can be introduced when a short time period is used for the objective function. For the Boughton model, operated with a daily time period, it is tempting to use the daily deviations between observed and calculated runoff in the objective function. However, the result will be as shown in Fig. 9.2.

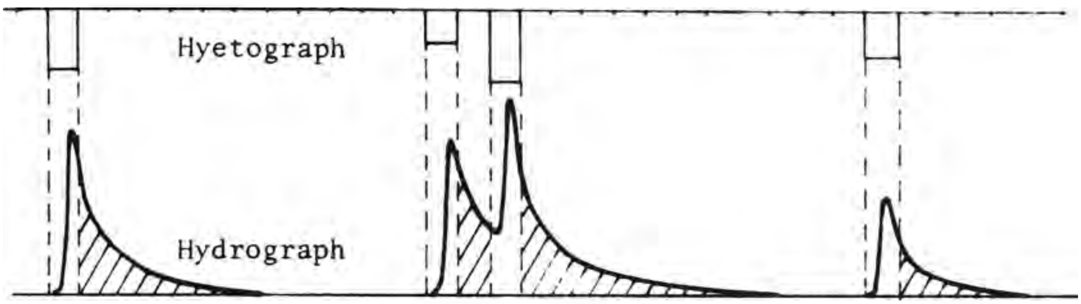


FIGURE 9.2 EFFECT OF USING SHORT TIME PERIODS.

The model will produce no calculated runoff at the times indicated by the shaded areas of the observed runoff hydrograph. The daily runoff amounts in these areas will therefore form a constant base figure in the objective function which is unaffected by changes in the parameters. Optimisation will attempt to minimise the deviations between the modelled runoff and the unshaded areas of the observed runoff hydrograph, and the model will then be forced to reproduce a smaller amount of runoff than the amount actually produced by the catchment. In models which use a much shorter time period than one day for calculations, a similar effect will probably still be obtained if the same time period is used for evaluating the objective function.

For all models, then, it appears that the time period used for evaluating the objective function should be substantially greater than that used in the model calculations and, where an event overlaps two time periods, that judicious re-allocation of the observed runoff amounts between the two time periods should be made so that the model is not forced to produce from a given amount of rainfall a runoff quantity which is different from that produced by the catchment.

9.1.2 Variable Time Periods (Events)

The time periods may be selected to encompass the individual rainfall-runoff events. Intuitively, an objective function which is based on the deviations between observed and calculated total runoff quantities for each runoff event is preferred to a function which uses the deviations between the totals for fixed time periods such as a month. This is because several events may occur within the month and the individual positive and negative deviations for these events may cancel out when the monthly deviation is calculated. However, some results obtained in this project indicate that the differences between the sets of parameter values which optimise the two different objective functions will be small.

For catchments which rarely have more than one event in any month the two objective functions will be practically the same. This was so for Middle Ck. (Site 2) at Pokolbin, as described in Section 5. Under these circumstances the optimum parameter values for the two objective functions would be expected to be very similar.

For the Lidsdale No. 2 catchment 10 of the 28 months of data contain more than one event. When the two-parameter response surfaces shown in Figs. 7.13, 7.15, 7.16 and 7.20 (for the objective function based on monthly deviations) were prepared for this catchment, as described in sub-section 7.2.2, the objective function based on event deviations was also evaluated at each node of the grids of parameter values to enable the response surfaces for this function to be plotted if required. In addition to the grids of FO-KF, VSMAX-USMAX, USMAX-PV and DSMAX-SSMAX combinations evaluated to plot the above surfaces, the two objective functions were also evaluated on a grid of EVPMAX-VSMAX parameter values. For all of the above parameter combinations, comparison of the values of the two objective functions revealed that the two response surfaces were of the same shape. The position of the valley floor for event deviations was almost always within one grid spacing of the valley floor for monthly deviations and never more than two grid spacings away. The grid spacings were of the order of 5 to 10 percent of the parameter values at the minimum points of the surfaces.

It appears that the model cannot be forced into better reproduction of the observed runoff quantities simply by adopting event deviations instead of monthly deviations in the objective function. Where there are compensating errors in the calculated runoff quantities for events in the same month the numerical value of the objective function based on event deviations will simply be higher than for the function based on monthly deviations, but the minimum value of each function will occur at about the same parameter values. It is probable that this conclusion would also be true for other models, including those which use a shorter time period than one day for calculations.

There are some practical difficulties in using event deviations in the objective function. The start and end of an event may be defined in a number of ways and extra programming is required to evaluate the deviations over events. Because of these difficulties and the findings described above there does not appear to be an advantage in using event deviations rather than fixed time period deviations in the objective function.

9.2 FORMING THE OBJECTIVE FUNCTION FROM THE DEVIATIONS

The deviations, or functions of the deviations, are usually added together according to the following general equation:-

$$F = \sum_{i=1}^{k} \left| Q_{o_i} - Q_{c_i} \right|^j$$

where Q_{o_i} = observed total runoff in time period i,

Q_{c_i} = calculated " " " " " ",

k = number of time periods in the data record,

and the vertical lines signify the absolute value of the enclosed expression.

Different objective functions and response surfaces are obtained by selecting different values for the exponent, j. It is commonly asserted that if optimisation is performed for an objective function with j = 2 (i.e., the objective function consists of the sum of squares of the deviations) or with some higher value, then the optimum parameter values will bias the model output to give good reproduction of the large events and poor reproduction of the small events. Conversely, if j = ½ it is asserted that the optimum parameter values will result in poor reproduction of the large events and good reproduction of the small events. These assertions imply that there will be significant differences between the parameter values which optimise the different objective functions.

In sub-sections 8.1.1 and 8.1.2 the optimum parameter values for one event in a simple store and for a number of events in the

Interception Store were derived analytically using objective functions in which j = 2. If these analyses are repeated using objective functions in which (a) j = 1, and (b) j = ½, then the same answers will be obtained for the optimum parameter values as were obtained with j = 2. Figs 8.2 and 8.6 from the above sub-sections are reproduced here in Figs. 9.3 and 9.4 with the response surfaces for the other objective functions also shown.

Assuming that the features of the response surfaces shown in these figures will also be present in the response surfaces for more complicated models, the figures show that:-

(i) Changing the exponent j in the objective function merely changes the vertical "scaling" of the response surface without altering the position of the minimum point. This provides substantial evidence that the optimum parameter values and, consequently, the reproduction by the model of the large and small events, are independent of the particular function of the deviations which is minimised.

(ii) The use of j = ½ (and probably any value between zero and one) gives rise to a much more difficult response surface on which to locate the minimum than when j is greater than one. The surface is very unsatisfactory because

- (a) it is relatively flat,
- (b) it may have unwanted secondary minima at each discontinuity, and
- (c) for satisfactory operation of the descent methods, the surface should be concave upwards in the area of the global minimum and the function and its partial derivatives should be continuous at the global minimum. This surface does not meet these requirements.

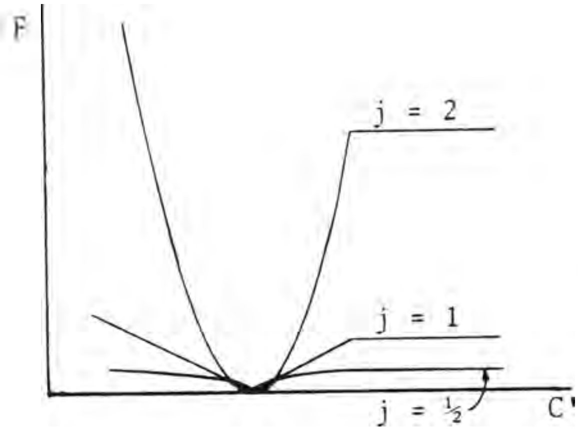
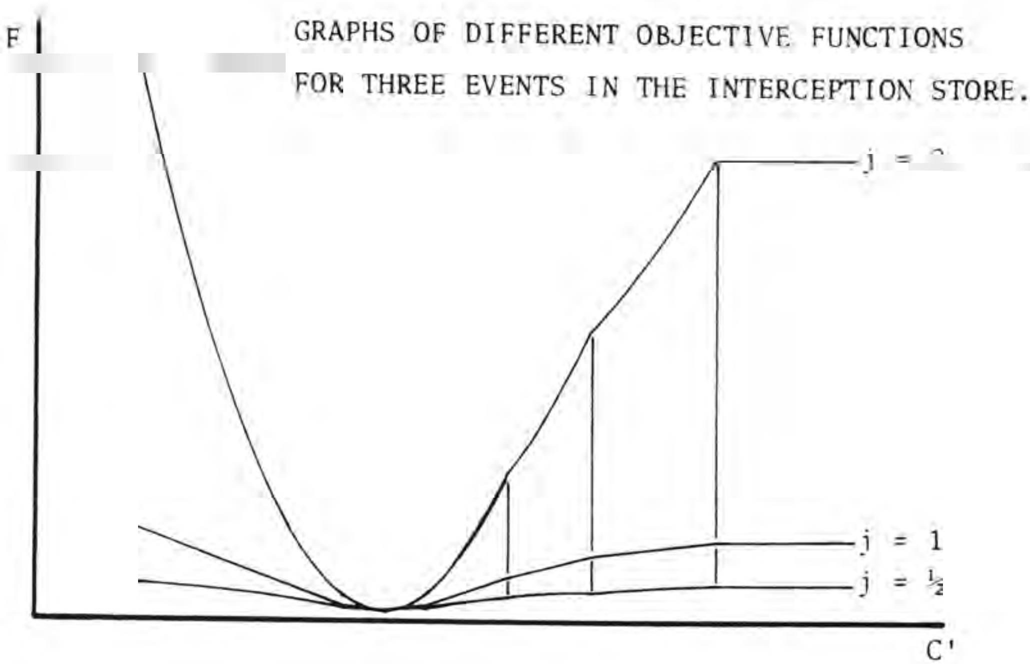


FIGURE 9.3.

GRAPHS OF DIFFERENT OBJECTIVE FUNCTIONS FOR SIMPLE STORE OF FIG. 8.1.

FIGURE 9.4.

GRAPHS OF DIFFERENT OBJECTIVE FUNCTIONS FOR THREE EVENTS IN THE INTERCEPTION STORE.



The conclusions above must be qualified by pointing out that, for the two stores analysed, the output was a linear function of the parameter value. The conclusions may not be valid where the model output is a non-linear function of the parameter, and may thus not apply to, say, the infiltration parameters. However it appears that an objective function consisting of the sum of squares of the deviations is as good as or better than other functions.

9.3 TRANSFORMING THE OBSERVED AND CALCULATED FLOWS

Different objective functions may be formed by transforming the observed and calculated total flows within each time period before calculating the deviations. For example, the objective function could be made up from the deviations between, say, the squares of the observed and calculated flows, or the logs of these flows. Using the sum of squares of these deviations, the general equation for the objective function would be:-

$$F = \sum_{i=1}^{k} \left({Q_o}^j_{(i)} - {Q_c}^j_{(i)} \right)^2$$

where ${Q_o}_{(i)}$ = observed total runoff in time period i,

${Q_c}_{(i)}$ = calculated total runoff in time period i, and

k = number of time periods in the data record.

Some indication of the effect of using such an objective function may be obtained by repeating the analysis in sub-section 8.1.2 for a number of events in the Interception Store.

In the area of the minimum of the objective function,

$$Q_{c_{(i)}} = P_i - C'$$

where P_i = observed inflow in time period i, and

C' = parameter representing theccapacity of the store, and with optimum value of C.

$$\therefore \quad F = \sum_{i=1}^{k} (Q_{o_{(i)}}^{\;j} - (P_{(i)} - C')^j)^2$$

Differentiating with respect to C' :-

$$\frac{dF}{dC'} = \sum_{i=1}^{k} 2(Q_{o_{(i)}}^{\;j} - (P_{(i)} - C')^j) \cdot (-j)(P_{(i)} - C')^{j-1} \cdot (-1)$$

$$= \sum_{i=1}^{k} 2j(Q_{o_{(i)}}^{\;j} - (P_{(i)} - C')^j) \cdot (P_{(i)} - C')^{j-1}.$$

The minimum value of the objective function F occurs at the value of C' which makes the above expression equal to zero. It seems reasonable to expect that this value of C' will be a function of j. Therefore the optimum value of the parameter depends on the particular transformation which is applied to the flows before calculating the deviations.

This conclusion was checked numerically using synthetic data for three events. These data contained errors so that a single value of C' could not satisfy the equation $Q = P - C'$ for each event and a best fit estimate of C' was required. The data are shown in Table 9.1, which also shows the values of C which would satisfy the individual events.

TABLE 9.1

SYNTHETIC TEST DATA

EVENT	INFLOW P	OUTFLOW Q	VALUE OF C CONSISTENT WITH P & Q DATA
1	15	5	10
2	91	79	12
3	39	31	8

The best fit estimate of C would be expected to lie between 8 and 12 regardless of the chosen objective function. Three different objective functions were evaluated and graphed for values of C between 8 and 12. The functions were:-

$$F1 = \sum_{i=1}^{3} (Q_i^{\frac{1}{2}} - (P_i - C)^{\frac{1}{2}})^2$$

$$F2 = \sum_{i=1}^{3} (Q_i - (P_i - C))^2$$

$$F3 = \sum_{i=1}^{3} (Q_i^2 - (P_i - C)^2)^2$$

The graphs of these functions are shown in Fig. 9.5. They confirm the conclusion reached above that the optimum value of the parameter depends on the transformation applied to the observed and calculated flows. Furthermore, it appears that if the flows are squared the optimum parameter value will favour the reproduction of the

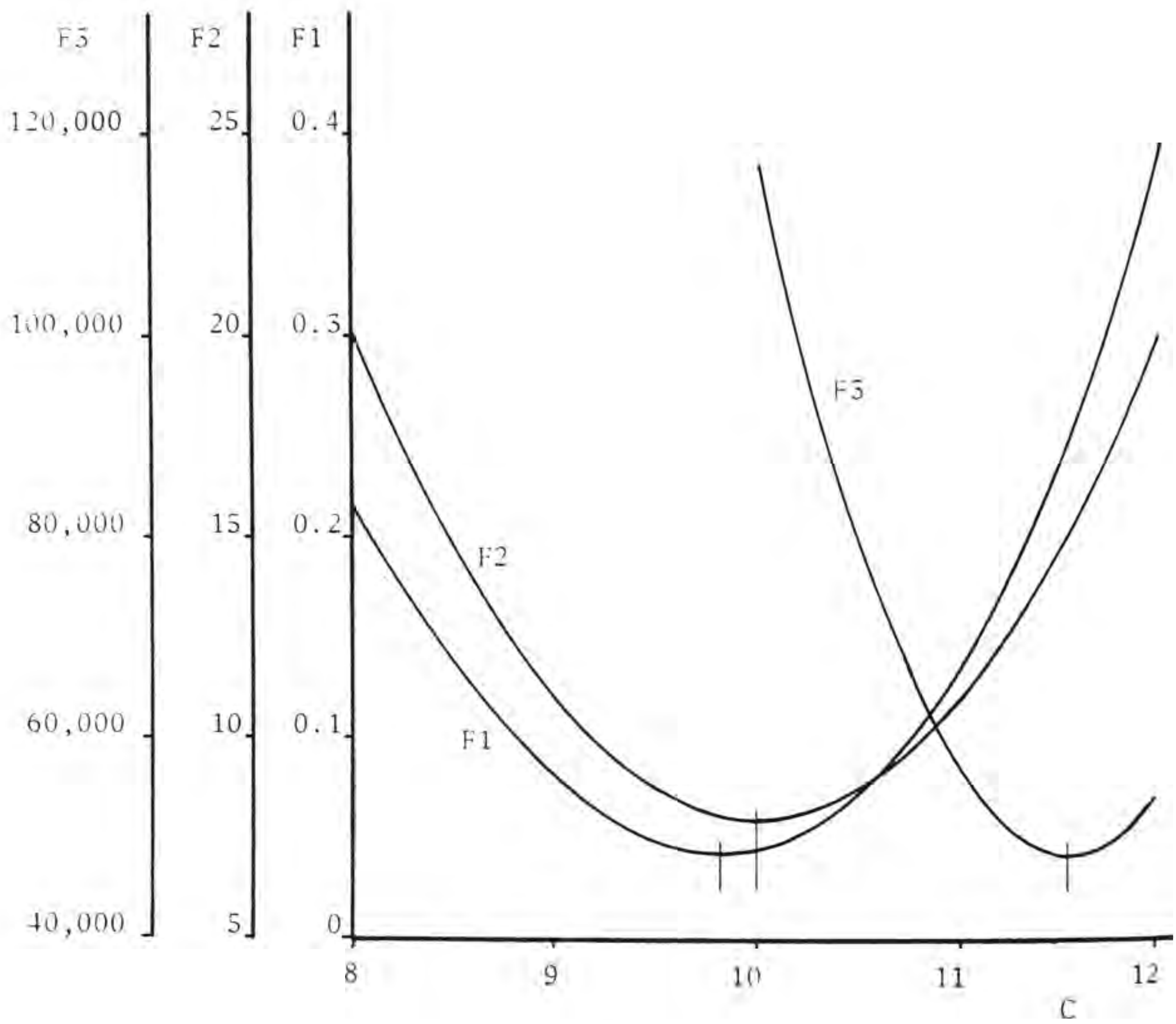


FIGURE 9.5
GRAPHS OF THREE OBJECTIVE FUNCTIONS
USING TRANSFORMED FLOW VALUES

large events while the reproduction of the small events will be favoured if a square root transformation is applied.

In arriving at the above conclusions it was only necessary to assume errors in the input and output data. Therefore it is probable that the conclusions would apply for both "good" and "poor" rainfall-runoff models whenever transformations of the observed and calculated flows are used in forming the objective function.

9.4 THE OBJECTIVE FUNCTION USED IN THIS PROJECT

The objective function chosen for use in this project was the sum of squares of the deviations between the monthly observed and calculated runoff amounts. It was thought that some optimisation should be attempted using other objective functions but there was not sufficient time for this. The studies described in this section indicate that optimisation with other objective functions would not have been easier. It appears that there would have been no advantage in using much shorter time periods such as days for calculating the deviations. If the square roots of the deviations had been used instead of the squares, the response surface would have been a more difficult one on which to locate the global minimum.

No transformation was applied to the flows before calculating the deviations. It appears that transformations may be used to produce optimum parameter values which bias the model output towards better reproduction of some events. The significance of such parameter values should be the subject of further investigation.

10. SUMMARY, RECOMMENDATIONS AND CONCLUSION

The main aims of the project were:-

(i) to select an appropriate model of the rainfall-runoff process,

(ii) to obtain optimum values for the parameters of the selected model for about 80% of the catchments in A.W.R.C. Research Project 68/1 for which records were available,

(iii) to correlate the optimum parameter values with measurable catchment characteristics, and

(iv) to test these relationships on the remaining catchments.

The Boughton model was selected for use in the project as it was already well known and its use of daily data makes it widely applicable. The Steepest Descent, Simplex and Davidon methods were used to search for the optimum values of the parameters of this model for the four Pokolbin catchments and the Lidsdale No. 2 catchment. Many difficulties were encountered in these searches and an optimum set of parameter values was not found for any of the catchments. Consequently no work under (iii) and (iv) above was possible.

Most of the project time was spent in examining the difficulties encountered when searching for the optimum model parameter values, and a useful understanding of many of these problems has been achieved.

Examination of previously published work on rainfall-runoff models indicates that similar problems have been encountered in these studies and that optimum parameter values have not generally been found. It is therefore believed that the insights achieved in this project have wide applicability.

Consequently, although the aims of the project were not achieved, the findings should be valuable in future studies aimed at the determination of optimum parameter values for rainfall-runoff models which use daily data and also for those which use data at shorter time intervals. The insights obtained and actions taken with respect to some features of the model and of the optimisation problem are summarised in the following sub-sections.

10.1 FEATURES OF THE MODEL

Most of the work discussed in this sub-section would not be peculiar to the Boughton model. Many rainfall-runoff models are built up with components and functions similar to those used in the Boughton model. Some of the difficulties described below could therefore be expected when working with other models.

(i) Evapotranspiration and Infiltration Functions

In the original model calculations the daily amounts for these processes were determined by functions of the relevant store contents at the start of the day. These calculations occasionally gave unrealistic results such as "over-filling" and "over-emptying" of some stores. Also, it was thought that the infiltration amount should be related to the capacity of the relevant store as well as to the contents. The infiltration function was changed to accomplish this and then both functions were regarded as giving instantaneous values for the processes, the daily amounts being found by integration. The changes eliminated the unrealistic results and also assisted the algebraic analysis of the operation of the model.

(ii) Initial Store Contents and "Warm-up" Period

The operation of the model was found to be independent of the assumed initial contents of the moisture stores after the first time when they overflowed. For the Interception, Upper Soil and Drainage Stores this occurs during the first rainfall-runoff event where the calculated runoff by the model is not zero. For these three stores, then, the "warm-up" period need only encompass the first runoff-producing event, regardless of the assumed initial store contents. The Lower Soil Store only fills during prolonged rainfall. If such a period occurs near the start of the data record then the "warm-up" period should encompass this event. However, if the Lower Soil Store does not approach the full state for some years, it appears that the "warm-up" period should be selected for the upper three stores of the model and that the initial contents of the Lower Soil Store should be regarded as another parameter.

(iii) Lower Soil Store Depletion Factor

A constant value of 0.999 was adopted for the factor throughout the work of this project. This value was used by Boughton and other workers. Application of the factor removes a relatively large amount of water from the model over the period of the data record. Therefore, small changes to the value of the factor could be expected to have a marked influence on the amount of runoff calculated by the model and the depletion factor should be regarded as another model parameter to be optimised.

(iv) Inter-action between Parameters and Indifference to Parameters

Inter-action between parameters produces long, flat-bottomed valleys in the response surface, and indifference to parameters creates relatively flat areas on the response surface. Optimisation of such parameters is difficult because of the insensitivity of the objective function to large changes in the parameter values. In this project the algebraic analysis of the model revealed the possibility of inter-action between the parameters VSMAX, USMAX and DSMAX, while numerical studies confirmed this and revealed inter-actions between VSMAX, USMAX and PV, and between VSMAX, USMAX and EVPMAX. Numerical studies also revealed indifference of the objective function to a wide range of combinations of FO and KF values. The sets of parameter values obtained from the work with Lidsdale No. 2 catchment indicate that inter-action between EVPMAX and some other parameters, probably DSMAX and SSMAX, is present. Thus it appears that almost all of the parameters of the Boughton model take part in inter-actions and that it would be difficult to find the optimum values for the parameters.

(v) Unexpected Parameter Values

For the Lidsdale No. 2 catchment the near-optimum values obtained for some of the parameters indicated that they were not operat-ing according to their intended function in the model. The values for VSMAX and DSMAX were higher than expected while that for USMAX was very low. The inconsistent values for USMAX and DSMAX throw doubt on the value for PV. Possible reasons for the unexpected values are inter-action between parameters, the use of a short data record for optimis-ation, and the use of a fixed value for the Lower Soil Store depletion factor. If such unexpected and inconsistent optimum parameter values were also obtained for other catchments it is most unlikely that the parameters could be correlated with measurable catchment characteristics. It may also then be necessary to review the structure of the model.

Item (iii) above is specifically related to the Boughton model, but the other items could be relevant for other models.

10.2 FEATURES OF THE OPTIMISATION PROBLEM

Findings obtained from the project on these features may be further sub-divided into those related to the objective function, to the nature of the response surface, and to the implementation of the optimising methods.

10.2.1 Objective Function

(i) Time Periods

There appears to be little advantage in using variable time periods which encompass individual rainfall-runoff events rather than fixed time periods such as months when calculating the deviations between the observed and modelled runoff volumes. Results obtained in this project indicated that there would only be small differences between the optimum parameter values for objective functions based on these different types of time periods. This may only be true for rainfall-runoff regimes similar to that of eastern New South Wales. The time period used in the objective function should be considerably longer than that used in the model calculations. For models which do not contain a runoff-routing procedure, runoff which was caused by rainfall in a preceding time period should be regarded as having occurred in that time period when calculating the deviations between the observed and modelled runoff.

(ii) The Form of the Objective Function

The objective function is usually computed as the sum of the absolute values of the deviations raised to some power. It appears that the optimum parameter values do not depend on the power to which the deviations are raised, i.e.,the minimum values of the objective functions formed by using different powers all occur at the same parameter values. However, if the power is 0.5 (i.e. the objective function consists of the sum of square roots of the absolute values of the deviations), and probably any value between zero and one, then the response surface will be a difficult one on which to locate the global minimum as it will be relatively flat and may contain a larger number of secondary minima. It is preferable to use a power greater than one, and an objective function consisting of the sum of squares of the deviations appears to be as good as any other.

It is possible to transform the observed and modelled runoff volumes before calculating the deviations. The optimum parameter values will then be a function of the transformation. If the flows are raised to a power greater than one then the optimum parameter values will give better reproduction of the large events, while use of a power between zero and one will tend to favour the small events.

The remarks above apply for both "good" and "poor" models of the rainfall-runoff process, as they are only based on the assumption that there will be errors in the rainfall and runoff data.

10.2.2 The Nature of the Response Surface

(i) Discontinuities

Theoretical and numerical results obtained from the work indicate that discontinuities are present in the response surface for the Boughton model and are probably also present in the response surfaces for other models. Discontinuities have an adverse effect on the performance of the descent methods of optimisation. Errors occur in the definition of the steepest descent direction from points which lie close to a discontinuity. In the conjugate gradient methods information about the response surface is gained at each iteration and this is used in defining the next search direction. When a discontinuity is crossed the surface changes abruptly, but the next search direction is still defined using the information gained about the old surface. The direct search optimisation methods, such as the Simplex method, do not appear to be adversely affected by the presence of discontinuities in the response surface.

(ii) Flat-bottomed Valleys - Scaling the Parameters

As mentioned previously, inter-dependence between the model parameters creates long, flat-bottomed valleys in the response surface which present difficulties to all optimisation methods. Large differences in the magnitudes of the parameter values also create a difficult response surface. Scaling of the parameters, i.e., optimising a transformed problem in which some of the parameters have been multiplied by chosen factors, may be used to obtain a surface in which the elongation of the contours has been reduced. However, the most satisfactory scaling for a particular problem is not obvious, and it may be necessary to experiment with the scaling during the course of the optimisation.

10.2.3 The Optimising Methods

(i) Apparent Optimum Points

Many times when using the Simplex method in this project, the simplex of points on the response surface contracted until the points were almost coincident. This is normally interpreted as convergence to a local minimum point on the response surface. Similarly, the Descent methods often reached a state where practically no movement was made in the descent direction at each iteration and it appeared that a minimum had been approached. Further movement from such points was then found to be possible using a different optimising method,by numerical studies, or by sub-optimisation with only some of the parameters varying. Points obtained after apparent convergence of the optimising methods should not therefore be regarded as optima until attempts to further the optimisation have been made by the above procedures.

(ii) Defining the Steepest Descent Direction at a point on the Response Surface

At each iteration of most of the Descent methods of optimisation, the steepest descent direction must be found from the current point on the response surface. This is done by finding the partial derivatives of the objective function with respect to each of the parameters. When there are no explicit equations for the partial derivatives in terms of the model parameters, they must be found by numerical methods. This was a troublesome aspect of the work of this project. Five different methods were used and it is thought that Method 5, which was used for much of the work with Lidsdale No. 2 catchment, is the only one which would define the steepest descent direction accurately in many situations. The method assumes that cross-sections of the response surface in each co-ordinate direction may be approximated by parabolas. Some results from the algebraic analysis carried out in the project indicate that the assumption could be valid for at least some of the parameters when using the same objective function as was used in this project. The assumption may not be valid when other objective functions are used. Also, Method 5 would be inaccurate if a discontinuity in the response surface was close to the point at which the steepest descent direction was being found.

(iii) The Quadratic Assumption in the Conjugate Direction Methods of Optimisation

The Conjugate Direction methods assume that, in the area of the minimum, the response surface may be approximated by a positive-definite quadratic form. If this assumption is true then the ultimate rate of convergence achieved by the methods should be good. The algebraic analysis indicated that the assumption may be valid for the Boughton model when using the objective function used in this project. If the assumption is not valid for areas away from the minimum, the methods should still be satisfactory. It is considered that the difficulty experienced with the Davidon method in this project and the fact that the method did not locate any local minima were due mainly to the presence of discontinuities and the inadequacies in the methods for finding the steepest descent directions rather than to the response surface not being of quadratic form.

(iv) The Difficulty of the Optimisation Problem and the Recommended Method

The problem of finding the optimum values for the parameters of rainfall-runoff models is difficult, as

(a) at this time, it is not possible to express the objective function or its partial derivatives as explicit functions of the model parameters, and

(b) there are discontinuities in the objective function and inter-dependencies between parameters.

These factors are not all present in many other optimisation problems. In verbal communication, workers in other fields (Electrical and Chemical Engineering) have considered this problem as difficult compared to those with which they were experienced.

The Simplex and Davidon methods were used for most of the optimisation work in this project. They are regarded by Kowalik and Osborne as the best of the Direct Search and Descent methods, respectively. Both methods appear to be adversely affected by inter-dependence between parameters, while the Davidon method appears to be more vulnerable to the effect of discontinuities. The Simplex method generally appeared to be more efficient and is recommended as the better method provided that contraction of the simplex is not regarded as convergence to a local minimum.

10.3 PERFORMANCE OF THE MODEL WITH NEAR-OPTIMUM PARAMETER VALUES

A number of sets of parameter values which appear to be near-optimum were found for the Lidsdale No. 2 catchment. The performance of the model with the two most widely separated sets of values was investigated. The period of data was the same as that used for optimisation with the addition of a subsequent year of additional data. There were only minor differences between the outputs from the model with the different sets of parameters. The model gave good reproduction of the observed catchment runoff during the period which was used for optimisation. Although the model produced zero runoff for many of the small events, the large events, which form the bulk of the catchment yield, were well reproduced. However the model output compares poorly with the observed catchment output in the additional one-year period. There are large errors in both under - and over - estimation of individual events by the model. This result indicates that the model may not be satisfactory for the synthesis of long periods of runoff data.

The fact that the model gave similar outputs with different sets of parameter values indicates that difficulty would be experienced in attempting to correlate the parameter values with catchment characteristics, while the poor reproduction by the model of the extra one-year period of data underlines the need to test the validity of a model and its parameter values on a period of data which was not used for optimisation.

10.4 ALGEBRAIC ANALYSIS

The algebraic analysis which was commenced in this project and is described in Section 8 can only be regarded as an elementary beginning to a full analysis of the operation of the model. However, it has already been beneficial in indicating the likelihood of parameter inter-dependence and some of the features to be expected in the response surface, e.g., discontinuities. It has also enabled a comparison of the different response surfaces which may be obtained with different objective functions to be made.

It is thought that this type of analysis has not been attempted before and that its continuation could lead to the following important results:-

(i) a full understanding of the structure and operation of the model,

(ii) indications of inter-dependencies between parameters and of indifference to some ranges of parameter values,

(iii) knowledge of the location of discontinuities on the response surface,

(iv) an explicit equation for the value of the objective function in terms of the data and of the model parameters and, by differentiating this equation, explicit equations for the partial derivatives of the objective function with respect to each parameter, and

(v) ultimately, a direct analytical method for finding the optimum parameter values.

Continuation of this analysis is therefore considered to be of much importance in future research on rainfall-runoff models.

10.5 DATA

The following points relating to data should be considered in future work.

10.5.1 Unrepresentative Data

For many catchments of the size used in this project, the nearest rain gauge may be some miles away, while others may only have one gauge on the catchment. Therefore for many events the rainfall data from these gauges may be unrepresentative of the average catchment rainfall, and such data should not be used for optimisation. Two catchments which were under consideration for use in this project were eliminated when it was found that the runoff quantities for many events were greater than the amounts of rain which fell onto the catchment according to the rainfall data.

10.5.2 Evaporation Data

The bulk of the precipitation onto catchments is removed again by evapotranspiration. It seems logical that this phase of the rainfall-runoff process must be modelled accurately to achieve good reproduction of the runoff quantities. To achieve this, estimates of potential evaporation and evapotranspiration are required, preferably for the same time intervals as those used in the model calculations. Ideally these estimates are obtained by energy balance methods using climatic data. However, such data would rarely be available for catchments of less than ten square miles. Alternatively, observations of pan evaporation may be used, and these may be available for a greater number of catchments. Further optimisation work with small rural catchments should be restricted to those for which good evaporation data are available.

10.5.3 Soil Moisture Data

In this project soil moisture data were used to check the variation of the contents of the soil moisture stores in the model and to revise the estimate of the initial contents of the Lower Soil Store.

More intensive study of these data could lead to useful modifications of the model. At present such data are only available for the Lidsdale catchments Nos. 2 and 6, and it would be useful to obtain similar data on other catchments. For the above two catchments, there are discrepancies and inconsistencies between the data records of rainfall, evaporation, runoff and soil moisture. These are most probably due to unrepresentativeness in both the rainfall and soil moisture data records.

10.5.4 Longer Data Records

The length of the Pokolbin data records used for optimisation was four years and seven months including two "warm-up" periods of six months each. For Lidsdale No. 2 catchment the length of the data record was two years and four months, including one "warm-up" period of six months. The tendency of some parameters to adopt unrealistic values during optimisation may be reduced if a much longer data record is used. Optimisation of the model parameter values should be attempted for a catchment having concurrent rainfall, evaporation and runoff data covering a period of, say, ten years or more.

10.5.5 Effect of Data Errors

It is thought that errors in the rainfall, evaporation and runoff data induce alterations in the shape of the response surface and move the position of its global minimum, these changes being relative to the surface which would be obtained with error-free data. The new surface would not necessarily be a more difficult one on which to search for the minimum, but the optimum parameter values for this surface would be different to those which would be obtained with error-free data (the true or correct values), and these true values could never be found. These conclusions are thought to be valid for the response surface of all rainfall-runoff models, and are supported by the results of Ibbitt (1972) and Dawdy and Bergmann (1969).

Ibbitt, using synthetic data, found that when errors were present in the rainfall, evaporation or runoff data, the optimising method led to parameter values which were different to those obtained using error-free data. Dawdy and Bergmann found that increasing errors in rainfall data led to greater differences in the values of the model parameters. The errors used in these studies were of the same order of magnitude as for the errors likely to be present in even good quality hydrologic data, and were not simply gross errors.

For any particular rainfall-runoff model, then, it is probable that data errors could lead to errors in the parameter values which are large enough to prevent correlation of these values with catchment characteristics. As errors are inevitable in hydrologic data, this problem is of practical importance and the effect of data errors requires further research.

10.6 RECOMMENDATIONS

The following practical recommendations are made for future work with rainfall-runoff models:-

(i) Correlation of parameter values with catchment characteristics should not be attempted until true optimum values have been obtained.

(ii) Optimisation should only be attempted for catchments which have a long period of concurrent rainfall, evaporation and runoff data. For catchments in eastern New South Wales, the length of data record should preferably be about ten years or more. The evaporation data should preferably be at the same time intervals as used for the model calculations, and not at longer than monthly intervals.

(iii) A number of different starting points and more than one optimising method should be used in the search for optimum parameter values for any particular catchment. The Simplex and Davidon optimising methods are satisfactory, the Simplex method being perhaps more efficient in the early stages of optimisation. New methods which may have been developed recently should also be investigated.

(iv) A set of parameter values should not be accepted as the optimum values until a number of attempts to make further improvements to the objective function have been made, e.g. by using another optimising method starting from this set of parameter values, or by numerical trials around the values.

(v) The effect of scaling the parameters on the efficiency of optimisation needs further research. A strategy for imposing the most efficient scaling on the parameters for optimisation is required. Until this is obtained, frequent experimentation with the scaling during optimisation may be beneficial.

(vi) Objective functions in which the deviations between the observed and calculated runoff quantities are raised to a power greater than one are preferred to those where the power lies between zero and one. The latter type of objective function gives a response surface which has the same location of the global minimum, but which is relatively flat and may contain a larger number of secondary minima.

(vii) The use of fixed time periods in the objective function instead of individual events appears to be satisfactory when there is only a small number of events (say 1 to 3) per time period. The time period should be substantially longer than that used for the model calculations.

(viii) More research is required into the effect of data errors on the optimum parameter values.

(ix) The algebraic analysis of the model which was commenced in this project should be pursued with the aim of writing an explicit equation for the model output in terms of the data and the parameters.

(x) For the top three stores of the Boughton model, the "warm-up" period need only be long enough to include the first event which fills these stores regardless of their initial contents. If the Lower Soil Store approaches the full state during a prolonged wet period early in the data record, then the "warm-up" period should include this wet period. Otherwise, the "warm-up" period should be chosen with regard to the top three stores and the initial contents of the Lower Soil Store regarded as a parameter. The "warm-up" period for other models should be selected on similar considerations.

(xi) For the Boughton model, the Lower Soil Store depletion factor should be regarded as another parameter of the model and an optimum value should be sought for this factor.

10.7 CONCLUSION

Rainfall-runoff models are potentially very useful hydrological tools. However, before they can be of general usefulness it is necessary that (a) the optimum values of the model parameters should be closely related to measurable physical properties of catchments, and (b) the accuracy with which a given model can synthesise the observed runoff from a catchment should be known. It is unlikely that any model can meet these conditions at this stage. Problems exist with regard to the models themselves, the methods used to search for the optimum values of their parameters, and their synthesis of catchment runoff in periods which were not used for optimisation. Those models which use short time periods (6 or 15 mins.) in their calculations would be expected to give the best reproduction of observed runoff quantities, but these models are still subject to the problems mentioned above and their more exacting data requirements lessen their widespread usefulness.

The complexities involved in deriving the optimum values for the parameters of rainfall-runoff models for a large number of catchments and then correlating these values with physical catchment characteristics do not appear to have been fully appreciated in the past. Many of these difficulties have been identified in the work of this project and an understanding of some of them has been gained. Further work along these lines will be necessary, leading ultimately to the derivation of truly optimum parameter values for a given model and a number of catchments. During this time, existing models may be modified to incorporate recent findings in such fields as infiltration and the movement of water in the unsaturated soil zone. Only after this work will it be possible to make a judgement on the potential usefulness of these models.

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APPENDIX A1. DESCRIPTIONS OF OPTIMISING METHODS

A1.1 THE SIMPLEX METHOD OF NELDER AND MEAD

When optimising a function of n parameters, a simplex of $(n + 1)$ sets of parameter values is used. Each set of parameter values is referred to as a point. In each iteration, the method attempts to generate a new point which has a lower value of the objective function than at the two highest points and this new point replaces the highest point in the simplex. The features of the method will be illustrated with reference to a function of two parameters. The simplex for such a problem consists of three points.

A1.1.1 The Initial Simplex

As the simplex is not necessarily regular at any iteration of the method there is no need for the simplex which is used at the start of the search to be regular. Usually, one point of the initial simplex is a set of estimates of the optimum parameter values. In this project the end point from a search by another method, or a set of parameter values estimated from physical considerations, was used as one point of the initial simplex. The other n points were generated by setting each parameter in turn to zero. (For some parameters a small positive value was used to avoid zerodivide errors.) For a two parameter problem, with initial parameter estimates of x_1' and x_2', the starting simplex formed in this way would consist of the points

$$(x_1' , x_2'),\ (0 , x_2'),\ \text{and}\ (x_1' , 0)$$

and would be as shown in Fig. A1.1.

After choosing the initial simplex, the objective function is evaluated at each point and the points with the highest, second highest and lowest values of the objective function are identified.

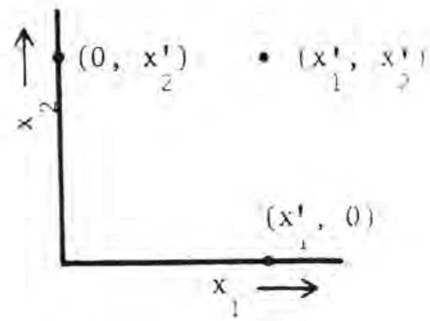


FIGURE A1.1
TYPICAL INITIAL SIMPLEX

A1.1.2 Reflection

Reflection is the basic step which is made at each iteration of the method. The highest point of the simplex is reflected around the centroid of all the other points and the objective function is evaluated at the reflected point (see Fig. A1.2).

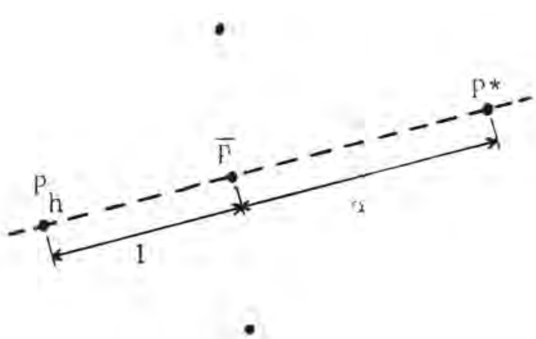


FIGURE A1.2
REFLECTION

Denoting the highest point by P_h, the centroid of the other points by $\overline{P}$ and the reflected point by P*, the co-ordinates of P* are given by

$$P^* = \overline{P} + \alpha(\overline{P} - P_h)$$

where α, known as the reflection coefficient, is a positive constant which determines the distance $\overline{P}\,P^*$ relative to the distance $\overline{P_h \overline{P}}$. The value 1 was used for α in this project.

Depending on the value of the objective function at the reflected point P*, one of three alternative branches is taken to complete the iteration.

(i) If the objective function at P* is lower than at all other points of the simplex then an expansion step is attempted.

(ii) If the objective function at P* is higher than at the second highest point of the simplex then a contraction step is taken.

(iii) When the objective function at P* lies between the values at the lowest and second highest points of the simplex the next iteration is commenced using a new simplex consisting of the point P* and all of the points in the old simplex except P_h.

A1.1.3 Expansion

When the objective function value at P* is lower than at all other points of the simplex it is probable that the direction $P_h \rightarrow P^*$ is favourable for further improvement. P* is then expanded to P** (see Fig. A1.3) by the equation

$$P^{**} = \overline{P} + \gamma(P^* - \overline{P})$$

where γ, the expansion coefficient, is greater than unity and is the ratio of the distance $\overline{P}$ P^{**} to $\overline{P}$ P^*. The value 2 was used for γ in this project.

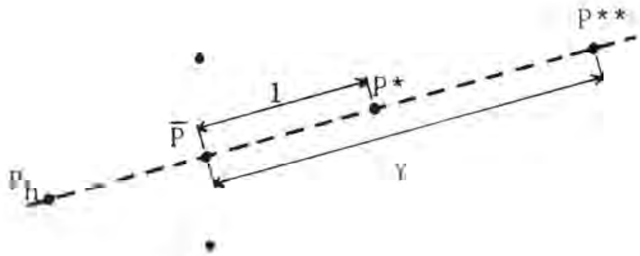


FIGURE A1.3
EXPANSION

If the value of the objective function at P^{**} is also lower than at all other points of the simplex then P_h is replaced by P^{**} and a new iteration is commenced.

If the objective function at P^{**} is higher than at the lowest point of the simplex, (a failed expansion) then P_h is replaced by P^* and a new iteration is commenced.

A1.1.4 Contraction

When the objective function value at P^* is higher than at the second highest point of the simplex there is no advantage in replacing P_h by P^* as a reflection back to P_h will occur in the next iteration. In these circumstances it is probable that the lower areas of the response surface lie within the area of the current simplex and that contraction of the simplex may be favourable.

A new P_h is defined as the lower of either the old P_h or P^* and the contracted point P^{**} (see Fig. A1.4) is then found by

$$P^{**} = \overline{P} - \beta(\overline{P} - P_h)$$

where β, the contraction coefficient, lies between 0 and 1 and determines the ratio of the distance $P^{**}\,\overline{P}$ to $P_h\,\overline{P}$. In this project the value 0.5 was used for β.

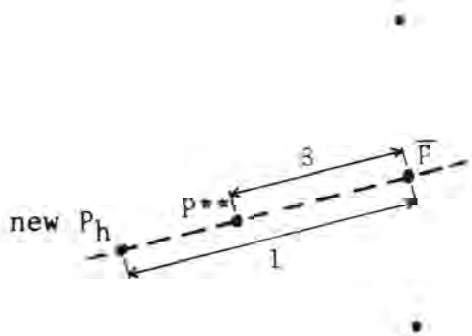


FIGURE A1.4
CONTRACTION

If the objective function at P** is lower than at P_h then P_h is replaced by P** and a new iteration is commenced

If the objective function at P** is higher than at P_h (a failed contraction) then all points of the simplex are moved half-way towards the lowest point and a new iteration is commenced.

The expansion and contraction moves in the method of Nelder and Mead enable the simplex to adapt itself to the local shape of the response surface. As the minimum is approached the simplex contracts until there are only small differences between the points.

A1.1.5 Behaviour of the Method in a Valley

In this project it was found that, whenever the simplex contracted until the points were almost coincident, further improvement of the objective function could be obtained by some other search technique. The points reached by the Simplex method were therefore not the required optima.

Whenever a simplex spans a steep-sided valley in the response surface, it is probable that reflection of the highest point produces a point which lies high up on the opposite side of the valley. A contraction move would then take place. As the range of downhill directions (along the valley) is very small compared to the range of uphill directions (across the valley), most iterations would result in a contraction of the simplex, which would then become so small that, when an expansion move down the valley did occur, the improvement would be very slight.

In this project it was found that the early iterations of the Simplex method contained a number of expansion moves but that the later iterations contained few expansion moves and a large number of contraction moves.

It therefore appears that the Simplex method led to points which lay on the floors of valleys in the response surfaces and was inefficient in moving along these valleys. However it should be noted that such valleys provide difficulties for all optimising methods and that, in general, the Simplex method achieved more rapid initial reductions in the objective function than the other methods used in this project.

A1.2 THE DAVIDON METHOD

This method is one of a number based on the use of conjugate directions. Proposed originally by Davidon (1959), the method was presented definitively by Fletcher and Powell (1964) and is also described by Kowalik and Osborne (1968). These descriptions assume a knowledge of matrix notation for vectors and functions and of matrix manipulations. In this appendix the basis of the method is explained in more general terms. The method assumes that the response surface may be approximated by a quadratic form in the area of the minimum, although the equation of the quadratic function which best approximates the objective function is not required explicitly. The usual method of finding the minimum of a quadratic form is not applicable to finding the minimum of more general functions such as the objective function used in this project. Conjugate directions are the basis of a method which may be used to find the minimum of a quadratic form and which may also be extended to search for the minimum of more general functions. In the remainder of this appendix, the usual method of finding the minimum of a quadratic form is first considered. Conjugate directions are then defined and their use in minimising a quadratic form is explained. Finally the use of conjugate direction methods to search for the minimum of more general functions is described.

A1.2.1 The Minimum of a Quadratic Form

The general equation for a quadratic form in n variables, $x_1, x_2, \ldots x_k \ldots x_n$, is:-

$$\begin{aligned} F = a_{11}x_1^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + \ldots\ldots + 2a_{1n}x_1x_n + b_1x_1 + c_1 \\ + a_{22}x_2^2 + 2a_{23}x_2x_3 + \ldots\ldots + 2a_{2n}x_2x_n + b_2x_2 + c_2 \\ + \ldots\ldots\ldots\ldots\ldots\ldots \\ + a_{nn}x_n^2 + b_nx_n + c_n \end{aligned}$$

In particular, for $n = 1$,

$$F = ax^2 + bx + c$$

which is the general equation for a parabola.

For $n = 2$, the general equation is

$$F = a_{11}x_1^2 + 2a_{12}x_1x_2 + b_1x_1 + c_1$$

$$+ a_{22}x_2^2 + b_2x_2 + c_2$$

while particular examples are

$$F = x_1^2 + x_2^2 \qquad \text{(a paraboloid of revolution)}$$

and $$F = x_1^2 - x_2^2 \qquad \text{(a hyperbolic paraboloid)}$$

The extreme values (maxima and minima) of quadratic forms may be either finite or infinite. Only those which have a finite minimum value (but which may have an infinite maximum value) would reasonably approximate the objective function used in this project. The location of the minimum value of such a quadratic form may be found by the usual methods of calculus. The function is differentiated with respect to each of the variables x_i, $i = 1, 2, \ldots n$. The partial derivatives are set equal to zero, giving n linear equations which are solved simultaneously for the required values of the variables x_i.

In matrix notation the general equation for the quadratic form is

$$F = x^T A x + b^T x + c$$

where x is a column vector whose elements are the x_i's, i.e.,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

x^T is the row vector obtained by transposing x

A is an $n \times n$ matrix which contains the a_{ii}'s and which is symmetrical (i.e., $a_{jk} = a_{kj}$). If A is also positive-definite, then the form will have a finite minimum value.

b^T is a row vector containing the b_i's.

c is a constant.

The inverse of the matrix A, i.e., the matrix A^{-1}, is used when solving the n simultaneous equations to locate the minimum of the quadratic form.

This method is obviously not suitable for finding the minimum of functions such as the objective function used in this project, as the equation of the function is unknown.

A1.2.2 Conjugate Directions

Conjugate directions are the basis of an alternative method for locating the minimum of a quadratic form. The method does not require the inverse matrix A^{-1} to be computed, and may be extended to more general functions.

A direction in n dimensional space may be represented by a vector having components v_1, v_2 ... v_n in the co-ordinate directions x_1, x_2 ... x_n. In matrix notation, the direction may be represented by the column vector

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

Two directions $\mathbf{u}$ and $\mathbf{v}$ are said to be conjugate with respect to the positive-definite matrix A if

$$\mathbf{u}^T A \, \mathbf{v} = 0$$

It can be shown that:-

(i) there exists at least one set of n independent vectors mutually conjugate with respect to A,

(ii) it is possible to generate a set of n conjugate directions from a given starting direction, and

(iii) the minimum of a quadratic form with positive-definite matrix A may be found from any given starting point by a descent search in which each of n conjugate directions is used as a descent direction only once. The order in which these directions is used is immaterial.

The matrix A of the quadratic form is used in generating the n conjugate directions. In the descent search, the minimum must be found along each direction and this point used as the starting point for the search along the next direction.

A1.2.3 Optimising Methods Based on Conjugate Directions

For general functions, no A matrix is available to generate the search directions. Optimising methods for general functions are required to generate search directions using such quantities as function values and partial derivatives at various points on the response surface. Several methods exist which define search directions in this way and which were designed to produce conjugate search directions when used for the minimisation of a quadratic form. They are therefore able to minimise a quadratic form in n iterations without using explicitly the matrix A of the quadratic form.

These methods may be used to search for the minimum of functions such as the objective function used in this project. Assuming that the function may be approximated by a quadratic form in the area of the minimum, the methods would be expected to have a fast rate of ultimate convergence as they only require a finite number of iterations (n) to minimise a quadratic form. The methods are described as being quadratically convergent and the Davidon method is one such method.

In the Davidon method a matrix H, an approximation of the inverse matrix A^{-1}, is used in conjunction with the steepest descent direction at the current point to define the search direction. After locating the minimum along this direction the H matrix is revised before proceeding to the next iteration. For the first iteration the unit matrix

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ \vdots & & & \end{bmatrix}$$

is used for H and the first search direction used is the steepest descent direction from the starting point. As the minimum is approached, H approaches A^{-1} where A is the matrix of the quadratic form which approximates the objective function.

APPENDIX A2. THE STEEPEST DESCENT DIRECTION AT A POINT ON THE RESPONSE SURFACE AND THE SCALING OF THE PARAMETERS

The Descent Methods of optimisation require the steepest descent direction at the current point on the response surface to be found at the start of each iteration. This direction is simply the opposite of the steepest ascent or gradient direction and is sometimes referred to as the negative gradient direction.

Let F be a function of n parameters $x_1, x_2 \ldots x_n$, i.e.,

$$F = F(x_1, x_2, \ldots x_n).$$

At any point P defined by a particular set of parameter values the gradient direction, denoted by grad F, is given by

$$\text{grad } F = i_1 \frac{\partial F}{\partial x_1} + i_2 \frac{\partial F}{\partial x_2} + \ldots i_k \frac{\partial F}{\partial x_k} \ldots + i_n \frac{\partial F}{\partial x_n}$$

where i_k is the unit vector in the direction of the x_k axis.

This may be expressed more simply as

$$\text{grad } F = \nabla F$$

where the operator $\nabla \equiv i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + \ldots + i_n \frac{\partial}{\partial x_n}$.

The gradient vector is therefore the sum of n component vectors. The kth component vector is in the direction of the x_k axis and its magnitude is equal to the partial derivative of F with respect to x_k, i.e., the slope of F in the x_k direction.

A movement away from P in the gradient direction is therefore made by adjusting the parameters simultaneously, the individual adjustments being in proportion to the corresponding partial derivatives.

The steepest descent direction is simply $-\nabla F$ and a movement away from P in this direction is made by making adjustments to each parameter which are equal in magnitude but opposite in sign to those required for a movement in the gradient direction.

The steepest descent direction at any point is therefore found by evaluating the partial derivatives of the objective function with respect to each of the parameters and then multiplying them by -1. For the Boughton model there are no explicit equations for the objective function or its derivatives in terms of the parameter values, so the slopes in the x_k directions have to be found by numerical methods. Five different methods were used during the course of this project. The methods, their inadequacies, and the improvements which were made are described below. For all methods it is assumed that the value of the objective function for the current parameter values is already known.

A2.1 METHOD 1

This method is only correct in special circumstances, as shown later. It is described here because it was used in the optimisation for the Pokolbin catchments, in the early work of this project by Mr. F. Bell, and by Boughton (1968). The steps in the method are:-

(i) for each parameter in turn, increase the value of the parameter by a fixed percentage, operate the model to find the resulting change in the objective function, and then return the parameter to its original value.

One percent increases were used in this project and Boughton (1968) used 10% increases.

(ii) for any parameter which caused the objective function to increase when its value was increased, assume that the change in the objective function will be of equal magnitude but opposite sign if that parameter is decreased by the same percentage.

(iii) calculate the adjustments to be made simultaneously to all of the parameter values in order to take a step in the steepest descent direction. For the parameter which caused the greatest change (+ve or -ve) in the objective function, the adjustment is equal to the increment used in step (i). For any other parameter the adjustment is a fraction of the increment used for that parameter in step (i), in proportion to the smaller change in the objective function. The adjustment for each parameter is in the direction necessary to reduce the objective function.

In algebraic terms, the kth component of the steepest descent vector is defined as

$$\frac{(\Delta F_k)}{\mathrm{abs}(\Delta F_{max})} \cdot \frac{y}{100} \cdot x_k$$

where ΔF_k is the change in the objective function caused by a y percent increase in the parameter x_k. ΔF_k is +ve if the objective function increased and vice versa.

ΔF_{max} is the ΔF_k which has the greatest magnitude.

This method could not define the steepest descent direction correctly in some situations even if it were theoretically correct. Fig. A2.1 illustrates the method for a two-parameter response surface, and Fig. A2.2 illustrates a situation where the direction defined by the method is incorrect. The error is due to the assumption that a 1% decrease in the value of a parameter will induce a change in the objective function of equal magnitude but opposite sign to the change caused by a 1% increase in the parameter value. Whenever it is found that the objective function is increased by a 1% increase in a parameter value it is necessary to also find the effect of a decrease in the parameter value and, if this causes the objective function to increase as well, the adjustment for that parameter must then be set to zero. Fig. A2.3 illustrates a situation where 1% increases and decreases to the parameter values are too coarse to find the descent direction. It would be necessary to use smaller parameter changes to find the required direction at this point and prevent the search from stopping prematurely.

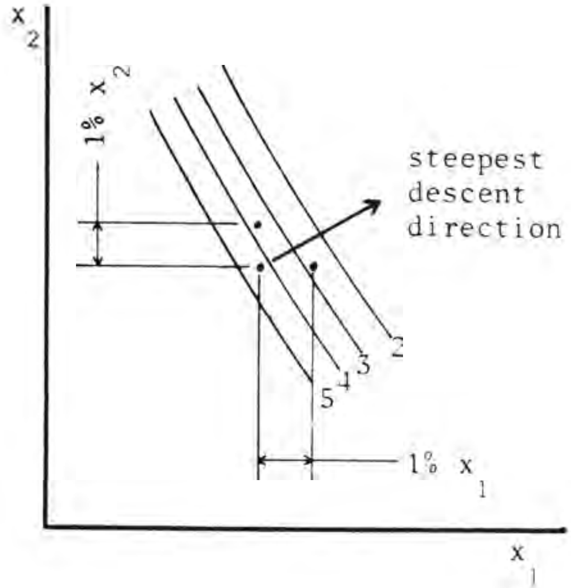


FIGURE A2.1
METHOD 1 ILLUSTRATED
FOR TWO-PARAMETER
RESPONSE SURFACE

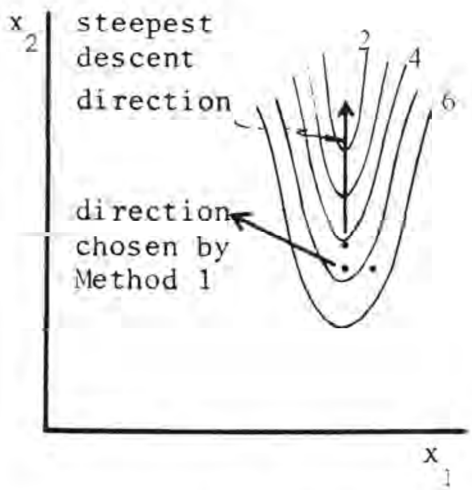


FIGURE A2.2
SITUATION WHERE METHOD 1
IS IN ERROR

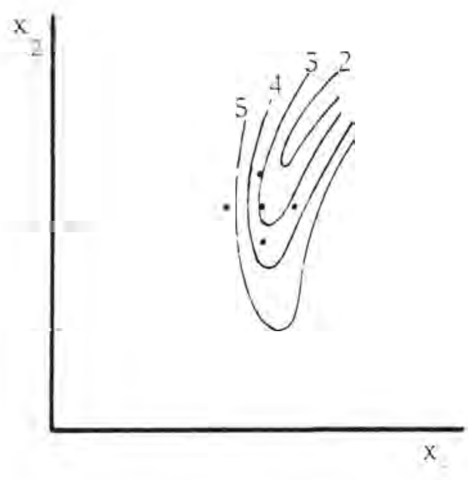


FIGURE A2.3
SITUATION WHERE METHOD 1
FAILS

A2.2 METHOD 2

This method was developed by modifying Method 1 to overcome the difficulties described above. The steps are:-

(i) as for Method 1, i.e., for each parameter in turn, increase the value of the parameter by a fixed percentage, operate the model to find the resulting change in the objective function, and then return the parameter to its original value.

(ii) for each parameter which caused the objective function to increase at step (i), reduce the value of the parameter by the same fixed percentage, operate the model to find the resulting change in the objective function, and then return the parameter to its original value.

(iii) calculate the adjustments to be made simultaneously to all of the parameter values in order to take a step in the steepest descent direction. If any parameter caused the objective function to increase at both of the steps (i) and (ii) the adjustment for that parameter is set to zero. The adjustments for the other parameters are calculated in the same way as for Method 1.

(iv) if the adjustments are set to zero for all of the parameters, repeat steps (i) to (iii) using smaller percentage changes at steps (i) and (ii), and if necessary repeat the steps again with still smaller changes.

In this project, 1% parameter changes were always used in the first attempt to find the descent direction at a given point. If 1% changes were too coarse, ½% changes were tried, then ¼, 1/8 and 1/16 percent changes. If a descent direction could not be defined using 1/16% parameter changes the search was abandoned.

In a general review of the optimising methods and their implementation which was undertaken after the work with the Pokolbin catchments it was found that Methods 1 and 2 are theoretically in error because they do not comply with the definition of the steepest descent direction given at the start of this appendix. By reference to the simple response surface shown in Fig. A2.4, the methods can be shown to be erroneous in all but special circumstances.

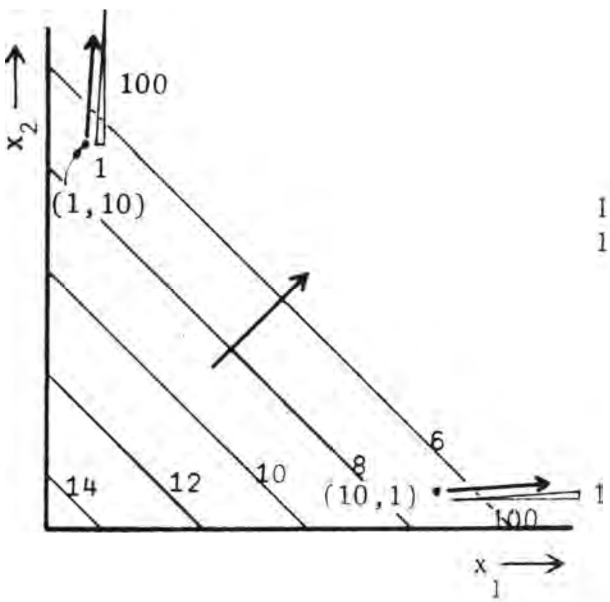


FIGURE A2.4
ILLUSTRATION THAT METHODS 1 AND 2 ARE THEORETICALLY IN ERROR

The steepest descent direction at all points on this surface is at 45° to the x_1 axis. When either of Methods 1 or 2 are used to find this direction at, say, the points (10,1) and (1,10), they will define directions which are considerably different to the true steepest descent direction. The methods will only find the correct direction at points on the line $x_1 = x_2$. At other points, the error will increase as the distance from this line increases.

The parameter adjustments derived by methods 1 and 2 are not in proportion to the corresponding partial derivatives of the objective function. They are derived from gross changes in the objective function instead of changes per unit increment in the parameter values. Furthermore the gross changes are found for varying parameter increments which depend on the current parameter values.

A2.3 METHOD 3

For this method, fixed increments to each parameter were nominated for finding the change in the objective function instead of increments related to the current parameter values. The features of Method 2 which were designed to overcome difficult situations on the response surface were retained. The steps in the method are:-

(i) for each parameter in turn, increase the value of the parameter by the nominated fixed increment, operate the model to find the resulting change in the objective function, divide this change by the parameter increment, and return the parameter to its original value.

(ii) for each parameter which caused theoobjective function to increase at step (i), reduce the value of the parameter by the same fixed increment and find the resulting change in the objective function. If the function also increases at this step, the slope is set to zero.

Otherwise find the slope as in step (i).
Return the parameter to its original value.

(iii) the slopes of the objective function with respect to each parameter constitute the components of the steepest descent vector. If the slopes are set to zero for all of the parameters, repeat steps (i) and (ii) using smaller increments and if necessary repeat the steps again with still smaller increments. In this project the increments were halved at each repetition of steps (i) and (ii) until increments 1/16th the size of the original increments had been used, after which the search was abandoned.

In algebraic terms the kth component of the steepest descent vector is defined as

$$-(\Delta F/\Delta x_k)$$

provided ΔF is negative for either a +ve or -ve Δx_k. Otherwise the kth component is zero.

When this method of defining the steepest descent vector was adopted, scaling of the parameters was found to be necessary because of difficulties caused by the relative magnitudes of the parameter values. Both scaling and the size of the parameter increments used in steps (i) and (ii) above are discussed later.

In optimisation work using Method 3 it was found after several iterations of the search technique that most of the components of the steepest descent vector were being set to zero. Consequently only two or three of the parameters were being adjusted at each iteration.

This implied that the search had descended to points in a valley in the response surface so that, for most parameters, small +ve or -ve increments caused movements up the sides of the valley. Typical cross-sections of the response surface at such a point would appear as in Fig. A2.5.

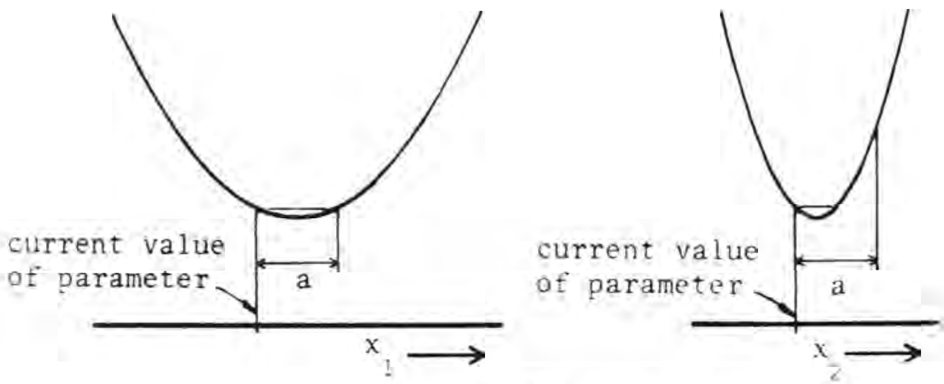


FIGURE A2.5
TYPICAL CROSS-SECTIONS OF THE
RESPONSE SURFACE

For parameter x_1, Method 3 would be able to define a non-zero slope if the increment used were less than "a", but for parameter x_2 a much smaller increment would be necessary. A weakness of the method is that if a non-zero component of the steepest descent vector is found using a given set of parameter increments, no attempt is made to define non-zero components for the other parameters using smaller increments.

A2.4 METHOD 4

This method was designed to overcome the weakness in Method 3 described above. The slope is found for each parameter by the procedure illustrated in Fig. A2.6. The objective function is evaluated at each of the points 1, 2, 3 etc. until a point is found at which it is lower than at the current parameter value. The corresponding change in the objective function and increment size are then used to determine the slope. In this project increments down to 1/16th of the original increments were used before setting the slope to zero.

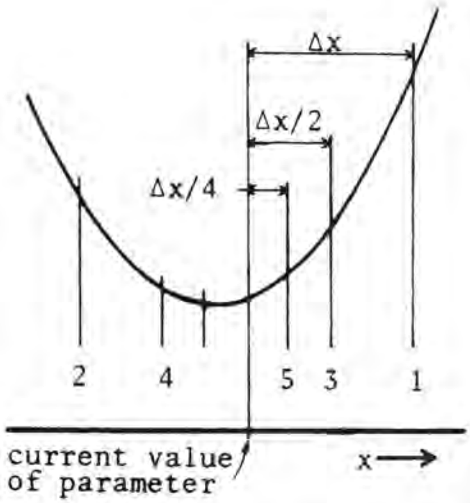


FIGURE A2.6
ILLUSTRATION OF METHOD 4

The above procedure is repeated for each parameter and the resulting slopes are the components of the steepest descent vector. If all the slopes are set to zero the search is abandoned.

The slopes obtained by Methods 3 and 4 would often be poor approximations of the true slopes, as shown in Fig. A2.7.

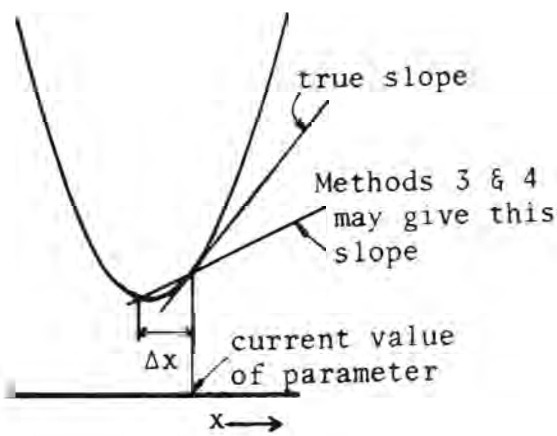


FIGURE A2.7
METHODS 3 AND 4 MAY GIVE POOR APPROXIMATIONS OF TRUE SLOPE

The last method used in this project was designed to give better approximations of the slopes than those given by Method 4.

A2.5 METHOD 5

This method will give theoretically correct values for the slopes in each co-ordinate direction if the basic assumption of the method is valid. The slope for each parameter is found by assuming that the cross sections of the response surface in each of the co-ordinate directions may be approximated by parabolas. Sub-section 9.2 of this report indicates that this assumption is probably valid for the objective function used in this project.

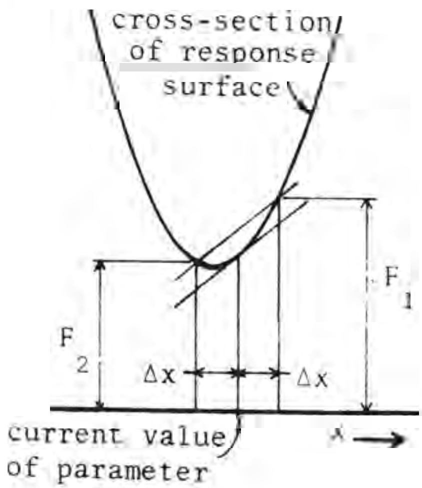


FIGURE A2.8
ILLUSTRATION OF METHOD 5

As shown in Fig. A2.8, the objective function is evaluated on each side of the current parameter value and the slope estimated as

$$\frac{F_1 - F_2}{2\Delta x}$$

The slopes found in this way for each parameter are taken as the components of the steepest descent vector.

This method requires two evaluations of the objective function to find the slope in each co-ordinate direction. Method 4 requires up to ten evaluations, the usual number being between four and eight. Method 5 would give better approximations of the slopes than Method 4 if the parabolic assumption is valid. Therefore Method 5 represents a considerable improvement over the other methods used in this project.

A2.6 THE EFFECT OF DISCONTINUITIES IN THE RESPONSE SURFACE

If a discontinuity were present in the response surface within the distance Δx of the current parameter value then the slope defined by Method 5 (or any of the other methods) would be incorrect. Discontinuities appear to be present in the response surfaces for the Boughton model (see sub-section 7.2.2(iv), 8.1.1, 8.1.2 and 8.4). These may have adversely affected the efficiency of the descent methods of optimisation in this project.

A2.7 SCALING THE PARAMETERS

Scaling is necessary when there are large differences between the model parameter values. This may be demonstrated by considering the response surface for a two-parameter model in which the expected optimum value for parameter x_1 lies in the range 500-1000 and the optimum value of x_2 lies in the range 0-1. (The parameters SSMAX and PV of the Boughton model have such values.) If plotted to a natural scale the response surface would occupy the area shown in Fig. A2.9. A long, flat-bottomed valley in which the lowest point is difficult to locate would almost certainly be present in this response surface. At every point on the surface the steepest descent direction would be predominantly in either the +ve or -ve x_2 direction, with only a very small component in either of the x_1 directions. This surface would be an extreme example of the surface with elongated contours shown in Fig. 4.5.

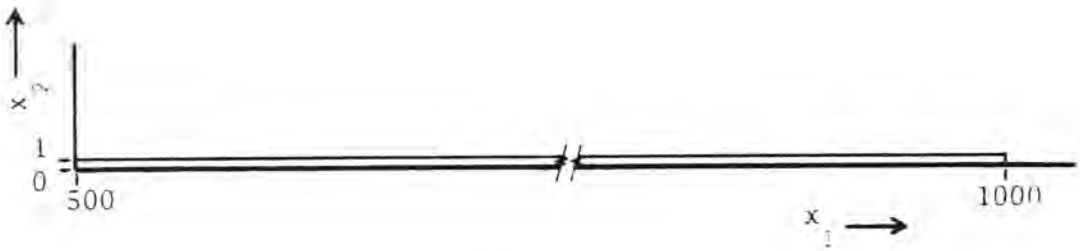


FIGURE A2.9
TYPICAL AREA OF RESPONSE SURFACE
WHEN PARAMETERS ARE NOT SCALED

A more satisfactory response surface having far less elongation in the contours would be obtained by optimising a transformed problem in which one of the parameters is scaled so that its expected optimum value lies within a range of about the same magnitude as that of the other parameter. Thus, the search for the optimum x_1 and x_2 values could be transformed into a search for optimum x_1 and x'_2 values where

$$x'_2 = x_2 \times 500.$$

The objective function value at any point on the $x_1 - x'_2$ response surface is found by operating the model with parameter values of x_1 and $x'_2/500$. The steepest descent direction from a point on the new response surface will have a smaller x_2 component than the steepest descent direction from the corresponding point on the old surface. This is because slopes in the x'_2 direction on the new surface are 1/500th of the equivalent slope on the old surface.

In this project the parameters were scaled for the optimisation runs for Lidsdale No. 2 catchment. Their transformed values were usually in the range of about 300-1200. A maximum increment size of 10 was used for finding the slopes in each co-ordinate direction.

Scaling may alternatively be used when the model parameters are of approximately the same magnitude but when inter-dependence between parameters gives rise to a long flat-bottomed valley. A change of scaling towards different magnitudes for the parameter values may then improve the shape of the response surface.

As scaling may be used in different ways the most satisfactory scaling for models having more than two or three parameters is not obvious. The descent path from any particular set of parameter values is related to the scaling that has been imposed on the response surface (see Fig. 4.5). Frequent experimentation with the scaling of the parameters during optimisation may therefore assist in locating the optimum point more rapidly. Further research work aimed at devising a systematic method of determining the best scaling for an optimisation problem is required.

APPENDIX A3. CATCHMENT DESCRIPTIONS

A3.1 POKOLBIN CATCHMENTS

The location of these catchments is shown in Fig. A3.1. Other descriptive material has been extracted from the Pokolbin Soil Survey Report contained in Volume 3 of the Final Report on A.W.R.C. Research Project 68/1 (Australian Water Resources Council, 1971). The soil survey was conducted by the Soil Conservation Service, N.S.W.

Fig. A3.2 shows the Land Systems in the Pokolbin area and the location of the sites where the soils were inspected and sampled. Fig. A3.3 shows soils distribution in terms of great soils groups and Northcote codings (Northcote, 1965). Descriptions of the soils at each inspection site are contained in the Soil Survey Report but are not repeated in this Appendix. The descriptions of the Land Systems are quoted below.

Land Systems:

Five land systems are represented in the Pokolbin area surveyed, and the following general descriptions are drawn directly from "General Report on the Lands of the Hunter Valley" (C.S.I.R.O. Land Research Series No. 8). The descriptions are provided as a guide to land forms, geology, soils and vegetation, applicable in the Pokolbin area.

Killarney Land System

Geology: Permian shale, sandstone, conglomerate.

Undulating lowlands with shallow valleys, and a small proportion of terraced alluvium mainly associated with Deep Creek. Soils are variable, mainly podsolics (red and yellow).

Vegetation: savannah woodland of box, gum and ironbark, mostly thinned or cleared. Shrubs rare. Ground cover of Themeda australis where protected, otherwise mainly wiry grasses such as Aristida and Danthonia spp., with some Medicago and Trifolium spp.

Land Use: mostly grazing and vineyards, with a strong swing towards the latter usage at the present time. Some dairying. Considerable improved pasture in the Middle Creek Catchment.

Glendower Land System

Geology: Permian shale, sandstone, conglomerate.

Moderately steep hills situated above the Killarney land system. Soils similar, but with areas of skeletal soils and "krasnozem" types (red friable clays), only generally shallow.

Vegetation similar to Killarney.

Land Use: a bigger proportion of grazing country with some uncleared timber. Vineyard development increasing.

Hunter Land System

Geology: quaternary alluvium.

Mainly old river terraces associated with the main creeks - Deep Creek is the only one with any significant area.

Soils variable - mainly uniform Um or Uf soils, sometimes with Gravel seams, immediately adjacent to the creek. Intergrading with Yellow Podsolic (Dy2 and 3) soils, away from the creek, and higher up the catchment.

Vegetation: mostly cleared and under cultivation or grassland of Cynodon, Aristida and Paspalum species. Scattered Eucalyptus spp. with Casuarina spp. on creek banks.

Land Use: mainly grazing with some small areas cultivated.

Cranky Corner Land System

Geology: Carboniferous lavas with some conglomerate and glacial beds.

Steep massive mountains and ravines with soils variable, Skeletal soils widespread with some podsolics and shallow red friable clays. Rock outcrops frequent.

Vegetation: tall mixed woodland, mainly gums and ironbarks with medium dense shrubs below and sparse but leafy grasses. Some rain forest in ravines.

Land Use: some grazing but mainly native timber.

Ogilvie Land System

Geology: Permian conglomerate, sandstone and shale.

Steep hills and escarpments with mainly skeletal soils and shallow podsolics.

Vegetation: wet or dry sclerophyll forest with smaller non-eucalypt trees frequent where sheltered; fairly dense mixed shrubs and dense ground cover of grasses and herbs.

Land Use: occasional grazing but mainly native timber.

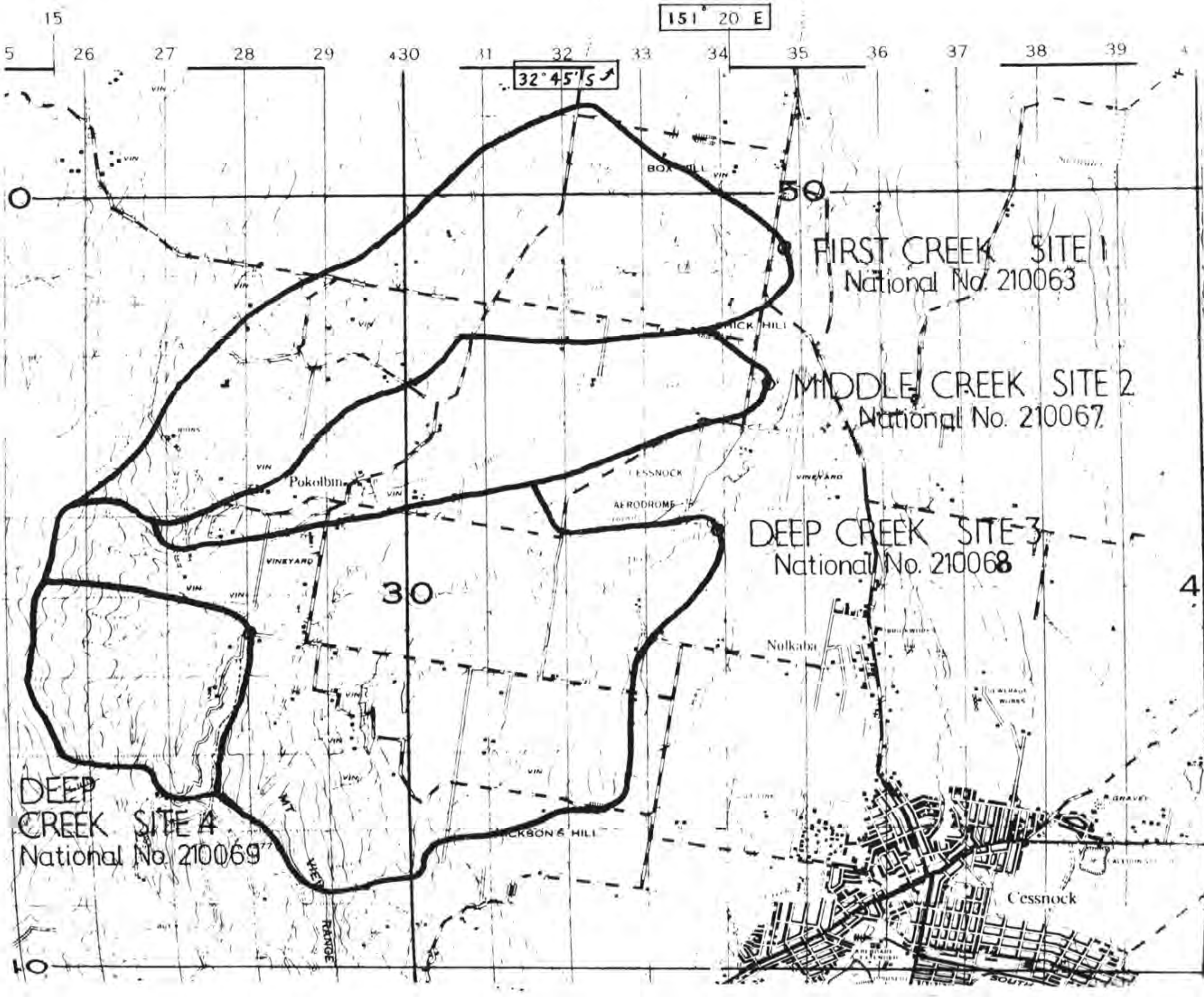
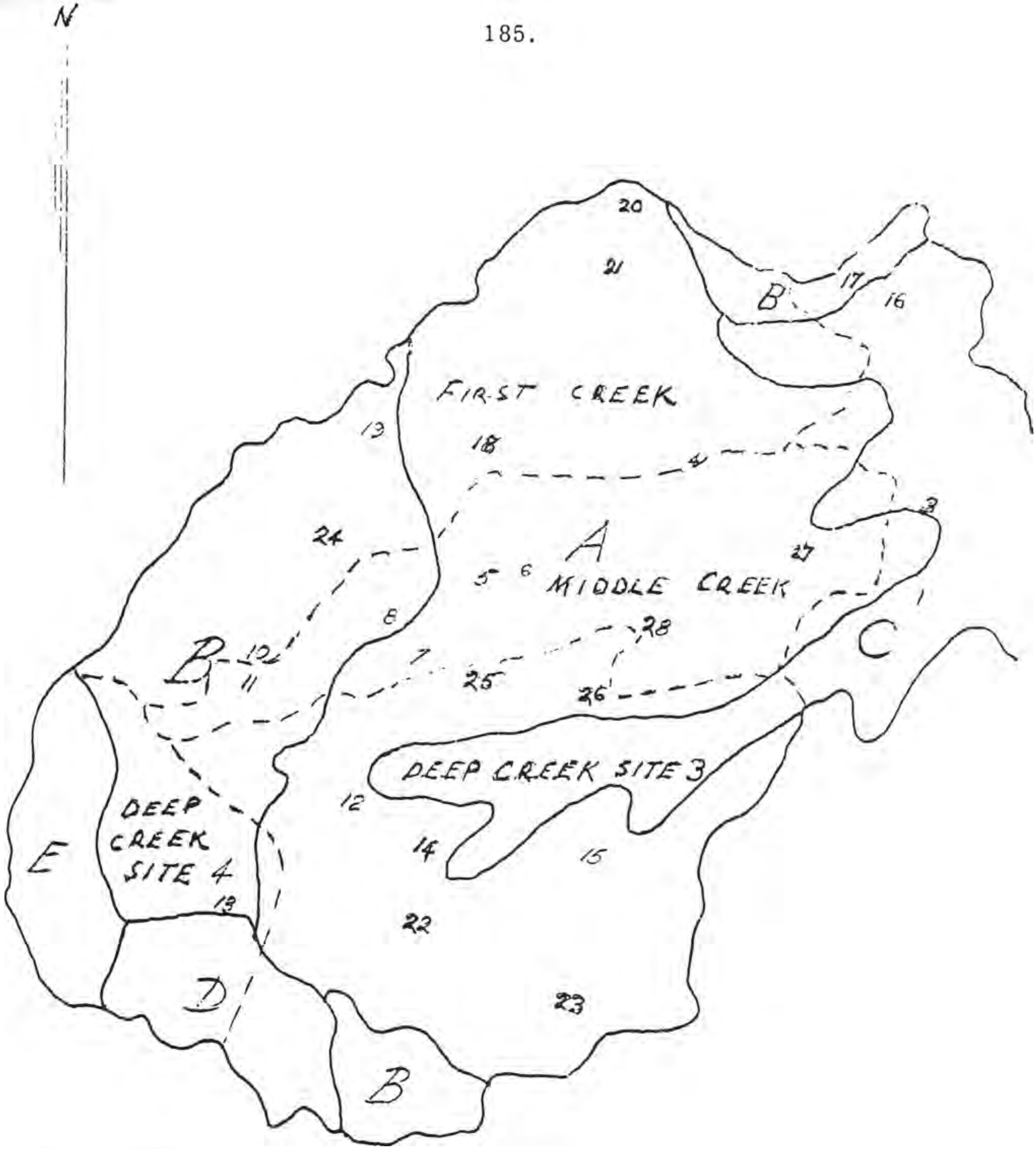


Fig. A3.1: Location of the Pokolbin Catchments.



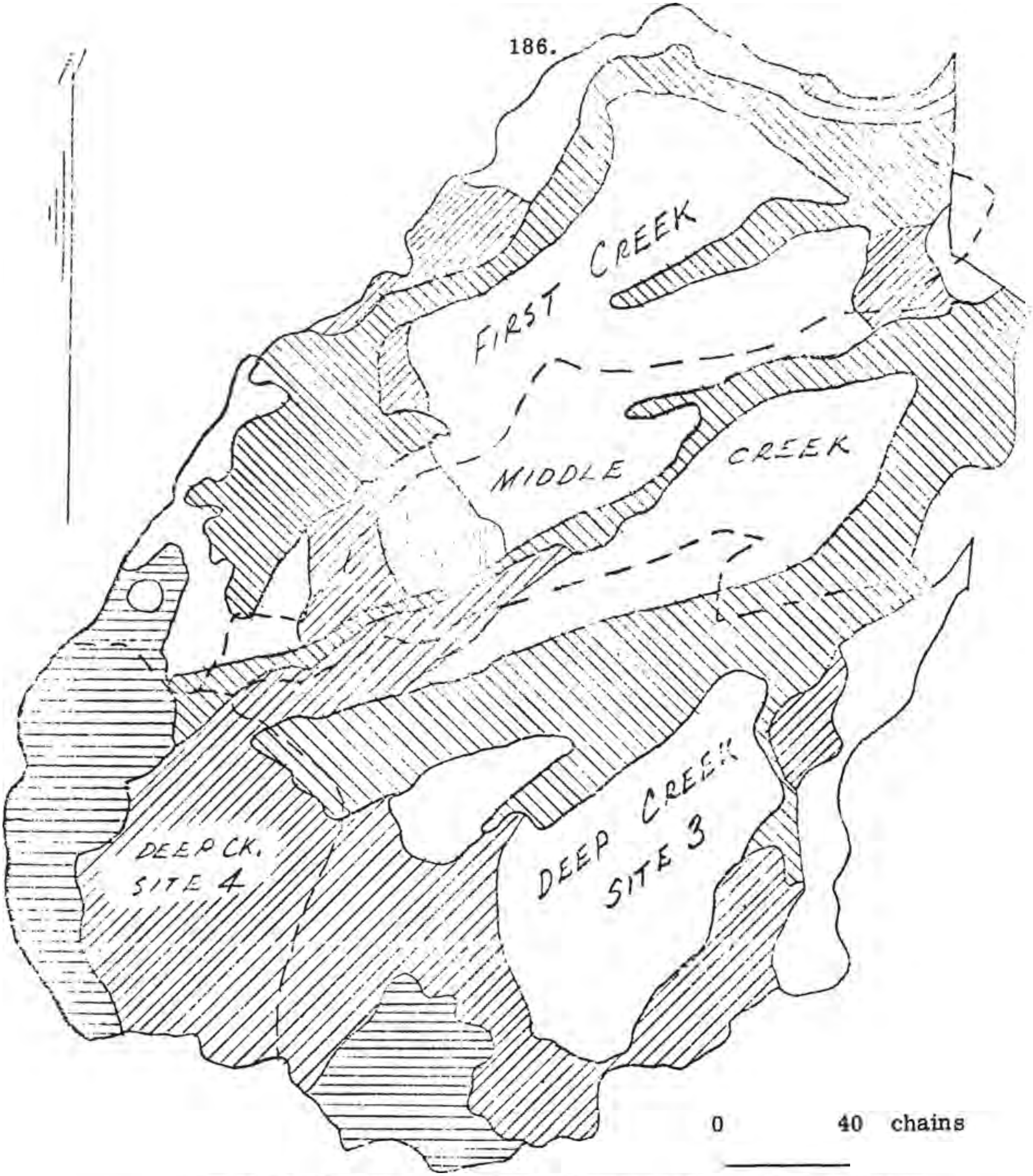
210063, -67, -68, -69
Pokolbin Area
Land Systems
Note:
Numbers refer to soil inspection sites

0 ——— 80 chains
Scale

Legend:

A - Killarney
B - Glendower
C - Hunter
D - Cranky Corner
E - Ogilvie

Fig. A3.2



Great Soil Group	Northcote Commonest	Coding Range
Yellow Podsolic	DY 2.31 - 2.41	DY 2.31 - 3.42
Red Podsolic	Dr 2.31 - 3.31	Dr 2.21 - 3.21
Krasnozem	Gn 4.12	Gn 4.11 - Gn 4.13
Yellow Podsolic and Skeletal	Shallow Dy 2	Uml - Ucl
Skeletal Soils	Ucl. 41	
Catchment boundary		

Fig. A3. 3 Pokolbin Area - Soils.

A3.2 LIDSDALE NO. 2 CATCHMENT

The maps and description of this catchment are reproduced from Smith (1972). Fig. A3.4 shows the location of the Lidsdale State Forest, Fig. A3.5 shows the layout of the experimental catchments in the forest and Fig. A3.6 shows the No. 2 catchment with the location of the soil moisture measuring stations indicated.

The following terms used in describing the catchment require definition:-

"Mean dominant height" is the average of the measured heights of the ten tallest trees per catchment.

"Mean basal area" is a measure of tree density, and is the sum of the cross sectional areas of the tree trunks at 4'3" above ground level, expressed in square feet per acre.

"Crown density index" is a measure of the projected area of the tree crowns, measured with an optical crownometer.

"Channel characteristics" are specified by the percentage, in terms of length, within each of the following groups:

Type I - entrenched channel with solid rock bottom.

Type II - entrenched channel with unconsolidated bottom.

Type III - broad waterway with no significant entrenchment.

Smith's description, which was based on information from Bell and Gatenby (1969), follows.

Area: 31.8 acres.

Vegetation: The forest consists of planted Radiata Pine (*Pinus radiata*, D.Don) with a mean dominant height of 88 feet, mean basal area 133 square feet per acre and a crown density index of 63 per cent. The original forest stand of native Eucalypt, including Brittle Gum (*Eucalyptus maculosa*) and Broadleaf Peppermint (E. dives) was felled and burnt prior to planting with Pinus radiata in 1935. Thinning, carried out between 1956 and 1960, produced a total yield of approximately 9,000 super feet per acre. Topographic features: The general aspect is South-East, with an average catchment slope of 15 per cent, average channel slope 12 per cent, and channel characteristics of 30 per cent Type I, 40 per cent Type II and 30 per cent Type III.

Geology and Soils: The parent material is variable, with Permian conglomerate, sandstone and siltstone in the North-Western section of the catchment, and a combination of both Permian and Devonian sediments in the remainder of the catchment.

The profile is gritty to stoney to a depth of approximately 70 inches, below which there is a fine compacted clay, relatively free of stones. The uppermost 50 inches of soil contains an estimated volume of 5 to 10 percent stone larger than 0.25 inches in diameter. This figure has been determined from mechanical soil analysis. The 50 to 70 inch layer usually contains a lower percentage of stones than the 0 to 50 inches soil strata.

The A horizon of the soil is approximately 12 inches deep, grading through this depth from a dark grey to a pale grey loam. The B horizon is a yellow mottled clayey soil of variable depth, extending to a depth between 40 and 70 inches, with an estimated average depth of 50 inches. Below the B horizon the soil grades into a fine compacted clay loam, extending to 90 inches at least.

Under the Northcote System (Northcote, 1965, 1966), the soil derived from the Permian parent material is classified as Dy 3.41, a hard setting loamy soil with a mottled yellow subsoil. There is an acid reaction trend and the A_2 horizon is bleached.

The soil derived from the Devonian parent material is classified as Dy 2.61, a hard setting loamy soil with a yellow clayey subsoil, an acid reaction trend, an unbleached A_2 horizon and an apedal subsoil.

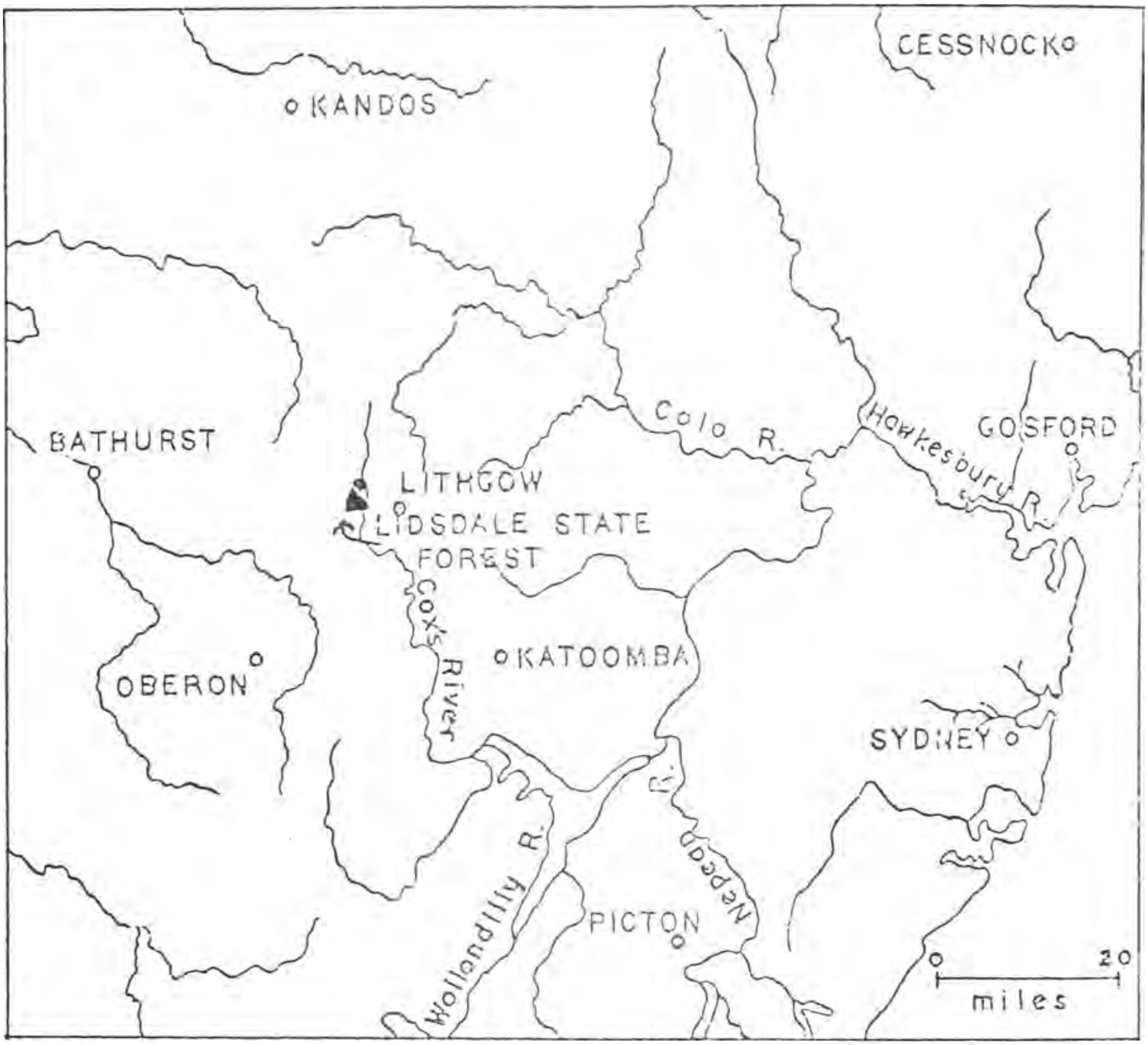


Fig. A3.4: Location of the Lidsdale State Forest.

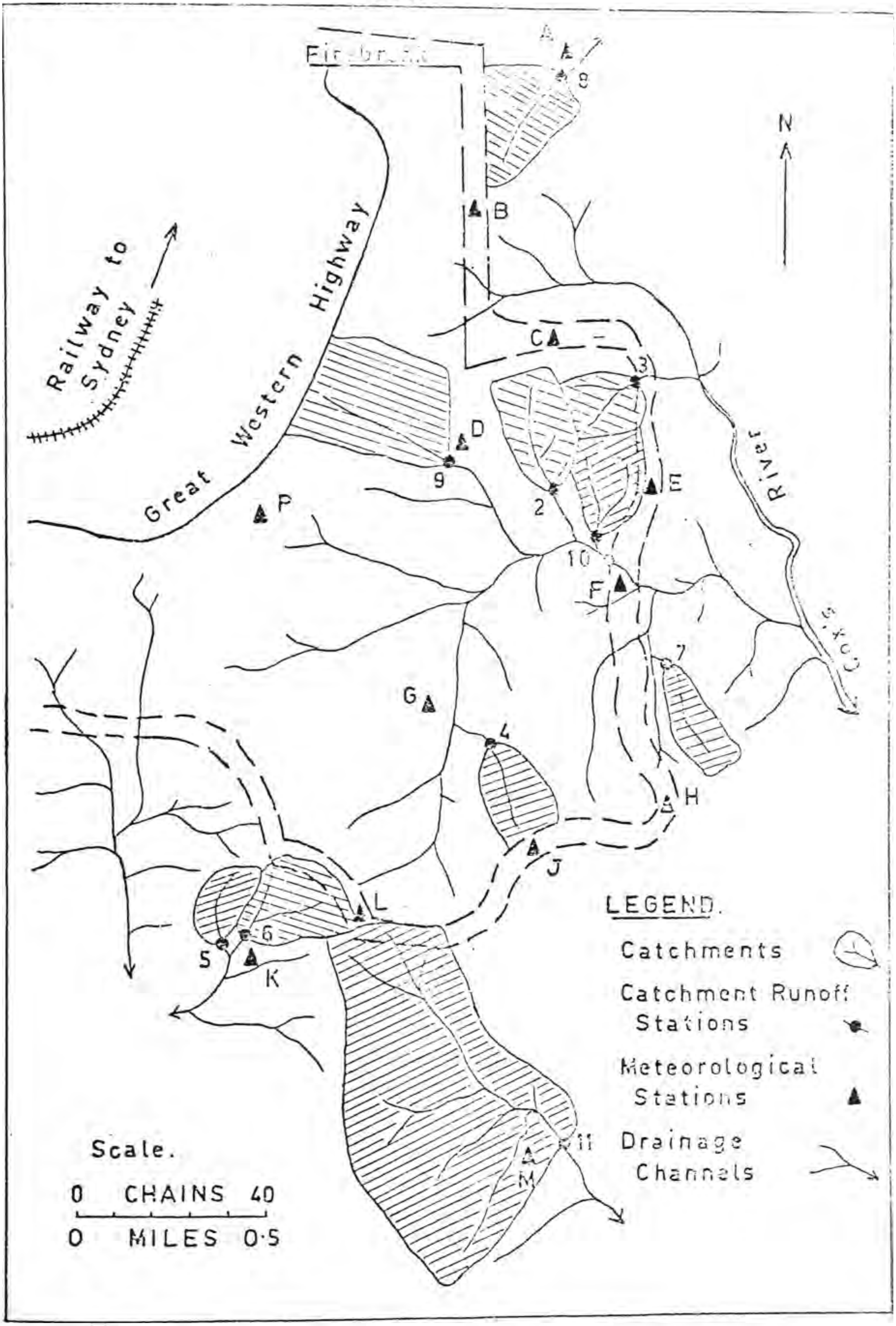


Fig. A3. 5: Experimental Catchments Lidsdale State Forest.

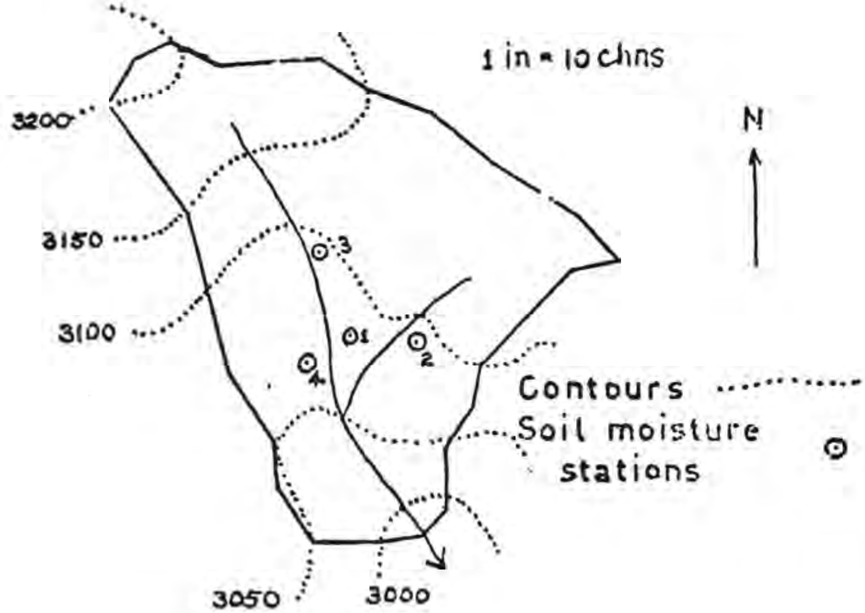


Fig. A3.6: Principal Features, Catchment No. 2.

APPENDIX A4. INTEGRATION OF THE EVAPOTRANSPIRATION AND INFILTRATION FUNCTIONS

A4.1 EVAPOTRANSPIRATION FUNCTION

At low levels of soil moisture S, where the e/t rate is a linear function of the moisture level, the governing equation was

$$\frac{dS}{dt} = kS \qquad \text{(equation 6.7)}$$

The change in moisture level caused by e/t over a period of time is found by integrating this equation.

$$\frac{dS}{dt} = kS$$

$$\therefore \frac{dS}{S} = kdt$$

$$\therefore \int \frac{dS}{S} = kt + B \qquad \text{where B is the constant of integration.}$$

$$\therefore \ln S = kt + B$$

$$\therefore \quad S = e^{kt + B} = e^{kt} \cdot e^{B} .$$

Initially, when $t = 0$, let $S = S_0$

then, $S_0 = e^0 \cdot e^B \qquad \therefore \quad e^B = S_0$

$$\therefore \quad S = e^{kt} \cdot S_0$$

Now when $t = 1$, i.e., after 1 day, let $S = S_1$

$$\therefore \quad S_1 = e^k \cdot S_0$$

A4.2 INFILTRATION FUNCTION

The assistance of Mr. D. Doran in integrating this function is gratefully acknowledged.

The infiltration rate is the time rate of change of storage in the Lower Soil Store, i.e.,

$$\frac{d(SS)}{dt} = FO.e^{-KF \cdot SS/SSMAX} - FO.e^{-KF} \qquad \text{(equation 6.17)}$$

$$\text{Substitute} \quad \left.\begin{aligned} S &= SS \\ K &= KF \\ \text{and } C &= SSMAX \end{aligned}\right\} \text{ for greater clarity.} \qquad \text{(A4.1)}$$

$$\text{Then } \frac{dS}{dt} = FO.e^{-K \cdot S/C} - FO.e^{-K}$$

$$= FO(e^{-K \cdot S/C} - e^{-K})$$

$$= FO.e^{-K}(e^{-K(S/C - 1)} - 1)$$

Inverting,

$$\frac{dt}{dS} = \frac{1}{FO.e^{-K}} \cdot (e^{-K(S/C - 1)} - 1)^{-1}$$

Integrating,

$$t + A = \frac{1}{FQ \cdot e^{-K}} \int (e^{-K(S/C - 1)} - 1)^{-1} \, dS$$

where A is the constant of integration.

$$\text{Let} \quad u = e^{-K(S/C - 1)}$$

$$\therefore \quad du = \frac{-K}{C} e^{-K(S/C - 1)} \, dS$$

$$\therefore \quad dS = \frac{-C}{K} (e^{-K(S/C - 1)})^{-1} \, du$$

$$= \frac{-C}{K} u^{-1} \, du.$$

Then

$$t + A = \frac{-C}{FO \cdot Ke^{-K}} \int (u - 1)^{-1} u^{-1} \, du$$

$$\therefore \frac{t + A}{B} = \int \frac{du}{u - 1} - \int \frac{du}{u}$$

$$\text{where} \quad B = \frac{-C}{FO \cdot Ke^{-K}}$$

$$\text{and R.H.S.} = \int \frac{u \, du - (u - 1) \, du}{(u - 1) \cdot u}$$

$$= \int \frac{du}{(u - 1) \cdot u}$$

$$= \int (u - 1)^{-1} \cdot u^{-1}\, du, \quad \text{as before.}$$

$$\therefore \frac{t + A}{B} = \ln(u - 1) - \ln(u)$$

$$= \ln\left(\frac{u - 1}{u}\right)$$

$$= \ln\left(1 - u^{-1}\right)$$

$$= \ln\left(1 - e^{K(S/C - 1)}\right) \qquad \text{(A4.2)}$$

$$\therefore e^{\frac{t + A}{B}} = 1 - e^{K(S/C - 1)}$$

$$\therefore e^{K(S/C - 1)} = 1 - e^{\frac{t + A}{B}}$$

$$\therefore K(S/C - 1) = \ln\left(1 - e^{\frac{t + A}{B}}\right)$$

$$\therefore S = \frac{C}{K} \ln\left(1 - e^{\frac{t + A}{B}}\right) + C$$

From equation A4.2, $A = B \ln(1 - e^{K(S/C - 1)}) - t$

At the start of the day, $t = 0$ and let $S = S_0$

$$\therefore A = B \ln\left(1 - e^{K(S_0/C - 1)}\right)$$

$$\therefore S = \frac{C}{K} \ln\left(1 - e^{\frac{t + B \ln(1 - e^{K(S_0/C - 1)})}{B}}\right) + C$$

$$= \frac{C}{K} \ln\left(1 - e^{t/B}\left(1 - e^{K(S_0/C - 1)}\right)\right) + C$$

For S after one day (S_1), put t = 1 (and substitute for B)

$$\therefore S_1 = \frac{C}{K} \ln \left(1 - e^{-FO \cdot Ke^{-K}/C} \cdot \left(1 - e^{K(S_0/C - 1)}\right)\right) + C$$

Let $D = e^{-FO \cdot Ke^{-K}/C}$

Then $S_1 = \frac{C}{K} \ln \left(1 - D + De^{K(S_0/C - 1)}\right) + C$

S_1 is the storage after one day provided infiltration has proceeded at the potential rate throughout the day. The total infiltration amount for the day is

$$FT = S_1 - S_0$$

$$= C\left(1 + \frac{1}{K} \ln \left(1 - D + De^{K(S_0/C - 1)}\right)\right) - S_0$$

Re-substituting from equations A4.1, and using F to denote the potential daily infiltration amount rather than the instantaneous rate as before,

$$F = SSMAX\left(1 + \frac{1}{KF} \cdot \ln \left(1 - D + De^{KF(SS/SSMAX - 1)}\right)\right) - SS$$

where $D = e^{-FO \cdot KF . e^{-KF}/SSMAX}$

and SS = storage at the start of the day.