Mutual effects of satellite signal quality and satellite geometry on positioning quality

Author:
Tabatabaei Balaei, Asghar; Motella, Beatrice; Dempster, A.G; Rizos, C

Publication details:
Proc ION-GNSS 2007

Event details:
20th Int. Tech. Meeting of the Satellite Division of the U.S. Inst. of Navigation
Fort Worth, Texas

Publication Date:
2007

DOI:
https://doi.org/10.26190/unsworks/688

Mutual effects of satellite signal quality and satellite geometry on positioning quality

A. T. Balaei B. Motella A. G. Dempster

Asghar Tabatabaei Balaei is PhD student in the School of Surveying and Spatial Information Systems, University of New South Wales, Australia. He received B.Sc and M.Sc in Electrical Engineering from Sharif University of Technology, Iran in 1998 and 2000. His current research activities are focused on the characterization and detection of sources of interference in satellite navigation systems.
Beatrice Motella is Ph. D. student in Electronic and Communication Engineering. She is member of the NavSAS (Navigation Signal Analysis and Simulation) at the Electronic Department of Politecnico di Torino (Italy). She received the Master's Degree in Communications Engineering at the same University. Her research activity is focused on the Radio Frequency Interference (RFI) for the radio-navigation systems.
Andrew G. Dempster (M'95–SM'04) is Director of Research in the School of Surveying and Spatial Information Systems at the University of New South Wales. He was project manager and system engineer on the first GPS receiver to be designed in Australia and has been teaching satellite navigation for over a decade. His current research interests are in signal processing for satellite navigation, new receiver techniques and new location technologies.

Abstract - The accuracy of positioning, using the measured pseudoranges from the receiver to each of the satellites, depends on several different factors. The position evaluation in the GPS receiver estimates four quantities (x, y, z and time) using four or more pseudoranges. In GPS text books, it is easy to find the nonlinear relationship between the measured pseudoranges, the satellite position at transmit time, the receiver position at receive time, the bias in the receiver clock and the composite of errors that can be estimated from a budget [1]. It can be seen that in the linearized version of this equation about a nominal point, the position accuracy is decided by two factors, the measurement quality and the user-to-satellite geometry. These factors separately are extensively discussed in [1] in two separate chapters. In this work we will make a quantitative comparison of the effect of each one of these two factors and investigate their mutual effects on the positioning accuracy. The aim is to establish if the pseudorange error is large because of the poor satellite signal quality (low C/No), under which circumstances we can achieve better position accuracy by eliminating that satellite, noting the fact that eliminating the satellite will affect the geometry.

1 Introduction

GPS is a satellite-based radio navigation system, which uses line-of-sight ranges between the navigation satellites and receivers to derive position solutions. GPS signals therefore are subject to severe degradation in the presence of diffraction and multipath [1].
GPS position solutions in this case would become unreliable especially in

urban areas due to significantly larger position errors. Such deteriorated solutions should be reliably identified and should not be used for navigation in applications.
For land vehicle navigation and other low-end applications, low-cost GPS chipset receivers are typically used with code or pseudorange measurements as the principal observable for position determination. The code-based GPS positioning accuracy is mainly determined by two factors, namely, the accuracy of pseudorange measurements and the geometric configuration of the observed satellites [5]. In other words, the GPS positioning accuracy will vary over time dependent on the satellite geometry and the influence of the measurement errors. As for the satellite geometric configuration, its strength can be evaluated by the dilution of precision (DOP). The accuracy of the pseudorange measurements is dependent on a number of error sources including the satellite orbital and clock errors, the atmospheric delay errors, multipath and measurement noise.
So it is usually difficult to evaluate the position accuracy in real-time applications especially under severe signal degradation environments such as in urban areas [8]. Various statistical methods have been proposed for the assessment of the GPS position solution accuracy but they are not considered very robust because the obtained statistical values are often too optimistic. Generally speaking, a simple examination of the DOP number and the carrier-to-noise (C/N0) value is not adequate to obtain a reliable assessment of the actual accuracy of the GPS positioning solutions. In [3], [4] and [7] the authors have used a fuzzy system or neuro-fuzzy soft computing to derive a quality indicator for GPS code-based positioning solutions using the C/No and DOP value. But the resultant performance was still limited because the simple use of C/N0 is not sufficient to reliably assess the pseudorange errors.
In [9], the problem of reliable assessment of GPS signal quality is investigated. As was proposed in [2], a difference between the measured and expected C/No values is used as an indicator of signal quality. The expected C/No value is determined by the highest C/N0 value at a certain elevation under an open sky environment.
In this paper, the prediction of the C/No based on the formula presented by the author in [6] is used as the satellite signal quality. CW interference is used which is shown to be an effective way to deteriorate the signal quality of each satellite at a time. The relationship between the signal quality and the positioning quality is then analyzed based on the effect of that particular satellite on the user-satellite DOP.

2 Positioning Evaluation in the GNSS Receivers

The accuracy of positioning, using the measured pseudoranges from the receiver to each of the satellites depends on different factors. The final stage in the position evaluation in the GPS receiver, is basically the estimation of four quantities (x, y, z and time) using four or more measured distances from the receiver to the satellites with known positions. In other words we have the following nonlinear estimation problem [1].

$$\rho_i = \left| r_i - r_u \right| + c.b_u + \varepsilon_{\rho_i} \qquad (1)$$

Where r_i is the satellite position at transmit time; r_u is the receiver position at receive time; b_u is the bias in the receiver clock, c is the speed of light and ε_{ρ_i} is the composite of errors. [1] provides an error budget for ε_ρ under various conditions. The states to be estimated are r_u and b_u. The linearized version of the above equation about a nominal point ($\hat{r}_u$, $\hat{b}_u$) is as follows:

$$\Delta\rho_i = G.\begin{bmatrix}\Delta r_u \\ c.\Delta b_u\end{bmatrix} + \Delta\varepsilon_{\rho_i} \qquad (2)$$

Where

$\hat{1}_i = \frac{r_i - \hat{r}_u}{|r_i - \hat{r}_u|}$, $\Delta r_u = \hat{r}_u - r_u$, $\Delta b_u = \hat{b}_u - b_u$, $\Delta\varepsilon_{\rho_i} = \hat{\varepsilon}_{\rho_i} - \varepsilon_{\rho_i}$ and $G = \begin{bmatrix}-\hat{1}_i^T & 1\end{bmatrix}$.

Or briefly: $\Delta\rho = G\Delta x + \Delta\varepsilon_\rho$.

$\Delta\varepsilon_\rho$ is assumed to be zero mean, so that the least squares solution to the set of normal equations is given by

$$\Delta\hat{x} = (G^T G)^{-1} G^T \Delta\rho \qquad (3)$$

It is easy to see in this equation that the position accuracy is decided by two factors, the measurement quality and the user-to-satellite geometry. Each of these factors is separately discussed in [1]. In the following section the mutual effect of these parameters is investigated on quality of the positioning evaluation in the receiver.

3 Mutual Effect of The Signal Quality And Satellite Geometry

In this section we will make a quantitative comparison of the effect of the signal quality and the satellite geometry on the positioning accuracy. The aim is to find out if $\Delta\varepsilon_{p_i}$ of the measured pseudorange of a satellite is large because of the poor signal quality (low C/No), under which circumstances we can achieve better position accuracy by eliminating that satellite, noting the fact that eliminating the satellite will affect the geometry.

To achieve this goal, we simplify the scenario. The assumption is that there are 5 satellites available to the receiver and only one of these satellites has its signal quality is affected. It is discussed in the previous sections that this scenario can be realized in the presence of CW interference. The position error covariance is studied in this investigation: $\mathrm{cov}(\Delta x) = E(\Delta x \Delta x^T)$ where *E(.)* operates as an expected value operator. From (3) we have:

$$\mathrm{cov}(\Delta x) = E((G^T G)^{-1} G^T \Delta\rho\Delta\rho^T G (G^T G)^{-1}) \qquad (4)$$

At this stage two different cases are considered; 4 satellites all having the same pseudorange error (ε) and 5 satellites one of which has an attenuated signal which has a larger pseudorange error (η). Without loss of generality we can assume here that the pseudorange error in the first case for each satellite is 1 m and that of the second case to be $\xi = \eta / \varepsilon$. Then for the two cases we will have:

$$\mathrm{cov}_4(\Delta x) = (G_4^T G_4)^{-1}(G_4^T W_4(\xi) G_4)(G_4^T G_4) \qquad (5)$$

where $W_5(\xi) = I_4$

and

$$\mathrm{cov}_5(\Delta x) = (G_5^T G_5)^{-1}(G_5^T W_5(\xi) G_5)(G_5^T G_5) \qquad (6)$$

where $W_5(\xi) = \begin{bmatrix} I_4 & 0 \\ 0 & \xi^2 \end{bmatrix}$.

G_4 and G_5 represent the G matrix respectively for the cases of 4 and 5 satellites.

The difference between the above two quantities comes from the difference between G_4 and G_5 on one hand and W_4 and W_5 on the other hand. In the scenario that is explained in the following of this section, the effect of W and G are studied on the covariance of the position error.

The data used for this scenario is a set of real data collected with a GPS software receiver NordNav-R30 at the University of the New South Wales on the 6th

November 2006. 6 satellites (1, 11, 20, 23, 25 and 31) are acquired by the receiver. To use in this experiment, in the first step satellite 11 is removed from the positioning evaluation. There is a huge change in the geometry for this satellite configuration: the horizontal dilution of precision (HDOP) varies from 9 to 3 over the period of the whole experiment which is about 38 minutes. This geometry variation is used in this experiment. In Figure **1**, the position error is examined at in different epochs (different geometries). In each of these epochs, the pseudorange error has been increased from 1 to 20. It is clearly recognized that the variance of position error changes linearly proportional with the pseudorange error and also with HDOP.

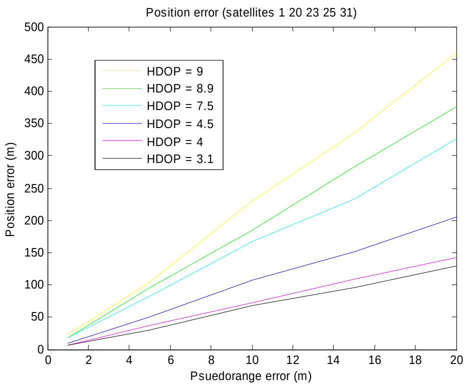


Figure 1 position error vs. pseudorange error for different HDOP values

To compare these covariance matrices ((5) and (6)), one way is to compare their determinants. This comparison will allow us to have a quantitative analysis of the effect of geometry and the signal quality or the pseudorange error. To do that, two satellite sets of (1, 23, 25, 31) and (1, 11, 23, 25, 31) are chosen. The point about these two sets is that in some parts of the data (the starting epochs of the data) satellite 11 plays a fundamental rule in providing good geometry in the constellation. In this experiment, as explained earlier, only the pseudorange error of the satellite 11 is changed. By using ((5) and (6), the amount of pseudorange error of satellite 11, which makes the position error for the two configurations equal, is found to be $\xi_0 = 22\,m$. This is also achieved by experiment. Figure 2 shows the position error for the two cases. It is clearly seen that the position errors for the two configurations (4 satellites and 5 satellites) become equal at $\xi_0 = 23.1\,m$ which is very close to what is calculated theoretically.

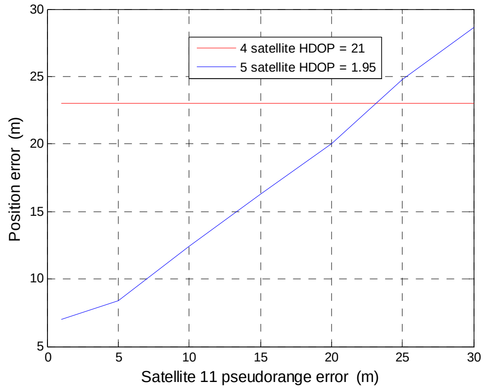


Figure 2 Position error vs. satellite 11 pseudorange error for the 4 and 5 satellite configuration

In another scenario, another two satellite sets are chosen which have very similar and good geometries. These two sets are (1, 11, 20 and 23) and (1, 11, 20, 23 and 25). The satellite for which the pseudorange error has been increased is satellite 25. Again using (5) and (6), the pseudorange error at which the two position errors become equal is calculated. This error is found to be $\xi_0 = 1.45\,m$. Figure 3 shows that this value was found in experiment to be 1.65 *m*.

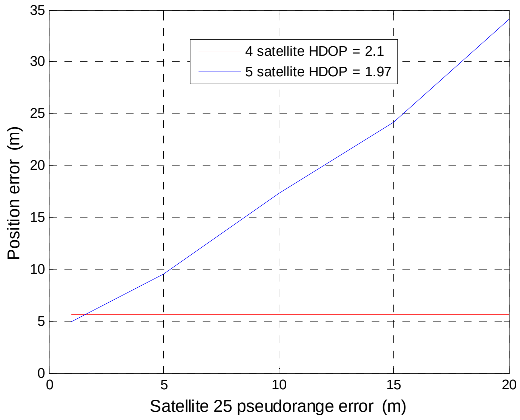
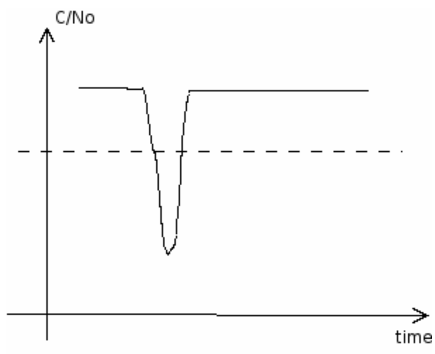


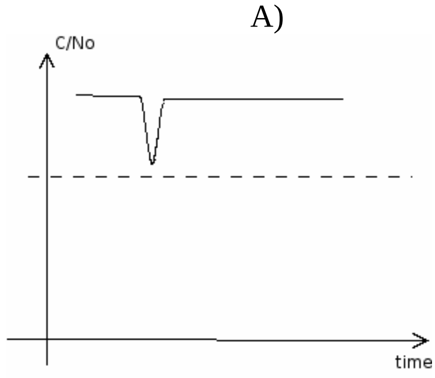
Figure 3 Position error vs satellite 25 pseudorange error for the 4 and 5 satellite configuration

4 Experiments

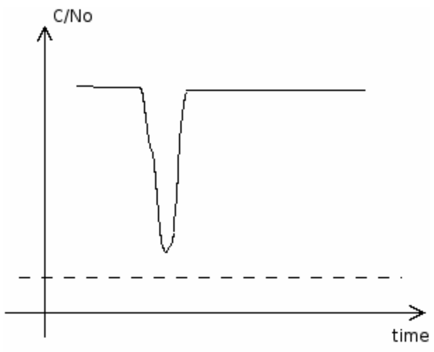
This section aims at showing, by means of some experiments based on real data collections, the concepts presented so far. The theoretical analysis discussed in the previous sections regarded how the position error can be affected both by the user-to-satellite geometry and the received signal quality. The current section will show how the two factors can influence the accuracy in the position estimation in a real scenario.
Different situations will be considered and the decision to apply the 'exclusion zones' algorithm [10] will be discussed. As we saw, the application of the algorithm depends also on the relationship between the C/No and the threshold fixed by the satellites geometry.
Figure 4 shows three different situations in a qualitative way. The threshold will be higher when the satellite constellation presents the same horizontal dilution of precision (HDOP) before and after excluding the satellite affected by the interference (Case A and B), while it will get lower with the HDOP degradation increasing (Case C).
If the HDOP does not change heavily after excluding one satellite (Case A and B), the application of the 'exclusion zone' algorithm depends on the effect the interference could have on the position error. If it is strong enough to make the C/No overtake the threshold (Case A), the algorithm will be applied. Otherwise, if the degradation due to the interference is weaker and the C/No stays above the threshold, the satellite will be still used for position velocity time (PVT) evaluation (Case B).
On the other hand, if the dilution of precision changes significantly after keeping out the damaged satellite, a stronger interference will be needed to activate the 'exclusion zone' algorithm (Case C).



(Case A)



(Case B)



(Case C)

Figure 4 Three case studies for different combination of threshold and signal quality

The data we are going to analyze was collected in Turin (Italy) on the 4th of September 2007 using a commercial software receiver, NordNav R-30. It allows the recording of digital samples at the Intermediate Frequency (IF) and the injection of simulated interference signals.
Let's analyze more in detail the cases A, B, and C by the post process of the received signal.

Case A and B

In the first two cases we analyzed the situation when the HDOP does not significantly change by excluding one satellite. In order to better underline the potentialities of the exclusion zone algorithm [10], a situation with 5 satellites in view is considered. The initial constellation is composed of PRN 2, 5, 12, 14 and 30.
The interferer is modeled as a sinusoidal continuous wave and the satellite it affects is the PRN 2. By excluding it from the PVT computation, the HDOP suffers a small change. Figure 5 shows the trend of the horizontal dilution of precision for the 5 and 4 satellite constellations, obtained respectively including and excluding satellite 2. The value of the HDOP is comparable between the two cases during the all 10 minutes of data collection.

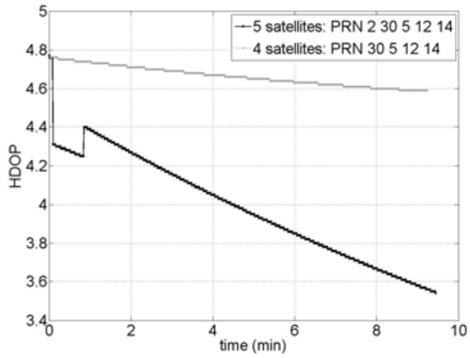


Figure 5 Horizontal dilution of precision during 10 minutes of data collection in two cases: 5 satellites, 4 satellites excluding PRN 2

In order to analyze the Cases A and B, we processed the data twice: the first time the interferer power is double the the second value. In both cases satellite 2 is affected by the interferer, but the 'exclusion zone' algorithm acts in a different way.
Let's consider the first processing (corresponding to Case A), with a high level of interferer power. In this case the carrier to noise ratio degradation of PRN 2 due to the presence of the interference is plotted in Figure 6: the C/No is shown versus time, with and without interference.

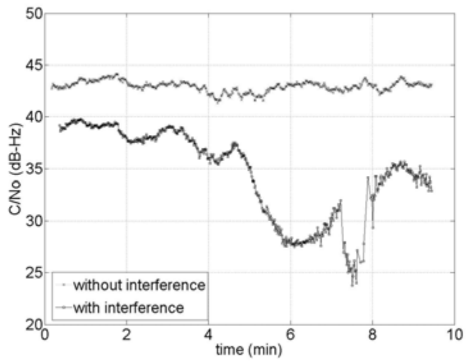


Figure 6 Carrier to noise ratio during 10 minutes of data collection for PRN 2 (with and without interference). Case A.

It is easy to observe how the trend of the carrier to noise ratio of PRN 2 is strongly compromised by the presence of the interference. During the time interval from minute 5 to minute 9, the quality of the signal from satellite 2 can seriously compromised the accuracy of the position estimation. This fact is confirmed by Figure 7, where the position estimation is compared in three different situations:

- PVT computed using 5 satellites when the interference is off (blue line). This is the ideal case, used for comparison;
- PVT computed using 5 satellites in presence of interference (black line). The relationship between the degradation of the PRN 2 signal quality and the loss of accuracy in the position estimation is clearly observable between minutes 7 and 9. During this time interval in fact the maximum position error increases by one order of magnitude.
- The application of the ‘exclusion zone’ [10] removes the satellite affected by the interference from the PVT computation. The pink line shows the position error evaluated by using 4 satellites and excluding satellite 2.

The application of the ‘exclusion zone’ algorithm [10] mitigates the presence of the interference, providing a position estimation completely comparable with the non interfered one.

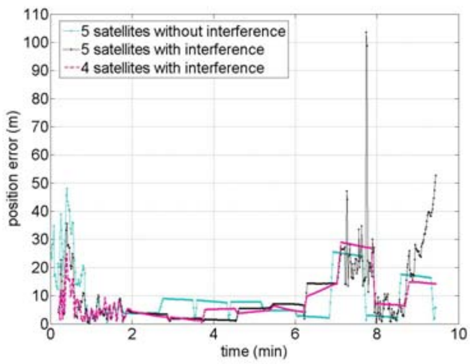


Figure 7 Position evaluation in three different situations: 5 satellites without interference (blue line), 5 satellites with interference (black line), 4 satellites with interference (pink line). The position estimation computed using 4 satellites corresponds to the application of the ‘exclusion zone’ algorithm. Case A

After halving the power of the interference, a new data processing has been done (corresponding to the Case B). Also in this case the signal quality of satellite 2 is damaged, but the effect on the C/No is not pronounced (Figure 8).

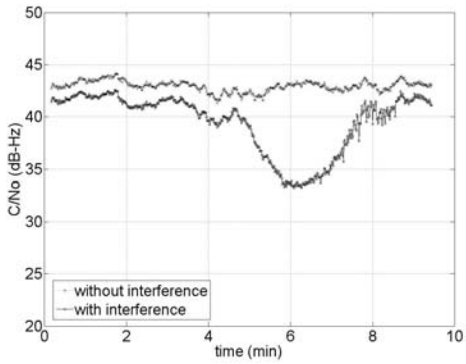


Figure 8 Carrier to noise ratio during 10 minutes of data collection for PRN 2 (with and without interference). Case B

The interferer is not strong enough to trigger the application of the ‘exclusion zone’ algorithm. The presence of the interference in fact does not significantly change the position accuracy level, as shown by Figure 9.

The position error is plotted versus time for the cases with and without interference and it is easy to observe that the two estimations are equivalent.

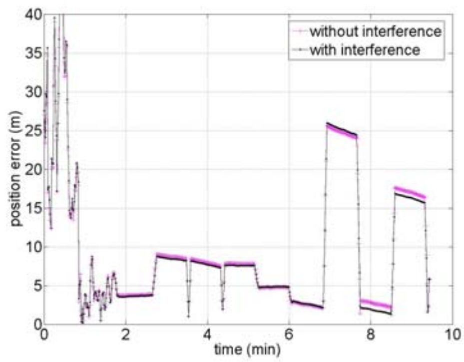


Figure 9 Position evaluation in two different situations: 5 satellites without interference (pink line), 5 satellites with interference (black line). Case B

Case C

The situation where the horizontal dilution of precision changes significantly by excluding one satellite has also been considered. In this case the initial constellation is PRN 1, 30, 5, 12, and 14 and the interfered satellite signal is PRN 14. The trend of the C/No of that satellite is shown in Figure 10. Its degradation in the presence of interference is clearly visible.

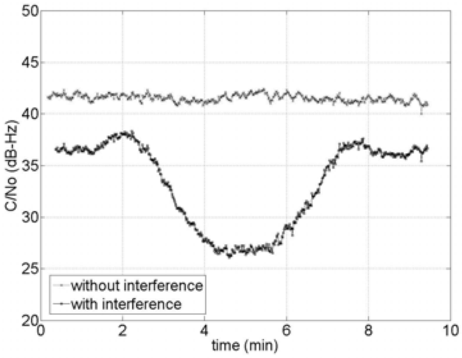


Figure 10 Carrier to noise ratio during 10 minutes of data collection for PRN 14 (with and without interference)

The HDOP is completely different with and without the inclusion of the PRN 14 in the constellation. The trend of the HDOP during the 10 minutes is shown in Figure 11 for the 5 and 4 satellites constellation.

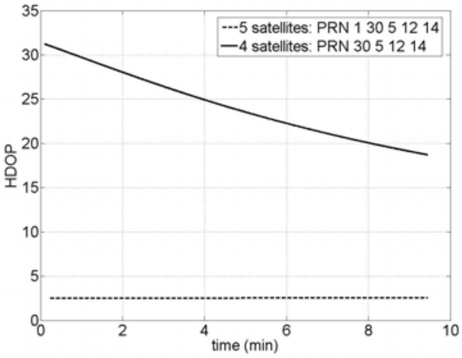


Figure 11 Horizontal dilution of precision during 10 minutes of data collection in two cases: 5 satellites, 4 satellites excluding PRN 14

In this case there is no reason to exclude the satellite from the position estimation. Considering only 4 satellites in fact, the dilution of precision would be too high and the receiver would be no longer able to output the position.

5 SUMMARY

In this paper the importance of satellite signal quality and satellite geometry were investigated in a comparison approach. C/No is considered as an indicator for the signal quality and by proposing a few experimental scenarios, it is shown that if there exist a precise relationship between C/No and pseudorange accuracy, then it is possible to indicate a certain level of signal quality of a particular satellite with and without which, we can achieve the same level of position accuracy. This level is a critical level in which the quality of the signal of that particular satellite worth as much as its presence in the positioning evaluation in terms of adding value to the user-satellite DOP.

References

[1] Parkinson BW, Spilker JJ (1996) Global positioning system: theory & applications. Progress in astronautics and aeronautics, vol 163. American Institute of Aeronautics and Astronautics, Washington

[2] Brunner FK, Hartinger H, Troyer L (1999) GPS signal diffraction modelling: the stochastic SIGMA-D model. J Geodesy 73:259–267

[3] Ghalehnoe A, Mohammadi K, Mosavi MR (2002) Improve determining the location of a moving body on map by low cost GPS receiver. In: 2002 student conference on research and development proceedings, Shah Alam, 16–17 July 2002

[4] Lin C-J, Chen Y-Y, Hang F-R (1996) Fuzzy processing on GPS data to

improve the position accuracy. Soft computing in intelligent information systems and information processing. In: Proceedings of the 1996 Asian Fuzzy Systems symposium, Kenting, 11–14 December 1996

[5] Seeber G (1993) Satellite geodesy: foundations, methods, and applications. Walter de Gruyter, Berlin

[6] Tabatabaei Balaei A, Barnes J, Dempster A G (2006) A Novel Approach in the Detection and Characterization of CW Interference on the GPS Signal Using the Receiver CN0 Estimation. Proceeding of IEEE/ION PLANS, San Diego pp. 1120 - 1126

[7] Wang J-H, Gao Y (2003) Evaluating the accuracy of GPS positions under severe signal-degradation using adaptive-network-based fuzzy inference systems (ANFIS). In: The 50th CASI annual general meeting and conference, Montra´l, Quebec, 28–30 April 2003

[8] Wells D (1987) Guide to GPS positioning. Canadian GPS Associates, Fredericton, Canada

[9] Wang, J-H. and Y. Gao (2004). Identification of GPS Positioning Solutions Deteriorated by Signal Degradations Using a Fuzzy Inference System. *GPS Solutions*, Vol. 8, No. 4, December, 2004, 245-250. (published)

[10] Tabatabaei Balaei A, Dempster A. G, Ngoc T, Barnes J, "Exclusion Zones for GNSS signals when reconfiguring receiver hardware in the presence of narrowband RFI", Proc IAIN/GNSS 2006, Jeju, Korea, 18-20 Oct 2006, pp 347-352