

Design and optimisation of acid fracture treatment in low permeability carbonate gas reservoirs

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**Publication Date:** 2017

DOI: https://doi.org/10.26190/unsworks/19592

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# DESIGN AND OPTIMISATION OF ACID FRACTURE TREATMENT IN LOW PERMEABILITY CARBONATE GAS RESERVOIRS

By

## Mohammed N Al Dahlan

A dissertation submitted to The University of New South Wales in partial fulfilment of the requirements for the Degree of Doctor of Philosophy

# April 2017

School of Petroleum Engineering The University of New South Wales, Sydney 2052, NSW, AUSTRALIA

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Abstract 350 word	s maximum: (PLEASE TYPE)

This study presents an innovative modelling scheme that can effectively optimise acid fracturing treatment. The scheme consists of a fracture geometry model, a thermo-kinetic model, a production model, an economic model, and an optimisation algorithm. Acid penetration distance was found to increase with the increase in injection rate, decrease in leakoff rate, and decrease in formation temperature. Fracture width increases with the increase in injection rate, and acid concentration. Fracture conductivity is estimated from the gain in fracture width and fracture half-length is defined by the acid penetration distance.

Fracture conductivity and half-length are used in the production model to estimate cumulative gas production over a period. Then, the economic model estimates treatment cost and net present value. An optimisation algorithm, based on the combined features of Genetic Algorithm and Evolutionary Operation, is used to maximise an objective function by manipulating free design variables.

The effects of three objective functions – maximum NPV, maximum cumulative production and maximum NPV with minimum treatment cost on optimum acid fracturing design were investigated. A design based on maximum net present value yields almost the same cumulative production as that for maximum cumulative production but less treatment cost. In addition, a design based on maximum net present value with minimum treatment cost results in up to 19% saving in treatment cost with less than 1% loss in net present value.

The effects of reservoir permeability, formation temperature and rock embedment strength on optimum acid fracturing design study indicates: The increase in formation permeability results in an increase in both treatment cost and net present value. Increase in formation temperature results in decrease in both treatment cost and net present value increases with the increase in rock embedment strength.

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# ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude to Professor Sheikh Rahman and Dr. Mohammad Ali Aghighi from the School of Petroleum Engineering and to the University of New South Wales for its careful supervision, constant guidance, constructive comments, and enthusiastic encouragement. I express my special thanks to Professor Hisham A. Nasr-El-Din, Department of Petroleum Engineering, Texas A&M/College Station, USA, for his careful review of the thermo-kinetic aspects and his constructive comments. I also express my special thanks to Dr. M. Khalilur Rahman from the School of Oil & Gas Engineering, UWA/Perth, Australia for providing the source code of the optimisation algorithm, used in acid fracturing optimisation in this thesis. Thanks are also due to the Saudi Aramco for providing me with the scholarships throughout this study. Last but not least, I thank my parents for their continuous prayers for me, and my wife and my sons for their continuous love and patience during this work.

# ABSTRACT

This study presents an innovative modelling scheme that can effectively optimise acid fracturing treatment. The scheme consists of a fracture geometry model, a thermo-kinetic model, a production model, an economic model, and an optimisation algorithm. Acid penetration distance was found to increase with the increase in injection rate, decrease in leakoff rate, and decrease in formation temperature. Fracture width increases with the increase in injection time and acid concentration.

Fracture conductivity and half-length are used in the production model to estimate cumulative gas production over a period. Then, the economic model estimates treatment cost and net present value (NPV). An optimisation algorithm, based on the combined features of Genetic Algorithm and Evolutionary Operation, is used to maximise an objective function by manipulating free design variables.

The effects of three objective functions – maximum NPV, maximum cumulative production and maximum NPV with minimum treatment cost on optimum acid fracturing design were investigated. A design based on maximum NPV yields almost the same cumulative production as that for maximum cumulative production but less treatment cost. In addition, a design based on maximum NPV with minimum treatment cost results in up to 19% saving in treatment cost with less than 1% loss in NPV.

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## **CHAPTER 1**

# INTRODUCTION

Acid and propped fracturing treatments are common technologies used to enhance hydrocarbons recovery from carbonate reservoirs (Schechter 1992). These two techniques have advantages and limitations; however, acid fracturing should be the primary consideration in carbonate formation (Economides and Nolte 2000). Generally, acid fracture is limited to penetration distance inside the fracture but results in high fracture conductivity. On the other hand, propped fracture has a longer fracture halflength but is limited in fracture conductivity. The advantages of acid fracturing over propped are: (1) lower net pressure, which minimises fracture height growth; (2) higher conductivity; and (3) no screenout (proppant build up close to wellbore) or proppant flowback problems (Economides and Nolte 2000).

The majority of oil and gas fields in the Middle East are (mainly) carbonate reservoirs. Carbonate formation (limestone and dolomite) dissolves easily in acids such as hydrochloric, formic, and acetic acids. Therefore, these acids are used widely in wells treatment, including matrix and acid fracturing. They are used in acid fracturing to etch the fracture surfaces, creating rough surfaces where many channels to act as fluid production pathways. The use of hydrochloric acid (HCI) as a fracturing fluid has been considered in theory and applied in the fields as a practical technique for stimulating limestone and dolomite formations. Modelling of acid fracturing history goes back to the early 1970s when Williams and Nierode (1972) presented their first model. However, very little research comparing proppants fracturing modelling has been conducted; this is due to the complexity of acid fracture geometry growth modelling. The shortage of sufficient kinetics and mass transfer data at fracturing conditions add to the complexity. This results in the thought that acid fracturing is not predictable and an economically unsuccessful technique, which results in more fracturing treatments using proppants and non-reactive fluids than acid fracturing (Economides and Nolte 2000).

The use of a modelling scheme that can effectively estimate and optimise acid fracturing geometry is an essential step in designing an efficient and economical treatment. An efficient acid fracturing modelling scheme should consist of fracture geometry, thermo-kinetic, production, economic, and optimisation models. The fracture geometry model determines initial fracture geometry, which is used by the thermo-kinetic model to estimate fracture growth by acid-rock reaction. The outputs from the thermo-kinetic model include penetration distance, fracture gained width by acid-rock reaction, acid concentration, and temperature profiles of the acid as it flows inside the fracture. Fracture conductivity is estimated from the gain in fracture width by acid reaction. The acid fracture half-length is defined by acid penetration distance inside the fracture. Fracture conductivity and half-length by acid fracturing are fed to the production model to estimate cumulative gas production over the production time. The economic model estimates treatment cost and net present value. The three models; acid fracture geometry, production, and economic are integrated into the optimisation scheme to arrive at an optimum treatment design through rigorous parametric analysis.

The chemical and physical properties of reservoir formation and treatment fluids is an important and essential factor in designing a fracturing treatment to enhance oil and gas production. Chemical compositions of the formation play a major role in selecting treatment fluids, carbonate formation is suitable for acid fracturing because it dissolve in acid. Physical properties of the formation are also major factors in selecting a treatment method (acid or proppant fracturing), formation with high rock strength is suitable for acid fracturing (Schechter 1992). Appendix A presents the mechanical properties of main sedimentary rocks found in hydrocarbons reservoirs and Appendix B presents the chemical compositions and thermal properties of these rocks.

This chapter will outline basic information of the chemical and physical properties of reservoir formation and acid treatment. It will focus on basic physical, chemical, and thermal properties and on reaction kinetic for hydrochloric acid treatment of carbonate formation. Hydrochloric acid is more

commonly used in carbonate formation acid treatments than other acids. Therefore, reaction kinetic of this acid will be discussed in this chapter. The main part of the chapter will summarise previous acid fracturing models. In addition, the objective of this study and the outline of the rest of the thesis will be presented in this chapter.

# **1.1. Hydrochloric Acid Reaction with Reservoir Minerals**

Different types of acids are used in well treatment depending on formation type, temperature, and treatment objective. However, treatments of carbonate formation to enhance hydrocarbons recovery are usually carried out using hydrochloric acid (HCI). The common use of HCl in well treatment results from its reasonable cost, soluble reaction products, and its high dissolving power (Schechter 1992). Usually, 5-10 wt% HCl is used for acid wash of perforated and openhole zone and for scale removal from well's production tubular. In matrix stimulation and acid fracturing treatments, 15-28 wt% HCl is commonly used. *Regular acid* is the common name in oil fields for the 15 wt% HCl. Appendix C presents a summary of available acid systems being used in matrix and acid fracturing treatments of carbonate formation.

Reaction of HCI with limestone and dolomite rocks produces calcium and magnesium chloride (which are soluble in water), carbon dioxide gas, and water, as follows (Laidler 1965):

Reaction of HCI-limestone;

$$CaCO_3 + 2HCI \rightarrow CaCl_2 + H_2O + CO_2$$
 1-1

Reaction of HCI-dolomite;

$$CaMg(CO_3)_2 + 4HCI \rightarrow CaCl_2 + MgCl_2 + 2H_2O + 2CO_2 \qquad 1-2$$

The dissolving power of acid when it reacts with carbonate formation is defined as mass or volume of rock dissolved per unit mass or unit volume of acid reacted. Appendix D details the theoretical dissolving power of HCI acid when it reacts with limestone and dolomite formation.

Reactions of acids with carbonate formation are heterogeneous reactions because they take place between the matrix solid surface and acid solution rather than in the bulk phase. Three main steps take place during acid-rock reaction: The first step involves mass transport of acid by convection and diffusion from bulk solution to the rock surface. This step is controlled by mass transfer rate. Second step, acid reacts with the rock surface which is controlled by acid reaction rate. The last step is when reaction products are transported away from the rock surface to the bulk solution by mass transfer, convection and diffusion. This step is negligible by assuming reaction products transfer away from rock surfaces is equivalent to that off acid mass transfer to rock surfaces (Williams et al. 1979). In addition, hydrochloric acid is strong acid; it reacts completely with carbonate regardless of its reaction by-products concentration.

The mass transfer and reaction rate of HCI acid with formation rocks are the two important factors in controlling acid spending rate. One of these two steps is usually slow in comparison to the other step. The slower step controls the consumption rate of acid and is indicated as the controlling step. The penetration distance of acid in the formation is a critical factor in designing acid fracturing treatments where fracture length is defined by acid penetration distance (Williams et al. 1979).

### 1.1.1. Reaction Kinetics of Hydrochloric Acid with Carbonate

The reaction rate of heterogeneous reactions such as acid reaction with formation rock in is primarily a function of acid concentration near the formation rock surface (Laidler 1965). This concentration is called *acid wall concentration* ( $C_w$ ) to differentiate it from acid bulk

concentration (C<sub>A</sub>). Even if a acid concentration of the bulk phase varies from point to point, acid concentration at fracture wall is used in reaction rate calculation. The reaction rate of acid-rock is expressed normally as mole of reactants or products per unit wetted rock surface area per unit time. Therefore, reaction rate of HCI-carbonate can be expressed as kg-mole of HCI/ft<sup>2</sup>.min reacted or kg-mole  $CO_2/ft^2$ .min produced. The reaction rate is negative by convention when referring to the reacting group. Strong acid such as HCI is highly dissociated, and its reaction goes to completion and its reaction rate is expressed as follows (Williams et al. 1979):

$$-r_A = k_r C_w^{\alpha}$$
 1-3

Where  $C_w$  is the concentration of acid at rock surface (fracture walls in acid fracturing),  $k_r$  is reaction rate constant, and  $-r_A$  is the reaction rate. Reaction rate is expressed in kg-mole/ft<sup>2</sup>.min if the concentration is expressed in kg-mole/ft<sup>3</sup> and reaction constant in ft/min. The exponent  $\alpha$  is a constant called *reaction order*. It relates the acid concentration to the reaction rate. If the reaction order is one, then doubling acid concentration at the rock surface will double the reaction rate. Reaction order can be a fraction, less than one. Nirode and Williams (1971) and Lund et al. (1975) studied the reaction rate of HCI with limestone and found its reaction order to be 0.63. For HCI-dolomite reaction, the reaction order can be calculated as follows (Lund, Fogler et al. 1973):

$$\alpha = \frac{6.32 \times 10^{-4} T}{1 - 1.92 \times 10^{-3} T}$$
 1-4

where T is reaction temperature in Kelvin's.

Reaction constants ( $k_r$ ) of HCI-limestone and HCI-dolomite can be measured experimentally or calculated, respectively, as follows (Lund et al. 1975):

$$k_{r,HCl-\lim estone} = 7.314 \times 10^7 \exp(-\frac{E}{RT})$$
1-5

$$k_{r,HCl-dolomite} = 4.48 \times 10^5 \exp(-\frac{E}{RT})$$
 1-6

where E is the activation energy, its values for HCI-limestone reaction equals  $15x10^3$  kcal/kg-mole and for HCI-dolomite reaction is  $7.9x10^3$ . Gas constant R is 1.987 kcl/kg-mole-°K, and T is reaction temperature in Kelvin's.

Reaction constant is a function of temperature and can be calculated at a different reaction temperature by Arrhenius equation as follows:

$$k_{r,T} = k_{r,T_o} Exp(-\frac{E}{R}(\frac{1}{T} - \frac{1}{T_o}))$$
1-7

where  $k_{r,To}$  is reaction constant at a reference temperature  $T_o$  and T is reaction temperature in Kelvin's.

#### 1.1.2. Diffusion Rate and Mass Transfer Coefficient of HCI Acid

As previously discussed, for a heterogeneous reaction to take place, reactant ions ( $H^+$ ) in acid solution must first transfer to the solid surface. If the mass transfer is slower than the reaction rate, then the reaction is mass transfer limited; vice versa, if the reaction rate is slower than mass transfer, then the reaction is reaction rate limited. Most hydrocarbons reservoirs are at high temperature, and reaction rate of HCl with carbonate is instantaneous. Therefore, mass transfer is slower and controls the acid spending rate in most cases (Williams et al. 1979).

The transfer of ions in solution takes place by two mechanisms: diffusion and convection. Diffusion is the process by which ions move randomly because of concentration gradient. As  $H^+$  ions of acid consumed on the carbonate solid surface by reaction,  $H^+$  ions from the acid bulk solution replace them to

balance the solution concentration. On the other hand, mass transfer by convection mechanism is caused by the fluid movement or density gradient. Mass transfer by fluid flow is called *forced convection*, while the one caused by the density gradient is called *natural convection*. Diffusion rate can be represented by Fick's law, as follows (Schechter 1992):

$$J_A = -D_A \frac{\partial C_A}{\partial y}$$
 1-8

where  $J_A$  is the flux of H<sup>+</sup> ions across a plane perpendicular to *y*. It has a unit of mole per unit area per unit time. The term  $D_A$  is the diffusion coefficient, which is measured experimentally.

An approximate dependence of the diffusion coefficient on temperature can be determined using the Stokes-Einstein equation, as follows (Smith 2004):

$$\frac{D_{A@T_1}}{D_{A@T_2}} = \frac{T_1}{T_2} \times \frac{\mu_{T_2}}{\mu_{T_1}}$$
1-9

where:

 $T_1$  and  $T_2$  denote absolute temperatures in Kelvin's  $D_{A@T}$  is the diffusion coefficient of acid, ft<sup>2</sup>/min, at temperature T  $\mu_T$  is the dynamic viscosity, cp, of acid at temperature T

When an acid is injected inside a fracture of width w (ft), apparent mass transfer coefficient (K<sub>g</sub>) is used to describe mass transfer. Apparent mass transfer coefficient is called *mixing coefficient* in some references. It has a unit such as velocity and reaction rate constant (ft/min) and can be calculated as follows (Lo and Dean 1989):

$$K_{g} = \frac{N_{Sh}.D_{A}}{2w}$$
 1-10

where  $N_{Sh}$  is the Sherwood number, a dimensionless number used in masstransfer operation. The Sherwood number represents the ratio of convective to diffusive mass transport, calculated as follows:

For turbulent flow,  $N_{Re} > 7000$ ;

$$N_{\rm Sh} = 0.026 N_{\rm Re}^{5/4} N_{\rm Sc}^{1/3}$$
 1-11

For transitional flow,  $7000 > N_{Re} > 1800$ ;

$$N_{\rm Sh} = 0.001104 N_{\rm Re}^{1.1532} N_{\rm Sc}^{1/3}$$
 1-12

For laminar flow,  $N_{Re} < 1800$ ;

$$N_{\rm Sh} = 6.26 N_{\rm Sc}^{-1/3}$$
 1-13

The Reynolds number ( $N_{Re}$ ) is a dimensionless number that measures the ratio of inertial forces to viscous forces. The Schmidt number ( $N_{Sc}$ ) is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity, and it is used to characterise fluid flows where there are simultaneous momentum and mass diffusion convection processes. The Reynolds number and the Schmidt number are defined, respectively, as follows:

$$N_{Re} = \frac{2w\overline{v}_{x}}{(\mu/\rho)}$$
 1-14

$$N_{\rm Sc} = \frac{(\mu/\rho)}{D_{\rm A}}$$
 1-15

The term *w* is fracture width and  $\overline{v}_x$  is average fluid velocity along the fracture,  $\mu$  is acid viscosity, and  $\rho$  is acid density.

Appendix E presents HCI reaction rate constant and diffusion rate coefficient as a function of temperature. In addition, the acid apparent mass transfer coefficient using equations 1.10 to 1.15 is compared with its reaction rate constant at different flow rate and temperatures to determine when HCI reaction with carbonate is mass transfer limited and when it is reaction rate limited.

#### **1.2.** Thermal Conductivity and Heat Transfer Coefficient

Thermal conductivity of a material defines its ability to transfer heat. Materials with high thermal conductivity, such as metals, transfer heat easily and fast, while those with low conductivity, such as wood, work as insulators. Thermal conductivity  $k_{th}$  (Btu/ft.min.°F), of a material is determined from the heat flux,  $Q_w$ , transmitted through its thickness  $\Delta w$  due to a temperature difference  $\Delta T$ , under steady state conditions when the heat transfer is dependent only on the temperature gradient. It is defined by Fourier's law, as follows (Schechter 1992):

$$Q_w = -k_{\rm th} \frac{\partial T}{\partial w}$$
 1-16

where  $Q_w$  is heat flux in Btu/sq.ft.min, T is temperature in  ${}^{\circ}F$ , and w is thickness in ft.

When an acid is injected inside a fracture of width w, heat transfer coefficient ( $K_h$ ) is calculated as follows (Lee and Roberts 1980):

$$K_{h} = \frac{N_{Nu}.k_{th}}{2w}$$
 1-17

where  $N_{Nu}$  is the Nusselt number, a dimensionless ratio of convective to conductive heat transfer, defined as follows:

For turbulent flow, 
$$N_{Re} > 7000$$
;  
 $N_{Nu} = 0.026 N_{Re}^{5/4} N_{Pr}^{1/3}$  1-18

For transitional flow, 7000 > 
$$N_{Re}$$
 > 1800;  
 $N_{Nu}$  = 0.001104  $N_{Re}^{1.1532} N_{Pr}^{1/3}$  1-19

For laminar flow, 
$$N_{Re} < 1800$$
;  
 $N_{Nu} = 6.26 N_{Pr}^{1/3}$  1-20

 $N_{Re}$  is the Reynolds number, defined as before in Equation 1.14, and  $N_{Pr}$  is the Prandlt number, a dimensionless number approximating the ratio of momentum diffusivity (viscosity) and thermal diffusivity, defined as follows:

$$N_{sc} = \frac{(Cp\mu)}{k_{th}}$$
 1-21

The term *w* is fracture width and  $\overline{v}_x$  is average fluid velocity along the fracture,  $\mu$  is acid viscosity,  $\rho$  is acid density and *Cp* is acid specific heat capacity. Appendix F presents the heat transfer coefficient of injected HCI acid inside a fracture function of injection rate and fluid temperature.

## 1.3 Available Acid Fracturing Models

Five main acid fracturing models have been developed and published by the industry. A summary of four of these models and the differences among them have been studied and published by Yan et al. (1993). The fifth model was developed and published by Remero (1998). Figure 1.1 presents schematic of fracture showing direction of fluid flow, fracture height, length, and width for these models. An overview of these models is presented in the following sections:



Figure 1-1 Fracture geometry showing direction of fluid flow, fracture height, length, and width.

### 1.3.1 First Model: Williams and Nierode, 1972

This model was developed by Williams and Nierode in 1972. It solves the convection-diffusion equation analytically in one-dimensional space. The assumptions of this model, as summarised by Yan Li et al. (1993), are: (1) flow of acid is steady state; (2) reaction rate is infinite, concentration at fracture walls is zero; (3) flow along the fracture is laminar and incompressible; (4) acid viscosity is constant; (5) leakoff rate is constant; and (6) there are no external forces such as gravity effect.

The calculation of acid penetration desistance and fracture conductivity as given by Williams et al. (1979) is as follows: An average fluid leakoff velocity - which is the velocity at which a fracturing fluid flows from the fracture into the surrounding formation-  $\bar{v_l} = (\pi C_l / 2\sqrt{t})$  is calculated, where  $C_l$  is fluid leakoff coefficient, ft/min<sup>1/2</sup>, and *t* is total injection time in minutes. The Reynolds

number for acid flow into the fracture is defined as  $N_{\rm Re} = (2\rho \overline{v} w / \mu)$ , where  $\overline{v}$ is average injected fluid flow velocity, for flow into one wing defined as  $\overline{v} = (q_i/2hw)$ . The term  $q_i$  is volumetric injection rate, w is fracture width, h is fracture height,  $\rho$  is acid density and  $\mu$  is acid viscosity. Substituting for average velocity into the Reynolds number equation,  $N_{\rm Re} = (\rho q_i / \mu h)$ . The effective diffusion coefficient of acid,  $D_A$ , is read from a graph as function of the Reynolds number and formation temperature. both Then, a dimensionless acid penetration distance,  $L_D$ , value is read from a graph as a function of the Peclet number. The Peclet number is given as  $N_{P_e} = (\overline{wv_I}/2D_A)$ . From that, the acid penetration distance is calculated as  $L_A = (L_D/2)(N_{\text{Re}}/N_{\text{Re}}^*)$ . Where  $N_{\text{Re}}^* = (2\rho \overline{v_l} w / \mu)$ , is the fluids leakoff Reynolds number. The fracture conductivity is then calculated using the fracture length ( $L_A$ ), total volume of injected acid ( $Q_i$ ), volume dissolving power of acid (VDP), and rock embedment strength (RES) as follows:

$$wk_f = C_1 \exp(-C_2 \sigma)$$
 1-22

where  $wk_f$  is fracture conductivity in millidarcy-inches and  $\sigma$  is closure stress in psi,  $C_1$  is defined as follows:

$$C_1 = 0.265 (wk_{fth})^{0.822}$$
 1-23

where  $wk_{fth}$  is fracture theoretical conductivity, defined as follows:

$$wk_{fth} = 9.36 \times 10^{13} \left( \frac{w}{12} \right)$$
 1-24

Further, w is fracture average width, defined as follows:

$$w = \frac{(VDP) \times Q_i}{2L_A h(1 - \phi)}$$
 1-25

*VDP* is acid volume dissolving power,  $Q_i$  is total volume of acid injected,  $L_A$  is acid penetration distance inside the fracture, *h* is fracture height, and  $\phi$  is formation porosity.

 $C_2$  is defined as follows:

for RES < 20,000 psi  

$$C_2 = (19.9 - 1.3 \ln(RES)) \times 10^{-3}$$
  
for RES > 20,000 psi  
 $C_2 = (3.80 - 0.28 \ln(RES)) \times 10^{-3}$   
1-26

#### 1.3.2 Second Model: Roberta and Guin, 1975

This model was developed by Roberta and Guin in 1975. The basic assumptions of this model are the same as those of the first model, except that both mass transfer and reaction rates control acid reaction. The mathematical modelling started with the general 2D continuity and convection-diffusion equation of acid in a fracture as follows:

2D continuity equation:

$$\frac{dv_x}{dx} + \frac{dv_y}{dy} = 0$$
1-27

2D convection-diffusion equation:

$$v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} = D_A \frac{\partial^2 C}{\partial y^2}$$
 1-28

The term *C* is the acid concentration,  $v_x$  and  $v_y$  are velocities in the *x* and *y*- directions in the fracture with fracture length along x-axis and width along y-axis, Figure 1.1, and  $D_A$  is acid diffusion coefficient.

The boundary conditions include concentration at wellbore and at fracture centre, defined as follows:

$$C(t, x = 0) = C_i$$

$$\frac{dC}{dy} = 0, \text{ and } y = 0, \text{ at fracture center}$$
1-29

where  $C_i$  the initial acid concentration.

Rate of reaction equals rate of mass transfer to fracture wall, as follows:

$$-D_A \frac{\partial^2 C_A}{\partial y^2} = k_r C^{\alpha}, at \quad y = \pm \frac{1}{2} w.$$
 1-30

here, *w* is the fracture width,  $k_r$  is acid reaction rate constant, and  $\alpha$  is acid reaction order.

Defining average velocities,  $v_x^{\overline{w}}$ ,  $\overline{v}_l$  in the *x*-direction and toward fracture wall *y*-direction, and average acid concentration  $\overline{C}$  along an interval of length dx and half width  $\frac{1}{2}w$ , and  $C_w$  as the acid concentration at the fracture wall in that interval; convection-diffusion Equation 1.28, after integration from 0 to  $\frac{1}{2}w$ , becomes as follows:

$$\frac{1}{2}w \cdot \overline{v}_{x} \frac{\partial C_{A}}{\partial x} + \overline{v}_{l}C_{w} = D_{A} \frac{\partial^{2}C_{A}}{\partial y^{2}}$$
 1-31

Further, the continuity equation 1.27 becomes as follows:

$$\frac{1}{2}w\frac{d\overline{v}_x}{dx} + \overline{v}_l = 0$$
1-32

The mass-transfer coefficient  $K_g$  is defined as follows:

$$K_{g}(C_{w}-\overline{C}) = D_{A}\frac{\partial^{2}C_{A}}{\partial y^{2}} = k_{r}C_{w}^{\alpha}$$
1-33

#### 1.3.3 Third Model: Lo and Dean, 1989

This is a numerical model developed by Lo and Dean in 1989. It solves for fracture geometry and acid transport by convection and diffusion simultaneously. The model consists of two models: a geometry model and an acid transport reaction model. In Lo and Dean's model, the two-dimension (2D) convection-diffusion equation is solved by a one-dimension (1D) approximation averaged over the fracture width. The assumptions of this model are the same as for the previous model.

The mathematical modelling started with the general 2D continuity and convection-diffusion equations of acid in a fracture, as shown before in Equations 1.27 and 1.28. An average fracture width  $\overline{w}$  is defined across the cross-section of the elliptical fracture. Since the cross-sectional shape diverges slowly with respect to *z*-direction, then the 2D equation is defined in the domain  $-\overline{w}/2 < y < \overline{w}/2$ . This domain approximation is expected to be suitable in the region away from the fracture tip. Therefore, the solution for a cross-section of the fracture can be approximated by the solution of parallel plates, with  $\overline{w}$  being the spacing between these plates (Lo and Dean 1989).

The average concentration is defined as  $\overline{C} = \frac{1}{\overline{wv}} \int_{-\overline{w}/2}^{+\overline{w}/2} Cvdy$  where  $\overline{v}$  is the average velocity across channel defined as  $\overline{v} = \frac{1}{2\overline{w}} \int_{-\overline{w}/2}^{+\overline{w}/2} y_x dy$  and  $v_x$  is velocity in the *x*-direction. Applying this averaging to the continuity and diffusion-convection equations (Equations 1.27 and 1.28) and using the boundary condition acid concentration at fracture walls,  $C_w=0$  at  $y = -\overline{w}/2$  and  $+\overline{w}/2$  gives a 1D concentration profile along fracture half-length, as follows (Lo and Dean 1989):

$$\partial \overline{C} / \partial x = (2/\overline{vw})[v_l - (D_A/\overline{w})N_{Nu}]\overline{C}$$
 1-34

where:

 $D_A$  is diffusion coefficient  $v_l$  is leakoff velocity  $N_{Nu}$  is Nusselt number defined as :  $N_{Nu} \approx 4.10 + 1.26N_{Pe} + 0.04N_{Pe}^2$  for  $N_{Pe}$  of 1  $N_{Nu} \approx 2N_{Pe}$  for  $N_{Pe} > 1.0$  $N_{Pe} = v_l \overline{w}/2D_A$ 

#### 1.3.4 Fourth Model: A. Settari, 1993

This model is a 2D numerical model developed by Settari in 1993. The basic assumptions of this model are: (1) 2D unsteady-state convection-diffusion mass transfer with the mass transfer in the z-direction negligible, Figure 1.1; (2) the effect of diffusion along the fracture length is negligible because acid transport by convection is much larger than by diffusion; (3) acid viscosity is not a function of concentration; (4) acid is an incompressible, single-phase fluid; and (5) the reaction has no effect on velocity profiles.

The acid transport model is developed by assuming a constant fracture height but a variable width. It starts with a general continuity equation, as follows:

$$\frac{dv_x}{dx} + \frac{dv_y}{dy} = \frac{\partial A}{\partial t} - q_i$$
 1-35

Where *A* is area equals *dxdy*, and *dA/dt* is a change in volume by the change in fracture width *w* with time, the term  $q_i$  is the injection rate. Velocity in the ydirection equals leakoff velocity at fracture walls,  $v_y|_{y=w/2} = v_l$  leakoff velocity Integrating Equation 1.35 across *y* and *z* becomes as follows:

$$\frac{\partial(\overline{v}_{x}A)}{\partial x} - 2q_{l} = \frac{\partial A}{\partial t} - q_{i}$$

$$\overline{v}_{x} = average \ velocity$$

$$q_{l} = leakoff \ rate \ at \ one \ of \ the \ fracture's \ wall$$
1-36

Assuming constant width vertically  $A = \gamma (hw)$ ,  $\gamma$  is shape coefficient and  $q_l = hv_l$ where *h* is fracture height. The diffusivity equations, neglecting diffusion in the x-direction is given as follows:

$$-\frac{\partial v_{x}C}{\partial x} - \frac{\partial v_{y}C}{\partial y} + D_{A} \frac{\partial^{2}C}{\partial y^{2}} = \frac{\partial C}{\partial t} - q_{i}C_{i}$$
1-37

The terms *C* is acid concentration,  $q_i$  is acid volumetric injection rate,  $C_i$  is initial acid concentration, and  $D_A$  is acid diffusion coefficient.

The boundary conditions are: acid concentration at wellbore is initial acid concentration,  $C(t, x=r_w) = C_i$  and at fracture walls  $(y=\pm w/2)$ , acid transferred to fracture wall equals acid reacted plus leaked off, as follows:

$$-(-C_w v_l + D_A \frac{\partial C}{\partial y}) = k_r (1 - \phi)(C_w - C_{eq})^{\alpha} + v_l C_l$$
1-38

Where the terms  $C_w$  is acid concentration at the wall,  $C_{eq}$  is equilibrium concentration (which is zero for HCl),  $C_l$  is leakoff acid concentration,  $C_l \leq C_w$ . The terms  $k_r$  and  $\alpha$  are acid reaction rate constant and reaction order, respectively. The acid diffusion coefficient term  $D_A$  is replaced by apparent mass transfer coefficient  $K_g$ , which was proposed by Roberts and Guin (1975), as follows:

$$K_{g}(\overline{C} - C_{w}) = -D_{A}(\partial C / \partial y)\Big|_{y=\pm w/2}$$
1-39
The final form of the diffusivity equation is as follows:

$$-\frac{\partial \overline{V}_{x} A \overline{C}}{\partial x} = \frac{\partial A \overline{C}}{\partial t} + 2[q_{l} - q_{l}C_{w} + hK_{g}(\overline{C} - C_{w})] - q_{i}C_{i}$$
 1-40

With the boundary condition, as follows:

$$K_{g}(\overline{C} - C_{w}) = k_{r}(1 - \phi)(C_{w} - C_{eq})^{n}$$
 1-41

Equations 1.40 and 1.41 are solved simultaneously for averaged acid concentration along the fracture half-length and at fracture wall. The mass transfer coefficient  $K_g$  is determined by the correlations proposed by Lee and Roberts (1980), see Appendix E,

#### 1.3.5 Fifth Model: Remero, 1998

This is a 3D numerical model developed by Remero et al. in 1998. The model coupled 2D fluid flow along fracture length and height with 3D acid diffusion-convection equation. The 2D fluid flow along fracture length and height is as follows:

$$\frac{d(wv_x)}{dx} + \frac{d(wv_z)}{dz} = -\frac{\partial w}{\partial t} - 2v_l$$
 1-42

where w is fracture width:

Horizontal velocity along fracture v<sub>x</sub> is defined as  $v_x = -\frac{w^2}{12\mu}\frac{\partial p}{\partial x}$ 

Leakoff velocity toward fracture wall is  $v_l$ 

Vertical velocity in the fracture v<sub>z</sub> is defined as 
$$v_z = -\frac{w^2}{12\mu} \left( \frac{\partial p}{\partial z} + \rho g \right)$$

Acid concentration is calculated by solving the 3D diffusion-convection:

$$-\frac{\partial v_{x}C}{\partial x} - \frac{\partial v_{y}C}{\partial y} - \frac{\partial v_{z}C}{\partial z} + D_{A}\frac{\partial^{2}C}{\partial y^{2}} = \frac{\partial C}{\partial t}$$
1-43

Where *C* is acid concentration,  $v_i$  is velocity in the *i*-direction, and  $D_A$  is acid diffusion coefficient. The mass transfer by diffusion along the fracture, *x*-direction, is neglected since it is small compared to the mass transfer by convection. The mass transfer by diffusion vertically, *z*-direction, is neglected since it is very small and caused by gravity forces.

The boundary conditions are as follows:

Concentration at wellbore;

$$C(x=0, y, z) = C_i$$
, initial acid concentration 1-44

Concentration at fracture wall;

$$k(1-\phi)(C_{(x,w/2,z)} - C_{eq})^{\alpha} = -D_{A}(\partial C/\partial y)\Big|_{x,z,y=\pm w/2}$$
 1-45

The velocities terms are defined analytically from the fluid flow between porous parallel plates as follows:

$$v_x = \frac{2n+1}{n+1}\overline{v}_x \left(1 - \left(\frac{2y}{w}\right)^{\frac{n+1}{n}}\right)$$
 1-46

$$v_{y} = \frac{2n+1}{n+1} v_{l} \left( \left( \frac{2y}{w} \right) - \frac{n}{2n+1} \left( \frac{2y}{w} \right)^{\frac{n+1}{n}} \right)$$
 1-47

$$v_z = \frac{2n+1}{n+1}\overline{v}_z \left(1 - \left(\frac{2y}{w}\right)^{\frac{n+1}{n}}\right)$$
 1-48

where *n* is fluid power law exponent, equals one for Newtonian fluid and the term  $\overline{v}_i$  is average velocity in *i*-direction.

# 1.4 Study Objectives

The main objectives of the present study are: (1) to develop a thermo-kinetic model that can estimate acid fracture geometry (length and width) and conductivity; (2) to study the effects of different parameters on the acid fracture geometry and conductivity; (3) to use a multi-objectives optimisation model to improve acid fracturing treatment design in low permeability gas formation; (4) to study the effects of objective functions and reservoir properties on the optimum acid fracture design; and (5) to conduct case study on actual gas field to demonstrate the benefits of the proposed scheme in terms of design optimisation and fracture geometry estimation.

To achieve these objectives, a scheme of thermo-kinetic will be developed to address the first two objectives. Then acid fracture geometry, production and economic models will be built into an optimisation algorithm to optimise acid fracturing treatment design in tight-gas reservoirs. The thermo-kinetic model will consist of thermal and reaction kinetic models. The thermal model will be capable of estimating temperature changes of acid at the wellbore and inside the fracture; since reaction temperature is the main factor that affects acid mass transfer and reaction rate. Acid temperature changes during injection inside the well tubing because of friction and heat exchange with the surroundings. Inside the fracture, it changes because of the heat of the reaction and heat exchange with the formation. The kinetic model will estimate acid penetration distance and the amount of rock dissolved from the fracture surfaces. Acid penetration distance defines fracture half-length by acid fracturing and fracture width is created by acid dissolving rock from the fracture surfaces etching a rough surface that provides pathways for hydrocarbons production. Fracture conductivity is to be calculated from the fracture gained width by the acid-rock reaction.

The effects of many parameters on the acid fracture geometry and conductivity will be investigated by changing a parameter over its field range and keeping the other parameters constant. The tested parameters will include injection rate, injection time, leakoff velocity, acid concentration, initial fracture width, and formation initial temperature. The fracture conductivity and half-length will be used in a production model to estimate gas cumulative production. Then, the economic model will be used to estimate treatment cost and net present value.

An optimisation model will be used to maximise an objective function by manipulating free design variables. The objectives functions are: maximum net present value, maximum cumulative production or maximum net present value with minimum treatment cost. The free design variables will include injection rate, injection time, and acid initial concentration. The effects of many parameters on optimum acid fracturing treatment design will be investigated by changing a parameter over its field range. These tested parameters include reservoir permeability, formation temperature, and rock embedment strength.

# 1.5 Thesis Outline

Chapter 1 outlines basic chemical and physical properties of reservoir formation and treatment acid, it focuses on carbonate formation and hydrochloric acid basic physical, chemical, thermal properties, and reaction kinetic. The main part of the chapter summarises previous acid fracturing models. In addition, it states the objective of this study.

Chapter 2 details the thermo-kinetic model development. This model consists of two thermal models: mass transfer and reaction kinetic model. The first thermal model estimates the injected acid temperature at the well bottom hole to account for acid heat exchange with the surroundings during injection inside the wellbore tubing. The second thermal model estimates the acid temperature in the fracture as it changes due to the exothermic reaction of acid with carbonate and heat exchange with the surrounding formation. The mass transfer and reaction kinetic models estimate acid spending profile as it flows inside the fracture and reacts with its surfaces. From this model, the gain in fracture width defined by the amount of rock dissolved and fracture half-length defined by acid penetration is determined, after which, the fracture conductivity is calculated.

Chapter 3 presents the results and discussion of the thermo-kinetic model sensitivity analysis. In this chapter, the effect of different parameters on acid fracture geometry and conductivity was investigated. The parameters that were examined here are the injection rate, initial fracture average width, injection time, reaction temperature, initial acid concentrations, and leakoff velocity. The results are presented in graph format. These graphs show acid penetration distance inside a fracture, acid concentration profile along the fracture, fracture width gained by the acid, and acid temperature profile function of the tested parameters. Most of the figures are given in both linear and semi-log plot in order to give a clear reading of the model output: the semi-log plot presents model results better at fracture length closer to the wellbore and the linear plot presents these results better away from the wellbore.

Chapter 4 presents formulations, sensitivity analysis and application of the optimisation models for acid fracturing. Critical treatment parameters affecting acid fracture geometry were investigated in Chapter 3; these treatment parameters—injection rate of acid, injection time, and acid initial concentration—can be manipulated to design an optimum acid fracture treatment to maximise a design objective function. The objective functions of the optimisation model include maximum net present value, maximum cumulative production, and maximum net present value with minimum treatment cost.

Chapter 5 presents the conclusions and recommendations from this study and areas of research for future studies recommendation.

## **CHAPTER 2**

# ACID FRACTURING MODEL DEVELOPMENT

To learn if improvements can be made to previous models, here is a summary of these models assumptions: The first model solves the convection-diffusion equation analytically in a one-dimensional space. The assumptions of this model are: (1) flow of fluid is steady state; (2) reaction rate is infinite and concentration at fracture walls is zero; (3) flow along the fracture is laminar and incompressible; (4) acid viscosity is constant; (5) leakoff rate is constant; and (6) there are no external forces, such as gravity effect (Williams and Nierode 1972). The second model's basic assumptions are the same as those of the first, except that acid reaction is assumed to be controlled by both mass transfer and reaction rate (Roberts and Gun 1975).

The third model solves for fracture geometry and acid transport by convection and diffusion simultaneously. The acid model solves the two dimensions convection-diffusion equation by a one-dimension approximation averaged over the fracture width. The assumptions of this model are: (1) a steady-state concentration profile forms downstream; (2) zero acid concentration along the fracture walls; and (3) constant leakoff velocity (Lo and Dean 1989).

The fourth model's basic assumptions are: (1) 2D unsteady-state convectiondiffusion equation; (2) the z-variation (vertical diffusion) in the equation is negligible; (3) the effect of diffusion along the fracture length is negligible because acid transport by convection is much larger than by diffusion; (4) acid viscosity is not a function of concentration; (5) acid is an incompressible, single phase fluid; and (5) the reaction has no effect on velocity profiles (Settari 1993). The fifth model's assumptions are the same as for the third model. It couples 2D fluid flow along fracture length and height with 3D acid diffusion-convection equation. The mass transfer by diffusion along the fracture, *x*-direction, is neglected since it is small compared to mass transfer by fluid flow in that direction. The mass transfer by diffusion vertically, *z*-direction is neglected. The continuity equation in 2D is solved to account for the fluid density differences caused by concentration changes and dissolved rock (Remero et al. 1998).

In summary, first two models solve the diffusion convection equation analytically and assume zero acid concentration at boundaries. The third model solves the 2D diffusion-convection equation using a 1D approximation. It assumes zero acid concentration at fracture walls, an infinite reaction. The former three models assumed steady-state concentration profile. The fourth and fifth models did not account for the effect of acid temperature changes on diffusion rate and mass transfer of acid inside the fracture by including thermal models. In addition, none of the models included a thermal model to estimate acid temperature changes during injection in the well tubular, which estimates acid temperature at fracture entrance at wellbore.

## 2.1 Model Scheme

This study's model scheme as presented in Figure 2.1 consists of: two thermal models and a mass transfer-reaction kinetic model. The first thermal model estimates the injected acid temperature inside the wellbore as it exchanges heat with the surroundings. The second thermal model estimates the acid temperature inside the fracture as it changes due to the exothermic reaction of acid with carbonate and heat exchange with the surrounding formation. The mass transfer -kinetic model estimates; the acid concentration profile as it flows inside the fracture and reacts with its walls, gain in fracture width by acid-rock dissolution and acid penetration distance. The model scheme updates the changes of injected acid reaction rate and mixing coefficient for each time step along the fracture resulting from changes in acid temperature. In addition, the acid flow velocity is updated to account for the changes of fracture width by acid reaction. This scheme will be called the thermo-kinetic model through the rest of the thesis. The models details are discussed in the following sections.



Figure 2-1 Acid fracturing model scheme for the sensitivity analysis study

# 2.2 Thermal Models

To estimate temperature of injected acid entering into the fracture, injected acid temperature at the wellbore bottomhole needs to be calculated first. The injected fluid temperature changes during injection in the well tubular due to the heat exchange with formation, in addition to heat from friction (which is small and negligible). As the injected acid enters the fracture, it reacts with the formation materials at fracture wall surfaces and generates heat because the acid reaction with carbonate is an exothermic reaction. In addition, injected acid gains heat from the formation by heat exchange. Since acid reaction rate and mass transfer rate are temperature dependent, it is essential to predict the acid temperature as it penetrates inside the fracture

and reacts. Therefore, the two thermal models to estimate the injected acid temperature inside the wellbore and in the fracture are used as follows:

## 2.2.1 Thermal Model inside the Wellbore

By the law of energy conservation, the amount of heat injected plus gained or lost to the surround plus generated by reaction should equal to the amount of heat stored. To estimate the temperature of acid entering into the fracture, the acid temperature at the wellbore bottomhole need to be estimated first. Acid surface temperature is usually known or even controlled. Acid's temperature inside the well tubular changes during injection by friction and heat exchange with the tubular and surrounding formations. To estimate this change, several numerical models have been developed (Raymond 1969; Sump and Williams 1973). These models were developed mainly to predict mud and cement temperatures; however, with slight modification, they can be used to predict injected acid temperature.

An analytical model used in this study was developed by Schechter in 1992. It takes into account injected fluids heat exchange with well surrounding formation, which controls the injected fluid temperature.

This analytical model is as follows (Schechter 1992):

$$T(z,t) = T_{f.surf.temp} e^{-\alpha z} + (T_{ATemp} - \frac{g_G}{\alpha})(1 - e^{-\alpha z}) + g_G z$$

$$\alpha = \frac{2\pi r_w}{q_f.\rho_f.Cp_f} \sqrt{\frac{k_{th.m}.\rho_m.Cp_m}{\pi t}}$$
2-1

$$\label{eq:Where:} \begin{split} \text{T}(z,\,t) \text{ is fluid temperature at depth } z \text{ and injection time t} \\ \text{T}_{f.surf.temp} \text{ is injected fluid surface temperature, } ^{\circ}\text{F} \\ \text{T}_{A.Temp} \text{ is ambient temperature, } ^{\circ}\text{F} \\ \text{g}_{G} \text{ is geothermal gradient, } ^{\circ}\text{F}/\text{ft} \\ \text{K}_{th.f} \text{ is thermal conductivity of fluid, Btu/ft-min-}^{\circ}\text{F} \end{split}$$

Cp<sub>f</sub> is heat capacity of injected fluid, Btu/lb.°F Cp<sub>m</sub> is heat capacity of formation matrix, Btu/lb.°F K<sub>fh.m</sub> is formation thermal conductivity, Btu/min.ft.°F  $\rho_f$  is density of injected fluid, lb/ft<sup>3</sup>  $\rho_m$  is density of formation matrix, lb/ft<sup>3</sup> t is injection time , minutes r<sub>w</sub> is wellbore tubing radius, ft z is formation depth, ft

The injected fluid temperature when arriving at depth z and after injection time *t* is a function of geothermal gradient, which indicates formation initial temperature at depth z, thermal conductivity of injected fluid, and formation. The thermal conductivity indicates how easily heat will transfer from the higher temperature to the lower one; heat capacity of injected fluid and formation matrix defines the capability of these substances to store heat, and injection time defines the heat exchange time. The thermal resistance of pipe and casing is very small (conductivity is high) compared to formation rock and injected fluid thermal resistivity. Therefore, only formation matrix and injected fluid conductivity was accounted for in this model. Appendix G presents the results of applying this model when an HCl acid is injected inside a wellbore. It details the effect of formation temperatures and acid injection rates on acid temperature at the well bottom hole.

## 2.2.2 Thermal Model inside the Fracture

When acid is injected into a fracture, as in Figure 2.2, the acid reacts with the formation materials at fracture surface and generates heat since acid's reaction with carbonate is an exothermic reaction. In addition, injected acid gains or losses heat into the formation by heat exchange. Since HCI acid reaction and diffusion rates are temperature dependent, it is essential to predict its temperature as it penetrates inside the fracture and reacts. Whitsitt and Dysart (1969) developed a model for temperature distribution of non-reactive fracture fluid. This model was modified by Lee and Roberts (1980) to

allow for the heat of reaction in the energy balance equations, as follows (Schechter 1992):

$$\frac{\pi}{4}\rho.Cp.h.w.\frac{\partial T}{\partial t} + q\rho Cp \frac{\partial T}{\partial x} + 2h.Mr \cdot \Delta H_r + 2h.H_f = 0 \qquad 2-2$$

The term with  $\partial T/\partial t$  is the stored heat, and  $\partial T/\partial x$  is the change in fluid temperature along the interval  $\partial x$ . The other terms are; acid density  $\rho$ , acid specific heat capacity Cp, fracture height h, fracture width w, injection rate q, mass of rock reacted Mr, heat of reaction  $\Delta H_r$ , and heat flux from or to the formation by heat exchange H<sub>f</sub>.

Injected acid temperature at fracture entrance from the wellbore is acid temperature at well bottomhole estimated by the thermal model inside the wellbore, as discussed previously. Injected acid temperature at fracture centre is not changing with respect to fracture wall direction; it is changing only along the fracture x-axis as follows:

$$\partial T / \partial y = 0 \ at \ y = 0$$
 2-3

Heat transfers into or out off the injected acid equals the heat of reaction plus heat flux from the formation, as follows:

$$M_{r} \cdot \Delta H_{r} + H_{f} = K_{h} (T_{w} - T)$$
2-4

The terms  $\Delta H_r$  and  $H_f$  are heat generated by reaction and heat lost or gained by injected acid to or from the formation, respectively. Mass of acid reacted is  $M_r$  and  $K_h$  is injected fluid heat transfer coefficient estimated, as detailed in Appendix F.

The heat gained or lost by fracturing fluid through fracture walls to the formation ( $H_f$ ) is calculated as follows (Dysart GR and Whitsitt 1967; Whitsitt and Dysart 1969):

$$H_{f} = \sqrt{\frac{\rho_{m}.Cp_{m}.k_{th.m}}{\pi.t}}.(T - T_{w})(e^{-\zeta^{2}} - \sqrt{\pi}.\zeta.erfc(\zeta))$$

$$\zeta = \frac{v_{l}.Cp}{2(1 - \phi)}\sqrt{\frac{t}{\rho_{m}.Cp_{m}.k_{th.m}}}$$
2-5

Where:  $\rho$  is injected fluid density, lb/ft<sup>3</sup>  $\rho_m$  is formation density, lb/ft<sup>3</sup> Cp is injected fluid thermal capacity, Btu/lb.°F Cp<sub>m</sub> is formation thermal capacity in, Btu/lb.°F T is injected fluid temperature at wellbore, °F T<sub>w</sub> is injected fluid temperature at fracture wall, °F k<sub>th,m</sub> is formation heat conductivity, Btu/ft-min-°F v<sub>l</sub> is fluid leakoff velocity, ft/min *t* is injection time, minutes ø is porosity in fraction *erfc* is complementary error function

The numerical formulation of the thermal model inside the fracture, Equation 2-2, using implicit forward finite difference methods that is unconditionally stable is as follows:

Discretisation of coordinates:  $(x, t) \rightarrow (x_i, t_n)$ 

$x_i = x_0 + i\Delta x$	
$t_i = t_0 + n\Delta t$	2-6
$T(x_i; t_n) = T_i^n$	

Here, *i* is grid number (space step) along the fracture,  $\Delta x$  is grid length, *n* is the time step number, and  $\Delta t$  is injection time interval. The finite difference approximation for 1st derivative is as follows:

$$\frac{\partial T}{\partial x} = \frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta x} + O(\Delta x) \text{ and } \frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^{n}}{\Delta t} + O(\Delta t)$$
 2-7

Applying finite difference approximation in Equation 2-7 to fluid temperature Equation 2-2 as follows:

$$q \rho Cp \frac{T_{i+1}^{n+1} - T_{i}^{n+1}}{\Delta x} + 2h \cdot M r_{i}^{n} \Delta H_{r} + 2h \cdot H_{f}^{n} + \frac{\pi}{4} \rho Cp \cdot h w \cdot \frac{T_{i}^{n+1} - T_{i}^{n}}{\Delta t} = 0$$
2-8

Rearrange the above equation as follows:

$$T_{i+1}^{n+1} - T_{i}^{n+1} = -\frac{2h\Delta x}{q\rho Cp} \cdot Mr_{i}^{n}\Delta H_{r} - \frac{2h\Delta x}{q\rho Cp} \cdot H_{fi}^{n} - \frac{\pi}{4}\frac{\Delta x}{\Delta t}\frac{\rho Cp \cdot hw}{q\rho Cp} (T_{i}^{n+1} - T_{i}^{n})$$
2-9

Since injection rate is defined as fracture cross-section multiplied by average velocity as follows:

$$q = \overline{v_x} h w$$
 2-10

Using the injection rate definition Equation 2-10 in Equation 2-9 as follows:

$$T_{i+1}^{n+1} - T_{i}^{n+1} = -\frac{2\Delta x}{v_{x}w \ \rho Cp} \cdot Mr_{i}^{n} \Delta H_{r} - \frac{2\Delta x}{v_{x}w \ \rho Cp} \cdot H_{fi}^{n} - \frac{\pi}{4v_{x}} \frac{\Delta x}{\Delta t} (T_{i}^{n+1} - T_{i}^{n})$$
2-11

Define A and B as follows:

$$A = \frac{2}{v_x \rho C p} \frac{\Delta x}{w}$$
  
$$B = \frac{\pi}{4v_x} \frac{\Delta x}{\Delta t}$$
  
2-12

Replace A and B into Equation 2-11 and rearrange as follows:

$$T_{i+1}^{n+1} - T_{i}^{n+1} = -A \left( M r_{i}^{n} \Delta H_{r} + H_{fi}^{n} \right) - B \left( T_{i}^{n+1} - T_{i}^{n} \right)$$
 2-13

The final shape of the numerical equation is as follows:

$$(B - 1)T_{i}^{n+1} + T_{i+1}^{n+1} = BT_{i}^{n} - A(Mr_{i}^{n}\Delta H_{r} + H_{fi}^{n})$$
 2-14

Solving the M x N matrix of Equation 2-14 looks as follows:

$$= \begin{pmatrix} B_{0}^{n} T_{0}^{n} - A_{0}^{n} (Mr.\Delta Hr_{0}^{n} + H_{f_{0}}^{n}) \\ B_{1}^{n} T_{1}^{n} - A_{1}^{n} (Mr.\Delta Hr_{1}^{n} + H_{f_{1}}^{n}) \\ B_{2}^{n} T_{2}^{n} - A_{2}^{n} (Mr.\Delta Hr_{2}^{n} + H_{f_{2}}^{n}) \\ \vdots \\ B_{M-2}^{n} T_{M-2}^{n} - A_{M-2}^{n} (Mr.\Delta Hr_{M-2}^{n} + H_{f_{M-2}}^{n}) \\ B_{M-1}^{n} T_{M-1}^{n} - A_{M-1}^{n} (Mr.\Delta Hr_{M-1}^{n} + H_{f_{M-1}}^{n}) \\ B_{M}^{n} T_{M}^{n} - A_{M}^{n} (Mr.\Delta Hr_{M}^{n} + H_{f_{M}}^{n}) \end{pmatrix}$$
2-15

With the following initial and boundary conditions:

• Temperature of acid at fracture mouth is the one calculated by the thermal model inside the wellbore.

$$T(x = 0, t) = T_0^{n \in N} = T_{BHT}$$
 2-16

• Temperature of acid at fracture walls is calculated by the heat balance equation 2-4 as follows:

$$K_{h}(T_{wi}^{n} - T_{i}^{n}) = Mr_{i}^{n}\Delta H_{r} + H_{fi}^{n}$$
2-17

Equations 2-14 and 2-17 can be solved with the conditions of Equation 2-16 for acid average bulk and at wall temperatures of each grid along the fracture. A total grid number of 100 (space steps) along the fracture, with  $\Delta x$  of 10 ft grid length, and 300 time steps of  $\Delta t$  equals 0.10 minute used in this implicit scheme for the sensitivity analysis presented in chapter 3.



Figure 2-2. Fracture geometry for acid injection formulation showing direction of fluid flow, fracture height, length, and width.

## 2.3 Reaction Kinetic Model

Consumption of HCI acid when reacting with carbonate formation can be mass transfer limited or reaction rate limited as discussed in Appendix E. The consumption rate of acid controls how deep live acid can penetrate in a fracture before being totally spent, which defines fracture geometry width and length. Therefore, mass transfer of acid inside the fracture must be included in the model. The general form of mass balance transfer in 3-D is as follows (Schechter 1992):

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z} + D_A \frac{\partial^2 C_A}{\partial x^2} + D_A \frac{\partial^2 C_A}{\partial y^2} + D_A \frac{\partial^2 C_A}{\partial z^2}$$
 2-18

The term  $\partial C_A / \partial t$  is the accumulated acid inside the fracture. The terms with velocities v<sub>x</sub>, v<sub>y</sub>, and v<sub>z</sub> are acid mass transfer by forced convection toward x, y, and z directions, respectively. All terms with diffusivity constant,  $D_A$ , are acid mass transfer by diffusion.

If the fracture length is along the x-axis, its width is along the y-axis and its height is along z-axis, as presented before in Figure 2.2. Then, the shape of the mass transfer equation inside a fracture is reduced, as discussed in Appendix H to:

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + 2v_y \frac{\partial C_A}{\partial y} + 2D_A \frac{\partial^2 C_A}{\partial y^2}$$
 2-19

The term  $v_x$ ,  $v_y$  and  $D_A$  are as defined before in equation 2.18. The mass transfer in the y-direction is multiplied by two to account for the two fracture surfaces.

The mass transfer toward fracture walls by diffusion (y-direction) is replaced by apparent mass transfer coefficient  $K_{g}$ , which was proposed by Roberts and Guin (1975), as follows:

$$K_{g}(C_{A} - C_{w}) = D_{A}(\partial^{2}C_{A}/\partial y^{2})$$
 2-20

where  $C_A$  is acid bulk concentration and  $C_w$  is acid concentration at fracture wall. For apparent mass transfer coefficient calculation, see Appendix E.

Substituting Equation 2.20 into Equation 2.19, the mass transfer inside a fracture equation becomes as follows:

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + 2v_y \frac{\partial C_A}{\partial y} + 2K_g (C_A - C_w)$$
 2-21

With the following boundary conditions:

<u>Concentration of acid at wellbore (x=r<sub>w</sub>)</u>

If  $r_w$  is wellbore radius and t is injection time, acid concentration at the wellbore is initial injected acid concentration.  $C_A = C_i$  for all t > 0 and  $x = r_w$ .

<u>Concentration of acid at fracture wall (x>r<sub>w</sub>) has two case scenarios</u>

**I.** If acid reaction rate at fracture surfaces is higher than its mass transfer rate to the fracture surfaces by diffusion and convection - as in the case of HCl reaction with limestone at temperature greater than 200 °F with low leakoff rate - then the reaction at fracture walls is mass transfer limited, the mass of acid transfers toward fracture walls will react instantaneously as it arrives at the fracture's two surfaces (at y = +w/2 and y = -w/2) and can be expressed as follows:

$$k_{r}C_{w}^{\alpha} = v_{l}(C_{A} - C_{w}) + K_{g}(C_{A} - C_{w})$$
2-22

where  $C_w$  is the acid concentration at fracture wall,  $k_r$  is reaction rate constant, and  $\alpha$  is reaction order. Equations 2.20 and 2.22 can be solved for acid concentrations inside the fracture and at fracture walls. Equation 2.22 is solved iteratively if reaction order is not 1.

**II.** If mass transfers of acid to the fracture walls by diffusion and convection is faster than acid reaction rate - as in the case of HCI reaction with carbonate at temperature less than 200 F when the flow is laminar - then the amount of acid reacts at the fracture surface plus the amount leaks off equals the amount transfers to the fracture walls. This can be expressed as follows:

$$k_{r}C_{w}^{\ \alpha} + v_{l}C_{w} = v_{l}(C_{A} - C_{w}) + K_{g}(C_{A} - C_{w})$$
2-23

where  $C_w$  is the acid concentration at fracture wall,  $k_r$  is reaction rate constant, and  $\alpha$  is reaction order and  $v_l$  is leakoff velocity. Equations 2.20 and 2.23 can be solved for acid concentrations inside the fracture and at fracture walls.

The assumptions used in this model are: (1) 2D unsteady-state convectiondiffusion equation; (2) acid transport by convection is much larger than by diffusion along the fracture length, diffusion is negligible along fracture length; (3) acid is an incompressible, single phase fluid; (4) leakoff acid concentration  $C_l$  is less than of equal acid concentration  $C_w$  at fracture wall; (5) Newtonian fluid, acid viscosity is function of fluid temperature but not flow rate; (6) reaction and diffusion rate is function of fluid temperature; and (7) homogenous leakoff velocity.

The numerical formulation using implicit finite difference methods for this kinetic model is discussed in Appendix H.

# 2.4 Validation of Developed Thermal and Kinetic Models

The kinetic model developed in this study was validated by two means. First, by mass balance where the mass of rock dissolved estimated by the model, equations 2-22 and 2-23 is compared to the theoretical amount as in Appendix D. It was found that these two values are in excellent match. The estimated dissolved rock from the model varies from 99.8 to 97 % of the theoretical one as the gridding along the fracture length was increased from 1.0 to 10 ft, respectively. The time step was kept constant 0.10 minutes.

The second method for kinetic model validation was comparing its result for HCI concentration profile and acidized width to that published by the Settari model (1993). The model input data used by Settari and used in this model are presented in Table 2.1. Figure 2.3 presents the acid concentration profile, penetration distance, and gained width by acid from Settari model, while Figure 2.4 presents the result from this developed model. The penetration distance and acidized width by the two models are in excellent match. Penetration distance was 170 ft and acidized width at wellbore was 0.055 inches. However, there is a slight difference in acid concentration profile where it decreased faster in Settari's model. This is a result of homogenous leakoff assumption in this model.

The developed thermal model was validated by comparing its result with Lee and Robert's thermal model result for HCl reaction with limestone and dolomite at formation temperature of 66 °C (151 °F) (Lee and Roberts 1980). Their model input data is presented in Table 2.2 which was used in this model for the comparison. Figure 2.5 presents the result of their model, while Figure 2.6 presents the result from this developed model for the same kinetic data and reaction conditions.

The temperature profile for the HCl reaction with limestone and dolomite are in excellent match when looking at the maximum acid temperature and the distance from the wellbore at which injected acid reaches this temperature. Maximum temperature of acid from this thermal model was 90 °C at 100 ft

from the wellbore when reacted with limestone and 87 °C at 240 ft when reacted with dolomite. Maximum temperature values from Lee and Robert's model are the same except at 90 ft and 250 ft from the wellbore. The acid temperature profile from this study thermal model increased by the heat of reaction and dropped back to the formation's original temperature faster than the case of Lee and Robert's model. An explanation for this is that, in their model, they did not account for the change in acid temperature at well bottomhole during injection and the change in acid heat transfer coefficient with the change in acid temperature.

## 2.5 Fracture Conductivity by Acid Fracturing

Nierode and Krunk (1973) published a correlation relating rock embedment strength to closure stresses to provide fracture conductivity following acid fracturing treatment. This correlation was re-evaluated by Nasr-El-Din et al. (2006) and they reported two different correlations for limestone and dolomite formation as follows:

The ideal fracture conductivity ( $wk_{fi}$ ) of a fracture with an average fracture width  $w_A$  is defined as follows:

$$wk_{fi} = 9.36 \times 10^{13} \left( \frac{w_A}{12} \right)$$
 2-24

The actual fracture conductivity (wk<sub>f</sub>) is defined as follows:

$$wk_f = C_1 \exp(-C_2 \sigma_{CL})$$
 2-25

where  $wk_f$  is fracture conductivity in millidarcy-inches,  $\sigma_{CL}$  is closure stress in psi. The terms C<sub>1</sub> and C<sub>2</sub> are defined as follows:

#### For limestone formation

$$C_{1} = 0.165(wk_{fi})^{0.8746}$$
2-26  
for RES < 20,000 psi  

$$C_{2} = (26.567 - 8.6341\ln(RES)) \times 10^{-3}$$
2-27  
for RES > 20,000 psi  

$$C_{2} = (2.9795 - 0.202\ln(RES)) \times 10^{-3}$$

#### For dolomite formation

$$C_1 = 13.26(wk_{fi})^{0.5592}$$
 2-28

for RES < 20,000 psi  

$$C_2 = (8.6383 - 0.7479 \ln(RES)) \times 10^{-3}$$
  
for RES > 20,000 psi  
 $C_2 = (2.3147 - 0.1513 \ln(RES)) \times 10^{-3}$ 

Fracture conductivity index is the ratio of fracture conductivity to formation conductivity ( $wk_{fracture}/Lk_{formation}$ ). Where  $wk_{fracture}$  is fracture conductivity estimated using the previous correlation, *L* is fracture half-length penetrated by acid and  $k_{formation}$  is formation permeability. Fracture conductivity index defines the efficiency of a fracture. If the index is less than one, then the amount of fluid that can flow from formation into the fracture is greater than the capability of the fracture, the fluid flow inside the fracture is chocked and flow is turbulent. However, if this index is greater than one, then this fracture can allow more fluid to flow inside it more than that the formation can supply.

The conductivity of the fracture ( $wk_{fracture}$ ) gained by acid is function of fracture width, rock embedment strength (*RES*) and closure stresses. The increase in injected acid volume would increases fracture conductivity by dissolving more rock from fracture wall surfaces and increasing fracture gained width. However, treating formation with very large acid volume causes

loss in rock embedment strength and decreases fracture conductivity. The reported loss in *RES* is up to 38 per cent. This loss in rock embedment strength is function of rock type, contact time, acid type and strength (Pournk 2008; Nasr-El-Din et al. 2008). The main problem with *RES* loss by acid is repeatability where none of the results could be repeated, as noticed by Pournk (2008) and Nierode and Krunk (1973). Therefore, based on 50 % loss in rock embedment strength, fracture conductivity in a limestone formation was estimated using Equations 2.24 to 2.27. An average fracture width was calculated using the relation average width =  $(\pi/5)^*$ width at wellbore. Fracture conductivity index (FCI) is function of fracture conductivity, formation permeability and acid fracture length.

The effects of formation permeability and formation embedment strength on fracture conductivity index (FCI) versus closure stress of fracture shown in Figure 2.7 are presented by Figures 2.8 and 2.9. Even defining an optimum FCI is not agreed on, a FCI less than one means formation can deliver more fluid than the fracture can handle and will cause turbulence flow inside the fracture which results in emulsions and erosion of fracture surfaces. While FCI greater than one means the fracture can handle more fluid than that the formation can deliver. However, if FCI is very high, i.e. greater than 100, it means the fracture is ether wider than needed or its length is shorter than wanted.

Figure 2.8 shows the FCI of the same fracture geometry presented by Figure 2.7 versus closure stress and function of formation permeability. FCI is high in low permeability formation (k=1 md) compared to moderate and high permeability formation. In addition, FCI decreases as closure stress increase. It is greater than 100,000 at 1000 psi closure stress and decreases to 400 as closure stress increases to 10,000 psi for the 1 md permeability formation.

The effect of rock embedment strength on fracture conductivity is presented by Figure 2.9. Formation with higher rock embedment strength will hold better against closure stress. Therefore, FCI is greater with higher rock embedment strength. The effect of rock embedment strength increases as closure stress increase. Doubling the rock embedment strength from 100 to 200 kpsi resulted in almost two and four times increase in FCI at 5000 and10000 psi closure stress, respectively. Formation with low rock embedment strength is not recommended for acid fracturing jobs, as seen from Figure 2.9, formations with rock embedment strength of 15 kpsi would collapse under 4500 psi closure stress and higher, FCI less than one.

Variable	Value, Field units		
Acid strength	10	wt. %	
Injection rate	20	bbl/min	
Acid volume	200	bbl	
Average leakoff velocity	0.0010	ft/min	
Acid density	62.4	lb/ft <sup>3</sup>	
Fracture height	100	ft	
Formation density	169.2	lb/ft <sup>3</sup>	
Diffusion coefficient @ 60 °F	6.6E-6	ft²/min	
Reaction constant @ 60 °F	1.97E-3	ft/min	
Reaction order, $\alpha$		0.5	

Table 2.1 – Model Input Data for Comparison with Settari Kinetic Model (1993)

Variable	Value, SI units		Value, Field units				
Acid strength	28	wt. %	28	wt. %			
Injection rate	1.59	m³/min	10.0	bbl/min			
Acid specific heat	1.0	cal/g.ºC	1.0	Btu/lb.°F			
Acid volume	151.4	m³	952	bbl			
Wellbore temperature	27	°C	80.6	°F			
Fracture width	0.635	cm	0.26	in			
Fracture height	15	m	50	ft			
Bottom hole temp, case 1	66	°C	151	°F			
Bottom hole temp, case 2	149	°C	300	°F			
Formation specific heat	0.22	cal/g.ºC	0.22	Btu/lb.°F			
Formation density	2.6	g/cm <sup>3</sup>	162.3	lb/ft <sup>3</sup>			
Formation thermal conductivity	5.78E-03	cal/s.cm.ºC	2.33E-02	Btu/ft-min-⁰F			
Acid thermal conductivity	1.24E-03	cal/s.cm.ºC	5.00E-03	Btu/ft-min-⁰F			
Heat of reaction	9500	cal/g.mol	17089	Btu/lb-mol			
Reaction constants, k <sub>r</sub>							
HCI-Limestone @ 100 °F	5.53E-05	cm/s	1.09E-04	ft/min			
HCI-Dolomite @ 200 °F	3.81E-04	cm/s	7.50E-04	ft/min			
Reaction order, α							
HCI-Limestone	0.441						
HCI-Dolomite	0.669						

## Table 2.2 – Model Input Data for Comparison with the Lee and Robert Thermal Model Result (1980)



Figure 2-3. HCI concentration and acidised width vs. distance along fracture (Settari 1993).



Figure 2-4. HCl acid concentration and acidised width vs. distance along fracture.



Figure 2-5. Temperature profile vs. distance along the fracture (Lee and Roberts 1980).



Figure 2-6. Temperature profile vs. distance along the fracture.



Figure 2.7 - Fracture width profile used for fracture conductivity index analysis



Figure 2.8 – Fracture conductivity index versus closure stress and function of formation permeability, fracture height = 100 ft, reservoir pressure = 4300 psi, closure pressure = 1000 - 10000 psi, rock embedment strength = 100 kpsi, formation permeability = 1.0 - 100 md



Figure 2.9 – Fracture conductivity index versus closure stress and function of formation embedment strength; reservoir pressure = 4300 psi, closure pressure = 1000-10000 psi, rock embedment strength = 15 - 100 kpsi, formation permeability = 100 md

# CHAPTER 3

# **RESULTS AND DISCUSSION**

## 3.1. Parameters Sensitivity

After the development and validation of the thermo-kinetic model in chapter 2 it was used to study the effects of different parameters on acid penetration distance, acid concentration, fracture width gained by the acid and acid temperature profiles along the fracture half length. The parameters were examined are; acid injection rate, injection time, initial acid concentrations, leakoff velocity, Initial fracture average width, and bottomhole temperature. The effects of these parameters on acid fracture geometry over their range encountered in Middle East oil and gas fields have not been studied. The determination and understanding of these parameters effects on acid fracture geometry is an essential guideline for acid fracturing treatment design.

To study the effects of one of the parameters, it was changed over the expected field range while the other parameters were kept constant. An ideal case of zero leakoff velocity is compared in each study with leakoff velocity of 0.001 ft/min. Table 3.1 presents tested parameters with their field range used in the model and Table 3.2 presents the kinetic and thermal data of HCI acid and carbonates formation. The ranges of tested parameters were selected to mimic most of their ranges that would be encountered in the oil and gas fields. To study the profile of acid fracture geometry with time, gained width by acid, and penetration distance, the injection time was varied from 0.10 to 30 minutes with 0.10-minute time intervals. The 30 minutes was enough to study the effects of injection time and to determine maximum acid penetration distance. This injection time was increased to 90 minutes in the design and optimisation (Chapter 4) to cover data from field experience (Nasr-El-Din, Al-Driweesh et al. 2003). The injection rate range was from 20 to 60 bbl/min into the fracture two wings. An average maximum injection rate of 45 bbl/min is common field practice (Economides and KG Nolte 2000) however up to 55 bbl/min injection rate was reported by Nasr-El-Din et al.

(2003). The leakoff velocity used range from ideal case of zero leakoff to a very high rate of 0.003 ft/min. Settari (1993) reported HCl acid leakoff velocity of 0.001 ft/min; three floods of that rate were used as a maximum leakoff rate velocity to study its effect in acid facture geometry. An average initial fracture width up to 0.30 inch was studied. A common reservoir's temperature varies between 150 to 250 °F (Schechter 1992). However, in this study, the tested reservoir temperature range was from 100 to 300 °F. The lowest temperature was selected to investigate the case of water injector wells that have been under injection for a long time where their bottom hole temperature can be low and the highest range was selected to investigate the case of high temperatures gas wells where their bottom hole temperatures can be up to 280 °F (Nasr-El-Din, Al-Driweesh et al. 2003). The most common acid fracturing fluid concentration is 15, 21, and 28 wt % HCl (Economides and KG Nolte 2000). Therefore, the range of acid concentration in this study was selected to be between 10 and 30 wt %.

Figures 3.1 to 3.27 present the different parameters sensitivity analysis results from the HCI reaction thermo-kinetic model with limestone. In these figures, the legend with capital letter *L* is indication of leakoff case and those legends without letter *L* is for the ideal case of zero leakoff. For example, the terms *C-tinj-L*, C is acid concentration profile in wt %, *tinj* is injection time in minutes, the parameter under study, and L indicates the case of leakoff. In case of no leakoff, *C-tinj* simple is used in the figure for concentration. Similar legend simples are used for fracture width (*w-tinj-L* and *w-tinj*), acid bulk temperatures (*T-tinj-L* and *T-tinj*), and acid temperatures at fracture wall (Tw-tinj-L and Tw-tinj). In the legend, the term *tinj* is replaced with parameter under study; for example, for leakoff velocity effect study the term  $u_y$  is used, injection rate is *q*, initial average fracture width is *wi*, initial acid concentration is *Ci*, and formation original temperature is BHT.

In most of the figures, acid concentration, fracture gained width by acid reaction, and acid temperature profiles are given in both linear and semi-log plot. The reason is to give a clear reading of the model output: Semi-log plot presents model results clearer closer to the wellbore in the first 100 ft from the wellbore, while the linear plot presents these profiles clearer away from the wellbore.

## 3.1.1. Effects of Injection Time

Figure 3.1 (a & b) presents the concentration profile of injected 15 wt % HCl acid along fracture half length in limestone formation as a function of time with and without leakoff. The remaining parameters are given in the figure title. Increasing the injection time means increasing the volume of injected acid, which results in deeper acid penetration. The loss of acid by the leakoff results in faster acid concentration curve with leakoff case from that without leak off is a function of injection time. With the increase in injection time, acid concentration curve deviation increases. Moreover, the acid concentration profile of the leakoff case is always less than or equals to that without leakoff.

Figure 3.2 (a & b) presents the fracture width gained by the acid-rock reaction as function of injection time. The increase of injection time results in more fracture width gain by rock dissolution since increasing the injection time is an increase in reaction time and injected acid volume. Leakoff results in more rock dissolved closer to the wellbore compared to the case of no leakoff. The leakoff forces more acid toward the fracture surface, forced convection, which causes faster acid consumption by acid loss into the formation and reaction with the rock closer to the wellbore. Therefore, less acid concentration away from the wellbore is noticed in the case of leakoff as presented in Figure 3.1. This leads to less gain in fracture width away from the wellbore compared to the cases without leakoff, as in Figure 3.2. In conclusion, leakoff has a negative impact on acid fracturing, it results in less acid penetration distance (shorter fracture length), larger fracture width close to the wellbore and thinner deep in the fracture.

Figures 3.3 and 3.4 present the injected acid bulk and at fracture wall's temperature profiles, along the fracture half length, function of injection time. As acid is being injected, injected acid exchanges heat with the well tubular and surrounding formation, as discussed in the thermal model inside the

wellbore section. Acid arrives at fracture opening at lower temperatures with the increase in injection time because the wellbore and surroundings formation have been cooled down by previously injected acid (see Appendix G). Acid bulk temperature and at fracture walls are different by a few degrees in the early injection time. As injection time increases and more acid arrives into the fracture at lower temperatures, the difference in acid bulk and at wall temperature is less than 1 degree. Temperature of injected acid increases back to formation temperature by heat of reaction generated and heat exchange with the formation. It can be seen that the temperature of acid in the case of leakoff is slightly higher than that of acid with no leakoff in the first 10 to 200 ft of the fracture half length. In the case of leakoff, more acid is reacting closer to the wellbore, which results in more heat generated and a higher temperature. However, after the first 200 ft from the wellbore, there is less acid with leakoff case than without leakoff (see acid concentration Figure 3.1). Therefore, in the case of no leakoff injected acid starts generating more reaction heat, which results in a slightly higher temperature in the fracture half length 200 to 400 ft. As the acid flows deeper into the fracture half length and is consumed, in both cases-with and without leakoff-the temperatures come to the same values by heat exchange with the formation.

## 3.1.2. Effects of Leakoff Velocity

To investigate the effects of leakoff velocity on acid penetration and fracture gained width by the acid reaction, different leakoff velocities were tested and the other treatment parameters were kept constant. Leakoff velocity was increased from zero (the no leakoff ideal case) to very high leakoff velocity of 0.003 ft/min. Figure 3.5 (a & b) presents acid concentration profiles along the fracture half length. It is clear that increasing the leakoff velocity results in more acid loss and consumption at shorter distance along the fracture. The acid penetration distance is a strong function and inversely proportional to leakoff velocity. Acid penetration distance ends shortly, as seen from the drop in acid concentration profile with the leakoff rate increase.

Increasing the leakoff velocity results in more rock dissolved closer to the wellbore, which causes a wider fracture at the wellbore, shown in Figure 3.6 (a & b). However, the gained fracture width by acid reaction drops sharply with leakoff velocity increase. Decreasing the leakoff velocity produces a narrower fracture width at the wellbore compared to the case with leakoff, but it has deeper penetration distance. Therefore, for the same amount of injected acid, controlling the leakoff would result in longer acid penetration and a better fracture width profile. Leakoff rate of acid can be reduced by increasing acid viscosity using gelled acid, emulsified, or visco-elastic acid systems (Nasr-El-Din, Al-Driweesh et al. 2003).

Figure 3.7 (a & b) presents the acid temperatures ( $T_w$  at fracture wall and T bulk temperature) profile along the fracture half-length as function of acid leakoff velocities. The increase in acid leakoff forces more acid to flow toward the fracture walls, where it reacts and generates heat. Therefore, the temperature of injected acid is higher closer to the wellbore with higher leakoff rate compared to lower or zero leakoff rates. This increase in acid temperature results in an increase in acid reaction since reaction rate is a function of temperature and increases with temperature increase. Thus, an increase in leakoff rate results an increase in acid consumption closer to the wellbore by two mechanisms: the first one is forced convection of acid temperature. Leakoff rate should be minimised to allow deeper acid penetration.

## 3.1.3. Effects of Injection Rates

To study the effect of acid injection rate on fracture geometry, the injection rates were increased from 15 to 30 bbl/min, and the total volume of injected acid was kept constant at 150 barrels. Figures 3.8 to 3.10 present the effects of injection rates on acid concentration, fracture gained width by acid reaction, and acid temperature inside the fracture. The other parameters are as presented in figure titles. Increasing the acid injection rate results in an increase in penetration distance as indicated by the acid concentration

profile, Figure 3.8 (a & b). As acid injection rate increases, acid flow velocity inside the fracture increases, enabling the acid to penetrate deeper in the fracture before it reacts completely. The effect of leakoff velocity is also presented and, as previously discussed, leakoff always results in faster acid consumption and less acid penetration.

At lower injection rates, the acid has a relatively longer time to travel to the fracture wall surface where it will react. Therefore, fracture width gained by acid reaction is wider closer to the wellbore at lower acid injection rate, Figure 3.9 (a & b). However, the gain in fracture width by acid reaction is less deep in the fracture at lower injection rates. This can be explained: By the time acid flows deeper in the fracture at lower injection rates acid concentration is already low compared to acid concentration at the same distance for a higher injection rate, Figure 3.8 (a & b). And therefore, it has a lower dissolving power. Therefore, for the same amount of acid injected, a lower injection rate will result in a wider fracture width at the wellbore, a sharp drop in that gained width, and less penetration distance. Vice versa, increasing the injection rate will give a better width profile and a deeper penetration distance. A similar scenario can be seen in the case with leakoff. The presence of leakoff results in a wider fracture width at wellbore, a sharper drop in that fracture width, and shorter acid penetration distance.

Figure 3.10 (a & b) presents the temperature of injected acid along the fracture half length. Injection rates initially affect the temperature at which the acid will enter the fracture at the wellbore. At a lower injection rate, acid will reach fracture opening at the wellbore at a higher temperature as seen previously in the thermal model inside the wellbore (Appendix G). Therefore, the acid temperature as it arrives at the fracture opening at the wellbore is different, as presented in Figure 3.10 (a & b). In addition to acid arrival at slightly higher temperatures at lower injection rates, which makes its reaction rate faster, injection at lower injection rates will give acid a relatively longer time to transfer to the fracture walls, where it will react and generate heat. Therefore, acid temperature is higher at lower injection rate by different mechanisms: First, acid arrives at the fracture opening at a relatively higher

temperature than when injected at a higher rate. Second, acid has a relatively longer time to move toward to the fracture wall surfaces and react, by which it generates heat and increases its temperature even more. Thus, acid should be injected at the maximum allowable injection rate to penetrate deeper inside the fracture and create a better width profile.

## 3.1.4. Effects of Fracture Initial Average Width

The effects of fracture initial average width on acid concentration, gained width and temperature are presented in Figures 3.11 to 3.13 for cases with and without leakoff. The fracture initial average width was increased from 0.10 to 0.30 inch, leakoff velocities of zero and 0.001 ft/min were considered. The other parameters are given in the figures titles. If the acid injection rate is kept constant, increasing the initial fracture width will result in reducing the acid flow velocity inside the fracture, giving the acid a relatively longer time to move towards the fracture surface and react. Doubling the initial fracture width from 0.10 to 0.20 inch, for the same injection rate, results in reducing the acid flow velocity by half inside the fracture.

Figure 3.11(a & b) presents acid concentration profiles along the fracture as function of initial fracture average width. It shows faster acid spending with the increase in initial fracture average width. Doubling the initial width from 0.10 to 0.20 inches reduces the acid penetration distance inside the fracture half length, as indicated from acid concentration; penetration distance was reduced from 700 to 500 ft for the same injection rate of 15 bbl/min. This reduction in acid penetration distance is caused by more acid reacting closer to the wellbore and creating a wider fracture, shown in Figure 3.12 (a & b). As more acid reacts closer to the wellbore. This is noticed from acid temperature profiles, shown in Figure 3.13 (a & b). Since acid flow velocity inside the fracture is reduced by the increase in initial fracture width, acid temperature increases faster by having relatively longer time to react, generate, and exchange heat with its surroundings. Therefore, acid temperature profile is higher in the case of a wider initial fracture width. Thus,

a relatively narrow initial fracture width is recommended for deeper acid penetration inside the fracture and better gained fracture width.

## 3.1.5. Effects of Formation Initial Temperature

The effects of fracture initial temperature on injected acid concentration, gained width and acid temperature are presented in Figures 3.14 to 3.16. Figure 3.14 (a) presents the acid concentration profile when injected for 10 minutes into a fracture at different initial bottomhole formation temperature (BHT). The increase in fracture initial temperature results in faster acid reaction and sharp decrease in acid concentration. For the acid injection into a fracture at 100 °F initial formation temperature and after 10 minutes of acid injection, acid spent at almost 380 ft inside the fracture. On the other hand, when the acid was injected into a fracture of 400 °F initial formation temperature and for 10 minutes of injection, acid spent at almost 380 ft inside the fracture.

The acid concentration profile at different formation temperatures, in the case of no leakoff and leakoff are presented in Figure 3.14 (b), leakoff causes the acid to spend faster. However, as formation temperature increases, the effect of leakoff decreases, as indicated by the acid concentration profiles. The deviation between acid concentration profiles with leakoff and without leakoff, at the same initial formation temperature, decreases with formation initial temperature increase.

The fast reaction of acid at higher formation temperature, as seen from the concentration profile, causes wider fracture width closer to the wellbore but shorter acid penetration distance, shown in Figure 3.15 (a & b). The increase in formation temperature from 100 to 300 °F results in increasing fracture width at the wellbore from 0.05 to 0.30 inches, Figure 3.15 (a) and reducing acid penetration from 380 ft to 120 ft, Figure 3.15 (b). Therefore, the increase in formation temperature with the increase in leakoff velocity results in wider but shorter acid fracture half length. A pre acid injection conditioning of
fracture to lower its temperature is recommended to allow the acid deeper penetration distance inside the fracture and better gained width profile.

Figure 3.16 (a & b) presents the temperature profile of injected acid along the fracture half length when injected into a fracture at different initial formation temperature. A formation temperature is proportional to its depth and geothermal gradient. The increase in formation temperature causes the acid to arrive at the fracture mouth in that formation at a higher temperature as seen from acid temperature profile, Figure 3.16 (a). This is caused by heat exchange during injection inside the well tubular (see Appendix G). The reaction of acid is faster at a higher temperature since its reaction rate is strong function of temperature. In the case of formation at low temperatures, such as the ones at initial formation temperature of 100 and 150 °F, the acid temperature increases by heat of reaction to a temperature higher than formation temperature, and then it drops back to formation temperature by heat flux into formation. While in the case of formation at moderate to high temperatures, acid temperature increases gradually by both heat of reaction and heat exchange with the formation to the formation original temperature.

## 3.1.6. Effects of Injected Acid Initial Concentration

Increasing acid concentration would increase acid reaction rate since the reaction rate equals reaction constant multiplies by concentration to the power of reaction order ( $\alpha$ ) as discussed in the Reaction Kinetics Section 1.1.1. For a first order reaction, doubling the concentration would double the reaction rate, and for a lower order reaction—such as a reaction order of half doubling the concentration—would increase the reaction rate by half. When acid reaction rate increases, the acid spends faster close to the wellbore, which result in a shorter fracture half-length and wider width at the wellbore.

Figure 3.17 (a & b) presents injected acid concentration profile along the fracture when injected acid concentrations vary from 5 to 30 wt %. The increase in acid initial concentration results in faster acid spending. In the first 10 ft away from the wellbore, the 30 wt % HCl acid decreased to 25.5 wt %.

Therefore, 15 % of the initial injected acid reacted. On the other hand, the 25 wt %HCl acid is reduced to 22.5 wt % at the same distance from the wellbore. This indicates only 10 % of the initial acid reacted. The increase in injected acid initial concentration causes more gained fracture width closer to the wellbore but less penetration distance, shown in Figure 3.18 (a & b), because of higher acid dissolving power and faster acid spending.

To investigate acid penetration distance further, injection time was increased to 30 minutes. The results are presented in Figure 3.19 for the concentration profiles and in Figure 3.20 for the gained width profile. These figures show there is a maximum penetration distance that an acid would penetrate regardless of its initial concentration. For the tested condition, all acids would be consumed at fracture half-length of almost 400 ft. The clear effect of initial concentration would result in wider fracture, shown in Figure 3.20. The effect of leakoff on acid concentration profile and gained fracture width, compared to the non leakoff case, function of initial acid concentration are presented by Figures 3.21 to 3.22. Leakoff, as seen previously, always results in faster acid spending, less acid penetration and wider fracture width at the wellbore.

The increase in acid concentration causes an increase in its dissolving power, the amount of rock that can be dissolved increases, (see Appendix D). Therefore, more reaction heat is generated. This increase in heat generation causes a higher increase in acid temperature inside the fracture. Thus, the increase in initial acid concentration causes faster acid consumption by two mechanisms: first by definition of reaction rate and then by heat of reaction generation.

Temperature profiles of injected acid inside the fracture, function of initial acid concentration, with and without leakoff, are presented in Figure 3.23 (a & b). The injection of acid at high initial concentration results in a higher acid temperature inside the entire fracture half length due to heat of reaction generation compared to injected acid with lower initial concentration. The temperature of the 30 wt% injected HCI acid increased from almost 110 °F at

wellbore to 180 °F at 100 ft inside the fracture. On the other hand, when the injected acid initial concentration was 15 wt%, it is temperature at the same distance, 100 inside the fracture was almost 138 °F.

Leakoff causes more acid to move toward the fracture surfaces where it reacts and generates heat. Therefore, temperature of injected acid with leakoff case is higher closer to wellbore, Figure 3.23 (a & b). However, as injected acid move away from the wellbore inside the facture, there will be more acid in the case of zero leakoff, which will react and generate more heat than the acid with leakoff, which is already less in concentration because of its faster reaction closer to wellbore. Far away from wellbore, all acid temperature goes back to formation original temperature by heat exchange with the nearby formation. Therefore, to create a shorter and wider acid fracture, the injected acid concentration should be high, and to create narrow and longer fracture the acid concentration should low.

## 3.2. Lithology Effects on Acid Fracture Geometry

Acid reacts faster with limestone formation than it does with dolomite because the reaction rate of HCI with limestone is faster than that with dolomite, as detailed in Appendix E. The thermo-kinetic model was used to study the effect of lithology on acid spending and fracture geometry. The model kinetic data are as presented in Table 3.2. The results from the model for injected 15 wt % HCl into a fracture in dolomite and limestone formation are presented in Figures 3.24 to 3.27. All other parameters are as in the figures title. Figures 3.24 (a & b) and 3.25 (a & b) present acid concentration profile when the acid is injected into the fracture in dolomite and limestone formation, respectively. There is a difference in these acid concentration profiles with respect to lithology; for example, at the end of 10 minutes injection time and at 100 ft from the wellbore, acid concentration is approximately 11.5 wt % in the dolomite formation fracture, shown in Figure 3.24 (a & b). On the other hand, acid concentration decreased to approximately 9 wt % when injected into the limestone formation fracture, shown in Figure 3.25 (a & b). The acid penetration distance is a function of formation lithology. In the case of dolomite formation and after 10 minutes of injection time, acid penetrated up to almost 600 ft, shown in Figure 3.24 (a & b). In the case of limestone formation, injected acid is spent at almost 480 ft, seen in Figure 3.25 (a & b).

Figure 3.26 (a & b) presents the temperature profile of injected acid in dolomite and limestone formation, respectively. Temperature of injected acid increases by heat of reaction generated and heat exchange with the formation. The change in acid temperature is faster and slightly higher in the case of acid reaction with limestone, Figure 3.26 (b). The amount of heat generated depends on the amounts of acid reacted and formation rock dissolved. Since more acid reacts in the case of limestone formation compared to dolomite for the same reaction time, as seen previously from the change in concentration profiles, more heat is generated; this increases the acid temperature in limestone formation faster than in dolomite formation.

Figure 3.27 (a & b) presents the increase in fracture width by acid reaction. The volume-dissolving power of HCl acid is lower when reacting with dolomite formation compared to limestone, as detailed in Appendix D. Its volume-dissolving power in dolomite formation is 88 % of its volume-dissolving power in limestone. In addition, acid reaction rate with dolomite is slower than its reaction rate with limestone. Therefore, for the same volume of acid injected and the same reaction time, less acid would react and would dissolve less formation rock from the fracture surfaces in the case of dolomite formation. The fracture gained width by acid reaction after 10 minutes of injection in dolomite formation is approximately 0.045 inch at the wellbore, seen in Figure 3.27 (a). This fracture width decreases to almost nil at fracture half-length of 600 ft. However, after the same injection time, the gained fracture width at the wellbore in limestone formation is 0.054 inch, Figure 3.27 (b). This gain in fracture width decreases deeper inside the fracture and finishes at fracture half-length of almost 480 ft.

In conclusion, the fracture geometry in carbonates formation by acid fracturing is strong function of lithology. In the case of limestone formation,

acid reacts faster, which makes it penetrate less and create a wider fracture. While in the dolomite formation, it reacts slower, which enables it to penetrate deeper in the fracture before it is spent completely. Therefore, for the same acid volume and treatment conditions, acid creates a narrower and longer fracture in dolomite formation compared to the one in limestone formation.

Parameter, unit	Range of study	
Injection time, minutes	0.1-30	
Injection rate into one wing, bbl/min	10-30	
Leakoff velocity, ft/min	0.0-0.003	
Initial average fracture width, in	0.10-0.30	
Bottom hole temperature, °F	100-400	
Acid initial concentration, wt%	10-30	
Porosity, %	15	
Reservoir average permeability, md	1-100	
Formation depth , ft	6000-12000*	
Average reservoir pressure, psi	2500-5000*	
Rock embedment strength. kpsi	15-500	
Treatment zone height, ft	100	
Injected fluid surface temperature, °F	70*	
Ambient temperature, °F	80*	

 Table 3.1 – Parameters Range Used in the Study for the

 Thermo-Kinetic Model (Economides and KG Nolte 2000)

• Average Middle East Data

Table 3.2 – Kinetic and Thermal Data	(Lee and Roberts 1980; Settari 199	3)
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Variables	Values	
	Limestone	Dolomite
HCI reaction rate constant, k <sub>r</sub> , ft/min	8.13E-4	2.461E-3
	@ 100 °F	@ 200 °F
HCl reaction order, α	0.441	0.669
Activation energy, E, Btu/lb.mol	41372	72671
Heat of reaction, $\Delta H_r$ , Btu/lb	170.72	92.728
HCI diffusion rate coefficient, D <sub>A</sub> , ft <sup>2</sup> /min	6.60E-6 @ 60 °F	
HCI heat capacity, Btu/lb.°F	1.0	
HCl thermal conductivity, , Btu/min.ft.°F	233E-3	
Formation heat capacity, Btu/lb.°F	0.22	
Formation thermal conductivity, Btu/min.ft.°F	5.0E-3	





Figure 3.1 – Acid concentration along the fracture as function of injection time with & without leakoff; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{\circ}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 30 min; (a) semi-log for concentration closer to wellbore, (b) linear for concentration away from wellbore.





Figure 3.2 – Fracture width gained by acid as function of injection time; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{\circ}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 30 min; (a) semi-log for width closer to wellbore, (b) linear for width away from wellbore.



Figure 3.3 – Acid bulk temperature as function of injection time; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 30 min.



Figure 3.4 – Acid temperature at fracture wall as function of injection time; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{\circ}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 30 min.





Figure 3.5 – Acid concentration as function of leakoff velocity; injection rate = 15 bbl/min, leakoff velocity =0.0 - 0.003 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for concentration closer to wellbore, (b) linear for concentration away from wellbore.





Figure 3.6 – Fracture width gained by acid as function of leakoff velocity; injection rate = 15 bbl/min, leakoff velocity =0.0 - 0.003 ft/min, bottom hole temperature =  $200 \,^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for width closer to wellbore, (b) linear for width away from the wellbore.





Figure 3.7 – Acid temperature along the fracture as function of leakoff velocity; injection rate = 15 bbl/min, leakoff velocity =0.0 - 0.003 ft/min, bottom hole temperature = 200 °F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for temperature closer to wellbore, (b) linear for temperature away from the wellbore.





Figure 3.8 – Acid concentration along the fracture as function of injection rate; injection rate = 15 - 30 bbl/min, leakoff velocity =0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration =  $15 \,$  wt%, injection time =  $15 - 30 \,$  min; (a) semi-log for concentration closer to wellbore, (b) linear for concentration away from the wellbore.





Figure 3.9 – Fracture width gained by acid as function of injection rate; injection rate = 15 - 30 bbl/min, leakoff velocity =0.0 & 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 15 - 30; (a) semi-log for width closer to wellbore, (b) linear for width away from the wellbore.





Figure 3.10 – Acid temperature along the fracture as function of injection rate; injection rate = 15 - 30 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time =  $15 - 30 \,$  min; (a) semi-log for temperature closer to wellbore, (b) linear for temperature away from wellbore.





Figure 3.11 – Acid concentration along the fracture as function of initial average fracture width; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.10 - 0.30 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for concentration closer to wellbore, (b) linear for concentration away from the wellbore.





Figure 3.12 – Fracture width gained by acid as function of initial average fracture width; injection rate = 15 bbl/min, leakoff velocity =0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{\circ}$ F, initial fracture average width = 0.10 - 0.30 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for width closer to wellbore, (b) linear for width away from the wellbore.





Figure 3.13 – Acid temperature as function of initial average fracture width; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature =  $200 \,^{0}$ F, initial fracture average width = 0.10 - 0.30 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 30 min; (a) semi-log for temperature closer to wellbore, (b) linear for temperature away from the wellbore.





Figure 3.14 – Acid concentration along the fracture as function of fluid reaction temperature; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 100 - 400 °F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 10 min; (a) with leakoff, (b) compared leakoff case with no leakoff.





Figure 3.15 – Fracture width gained by acid as function of initial formation temperature; injection rate =15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature = 100 - 400 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration =15 wt%, injection time = 10 min; a) semi-log for width closer to wellbore, b) linear for width away from wellbore.





Figure 3.16 – Acid temperature as function of initial formation temperature; injection rate = 15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature =  $100 - 400^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 10 min; (a) semi-log for temperature closer to wellbore, )b) linear for temperature away from the wellbore.





Figure 3.17 – Acid concentration along the fracture as function of initial concentration; injection rate = 15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 -30 wt%, injection time = 10 min; (a) semi-log for concentration closer to wellbore, (b) linear for concentration away from the wellbore.





Figure 3.18 – Fracture width gained by acid as function of initial concentration; injection rate = 15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature =  $200 \, {}^{\circ}F$ , initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 -30 wt%, injection time = 10 min; (a) semi-log for width closer to wellbore, (b) linear for width away from the wellbore.



Figure 3.19 – Acid concentration as function of initial concentrations; injection rate = 15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 -20 wt%, injection time = 30 min.



Figure 3.20 – Fracture width gained by acid as function initial concentrations; injection rate = 15 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 -20 wt%, injection time = 30 min.



Figure 3.21 – Acid concentration along the fracture as function of initial concentration and leakoff velocity; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 - 30 wt%, injection time = 10 min.



Figure 3.22 – Fracture width gained by acid as function of initial concentration and leakoff velocity; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 200 °F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 - 30 wt%, injection time = 10 min.





Figure 3.23 – Acid temperature as function of initial concentrations; injection rate = 15 bbl/min, leakoff velocity = 0.0 & 0.001 ft/min, bottom hole temperature = 200 <sup>0</sup>F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 5 - 30 wt%, injection time = 10 min; (a) semi-log for temperature closer to wellbore, (b) linear for temperature away from the wellbore.





Figure 3.24 – Acid concentration profile in dolomite formation as function of injection time; injection rate = 20 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature =  $200 \,^{\circ}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 10 min; (a) concentration along fracture half length, (b) concentration after the first 10 ft away from the wellbore.





Figure 3.25 – Acid concentration profile in limestone formation as function of injection time; injection rate = 20 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature = 200 0F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 10 min; (a) concentration along fracture half length, (b) concentration after the first 10 ft away from the wellbore.





Figure 3.26 – Acid bulk temperature along fracture half length as function of injection time; injection rate = 20 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature =  $200 \,^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 1 - 10 min; (a) fracture in dolomite formation, (b) fracture in limestone formation.





Figure 3.27 – Fracture width gained by acid along fracture half length; injection rate = 20 bbl/min, leakoff velocity = 0.001 ft/min, bottom hole temperature =  $200 \, {}^{0}$ F, initial fracture average width = 0.20 in, fracture height = 100 ft, acid initial concentration = 15 wt%, injection time = 10 min; (a) fracture in dolomite formation, (b) fracture in limestone formation.

## **CHAPTER 4**

## **OPTIMISATION OF ACID FRACTURING**

The main goal of an optimisation algorithm is to minimise or maximise an objective function. In acid fracturing, the objective function of an optimisation model is usually maximum net present value (NPV), the present value of future cash flows minus the purchase price. However, other objective functions can be set as the goal of optimisation. Aside from maximum NPV, these objective functions include in acid fracturing maximum cumulative production and maximum net present value with minimum treatment cost, in case there is a shortage in cash money at the time of treatment. There are many optimisation techniques available that have been reviewed by Rahman (2002) and Karen (2005). While optimisation of proppant fracturing has been well developed (Economides and Nolte 2000; Rahman 2002; Karen 2005), little attention has been given to the optimisation of acid fracturing (Kalfayan 2007). All previous acid fracturing optimisation work is based mainly on laboratory experiments (Walter and Keeney 1978; Domelen 1992; Navarrete et al. 1998) and field experience (Olsen and Karr 1986; Al-Qahtani and Rahman 2001; Xugang et al. 2003) rather than in a mathematical approach.

Different parameters can affect acid fracture geometry, including acid injection rate, injection time, leakoff rate, initial concentration, and formation initial temperature. The effects of these parameters have been identified in the sensitivity analysis in Chapter 3. Acid injection rate and leakoff optimum design for acid fracturing were identified from the sensitivity analysis to be maxim and minimum achievable, respectively. However, the injection time and acid concentration can be manipulated to design an optimum acid fracture treatment. For a given reservoir, these treatment parameters need to be designed such that a favourable acid fracture geometry is created to maximise the design objective function. The optimum design must satisfy various operational limitations (pump capacity, tubular strength, pressure rating of surface equipment, etc.).

This chapter presents the optimisation model formulations of acid fracturing in gas reservoir. The main objectives of the chapter are to: (1) present the proposed model formulations; (2) demonstrate the benefits and capabilities of the model to investigate various designs based on different objective functions; (3) perform sensitivity analysis of the effect of reservoir properties on optimum treatment parameters design; and (4) conduct case study using gas wells data from the Middle East.

# 4.1 Acid Fracturing Optimisation Scheme

The overall problem of acid fracturing optimisation is formulated within the framework of the optimisation algorithm in the following sections, which presents the optimisation algorithm formulation, objective functions, design constrains, free design variables, net present value and treatment cost formulation in the economic model, acid fracture geometry, and production model. Figure 4.1 presents this acid fracturing optimisation scheme.

## 4.1.1 Optimisation Algorithm Formulation

An optimisation algorithm was developed by Rahman et al. (2001) for propped hydraulic fracturing integrating with a procedural optimisation algorithm. The updated full description of the optimisation algorithm can be found in Rahman (2006). The general formulation of this method is as follows:

Find

$$\underline{\mathbf{x}} = \begin{cases} \begin{array}{c} x_1 \\ x_2 \\ \mathbf{M} \\ \mathbf{M} \\ x_i \\ \mathbf{M} \\ x_N \end{array} \end{cases}$$

$$4 - 2$$

Subject to bound constraints

$$\left.\begin{array}{ccc}
l_{1} \leq x_{1} \leq u_{1} \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
l_{N} \leq x_{N} \leq u_{N}
\end{array}\right\}$$
4-2

And design constraints

$$\begin{array}{c}
C_{l1} \leq C_{1}(\underline{x}) \leq C_{u1} \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
C_{lM} \leq C_{M}(\underline{x}) \leq C_{uM}
\end{array}$$
4-3

To minimise

$$Z = f(\underline{x}) \tag{4-4}$$

Where,  $\underline{x}$  stands for the vector of free design variables,  $l_i$ 's and  $u_i$ 's are constants' lower and upper limits of free design variables  $x_i$ 's, and  $C_{l_i}$ 's and  $C_{u_i}$ 's are ranges of design constraints,  $C_i(\underline{x})$ 's; N and M are the total number of free design variables and design constraints, respectively.

This optimisation procedure consists of six major steps: (1) generates 'vertices' and forms of a 'compound'; (2) shifts a vertex of a 'compound'; (3) tests for collapse of a 'compound'; (4) deals with a collapsed 'compound'; (5) moves a 'compound'; and (6) terminates the process by convergence tests.

To solve a multi-objective optimisation a general equation applicable to any engineering problem is as follows:

To minimise

$$Z = \sum_{i=1}^{I} \frac{\left(f_i(\underline{x}) - T_i\right)}{D_i} P_i$$
4-5

where  $f_i(\underline{x})$  is the objective function for i-th objective, T<sub>i</sub> is the target value for the i-th objective, D<sub>i</sub> is dividing factor for i-th objective equation, and P<sub>i</sub> is the priority to achieve the i-th objective.

The maximisation of an objective function can be achieved by minimising the negative function for that objective. If a particular objective has no target value, then that objective requires minimisation which is achieved only by equating the corresponding  $T_i$  to zero. Any of the objectives can be excluded by equating the corresponding priority factor to zero.

#### 4.1.2 Objective Functions

The optimisation model was designed for three objective functions: maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost. The maximum NPV objective function is the main design for acid fracturing, which aims to maximise the revenue. The objective function of maximum cumulative production is to maximise cumulative production over a period of time from a fractured well by increasing the fracture conductivity at minimum cost. The third objective function aims to maximise net present value at minimum treatment cost in case of cash shortage at the time treatment needed.

The global optimisation Equation 4.5 is used to formulate various design objective functions for acid fracturing optimisation, as follows:

 Maximise net present value (NPV) over a number of production years. Since there is only one objective to maximise without any target value, *I* = 1, *T*<sub>1</sub> = 0, *P*<sub>1</sub> = 1 and *D*<sub>1</sub> = 1. To maximise NPV, a negative NPV is minimised in Equation 4.5. The optimisation statement is as follows:

minimise 
$$Z = minimise(-NPV)$$
 4-6

 Maximise cumulative production (REC) by maximising total production, G<sub>ρ</sub>, over a number of production years. Similar to NPV maximisation statement, the G<sub>ρ</sub> maximisation statement is as follows:

minimise 
$$Z = \min(-G_n)$$
 4-7

3. Maximise NPV and minimise treatment cost (TC). There are two explicit design objectives in this case: maximise NPV and minimise treatment cost. Therefore, I = 2, values of  $D_1$  and  $D_2$ , are adjusted such that the value of both terms at the right hand side of Equation 4.5 approaches to 0.5,  $P_1$  and  $P_2$  are set to 1 to assign equal priorities to maximise NPV and minimise treatment cost. To maximise NPV, a negative NPV is minimised in Equation 4.5; to minimise TC, a positive TC is minimised, which makes the optimisation statement as follows:

minimise 
$$Z = \text{minimise}\left[\frac{-NPV}{D_1}P_1 + \frac{TC}{D_2}P_2\right]$$
 4-8

#### 4.1.3 Free Design Variables

The effects of different parameters on acid fracture geometry have been studied in detail in Chapter 3. Of those parameters, acid injection rate, injection time, and concentration can be manipulated to achieve an optimum design. The injection rate was kept constant at maximum allowable injection of 45 bbl/min. Increasing the injection rate results in an increase in maximum acid penetration distance inside the fracture, which is an increase in fracture length. Increasing the injection rate does not cause an increase in treatment cost. The increase in the injection time results in higher fracture conductivity through the increase in acid volume, creating a wider fracture. However, this increase in acid volume causes an increase in its dissolving power. This increase in dissolving power gives higher fracture conductivity by creating a wider fracture. However, increasing the acid concentration does

not improve penetration length. In addition, treatment cost increases with the increase in acid concentration as acid price is a function of its concentration.

The cost of acid fracturing treatment is a direct function of acid volume and acid concentration but *not* a direct function of injection rate or injection time. The increase of fracture conductivity by acid treatment is function of injected acid volume, injection rate, and acid concentration. Fracture length is a function of injection rate and injection time. However, there is a maximum penetration distance by acid as function of formation temperature; after this distance is reached, the continuous injection of acid would result only in an increase in fracture width and treatment cost.

These interconnected relations between acid injection rate, injection time, and acid concentration with fracture geometry and treatment cost allows for designing variables that can be manipulated to achieve an optimum acid fracturing treatment. While the design variables in hydraulic fracturing used by Rahman (2002) include pumping rate, pumping time, end-of-job proppant concentration, and fracturing fluid viscosity; however, in acid fracturing, no proppant is used, and acid viscosity is close to water viscosity, acid injection rate should be maximum to allow for maximum acid penetration distance. Therefore, the injection time and acid initial concentration are modelled as free design variables. Their design ranges are as follows (Economides and Nolte 2000):

- 1. acid injection time; 10  $\leq t_i$ , minutes  $\leq$  90
- 2. acid initial concentration;  $15 \le C_i$ , wt%  $\le 28$

## 4.1.4 Design Constraints

Design and fracture growth control constraints are less in acid fracturing compared to hydraulic fracturing for the following reasons:

1. There is no risk of uncontrolled fracture geometry growth in acid fracturing

because fracture length created by acid fracturing is controlled by reaction kinetic; this is unlike hydraulic fracturing, where the length can be extended to thousands of feet.

- In acid fracturing, there is a maximum penetration length; several hundred feet can be reached by the acid before it reacts completely. This maximum length can be determined by the thermo-kinetic model.
- 3. Acid is not reacting with the confining layers; therefore, fracture height equals the pay-zone height. In addition, lower net pressure is encountered in acid fracturing since the acid's viscosity is low compared to proppants fracturing; this is unlike hydraulic fracturing, where fracture height can extend to the confining layer and needs to be controlled.
- 4. There is no risk of proppant screen-out or flow back in acid fracturing since no proppants are involved in acid fracturing. Fracture conductivity by acid fracturing is provided by the channels and rough surfaces created by dissolved rock from the fracture surfaces; in hydraulic fracturing, proppants provide this conductivity.

Design constraints are formulated based on operational limitations (see Table 4.1) and fracture growth control requirements for the initial fracture geometry generated using 2D PKN-C model (see Appendix J), as follows:

#### Operational Limitations and Fracture Growth Control Requirements

- 1.  $1.0 \le C_{1} \le 10$ : where  $C_{1} = (HP_{av} \times P_{eff})/HP_{reqd}$ ;  $HP_{av}$  is the horsepower available from the pump to be used and  $P_{eff}$  is the pump efficiency factor. This constraint bound ensures that the horsepower required,  $HP_{reqd}$  (see Appendix I) to deliver net fracture pressure,  $p_{net}$ , is within the capacity of the pump. The fracture net pressure is estimated as a function of design variables and formation properties. The upper bound is chosen arbitrarily to meet the optimisation algorithm requirement. This is also true for below constraints, unless mentioned otherwise.
- 1.0 ≤ C<sub>2</sub> ≤ 10: where C<sub>2</sub> = P<sub>burst</sub>/(P<sub>surf</sub> x SF); P<sub>burst</sub> is the burst strength of the tube in use, P<sub>surf</sub> is the pressure developed at the surface, and SF is a safety factor. The constraint bound ensures that the pressure developed
inside the tube at the surface level is below the burst strength of the tube during injection.  $P_{surf}$  is calculated based on the required net fracture pressure accounting for static head and dynamic frictional loss (see Appendix I).

- 1.0 ≤ C<sub>3</sub>(<u>x</u>) ≤ 15: where C<sub>3</sub>(<u>x</u>) = P<sub>seqp</sub>/P<sub>surf</sub>, P<sub>seqp</sub> is the minimum pressure within the rated pressures of various surface equipment in the injection line. The lower bound of this constraint ensures that the pressure developed at the surface does not exceed the pressure capacity of the critical equipment.
- 1.25 ≤ C<sub>4</sub> ≤ 50: where C<sub>4</sub> = L<sub>f</sub>/h<sub>f</sub>. This constraint lower bound ensures that the fracture half-length is always greater than the fracture height, which is a basic assumption in the 2D PKN-C (see Appendix J) fracture geometry model.
- 5.  $1.0 \le C_5 \le 10.0$ : where  $C_5 = P_{fcr}/P_{treat}$ ;  $P_{fcr}$  is the formation critical pressure and  $P_{treat}$  is fracture treatment pressure. The lower bound ensures the treatment pressure is below the formation critical pressure to avoid uncontrolled fracture growth.
- 1.0 ≤ C<sub>6</sub> ≤ 10.0: where C<sub>6</sub> = Δσ<sub>h</sub>/p<sub>net</sub>; Δσ<sub>h</sub> is the difference between the minimum horizontal stresses in the pay-zone and the bounding layers. The lower bound is to ensure fracture net pressure does not induce excessive fracture height growth into the bounding layers.
- 7.  $1.0 \le C_7 \le 10.0$ : where  $C_7 = \Delta \sigma / (0.7 \times p_{net})$ ;  $\Delta \sigma$  is the difference between maximum and minimum horizontal in-situ stresses in the pay-zone. The lower bound ensures the fracture net pressure does not initiates secondary fractures.

# 4.2 Acid Fracture Geometry

Initial fracture geometry is needed as an input for the acid fracture model. This initial fracture geometry is generated using 2D PKN-C model (see Appendix J). The input from the 2D PKN-C model to the acid fracture model includes fracture initial width, height, and length. The fracture geometry generated using the 2D PKN-C model is based on acid maximum penetration distance that can be estimated using Terill's equation (Eq. 4.9) at maximum acid injection rate. Acid fracture geometry (length and width) is then estimated as a function of the design variables. From the estimated fracture width and length, fracture conductivity is calculated as discussed in Chapter 2 and used in the production model. The acid fracture geometry is determined as follows:

Fracture half-length by acid is estimated from the Terill's (1965) equation. The mean acid concentration function of fracture half-length, x is as follows (Schechter 1992):

$$\frac{\bar{c}}{c_o} = \sum G_n \left(1 - \frac{N_{\text{Re}^*}}{N_{\text{Re}}} \xi\right)^{2\lambda_n^2 / 3N_{Pe}}$$
4-9

Here  $\overline{c}$  and  $c_o$  are average acid concentration at x and initial acid concentration, respectively. The terms N<sub>Re</sub> and N<sub>Re</sub><sup>\*</sup> are the Reynolds numbers for acid flow along the fracture and leakoff flow towards fracture walls defined as follows (Schechter 1992):

$$N_{\rm Re} = \frac{\overline{v_x} \, \overline{w} \, .\rho}{2\mu} = \frac{q \, .\rho}{4.h.\mu} \tag{4-10}$$

$$N_{\rm Re^*} = \frac{\overline{v_l} \, \overline{w} \, . \rho}{2\mu} \tag{4-11}$$

where *q* is acid injection rate, h and w are fracture height and average width,  $\overline{v_l}$  is average leakoff velocity and equals 0.001 ft/min (Settari 1993),  $\overline{v_x}$  is acid average velocity along the fracture equals injection rate into one wing divided by fracture cross section area, and  $\rho$  and  $\mu$  are acid density and viscosity function of acid concentration, as discussed in Appendix D. N<sub>Pe</sub> is the Peclet Number, defined as follows (Schechter 1992):

$$N_{Pe} = \frac{\overline{v_{I}} \, \overline{w}}{2D_{A}} \tag{4-12}$$

where  $D_A$  is diffusion coefficient equals  $6.01 \times 10^{-6}$  ft<sup>2</sup>/min at 60 °F (Settari 1993).

The term  $\xi$  is fracture half-length ratio to average width,  $\xi = 2x / \overline{w}$ , G<sub>n</sub> and  $\lambda_n$  are defined as follows (Schechter 1992):

$$G_{n} = \sum_{i=0}^{3} g_{i,n} N_{Pe}^{i} + \sum_{i=1}^{2} h_{i,n} N_{Re^{*}}^{i}$$

$$4-13$$

$$\lambda_{n} = \sum_{i=0}^{3} \overline{g}_{i,n} N_{Pe}^{i} + \sum_{i=1}^{2} \overline{h}_{i,n} N_{Re^{*}}^{i}$$

The terms  $g_{i,n}$ ,  $\overline{g}_{i,n}$ ,  $h_{i,n}$ ,  $\overline{h}_{i,n}$  are constants with values as presented in Table 4.2.

With the assumption of steady-state and by material balance, the flow into one wing of the fracture equals the fluid loss, as follows (Schechter 1992):

$$q/2 = \overline{v_x} \cdot \overline{w} \cdot h = 2\overline{v_1} \cdot h \cdot L$$
 4-14

where q is total injection rate, h is fracture height, and L is fracture half-length.

In addition, the Reynolds number terms in Equation 4.9 can be replaced by their definitions in Equations 4.10 and 4.11, as follows:

$$\frac{N_{\text{Re}^*}}{N_{\text{Re}}}\xi = \frac{\frac{\overline{v_l} \cdot \overline{w} \cdot \rho}{2\mu}}{\frac{q \cdot \rho}{4.h \cdot \mu}} (\frac{2x}{\overline{w}}) = \frac{\overline{v_l} \cdot \overline{w} \cdot \rho}{2\mu} \frac{4.h \cdot \mu}{q \cdot \rho} (\frac{2x}{\overline{w}}) = \frac{4.h \overline{v_l}}{q} x$$
 4-15

Replacing the term q in Equation 4.15 with its value from Equation 4.14 and cancelling the equal terms, the Reynolds number term become as follows:

$$\frac{N_{\text{Re}^*}}{N_{\text{Re}}}\xi = \frac{x}{L}$$
4-16

Using Equation 4.16 in Equation 4.9 as follows:

$$\frac{\bar{c}}{c_o} = \sum G_n (1 - \frac{x}{L})^{2\lambda_n^2 / 3N_{Pe}}$$
4-17

The mean acid concentration is now a function of fractional distance along the fracture. Solving Equation 4.17 for the fractional distance ratio along the fracture x/L with  $\overline{c}/c_o = 0.1$ , the acid penetration distance then can be estimated using Equation 4.14 in the following form:

$$L = \frac{q}{4\overline{v_{l}}.h}$$
 4-18

The average fracture width is calculated as follows (Williams, Gidley et al. 1979):

$$\overline{w} = \frac{X V}{2(1-\phi).h.L}$$
 4-19

where X is acid volume dissolving power as discussed in Appendix D, V is total acid volume injected into the fracture two wings, and  $\emptyset$  is formation

porosity. Equations 4.17, 4.18, and 4.19 can be solved for acid fracture geometry.

# 4.3 Production Model

A production model is needed to determine gas cumulative production from the acid fractured well. The production model for a fracture well in gas reservoir was reviewed by Rahman M. M., et al. (2002). To estimate the cumulative production from a well, a constant production rate is needed. However, production rate changes with time as reservoir and bottom hole flowing pressures change during production. If the well can be produced with a constant bottomhole flowing pressure, then adjusted production rates can be estimated with a regular time interval. These production rates will vary with time due to declining reservoir pressure with production and can be used to determine the cumulative production (Agarwal, Carter et al. 1979).

Assuming the well is producing at a constant bottom hole flowing pressure, since reservoir pressure will decline with production, production rate under this condition will decline as well. To adjust for constant flow rate, the total production life is defined as cumulative of a small time interval. After each cumulative period, the average reservoir pressure and gas properties are evaluated as functions of cumulative production up to that period and then used to estimate the constant production rate during these small time intervals, as follows.

The small time interval is defined as  $\Delta t$ , and successive time steps are indexed as i = 0, 1, 2, 3,..., n. At the beginning of production, i = 0, and initial reservoir condition parameters are used for production with bottom hole pressure. During  $\Delta t$  period in hour, the cumulative production during this period will be qg x ( $\Delta t/24$ ) where qg is average gas production rate over the period  $\Delta t$ . Then average reservoir pressure can be calculated, which will be less than the initial pressure due to production. This new calculated average reservoir pressure will be used to calculate the average gas properties. At the next time step, i = 1, average pressure and gas properties calculated for the previous time step are used to calculate a production rate that is constant over the next  $\Delta t$ . The procedure will be repeated, and the total cumulative production,  $G_p$ , can be estimated over the total production period. The transient production rate is used until the pseudo-steady state condition becomes active, after which the pseudo-steady state condition is used. Transition between these conditions and the relationships for average gas properties in every time step are presented in Appendix K.

# 4.4 NPV and Treatment Cost

Maximise net present value, NPV, over a number of years can be set as an objective function used to formulate acid fracturing design optimisation. The NPV is formulated as follows:

$$NPV = \sum_{n=1}^{NY} \frac{R_n}{(1+i)^n} - TC$$
 4-20

where,  $R_n$  is the revenue generated (US\$) at year *n*, *NY* is the total number of years during which revenue to be generated, *i* is the discount rate, and *TC* is the treatment cost (US\$). The revenue,  $R_n$  in the year *n* is calculated as the product of the total production,  $G_{pn}$  (Mscf) at the *n*-th year and an average gas price (US\$/Mscf). The cumulative production  $G_p$  is estimated using the production model, as discussed in Appendix K.

The treatment cost is a direct function of the total volume of the fracturing pad fluid and acid, pumping cost, and fixed cost. The fixed cost is included to cover equipment hire and other treatment expenses. The pad fluid is used to initiate and propagate the fracture in front of the acid. It is designed for acid maximum penetration length (which can be estimated by the acid fracture geometry model), while the acid is used to dissolve rock formation from fracture surfaces creating conductive channels. These channels provide conductive pathways for the reservoir fluids production. The formulation of total treatment cost is as follows:

$$TC = P_{pf} \times V_{pf} + P_{acid} \times V_{acid} + P_{pump} \times HP_{av} + FxC$$

$$4-21$$

Here, *TC* is the treatment cost (US\$),  $P_{pf}$  is the price of fracturing pad fluid (US\$/gal),  $V_{pf}$  is the total volume of pad fracturing fluid (gal),  $P_{acid}$  is the price of acid (US\$/gal),  $V_{acid}$  is the total volume of fracturing acid (gal),  $P_{pump}$  is the pumping price (US\$/hp), HP<sub>av</sub> is available horse power, and *FxC* are the fixed and miscellaneous costs. Economic data for the model is presented in Table 4.3. The fracturing acid price is related to its concentration where 15 and 28 % HCl prices are 2.24 US\$/gal and 4.46 US\$/gal, respectively (Economides and Nolte 2000). Fracturing pad fluid price (US\$/gal), is related to viscosity ( $\mu$ ) in cp, as follows (Rahman 2002):

$$P_{pf} = 0.001\mu + 0.6721$$
 4-22

## 4.5 Results and Discussion

An example of a gas well located in a low permeability carbonate formation is used for sensitivity analysis to illustrate the application of the multi-objective optimisation model. The model input pumping limitations are as presented in Table 4.1, and the economics data are as presented in Table 4.3. In addition, reservoir properties, gas properties, and well data are presented in Table 4.4. The sensitivity analysis includes two sections: The first section is to study the effects of objective functions on the optimum design. These objective functions include maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost. In the second section, the effects of reservoir properties including permeability, temperatures, and rock strength on the optimum design are investigated.

# 4.5.1 Effects of Objective Functions on Optimum Design

Three objective functions—maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost—were investigated by the multi-objective optimisation model for a low permeability gas reservoir. The gas reservoir has an average permeability of 1.0 mD, temperature of 200 °F, and rock embedment strength of 100 kpsi with other properties, as presented in Table 4.4. The optimum design variables—injection rate, injection time and acid concentration—versus the three objective functions are presented in Figure 4.2. The acid fracture geometry, cumulative production, testament cost, and net present value that were obtained from these optimum designs are presented in Table 4.5. In addition, the percentage change in optimum designs for two objective functions—maximum cumulative production and maximum NPV with minimum treatment cost—was compared with that of maximum NPV, and results are presented in Table 4.5.

The optimum design for objective function of maximum NPV (Max. NPV) is obtained by injecting 23 wt % HCl acid at the maximum injection rate of 45 bbl/min for 26 minutes injection time. In addition, designing for objective function of maximum cumulative production (Max. Cum. Prod.) is obtained by injecting 27 wt % HCl acid at maximum operational rate of 45 bbl/min with injection time of 34 minutes. Optimum design for objective function of maximum NPV with minimum treatment cost (Max. NPV - Min. Treat. Cost) is obtained by the injection of 15 wt % HCl acid at the same maximum operational injection rate of 45 bbl/min for 16 minutes injection time. Therefore, optimum design variables are strong functions of the objective functions.

It can be seen from the results in Table 4.5 that the optimum fracture halflength is 347 ft, and this value is the same for all cases. This length is maximum acid penetration distance obtained by injecting at maximum operational injection rate. Optimum fracture width is 0.195 inch for the objective function of maximum NPV. However, this width is increased by 58 % (0.308 inch) when designing for an objective function of maximum cumulative production by injecting larger acid volume (648 gal/ft) with higher concentration (27 wt %). When designing for maximum NPV with minimum treatment cost, the fracture width is reduced by 57 % (0.084 inch) by injecting less acid volume (309 gal/ft) at lower concentration (15 wt %).

The cumulative productions obtained for the two designs—maximum NPV and maximum cumulative production—are almost the same (3.30E+8 MSCF). When designing for maximum cumulative production, a few thousands of standard cubic feet increase in cumulative production over the 10 years production period can be observed. However, there is a 0.61 % decrease in cumulative production when designing for maximum NPV with minimum treatment cost compared to the design for maximum NPV. Cumulative production of 3.28E+8 MSCF is obtained with maximum NPV with minimum treatment cost as objective function.

Changing objective function from maximum NPV to maximum cumulative production resulted in an increase in the treatment cost by 8 % from US\$ 424,972 to US\$ 458,411. This increase in treatment cost is a result of increasing both acid volume and concentration. In addition, NPV decreased from US\$ 2.3040E+08 to 2.3037E+08. However, optimising for maximum NPV with minimum treatment cost as the objective function was obtained by 19 % savings (\$8.21E+4) in treatment cost and only 0.57 % loss in NPV compared to maximum NPV objective function design. This decrease in treatment cost is the result of injecting less acid volume at lower concentration (see Table 4.5).

## 4.5.2 Effects of Reservoir Properties on Optimum Design

The effects of reservoir properties on the optimum design are investigated in this section. A sensitivity analysis of the effects of reservoir permeability, temperature, and rock embedment strength on acid fracture treatment design were examined for the three objective functions—maximum NPV, maximum cumulative production, and maximum NPV—with minimum treatment cost.

#### Effects of Reservoir Permeability

To study the effects of reservoir permeability on acid fracture treatment design, permeability values of 0.01 mD for a very low permeability formation, 0.1 to 1.0 mD for moderate formation, and 10.0 mD for relatively permeable formation were examined. This range of permeability was selected based on published data of low permeability gas reservoirs from the Middle East (Nasr-El-Din et al. 2003; Rahim and Petrick 2004). The reservoirs were assumed to be at 200 °F, have rock embedment strength of 100 kpsi, closure stress of 5500 psi, and the other properties, as presented in Table 4.4.

Figure 4.3 presents the optimum values of the free design variables, injection rate, injection time, and acid concentration as a function of reservoir permeability for the three objective functions. The obtained acid fracture geometry, cumulative production, testament cost, and net present value from these optimum designs are presented in Tables 4.6 to 4.8. The injection is the maximum operational injection rate of 45 bbl/min. Acid penetration distance inside the fracture, which defines fracture length by acid, is a strong function of injection rate, where fracture length by acid increases with the increase in injection rate, as discussed in Chapter 3. The optimum injection time increases with the increase in formation permeability. It increased from 10 to 41 minutes as formation permeability increased from 0.01 to 10 mD for objective function of maximum NPV. As the formation permeability increases, a wider fracture is needed to account for the increase in gas flow from the formation into the fracture. A wider fracture is created by injected more acid volume, which needs longer injection time. Therefore, the optimum injection time increases as formation permeability increases.

A similar scenario is noticed when the objective function was maximum cumulative production where optimum injection time increases with an increase in permeability (see Figure 4.3). Injection time increased from 13 to 55 minutes as formation permeability increased from 0.01 to 10 mD, which are higher than that of maximum NPV as an objective function. When designing for maximum cumulative production as the objective function, a wider fracture width is needed to increase the production. To create a wider fracture, acid volume is increased by increasing injection time. For maximum NPV with minimum treatment cost as objective function, the optimum injection time increased from 10 to 31 minutes as formation permeability increased from 0.01 to 10 mD. This injection time is lower than the optimum injection time, which was obtained for maximum NPV as objective function.

The optimum acid concentration for maximum NPV as objective function decreased from 24 to 22 wt % HCl as the formation permeability increased from 0.01 to 10 mD. A similar scenario is noticed when designing for maximum cumulative production where optimum acid concentration decreased from 28 to 26 wt %. In the case of maximum NPV with minimum treatment cost, the optimum acid concentration was between 15 and 24 wt % HCl. Injected acid volume controls the fracture width in addition to acid concentration. A wider fracture would be created by injecting more acid volume. Acid volume dissolving power—defined as the volume of formation rock dissolves per volume of acid reacts—increases with acid concentration increase, as discussed in Appendix D. Thus, a wider fracture would be created by a higher acid concentration that is needed for formation of higher permeability.

The acid volume ratio to fracture height is defined as total volume of injected acid in gallons divided by fracture height in foot. These acid volume ratios calculated from the optimum injection time and rate are presented in Tables 4.6 to 4.8. It can be seen that the optimum volume ratio of injected acid increases with a formation permeability increase, and a wider fracture has been created. For a maximum NPV objective function, the acid volume ratio increased from 189 to 777 gal/ft as formation permeability increased from 253 to 1037 gal/ft as formation permeability increased from 253 to 1037 gal/ft as formation permeability increased from 0.01 to 10 mD, (see Table 4.6). Acid volume ratio increased from 253 to 1037 gal/ft as formation permeability increased from 0.01 to 10 mD when designing for maximum cumulative production objective function, (see Table 4.7). In the case for maximum NPV with minimum treatment cost objective function, the optimum acid volume ratio increased from 268 to 582 gal/ft, except at formation permeability of 0.10 mD, which was 189 gal/ft, (see Table 4.8). To investigate further the optimum acid volume ratio,

equivalent volume of 15 wt % of HCl is used as a base for comparison. The equivalent 15 wt % HCl volume ratio to the optimum acid volume ratio is defined as follows:

$$V_{15wt \% HCl} = \frac{V_{Optimum} (gal) \times \rho_{Optimum} (lb / gal) \times C_{Optimum} (wt\%)}{\rho_{15wt \%} (lb / gal) \times 15(wt\%)}$$
4-23

An equivalent volume ratio of 15 wt % HCl to the optimum acid volume ratio is calculated using the above equation and presented in Tables 4.6 to 4.8. There is an increase in equivalent acid volume ratio for all design objective functions with the increase in formation permeability. For maximum NPV, acid volume ratios increased from 312 to 1178 gal/ft of 15 wt % HCl as formation permeability increased from 0.01 to 10 mD (see Table 4.6). For maximum cumulative production, the acid volume ratio increased from 502 to 1894 gal/ft with formation permeability increase (see Table 4.7). The increase in acid volume ratio for maximum NPV with minimum treatment cost was from 302 to 735 gal/ft for 15 wt % of HCl as formation permeability increased from 0.01 to 10 mD (see Table 4.8).

The optimum fracture half-length for all tested permeability range and for the three objective functions is the maximum acid penetration distance which has been found to be 347 ft, (see Tables 4.6 to 4.8), The optimum fracture width for the maximum NPV increases with the increase in reservoir permeability, (see Table 4.6). It increased from 0.08 in to 0.304 inches as the reservoir permeability increased from 0.01 to 10 mD. When designing for maximum cumulative production as the objective function, the optimum fracture width also increases with the increase in reservoir permeability (see Table 4.7). However, these optimum fracture widths are higher than the ones in the case of maximum NPV. They increased from 0.01 to 10.0 mD. When designing for maximum NPV with minimum treatment cost as the objective function, the optimum fracture width increases with the increase in reservoir permeability (see Table 4.7). However, these optimum fracture widths are higher than the ones in the case of maximum NPV. They increased from 0.01 to 10.0 mD. When designing for maximum NPV with minimum treatment cost as the objective function, the optimum fracture width increases with the increase in reservoir permeability (see Table 4.8). However, these optimum fracture widths are lower than the

ones in the case of maximum NPV as objective function, except for 0.01 mD. They increased from 0.08 in to 0.193 inches as the reservoir permeability increased from 0.1 to 10.0 mD.

The cumulative production over 10 years increased with the increase in formation permeability (see Tables 4.6 to 4.8). The cumulative productions are slightly higher for maximum cumulative production as objective function compared to that for maximum NPV for both formation permeabilities of 0.01 and 0.1 mD (see Tables 4.6 and 4.7). For higher formation permeability of 1.0 and 10 mD, both objective functions have the same cumulative production. It has been noticed that the increase in cumulative production when compared with maximum NPV design. The cumulative production for maximum NPV with minimum treatment cost is the same as that when designing for maximum NPV in formation of permeability 0.01 mD because of similar optimum fracture geometry (see Tables 4.6 and 4.8). However, for the higher permeability formation, the cumulative production for maximum NPV with minimum treatment cost design is less than that for the treatment design for maximum NPV.

The treatment cost and NPV for the three objective functions for different reservoir permeability are presented in Tables 4.6 to 4.8. The treatment cost and NPV increase with the increase in formation permeability for the three objective functions. The treatment cost has been found to be highest for maximum cumulative production. In the case of maximum cumulative production, the treatment cost increased by 8 % when compared with that for maximum NPV. For maximum NPV with minimum treatment cost, the loss in NPV was up to 0.57 % when compared with that of maximum NPV as objective function. In addition, treatment cost for maximum NPV with minimum treatment cost decreased up to 19 % when compared to that of maximum NPV. Thus, for all tested reservoir permeabilities, the maximum NPV as the objective function has been found to be optimum. When the treatment cost is considered in the design, the optimum design should be

based on maximum NPV with minimum treatment cost. However, the NPV will be reduced by 0.6% due to loss in production.

### **Effects of Reservoir Temperature**

The reaction of HCI acid with carbonate formation is a strong function of formation temperature, as discussed in Chapter 3. To study the effects of reservoir initial temperature on the acid treatment design, reservoir temperature was varied from 150 to 300 °F. This temperature range covers most of actual carbonate formations in the Middle East (Nasr-El-Din et al. 2003). The reservoir was assumed to have 1.0 mD permeability, rock embedment strength of 100 kpsi, closure stress of 5500 psi, and other properties as presented in Table 4.4. Figure 4.4 presents the optimum free design variables versus reservoir temperature for the three objective functions. Tables 4.9 to 4.11 present the acid fracture geometry, cumulative production, testament cost, and net present value for different reservoir temperatures. The injection rate was kept at 45 bbl/min (see Figure 4.4).

The optimum injection time and acid concentration are functions of reservoir temperature (see Figure 4.4). The optimum injection time decreases with increase in formation temperature. As the formation temperature increases, the fracture length that can be penetrated by the acid before completely spent decreases because reaction rate and mass transfer of acid are strongly influenced by temperature, as discussed in Chapter 3. Acid fracture half-length decreased from 533 to 184 ft as formation temperature increased from 150 to 300 °F (see Tables 4.9 to 4.11). Thus, the optimum injection time decreases with temperature increase.

When designing acid fracture treatment for maximum NPV, optimum acid concentration varies between 16 and 24 wt%HCl. In addition, optimum injection time decreased from 57 to 15 minutes as formation temperature increased from 150 to 300 °F (see Figure 4.4). Since the optimum injection time decreased with temperature increase, the acid volume ratio also decreased from 1081 to 288 gal/ft (see Table 4.9). This decrease in optimum acid volume resulted in treatment cost decrease. The optimum average

fracture width is 0.195 inch for all tested temperatures since the formation permeability remained the same at 1.0 mD. The cumulative production over 10 years is lower at higher formation temperature because of shorter fracture length, which resulted in NPV increase from US\$ 3.0893E+08 to US\$ 1.5327E+08.

Similar scenario is noticed for maximum cumulative production as the objective function where optimum injection time decreases with temperature increase (see Figure 4.4). It decreased from 66 to 20 minutes as formation temperature increased from 150 to 300 °F. These optimum injection times are higher than that for maximum NPV design. In addition, optimum acid concentration has been found to be higher for maximum cumulative production design than that for maximum NPV (see Tables 4.9 and 4.10). When designing for maximum cumulative production, a wider fracture is needed that requires larger acid volume and higher acid concentration. The optimum acid volume ratio (gallons of acid per feet of fracture height) decreased from 1239 to 377 gal/ft as formation temperature increased from 150 to 300 °F. However, the optimum fracture width was the same at 0.310 inch for tested reservoir temperatures (see Table 4.10). The treatment cost for maximum cumulative production was higher when compared with that for maximum NPV. It increased by 8 %. The increase in cumulative production over 10 years is negligible compared to that of maximum NPV. This results in an average of 0.015% loss in NPV.

When designing for maximum NPV with minimum treatment cost as the objective function, optimum injection time also decreases with temperature increase (see Table 4.11). It decreased from 26 to 10 minutes as formation temperature increased from 150 to 300 °F. This optimum injection time is lower than that for maximum NPV design (see Table 4.9). In addition, optimum acid concentration is lower; it is between 15 and 18 wt % HCI. The optimum acid volume ratio decreased from 494 to 189 gal/ft as the reservoir temperature increased from 150 to 300 °F. Optimum fracture width was between 0.10 and 0.08 inch, which is almost half of that for maximum NPV design (see Tables 4.9 and 4.11). The cumulative production for maximum

NPV with minimum treatment cost over 10 years is less than that for maximum NPV design cumulative production.

The treatment cost decrease with temperature increase because of decrease in optimum acid volume (see Tables 4.9 to 4.11). In addition, the NPV decreased with temperature increase because of the decrease in cumulative production. For the case of maximum cumulative production, the treatment cost increased by 8 % when compared with that for maximum NPV design. This resulted in average loss in NPV of 0.015 %. When designing for maximum NPV with minimum treatment cost as the objective function, the treatment cost decreased between 2 and 28 % of that for maximum NPV design. However, the loss in NPV was between 0.46 and 0.65 % when compared with that for maximum NPV due to loss in production. Thus, maximum NPV should be the objective function for acid fracture treatment design to maximise NPV and cumulative production. When maximum NPV with minimum treatment is considered, it will result in up to 28 % saving in treatment cost (saving of US\$ 1.60E+5). However, it will lead to up to 0.65 % loss in NPV (loss of US\$ 2.02E+6).

### Effects of Reservoir Rock Embedment Strength

The rock embedment strength is a mechanical property of the formation porous media. It defines the formation strength to holds the acid fracture open against closure stresses. When an acid is injected inside a fracture in carbonate formation, it creates etched rough surfaces. If the formation rock embedment strength is high enough to hold against closure stresses, then conductive pathways will stay open for hydrocarbon fluids to flow. Acid fractures of the same geometry would have different conductivity if they are created in formations of different rock embedment strength. The higher the formation rock embedment strength, the more conductive the fracture. Details can be found in Chapter 2.

To study the effects of reservoir rock embedment strength on acid fracture treatment design, its value was varied from relatively soft formation of 30 kpsi embedment strength to hard formation of 200 kpsi. Any formation with

embedment strength less than 20 kpsi is not recommended for acid fracturing (Schechter 1992; Economides and Nolte 2000). The reservoir is assumed to be at 200 °F with permeability of 1.0 mD and a closure stress of 5500 psi. The other properties are presented in Table 4.4. Figure 4.5 presents the optimum free design variables versus rock embedment strength for the three objective functions. The obtained acid fracture geometry, cumulative production, testament cost, and net present value from these optimum designs are presented in Tables 4.12 to 4.14. The injection rate for the tested rock embedment strength was kept at maximum operational injection rate of 45 bbl/min (see Figure 4.5).

The optimum injection time and acid concentration are functions of rock embedment strength and objective functions. When designing for maximum NPV design, optimum injection time decreases with the increase in formation rock embedment strength. It decreased from 29 to 24 minutes as formation rock embedment strength increased from 30 to 200 kpsi. This resulted in acid volume ratio decrease from 549 to 454 gal/ft (see Table 4.12). However, optimum acid concentration increased from 20 to 24 wt % HCl as formation rock embedment strength increased from 30 to 200 kpsi. The equivalent volume of 15 wt % HCl was calculated using Equation 4.23; it shows that the optimum acid volume ratio increased from 748 to 762 as formation rock embedment strength increased from 30 to 200 kpsi. The optimum average fracture width was 0.195 inch, and the optimum fracture half-length was 347 ft for the tested rock embedment strength (see Table 4.12). The cumulative production over 10 years is greater with higher formation rock embedment strength because of higher conductivity. The treatment cost remains almost the same for tested rock embedment strength. The NPV increases with the increase in rock embedment strength because of the increase in cumulative production. It increased from US\$ 2.2891E+08 to US\$ 2.3134E+08 as formation rock embedment strength increased from 30 to 200 kpsi.

When designing an acid fracture treatment for maximum cumulative production, it was found that the optimum injection time and acid concentration are not strong functions of rock embedment strength (see Figure 4.5). The optimum injection time was almost 34 minutes for the tested rock embedment strength except for formation with embedment strength of 30 kpsi, where it was slightly lower at 32 minutes. In addition, the optimum acid concentration was 27 wt % HCl for formation of rock embedment strength greater than 50 kpsi and 28 wt % HCl for the 30 kpsi rock embedment strength. The equivalent volume of 15 wt % HCl was 1217 gal/ft for the formation with rock embedment strength greater than 50 kpsi and 1193 gal/ft for rock embedment strength of 30 kpsi (see Table 4.13). This amount of injected acid resulted in fracture width of 0.308 inch and fracture half-length of 347 ft for all cases. The cumulative production is higher from formation with high rock embedment strength. It increased from 3.276E+08 to 3.311E+08 MSCF as the rock embedment strength increased from 30 to 200 kpsi. This increase in cumulative production resulted in an increases in NPV from US\$ 2.2887E+08 to US\$ 2.3130E+08.

When designing for maximum NPV with minimum treatment cost as the objective function, optimum injection time increases with rock embedment strength increase, except for formation of 200 kpsi embedment strength (see Figure 4.5). It increased from 12 to 16 minutes as formation rock embedment strength increased from 30 to 100 kpsi, for 200 kpsi rock embedment strength formation, the injection time remained 10 minutes. In addition, optimum acid concentration decreases with rock embedment strength, increase except for formation of 200 kpsi. It decreased from 20 to 15 wt % HCI as formation rock embedment strength increased from 30 to 100 kpsi while the acid concentration is 27 wt % HCl for the 200 kpsi. The effects of rock embedment strength on acid volume ratio was studied by using a reference of acid volume ratio of 15 wt % HCl, and the results are presented in Table 4.14. The acid volume ratio increased from 308 to 361 gal/ft as the formation rock embedment strength increased from 30 to 200 kpsi. This increase in acid volume ratio resulted in an increase in fracture average width from 0.08 to 0.091 inch. The fracture half-length has been found to be 347 ft, which is the same for all tested rock embedment strength tested in this study. The cumulative production over 10 years increases with the increase in rock embedment strength.

Tables 4.12 to 4.14 presents the treatment cost and NPV as a function of formation rock embedment strength for the three objective functions. The treatment cost is minimum for maximum NPV with minimum treatment cost design and maximum for maximum cumulative production design. When designing for maximum NPV with minimum treatment cost, the treatment cost decreased. These decreases range between 18 and 20 % of that for maximum NPV design. This has resulted in treatment cost saving between US\$ 7.5E+4 to US\$ 8.5E+4. However, the NPV also decreased: The decreases in NPV range between US\$ 1.14E+6 and US\$ 1.33E+6 of that for maximum NPV design which is due to decrease in production. Thus, maximum NPV should be used as the objective function.

### 4.6 Case Study to Low Permeability Gas Reservoir

Khuff formation is low permeability gas reservoir in the Ghawar field in Saudi Arabia. This formation is a carbonate of depth between 11,000 and 12,000 ft. Its temperature varies between 250 and 280 °F. The Khuff formation has porosity between 1 and 15 % and average permeability of 1.0 mD. The payzone height varies between 40 and 70 ft. The reservoir properties from the well data are presented in Table 4.15. Two gas wells in Khuff formation were subjected to acid fracturing treatments. These two wells-W-2 and W-4—were treated by acid fracturing to eliminate formation damage induced during drilling and maximise their production. The treatments fluids, pumping schedule, and after-treatment production rates are summarised in Table 4.16 (Nasr-El-Din, Al-Driweesh et al. 2003). The two wells were treated with the same acid volume, 55600 gal, of 28 wt % HCl. (see Table 4.16). The acid injection rates were 47 and 43 bbl/min, and the injection times were 34 and 35.4 minutes for the two wells W-2 and W-4, respectively. The acid volumes to fracture heights ratio were almost 800 gal/ft for wells W-2 and W-4, which have pay-zone height of 69 and 68 ft, respectively. Well W-2 mainly penetrated limestone formation while well W-4 mainly penetrated dolomite formation with 30 % limestone.

The results of these treatments are summarised in Table 4.16. From these results, it can be seen that acid fracturing is an effective treatment for carbonate formation. Effective fracture lengths were achieved with excellent fracture conductivities. When wells W-2 and W-4—which have almost the same reservoir properties and pay-zone height but different lithology—were stimulated with almost same acid fracturing treatment design (see Table 4.16), the fracture half-length of well W-4 is slightly longer at 433 ft than that of well W-2 at 413 ft. This is expected since well W-4 formation is mainly dolomite and acid can penetrate a longer distance before it completes the reaction, as discussed in Chapter 3, Section 3.3. The dissolving power of HCl acid in dolomite formation is 88 % of its dissolving power in limestone formation, Appendix D. Therefore, the fracture width of well W-4 fracture is expected to be 88 % of that encountered in well W-2 as it was treated with the same acid volume ratio. However, the actual width was almost double in the well W-2 fracture.

From the above discussion, it is clear that the actual treatment data agrees grossly with the predicted design geometries. Two analyses were conducted to investigate this further, first by calculating the theoretical fractures' widths using Equation 4.19, and second by estimating fracture geometry using the thermo-kinetic model discussed in Chapter 2. The theoretical fractures' widths using Equation 4.19 and the field data (acid volume and concentration, fracture half-length and average porosity) are presented in Table 4.17. The theoretical average fracture widths are 0.333 and 0.283 inch for wells W-2 and W-4, respectively. Therefore, the measured average fracture widths from the field data are 63 and 39 % of the theoretical ones for the wells W-2, and W-4, respectively. These differences can be explained as follows: The measured lengths could have been less than the actual fracture half-length and/or the acid leakoff could have been greater than the estimated value.

The thermo-kinetic model was used to estimate theoretical fracture width and half-length using field data presented in Table 4.18. The model input reaction

kinetic, mass transfer and thermal data are detailed in Table 2.2 of Chapter 2. It was found that for well W-2, the facture half-length and average width was estimated to be 440 ft and 0.243 inch, respectively. The measured fracture half-length and widths from the field data are 94 and 84.4 %, respectively, of the theoretical values. In addition, the fracture half-length and average width for well W-4 was estimated to be 500 ft and 0.191 inch, respectively. The measured fracture half-length and average width from the field data are 87 and 58 % of the ones estimated by the thermo-kinetic model.

Again, the predicted values for the acid fracture geometries for wells W-2 and W-4 agrees well with that of actual values. The fracture geometry of well W-2 is close to the estimated geometry by the thermo-kinetic model. This indicates an efficient acid fracturing treatment and accurate determination of fracture geometry after the treatment. The fracture half-length of well W-4 is close to the estimated half-length by the thermo-kinetic model. This fracture half-length indicates an efficient acid fracturing penetration distance inside the fracture. However, the field measured width is 58 % of the one estimated by the thermo-kinetic model, which indicates inaccurate field measurement of fracture average width or a large loss of acid by leakoff.

After analysing acid fracturing geometry from field data, the optimisation model was used to design optimum acid fracture treatments for the two wells with net present value as the objective function. The optimisation results are presented in Table 4.19. The injection rate is the maximum operational pumping rate, 45 bbl/min for the two wells. This is to ensure maximum acid penetration distance, as discussed before. The optimum acid concentration is 21 wt % for the two wells. The optimum injection times are 17 and 20 minutes for wells W-2 and W-4, respectively. These optimum times are 50 and 56 % of the field injection time for wells W-2 and W-4, respectively. These optimum injection times would result in up to 56 % saving in acid volume and cost.

The optimum injection times are based on fully efficient acid treatment. However, based on field data for the two wells, the best acid treatment efficiency was found to be 60 %. Therefore, to account for acid efficiency, these optimum injection times can be increased by 40 % to 29 and 34 minutes for wells W-2 and W-4, respectively. These optimum acid (21 wt % HCI) injection time and rate will results in acid volume ratio to fracture height of 796 and 940 gal/ft for wells W-2 and W-4, respectively. The acid (28 wt % HCI) volume ratio used in the field were 806 and 818 gal/ft. when converted to 21 wt % HCI, using Equation 4.23, and the acid volume ratio are 1109 and 1125 gal/ft for wells W-2 and W-4, respectively. Thus, with the 40 % increase in optimum injection time, the saving in acid volume are 313 and 185 gal/ft for wells W-2 and W-4, respectively. This would result in cost saving of acid of 28 and 16 %.

The acid fracture geometry predicted for well W-2 by the optimisation model has average width of 0.151 inch and half-length of 337 ft. For well W-4, the predicted acid fracture has width of 0.149 inch and half-length of 353 ft. Acid volume dissolving power in dolomite formation is lower than its dissolving power in limestone. Therefore, fracture width predicted for well W-4 is less than that for well W-2 because well W-4 mainly penetrated dolomite formation. The small increase in fracture half-length for well W-4 is because acid can penetrate deeper in dolomite than in limestone. The thermo-kinetic model was used to estimate the fractures geometries that would be created in these two wells using optimum injection rate, injection time, and acid concentration. The acid fractures geometries from the thermo-kinetic model are presented in Table 4.20. The average acid fracture width is calculated in the thermo-kinetic model by the relation  $(\pi/5)$  multiplied by fracture width at the wellbore and by using Equation 4.19. Both values are presented in Table 4.20. The average fractures' widths are within 93 to 97 % of the widths predicted by the optimisation model for the two wells. In addition, fractures' half-lengths from the thermo-kinetic model are 7 to 8 % longer than those predicted by the optimisation model. Thus, fracture geometry predicted by the optimisation mode is almost identical to the thermo-kinetic model.

# 4.7 Closure

In this chapter, a multi-objectives acid fracture optimisation model was developed. This model combines for the first time acid fracture geometry, production and economic models in multi-objectives optimisation algorithm that is based on combined feature of Genetic Algorithm and Evolutionary Operation. The objective functions included maximum net present value, maximum cumulative production, and maximum net present value with minimum treatment cost. A sensitivity analysis was conducted to compare the designs of these three objective functions. This analysis indicates that the design for maximum NPV as the objective function would save 8 % in treatment cost and results in almost the same cumulative production of maximum NPV with minimum treatment cost there is up to US\$ 8.2E+5 (19 %) saving in treatment cost but at the expense of up to US\$ 1.32E+6 (0.60 %) losses in NPV when compared to the design for maximum NPV.

In addition, sensitivity analysis was conducted to determine the effects of reservoir properties including permeability, temperature, and rock embedment strength on the optimum design for the three objective functions. The results from this analysis indicate that the injection rate should be the maximum operational pumping rate. In addition, optimum acid volume increases with the increase in formation permeability. The NPV increases also with the increase in formation permeability because of the increase in cumulative production. The effect of formation temperature on optimum design analysis indicates that the optimum injection time decreases with the increase in formation temperature because of the shorter fracture length that can be achieved as temperature increase. In addition, the NPV decreases with the increase in formation temperature because of shorter fracture length. The effect of rock embedment strength on optimum design is not as strong as the other tested properties. There is a slight increase in optimum acid volume with the increase in rock embedment strength. However, the cumulative production and the net present value increase with the increase in rock embedment strength.

A case study based on two fractured gas wells' data was conducted to demonstrate the capability of this optimisation model. The optimisation model predicted the optimum acid volume of 50 % less than the volumes used in the field. When 60 % acid efficiency factor is considered, the predicted acid volumes by the optimisation model could save up to 28 % of the fracturing acid cost. The optimum design variables were used in the thermo-kinetic model to compare fractures geometries from both models. The fracture geometry predicted by the thermo-kinetic model is in agreement with the optimisation model. The average fractures' widths from thermo-kinetic model were within 93 to 97 % of the widths predicted by the optimisation model and fractures' half-lengths were 7 to 8 % higher of those predicted by the optimisation model.

Table 4.1 -	· Pumping	Power	and	Limitation
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Variables	Value
Available horse power of pump, hp	12000
Surface equipment pressure loss, psi	1000
Burst pressure of the tubing, psi	13000
Max. operating pressure of surface equipment, psi	14000
Pump efficiency	0.85

	Coefficients for Calculation of $\lambda_n$						
n	$g_{0,n}$	$g_{_{1,n}}$ x 10	$g_{2,n} \times 10^3$	$g_{3,n} \times 10^4$	$h_{1,n}   \mathrm{x}  \mathrm{10}^3$	$\overline{h}_{2,n} \ge 10^3$	
0	1.682310	-2.266930	6.754400	-1.840800	6.759300	-4.627400	
1	5.670530	-0.696000	17.293100	-2.930400	1.003200	-3.437600	
2	9.668420	-0.395870	10.774500	-0.556400	5.702800	-0.470500	
3	13.667720	-0.276620	7.937500	-0.135800	9.150000	-0.566800	
4	17.667400	-0.213050	6.343100	-0.037300	12.44960	-0.711690	
		Coe	fficients for Cal	culation of G <sub>n</sub>			
n	$\overline{g}_{0,n}$ x 10	$\overline{g}_{1,n} \ge 10^4$	$\overline{g}_{2,n} \ge 10^4$	$\overline{g}_{3,n}$ x 10 <sup>5</sup>	$\overline{h}_{\!\!1,n}$ x 10 <sup>4</sup>	$\overline{h}_{2,n} \ge 10^4$	
0	9.103780	-2.382790	14.929800	-8.970170	-7.081880	-1.183920	
1	0.531260	1.889090	-12.537500	8.134820	4.015380	0.351480	
2	0.152720	0.390350	-1.660700	0.680785	1.039400	0.515400	
3	0.068070	0.073300	-0.417200	0.111312	0.586390	0.141225	
4	0.037390	0.019010	-0.150300	0.027559	0.352770	0.056322	

Table 4.2 – Coefficients for Calculation of  $\lambda_n$  and  $G_n$  (Schechter 1992)

Table 4.3 –	Economic	Model	Data
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Variables	Value
Price of HCI acid, US\$/gal	$f(C_i)$
Price of pad fluid, US\$/gal	<i>f</i> (µ)
Price of pumping, US\$/hp	20
Price of gas, US\$/MSCF	1.0
Misc. Cost (fluid tank, blender, transportation, etc.), US\$	10000
Discount rate (10%)	0.1
Number of production days	3650

 Table 4.4 – Reservoir Properties, Well and Reservoir Fluids Data for Optimisation

 Model Sensitivity Analysis

Variables	Value
Wellbore radius, ft	0.492
Inside diameter of the tubing, ft	0.333
Pay-zone height ft	100
Porosity, %	10%
Total vertical depth, ft	7500
Young's modulus, Pascal	4.83E+09
Poisson ratio, dimensionless	0.20
Compressibility of rock, psi <sup>-1</sup>	8.60E-06
Formation critical pressure, psi	15000
Min. In-situ stress in the pay-zone, psi	5500
Min. In-situ stress in the confining zone (shale), psi	6000
Max. In-situ stress in the pay-zone, psi	5800
Skin factor before fracture, dimensionless	0
Initial reservoir pressure, psi	5500
Flowing bottom hole pressure, psi	1000
Drainage radius, ft	2979
Connate water saturation, fraction	0.20
Compressibility of water, psi <sup>-1</sup>	3.00E-06
Slope of gas expansion factor vs pressure	0.0226
Gas factor at 4400 psi	0.85

Variables	Max. NPV	Max. Cum. Prod. & % Change to Max. NPV Design		Max. NPV- Min. Trea Cost &% Change to Max. NPV Design	
Optimum injection time, minutes	26	34	33.5%	16	-36.4%
Optimum injection rate, bbl/min	45	45	0.0%	45	0.0%
Optimum acid concentration, wt%	23	27	18.3%	15	-32.3%
Acid volume/height, gal/ft	485	648	33.5%	309	-36.4%
Average fracture width, in	0.195	0.308	57.8%	0.084	-56.9%
Fracture half length, ft	347	347	0.0%	347	0.0%
Cumulative production, 10 <sup>+8</sup> MSCF	3.297555	3.297556	4E-6%	3.277495	-0.61%
Cost of acid, US\$	175515	277050	57.8%	75564	-56.9%
Treatment cost, US\$	424972	458411	7.9%	342879	-19.3%
NPV, 10 <sup>+8</sup> US\$	2.3040	2.3037	0.015%	2.2908	-0.57%

 Table 4.5 – Comparing the Optimum Designs for the Three Objective Functions

 (Max. NPV, Max. Cumulative Production and Max. NPV with Minimum Treatment Cost)

#### Table 4.6 – Effects of Reservoir Permeability on Optimum Design for Maximum NPV

Reservoir Permeability, mD	0.01	0.10	1.0	10
Optimum acid injection time, min	10	16	26	41
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	189	303	485	777
Optimum acid concentration, wt%	23.8	23.2	22.6	22.0
Equivalent 15 wt% acid volume ratio	312	486	757	1178
Optimum average fracture width, in	0.080	0.125	0.195	0.304
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	3.616E+06	3.551E+07	3.298E+08	1.734E+09
Treatment cost, US\$	339342	379751	424972	475577
NPV, US\$	2.192E+06	2.447E+07	2.304E+08	1.213E+09

Reservoir Permeability, mD	0.01	0.10	1.0	10
Optimum acid injection time, min	13	21	34	55
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	253	405	648	1037
Optimum acid concentration, wt%	28	27	27	26
Equivalent 15 wt% acid volume ratio	502	782	1217	1894
Optimum average fracture width, in	0.126	0.197	0.308	0.480
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	3.645E+06	3.570E+07	3.298E+08	1.734E+09
Treatment cost, US\$	366044	409632	458411	512998
NPV, US\$	2.185E+06	2.446E+07	2.304E+08	1.213E+09

Table 4.7 – Effects of Reservoir Permeability on Optimum Design for Maximum Cumulative Production

# Table 4.8 – Effects of Reservoir Permeability on Optimum Design for Maximum NPV with Minimum Treatment Cost

Reservoir Permeability, mD	0.01	0.10	1.0	10
Optimum acid injection time, min	14	10	16	31
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	268	189	309	582
Optimum acid concentration, wt%	16.8	23.8	15.3	18.6
Equivalent 15 wt% acid volume ratio	302	312	315	735
Optimum average fracture width, in	0.080	0.080	0.084	0.193
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir closure stress, Psi	5500	5500	5500	5500
I Cumulative production, MSCF	3.616E+06	3.543E+07	3.277E+08	1.733E+09
Treatment cost, US\$	339342	339342	342879	440886
NPV, US\$	2.192E+06	2.446E+07	2.291E+08	1.212E+09

Reservoir Temperature, °F	150	200	250	300
Optimum acid injection time, min	57	26	17	15
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	1081	485	326	288
Optimum acid concentration, wt%	16	23	24	20
Equivalent 15 wt% acid volume ratio	1124	757	538	397
Optimum average fracture width, in	0.195	0.195	0.195	0.195
Optimum fracture half length, ft	533	347	246	184
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	1000	100	100	100
Reservoir permeability, mD	1.00	1.00	1.00	1.00
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	4.421E+08	3.298E+08	2.635E+08	2.194E+08
Treatment cost, US\$	554449	424972	353937	311009
NPV, US\$	3.0893E+08	2.3040E+08	1.8411E+08	1.5327E+08

Table 4.9 – Effects of Reservoir Temperature on Optimum Design for Maximum NPV

# Table 4.10 – Effect of Reservoir Temperature on Optimum Design for Maximum Cumulative Production

Reservoir Temperature, °F	150	200	250	300
Optimum acid injection time, min	66	34	22	20
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	1239	648	421	377
Optimum acid concentration, wt%	21	27	28	24
Equivalent 15 wt% acid volume ratio	1823	1217	833	638
Optimum average fracture width, in	0.31	0.31	0.31	0.31
Optimum fracture half length, ft	533	347	246	184
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	100	100	100	100
Reservoir permeability, mD	1.00	1.00	1.00	1.00
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	4.421E+08	3.298E+08	2.635E+08	2.194E+08
Treatment cost, US\$	598077	458411	381787	335483
NPV, US\$	3.0888E+08	2.3037E+08	1.8408E+08	1.5324E+08

Reservoir Temperature, °F	150	200	250	300
Optimum acid injection time, min	26	16	10	10
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	494	309	189	189
Optimum acid concentration, wt%	15	15	18	15
Equivalent 15 wt% acid volume ratio	494	315	229	189
Optimum average fracture width, in	0.09	0.08	0.09	0.10
Optimum fracture half length, ft	533	347	246	184
Fracture height, ft	100	100	100	100
Rock embedment strength, kpsi	100	100	100	100
Reservoir permeability, mD	1.00	1.00	1.00	1.00
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	4.390E+08	3.277E+08	2.621E+08	2.184E+08
Treatment cost, US\$	395197	342879	316597	304548
NPV, US\$	3.0691E+08	2.2908E+08	1.8315E+08	1.5257E+08

Table 4.11 – Effect of Reservoir Temperature on Optimum Design for Maximum NPV with Minimum Treatment Cost

# Table 4.12 – Effects of Reservoir Rock Embedment Strength on Optimum Design forMaximum NPV Objective Function

Rock Embedment Strength, kpsi	30	50	100	200
Optimum acid injection time, min	29	27	26	24
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	549	511	485	454
Optimum acid concentration, wt%	20	21	23	24
Equivalent 15 wt% acid volume ratio	748	753	757	762
Optimum average fracture width, in	0.195	0.195	0.195	0.195
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir permeability, mD	1.000	1.000	1.000	1.000
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	3.276E+08	3.285E+08	3.298E+08	3.311E+08
Treatment cost, US\$	424964	424971	424972	424972
NPV, US\$	2.2891E+08	2.2952E+08	2.3040E+08	2.3134E+08

Rock Embedment Strength, kpsi	30	50	100	200
Optimum acid injection time, min	31.9	34.4	34.3	34.1
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	603	651	648	645
Optimum acid concentration, wt%	28.0	26.6	26.7	26.8
Equivalent 15 wt% acid volume ratio	1193	1216	1217	1217
Optimum average fracture width, in	0.308	0.308	0.308	0.308
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir permeability, mD	1.000	1.000	1.000	1.000
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	3.276E+08	3.285E+08	3.298E+08	3.311E+08
Treatment cost, US\$	458412	458412	458411	458411
NPV, US\$	2.2887E+08	2.2949E+08	2.3037E+08	2.3130E+08

 Table 4.13 – Effects of Reservoir Rock Embedment Strength on Optimum Design for

 Maximum Cumulative Production

Table 4.14 – Effects of Reservoir Rock Embedment Strength on Optimum Design for Maximum NPV and Minimum Treatment Cost

Rock Embedment Strength, kpsi	30	50	100	200
Optimum acid injection time, min	12	13	16	10
Acid injection rate, bbl/min	45	45	45	45
Optimum acid volume/height, gal/ft	220	249	309	190
Optimum acid concentration, wt%	20	18	15	27
Equivalent 15 wt% acid volume ratio	308	304	315	361
Optimum average fracture width, in	0.080	0.080	0.084	0.091
Optimum fracture half length, ft	347	347	347	347
Fracture height, ft	100	100	100	100
Reservoir temperature, °F	200	200	200	200
Reservoir permeability, mD	1.000	1.000	1.000	1.000
Reservoir closure stress, Psi	5500	5500	5500	5500
Cumulative production, MSCF	3.259E+08	3.266E+08	3.277E+08	3.291E+08
Treatment cost, US\$	339343	339344	342879	349338
NPV, US\$	2.2776E+08	2.2827E+08	2.2908E+08	2.3000E+08

Parameters	Well W-2	Well W-4
Formation type	Limestone	70% Dolomite
Formation depth, ft	11,400	11,200
Reservoir pressure, psi	7,494	7,461
Temperature, °F	259	259
Porosity, %	1.2-12.6	1.5-15
Average porosity, %	7.0	8.3
Permeability, mD	0.07-2.2	0.01-2.8
Average permeability, mD	1.1	1.4
Pay-zone height, ft	69	68
Young's modulus, psi	5.5E+6	5.8E+6
Poisson ratio	0.29	0.29
Min. in-situ stress, psi	11,645	11,819
Fracture gradient, psi/ft	1.02	1.06

Table 4.15 – Case Study Gas Wells Data in Carbonate Reservoir (Nasr-El-Din et al. 2003)

Table 4.16 – Acid Fracture Treatments Conducted on the Gas Wells and their Results (Nasr-El-Din et al. 2003)

Treatment Variables	Well W-2	Well W-4
Total volume of acid, gal	55600	55600
Acid concentration, wt%	28	28
Acid average injection rate, bbl/min	47.0	45.0
Acid injection time, min	33.9	35.4
Acid volume/height, gal/ft	806	818
Pumping horsepower, hp	16,381	18,900
Treatment Result	Well W-2	Well W-4
Fracture half-length, ft	413	433
Fracture average width, inch	0.21	0.11
Fracture conductivity, md-ft	48,475	12,972
Production rate, MMscfd	53.27	43
Flowing wellhead pressure, psi	3,095	3,421

Variables	Well W-2	Well W-4		
Total volume of acid, cubic ft	8957	8957		
Acid concentration, wt%	28	28		
Fracture half-length, ft	413	433		
Average porosity, %	7.0	8.3		
Field fracture average width, inch	0.21	0.11		
Fracture Average Width using Equation 4.19 and Field Fracture Half-Length				
Theoretical fracture average width, inch	0.333	0.283		
Field fracture width/ Theoretical, %	63	39		

 Table 4.17 – Theoretical Acid Fracture Geometry based on Field

 Injection Rates, Time and Acid Concentration

Table 4.18 – Acid Fracture Geometry based on Field Injection Rates, Time and Acid Concentration using the Thermo-Kinetic Model

Variables	Well W-2	Well W-4	
Acid average injection rate, bbl/min	47.0	45.0	
Acid injection time, min	33.9	35.4	
Acid concentration, wt%	28	28	
Average porosity, %	7.0	8.3	
Bottom hole static temperature, °F	259	259	
Fracture height, ft	69	68	
Formation depth, ft	11,400	11,200	
Fracture Geometry using Thermo-Kinetic Model			
Fracture half-length, ft	440	500	
Field fracture half length / theoretical, %	94	87	
Fracture average width, inch	0.243	0.191	
Field fracture width / theoretical, %	86	58	

Variables	Well W-2	Well W-4
Acid injection time, min	17	20
Acid injection rate, bbl/min	45	45
Acid volume/height, gal/ft	477	564
Acid concentration, wt%	21	21
Average fracture width, in	0.151	0.149
Fracture half-length, ft	337	353
Fracture height, ft	69	68
Rock embedment strength, kpsi	100	100
Reservoir temperature, °F	259	259
Reservoir permeability, mD	1.10	1.40
Reservoir closure stress, psi	11,645	11,819
Treatment Cost, \$	347488	355049
Cumulative production, MSCF	3.59E+08	4.38E+08
Net present value NPV, \$	1.60E+08	1.94E+08

Table 4.19 – Optimum Treatment for the Three Gas Wells

Table 4.20 - Fracture Geometry by the Thermo-kinetic Model usingOptimum Treatment Design for the Three Gas Wells

Variables	Well W-2	Well W-4
Acid injection time, min	17	20
Acid injection rate, bbl/min	45	45
Acid volume/height, gal/ft	477	564
Acid concentration, wt%	21	21
Fracture width at wellbore, in	0.230	0.230
Average fracture width <sup>1</sup> , in	0.145	0.145
Average fracture width <sup>2</sup> , in	0.141	0.139
Average fracture width / optimum	93-96%	93-97%
Fracture half-length, ft	360	380
Average fracture half-length / optimum	107%	108%
Fracture height, ft	69	68
Reservoir temperature, °F	259	259

1. Calculating in the model using the relation average width =  $(\pi/5)$  width at wellbore.

2. Calculating in the mode using equation 4.19.



Figure 4.1 – Acid Fracturing Optimisation Scheme Flow Chart



Figure 4.2 — Optimum design variables (injection time, injection rate, and concentration) for the three objective functions (max. NPV, max. cumulative production, and max. NPV with minimum treatment cost).



Figure 4.3 – Optimum treatment parameters (injection time, injection rate, and acid concentration) for different reservoir permeability for the three objective functions (maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost).


Figure 4.4 – Optimum treatment parameters (injection time, injection rate, and acid concentration) for different reservoir temperature for the three objective functions (maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost).



Figure 4.5 – Optimum treatment parameters (injection time, injection rate, and acid concentration) for different reservoir rock embedment strength for the three objective functions (maximum NPV, maximum cumulative production, and maximum NPV with minimum treatment cost).

#### **CHAPTER 5**

#### **CONCLUSIONS AND RECOMMENDATIONS**

This study presented an innovative modelling scheme that can be used effectively to model, design, and optimize acid fracturing treatment in carbonate gas reservoirs. The scheme consists of a fracture geometry model, a thermo-kinetic model, a production model, an economic model, and optimisation algorithm. Acid kinetic, physical, mechanical, and economic data for this study were derived from published sources. A typical Saudi Aramco gas carbonate reservoir was used as a case study.

The thermo-kinetic model consists of thermal and reaction kinetic models. The thermal model is capable of estimating temperature changes of acid at the wellbore and inside the fracture. Acid temperature changes during injection inside the well tubing because of heat exchange with the surroundings. Inside the fracture, it changes as a result of heat of reaction and heat exchange with the formation. The kinetic model is capable of estimating acid penetration length and the amount of rock dissolved from the fracture surfaces. Acid penetration length defines fracture half length by acid fracturing. Fracture width is created by acid reaction with the fracture surfaces, creating etched rough surfaces that provide the pathways for hydrocarbons production. The effects of many parameters on the acid fracture geometry were investigated. These tested parameters included injection rate, injection time, leakoff rate, acid concentration, and formation temperature.

Fracture conductivity is estimated from the gain in fracture width, rock embedment strength, and closure stresses. Fracture half length is defined by acid penetration distance. Fracture conductivity and half length are used in the production model to estimate cumulative gas production over period. Treatment cost and net present value are estimated by the economic model. An optimization model based on combined feature of Genetic Algorithm and Evolutionary Operation was used to maximize an objective function by manipulating free design variables. The design objectives functions include maximum net present value, maximum recovery, and maximum net present value with minimum treatment cost. The free design variables are injection time, and acid initial concentration. The effects of objective functions, reservoir permeability, formation temperature, and rock embedment strength on optimum acid fracturing design were investigated. This study was used to evaluate and optimize acid fracturing treatment conducted on two gas wells in carbonate formation in Saudi Arabia.

## 5.1 Conclusions

Based on this study, the following are concluded:

- 1. Fracture geometry by acid fracturing is a function of treatment parameters, formation lithology, and properties. Fracture half-length increases with the increase in injection rate and decrease in leakoff rate and formation temperature. It was also observed that acid fracture half-length is longer in dolomite formation than in limestone. Fracture width increases with the increase in injection time, leakoff rate, acid concentration, and formation temperature. In limestone formation, a relatively wider but shorter fracture is created compared to dolomite formation at the same treatment conditions. Fracture conductivity increase with the increase in formation rock embedment strength.
- 2. Increasing the acid injection rate increases the penetration distance which, in turn, increases fracture length. For the same volume of acid injected, lower injection rate will result in a wider width at wellbore and less acid penetration distance, and vice versa.
- **3.** Increasing the acid injection time results in an increase in acid fracture width and length. However, depending on other treatment parameters,

there is a maximum penetration depth at which injecting more acid will result only in more gain in fracture width but not in fracture half-length.

- 4. The acid penetration distance is inversely proportional to leakoff rate. Increase in leakoff rate results in more acid consumption at shorter distance closer to the wellbore and more rock dissolved, which causes a wider fracture at the wellbore and less acid penetration distance.
- 5. Increase in reservoir temperature results in faster acid consumption closer to the wellbore and decreases acid penetration distance. This results in a wider fracture closer to wellbore but shorter acid fracture half-length. Increase in formation temperature from 150 to 300 °F results in reducing the acid penetration distance by three times and increasing fracture width at the wellbore by six times.
- 6. Increasing injected acid concentration results in an increase in gained fracture width; the higher the acid concentration, the wider the fracture. Increase in acid concentration, however, does not yield a longer fracture half-length
- 7. The conductivity of acid fracture is a function of fracture width and rock embedment strength. The increase in injected acid volume increases fracture conductivity by increasing the fracture width.
- 8. Formation permeability affects fracture conductivity index. In low permeability formation, a longer fracture with narrower width is more desirable; in relatively higher permeability formation, a wider and shorter fracture is more desirable.
- 9. The optimum design depends on the specified design objective function. Designing for maximum net present value as the objective function results in cumulative production almost identical to that for a maximum cumulative production but with an 8 % saving in the treatment cost. The optimum design using maximum net present value with

minimum treatment cost will result in up to 19 % saving in treatment cost with only 0.6 % loss in NPV.

- 10. The acid injection rate was kept at maximum operational pumping rate for the three objective functions and tested formation properties. Optimum injection time and acid concentration depends strongly on objective functions and reservoir properties. Rock embedment strength has less effect on optimum design compared to the effect of reservoir permeability and temperature.
- 11. The optimum injection time increases with increase in formation permeability. The optimum injection time decreases with increase in formation temperature because of a shorter fracture length that can be achieved as temperature increases. Optimum injection time is not a strong function of rock embedment strength.
- 12. Cumulative production and net present value increase with the increase in reservoir permeability and rock embedment strength. In addition, the cumulative production and NPV decrease with the increase in formation temperature.
- 13. The effect of rock embedment strength on optimum design is not as strong as the reservoir permeability and temperature effects. There is a slight increase in optimum acid volume with the increase in rock embedment strength.
- 14. Reservoirs with higher rock embedment strength are better for acid fracturing treatment. Higher rock embedment strength would result in an increase in cumulative production and NPV.

## 5.2 Recommendations

Based on this study, the following are recommended:

- The volume of injected acid, its concentration, and injection time should be determined using an optimisation model. If the injected acid volume is less than required, then fracture conductivity would be low because of a thin and short fracture. However, if the acid volume is too large, it would increase the treatment cost and reduce rock embedment strength.
- 2. Depending on the operation limitations, acid should be injected at maximum allowable injection rate to increase the penetration distance. For the same volume of acid injected, lower injection rate results in a wider fracture width at the wellbore and a shorter fracture length, and vice versa; increasing the injection rate gives a better fracture width profile and a deeper penetration.
- 3. The leakoff rate must be minimised to minimise acid loss and consumption at shorter distance. The penetration distance is a strong function of leakoff rate. Leakoff rate could be reduced by increasing acid viscosity using gel, emulsified, or viscoelastic surfactant.
- 4. Acid fracturing is recommended for formation with high rock embedment strength. Rock embedment strength affects fracture conductivity by holding against closure stress. Higher rock embedment strength formation would result in an increase in cumulative production and NPV.
- Optimum injection time and acid concentration are functions of formation properties and objective functions; they should be determined using the optimisation model.
- 6. Optimisation of acid fracturing treatment should be conducted with maximum net present value as the objective function. This design would maximise NPV and the cumulative production.

## 5.3 Direction for Future Work

Following the work conducted in this thesis, the following areas are considered worthy further research study. The thermo kinetic model could be improved by accounting for retarded systems like geld and emulsified acid and organic acids like formic and acetic acids that are commonly used in carbonates formation stimulation. In addition, modelling of acid fracturing in sandstone formation has got no attention at all and research study in this area is essential. Kalfayan L.J. (2007) presented experimental results of successful acid fracturing in sandstone formation using mixture of HCI and HF acids. This work proved that sandstone fracture surfaces can be etched similar to carbonates formation and opened a window to the future for sandstone acid fracturing technology.

## NOMENCLATURE

C	Injected acid concentration
C	
Cn	
то	
10	Treatment cost
d	Diameter
$D_A$	Diffusion constant
D <sub>e</sub>	Effective diffusion constant
E	Activation energy
E	Young's modulus
FxC	fixed and miscellaneous costs
<b>g</b> G	Geothermal gradient
G <sub>pn</sub>	Total production
i	Discount rate
$J_{A}$	Flux of molecular or ion A
K <sub>1c</sub>	Fracture toughness
Kg	Apparent mass transfer coefficient
K <sub>h</sub>	Apparent heat transfer coefficient
k <sub>r</sub>	Reaction rate constant
MDP	Mass dissolving power
MW	Molecular weight
N <sub>Pr</sub>	Prandlt number
NPV	Net present value
N <sub>Re</sub>	Reynolds number
N <sub>Sc</sub>	Schmidt number
N <sub>Sh</sub>	Sherwood number
NY	Total number of years
P <sub>A</sub>	Price of acid
Pe	Reservoir pressure
$P_{ ho f}$	Price of fracturing pad fluid
P <sub>pump</sub>	Pumping price
q	Injected fluid rate into one wing

R	Gas constant
r <sub>A</sub>	Reaction rate
RES	Rock embedment strength
R <sub>n</sub>	Revenue generated
To	Injected fluid temperature at surface
Ts	Average annual surface temperature
Tw	Injected fluid temperature at fracture wall
V <sub>A</sub>	Total volume of fracturing acid
VDP	Volume dissolving power
VL	Fluid leak off velocity
$\overline{V_i}$	Average velocity in the <i>i</i> -direction
V <sub>tpf</sub>	Total volume of pad fracturing fluid
W	Load applied for RES
z	Formation depth
$\Delta h_r$	Heat of reaction
Vi	Flow velocity in the <i>i</i> -direction

# Greek symbols

β	non-Darcy flow coefficient, ft-1
Δ	difference
$\phi$	Porosity in fraction
μ	viscosity, cp
ω	fracture width, ft
σ <sub>V</sub>	Overburden stress
V	Poisson's ratio
δ <sub>ij</sub>	Kronecker delta, defined $\delta_{ij} = 1$ if $i = j$ , and $\delta_{ij} = 0$
ρ	Injected fluid density
ρ <sub>m</sub>	Formation density
$\sigma_{CL}$	Closure pressure
$\sigma_{ii}$	Compressive stresses

# Subscripts and superscripts

A acid

е	effective
f	fracture
i	initial
wf	wellbore

# **SI Metric Conversion Factors**

bbl x 1.589 873 E-01= m<sup>3</sup> cp x 1.0 E-03= Pa\*s ft x 3.048 E-01= m ft<sup>3</sup> x 2.831 685 E-02= m<sup>3</sup> in. x 2.54 E+00= cm lbf x 4.448 222 E+00= N lbm x 4.535 924 E-01= kg mD x 9.869 233 E-04=  $\mu$ m<sup>2</sup> psi x 6.894 757 E+00= kPa

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## **APPENDIX A**

## MECHANICAL PROPERTIES OF FORMATION MATERIALS

Physical properties of rock are needed in order to predict the characteristics of hydraulically induced fractures. The required physical properties of rocks include Young's modulus (*E*), Poisson's ratio (v), fracture toughness (K<sub>1</sub>c), and rock embedment strength. In solid mechanics, *Young's Modulus* is a measurement of the stiffness of a given material. It is also known as modulus of elasticity, elastic modulus, or tensile modulus. It is defined as the ratio of rate change of stress with strain and can be determined experimentally from the slope of a stress-strain curve. For many materials, Young's modulus is a constant over a range of strains. Such materials are called 'linear' and are said to obey Hooke's law. When a sample of material is compressed, it tends to get thicker toward the other directions. *Poisson's ratio* is a measurement of this tendency. Most practical engineering materials have Poisson's ratio values between 0.0 and 0.5.

*Fracture toughness* is a property that describes the ability of a material containing a crack to resist fracture. It is denoted K1c and has the units of MPa $\sqrt{in}$ . The subscript '1c' denotes Mode I crack opening or plain strain to distinguish it from shear cracking Mode II, or tear cracking, Mode III. Fracture toughness is a quantitative way of expressing a material's resistance to brittle fracture when a crack is present. If a material has a large value of fracture toughness, it will probably undergo ductile fracture. Brittle fracture is characteristic of materials with a low fracture toughness value.

*Rock embedment strength (RES)* is another important mechanical property of porous media for both propped and acid fracturing. RES is measured by a high-speed steel ballpoint of 0.05-inch in diameter (Jaeger and Cook 1977). A core sample of 3.5 inch diameter and 6 inches long is loaded in the testing machine. The ballpoint is brought into contact with the rock surface and then embedded into the rock to a depth 0.0125 inch. The load needed to achieve

that depth is recorded. Then RES is calculated by the relation in Equation A-1 (Schechter 1992).

$$RES = \frac{4W}{\pi d^2}$$
A-1

where W is the load and d is the diameter of the indentation of the ballpoint. Embedment strength data for various formations can be found in hydraulic fracturing monographs.

Castle et al. (2005) reported the range of published physical properties for different formation. Summary of these data is presented in Table A-1. Young's Modulus for limestone ranges from 0.31 to 14.1 Mpsi with an average of 6.27 Mpsi. Poisson's ratio values for limestone ranges from 0.0 to 0.5. The distribution of the limestone elastic modulus, *E'* defined as in Equation A-2, indicates that 40 per cent of the values are between 4.6 to 8.9 Mpsi and that 21 per cent are less than 2.5 Mpsi. Fracture toughness of limestone ranges from 325 to 1810 psi $\sqrt{$ in with an average of 1003 psi $\sqrt{$ in (Robertson 1959; Somerton, Esfandiari et al. 1969; Schmidt 1976; Atkinson and Meredith 1987; Hatheway and Kiersch 1989; Meredith 1989; Ouchterlony 1989; Castle, Ronald et al. 2005).

$$E' = E/(1 - v^2)$$
 A-2

For sandstone, Young's Modulus ranges from 0.06 to 8.0 Mpsi with an average value of 2.1 Mpsi. Poisson's ratio ranges from 0.06 to 0.36 with an average value of 0.16. Values for the elastic modulus of sandstone range from 0.06 to 8.2 Mpsi with an average value of 2.2 Mpsi. The distribution of the sandstone elastic modulus is 39 per cent of the reported values in the range of 1.2 to 2.4 Mpsi. The sandstone K₁c ranges from 193 to 2345 psi√in with an average of 1021 psi√in (Baidyuk 1967; Somerton et al. 1969; Atkinson and Meredith 1987; Hatheway and Kiersch 1989; Matsuki 1989; Ouchterlony 1989; Chen and Zhang 2004).

Young's Modulus of shale ranges from 0.06 to 9.9 Mpsi with an average value of 2.5 Mpsi. The reported values of Poisson's ratio for shale range are from 0.01 to 0.37 with an average value of 0.16. Elastic modulus ranges from 0.06 to 10.0 Mpsi with an average value of 2.5 Mpsi, 65 per cent of the reported values less than 2.05 Mpsi. The shale fracture toughness ranges from 220 to 1177 psi√in with an average of 721 psi√in (Atkinson and Meredith 1987; Hatheway and Kiersch 1989; Chen and Zhang 2004).

Variables	Limestone	Sandstone	Shale
Young's Modulus E, Mpsi	0.31 - 14.1	01 - 8.0	0.06 - 9.9
	Average 6.3	Average 2.1	Average 2.5
	0.01 - 0.32	0.01 - 0.36	0.01 - 0.37
Poisson's ratio	Average 0.18	Average 0.16	Average 0.16
	0.3 – 15.4	0.06 - 8.2	0.10 - 10.0
Elastic modulus E' Mosi	Average 6.6	Average 2.1	Average 2.5
	40% 4.6 - 8.9	39% 1.2 - 2.4	65% < 2.05
	21% < 2.5		
Fracture toughness K <sub>1C</sub> .	325 – 1810	193 – 2345	220 – 1177
psi√in	Average 1003	Average 1021	Average 721

Table A.1 – Ranges of Published Physical Properties of Formation (Castle 2005)

## **APPENDIX B**

# COMPOSITIONS AND THERMAL PROPERTIES OF FORMATION ROCK

#### **Rock Compositions and Mineralogy**

There are four major groups of minerals in sedimentary rock based on their origin. The *detrital* minerals like quartz, orthoclase, and microcline. *Plagioclase* are formation rocks that survive weathering and transportation. The secondary minerals, formed during weathering and transportation, are clays. *Calcite* forms the third group, which are precipitated minerals directly from solutions by chemical or biochemical reactions. The last group is the *authigenic* minerals that are formed in sediments during and after deposition like dolomite. Carbonate formation consists of limestone and dolomite; it also contains small amounts of iron, manganese, barium, and strontium in addition to silicate impurities. The composition of these major minerals found in sedimentary rock is presented in Table B.1.

Minerals	Formulae
Quartz	SiO <sub>2</sub>
Orthoclase (feldspar)	KAISi <sub>3</sub> O <sub>8</sub>
Microcline	KAIS <sub>3</sub> O <sub>8</sub>
Plagioclase	NaAlSi <sub>3</sub> O <sub>8</sub> -CaAl <sub>2</sub> Si <sub>2</sub> O <sub>8</sub>
Muscovite	KAl <sub>3</sub> Si <sub>3</sub> O <sub>10</sub> (OH) <sub>2</sub>
Biotite	J(Mg, Fe) <sub>3</sub> AlSi <sub>3</sub> O <sub>10</sub> (OH) <sub>2</sub>
Kaolinite	$AI_4Si_4O_{10}(OH)_8$
Smectite	(AI, Mg, Fe)₄(AI, Si) <sub>8</sub> O <sub>20</sub> (OH)₄·nH <sub>2</sub> O
Illite	K <sub>0</sub> - <sub>2</sub> Al <sub>4</sub> (Al, Si) <sub>8</sub> O <sub>20</sub> (OH) <sub>4</sub>
Chlorite	(Mg, Al, Fe) <sub>3</sub> (Al, Si) <sub>4</sub> O <sub>10</sub> (OH) <sub>2</sub> ·(Mg, Al) <sub>3</sub> (OH) <sub>6</sub>
Calcite	CaCO <sub>3</sub>
Dolomite	CaMg(Ca <sub>3</sub> ) <sub>2</sub>

Table B.1 – Chemical Formulae of Major Minerals Found in Sedimentary Rock (Schechter 1992)

#### Heat Capacity and Thermal Conductivity of Sedimentary Rock

Heat capacity (Cp) is a measurement of an object ability to store heat. The heat capacity of a material is defined as the amount of heat (kcal) needed to increase the temperature of 1kg mass by  $1^{\circ}C$  at constant pressure. The heat capacity of sandstone and limestone can be measured experimentally or approximated by the following equations (Semerton 1958):

Heat capacity of sandstone;

$$Cp = 0.1812 + 1.4582 \text{ E} - 4 \times \text{T} - \frac{1.495 \text{ E} + 3}{\text{T}^2}$$
B-1

Heat capacity of carbonates;

$$Cp = 0.1968 + 1.189 \text{ E} - 4 \times \text{T} - \frac{3.076 \text{ E} + 3}{\text{T}^2}$$
B-2

where T is formation temperature in  ${}^{\circ}K$  and Cp is heat capacity in Btu/lb.  ${}^{\circ}F$ .

This heat capacity is for rock martial; for rock filled with fluid, fluid heat capacity must be introduced. Bulk heat capacity is presented as follows:

$$\rho_{\text{bulk}} \cdot C \rho_{\text{bulk}} = (1 - \phi) \rho_{\text{rock}} \cdot C \rho_{\text{rock}} + (\phi) \rho_{\text{fluid}} \cdot C \rho_{\text{fluid}}$$
B-3

where  $\rho$  is density and  $\phi$  is porosity in fraction.

Thermal conductivity of a material defines the ability of a material to transfer heat. Thermal conductivity of many formations were reported and found to have no clear correlation. Therefore, it should be measured at formation conditions using core sample and formation fluid. Birch and Clark (1940) reported that thermal conductivity of minerals decreases clearly with temperature increase. Part (1982) collected extensive thermal data and reported these properties in the thermal recovery process monograph. These data can be used in the absence of measured ones (Part 1982).

## **APPENDIX C**

## ACID SYSTEMS USED IN CARBONATE FORMATION

Different systems of acid are used in well treatments of carbonates formation. They can be classified as mineral acid (HCI), organic acids (formic and acetic acid), mixed acids (HCI-formic, HCI-acetic, HF-formic), and retarded acids (gelled and emulsified) (Williams, Gidley et al. 1979).

## **Minerals Acids**

Treatments of carbonates formation are usually carried out by hydrochloric acid (HCI). Usually, 5 to 10 wt per cent HCl are used for acid wash of perforated and openhole zone and for scale removal from well tubulars. In matrix stimulation and acid fracturing treatments, 15 to 28 wt per cent HCl is commonly used. Regular acid is a common name in oil fields for the 15 wt per cent HCl. The disadvantage of HCl is its high corrosivity on tubulars. Its corrosivity is difficult to control especially at high temperature (> 120 °C). The difficulty in controlling the corrosion rate increases with the increase in acid concentration, especially above 21 wt per cent (Williams, Gidley et al. 1979). The corrosion inhibitors in early years of acid treatment jobs were not good enough to control corrosion of tubulars at concentration over 15 wt per cent. However, the improvement in corrosion inhibitors today makes the use of HCl as high as 30 wt per cent possible. The common use of HCl acid in well treatment resulted from its reasonable cost, soluble reaction products, and high dissolving power (Schechter 1992)

## **Organic Acids**

The low corrosion rate of organic acid (formic and acetic acid) and the ease of inhibiting them at high temperature compare to HCl is an advantage use of these acids over that of HCl. In addition, the reaction rate of these acids is much slower than that of HCl, and they can penetrate more in formation before they are spent or reach equilibrium. However, their dissolving power is low compare to HCl, and they can cause precipitation and emulsion in sensitive formation. In addition, these acids react incompletely in the presences of their reaction products; because of that, they are called 'weak acids' (Williams et al. 1979; Schechter 1992).

## **HCI and Organic Acids Mixtures**

A mixture of HCl and organic acids is commonly used in carbonate formation stimulation. These mixtures are used in high temperature formation where controlling corrosion is critical and expensive when a mineral acid is used by itself. The dissolving power of these mixtures is increased by using HCl, while their corrosivity is reduced by adding organic acid. These mixtures can be used as a preflush in sandstone acidising prior to HF acid treatment to dissolve carbonates and minimise HF precipitated products (Williams et al. 1979).

## **Retarded Acids**

The rate of HCl spending in the formation can be controlled physically to some limit by decreasing diffusion rate of hydronium ions ( $H^+$ ) from the solution to the rock surface where reaction take place. This can be done by increasing acid solution viscosity using a gelling agent or by emulsifying acid droplet in a hydrocarbons liquid, acid-in-oil emulsion (Schechter 1992). These systems proved to be very effective in matrix stimulation and acid fracturing of carbonates formation (Nasr-El-Din et al. 2003).

#### **APPENDIX D**

#### **HCI-CARBONATE DISSOLVING POWER**

Reaction of HCI with limestone and dolomite rocks produces calcium and magnesium chloride, which are soluble in water, carbon dioxide gas, and water as presented by Equations D-1 and D-2 (Williams et al. 1979). Mass dissolving power (MDP) of acid is defined as mass of rock dissolved per mass of acid reacted. The dissolving power is an important factor in designing an acid treatment job. It can be calculated easily from the stoichiometry equation by knowing the molecular weight of reactants that are presented in Table D.1 and acid concentration in wt per cent (Williams 1979).

$$CaCO_3 + 2HCI \rightarrow CaCl_2 + H_2O + CO_2 \qquad D-1$$

$$CaMg(CO_3)_2 + 4HCI \rightarrow CaCl_2 + MgCl_2 + 2H_2O + 2CO_2 \qquad D-2$$

Table D.1 – Molecular Weight of HCI-carbonates Reactants and Products (Williams et al. 1979)

Materials	Chemical Formula	Molecular Weight
Hydrochloric acid	HCI	36.47
Limestone	CaCO <sub>3</sub>	100.09
Dolomite	CaMg(CO <sub>3</sub> ) <sub>2</sub>	184.3
Calcium chloride	CaCl <sub>2</sub>	110.99
Magnesium chloride	MgCl <sub>2</sub>	95.3
Carbon dioxide	CO <sub>2</sub>	44.01
Water	H <sub>2</sub> O	18.02

Mass dissolving power of *HCI* when it reacts with *limestone* is presented by Equation D-3 and by Equation D-4 when it reacts with *dolomite* (Williams 1979).

$$MDP = \frac{1moleCaCO_3}{2moleHCl} \times \frac{MW_{CaCO_3}}{MW_{HCl}} \times \frac{wt\%Acid}{100}$$
D-3

$$MDP = \frac{1moleCaMg(CO_3)_2}{4moleHCl} \times \frac{MW_{CaMg(CO_3)_2}}{MW_{HCl}} \times \frac{wt\%Acid}{100}$$
D-4

where *MW* is molecular weight in g/mole.

Volume dissolving power (VDP) is the volume of rock dissolved per volume of acid reacted. For HCI-limestone and HCI-dolomite reactions, the VDP is presented by Equation D-5 and D-6, respectively (Williams, Gidley et al. 1979). Limestone has a density of 2.71 g/cm<sup>3</sup> and dolomite has a density of 2.87 g/cm<sup>3</sup>. Figures D-1 presents HCI acid specific gravity, MDP, and VDP when reacting with limestone and dolomite generated, using Equations D-3 to D-6 at different acid concentration. The VDP plotted is for zero porosity and should be corrected for formation porosity by dividing the values from the figure by (1.0-porosity).

$$VDP = \frac{1moleCaCO_3}{2moleHCl} \times \frac{MW_{CaCO_3}}{MW_{HCl}} \times \frac{wt\%Acid}{100} \times \frac{AcidDensity}{RockDensity} \times \frac{1}{(1-\phi)}$$
D-5

$$VDP = \frac{1moleCaMg(CO_3)_2}{4moleHCl} \times \frac{MW_{CaMg(CO_3)_2}}{MW_{HCl}} \times \frac{wt\%Acid}{100} \times \frac{AcidDensity}{RockDensity} \times \frac{1}{(1.0-\phi)}$$
D-6



Figure D.1 - HCl acid specific gravity mass dissolving power (MDP) and volume dissolving power (VDP) when reacts with limestone and dolomite vs. concentration generated using equations D-3 to D-6.

#### **APPENDIX E**

## ACID REACTION AND DIFFUSION RATES

Reaction rate constant is function of temperature and can be calculated at different temperatures by Arrhenius Equation, as follows:

$$k_{r,T} = k_{r,T_o} Exp(-\frac{E}{R}(\frac{1}{T} - \frac{1}{T_o}))$$
 E-1

An approximate dependence of the diffusion coefficient on temperature can often be estimated using Stokes-Einstein, as follows:

$$\frac{D_{A @T_1}}{D_{A @T_2}} = \frac{T_1}{T_2} \times \frac{\mu_{T_2}}{\mu_{T_1}}$$
E-2

Using the kinetic and physical data for 15 wt per cent HCl, which is presented in Table E.1, reaction rate and diffusion rate constants of hydronium ions ( $H^+$ ) function of temperature was generated using Equations E-1 and E-2. They are presented in Figure E.1. The reaction rate of HCl acid with dolomite is less than that with limestone at all tested temperatures.

Table E.1 – HCI Kinetic and Physical Data (Lee and Roberts 1980; Settari 1993)

Variables	Values	
Valiables	Limestone	Dolomite
HCI reaction rate constant, k <sub>r</sub> , ft/min	8.13E-4 @ 100 °F	2.461E-3 @ 200 °F
HCl reaction order, $\alpha$	0.441	0.669
Activation energy, E, cal/mol	5497	9675
Gas constant, R, cal/mol.J	1.987	
HCI diffusion rate coefficient, D <sub>A</sub> , ft <sup>2</sup> /min	6.60E-6 @ 60 °F	
HCI density, lb/ft <sup>3</sup>	67	.0
HCI Viscosity, cp @ 68 °F	1.2	26
HCI concentration, wt%	15	



Figure E.1 – HCI reaction rate constant (kr) when reacts with limestone and dolomite and its diffusion rate constant versus temperature

The apparent mass transfer coefficient ( $K_g$ ) can be calculated as follows (Lo and Dean 1989):

$$K_{g} = \frac{N_{sh}.D_{A}}{2w}$$
E-3

where  $N_{Sh}$  is Sherwood number calculated as follows:

For turbulent flow,  $N_{Re} > 7000$  $N_{Sh} = 0.026 N_{Re}^{5/4} N_{Sc}^{1/3}$  E-4

For transitional flow, 7000 > 
$$N_{Re}$$
 > 1800  
 $N_{Sh} = 0.001104 N_{Re}^{1.1532} N_{Sc}^{1/3}$  E-5

For laminar flow,  $N_{Re} < 1800$  $N_{Sh} = 6.26 N_{Sc}^{-1/3}$  E-6 N<sub>Re</sub> is the Reynolds number, defined as follows:

$$N_{Re} = \frac{2w\overline{v}_{x}}{(\mu/\rho)}$$
 E-7

N<sub>Sc</sub> is the Schmidt number, defined as follows:

$$N_{Sc} = \frac{(\mu/\rho)}{D_A}$$
 E-8

The term w is fracture width and  $\overline{v}_x$  is average fluid velocity along the fracture,  $\mu$  is acid viscosity and  $\rho$  is acid density.

If 15 wt per cent HCl acid is injected into a fracture that has an average width of 0.1 inch and height of 100 ft, and the injection rate is increased from 1 to 20 bbl/min, and fluid temperature is increased from 100 to 390  $^{\circ}$ F, then the acid flow regimes inside the fracture using Equation E-7 is as presented in Figure E-2.



Figure E.2 – Reynolds number when 15 wt% HCl acid is injected into one wing of a fracture that has an average width of 0.1 inch and height of 100 ft and the injection rate increased from 1 to 20 bbl/min and fluid temperature is increased from 100 to 390 °F.

By using Equations E-3 to E-8, apparent mass transfer coefficient (mixing coefficient) was estimated for the previous injection conditions and presented in Figure E-3. Sine HCI acid reaction rate with limestone is fast, even at low temperatures (Figure E-1). Its reaction is a reaction rate limited only up to 130 °F for all tested injection rates. At temperature higher than 130 °F, its reaction rate is greater than the mass transfer coefficient (mass transfer to fracture surfaces is slower than reaction rate), and the reaction becomes mass transfer limited.

In contrast, HCI acid reaction rate with dolomite is slower than that with limestone, shown in Figure E-1. Therefore, HCI reaction with dolomite is reaction rate limited up to 180 °F for injection rate up to five bbl/min, shown in Figure E-3. As the injection rate increased to 10 bb/min, the reaction is reaction rate limited up to 220 °F. At temperature higher than 220 °F, reaction rate constant become higher than mixing coefficient (mass transfer to fracture surfaces is slower than reaction rate); therefore, the reaction is mass transfer limited.

In summary, the reaction rate of HCI with limestone is reaction rate limited at temperature up to 130 °F for the tested injection rates but up to 220 °F when reacting with dolomite. Therefore, temperature and injection rate control spending mechanism of HCI acid where reaction rate of HCI acid is strong function of temperature and mass transfer (mixing coefficient) is strong function of temperature and flow rate.



Figure E.3 – HCl reaction rate constant (kr) with limestone, dolomite and apparent mass transfer coefficient (mixing coefficient) when 15 wt% HCl acid is injected into one wing of a fracture that has an average width of 0.1 inch and height of 100 ft and the injection rate increased from 1 to 10 bbl/min and fluid temperature is increased from 100 to 390  $^{\circ}$ F.

#### **APPENDIX F**

#### HEAT TRANSFER COEFFICIENT

When an acid is injected inside a fracture of average width w and height h, heat transfer coefficient ( $K_h$ ) is calculated as follows (Lee and Roberts 1980):

$$K_{h} = \frac{N_{Nu} \cdot k_{th}}{2w}$$
 F-1

where:  $N_{Nu}$  is the Nusselt number, calculated as follows:

For turbulent flow, 
$$N_{Re} > 7000$$
;  
 $N_{Nu} = 0.026 N_{Re}^{5/4} N_{Pr}^{1/3}$  F-2

For transitional flow, 7000 > 
$$N_{Re}$$
 > 1800  
 $N_{Nu}$  = 0.001104  $N_{Re}^{1.1532} N_{Pr}^{1/3}$  F-3

For laminar flow, 
$$N_{Re} < 1800$$
  
 $N_{Nu} = 6.26 N_{Pr}^{1/3}$  F-4

 $N_{Re}$  is the Reynolds number, defined as follows:

$$N_{Re} = \frac{2w\overline{v}_{x}}{(\mu/\rho)}$$
F-5

N<sub>Pr</sub> is the Prandlt number, defined as follows:

$$N_{Sc} = \frac{(Cp\mu)}{k_{th}}$$
F-6

The term *w* is fracture width and  $\overline{v}_x$  is average fluid velocity along the fracture,  $\mu$  is acid viscosity and  $\rho$  is acid density.

When a regular 15 wt per cent HCl acid at temperatures varies from 100 to 320 °F is injected into a fracture of average width 0.1 inches and height of 100 ft at injection rates from 1 to 20 bbl/min, the relationship between injected fluid heat transfer coefficient, injection rate, and temperature was

calculated using the above equations and presented in Figure F.1. Acid thermal data, density and viscosity are presented in Table F.1. Heat transfer coefficient is function of fluid flow rate and its temperature. However, it is more dependent on fluid flow rate at temperatures up to 200 F. At higher temperatures, it value increases sharply with both increase in fluid flow rate and temperature.

Variables	Values
HCI heat capacity, Btu/lb.°F	1.0
HCI thermal conductivity, Btu/min.ft.°F	23.3E-3
HCI density, lb/ft <sup>3</sup>	67.0
Viscosity, cP @ 68 °F	1.26

Table F.1 – Injected 15 wt% HCI Acid Physical Data (Perry and Maloney 1984; Settari 1993)



Figure F.1 – Acid heat transfer coefficient function of temperature versus injection rate when a regular 15 wt% HCl acid at temperatures varies from 100 to 320  $^{\circ}$ F is injected into a fracture of average width 0.1 inches and height of 100 ft at injection rates from 1 to 20 bbl/min.
## **APPENDIX G**

# INJECTED ACID TEMPERATURE AT WELL BOTTOM HOLE

The thermal model inside the wellbore, discussed in Chapter 3, was used to investigate the effect of injection rate (q) and initial bottom hole temperature (BHT) on injected acid temperature profile at well bottom hole. The input to the model is presented in Table G.1, and the tested initial formation temperature is presented in Table G.2. The results of this model are presented in Figures G.1 and G.2.

Figure G.1 presents injected fluid bottom hole temperature (FBHT) versus injection time function of formation original temperature. It shows the effect of initial formation temperature on fluid temperature as it arrived at depth of 12000 ft when injected at 15 bbl/min into a well of radius 0.333 ft. Geothermal gradient versus equivalent formation temperature at depth of 12000 ft is calculated ( $T_{Ambient}$  + Depth x geothermal gradient) and presented in Table G.2. The increase in formation initial temperature results in an increase in injected fluid temperature at well bottom hole, for the same injection time and rate.

Figure G.2 presents the effect of injection rate on injected acid temperature profile at bottom hole when the same fluid is injected into the same formation at different injection rates. The increase in injection rate results in shorter contact time for fluid to exchange heat with the formation. This results in drop in fluid temperature profile. Even for the same volume of fluid injected for example after 10 minutes of 30 bbl/min injection rate and 20 minutes of 15 bbl/min, fluid temperature at well bottom hole is higher at lower injection rate.

Variables	Values
HCI heat capacity, Btu/lb.ºF	1.0
HCI thermal conductivity, Btu/min.ft. °F	23.3E-3
Formation heat capacity, Btu/lb. °F	0.22
Formation thermal conductivity, Btu/min.ft. °F	5.0E-3
Injected fluid surface temperature, °F	70
Ambient temperature, °F	80
Density of injected fluid, lb/ft <sup>3</sup>	67.0
Density of limestone formation, lb/ft <sup>3</sup>	169.2
Density of dolomite formation, lb/ft <sup>3</sup>	179.2
Wellbore tubing radius, ft	0.333
Formation depth, ft	12000

Table G.1 – Thermal Model inside the Wellbore Input Data(Roberts and Gun 1975)

Geothermal Gradients, °F/ft	BHT @ 12000 ft
0.006	152
0.010	200
0.014	248
0.018	296
0.020	320
0.022	344

Table G.2 – Formation Temperature versus Geothermal Gradients



Figure G.1 – Injected acid temperature at bottom hole (FBHT) function of formation initial temperature (BHT) and injection time,  $T_{f.surf.temp} = 70$  °F,  $T_{A.Temp} = 80$  °F, z =12000 ft, q = 15 bbl/min and  $r_w = 0.333$  ft.



Figure G.2 – Injected fluid temperature at bottom hole (FBHT) function of injection rate and injection time,  $T_{f.surf.temp} = 70$  °F,  $T_{A.Temp} = 80$  °F, z =12000 ft,  $r_w = 0.333$  ft and BHT = 200 °F.

## **APPENDIX H**

### MASS TRANSFER OF ACID INSIDE A FRACTURE

The general form of mass transfer in 3-D during acid injection inside a fracture is presented by Equation H-1 (Schechter 1992). Figure H.1 presents a fracture with its length is along the x-direction, its width w = 2y, is along the *y*-direction and its height, *h*, is along the *z*-direction. If an acid is injected into this fracture, and then from the mass balance Equation H -1, the term  $\partial C_A / \partial t$  is the accumulated acid inside the fracture. The terms with velocity  $v_x$ ,  $v_y$ , and  $v_z$  are acid mass transferred by forced convection toward x, y, and z direction, respectively. All terms with diffusivity constant,  $D_A$ , are acid mass transferred by diffusion.

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z} + D_A \frac{\partial^2 C_A}{\partial x^2} + D_A \frac{\partial^2 C_A}{\partial y^2} + D_A \frac{\partial^2 C_A}{\partial z^2}$$
H-1

where  $v_x$  is fluid flow velocity along the fracture. The velocity  $v_y$  is the fluid flow velocity toward fracture walls and velocity in *z*-direction  $v_z$  can be assumed to be zero, no flow in the *z*-direction.

The mass transfer by diffusion  $(D_A \frac{\partial^2 C_A}{\partial x^2})$  in the x-direction, along the fracture length is small compared to mass transfer by forced convection (fluid flow), and can be neglected. Therefore, diffusion term in the x-direction is negligible. Mass transfer by diffusion in the z-direction  $(D_A \frac{\partial^2 C_A}{\partial z^2})$  is caused by natural convection which are gravity forces and can be neglected. Mass transfer in the y-direction is both convection  $(v_y \frac{\partial C_A}{\partial y})$  and diffusion  $(D_A \frac{\partial^2 C_A}{\partial y^2})$ control. In the case of zero leakoff where v<sub>1</sub> is zero, only mass transfer by diffusion plays a role in transporting reactant to the fracture surface. Therefore, the final shape of the mass transfer equation inside a fracture Equation H-1 becomes as in Equation H-5 The mass transfer in the y-direction is multiplied by two to account for the two surfaces of the fracture (Schechter 1992).

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + 2v_y \frac{\partial C_A}{\partial y} + 2D_A \frac{\partial^2 C_A}{\partial y^2}$$
H-2



Figure H.1 – Fracture geometry for acid injection formulation showing direction of fluid flow, fracture height, length, and width.

The mass transfer toward fracture walls by diffusion (y-direction) is replaced by apparent mass transfer coefficient  $K_g$  that was proposed by Roberts and Guin (1975), as follows:

$$K_g(C_A - C_w) = D_A(\partial^2 C_A / \partial y^2)$$
 H-3

where  $C_A$  is acid bulk concentration and  $C_w$  is acid concentration at fracture wall. Apparent mass transfer coefficient calculation was discussed in Appendix E.

The final shape of the mass transfer equation inside a fracture is reduced as:

$$\frac{\partial C_A}{\partial t} = v_x \frac{\partial C_A}{\partial x} + 2v_y \frac{\partial C_A}{\partial y} + 2K_g (C_A - C_w)$$
 H-4

The velocities terms for Newtonian fluid are defined analytically from the flow between porous parallel plates as follows:

$$v_x = \frac{3}{2}\overline{v_x} \left( 1 - \left(\frac{2y}{w}\right)^2 \right)$$
 H-5

$$v_{y} = \frac{3}{2} v_{l} \left( \left( \frac{2y}{w} \right) - \frac{1}{3} \left( \frac{2y}{w} \right)^{2} \right)$$
 H-6

where  $\overline{v_x}$  is average acid flow velocity along the fracture equals injection rate divided by fracture cross sectional area, and v<sub>l</sub> is leakoff velocity.

The boundary conditions are:

#### <u>Concentration of acid at wellbore (x=r<sub>w</sub>)</u>

If  $r_w$  is wellbore radius and t is injection time, acid concentration at the wellbore is initial injected acid concentration.  $C_A = C_i$  for all t > 0 and  $x=r_w$ .

#### Concentration of acid at fracture wall (x>rw) has two case scenarios

**I.** If reaction rate is higher than mass transfer rate of acid to the fracture surfaces by diffusion and convection—as in the case of HCl reaction with limestone at temperature greater than 200 °F at low leakoff rate—then the reaction at fracture walls is mass transfer limited and mass of acid transfers toward fracture walls will react instantaneously as it arrives at the fracture's two surfaces (at y = +w/2 and y = -w/2) and can be expressed as follows:

$$k_r C_w^{\ \alpha} = v_l (C_A - C_w) + K_g (C_A - C_w)$$
 H-7

where  $C_w$  is the acid concentration at fracture wall,  $k_r$  is reaction rate constant, and  $\alpha$  is reaction order. Equations H-4 and H-7 can be solved for acid concentrations inside the fracture and at fracture walls. Equation H-7 is solved iteratively if reaction order is not 1.

**II.** If mass transfers of acid to the fracture walls by diffusion and convection is faster than acid reaction rate—as in the case of HCI reaction with dolomite at temperature less than 200 F when the flow is laminar—then the amount of acid reacts at the fracture surface plus the amount leakoff equals the amount transferred to the fracture walls. This can be expressed as follows:

$$k_{r}C_{w}^{\ \alpha} + v_{l}C_{w} = v_{l}(C_{A} - C_{w}) + K_{g}(C_{A} - C_{w})$$
 H-8

where  $C_w$  is the acid concentration at fracture wall,  $k_r$  is reaction rate constant,  $\alpha$  is reaction order ,and  $v_l$  is leakoff velocity. Equations H-4 and H-8 can be solved for acid concentrations inside the fracture and at fracture walls.

The numerical formulation using implicit forward finite difference methods, which is unconditionally stable for the acid mass transfer and reaction inside the fracture is as follows:

Discretisation of coordinates:  $(x, t) \rightarrow (x_{j}; t_n)$ 

$$\begin{split} \mathbf{x}_{i} &= \mathbf{x}_{0} + i\Delta \mathbf{x} \\ \mathbf{y}_{j} &= \mathbf{y}_{0} + j\Delta \mathbf{y} \\ \mathbf{t}_{n} &= \mathbf{t}_{0} + \mathbf{n}\Delta \mathbf{t} \\ \mathbf{C}(\mathbf{x}_{i};\mathbf{y}_{j} \ ; \ \mathbf{t}_{n}) &= \mathbf{C}_{i,j}^{n} \end{split} \tag{H-9}$$

where *i* is grid number (space step) along the fracture X-axis,  $\Delta x$  is grid length, j is grid number (space step) across the fracture width Y-axis,  $\Delta y$  is grid length, and *n* is the time step number with  $\Delta t$  time interval length

Finite difference approximation for 1<sup>st</sup> derivative:

$$\frac{\partial C}{\partial x} = \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}}{2\Delta x} + O(2\Delta x)$$
$$\frac{\partial C}{\partial y} = \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y} + O(2\Delta y)$$
H-10
$$\frac{\partial C}{\partial t} = \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} + O(\Delta t)$$

Applying finite difference approximation into mass transfer equation H-4 as follows:

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = (v_{x})_{i,j}^{n} \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}}{2\Delta x} + 2(v_{y})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y} + 2(K_{g})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y}$$
H-11

If the fracture height is h, the fracture width w is divided into equivalent unit  $\Delta y$ , and the fracture length L is divided into equivalent unit lengths of  $\Delta x$ , then from mass balance in the grid, volume h. $\Delta x$ . $\Delta y$  is as follows:

The mass transfer equation inside a fracture becomes as follows:

$$h.\Delta x \ \Delta y \ \cdot \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = h.\Delta x \ \cdot \Delta y \ \cdot (v_{x})_{i,j}^{n} \ \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}}{2\Delta x} + h.\Delta x \ \cdot \Delta y \ \cdot 2(v_{y})_{i,j}^{n} \ \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y} + h.\Delta x \ \cdot \Delta y \ \cdot 2(K_{g})_{i,j}^{n} \ \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y}$$
H-12

Rearrange as follows:

$$h \cdot \Delta x \cdot \Delta y \cdot \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = h \cdot \Delta y \cdot (v_{x})_{i,j}^{n} \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}}{2} + h \cdot \Delta x \cdot (v_{y})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2} + h \cdot \Delta x \cdot \Delta y \cdot 2(K_{g})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y}$$
H-13

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = \frac{h \cdot \Delta y}{h \cdot \Delta x} \cdot \Delta y} (v_{x})_{i,j}^{n} \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}}{2} + \frac{h \cdot \Delta x}{h \cdot \Delta x} \cdot \Delta y} (v_{y})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{1} + \frac{h \cdot \Delta x}{h \cdot \Delta x} \cdot \Delta y} \cdot 2(K_{g})_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta y}$$
H-14

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} = \frac{1}{2\Delta x} (v_{x})_{i,j}^{n} (C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}) + \frac{1}{\Delta y} (v_{y})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}) + \frac{1}{\Delta y} (K_{g})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1})$$
H-15

$$C_{i,j}^{n+1} - C_{i,j}^{n} = \frac{\Delta t}{2\Delta x} (\nu_{x})_{i,j}^{n} (C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}) + \frac{\Delta t}{\Delta y} (\nu_{y})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}) + \frac{\Delta t}{\Delta y} (K_{g})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1})$$
H-16

Define  $\chi$  and  $\beta$  as follows:

$$\chi = \frac{\Delta t}{2\Delta x}, \qquad \beta = \frac{\Delta t}{\Delta y}$$
 H-17

Replace equation H-17 into H-16 as follows:

$$C_{i,j}^{n+1} - C_{i,j}^{n} = \chi \cdot (v_{x})_{i,j}^{n} (C_{i+1,j}^{n+1} - C_{i-1,j}^{n+1}) + \beta \cdot (v_{y})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}) + \beta \cdot (K_{g})_{i,j}^{n} (C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1})$$
H-18

Open the brackets in the previous equation and rearrange as follows:

$$C_{i,j}^{n+1} - C_{i,j}^{n} = \chi \cdot (v_{x})_{i,j}^{n} C_{i+1,j}^{n+1} - \chi \cdot (v_{x})_{i,j}^{n} C_{i-1,j}^{n+1} + \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j+1}^{n+1} - \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j-1}^{n+1}$$

$$C_{i,j}^{n+1} - \chi \cdot (v_{x})_{i,j}^{n} C_{i+1,j}^{n+1} + \chi \cdot (v_{x})_{i,j}^{n} C_{i-1,j}^{n+1} - \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j+1}^{n+1} + \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j-1}^{n+1} = C_{i,j}^{n}$$

$$\chi \cdot (v_{x})_{i,j}^{n} C_{i-1,j}^{n+1} + C_{i,j}^{n+1} - \chi \cdot (v_{x})_{i,j}^{n} C_{i+1,j}^{n+1} - \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j+1}^{n+1} + \beta \cdot ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n}) C_{i,j+1}^{n+1} = C_{i,j}^{n}$$

After the above rearrangement, the equation become as follows:

$$\chi.(v_{x})_{i,j}^{n}C_{i-1,j}^{n+1} + C_{i,j}^{n+1} - \beta.((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n})C_{i,j+1}^{n+1} + \beta.((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n})C_{i,j-1}^{n+1} - \chi.(v_{x})_{i,j}^{n}C_{i+1,j}^{n+1} = C_{i,j}^{n}$$
H-20

Simplifying the shape of the above equation more as follows:

Define A and B as:

$$A_{i,j}^{n} = \chi (v_{x})_{i,j}^{n}, \qquad B_{i,j}^{n} = \beta ((v_{y})_{i,j}^{n} + (K_{g})_{i,j}^{n})$$
H-21

Therefore, the final shape of the mass transfer Equation H-20 is as follows:

$$A_{i,j}^{n}C_{i-1,j}^{n+1} + B_{i,j}^{n}C_{i,j-1}^{n+1} + C_{i,j}^{n+1} - B_{i,j}^{n}C_{i,j+1}^{n+1} - A_{i,j}^{n}C_{i+1,j}^{n+1} = C_{i,j}^{n}$$
H-22

In the above equation, at i =1 all points are at the fracture mouth where  $C_{i-1,j=1..Z-1}^{n=1..M-1} = C_{inj}$ , acid initial concentration. Therefore, the term  $A_{i,j}^n C_{i-1,j}^{n+1}$  of Equation H-22 becomes  $A_{i,j}^n C_{inj}$ , moving it to the right-hand side, Equation H-22 in matrix form is as follows:

$$\begin{pmatrix} B_{i,j}^{n} & 1 & -B_{i,j}^{n} & \dots & -A_{i,j}^{n} \\ A_{i,j}^{n} & B_{i,j}^{n} & 1 & -B_{i,j}^{n} & \dots & -A_{i,j}^{n} \\ A_{i,j}^{n} & B_{i,j}^{n} & 1 & -B_{i,j}^{n} & \dots \\ & & & \ddots & \ddots & \ddots & \ddots \\ & & A_{i,j}^{n} & B_{i,j}^{n} & 1 & -B_{i,j}^{n} \\ & & & A_{i,j}^{n} & B_{i,j}^{n} & 1 \\ & & & A_{i,j}^{n} & B_{i,j}^{n} \end{pmatrix} \begin{bmatrix} C_{1}^{n+1} \\ C_{12}^{n+1} \\ \vdots \\ \vdots \\ C_{M,Z-2}^{n} \\ C_{M,Z-1}^{n} \\ C_{M,Z}^{n} \end{bmatrix}$$

$$= \begin{pmatrix} C_{10}^{n} - A_{i,j}^{n} C_{inj} \\ C_{11}^{n} - A_{i,j}^{n} C_{inj} \\ C_{12}^{n} - A_{i,j}^{n} C_{inj} \\ \vdots \\ C_{M,Z-1}^{n} \\ C_{M,Z-1}^{n} \\ C_{M,Z-1}^{n} \\ C_{M,Z-1}^{n} \\ C_{M,Z}^{n} \end{pmatrix}$$
H-23

## For mass transfer limited case; $k_r > K_g + v_l$

The initial and boundary conditions are as follows:

Concentration of acid at fracture mouth is the initial concentration for all *j* as follows:

$$C(x = 0, y, t) = C_{1, i \in Z}^{n \in N} = C_{init}$$
 H-24

Concentration of acid at fracture walls is calculated as follows:

$$k_{r}(C_{w}^{\alpha})_{i,j=Z}^{n} = v_{l}(C_{i,0}^{n} - (C_{w})_{i,j=Z}^{n}) + K_{g}(C_{i,0}^{n} - (C_{w})_{i,j=Z}^{n})$$
 H-25

Equations H-23 and H-26 are solved for acid concentrations. Space interval along the fracture length of 10 ft, 10 space grids along fracture half width and injection time step of 0.10 minute are used. The acid penetration distance is determined each time step by the length at which first space interval with acid concentration less than 1 per cent of initial injected acid concentration.

### <u>Reaction rate limited case; $k_r < K_g + v_l$ </u>

The initial and boundary conditions are as follows:

Concentration of acid at fracture mouth is the initial concentration for all j as follows:

$$C(x = 0, y, t) = C_{1, j \in \mathbb{Z}}^{n \in \mathbb{N}} = C_{init}$$
 H-26

Concentration of acid at fracture walls is calculated as follows:

$$k_{r}(C_{w}^{\alpha})_{i,j=Z}^{n} + v_{l}(C_{w})_{i,j=Z}^{n} = v_{l}(C_{i,0}^{n} - (C_{w})_{i,j=Z}^{n}) + K_{g}(C_{i,0}^{n} - (C_{w})_{i,j=Z}^{n})$$
 H-27

Equations H-23 and H-27 are solved for acid concentrations. Similar spaces and time steps are used as above. From the acid concentration profile, the amount of acid reacted is estimated, and the amount of rock dissolved is calculated. Then, the gain in fracture width by acid is determined. For the new time step, the new fracture width is used in the gridding.

## **APPENDIX I**

# PUMP CAPACITY AND SURFACE PRESSURE CALCULATION

To initiate and propagate a fracture and keep it open during pad fluid and acid injection, the pump has to deliver the required net fracture pressure,  $p_{net}$ . The surface pressure needed to be delivered by the pump to maintain  $p_{net}$  in the fracture can be computed considering the static head pressure and dynamic friction loss. Then, the pump capacity required to deliver  $p_{net}$  is estimated. The designed pump capacity must not exceed the available pump capacity, and the designed surface pressure must not exceed the tubing strength and the pressure rating of other equipment. The estimated pump capacity and required surface pressure are constrained not to exceed operation limitation. The required pump capacity calculation is as follows:

#### Hydrostatic Head

Hydrostatic head (psi) in the tubing while pumping fracturing fluid is estimated as follows:

$$p_{head} = 0.052 H \rho_f$$
 I-1

where *H* is length of the tubing (ft) from surface to the fracture centre, and  $\rho_f$  is average density of fracturing fluid (ppg) (Bourgoyne et al. 1991).

#### Frictional Pressure Loss in the Tubing

The frictional pressure loss in the tubing is estimated as follows:

$$\Delta P_{fric.loss} = f.\overline{v}^2 \cdot \frac{H}{25.8(d)}$$

where  $\Delta P_{fric.loss}$  is frictional pressure drop (psi), *f* is friction factor (dimensionless),  $\overline{v}$  is average velocity in the tubing (ft/sec), and *d* is internal diameter of the tubing (inch).

The average velocity,  $\overline{v}$ , in the tubing is given as follows:

$$\overline{v} = \frac{q_i}{2.448.d^2}$$
 I-3

where  $q_i$  is the injection rate (gal/min) and *d* is internal diameter of the tubing (inch). For laminar flow when Reinhold's number N<sub>Re</sub> < 2300, the frictional factor is calculated as follows:

$$f = \frac{16}{N_{\text{Re}}}$$

For turbulent flow, which is usually the case during injection of fracturing fluid, the frictional factor f can be obtained by solving the following equation (Haaland 1983):

$$\sqrt{\frac{1}{f}} = -1.8 \log_{10} \left[ \left( \frac{(\varepsilon/d)}{3.7} \right)^{1.11} + \frac{6.9}{N_{\text{Re}}} \right]$$
 I-5

where  $\epsilon/d$  is the relative roughness (dimensionless) and  $N_{Re}$  is the Reinhold's number (dimensionless), calculated as follows:

$$N_{\rm Re} = 928(\rho_f)(\bar{v})\frac{d}{\mu_e}$$
 I-6

Here  $\rho_{\rm f}$  is density of fracturing fluid (ppg,  $\bar{\nu}$  is average velocity in the tubing (ft/sec), *d* is internal diameter of the tubing (inch), and  $\mu_{\rm e}$  is the effective viscosity (apparent Newtonian viscosity) equals to fluid viscosity if it is Newtonian (cp). If the fluid is non-Newtonian and follows power low, the

effective viscosity  $\mu_e$  (Pa.s) is calculated as follows (Dodge and Metzner 1959):

$$\mu_e = K(d)^{1-n} \frac{\left(3 + n^{-1}\right)^n}{96(\overline{\nu})^{1-n} \left(0.0416\right)^n}$$
 I-7

where *n* is power law index (dimensionless) and *J* is fluid consistency index (mPa-sec<sup>n</sup>).

### **Required Pump Capacity and Surface Pressure**

The treatment pressure (psi) required to deliver  $p_{net}$  is as follows (Smith and Shlyapobersky 2000):

$$p_{treat} = p_{net} + \sigma_h$$
 I-8

where  $p_{net}$  is the net wellbore fracture pressure estimated using the 2D PKN-C model, and  $\sigma_h$  is the minimum in-situ stress in the pay-zone to be fractured. The pressure required (psi) at the surface,  $p_{surf}$  to develop the treatment pressure, is calculated as follows:

$$p_{surf} = p_{treat} + \Delta p_{fric.loss} + \Delta p_{surf.eq.loss} - p_{head}$$
 I-9

where  $\Delta p_{surf.eq.loss}$  is the pressure loss in the surface equipment.

The pump capacity (horsepower) required to develop the above surface pressure is as follows (Bourgoyne Jr., KK Millheim et al. 1991):

$$HP_{reqd} = \frac{q_i \cdot p_{surf}}{1714}$$
 I-10

# **APPENDIX J**

# THE 2D PKN-C FRACTURE MODEL

A 2D PKN-C fracture model was developed by Nordgren (1972) by integrating the Carter Equation II (Howard and Fast 1957) into the original PKN model (Perkins and Kern 1961) for material balance with fluid leakoff at a constant injection rate. The main assumptions of 2D PKN-C model are: (1) the fracture height is constant; (2) the fracture length is greater than the fracture height; (3) there is no flow in vertical direction; (4) the pressure in a vertical cross section of the fracture is constant; and (5) the fracture has an elliptical shape, as presented in Figure J-1.



Figure J.1 – The PKN fracture model geometry.

Fracture width can be expressed as a function of the local pressure. The fracturing fluid pressure in vertical cross-sections perpendicular to the direction of propagation is considered the net fracture pressure,  $p_{net}$ , which is a function of the lateral coordinate. This net fracture pressure creates an

elliptical cross-section with maximum width at the wellbore, as follows (Valko and Economides 1995):

$$w_0 = \frac{2h_f p_{net}}{E'} \qquad \qquad J-1$$

where

$$E' = \frac{E}{\left(1 - v^2\right)} \qquad \qquad J-2$$

Here,  $h_f$  is the fracture height, E' is the plane strain modulus, E is the elastic modulus, and v is the Poisson's ratio. The maximum width,  $w_o$ , is a function of the lateral coordinate. At the wellbore, it is denoted by  $w_f$ .

The fluid pressure gradient during propagating in  $x_f$  direction is given by the flow resistance in a narrow elliptical flow channel. For Newtonian flow behaviour with constant viscosity, pressure drop is given as follows:

$$\frac{\Delta P}{x_f} = \frac{64\mu}{\pi w_0^3 h_f} \times \frac{q_i}{2}$$
 J-3

Here,  $\mu$  is the viscosity of the fracturing fluid, and  $q_i$  is the injection rate into fracture two wings. Thus, half of  $q_i$  is the flow rate in one wing of the fracture. The injection rate is assumed constant, and the fluid flow in the fracture is laminar.

Perkins and Kern (1961) assumed that the net pressure is zero at the fracture tip. The elliptical cross-section has maximum width at the centre, which gives maximum fracture width at the wellbore. Thus, integrating the above equation between the wellbore and the tip and further simplifying, the Perkins-Kern width equation for maximum width at the wellbore is as follows:

$$w_f = 3.57 \left(\frac{\mu q_i x_f}{2E'}\right)^{\frac{1}{4}}$$
 J-4

The average width is calculated by multiplying the maximum fracture width at the wellbore by a constant shape factor ( $\gamma$ ). Because of elliptic shape of the fracture along the vertical direction and the lateral variation of the width,  $\gamma$  is found to be  $\pi/5$ . Therefore, the average fracture width  $\overline{w}$  is as follows:

$$\overline{w} = 2.24 \left(\frac{\mu q_i x_f}{2E'}\right)^{\frac{1}{4}}$$
 J-5

This relationship between treatment parameters, rock properties, and fracture width at the wellbore for no-leakoff situation was modified by Valko and Economides (1995) to the following improved expression for the fracture width at the wellbore,  $w_f$ , as follows:

$$w_f = 3.27 \left(\frac{\mu q_i x_f}{2E'}\right)^{\frac{1}{4}}$$
 J-6

The basic solution for estimating the extent of the fracture area—taking into account the effect of fluid leaking into the formation and fracture propagation—is derived from Carter Equation (Howard and Fast 1957). Fracture width and height are assumed constant, and fracture length is a variable. Injection rate is also assumed constant. The leak-off flow rate, normal to the fracture faces, corresponding to the given surface element is as follows:

$$\frac{\partial V_l}{\partial t} = \frac{C_l}{\sqrt{t-\tau}} \partial A \qquad \qquad J-7$$

The term  $\tau$  is the opening time at which filtration starts. The actual time is denoted by *t*, *C*<sub>*l*</sub> is the overall fluid leak-off coefficient, *V*<sub>*l*</sub> is the volume being leaked, and *A* is the fracture surface area. If the growth rate of fracture surface area  $dA/d\tau$  is known, then the leak-off flow rate through the two fracture surfaces of one wing is the summation of the different flow rates

along the surface elements, and the leak-off flow rate from one wing (for two fracture surfaces) is as follows:

$$2\int_{0}^{t} \frac{C_{L}}{\sqrt{t-\tau}} \left(\frac{dA}{d\tau}\right) dA \qquad \qquad J-8$$

The fluids are partly leaking into the formation and partly engaged in fracture growth, with  $S_p$  being the spurt loss and *w* the constant fracture width. The volumetric fracture growth rate and the spurt loss rate at the new fracture surface are as follows:

$$w\frac{dA}{dt} + A\frac{dw}{dt} + 2S_p \frac{dA}{dt}$$
 J-9

Carter formulated the material balance in terms of flow rates. At any injection time *t*, the injection rate entering one wing of the fracture is equal to the sum of the different leakoff rates plus the growth rate of the fracture volume. With  $q_i$  be the total injection rate, it can be balanced as follows (Howard and Fast 1957):

$$\frac{q_i}{2} = 2 \int_0^t \frac{C_L}{\sqrt{t-\tau}} \left(\frac{dA}{dt}\right) d\tau + \left(w + 2S_p\right) \frac{dA}{dt} + A \frac{dw}{dt}.$$
 J-10

Carter solved a simplified version of the material balance and obtained an analytical solution for the constant injection rate, neglecting the width increase during the fracture growth. Carter material balance solution was used in the PKN-C model to estimate fracture half-length,  $X_f$  for any given injection time, *t* and fracture average width,  $\overline{w}$  as follows:

$$X_{f}(t) = \frac{\left(\overline{w} + 2S_{p}\right)}{4C_{L}^{2}\pi} \frac{q_{i}}{2} \left[ \exp\left(\beta^{2}\right) erfc(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1 \right]$$
 J-11

where

When a non-Newtonian fracturing fluid is used with the power law exponent n and the consistency index J (Pa-sec<sup>n</sup>), the maximum width at the wellbore in terms of power law parameters can be expressed as follows (Valko and Economides 1995):

$$w_{f} = 9.15^{\left(\frac{1}{2n+2}\right)} 3.98^{\left(\frac{n}{2n+2}\right)} \left[\frac{1+2.14n}{n}\right]^{\left(\frac{n}{2n+2}\right)} K^{\left(\frac{1}{2n+2}\right)} \left(\left(\frac{q_{i}}{2}\right)^{n} \frac{h_{f}^{1-n} x_{f}}{E'}\right)^{\left(\frac{1}{2n+2}\right)}$$
J-13

Using the shape factor ( $\pi$ /5) for the PKN model, the average width along the fracture length is given by:

$$\overline{w} = \frac{\pi}{5} w_f \qquad \qquad J-14$$

The net fracture pressure,  $p_{net}$ , is calculated as follows:

$$p_{net} = \frac{E'}{2h_f} w_f \qquad \qquad J-15$$

Here  $p_{net}$  is inversely proportional to the fracture height and directly proportional to fracture width at the wellbore. The fracture treatment pressure at the wellbore with  $\sigma_1$  as the minimum horizontal in-situ stress in the payzone is then given as follows:

$$p_{treat} = \sigma_1 + p_{net} \qquad \qquad J-16$$

## **APPENDIX K**

## **GAS PRODUCTION FROM A FRACTURED WELL**

A production model is needed to determine gas cumulative production from the acid fractured well and the recovery factor. The production model for a fracture well in gas reservoir was reviewed by Rahman M.M. et al (2002), as detailed in this appendix. To estimate the cumulative production from a well, a constant production rate is needed. However, production rate changes with time as reservoir and bottom hole flowing pressures change during production. If the well can be produced with a constant bottom hole flowing pressure, then adjusted production rates can be estimated with a regular time interval. These production rates will vary with time due to declining reservoir pressure with production and can be used to determine the cumulative production (Agarwal, Carter et al. 1979).

Assuming the well is producing at a constant bottom hole flowing pressure, since reservoir pressure will decline with production, production rate under this condition will decline, too. To adjust for constant flow rate, the total production life is defined as cumulative of a small time interval. After each cumulative period, the average reservoir pressure and gas properties are evaluated as functions of cumulative production up to that period and then used to estimate the constant production rate during these small time intervals, as follows.

The small time interval is defined as  $\Delta t$  and successive time steps are indexed as *i* = 0, 1, 2, 3,..., n. At the beginning of production, *i* = 0, and initial reservoir condition parameters are used for production with bottomhole pressure. During  $\Delta t$  period in hour, the cumulative production during this period will be  $q_g \propto (\Delta t/24)$  where  $q_g$  is average gas production rate over the period  $\Delta t$ . Then average reservoir pressure can be calculated, which will be less than the initial pressure due to production. This new calculated average reservoir pressure will be used to calculate the average gas properties. At the next time step, i = 1, average pressure and gas properties calculated for the previous time step are used to calculate a production rate that is constant over next  $\Delta t$ . The procedure will be repeated, and the total cumulative production,  $G_p$ , can be estimated over the total production period. The transient production rate is used until the pseudo-steady-state condition becomes active, after which the pseudo-steady-state condition is used. Transition between these conditions and the relationships for average gas properties in every time step are presented in the following sections.

## **Reservoir Flow Conditions**

There are three flow regimes valid at different times after the start of production from a well. These flow regimes are transient, pseudo-steadystate, and steady-state conditions. Production of gas from newly fractured or perforated well is in a transient flow condition changing in time to a pseudosteady-state flow. The transient effect of a fractured well in moderate to high permeability reservoir is of less importance since its period is very short. However, in low permeability formation, transient flow is important as it continues for a significantly long period. Steady-state condition occurs, after the transient flow period, in a reservoir that has outer boundaries open to aquifer or water flooding. If production from the reservoir is closely balanced by fluid entry across the boundary, the reservoir pressure at the boundary remains constant. Therefore, the pressure distribution within the reservoir does not change with time.

Transient flow regime occurs in the reservoir once a pressure disturbance is created in the reservoir, such as starting production or changing the rate of production. During this flow condition, the pressure responsive happens around the well and does not arrive at the reservoir boundary. Therefore, the reservoir looks infinite acting. The pressure in this region is a function of time and radial distance from the wellbore. When a well is opened to produce after perforation or fracturing, the pressure in the wellbore usually decreases with production time, and the drainage area from which the gas is producing increases. The pressure response to this disturbance moves further out into the reservoir until it reaches the boundary.

Pseudo-steady-state condition occurs in reservoir that has been producing for enough time to overcome the transient response period and the effect of the outer boundary has been felt. This condition continues in a depletion type reservoir which is surrounded by impermeable boundaries. Because there is no flow from nearby formations, the reservoir pressure declines proportionally with production. For a well producing at constant rate, the pressure decline rate during the production with respect to time remains constant all over the reservoir. For gas production from a low permeability reservoir the following equation was suggested by Earlougher (1977) to estimate the time when the pseudo-steady state regime starts:

$$t_{pss} = \frac{\phi\mu c_t A t_{DA}}{0.000264k}$$
K-1

The term  $t_{pss}$  in the equation is the time in hour at which the pseudo-steady state regime begins, *A* is the drainage area (ft<sup>2</sup>),  $c_t$  is the total compressibility at initial reservoir condition in psi<sup>-1</sup>,  $\emptyset$  is formation porosity,  $\mu$  is gas viscosity in cp and k is formation permeability in millidarcy (md). The non-dimensional pseudo-steady state time,  $t_{DA}$  depends on drainage area shape and well location it has a value of 0.1 for a regular shape such as a circle or a square with a well in the centre.

## **Production Equations for Gas Flow from Fractured Well**

The gas flow rate from a well during transient period is estimated based on Darcy's law and the continuity principle. The equation for gas flow rate in the transient condition is derived using the mass conservation law from the following general expression (Dake 1978):

$$\phi \frac{\partial \rho}{\partial t} = \nabla \left( \rho \frac{k}{\mu_s} \nabla p \right)$$
 K-2

Here  $\phi$  is the porosity,  $\rho$  is the density of gas, *t* is the flow time,  $\mu_g$  is the viscosity of gas, *k* is the formation permeability, and p is the pressure.

Considering radial co-ordinates and real gas law, Equation K.2 can be written as follows:

$$\frac{\phi}{k}\frac{\partial}{\partial t}\left(\frac{p}{Z_g}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{p}{\mu_g Z_g}r\frac{\partial p}{\partial r}\right)$$
 K-3

where  $Z_g$  is the gas deviation factor (*Z*-factor) and *r* is an arbitrary radial distance from the wellbore centre. Other terms are as defined in the above equation. The basic assumptions for radial fluid flow from the well surrounding area are: homogeneous and isotropic reservoir, well producing from the entire formation thickness, and the formation is saturated with a single-phase fluid.

By differentiate and re-arranging equation K.3, it becomes as follows:

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r} \frac{\partial p^2}{\partial r} = \frac{\phi \mu_g}{kp} \frac{\partial p^2}{\partial t}$$
 K-4

The compressibility of ideal gas,  $c_g$ , can be approximated by 1/p. therefore, for ideal gas, Equation K.4 can be expressed as follows:

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r} \frac{\partial p^2}{\partial r} = \frac{\phi \mu_s c_s}{k} \frac{\partial p^2}{\partial t}$$
 K-5

The gas properties change as functions of pressure. Therefore, Al-Hussainy and Ramey (1966) developed a pressure drawdown solution of the above diffusivity equation using a real gas pseudo-pressure function. This real gas pseudo-pressure function, m(p) is defined as follows:

$$m(p) = 2\int_{p_0}^{p} \frac{p}{\mu_g Z_g} dp$$
 K-6

The differential pseudo-pressure  $\Delta m(p)$  is defined as  $m(p_i) - m(p_{wf})$ . This is the driving force in the reservoir where  $p_i$  is the initial pressure and  $p_{wf}$  is the bottom hole flowing pressure. The product of initial gas properties,  $\mu_g Z_g$  and the product of average gas properties,  $\overline{\mu}_g \overline{Z}_g$  remains constant at low pressure. Therefore,  $\Delta m(p)$  is given as follows:

$$2\int_{P_{wf}}^{P_i} \frac{p}{\mu_g Z_g} dp \approx \frac{p_i^2 - p_{wf}^2}{\overline{\mu}_g \overline{Z}_g}$$
 K-7

The gas average properties  $\overline{\mu}_{g}$  and  $\overline{Z}_{g}$  are evaluated at an average pressure within the drainage area of the well. At high pressures when both  $p_{i}$  and  $p_{wf}$  are higher than 3000 psi,  $\Delta m(p)$  is approximated as follows (Dake 1978):

$$2\int_{P_{wf}}^{P_i} \frac{p}{\mu_g Z_g} dp \approx 2 \frac{\overline{p}}{\overline{\mu}_g \overline{Z}_g} \left( p_i - p_{wf} \right)$$
 K-8

where  $\overline{p}$  is the average reservoir pressure. For real gas, the diffusivity equation was then derived by replacing  $p^2$  in Equation K.5 by the pseudo-pressure m(p) as follows:

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r} \frac{\partial m(p)}{\partial r} = \frac{\phi \mu_g c_g}{k} \frac{\partial m(p)}{\partial t}$$
 K-9

The real gas pseudo-pressure drawdown at a constant flow rate in the transient phase is then expressed as follows for field units (Economides, Hill et al. 1994; Lee and Wattenbarger 1996):

$$m(p_{i}) - m(p_{wf}) = \frac{57,910q_{g}p_{sc}T}{khT_{sc}} \left(\log t + \log \frac{k}{\phi \overline{\mu}_{g}\overline{c}_{t}r_{w}^{2}} - 3.23 + 0.869s'\right)$$
 K-10

Here,  $q_g$  is the gas flow rate in Mscf/D, T is the reservoir temperature,  $p_{sc}$  and  $T_{sc}$  are pressure and temperature at standard conditions, respectively, h is the pay-zone thickness,  $\overline{c}_t$  is the total system compressibility, and s' is the total skin factor that accounts for the formation damage or stimulation skin, s, and a non-Darcy flow effect,  $Dq_g$ .

For  $p_{sc} = 14.7 psia$  and  $T_{sc} = 520^{\circ} R$ , Equation K.10 becomes as follows:

$$m(p_i) - m(p_{wf}) = \frac{1637q_g T}{kh} \left( \log t + \log \frac{k}{\phi \overline{\mu}_g \overline{c}_t r_w^2} - 3.23 + 0.869s' \right)$$
 K-11

Russell et al. (1966) used different approach and developed the pressuresquared formula which relates the pressure drawdown to the flow rate of gas. Lee and Wattenbarger (1996) argued that at low pressures (e.g., below 2000 psia), the product  $\mu_g Z_g$  and  $\overline{\mu}_g \overline{Z}_g$  are approximately equal and constant for some gases. Thus, the pseudo-pressure difference can be conveniently defined by Equation K.7. The substitution of Equation K.7 into Equation K.11 gives the pressure-squared formula presented by Lee and Wattenbarger (1996) as follows:

$$p_{i}^{2} - p_{wf}^{2} = \frac{1637q_{g}T\overline{Z}_{g}\overline{\mu}_{g}}{kh} \left(\log t + \log \frac{k}{\phi\overline{\mu}_{g}\overline{c}_{t}r_{w}^{2}} - 3.23 + 0.869s'\right)$$
 K-12

The pressure-squared and the pseudo-pressure solutions give equal results in many cases (AI-Hussainy and Ramey 1966; Ramey and Wattenbarger 1968; Aziz, Ko et al. 1976). A degree of inaccuracy is generally involved in the pressure-squared relationship when both the reservoir pressure and the pressure drawdown are high (Dake 1978). The pressure-squared relationship is used widely in the industry more than the pseudo-pressure solutions, it is more convenient to use especially if gas PVT data for a reservoir is not available.

The pressure-squared relationship is used in this study to calculate the production rate during the transient flow period in the following form:

$$q_{g} = \frac{kh[p_{i}^{2} - p_{wf}^{2}]}{1637\overline{\mu}_{g}\overline{Z}_{g}T} \left[\log t + \log \frac{k}{\phi\overline{\mu}_{g}\overline{c}_{t}r_{w}^{2}} - 3.23 + 0.869s'\right]^{-1}$$
 K-13

When a well is fractured its radius,  $r_w$  in Equation K.13 is replaced by the effective wellbore radius,  $r'_w$  to account for the effect of fracture. Lee and Wattenbarger (1996) developed an expression for the effective wellbore radius based on the works of Van Everdingen (1953) and Mathews and Russell (1967) as follows:

Here  $s_f$  is pseudo-skin determined by graphical method proposed by Cinco-Ley and Samaniego (1981). They considered a bilinear flow characteristic in a finite-conductivity fracture to represent a fractured well reasonably more than an infinite-conductivity fracture described by Gringarten and Ramey (1974). Valko et al. (1997) developed an equation that provides a simple and accurate curve fit of the graphical data proposed Cinco-Ley and Samaniego (1981), as follows:

$$F = \frac{1.65 - 0.328u + 0.116u^2}{1 + 0.18u + 0.064u^2 + 0.005u^3}$$
 K-15

where  $u = \ln(F_{CD})$  K-16

And the value of non-dimensional fracture conductivity ( $F_{CD}$ ) is estimated as follows:

$$F_{CD} = \frac{wk_f}{kx_f}$$
 K-17

Here,  $wk_f$  is the fracture conductivity in md-ft calculated as a function of fracture width, rock embedment strength and closure stress (Chapter 2, Section 2.3), *k* is formation permeability in md and  $x_f$  is the fracture half-length in ft.

From the estimated non-dimensional fracture conductivity,  $F_{CD}$ , the value of F is calculated then the pseudo-skin,  $s_f$  is calculated using the following relationship:

$$F = s_f + \ln \frac{x_f}{r_w}$$
 K-18

The basic assumption of Darcy's fluid flow equation in a porous media is that the pressure drop in the direction of flow is proportional to the fluid velocity as the flow is at low to moderate rates. When a fractured well is opened to start gas production, the flow turbulence is very high. The flux distribution along the fracture becomes irregular. Therefore, the pressure drop in the fracture is very high due to viscous and inertial force effects. These viscous and inertial force effects are called 'non-Darcy effects' because they deviate the flow from Darcy's law.

The influence of these non-Darcy effects on gas production from a fractured well can be significant (Fligelman, Cinco-Ley et al. 1989; Rangel-German and Samaniego 2000). These effects are taken into account by adding a non-Darcy component to the usual Darcy's equation. In the pressure drawdown solution of flow equation, non-Darcy component is included in the total skin, s' factor in Equations K.10 and K.13. This total skin factor that includes the non-Darcy effects is expressed as follows (Dake 1978):

$$s' = s + Dq_g \tag{K-19}$$

Here *s* is the formation damage or stimulation skin factor, and *D* is the non-Darcy flow co-efficient constant (D/Mscf). The term  $Dq_g$  accounts for turbulence skin effect or rate-dependent pseudo skin in high production rate wells.

Economides et al. (1994) developed an empirical relationship for non-Darcy flow coefficient, given as follows:

$$D = \frac{6 \times 10^{-5} \, \gamma k_s^{-0.1} h}{\mu_{g,wf} r_w h_{perf}^2}$$
K-20

where  $\gamma$  is gas specific gravity,  $k_s$  is the near-wellbore permeability in *mD*, *h* and  $h_{perf}$  are total and perforated pay-zone thickness, respectively, both in ft, and  $\mu_g$  is gas viscosity in cp, estimated at the bottom hole flowing pressure.

Lee and Wattenbarger (1996) developed another equation for non-Darcy coefficient presented as follows:

$$D = \frac{2.715 \times 10^{-15} \,\beta k M p_{sc}}{h r_w T_{sc} \mu_{g,wf}}$$
K-21

where *M* is gas molecular weight equals  $28.96\gamma$  multiplies by gas specific gravity ( $\gamma$ ) to air,  $\mu_{g,wf}$  is gas viscosity evaluated at wellbore flowing pressure  $P_{wf}$ , h is pay-zone height,  $r_w$  is wellbore radius,  $T_{sc}$  is temperature at standard conditions, and  $\beta$  is a turbulence factor that can be determined from the following equation (Jones 1987):

$$\beta = 1.88 \times 10^{10} k^{-1.47} \phi^{-0.53}$$
 K-22

By substituting the total skin factor s' and the effective wellbore radius  $r'_w$  values into Equation K.13, the transient flow rate equation for a fractured well becomes as follows:

$$q_{g} = \frac{kh \left[ p_{i}^{2} - p_{wf}^{2} \right]}{1637 \overline{\mu}_{g} \overline{Z}_{g} T} \left[ \log t + \log \frac{k}{\phi \overline{\mu}_{g} \overline{c}_{t} {r_{w}'}^{2}} - 3.23 + 0.869 \left( s + Dq_{g} \right) \right]^{-1}$$
 K-23

Reservoir pressure at the outer boundary declines at a constant rate with time when the reservoir is surrounded by no-flow boundaries. These no-flow boundaries can be natural limits, such as faults, or artificially induced limits by production from adjoining wells. This flow condition is known as 'pseudo-steady-state', which succeeds the transient flow period. Following the approach taken by Russell et al.(1966) and using the basic radial diffusivity Equation K.5, the gas flow equation form a fracture well at pseudo-steady-state condition is as follows (Dake 1978):

$$\overline{p}^{2} - p_{wf}^{2} = \frac{1422q_{g}\overline{\mu}_{g}\overline{Z}_{g}T}{kh} \left( \ln \frac{r_{e}}{r_{w}} - \frac{3}{4} + s \right)$$
 K-24

Here  $\overline{p}$  is the average reservoir pressure,  $\overline{\mu}_{g}$  and  $\overline{Z}_{g}$  are gas average viscosity and average z-factor evaluated at the average of  $\overline{p}$  and wellbore flowing pressure,  $p_{wf}$ . Economides et al. (1994) rearranged the above equation by taking the term  $\frac{3}{4}$  into logarithm, as follows:

$$\overline{p}^{2} - p_{wf}^{2} = \frac{1424q_{g}\overline{\mu}_{g}\overline{Z}_{g}T}{kh} \left( \ln \frac{0.472r_{e}}{r_{w}} + s \right)$$
 K-25

Valko et al. (1997) modified Equation K.25 for the fracture effect with F as defined in Equation K.15, to estimate the gas flow rate as follows:

$$q_{g} = \frac{kh(\bar{p}^{2} - p_{wf}^{2})}{1424\mu_{g}Z_{g}T} \times \frac{1}{\ln\left(\frac{0.472r_{e}}{x_{f}}\right) + F}$$
 K-26

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### **Average Gas Properties and Cumulative Gas Production**

The reservoir pressure declines with production when it is confined by impermeable strata and there is no or insignificant water influx from the adjoining aquifer. Reservoir volume occupied by gas does not change much with production, except for insignificant reduction because of connate water expansion and void volume compaction with the reservoir pressure decline. The real gas state in the reservoir is expressed as follows:

$$E_i = 35.37 \frac{p_i}{Z_i T}$$
 K-27

where  $E_i$  is the gas initial expansion factor (scf/rcf),  $p_i$  is the reservoir initial pressure (psia),  $Z_i$  is the initial Z-factor, and T is the reservoir temperature (°R). The initial gas in place, G is calculated as follows:

$$G = V\phi(1 - S_{wc})E_i$$
 K-28

where *V* is the net bulk volume of the reservoir,  $S_{wc}$  is the connate water saturation,  $E_i$  is gas initial expansion factor, and  $\emptyset$  is formation porosity.

From material balance, the state of the depletion reservoir at any stage of gas production at an isothermal condition is as follows (Dake 1978):

$$\frac{G_p}{G} = 1 - \left[1 - \frac{\left(C_w S_w + C_f\right)\Delta p}{1 - S_{wc}}\right]\frac{E}{E_i}$$
K-29

Here, *E* is the gas expansion factor (scf/rcf),  $G_p$  is the cumulative gas production (*scf*),  $C_w$  is the compressibility of water (psi<sup>-1</sup>),  $S_w$  is the water saturation,  $C_f$  is the pore compressibility (psi<sup>-1</sup>), and  $\Delta p = p_i - \overline{p}$ , where  $\overline{p}$  is the average reservoir pressure (*psia*). The gas expansion factor can be estimated from standard PVT analysis as a linear function of reservoir pressure as follows:

$$E = E_i + a_e (\overline{p} - p_i)$$
 K-30

The term  $a_e$  is the slope of experimental *E* vs. *p* data. Substituting Equation K.30 into Equation K.29, to obtain the following equation (Guo and Evans 1993):

$$A(\overline{p} - p_i)^2 + B(\overline{p} - p_i) + C = 0$$
 K-31

where

$$(A)^{i} = a_{e} \left( \frac{C_{w} (S_{w})^{i-1} - C_{f}}{1 - S_{wc}} \right)$$
 K-32

$$(B)^{i} = a_{e} + E_{i} \left( \frac{C_{w} (S_{w})^{i-1} - C_{f}}{1 - S_{wc}} \right)$$
 K-33

$$(C)^{i} = E_{i} \left( \frac{(G_{p})^{i-1}}{G} \right)$$
 K-34

The *i* superscripts represent time steps and the solution of Equation K.31 for time step *i* production is as follows:

(for  $A \neq 0$ )

$$(\overline{p})^{i} = p_{i} + \frac{-(B)^{i} \pm \sqrt{[(B)^{i}]^{2} - 4(A)^{i}(C)^{i}}}{2(A)^{i}}$$
 K-35

(for A = 0, B  $\neq$  0)

$$\left(\overline{p}\right)^{i} = p_{i} - \frac{\left(C\right)^{i}}{\left(B\right)^{i}}$$
 K-36

Values of  $\overline{Z}_{s}$ ,  $\overline{\mu}_{s}$  and  $\overline{c}_{i}$  are adjusted at the *i*-th time step as follows:

$$(\overline{Z}_{g})^{i} = 35.37 \frac{((\overline{p})^{i} + p_{wf})}{2(E)^{i}T}$$
 K-37

$$\left(\overline{\mu}_{g}\right)^{i} = 4.0 \times 10^{-6} \left(\frac{(\overline{p})^{i} + p_{wf}}{2}\right) + 0.0107$$
 K-38

$$(\overline{c}_t)^i = (S_g)^i (c_g)^i$$
 K-39

in which

$$(E)^{i} = E_{i} + a_{e} \left\{ \left[ \frac{(\overline{p})^{i} + p_{wf}}{2} \right] - p_{i} \right\}$$
 K-40

$$\left(S_{g}\right)^{i} = 1 - \left(S_{w}\right)^{i}$$
 K-41

$$(S_{w})^{i} = \frac{S_{wc} \left[ 1 + C_{w} \left( p_{in} - (\overline{p})^{i} \right) \right]}{1 - \left( \frac{C_{w} (S_{w})^{i-1} - C_{f}}{1 - S_{w}} \right) \left[ P_{i} - (\overline{p})^{i} \right]}$$
K-42
$$(c_{g})^{i} = \frac{1}{(\overline{p})^{i}}$$
K-43

One day (24 hours) is used as the time interval,  $\Delta t$  for time-step production estimation. After fracturing the rate equation is used with the reservoir initial pressure and gas initial properties, then average values are calculated according to above formulations for each 24-hour production period. The cumulative gas production,  $G_p$  and the recovery factor, *RF* at the end of *i*-th time step are as follows:

$$\left(G_{p}\right)^{i} = \left(G_{p}\right)^{i-1} + \left(q_{g}\right)^{i} \Delta t$$
K-44

$$\left(RF\right)^{i} = \frac{\left(G_{p}\right)^{i}}{G}$$
 K-45