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A NOTE ON SELECTION OF RESISTANCE COEFFICIENTS FOR EMPIRICAL PIPE FLOW FORMULAS
by
T.R. FIETZ

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## A NOTE ON SELECTION OF RESISTANCE COEFFICIENTS

## FOR EMPIRICAL PIPE FLOW FORMULAS

## by

T.R. Fietz

Abstract:

Methods of finding resistance coefficients for the Hazen-Williams and Manning formulas are discussed. Coefficients are selected to maximise agreement with the Colebrook-White equation, using pipe data from Australian Standard codes.

Notation

| $c^{1}$ | Resistance coefficient in an empirical formula |
| :---: | :---: |
| $C_{1}$ | A parametric value for $C_{\text {hw }}$ |
| $\mathrm{C}_{\mathrm{hw}}$ | Hazen-Williams resistance coefficient |
| d | Pipe diameter (bore) |
| $\mathrm{d}_{\mathrm{hw}}$ | Pipe diameter from Hazen-Williams formula |
| $\mathrm{d}_{\mathrm{m}}$ | Pipe diameter from Manning formula |
| $\left(\frac{d f}{d \mathbb{R}}\right)_{C W}$ | Slope of $\frac{k}{d}=$ constant line on Moody Chart |
| $\left(\frac{\mathrm{df}}{\mathrm{~d} \mathbb{R}}\right)$ | Slope of $\mathrm{C}_{\mathrm{hw}}=$ constant line on Moody Chart |
| $f$ | Darcy friction factor |
| $\mathrm{f}_{\mathrm{cw}}$ | Friction factor from Colebrook-White equation |
| $f_{h w}$ | Friction factor from rearranged Hazen-Williams formula |
| $\mathrm{f}_{\mathrm{m}}$ | Friction factor from rearranged Manning formula |
| $\mathrm{f}_{\mathbf{r}}$ | Friction factor for wholly rough wall turbulent flow |
| ${ }^{\text {f }}$ | Friction factor at tangent point |
| F | Function of |
| $g$ | Gravitational acceleration |
| $\mathrm{h}_{\mathrm{f}}$ | Frictional head loss |
| k | Equivalent sand grain roughness |
| $\mathrm{K}_{1}$ | A parametric value of $\frac{k}{d}$ |
| 2 | Pipe length |
| n | Manning's n |

```
N
Q Discharge
    Discharge from Hazen-Williams formula
    Discharge from Manning formula
    Reynolds Number = vd
    Reynolds Number at tangent point
    Hydraulic radius
    Energy (friction) gradient = 㬏
    Energy gradient from Darcy-Weisbach and Colebrook-White equations
    Energy gradient from Hazen-Williams formula
    Energy gradient from Manning formula
    Mean velocity = 4Q 
    R
    S exponent in an empirical formula
    Function of
    Kinematic viscosity
```

The Darcy-Weisbach formula, with friction factor $f$ given by the ColebrookWhite equation, is generally recognised as the preferable, universal method for predicting frictional head loss in water supply pipes.

Older empirical formulas, such as Hazen-Williams and Manning, still persist, however, in textbooks, handbooks and standard codes.

Their main advantage compared to the preferred formula is ready manipularLion to find head loss, discharge, or diameter for single pipe problems. Also they are easily differentiated, which saves computation in pipe network analysis.

The empirical formulas rely on specifying a resistance coefficient for a particular pipe material; $C_{h w}$ is used in the Hazen-Williams formula and $n$ in the Manning formula.

This report discusses selection of the empirical coefficients $C_{h w}$ and $n$ in order to maximise the agreement between the empirical formulas and the arefared formula. Firstly, areas of agreement are defined and charts are presented to relate $C_{h w}$ and $n$ to diameter $d$ and equivalent sand grain roughness $k$ for single pipes. Secondly, typical values of $C_{h w}$ and $n$ for common pipe materials are derived for a range of pipe diameters and flowrates. The methods for selecting $C_{h w}$ and $n$ are applied to pipe data shown in Australian Standard Codes.

## BASIC EQUATIONS

Frictional (or surface resistance) head loss for steady flow under pressure in prismatic circular pipes is given by the Darcy-Weisbach equation:

$$
\begin{equation*}
h_{f}=\frac{f l v^{2}}{d 2 g} \tag{1}
\end{equation*}
$$

The friction factor $f$ is a function of the relative roughness $\frac{k}{d}$ and the Reynolds Number $\mathbb{R}$, i.e. $f=\phi\left(\frac{k}{d}, \mathbb{R}\right)$. The most popular expression for $\phi$ is given by the Colebrook-White equation (Ref. 8):

$$
\begin{equation*}
\frac{1}{\sqrt{f}}=-2 \log _{10}\left(\frac{k}{3.7 d}+\frac{2.51}{R \sqrt{f}}\right) \tag{2}
\end{equation*}
$$

Plotting $\log f$ vertically against $\log \mathbb{R}$ horizontally for parametric values of $\frac{k}{d}$ gives the Moody Chart (Ref. 13).

At high $\mathbb{R}$ values equation (2) is asymptotic of the "Rough Law":

$$
\begin{equation*}
\frac{1}{\sqrt{f}}=1.74+2 \log _{10}\left(\frac{d}{2 k}\right) \tag{3}
\end{equation*}
$$

Earlier empirical expressions for frictional head loss are often in the form (Ref. 1):

$$
\begin{equation*}
V=C^{\prime} R_{h}^{\alpha} S^{\beta} \tag{4}
\end{equation*}
$$

The most popular empirical formulas used in Australia (Ref. 4) are the Hazen-Williams formula:

$$
\begin{equation*}
V=0.849 C_{h w} R_{h}^{0.63} S^{0.54} \tag{5}
\end{equation*}
$$

and the Manning formula:

$$
\begin{equation*}
V=\frac{1}{n} R_{h}^{0.67} S^{0.5} \tag{6}
\end{equation*}
$$

Using $Q=\frac{J}{4} d^{2} V$ and $R_{h}=\frac{d}{4}$ in equation (5) gives alternative forms of the Hazen-Williams formula:

$$
\begin{align*}
& Q=0.2784 C_{h w} d^{2.63} S^{0.54}  \tag{7}\\
& S=\frac{6.824 V^{1.852}}{C_{h w}^{1.852} d^{1.167}}  \tag{8}\\
& S=\frac{10.676 Q^{1.852}}{C_{h w}^{1.852} d^{4.87}} \tag{9}
\end{align*}
$$

Similarly, alternative forms of the Manning formula are:

$$
\begin{align*}
& Q=\frac{0.3102 d^{2.67} S^{0.5}}{n}  \tag{10}\\
& S=\frac{6.409 n^{2} V^{2}}{d^{1.34}}  \tag{11}\\
& S=\frac{10.39 n^{2} Q^{2}}{d^{5.34}} \tag{12}
\end{align*}
$$

Lamont (Ref. 12) has made a comprehensive review of the above and many
other empirical equations.

OPERATING ZONES ON THE MOODY CHART
$k$ and $d$ ranges for some clean water supply pipes are shown in Table 1. The $k$ and data in Table 1 have been used with a velocity range of 0.2 to 6 mes ${ }^{-1}$ to plot operating zones on the Moody chart. These are shown as Figures 1 and 2 , and will be used subsequently for assessing the empirical formulas.

METHODS OF COMPARISON OF PIPE FLOW EQUATIONS

In the following comparisons the Darcy-Weisbach equation (1) [in conjuncLion with the Colebrook-White equation (2)] has been taken as the preferred formula for pipe flow calculations. Results from the Hazen-Williams and Manning equations are compared with those from the preferred formula.

The first problem is to examine single pipes of given $d$ and $k$ to see if there is a velocity, or range of velocities, where the empirical formulas are in agreement with the preferred formula. By considering several pipe diameters a line, or zone, of agreement may be mapped on the Moody chart for the Hazen-Williams and Manning formulas. The errors involved in operating away from these lines, or zones, of agreement may also be shown on the Moody chart.

The second problem is to select a typical (or single) value of either $C_{h w}$ or $n$ for a pipe material so as to maximise the agreement with the preferred formula over the available range of pipe diameters and the normal range of velocities. Most of the typical $C_{h w}$ and $n$ values given in textbooks, handbooks and codes fall in this category.

FINDING EMPIRICAL COEFFICIENTS FOR A SINGLE PIPE
Using the Hazen-Williams Formula
Equation (8), with extended constants and exponents, may be manipulated to obtain $\frac{\mathrm{V}^{2}}{2 \mathrm{gd}}$ as a variable group:

$$
\begin{equation*}
S=\frac{6.824263 \times 2 \mathrm{~g}}{C_{h w}^{1.85185}\left(V_{d}\right)^{0.14815} d^{0.01852}}\left(\frac{V^{2}}{2 g d}\right) \tag{13}
\end{equation*}
$$

Using $S=\frac{h f}{2}$ in equation (1):

$$
\begin{equation*}
S=f\left(\frac{V^{2}}{2 g d}\right) \tag{A}
\end{equation*}
$$

Taking $\mathrm{g}=9.8 \mathrm{~m} \cdot \mathrm{~s}^{-2}$, then equations (13) and (14) are equivalent when:

$$
\begin{equation*}
f=\frac{133.7556}{C_{\text {how }}^{1.85185} V^{0.14815} d^{0.1667}} \tag{15}
\end{equation*}
$$

A Reynolds Number $\mathbb{R}$ may be introduced when combining equations (13) and (14):

$$
f=\frac{133.7556}{C_{h w}^{1.85185} d^{0.01852}}\left(\frac{\nu}{V d}\right)^{0.14815}\left(\frac{1}{v^{0.14815}}\right)
$$

Taking $\nu=1.0038 \times 10^{-6} \mathrm{~m}^{2} . \mathrm{s}^{-1}$ for water at 20 degrees Celsius gives:

(17)

Equation (17), with slight variations in coefficient and exponents, has been derived previously by numerous authors (Refs. 9, 12, 5, 14, 6, 15).

Equation (17) plots as a family of straight lines of slope -0.14815 on the $\log f$ vs. $\log \mathbb{R}$ Moody Chart. Note that variation of the water temperature will affect the numerator and shift the location of the plotted lines. Equation (17) is shown plotted on the Moody Chart in Figure 3, for Cow from 80 to 160 and $d=0.3 \mathrm{~m}$. Also plotted near the $C_{h w}=80$ line are lines for $\mathrm{d}=0.05 \mathrm{~m}$ and $\mathrm{d}=2 \mathrm{~m}$ which show that the diameter has a slight effect on the location of a $C_{h w}=$ constant line.

Diskin (Ref. 9) has shown that the Hazen-Williams formula and ColebrookWhite equation are in agreement at the point where a $C_{h w}=$ constant line is tangential to a $\frac{k}{d}=$ constant line on the Moody Chart. ${ }^{\text {nw }}$ On Figure 4 (A) the tangent point is at $T$ where the slope of the $\frac{k}{d}=$ constant line is equal to the slope of the $C_{h w}=$ constant line.

From the Colebrook-White equation (1), for a specified value of $\frac{k}{d}$, the slope is:

$$
\begin{equation*}
\left(\frac{d f}{d \mathbb{R}}\right)_{c w}=-\left(0.062 \frac{k}{d} \frac{\mathbb{R}^{2}}{f}+0.5756 \frac{\mathbb{R}}{f^{1.5}}+\frac{\mathbb{R}}{2 f}\right)^{-1} \tag{18}
\end{equation*}
$$

For any $\mathbb{R}$ value, $f$ is found from the Colebrook-White equation by Yao's method (Ref. 16).

From equation (16) the slope is:

$$
\begin{equation*}
\left(\frac{d f}{d \mathbb{R}}\right)_{h w}=\frac{-133.7556 \times 0.14815}{C_{h w}^{1.85185} d^{0.01852} v^{0.14815} \mathbb{R}^{1.14815}} \tag{19}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\frac{d f}{d \mathbb{R}}\right)_{h w}=\frac{-f_{\times} 0.14815}{\mathbb{R}} \tag{20}
\end{equation*}
$$

The tangent point $T$ was found by applying the method of bisection ior interval halving) (Ref. 7) about $\mathbb{R}$ to the equation:

$$
\begin{equation*}
F(\mathbb{R})=\left(\frac{d f}{d \mathbb{R}}\right)_{c w}-\left(\frac{d f}{d \mathbb{R}}\right)_{h w}=0 \tag{21}
\end{equation*}
$$

Note that $C_{h w}$, the coefficient being sought, is required to find the derivative from equation (19). The solution may be simplified by using the Colebrook-White $f=f_{c w}$ in equation (20), as $f_{c w}$ and the Hazen-Williams $f=$ $\mathrm{f}_{\mathrm{hw}}$ converge as the tangent point is approached.

The tangent point occurs at $\left(\mathbb{R}_{T}, f_{T}\right)$ and at any other $\mathbb{R}$ value $f_{c w}$ exceeds $f_{h w}$. The useful range of $\mathbb{R}$ values may be extended by specifying an arbitrary difference in $f_{h w}$ and $f_{c w}$. In this case $f_{h w}$ is allowed to be within $-2.5 \%$ of $f_{c w}$. The $C_{h w}=C_{1}$ line is then taken through the point ( $\mathbb{R}_{T}$, $1.025 \mathrm{f}_{\mathrm{T}}$ ) as shown on Figure $4(\mathrm{~A})$. The value for $\mathrm{C}_{1}$ is found by putting f $=1.025 \mathrm{f}_{\mathrm{T}}$ and $\mathbb{R}=\mathbb{R}_{\mathrm{T}}$ in a rearranged equation (16):

$$
\begin{equation*}
C_{h w}=\frac{14.067}{f^{0.54} d^{0.01} \mathbb{R}^{0.08} v^{0.08}} \tag{22}
\end{equation*}
$$

The above method has been applied for a range of $d$ and $k$ values to produce Figure 5, which may be used to find a $C_{h w}$ value for a single pipe of known $d$ and $k$. Pipe velocity $V$ for $\mathbb{R}_{T}$ at the tangent point is also plotted for the range 0.2 to $6 \mathrm{~m} . \mathrm{s}^{-1}$.

To show the difference between $f_{h w}$ and $f_{c w}$ when operating away from the tangent point, $f_{h w}$ is found from equation (16) for a given $\mathbb{R}$ value as shown on Figure $4(\mathrm{~A})$. The ratio $\frac{\mathrm{f}_{\mathrm{hw}}}{\mathrm{f}_{\mathrm{cw}}}$ is then plotted at the point $\left(\mathbb{R}, \mathrm{f}_{\mathrm{cw}}\right)$, and from numerous points the contour lines of Figure 6 have been obtained.

Equation (22) for finding $C_{h w}$ is a rearranged form of equation (16), used for finding $f_{h w}$. The diameter effect is eliminated so Figure (6) applies to all pipe diameters. Comparison of Figure 6 with Figures 1 and 2 indicate the degree of agreement between the Hazen-Williams formula and the preferred formula for different pipes.

## Using the Manning Formula

Following the argument of Barnes et al (Ref. 6), equations (11) and (14) are equivalent when:

$$
\begin{equation*}
\frac{6.409 n^{2} V^{2}}{d^{1.34}}=\frac{f V^{2}}{d 29} \tag{23}
\end{equation*}
$$

Taking $g=9.8 \mathrm{~m}_{\mathrm{s}} \mathrm{s}^{-2}$ gives:

$$
\begin{equation*}
f=\frac{125.616 n^{2}}{d^{0.34}} \tag{24}
\end{equation*}
$$

Equation (24) plots as a family of horizontal lines on the Moody Chart, so the Manning equation agrees with the preferred formula in the zone of wholly rough wall turbulent flow, where equation (3) applies. Rearranging equation (3):

$$
f=0.25\left[\log _{10}\left(3.707 \begin{array}{l}
d  \tag{25}\\
k
\end{array}\right)\right]^{-2}
$$

For equations (24) and (25) to be equivalent:

$$
\begin{equation*}
\frac{125.616 n^{2}}{d^{0.34}}=0.25\left[\log _{10}\left(3.707 \frac{d}{k}\right)\right]^{-2} \tag{26}
\end{equation*}
$$

Giving:

$$
\begin{equation*}
n=\frac{d^{0.17}}{22.416}\left[\log _{10}\left(3.707 \frac{d}{k}\right)\right]^{-1} \tag{27}
\end{equation*}
$$

If $f$ from equation (25) is called $f_{r}$ then equation (27) becomes:

$$
\begin{equation*}
n=\frac{f_{r}^{0.5} d^{0.17}}{11.208} \tag{28}
\end{equation*}
$$

For given $d$ and $k$, and $\frac{k}{d}=K_{1}$, the value of Manning's $n$ is $N_{1}$ for agreement
of the Manning and the preferred formula. $N_{1}$ is found from equation (27). The corresponding curves are shown plotted on Figure 4(B).

For flow in the transition zone the Manning friction factor $f_{m}=\mathbf{f}_{\mathbf{r}}$ underestimates $f_{c w}$ from the preferred formula. The value of $\frac{f_{m}}{f_{c W}}$ has been determined for numerous points on the Moody chart, and these points were used to plot the contour lines of Figure 7. Comparison of Figures 1 and 2 with Figure 7 Indicates the degree of agreement between the Manning and the preferred formulas for various pipes.

Equation (27) has been applied to a d range from 0.05 to 2 m and for numerous $k$ values to produce Figure 8. Also shown (as broken lines) are velocity contours for $\frac{\mathrm{f}_{\mathrm{m}}}{\mathrm{f}_{\mathrm{cW}}}=0.99$ indicating the lower limit of velocity for various $d$ and $k$ values. Note that Figure 8 is an alternative to the chart of Barnes et al (Ref. 6, Figure 3.8) which plots $n$ against $d$ (both linear scales) for parametric values of $k$.

FINDING EMPIRICAL COEFFICIENTS FOR AN OPERATING ZONE

## Using the Hazen-Williams Formula

The operating zone is covered by a mesh of points as shown partially in Figure 9. The $V$ range is from 0.2 to $6 \mathrm{~m} . \mathrm{s}^{-1}$, in increments of $0.25 \mathrm{~m} \cdot \mathrm{~s}^{-1}$, giving 24 values. The d range is given in Table 1 for various pipes and this was divided to give 23 values. The total number of grid points is then 552.

For a given pipe material of known $k$ value, $a$ value is chosen for $C_{h w}$ and water temperature is taken as 20 degrees Celsius. If friction gradient $S$ is the pertinent variable then the following procedure is used to compare the Hazen-Williams and the preferred formulas:
(i) At any mesh point compute $\mathbb{R}=\frac{V d}{V}$ and $\frac{k}{d}$. Use Yao's method (Ref. 16) to find $f$ from equation (2), then $S_{c w}$ from equation (14).
(ii) Calculate $S_{h w}$ from equation (8).
(iii) Find $\frac{S_{h w}}{S_{c w}}$ and increment the count for the appropriate range, e.g. from 0.9 to 0.95.
(iv) Repeat for all mesh points then the number of points in any range divided by the total number of points represents the proportion of the chart area covered by that range, e.g. for
the range $\frac{S_{h w}}{S_{w}}=0.95$ to 1.05 , a chart area of $45 \%$ indicates that over $45 \%$ of the operating zone the Hazen-Williams and the preferred formulas agree within $\pm 5 \%$ for finding the friction gradient.

An alternative method is to plot contour lines on the operating zone mesh and to measure areas by planimeter. Contour lines for one pipe material are shown on Figure 9.

Various $C_{h w}$ values were tried for each pipe in Table 1 to maximise agreement with the preferred formula. The effect of varying $C_{h w}$ on the $\pm 5 \%$ and $\pm 10 \%$ agreement ranges for one pipe is shown in Figure 10(A).

The other dependent variables of interest are $d$ and $Q$. To find $d_{h w}$ for $a$ given mesh point ( $\mathrm{d}, \mathrm{V}$ ), and specified $\mathrm{C}_{\mathrm{hw}}$ value, the procedure is:
(i) Compute $\mathbb{R}=\frac{V d}{\nu}$ and $\frac{k}{d}$, and find $f$ by $Y a 0^{\circ} s$ method (Ref. 16). Find $S_{\text {cw }}$ from equation (14).
(ii) Find $d_{h w}$ from a rearranged equation (5) with $\mathbb{R}_{h}=\frac{d}{4}$ :

$$
\begin{equation*}
d_{\text {hw }}=\frac{5.1866}{S_{\text {cw }}^{0.857}}\left(\frac{V}{C_{\text {Lw }}}\right)^{1.5873} \tag{29}
\end{equation*}
$$

To find $Q_{h w}$ for a given mesh point ( $\mathrm{d}, \mathrm{V}$ ) and specified $C_{h w}$ value, the procedure is:
(i) Compute $\mathrm{S}_{\mathrm{cw}}$ as above.
(ii) Find $Q_{h w}$ from equation (7):

$$
\begin{equation*}
Q_{h w}=0.2784 C_{h w} d^{2.63} S_{c w}^{0.54} \tag{30}
\end{equation*}
$$

$\frac{d_{h w}}{d}$ may be expressed as a function of $\frac{S_{h w}}{S_{c w}}$ by rearranging equation (8) to get d:

$$
\begin{equation*}
d=\frac{5.1866}{S_{h w}^{0.857}}\left(\frac{V}{C_{h w}}\right)^{1.5873} \tag{31}
\end{equation*}
$$

Dividing equation (29) by equation (31) gives $\frac{d_{h w}}{d}$ :

$$
\begin{equation*}
\frac{d_{h w}}{d}=\left(\frac{S_{h w}}{S_{c w}}\right)^{0.857} \tag{32}
\end{equation*}
$$

Applying a similar argument to a rearranged equation (9) and equation (30) gives $\frac{Q_{h w}}{Q}$ :

$$
\begin{equation*}
\frac{Q_{h w}}{Q}=\left(\frac{S_{h w}}{S_{c w}}\right)^{-0.54} \tag{33}
\end{equation*}
$$

Equations (32) and (33) show that $\frac{S_{h w}}{S_{c W}}$ is more sensitive to change in $C_{h w}$ than the $Q$ and $d$ ratios. For investigating the effect of $C_{h w}$ change only the $S$ ratio needs to be examined. On Figure 9 the contour lines for the three ratios $\frac{S_{h w}}{S_{c w}}, \frac{Q_{h w}}{Q}, \frac{d_{h w}}{d}$ would coincide at 1.0 . At other contours of $\frac{S_{h w}}{S_{c w}}$ the corresponding values for $\frac{d_{h w}}{d}$ and $\frac{Q_{h w}}{Q}$ are found from equations (32) and (33) and shown in the table on Figure 9.

The results of trying various $C_{h w}$ values for the pipes in Table 1 are presented in Table 2. The explanatory diagram for the points $A, B$, etc. used in the "Notes" columns of Table 2, is shown as Figure 11.

## Using the Manning Formula

A similar procedure to that used above for the Hazen-Williams formula has been adopted. Additional equations required are shown below.

Diameter $d_{m}$ [corresponding to equation (29)] is found by rearranging aquasion (6) with $R_{h}=\frac{d}{4}$ :

$$
\begin{equation*}
d_{m}=\frac{4\left(V_{n}\right)^{1.49254}}{S_{c w}^{0.74627}} \tag{34}
\end{equation*}
$$

Flow rate $Q_{m}$ [corresponding to equation (30)] comes from equation (10):

$$
\begin{equation*}
Q_{m}=\frac{0.3102 d^{2.67} S_{c w}^{0.5}}{n} \tag{35}
\end{equation*}
$$

The diameter and flowrate ratios [corresponding to equations (32) and (33) respectively] are:
and:

$$
\begin{equation*}
\frac{d_{m}}{d}=\left(\frac{S_{m}}{S_{c w}}\right)^{0.746} \tag{36}
\end{equation*}
$$

$$
\begin{equation*}
\frac{Q_{m}}{Q}=\left(\frac{S_{m}}{S_{c w}}\right)^{-0.5} \tag{37}
\end{equation*}
$$

The effect of varying $n$ on the $\pm 5 \%$ and $\pm 10 \%$ agreement ranges for one pipe is shown in Figure $10(B)$. The results of trying various $n$ values for the pipes in Table 1 are shown in Table 3.

## CONCLUSIONS

Finding Empirical Coefficients for a Single Pipe Using the Hazen-Williams Formula

For a single pipe of given $d$ and $k$ values (and water temperature 20 degrees Celsius) $\mathrm{C}_{\mathrm{hw}}$ may be found from Figure 5.

The $C_{h w}$ values on Figure 5 correspond to those along the $\frac{f_{h w}}{f_{c w}{ }_{f}}=1.025$ contour line on Figure 6. For operation away from this line the $\frac{\mathrm{f}_{\mathrm{hw}}}{\mathrm{f}_{\mathrm{cw}}}$ values of Figure 6 apply.

Comparison of Figure 6 with Figures 1 and 2 indicates that the pipes with operating zones closer to the smooth pipe curve may utilise the HazenWilliams formula with an error in $f$ of $-5 \%$ to $+2.5 \%$ compared to the preferred formula. For the rougher (high $k$ value) pipes the error in $f$ becomes unacceptable over much of the operating zone.

## Using the Manning Formula

The Manning formula agrees with the preferred formula when the flow is wholly rough wall turbulent. For known values of $d$ and $k$ Manning's $n$ may be found from Figure 8 to give agreement in this zone of flow.

For operation in the transition zone the Manning formula underestimates $f$ and the $\frac{f_{m}}{f_{c w}}$ contours of Figure 7 apply.

Comparison of Figure 7 with Figures 1 and 2 indicates that it is only for rougher (high $k$ value) pipes of poor quality that the Manning formula may be used. For these pipes an error in $f$ of $-5 \%$ compared to the preferred formula may result over part of their operating zone.

## Finding Empirical Coefficients for an Operating Zone

For a V range of 0.2 to $6 \mathrm{~m} . \mathrm{s}^{-1}$, and the d ranges shown in Table 1 for the various pipes, the recommended $C_{h w}$ and $n$ values (extracted from Tables 2 and 3) are shown in Table 4.

In Tables 2 and 3 the point $A$ conditions (also shown on Figure 11) correspond to maximum $V$ (i.e. $6 \mathrm{~m} . \mathrm{s}^{-1}$ ), and maximum $d$ for the particular
pipe (from Table 1). Maximum $V$ and maximum $d$ may be used as a rough criterion for finding $C_{h w}$ and $n$, as follows:
(i) To find $C_{h w}$ for given $V, d$ and $k$, calculate $\mathbb{R}$ and $\frac{d}{k}$ and find $f$ from the Moody Chart, or use Yao's method (Ref 16). Then find $C_{h w}$ from equation (22).
(ii) To find $n$ for given $V, d$ and $k$ use equation (27).

## Application to Other Empirical Formulas and Pipe Data

The methods demonstrated above for a single pipe may be applied to any other empirical formula in the general form of equation (4). Such equations may be reduced to a form suitable for plotting on the Moody Chart, giving a family of straight lines which attempt to approximate portion of the chart (Ref. 12).

The method used above for finding a single value of an empirical coefficient for a given operating zone may be used for any pipe or range of $V$ and d. The operating zone should be specified when a single value of an empirical coefficient is quoted for a pipe material.

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TABLE 1: DATA FOR TYPICAL CLEAN WATER SUPPLY PIPES

| Material | $\begin{aligned} & \quad d(\mathrm{~m}) \\ & \text { Range } \end{aligned}$ | Ref. | Quality | Roughness and Coefficients |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | k (mm) | $\mathrm{C}_{\mathrm{hw}}$ | n | Ref. |
| Smooth, Brass <br> Copper, UPVC, Polyethylene | 0.015 to 0.25 | 11 | Good | 0.003 | 160 | 0.008 | 4 |
|  |  |  | Poor | 0.015 | 155 | 0.009 | 4 |
| Asbestos Cement | 0.05 to 0.6 | 10 | Good | 0.015 | 155 | 0.008 | 4 |
|  |  |  | Poor | 0.06 | 145 | 0.011 | 4 |
| Concrete, spun | 0.1 to 1.83 | 2 | Good | 0.03 | 150 | 0.009 | 4 |
|  |  |  | Poor | 0.15 | 140 | 0.012 | 4 |
| Cast Iron, uncoated | 0.08 to 0.625 | 3 | Good | 0.15 | 140 | 0.010 | 4 |
|  |  |  | Poor | 0.6 | 125 | 0.013 | 4 |

## FORMULA TO PIPE OPERATING ZONES

$V$ range 0.2 to $6 \mathrm{~m} \cdot \mathrm{~s}^{-1}$, Water at 20 deg . C.

| Material <br> \& Quality | d (m) <br> Range | $\begin{gathered} k \\ (\mathrm{~mm}) \end{gathered}$ | $\mathrm{C}_{\text {hw }}$ | Notes (see Fig.1l) | \% Area for 0.95 to 1.05 | $\begin{aligned} & \mathrm{S}_{\mathrm{cW}} \text { Range } \\ & 0.9 \text { to } 1.1 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Smooth, <br> Good | 0.015 to 0.25 | 0.003 | 150 | Mean of pts. A \& B Pts. A \& C coincide | 38.5 | 97.7 |
|  |  |  | 152.5 |  | 89.3 | 95.8 |
|  |  |  | 155.8 |  | 76.8 | 91.5 |
|  |  |  | 156 |  | 75.5 | 91.3 |
|  |  |  | 160 | AS Code (Ref. 4) | 10.5 | 79 |
| Smooth, <br> Poor | 0.015 to 0.25 | 0.015 | 14.3 .1 | Mean of Pts. A \& B | 25.3 | 93.3 |
|  |  |  | 145 |  | 56.3 | 98.9 |
|  |  |  | 148.1 | Pts. A \& C coincide | 86.6 | 94.4 |
|  |  |  | 150 |  | 78.6 | 92.9 |
|  |  |  | 155 | AS Code (Ref. 4) | 4.5 | 74.2 |
| Asbestos <br> Cement, <br> Good | 0.05 to 0.6 | 0.015 | 146.5 | Mean of Pts. A \& B Pts. A \& C coincide | 60 | 99.8 |
|  |  |  | 148.8 |  | 97.1 | 99.5 |
|  |  |  | 150 |  | 93.7 | 99.5 |
|  |  |  | 155 | AS Code (Ref. 4) | 32.6 | 91.8 |
| Asbestos <br> Cement, <br> Poor | 0.05 to 0.6 | 0.06 | 131.6 | Mean of Pts. $A \& B$ Pt. A | 16.6 | 48.3 |
|  |  |  | 136 |  | 51.8 | 79 |
|  |  |  | 140 |  | 56 | 93.7 |
|  |  |  | 145 | $\begin{aligned} & \text { AS Code (Ref. 4) } \\ & \text { Pt. C } \end{aligned}$ | 33.3 | 65.2 |
|  |  |  | 146.8 |  | 22.4 | 50.5 |


| Concrete, <br> Good | 0.1 to 1.83 | 0.03 | $\begin{aligned} & 143.6 \\ & 145 \\ & 147.5 \\ & 150 \\ & 150.9 \end{aligned}$ | ```Pt. A Mean of Pts. A & C AS Code (Ref. 4) Pt. C``` | $\begin{aligned} & 53.9 \\ & 68.2 \\ & 88.3 \\ & 61.7 \\ & 51.8 \end{aligned}$ | $\begin{aligned} & 93 \\ & 99.9 \\ & 99.9 \\ & 97.8 \\ & 96.5 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Concrete, Poor | 0.1 to 1.83 | 0.15 | $\begin{aligned} & 122.2 \\ & 127.1 \\ & 130 \\ & 140 \\ & 147 \end{aligned}$ | ```Mean of Pts. A & B Pt. A AS Code (Ref. 4) Pt. C``` | $\begin{gathered} 8 \\ 41.3 \\ 49.9 \\ 20.3 \\ 6.4 \end{gathered}$ | $\begin{aligned} & 32.3 \\ & 63.4 \\ & 73.6 \\ & 43.7 \\ & 16.4 \end{aligned}$ |
| Cast Iron, Good | 0.08 to 0.625 | 0.15 | $\begin{aligned} & 124.9 \\ & 127.5 \\ & 134.3 \\ & 140 \\ & 143.6 \end{aligned}$ | Pt. A <br> Mean of Pts. A \& C <br> AS Code (Ref. 4) <br> Pt. C | $\begin{gathered} 43.8 \\ 46.5 \\ 26.1 \\ 14.2 \\ 6 \end{gathered}$ | $\begin{aligned} & 67.1 \\ & 74.3 \\ & 55.3 \\ & 28 \\ & 16 \end{aligned}$ |
| Cast Iron, Poor | 0.08 to 0.625 | 0.6 | $\begin{aligned} & 100.1 \\ & 106.4 \\ & 110 \\ & 125 \\ & 132.2 \end{aligned}$ | Mean of Pts. A \& B Pt. A <br> AS Code (Ref. 4) <br> PE. C | $\begin{gathered} 12.3 \\ 37.7 \\ 32.3 \\ 5.6 \\ 0 \end{gathered}$ | $\begin{gathered} 27.6 \\ 60.3 \\ 61.5 \\ 13.2 \\ 0 \end{gathered}$ |

TABLE 3: RESULTS OF APPLYING MANNING FORMULA TO PIPE OPERATING ZONES

V range 0.2 to $6 \mathrm{~m} \cdot \mathrm{~s}^{-1}$. Water at $20 \mathrm{deg} . C$.


| Concrete, <br> Good | 0.1 to 1.83 | 0.03 | .009 | AS Code (Ref. 4) | 21.4 | 45.3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | .0094 |  | 35.2 | 58.7 |
| Concrete, <br> Poor | 0.1 to 1.83 | 0.15 | .01 |  | 33.5 | 54.5 |

TABLE 4: RECOMMENDED EMPIRICAL COEFFICIENTS FOR PIPE OPERATING ZONES
$V$ range 0.2 to $6 \mathrm{~m} \cdot \mathrm{~s}^{-1}$. Water at 20 deg. $C$.

| Material | d (m) | k (mm) | Coefficient |  |
| :---: | :---: | :---: | :---: | :---: |
| \& Quality | Range | $k$ (min) | $C_{\text {hw }}$ | n |
| Smooth, | 0.015 to 0.25 |  |  |  |
| Good |  | 0.003 | 152.5 | . 0075 |
| Poor |  | 0.015 | 148 | . 0078 |
| Asbestos | 0.05 to 0.6 |  |  |  |
| Cement, |  |  |  |  |
| Good |  | 0.015 | 149 | . 0084 |
| Poor |  | 0.06 | 140 | . 0091 |
| Concrete, | 0.1 to 1.83 |  |  |  |
| Good |  | 0.03 | 14.7 .5 | . 0094 |
| Poor |  | 0.15 | 130 | . 0107 |
| Cast Iron, | 0.08 to 0.625 |  |  |  |
| Good |  | 0.15 | 127.5 | . 0099 |
| Poor |  | 0.6 | 106.5 | . 0113 |




FIG. 2. OPERATING ZONES FOR CIEAN PIPES


(A) HAZEN-WILLIAMS FORMULA

(B) MANNING FORMULA

FIG.4. FITTING EMPIRICAL FORMULAS TO COLEBROOK-WHITE EQUATION FOR SINGLE PIPES


FIG. 5. HAZEN-WILLIAMS COEFFICIENT $C_{h w}$ for a single pipe



FIG.7. MANNING FORMULA
$\frac{f_{m}}{f_{c w}}=$ CONSTANT CONTOURS


FIG. 8. MANNING'S $n$ FOR A SINGLE PIPE


| $\frac{S_{h W}}{S_{C W}}$ | 0.9 | 0.925 | 0.95 | 0.975 | 1.0 | 1.01 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{Q_{h W}}{Q}$ | 1.059 | 1.043 | 1.028 | 1.014 | 1.0 | 0.995 |
| $\frac{d_{h W}}{d}$ | 0.914 | 0.935 | 0.957 | 0.979 | 1.0 | 1.009 |

FIG. 9. EMPIRICAL FORMULA APPLIED TO AN OPERATING ZONE

(B) MANNING FORMULA

FIG. 10. FINDING EMPIRICAL COEFFICIENTS


FIG.11. POINTS FOR TESTING EMPIRICAL COEFFICIENTS FOR AN OPERATING ZONE

