Numerical analysis of one dimensional soil water systems. April 1992.

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Publication details:
Report No. UNSW Water Research Laboratory Report No. 179
0858244039 (ISBN)

Publication Date:
1992

DOI:
https://doi.org/10.4225/53/5796a62442ea6

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BIBLIOGRAPHIC DATA SHEET

Report No. 179	Report Date: April 1992	I.S.B.N. 0/85824/403/9

Title

NUMERICAL ANALYSIS OF ONE DIMENSIONAL SOIL WATER SYSTEMS

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Sponsoring Organisation

Supplementary Notes

Abstract

A computer-based numerical approach for the analysis of the movement of water in unsaturated one dimensional soil water systems is described. The development of the relevant finite difference equations and the form of the solution analogue are summarized together with details of the initial conditions and boundary conditions. The program structure is then outlined and the analysis of the treatment of nonhomogeneous profiles is briefly described. The latter chapters in the report cover input data presentation, parameter definition and three examples of typical soil water problems.

Distribution Statement

For general distribution.

Descriptors

Numerical analysis ; Soil water systems ; Unsaturated flow ; Infiltration ; Drainage.

Identifiers

Number of Pages: 70	Price: On Application

TABLE OF CONTENTS

1. GENERAL APPROACH

1.1 Introduction

The analysis of problems involving the movement of water in unsaturated soils can be divided into two distinct approaches. These can be conveniently classified as the quasi-analytical approach and the computer-based numerical approach. Philip (1969) presented a comprehensive review of the quasi-analytical methods, particularly in regard to the theory of infiltration. Recently, Broadbridge and White (1988) and Sander et al. (1988) have presented analytical solutions for constant flux infiltration using algebraically - expressed soil water parameters of realistic form. The quasi-analytical approach allows a conceptual framework of water movement in unsaturated soils to be presented and general statements to be formulated concerning the physics of the phenomenon. However, the approach is necessarily restrictive in regard to the range of the initial and boundary conditions and medium properties that can be handled. Such restrictions eliminate from precise analysis a wide range of problems that simulate field conditions.

By contrast, the computer-based numerical approach allows problems of a complex nature that simulate field conditions to be readily solved. Lack of profile homogeneity, variations in initial water content, the effect of moving boundaries, intermittency in the surface flow pattern with its attendant hysteresis complications and the effects of pore air compression in certain systems can all be handled without difficulty. In addition, from a field system viewpoint, the numerical approach has significant contributions to make in the understanding and assessment of unsaturated flow behaviour and in the management sphere. In this latter area a specially - tailored variant of the numerical analysis could be included in an overall management system where field sensors monitor different hydrologic processes and where periodic calculations are required for the programming of management practice. The main disadvantage with the approach is its limitation in readily providing general physical statements.

Recent advances in computer technology and the developments currently emerging in the industry, together with the ready access to powerful personal computers have stimulated the authors to make available for more general circulation a numerical package that has been in use for many years in the School of Civil Engineering for research studies involving the hydrology of the unsaturated zone.

1.2 Program Development

At the outset the authors wish to acknowledge the many contributions made by their colleagues over the years to the numerical analysis presented in the report. The program had much of its genesis in the co-operative research of F.D. Whisler and K.K. Watson at the U.S. Water Conservation Laboratory, Phoenix, Arizona, in the late sixties. Since that time apart from the work of the authors significant contributions, usually in extending the basic program to enable it to simulate a new set of physical parameters, have been made by Mr. R.N. Ayers, Dr. S.J. Perrens, Dr. S.N. Webb, Dr. S.J. Lees, Mr. D.G. Doran and Dr. G.A. Diment. More recently, a significant parallel contribution has been made by one of the authors (Dr. M.J. Jones) in coupling a program for solute movement with the soil water program. Size constraints place limitations on the material that can be covered in any one report. Accordingly, it is planned that two Water Research Laboratory Reports will follow this present report. The first will cover several subroutines relating to specific processes not included in this report, while the second will describe the equations and background material relating to the movement of solutes in unsaturated systems together with necessary subroutine details.

1.3 One Dimensional Flow Equation

The equation of continuity for a rigid soil water system is

$$\partial\theta/\partial t = - \nabla \bullet \bar{\upsilon} \qquad (1.1)$$

where $\bar{\upsilon}$ = macroscopic velocity vector and θ volumetric water content.

Combining this with Darcy's Law gives

$$\partial\theta/\partial t = \nabla\bullet(K\nabla H) \tag{1.2}$$

where H = hydraulic head.

This equation may be written as

$$\partial\theta/\partial t = \nabla\bullet(K\nabla h) + (\partial K/\partial z) \tag{1.3}$$

where h is the pressure head, t is time, and z is the vertical ordinate positive upward.

For one dimensional flow in the vertical direction Equation (1.3) reduces to

$$\partial\theta/\partial t = \partial[K(\partial h/\partial z)]/\partial z + (\partial K/\partial z) \tag{1.4}$$

It must be remembered that K and h are functions of water content; in addition, h depends on the wetting and draining history.

It is convenient to convert Equation (1.4) into a pressure-head form for analysis since it has decided advantages in simulating many soil water systems. For vertical isothermal flow in a homogeneous porous medium Equation (1.4) then becomes

$$C(h)\,\partial h/\partial t = \partial[K(h)\partial h/\partial z]/\partial z + \partial K(h)/\partial z \tag{1.5}$$

where C(h) is the specific water capacity defined by

$$C(h) = d\theta/dh \tag{1.6}$$

In this report implicit finite difference techniques have been used in the numerical solution of Equation (1.5). The finite difference equations are detailed in the next chapter. The particular numerical method involved is that presented by Whisler and Klute (1967) and Whisler and Watson (1969). The nonlinearity that exists in Equation (1.5) is accounted for by using an iterative procedure whereby the parameters are evaluated implicitly at each iteration and then time averaged with those of the previous time step to produce the coefficients of a set of simultaneous linear equations. These equations are then evaluated using the Crank-Nicolson

implicit central difference method. The new value of the dependent variable h is used to re-evaluate the coefficients and iteration is continued until a satisfactory convergence in h is achieved.

The numerical analysis of this report has been evaluated during its evolution against several analytical and quasi-analytical bench marks (e.g.Watson et al (1989), Watson and Sardana (1990), Watson and You (1980)) and has always been found to give accurate results when appropriate convergence parameters have been chosen. It has also been checked against experimental data with satisfactory results being achieved. However, the degree of flexibility inherent in the program options, enabling a wide range of physical systems to be simulated, does mean that certain 'non-tried' combinations could arise.

Chapters 2 and 3 of this report detail the finite difference equations utilized and outline the range of possible initial and boundary conditions available. This background detail is followed in Chapter 4 by the general program structure and the significance of its various components. In Chapter 5 the necessary theoretical material related to the modelling of nonhomogeneous profiles is presented so that both the detailed structure of the appropriate subroutine is made clear and the required input data format (Chapter 6) can be described. The extensive set of parameters used in the program is defined in Chapter 7. Chapter 8 provides examples of the use of the program and the results obtained. Chapter 9 lists the References. A listing of the program has not been included in this report; however, details of the availability and cost of such program documentation can be obtained by writing to the Dr. K. K. Watson, Department of Water Engineering, School of Civil Engineering, The University of New South Wales, P.O. Box 1, Kensington, N.S.W., 2033.

2. FINITE DIFFERENCE EQUATIONS

2.1 Flow Equation Approximations

In using the finite difference approach to solve Equation (1.5) by numerical means the usual procedure is adopted in which a grid of points is superimposed upon the flow domain defined by $t \geq 0$, $-L \leq z \leq 0$ where $|L|$ is the thickness of the profile. The z-axis is divided into N intervals and, for convenience in simulating problems such as those involving a falling or rising water table, the nodes are numbered from the surface downwards. In addition, the z-axis is defined as positive upwards with z=0 at the soil surface. The finite difference grid used in this study is shown diagrammatically in Figure 2.1. In terms of this figure the mesh points are defined as

$$t_n = n\Delta t, \quad n = 0,1,2 \ldots\ldots \tag{2.1}$$

$$z_i = -(i-1)\Delta z, \quad i = 1,2, \ldots N+1 \tag{2.2}$$

$$\text{where } \Delta z = L / N \tag{2.3}$$

In keeping with the benefits to be gained from the implicit approach Equation (2.1) should not be interpreted as implying that Δt remains constant throughout a simulation.

The soil water pressure head values at the intermediate time $(n+\frac{1}{2})$ are defined as

$$h_{(i+1,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i+1,n)} + h_{(i+1,n+1)}\right] \tag{2.4}$$

$$h_{(i,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i,n)} + h_{(i,n+1)}\right] \tag{2.5}$$

$$h_{(i-1,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i-1,n)} + h_{(i-1,n+1)}\right] \tag{2.6}$$

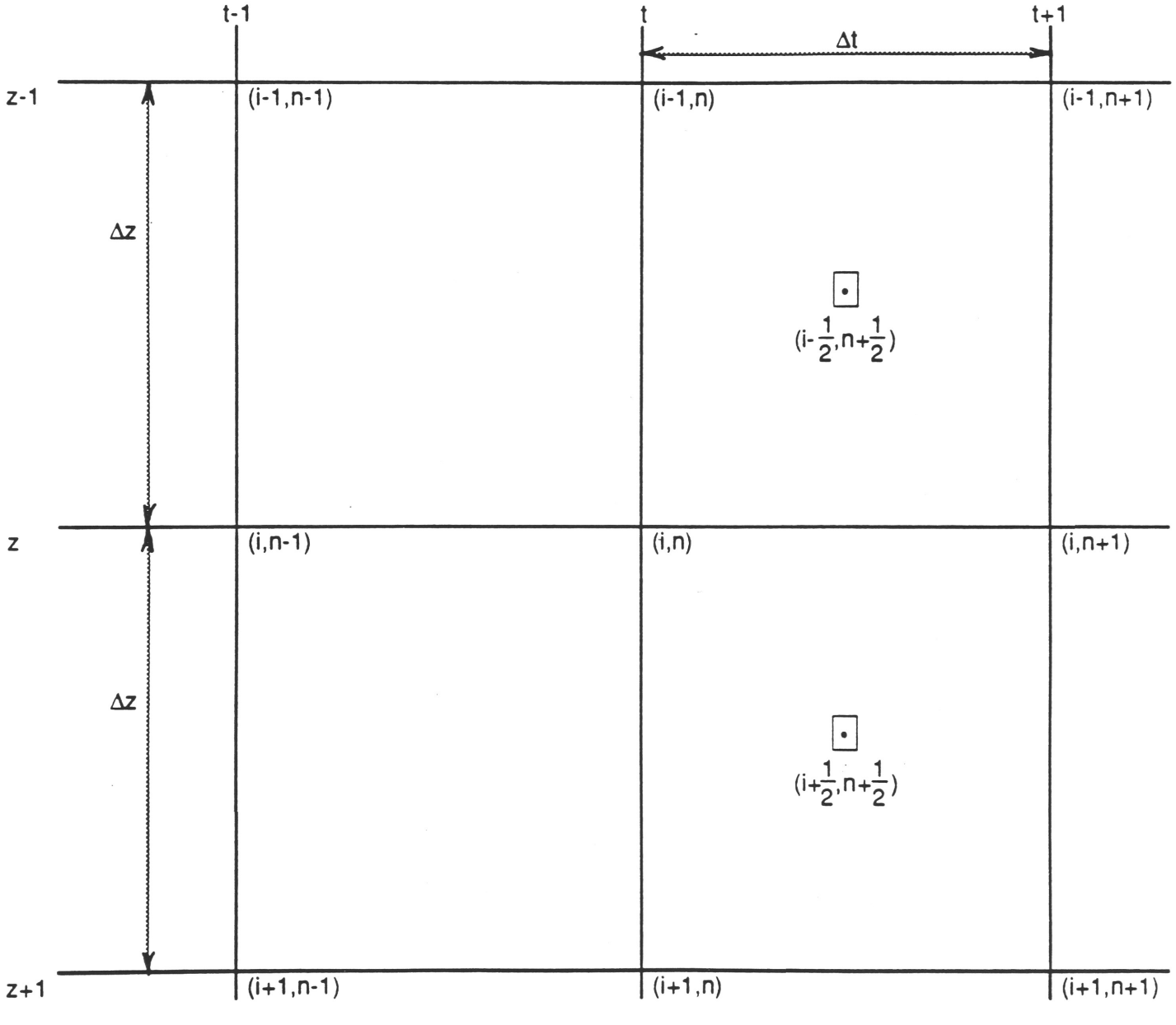

Figure 2.1 Finite difference grid.

It then follows that

$$\frac{\partial h}{\partial z}_{(i+\frac{1}{2},n+\frac{1}{2})} \approx \frac{1}{2\Delta z}\left[h_{(i+1,n)} + h_{(i+1,n+1)} - h_{(i,n)} - h_{(i,n+1)}\right] \tag{2.7}$$

$$\frac{\partial h}{\partial z}_{(i-\frac{1}{2},n+\frac{1}{2})} \approx \frac{1}{2\Delta z}\left[h_{(i,n)} + h_{(i,n+1)} - h_{(i-1,n)} - h_{(i-1,n+1)}\right] \tag{2.8}$$

The hydraulic conductivity values are defined in terms of the arithmetic averages of the values at the surrounding nodes giving

$$K_{(i+\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}\left[K_{(i+1,n)} + K_{(i+1,n+1)} + K_{(i,n)} + K_{(i,n+1)}\right] \tag{2.9}$$

$$K_{(i-\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}\left[K_{(i,n)} + K_{(i,n+1)} + K_{(i-1,n)} + K_{(i-1,n+1)}\right] \tag{2.10}$$

The finite difference expression for the left hand side of Equation (1.5) is written as

$$C\frac{\partial h}{\partial t}_{(i,n+\frac{1}{2})} \approx \frac{1}{\Delta t} C_{(i,n+\frac{1}{2})}\left[h_{(i,n+1)} - h_{(i,n)}\right] \tag{2.11}$$

The finite difference form of Equation (1.5) then becomes

$$\begin{aligned}
&\frac{1}{\Delta t} C_{(i,n+\frac{1}{2})}\left[h_{(i,n+1)} - h_{(i,n)}\right] \\
&= \frac{1}{\Delta z}\left[\frac{1}{2\Delta z} K_{(i+\frac{1}{2},n+\frac{1}{2})}\left[h_{(i+1,n)} + h_{(i+1,n+1)} - h_{(i,n)} - h_{(i,n+1)}\right]\right. \\
&\quad \left. - \frac{1}{2\Delta z} K_{(i-\frac{1}{2},n+\frac{1}{2})}\left[h_{(i,n)} + h_{(i,n+1)} - h_{(i-1,n)} - h_{(i-1,n+1)}\right]\right] \\
&\quad + \frac{1}{\Delta z}\left[K_{(i+\frac{1}{2},n+\frac{1}{2})} - K_{(i-\frac{1}{2},n+\frac{1}{2})}\right]
\end{aligned} \tag{2.12}$$

The terms may then be collected and arranged in general form to give

$$A_i\, h_{(i-1,n+1)} + B_i\, h_{(i,n+1)} + C_i\, h_{(i+1,n+1)} = D_i \tag{2.13}$$

with all the terms in n being contained in the right hand side, D_i.

Thus

$$A_i = -\frac{1}{2\Delta z^2} K_{(i-\frac{1}{2},n+\frac{1}{2})} \tag{2.14}$$

$$C_i = -\frac{1}{2\Delta z^2} K_{(i+\frac{1}{2},n+\frac{1}{2})} \tag{2.15}$$

$$B_i = \frac{1}{\Delta t} C_{(i,n+\frac{1}{2})} - A_i - C_i \tag{2.16}$$

$$D_i = -A_i h_{(i-1,n)} + \left[\frac{1}{\Delta t} C_{(i,n+\frac{1}{2})} + A_i + C_i\right] h_{(i,n)} - C_i h_{(i+1,n)} + 2\Delta z [A_i - C_i] \tag{2.17}$$

When the equation is expanded for each of the N-1 internal node points of the finite difference grid of N intervals in the z direction, we have the following set of tridiagonal simultaneous equations

$$\begin{aligned}
A_2h_1 + B_2h_2 + C_2h_3 &= D_2 \\
A_3h_2 + B_3h_3 + C_3h_4 &= D_3 \\
A_4h_3 + B_4h_4 + C_4h_5 &= D_4 \\
&\cdots\cdots\cdots\cdots \\
A_Nh_{N-1} + B_Nh_N + C_Nh_{N+1} &= D_N
\end{aligned} \tag{2.18}$$

2.2 Solution of Tridiagonal Equations

The solution of Equations (2.18) involves the solution of a set of N-1 equations in N+1 unknowns and, accordingly, requires the application of top and bottom boundary conditions to eliminate the terms in h_1 and h_{N+1} respectively. The algorithm used in solving this set of tridiagonal equations is often known as the Thomas algorithm. This algorithm arrives at a convenient and ready solution by firstly considering the set of equations in a forward direction (the preparatory sweep) and developing equations for the coefficients of the individual pressure

head terms. With these established and utilizing the boundary condition at node N+1 it is then possible to consider the equations in reverse order (the solution sweep) to determine the nodal values.

2.2.1 Preparatory Sweep

For the first equation in the set of Equations (2.18) the term A_2h_1 is eliminated by applying the top boundary condition giving

$$B_2h_2 + C_2h_3 = D_2 \tag{2.19}$$

$$\begin{aligned} h_2 &= \frac{D_2}{B_2} - \frac{C_2}{B_2} h_3 \\ &= Z_2 - Y_2h_3 \end{aligned} \tag{2.20}$$

where

$$Z_2 = \frac{D_2}{B_2}$$

$$Y_2 = \frac{C_2}{B_2}$$

The second equation can then be written as

$$A_3(Z_2 - Y_2h_3) + B_3h_3 + C_3h_4 = D_3$$

giving

$$(B_3 - A_3Y_2)\, h_3 = D_3 - A_3Z_2 - C_3h_4 \tag{2.21}$$

$$\begin{aligned} h_3 &= \frac{D_3 - A_3Z_2}{B_3 - A_3Y_2} - \frac{C_3}{B_3 - A_3Y_2} h_4 \\ &= Z_3 - Y_3h_4 \end{aligned} \tag{2.22}$$

where

$$Z_3 = \frac{D_3 - A_3 Z_2}{B_3 - A_3 Y_2}$$

$$Y_3 = \frac{C_3}{B_3 - A_3 Y_2}$$

Similarly for the third equation

$$h_4 = Z_4 - Y_4 h_5 \tag{2.23}$$

where

$$Z_4 = \frac{D_4 - A_4 Z_3}{B_4 - A_4 Y_3}$$

$$Y_4 = \frac{C_4}{B_4 - A_4 Y_3}$$

Therefore in general we may write

$$h_i = Z_i - Y_i h_{(i+1)} \tag{2.24}$$

where

$$Z_i = \frac{D_i - A_i Z_{(i-1)}}{B_i - A_i Y_{(i-1)}} \tag{2.25}$$

$$Y_i = \frac{C_i}{B_i - A_i Y_{(i-1)}} \tag{2.26}$$

2.2.2 Solution Sweep

For the last (Nth) equation the term $C_N h_{N+1}$ is eliminated by the application of the bottom boundary condition giving

$$h_N = Z_N \tag{2.27}$$

With h_N known

$$h_{(N-1)} = Z_{(N-1)} - Y_{(N-1)}h_N \tag{2.28}$$

or in general

$$h_{(i-1)} = Z_{(i-1)} - Y_{(i-1)}h_i \tag{2.28}$$

The solution then progresses backwards to give the values of h between h_N and h_2. Since the boundaries are located at nodes 1 and N+1, the respective boundary condition equations must be solved using the values of h just determined to give new values of h_1 and h_{N+1}. Once a solution is obtained it is then necessary to proceed through the iteration process.

For each incremented time step (n+1), several iterations are carried out until convergence between two successive solutions is achieved. A solution is accepted when the maximum of the difference between any h value at successive iterations does not exceed a preset limit. For each iteration the values of the parameters C(h) and K(h) are recomputed if a change in h is evident at that node. This change is limited to a very small value to ensure accurate coefficients, but prevents time consuming recalculations if only a negligible change in h is present.

Whilst it is desirable to have as large a time step as possible to speed the overall analysis, there are two limitations on the size that Δt may take. Since the actual equation solution and the requisite coefficient calculations are comparatively slow it is desirable to minimize the number of iterations. This can be achieved by balancing the size of the time increment against the resulting number of iterations to achieve an overall optimal progression. Secondly if the time step is too large (even though convergence was quickly achieved), the resulting change in h may also be so large as to invalidate the approximation used in computing the parameters C(h) and K(h).

2.3 Grid Refining Approximations

In many environmental soil water systems the conditions existing at various interfaces within the system (e.g. evaporation from the soil surface) are of critical importance in assessing the movement of soil water. It is essential that a grid spacing, which may be satisfactory for the major part of a given profile, not become limiting from an accuracy viewpoint in an interface region. Generally, it is computationally inefficient to use a uniform grid, of a sufficiently fine spacing for the interface region, throughout the entire profile. The alternative procedure is the utilization of a grid-refining capability in the critical regions. Such a procedure has been developed as a subroutine in the soil water program.

It will be apparent immediately that a new set of finite difference equations must be established for the nodal sequence involving the juxtaposition of the refined and non-refined grid areas. The finite difference grid for this nodal sequence is given in Figure 2.2. The following finite difference expressions can then be formulated

$$h_{(i+1,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i+1,n)} + h_{(i+1,n+1)}\right] \tag{2.29}$$

$$h_{(i,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i,n)} + h_{(i,n+1)}\right] \tag{2.30}$$

$$h_{(i-1,n+\frac{1}{2})} = \frac{1}{2}\left[h_{(i-1,n)} + h_{(i-1,n+1)}\right] \tag{2.31}$$

The spatial differential expressions may then be written

$$\frac{\partial h}{\partial z}_{(i+\frac{1}{2},n+\frac{1}{2})} \approx \frac{1}{2P_i}\left[h_{(i+1,n)} + h_{(i+1,n+1)} - h_{(i,n)} - h_{(i,n+1)}\right] \tag{2.32}$$

$$\frac{\partial h}{\partial z}_{(i-\frac{1}{2},n+\frac{1}{2})} \approx \frac{1}{2Q_i}\left[h_{(i,n)} + h_{(i,n+1)} - h_{(i-1,n)} - h_{(i-1,n+1)}\right] \tag{2.33}$$

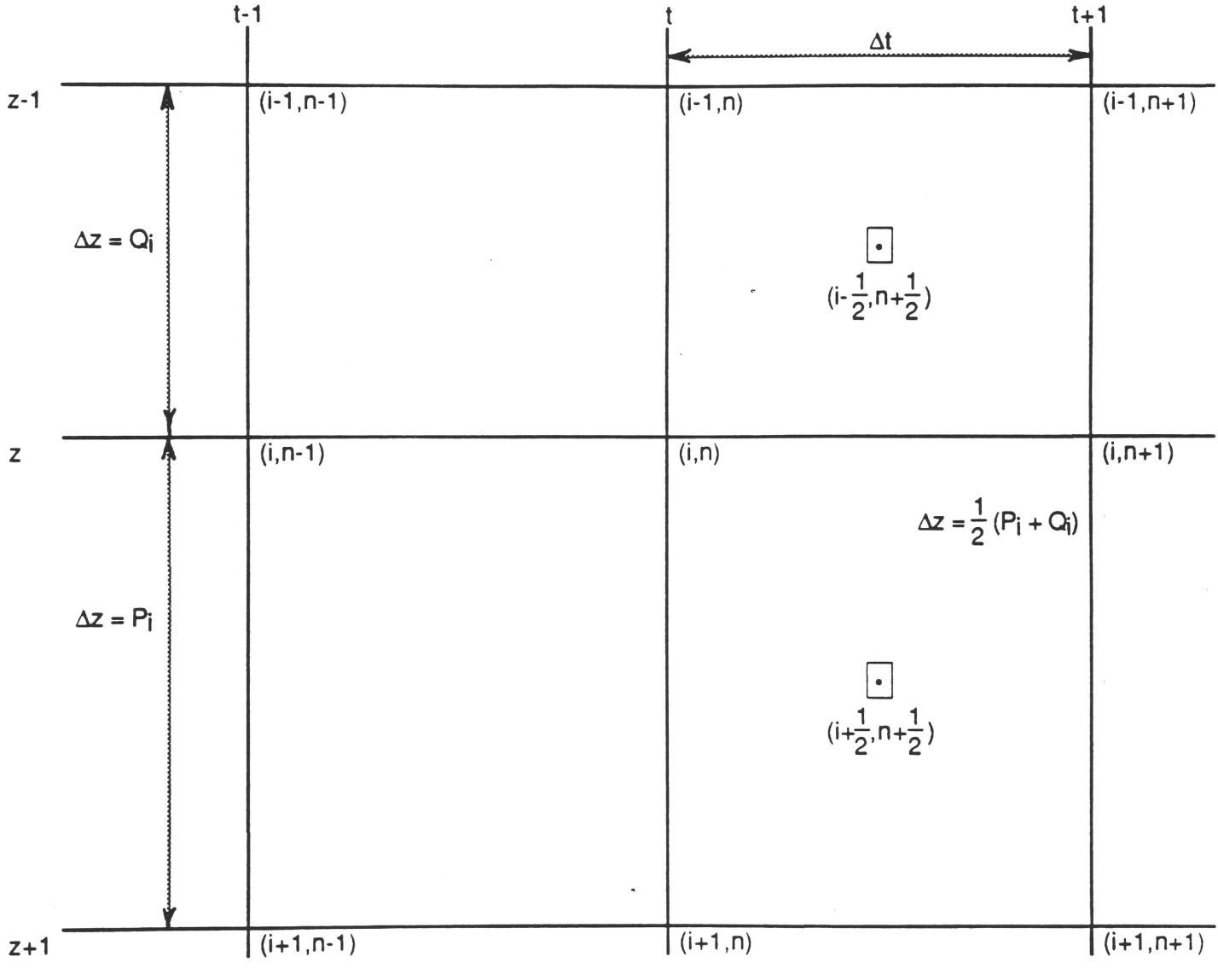

Figure 2.2 Finite difference grid used with grid refining.

As previously

$$K_{(i+\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}[K_{(i+1,n)} + K_{(i+1,n+1)} + K_{(i,n)} + K_{(i,n+1)}] \quad (2.34)$$

$$K_{(i-\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}[K_{(i,n)} + K_{(i,n+1)} + K_{(i-1,n)} + K_{(i-1,n+1)}] \quad (2.35)$$

and

$$C\frac{\partial h}{\partial t}_{(i,n+\frac{1}{2})} \approx \frac{1}{\Delta t} C_{(i,n+\frac{1}{2})}[h_{(i,n+1)} - h_{(i,n)}] \quad (2.36)$$

The finite difference form of Equation (1.5) for the significant refined grid nodal sequence then becomes

$$\frac{1}{\Delta t} C_{(i,n+\frac{1}{2})}[h_{(i,n+1)} - h_{(i,n)}]$$

$$= \frac{1}{\frac{1}{2}(P_i + Q_i)}\left[\frac{1}{2P_i} K_{(i+\frac{1}{2},n+\frac{1}{2})}[h_{(i+1,n)} + h_{(i+1,n+1)} - h_{(i,n)} - h_{(i,n+1)}]\right.$$

$$\left. - \frac{1}{2Q_i} K_{(i-\frac{1}{2},n+\frac{1}{2})}[h_{(i,n)} + h_{(i,n+1)} - h_{(i-1,n)} - h_{(i-1,n+1)}]\right]$$

$$+ \frac{1}{\frac{1}{2}(P_i + Q_i)}[K_{(i+\frac{1}{2},n+\frac{1}{2})} - K_{(i-\frac{1}{2},n+\frac{1}{2})}] \quad (2.37)$$

As previously this may be written

$$A_i\, h_{(i-1,n+1)} + B_i\, h_{(i,n+1)} + C_i\, h_{(i+1,n+1)} = D_i \quad (2.38)$$

with the coefficients now being defined as follows.

$$A_i = -\frac{1}{Q_i(P_i + Q_i)} K_{(i-\frac{1}{2},n+\frac{1}{2})} \quad (2.39)$$

$$C_i = -\frac{1}{P_i(P_i + Q_i)} K_{(i+\frac{1}{2},n+\frac{1}{2})} \quad (2.40)$$

$$B_i = \frac{1}{\Delta t} C_{(i,n+\frac{1}{2})} - A_i - C_i \tag{2.41}$$

$$D_i = - A_i \, h_{(i-1,n)} + \left[\frac{1}{\Delta t} C_{(i,n+\frac{1}{2})} + A_i + C_i\right] h_{(i,n)}$$

$$- C_i \, h_{(i+1,n)} + 2[Q_i \, A_i - P_i \, C_i] \tag{2.42}$$

When the above coefficient equations are compared with those for the uniform grid developed earlier it may be seen that the coefficients A_i and C_i are changed to the extent that $Q_i(P_i + Q_i)$ and $P_i(P_i + Q_i)$ replace $2\Delta z^2$ respectively. Coefficient B_i is unaltered and D_i only changed in the last term with Δz being replaced by Q_i or P_i. These changes are incorporated very simply in the relevant sections of the computer program.

It is also necessary to incorporate an algorithm to control those finite difference grids which require refining. To facilitate programming, only those about an interface are refined, with the distance from the interface over which refining applies being independently variable both above and below. Only a multiple of the original finite difference grid may be refined with the new refined grid size being the same for each interface. The soil surface and the base are incorporated within the definition of an interface and refining could be achieved at the surface of a homogeneous profile, for example, by simply specifying the required depth below an interface and the desired refined size. If it is desired to study any position in the profile in greater detail it is only necessary to specify an interface between two layers of the same soil and then refine about this artificial interface. The grid refining algorithm is contained in subroutine REFINE (see Chapter 4).

Although it is theoretically possible to have large changes in grid size it has been found advisable to limit the ratio of the original grid to the refined grid to 10:1, and preferably not to exceed 5:1. This restriction is necessary because the hydraulic conductivity is averaged across the refined grid on one side of the interface and across the original grid on the other side. If the change is too abrupt the averaging associated with the larger grid size tends to damp out the significance of the refined value.

3. INITIAL CONDITIONS AND BOUNDARY CONDITIONS

3.1 Introduction

The effectiveness of a comprehensive soil water program in modelling a wide range of naturally occurring water movement processes in the unsaturated zone depends to a large extent on the ability of the program structure to define readily the varying types of initial conditions that may be encountered together with the upper and lower boundary conditions. In the program discussed in this report initial conditions and boundary conditions for a variety of physical systems, that have been studied during the development of the package, have been organized so as to provide a simple mechanism for their selection for any particular simulation. In this chapter the initial conditions will be detailed first followed by the set of available boundary conditions.

3.2 Initial Conditions

The initial specification of the dependent variable h over the solution domain is a necessary prerequisite for the initiation of the solution process. In addition, where a purely monotonic process is not being specified the immediate wetting or draining history that lies 'behind' the initial h distribution must be known. These requirements are reasonably demanding and have, for convenience, been gathered together in the program subroutine INPUT (see Chapter 4). Five different input mechanisms are included in the program. The first is designed to cater for extensive timewise simulations where more than one computer 'run' is necessary; it reads in as continuation data (with all relevant data being specified) the information that was calculated at the termination of the previous run. The second mechanism allows a particular known set of h and θ data (perhaps field-derived data) to be read and used as the initial condition for the simulation. The remaining three input mechanisms have been chosen to reflect input conditions that represent fairly standard 'starting' points.

The first of these is described as the uniformly 'dry' profile and represents the initial condition that is commonly used in quasi-analytical studies namely that of uniform initial water content

(and pressure head). Implicit in this condition is the development of a purely monotonic flow system prior to any boundary condition change that may be introduced. The next input mechanism considers a profile which has been drained to equilibrium, this being defined in terms of zero hydraulic head through the system. Such an initial condition would usually then have imposed upon it an infiltration event resulting in wetting-up along a set of primary wetting scanning curves. Finally, the case of a profile initially saturated above a water table is available. The pressure head distribution for this condition is defined within the subroutine as being of linear form between the boundary values of the air entry value at the surface and h=0 at the water table.

3.3 Boundary Conditions

The specification of both top and bottom boundary conditions is necessary throughout the solution process, firstly to eliminate terms in h_1 and h_{N+1} and then for the actual determination of h_1 and h_{N+1}. The head value at a boundary may be given directly by specification, or indirectly either as a flux rate or as a 'feed-back' value from a calculation already carried out by the program.

To enable a variety of flow systems to be studied six top and six bottom boundary conditions are incorporated in subroutine BCOND (see Chapter 4). An identifying number is given to each boundary condition and by a switching process (incorporated in the MAIN program) the boundary conditions may be changed during the course of any analysis. This flexibility in boundary condition changing provides a powerful method for simulating the changes with time that occur in naturally-occurring systems.

As noted in the previous chapter it is necessary to formulate a boundary condition in the form of two equations. The first, the preparation equation, is used to eliminate the term in either h_1 or h_{N+1} and thus prepare the set of tridiagonal equations. The second, the solution equation, is used to determine a solution for h_1 or h_{N+1} after the tridiagonal equations have been solved.

3.3.1 Top Boundary Conditions

Six top boundary conditions are included in the program these being zero surface flux (i.e. drainage/redistribution), surface flux, surface ponding, air pressure transform, applied air pressure and time-dependent surface ponding or flux. The derivation of the relevant equations for most of these conditions is given below.

(a) Surface Flux (NTOPBC=1)

The equation of flow in the vertical direction, Equation (1.5), may be expressed alternatively as

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\left[\frac{\partial h}{\partial z} + 1\right]\right) \tag{3.1}$$

The finite difference grid to be used in the following analysis for surface nodes is shown in Figure 3.1.

Let the flux at the surface R_1 be positive in the outflow direction in accord with z being positive upwards. The following finite difference expressions can then be formulated

$$h_{(2\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}\left[h_{(3,n)} + h_{(3,n+1)} + h_{(2,n)} + h_{(2,n+1)}\right] \tag{3.2}$$

$$h_{(1\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}\left[h_{(2,n)} + h_{(2,n+1)} + h_{(1,n)} + h_{(1,n+1)}\right] \tag{3.3}$$

It follows that

$$\frac{\partial h}{\partial z}_{(2,n+\frac{1}{2})} \approx \frac{1}{4\Delta z}\left[h_{(3,n)} + h_{(3,n+1)} - h_{(1,n)} - h_{(1,n+1)}\right] \tag{3.4}$$

also

$$K_{(2,n+\frac{1}{2})} = \frac{1}{2}\left[K_{(2,n)} + K_{(2,n+1)}\right] \tag{3.5}$$

$$C_{(1\frac{1}{2},n+\frac{1}{2})} = \frac{1}{4}\left[C_{(2,n)} + C_{(2,n+1)} + C_{(1,n)} + C_{(1,n+1)}\right] \tag{3.6}$$

and

$$\frac{\partial h}{\partial t}_{(1\frac{1}{2},n+\frac{1}{2})} \approx \frac{1}{2\Delta t}\left[h_{(2,n+1)} + h_{(1,n+1)} - h_{(2,n)} - h_{(1,n)}\right] \tag{3.7}$$

From Darcy's law

$$K\left[\frac{\partial h}{\partial z} + 1\right]_{(1,n+\frac{1}{2})} = -R_1 \tag{3.8}$$

Equation (3.1), in finite difference terms across the first grid spacing, becomes

$$C_{(1\frac{1}{2},n+\frac{1}{2})} \frac{\partial h}{\partial t}_{(1\frac{1}{2},n+\frac{1}{2})} = \frac{1}{\Delta z}\left[K\left[\frac{\partial h}{\partial z} + 1\right]_{(2,n+\frac{1}{2})} - K\left[\frac{\partial h}{\partial z} + 1\right]_{(1,n+\frac{1}{2})}\right] \tag{3.9}$$

and on substituting

$$\frac{1}{2\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} \left[h_{(2,n+1)} + h_{(1,n+1)} - h_{(2,n)} - h_{(1,n)}\right]$$

$$= \frac{1}{\Delta z}\left[\frac{1}{4\Delta z} K_{(2,n+\frac{1}{2})} \left[h_{(3,n)} + h_{(3,n+1)} - h_{(1,n)} - h_{(1,n+1)} + 4\Delta z\right] + R_1\right] \tag{3.10}$$

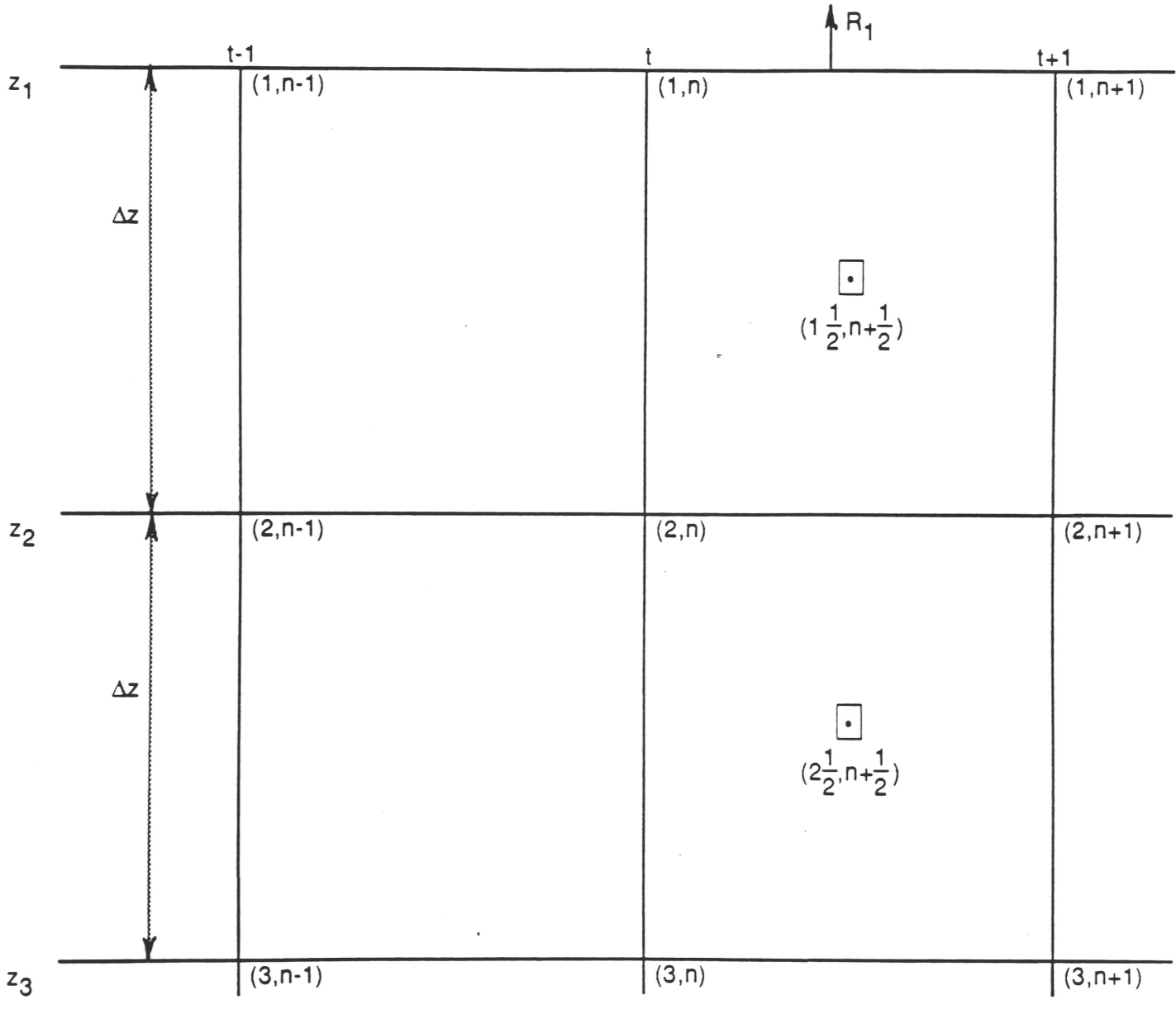

Figure 3.1 Finite difference grid used for surface flux conditions.

To establish the boundary condition equations the h terms are arranged in the form

$$h_{(1,n+1)} = Uh_{(2,n+1)} + Vh_{(3,n+1)} + W \tag{3.11}$$

giving

$$\left[\frac{1}{\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} + \frac{1}{2\Delta z^2} K_{(2,n+\frac{1}{2})}\right] h_{(1,n+1)}$$

$$= -\frac{1}{\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} h_{(2,n+1)} + \frac{1}{2\Delta z^2} K_{(2,n+\frac{1}{2})} h_{(3,n+1)}$$

$$+\frac{1}{\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} [h_{(2,n)} + h_{(1,n)}] + \frac{1}{2\Delta z^2} K_{(2,n+\frac{1}{2})} [h_{(3,n)} - h_{(1,n)}]$$

$$+\frac{2}{\Delta z} [K_{(2,n+\frac{1}{2})} + R_1] \tag{3.12}$$

Let $$L = \frac{1}{\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} + \frac{1}{2\Delta z^2} K_{(2,n+\frac{1}{2})} \tag{3.13}$$

Then $$U = -\frac{1}{L\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} \tag{3.14}$$

$$V = \frac{1}{L2\Delta z^2} K_{(2,n+\frac{1}{2})} \tag{3.15}$$

$$W = \frac{1}{L}\left[\frac{1}{\Delta t} C_{(1\frac{1}{2},n+\frac{1}{2})} [h_{(2,n)} + h_{(1,n)}]\right.$$

$$+ \frac{1}{2\Delta z^2} K_{(2,n+\frac{1}{2})} [h_{(3,n)} - h_{(1,n)}]$$

$$\left. + \frac{2}{\Delta z} [K_{(2,n+\frac{1}{2})} + R_1]\right] \tag{3.16}$$

Now the first tridiagonal equation is

$$A_2h_{(1,n+1)} + B_2h_{(2,n+1)} + C_2h_{(3,n+1)} = D_2 \tag{3.17}$$

Substituting Equation (3.11) we have

$$A_2(Uh_{(2,n+1)} + Vh_{(3,n+1)} + W) + B_2h_{(2,n+1)} + C_2h_{(3,n+1)} = D_2 \quad (3.18)$$

which when rearranged becomes

$$(B_2 + A_2U)\ h_{(2,n+1)} + (C_2 + A_2V)\ h_{(3,n+1)} = D_2 - A_2W \quad (3.19)$$

Thus the boundary condition preparation equations are

$$B_2 = B_2 + A_2U \quad (3.20)$$

$$C_2 = C_2 + A_2V \quad (3.21)$$

$$D_2 = D_2 - A_2W \quad (3.22)$$

and the solution equation is

$$h_{(1,n+1)} = Uh_{(2,n+1)} + Vh_{(3,n+1)} + W \quad (3.23)$$

It should be noted that the surface flux is not restricted to inflow only. An evaporative condition can be specified by defining a positive R_1 value.

(b) Zero Flux at the Surface (NTOPBC=0)

A zero flux or drainage/redistribution boundary condition is a special case of the more general surface flux condition, with $R_1 = 0$. It is treated as a separate condition to facilitate data preparation and to clearly distinguish between drainage/redistribution and a non-zero applied flux case.

(c) Surface Ponding (NTOPBC=2)

If the depth of ponding above the surface is defined as h_{sp}, then the solution equation is

$$h_{(1,n+1)} = h_{sp} \quad (3.24)$$

The first tridiagonal equation becomes

$$A_2h_{sp} + B_2h_{(2,n+1)} + C_2h_{(3,n+1)} = D_2 \quad (3.25)$$

Since h_{sp} is constant at (n+1)

$$B_2h_{(2,n+1)} + C_2h_{(3,n+1)} = D_2 - A_2h_{sp} \quad (3.26)$$

The preparation equation is then simply

$$D_2 = D_2 - A_2h_{sp} \quad (3.27)$$

(d) Air Pressure Transform (NTOPBC=3)

This boundary condition enables a time-dependent air pressure build up to be applied as a top boundary condition. Details will be given in a following report. (3.29)

(e) Applied Air Pressure (NTOPBC=4)

This facility is incorporated in the program to allow an air pressure to be applied to the top boundary of the system. Details will be given in a following report.

(f) Variable Surface Ponding/Flux (NTOPBC=5)

This boundary condition is used to apply a hyetograph of rainfall to the soil surface and to calculate any resultant runoff. It entails the use of the surface flux and surface ponding boundary condition equation in conjunction with a preset depth of depression storage. Details will be given in a following report.

3.3.2 Bottom Boundary Conditions

The six bottom boundary conditions are contained within the same subroutine these being semi-infinite profile, stationary water table, air pressure transform, applied water pressure, moving water table and impervious boundary. The details for the preparation and solution equations are given below for three of these boundary conditions.

(a) Semi-Infinite Profile (NBOTBC=1)

For a semi-infinite condition to hold it is assumed that the bottom boundary is far enough from the nodes at which flow is occurring so that the pressure head at the boundary remains constant at its initial value. A test is incorporated within the MAIN program to terminate the analysis if the pressure head two nodes above the boundary changes by more than a preset limit.

If the initial pressure head at the bottom boundary is stored as h_{con} then the solution equation is

$$h_{(N+1,n+1)} = h_{con} \tag{3.30}$$

Since the last tridiagonal equation is

$$A_N h_{(N-1,n+1)} + B_N h_{(N,n+1)} + C_N h_{(N+1,n+1)} = D_N \tag{3.31}$$

then on substituting Equation (3.30) and rearranging, we obtain

$$A_N h_{(N-1,n+1)} + B_N h_{(N,n+1)} = D_N - C_N h_{con} \tag{3.32}$$

Thus the preparation equation is

$$D_N = D_N - C_N h_{con} \tag{3.33}$$

(b) Stationary Water Table (NBOTBC=2)

A water table is defined as the position at which the pressure head equals atmospheric pressure. Since for this study atmospheric pressure is the zero pressure datum then the solution equation is

$$h_{(N+1,n+1)} = 0 \tag{3.34}$$

Since the $C_N h_{(N+1,n+1)}$ term in Equation (3.31) is therefore zero, no preparation equation is required.

(c) Air Pressure Transform (NBOTBC=3)

Details will be presented in a following report.

(d) Applied Water Pressure (NBOTBC=4)

Details will be presented in a following report.

(e) Moving Water Table (NBOTBC=5)

This rather specific boundary condition is discussed in detail in a following report.

(f) Impervious Boundary (NBOTBC=6)

We may approximate the fact that there will be zero flux across an impermeable bottom boundary by assuming that the flux through the bottom finite difference grid will be zero. Thus, using Darcy's law

$$q_{(N+\frac{1}{2},n+1)} = - K(h) [\frac{\partial h}{\partial z} + 1]_{(N+\frac{1}{2},n+1)} \tag{3.39}$$

and if

$$q_{(N+\frac{1}{2},n+1)} = q_{(N+1,n+1)} = 0 \tag{3.40}$$

then necessarily

$$\frac{\partial h}{\partial z}_{(N+\frac{1}{2},n+1)} = -1 \tag{3.41}$$

This may be approximated as

$$h_{(N+1,n+1)} - h_{(N,n+1)} = -\Delta z \tag{3.42}$$

yielding the solution equation

$$h_{(N+1,n+1)} = h_{(N,n+1)} - \Delta z \tag{3.43}$$

Substituting into the last tridiagonal Equation (3.31), we have

$$A_N h_{(N-1,n+1)} + B_N h_{(N,n+1)} + C_N (h_{(N,n+1)} - \Delta z) = D_N \tag{3.44}$$

Thus the preparation equations are

$$B_N = B_N + C_N \tag{3.45}$$

$$D_N = D_N + C_N \Delta z \tag{3.46}$$

4. PROGRAM STRUCTURE

4.1 Introduction

The following information is required to operate the program:

(i) profile depth, number of nodes, and the region and size of any refining;

(ii) initial profiles such that h, θ, C and K are specified at each node;

(iii) boundary conditions at the top and bottom of the profile;

(iv) relationships $K(\theta)$, $\theta(h)$ and $C(h)$ for the soils in the profile;

(v) times of any output, change of boundary condition or termination of the run; and

(vi) test tolerances to ensure that the solution converges and that the approximations in the solution scheme remain valid.

The principal features of the computer program used in this study are shown in Figure 4.1. The main program contains the iterative solution scheme, a test for convergence of the solution and the accounting necessary to output results, change boundary conditions and terminate the run at required times. All auxiliary functions are performed in subroutines, some of which are only called when special conditions apply.

The purpose of each subroutine relevant to this report is briefly described below. As noted previously, following reports will describe the more specialized subroutines.

4.2 REFINE

A uniform vertical grid is created at the start of the program by spacing the specified number of nodes evenly between the surface and the base of the profile. If a finer grid is required over a particular region then the subroutine REFINE inserts additional nodes in the coarser uniform grid, and sets P_i and Q_i for each node in the refined grid.

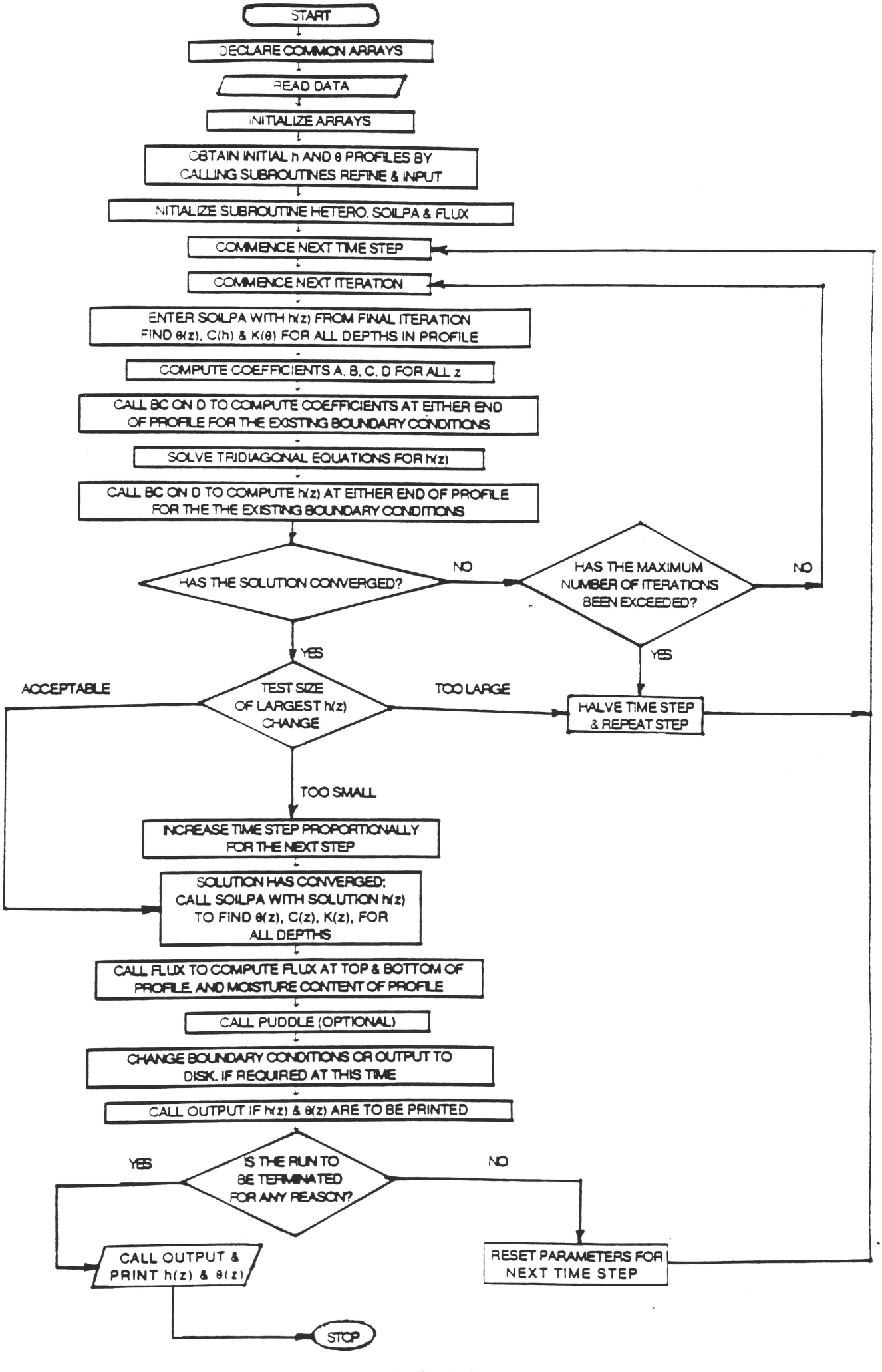

Figure 4.1 Simplified flow diagram showing the principal elements in the computer program solving the unsaturated flow equation

4.3 INPUT

The INPUT subroutine sets up the initial profiles with which to start the run. A semi-infinite profile assumes uniform values of the parameters h, θ, C and K, down the profile. But if the specified profile has either drained to, or become saturated above a water table, the appropriate parameter values are a simple function of the boundary draining curve and the height of each node above the water table. Alternatively, if the parameters are saved into a file at the end of a run, they may be read directly as the initial profiles for a subsequent run. This facility largely overcomes computer time limitations.

4.4 OUTPUT

The first call to OUTPUT prints the initial data as a check. Subsequent calls are made to either of two internal entry points: one to write results into a file, the other to write via the line printer. OUTPUT is called from the main program to print results at a regular sequence of times, at intervals of a specified number of time steps or when the pressure profile has altered by a certain amount from that at the previous print out. The times at which results are to be written into a file are specified in the input data. Results are also printed and written into a file when the run is about to terminate.

4.5 SOILPA

The subrouting SOILPA is called by each node in turn and returns the values of θ, C and K corresponding to the latest value of h. θ is found by applying h to the h(θ) relationship, C is computed as the slope (dθ/dh) of the moisture characteristic and K is calculated from the K(h) relationship.

4.6 HETERO

In a heterogeneous soil the gradation in pore size causes K to vary as a function of depth. If required, HETERO simulates a heterogeneous profile by scaling pressure values to reflect heterogeneity before a call is made to the SOILPA subroutine containing homogeneous soil

data. The parameters returned by SOILPA are thereby distorted in value to represent the effect of a heterogeneous profile.

4.7 BCOND

The purpose of BCOND is to incorporate the boundary conditions which are essential to the solution. It is called twice during the iteration loop. The first call determines the coefficients at the top and bottom nodes of the profile. Then after the tridiagonal equations have been solved, BCOND is again called to compute h at the top and bottom nodes.

As previously discussed the surface boundary conditions which may be nominated are:

(i) constant specified flux of water through the surface;

(ii) constant depth of excess water on the surface ponding;

(iii) variable depth of ponding being augmented by rainfall or drawn down by infiltration or evaporation; and

(iv) variations in air pressure.

The possible boundary conditions at the base of the profile are:

(i) semi-infinite profile;

(ii) rising, falling or stationary water table;

(iii) impervious lower boundary; and

(iv) variations in air pressure.

4.8 FLUX

The FLUX subroutine computes net flux into the profile in two independent ways. At the completion of each time step the total water content of the profile is evaluated as the area under the $\theta(z)$ profile. The net flux during the time step is then the increment in total θ divided by the time step. The answer can be compared with the estimate of net flux which is found by computing the difference between the fluxes through the top and bottom boundaries: these being the product of the existing K and hydraulic head gradient at each boundary.

4.9 PUDDLE

PUDDLE is only called when one of two special boundary conditions is operating: variable ponding at the surface or a moving water table at the base. As noted, detailed information will appear in a later report.

5. NONHOMOGENEOUS PROFILES

5.1 Introduction

In this chapter a complexity is introduced into the soil system in the form of profile nonhomogeneity. Two categories of nonhomogeneity are considered with both being included in the soil water program. The first is the common and relatively easily-handled phenomenon of soil layering. Since homogeneity is assumed within each soil layer the main issue to be faced is that of defining the position of the layers and ensuring that the program selects the appropriate soil parameters for each layer during the solution process. The second category considers soil profiles in which the hydrologic characteristics of the soil vary spatially in some continuous manner. To assist in both simplifying and giving order to such a system, the concept of scale heterogeneity has been used in defining the hydrologic characteristics.

5.2 Soil Layering

Many authors have discussed the numerical analysis of stratified profiles including Hanks and Bowers (1962), Whisler and Klute (1966), Watson (1971) and Dane and Wierenga (1975). The reason for this interest in layered profiles is a natural by-product of the frequency with which layering is encountered in field soils, even if not with the well defined changes found in most numerical studies.

In order to make the computer program as flexible as reasonably practicable, a means of incorporating layered profiles is included in the MAIN program. The current program is currently dimensioned for four layers but any number of layers could be specified. Each layer is defined by a vertical ordinate, z_{lay}, to the top of the layer. This results in the top layer of a layered system or a homogeneous profile being defined by $z_{lay} = 0$.

The various parameters controlling the heterogeneity of each layer and its soil water characteristics are read in at the beginning of the program. A vector is then defined to record in what layer any node in the profile is positioned. This vector is used to determine which soil

parameter subroutine SOILPA should be called to obtain the correct values of K(h) and C(h). It is thus necessary to have a SOILPA subroutine for each layer of a different material and these are designated SOILP1, SOILP2, etc. Since binary codes of SOILPA were previously compiled little additional programming is required.

The flexibility of such a layering scheme allows not only various soil types to be used but also different specifications for the 'same' soil. As an example, in a finite depth drainage problem a perched water table may be simulated by using a lower layer of the same soil with a reduced saturated hydraulic conductivity. This is possible since, if the layer does not desaturate, there is no need to access the unsaturated hydrologic characteristics. The authors made use of this facility of the program to produce part of the data for the paper by Watson et al. (1978).

5.3 Scale Heterogeneity

A concept which is very useful in studies involving heterogeneity is that of scale heterogeneity, enunciated by Philip (1967) and applied to numerical studies by Watson and Whisler (1972), Whisler et al. (1972) and Watson et al. (1973). A scale heterogeneous medium is one in which the internal geometry is everywhere geometrically similar but where the characteristic length scale is free to vary with position.

Let the suffixes i and d refer to two positions within a scale heterogeneous medium and let the characteristic length scales at these positions be λ_i and λ_d respectively. It follows from considerations of dimensional analysis that for a specified volumetric water content θ

$$h_i(\theta)\,\lambda_i = h_d(\theta)\,\lambda_d \tag{5.1}$$

and

$$K_i(\theta)/\lambda_i^2 = K_d(\theta)/\lambda_d^2 \tag{5.2}$$

If we consider the length scale λ_d as defining the experimental soil water characteristics of a given soil we can use these characteristics as a datum to evaluate those characteristics at any position of length scale λ_i. Thus if we define a scale factor

$$\lambda_i^* = \lambda_i/\lambda_d \tag{5.3}$$

then the spatial variation of the soil properties $h(\theta)$ and $K(\theta)$ is wholly embodied in λ_i^*.

Therefore to represent any point i within the soil it is only necessary to know the scale factor λ_i^* and the datum set of experimental soil characteristics.

To establish λ_i^* we require a datum characteristic length scale, λ_d, for the soil used in obtaining the experimental soil water characteristics. For example, the average grain size of a uniform medium sand could be used giving $\lambda_d = 0.02$ cm approximately.

The variable characteristic length scale, λ_i, may be defined quite generally to give various patterns of scale heterogeneity. One approach is to define λ_i indirectly by specifying a linear variation of the saturated hydraulic conductivity down the profile. If the saturated hydraulic conductivity is K_t at $z = 0$ and K_b at $z = -L$, then at any node i

$$K_i = K_t - (K_t - K_b)\ z_i/z_{-L} \tag{5.4}$$

It follows from Equation (5.2) that

$$\lambda_i = \sqrt{K_i\ \lambda^2_d/K_d} \tag{5.5}$$

Although such a variation is unlikely to occur precisely in field situations it is a convenient and useful approximation and is readily adaptable to numerical analysis.

The general equation for one dimensional flow in a rigid heterogeneous medium under isothermal conditions may be written

$$C(h,z)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}[K(h,z)\frac{\partial h}{\partial z}] + \frac{\partial K(h,z)}{\partial z} \tag{5.6}$$

The concept of scale heterogeneity provide us with an interconnection between the functions and the dependent variable h in terms of λ_i^*. Combining Equation (5.1) with Equation (5.3), then for a specified water content θ

$$h_i(\theta) = \frac{1}{\lambda_i^*} h_d(\theta) \tag{5.7}$$

Similarly Equation (5.2) yields

$$K_i(\theta) = \lambda_i^{*2} K_d(\theta) \tag{5.8}$$

In addition, we require a specification for C(h,z). This can be written as

$$C_z(h_z) = \frac{d\theta}{dh_z} \tag{5.9}$$

For a given increment of water content, $d\theta$, we have in terms of the positions i and d

$$C_i(h_i)\, dh_i = C_d(h_d)\, dh_d \tag{5.10}$$

Using Equation (5.7)

$$C_i(h_i) = \frac{dh_d}{1/\lambda_i^{*} dh_d} C_d(h_d)$$

$$= \lambda_i^{*} C_d(h_d) \tag{5.11}$$

Equations (5.7), (5.8) and (5.11) allow the ready introduction of a spatial variation in the soil water characteristics at each elevation considered in the solution of the flow Equation (5.6). For a soil of characteristic scale length λ_d the experimental data is tabulated for $h_d(\theta)$, $C_d(h_d)$ and $K_d(\theta)$, that is, a SOILPA subroutine is set up exactly as for a homogeneous profile and is used as the datum for determining the soil parameters at any node.

6. INPUT DATA

6.1 Introduction

The input data are processed to the main program from a file containing records as specified below. The length and time units used are centimetre and minute respectively. It is appreciated that the S.I. units of metre and second would be more 'standard'. However, since such a change would require the rewriting of complex data sets which will appear in the later reports, the units as originally used have been retained. Within the compass of this report many data components will have zero values.

6.2 Details of Input Data

Record 1: (Profile Details) N, CM, AIRFAC, DLTINL, IHORIZ

N	Number of equal intervals into which the column is divided
CM	Length of column
AIRFAC	Factor by which CM is multiplied to simulate extra length of column as far as air is concerned
DLTINL	Initial value of time step
IHORIZ	Control parameter for direction of flow

Record 2: (Grid Specifications) IRFINE, MAXLAY, REFSIZ, RUPPER, RLOWER, VELWTM,DETSTR

IRFINE	Control parameter for grid refining
MAXLAY	Maximum number of soil layers in the soil profile
REFSIZ	Size of the refined grid
RUPPER	Distance above a layer interface over which the refining is to apply
RLOWER	Distance below a layer interface over which the refining is to apply

VELWTM Velocity of water table movement

DETSTR Depth of detention storage

Record 3: (Details of solution limits) MAXITR, TOLITR, TOLDIF, TOLLKS, DIVERG, FINTIM

MAXITR Maximum number of iterations permitted before iteration excess signalled

TOLITR Tolerance limit to which successive iterations must reach to achieve convergence

TOLDIF Tolerance value to limit the maximum soil water potential difference between successive timesteps

TOLLKS Tolerance limit beyond which soil water potential values cannot move without necessitating another look up of soil parameters

DIVERG Permitted value by which the second lowest node pressure can diverge from initial value before a semi-infinite lower boundary approximation becomes invalid and run terminates.

FINTIM End of complete simulation time

Record 4: (Hysteresis Details) DRNREV, REWREV, AIRLIM

DRNREV Test limit for reversal to drainage

REWREV Test limit for reversal to rewetting

AIRLIM Length of column as far as air is concerned

Record 5: (Input and Output Controls) MECINP, MECPRT, NUMSTP, CUMPRT, PRTINL, ISOLUT

MECINP Input mechanism specification for water flow

MECPRT Print mechanism specification

NUMSTP Number of time steps specified to control printout.

CUMPRT Value of cumulative pressure change at any node, used to trigger printout of result

PRTINL Initial printout time specified for each boundary condition

ISOLUT Control parameter for simultaneous flow of water and solute

Record 6: (Heterogeneity Details) KINDHE, CONTOP, CONBOT, CONSAT, CAPSAT, AMDAD

KINDHE Control parameter for heterogeneity

CONTOP Saturated conductivity at the top end of a particular soil layer when using scale heterogeneity

CONBOT Saturated conductivity at the bottom end of a particular soil layer when using scale heterogeneity

CONSAT Saturated conductivity of a particular soil (datum value for scale heterogeneity)

CAPSAT Saturated specific capacity for a particular soil

AMDAD Scale parameter (Lambda) at datum for a particular heterogeneous soil.

Record 7: (Initial Condition Details) HINL, HWEV, HAEV, THTINL, THTSAT, THTFUL, ZLAYER

HINL Initial uniform soil water potential

HWEV Water entry value for a particular soil

HAEV Air entry value for a particular soil

THTINL Initial moisture content profile values

THTSAT Resaturated moisture content of a particular soil

THTFUL Fully saturated moisture content of the particular soil

ZLAYER Distance from surface to the top of a particular soil

Record 8 : (Boundary Condition and Punch Output Numbers) MAXPUN, MAXBCC, NUMPUN, NUMBCC

MAXPUN Maximum number of punch times specified for a particular simulation

MAXBCC Maximum number of boundary condition change times in any given simulation run

NUMPUN Number of the punch time now operative

NUMBCC Number of the boundary condition now operative

Record 9: (Punch Output Times)

Successive values of punch times are specified at which h & θ profiles are outputted.

Record 10*: (Boundary Condition Details) BCCT, NTOP, NBOT, SFLX, ASPD, APPT, APPB

BCCT Duration of boundary condition

NTOP Control parameter for upper boundary condition

NBOT Control parameter for lower boundary condition

SFLX Water flux through surface

ASPD Constant depth of ponding on surface

APPT Applied pressure at top of profile

APPB Applied pressure at base of profile

*This record is repeated for each additional boundary condition until all MAXBCC conditions have been specified.

6.3 Sample Input Data File

A sample input file is shown in Figure 6.1. Free format is used in specifying the input data.

```
   100     100.0     1.0000  0.0000001         0
     0         1     0.0000     0.0000    0.0000    0.0000    0.0000
     7    0.0500     2.0000     0.0050   20.0000  200.0000
 0.02000   0.5000     100.0
     3         1         10   100.0000   200.000         0
     0    0.0700     0.0700     5.E-06    0.0200
     -   -60.00     -80.00     0.1200    0.3800    0.3800    0.0000
282.6670
     7         1          1          1
1.0000    5.0000    10.0000    20.0000   50.0000  100.0000  200.0000
200.0000        1          1    -0.0500    0.0000    0.0000    0.0000
-80.0000  -60.0000     0.3800     0.0800    1.3000    0.2400    0.0700         1
```

Figure 6.1 Input Data File for Example 1 (refer p.60).

7. PARAMETER DEFINITION

For the sake of simplicity the following parameter listing is complete, in that it also includes parameters for those subroutines (e.g. hysteresis, solute movement) which will appear in the later reports.

A	Coefficient in tridiagonal equations
ABS	System function to determine absolute value
AIRFAC	Factor by which CM is multiplied to simulate extra length of column as far as air is concerned
AIRLEN	Column length when simulating air effects (cm)
AIRLIM	Length of column as far as air is concerned (cm)
ALOG	System routine to take the natural logarithm of a number
AMDA	Scaling factor for heterogeneous soils (applies only when KINDHE = 1)
AMDAD	Scale parameter (lambda) at datum for particular heterogeneous soil
AMDCON	Temporary constant used in determining AMDA
AMDFAC	Value of AMDA relative to datum
AMTWTM	Amount of water table movement in current timestep (cm)
AN	Floating point value for N
APC	A + C
APPB	Input storage vector for successive values of APPBOT
APPBOT	Applied pressure at the distal (bottom if vertical) end of the column (cm)

APPT	Input storage vector for successive values of APPTOP
APPTOP	Applied pressure at the proximal (top if vertical) end of the column (cm)
B	Coefficient in tridiagonal equations
BCCT	Input storage vector for successive values of BCCTIM
BCCTIM	Boundary condition change time (min)
BG	Temporary variable used in subroutine SOLUTE
C	Coefficient in tridiagonal equations
CAPC	Current value of soil water specific capacity (1/cm)
CAPDLT	Average soil water specific capacity divided by DLT
CAPP	Soil water specific capacity at previous time step (1/cm)
CAPSAT	Saturated specific capacity for a particular soil
CAPSCL	Scale factor applied to capacity values with scale heterogeneous soils (applies only when KINDHE = 1)
CINFL	Input storage vector for successive values of CUINFL
CM	Length of the column (cm)
CONAVE	Simple average hydraulic conductivity value (cm/min)
CONBOT	Saturated hydraulic conductivity at the distal end of the particular layer of soil when using scale heterogeneity (cm/min)
CONC	Current value of soil water hydraulic conductivity (cm/min)
CONCAP	Sum of CONDEL and CAPDEL

CONDEL	Average hydraulic conductivity divided by DLZ
CONP	Soil water hydraulic conductivity at previous timestep (cm/min)
CONSAT	Saturated hydraulic conductivity of a particular soil (datum value for scale heterogeneity) (cm/min)
CONSCL	Scale factor for hydraulic conductivity values with scale heterogeneous soils
CONTOP	Saturated hydraulic conductivity at the proximal end of the particular soil layer when using scale heterogeneity (cm/min)
CONZAT	Saturated hydraulic conductivity as it varies with z for scale heterogeneous soils (cm/min)
CP	Solute concentration at start of solute timestep (meq/litre)
CU	Solute concentration at end of solute timestep (meq/litre)
CUI	Initial value of solute concentration (meq/litre)
CUINFL	Value of solute concentration maintained at proximal boundary (meq/litre)
CUINL	Constant initial solute concentration value (meq/litre)
CUMASS	Total mass of solute present in the soil column (meq/sq m)
CUMDAY	Time from the start of the simulation (days)
CUMDIF	Cumulative value of DIFMAX (cm)
CUMHRS	Time from the start of the simulation (hours)
CUMPRT	Value of CUMDIF used to trigger printout of result (cm)
CUMRUN	Cumulative runoff (cm)

CUMTIM	Time from the start of the simulation (mins)
D	In subroutine SW1P1D, this variable is one of the coefficients in the tridiagonal equations. In subroutine SOLUTE this variable is the hydrodynamic dispersion coefficient (sq cm/min)
DAYS	Elapsed time for particular boundary condition (days)
DCP	Temporary variable in subroutine SOLUTE
DD	Temporary variable in subroutine SOLUTE
DDCP	Temporary variable in subroutine SOLUTE
DENAM	Temporary variable in subroutine SOLUTE
DENOM	Denominator term tridiagonal equation solution process
DEP	Value of effective hydrodynamic dispersion coefficient for the previous timestep (sq cm/min)
DETSTR	Depth of detention storage (cm)
DHZ	Gradient of soil water potential used in subroutine SOLUTE to calculate the Darcy flux
DIFCPI	Difference between current and previous iteration soil water potential values (cm)
DIFCPL	Difference between current and previous lookup soil water potential values (cm)
DIFCPT	Difference between current and previous time step soil water potential values (cm)
DIFMAX	Maximum difference at any node between current and previous timestep soil water potential values (cm)

DIVERG Permitted variation of the NM1 soil water potential value from its initial value (cm)

DIVNOW Actual variation of the NM1 node soil water potential value from its initial value

DLT Timestep for the water flow equation (min)

DLTFAC Factor by which DLT is multiplied to obtain DLT for the next timestep to optimize the movement in the profile

DLTINL The initial value of DLT (min)

DLTSML Value of DLT when SMLTIM encountered. It holds the DLT value which would ordinarily have applied for use in subsequent upgrades of DLT (min)

DLTSOL Timestep for the solute flow equation (may be smaller than DLTUSD if explicit finite difference model of solute flow is used) (min)

DLTUSD Value of DLT used in current timestep (min)

DLZ Spacestep (this will be negative, since the convention used measures upward from the soil surface) (cm)

DLZHA 0.5 x DLZSQ (sq cm)

DLZSQ DLZ x DLZ (sq cm)

DLZSQU DLZ squared (sq cm)

DLZIN Inverse of twice DLZ (1/cm)

DN Temporary variable used in subroutine SOLUTE

DO Constant value estimate for the molecular diffusion component of the hydrodynamic dispersion coefficient (sq cm/min)

DRNREV Test limit for reversal to drainage (%)

EXP System routine to compute the exponential value

EXSBOT Non Darcy flow at bottom of profile

EXSTOP Non Darcy flow at top of profile

FINTIM End of complete simulation time (min)

FLXBOT Soil water flux at the distal end of the column (cm/min)

FLXDIF Difference between proximal and distal soil water flux values when computed using the Darcy flux approach (cm/min)

FLXINC Incremental change in water content of the soil profile by Darcy flux method (equal to the product of FLXDIF and DLT) (cm)

FLXTOP Soil water flux at the proximal end of the column using the Darcy flux approach (cm/min)

HAEV Air entry value for a particular soil (cm)

HCONST Initial soil water potential at the distal end of the column (node NP1) (cm)

HCU Current soil water potential value (cm)

HDIVERG Initial soil water potential two nodes from the distal end of the column (node NM1) (cm)

HEADC Current value of HCU passed in the parameter list to subroutine(s) SOILP1 (2,3,4) (cm)

HEADP Current value of HPT passed in the parameter list to subroutine(s) SOILP1 (2,3,4) (cm)

HFRONT The air entry value plus the applied air pressure at the proximal end of the column (cm)

HINDEX Hysteresis index parameter. Positive sign indicates rewetting, negative sign indicates draining. Integer value gives the number of the first curve for interpolation, the fractional part that portion of the curve

HINL Constant initial soil water potential value (cm)

HOLDCU Temporary storage for HCU (cm)

HOLDPT Temporary storage for HPT (cm)

HOURS Elapsed time for a particular boundary condition (hours)

HPI Soil water potential at previous iteration (cm)

HPL Soil water potential at previous lookup. For increased computer efficiency, soil parameters are only looked up if the soil water potential varies more than a specified amount (set via parameter TOLLKS) (cm)

HPT Soil water potential at previous timestep (cm)

HSCL Scale factor for soil water potential values with heterogeneity

HSGNTH Vector used to preserve the sign carried by THTC when in subroutine SOLUTE

HSPD Input storage vector for successive values of HSPOND

HSPOND Depth of surface ponding (cm)

HWEV Water entry value for a particular soil (cm)

HYH Total hydraulic potential (soil water potential plus gravitational potential) (cm)

I General 'do loop' counter

IFIX — System routine to convert real numbers to integers

IHORIZ — Control parameter for direction of flow:

0 = Vertical flow

1 = Horizontal flow

INT — System routine to take the integer portion of a real number

ITIME — Counter used in subroutine SOLUTE to register the number of valid solute timesteps

IRFINE — Control parameter for grid refining:

0 = uniform grid (no refining)

1 = refined grid

ISOLUT — Control parameter for simultaneous flow of water and solute:

0 = no solute is present

1 = solute is present in the simulation

ITREXS — Counter for the number of times the number of iterations (NUMITR) exceeds the maximum allowable (MAXITR) before the solution converges

ITRREP — Internal switch to trigger iteration repeats

J — A 'do loop' counter

JUSPUN — Internal switch to prevent extra OUTPUN call at each boundary condition change

K — A 'do loop' counter

KAPPAD Internal switch to trigger the storage and return of HCU and HPT when applied pressure at proximal end is operative (NTOPBC = 4)

KINDHE Control parameter for heterogeneity:

0 = homogeneous

1 = scale heterogeneous

KUTLKS Internal switch to minimise looking up of the soil parameters

LAYZIN Number of the soil layer at a particular node

LTHAN Identifying mark (<<<<<) for each timestep with data output in subroutine OUTPUN

MAXBCC Maximum number of boundary condition change times in any given simulation run (set by dimension statement)

MAXITR Maximum number of iterations permitted before iteration excess signalled

MAXLAY Maximum number of soil layers specified for the simulation (maximum limited to four)

MAXPUN Maximum number of punch times specified for a particular simulation (set by dimension statement)

MDIFXS Counter for the number of times the maximum soil water potential differences between time steps is excessive (as determined from TOLDIF)

MECINP Input mechanism specification for water flow:

1 = continuation data

2 = card data (profiles only)

3 = initially drained constant potential value (semi-infinite)

4 = initially drained to equilibrium above a water table

5 = initially saturated above a water table

MECPRT Print mechanism specification:

<0 = step number control

=0 = cumulative difference control

>0 = specified time control (a built in sequence)

MECSOL Input mechanism specification for solute flow:

1 = continuation data when using IMPLICIT solute version

2 = continuation data when using EXPLICIT solute version

3 = initially constant solute concentration profile

N Number of intervals into which column is divided

N1 1

N2 2

N3 3

NBGEXS Internal switch triggered when the iteration limit (MAXITR) is exceeded before the solution converges, which forces reiteration

NBOT Input storage vector for successive values of NBOTBC

NBOTBC Distal boundary condition type, selected from:

1 = semi-infinite profile

2 = water table

3 = air pressure

4 = applied pressure

5 = moving water table

6 = impervious boundary

NCOND Specifies the system device (unit) number allocated for file CONDAT

ND Value of N for previous time step (when using variable grid)

ND1 ND + 1

NDATA Number of data points given to specify soil characteristics in subroutine SOILP1 when using hyperbolic spline relationships

NDARCB Non Darcy flow parameter

NDARCT Non Darcy flow parameter

NM1 N - 1

NMFIF N - N/5

NN Storage for N to maintain original value

NOWBCC Internal switch to indicate that a boundary condition is about to change under BCCTIM control

NOWDUN Internal switch to indicate that PUNTIM has been called for the current timestep

NOWPRT Internal switch to indicate that PRTTIM is about to take place now

NOWPUN Internal switch to indicate that PUNTIM is to take place now under PUNTIM control

NP1 N + 1

NPRT Logical unit number for printer

NPUN Logical unit number for card punch

NREAD Logical unit number for input data file

NTBCP1 NTOPBC + 1

NTOP Input storage vector for successive values of NTOPBC

NTOPBC Proximal boundary condition type, selected from:

0 = redistribution or drainage

1 = constant surface flux

2 = constant surface ponding

3 = air pressure

4 = applied pressure

5 = variable surface ponding

NUMBCC Number of BCCTIM now operative

NUMITR Number of iterations used for the current timestep

NUMLAY The number of the soil layer that the node is in

NUMLKS The number of node points for which soil parameters have been determined for the current timestep

NUMPTC	Number of nodes changed (added or removed from space grid by water table movement) during timestep
NUMPUN	Number of the punch time now operative
NUMSIG	Internal counter for the number of timesteps to trigger step number control
NUMSTP	Number of time steps specified to control printout
NUMTIM	Total number of timesteps since the start of the simulation
NUMXPT	Number of extra nodes to be added in each old grid interval (DLZ) when grid is being refined
P	Distance from node (I-1) to node I when using a refined grid (cm)
PAIRXS	Air pressure enclosed in a bounded column in excess of air pressure (cm)
PP	Temporary storage for P
PRTADD	Internal counter used to automatically determine PRTTIM
PRTINL	Initial printout time specified for each boundary condition (min)
PRTMUL	Factor used in the automatic determination of PRTTIM
PRTTIM	Sequence of printout times - automatically follows
PUNT	Input storage vector for successive values of PUNTIM
PUNTIM	Specified times for 'punchouts' (min)
QM	Darcy flux at proximal boundary, used in subroutine SOLUTE
REFSIZ	Size of the refined grid (cm)
REWREV	Test limit for reversal to rewetting (%)

RLOWER	Distance below a layer interface over which refining is to apply (cm)
RUNOFF	Surface runoff as a result of excess flux (cm)
RUNRAT	Runoff rate for a particular timestep (cm/min)
RUPPER	Distance above a layer interface over which refining is to apply (cm)
SFLX	Input storage vector for successive values of SURFLX
SIGMA	Spline tension factor, used when specifying soil characteristics by hyperbolic spline functions
SIGN	System routine - transfers the sign of one variable to another variable
SMLTIM	Smallest of BCCTIM, PUNTIM and PRTTIM, but which is greater than the current value of TIME (min)
SOLITR	Counter for the number of solute timesteps used to date
SOLMAX	Maximum allowable solute timestep (DLTSOL) using stability criteria, applicable only to the explicit solute model (min)
SQRT	System function - determines the square root of the variable
STIME	Total simulation time in subroutine SOLUTE, always <= TIME (min)
SURFLX	Value of the applied surface flux (cm/min)
TAIRXS	Temporary value of PAIRXS (cm)
TCU	Temporary storage, used when computing CUMASS in subroutine SOLUTE
TEMP1	Temporary storage used when computing moisture profile in subroutine FLUX
TEMP2	Temporary storage used when computing moisture profile in subroutine FLUX

THETAC Current value of THTC passed to subroutine SOILP1 (2,3,4) (cc/cc)

THETAP Current value of THTP passed to subroutine SOILP1 (2,3,4) (cc/cc)

THTC Current value of moisture content (cc/cc)

THTEMP Temporary storage for the volume of water within the moisture content vs depth profile at the end of each timestep (cm)

THTFLX Difference between proximal and distal end fluxes when calculated by the moisture content profile method (cm/min)

THTFUL Fully saturated moisture content of the particular soil (cc/cc)

THTINC Incremental change in moisture content when using the moisture content profile method (cm)

THTINL Initial moisture content profile values (cc/cc)

THTP Moisture content at previous time step (cc/cc)

THTSAT Resaturated moisture content of the particular soil (cc/cc)

THTSTR Volume of water within the soil profile at the commencement of the simulation (cm)

TIMDIF Difference between SMLTIM and either PUNTIM or BCCTIM (min)

TIME Elapsed time for particular boundary condition (min)

TOLDIF Tolerance value to limit the maximum soil water potential difference between successive timesteps (cm)

DLT is changed according to the magnitude of this maximum difference:

a. exceeds 2x TOLDIF - reiteration necessary, DLT halved

b. equals TOLDIF - DLT unaltered

c. less than 0.5 x TOLDIF - DLT doubled

d. between a and b - DLT reduced proportionately

e. between b and c - DLT increased proportionately

TOLITR Tolerance limit to which successive iterations must reach to achieve convergence (cm)

TOLLKS Tolerance limit beyond which soil water potential values cannot move without necessitating another look up of soil parameters (cm)

TOTFLX Total amount of water to enter (or leave) the profile since the start of the simulation, using the Darcy flux method (cm)

TOTHTC Total amount of water to enter (or leave) the profile since the start of the simulation, using the moisture content profile method (cm)

TOTHTP Previous timestep value of TOTHTC (cm)

TOTSOL Total mass of solute present in soil profile (meq/sq cm)

TTC Temporary storage, used when calculating CUMASS in subroutine SOLUTE

TTH Temporary storage, used when calculating CUMASS in subroutine SOLUTE

TTP Temporary storage, used when calculating CUMASS in Subroutine SOLUTE

T1 Temporary storage, used when calculating SOLMAX in subroutine SOLUTE

U Coefficient used in top boundary condition equation in subroutine BCOND

UNSATB Unsaturated water content at bottom of profile

UNSATT Unsaturated water content at top of the profile

V Coefficient used in top boundary condition equation in subroutine BCOND. In subroutine SOLUTE, it is the current (water) timestep value of pore water velocity (cm/min)

VELWTM	Velocity of the water table movement - may be positive (rising) or negative (falling) (cm/min)
VP	Pore water velocity at the previous water timestep (cm/min)
VXT	Temporary storage, holds an approximation to the first derivative with time of the pore water velocity (V and VP) in subroutine SOLUTE
VXZ	Temporary storage, holds an approximation to the first space derivative of the pore water velocity VP in subroutine SOLUTE
W	Coefficient used in top boundary condition equation in subroutine BCOND
WTMPPN	Distance water table has moved during current timestep; measured from the position of the second lowest node (ND) in the grid which was applicable for the previous timestep (cm)
X	Dummy variable used in subroutine SW1P1D as a call list parameter. In subroutine SOLUTE it is the 'reduced' pore water velocity
YY	Temporary storage for B in solution process
Z	Distance of any point from the wetting face (convention based on positive upwards from the surface with vertical flow) (cm)
ZLAM	Usually termed the 'dispersion length', used in the definition of the solute hydrodynamic dispersion coefficient (D) (cm)
ZLAYER	Distance to the proximal end (top) of any soil layer (cm)
ZLOWER	Z value at a distance RLOWER below interface (cm)
ZSATFR	Non darcy flow from 'Z' depth
ZSATWT	Non darcy flow to 'Z' depth

ZUNIFM	Z values of the uniform grid prior to refining (cm)
ZUPPER	Z value at a distance RUPPER above interface (cm)
ZZ	Temporary storage for D in solution process

8. EXAMPLES

8.1 Introduction

In this chapter three examples illustrating the simulation of three basic flow configurations will be outlined. The input data as required for the execution of these examples has been included on the floppy disc containing the program listing as previously mentioned. In order to simplify both the specification of the hydrologic characteristics of the porous materials used in the examples and the form of the SOILPA subroutine, the equations detailed by Brooks and Corey (1964) have been utilized. These may be written as follows:

$$\left(\frac{\theta - \theta_R}{\theta_{sat} - \theta_R}\right) = \left(\frac{h(\theta)}{h_{AEV}}\right)^{-\alpha} \qquad \text{for } h \leq h_{AEV}$$

$$\theta = \theta_{sat} \qquad \text{for } h > h_{AEV}$$

$$\left(\frac{\theta - \theta_R}{\theta_{sat} - \theta_R}\right) = \left(\frac{K(\theta)}{K_{sat}}\right)^{\gamma}$$

where θ_R = residual water content

h_{AEV} = air entry value.

For wetting-up conditions the air entry value is replaced by the water entry value, h_{WEV}. The input data for SOILPA, when the Brooks and Corey equations are used, requires an additional input for the soil in question as follows:

Record: (Soil data) HAEV, HWEV, THTSAT, THTDRY, ALPHA, GAMMA, CONSAT, NTOPBC

HAEV Air entry value

HWEV Water entry value

THTSAT Resaturated water content

THTDRY Residual water content

ALPHA A constant soil parameter

GAMMA A constant soil parameter

CONSAT Saturated hydraulic conductivity

NTOPBC Proximal boundary condition type

8.2 Example 1 Constant Flux Infiltration

This example studies the constant flux infiltration of water into a soil profile of uniform initial water content θ_i under semi-infinite conditions. The input data are listed below.

θ_{sat} = 0.38 cm^3cm^{-3}

θ_i = 0.12 cm^3cm^{-3}

θ_R = 0.08 cm^3cm^{-3}

h_{AEV} = -80.0 cm of water (not used in analysis of this example)

h_{WEV} = -60.0 cm of water

K_{sat} = 0.07 cm min^{-1}

L = 100 cm

Δz = -1.0 cm

α = 1.3

γ = 0.24

surface flux = -0.05 cm min^{-1}

For $h > h_{AEV}$ (or $h > h_{WEV}$) the specific water capacity is assumed to be 10^{-6} cm^{-1}.

Figures 8.1 and 8.2 illustrate the $\theta(z)$ and $h(z)$ profiles respectively for times from the onset of infiltration of 1, 5, 10 20, 50, 100 and 200 min. The printed output provides firstly a listing of the initial data and then, for each time step, the values of the relevant parameters calculated for that time.

8.3 Example 2 Gravity Drainage

The gravity drainage of an initially saturated profile to a stationary water table is detailed in this example. The porous material specified in this case is coarser than that described in Example 1; it exhibits the characteristics of a medium sand. The initial condition is assumed to be a straight line relationship with h = 0 at the water table and h = h_{AEV} at the surface.

The input data are as follows:

θ_{sat} = 0.35 cm^3cm^{-3}

θ_i = 0.35 cm^3cm^{-3}

θ_R = 0.05 cm^3cm^{-3}

h_{AEV} = -40.0 cm of water

h_{WEV} = -20.0 cm of water (not used in analysis of this example)

K_{sat} = 1.0 cm min^{-1}

L = 100 cm

Δz = -1.0 cm

α = 7

γ = 0.28

The $\theta(z)$ and h(z) profile for times of 1, 5, 10, 20, 50, 100 and 200 min are given in Figures 8.3 and 8.4. It may be noted that at t = 100 min the draining capillary fringe has decreased almost to its equilibrium thickness of 40 cm this being equal to $|h_{AEV}|$.

8.4 Example 3 Constant Flux Infiltration into a Stratified Profile

Constant flux infiltration into a stratified profile is now considered. The profile consists of 30 cm of the porous material described in Example 1 over 30 cm of the porous material detailed below:

θ_{sat} = 0.35 cm^3cm^{-3}

θ_i = 0.06323 cm^3cm^{-3}

θ_R = 0.06 cm^3cm^{-3}

h_{AEV} = -80.0 cm of water (not used in analysis of this example)

h_{WEV} = -40.0 cm of water

K_{sat} = 0.28 cm min^{-1}

α = 2.3

γ = 0.25

surface flux = -0.06 cm min^{-1}

L = 60 cm

Δz = -1.0 cm

The θ_i value of 0.06323 cm^3cm^{-3} is specified to give a uniform $h_i(z)$ down the stratified profile. To enable conditions in the interface region to be modelled accurately during the passage of the wet front and to obtain accurate profiles in that region, the grid has been refined for a 3 cm range on each side of the interface. For this region Δz = -0.2 cm.

The $\theta(z)$ and $h(z)$ profiles are given in Figures 8.5 and 8.6 for times of 20, 50, 70, 85, 95, 100, 110, 140 and 200 min. The sharply-defined changes at the interface in Figures 8.5 and 8.6 would require plotting to an enlarged scale for precise definition.

Many other examples could be presented but space limitations preclude their inclusion in this report. However, the typical examples given above should enable the general operation of the numerical analysis to be understood together with the output data format.

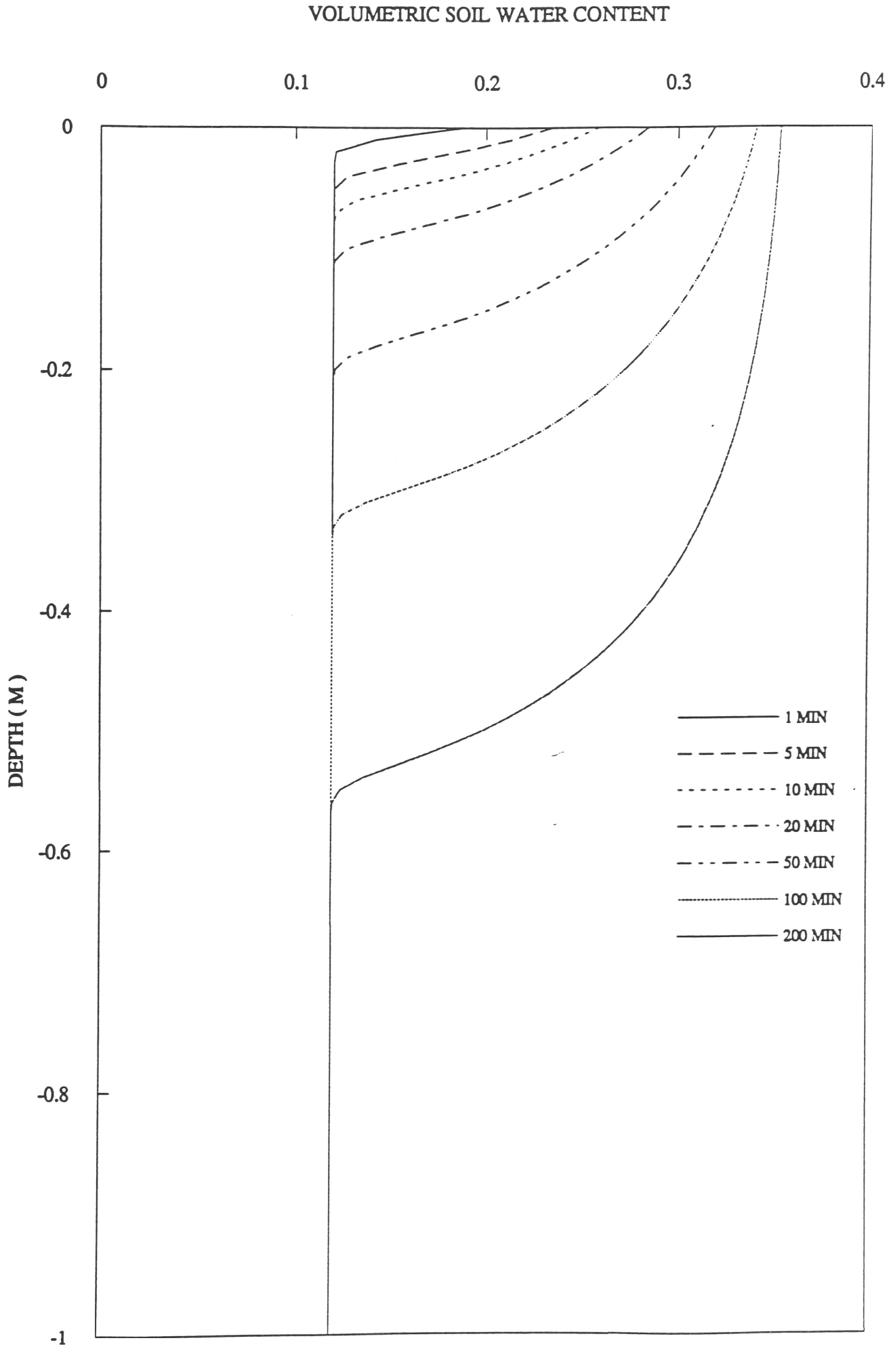

Figure 8.1 $\theta(z)$ profile for Example 1

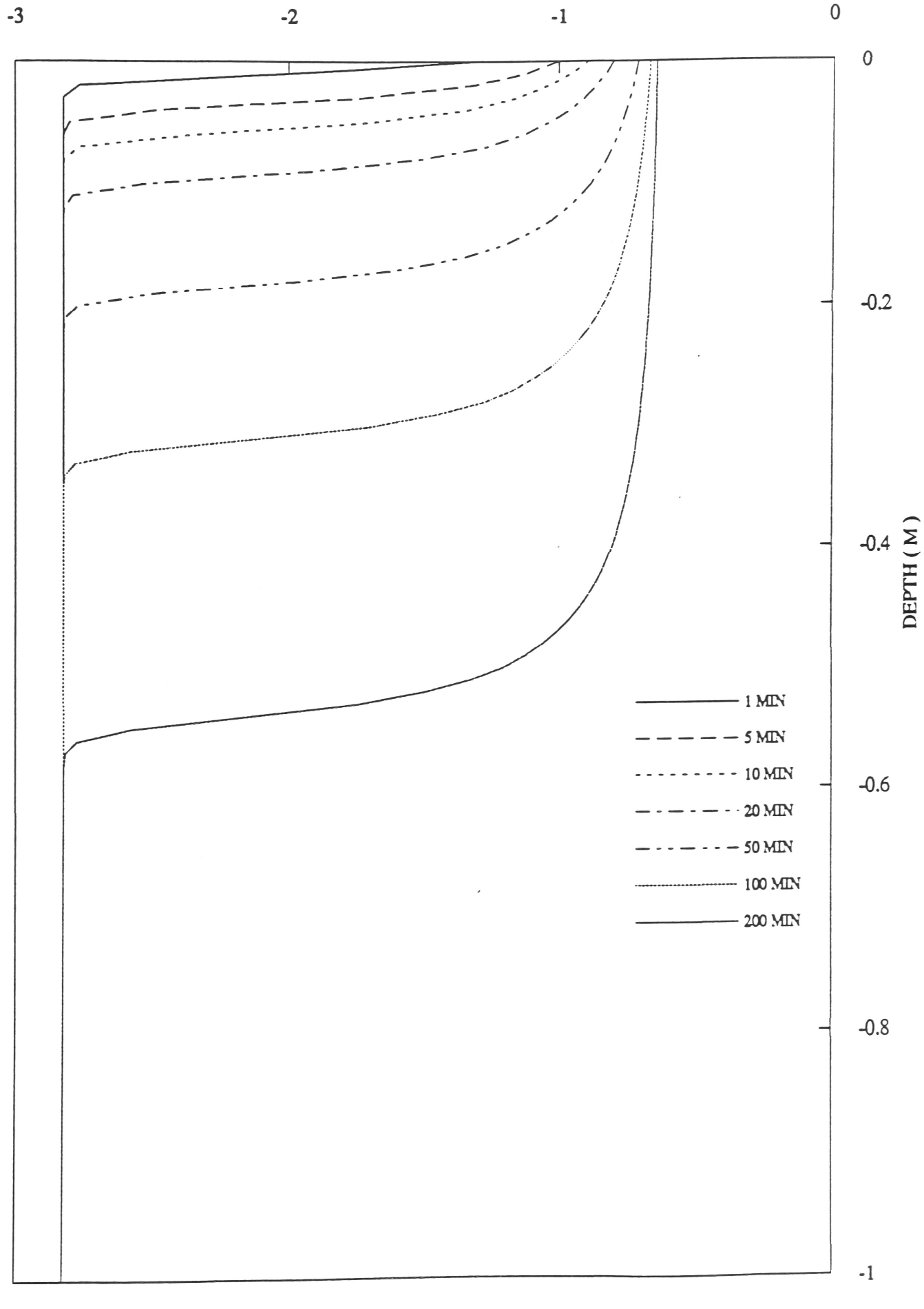

Figure 8.2 h(z) profiles for Example 1

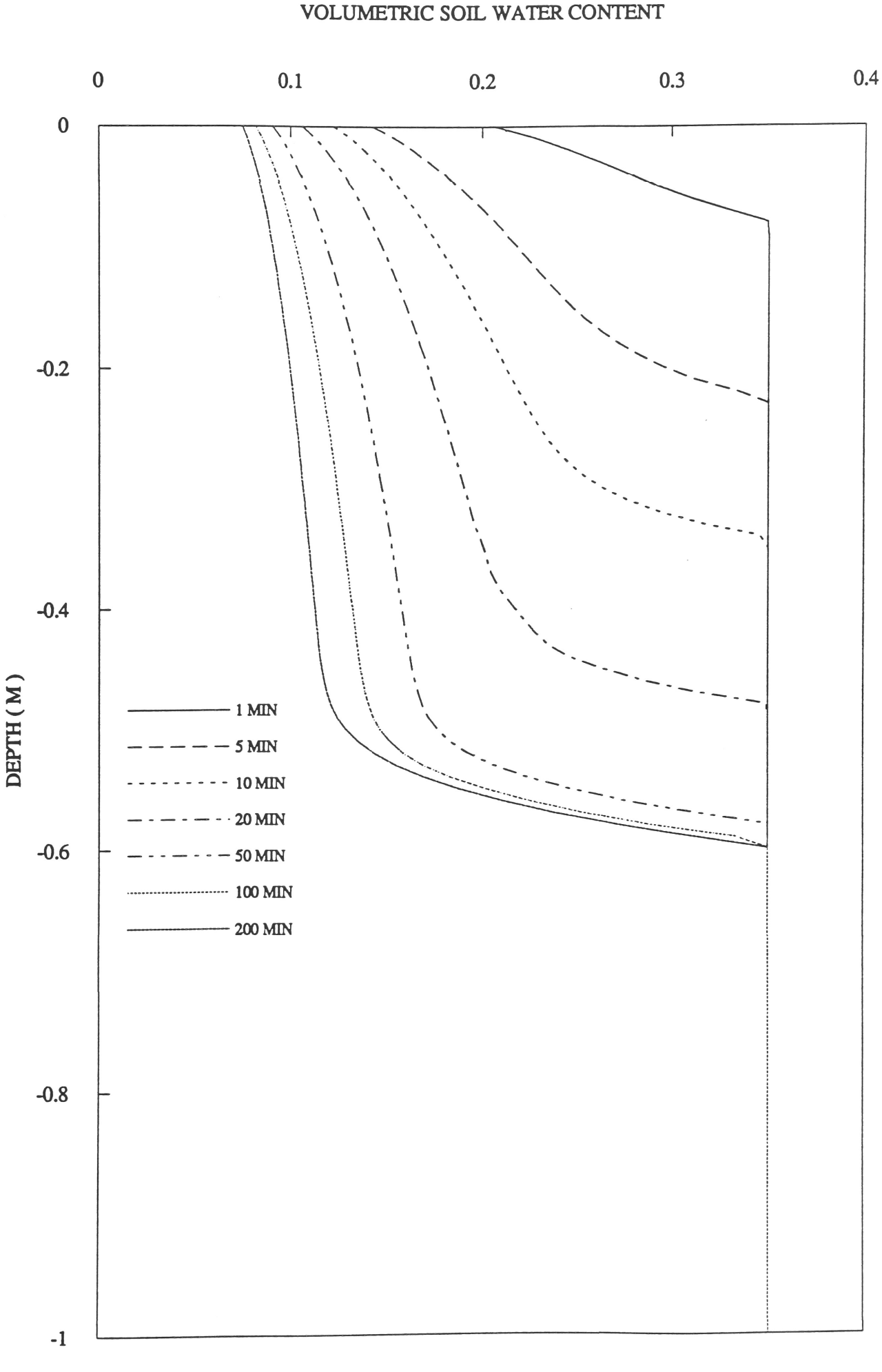

Figure 8.3 θ(z) profiles for Example 2

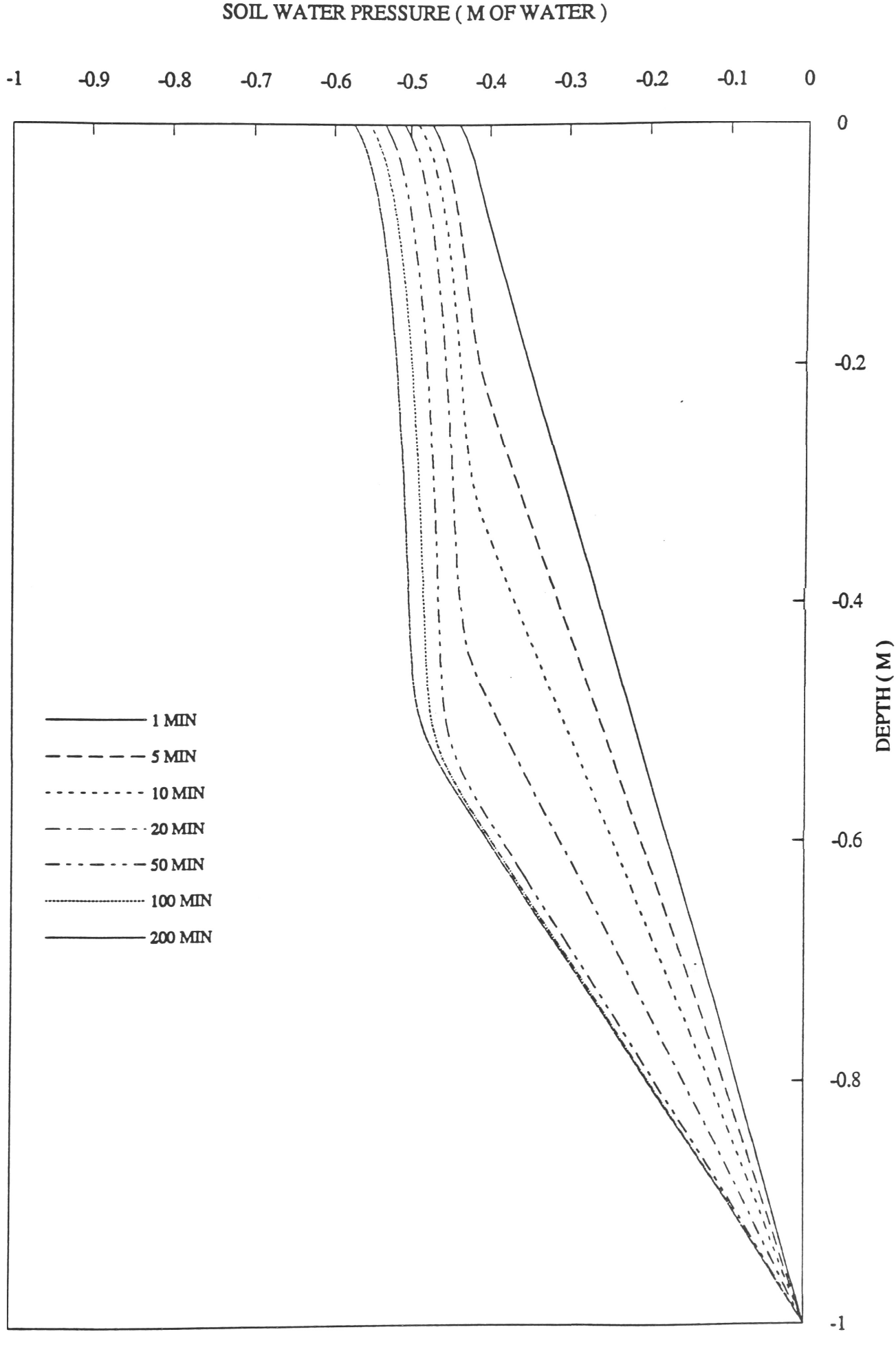

Figure 8.4 h(z) profiles for Example 2

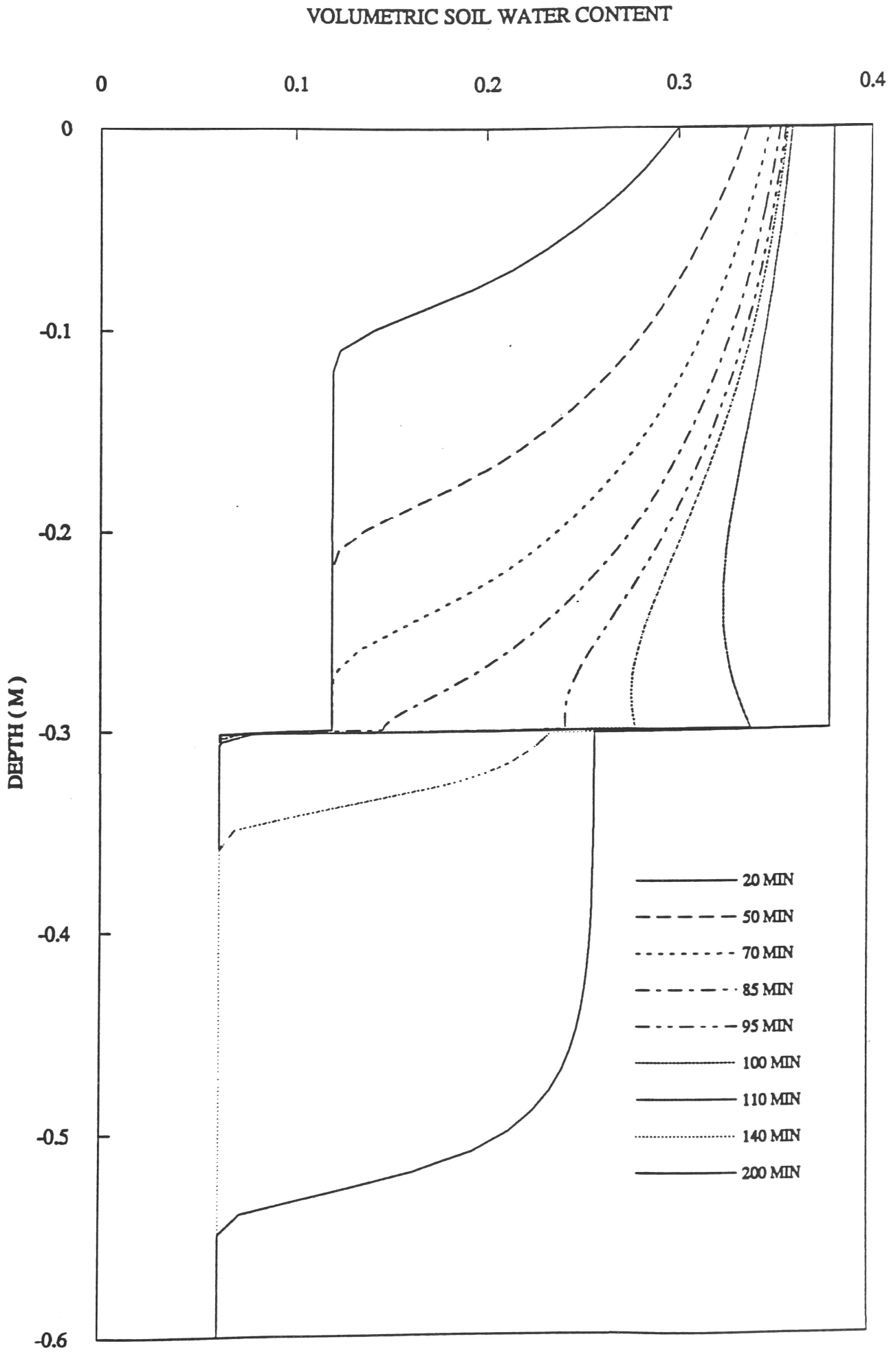

Figure 8.5 θ(z) profiles for Example 3

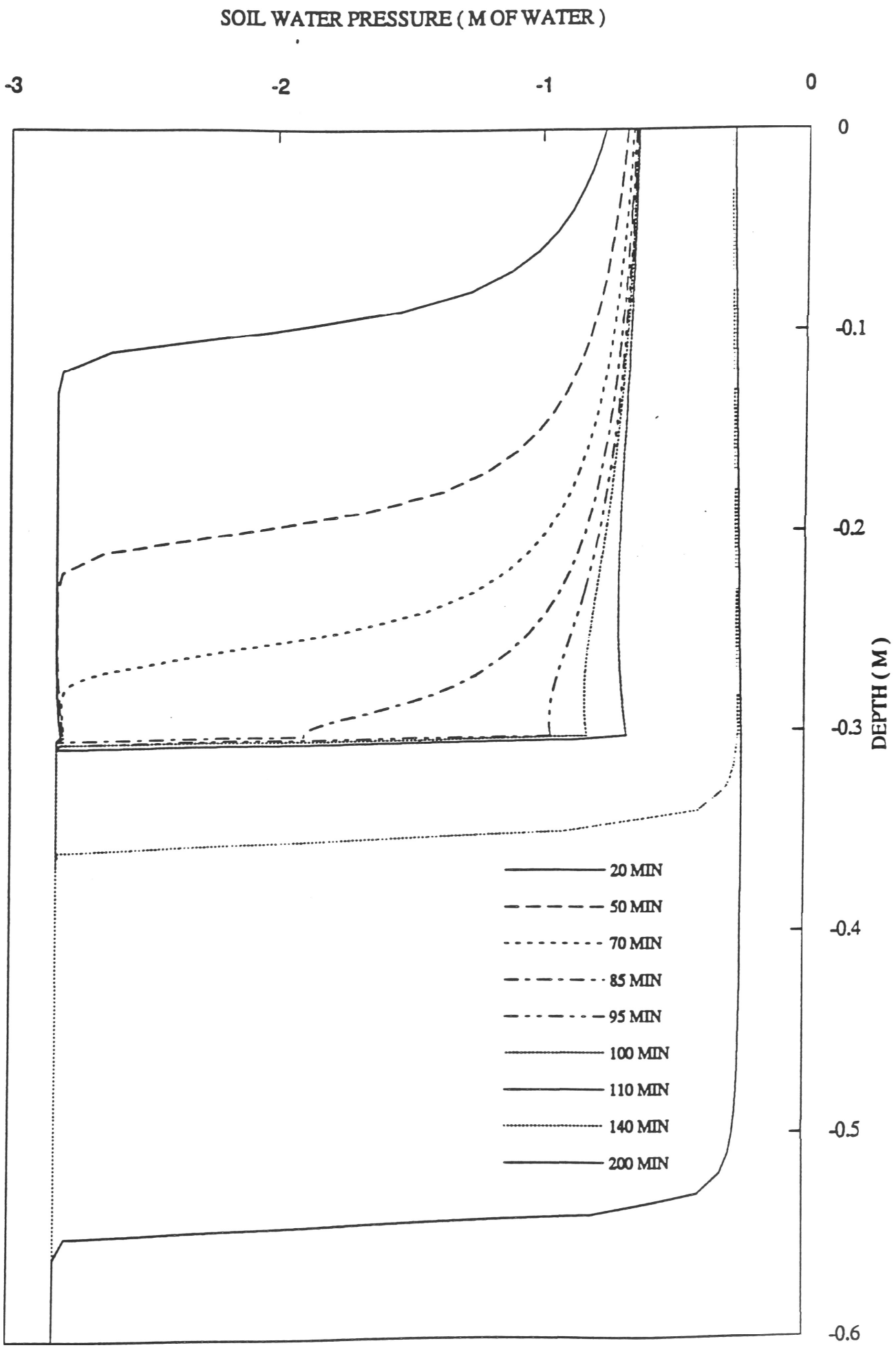

Figure 8.6 h(z) profile for Example 3

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