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by

T.R.Fietz

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Preface

The work reported herein formed part of the programme of unsponsored research carried out in the Water Engineering Department of the School of Civil Engineering by Mr. T.R.Fietz, Senior Lecturer, as a contribution to an evaluation of methods which might be applied to analysis of small pipe networks.

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Officer-in-Charge.

Abstract

The linear theory of Wood and Charles (Proc. A.S.C.E., J. Hyds. Div., July 1972) is generalised to analyse steady flow in water supply networks of pipes, pumps and reservoirs. The method is efficient and applicable to networks up to 50 lines.

Table of Contents

1. Introduction

1.1 Contents

This report describes an extension of the linear theory for analysis of steady flow in water supply networks as presented by Wood and Charles (Ref. 1). The original theory is generalised to include pumps and reservoirs in the network and to use a comprehensive pipe head loss formula.

The development of a computer program is outlined and a small test network is analysed. The linear theory is compared with simple loop method.

1.2 Network Definition

The network consists of lines which join at the nodes. A line may be a pipe or a pump and three empirical coefficients are required to describe the head change-discharge relation for any line. One end node for a line is arbitrarily called the upstream node and flow is positive when from the upstream to the downstream node.

Nodes are either reservoirs, junctions with no external flow, or junctions with outflow or inflow.

A closed loop is a non-intersecting path through the network which returns to its starting point. An open loop is a non-intersecting path joining two reservoirs. The direction of a loop through a line is positive when the loop passes from the upstream to the downstream node of the line.

A directed minimum resistance tree is a spanning subtree connecting all nodes and composed of the lowest resistance lines. When adding a line to the tree to include a new node the direction of the line is positive when the new node is the downstream node of that line.

2. The Linear Theory

2.1 Basis of the Method

For a network consisting of N_n nodes including N_r reservoirs there are $N_n - N_r$ junction nodes where continuity of line flows must be satisfied. At a junction node k:-

$$\sum_{j=1}^{n_{p_k}} Q_j + S_k = 0 \tag{1}$$

where n_{p_k} is the number of lines connected at node k, Q_j is the flow in a line, and S_k is the external inflow or outflow (i.e. demand). Flows towards the node are taken as positive.

For a "fully floating" case where the head at none of the nodes is specified then N_r must be taken as 1, that is one of the nodes is ignored.

The additional simultaneous equations required to find the line flows are obtained from conservation of energy around closed or open loops. The number of independent loop equations N_l is (Ref. 2):-

$$N_l = N_p + N_r - N_n \tag{2}$$

where N_p is the number of lines in the network.

For loop l conservation of energy requires:-

$$(X_l - Y_l) - \sum_{j=1}^{n_{pl}} h_{f_j} + \sum_{j=1}^{n_{ul}} h_{p_j} = 0 \tag{3}$$

where X_l and Y_l are the heads at the nodes at the beginning and end of the loop respectively; n_{pl} and n_{ul} are the number of pipe lines and pump lines respectively; and hf_j and h_{p_j} are the head changes through a pipe line and a pump line respectively. For a closed loop X_l minus Y_l is zero. Unlike the simple loop method (Ref. 3) the selection of loops does not appear to affect the convergence of the solution.

The node continuity equations (e.g. equation (1)) are linear in the line flows. If the loop energy equations (e.g. equation (3)) can be made linear in the line flows then any conventional method of solving simultaneous linear equations directly may be used. Linearisation of the loop energy equations is achieved by basing the current value on a previous estimate of the line flow. The solution procedure is therefore iterative and requires several solutions of a set of simultaneous linear equations.

2.2 Head Change through a Line

2.2.1 Sign of Head Change

The head change terms h_{f_j} and h_{p_j} in equation (3) are with respect to the direction of the loop through the line j. For pipe line j in loop l:-

$$h_{f_j} = T_{l_j} \; f_j \; (Q_j) \tag{4}$$

where T_{lj} is the direction of loop l through line j, being set as +1 when coinciding with + Q_j, that is when passing from the upstream to the downstream node, and - 1 for the converse. f_j (Q_j) is the head loss function for pipe j as a function of the discharge Q_j.

For pump line j in loop l:-

$$h_{p_j} = T_{l_j} \; p_j \; (Q_j) \tag{5}$$

where p_j (Q_j) is the head rise function for pipe j.

2.2.2 Head Loss Functions for Pipes

Head loss functions for pipes are usually exponential or comprehensive. The Hazen-Williams formula is an example of the exponential type. For a pipe j:-

$$f_j(Q_j) = \text{Sign}\,(Q_j)\, K_{hw} \quad \frac{\left(|Q_j|\right)^{1.852} \; l_j}{d_j^{\,4.87} \; C_{hw_j}^{\,1.852}} \tag{6}$$

where l_j is the length; d_j the diameter and C_{hw_j} the Hazen-Williams coefficient. K_{hw} is a constant depending on the unit system used. Derivation of equation (6) is given in Appendix A.

A comprehensive formula applying over the whole of the range of turbulent flow is preferred. One obtained by combining the Darcy-Weisbach equation with an explicit approximation for the friction factor is:-

$$f_j(Q_j) = \text{Sign}\,(Q_j) \left\{ A_j \;\; (\,|\,Q_j\,|\,)^2 \; + B_j \; (\,|\,Q_j\,|\,)^{2-C_j} \right\} \tag{7}$$

where A_j, B_j and C_j are empirical coefficients for pipe j, details of which are given in Appendix A. Equation (7) is shown plotted in Figure 1(A).

2.2.3 Head Rise Functions for Pumps

It is convenient to fit a curve to the experimentally obtained head rise-discharge curve for a rotodynamic pump. Using a second degree polynomial (Ref. 3) for pump j:-

$$p_j\,(Q_j) = A_j\,Q_j^2 + B_j\,Q_j + C_j,\quad Q_j \geq 0 \tag{8a}$$

$$p_j\,(Q_j) = |A_j\,Q_j^2| + |B_j\,Q_j| + C_j,\quad Q_j < 0 \tag{8b}$$

where A_j, B_j and C_j are coefficients for pump j.

Alternatively, for the "flat" curves of water supply pumps, the tangent at the design point is a reasonable approximation:-

$$p_j\,(Q_j) = A_j - B_j\,C_j + C_j\,Q_j \tag{9}$$

where A_j, B_j and C_j are the head, discharge and tangent slope respectively at the design point. Computation is simpler using equation (9) compared with that using equation (8). Equation (9) is adequate for network synthesis problems where the pump performance curve is not known in detail in advance.

Equations (8) and (9) are shown plotted in Figure 1(B).

2.3 Linearisation of the Loop Energy Equations

2.3.1 Pipe Line Head Loss Terms

If we have a previous estimate Q_j^o for the flow in line j, with corresponding head loss $f_j\,(Q_j^o)$, then the head loss $f_j\,(Q_j^1)$ for the current flow Q_j^1 is approximately:

$$f_j\,(Q_j^1) = f_j\,(Q_j^o) + f'\,(Q_j^o)\,(Q_j^1 - Q_j^o) \tag{10}$$

where $f'\,(Q_j^o)$ is the slope of the tangent (always + ve) to the head loss curve at flow Q_j^o as shown in Figure 1(A). From the comprehensive equation (7), $f'\,(Q_j^o)$ is:-

$$f_j'\,(Q_j^o) = 2A_j\,|Q_j^o| + (2 - C_j)\,B_j\,(|Q_j^o|)^{1-C_j} \tag{11}$$

Using equation (4), (7), (10) and (11), a term $-h_{f_j}$ in equation (3) becomes:-

$$-h_{f_j} = -T_{1j} f_j (Q_j^1) = -T_{1j} \text{ Sign } (Q_j^o) \left\{ A_j (|Q_j^o|)^2 + B_j (|Q_j^o|)^{2-C_j} \right\}$$

$$+ T_{1_j} Q_j^o \left\{ 2A_j |Q_j^o| + (2-C_j) B_j (|Q_j^o|)^{1-C_j} \right\}$$

$$- T_{1_j} Q_j^1 \left\{ 2A_j |Q_j^o| + (2-C_j) B_j (|Q_j^o|)^{1-C_j} \right\} \tag{12}$$

The first two terms on the right hand side of equation (12) are functions of the known flow Q_j^o, while the third is linear in the current flow Q_j^1. Equation (12) is therefore in a form suitable for solution of linear simultaneous equations.

An equation which is linear in Q_j^1 may also be derived by using equation (6) as the pipe head loss expression. Defining a pipe coefficient D_j for pipe j:-

$$D_j = \frac{K_{hw} \; l_j}{d_j^{4.87} \; C_{hw_j}^{1.852}} \tag{13}$$

then $-h_{f_j}$ is given by:-

$$-h_{f_j} = -T_{1j} \; f_j (Q_j^1) = -T_{1j} \text{ Sign } (Q_j^o) D_j (|Q_j^o|)^{1.852}$$

$$+ T_{1j} \; Q_j^o \left\{ 1.852 D_j (|Q_j^o|)^{0.852} \right\}$$

$$- T_{1j} \; Q_j^1 \left\{ 1.852 D_j (|Q_j^o|)^{0.852} \right\} \tag{14}$$

In the derivation of Wood and Charles (Ref. 1) the first two terms on the right hand side are ignored so that $-h_{f_j}$ becomes:-

$$-h_{f_j} = -T_{1j} \; f_j (Q_j^1) = -T_{1j} \; Q_j^1 \left\{ 1.852 D_j (|Q_j^o|)^{0.852} \right\} \tag{15}$$

The improvement obtained by including the extra terms in equation (14) is shown graphically in Figure 1(A). When the additional terms are retained then there is no need to apply Wood and Charles' (Ref. 1) smoothing equation to prevent oscillation of the solution and the dependence on good initial estimates for starting the iterative solution is reduced.

2.3.2 Pump Line Head Rise Terms

The counterpart of equation (10) for a pump line j is:-

$$p_j (Q_j^1) = p_j (Q_j^o) + p_j' (Q_j^o) (Q_j^1 - Q_j^o) \tag{16}$$

where $p_j'(Q_j^o)$ is the slope of the tangent to the head rise curve at flow Q_j^o as shown in Figure 1(B). Using equation (8) as the head rise function for a pump, the tangent slope is given by:-

$$p_j'(Q_j^o) = 2A_jQ_j^o + B_j, \quad Q_j^o \geq 0 \tag{17a}$$

$$p_j'(Q_j^o) = -\left\{|2A_jQ_j^o| + |B_j|\right\}, \quad Q_j^o < 0 \tag{17b}$$

For the pump curve shown in Figure 1(B) the tangent slope is zero at $Q_j = -B_j/(2A_j)$. There is a remote possibility that this could cause a singular element to appear when solving the simultaneous linear equations. This possibility has been ignored in this report.

The counterpart of equation (12) for a pump line j is:-

$$\begin{aligned} h_{p_j} = T_{1j}\, p_j(Q_j^1) &= T_{1j}\left\{A_j (Q_j^o)^2 + B_j Q_j^o + C_j\right\} \\ &- T_{1j}\, Q_j^o\left\{2A_j Q_j^o + B_j\right\} \\ &+ T_{1j}\, Q_j^1\left\{2A_j Q_j^o + B_j\right\}, \quad Q_j^o \geq 0 \end{aligned} \tag{18a}$$

$$\begin{aligned} h_{p_j} = T_{1j}\, p_j(Q_j^1) &= T_{1j}\left\{\left|A_j (Q_j^o)^2\right| + |B_jQ_j^o| + C_j\right\} \\ &- T_{1j}\, Q_j^o\left\{-\left(|2A_j Q_j^o| + |B_j|\right)\right\} \\ &+ T_{1j}\, Q_j^1\left\{-\left(|2A_j Q_j^o| + |B_j|\right)\right\}, \quad Q_j^o < 0 \end{aligned} \tag{18b}$$

Alternatively, using the linear equation, equation (9), where $p_j'(Q_j^o) = C_j$ for pipe j, then h_{p_j} is given by:-

$$h_{p_j} = T_{1j}\, p_j(Q_j^1) = T_{1j}(A_j - B_j C_j) + T_{1j}\, Q_j^1 C_j \tag{19}$$

2.4 Method of Solution

2.4.2 Necessity for an Iterative Solution

There is a set of N_p simultaneous equations which are linear in the line flows. In matrix notation:-

$$EQ = F \tag{20}$$

where E is the system matrix and F is the matrix of constants. The solution vector Q contains the unknown line flows, that is Q_j^1 for line j.

For the node continuity subset the submatrix of E is a connectivity matrix containing elements +1, -1 or 0. The submatrix of F contains the fixed external node outflows or inflows. These submatrices remain constant from iteration to iteration.

For the loop energy subset the submatrix of E contains terms such as the multiplicand of Q_j^1 in equation (12). For a loop l corresponding to equation i the element in the submatrix of F is the sum of the known value terms, such as the first two terms on the right hand side of equation (12) (with appropriate sign change), for all of the lines in loop l. The F element also includes the head difference between the ends of the loop, again with appropriate sign change. The submatrices of E and F for the loop energy subset change from iteration to iteration. Methods of starting the iterative procedure are discussed in Sections 2.5 and 2.6 below.

2.4.2 Limitation on Network Size

For networks up to about 50 lines solution of the linear simultaneous equations (equation (20)) may be by a direct method, for example Gauss-Jordan with maximum pivot strategy. For computers with built-in matrix operations the solution is readily obtained by matrix inversion:-

$$Q = E^{-1}F \tag{21}$$

For networks of more than about 50 lines an iterative procedure, such as Gauss-Seidel, would be required to solve the linear simultaneous equations. Unfortunately the node continuity subset of the equations does not exhibit diagonal dominance (being composed of elements of value +1, -1 or 0) so that iterative methods cannot be used.

2.4.3 Convergence Criteria

Convergence of the solution is checked after each iteration using an absolute or relative criterion based on line flows. For the absolute criterion:-

$$\text{Any } \left| Q_j^1 - Q_j^o \right| \leq Q_a \tag{22}$$

where Q_a is a suitable small value of line flow.

For the relative criterion:

$$\text{Any } \left| \frac{Q_j^1 - Q_j^o}{Q_j^o} \right| \leq Q_r \tag{23}$$

where Q_r is a suitable small value.

Because of the accumulation of errors in direct methods of solution of the equations, care must be taken not to make Q_a or Q_r too small, otherwise the solution will never satisfy the convergence criterion, particularly for networks approaching the upper limit of about 50 lines.

2.5 Starting the Iterative Procedure using Initial Estimates for E and F Matrices

2.5.1 Closed Loop of Pipe Lines Only

Wood and Charles (Ref. 1) take E_{ij} for pipe j in equation i for loop l as:-

$$E_{ij} = - T_{lj} \; D_j \tag{24}$$

The effect of this is to divide the head loss in pipe j (as given by equation (15)) by $1.852 \, (|Q_j|)^{0.852}$, or to assume that the pipe flow is laminar.

Alternatively, taking the wholly rough turbulent element $A_j \, (|Q_j|)^2$ from equation (7) as the pipe head loss, the appropriate E element for pipe j in equation i for loop l is:-

$$E_{ij} = - T_{lj} \, A_j \tag{25}$$

For a closed loop l the element F_i is zero.

Equations (24) or (25) may be used for pipe lines in the following loop situations providing due allowance is made for dividing the pipe head losses by their discharges.

2.5.2 Closed Loop with Pump Lines

For a pump line in a closed loop the pump is assumed to be working near the design point and the head rise through the pump is divided by the design discharge Q_{jD}.

If the linear approximation given by equation (9) is used as the pump head rise function then the E element for pump j in equation i for loop l is:-

$$E_{ij} = T_{lj} \; \frac{C_i}{B_j} \tag{26}$$

The contribution to the F element is:-

$$\Delta F_i = - T_{lj} \; \left(\frac{A_j}{B_j} - C_j \right) \tag{27}$$

If the parabolic approximation given by equation (8) is used as the pump head rise function then the design discharge must first be found. Taking the design discharge for a water-supply pump as 0.6 of the maximum discharge, then for pump j the design discharge Q_{jD} is given by

$$Q_{jD} = 0.6\left\{- \frac{B_j}{2A_j} - \text{Sign}\,(A_j)\left(\left|\left(\frac{B_j}{2A_j}\right)^2 - \frac{C_j}{A_j}\right|\right)^{0.5}\right\} \tag{28}$$

and H_{jD} is the head at the design point, found by putting $Q_j = Q_{jD}$ in equation (8a).

Dividing H_{jD} by Q_{jD}, then from equation (18a) the starting value of the E element for pump j in equation i is:-

$$E_{ij} = T_{lj}\,(2A_j + \frac{B_j}{Q_{jD}}) \tag{29}$$

The contribution to the F element of equation i is:-

$$\Delta F_i = - T_{lj}\,\frac{H_{jD}}{Q_{jD}} + T_{lj}\,(2A_j\,Q_{jD} + B_j) \tag{30}$$

2.5.3 Open Loop of Pipe Lines Only

Here it is necessary to divide the head difference between the ends of the loop by a convenient discharge value so that the head difference term in equation (3) is not excessive relative to the pipe head loss terms. A convenient discharge is found by assuming that the pipe with the minimum resistance accounts for all of the head loss in the loop.

Using the Hazen-Williams formula as the pipe head rise function then from equation (3) the required discharge Q_{ol} for open loop l is given by:-

$$Q_{ol} = \left(\frac{|X_l - Y_l|}{(D_j)_{min.}}\right)^{0.54} \tag{31}$$

where $(D_j)_{min.}$ is the minimum pipe coefficient (from equation (13)) of the pipes in the loop.

Using the comprehensive head loss function (equation (7)) with the transition term involving B_j ignored, the discharge Q_{ol} is given by:-

$$Q_{o_l} = \left(\frac{|X_l - Y_l|}{(A_j)_{min.}}\right)^{0.5} \tag{32}$$

where $(A_j)_{min.}$ is the minimum pipe coefficient of the pipes in loop 1.

The E element for pipe j in equation i for open loop 1 is given by either equation (24) or (25), and the F element is:-

$$F_i = - \frac{(X_1 - Y_1)}{Q_{o1}} \tag{33}$$

2.5.4 Open Loop with Pump Lines

The head difference between the ends of the loop is divided by the largest design discharge $(Q_{jD})_{max.}$ of the pump lines in the loop. The additional contribution to the F element for equation i is then:-

$$\Delta F_i = - \frac{(X_1 - Y_1)}{(Q_{jD})_{max.}} \tag{34}$$

The E elements and the other contributions to the F element are as given in Sections 2.5.1 and 2.5.2 above.

2.6 Starting the Iterative Procedure using Initial Estimates for Line Discharges

2.6.1 Alternative Starting Procedure

As an alternative method to estimating the elements in the loop energy subset of the E and F matrices directly, the line discharges may be initialised to start the iterative procedure.

The amount of computation for initialising line discharges is less than that required for initialising the E and F subset but additional iterations are required to reach final convergence.

2.6.2 Procedure for Initialising Line Discharges

A directed minimum resistance tree is constructed for the network, starting at a reservoir node. Prim's (Ref. 4) method is convenient for manual construction of the tree. Pipe line resistances are taken as the coefficient A_j of equation (7) or D_j of equation (13). Pump line resistances are taken as the slope of the tangent to the design point. This slope is equal to the coefficient C_j when equation (9) is used as the pump head rise function. Alternatively when equation (8) is used the tangent slope may be readily derived from the actual performance curve.

Starting with the last line added to the tree, the initial discharges are found as follows:-

(a) If the node at the free end of the line (i.e. the unconnected node in the tree) is a reservoir node the flow is assumed to be from or into the reservoir through the line to satisfy the current external flow at the fixed end node. The external flows at the nodes (usually - ve demands) are supplied in the initial node data for the network. The external flow at the fixed end node is then set equal to zero.

(b) If the free end node is an ordinary junction node of the network the line flow is assumed to be towards the free end to satisfy the external flow at the free end. The free end external flow is set equal to zero and the fixed end node external outflow is incremented by the line flow.

(c) The initial flows in the high resistance lines which do not appear in the directed tree are taken as zero.

The above procedure is an extension of that outlined by Epp and Fowler (Ref.5) who dealt with ordinary junction nodes only.

3. Programming the Linear Theory

The method has been programmed for use on a small time-sharing computer. It is convenient to split the procedure into three sub-programs which communicate through common variables.

The first sub-program is for data input. For a pipe line the upstream node number, the downstream node number, the length, the diameter, and the wall roughness (or Hazen-Williams coefficient) are required. For a pump line the end node numbers and the three head rise function coefficients are required. For a node the head and fixed external flow are supplied. The head is set at zero for an ordinary junction node while the external flow is always zero for a reservoir node. For a loop the end node numbers, the number of lines in the loop, and each line number in the loop is required. The line number is signed according to the direction of the loop through the line (Ref.1). When the line discharge method of starting the iterative procedure (Section 2.6) is used then a directed minimum resistance tree is also supplied as data. The tree may also be used subsequently for calculating node heads after the line flows have been found (Ref.3).

The second sub-program finds values for initialising the iterative procedure. The elements of the E and F matrices for the node connectively subset are found and then either the E and F elements for the loop energy subset (Section 2.5) or, alternatively, the initial line discharges (Section 2.6).

The third sub-program performs the iterative solution and checks for convergence. The programming is simplified if the computer has built-in matrix operations.

The maximum number storage required for a network of N_p lines is about 40 N_p for the third sub-program. This compares unfavourably with 8 N_p for the simple loop method with similar chaining of sub-programs (Ref. 3).

4. Application to a Test Network

The program has been used to analyse the small test network shown in Figure 2. The line, loop and minimum resistance tree details are shown in Table 1. The comprehensive formula (equation (7)) has been used as the pipe head loss function and the linear approximation (equation (9)) as the pump head rise function.

The results obtained are compared with those from a simple loop method analysis of the same network in Table 2. The results are practically identical. For a similar convergence criterion the linear method required 6 iterations using initial estimates for the E and F matrices or 7 iterations using initial estimates for line flows. The loop method required 9 iterations and the time per iteration was at least twice that for the linear method.

5. Conclusions

(a) The linear theory is an efficient method of analysis of steady flow in small water supply networks of pipes, pumps and reservoirs up to 50 lines.

(b) The Hazen-Williams or comprehensive formulas may be used as the pipe head loss function.

(c) A parabolic or straight line approximation may be used as the pump head rise function. The latter requires less computation and is preferable for network synthesis.

(d) The linear theory requires more computer storage than the simple loop method but the computing time is less.

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Notation

a, b, c	Coefficients in friction factor approximation for a pipe
A, B, C	Empirical coefficients in head loss function for a pipe or head rise function for a pump
A_j, B_j, C_j	A, B, C for line j
$(A_j)_{min}$	Minimum A_j for pipes in a loop
C_{hw}	Hazen-Williams coefficient for a pipe
C_{hw_j}	C_{hw} for pipe j
d	Pipe diameter
d_j	Diameter of pipe j
D_j	Coefficient for pipe j using Hazen-Williams formula
$(D_j)_{min.}$	Minimum D_j for pipes in a loop
e	Nikuradse wall roughness for a pipe
e/d	Relative roughness of a pipe
E	System matrix
E_{ij}	E element for line j in equation i
f_j	Head loss function for pipe j
f_j'	Slope of tangent to f_j at a particular discharge
F	Constants matrix
F_i	F element in equation i
g	Gravitational acceleration
h_f	Head loss in a pipe
h_{f_j}	Head loss in pipe j in a loop
h_{p_j}	Head rise through pump j in a loop

H_{j_D}	Head at design point for pump j
K_{hw}	Unit system constant for Hazen-Williams formula
l	Pipe length
l_j	Length of pipe j
n_{p_l}	Number of pipes in loop l
n_{p_k}	Number of lines connected at node k
n_{u_l}	Number of pumps in loop l
N_l	Number of loops in network
N_n	Number of nodes in network
N_p	Number of lines in network
N_r	Number of reservoir nodes in network
p_j	Head rise function for pump j
p_j'	Slope of tangent to p_j at a particular discharge
$\mathbf{Q}$	Solution vector for line flows
Q	Flow in a line
Q_a	Maximum absolute value of change in any line flow for convergence
Q_{o_l}	Discharge for adjusting head difference between the ends of open loop l
Q_j	Flow in line j
Q_{j_D}	Discharge at design point for pump j
$(Q_{j_D})_{max.}$	Maximum Q_{j_D} for pumps in a loop
Q_j^o	Flow in line j from previous iteration
Q_j^1	Flow in line j for current iteration

Q_r	Maximum absolute value of relative variation in any line flow for convergence
S_k	External inflow or outflow at node k
T_{lj}	Direction of loop l through line j
V	Mean velocity in a pipe
X_l	Head at node at beginning of loop l
Y_l	Head at node at end of loop l
ν	Kinematic viscosity of liquid
ΔF_i	Contribution to F_i

Appendix A - Head Loss Functions for Pipes

A1: The Hazen-Williams Formula

A1.1: Using English-Engineering Units

For English Engineering units the Hazen-Williams formula (Ref. 6) is:-

$$V = 1.318\, C_{hw} \left(\frac{d}{4}\right)^{0.63} \left(\frac{h_f}{l}\right)^{0.54} \tag{35}$$

where V is the mean velocity in ft. s^{-1}, C_{hw} is the Hazen-Williams coefficient for the particular pipe material; d is the diameter in ft; h_f is the head loss in ft. over the length l in ft.

Rearranging equation (35) to make the head loss the dependent variable gives

$$h_f = 4.7273 \frac{Q^{1.85185}\; l}{d^{4.8704}\; C_{hw}^{1.85185}} \tag{36}$$

where Q is the discharge in ft.^{3} s^{-1}, and 4.7273 is the unit system constant K_{hw}.

When the exponents are corrected to the third decimal place K_{hw} changes slightly. For a typical pipe (V = 6 ft. s^{-1}; d = 1 ft; C_{hw} = 100) the head loss is given by:-

$$h_f = 4.7295 \frac{Q^{1.852}\; l}{d^{4.87}\; C_{hw}^{1.852}} \tag{37}$$

For practical purposes K_{hw} = 4.73 in equation (37).

A1.2: Using SI Units

To convert equations (36) and (37) for use in SI units the dimensions of the Hazen-Williams coefficient C_{hw} are required. Vennard (Ref. 6) contends that C_{hw} is a measure of relative roughness e/d and not absolute roughness e. The use of smaller C_{hw} values for increasing diameters of pipes of the same material tends to support this argument.

Taking C_{hw} as a measure of relative roughness, and hence as dimensionless, gives a unit system constant K_{hw} = 10.67 for use in equation (36), where h_f is in m; Q is in $m^3 s^{-1}$; l is in m; and d is in m. In equation (37) K_{hw} = 10.685 for SI units.

A2: A Comprehensive Formula

By combining the Darcy-Weisbach expression for pipe head loss with an explicit approximation (Ref. 7) for the Colebrook-White friction factor formula a comprehensive formula for pipe head loss is obtained (Ref. 3):-

$$h_f = AQ^2 + BQ^{2-C} \tag{38a}$$

where

$$A = a\left(\frac{81}{g\pi^2 d^5}\right) \tag{38b}$$

$$B = b\left(\frac{81}{g\pi^2 d^5}\right)\left(\frac{4}{\pi d \nu}\right)^{-c} \tag{38c}$$

$$C = c \tag{38d}$$

and g is gravitational acceleration and ν is the kinematic viscosity of the liquid.

The coefficients a, b, c are functions of the relative roughness e/d of the pipe, and are given by (Ref. 7):-

$$a = 0.094\,(e/d)^{0.225} + 0.53\,\frac{e}{d} \tag{39a}$$

$$b = 88\,(e/d)^{0.44} \tag{39b}$$

$$c = 1.62\,(e/d)^{0.134} \tag{39c}$$

The pipe coefficient C in equation (40) is dimensionless but the values of A and B depend on the unit system used. The advantages of equation (40) over the Hazen-Williams formula are that it applies over the entire range of turbulent flow and that it may be used for liquids other than water.

Table 1: Details of Test Network

Pipe Lines

Line No.	Length l (m)	Dia. d (mm)	Roughness e ($mm \times 10^{-2}$)
2	2438.4	914.4	1.524
3	609.6	609.6	1.524
4	914.4	457.2	1.524
5	1524	609.6	1.524
6	1524	609.6	1.524
7	914.4	304.8	1.524
8	1219.2	609.6	1.524
9	3657.6	914.4	60.96
10	1828.8	304.8	1.524
11	1219.2	609.6	1.524
12	914.4	304.8	1.524
13	1219.2	304.8	1.524

Pump Line

Line No.	A	B	C
1	67.78	0.54015	-107

Loops

Loop No.	Directed Lines in Loop
1	-5, 7, 8, 6, -4, -3
2	-10, 12, 11, -8
3	1, 2, 3, 4, -6, 9
4	13, 5, 3, 4, -6, 9

Minimum Resistance Tree

Directed Lines in Tree: 1, 2, 3, -5, 4, -6, 9, -8, -11, -12, -13

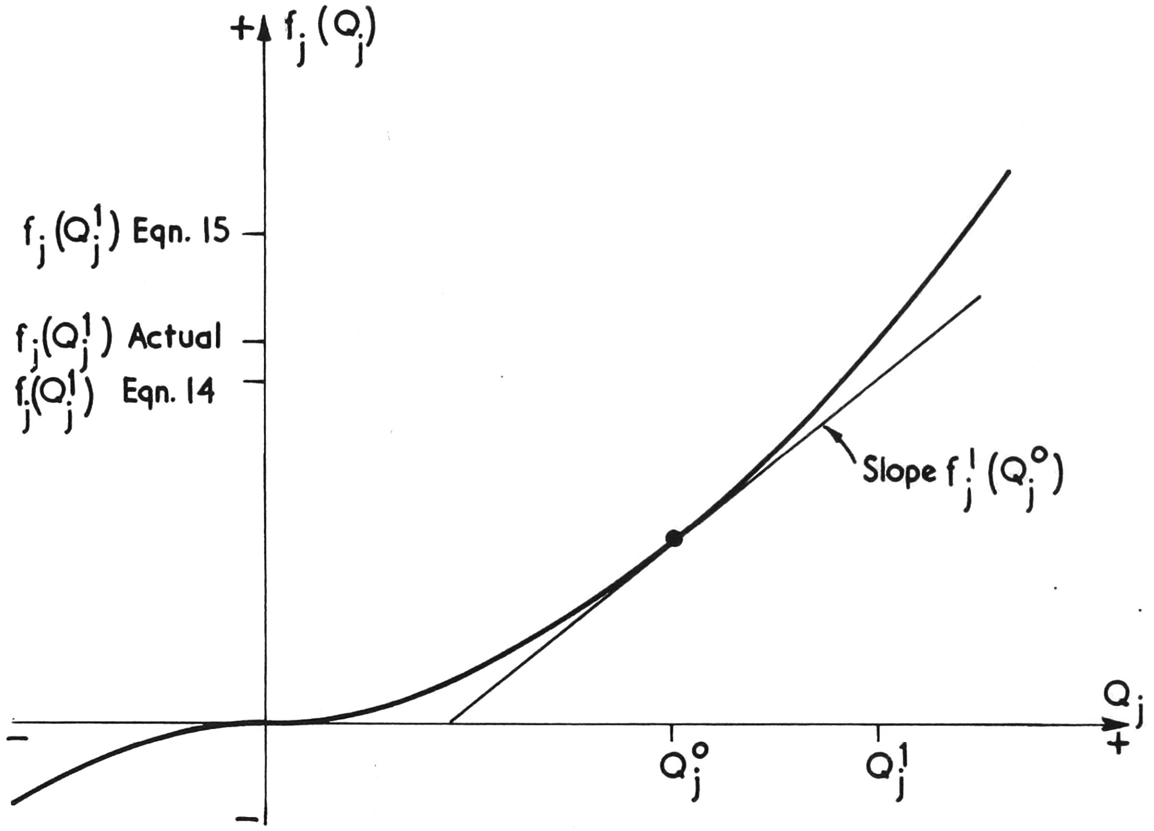
Table 2: Summary of Results of Network Analyses

Line No.	Line Flow from Simple Loop Method ($m^3s^{-1} \times 10^{-3}$)	Line Flow Variation, Linear Method, Initial E & F Estimates ($m^3s^{-1} \times 10^{-6}$)	Line Flow Variation, Linear Method, Initial Line Flow Estimates ($m^3s^{-1} \times 10^{-6}$)
1	771.286	-0.54	+0.25
2	771.286	-0.54	+0.25
3	337.970	-0.25	+0.19
4	148.550	-0.25	+0.17
5	- 227.607	-0.46	+0.26
6	230.290	+0.25	-0.17
7	47.731	+0.02	+0.24
8	-266.178	-0.08	-0.09
9	-807.340	+0.08	-0.20
10	- 86.608	0	0
11	-310.872	-0.01	-0.02
12	-121.452	-0.01	-0.02
13	9.544	+0.46	-0.04

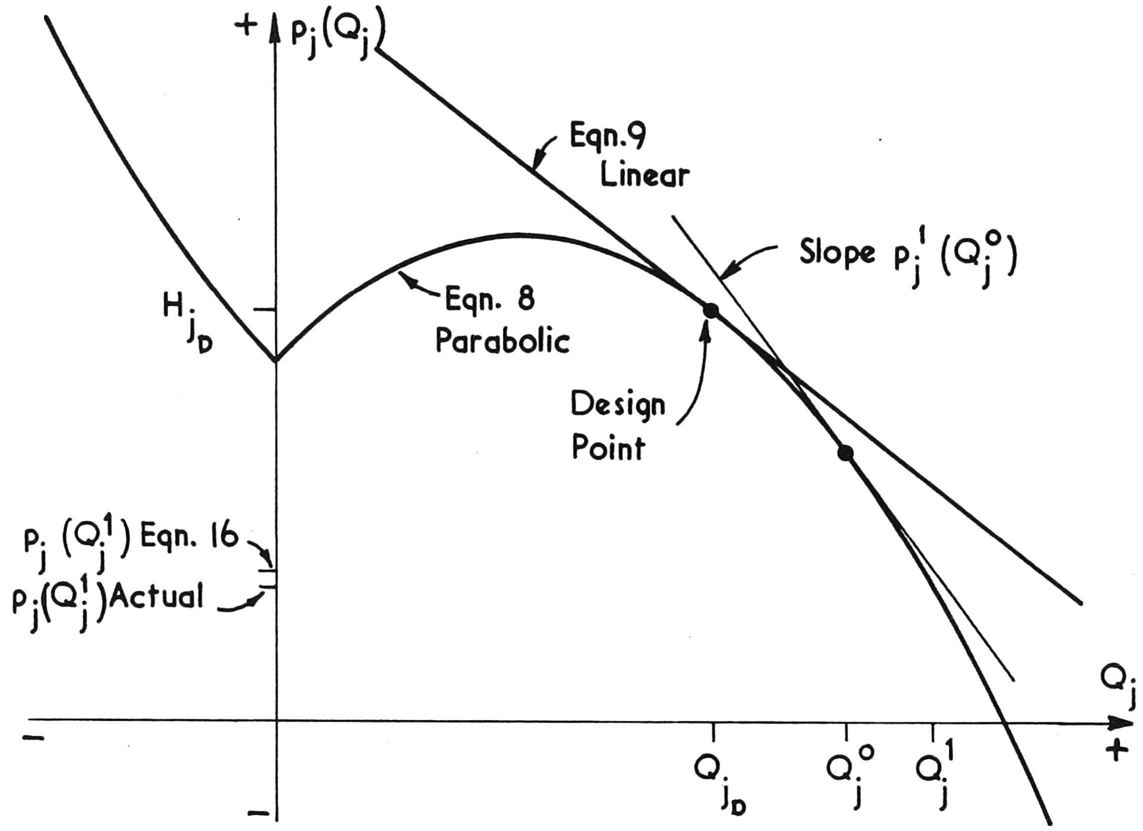
Notes (1) Flow variation = |Flow| - |Flow from Simple Loop Method|

(2) Simple loop method convergence criterion: loop flow correction changing by less than 1×10^{-6} m^3s^{-1}

(3) Linear method convergence criterion: line flow changing by less than 1×10^{-6} m^3s^{-1}



(A) HEAD LOSS FUNCTION FOR A PIPE



(B) HEAD RISE FUNCTION FOR A PUMP

FIGURE 1 : HEAD-DISCHARGE CURVES FOR PIPES AND PUMPS

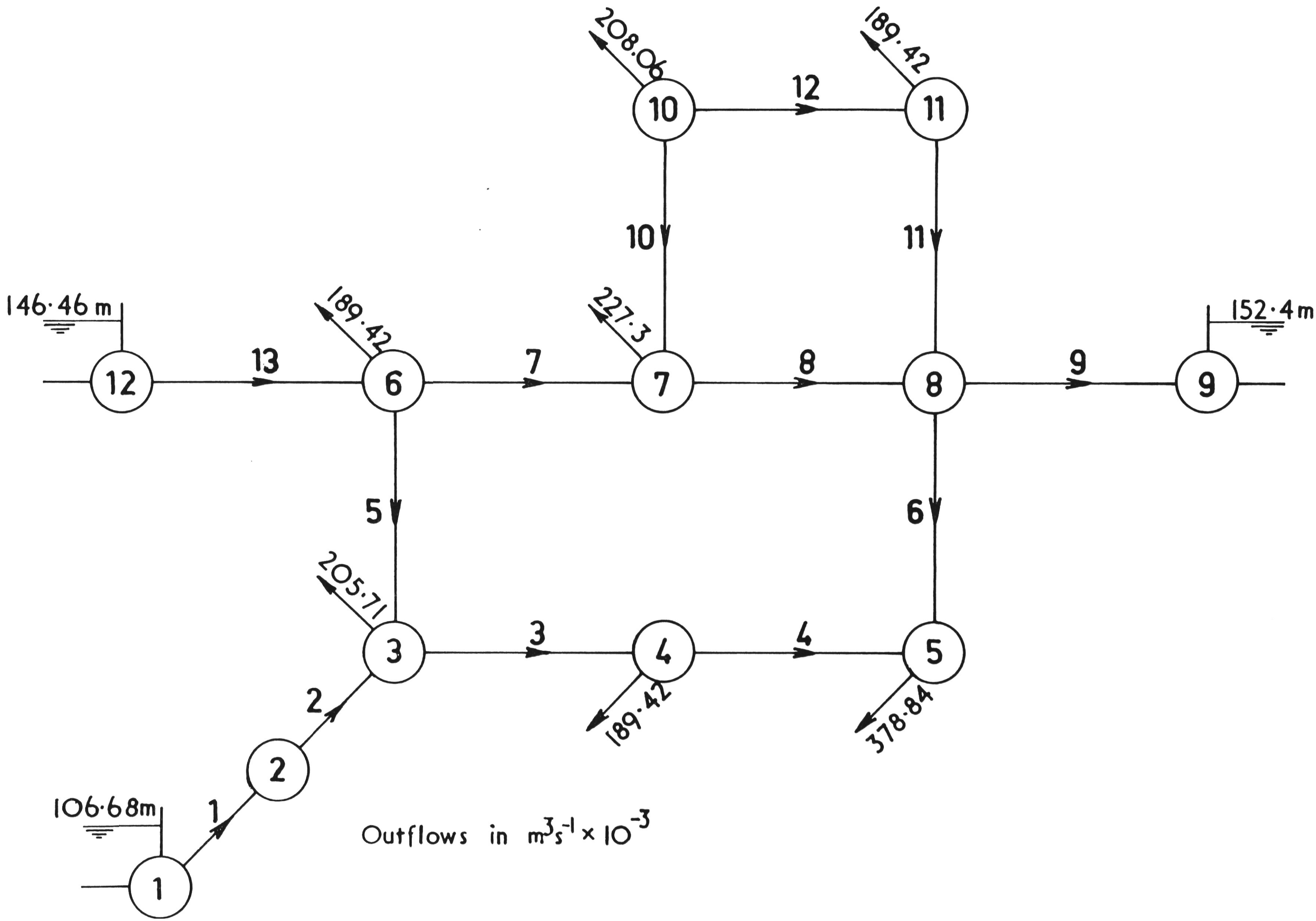


FIGURE 2 : TEST NETWORK