Essays in economic measurement : addressing quality change and new and disappearing goods in price measurment

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Essays in Economic Measurement:

Addressing Quality Change and New and Disappearing Goods in Price Measurement

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By

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DECLARATION OF ORIGINALITY

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where due acknowledgement is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis. I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.

Daniel Melser

ABSTRACT

This thesis primarily focuses on the problem of quality change and new and disappearing goods in the context of the measurement of price change. Changes in the nature of goods and in their availability over time (and space) pose great difficulties for price measurement. If the quality or availability of goods changes then in determining the effect on the cost of living we must compare and quantify the value of these changes. This is conceptually and practically a difficult task. In this thesis both theoretical and empirical arguments are used in discussing various aspects of this topic. Chapter 2 surveys the measurement problem, the different conceptual approaches are outlined and the various practical adjustment procedures are discussed and reviewed. Chapter 3 provides a novel result; the hedonic regression time-dummy method fails to satisfy the monotonicity axioms from index theory. This is interesting as the hedonic regression time-dummy method is an important approach in the literature and has frequently been used to calculate quality-adjusted price indexes. Chapter 4 undertakes an empirical investigation of the effects of new and disappearing goods in a scanner data set. The results indicate that the failure to account for new and disappearing goods leads to a price index with a significant upward biased. Chapter 5 attempts a reconciliation between the stochastic and economic approaches to index numbers. Given this different perspective on the stochastic approach some new methods for estimating economic-stochastic price indexes, and even quantity indexes, naturally arise. This discussion is in the context of the usefulness of the stochastic approach for cases where there are changes in the domain of goods. The thesis, in providing a review of the quality change and new and disappearing goods literature, shows that we now have a wide range of tools for dealing with these problems which makes us optimistic about the future.

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I was introduced to index numbers as a young *Economic Statistician* working on the Consumers Price Index at Statistics New Zealand. This experience prompted my interest in this area and the discussions I had at this time with Vince Galvin, Chris Pike, Phillipa O'Brien and Lyall Payne were invaluable in understanding both the conceptual and practical difficulties of price measurement. While leaving this job was difficult it was made easier by the financial support of UNSW who awarded me a scholarship for which I am grateful.

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CHAPTER 1

1. Introduction

The economic theory of index numbers is an interesting and rewarding area in which to research. It involves the application economic ideas and methodology, regarding price and volume measurement, to the practical issues of collecting, calculating, organizing and interpreting data. This task goes back many years and constitutes one of the core functions of economists. Boskin et al. (1997, p. 78) emphasize the importance of this topic,

> Measuring prices and their rate of change accurately is central to almost every economic issue, from the conduct of monetary policy to measuring economic progress over time and across countries to the cost and structure of indexed spending and taxes.

For an area which has been so thoroughly studied, where detailed discussions of the issues date back to great economists such as Alfred Marshall, John Maynard Keynes and Irving Fisher, there is still a lot of complex and unresolved issues.

The general focus of this thesis is on the problem of quality change and new and disappearing goods in price measurement. The quality change and new and disappearing goods problem relates to the difficulty in defining and measuring price change when the goods change either in terms of their availability or performance. How do we determine pure price change when we have prices for goods which are different? We cannot simply compare the price of one good with another as in the conventional static framework. This problem has particularly risen to prominence in recent times given the dynamic nature of modern business in developing new varieties of products, though it was recognized as far back as Sidgwick (1887) and Marshall (1887). It is not an easy problem to solve, Shapiro and Wilcox (1996, p. 40) called quality change "…the house-to-house combat of price measurement", while the National Research Council (Schultze and Mackie, 2002, p. 106) noted that, “Quality change has typically been considered the least tractable problem associated with the Consumer Price Index.” It seems likely that a single complete solution to this problem will never be found and instead instances of quality change and new and disappearing goods will have to be dealt with on a case-by-case basis. The goal in dealing with this problem then is to develop our understanding and to broaden our tool-box of methods available for dealing with quality change and new and disappearing goods. While this sounds fatalistic this thesis is certainly not pessimistic in tone. In fact, in recent times, much progress has been made in developing techniques that convincingly deal with these problems – though there is certainly still a great deal more research to do.

In this thesis we hope to progress research in a number of areas, both practically, in providing estimates of the bias in indexes which fail to properly account for new and disappearing goods, and theoretically, in discussing and questioning a particular approach to hedonic regression as well as interpreting and outlining alternative stochastically derived price indexes.

Chapter 2, “Quality Change and New and Disappearing Goods: A Survey and Discussion of the Measurement Challenges,” provides, as the title suggests, a survey and discussion of the measurement problem. While the paper is lengthy, it is by no means a complete survey of the burgeoning literature in this area. The chapter looks at different conceptual approaches to accounting for quality change and new and disappearing goods in price indexes. We look at various methods that are used by statistical agencies to deal with these issues. However, in many ways these conventional methods are inadequate which prompts us to turn to three alternative methods. These methods are; hedonic regression, estimating reservation prices, and a new approach using the Constant Elasticity of Substitution (CES) cost function. These methods are more complex than those traditionally used by statistical agencies but represent the most promising options in systematically accounting for quality change and new and disappearing goods. These approaches are briefly outlined as they are used to varying degrees in the later chapters of the thesis. Particular attention is given to hedonic regression, which is by far the most widely used of these ‘frontier’ methods. In summary, the aim of this chapter is to provide a sketch of the problem and methodological approach as well as outlining some of the tools that will be adopted throughout the thesis.

Chapter 3, "The Hedonic Regression Time-Dummy Method and the Monotonicity Axioms," turns to a specific issue with regard to constructing quality-adjusted price indexes using hedonic regression. That is, whether the hedonic regression time-dummy method, where the price index is calculated directly from a time-dummy variable in the regression, satisfies the monotonicity axioms. These axioms arise naturally out of the test or axiomatic approach to index numbers, which regards a price index as a function of independent variables, and specifies reasonable properties that this function should satisfy. The startling result of a simple numerical example, and an application to a well known hedonic data set, is that this method need not satisfy monotonicity. This implies that this approach to price measurement is somewhat questionable. Interestingly, however, there may be an economic explanation for this failure of monotonicity along the lines of that proposed by Reinsdorf and Dorfman (1999) in the context of exact cost-of-living indexes. Despite this explanation there is still a concern that the hedonic time-dummy method can produce counter-intuitive results.

Chapter 4, "Accounting for the Effects of New and Disappearing Goods Using Scanner Data," is a project that arose as a result of having access to a large scanner data set of supermarket products from the Australian Bureau of Statistics. In this chapter we undertake an application of the approach suggested by Feenstra (1994) and Balk (1999) using the CES cost function to account for new and disappearing goods across time. While the CES approach is primarily used we also adopt a simplified version of the reservation price method and discuss why hedonic methods are not useful in accounting for the welfare gains, or losses, from changes in the variety or goods. The results indicate that the basic matched-goods price index is significantly upwardly biased. This bias results from the strong empirical regularity that the expenditure share on new goods is larger than the expenditure share on disappeared goods which indicates that there are likely to be significant gains to consumers from improvements in the range of products across time. These are not properly accounted for in official indexes or indeed any matched-model price indexes.

Chapter 5, "The Economic and Stochastic Approaches to Index Numbers," is different from the previous chapters in that the sole focus does not relate to the issues of quality change and new and disappearing goods. The primary goal of this paper is to provide a reconciliation between the stochastic and economic approaches to index numbers. This is particularly important given that versions of the stochastic approach are increasingly popular given their ability to be used in instances where there are new and disappearing goods. The paper briefly surveys the basic stochastic approach for estimating prices indexes and shows how this approach can be justified by recourse to economic theory. This naturally leads to an analogous motivation for stochastic quantity indexes. The discussion is then extended to more conventional economic methods of estimating price indexes using parametric representations of the cost function. A synthesis of these two approaches is attempted, using the CES cost function, which has the advantage of the direct estimation of the price index, as in the basic stochastic approach, and the benefits of a firm economic foundation.

Chapter 6 provides a brief summary and conclusion of the main points of the thesis by emphasizing the primary results. With an eye towards future research, we discuss how some of the results in this thesis may be applied, extended or replicated.

1.1. Author's Note

As can be seen from the outline of the chapters in the previous section there is some relationship between the chapters of this thesis in terms of the focus on issues of quality change and new and disappearing goods. However, the similarity should not be overemphasized. Each of the Chapters 2 to 5 were written independently and constitute self-contained documents. This means that there is not a natural flow, as in a book, from one chapter to another and has led to a small amount of repetition, which it is hoped will not distract the reader. Rather than focusing on a particular issue in great detail the aim of this thesis was to make a contribution to economic measurement in the specific areas where it was thought a contribution could be made and needed to be made. In this sense the chapters of this thesis constitute a series of essays or papers on various measurement issues. It is hoped that this meets the expectations of the reader.

CHAPTER 2

2. Quality Change and New and Disappearing Goods: A Survey and Discussion of the Measurement Challenges

Abstract*

How to adjust price indexes for changes in the quality and availability of goods is a major problem in economic measurement. This chapter provides a review and discussion of the conceptual and practical difficulties in this area within an economic framework. We look at different ways of approaching the problem and focus on the methods used by statistical agencies. The chapter also reviews alternative approaches to accounting for quality change and new and disappearing goods; hedonic methods, reservation price estimation, and an approach using the Constant Elasticity of Substitution cost function. The advantages and disadvantages of these methods are discussed. Particular emphasis is given to the discussion of hedonic methods as this technique is increasingly being adopted by statistical agencies. This survey of the literature emphasises that we have made much progress in solidifying the basis on which quality adjustments are made.

* I would like to especially thank Lorraine Ivancic, Kevin Fox and Robert Hill for their comments on this chapter.

Henry Sidgwick, writing in 1887, emphasising the importance of accounting for changes in the quality of products,

"And we have to deal similarly with a further source of inexactness introduced into this calculation by the progress of the industrial arts. The products of industry keep changing in quality; before we can say whether any kind of thing – e.g. cloth – has really grown cheaper or dearer we must compare the quality – that is, the degree of utility – of the article produced at the beginning of the period with that of the more recent ware."

Sidgwick (1887, p. 63).

2.1. Introduction: A Challenge for Measurement

Shoppers in developed countries are faced with a dizzying array of products, many of which may not have even been imagined a decade or so ago. Moreover change is often so rapid that it seems impossible to know exactly what type of products will be available in years to come.

These rapid changes in the selection and performance of products are likely to be advantageous for consumers. However, they create great difficulties in measuring aggregate price and quantity change. Indeed one of the most pressing problems in the measurement of price change is the problem of adequately accounting for the changes in the quality and availability of different products over time. It is easy to see how these problems manifest themselves. If a product changes in terms of its performance then this is likely to be reflected in its price. However, in determining changes in consumers' *real consumption* it is essential to include the increase in the performance of the good as a *quantity* or *real* change rather than a *price* change because consumers are materially better off when the good increases in quality (or worse off when quality falls). It is for this reason that we want to separate the quality change effect, that is the change in price that is related to changes in the performance of the product, from the *pure price* effect.

The task of adequately accounting for quality change and new and disappearing goods is far from easy. There have been numerous studies examining the effects of these factors on price indexes. Perhaps the most notable was that of the Boskin Commission (Boskin et al., 1997), which estimated the bias due to the effect of quality change and new and disappearing goods to be of the order of 0.6 percentage points for the US Consumer Price Index (CPI) in 1996.[1] While the Boskin Commission is the most recent and comprehensive review of the bias in official indexes from quality change and new and disappearing goods, there have also been others. Shapiro and Wilcox (1996), drawing strongly on Lebow, Roberts and Stockton (1994), estimated that the US CPI was overstated by around 0.2 percentage points annually due to new goods. For quality change, which they famously called the "…house-to-house combat of price measurement" (Shapiro and Wilcox, 1996, p. 40), they estimated that the upward bias was around 0.25 percentage points annually. These estimates of the bias from the failure to adequately reflect quality change and new and disappearing goods in official indexes provide strong motivation for research in this area. We outline the structure of the chapter below.

a. Outline

This chapter provides an introduction to the major topic of this thesis, quality change and new and disappearing goods, and undertakes a wide-ranging survey of various aspects of the issue. However, given that this area is so large and much has been written, especially in recent times, the review is inevitably partial. In the remainder of this section we outline the measurement problem more rigorously and discuss different ways of conceptualising quality change and changes in the availability of goods. In Section 2.2 we firmly ground the discussion in practical issues by outlining how the quality change and new and disappearing goods problem arises, and is dealt with, in the context of statistical agencies. What becomes apparent from this discussion is some of the limitations of the

[1] The Boskin Commission was convened as a result of a statement by the Governor of the Federal Reserve Alan Greenspan (Greenspan, 1997) that he believed the US CPI was overstated by between 0.5 and 1.5 percentage points annually.

conventional approaches to this issue. This leads us to consider alternatives in the later sections. In Section 2.3 hedonic methods are discussed. The discussion touches on the economic theory of hedonic models, how various hedonic approaches can be applied to derive price indexes, and some of the advantages and disadvantages of hedonic methods. Section 2.4 discusses a very general approach to new and disappearing goods, that of estimating reservation prices for goods prior to their introduction or post their disappearance. The final method, which we briefly discuss in Section 2.5, is the relatively new Constant Elasticity of Substitution (CES) cost function approach. Section 2.6 concludes the discussion and outlines the relationship between this chapter and later chapters in the thesis.

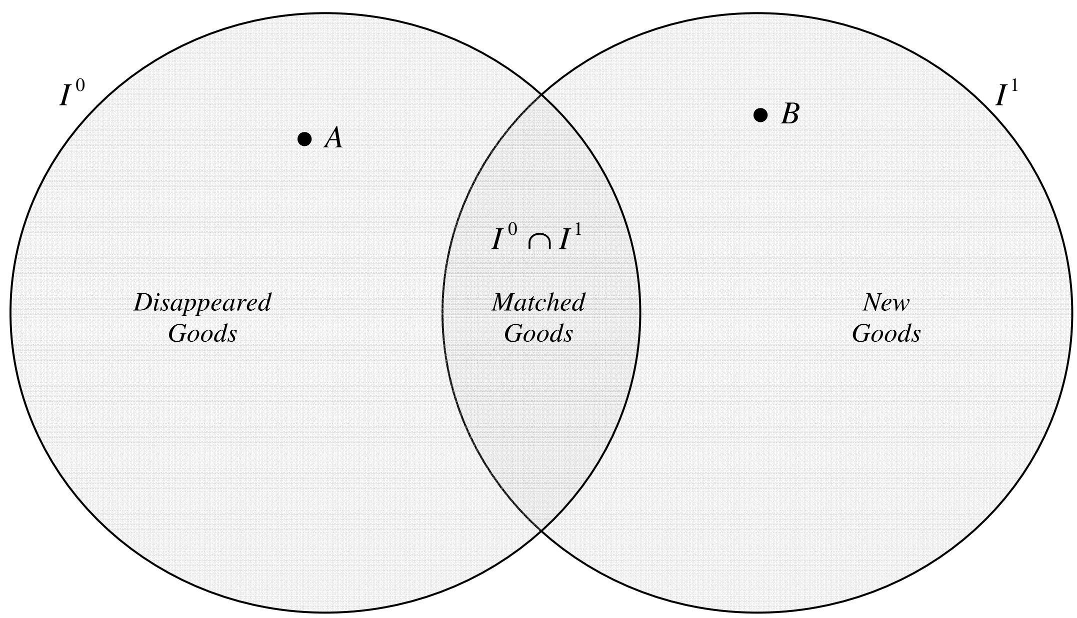
2.1.1. The Quality Change and New and Disappearing Goods Problem

One of the features of modern dynamic market economies is that new goods arrive on the market, and old goods disappear from the market, at great rapidity. While these new goods may be similar to the existing goods, often the changes are more substantial. New products may provide services in more efficient ways than older products or different services altogether. This poses a measurement problem as in the economic cost-of-living approach to price measurement our goal is to measure the effect of these changes, along with price changes, on the cost of living in one period compared with another. We seek an answer to the question, what is the cost in the current period compared with the previous period or obtaining some level of utility, given the quality, availability and price of the goods in these periods (Hausman, 1999, p. 188)? This cost-of-living approach is well established in the literature as the economically ideal way of conceptualising and measuring price changes, and it is the approach adopted here.

The economic approach provides a very useful approach to conceptualising quality change and new and disappearing goods. Interestingly, while there are other conceptual frameworks for price index construction they are likely to deal with quality change and new and disappearing goods in similar ways to the cost-of-living approach. For example, the National Research Council (Schultze and Mackie, 2002, p.109) writes, "The manner in which quality change and new goods problems arise depends to some extent on the index's underlying conceptual structure whether a cost-of-goods index (COGI) or a cost-of-living index (COLI) though procedures for dealing with these problems are essentially the same in both cases."

To make the measurement challenge concrete let us consider the situation diagrammatically. In Figure 2.1 we depict the set of goods available in each period. We use I^t, for periods $t = 0,1$, to denote those goods which were available and purchased by the consumer in the relevant time period.

Figure 2.1. The Dynamic Universe of Goods



We can see that the goods fit into three categories; *Disappeared Goods*, *New Goods* and *Matched Goods*. Given the cost-of-living framework, the crux of the measurement problem is how to account for the changes in the choice set and the relative pricing of these choice sets. Suppose we have a good *A* (depicted in Figure 2.1) which is available in period 0 but not period 1, and a good *B* which is available in period 1 but not period 0. For illustrative purposes let us further suppose that goods *A* and *B* are different models of *personal computer*. Then, in order to calculate the cost-of-living index we need to compare the ‘quality’ or the ‘utility yielding ability’ of these two goods. The heart of the measurement problem is finding a convincing basis on which to make this comparison so that the changes in the quality and availability of goods can be reflected in the cost-of-living index.

There are two different ways of looking at this measurement problem. The most general way of conceptualising this problem is simply to think of the goods *A* and *B* as different utility-yielding commodities and the changes from I^0 to I^1 as simply being changes in the availability of these commodities. Pollak (1989, p. 153) called this the “goods” approach.

Another way of viewing the problem is to think of the appearance of *B* and disappearance of *A* as representing changes in the ‘quality’ of the provision of some underlying service, in the case of *A* and *B* this service is *computing*. This approach views consumers as deriving utility from various generally defined services such as; *computing*, *transport*, and *entertainment*, and we regard different goods as inputs into the consumer’s production of these services.[2] This way of conceptualising the measurement problem allows us to compare goods *A* and *B* at a more detailed level. In the “goods” approach we compare *A* and *B* by determining the difference between them in utility yielding ability. However, in this second approach we can compare *A* and *B* on the basis of their ability to provide one of the desired services. An important and very much analogous idea is that instead of regarding the *goods* as being inputs into the production function for these services we could regard the *characteristics*, or essential attributes, of the goods as the important features that enter the production function (Lancaster, 1966). This is what Pollak (1989, p. 153) calls the “characteristics” approach. This approach is particularly useful as it gives a basis upon which to compare goods *A* and *B*. We can ask, what are the characteristics of goods *A* and *B* and how do they compare? Another reason why this is a particularly useful approach is that the characteristics of goods are likely to be relatively stable over time.

[2] In this conception there are no wholly new goods, just different ways of providing the basic services that consumers desire.

The appearance and disappearance of goods is clearly problematic for the construction of price indexes. In the following section we show how the basic cost-of-living approach must be broadened to include these effects. Adopting different conceptions of the problem, the "goods" and "characteristics" approaches, we outline how we can derive indexes which have a meaningful relationship to the ideal cost-of-living index.

2.1.2. The Cost-of-Living Index

In this section we consider the measurement of the cost-of-living index under various conceptual frameworks incorporating quality change and new and disappearing goods. We will begin the discussion by considering the conventional consumer cost minimisation problem without changes in the quality or availability of goods. This establishes a basis for contrasting with alternative formulations which incorporate these effects.

a. The Conventional Cost-of-Living Index

We use p_i^t and x_i^t to denote the price and quantity of good i for period t, p^t and x^t represent the respective vectors, $U(.)$ is the utility function with $\overline{U}$ representing a specific utility level. The consumer's cost minimisation problem is written below with the standard assumption that the set of available goods is fixed across time, $I^0 = I^1 = I$.

$$C(p^t, \overline{U}) = \min{}_x \left\{ \sum_{i \in I} p_i^t x_i \ : U(x) \geq \overline{U} \right\}, \qquad t = 0,1 \tag{2.1}$$

The well-known Konus (1924) cost-of-living index is the ratio of cost functions. There are two main approaches to the problem of measuring the cost-of-living index. The first is the derivation of general bounds on various versions of this index and the second is the calculation of exact indexes, under assumptions on the form of the cost function.

Turning first to bounds, as is well known the Laspeyres (P^L) and Paasche (P^P) Price Indexes provide bounds on the change in the cost-of-living index from various perspectives. These are shown below.[3]

$$\frac{C(p^1,U^0)}{C(p^0,U^0)} \leq \frac{\sum_{i\in I} p_i^1 x_i^0}{\sum_{i\in I} p_i^0 x_i^0} \equiv P_{1,0}^L \qquad (2.2)$$

$$\frac{C(p^1,U^1)}{C(p^0,U^1)} \geq \frac{\sum_{i\in I} p_i^1 x_i^1}{\sum_{i\in I} p_i^0 x_i^1} \equiv P_{1,0}^P \qquad (2.3)$$

These indexes are particularly useful as they are easily calculated and have a known relationship with a conceptually ideal measure of price change. However, the problem with these indexes is that they are biased.

An alternative approach to index numbers, exemplified by Diewert (1976), is the exact approach. In this approach we hypothesise a functional form for the cost function in (2.1) and use economic theory to derive the form of the price index from observable information. In a very important paper Diewert (1976) showed that the Fisher Price Index (P^F), the geometric mean of the Laspeyres and Paasche Indexes, is exact for a homogeneous quadratic functional form.

$$P_{1,0}^F \equiv (P_{1,0}^L \times P_{1,0}^P)^{\frac{1}{2}} \qquad (2.4)$$

[3] Note that if instead of looking at an input price index, such as the cost-of-living index, we focused on an output price index, such as for a revenue maximising firm, then the Laspeyres Price Index would provide a lower bound while the Paasche Price Index would provide an upper bound.

Diewert (1976) termed the Fisher Price Index *superlative* because this homogeneous quadratic functional form is flexible in the sense that it can provide an arbitrary approximation to a linearly homogeneous function to the second order. As a result the Fisher Price Index, unlike the Laspeyres and Paasche Indexes, can account for substitution and hence is not biased.

b. The Cost-of-Living Index Under the "Goods" Approach

The most general formulation of the quality change and new and disappearing goods problem is that of the "goods" approach, where products are regarded as separate entities. This is a natural extension of the conventional framework above where we generalise the cost function in (2.1) by allowing for changes in the set of goods, $I^0 \neq I^1$.

$$C(p^t, \overline{U} \mid I^t) = \min{}_x \left\{ \sum_{i \in I^t} p_i^t x_i \ : U(x) \geq \overline{U} \right\}, \qquad t = 0,1 \tag{2.5}$$

The cost-of-living index in this framework is indeed very general, representing changes in prices as well as changes in the availability of goods. However, in general, it is not possible to calculate either an upper or lower bound on this changing-domain-of-goods cost-of-living index (Pollak, 1989, p. 165). The bounds in the conventional cost-of-living index worked because we were able to compare the cost of purchasing a fixed bundle of goods, which gives a constant level of utility, across time. However, this approach is not possible in the case where the domain of goods is changing. There may be some goods which are missing in period 1 relative to period 0. Then in endeavouring to calculate the Laspeyres Index we cannot ensure that the consumer reaches the base utility level as they do not have access to all period 0 goods. An analogous problem arises in attempting to calculate the Paasche Index.

The problem of missing prices seems to pose insurmountable problems to index construction in the sense of calculating anything that has a meaningful relation to the ideal cost-of-living index. However, as we will see in Section 2.5, for a particular functional form for the cost function in (2.5) it is actually possible to calculate the *exact* cost-of-living index even when the domain of goods is changing. We will postpone further discussion of this case until then. However, generally speaking the changing domain of goods cost-of-living index is too general an approach to the new and disappearing goods problem and except in very special cases very little can be said about it.

c. The Cost-of-Living Index Under the "Characteristics" Approach

The situation is a little more encouraging in the case of the "characteristics" approach. The fundamental difference between the "characteristics" approach and the "goods" approach is that in the former the goods are thought of as bundles of some common characteristics. These characteristics are the important utility-yielding features of goods which consumers desire. To formalise this notion let us re-specify the cost function so that consumption choices depend on characteristics but noting that it is still goods (i.e. bundles of characteristics) which is the consumer's choice variable.

$$C(p^t, \overline{U} \mid I^t, Z^t) = \min_{x_{z^t}} \left\{ \sum_{i \in I^t} p_i^t x_{i,z_i^t} : U(x_{z^t}) \geq \overline{U} \right\}, \quad t = 0,1 \tag{2.6}$$

Here we have added two new pieces of notation. Firstly, we have denoted the index set of *characteristics* by Z^t. This represents the set of characteristics available in period t, and is the counterpart of I^t in characteristics-space. Secondly, we have introduced the vector of quality characteristics represented by z_i^t for each good i in period t. We have indicated the dependence of the choice of consumption quantities on the characteristics by including the characteristics vector as a subscript. Note that the consumer takes the characteristics of each product as given and chooses products based upon both a good's characteristics and its price.

In this framework there is more likelihood that we can calculate indexes of economic interest, particularly bounds on the cost-of-living index. To see this let us make a natural assumption. First, let us suppose that the characteristics of the goods are measured in such a way that more of a characteristic is better. Then let us assume that a unit of good i is weakly preferred to a unit of good j if $z_i \geq z_j$, which means that good i has no less of each and every characteristic than good j.[4] In this case we are able to justify, under certain conditions, the following Modified Laspeyres Index ($P_{1,0}^{ML}$) upper bound on the cost-of-living index between periods 0 and 1. Note for this bound we have allowed the set of available goods to change but require the set of characteristics to be fixed at those available in period 0.

$$\frac{C(p^1, U^0 \mid I^1, Z^0)}{C(p^0, U^0 \mid I^0, Z^0)} \leq \left(\frac{\sum_{i \in I^0, i \in I^1} p_i^1 x_{i,z_i^0}^0 + \sum_{i \in I^0, i \notin I^1} \tilde{p}_{i,\tilde{z}_i^0}^1 x_{i,z_i^0}^0}{\sum_{i \in I^0} p_i^0 x_{i,z_i^0}^0} \right) \equiv P_{1,0}^{ML} \tag{2.7}$$

This bound includes some unusual notation which requires explanation. As we discussed above the problem with calculating a conventional Laspeyres Index when there was a changing domain of goods was that we had no way of being sure what bundle obtained period 0 utility other than the period 0 bundle. However, the period 0 bundle of goods is not available in period 1. The way that we can get around this problem in the "characteristics" case is to use the fact that we don't have to match goods but can instead match characteristics. To illustrate the notation and identify what (2.7) represents let us consider an example where we have some good A that was available in period 0 but not in period 1, so the conventional Laspeyres bound in (2.2) cannot be calculated. However, using the dependence of utility on the

[4] This assumption can be thought of as a free disposability condition where consumers can costlessly dispose of any surplus characteristics if they wish.

characteristics of the good, we may be able to find a good *B* in period 1 which dominates *A* with regard to characteristics, $z_B^1 \geq z_A^0$, and hence provides more characteristics-utility than good *A* under the assumption made above. Then, instead of including the price of good *A* in the index in period 1, which does not exist, we can enter the price of good *B*. In (2.7) this process is denoted by the inclusion of the price, $\tilde{p}_{i,z_i^0}^1$ in the index for those items which existed in period 0 but are missing in period 1. Essentially we are finding an upper bound on the cost of obtaining the characteristics that were available in the base period.

It is possible that there may be no goods in period 1 which characteristics-dominate the disappeared goods from period 0. In this case we cannot calculate the bound in (2.7). However, this case appears unlikely as in practice most changes in characteristics over time tend to constitute increases or improvements in characteristics. Indeed the good *A* which disappeared in period 1 is likely to have disappeared because it was superseded by a superior quality product yielding greater characteristics-utility.

If we follow analogous reasoning to that in the previous paragraphs we can suggest a Modified Paasche Index ($P_{1,0}^{ML}$) lower bound on the relevant Konus cost-of-living index.

$$\frac{C(p^1, U^1 \mid I^1, Z^1)}{C(p^0, U^1 \mid I^0, Z^1)} \geq \left(\frac{\sum_{i \in I^1} p_i^1 x_{i,z_i^1}^1}{\sum_{i \in I^1, i \in I^0} p_i^0 x_{i,z_i^1}^1 + \sum_{i \in I^1 . i \notin I^0} \tilde{p}_{i,\tilde{z}_i^1}^0 x_{i,z_i^1}^1} \right) \equiv P_{1,0}^{MP} \qquad (2.8)$$

To calculate this index we need to find some goods in period 0 which characteristics-dominate those newly available goods in period 1. In (2.8) we denote the prices of these goods by $\tilde{p}_{i,\tilde{z}_i^1}^0$. However, in contrast to the Modified Laspeyres Index, there is likely to be some difficultly in finding base period varieties which characteristic-dominate the new varieties because, as argued above, changes in varieties over time usually lead to increases in characteristics rather than decreases. In this case (2.8) may not be calculable.

With the use of the characteristics approach we have shown that it is possible to derive bounds on the cost-of-living index which under some circumstances are calculable. While bounds may be derivable it does not seem possible to calculate exact price indexes in this case without putting a great deal more structure on the relationship between characteristics and utility. In the next section we discuss another application of the "characteristics" approach under which exact indexes may be justified.

(i). Extending the "Characteristics" Approach: the Price-Characteristics Function

One natural extension of the "characteristics" approach is to regard market prices as a function of the characteristics of the goods. This is known as the *hedonic hypothesis* and will be discussed in greater detail in Section 2.3. Here we suppose that a price-characteristics function exists, and we explore the consequences of this for calculating bounding and exact price indexes. We rewrite the cost function to reflect this price-characteristics function, $p^t(z)$.

$$C(p^t(z),\overline{U} \mid I^t, Z^t) = \min{}_x \left\{ \sum_{i \in I^t} p_i^t(z_i^t) x_{i,z_i^t} : U(x_{i,z_i^t}) \geq \overline{U} \right\}, \quad t = 0,1 \qquad (2.9)$$

With this cost function we have yet more possibilities for indexes which approximate the cost-of-living index. If the price-characteristics function is known then the following Hedonic Laspeyres ($P_{1,0}^{HL}$) and Hedonic Paasche ($P_{1,0}^{HP}$) bounds on the cost-of-living index seem reasonable. These can be justified on the basis that we can use the price-characteristics function to calculate the price of a bundle of characteristics in a period even when it was not available.[5]

$$\frac{C(p^1(z),U^0 \mid I^1,Z^0)}{C(p^0(z),U^0 \mid I^0,Z^0)} \leq \left(\frac{\sum_{i\in I^0, i\in I^1} p_i^1 x_{i,z_i^0}^0 + \sum_{i\in I^0, i\notin I^1} p_i^1(z_i^0) x_{i,z_i^0}^0}{\sum_{i\in I^0} p_i^0 x_{i,z_i^0}^0} \right) \equiv P_{1,0}^{HL} \tag{2.10}$$

$$\frac{C(p^1(z),U^1 \mid I^1,Z^1)}{C(p^0(z),U^1 \mid I^0,Z^1)} \geq \left(\frac{\sum_{i\in I^1} p_i^1 x_{i,z_i^1}^1}{\sum_{i\in I^1, i\in I^0} p_i^0 x_{i,z_i^1}^1 + \sum_{i\in I^1, i\notin I^0} p_i^0(z_i^1) x_{i,z_i^1}^1} \right) \equiv P_{1,0}^{HP} \tag{2.11}$$

We could naturally take the geometric mean of these two indexes to obtain the Fisher Hedonic Price Index. It could be argued that this index is exact in the sense that it could be derived from a particular parametric representation of the cost function with prices replaced by the price-characteristics function.

An important caveat to this discussion is that in practice we do not know $p^t(z)$ as only points along this function are observable in the market. However, as we discuss later in Section 2.3, we may be able to estimate the function adequately for some goods. Given $\hat{p}^t(z)$, an estimate of $p^t(z)$, we could then calculate approximations to the indexes in (2.10) and (2.11).

As a final point note that the bounds derived in (2.7) and (2.8) will not be as tight as those in (2.10) and (2.11). The earlier bounds can be considered non-parametric hedonic bounds on the prices of bundles of characteristics.

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[5] Pakes (2003, pp. 1594-1595, App.) formally derives the conditions for these bounds to hold. However, if we assume that every possible combination of characteristics is available each period, but that each combination is not necessarily consumed each period, then this proves sufficient to justify these bounds.

2.1.3. Summary

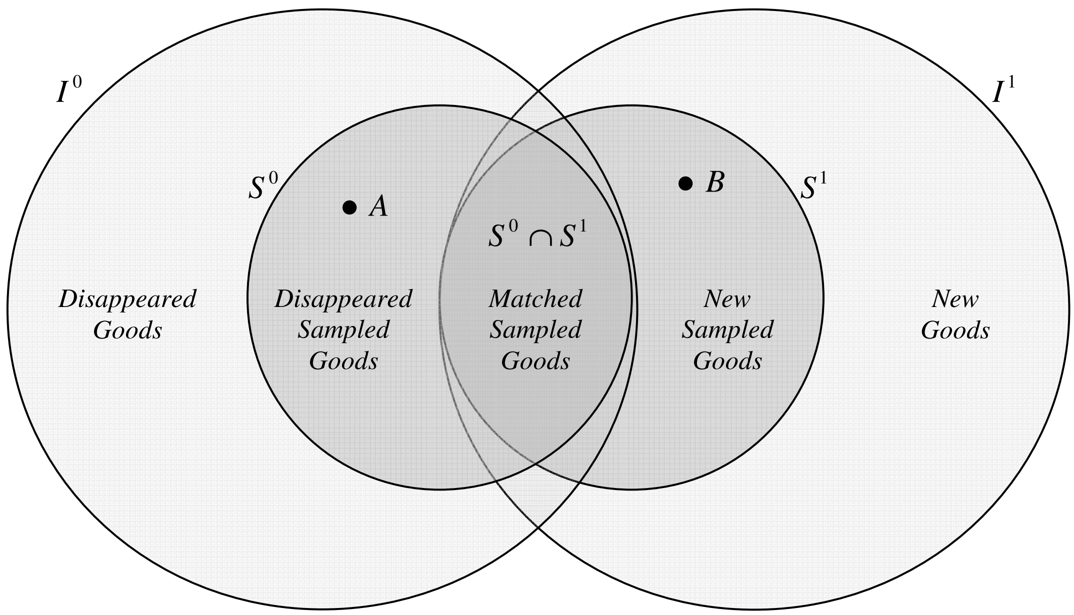
In this section we have outlined various ways of conceptualising the quality change and new and disappearing goods problem. Using these different methods we were able to approach the problem in different ways. The most general "goods" approach, which regarded goods as different utility-providing entities, proved difficult in terms of calculating bounds on the cost-of-living index. In contrast the "characteristics" approach, where goods are linked by the features they possess, proved more amenable to the calculation of bounds. In later sections we will adopt different versions of these approaches in endeavouring to determine the cost-of-living index. First, however, let us turn to more practical aspects of the discussion and look at the problem from a statistical agency perspective.

2.2. A Statistical Agency Perspective

The preceding discussion was primarily concerned with different ways of viewing the quality change and new and disappearing goods problem. In this section we ground the discussion in the context of statistical agency practice. The most important difference is that statistical agencies monitor only a sample of goods rather than the complete population of goods.

The basic approach used by most statistical agencies is what is called the *matched-model* method. This involves pricing the same item repetitively through time. This item will usually be described in detail on the pricing sheet of the survey statistician. As the price quote is collected at the same geographical location in the same store usually at the same time of the month this procedure controls very effectively for any differences in the services associated with the purchase of the product. As Triplett (1990, p. 36) notes the main advantage of this approach is that it, "…assures that any difference between periods reflects solely price change rather than a change in what was bought." While the matched model approach is generally desirable there are of course problems in maintaining a fixed sample if there is turnover in the available set of products. While every effort is often made to sample the same good across time often goods will disappear from the market and new replacement goods will have to be introduced to the sample. To clarify this we modify Figure 2.1 to reflect the fact that each period the statistical agency has available a sample of prices from the two periods. We denote the index set of goods sampled by S^t where $S^t \subseteq I^t$ $t = 0,1$, as depicted in Figure 2.2.

Figure 2.2. The Statistical Agency Sample of Goods



It can be seen from Figure 2.2 that the statistical agency faces a problem very much analogous to that discussed above for the population of goods where we have non-matching sets of *sampled products* rather differences in the product universes. We will discuss the difficulties that this raises in a moment. We first turn to another problematic area in the context of sampling, the issue of the representativity of the sampled goods.

a. Representativity

An important issue with regard to sampling is that of obtaining and maintaining a representative set of goods and services. The aim of sampling is to obtain a fair representation of the universe of products in each period.[6] This raises important issues about manipulating the sample so that it adequately reflects trends in the population of items over time. For example, we would like to ensure that it is not biased towards say the low technology spectrum of a product range or that it does not over represent established brands compared with new brands. A difficulty often arises in practice when new types of goods appear. Statistical agencies are often slow to incorporate these goods into the price index. A notable example of this was the delayed introduction of *cellular phones* in the US CPI. As noted in Hausman (1999, p. 188) cellular phones were not introduced into the US CPI until 1998 when there were over 55 million users, although they were available in the US as far back as 1983. This causes a concern, as Hausman (1999) stresses, because new goods often fall in price after their introduction. Hausman (1999, p. 190) estimated that cellular telephone prices had fallen by over 50 percent prior to their introduction to the US CPI in 1998.

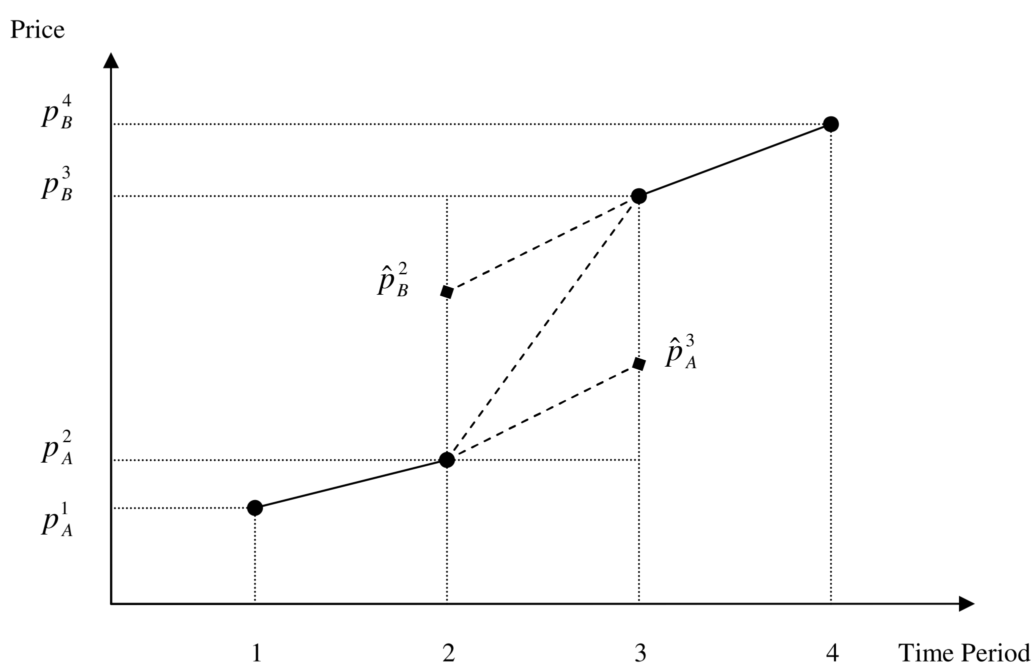
Note that some non-representativity in the statistical agency sample may arise as a by product of the statistical agencies desire to match the prices of the same goods over time. If the goods are repetitively sampled period after period then this is likely to lead to a gradual outdating of the sample. These issues are extremely important and call for the careful monitoring and design of sampling frameworks, however, we will not discuss them further in this chapter as the solutions are relatively straightforward, though perhaps costly; introduce new items more speedily into the index and monitor the market shares of sampled items.

[6] Lane (2000) discusses the sampling methods used in the US CPI in detail.

b. Non-Matched Goods

The other problem that arises is that some goods sampled in period 0 are not available in period 1 or equivalently the model or variety of the good available in period 0 may be of different quality compared with the variety in period 1. This is very much the mirror image of the problem that arose in the case of the universe of products. Here we need to consider how we should treat the non-matched sampled goods? Indeed, this is how the issue of quality change and new and disappearing goods manifests itself in the statistical agency context. This is the primary issue of interest in this chapter. To illustrate our discussion of possible solutions we will consider a simple example of the problem shown in Figure 2.3. In periods 1 and 2 we record the price of good *A* while in periods 3 and 4, due to the disappearance of good *A*, the price of good *B* is recorded. The primary question that the statistical agency faces is, how should the available price information be used to calculate an index from periods 2 to 3?

Figure 2.3. An Illustration of the Problem



This question is the basic question of how to adjust for quality change and new and disappearing goods. In the following section we discuss the methods used by statistical agencies for dealing with this problem.

2.2.1. A Survey of Statistical Agency Methods

There are a number of techniques used by statistical agencies to adjust for missing prices. These are surveyed in this section where we acknowledge the contribution made by others in this area (Moulton and Moses, 1997; Armknecht and Maitland-Smith, 1999; Hoven, 1999; Greenlees, 2000; ILO, 2004). We start by outlining a conceptual framework from economic theory for thinking about adjusting for quality change.

a. A Conceptual Framework for Quality Change

The methods used by statistical agencies provide practical solutions to very difficult problems. They involve various ways of comparing the prices of goods *A* and *B* in Figure 2.2. In terms of Figure 2.3, we can think of the quality adjustment procedure as being an effort to estimate the notional price of either good *A* in period 3, $\hat{p}_A^3$, or good *B* in period 2, $\hat{p}_B^2$. The methods used in this regard are often based upon our intuition of what is appropriate. Here we endeavour to better understand the economic assumptions and implications of these methods by developing an economic framework for thinking about quality change.

Our basic framework is centred on the cost-of-living approach to economic measurement which, when there is quality change or new and disappearing goods, requires us to compare the utility giving capabilities of the relevant goods. One approach to comparing the quality of two goods within a utility-based framework is to focus on how consumers make choices between goods at the margin. We can ask, what is the marginal contribution to utility of good *A* compared with good *B*? This approach was noted in passing by the National Research Council (Schultze and Mackie, 2002, p. 110). In this case our measure of quality of a good is its marginal utility. Then in order to compare the quality of goods *A* and *B* we can use the well known first-order condition for the utility maximising consumer. For period t the consumer equates relative prices to relative marginal utilities which gives the condition below.

$$p_B^t = p_A^t \frac{U_B^t}{U_A^t}, \qquad U_i^t \equiv \frac{\partial U(x^t)}{\partial x_i^t},\ i = A, B \tag{2.12}$$

This relation will prove particularly useful in the following discussion as it allows us to evaluate the economic assumptions that are being made in using a particular quality adjustment method. The ratio of marginal utilities is an intuitively appealing measure of the quality difference between two goods as it can be seen that, in equilibrium, quality-adjusted prices, $p_B^t / U_B^t = p_A^t / U_A^t$, are equal. However, note that the marginal utility has a somewhat unusual feature for a measure of the quality of a good, it is time dependent. This is because, in general, the marginal utility of a good depends on the consumption of that good as well as the consumption of all other goods.[7] An advantage of using marginal utility to measure quality is that it can be seen that relative qualities are directly reflected in relative prices. We do not require any information about the quantities of the goods consumed to judge quality.

From Figure 2.3 the goal of the statistical agency is to estimate the price change for good *A* or *B* in moving from period 2 to period 3. By using (2.12) for $t = 3$, where we use the *notional* price and marginal utility of good *A* in period 3, $\hat{p}_A^3$ and $\hat{U}_A^3$, and divide both sides by p_A^2, we arrive at the equation below.[8]

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[7] The observation that the 'quality' of a good can change over time despite the constancy of the characteristics of the good was made by Fisher and Shell (1972, p. 26) and more recently Hoven (1999, p. 16). Fisher and Shell (1972, p. 26) write, "…in general, the price adjustment [for quality change] which must be made will depend on all prices and purchase of all commodities and not simply on the physical characteristics of the quality change."

[8] We can construct a similar condition from the perspective of good *B* where marginal utilities are compared in period 2.

$$\frac{p_B^3}{p_A^2} = \frac{\hat{p}_A^3}{p_A^2}\frac{U_B^3}{\hat{U}_A^3} \tag{2.13}$$

This is a useful way of thinking about the change from pricing *A* to *B* in that the price change on the LHS is decomposed on the RHS into a pure-price change, $\hat{p}_A^3 / p_A^2$, and a quality change, $U_B^3 / \hat{U}_A^3$. With this framework in mind we now turn to the various quality adjustment methods used by statistical agencies.

b. Direct Substitution

A common method for dealing with a missing item is simply to try and find a replacement that is identical to the original item, or at least identical in the important dimensions. In terms of Figure 2.3 this means selecting a replacement product *B* which is very similar to the original product *A*.[9] Using this method the prices, p_A^2 and p_B^3 can be directly compared to calculate the pure price change. In terms of economic theory and equation (2.12) the implicit assumption made is that the marginal utility of the two goods is the same, $U_B^3 = \hat{U}_A^3$.

This approach seems acceptable in cases where the replacement item is, for all practical purposes, identical. In the cases where the replacement item is not of exactly the same quality as the original item then the comparison of prices over time introduces an error, of unknown sign and magnitude, from not comparing like goods. The error arises from a faulty apportioning of the price difference, p_B^3 / p_A^2, in (2.13) between quality change and pure price change.

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[9] Note, however, that the systematic selection of replacement items which are as similar as possible to the disappeared item is problematic. The disappearance of the original item is likely due to the fact that the item is of relatively low quality and unpopular. If the next most similar item is selected then it is also likely to have these characteristics. This has prompted some to call this the practice of replacing an obsolete item with the next most obsolete item (ILO, 2004, p. 137; Schultze and Mackie, 2002, p. 136, n. 33).

c. Substitution with Overlap

Sometimes a price for the replacement good may have been observed in the previous period. For example, we may have recorded the price of good *B* in period 2 of Figure 2.3. In this case we calculate the price movement based upon the price movement of good *B* from periods 2 to 3. This is a quality adjustment method in the sense that we replace good *A* with good *B* by using the price ratios of the goods in period 2, when they are both observed, as an indicator of their relative quality. This is certainly the precisely correct approach in our economic framework as can be seen in equation (2.12) for $t = 2$ the price ratio for the goods *A* and *B* is equal to the ratio of marginal utilities, our measure of relative qualities.

In practice, however, the overlap method is likely to be only applicable in rare instances where this additional price information is available. The overlap method is routinely used by statistical agencies when samples of goods and services are being rotated into and out of the index to preserve representativity.

(i). Evaluating the Overlap Method

In the recently published ILO Manual (2004) on CPIs there is extensive discussion of the various quality adjustment methods. One of the most interesting and important points made by the Manual is the scepticism with which the Overlap method is regarded (ILO, 2004, p. 107).[10] The argument here is that at the time when the overlap price is recorded either the old-disappearing good *A* or the new-appearing good *B* may be exhibiting unusual prices. This could be caused, for example, by a run-out sale for good *A*, which is being removed from the market, where the price of *A* is relatively low but a large quantity of the product is being sold. The implicit argument of ILO (2004) is that in this case the ratio of marginal utilities is not a good measure of relative qualities. One approach then is to modify our measure. We could do this by not looking at quality at the margin but instead compare the contribution of the goods to overall utility. If we multiply both sides of

[10] This method was also criticised in Nordhaus (1998) and Hoven (1999).

(2.12) by x_A^t / x_B^t and rearrange we obtain the equation below which compares *utility shares* of the goods – an appealing measure of quality.

$$\frac{s_B^t}{s_A^t} = \frac{u_B^t}{u_A^t}, \qquad u_i^t \equiv \frac{U_i^t x_i^t}{U^t}, \; s_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i=I^t} p_i^t x_i^t}, \; i = A, B \tag{2.14}$$

This relation is a version of Wold's Identity (Wold, 1944), under the assumption of linear homogeneity of degree one of the utility function, and states that the relative utility shares of the two goods is equal to the relative expenditure share. In this case we look at quality by not just looking at price differences but also looking at quantity differences. If there was a run-out sale for good *A* in the period prior to the disappearance of this product then its price would be relatively low but the quantity sold would be relatively high. From (2.14) these two effects need to be balanced in a comparison of the quality of *A* with *B*. Note that (2.14) gives an intuitively appealing rule for ensuring that the existing and replacement items are of similar quality. If the expenditure shares of the replacement and existing items are similar then they are judged of similar quality. However, applying this quality measure requires more information than when we compare quality at the margin.

d. Direct Quality Adjustment

Another way to incorporate the price of a newly sampled good into the index is to make a direct comparison of the quality difference between the disappeared good and the new good. Suppose we estimate the quality difference between the goods *A* and *B* to be equal to some scalar, ψ. This value can be used to adjust prices accordingly, $\hat{p}_B^2 = p_A^2 \times \psi$. For this adjustment to be correct in terms of our economic framework the quality scalar must be equal to our economic quality estimator, either the ratio of marginal utilities or the ratio of utility shares.

What is interesting from an economic perspective is that we would not expect ψ to be fixed across time.

The main difficulty with this approach is in formulating an adequate estimate of the quality difference. One approach is purely subjective – a commodity specialist is asked to estimate, as best as possible, the perceived quality difference. This approach clearly suffers from a reproducibility problem. Another approach, which will be discussed in detail in Section 2.3, is to use hedonic methods. There are two other methods of direct quality adjustment which have been used by statistical agencies and we discuss these in greater detail.

(i). Estimating the Cost of Quality Change

One approach to estimating the value of a quality change between two goods is to use information on the cost differences of producing the two goods. This cost difference is then used as an estimate of the difference in quality of the products (ILO, 2004, pp. 114-116).

Moulton and Moses (1997, p. 327) report that the use of cost information to assess the value of quality change has most frequently been applied in the US CPI for new and used cars and motor fuel. One version of this approach, which seems to be particularly useful is where manufacturers bring out a new model with a feature included which was previously available as an option. If the cost of the option on the previous model is readily available then this price can be used to estimate the cost of the quality change. This approach has the advantage that it is based upon an actual market-option price. The UK Office of National Statistics (ONS) uses a version of this method where they adjust for the bundling of characteristics by using only a fraction of the option cost. Ball and Allen (2003, p. 2) write,

> In option costing the retail cost associated with a change in specification is obtained from the cost of purchasing the change separately or as an added option. Fifty per cent of this is added to the price of the original model to give a price comparison that is independent of any changes in quality. Fifty per cent of the cost is applied for a number of reasons. In part this is because 100 per cent option cost

> could lead to an over adjustment for quality change, due to the fact that the cost of buying features separately is generally greater than buying them as a package.

Clearly the figure 50 percent is somewhat arbitrary. Why not 60 percent? However, it does seem valid to adjust for bundling in some way (ILO, 2004, p. 115).

Another version of this approach is to obtain information on the production cost of any changes in specifications from the producers of the good. This is most appropriate in the context of Producer Price Indexes rather than CPIs. In the latter we are interested in the effect of the change on consumers' utility rather than producers' cost. For this reason, when using this method in the context of the CPI, we should also adjustment the cost of the changes in specification by the producer's markup as well as any indirect taxes (Greenlees, 2000, p. 62).

Triplett (1990, pp. 216-218) gives some examples of the difficulties involved in using this approach to assess quality change for automobiles. Firstly, obtaining the required cost information is often very difficult or impossible. It may not be possible to separate the cost of specific changes to the vehicle from overall costs. Furthermore, producers may regard the information as proprietary or confidential (Armknecht and Maitland-Smith, 1999, p. 19). Secondly, the manufacturer may not be completely honest or may overestimate the costs associated with the addition of a feature. Thirdly, significant knowledge may be required to understand the changes and to simply assess whether it has increased or decreased quality.

It is interesting to note that Triplett (1997) has argued that this method may in fact over adjust for quality change. In the case of automobiles Triplett (1997, p. 27) writes,

> …it has long been suspected that the use of manufacturers' cost data for quality adjustment in the CPI tends to over adjust for the value of quality change, particularly in the case of automobiles…. In my experience in the BLS, the auto manufacturers never overlooked quality changes when they submitted costs to the BLS. Rather manufacturers tried to attribute too much price change to quality improvements – I recall one auto manufacturer's contention that removing the 90

and 100 numerals from a speedometer ought to qualify as improved quality in an automobile.

While using cost information to estimate changes in quality may be useful in some areas it is unlikely to be able to deal adequately with quality change more generally as the required information will not always be readily available.

(ii). Quantity-Quality Adjustment

One common form of product change is when the package size of a good changes. A natural approach, and one that is often used in practice, is to account for this type of quality change by adjusting the price by the scalar of the package size change (Dalén, 1998, p. 8; Greenlees, 2000, p. 61; Schultze and Mackie, 2002, p. 107; ILO, 2004, pp. 113-114). This was, in fact, just the adjustment undertaken in the US CPI when cans of tomato soup went from 16 ounces to 14.5 ounces (Kokoski et al., 2001, p. 5).

In effect this approach amounts to comparing per unit prices of the good and is economically justified in our framework only if the ratio of marginal utilities is equal to the relative package sizes. Whether this is a correct approach depends on whether purchasers value the extra units of the good at the same rate as existing units in which case per unit prices would be fixed. However, we have reason to doubt that per unit prices are indeed fixed as casual observation reveals violations of this in the market with discounts often available for larger package sizes. In this section we propose a model, drawing on hedonic techniques discussed in Section 2.3, for testing this assumption.[11]

1). A Model for Testing the Constancy of Unit Prices

In testing the effects of package size on price let us suppose that the price of a product i is a function of, (a) the particular features of the product, $A_{i,k}$ where there are $k = 1,...,K$ characteristics which in this context are dichotomous

[11] Diewert (2003a) suggested using package size information in hedonic regressions.

attributes, (b) inflation factors, D^t, representing differences in price level over time periods, $t = 1,...,T$, and (c) the package size of the good in a given time period, Q_i^t. In order to apply this general approach we hypothesise the following Cobb-Douglas price-characteristics function, where ε_i^t is a random error term.

$$p_i^t = A_0\left(\prod_{k=1}^{K} A_{i,k}\right)D^t(Q_i^t)^{\alpha}\varepsilon_i^t, \quad i = 1,...,N,\ t = 1,...,T \tag{2.15}$$

Of most interest is the impact of package size on the price. If the parameter $\alpha = 1$ then prices will increase one-for-one with package size and unit prices are constant.[12]

To estimate the model in (2.15) let us first divide price by the package size to obtain an equation in unit prices. We take the logarithms of the equation in order to obtain a linear regression model and use dummy variables for the quality-characteristics and time.

$$\log\left(\frac{p_i^t}{Q_i}\right) = \beta_0 + \sum_{k=1}^{K} \beta_k a_{i,k}^t + \sum_{\tau=2}^{T} \delta_\tau d_\tau^t + \theta\log(Q_i^t) + u_i^t, \quad i = 1,...,N,\ t = 1,...,T \tag{2.16}$$

Here an arbitrary normalisation on the time-dummy for the first period has been imposed. We have a convenient form for the equation where $\theta = \alpha - 1$, so a conventional T-Test on θ will be a test of whether the package size has a statistically significant effect on price. Simply, if package size is uncorrelated with unit-price then θ should equal zero.

[12] Note that in this model we have assumed that the parameters are fixed over time so that the affect of package size on price is not time dependent.

2). Some Empirical Evidence on Package Size and Product Unit Cost

In implementing the model above we use a scanner data set for *Soft Drinks*. The data set is based upon information from over 100 stores in one of the cities of Australia. It includes information on the price of the product as well as a description of the product, brand name and package size. This data set is extremely detailed in recording a very large number of transactions at a weekly frequency over a 15-month period. We use just under 850,000 observations in our regression estimation and focus only on soft drink bottle sizes between 1 and 2 litres (inclusive). Brand variables such as; Coca-Cola (the base brand), Pepsi, and Schweppes, are used along with a range of indicators for the flavour and type of soft drink. A complete list of the variables is given in Table 2.2, Section 2.7.1 of the Appendix, along with the results of the regression. The primary point of interest is the value of the parameter θ. Here we estimate $\hat{\theta} = -0.2021$ ($\hat{\alpha} = 0.7979$) which is highly significant with a T-Statistic of -179.5 and a P-Value of less than 0.0001. This provides strong statistical evidence that $\theta \neq 0$ ($\alpha \neq 1$).

Not only are the results statistically significant but they are also economically significant. Suppose the package size doubles, then the conventional statistical agency approach would be to halve the price. However, if we use our estimate of $\hat{\alpha} = 0.7979$ the correct adjustment is not 0.5 but is equal to $1/2^{\hat{\alpha}} = 1/1.7386 = 0.5752$. Therefore, if the price was equal to \$2.00 then rather than obtaining an adjusted price of \$1.00 we would instead obtain a price of \$1.15. This difference seems large enough to raise concerns about the use of the conventional method. Indeed, given that in general larger package sizes are associated with lower per unit prices, the conventional method will over adjust for quantity-quality change when package sizes increase and lead to a *downward* bias in the index. The bias is reversed when package sizes decrease.

e. Price Imputation Methods

While the methods discussed in the previous section focused on quantifying quality change so that pure price movements could be calculated we could just as easily focus on estimating the pure price movements of the missing goods *implicitly*. There are a whole class of methods that do this, known as imputation methods. Essentially they estimate the pure price movement for one good from the pure price movement of other related goods.[13] In terms of Figure 2.2 this means that statistical agencies use the prices of *Matched Sampled Goods* to represent the price movements of the *New* and *Disappeared Sampled Goods*.

In the context of Figure 2.3, suppose there is some collection of goods $j = 1,...,J$ which are available in both periods 2 and 3. To determine the pure price movement of either good A or good B, between periods 2 and 3, we could use the movement of these goods. We illustrate below by showing how we could estimate the pure price movement for good A by taking an arithmetic average of the goods $j = 1,...,J$ where w_j is a weight which sums to one.[14]

$$\frac{\hat{p}_A^3}{p_A^2} = \sum_{j=1}^{J} w_j \left(\frac{p_j^3}{p_j^2} \right) \tag{2.17}$$

We can think of this approach in two different but equivalent ways. Firstly, imputing a price from goods $j = 1,...,J$ is like estimating the price for the relevant missing good from these items and then using it in the index calculation. In the case of good A we estimate the missing price by,

$$\hat{p}_A^3 = p_A^2 \left(\sum_{j=1}^{J} w_j \left(\frac{p_j^3}{p_j^2} \right) \right) \tag{2.18}$$

Secondly, using this imputations approach is equivalent to dropping the missing good from the index for the relevant period, reapportioning the weight, and

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[13] Armknecht and Maitland-Smith (1999) provide a more detailed discussion of imputation methods than is pursued here.

[14] Note that we could just as easily have used a geometric mean. The example is purely illustrative.

calculating the index over the goods $j=1,...,J$. That is, we calculate a *matched-goods* index between periods with rescaled weights.

Economically we can get some insight into the assumptions that are implicit in the imputations approach if we refer to our economic framework. If we measure the quality of goods using marginal utility then in order for (2.17) and (2.18) to be true, the changes in marginal utility for the good A must be the same as for the weighted average of the J goods. This is certainly true if $\hat{U}_A^3 / U_A^2 = U_j^3 / U_j^2$ $\forall j=1,...,J$.[15] This is an interesting condition. It does not say that the quality or marginal utility of the goods needs to be the same but that the *growth* in marginal utility must be the same.

There are in fact many different versions of this basic method as we can choose different reference sets of goods, $j=1,...,J$, over which to take the imputation average. For example, suppose we have a non-matched price for an apple quote in the CPI. Then we could impute this missing price from the entire CPI or perhaps from the 'food price index', 'fruit price index', 'apple price index', a specific 'regional apple price index', or even another specific quote for apples. One version of this method, called *class-mean imputation*, is extensively used in the US CPI and imputes the price for the missing item from *other missing items which have been matched with a direct replacement*. This method was implemented to guard against the possibility that changes in the model of a good may be associated with changes in prices by producers. This class-mean imputation method is regarded as an "improvement" on the basic imputations approach by the ILO (2004, p. 111).

(i). Evaluating Price Imputations Methods

Various forms of the imputation approach are frequently used to deal with quality change and new and disappearing goods. However, this approach has been

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[15] To see that $\hat{U}_A^3 / U_A^2 = U_j^3 / U_j^2$ is the condition required for this approach to be correct we can use (2.12) for goods *A* and *j* for periods 2 and 3. This gives the following relationship, from which it can be seen that the change in marginal utilities must be equal for the imputations method to be valid, $\hat{p}_A^3 = p_A^2 \times (p_j^3 / p_j^2) \times (\hat{U}_A^3 / U_A^2) \times (U_j^2 / U_j^3)$.

the subject of widespread criticism. The first problem with price imputation methods is that in rapidly changing sectors there may be very few or even no matched-sampled prices between the comparison periods. The case of no comparable prices is obviously extreme but could possibly occur in some product areas, such as for computers, or if the comparison periods were far apart. However, more generally the imputations technique will lack credibility if there are only very few matched sampled prices (i.e. where $S^0 \cap S^1$ in Figure 2.2 is small relative to the set of all sampled prices, $S^0 \cup S^1$).

A second and perhaps more serious problem with the imputations method is that the price movement of continuous varieties may be a faulty proxy for new and disappearing varieties (Hulten, 1997; Triplett, 1997; ILO, 2004). In fact Triplett (1997, pp. 28-34) argues that imputations methods are *biased downward* as retailers and manufacturers often introduce price *increases* when introducing new models. More generally the imputations methods, which as argued above can be viewed as a matched model approach, will be inadequate if there are systematic differences in quality-adjusted prices between those goods that are appearing and disappearing and those that are constant through time. As Triplett (1997, p. 31) writes,

> The matched model method would work fine so long as price changes for unchanged items paralleled the true price changes for models that were changed, in which case we would not need to worry much about quality change. But prices of changed and unchanged models don't move in parallel; quality change is a problem for measuring prices because prices are more likely to change when models change.

In terms of our framework Triplett (1997) seems to be arguing that in fact the marginal utility of new or disappeared products may systematically differ from existing products. This is a particularly serious criticism. However, it is hard to check this empirically because of the difficulty in knowing what the 'true' quality-adjusted prices are. Such concerns mean that the imputations method should be used with caution (ILO, 2004, p. 111).

2.2.2. The Effects of the Adjustment Methods

In an interesting paper Moulton and Moses (1997) discuss and quantify the impact of quality change in the US CPI. In identifying how many items disappear from the sample they indicate that of a total of around 80,000 price quotations recorded each month approximately 4 percent represent replacement items (Moulton and Moses, 1997, p. 323). Over a whole year the problem is more serious. Moulton and Moses (1997, p. 323) write,

> Approximately 30 percent of all sample items scheduled to remain in the sample for the full year (that is, not scheduled for regular sample rotation) need to be replaced at some time during the year.

The fact that around 30 percent of items disappear in a given year means that the adjustment methods for quality change and new goods are extremely important in determining the overall movement of the index.

Moulton and Moses (1997, pp. 335-337) also report the frequency with which the various quality adjustment methods described above are used. Part of their table is reproduced below in Table 2.1.

Table 2.1. Product Replacements by Replacement Method for All Items
Percentage of all Quotes (Percentage of Missing Quotes)

Year	Total	Replacement Methods				
		Direct Substitution	Overlap Method	Imputations Method	Class-Mean Imputation	Direct Quality Adjustment
1983	3.85	1.56 (41)	0.23 (6)	1.74 (45)	…	0.32 (8)
1984	3.95	1.70 (43)	0.23 (6)	1.71 (43)	…	0.30 (8)
1995	3.90	2.54 (65)	0.05 (1)	0.57 (15)	0.32 (8)	0.41 (11)

Source: Moulton and Moses (1997, p. 336, Tab. 4).

The table is particularly interesting as it shows that *Direct Substitution* was generally the most commonly adopted procedure. Furthermore, its use seems to have grown over time. The next most important method was the *Imputations Method* where a price is imputed from a stratum of similar goods. Together these two methods account for over 80% of adjustments. What is interesting to note, however, is the move away from the *Imputations Method* in more recent times. This is likely to be in response to some of the concerns with this approach discussed above. The *Overlap Method* was rarely used as the information was not generally available, while *Direct Quality Adjustment* was used in some cases, most notably for automobiles. The *Class Mean Imputation* method is of more recent vintage.

In another of the important aspects of the Moulton and Moses (1997) paper they present some calculations of the effect of quality adjustments on the overall index movement. These are reproduced in Table 2.2 below.

Table 2.2. Pure Price Effects of Unchanged and Replacement Items

Percentage of all Quotes (Percentage of Relevant Quotes)

Year	All Quotes	Matched Items	Non-Matched Items	Replacement Methods				
				Direct Substitution	Overlap Method	Imputations Method	Class-Mean Imputation	Direct Quality Adjustment
1983	2.99	1.16 (39)	1.83 (61)	0.62 (34)	0.89 (49)	0.12 (7)	…	0.20 (11)
1984	3.40	0.14 (4)	3.26 (96)	1.37 (42)	1.38 (42)	0.16 (5)	…	0.35 (11)
1995	2.16	1.07 (50)	1.09 (50)	0.60 (55)	0.10 (9)	0.02 (2)	0.18 (17)	0.19 (17)

Source: Moulton and Moses (1997, p. 338, Tab. 5).

They show that in 1983 replacement items accounted for 61 percent of the total movement of the index, 96 percent in 1984 and 50 percent in 1995. These are extraordinarily high proportions of the overall movement considering that replacement items account for around 4 percent of all items in a given month. This shows that adjustment methods have a very important effect on overall index movement and provide further motivation for our study of these methods.

2.2.3. Summary

In this section the various approaches to new and disappearing goods and quality change used by statistical agencies were discussed. While these methods have a reasonable basis in economic theory they are often only applicable in certain areas, and provide short term fixes for more serious underlying problems. In the following three sections we explore some alternative methods for dealing with quality change and new and disappearing goods.

2.3. Hedonic Regression

One method for dealing with quality change and new and disappearing goods which is becoming increasingly important is hedonic regression. The National Research Council (Schultze and Mackie, 2002, p. 122) writes, "Hedonics currently offers the most promising technique for explicitly adjusting observed prices to account for changing product quality." The use of hedonic methods has been strongly advocated for many years. In 1961 the Price Statistics Review Committee in the US (more commonly known as the Stigler Commission) recommended the investigation of the use of hedonic methods for quality adjustment. More recently, again in the context of the US CPI, the Boskin Commission (Boskin et al., 1997, p. 80) recommended that, "…more use should be made of hedonic statistical methods to adjust for quality change."

While this method is gaining acceptance, it is more complex than the methods discussed in Section 2.2. We begin this section with a brief introduction to hedonic methods. In Section 2.3.2 we discuss the theoretical basis of hedonic regression. In Section 2.3.3 a more detailed discussion of the practice of hedonic methods is undertaken, particularly of methods for obtaining price indexes from hedonic regression. Section 2.3.4 discusses some of the results from hedonic regressions and the extent of their use by statistical agencies. The axiomatic approach to hedonic methods is briefly introduced in Section 2.3.5 while we spend sometime detailing the advantages and disadvantages of hedonic methods in Section 2.3.6.

2.3.1. An Introduction to Hedonic Methods

The hedonic approach is based on the notion that there exists a relationship between the prices and characteristics of a good. The basis for this belief is what Triplett (1991, p. 630) calls the *hedonic hypothesis*, which more formally states that, "…heterogeneous goods are aggregations of characteristics and economic behaviour relates to the characteristics." As noted in Section 2.1, the hedonic hypothesis is very useful in that it provides a way of linking the prices of the *New Goods* in Figure 2.1 to those of the *Disappeared Goods*, via the characteristics of these goods. This linking via the characteristics is advantageous because the characteristics of the goods are generally more stable over time than the goods themselves. Most importantly, as we observe both prices and characteristics in the market the hedonic function can, at least in principle, be estimated from this data. Once we have an approximation to this function then the prices of various combinations of characteristics can be calculated. The intuitive reasonableness of the price-characteristics function and the estimation of it is illustrated well by Triplett (1986, p. 37) in the following quote,

> Suppose that grocers, rather than placing their wares on shelves with unit prices marked on them, loaded various assortments of groceries into grocery carts, attaching prices to each of the preloaded carts. Buyers would select a preloaded cart and pay the specified price for the collection of groceries that it contains. Suppose further that a hedonic function were estimated on the grocery cart data. The dependent variable (which in hedonic regressions is normally the price of models of some product, such as automobiles) in this regression consists of the prices charged for the various preloaded carts of groceries.

It seems that in the simple *grocery cart* case hedonic regression will give sensible results. The hedonic coefficients will reflect the effect on the grocery cart of adding a particular item.[16]

In general the hedonic approach hypothesises a relationship between the price of the good and the characteristics of that good. Here $f^t(.)$ is a monotonically increasing function of the price so that it can be inverted (often the natural logarithm), and $g_i^t(.)$ is some aggregator function for the characteristics vector z_i^t, whose parameters are the object of estimation.

$$f^t(p_i^t) = g^t(z_i^t), \quad i = I^t, \ t = 1,...,T \tag{2.19}$$

A pressing question with regard to this model is, where does this relation come from and what does it mean? Fortunately there is a literature on the economic theory of hedonic regression which provides some guidance on these issues. We briefly survey this literature in the next section.

2.3.2. The Economic Theory of Hedonics

This literature review focuses on four influential theoretical papers on hedonic regression; Rosen (1974), Feenstra (1995), Diewert (2003a) and Pakes (2003). These papers have helped to demystify the hedonic approach in terms of economic theory. In many ways the theory of the hedonic approach followed the practice. The study of hedonics was given impetus by the pioneering work of Zvi Griliches in the 1960s and 1970s. This is certainly not to say that the theory is underdeveloped but the fact that practical approaches to hedonic methods came well before the theory had a significant effect on the direction of the theory. Much of the theoretical discussion has focused around constructing economic justifications for

[16] Note, however, that the hedonic approach will only give exact results in the grocery cart example if the hedonic function has price as the dependent variable. If instead log-prices is the dependent variable then the results will only be approximately correct.

standard hedonic practice. Indeed perhaps the primary question that the theory has tried to address is, under what conditions can the results of a standard hedonic regression be regarded as reflecting consumer valuations and hence legitimately be used for quality adjustment in CPIs? This question along with others is addressed below.

a. The Market Equilibrium Approach: Rosen (1974) and Feenstra (1995)

The meaning of the hedonic function was first discussed and outlined by Rosen (1974). In a formal framework, Rosen (1974) showed that the hedonic price function can be thought of as an envelope function relating the market equilibrium in prices and characteristics. This envelope function is traced out by the equilibrium of consumers' *bid functions* and producers' *offer functions*. As Rosen's (1974) model still provides the basic theoretical justification and insight into the meaning of hedonic regression we outline it here.

First, consider a consumer with individual characteristics α who maximises their utility. It is assumed that the consumer purchases one unit of the hedonic product and chooses the *vector* of characteristics, z, embodied in this product as well the quantity consumed of the outside good x. The price of x is normalised to one and the consumer faces the market hedonic price function, $p(z)$. This problem is shown below.

$$U(z^*, x^* : \alpha) = \max._{z,x}\{U(z, x : \alpha) : p(z) + x = Y\} \tag{2.20}$$

The consumer's bid function, $\theta(z, U, Y : \alpha)$, shows the maximum amount the consumer is willing to pay for the variety with characteristics vector z, given U, Y and α, and can be interpreted as the consumer's inverse demand curve for characteristics. For a utility maximum the bid function must approximate the hedonic function to the first order.[17]

$$\theta(z^*, U^*, Y : \alpha) = p(z^*) \tag{2.21}$$

$$\frac{\partial \theta(z^*, U^*, Y : \alpha)}{\partial z_k} = \frac{\partial p(z^*)}{\partial z_k}, \quad k = 1, ..., K \tag{2.22}$$

Rosen (1974) went on to add a supply-side to the model where there are heterogeneous firms which have different characteristics represented by the vector β. These firms maximise profit by choosing the vector of characteristics to supply for a given product, and the quantity, M, to produce, which depends on the market hedonic price function, $p(z)$. The problem is specified below.

$$\pi(z^*, M^* : \beta) = \max{}_{z,M} \{ p(z)M - C(M, z : \beta) \} \tag{2.23}$$

Define an offer function as, $\phi(z, \pi : \beta)$, which represents the minimum amount that a firm is willing to receive for a product embodying the characteristics vector z, given π and β. This can be interpreted as the producer's inverse supply function. As the hedonic price function represents the maximum amount that a firm can receive then the offer function should approximate the hedonic price function to the first order at the optimum.

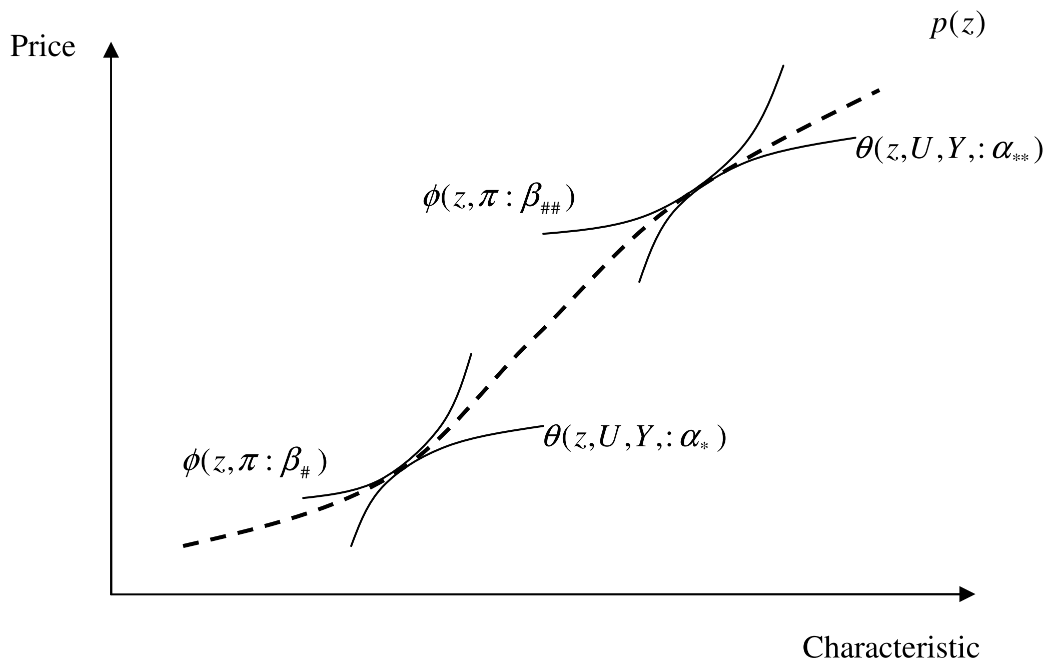
$$\phi(z^*, \pi^* : \beta) = p(z^*) \tag{2.24}$$

$$\frac{\partial \phi(z^*, \pi^* : \beta)}{\partial z_k} = \frac{\partial p(z^*)}{\partial z_k}, \quad k = 1, ..., K \tag{2.25}$$

[17] It can also be shown (Rosen, 1974, p. 38) that, under normal assumptions, the bid function is increasing and concave at the optimum.

It can readily be seen that the hedonic price function is the envelope of the tangency of consumers' bid functions and producers' offer functions. Consumers and producers are distributed in some way along the hedonic function according to preferences and technology. The case of two consumers who are matched with two producers for a hedonic good with a single characteristics dimension is considered in Figure 2.4.

Figure 2.4. Rosen's Hedonic Equilibrium



An important point, and one emphasised by Rosen, is that, as in the classical supply and demand case, the equilibrium of the bid and offer curves in general reveals no information about the form of the underlying curves. Indeed one of the great practitioners of hedonic methods, Zvi Griliches (1990, p. 189) writes,

> What is being estimated [by the hedonic function] is actually the locus of intersections of the demand curves of different consumers with varying tastes and the supply functions of different producers with possibly varying technologies of

production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

This is an important theoretical consideration as what we desire, from the perspective of calculating a CPI, is the consumer's valuation of the characteristics.

The pioneering work of Rosen (1974) raised a number of questions. Of most interest is under what conditions can we regard the hedonic price function as representing consumer preferences? From the discussion above we can see that, in general, the hedonic price function $p(z)$ will not equal the bid function (or the offer function for that matter). There is a special case where the hedonic function does trace out the consumers' bid function. This is when all consumers are identical and there is a single unique bid function. However, it seems reasonable in terms of Rosen's model to regard the hedonic function as a useful and meaningful *approximation* to the aggregate consumer's bid function. Note, however, that the hedonic function always lies above the individuals bid functions so can be thought of as an upper bound on the aggregate bid function. As a result of the approximate relationship between the consumer bid function and the market hedonic function, Rosen's model has often been used to justify the use of hedonic methods in CPIs.

In a more recent paper Feenstra (1995) further explores some of these issues, particularly how individuals with different preferences can be aggregated so that a 'representative consumer' framework can be adopted. Interestingly, for the representative consumer framework to be valid the form of heterogeneity of preferences is quite restrictive. Even for Feenstra's *General Utility Function* the form of heterogeneity is reflected in the fact that consumers receive a random draw of an additive error term. Cases where different consumers have more sophisticated assessments of quality are likely to be too complicated to model. Feenstra also shows how firms' choice of product characteristics, prices and costs are related. Within this framework Feenstra (1995) proceeds to derive readily calculable bounds on the cost-of-living index for various preference formulations when the characteristics of goods change over time.

b. A Consumer Theory Approach: Diewert (2003a)

A recent, and valuable, contribution to the debate is that of Diewert (2003a) who outlines how a hedonic regression can be justified within a conventional consumer theory model. The production side is ignored unlike in the previous models. As this model is so useful and we outline it below.

Consider a typical cost minimisation problem where the consumer chooses between a hedonic good and an outside good, x. Here we have the hedonic aggregator sub-utility function, $f(z)$ which is the consumer's 'quantity' measure for the hedonic good over the characteristics vector z. The hedonic price function, $p(z)$ measures the price of a unit of $f(z)$. We assume that the consumer purchases a single hedonic good and hence faces the cost minimisation problem shown below.

$$C(p(z^*), p_x, \overline{U}) = \min._{x,z}\{p(z) + p_x x : U(x, f(z)) \geq \overline{U}\} \qquad (2.26)$$

The (rearranged) first order condition for cost minimisation, where the consumer chooses the quantity of the hedonic good $f(z)$ and the outside good x, is shown below.

$$p(z^*) = p_x \frac{\left(\dfrac{\partial U(x^*, f(z^*))}{\partial f(z)}\right)}{\left(\dfrac{\partial U(x^*, f(z^*))}{\partial x}\right)} \qquad (2.27)$$

Let us specialise this framework a little, following Diewert (2003a), and assume that the utility function takes the following *additively separable* functional form, where a_z and a_x are preference parameters that are assumed fixed.

$$U(x, f(z)) = a_z f(z) + a_x x \qquad (2.28)$$

Using this functional form we can simplify (2.27). Also let us multiply both the LHS and RHS of (2.27) by the hedonic quantity aggregator, $f(z)$. On the LHS this gives the total value of the bundle of characteristics purchased, that is the market price of the hedonic good, which at the optimum we denote $P(z^*)$.

$$P(z^*) \equiv p(z^*)f(z^*) = \left(p_x \frac{a_z}{a_x} \right) f(z^*) \tag{2.29}$$

This equation is the basis of Diewert's (2003a) hedonic model and gives a justification for a hedonic regression. All that is required to implement this model is a functional form for the characteristics aggregator function. The hedonic regression is then the process of estimating or cardinalising the parameters of the sub-utility function over characteristics.

Diewert's (2003a) formulation is particularly useful as it has the distinct advantage of being rationalised with reference to consumer preferences alone so the hedonic function represents the user value of characteristics. The model has been criticised by some, most notably Hausman (2003, p. 35), on the grounds that the assumptions made are too restrictive. However, this point is made by Hausman (2003) not as a specific criticism of Diewert's (2003a) model, but of the hedonic approach in general.

c. A Producer View: Pakes (2003)

Pakes (2003) takes the polar approach to Diewert (2003a) and looks at the hedonic function from the producer's perspective. He believes that prices are a function of the marginal cost of producing the characteristics embodied in the good as well as the particular markup associated with the good. Importantly, this markup will depend on the demand elasticities, the degree of competition in the market and many other factors. In this sense Pakes calls the hedonic function a 'reduced form' as it reflects consumer preferences as well as production-side factors.

One of Pakes' (2003) points is to argue that, because of the complex factors involved in determining market price, we need not always expect intuitively appealing signs on the characteristics variables in our hedonic regressions. The sign on a characteristic which the consumer unambiguously deems good could be negative due, say, to a very low markup for a particular variety which has a large quantity of this characteristic. The possibility that 'good' characteristics would have negative signs is clearly undesirable from the perspective of measuring the consumer valuation of a characteristic. This has led to some scepticism regarding hedonics (Hulten, 2003).

d. Summary of the Economic Theory of Hedonic Methods

The survey of the economic theory of hedonic regression has shown that the meaning and interpretation of the hedonic function is contentious. This raises perhaps the most controversial aspect of hedonics, can the hedonic function be interpreted as representing the consumer valuations of characteristics and hence be legitimately used to quality adjust in a CPI? In Diewert's (2003a) model the answer is clearly, yes. However, in the other models, particularly that of Pakes (2003), the fact that the hedonic function can represent supply-side and market features is emphasised. Hausman (2003, p. 37) also stresses this point and writes,

> The coefficients in a hedonic regression on these attributes will mix together factor input prices, markups that vary by firm and the utility that consumers derive from various attributes, all of which may vary across time. Thus hedonic regressions are not structural econometric equations.

This interpretation of hedonic regression is echoed by Nerlove (2001, p. 431) who notes,

> …just as in the case of ordinary demand analysis, using data on prices and quantities, there is an unresolved identification problem involved in trying to draw

> inferences about consumer' preferences of producer' production possibilities from such hedonic price functions.

The objection to the interpretation of the hedonic function as representing consumer preferences is a strongly held and dates back to Rosen's (1974) hedonic model. It undermines the use of hedonic regressions for the job which they were designed, that is to show how consumers' perceptions of quality are reflected in price differences. However, not all writers have agreed with this opinion. In an interesting defence of hedonic regressions as a reasonable representation of consumer preferences Diewert (2003b, p. 30) writes,

> The situation is similar to ordinary general equilibrium theory where equilibrium price and quantity for each commodity is determined by the interaction of consumer preferences and producer's technology sets and market power. However, there is a big branch of applied econometrics that ignores this complex interaction and simply uses information of the prices that consumers face, the quantities that they demand and perhaps demographic information in order to estimate systems of consumer demand functions. Then these estimated demand functions are used to form estimates of consumer utility functions and these functions are often used in applied welfare economics. What producers are doing is entirely irrelevant to these exercises in applied econometrics with the exception of the prices that they are offering to sell at.

Clearly, there are some important differences of opinion in interpreting what a hedonic regression represents. However, despite these conceptual difficulties it seems reasonable to regard the hedonic function as an *approximation* to the willingness-to-pay of consumers' for various characteristics. It is in this approximate sense that the hedonic price function provides a legitimate method for adjusting prices for changes in the quality of goods.

2.3.3. The Various Hedonic Methods

There are a number of different ways in which hedonic regressions can be estimated and hence how the price indexes can be derived. Indeed the question of which hedonic method is 'best' has been one that has preoccupied many researchers in this area. In this section we discuss three methods using Triplett's (1989) taxonomy.

a. The Time-Dummy or Direct Method

The initial approach used by Court (1939) in estimating the price index from the hedonic regression was the time-dummy or direct method. Let us specialise the regression model (2.19) above with the logarithm of price as the dependent variable, $f^t(p_i^t) = \ln(p_i^t)$, and impose the following functional form for the quality characteristics aggregator, $g^t(z_i^t) = g(z_i^t) + \delta^t$, where δ^t is a time specific effect. This gives the following hedonic model.

$$\ln(p_i^t) = g(z_i^t) + \delta^t \tag{2.30}$$

This assumption about the functional form of the characteristics aggregator is important as we are assuming that the relationship between characteristics and log-price is fixed with all time effects entering additively. The advantage of this approach is that if we compare the relative of estimated prices in two periods, 0 and 1, this is independent of the reference vector of quality characteristics and a natural measure of inflation between the two periods. We call this the Hedonic Time-Dummy Price Index (P^{TD}).

$$P_{1,0}^{TD} \equiv \frac{\hat{p}_i^1(z_i^*)}{\hat{p}_i^0(z_i^*)} = \frac{\exp(\hat{g}(z_i^*) + \hat{\delta}^1)}{\exp(\hat{g}(z_i^*) + \hat{\delta}^0)} = \exp(\hat{\delta}^1 - \hat{\delta}^0) \tag{2.31}$$

A further advantage of this approach is that it is computationally very straightforward. A regression is run with data pooled over at least two time periods with the form and parameters of the characteristics aggregator function held fixed and the time-dummy variables included. Griliches (1971, p. 7) has the following justification for the use of the time-dummy approach.

> The justification for this [method] is very simple and appealing: We allow as best we can for all of the major differences in specifications by "holding them constant" through regression techniques. That part of the average price change which is not accounted for by any of the included specifications will be reflected in the coefficient of the time dummy and represents our best estimate of the "unexplained-by-specification-change average price change."

In a recent contribution, Haan (2003) notes that there is a close relationship between the geometric mean matched-model price indexes and those calculated using the time-dummy method. If all models are matched in the sense that their characteristics are unchanged between periods then the time-dummy price index is simply equal to an unweighted geometric mean of the matched prices (Haan, 2003, p. 5).

b. The Indirect or Imputations Index Method

Many writers have criticised the time-dummy regression model as it fixes the parameters and functional form of the characteristics aggregator function over time (Silver et al., 2001; Schultze and Mackie, 2002). They argue that there is no basis in the theory or practice of hedonic methods which leads us to think that this should necessarily be the case. There are many reasons why the functional form or parameters could change, such as changes in firms' costs of producing the characteristics or income effects for consumers. In a flexible measurement framework it would be desirable if we could accommodate these changes. Most recently, the report of the National Research Council (Schultze and Mackie, 2002) has been strongly critical of the dummy-variable approach of fixing the characteristics aggregator over time. They write (Schultze and Mackie, 2002, pp. 127-128),

> The key problem with the time dummy approach is that, for product areas in which quality change bias is likely to be an issue, the relationship between price and characteristics often changes rapidly. As an example, it is unlikely that consumers value, on the margin, a 10 percent increase in computer hard drive memory the same now as a year or two ago. If regression coefficients assumed to be constant over time are in fact not constant, the estimated time dummies will reflect a mixture of pure price changes and quality changes, and the resulting index will be biased. More generally, there is neither theoretical support nor much empirical evidence for the assumption that prices of all varieties of particular products generally move proportionately over time.

This criticism is important. If there is evidence of changes in the relationship between characteristics and price over time, such as that found by Berndt and Rappaport (2001), then this should be reflected in the estimation approach.[18] If we adopt the more general approach and allow the parameters, and perhaps the functional form of the characteristics aggregator, to vary over time then we require a new method for deriving the price index from the estimated hedonic function. Generally speaking, for the two periods being compared, $t = 0,1$, these indexes are some function of actual prices, p^t, predicted prices, $\hat{p}^t(z) = (f^t)^{-1}(\hat{g}^t(z))$, and the quantities of the models sold, x^t, if such information is available. The general form of the Hedonic Imputations Price Index ($P_{1,0}^{IMP}$) is shown in (2.32).

$$P_{1,0}^{IMP} = I(p^1, p^0, \hat{p}^1(z), \hat{p}^0(z), x^t, x^{t-1}) \tag{2.32}$$

[18] However, the lack of stability in the parameters may be indicative of serious problems with the model. In fact the stability of the parameters over time is often regarded as evidence for the correct model specification (Kokoski, Waehrer and Rozaklis, 2001, p. 14).

Clearly the number of possible imputations indexes is large. However, most of the actual imputations indexes suggested and used in practice have been natural extensions of well known index numbers. For example, Diewert (2003b, p. 13-14) has defined a useful set of imputations indexes, the Hedonic Laspeyres and Paasche Indexes which we defined previously in (2.10) and (2.11). The Hedonic Fisher Index is an appealing index which is the geometric mean of these two indexes. It is straightforward to use the imputations approach to calculate the analogue of other well known indexes such as the Tornqvist (Silver and Heravi, 2001).

Note that with the estimated hedonic function, $\hat{p}^t(z) = (f^t)^{-1}(\hat{g}^t(z))$, what we can do is estimate any missing prices of interest. In practice statistical agencies have most frequently used hedonic methods to undertake *direct quality adjustment* on a case-by-case basis as discussed in Section 2.2 above (Schultze and Mackie, 2002, p. 135). In terms of Figure 2.3 we can use the hedonic price-characteristics function to estimate an overlap price, for good *A* and/or *B*, which can be used in the index. This use of hedonic methods is called "patching" by the ILO (2004, p. 119). However, if hedonic methods are restricted in their application only to those items which require one-on-one replacement then the affect on the CPI is likely to be minimal (Schultze and Mackie, 2002, pp. 135, 140).

c. The Characteristics Index Method

The other type of index that we can derive from hedonic methods is the characteristics index. Here again we estimate a regression each period and obtain the estimated hedonic price-characteristics function, $\hat{p}^t(z) = (f^t)^{-1}(\hat{g}^t(z))$. However, rather than comparing the prices of *models*, bundles of characteristics, we instead compare the prices of *characteristics*. An example of such an index is where we evaluate the estimated price-characteristics function at the average of each period's characteristics vector, $\bar{z}^t \;\; t = 0,1$, and take the geometric mean of these two indexes in the spirit of the Fisher Price Index (Schultze and Mackie, 2002; Diewert, 2003a).

$$P^{CHAR} = \left(\frac{\hat{p}^1(\bar{z}^1)}{\hat{p}^0(\bar{z}^1)} \right)^{\frac{1}{2}} \left(\frac{\hat{p}^1(\bar{z}^0)}{\hat{p}^0(\bar{z}^0)} \right)^{\frac{1}{2}} \tag{2.33}$$

$$= \left(\frac{(f^1)^{-1}(\hat{g}^1(\bar{z}^1))}{(f^0)^{-1}(g^0(\bar{z}^1))} \right)^{\frac{1}{2}} \left(\frac{(f^1)^{-1}(g^1(\bar{z}^0))}{(f^0)^{-1}(g^0(\bar{z}^0))} \right)^{\frac{1}{2}} \tag{2.34}$$

Indexes of this general kind were used by Silver, Ioannidis and Webb (2001) over specific categories of goods. However, characteristics-type indexes have not been used very frequently despite their close relationship to the economic theory of hedonic methods (Triplett, 1989, p. 163). The likely reason for this is that the calculation of price indexes over characteristics is quite different from the more conventional approach of calculating price indexes over goods.

2.3.4. The Practice and Empirical Results of Hedonic Methods

Hedonic methods have a long pedigree. Waugh (1928) provides the first known use of what is now called hedonic regression looking at how vegetable characteristics affected vegetable prices. Somewhat more influential was the paper by Court (1939), who coined the term ‘hedonic’, and was looking at the prices of automobiles for the General Motors Corporation. Despite these early uses of hedonic techniques the method fell into disuse and it was Zvi Griliches who rejuvenated and popularized hedonics in a succession of papers during the 1960s and 1970s. Griliches (1961), who also looked at automobiles, was perhaps the most influential of these papers. The paper was research commissioned and published in conjunction with the Price Statistics Review Committee (the Stigler Commission) in the US.[19]

More recently the major practitioner of hedonic methods has been Mick Silver and his co-authors who have applied hedonic techniques using very detailed

[19] For a fuller discussion of the history of hedonic methods see Berndt (1991).

scanner-data. Silver (1999) focuses on quality-adjusted prices for televisions in the UK while Silver, Ioannidis and Webb (2001) consider VCRs. Silver and Heravi (2001) undertake one of the most comprehensive hedonic investigations, for televisions in the UK. Also in the areas of electronics, Kokoski, Waehrer and Rozaklis (2001) have investigated hedonic methods for audio products in the US CPI. The focus on consumer electronics is particularly important as it is one of the areas where the Boskin Commission indicated that official indexes most seriously failed to reflect the effects of quality change and new and disappearing goods.

Perhaps the most common application of hedonic methods has been to personal computers – an area which has seen extraordinarily rapid technological change. The early studies of Chow (1967) and Cole et al. (1986) were influential – the latter especially in regard to the construction of the US CPI component for computers. One of the best recent set of studies on computers is that undertaken by Berndt and Rappaport (2001), who look at computer prices in the US from 1976 to 1999. This followed an earlier paper by Berndt, Griliches and Rappaport (1995) looking at computers in the early 1990s. Both these papers find large average price falls, with Berndt and Rappaport (2001, p. 271, Table 1) finding an average annual growth rate in prices of around -30 percent from 1976 to 1999. Other studies confirm that hedonically calculated computer prices have fallen significantly. Pakes (2003) discusses the theory and provides an application to computers in the US while Shiratsuka (1995) looks at computer prices in Japan. There have also been numerous statistical agency investigations of hedonic methods for computers (Okamoto and Sato, 2001; Holdway, 2001; ABS, 2001; Lim and Mckenzie, 2002). This extensive empirical research has provided an impetus to statistical agencies to adopt hedonic methods.

a. Statistical Agencies Use of Hedonic Methods in the CPI

As can be seen from the discussion above, hedonic regression is a powerful tool for dealing with new and disappearing product varieties which is likely to be particularly useful for the compilation of official price indexes. Despite this it is interesting that, other than in the US, the hedonic method is not widely used by statistical agencies. Let us start by surveying the use of hedonics in the US statistical system.

The use of hedonics by US statistical agencies, both in the CPI and other areas, is extensive. Moulton (2001, p. 1) of the US Bureau of Economic Analysis writes,

> …currently 18 percent of the final expenditures in gross domestic product is deflated using price indexes that use hedonic methods.[20]

However, hedonic methods took a long time to gain acceptance in the US. Triplett (1990, pp. 207-208) reports that the first use of hedonic methods in official statistics in the US took place in 1968 while the second application occurred around 20 years later in 1988. In the US hedonic methods are now widely used in the CPI. Since 1987 hedonic adjustments for aging have been used in the CPI Rents Index while it has been used in the Apparel Index since 1991 (Moulton, 2001, p. 6). In 1998 hedonic methods were introduced for the Computer Index and in 1999 for the Televisions Index (Moulton, 2001, p. 7). More recently, in 2000, hedonic methods have been applied in the US CPI for a wide variety of categories including: clothes washers and dryers, microwave ovens, 12 types of audio products, camcorders, college textbooks, DVD players, refrigerators and videocassette recorders (Moulton, 2001, p. 7; Schultze and Mackie, 2002, p. 131). Fixler et al. (1999, p. 2) noted that, in 1999, 2.9 percent of the weight of the US CPI was subject to hedonic quality adjustment.

Other statistical agencies have not been as quick as the US to adopt hedonic methods. For example, in the UK Retail Price Index, the main measure of headline inflation, hedonic methods were only just recently incorporated, in March 2004, for adjustments to personal computers and digital cameras (Ball and Allen, 2003; Ball et al., 2004). Prior to this hedonic methods were used in the Harmonised CPI in the

[20] To be more precise, hedonic methods are used in *product groups* which make up 18 percent of expenditures in gross domestic product.

UK, an index that conforms to EU guidelines, to derive the Computer Price Index. Currently, Ball and Allen (2003, Tab. 1) report, Canada, Germany and Sweden, as well as the US and the UK, use hedonic methods in their Personal Computer Price Indexes. Sweden and France use hedonics for clothing and France also uses hedonics for dishwashers, televisions and books. In Finland hedonics are used in indexes for used cars and owner-occupied dwellings. Interestingly, in Australia hedonic methods are not currently used anywhere in the CPI though they will soon be introduced in the Producer Price Index for computers (Berger, 2003).

2.3.5. The Axiomatic Approach and Hedonic Regression

A common approach to the assessment of various competing index calculation methods is the axiomatic approach to index numbers. This approach bases the assessment of index methods on the degree to which various tests or axioms are satisfied.[21] In an interesting development this approach has recently been applied to hedonic regression methods. Diewert (2003b, p. 32) outlines the motivation for this development.

> The theory of hedonic regressions has left a great deal of leeway open to the empirical investigator with respect to the details of implementation of the models. Our strategy in this review of the issues has been to use some of the ideas that are present in the test approach to index number theory in an attempt to narrow down some of these somewhat arbitrary choices.

Diewert's paper provides guidance on a number of issues to practitioners of hedonic regression. We summarise some of his conclusions in this section.

In terms of the functional form of the regression Diewert (2003b) argues that the regression of *log-prices* on characteristics is more likely to have homoscedastic errors than the regression of *prices* on characteristics. This supports

[21] For a survey and illustration of the axiomatic approach see Diewert (1992, 2001).

conventional practice which has invariably seen the log of price used as the explanatory variable.

While the weighting of hedonic price equations in the estimation has been infrequently used by practitioners, it is nevertheless a natural extension. It seems vital to reflect the differences in importance of various models in the estimation. If weighting is to be used, Diewert (2003b) advocates the use of *expenditure share* weights when estimating the hedonic function.

As the time-dummy hedonic method discussed above provides a relatively self-contained index calculation method it is straightforward to test whether it satisfies various intuitively appealing axioms. Diewert (2003b) shows that the time-dummy method has a number of highly desirable properties from an axiomatic perspective. These are summarised below.

1. *The time-dummy method satisfies the time reversal test*: If $P_{1,0}^{TD}$ is the time-dummy derived price index then if we reverse the time periods and calculate the index $P_{0,1}^{TD}$ then the following relation holds, $P_{1,0}^{TD} = 1/P_{0,1}^{TD}$. (Diewert, 2003b, pp. 24-25).
2. *The time-dummy method is invariant to which time period the dummy variable is ascribed to*: If we compare two periods 0 and 1 and use a zero-one dummy variable to represent differences in price levels it does not matter whether the variable takes the value one when a price is from period 0 or one when it is from period 1, the resulting index will be the same. (Diewert, 2003b, p. 27).
3. *The time-dummy method satisfies the identity test*: If the prices, models (i.e. the characteristics associated with each price) and the quantity sold are the same in both periods then the price index does not change. (Diewert, 2003b, p. 35).
4. *The time-dummy method satisfies the homogeneity tests*: If the price level of one of the periods compared increases by a constant proportion, and as long as any weights used in the estimation are

homogeneous of degree zero in prices, then the price index will also increase by this proportion. (Diewert, 2003b, pp. 36-37).

The fact that the time dummy method satisfies a number of important index number tests adds to the credibility of this approach to index numbers. However, this axiomatic approach to the time-dummy method can be taken further. In Chapter 3 of this thesis we test whether the time-dummy method satisfies the monotonicity axioms. One example of these axioms states that if one of the prices increases, for say the later period, then the index should also increase. Interestingly, the paper shows that this axiom is not satisfied by the time-dummy hedonic regression method. The implications of this are further developed in the accompanying chapter.

2.3.6 The Advantages and Disadvantages of Hedonic Methods

As a means of evaluating hedonic methods we address some of the advantages and disadvantages of hedonic methods that have not already been mentioned. Drawing on a range of sources (Triplett, 1990; Moulton, 2001; Schultze and Mackie, 2002; ILO, 2004) we begin by briefly outlining some of the advantages of hedonic regression and then turn to some areas where hedonics is perceived to have difficulties.

a. Some Advantages of Hedonic Methods

As can be seen from the preceding discussion there are a great deal of advantages in using the hedonic regression method, over the methods discussed in Section 2.2.1, to account for quality change. The method has a theoretical basis in economics, it is not too complex to implement, and it is flexible in that it can be used in a number of different ways to obtain a quality-adjusted price index. From a practical perspective an important feature of hedonic methods is that their application generally produces plausible results. We discuss some further benefits below.

(i). Hedonic Methods are more Powerful than Conventional Methods

Compared with the methods discussed in Section 2.2.1 hedonic methods are generally more powerful in terms of estimating and adjusting for quality change. Consider, for example, the various imputations methods described above. Here we used information on the prices of matched goods to 'guess' the price movement of non-matched goods. The hedonic method is clearly more general than this as even if there are no matched prices we can still estimate the hedonic function and calculate a hedonic price index. This emphasises the advantages of the hedonic method in sectors where there is a high degree of technical change and very few matches may be available.

(ii). All Relevant Information is Used

One of the most frequently cited benefits of the hedonic method is that it uses all the price information available in a non-trivial way unlike, for example, price imputation methods. In estimating the hedonic regression all prices, even those that are not matched, are included. In terms of Figure 2.2 the hedonic index will be influenced by the whole set of sampled prices $S^0 \cup S^1$ as opposed to the imputations based price index which draws upon the prices from $S^0 \cap S^1$. This aspect of hedonics is particularly important if the quality-adjusted prices of non-matched items are systematically different from that of matched items.

(iii). Standard Errors are Produced

Because the parameters of the hedonic function are estimated econometrically, standard errors are produced. In this case, another advantage of the hedonic approach is that the standard errors can be used to produce confidence intervals around the estimated price index or to test hypotheses about price change.

b. Some Disadvantages of Hedonic Methods

(i). Increased Data and Technical Requirements

One area that is likely to be of concern to statistical agencies is the greater data and technical requirements that are required to implement hedonic methods when compared with, for example, the imputations method (ILO, 2004, pp. 120-121). As Griliches (1990, p. 191) has written, "Hedonic Methods are difficult. They require more data and more analysis and judgement."

Let us first turn to the data issues. To adequately implement hedonic methods statistical agencies need detailed data on the prices as well as the quality-characteristics of the good under study. The National Research Council (Schultze and Mackie, 2002, p. 124) writes, "In order to produce meaningful results, one generally needs data on more product models than are represented in a typical price index's sample of items." Importantly, these rich data sets are increasingly available given the use of barcode scanning machines by retailers. Scanner data sets typically include data on the prices, characteristics and quantity sold of a good in a particular outlet. Perhaps a more pressing concern with regard to hedonic methods is the requirement for a suitable level of technical expertise in econometrics and regression analysis. These skills are required in order to determine the appropriate hedonic technique and form for the regression, to assess the adequacy of the resulting parameter estimates and to update and monitor the method as problems arise.

(ii). The Reproducibility Problem

Another difficulty that has been noted is that of the reproducibility of results. That is, suppose two different statistical agencies, skilled in hedonic methods, were asked to estimate a hedonic price index for a particular good in a particular market. It is almost certain that the price indexes would be different. The problem arises because there are so many choices that must be made in the calculation of hedonic indexes. For example, we must; (1) obtain an adequate data set, (2) choose a functional form for the hedonic regression, (3) decide what quality characteristics should be included in the regression, and (4) determine how the index should be derived from the hedonic function. All these choices are likely to have an important effect upon the resulting index number. It is hoped, however, that further research in this area will help to establish best practice guidelines which will mitigate this reproducibility problem.

(iii). Some Goods are too Complex for Hedonic Regression

It seems unlikely that every case of quality change can be adequately dealt with by hedonic methods. For some goods the utility-giving characteristics are likely to be difficult if not impossible to measure. Consider having to implement a hedonic model for a hip replacement. How do we objectively measure the characteristics associated with the service rendered by a hip replacement? Hausman (2003, p. 37) discusses this problem in the context of medical services, an area where quality change is particularly pronounced and writes, “In many instances of medical care and services, identifying the key product attributes for a hedonic regression seems implausible.” A related problem is that of specifying an appropriate price-characteristics functional form. The relationship between the characteristics and price may be so complex as for it to be practically impossible to model. Triplett (1990, p. 220) discusses this with regard to the added complexity of cars compared with computers, and writes,

> One may not care to argue that, from an engineering standpoint, the automobile is more complicated than, say, a computer or an airplane however, the way automobile characteristics enter the utility function – what the automobile does for its user – is in fact very complicated indeed, and very hard to model, and for this reason the appropriate set of variables is hard to determine.

There certainly seem to be some limitations in the use of hedonics due to our inability to always adequately define and measure the important characteristics of a good. It may be that the use of flexible functional forms, which have not often been applied, are necessary to better represent the ‘true’ hedonic function accurately.

(iv). Heterogeneity in the Assessment of Product Characteristics

Heterogeneity in preferences may lead to serious problems in our ability to adequately represent the 'true' price-characteristics function. To see this let us illustrate with a simple example for the case of *ice cream*. Some consumers may have tastes such that ice cream that is high in fat is desired – because they prefer the taste and are not worried about their weight. Other consumers may be more health conscious and dislike high-fat ice cream. Due to these differences in taste an aggregate hedonic function, relating price and fat-content, for ice cream may not be meaningful. It will recover *average* relationships rather than the underlying heterogeneous preferences of the population. To continue our example consider the case where half the population likes high-fat ice cream and is willing to pay a premium for it while the other half is willing to pay a premium for low-fat ice cream. The coefficient on fat-content in a hedonic regression will then reflect the balance of these two effects which may cancel out in the aggregate leading to a zero coefficient. Now, if a 'super-fatty' variety of ice cream is introduced the predicted price from the hedonic regression will be no different from the high-fat and low-fat versions because the coefficient on fat-content is zero. However, those who regard high-fat content as a desirable characteristic are likely to be willing to pay a premium for this new product. In this case the hedonic function would fail as a predictor of consumer values.

The problem arises because there are essentially two different markets, one where high-fat ice cream is desirable and the other where it is not. The naive application of hedonic methods would simply be to aggregate these two markets. However, as we have seen above this may lead to erroneous results.[22] One solution is to apply hedonic methods to product areas where there is not a great deal of heterogeneity in preferences. It is interesting to note that for computers, where hedonic methods have been most extensively applied, there is unlikely to be a great deal of heterogeneity in preferences. While different consumers will prefer different

[22] This point is very similar in principle to that of Pakes (2003) who noted that the price of characteristics in sub-markets, defined by the degree of monopolistic power, could in principle be quite different because of this price discrimination by firms.

mixes of features, the identification of the desirable features of a computer is unambiguous.

(v). Hedonic Methods do not Account for Product Variety

One feature of hedonic methods is that they focus on quantifying the relationship between characteristics and prices. In this sense they cannot account for the effects of changes in the availability of characteristics or goods over time (Schultze and Mackie, 2002; Hausman, 2003; Pakes, 2004). For example, if new goods appear with novel characteristics then this will generally lower the cost-of-living as long as these goods are desired by some consumers. However, the hedonic regression method, which is a "pure price" approach (Hausman, 2003, p. 35), does not take account of the gains (or losses) from changes in the choice set. This is an important feature of hedonic methods and in the cost-of-living approach we would ideally like to measure changes in product variety as well as changes in product quality.

2.3.7. Summary

The discussion above gives some sense of the attractiveness of hedonic methods for producing quality-adjusted price indexes. However, it is also hoped that a sense of the complexity of the method has been conveyed, as well as the fact that there are a number of unresolved empirical and theoretical issues. Hedonics do represent one of the best possible approaches to the treatment of quality change but given the wide range of concerns about the approach some have called for caution in the application of this method. The National Research Council (Schultze and Mackie, 2002, pp. 133-134) writes,

> Given these ongoing concerns [about hedonics], we are still quite uncomfortable with extending the application of hedonic models, in their current state of development, to additional index categories for use in the [US] CPI.

Rather gloomily, the National Research Council (Schultze and Mackie, 2002, p. 140) conclude,

> Hedonic methods are not a cure-all for indexing problems related to quality change.... The main thing to be said for hedonic methods is that there is nothing better for dealing with certain aspects of the quality change problem. This is not an elegant defense, but it is a powerful one.

It is in this context that we must still look for alternative methods for assessing the effects of new goods and quality change. We explore two such alternatives in Sections 2.4 and 2.5.

2.4. The Estimation of Reservation Prices

An alternative approach to dealing with new and disappearing goods is what we will term the *Reservation Price Approach*. This approach is more general than hedonic methods in that we adopt a "goods" approach, which treats each good as a separate entity, and we do not make any assumptions about the characteristics of the goods or their relationship to price. The approach is closely aligned with economic theory and dates back to Hicks (1940).

In the case where we have new goods appearing and old goods disappearing from the market, as in Figure 2.1, the Hicks (1940) approach has a ready answer to how we should calculate price changes for these goods. The price that a new good in period 1 should be compared with is the *reservation price* for this good in the period prior to its introduction. The reservation price is the (minimum) price that would have driven demand for the good to zero.[23] The Hicksian reservation price method regards the zero consumption of goods in the period prior to their introduction essentially as an equilibrium outcome. To see this more clearly

[23] In fact we could in theory use any price greater than or equal to the minimum price which would drive demand to zero. However, in order to derive a lower bound on the change in the cost-of-living we take the minimum of these prices (Fisher and Shell, 1972, p.24).

consider the cost function below, defined over all the goods that are available in periods 0 and 1.

$$C(p^t, \overline{U} \mid I^0 \cup I^1) = \min{}_x \left\{ \sum_{i \in I^t} p_i^t x_i + \sum_{i \in I^0 \cup I^1, i \notin I^t} \overline{p}_i^t x_i \; : U(x) \geq \overline{U} \right\}, \; t = 0,1 \quad (2.35)$$

We know that for those goods which were not present in the relevant time period the consumption was zero (i.e. for $i \in I^0 \cup I^1, i \notin I^t$ we have $x_i^t = 0$). The reservation price approach asks what is the price, $\overline{p}_i^t$, for these goods which would have generated this demand. This reservation price is the appropriate price to include in the cost-of-living index in this approach.

This approach is firmly embedded within the cost-of-living approach and is desirable from this perspective. This general framework has been extensively advocated and applied by Hausman (1981, 1997, 1999, 2003). The difficulty with this approach is in adequately estimating the reservation price.

2.4.1. A Sketch of the Reservation Price Method

This section briefly outlines the reservation price approach, drawing heavily upon Hausman (1981). As in Hausman's paper we will consider the case where there are two goods, *A* and *B*, and where *A* is a newly introduced good and *B* is continuously available. The first step is to hypothesise a functional form for the Marshallian demand function for good *A* and estimate this function over the post introduction period, say $t = 1,...,T$. In equation (2.36) the function depends upon the prices of both goods p_A^t, p_B^t, and income, Y^t.

$$x_A^t = x_A(p_A^t, p_B^t, Y^t), \qquad t = 1,...,T \quad (2.36)$$

The difficult part from an econometrics perspective involves separating out movements along the demand curve from movements of the demand curve. However, once the demand function has been estimated it is often possible to solve Roy's Identity (Roy, 1947) for the indirect utility function.[24]

$$x_A(p_A^t, p_B^t, Y^t) = \frac{\left(\dfrac{\partial V(p_A^t, p_B^t, Y^t)}{\partial p_A^t}\right)}{\left(\dfrac{\partial V(p_A^t, p_B^t, Y^t)}{\partial Y^t}\right)} \tag{2.37}$$

With an expression for the indirect utility function, $V(p_A^t, p_B^t, Y^t)$, it is straightforward to invert this function and derive the cost function, $Y^t = C(p_A^t, p_B^t, U^t)$, as the indirect utility function must be monotonically increasing in Y^t. With the cost function available we can calculate the cost-of-living index. Using the estimated demand function we can then calculate the reservation price for good *A*. This price and the cost function can be readily used to calculate the effects of the introduction of the new good on the cost-of-living. This approach has been used extensively by Hausman (1997, 1999). One important contribution made by those who have used this approach has been the demonstration that even run-of-the-mill new goods, such as new cereals (Hausman, 1997), can create relatively large welfare gains.

2.4.2. A Discussion of this Approach

The reservation price method has a strong basis in economic theory and hence has been strongly advocated by some, most notably Hausman (1981, 1997, 1999, 2003). However, this approach is controversial. The primary controversy revolves around how the Marshallian demand function is estimated and efforts to

[24] It may not be possible to solve this partial differential equation (or solve it uniquely) if the functional form for the Marshallian demand function is not chosen carefully.

separate out movements of the demand curve from movements along the demand curve. Bresnahan (1997, p. 246), the discussant on Hausman's (1997) paper was somewhat critical of the econometric approach to estimation and writes, "The reader was, unfortunately, left unconvinced by key econometric assumptions…". The disagreement and uncertainty surrounding such estimation seems to be the primary concern of the National Research Council (Schultze and Mackie, 2002, p. 159) with regard to this method who write,

> …Hausman demonstrated that a choke [or reservation] price could be estimated for a specific new good. However, there is no clearly acceptable technique for consistently estimating demand curves for new goods or services in such a way that choke prices can be confidently ascertained.

Due to the contestability surrounding the estimation of demand curves this method suffers from a serious reproducibility problem which limits its application to official indexes. However, this is not to say that the method is not valid – only that there are many different valid ways of applying it in practice. In the next section we discuss a simplified version of this approach.

2.4.3. A Simplified Reservation Price Approach

While the reservation price approach is conceptually very appealing it suffers from the criticism that the results are not robust to the estimation approach. As the formal estimation methods are very complex it seems worthwhile considering a version of the approach which preserves the basic idea of calculating reservation prices but limits the burden of estimation. A simplified reservation price approach can be used to obtain a reasonable approximation to the more elaborate method.

Hausman (2003) has argued that we can obtain an approximation to the reservation price of a good if we know the price elasticity of demand, which is defined as $\varepsilon_i^t \equiv -d\ln(x_i^t)/d\ln(p_i^t)$ for good i in period t. To see this suppose that we want to calculate the reservation price for good A which is new in period 1. The demand curve for this good is given by $x_A^1 = x_A(p_A^1, p_{-A}^1, Y^1)$, where p_{-A}^1 is the price of all goods other than *A*. From this we can take a first order approximation to this curve around the point p_A^1, x_A^1 and p_{-A}^1.

$$x_A - x_A^1 \cong \frac{\partial x_A(p_A^1, p_{-A}^1, Y^1)}{\partial p_A}(p_A - p_A^1) \tag{2.38}$$

Using this relationship it is straightforward to find the reservation price for good *A*, $\hat{p}_A^1$, derived from this linear approximation to the demand curve.

$$\hat{p}_A^1 \cong p_A^1\left(1 - \frac{1}{\left(\frac{\partial x_A(p_A^1, p_{-A}^1, Y^1)}{\partial p_A}\frac{p_A^1}{x_A^1}\right)}\right) = p_A^1\left(1 + \frac{1}{\varepsilon_A^1}\right) \tag{2.39}$$

Thus as long as we are able to obtain an estimate of the price elasticity of demand for good *A* then we can obtain an approximation to the reservation price for this good. The extent of the approximation to the actual reservation price will depend on the linearity of the demand curve. It is likely that the estimated reservation price from this method will tend to underestimate the true reservation price as for most new goods there will be some consumers who are willing to pay a great deal for its services implying the demand curve is convex to the origin. At the very least this method provides an intuitively appealing ‘back-of-the-envelope’ estimate of the reservation price which can be used to quantify the impact of new and disappearing goods. This approach will be applied in Chapter 4.

2.4.4. Summary

The reservation price approach to estimating the change in the cost-of-living from new and disappearing goods is controversial. However, it remains an attractive approach as it is firmly based in economic theory. For this reason Shapiro and Wilcox (1996, pp. 26-27) conclude,

> Although explicit modelling of demand may be of dubious practicality for widespread implementation in the CPI, strategic application in a few selected cases might be worthwhile.

The simplified method discussed above provides a readily calculable estimate of the order-of-magnitude effect of new and disappearing goods as long as an estimate of the price elasticity of substitution for the good is available along with price and quantity data.

2.5. The CES Cost Function Approach

The CES cost function approach is the final method we will discuss for dealing with new and disappearing goods. In a creative paper Feenstra (1994) adopted the most general "goods" approach, allowing for changes in the choice-set of goods available to the consumer, and showed how the exact cost-of-living index could be calculated if the cost function had the Constant Elasticity of Substitution (CES) form. The method was further developed and refined by Feenstra and Shiells (1994), Nahm (1998) and Balk (1999).

While this method is relatively new, it rates a mention in the ILO Manual on CPIs (ILO, 2004, p. 151). We provide a brief sketch of the method in the following section deferring a more detailed discussion until Chapter 4, which outlines the approach in greater detail and applies it to a scanner data set.

2.5.1. A Sketch of the Approach

We assume that consumers have preferences defined by a homothetic CES cost function. The cost function, however, is generalised to changing sets of goods, as in (2.5), and is shown below where the elasticity of substitution is defined as, $\sigma \equiv -d\log(x_i^t / x_j^t) / d\log(p_i^t / p_j^t)$ and a_i is a taste or quality parameter.

$$C(p^t, \overline{U} \mid I^t) = \left(\sum_{i \in I^t} a_i \, (p_i^t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \overline{U} \,, \qquad t = 0,1 \tag{2.40}$$

Feenstra (1994) showed that it was possible to exactly calculate the cost-of-living index between two periods, 0 and 1, even though the set of goods changed. The exact cost-of-living index takes the following form, as long as $\sigma > 1$.

$$\frac{C(p^1, \overline{U} \mid I^1)}{C(p^0, \overline{U} \mid I^0)} = \left(\frac{\sum_{i \in I^{1,0}} s_i^1}{\sum_{i \in I^{1,0}} s_i^0} \right)^{\frac{-1}{1-\sigma}} \times P_{1,0}^{CES} \,, \qquad s_i^t = \frac{p_i^t x_i^t}{\sum_{i \in I^t} p_i^t x_i^t} \,, \qquad t = 0,1 \tag{2.41}$$

Here $P_{1,0}^{CES}$ is a matched-model price index over goods $I^{1,0}$. Balk (1999) shows that there are a number of different forms that the matched-model price index $P_{1,0}^{CES}$ can take for CES preferences.

The cost-of-living index defined by (2.41) above has a number of intuitively appealing features. Firstly, the adjustment factor that is applied to the matched-model index is less than one if period 1 expenditure on continuous goods is less than period 0 expenditure on continuous goods. Therefore, if more is spent on new goods in period 1 than on disappeared goods in period 0 then the matched model price index should be rated downwards to reflect the greater importance of the gains in period 1 compared with the losses from period 0. The intuition is that expenditure shares reveal some information about the desirability of the new goods relative to the disappeared goods. Secondly, if the elasticity of substitution is very large, $\sigma \to \infty$, then all goods are very close substitutes. This means that the appearance and disappearance of goods does not matter and so the cost-of-living index is simply equal to the matched-model index as the adjustment will approximate 1.

a. A Generalisation of the Approach

It is possible to generalise these results somewhat. Instead of hypothesising a CES functional form let us suppose that the cost function takes the form shown in (2.42). We will call this functional form the 'Modified Quadratic Mean of Order r' functional form or just the Modified Quadratic functional form.[25]

$$C(p^t,\overline{U}\mid I^t)=\left(\sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}}a_{i,k}\,(p_i^t p_k^t)^{\frac{r}{2}}+\sum_{i\in I^t,i\notin I^{1,0}}a_i\,(p_i^t)^{\frac{r}{2}}\right)^{\frac{1}{r}}\overline{U}\,,\qquad t=0,1 \qquad (2.42)$$

The essential difference between the CES and the Modified Quadratic cost functions is that the latter allows interaction terms between those goods which are present in both periods. These interaction terms help to represent more complicated patterns of complementarity and substitutability. In contrast the CES cost function does not allow for any interaction between the prices of these goods. The cost-of-living index for the Modified Quadratic functional form is shown below and is very similar to that for the case of CES preferences.[26]

$$\frac{C(p^1,\overline{U}\mid I^1)}{C(p^0,\overline{U}\mid I^0)}=\left(\frac{\sum_{i\in I^{1,0}}s_i^1}{\sum_{i\in I^{1,0}}s_i^0}\right)^{\frac{-1}{r}}\left(\frac{\sum_{i\in I^{1,0}}\hat{s}_i^0\left(\frac{p_i^1}{p_i^0}\right)^{\frac{r}{2}}}{\sum_{i\in I^{1,0}}\hat{s}_i^1\left(\frac{p_i^1}{p_i^0}\right)^{\frac{-r}{2}}}\right)^{\frac{1}{r}},\quad \hat{s}_i^t=\frac{p_i^t x_i^t}{\sum_{i\in I^{1,0}}p_i^t x_i^t},\ t=0,1 \qquad (2.43)$$

---

[25] Note that this functional form is a restricted version of the 'Quadratic Mean of Order r' functional form of Diewert (1976).

[26] The derivation of this result involves a little algebra so is relegated to the Appendix, Section 2.7.2.

This result is interesting as it provides a somewhat more general framework for thinking about new and disappearing goods. Furthermore, it emphasises the assumptions that are required for this approach to work. Generally speaking, the indexes in the original CES approach, (2.41), and the generalised approach, (2.43), account for changes in the domain of goods by using relative expenditure shares as an indicator of the relative value of the choice sets. As noted by Feenstra and Shiells (1994, p. 257), without some separability in the cost function between changing goods (i.e. the sets of new and disappearing) and continuous goods the use of relative expenditure shares as representations of relative quality does not work.

2.5.2. Summary

The CES cost function approach has a number of advantages over the other competing approaches outlined in the previous sections. Firstly, it is able to account for new and disappearing goods in a simple manner – by an adjustment factor applied to a matched-model price index. In the other methods, adjustments are required at the basic level to the prices of missing items. Secondly, the amount of estimation required is limited. Using the approach outlined above, estimation is only required for the elasticity of substitution. This contrasts with the hedonic approach where we must estimate the hedonic price function, which is far from straightforward, and the reservation price approach to new and disappearing goods where there are very complex econometric issues. A number of procedures based on index and econometric methods exist for the estimation of the elasticity of substitution (Balk, 1999). These are discussed in greater detail in Chapter 4. Note also that the detailed data required to implement this approach is now available in scanner data sets. Overall this method shows a great deal of promise, but as it is relatively new it has yet to be widely applied. In Chapter 4 we progress research in this area by applying this method to a large scanner data set for Australia.

2.6. Conclusion and Looking Forward

This chapter has been rather lengthy and so we will make the conclusion brief. The quality change and new and disappearing goods problem is currently one of the most difficult aspects of price measurement. However, the problem is not as intractable as it first appears. In Section 2.1 we were able to justify reasonable bounds on the cost-of-living index using the "characteristics" approach. In Section 2.2 the statistical agency perspective was illustrative in clarifying how the problem of new and disappearing goods arose in a sampling context. Statistical agencies have traditionally used a range of approaches to adjust for quality change. Unfortunately some of these appear inadequate, such as that applied to cases of quantity-quality change. In Sections 2.3, 2.4 and 2.5 we discussed some of the more sophisticated ways of addressing quality change and new and disappearing goods. These methods show some promise in moving us towards robust solutions to the considerable conceptual and practical difficulties posed by quality change and new and disappearing goods.

The remainder of this thesis mainly deals with various aspects of the quality change and new and disappearing goods problem. Chapter 3 elaborates on Section 2.3.5 and examines the performance of the hedonic time-dummy method with regard to the monotonicity axioms. Chapter 4 estimates the bias due to new and disappearing goods in a scanner data set using the approaches of Section 2.4 and primarily Section 2.5. Chapter 5 examines the relationship between the stochastic approach to index numbers and the economic approach. In the latter part of this chapter there is a particular focus on stochastically estimated price indexes that account for new and disappearing goods.

In conclusion, given the focus on rather specialised and economic theory-driven approaches to quality change and new and disappearing goods, it is perhaps wise to end this chapter on a cautionary note. As Armknecht and Maitland-Smith (1999, p. 21) write, No matter… how sophisticated the models and procedures used to deal with it, adjustment for quality change is still an *art* for price index practitioners. [emphasis added].

2.7. Appendix

2.7. 1. Table for Package Size Regression

Table 2.3. Results of Package Size Hedonic Regression for Soft Drinks

Dependent variable is Ln(Price).

R^2	0.3847
Adjusted R^2	0.3847
No. Observations	849,557

Variable Categories	Variable	Parameter Estimates	Standard Error	T-Statistic	P-Value
Intercept	Intercept	-0.9320	0.0084	-111.0740	<0.0001
Brand and Product Characteristics	Pepsi	-0.0472	0.0013	-35.1870	<0.0001
	Schweppes	0.1251	0.0009	144.3409	<0.0001
	Fanta	0.1250	0.0020	62.6366	<0.0001
	Sprite	0.1531	0.0015	99.6887	<0.0001
	Solo	0.0856	0.0016	54.0890	<0.0001
	Lift	0.1413	0.0016	90.9415	<0.0001
	Tarax	-0.3440	0.0031	-111.4819	<0.0001
	Virgin	-0.0046	0.0051	-0.9120	0.3618
	Royal Crown	-0.3640	0.0034	-108.5844	<0.0001
	Sbrand	-0.3590	0.0007	-485.1795	<0.0001
	Kirks	0.0802	0.0008	96.3650	<0.0001
	Waterfords	-0.0166	0.0010	-16.5596	<0.0001
	Tristrams	-0.2389	0.0046	-52.3189	<0.0001
	Dr Pepper	-0.0047	0.0023	-2.0464	0.0407
	Seven Up	0.0643	0.0016	39.2581	<0.0001
	Cola	0.1126	0.0010	110.9351	<0.0001
	Lemonade	0.0162	0.0007	22.9888	<0.0001
	Raspberry	0.1016	0.0020	49.7200	<0.0001
	Orange	0.0241	0.0013	18.4680	<0.0001
	Diet	-0.0019	0.0008	-2.5329	0.0113
	Tonic	-0.0809	0.0024	-34.2662	<0.0001
	Ginger Beer	0.0329	0.0026	12.6939	<0.0001
	Ginger Ale	-0.0087	0.0015	-5.6748	<0.0001
	Soda Water	0.0497	0.0013	39.1571	<0.0001
	Caffeine Free	0.0536	0.0018	30.0487	<0.0001
	Pineapple	0.1779	0.0028	63.8935	<0.0001
	Mineral Water	0.0787	0.0010	76.4974	<0.0001
	Pepsi Max	0.0133	0.0021	6.3754	<0.0001
	Sarsaparilla	0.1392	0.0029	47.2428	<0.0001
	Passion Fruit	0.0590	0.0025	23.6741	<0.0001

Table 2.3 (contd.). Results of Package Size Hedonic Regression for Soft Drinks

Variable Categories	Variable	Parameter Estimates	Standard Error	T-Statistic	P-Value
Time Periods	2	0.0195	0.0026	7.4354	<0.0001
	3	0.0214	0.0026	8.1856	<0.0001
	4	-0.0192	0.0026	-7.3424	<0.0001
	5	-0.0007	0.0026	-0.2708	0.7865
	6	-0.0110	0.0026	-4.2101	<0.0001
	7	-0.0001	0.0026	-0.0200	0.9841
	8	0.0059	0.0026	2.2674	0.0234
	9	-0.0218	0.0026	-8.3682	<0.0001
	10	-0.0112	0.0026	-4.2947	<0.0001
	11	0.0047	0.0026	1.8154	0.0695
	12	-0.0048	0.0026	-1.8275	0.0676
	13	-0.0016	0.0026	-0.6102	0.5417
	14	-0.0301	0.0026	-11.4909	<0.0001
	15	-0.0116	0.0026	-4.4186	<0.0001
	16	-0.0160	0.0026	-6.0874	<0.0001
	17	-0.0077	0.0026	-2.9251	0.0034
	18	-0.0372	0.0026	-14.1243	<0.0001
	19	-0.0359	0.0026	-13.6619	<0.0001
	20	-0.0182	0.0026	-6.9091	<0.0001
	21	-0.0158	0.0026	-6.0089	<0.0001
	22	-0.0026	0.0026	-0.9717	0.3312
	23	-0.0068	0.0026	-2.5797	0.0099
	24	-0.0178	0.0026	-6.7595	<0.0001
	25	-0.0059	0.0026	-2.2481	0.0246
	26	0.0002	0.0026	0.0887	0.9293
	27	0.0030	0.0026	1.1551	0.2480
	28	0.0130	0.0026	4.9463	<0.0001
	29	-0.0064	0.0026	-2.4234	<0.0001
	30	-0.0214	0.0026	-8.1090	0.0154
	31	-0.0016	0.0026	-0.5901	0.5551
	32	0.0028	0.0026	1.0486	0.2943
	33	-0.0155	0.0026	-5.8717	<0.0001
	34	-0.0136	0.0026	-5.1646	<0.0001
	35	-0.0057	0.0026	-2.1682	0.0301
	36	-0.0222	0.0026	-8.4127	<0.0001
	37	-0.0128	0.0026	-4.8614	<0.0001
	38	-0.0033	0.0026	-1.2429	0.2139
	39	-0.0037	0.0026	-1.3964	0.1626
	40	-0.0068	0.0026	-2.6087	0.0091
	41	-0.0039	0.0026	-1.4715	0.1412
	42	0.0118	0.0026	4.5017	<0.0001
	43	0.0092	0.0026	3.5434	0.0004
	44	-0.0229	0.0026	-8.7709	<0.0001
	45	-0.0288	0.0026	-11.0603	<0.0001
	46	0.0046	0.0026	1.7462	0.0808
	47	-0.0728	0.0026	-27.9567	<0.0001
	48	-0.0715	0.0026	-27.4177	<0.0001

Table 2.3 (contd.). Results of Package Size Hedonic Regression for Soft Drinks

Variable Categories	Variable	Parameter Estimates	Standard Error	T-Statistic	P-Value
Time Periods	49	-0.0400	0.0026	-15.2861	<0.0001
	50	0.0144	0.0026	5.5358	<0.0001
	51	-0.0123	0.0026	-4.7334	<0.0001
	52	-0.0160	0.0026	-6.1410	<0.0001
	53	-0.0290	0.0026	-11.0868	<0.0001
	54	-0.0069	0.0026	-2.6397	0.0083
	55	-0.0078	0.0026	-2.9865	0.0028
	56	0.0033	0.0026	1.2656	0.2057
	57	-0.0420	0.0026	-16.0773	<0.0001
	58	-0.0295	0.0026	-11.3479	<0.0001
	59	-0.0091	0.0026	-3.5083	0.0005
	60	-0.0308	0.0026	-11.7974	<0.0001
	61	-0.0148	0.0026	-5.6788	<0.0001
	62	0.0162	0.0026	6.1881	<0.0001
	63	-0.0607	0.0026	-23.2603	<0.0001
	64	-0.0361	0.0026	-13.7651	<0.0001
	65	0.0264	0.0026	10.0628	<0.0001
Package Size	Ln(Pkg. Size)	-0.2021	0.0011	-179.4864	<0.0001

2.7.2. A Generalisation of the CES Cost Function Approach

Let us rewrite the Modified Quadratic cost function in the following way where we define the interaction (i.e. off diagonal) terms to be equal to zero for those goods not available in both period 0 and 1.

$$C(p^t, \overline{U} \mid I^t) = \left(\sum_{i \in I^t} \sum_{k \in I^t} a_{i,k} (p_i^t p_k^t)^{\frac{r}{2}} \right)^{\frac{1}{r}} \overline{U}, \qquad t = 0,1 \tag{2.44}$$

Where $a_{i,k} = 0$ either, if $k \notin I^{1,0}, i \neq k$ (2.45)

or, if $i \notin I^{1,0}, i \neq k$

If we apply Shephard's Lemma to the modified quadratic functional form then we get the relationship shown in (2.46).

$$s_i^t = \frac{\sum_{k\in I^t} a_{i,k}(p_i^t p_k^t)^{\frac{r}{2}}}{\sum_{i\in I^t}\sum_{k\in I^t} a_{i,k}(p_i^t p_k^t)^{\frac{r}{2}}}, \quad t = 0,1 \tag{2.46}$$

Now for $t=0$ let us multiply both sides of (2.46) by $(p_i^1 / p_i^0)^{\frac{r}{2}}$, and sum over common goods ($I^{1,0}$) only.

$$\sum_{i\in I^{1,0}} s_i^0 \left(\frac{p_i^1}{p_i^0}\right)^{\frac{r}{2}} = \frac{\sum_{i\in I^{1,0}}\sum_{k\in I^0} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}}}{\sum_{i\in I^0}\sum_{k\in I^0} a_{i,k}(p_i^0 p_k^0)^{\frac{r}{2}}} \tag{2.47}$$

Let us rewrite the numerator of the RHS of (2.47).

$$\sum_{i\in I^{1,0}}\sum_{k\in I^0} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}} = \sum_{i\in I^{1,0}} (p_i^1)^{\frac{r}{2}}\left(\sum_{k\in I^{1,0}} a_{i,k}(p_k^0)^{\frac{r}{2}} + \sum_{k\in I^0, k\notin I^{1,0}} a_{i,k}(p_k^0)^{\frac{r}{2}}\right) \tag{2.49}$$

Then for those elements where $k \notin I^{1,0}$ and $i \in I^{1,0}$ then $i \neq k$, so from (2.45) above we have $a_{i,k} = 0$. This implies that the second term in brackets on the RHS of (2.49) is equal to zero which gives (2.50).

$$\sum_{i\in I^{1,0}}\sum_{k\in I^0} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}} = \sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}} \tag{2.50}$$

Using this in (2.47) we have the following equation.

$$\sum_{i\in I^{1,0}} s_i^0\left(\frac{p_i^1}{p_i^0}\right)^{\frac{r}{2}} = \frac{\sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}}}{\sum_{i\in I^0}\sum_{k\in I^0} a_{i,k}(p_i^0 p_k^0)^{\frac{r}{2}}} \tag{2.51}$$

We can derive an analogous result for $t=1$.

$$\sum_{i\in I^{1,0}} s_i^1\left(\frac{p_i^1}{p_i^0}\right)^{\frac{-r}{2}} = \frac{\sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}} a_{i,k}(p_i^0 p_k^1)^{\frac{r}{2}}}{\sum_{i\in I^1}\sum_{k\in I^1} a_{i,k}(p_i^1 p_k^1)^{\frac{r}{2}}} \tag{2.52}$$

Note that the denominator of the RHS of (2.51) and (2.52) are equal to the unit cost functions for the respective periods so by rearranging and combining (2.51) and (2.52) we can write the cost-of-living index in the following form.

$$\frac{C(p^1,\overline{U}\mid I^1)}{C(p^0,\overline{U}\mid I^0)} = \left(\frac{\sum_{i\in I^{1,0}} s_i^0\left(\frac{p_i^1}{p_i^0}\right)^{\frac{r}{2}}}{\sum_{i\in I^{1,0}} s_i^1\left(\frac{p_i^1}{p_i^0}\right)^{\frac{-r}{2}}}\right)^{\frac{1}{r}} \left(\frac{\sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}}}{\sum_{i\in I^{1,0}}\sum_{k\in I^{1,0}} a_{i,k}(p_i^1 p_k^0)^{\frac{r}{2}}}\right) \tag{2.53}$$

Clearly the terms on the far-RHS of (2.53) cancel and we can use the definition of matched expenditure shares, $\hat{s}_i^t$, to obtain the expression in the text and shown below.

$$\frac{C(p^1,\overline{U}\mid I^1)}{C(p^0,\overline{U}\mid I^0)}=\left(\frac{\sum_{i\in I^{1,0}} s_i^1}{\sum_{i\in I^{1,0}} s_i^0}\right)^{-\frac{1}{r}}\left(\frac{\sum_{i\in I^{1,0}} \hat{s}_i^0\left(\frac{p_i^1}{p_i^0}\right)^{\frac{r}{2}}}{\sum_{i\in I^{1,0}} \hat{s}_i^1\left(\frac{p_i^1}{p_i^0}\right)^{\frac{-r}{2}}}\right)^{\frac{1}{r}}, \quad \hat{s}_i^t \equiv \frac{s_i^t}{\sum_{i\in I^{1,0}} s_i^t}, \quad t=0,1 \quad (2.54)$$

CHAPTER 3

3. The Hedonic Regression Time-Dummy Method and the Monotonicity Axioms

Abstract*

This chapter shows that the well-known and much applied hedonic regression time-dummy method, used to construct quality-adjusted price indexes, fails the monotonicity axioms from index theory. The chapter outlines the hedonic time-dummy method and defines the monotonicity axioms in this context. A simple numerical example is used to illustrate the failure of monotonicity. The reasons for this failure are identified and discussed. The frequency of the violation of monotonicity is considered in general and investigated for a particular data set. The chapter concludes by considering the seriousness of the failure of monotonicity and briefly discusses an alternative hedonic method that satisfies monotonicity.

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Irving Fisher, writing in 1927, outlining the axiomatic approach to index numbers,

"The multiplicity of formulae for computing index numbers has given the impression that there must be a corresponding multiplicity in the results of these computations, with no clear choice between them. But this impression is due to a failure to discriminate between index numbers which are good, bad, and indifferent. By means of certain tests we can make this discrimination. The most important tests are all embraced under the single head of fairness. The fundamental purpose of an index number is that it shall fairly represent, so far as one single figure can, the general trend of the many diverging ratios from which it is calculated. It should be the "just compromise" among conflicting elements, the "fair average," the golden mean." Without some kind of fair splitting of the differences involved, an index number is apt to be unsatisfactory, if not absurd."

Fisher (1927, pp. 9-10).

3.1. Introduction

One common approach to constructing a quality-adjusted price index is the hedonic regression *time-dummy* method, sometimes called the *direct* hedonic approach. Here, a pooled regression is estimated with the logarithm of price explained by a set of quality characteristics and time-dummy variables. The price index is calculated directly from the time-dummy coefficients in the regression. As we will see, this method has frequently been used by practitioners in the economic measurement literature.

In this chapter the validity of this approach is questioned in the context of its failure to satisfy the monotonicity axioms – an important set of axioms in index number theory. These axioms are derived from the axiomatic or test approach to index numbers which treats an index formula as a function of independent variables and then specifies 'reasonable' properties that this function should possess. One of these reasonable properties is *monotonicity*, of which there are two versions. The first states that the price index, which compares two periods, must increase if the prices of the later period increase, holding other factors fixed. The second states that the price index should decrease if the prices of the earlier period are increased, holding other factors fixed. In the axiomatic approach the satisfaction of the monotonicity axioms is regarded as a fundamental requirement for a price index to be a credible measure of price change.

In this chapter it is shown that the hedonic regression time-dummy method of price index construction in fact does not satisfy these monotonicity requirements. This brings into question the use of this method for constructing quality-adjusted price indexes.

In the next section the hedonic regression time-dummy method for constructing price indexes is discussed. Sections 3.3 and 3.4 outline the axiomatic approach to index numbers and define the monotonicity properties in the somewhat unusual context of the time-dummy hedonic method. Section 3.5 shows that monotonicity fails with a simple numerical example while Section 3.6 discusses the reasons behind this failure. Section 3.7 gives an empirical example of the failure of monotonicity for a well-known data set. Sections 3.8 and 3.9 discuss the economics of the failure of monotonicity and suggest alternative methods while Section 3.10 concludes.

3.2. The Hedonic Regression Time-Dummy Method

The time-dummy method for calculating quality-adjusted price indexes estimates a regression, using panel data, relating the prices and quality characteristics of a good over time. For some good of interest, we have the following hedonic regression model.

$$\ln(p_i^t) = \beta_0 + \sum_{k=1}^{K} \beta_k z_{i,k}^t + \sum_{\tau=2}^{T} \delta_\tau d_{i,\tau}^t + \varepsilon_i^t, \quad i = 1,\dots,N, \; t = 1,\dots,T \tag{3.1}$$

The set of observations on the varieties, or models, of the good are indexed $i = 1,\dots,N$, and the time period by $t = 1,\dots,T$, however, note that the number of models in each period can differ. In the regression model the logarithm of price,

$\ln(p_i^t)$, is explained by an intercept, a set of quality-characteristics, $z_{i,k}^t \quad k = 1,...,K$, and time-dummy variables, $d_{i,\tau}^t \quad \tau = 2,...,T$, with an arbitrary normalization on the first time-dummy.

After estimating this regression, the quality-adjusted price index can be calculated very simply and directly by taking the exponent of the time-dummy coefficients of interest. The justification for this is straightforward. If we compare the relative estimated price of a model, between say period t and period s, for an arbitrary quality configuration, $\bar{z}$, then this ratio is equal to the relative exponents of the time-dummy variables as shown in (3.2).[27]

$$P_{t,s}^{TD} = \frac{\hat{p}^t(\bar{z})}{\hat{p}^s(\bar{z})} = \frac{\exp\left(\hat{\beta}_0 + \sum_{k=1}^{K} \hat{\beta}_k \bar{z}_{i,k} + \hat{\delta}_t\right)}{\exp\left(\hat{\beta}_0 + \sum_{k=1}^{K} \hat{\beta}_k \bar{z}_{i,k} + \hat{\delta}_s\right)} = \frac{\exp(\hat{\delta}_t)}{\exp(\hat{\delta}_s)} \tag{3.2}$$

The simplest case is where s is the base period where the coefficient has been normalized to zero. Then the estimated price index for period t relative to the base period is simply the exponent of the time-dummy coefficient for period t. An important and desirable feature of this index is that it does not depend upon the particular value of the quality-characteristics vector chosen, $\bar{z}$.[28]

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[27] It is known in the econometrics literature that exponent of the time-dummy variable in (3.2) is a biased estimate of the equivalent population parameter as the estimated coefficient is random while the transformation is nonlinear. This has to some extent been neglected in much of the empirical applications of the hedonic regression time-dummy approach. However, in this chapter we will not dwell on this problem as it does not influence any of our main points. The literature and the solution to the problem are most recently discussed in Garderen and Shah (2002).

[28] Note also that we would get the same results if the intercept was excluded from the model (3.1) and a time-dummy for every time period was included. In the regression *without an intercept* the estimated time-dummy parameter for period t would be equal to $\hat{\delta}_t + \hat{\beta}_0$, where $\hat{\delta}_t$ is the time-dummy coefficient *with an intercept* included. As we are adding a constant to each of the estimated time-dummy parameters when we exclude an intercept these cancel when looking at relative exponents and the index is unchanged.

This method for constructing quality-adjusted price indexes is widely accepted in the index literature and has a long pedigree. The first use of the time-dummy method dates back to Court (1939) who examined automobile prices. It was rediscovered and popularized by Griliches (1961) and while other hedonic approaches have since been developed the time-dummy method still remains important. For instance, in a recent theoretical paper, Diewert (2003a) outlined a consumer theory hedonic model where the index could be determined using the time-dummy method.

The use of this particular hedonic technique for price index construction is also widespread in the empirical literature, often amongst other hedonic techniques. For example, Gordon (1990) applies the method to various durable goods while a number of researchers have used it to look at computer prices; Berndt and Rappaport (2001), ABS (2001), Berndt, Griliches and Rappaport (1995), Nelson, Tanguay and Patterson (1994), and Shiratsuka (1995). Okamoto and Sato (2001) look at computers as well as televisions and digital cameras, Silver (1999), and Silver and Heravi (2001) look at television prices, and Kokoski, Waehrer and Rozaklis (2001) look at consumer audio products. No doubt there are many other such applications of the time-dummy method that have escaped the author's attention.

The adoption of hedonic methods by statistical agencies has been particularly rapid in the past decade. Moulton (2001, p. 1) noted that,

> …currently [in the US] 18 percent of the final expenditures in gross domestic product is deflated using price indexes that use hedonic methods.

Hedonic methods are used in many different ways by statistical agencies. Interestingly, it appears that some official price indexes are calculated using the time-dummy method discussed above. With regard to the US, Moulton (2001, p. 2) writes,

> …the Federal Reserve's indexes for LAN routers and switches and the hedonic portion of the BEA [Bureau of Economic Analysis] software index… are calculated from the regression coefficients of indicator (or "dummy") variables for years…

From my review of websites and official papers, it is unclear to what extent other countries statistical agencies have adopted the time-dummy hedonic method to produce official statistics.

3.3. The Axiomatic Approach to Index Numbers and the Time-Dummy Method

Given the widespread acceptance of the time-dummy method for constructing hedonic price indexes it is interesting to know how it performs with regard to the *test* or *axiomatic* approach to index numbers. The axiomatic approach to index numbers has a long history and dates back at least until the start of the twentieth century. It is best illustrated by Irving Fisher (1927) in his classic book, *The Making of Index Numbers*. The axiomatic approach treats an index formula as a function of independent variables and specifies 'reasonable' properties, or tests, that this function *should* have with regard to these variables. One advantage of this approach is that it does not depend upon the assumption of optimizing agents which is fundamental to the economic approach to index numbers (Diewert, 1976).

There is a long list of axioms which are used to test index formulae (Diewert, 1992). In this chapter the focus is on a single set of axioms, the *monotonicity properties*. These properties are rigorously defined in the next section. First however, it is useful to compare, at a general level, the hedonic regression time-dummy method for constructing a price index with more conventional methods.

Most price index formulae are a function of the vectors of base and current period prices and quantities. Thus a conventional price index can be written as the following general function where p^r and q^r are price and quantity vectors for periods $r = t, t-1$.

$$P_{t,t-1} = I(p^t, p^{t-1}, q^t, q^{t-1}) \tag{3.3}$$

If there are N goods in each period then a conventional index takes $4N$ positive real numbers and transforms them into a single positive real number – the index value.[29]

Unlike conventional price indexes the hedonic regression time-dummy method is a function of the prices of at least two periods as well as the *quality-characteristics* of these goods in each period. Here let us consider the case where only two periods are compared (in regression (3.1) above $T = 2$) and let us also fix the number of models for which we have prices each period at N.[30] Both these simplifications are made in order to ease the exposition and they do not diminish any of the points made. Given this, we can write the general form of the hedonic regression time-dummy index in the following way.

$$P_{t,t-1}^{TD} = I(p^t, p^{t-1}, Z^t, Z^{t-1}) \tag{3.4}$$

Here we pool data over two time periods, t and $t-1$, and use Z^r to denote the matrix (with dimensions $N \times K$) of quality characteristics for the goods in each period $r = t, t-1$. It can be seen that the hedonic regression time-dummy method, in this case, transforms a set of $2 \times (N + N \times K)$ variables into a single positive real

[29] A price index may be a function of more than 4N variables if, for example, expenditure weights are derived from many periods. Also a price index can be a function of less than 4N variables. For example, the Laspeyres Price Index uses base quantities but not current quantities so is a function of just 3N variables.

[30] In general the number of goods used to construct the hedonic regression price index can change over time without causing problems. This is one of the benefits of the hedonic approach. Changing sets of goods poses greater problems for conventional price index methods.

number. With this definition for the hedonic regression time-dummy method we can proceed to defining the monotonicity axioms.

3.4. The Monotonicity Axioms for the Time-Dummy Method

There are two monotonicity axioms for price indexes. Diewert (2001) identifies Eichhorn and Voeller (1976) as the first to suggest these axioms, and they are regarded as fundamental in the price index literature. The first is called the *Monotonicity in Current Prices Axiom* and is shown in (3.5) below.

$$I(p_X^t, p^{t-1}, Z^t, Z^{t-1}) > I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^t > p^t \tag{3.5}$$

Here $p_X^t > p^t$ means that each element of the vector p_X^t is no smaller than the corresponding element of p^t and there is at least one element in p_X^t which is strictly larger than the corresponding element in p^t. In words (3.5) states that if the current (i.e. period t) price vector increases, holding other variables constant, then the index must also increase.

The base period twin to (3.5) is the *Monotonicity in Base Prices Axiom* shown in (3.6). This states that if the base price vector increases then the index should fall.

$$I(p^t, p_X^{t-1}, Z^t, Z^{t-1}) < I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^{t-1} > p^{t-1} \tag{3.6}$$

While these are the two versions of the monotonicity axioms that have been most widely applied we could perhaps weaken these axioms. Rather than requiring strict inequality we could include the case where the indexes are equal. Then (3.5) becomes the *Weak Monotonicity in Current Prices Axiom* shown below.

$$I(p_X^t, p^{t-1}, Z^t, Z^{t-1}) \geq I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^t > p^t \tag{3.7}$$

Similarly, for base prices (3.6) becomes the *Weak Monotonicity in Base Prices Axiom* shown in (3.8).

$$I(p^t, p_X^{t-1}, Z^t, Z^{t-1}) \leq I(p^t, p^{t-1}, Z^t, Z^{t-1}), \quad \text{if } p_X^{t-1} > p^{t-1} \tag{3.8}$$

These four monotonicity axioms are intuitively very appealing properties for price indexes to possess. Many regard the monotonicity axioms as being a fundamental requirement, that a credible price index should meet, and the failure of an index to satisfy these axioms as a serious shortcoming of the method. In Balk's (1995, p. 70) survey of the axiomatic approach to index numbers he distinguishes between "…axioms – which are more or less self-evident – and tests – about which more debate is possible." The monotonicity property is the *first axiom* that Balk lists. Indeed, as Reinsdorf and Dorfman (1999) have noted, it would be a daunting task to have to explain to an index user why a price index that does not satisfy the monotonicity axioms should be considered for use. However, in the next section, it is shown that the hedonic regression time-dummy method does not satisfy monotonicity.

3.5. A Simple Numerical Example

Let us consider a simple numerical example of the failure of the *Monotonicity in Current Prices Axiom* using artificial data. Suppose that we have five observations over two time periods, t and $t-1$, with two price series, *Prices A*; p_A^t and p_A^{t-1}, and *Prices B*; p_B^t and p_B^{t-1}. In order to test monotonicity in *current prices* we have set $p_B^t > p_A^t$, with the only difference between them being the final observation, which is strictly larger for p_B^t than for p_A^t. The base period prices for both series have been kept fixed, $p_B^{t-1} = p_A^{t-1}$. Suppose we also have information on the value of a single quality characteristic. This data along with the time-dummy variable is shown in Table 3.1.

Table 3.1. Artificial Data Set

Observation Number	Time Period	Prices A (p_A)	Prices B (p_B)	Quality Characteristic	Time-Dummy Variable
1	$t-1$	2	2	3	0
2	$t-1$	5	5	5	0
3	t	6	6	10	1
4	t	8	8	12	1
5	t	9	*10*	15	1

To test *Monotonicity in Current Prices*, we estimate the regression in (3.1) using least squares, for *Prices A* and *B*, and compare the resulting indexes. Using the data in Table 3.1 we estimate a regression of the natural logarithm of price explained by an intercept, the quality characteristic and a time-dummy. The parameter estimates are shown in Table 3.2 along with the price indexes, calculated as the exponent of the time-dummy variable.

Table 3.2. Estimation Results and Price Indexes

Prices	A	B
Intercept	0.63	0.56
Quality Characteristic	0.13	0.15
Time-Dummy	-0.21	-0.33
Price Index	0.81	0.72

As can be seen in Table 3.2, we have the somewhat perplexing result that the price index derived from *Prices B* is actually lower than that derived from *Prices A* even though the period t prices are higher for *Prices B*. This simple example shows that, in general, price indexes constructed using the hedonic regression time-dummy method are non-monotonic (or even weakly monotonic) in current prices – the axioms (3.5) and (3.7) above are violated. We could also construct an analogous example to illustrate that this index method fails monotonicity in base prices.

It is interesting to note here that this result is not an artefact of the linear least squares estimation method which is invariably used to estimate hedonic models. If we estimate the basic model underlying equation (3.1), for the data shown in Table 3.1, using different estimation techniques such as least absolute deviations, then we obtain very much analogous results. This point is developed in the Appendix in Section 3.11.1.

In the simple example above, the price index decreased even though the later period price vector increased. In theory, it is possible that the regression coefficient on the time-dummy variable can increase, decrease or remain constant, as current prices increase. In the next section we take a more detailed look at how the coefficient is estimated in the case of linear least squares and hence why monotonicity fails.

3.6. Monotonicity and Regression Coefficients

The results above indicate that the monotonicity axiom is violated by the hedonic regression time-dummy method. Considering the issue more generally we can illustrate how this problem arises for linear least squares estimation.

To aid illustration let us rewrite the hedonic regression in equation (3.1) above in matrix notation. Here let y be the vector of the dependent variable, the logarithm of price over at least two periods, $T \geq 2$. The full matrix of quality characteristics is represented by Z which also now includes an intercept, and D is the time-dummy vector (or matrix in the case where we have more than two periods). The estimators are represented by the vector β for the intercept and quality variables, and δ for the time-dummy variables.

$$y = Z\beta + D\delta + \varepsilon \tag{3.9}$$

Let us simplify the discussion a little by again assuming just two periods ($T = 2$) are compared in regression (3.9). We can use a fundamental result in econometrics, the Frisch-Waugh Theorem, to write the coefficient estimate of the time-dummy variable in a very simple way. The Frisch-Waugh Theorem tells us that the estimated time-dummy coefficient, $\hat{\delta}$, can be obtained by first estimating a regression of the quality-characteristics and intercept on the time-dummy variable, such as in (3.10) below, and then taking the residuals from this regression, u, and regressing the dependent variable y on these residuals.

$$D = Z\gamma + u \tag{3.10}$$

Then, using the Frisch-Waugh Theorem, we can write the estimated coefficient on the time-dummy variable as the following function of the errors from regression (3.1) and the dependent variable.

$$\hat{\delta} = (u^T u)^{-1} u^T y \tag{3.11}$$

The usefulness of (3.11) comes from the fact that it is linear in the dependent variable. If we differentiate (3.11), with respect to a particular model $i = 1,...,N$ from a particular period $r = t, t-1$, this gives the exact change in the coefficient for a change in the dependent variable due to the linearity of the expression.

$$\frac{d\hat{\delta}}{dy_i^r} = \frac{u_i^r}{u^T u}, \quad i = 1,...,N, \; r = t, t-1 \tag{3.12}$$

From (3.12) it is clear that if u_i^r, the error for model i in period r of the regression (3.10), is negative then the time-dummy coefficient will be decreasing in terms of that particular observation of the dependent variable while if the error is positive it will be increasing for that observation. This is because the denominator of (3.12) is always non-negative, $u^T u \geq 0$, and it will in fact be positive as long as the usual assumption, that there are no perfect linear relationships between the dependent variables, is satisfied in the original regression model (3.9).

In the two-period case, for the current period t and the base period $t-1$, we have the following conditions for the failure of monotonicity.

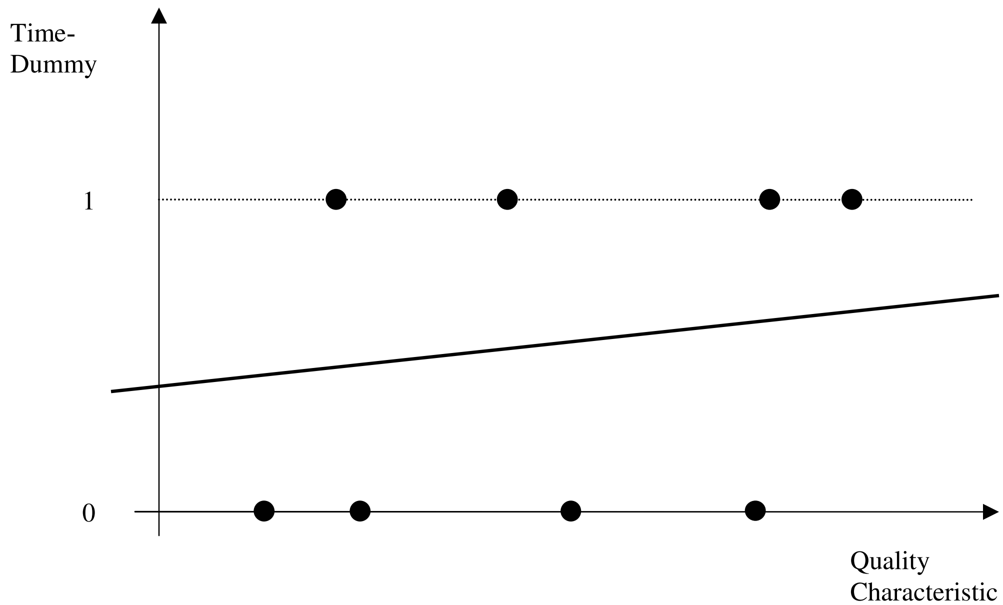
$$u_i^t < 0 \quad \Rightarrow \quad \frac{d\hat{\delta}}{dy_i^t} < 0 \quad \Rightarrow \quad \textit{Monotonicity in Current Prices Fails} \tag{3.13}$$

$$u_i^{t-1} > 0 \quad \Rightarrow \quad \frac{d\hat{\delta}}{dy_i^t} > 0 \quad \Rightarrow \quad \textit{Monotonicity in Base Prices Fails} \tag{3.14}$$

That is, for monotonicity to *fail*, we require that some of the error terms in the regression (3.10) are negative for current-period observations and positive for base-period observations. Clearly, as shown in the previous section, it is possible to construct examples where monotonicity fails but can more be said about the likelihood of this failure? Two general cases are considered below. Let us begin by discussing the case where monotonicity does not fail.

(i). Suppose that there is no relationship of any significance between the quality characteristics and the time-dummy variable. Then the estimated coefficients in the auxiliary regression (3.10) above will be close to zero and the regression intercept will be close to the average of the time-dummy variables. Calculating the error terms from the regression we will find that the errors for all current observations are positive while those for the base period are negative. Here monotonicity will not be violated. This is depicted in Figure 3.1 below for the case where there is a single quality characteristic.

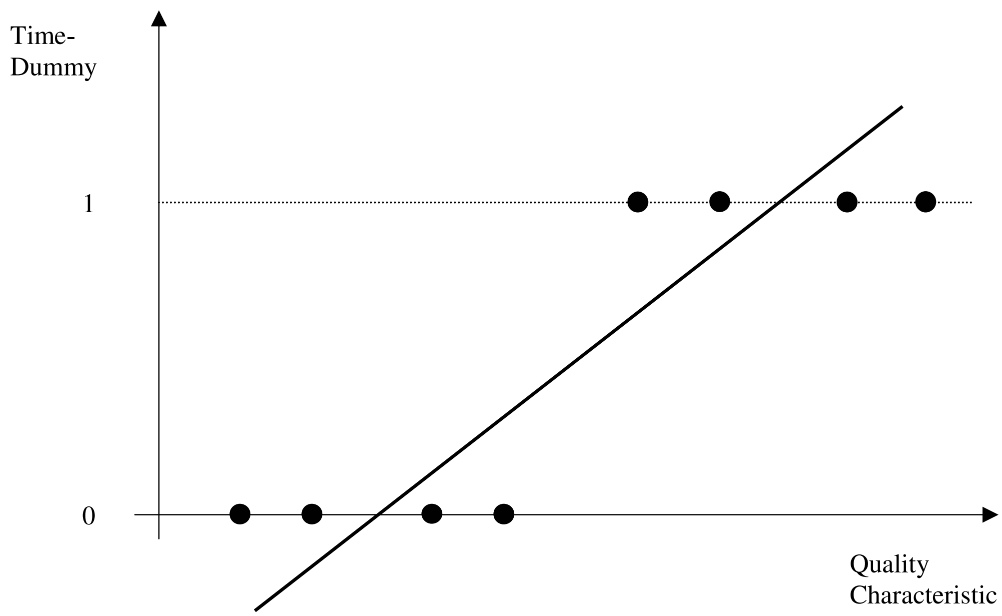
Figure 3.1. Quality Characteristic is Independent of Time



Note that the case depicted in Figure 3.1 seems somewhat unlikely in practice because the primary reason we use the hedonic regression technique to estimate price indexes is because there are systematic changes in the quality of goods over time. If there were not systematic changes in quality then it would be relatively easy to match prices for similar quality models over time and we could apply traditional index methods, in which case a hedonic regression would seem unwarranted. It then seems unlikely that in most of the cases where the hedonic regression time-dummy method is used that the quality characteristics and time would be unrelated. Let us now consider the polar case where monotonicity fails.

(ii). Suppose now that there is some linear relationship between the time-dummy variable and the quality characteristics, which will be reflected in regression (3.10). In general, it is possible that any particular pattern of errors could result. In the regression (3.10), monotonicity will fail if a current period observation lies below the regression hyperplane or a base period observation lies above the hyperplane. In Figure 3.2 we have depicted the case, for just one quality characteristic, where for both base and current-period observations have negative and positive errors implying that monotonicity in current and base prices is violated.

Figure 3.2. Quality Characteristic is Dependent on Time



As can be seen from Figures 3.1 and 3.2, one of the preconditions for the failure of monotonicity is that a relationship exists between the time-dummy variables and the quality characteristics. In practice it is difficult to predict the strength of such a relationship and hence the frequency of failures of monotonicity. To investigate this further we turn to an empirical application.

3.7. An Empirical Example

In this empirical example we use data from the classic Cole et al. (1986) study of computer prices. In particular, we use the *corrected* data on *hard disk drives* available in the textbook by Berndt (1991). This data is composed of 91 observations of list prices on 30 devices marketed by 10 vendors from 1972 to 1984 in the US (Cole et al., 1986, p. 44). We follow Cole et al. (1986) in using the *capacity* (c) and *speed* (s) of the hard drive as quality characteristics. Using this data we check monotonicity for two different applications of the hedonic regression time-dummy method, firstly for a pooled regression over all time periods, and secondly for adjacent-year pooled regressions.

We first look at monotonicity for a regression where we pool the data over all time periods, from 1972 to 1984, and estimate a regression with time-dummy variables for each year, with 1972 normalized to zero. This model was also estimated by Cole et al. (1986) – though with the original data. The results are shown in Table 3.4 of Section 3.11.2 of the Appendix. For the 91 observations, there was a single failure of monotonicity. The time-dummy coefficient for 1984 would have increased if a price observed in 1972, the base period, had increased. This implies a failure of the *Monotonicity in Base Prices Axiom*.

We now turn to the second case where we pool data over adjacent years and include a time-dummy variable for the later year. The results of this regression model are shown in Table 3.5 of Section 3.11.2 in the Appendix. One feature of the results is the instability in the parameter estimates over time, with some negative parameter estimates arising for *speed* in some years. This instability is likely due to there being only a small number of observations available to estimate these adjacent period models. This approach, however, proves illustrative as several failures of monotonicity are observed. The *Monotonicity in Base Prices Axiom* failed for 5 observations in the 1972–73 comparison and 2 observations in the 1977–78 comparison. For the 1972–73 comparison, an increase in the price associated with one of these 5 observations, from 1972, would increase the time-dummy coefficient for 1973. Similarly, in the 1977–78 comparison, a rise in the price for one of the 2 particular observations from 1977 would increase the time-dummy coefficient for 1978. In the 1980–81 comparison, the *Monotonicity in Current Prices Axiom* failed for a single observation. An increase in the price of this observation, from 1981, would lead to a fall in the time-dummy coefficient for 1981.

These two examples of the time-dummy method, applied to the Cole et al. (1986) data set, show that we observe violations of the monotonicity properties not only in artificial examples but in actual data sets. In the next section we consider how seriously we should regard the failure of monotonicity by the hedonic regression time-dummy method.

3.8. Discussion of the Failure of Monotonicity

The previous sections have outlined and illustrated the failure of the monotonicity axioms for the hedonic regression time-dummy method. An important question then is whether the failure of the monotonicity axiom is a serious shortcoming of this approach.

There are a number of well-used indexes that also fail the monotonicity axioms. The most notable casualty is the Tornqvist Index. This and the failure of the Sato-Vartia and Geometric Mean Indexes are discussed in Reinsdorf and Dorfman (1999). An interesting point made by Reinsdorf and Dorfman (1999) is that the failure of these indexes to satisfy the monotonicity axioms may not be the fault of the indexes themselves but of the axioms being applied in a manner which is inconsistent with economic theory. Let us briefly consider their argument.

In addressing the question of the importance of the failure of monotonicity it is useful to discuss the economics of the monotonicity axioms. The axiomatic approach to index numbers stands outside the economic approach nevertheless the monotonicity axioms can be given a strong economic interpretation. To see this consider the cost function from consumer theory defined below as the minimum cost of reaching a particular level of utility, $\overline{U}$, given the price vector, p.

$$C(p,\overline{U}) \equiv \min._x \{p^T x : U(x) \geq \overline{U}\} \tag{3.15}$$

If there are two price vectors, call them p_A and p_B, and $p_B > p_A$ in the sense defined earlier, then the following inequality must hold (see the Appendix, Section 3.11.3).

$$C(p_B,\overline{U}) \geq C(p_A,\overline{U}) \tag{3.16}$$

The economic approach to index numbers defines a price index (termed a cost-of-living index) as the relative cost of reaching a given level of utility under two different price regimes (Diewert, 1976). Dividing (3.16) through by some reference cost function we arrive at (3.17).

$$I(p_B, p_0, \overline{U}) \equiv \frac{C(p_B, \overline{U})}{C(p_0, \overline{U})} \geq \frac{C(p_A, \overline{U})}{C(p_0, \overline{U})} \equiv I(p_A, p_0, \overline{U}) \tag{3.17}$$

It can be seen that an economic price index *must* satisfy the *economic version* of the *Weak Monotonicity in Current Prices Axiom*. With similar reasoning, we could, of course, provide an analogous relationship for the *Weak Monotonicity in Base Prices Axiom*. It is important to note, however, that a cost-of-living index fixes preferences and the required utility level while the quantities of goods consumed are endogenous. However, the conventional formulation of the monotonicity axioms takes quantities as one of the fixed variables. Generally then a test of the monotonicity of an index formula will not be a test of the monotonicity property in the economic sense. This was the problem identified by Reinsdorf and Dorfman (1999) with regard to the Tornqvist, Sato-Vartia and Geometric Mean Indexes and led them to question the monotonicity axiom. In the case of the Sato-Vartia Index, Reinsdorf and Dorfman (1999, p. 57) write,

> If the goal is to estimate a cost of living index, it is the monotonicity axiom rather than the Sato-Vartia index that is suspect. By letting quantities make the same change when prices change to p_1 as they do when prices change to p_1^*, the monotonicity axiom implicitly assumes that commodities are more substitutable in one case than in the other.

The essence of their argument is that the economic application of monotonicity, which fixes preferences, is not the same as the way in which monotonicity is conventionally applied – with fixed quantities. How does this point relate to our finding that the time-dummy method fails monotonicity?

The economic theory of hedonic regressions is well developed (Rosen, 1974; Feenstra, 1995; Diewert, 2003a; Pakes, 2003) though it is far more contentious than the economic theory of index numbers. However, much can be gleaned from this literature. One pertinent feature of the economic models of hedonic regression is the interpretation of the quality-coefficients as the value that the market places upon the various characteristics. In general these values arise out of the optimization process of both buyers and sellers.[31] Given this economic interpretation, we see that the time-dummy method has a dual role in identifying, firstly, the market assessment of the value of characteristics, and secondly, the inflationary component of price change. Drawing on economic models of hedonics it could be argued that we should take account of this dual role when testing the monotonicity axioms. Most notably when we apply the monotonicity axioms we should recognize that the features of the market, which generate the quality characteristic coefficients, have not changed implying that these coefficients should not change either. Note, however, that in Section 3.5 we did not enforce this requirement when we applied the monotonicity axiom – we allowed the estimated coefficient on the quality characteristic to change along with the time-dummy coefficients. Suppose instead we applied monotonicity with the quality characteristic coefficients held fixed. In this case as the relevant price vectors increase the time-dummy will mirror these changes and monotonicity will consequently be satisfied. Looking at the time-dummy method this way, the failure of monotonicity arises in an economic sense because of confusion over whether the price changes are pure price change, and should be ascribed to the time-dummy, or whether they represent changes in the market assessment of characteristics and should be reflected in the quality coefficients.

Despite this interesting economic explanation for the failure of monotonicity, which parallels that of Reinsdorf and Dorfman (1999), the fact still remains that this property seems extremely important from an intuitive perspective. We can construct examples, as we have above, where the price index produced by the hedonic time-dummy method produces perverse and anti-intuitive results. In

[31] The economic theory of hedonic regression was discussed more fully in Chapter 2, Section 2.3.2.

this case it seems warranted to explore alternative hedonic methods where monotonicity does not fail. We briefly outline such a method in the next section.

3.9. An Alternative Hedonic Regression Method

There is one appealing hedonic approach that deserves strong consideration because it satisfies monotonicity and preserves the basic idea of calculating an index directly from estimated regression coefficients. This is called the generalised dummy variable method and is outlined in Diewert (2003a). Here the quality-characteristics space of the good is partitioned into categories – if there are K quality characteristics then a K-dimensional grid is imposed.[32] In the regression equation a dummy variable is used to represent the average log-price level for each of these categories and a time-dummy variable reflects inflationary changes in log-prices over time. This method satisfies monotonicity as the regression of the time-dummy variable on the quality dummy variables [i.e. equation (10) above] must produce non-negative errors for current period observations and non-positive errors for base period observations. This is because the estimated coefficients in this auxiliary regression are bounded between zero and one and as there is only ever a single non-zero variable for each observation the predictions are also bounded between zero and one.[33]

Interestingly, the well-known Country Product Dummy (CPD) method (Summers, 1973) is an example of the generalized dummy variable method that is

[32] Note that if the number of characteristics of the good, and the number of different configurations of these characteristics, is large then the dimension of the grid will be large. For example, if there are two different quality features and the first can take three different values while the latter can take two different values then there are potentially six different categorizations. In general if there are $k = 1,...,K$ characteristics and each characteristic has m_k different configurations then we have a maximum of $m_1 \times m_2 \times ... \times m_K$ different configurations.

[33] In Section 3.11.4 of the Appendix an alternative more formal proof is provided which shows that the Generalised Dummy Variable method satisfies monotonicity when the parameters are estimated using least squares.

applied in making spatial (as opposed to intertemporal) comparisons of prices. For this reason, the CPD method does not fail the monotonicity axioms discussed above.

3.10. Conclusion

This chapter has discussed the performance of the hedonic regression time-dummy method for constructing price indexes with regard to the monotonicity axioms from index number theory. It is shown that this method does not, in general, satisfy any of four versions of the monotonicity axioms. We show analytically how this failure arises for the least squares estimation technique. Moreover, we looked at a well-known historical data set on computer prices and indeed found instances where the monotonicity axioms were violated. The failure of the time-dummy method to satisfy monotonicity, an intuitively appealing criterion, calls into question the use of this method for the construction of price indexes. However, as we have discussed, it could be argued that an economic application of the monotonicity requirements for the time-dummy method implies that we should fix the quality characteristic parameters when we test monotonicity. With this restriction, the time-dummy method will not violate monotonicity. This argument is very much analogous to that given by Reinsdorf and Dorfman (1999) with regard to the failure of monotonicity by some indexes in the context of theory of the cost-of-living index. However, despite this economic explanation for the failure of monotonicity by the hedonic regression time-dummy method there must remain some concern amongst both index theorists and practitioners that such a fundamental requirement for a price index is not satisfied.

3.11. Appendix

3.11.1. Alternative Estimation Techniques

Here we consider whether the results of the numerical example in Section 3.5 are robust to the estimation technique. For the data in Table 3.1, let us compare the estimated parameters obtained using *linear least squares*, as in the text, with other methods of fitting the underlying model. One alternative approach is to apply *linear least absolute deviations* to equation (3.1). Another set of alternative approaches to estimating the parameters of the basic model in equation (3.1) arises if we reformulate the basic functional relationship shown in (3.1) as a nonlinear model as in (3.18) below. Here the *price level* is explained by an exponential function of the parameters with an additive error term.

$$p_i^t = \exp\left(\beta_0 + \sum_{k=1}^{K} \beta_k z_{i,k}^t + \sum_{\tau=2}^{T} \delta_\tau d_{i,\tau}^t\right) + e_i^t, \quad i = 1,\ldots,N,\ t = 1,\ldots,T \qquad (3.18)$$

This nonlinear regression model can be estimated using *nonlinear least squares* and *nonlinear least absolute deviations*.

We compare these three different methods of estimating the parameters of the underlying hedonic model with the results obtained for *linear least squares*. Table 3.3 reports the results of this exercise. Interestingly, these results confirm those of the linear least squares case where we observed that the time-dummy coefficient was lower for *Prices B* than *Prices A*. This indicates that the results are not an artefact of the linear least squares estimation technique.

Table 3.3. Alternative Estimation Results and Price Indexes

	Linear Least Squares (from the text)		Linear Least Absolute Deviations		Nonlinear Least Squares		Nonlinear Least Absolute Deviations	
Prices	A	B	A	B	A	B	A	B
Intercept	0.6327	0.5560	1.2040	1.0986	0.9453	0.8606	1.2040	1.0986
Quality Characteristic	0.1297	0.1488	0.0811	0.1022	0.0830	0.1049	0.0811	0.1022
Time-Dummy	-0.2090	-0.3335	-0.2231	-0.3285	0.0495	-0.1023	-0.2231	-0.3285
Price Index	0.8114	0.7164	0.8000	0.7200	1.0507	0.9027	0.8000	0.7200

3.11.2. Tables of Regression Results from the Empirical Example

Table 3.4. Pooled Regression

The model is estimated by pooling the data on hard disk drives from 1972 to 1984 with 1972 normalized to zero. Estimated coefficients are shown, with standard errors in parentheses, for the regression on Ln(Price).

R^2		0.84
Adjusted R^2		0.81
No. Observations		91
Intercept		9.43 (0.76)
Ln(Speed)		0.39 (0.13)
Ln(Capacity)		0.46 (0.08)
Time-Dummy Variables:	1973	0.02 (0.12)
	1974	-0.22 (0.12)
	1975	-0.31 (0.12)
	1976	-0.42 (0.11)
	1977	-0.42 (0.12)
	1978	-0.57 (0.17)
	1979	-0.77 (0.17)
	1980	-0.96 (0.16)
	1981	-0.97 (0.16)
	1982	-0.95 (0.17)
	1983	-1.10 (0.18)
	1984	-1.18 (0.19)

Table 3.5. Adjacent Period Regressions

The models are estimated by pooling the data on hard disk drives over adjacent time periods with a time-dummy variable included for the later period. Estimated coefficients are shown, with standard errors in parentheses, for the regression on Ln(Price).

Pooled Years		1972-73	1973-74	1974-75	1975-76	1976-77	1977-78
R^2		0.91	0.72	0.67	0.73	0.74	0.83
Adjusted R^2		0.89	0.63	0.57	0.67	0.69	0.78
No. Observations		17	13	14	17	19	14
Intercept		9.62 (1.99)	6.28 (6.62)	6.03 (5.33)	9.10 (1.60)	8.26 (1.17)	7.02 (1.03)
Ln(Speed)		0.53 (0.34)	0.23 (0.78)	0.25 (0.61)	0.44 (0.26)	0.13 (0.22)	-0.28 (0.21)
Ln(Capacity)		0.53 (0.20)	1.00 (0.83)	1.02 (0.70)	0.50 (0.18)	0.44 (0.11)	0.45 (0.09)
Time- Dummy Variables:	1973	-0.08 (0.14)	---	---	---	---	---
	1974	---	-0.23 (0.17)	---	---	---	---
	1975	---	---	-0.09 (0.17)	---	---	---
	1976	---	---	---	-0.13 (0.14)	---	---
	1977	---	---	---	---	0.06 (0.12)	---
	1978	---	---	---	---	---	-0.03 (0.10)

Table 3.5 (contd.). Adjacent Period Regressions

Pooled Years		1978-79	1979-80	1980-81	1981-82	1982-83	1983-84
R^2		0.70	0.87	0.86	0.94	0.98	0.91
Adjusted R^2		0.55	0.81	0.82	0.92	0.97	0.87
No. Observations		10	11	14	14	12	11
Intercept		-0.22 (12.77)	2.02 (2.52)	2.56 (3.56)	6.40 (2.96)	8.87 (1.70)	9.03 (3.61)
Ln(Speed)		-1.52 (1.59)	-1.07 (0.32)	-0.51 (0.48)	0.06 (0.40)	0.45 (0.23)	0.46 (0.50)
Ln(Capacity)		1.03 (1.28)	0.85 (0.26)	0.98 (0.35)	0.63 (0.29)	0.42 (0.16)	0.38 (0.35)
Time- Dummy Variables:	1979	-0.19 (0.06)	---	---	---	---	---
	1980	---	-0.16 (0.03)	---	---	---	---
	1981	---	---	0.01 (0.06)	---	---	---
	1982	---	---	---	0.02 (0.05)	---	---
	1983	---	---	---	---	-0.15 (0.06)	---
	1984	---	---	---	---	---	-0.08 (0.11)

3.11.3. Proof of Equation (3.16)

To see that the inequality shown in (3.16) holds consider the following argument. Suppose that x_A is the consumption bundle that satisfies the cost minimization problem (3.15) when prices are equal to p_A and similarly x_B is optimal given p_B, where $U(x_A) = U(x_B) = \overline{U}$. Then the following chain of inequalities must hold as $p_B > p_A$ and because x_A is optimal given p_A.

$$C(p_B, \overline{U}) = p_B{}^T x_B \geq p_A{}^T x_B \geq p_A{}^T x_A = C(p_A, \overline{U}) \qquad (3.19)$$

3.11.4. Proof that the Generalised Dummy Variable Method Satisfies Monotonicity

Consider the simplest example of the basic Generalised Dummy-Variable method where there is a panel of N goods over 2 time periods, $t = 0,1$. This gives the model below where the log-price is explained by a series of good-specific dummy variables and a time-dummy variable.

$$\ln(p_i^t) = \sum_{\varsigma=1}^{N} \alpha_\varsigma a_{i,\varsigma} + \delta^1 d_i^1 + \varepsilon_i^t, \quad i = 1,...,N, \; t = 0,1 \tag{3.20}$$

The most common way of estimating the parameters of this model is to use least squares which minimises the sum of squared residuals (SSR) shown below.

$$SSR = \sum_{i=1}^{N} (\ln p_i^0 - \alpha_i)^2 + \sum_{i=1}^{N} (\ln p_i^1 - \alpha_i - \delta^1)^2 \tag{3.21}$$

The solutions to the minimization of the SSR can be easily obtained and are shown below.

$$\hat{\delta}^1 = \frac{1}{N} \sum_{i=1}^{N} \ln\left(\frac{p_i^1}{p_i^0}\right) \tag{3.22}$$

$$\hat{\alpha}_i = \frac{(\ln p_i^1 - \hat{\delta}^1 + \ln p_i^0)}{2} \tag{3.23}$$

Clearly from (3.22) it can be seen that monotonicity in both current and base prices will be satisfied. If we differentiate (3.22) then it can be seen that $\hat{\delta}^1$ is increasing in current prices and decreasing in base prices.

$$\frac{\partial \hat{\delta}^1}{\partial p_i^1} = \frac{1}{Np_i^1} > 0 \tag{3.24}$$

$$\frac{\partial \hat{\delta}^1}{\partial p_i^0} = -\frac{1}{Np_i^0} < 0 \tag{3.25}$$

CHAPTER 4

4. Accounting for the Effects of New and Disappearing Goods Using Scanner Data

Abstract*

With the 'discovery' of scanner data by statistical agencies comes a wealth of new information upon which price index calculations can be based. But old problems, such as the appearance and disappearance of goods over time, are likely to be an important feature of such data. However, given that scanner data includes the prices and quantities of the population of transactions we have more information than is traditionally available to deal with the new and disappearing goods problem. We adopt a recently developed approach using the Constant Elasticity of Substitution cost function to provide a detailed empirical analysis of the effects of new and disappearing goods for an Australian scanner data set of supermarket products. Our results indicate that the failure to account for new and disappearing goods in the cost-of-living index leads to a significant upward bias.

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Alfred Marshall, writing in 1887, outlining the problem of new goods and advocating the chain method,

"*This brings us to consider the great problem how to modify our unit so as to allow for the invention of new commodities. The difficulty is insuperable, if we compare two distant periods without access to intermediate times, but it can be got over fairly well by systematic statistics. A new commodity almost always appears at first at something like a scarcity price, and its gradual fall in price can be made to enter year by year into readjustments of the unit of purchasing power, and to represent fairly well the increased power of satisfying our wants which we derive from the new commodity.*"

Marshall (1887, p.209).

4.1. Introduction

In this chapter we discuss and quantify the effects of new and disappearing goods on the cost-of-living index. This is done in the information-rich context of scanner data (sometimes called barcode or point-of-sale data). The availability of this new data source has the potential to greatly improve the way price change is measured as scanner data records the *population* of sales of items in a given store over a given time period. This means that both price and quantity data is available to index practitioners often at a very disaggregated level. This has led many authors to emphasize the advantages of scanner data over the data that is conventionally used by statistical agencies to compute price indexes (Diewert, 1993; Silver, 1995; Bradley et al., 1997; Dalén, 1997; Richardson, 2000; Schut, 2002; Silver and Webb, 2002).

As well as being of great benefit for the compilation of official statistics, scanner data can also be used to investigate enduring economic problems associated with index numbers. One such problem is the effect of new and disappearing goods on price indexes. In this chapter we undertake a detailed empirical investigation, using a large scanner data set, of the effects of non-matched goods on the cost-of-living index. The fact that we have both price and quantity data at a disaggregated level allows us to accurately estimate the differences between indexes which properly account for the effects of new and disappearing goods from those which do not.

In the next section we discuss the basic problem of quality change and new and disappearing goods in the context of the 'supermarket products' which are the focus of this study. Little research has been undertaken on quantifying the effects of new and disappearing goods on this product area. However, a number of methods have been suggested in the economics literature to account for non-matched goods. We briefly discuss three main approaches; estimating reservation prices, hedonic regression and an approach using the Constant Elasticity of Substitution (CES) cost function. We primarily use the last of these methods, which is outlined in greater detail in Section 4.3. As well as this primary approach we use a simplified version of the reservation price method to provide a cross-check on our results. In Section 4.4 we apply these two methods to a large scanner data set and discuss some of the important results. Section 4.5 concludes.

4.2. The Quality Change and New and Disappearing Goods Problem

One of the enduring problems of economic measurement is how to deal with changes in the quality and availability of goods over time. In fact this debate ranges back at least to Alfred Marshall in 1887 (Marshall, 1887, p. 209) who advocated the use of chained indexes to mitigate the effects of new and disappearing goods – as can be seen in the quote at the beginning of this chapter.

From an economic perspective the ideal measure of price change is the cost-of-living index which compares the minimum cost of obtaining a given level of utility under two price regimes. If there are differences in the quality or availability of goods under the two price regimes then this has an effect on utility which must be accounted for in the cost-of-living index (Gordon and Griliches, 1997). Given this goal of economic price measurement, it seems important to have an idea of the influence of new and disappearing goods on welfare. In this chapter we hope to advance empirical research in this area.

4.2.1. Estimates of the Bias from New and Disappearing Goods

The most comprehensive project quantifying the biases in official price indexes was that undertaken by the Boskin Commission (Boskin et al., 1996; Gordon and Griliches, 1997) who looked at the US Consumer Price Index (CPI). The Boskin Commission estimated that quality change and new goods constituted the largest source of bias in the US CPI. In total they estimated that the US CPI was overestimated by 0.6 percentage points in 1996 due to the failure to adequately account for quality change and new goods.

In this chapter we focus on one particular area of the CPI. We look at what we term 'supermarket products', in particular: *Biscuits*, *Bread*, *Butter*, *Cereal*, *Coffee*, *Detergent*, *Frozen Peas*, *Honey*, *Jams*, *Juices*, *Margarine*, *Oil*, *Pasta*, *Pet Food*, *Soft Drinks*, *Spreads*, *Sugar*, *Tin Tomatoes* and *Toilet Paper*. These products provide a selection of the goods available in supermarkets and mainly comprise 'processed food' products. The Boskin Commission did not look at this product area in particular detail, however, they concluded that the "Food at Home other than Produce" category, which covers most of the products above, had an annual upward bias of 0.3 percentage points from 1967 to 1996. The justification given for this bias estimate by Boskin et al. (1996) is interesting. They write,

> How much would a consumer pay to have the privilege of choosing from the variety of items available in today's supermarket instead of being constrained to the much more limited variety available 30 years ago? A conservative estimate of the value of extra variety and convenience might be 10 percent [approx. 0.3 percent annualized] for food consumed at home other than produce…

The noteworthy aspect of the quote from the Boskin Commission is that the primary reason they give for the upward bias of official indexes is the failure to properly account for change in the *variety* of available products. What is important then is the fact that the range of products available in supermarkets has increased substantially over recent decades. As noted by Koskimäki and Ylä-Jarkko (2003, p.

11), this increase in the range of products is likely to be a consequence of monopolistic competitors endeavouring to produce differentiated products so that substitution occurs within brands rather than between brands. The result of this behaviour is that an increasingly large set of niche-marketed products is available to consumers, which has an influence on their welfare and cost-of-living. Hausman (2003, p. 28) called this the “invisible hand of imperfect competition”. In the following sections we briefly discuss various ways of measuring these effects.

4.2.2. Estimating Reservation Prices

A diverse range of approaches have appeared in the economics literature for dealing with new and disappearing goods. The classic approach to the problem is derived from Hicks (1940) who saw it as one of missing prices. His solution for *new goods* was to estimate the reservation (or choke) price which would have driven demand for the good to zero in the period prior to its introduction. The reservation price can be used either in a conventional price index framework, or in a parametric framework, to look at the effect on welfare of the introduction of the good. An analogous approach can be used for disappeared goods.

This ‘reservation price’ method is very appealing and has a rigorous economic justification. Hausman (1997) adopts this approach and econometrically estimates a demand system for the introduction of a new brand of cereal in the US. Hausman (1997) finds that the price index for cereals was too high by between 20 to 25 percent due to the effect of new brands.[34]

While this approach is attractive it has the major disadvantage that it is technically very difficult to implement involving complex econometric estimation. These estimation methods are also contentious and as emphasized by Bresnahan (1997), the discussant on Hausman’s (1997) paper, the assumptions made in motivating the estimation can be important in influencing the results. This has led to

[34] This means that if the price change was 2.0 percent per year then the index would be over estimated by between 0.4 and 0.5 percentage points per year.

some suspicion of this approach. For example, the recent National Research Council report, *At What Price?* (Schultze and Mackie, 2002, p. 159) noted that,

> …there is no clearly acceptable technique for consistently estimating demand curves for new goods or services in such a way that choke prices can be confidently ascertained.

It appears that at present this method is quite controversial and not widely accepted. For this reason we will not adopt this version of the approach.

Recently, however, Hausman (2003) has suggested an alternative *approximate* reservation price method. This method is far simpler than the full econometric method and requires only the estimation of the price elasticity of demand, $\varepsilon_i^t \equiv -d\ln(x_i^t)/d\ln(p_i^t)$, where p_i^t and x_i^t are the price and quantity of good i in period t. Here, instead of using the compensated demand curve which is the theoretically correct approach, we take a linear approximation to the market demand curve. It can easily be shown that in this case the estimated reservation price, $\hat{p}_i^t$, can be calculated using the following formula.[35]

$$\hat{p}_i^t = p_i^t(1+1/\varepsilon_i^t) \tag{4.1}$$

Hausman (2003, p. 27) argues that this estimate of the reservation price provides a reasonable approximation, however, as we typically expect the demand curve to be convex to the origin then (4.1) will underestimate the 'true' reservation price. In the empirical section we use this method.

4.2.3. Hedonic Regression

Another popular approach to dealing with changing varieties of products is hedonic regression. The hedonic approach regards goods as being 'packages' of

[35] To see how this is derived consult Chapter 2, Section 2.4.3.

various utility-yielding characteristics which determine the price. A hedonic regression exploits the market relationship between the prices and characteristics of the good (Rosen, 1974). This approach is useful as it is often the case that the characteristics of a good are more stable than the various varieties (i.e. bundles of characteristics) produced. The hedonic function can be used to estimate the price of a good for any particular combination of characteristics and hence there are a number of ways in which it can be used to calculate price indexes (Silver, 1999; Diewert, 2003b).

Hedonic methods have most frequently been applied to areas where prices have changed rapidly due to technological factors such as computers (Berndt, Griliches and Rappaport, 1995; Berndt and Rappaport, 2001). It has not (to the best of my knowledge) been applied to supermarket commodities like those listed above. The reason for this is that hedonic methods will not measure the effects of changes in variety but this is just the aspect of the problem we are interested in, as emphasized by the Boskin Commission.

The hedonic regression approach to quality change and new and disappearing goods focuses entirely on how changes in prices relate to changes in characteristics where the characteristics are relatively stable across time. However, in our case, as emphasized by the quote from the Boskin Commission above, it is not a problem of accounting for *improvements* in the characteristics of products but rather one of accounting for the expansion in the *range* of available characteristics. Hedonic methods as presently constituted are not able to reflect these changes. To see this consider a case where prices for different varieties and characteristics are unchanging through time but an ever expanding range of varieties and characteristics is available. As long as some of these new varieties are desirable then the cost-of-living index should fall even though prices have not changed. The hedonic method will clearly not account for these changes. For this reason we will not explore this method further here and will instead turn to our primary method of accounting for new and disappearing goods. We outline this approach in more detail in the following section.

4.3. The CES Cost-of-Living Index with New and Disappearing Goods

In this chapter we primarily adopt a method of more recent vintage than the two alternatives discussed above. This method was initially proposed by Feenstra (1994), and developed, extended and refined by Nahm (1998) and Balk (1999). It is able to rigorously account for the effects of new and disappearing goods in a relatively simple framework. There have been only limited applications of this approach in the literature (Feenstra and Shiells, 1994; Haan, 2001; Opperdoes, 2001). Let us outline this method.

We consider the case of two periods, $t = 0,1$, where we denote the index set of goods available in each period by I^0 and I^1. We will also make use of the index set of goods which is common to both periods, $I^{1,0} \equiv I^1 \cap I^0$, $\overline{U}$ is some reference utility level, p^1 and p^0 are the price vectors, b_i are quality or taste parameters and σ is the elasticity of substitution, $\sigma \equiv -d\ln(x_i^t / x_j^t) / d\ln(p_i^t / p_j^t)$ for some goods i and j. The elasticity of substitution represents the extent to which consumers change their relative consumption of goods as relative prices change. It must be non-negative in order for consumers' (compensated) demand curves not to slope upwards.[36] With this terminology we can introduce the CES cost function over a changing domain of goods.

$$C(p^t, \overline{U} \mid I^t) = \left(\sum_{i \in I^t} b_i \, (p_i^t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \overline{U}, \qquad t = 0,1 \tag{4.2}$$

Note that when we adopt (4.2) the cost-of-living index will reflect not only price change but also changes in the availability of goods or consumption opportunities represented by I^0 and I^1. What is important is that we can represent the cost-of-living index exactly for the CES cost function over a changing set of goods. As

[36] To see this consult Section 4.6.1 of the Appendix.

Feenstra (1994) and Balk (1999) demonstrated the cost-of-living index has the following form where $\hat{P}$ is a price index over matched goods, $I^{1,0}$.[37]

$$\frac{C(p^1,\overline{U} \mid I^1)}{C(p^0,\overline{U} \mid I^0)} = \left(\frac{1-\sum_{i\in I^1, i\notin I^{1,0}} s_n^1}{1-\sum_{i\in I^0, i\notin I^{1,0}} s_n^0} \right)^{\frac{-1}{1-\sigma}} \times \hat{P}, \quad s_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i\in I^t} p_i^t x_i^t}, \; t = 0,1 \qquad (4.3)$$

The form of the cost-of-living index is relatively straightforward, it is calculated as a matched-goods price index $\hat{P}$ which is adjusted by a factor reflecting relative expenditure on new and disappeared goods, and the elasticity of substitution. The intuitive explanation for the form of the adjustment factor is that the expenditure shares for new and disappeared goods reflect their *importance* to consumers. The adjustment factor then compares the relative gain from new goods and the loss from disappeared goods and adjusts this ratio using the elasticity of substitution. It is interesting to note that no adjustment to the matched-goods price index occurs when, either, the expenditure shares on new and disappeared goods are equal, indicating that relative gains in consumption opportunities were equivalent to the losses, or as $\sigma \to \infty$, in which case all goods are very close substitutes and whether new goods appear or old goods disappear does not matter in terms of consumption opportunities.

Balk (1999) showed that the matched-goods price index, $\hat{P}$, had various representations. We will use three of these price indexes below. The first representation of $\hat{P}$ is the well known Lloyd-Moulton or Base-Weighted Price Index (P^{BW}).

[37] See Section 4.6.2 of the Appendix for a derivation of this result and those that follow. A slightly different representation of the cost-of-living index is presented.

$$\hat{P} = \left(\sum_{i \in I^{1,0}} \hat{s}_i^0 \left(\frac{p_i^1}{p_i^0} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \equiv P^{BW}, \quad \hat{s}_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i \in I^{1,0}} p_i^t x_i^t}, \; t = 0,1 \tag{4.4}$$

This index dates back to Lloyd (1975) and has attracted attention for its ability to reflect consumer's substitution behaviour while only requiring knowledge of base period expenditure shares.[38] It can be seen that the Lloyd-Moulton Index is equal to the Lapeyres Index when $\sigma = 0$. Furthermore, if we regard the Lloyd-Moulton Index as a function of independent variables then it can be shown that it is monotonically decreasing in σ (Hardy, Littlewood and Polya, 1952, p. 26, Th. 16) and by appropriate choice of σ it can produce any number between and including the maximum and minimum price relatives.

The second representation of $\hat{P}$ that we use is the equivalent current-weighted expression to (4.4). We call this index the Current-Weighted Price Index (P^{CW}) again first discussed by Lloyd (1975) and defined in (4.5) below. Note that when $\sigma = 0$ this index is equal to the Paasche Price Index.

$$\hat{P} = \left(\sum_{i \in I^{1,0}} \hat{s}_i^1 \left(\frac{p_i^1}{p_i^0} \right)^{-(1-\sigma)} \right)^{\frac{-1}{1-\sigma}} \equiv P^{CW} \tag{4.5}$$

Finally, the Sato-Vartia Price Index (P^{SV}) can also be derived from the CES functional form and is shown in (4.6). The weights for the Sato-Vartia Price Index

[38] This index was used by Shapiro and Wilcox (1997), for US data, to investigate whether a real-time price index could be calculated which reflected substitution. Shapiro and Wilcox (1997, p. 123) concluded that using the Lloyd-Moulton Index, "…it is possible to produce an approximation to the Tornqvist index that is both feasible in real time and quite accurate."

are rather complex and involve the normalized logarithmic mean of the expenditure shares in each period.[39]

$$\hat{P} = \prod_{i \in I^{1,0}} \left(\frac{p_i^1}{p_i^0} \right)^{\hat{w}_i^{1,0}} \equiv P^{SV} , \qquad \hat{w}_i^{1,0} = \frac{L(\hat{s}_i^1, \hat{s}_i^0)}{\sum_{i \in I^{1,0}} L(\hat{s}_i^1, \hat{s}_i^0)} \tag{4.6}$$

Interestingly, note that the Sato-Vartia Price Index does not depend on the elasticity of substitution, σ. This is important for later purposes.

4.3.1. A Restriction on the Elasticity of Substitution

A vital point to note regarding this approach to calculating the cost-of-living index is that the elasticity of substitution must be greater than one, $\sigma > 1$. Balk (1999) showed this by considering an example where $p_i^1 = p_i^0 \ \forall i \in I^{1,0}$ and where we have some newly appeared goods but no disappearing goods. Then using (4.3), and noting that under these assumptions the matched price index will equal one, the cost-of-living index for this particular case is shown below.

$$\frac{C(p^1, \overline{U} \mid I^1)}{C(p^0, \overline{U} \mid I^0)} = \left(1 - \sum_{i \in I^1, i \notin I^{1,0}} s_i^1 \right)^{\frac{-1}{1-\sigma}} \tag{4.7}$$

But this index must be no larger than 1 as the consumer now has a greater range of goods to choose from. It can be seen that this implies that we must have $\sigma > 1$.

[39] The logarithmic mean $L(a,b)$ is defined as $L(a,b) = (a-b)/(\ln a - \ln b)$ where $a \neq b$ and $L(a,a) = a$. Clearly we must have $a,b > 0$.

a. A Discussion of the Restriction on σ

Why do we have this restriction on the elasticity of substitution? Consider the following optimization argument. The cost function, by definition, is the minimum expenditure required to achieve a given level of utility. However, in looking at the effect of new and disappearing goods we are defining a *restricted* cost function where the consumption of some goods is constrained to zero in some periods. We can then write the modified cost function for period $t = 0,1$ in the following way.

$$C(p^t, \overline{U} \mid I^t) = \min{}_x \sum_{i \in I^1 \cup I^0} p_i^t x_i \qquad (4.8)$$

$$s.t. \quad U(x) \geq \overline{U}$$

$$x_i = 0, \ \forall i \notin I^t$$

However, this definition may cause problems if there are some goods i which are essential to consumption but are not common to both periods (i.e. where $i \notin I^{1,0}$). In this case it may be impossible to reach the reference utility level without some consumption of these goods and the constraints in the optimization problem may define a feasible set which is empty. To see that this is indeed the case for the CES functional form we can derive the CES utility function which is dual to the CES cost function.

$$U(x^t \mid I^t) = \left(\sum_{i \in I^t} b_i^{\frac{1}{\sigma}} (x_i^t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad t = 1,0 \qquad (4.9)$$

From inspection of (4.9) we can see that if $\sigma \leq 1$ then every good is essential to consumption, as was noted by Feenstra (1994). It is only when $\sigma > 1$ that the consumption of a good can equal zero without utility being either undefined or zero.

Economically this means that if $\sigma > 1$ then consumers can be compensated for the restricted (zero) consumption of some goods by increases in the consumption of other goods. This ability to compensate the consumer for the loss of some goods is vital in obtaining sensible answers to the effect of new and disappearing goods on the cost-of-living. If no compensation is possible then the cost-of-living index will be infinite if one of these 'essential' goods is lost. It seems reasonable that at the elementary level of aggregation where we will apply this theory that all goods are effectively replaceable. This is clearly not so plausible at higher levels of aggregation. Consider for example the goods 'food' and 'clothing'. Clearly if our consumption of these goods were restricted to zero then this certainly would be catastrophic for utility.

4.3.2. Estimating the Elasticity of Substitution

As the adjustment for new and disappearing goods shown in (4.3) depends on the elasticity of substitution we need to estimate this parameter to implement this approach in practice. Fortunately Balk (1999) outlined various ways in which the elasticity of substitution could be easily estimated. The basic idea of his approach is that all the CES matched-goods price indexes, (4.4) – (4.6), should be equal. This gives us three methods for estimating the elasticity of substitution.

The first method used to obtain $\hat{\sigma}$, an estimate of σ, is to find the value of $\hat{\sigma}$ which makes the Base and Current-Weighted Price Indexes equal as in (4.10) below.

$$\hat{P}^{BW} \equiv \left(\sum_{i \in I^{1,0}} \hat{s}_i^0 \left(\frac{p_i^1}{p_i^0} \right)^{1-\hat{\sigma}} \right)^{\frac{1}{1-\hat{\sigma}}} = \left(\sum_{i \in I^{1,0}} \hat{s}_i^1 \left(\frac{p_i^1}{p_i^0} \right)^{-(1-\hat{\sigma})} \right)^{\frac{-1}{1-\hat{\sigma}}} \equiv \hat{P}^{CW} \tag{4.10}$$

We will call this method the *Current v Base* method. It is particularly appealing as $\hat{\sigma}$ will be positive as long as the Laspeyres Index exceeds the Paasche Index. This is because, when $\hat{\sigma} = 0$, the LHS of (4.10) is equal to the Laspeyres Index while the RHS is equal to the Paasche Index. To lower the LHS and raise the RHS of (4.10) we increase $\hat{\sigma}$ (Hardy, Littlewood and Polya, 1952, p. 26, Th. 16) until equality is obtained.

The second and third methods suggested by Balk (1999) is to equate the Base and Current-Weighted Price Indexes, which both include the elasticity parameter, to the Sato-Vartia Price Index, which is independent of the elasticity of substitution.

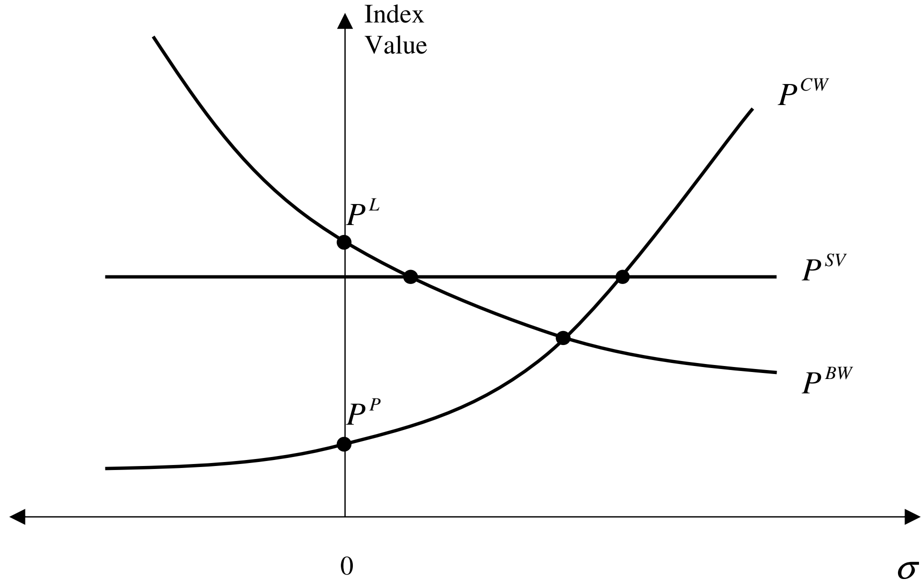
$$\hat{P}^{BW} \equiv \left(\sum_{i \in I^{1,0}} \hat{s}_i^0 \left(\frac{p_i^1}{p_i^0} \right)^{1-\hat{\sigma}} \right)^{\frac{1}{1-\hat{\sigma}}} = P^{SV} \tag{4.11}$$

$$\hat{P}^{CW} \equiv \left(\sum_{i \in I^{1,0}} \hat{s}_i^1 \left(\frac{p_i^1}{p_i^0} \right)^{-(1-\hat{\sigma})} \right)^{\frac{-1}{1-\hat{\sigma}}} = P^{SV} \tag{4.12}$$

This could potentially produce a negative estimate of σ even when the Laspeyres Index is greater than the Paasche Index as the Laspeyres and Paasche Indexes do not bound the Sato-Vartia Index.

To see how these three methods are related consider Figure 4.1 below which depicts the three indexes in σ-space.

Figure 4.1. Estimation of the Elasticity of Substitution



Interestingly, it can be seen that the two methods that use the Sato-Vartia Index give an estimate of σ which lies either side of that from the *Current v Base* method. For this reason it seems advisable to take an average of the two Sato-Vartia methods. However, the form of the average may influence the resulting estimate of σ. For this reason, and the fact that $\hat{\sigma}$ will be positive as long as the Laspeyres Index exceeds the Paasche Index, we prefer the *Current v Base* method, though we will consider both in the empirical section that follows.

4.4. An Empirical Application Using Scanner Data

Now that we have discussed the theory surrounding our primary approach to new and disappearing goods we can proceed to the application of these ideas to our scanner data set.

The data used in this study was purchased by the Australian Bureau of Statistics (ABS) for the purpose of investigating the use of scanner data in the Australian CPI. The data set includes observations from the start of February 1997 to the end of April 1998, 65 weeks of data in total, for 19 product categories as listed above: *Biscuits*, *Bread*, *Butter*, *Cereal*, *Coffee*, *Detergent*, *Frozen Peas*, *Honey*, *Jams*, *Juices*, *Margarine*, *Oil*, *Pasta*, *Pet Food*, *Soft Drinks*, *Spreads*, *Sugar*, *Tin Tomatoes* and *Toilet Paper*. These products represent a selection of the goods available in supermarkets and mainly comprise processed food items. The data set includes 100 stores belonging to four supermarket chains in one of the cities of Australia.[40] These stores accounted for around 80 percent of grocery sales in this city (Jain and Caddy, 2001, p. 4). The total value of sales for these products over the 65 week period was just over AU$600 million.

4.4.1. Aggregation Methods and Other Issues

The data basic form of the data was weekly unit-value prices, and the corresponding sales volume, for a *product code* in an *outlet*.[41] In order to ensure the robustness of the results various aggregation approaches were applied to derive the prices and quantities to be used in the index formulas. This is in the context of much research on scanner data which has have shown that the method of aggregation is often very important (Dalén, 1997; Reinsdorf, 1999; Jain and Caddy, 2001; Silver and Webb, 2002; Triplett, 2003; Koskimäki and Ylä-Jarkko, 2003).

Both *quarterly* and *monthly* aggregation of the weekly unit values was undertaken.[42] Additionally, we used both unique *product code* and *outlet* combinations as the definition of a good as well as aggregating *product codes* across *outlets*. It was anticipated that this latter approach would increase the extent

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[40] Note that while we know that the stores were from four supermarket chains, for commercial sensitivity reasons, we were not able to determine which supermarket came from which chain. This reduced the range of aggregation approaches that could be pursued.

[41] Formally the *product code* is called the Australian Product Number (APN) which is the equivalent of the Universal Product Code (UPC) in the US or the European Article Number (EAN).

[42] Producing price indexes at a quarterly and monthly frequency is standard international practice and it is consistent with the new ILO CPI Manual (ILO, 2004, p. 358), "…it is recommended that the index number time period be at least 4 weeks or a month." It should be noted, however, that one of the potential benefits of scanner data is that price indexes could be calculated more frequently. Feenstra and Shapiro (2001) have made some progress in this regard.

of matching and mitigate the effects of new and disappearing goods (Reinsdorf, 1999, p. 153). These various aggregation methods give four different approaches in total. At a monthly frequency we have *Month (Prod. Code, Outlet)*, which uses unique *product code* and *outlet* combinations and *Month (Prod. Code)* which uses only the *product code* as the definition of a good and aggregates over *outlets*. The corresponding quarterly indexes are *Quarter (Prod. Code, Outlet)* and *Quarter (Prod. Code)*. For each of these aggregation methods we implement the approach above using chained indexes. The primary reason for this is that chained indexes are more likely to mitigate the effects of new and disappearing goods because there is greater overlap in the goods available for time-periods which are adjacent than those that are more distant.[43] With these details out of the way we can proceed to the results of the empirical application. We start by discussing the estimation of the elasticity of substitution.

4.4.2. Estimating the Elasticity of Substitution

The results of the estimation of the elasticity of substitution are shown in Table 4.3 in Section 4.6.1 of the Appendix. We focus on the *Current v Base* method with the average and standard deviation of the estimated elasticity of substitution shown for each product category and aggregation method. However, what is interesting is that the difference between the *Current v Base* method and the *Average Sato-Vartia* method is relatively minor. This can be seen in the second part of Table 4.3 showing the average of absolute deviations between these two methods. These differences are relatively small compared with the volatility of the elasticity across time represented by the standard deviations of the estimates.

One interesting aspect of estimating the elasticity of substitution is the effect of aggregation. As we would have expected a priori, when we increase the level of aggregation the elasticity of substitution falls. What is notable, however, is that

[43] This is emphasized in ILO (2004, pp. 129-130). However, it should be noted that if there are seasonal products then adjacent periods may not be more similar than say periods separated by a year. Fortunately our data set does not include any products with strong seasonal patterns.

aggregation across time, from monthly to quarterly indexes, led to a far greater reduction in the elasticity than did aggregation across outlets. In a somewhat contradictory result a larger number of *negative*, and hence implausible, estimates of the elasticity of substitution occurred when we aggregated across outlets. These aggregation issues are discussed further below. We now move onto the effects on new and disappearing goods on the cost-of-living index but first discuss the estimate of the elasticity of substitution which we have used in the results in the following section.

In applying the adjustment for new and disappearing goods discussed above we used an estimate of the elasticity of substitution derived from the *Current v Base* method for each time period. However, when the elasticity of substitution fell below one we instead used the average estimate over all time periods. In the unusual case where the average estimated elasticity of substitution over all time periods was less than one, for the *Current v Base* method, we did not undertake the adjustment for new and disappearing goods.[44] With an estimate of the elasticity of substitution in hand we can now examine the effects of new and disappearing goods on the cost-of-living index.

4.4.3. The Effect of New and Disappearing Goods on the Cost-of-Living

The results of the application of the CES cost function approach to the problem of new and disappearing goods are startling and shown in Table 4.1. For each of the aggregation methods and for almost all of the goods, 69 out of 73, the matched-goods price index lies above the price index which reflects new and disappearing goods. The extent of the bias differs by aggregation method and product category, on average over all goods and aggregation methods the matched-goods price index was upwardly biased by 2.3 percentage points. The range of bias for the different aggregation methods varied from around 1.5 to 3 percent over the 65 week period or around 1.2 to 2.4 percent annually. This is significantly larger than the estimate by the Boskin Commission (Boskin et al., 1996, Tab. 2)

[44] This only occurred for *Jams*, *Pasta* and *Sugar* for the *Quarter (Prod. Code)* aggregation method.

mentioned earlier of an upward bias of 0.3 percentage points each year for the "Food at Home other than Produce" category.

The interesting feature of these results is that they imply a sizeable bias for the matched-goods method despite there being a large overlap of expenditures on common goods. As can be seen in Table 4.4 the average proportion of expenditure on *new* and *disappeared* goods is relatively small, usually less than 1 or 2 percent of total expenditure. However, as was noted in Section 4.3 it is the relative, not absolute, sizes of expenditure on new and disappeared products that is important. For the matched-goods price index to be upwardly biased it must be the case that expenditure on new goods is consistently larger than expenditure on disappeared goods. Indeed this seems to be a very strong empirical regularity in our data set. In the final section of Table 4.4 we show the percentage change in an index of the relative expenditure between current and base periods, on those goods which are common to both periods.[45] For all but 3 of our 76 comparisons, these indexes fell, and often quite significantly. It is this empirical regularity which is the driver of our estimate of an upward bias from omitting new and disappearing goods. However, while this is a strong feature of the data used in this study it may not arise in all such data sets. For example, in Reinsdorf's coffee data (Reinsdorf, 1999, p. 155, Tab. 3) there seems to be no systematic difference between the expenditure on new and disappeared varieties of coffee. In contrast in a scanner data study by Dalén (1997, p. 2, Tab. 1), which included data for four products categories, we do in fact see strong evidence that the proportion of expenditure on new goods is larger than that on disappeared goods.

An interesting question is, what is causing the disparity between expenditure on new and disappeared goods? One explanation is that there is an ever increasing number of products so that the number of newly introduced goods exceeds the number of goods withdrawn from the market. If this is the case then we would typically expect expenditure shares to follow a similar pattern. Table 4.5 compares the number of products available in each product category in the first and last of the time periods for each aggregation method. The results show that for most of the

[45] Note that this is a chained index of the adjustment in (4.3) without the elasticity exponent.

product categories the number of varieties of goods increased over time. However, this appears to be only a partial explanation. For example, consider the case of *Soft Drinks* where there were sizeable reductions in the product range over the period despite an upward bias in the matched-goods index for 3 out of 4 aggregation methods. This may indicate that complex factors, such as consumers' desire for variety, may be at play.

4.4.4. A Comparison with the Approximate Reservation Price Method

As outlined in Section 4.2.2 above an alternative method for determining the influence of new and disappearing goods on the cost-of-living index is to estimate reservation prices. It is interesting to compare the results from the *CES Cost Function Method* with the *Approximate Reservation Price Method*. In order to apply this latter method we require an estimate of the price elasticity of demand, ε_i^t, from (4.1). To ensure comparability with the *CES Cost Function Method* we used an estimate of ε_i^t derived from the CES functional form, $\hat{\varepsilon}_i^t = \hat{\sigma}(1 - s_i^t)$, where $\hat{\sigma}$ is the estimated elasticity of substitution and s_i^t is the expenditure share of the good.[46] Then in order to estimate the reservation price of a good i which is new in period 1 and hence absent in period 0 we use (4.1) to obtain $\hat{p}_i^1$ and then note that if ε_i^t is fixed over time (i.e. $\varepsilon_i^1 = \varepsilon_i^0$) then it can be shown that $\hat{p}_i^1 = \hat{p}_i^0 /(p_i^1 / p_i^0)$. However, the good i is new so p_i^0 does not exist in which case we use the overall price index to represent p_i^1 / p_i^0.[47] This price is then used in an index formula in a conventional fashion. A similar method is used for goods which were available in period 0 and disappeared in period 1.

In determining the effect of new and disappearing goods using this approximate reservation price method we compare a matched-goods Tornqvist

[46] To see this is indeed correct for the CES functional form see the Appendix, Section 4.6.1.

[47] That is, we use $\hat{p}_i^1 = \hat{p}_i^0 / P^{AT}$ to derive $\hat{p}_i^1$ where P^{AT} is defined in (4.14).

Price Index (P^T), shown in (4.13), with an Augmented Tornqvist Price Index (P^{AT}) which reflects new and disappearing goods, (4.14).

$$P^T \equiv \prod_{i \in I^{1,0}} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{\hat{s}_i^0 + \hat{s}_i^1}{2}} \tag{4.13}$$

$$P^{AT} \equiv \prod_{i \in I^{1,0}} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0 + s_i^1}{2}} \times \prod_{i \in I^0, i \notin I^{1,0}} \left(\frac{\hat{p}_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \times \prod_{i \in I^1, i \notin I^{1,0}} \left(\frac{p_i^1}{\hat{p}_i^0} \right)^{\frac{s_i^1}{2}} \tag{4.14}$$

The difference between the two indexes is that the Augmented Tornqvist Price Index includes the effect of new and disappearing goods through the use of the estimated reservation prices.

The results of this exercise are informative and help to reinforce our strong suspicion that the matched-goods price index is upwardly biased. The point estimate of the bias from this approach, over all product categories and aggregation methods, is 0.6 percentage points over the 65 weeks. The estimate ranges between 0.3 to 0.8 percentage points for the different aggregation methods. Interestingly, as suspected the linearization of the demand curve has led to an estimate of the bias from new and disappearing goods which is significantly less than that for the CES Cost Function Approach. Nevertheless, this 'conservative' estimate still implies significant bias, of the order of 0.4 to 1 percentage points annually. Let us now briefly turn to a somewhat different topic, the question of the effects of aggregation and index formula on the matched-goods price indexes.

4.4.5. Aggregation, Price Change and Index Formula

One of the most interesting and challenging features of scanner data is the sensitivity of the index numbers to the index formula and aggregation method. Because scanner data is so disaggregated, very large substitution effects are often observed in the data. This can best be seen by looking at the Paasche–Laspeyres spread which is shown in Table 4.2. The size of the differences between the Paasche and Laspeyres Indexes are large, the biggest reaching almost 230 percent for *Margarine* using *Month (Prod. Code, Outlet)* aggregation. What is interesting is that the Paasche–Laspeyres spread falls the most when we aggregate across time rather than across outlets. This reinforces what we found for our estimates of the elasticity of substitution. It appears that aggregation across time is more important than aggregation across outlets in reducing observed substitution effects.

Given that the Paasche–Laspeyres spread is large we emphasize, as have many other studies of scanner data (Dalén, 1997; Haan and Opperdoes, 1997; Reinsdorf, 1999), that the use of a superlative price index, which treat the periods being compared symmetrically, is essential. However, is the choice of superlative index formula important? In Table 4.2 we show the Fisher Price Index, the Tornqvist Price Index and the semi-superlative Sato-Vartia Price Index. The results from these indexes are fairly similar indicating that once the type of aggregation is chosen the choice of superlative index formula is not too important. A more problematic question is, what is the appropriate form of aggregation?

Here we argue that in undertaking aggregation one important criterion that should be considered in checking whether an aggregation method is valid is whether it produces economically meaningful results. That is, are the aggregated prices and quantities related in ways which are consistent with economic theory? One useful criterion for checking the economically meaningfulness of results is to see whether the estimated elasticities of substitution are non-negative. In terms of our framework this is equivalent to ensuring that the Laspeyres Index exceeds the Paasche Index. Indeed this is a fairly general test as under the assumption of cost minimization and stable homothetic preferences, which over short time spans seems reasonable, the Laspeyres (Input) Price Index should always exceed the Paasche (Input) Price Index.

This approach is useful in checking the plausibility of our aggregation methods as well as those used by others. Negative estimates of the elasticity of substitution arose in our study infrequently and only when we aggregated across outlets. This may indicate that the aggregation of product codes across different outlets is not reasonable. Consumers may not regard the same product code in a different store as very closely comparable and hence the normal economic rules relating the aggregated prices and quantities may breakdown. One of the most comprehensive recent papers on aggregation is that of Koskimäki and Ylä-Jarkko (2003), who use 16 different aggregation strategies in the calculation of Fisher and Laspeyres Price Indexes.[48] At the highest level of aggregation along both the product and spatial dimension the Fisher Price Index routinely exceeds the Laspeyres Index indicating that either very unusual economic behaviour is taking place or the aggregation strategy is inappropriate. In summary, while this approach to evaluating an aggregation method gives the problem some structure a great deal more research is required in this area before we can be more confident about the appropriate aggregation approaches.

4.5. Conclusion

The main purpose of this chapter has been to quantify the effects of new and disappearing goods on the cost-of-living index using a scanner data set. To this end we primarily adopted a particular approach to the measurement of this effect based on the CES cost function. The advantage of this approach as opposed to alternative methods, such as hedonic regression or the estimation of reservation prices, is that very little has to be estimated. Using the *CES Cost Function Approach* all that we require is an estimate of the elasticity of substitution which can be relatively easily obtained. These estimates were then used to determine the effects of new and disappearing goods on the cost-of-living index.

Most significantly, our results show that the matched-goods price index is upwardly biased due to the systematically larger expenditure on new goods than on disappeared goods. This upward bias appears to be larger than previously thought and on average is between 1.5 and 3 percent over the 65 week period under study. In annualized terms this amounts to an upward bias of 1.2 to 2.4 percent. Our use of

[48] This paper has been recently cited and discussed in the ILO Manual (ILO, 2004, pp. 359-360).

an approximate reservation price method confirmed these results though they indicated a smaller, but nevertheless still significant, upward bias for the matched-goods price index. A bias of this magnitude is too large to ignore and shows that the matched-goods approach is inadequate in a dynamic economic environment where the range and variety of products is constantly changing.

4.6. Appendix

4.6.1. The Elasticity of Substitution and the Demand Function

Here we briefly show that, for the CES functional form, we must have $\sigma \geq 0$ for the compensated demand function not to slope upwards. We also derive the form of the price elasticity of demand, ε_i^t. Using Shephard's Lemma we can derive the compensated demand curve.

$$x_i^t(p^t, \overline{U} \mid I^t) = \frac{\partial C(p^t, \overline{U} \mid I^t)}{\partial x_i^t} = b_i \, (p_i^t)^{-\sigma} \left(\sum_{i \in I^t} b_i \, (p_i^t)^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} \overline{U} \qquad (4.15)$$

Let us differentiate $x_i^t(p^t, \overline{U} \mid I^t)$ in order to determine the slope of the compensated demand function. With a bit of manipulation and using some of the definitions above we obtain the following expression.

$$\frac{\partial x_i^t(p^t, \overline{U} \mid I^t)}{\partial p_i^t} = \sigma \frac{x_i^t}{p_i^t}(s_i^t - 1) \qquad (4.16)$$

Given that x_i^t, p_i^t and s_i^t are positive with $s_i^t \leq 1$ we see that for the derivative to be non-positive, that is for the demand curve not to slope upwards, we must have $\sigma \geq 0$. Also from (4.16) we can easily see the form of the price elasticity of demand for the CES functional form.

$$\varepsilon_i^t = -\frac{\partial x_i^t(p^t, \overline{U} \mid I^t)}{\partial p_i^t} \frac{p_i^t}{x_i^t} = -\sigma(s_i^t - 1) \qquad (4.17)$$

4.6.2. Deriving the CES Cost-of-Living Index

The cost-of-living index over changing domains of goods for the CES cost function has the form shown below.

$$\frac{C(p^1,\overline{U} \mid I^1)}{C(p^0,\overline{U} \mid I^0)} \equiv \left(\frac{\sum_{i \in I^1} b_i \, (p_i^1)^{1-\sigma}}{\sum_{i \in I^0} b_i \, (p_i^0)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{4.18}$$

Following Balk (1999) let us briefly show how the exact CES cost-of-living index can be calculated. We will make use of Shephard's Lemma which, when applied to the CES cost function, gives the relationship shown in (4.19) between observable expenditure shares and the unobservable parameters of the cost function.

$$s_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i \in I^t} p_i^t x_i^t} = \frac{b_i \, (p_i^t)^{1-\sigma}}{\sum_{i \in I^t} b_i \, (p_i^t)^{1-\sigma}}, \qquad i \in I^t, \; t = 0,1 \tag{4.19}$$

Let us also define the expenditure shares over the set of matched goods, denoted by $\hat{s}_i^t$. Using (4.19) above it can be shown that the following relationship holds between the expenditure shares for matched goods and the parameters of the cost function.

$$\hat{s}_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i \in I^{1,0}} p_i^t x_i^t} = \frac{b_i \, (p_i^t)^{1-\sigma}}{\sum_{i \in I^{1,0}} b_i \, (p_i^t)^{1-\sigma}}, \qquad i \in I^{1,0}, \; t = 0,1 \tag{4.20}$$

Using these equations we are now able to derive the exact cost-of-living index over a changing domain of goods.

$$\frac{C(p^1,\overline{U} \mid I^1)}{C(p^0,\overline{U} \mid I^0)} = \left(\frac{\sum_{i \in I^1} b_i \, (p_i^1)^{1-\sigma}}{\sum_{i \in I^0} b_i \, (p_i^1)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{4.21}$$

$$= \left(\frac{\sum_{i \in I^1} b_i \, (p_i^1)^{1-\sigma}}{\sum_{i \in I^{1,0}} b_i \, (p_i^1)^{1-\sigma}} \frac{\sum_{i \in I^{1,0}} b_i \, (p_i^1)^{1-\sigma}}{\sum_{i \in I^{1,0}} b_i \, (p_i^0)^{1-\sigma}} \frac{\sum_{i \in I^{1,0}} b_i \, (p_i^0)^{1-\sigma}}{\sum_{i \in I^0} b_i \, (p_i^0)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{4.22}$$

$$= \left(\frac{\sum_{i \in I^{1,0}} s_i^1}{\sum_{i \in I^{1,0}} s_i^0} \right)^{\frac{-1}{1-\sigma}} \left(\frac{\sum_{i \in I^{1,0}} b_i \, (p_i^1)^{1-\sigma}}{\sum_{i \in I^{1,0}} b_i \, (p_i^0)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{4.23}$$

Equation (4.21) is the definition of the cost-of-living index and in (4.22) we simply multiply and divide the CES cost-of-living index by the same expressions. We can eliminate the first and third fractions in (4.22) by using (4.19) and we are left with a representation of the cost-of-living index which is a function of observable expenditure shares and unobservable parameters of the cost function relating to matched-goods. The second term on the RHS is equal to the Base-Weighted and Current-Weighted Price Indexes by substituting in equation (4.20) for $t = 0$ and $t = 1$ respectively. For the derivation of the Sato-Vartia Price Index see Feenstra (1994) or Balk (1999).

4.6.3. Tables of Results

Table 4.1. The Effects of New and Disappearing Goods on the Cost-of-Living Index

	CES Cost Function Method Adjustment for New and Disappearing Goods over 65 Weeks (%)				Approximate Reservation Price Method Adjustment for New and Disappearing Goods over 65 Weeks (%)			
Aggregation Method	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)
Biscuits	2.08	4.19	0.51	5.52	-1.57	1.18	-0.53	1.09
Bread	0.61	1.46	3.69	3.22	0.35	0.46	0.90	0.83
Butter	0.88	1.81	-1.74	0.85	0.29	0.61	-0.37	0.22
Cereal	6.08	5.89	3.77	5.62	1.72	1.61	0.90	1.11
Coffee	1.03	0.70	1.14	1.07	0.34	0.23	0.37	0.29
Detergent	1.83	1.32	0.93	6.07	0.59	0.42	0.22	0.79
Frozen Peas	1.71	1.46	3.14	5.49	0.51	0.41	0.53	0.62
Honey	0.06	0.35	0.53	1.52	0.05	0.12	0.09	-0.07
Jams	1.44	6.60	-4.27	#	0.42	1.09	0.22	1.35
Juices	3.27	1.99	2.90	3.69	0.87	0.54	0.67	0.84
Margarine	1.22	1.25	1.67	0.76	0.33	0.38	0.49	0.25
Oil	2.14	0.73	2.36	1.17	0.70	0.26	0.65	0.31
Pasta	3.35	3.22	3.89	#	0.71	0.48	0.47	1.17
Pet Food	0.99	2.78	1.79	5.37	0.33	0.81	0.52	1.48
Soft Drinks	3.97	1.30	-0.40	0.54	1.14	0.40	-0.10	0.11
Spreads	8.74	1.92	0.83	1.42	2.40	0.60	0.30	0.70
Sugar	-0.58	0.19	0.50	#	-0.15	0.04	-0.07	0.10
Tin Tomatoes	5.31	2.33	3.92	5.17	1.63	0.87	1.03	0.97
Toilet Paper	4.07	2.82	1.59	2.28	1.23	0.67	0.51	0.70
Average	2.54 [2.75]	2.23 [2.02]	1.41 [1.66]	[3.11]	0.63 [0.68]	0.59 [0.60]	0.36 [0.39]	0.68 [0.64]
Average (Exp. Wgt.)	2.77 [2.83]	2.50 [2.45]	1.65 [1.70]	[3.30]	0.58 [0.59]	0.69 [0.70]	0.35 [0.36]	0.75 [0.74]

Note: # indicates that the average elasticity was less than one so the CES Approach is invalid. The average excluding these goods is in square brackets (i.e. [.]).

Table 4.2. Price Change, Index Formula and Aggregation

	Fisher Price Index Change over 65 Weeks (%)				Paasche–Laspeyres Spread Difference between Laspeyres and Paasche over 65 Weeks (%)			
Aggregation Method	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)
Biscuits	-3.47	-4.94	-2.12	-3.67	42.60	13.68	5.37	-0.19
Bread	4.34	4.05	3.99	3.76	44.06	19.73	3.84	2.25
Butter	2.02	2.64	0.87	1.07	22.16	8.31	3.82	1.83
Cereal	0.46	0.42	0.09	-0.13	46.35	12.97	4.03	1.46
Coffee	10.59	11.13	10.16	10.08	90.88	28.04	6.02	2.59
Detergent	3.41	3.88	2.06	2.11	42.62	15.31	2.74	1.07
Frozen Peas	0.04	-0.27	0.52	0.42	35.81	15.24	2.65	1.07
Honey	4.93	4.82	4.20	4.12	10.98	4.66	1.53	0.53
Jams	-0.27	-0.26	-0.01	-0.42	31.74	11.81	2.89	1.04
Juices	-0.11	-0.50	0.68	0.50	51.09	21.24	5.22	2.72
Margarine	0.54	0.47	3.86	3.38	229.71	53.21	14.61	5.86
Oil	-12.32	-14.14	-8.73	-9.22	42.58	35.75	6.17	4.44
Pasta	-1.10	-0.45	-0.05	-0.25	53.09	20.20	3.40	1.06
Pet Food	-0.09	0.04	0.47	0.24	27.65	11.34	3.91	1.16
Soft Drinks	1.88	2.57	3.95	3.35	185.76	63.53	14.37	4.62
Spreads	6.19	6.67	4.33	4.22	17.78	7.56	2.12	0.90
Sugar	6.11	6.27	6.27	6.15	25.10	9.37	1.73	0.05
Tin Tomatoes	-1.25	-1.41	1.33	0.35	41.29	23.87	3.55	1.57
Toilet Paper	-3.05	-3.92	-0.14	-0.45	155.92	60.11	12.53	6.80
Average	0.99	0.90	1.67	1.35	63.01	22.94	5.29	2.15
Average (Exp. Wgt.)	0.69	0.55	1.60	1.17	79.89	28.42	6.81	2.53

Table 4.2 (contd.). Price Change, Index Formula and Aggregation

	Tornqvist Price Index Change over 65 Weeks (%)				Sato-Vartia Price Index Change over 65 Weeks (%)			
Aggregation Method	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)	Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)
Biscuits	-3.71	-5.54	-2.21	-4.35	-2.96	-4.48	-1.91	-2.96
Bread	4.61	4.27	4.00	3.81	4.85	4.63	4.07	3.92
Butter	1.86	2.64	0.76	1.06	1.81	2.63	0.72	1.02
Cereal	0.57	0.52	0.07	-0.16	0.76	0.65	0.09	-0.11
Coffee	10.82	11.14	10.22	10.10	11.33	11.19	10.31	10.13
Detergent	3.54	3.91	2.04	2.06	3.76	4.06	2.12	2.21
Frozen Peas	0.15	-0.17	0.51	0.43	0.27	-0.08	0.61	0.55
Honey	4.87	4.76	4.18	4.09	4.88	4.82	4.20	4.18
Jams	-0.25	-0.36	0.03	-0.55	-0.14	-0.17	0.10	-0.22
Juices	-0.07	-0.44	0.65	0.47	0.12	-0.29	0.76	0.61
Margarine	1.10	0.46	3.72	3.23	1.80	0.25	3.62	3.19
Oil	-11.88	-13.47	-8.61	-9.11	-11.31	-12.39	-8.47	-8.93
Pasta	-0.95	-0.51	-0.07	-0.27	-0.74	-0.46	0.04	-0.02
Pet Food	-0.02	0.04	0.48	0.20	0.13	0.14	0.55	0.36
Soft Drinks	2.12	2.52	3.85	3.31	3.18	2.53	3.82	3.38
Spreads	6.25	6.68	4.32	4.21	6.42	6.85	4.34	4.25
Sugar	6.42	6.28	6.22	6.11	6.92	6.53	6.33	6.18
Tin Tomatoes	-1.17	-1.18	1.31	0.31	-0.92	-1.30	1.41	0.65
Toilet Paper	-3.15	-3.96	-0.12	-0.54	-3.18	-3.53	-0.10	-0.34
Average	1.11	0.93	1.65	1.28	1.42	1.14	1.72	1.48
Average (Exp. Wgt.)	0.80	0.53	1.57	1.08	1.22	0.78	1.64	1.33

Table 4.3. Estimating the Elasticity of Substitution

	Current v Base Method Average and Standard Deviation of Estimated Elasticity								Average Sato-Vartia Method Average Absolute Deviations from Current v Base Method			
Aggregation Method	Month (Prod. Code, Outlet)		Month (Prod. Code)		Quarter (Prod. Code, Outlet)		Quarter (Prod. Code)		Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)
Statistic	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.	Avg.	Std. Dev.				
Biscuits	3.46*	0.99	2.96+	1.63	2.63*	1.02	1.63+	2.77	0.05	0.06	0.16	0.18
Bread	3.39	0.69	3.58+	1.05	2.30	0.42	2.45	0.55	0.02	0.05	0.05	0.04
Butter	4.01	0.45	3.87	0.94	3.43	0.76	3.70	1.05	0.07	0.05	0.05	0.10
Cereal	3.25	0.23	2.70	0.53	2.50	0.20	2.10	0.43	0.03	0.03	0.04	0.08
Coffee	5.66	0.46	4.66	0.65	3.73	0.41	2.63	0.46	0.06	0.02	0.03	0.02
Detergent	3.88	0.34	3.41	0.45	2.18	0.13	1.35*	0.49	0.04	0.03	0.06	0.08
Frozen Peas	3.44	0.30	3.36	0.58	2.28	0.46	1.68*	0.85	0.02	0.02	0.01	0.04
Honey	4.04	0.79	4.31	2.10	3.03	0.85	2.13+	2.20	0.08	0.07	0.03	0.11
Jams	3.17	0.58	2.72+	1.17	1.65	0.47	0.70*	0.91	0.06	0.04	0.07	0.19
Juices	3.14	0.32	3.38	0.56	2.53	0.30	2.50	0.36	0.05	0.03	0.08	0.10
Margarine	4.63	0.42	3.99	0.61	3.63	0.39	3.60	0.75	0.03	0.02	0.01	0.03
Oil	4.59	0.71	5.47	0.83	3.13	0.38	2.88	0.51	0.20	0.52	0.04	0.03
Pasta	2.16	0.32	2.51	0.61	1.43	0.17	1.00*	0.84	0.02	0.03	0.03	0.06
Pet Food	3.64	0.35	3.29	0.42	2.95	0.17	1.75*	0.97	0.04	0.04	0.01	0.09
Soft Drinks	4.17	0.65	4.06	0.65	3.28	0.46	2.35	0.70	0.11	0.04	0.06	0.06
Spreads	3.71	0.70	3.61	0.86	2.75	0.60	2.33*	1.54	0.03	0.14	0.05	0.15
Sugar	4.16	1.58	3.31+	1.92	2.00*	1.13	0.73+	1.02	0.26	0.04	0.38	0.03
Tin Tomatoes	3.95	0.41	3.97*	1.02	2.70	0.14	1.98+	1.73	0.03	0.06	0.18	0.01
Toilet Paper	5.21	0.91	4.91	1.36	4.05	0.61	3.25	0.90	0.09	0.06	0.05	0.11
Average	3.88	0.59	3.69	0.94	2.73	0.44	2.14	1.00	0.07	0.07	0.09	0.08
Average (Exp. Wgt.)	3.81	0.57	3.61	0.84	2.63	0.71	2.23	0.92	0.06	0.05	0.07	0.08

Note: * (+) indicates that the elasticity of substitution fell below one (zero) in at least one period.

Table 4.4. Expenditure Shares on New and Disappeared Goods

	Average Proportion of Current and Base Expenditure Shares (%)								Changes in Current Relative to Base Expenditure Shares Change over 65 Weeks (%)			
Aggregation Method	Month (Prod. Code, Outlet)		Month (Prod. Code)		Quarter (Prod. Code, Outlet)		Quarter (Prod. Code)		Month (Prod. Code, Outlet)	Month (Prod. Code)	Quarter (Prod. Code, Outlet)	Quarter (Prod. Code)
New or Dis. Goods	Dis.	New	Dis.	New	Dis.	New	Dis.	New				
Biscuits	1.66	2.20	0.06	0.76	3.97	4.58	0.29	2.38	-7.36	-9.41	-2.43	-8.14
Bread	0.50	0.77	0.11	0.36	0.78	2.12	0.34	1.66	-3.70	-3.57	-5.31	-5.20
Butter	0.59	0.77	0.01	0.41	1.47	1.05	0.12	0.43	-2.54	-5.52	1.73	-1.27
Cereal	1.15	2.07	0.01	0.78	1.67	2.95	0.08	1.49	-12.33	-10.43	-5.11	-5.55
Coffee	0.40	0.75	0.01	0.17	0.66	1.47	0.02	0.51	-4.86	-2.31	-3.24	-1.93
Detergent	0.75	1.15	0.02	0.28	1.82	2.13	0.04	0.72	-5.48	-3.66	-1.26	-2.68
Frozen Peas	0.75	1.04	0.02	0.23	1.57	2.14	0.10	0.70	-4.03	-2.89	-2.33	-2.41
Honey	0.53	0.61	0.02	0.12	1.08	1.15	0.11	0.57	-1.08	-1.34	-0.28	-1.84
Jams	1.33	1.56	0.03	0.54	2.46	2.85	0.08	1.33	-3.25	-6.84	-1.59	-4.90
Juices	0.81	1.24	0.02	0.29	1.73	2.63	0.17	1.32	-5.85	-3.68	-3.64	-4.55
Margarine	0.70	0.99	0.01	0.24	0.92	1.88	0.18	0.71	-4.07	-3.18	-3.85	-2.11
Oil	1.80	2.32	0.05	0.30	1.47	2.59	0.18	0.69	-7.23	-3.40	-4.50	-2.03
Pasta	1.12	1.40	0.08	0.25	2.23	2.68	0.27	0.90	-3.93	-2.46	-1.85	-2.47
Pet Food	1.14	1.37	0.09	0.52	2.27	3.17	0.23	1.74	-3.26	-5.97	-3.65	-5.91
Soft Drinks	2.03	2.75	0.02	0.26	2.02	1.93	0.09	0.24	-9.83	-3.34	0.34	-0.60
Spreads	0.91	2.17	0.01	0.37	1.21	1.83	0.10	0.83	-16.54	-5.05	-2.51	-2.93
Sugar	0.34	0.29	0.00	0.02	0.47	0.47	0.00	0.12	0.71	-0.26	-0.03	-0.46
Tin Tomatoes	1.33	2.31	0.02	0.54	3.06	4.64	0.16	2.12	-13.27	-7.12	-6.38	-7.63
Toilet Paper	1.37	2.58	0.01	0.53	2.25	3.49	0.06	1.51	-16.09	-7.11	-4.99	-5.72
Average	1.01	1.49	0.03	0.37	1.74	2.41	0.14	1.05	-6.53	-4.61	-2.68	-3.60
Average (Exp. Wgt.)	1.18	1.72	0.04	0.42	1.93	2.65	0.16	1.21	-7.29	-5.26	-2.92	-4.12

Table 4.5. Number of Products

Numbers of Products in First and Last Time Periods and the Percentage Change												
Aggregation Method	Month (Prod. Code, Outlet)			Month (Prod. Code)			Quarter (Prod. Code, Outlet)			Quarter (Prod. Code)		
Time Period	First	Last	Chg. (%)	First	Last	Chg. (%)	First	Last	Chg. (%)	First	Last	Chg. (%)
Biscuits	38,408	39,220	2.11	918	883	-3.81	41,777	42,537	1.82	992	1,009	1.71
Bread	11,587	11,048	-4.65	308	335	8.77	12,085	12,175	0.74	325	355	9.23
Butter	3,174	3,422	7.81	61	65	6.56	3,321	3,495	5.24	64	66	3.13
Cereal	17,769	19,533	9.93	389	476	22.37	18,577	20,613	10.96	399	491	23.06
Coffee	7,958	8,937	12.30	152	168	10.53	8,412	9,408	11.84	162	176	8.64
Detergent	7,310	6,956	-4.84	147	142	-3.40	7,514	7,430	-1.12	151	154	1.99
Frozen Peas	8,163	8,300	1.68	189	186	-1.59	8,651	8,636	-0.17	201	196	-2.49
Honey	3,841	3,887	1.20	97	90	-7.22	4,018	4,013	-0.12	102	93	-8.82
Jams	10,857	10,329	-4.86	312	282	-9.62	11,626	11,125	-4.31	328	303	-7.62
Juices	41,800	42,426	1.50	862	894	3.71	43,577	44,769	2.74	899	963	7.12
Margarine	4,514	4,872	7.93	80	89	11.25	4,625	5,128	10.88	81	90	11.11
Oil	8,220	9,116	10.90	232	251	8.19	9,049	9,900	9.40	250	273	9.20
Pasta	16,256	18,207	12.00	465	544	16.99	17,453	19,552	12.03	523	578	10.52
Pet Food	39,697	41,605	4.81	786	876	11.45	41,229	44,628	8.24	806	935	16.00
Soft Drinks	37,277	32,043	-14.04	774	681	-12.02	40,067	34,758	-13.25	812	732	-9.85
Spreads	4,191	4,615	10.12	84	89	5.95	4,668	4,924	5.48	89	93	4.49
Sugar	4,190	4,200	0.24	96	103	7.29	4,355	4,439	1.93	101	106	4.95
Tin Tomatoes	3,646	3,867	6.06	90	91	1.11	3,931	4,210	7.10	98	97	-1.02
Toilet Paper	6,564	6,835	4.13	129	134	3.88	6,835	7,138	4.43	135	138	2.22
Average	---	---	3.39	---	---	4.23	---	---	3.89	---	---	4.40
Average (Exp. Wgt.)	---	---	0.99	---	---	3.99	---	---	2.20	---	---	5.52

CHAPTER 5

5. The Economic and Stochastic Approaches to Index Numbers

Abstract*

This chapter discusses the relationship between the economic and stochastic approaches to index numbers. The primary point of this is to show that we can interpret the stochastic approach as a method for estimating the economic cost-of-living index, under certain circumstances. As well as surveying the stochastic approach we also look at the conventional economic approach to estimating preferences and technology, where a parametric functional form is hypothesised and estimable equations are derived using economic theory. Viewing the stochastic approach as a method for obtaining the cost-of-living index is illustrate and leads us to us to suggest an alternative stochastic approach closely linked to economic theory. In the discussion we place emphasis on applications of the stochastic approach to new and disappearing goods, an area where it is particularly useful.

* The author is grateful for comments received from Robert Hill.

John Maynard Keynes, writing in 1933, on the early stochastic approach of Jevons and Edgeworth,

"Nevertheless I venture to maintain that such ideas… are root-and-branch erroneous. The "errors of observation", the "faulty shots aimed at a single bull's-eye" conception of the index-number of prices, Edgeworth's "objective mean variation of general prices", is the result of a confusion of thought. There is no bull's-eye. There is no moving but unique centre, to be called the general price-level or the objective mean variation of general prices, round which are scattered the moving price-levels of individual things. There are all the various, quite definite, conceptions of price-levels of composite commodities appropriate for various purposes and inquiries which have been scheduled above, and many others too. There is nothing else. Jevons was pursing a mirage."

Keynes (1933, pp. 85-86).

5.1. Introduction

One of the interesting and challenging features of price and quantity index construction is the diversity of approaches to index numbers. There are at least three major and distinct approaches to the construction of index numbers: the *economic* approach, the *axiomatic* or *test* approach and the not-so-well known *stochastic* or *statistical* approach. In this chapter we are primarily interested in examining and relating the economic and stochastic approaches. While these two approaches to the construction of index numbers appear to be somewhat antithetical, the stochastic approach can be given a strong economic interpretation. Moreover it is shown that, under certain conditions, we can regard the stochastic price index as giving an estimate of the economic cost-of-living index.

This chapter is part of an ongoing renewal of interest in the stochastic approach to indexes numbers. In the last decade there has been renewed discussion of the stochastic approach. This was spurred by the book by Selvanathan and Rao (1994), who provide a thorough survey of the approach, and Diewert's (1995b) response to this book. More recently there has been an interesting array of papers concerning various aspects of the stochastic approach (Cecchetti, 1997; Wynne, 1997; Crompton, 2000; Feenstra and Reinsdorf, 2004). One reason that the stochastic approach has attracted particular attention in recent times has been because some formulations of the stochastic approach (i.e. the stochastic approach in price levels) is able to account for changing sets of commodities across time or space. This advantage means that this version of the stochastic approach is under consideration for use in the latest round of the International Comparisons Project (Hill, 2004). This possibility has prompted much research in this area (Diewert, 2002; Aizcorbe and Aten, 2004; Diewert, 2004; Rao, 2004).

5.1.1. Outline

In the two sections that follow, Sections 5.2 and 5.3, we outline the economic and stochastic approaches to index numbers respectively. In Section 5.4, with the stochastic approach in mind, we show how the stochastically estimated price index relates to the economic objective of price index estimation, the cost-of-living index. Moreover, given this economic view of the stochastic approach to index numbers the estimation of stochastic *quantity* indexes naturally arises. The more conventional approach to the economic estimation of preferences is discussed in Section 5.5. Here a functional form is specified for the cost function and economic optimising conditions are derived. From the estimation of these equations the parameters of the cost function, and associated standard errors, can be derived. Section 5.6 suggests an alternative economic-stochastic approach to estimating the cost-of-living index using the Constant Elasticity of Substitution (CES) cost function. This sits somewhere between the standard stochastic approach and the more conventional economic approach to preference estimation. Section 5.7 summarises and concludes.

5.2. The Economic Approach to Index Numbers

The economic approach to index numbers is perhaps the most important and rigorously defined approach to index numbers. This approach dates back to the time of Konus (1924) and draws heavily on consumer microeconomic theory and the assumption of optimization in this context.

5.2.1. The Cost-of-Living Index

The essence of the economic approach is the use of the relationship between prices and the cost of obtaining a given level of utility. This relationship is called the cost function and is shown below, for period t.

$$C(p^t,\overline{U}) = \min{}_x \left\{ \sum_{i=1}^{N} p_i^t x_i \ : U(x) \geq \overline{U} \right\} \tag{5.1}$$

Here p_i and x_i represent the price and quantity of good i with any superscripts representing price periods, the corresponding vectors have the subscript i removed. Also $\overline{U}$ represents a scalar utility level associated with the preference ordering represented by $U(x)$. An economic price index or *Konus cost-of-living index* is defined as the ratio of the minimum cost of obtaining a given level of utility under two alternative price regimes (Konus, 1924; Diewert, 1976).

There are number of important results regarding the cost-of-living index which we briefly review. Let us define the Laspeyres ($P_{1,0}^L$) and Paasche ($P_{1,0}^P$) Price Indexes, between periods 0 and 1, which have a well known bounding relationship between different versions of the cost-of-living index shown below.

$$\frac{C(p^1,U^0)}{C(p^1,U^0)} \leq \frac{\sum_{i=1}^{N} p_i^1 x_i^0}{\sum_{i=1}^{N} p_i^0 x_i^0} \equiv P_{1,0}^L \tag{5.2}$$

$$\frac{C(p^1,U^1)}{C(p^0,U^1)} \geq \frac{\sum_{i=1}^{N} p_i^1 x_i^1}{\sum_{i=1}^{N} p_i^0 x_i^1} \equiv P_{1,0}^P \tag{5.3}$$

The Paasche and Laspeyres Indexes above are particularly useful if the consumer's utility function is linearly homogeneous, $U(\lambda x) = \lambda U(x)$ where $\lambda > 0$. In this case the cost function can be written as in (5.4) (Diewert, 2001, pp. 48-49), where $c(p^t)$ is the *unit cost function* for period t, and defined below.

$$C(p^t,U) = c(p^t)U \tag{5.4}$$

$$c(p^t) = \min{}_x \left\{ \sum_{i=1}^{N} p_i^t x_i \ : U(x) \geq 1 \right\} \tag{5.5}$$

In this case there is a single unique cost-of-living index, which does not depend on the reference utility level chosen, and the Paasche and Laspeyres Price Indexes provide a lower and upper bound respectively to this cost-of-living index.

Perhaps the most important development in index numbers in the past 30 years has been the observation made by Diewert (1976), that for various flexible specifications of the unit cost function technology different price index formulae are *exact* and *superlative*. They are exact in the sense that the price index is exactly equal to the ratio of the cost functions assuming economic optimising conditions are satisfied. The description of an index as superlative arises from the fact that the representation of the cost function is flexible, in the sense defined by Diewert (1976), of being able to provide an approximation to an arbitrary linearly homogeneous function to the *second order*. Diewert (1976) showed that the Fisher

($P^F_{1,0}$) and Tornqvist ($P^T_{1,0}$) Price Indexes were exact and superlative for different representations of the cost function.[49]

$$P^F_{1,0} \equiv (P^P_{1,0} \times P^L_{1,0})^{\frac{1}{2}} \tag{5.6}$$

$$P^T_{1,0} \equiv \prod_{i=1}^{N} \left(\frac{p_i^1}{p_i^0} \right)^{\frac{1}{2}(s_i^0 + s_i^1)}, \quad s_i^t \equiv \frac{p_i^t x_i^t}{\sum_{i=1}^{N} p_i^t x_i^t},\ t = 0,1 \tag{5.7}$$

There are a number of other aspects of the economic approach to index numbers but we will not pursue these here. We have covered enough of the economic approach for our purposes and will proceed to a discussion of the stochastic approach.[50]

5.3. The Stochastic Approach to Index Numbers

The stochastic approach is somewhat different from the economic approach – it has a longer pedigree dating back to the time of Jevons and Edgeworth in the late 1800s. Here the price change or price level of each good or service is regarded as giving some information about the difference in prices between periods. The current interpretation of the stochastic approach is perhaps best summarised by Selvanathan and Rao (1994, p. 48), who write,

> Under the stochastic approach, each price relative is taken to be equal to the underlying price index which measures the overall price changes between the current and base periods, plus other components that are random and nonrandom. If we have N prices, then the price index can be estimated by taking some form of average of the N price relatives. The index

[49] The specific functional forms and the derivation of these indexes can be seen in Diewert (1976, 2001). However, we will define the Translog functional form in a later section.

[50] For a thorough survey of the economic approach to index numbers interested readers are referred to Diewert (1993).

number problem under the stochastic approach can be viewed as a signal extraction problem.

This approach to index numbers has some important advantages over other approaches to indexes numbers some of which we have already touched upon. We discuss these advantages below.

a. Advantages of the Stochastic Approach to Index Numbers

The major and enduring attraction of the stochastic approach is that along with the index number we also get a standard error. Balk (1996, p. 133) interprets the standard error as "…a measure of the strength of the signal (that is the common component) relative to the concomitant noise." The standard error is particularly useful for policy purposes in gauging the strength of inflationary changes. For example, Central Banks, who endeavour to control inflation by using monetary policy, may be more hesitant to raise interest rates for a given increase in price if the standard error of this change is high. A high standard error may indicate to the Central Bank that price changes are widely dispersed around the mean, which may imply that the general price trend is not strong.

As well as the standard error there are another two important advantages of the stochastic approach over the economic approach to index numbers. Firstly, some forms of the stochastic approach can accommodate changing sets of goods over time. Compared with the chained matched-model approach this means that more of the available price information is used in calculating the price indexes. This is one of the particularly useful features of the stochastic approach given that in modern economies there are pronounced changes in the sets of commodities available over time. Similarly, there are often significant differences between the set of commodities available in different countries or regions, meaning that this version of the stochastic approach is particularly useful in this context. Secondly, a closely related feature of the stochastic approach is that, unlike for the calculation of economic indexes, we do not require information on *all* goods purchased to estimate the price index. This is a distinct advantage as all price indexes in practice are constructed from samples of prices from the population of prices. In the economic approach, however, we have to adopt the fiction that we have the population of transactions. For example, to argue that the Laspeyres Price Index is indeed an upper bound to the cost-of-living index we must calculate the Laspeyres Index based upon the population of transactions, not just the sample. In the stochastic approach this fiction is dispensed with. With these advantages in mind, in the following section we give a brief outline of the stochastic approach to index numbers.

5.3.1. A Brief Outline of the Stochastic Approach

There are different types of stochastic approaches to index numbers. These are well discussed and surveyed in the book by Selvanathan and Rao (1994), *Index Numbers: A Stochastic Approach*. Diewert (1995b) provides his own discussion a critical response to the book. In this chapter we will not attempt to replicate these surveys but we will give a taste of the basic stochastic approach. We start by looking at the stochastic approach in price *relatives* and then consider the analogous model in price *levels*. Some extensions to this basic framework are considered.

a. The Stochastic Approach in Price Relatives

Let us begin by outlining the stochastic approach in relative prices. Suppose that we have a panel data set of prices for goods $i = 1,...,N$ over time periods $t = 0,1,...,T$. One example of the basic approach suggested by Selvanathan and Rao (1994), what Diewert (1995b) calls the "New Stochastic Approach to Index Numbers", is to hypothesise a model of the form shown in (5.8).[51] In this model the

[51] Balk (1980, p. 69) argues that a multiplicative regression model of prices, as in (5.8), is more plausible than a linear model because the influence of random components on price levels are large when a goods price is high and small when its price is low.

logarithm of the price relatives is related to the inflation rate, $\delta^{t,t-1}$, the object of estimation, and a random component $\varepsilon_i^{t,t-1}$.

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \delta^{t,t-1} + \varepsilon_i^{t,t-1}, \quad i = 1,\ldots,N,\ t = 1,\ldots,T \tag{5.8}$$

Note here that the model is specified in terms of price relatives so it is required that there are no missing observations. This model can be easily estimated using Ordinary Least Squares (OLS).[52] Another alternative is to use Weighted Least Squares (WLS) where we could account for differences in economic importance of the goods by weighting each good differently. For this model, estimated between periods 0 and 1, it can readily be seen that the estimated inflation rate is equal to the weighted mean of the logarithm of the price relatives where we have used some good-specific weight, $w_i^{1,0}$.

$$\hat{\delta}^{1,0} = \sum_{i=1}^{N}\left(\left(\frac{w_i^{1,0}}{\sum_{i=1}^{N} w_i^{1,0}}\right)\log\left(\frac{p_i^1}{p_i^0}\right)\right) \tag{5.9}$$

An unbiased estimator of the variance of this parameter is shown below (Selvanathan and Rao, 1994, p. 59).

$$Var(\hat{\delta}^{1,0}) = \frac{1}{N-1}\sum_{i=1}^{N} w_i^{1,0}\left(\log\left(\frac{p_i^1}{p_i^0}\right) - \hat{\delta}^{1,0}\right) \tag{5.10}$$

[52] Under the assumption that $\varepsilon_i^{t,t-1} \sim N(0,\sigma^2)$ then the OLS estimates would also be the Maximum Likelihood estimates.

Note that we need to take the exponent of $\hat{\delta}^{1,0}$ in order to determine the price index of interest. One of the particularly appealing features of this model is that it is readily seen that if we use the arithmetic average of the expenditure shares as weights, $w_i^{1,0} = 0.5 \times (s_i^0 + s_i^1)$, as Selvanathan and Rao (1994) suggest, then the estimator in (5.9) is equal to the log of the superlative Tornqvist Price Index defined above.

Interestingly, Selvanathan and Rao (1994) have shown that for various models (e.g. the linear price model), and various weighting strategies (e.g. using expenditure levels rather than shares), well known index numbers, such as the Laspeyres and Paasche Indexes, arise as estimators of the inflation parameter. The standard errors for these index estimators can also be readily derived.

b. The Stochastic Approach in Price Levels

We can also estimate price indexes in terms of price levels. In the context of new and disappearing goods this method will prove particularly useful. The most common application of the stochastic approach in levels is the Country Product Dummy (CPD) method due to Summers (1973). It is used in the context of making price comparisons between countries or regions where there may be missing observations. However, rather than thinking in terms of spatial comparisons we will continue to phrase our discussion in terms of intertemporal price comparisons. Hence we use what is naturally called the Time Product Dummy (TPD) method, which was applied by Balk (1980) to the measurement of seasonal fruit and vegetable prices. One common example of such a model, again a multiplicative model, is shown below.

$$\log(p_i^t) = \gamma_i + \delta^t + u_i^t, \quad i = 1,...,N, \; t = 0,1,...,T \tag{5.11}$$

In contrast with the stochastic approach in price relatives, we now do not require a complete panel of observations. That is, we do not now assume that every good i

exists in every time period t. In (5.11) the parameter δ^t is a measure of the price level, while γ_i represents the effect of good i on the price level and u_i^t is a random component. As the variables associated with δ^t and γ_i are represented by dummy variables, in order to avoid perfect collinearity, we must make one normalisation, say $\delta^0 = 0$. If we estimate the parameters of (5.11) using WLS for two time periods, say 0 and 1 where we have complete data, and where the weight for each good is fixed across time, $w_i^0 = w_i^1 = w_i^{0,1}$, then the estimated inflation parameter is the same as that for the stochastic approach in relatives shown in (5.9).

Diewert (2002) has undertaken an exercise for the stochastic approach in price levels, similar to that of Selvanathan and Rao (1994) for the stochastic approach in price relatives, showing that many well known index numbers result from different approaches to the WLS estimation of models of the general type shown in (5.11).[53]

c. Extensions of the Stochastic Approach

While we have outlined the basic stochastic approach to index numbers, there are some extensions of this approach which have been suggested in the literature. We discuss two extensions of the stochastic models above.[54] While these extensions are equally relevant to both the stochastic model in relatives and levels, in the interests of brevity, we just discuss these extensions as applied to the stochastic approach in price relatives.

[53] Interestingly, however, Diewert (2002) did not go the extra step taken by Selvanathan and Rao (1994) of deriving the standard errors for these indexes.

[54] Another extension, which we will not discuss here, is to relax the assumption of the independence of the error term in the models such as (5.8) and (5.11). The interested reader can find a discussion and some references in Rao (2004).

(i). The Good-Specific Price Trends Model

One extension of the basic stochastic approach is that suggested by Clements and Izan (1987), who allowed for good-specific price trends over time. They argued, this responded to the criticism of Keynes (1933), who noted that the early stochastic approach did not take account of "secondary" price levels, that is changes in the relative prices of goods over time. Clements and Izan (1987) incorporated relative price changes by including a good specific price trend term, β_i , in (5.8).

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \delta^{t,t-1} + \beta_i + \varepsilon_i^{t,t-1}, \quad i = 1,...,N\ ,\ t = 1,...,T \tag{5.12}$$

Here the parameter β_i is not identified so some restriction must be imposed upon it. Clements and Izan (1987) chose the following restriction where $\tilde{w}_i$ is a good specific weight that sums to one over all goods.

$$\sum_{i=1}^{N} \tilde{w}_i \, \beta_i = 0 \tag{5.13}$$

Clements and Izan (1987) show that if the weight, $\tilde{w}_i$, is chosen so that $w_i^{t,t-1} / \sum_{i=1}^{N} w_i^{t,t-1} = \tilde{w}_i$ for all time periods, then the estimator of inflation from model (5.12) is the same as that in (5.8). In general, when $w_i^{t,t-1} \neq \tilde{w}_i$, the estimators of inflation will be different for the two models. The fact that the inclusion of a good-specific price trend in the model can change the estimate of overall inflation seems questionable. However, as noted by Diewert (1995b, p. 20), there is no reason for thinking that the weights incorporated to reflect economic importance, $w_i^{t,t-1}$, and the weights required for the identifying restriction, $\tilde{w}_i$, need be the same. These difficulties, with the good-specific price trend approach, mean that it is more of an interesting theoretical generalization rather than a useful practical approach.

(ii). The Good-Specific Error Variance Model

Another interesting suggestion, by Diewert (1995b), is to allow for the fact that the variance of the error in a model like (5.8) or (5.11) may differ systematically across goods. Diewert (1995b) called this model the "neo-Edgeworthian model" as it is a formalisation of some ideas first espoused by Edgeworth. Using a model like (5.8) above, Diewert (1995b) supposes that the variance of the error term is good specific, and shows how the model can be estimated using the Maximum Likelihood approach with the assumption that $\varepsilon_i^{t,t-1} \sim N(0,\sigma_i^2)$. The optimal solutions for the estimator of the inflation rate and the variance parameters are characterised by a system of nonlinear equations shown below. These equations can be solved by an iterative procedure.

$$\hat{\delta}_*^{t,t-1} = \sum_{i=1}^{N} \left(\left(\frac{\frac{1}{\hat{\sigma}_{*i}^2}}{\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_{*i}^2}} \right) \log\left(\frac{p_i^t}{p_i^{t-1}} \right) \right), \quad t = 1,...,T \tag{5.14}$$

$$\hat{\sigma}_{*i}^2 = \frac{1}{T} \sum_{t=1}^{T} \left(\log\left(\frac{p_i^t}{p_i^{t-1}} \right) - \hat{\delta}_*^{t,t-1} \right)^2, \quad i = 1,..., N \tag{5.15}$$

What is particularly interesting about the estimator for inflation, $\hat{\delta}_*^{t,t-1}$, is that it is a variance weighted estimator. Those goods which have less volatile prices, so $\hat{\sigma}_{*i}^2$ is small, receive greater weight in the index. This is a highly appealing idea and an interesting development in the stochastic approach. It allows, in some sense, for the optimal weighting of the observations according to their ability to provide a signal of inflation.

However, it is readily verifiable that the asymptotic variance, the Cramer-Rao lower bound, of the estimated parameter $\hat{\delta}_*^{t,t-1}$ has the following form, as shown in Section 5.8.1 of the Appendix.

$$Asy.Var(\delta_*^{t,t-1}) = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{*i}^2} \right)^{-1}, \qquad t = 1,...,T \tag{5.16}$$

What is not so useful about this approach is that the asymptotic variance is the *same* for each of the estimated inflation parameters and so do not depend on t. The standard error of the inflation estimate can then not be usefully compared across time periods to indicate differences in the 'strength of the inflation signal'.

5.3.2. Summary

Our review of the stochastic approach has shown that there are two main stochastic approaches, in price relatives and price levels, and various extensions of these basic frameworks. Despite this the stochastic approach to index numbers is little used and controversial (Diewert, 1995b). In contrast the economic approach to index numbers is more widely accepted. One important area that has been neglected is in determining how the stochastic approach to index numbers is related to the economic approach.

This reconciliation serves a number of purposes. First, it provides an interpretation of the stochastic approach which has been lacking up until now. Second, it may give practitioners of the stochastic approach an idea of the types of stochastic models which are consistent with economic theory. Thirdly, it will clarify the economic assumptions that are being made in adopting the stochastic approach. Finally, the use of economic theory may suggest alternative methods which can be used to derive a set of estimable stochastic equations. In the next section we outline an economic motivation for the stochastic approach to index numbers.

5.4. An Economic Interpretation of Stochastic Price (and Quantity) Indexes

The primary economic motivation for the stochastic approach to index numbers has been found in the quantity theory of money. The quantity theory states that the prices of goods are closely related to the stock of money in the economy and that on average prices will move in a manner proportional to the changes in the stock of money. Keynes (1909, pp. 105-106) writes,

> The origin of the method is found in the doctrines of the quantity theory of money. If, through changes in the demand or supply of gold, there is a change in its purchasing power, while all other commodities maintain relatively to one another unaltered ratios of exchange, their gold prices will all rise or fall in the same proportion. In this case, however, the change in each will measure for us exactly the change in the purchasing power of gold.

In this case the price level then can be measured simply as some form of 'average', under various stochastic assumptions, of the change in prices. Edgeworth, as reported in Keynes (1933, p. 87), called a stochastic price index the "objective mean variation of general prices", Keynes (1909, p. 106) himself called it the "uniform ratio" while Diewert (1993, p. 197) called the index the "common factor of proportionality". However, while the quantity theory provides one motivation for the stochastic approach it can also be shown that the stochastic index number, under certain conditions, provides a measure of the economic cost-of-living index – the minimum cost of obtaining a given level of utility. In the next section we justify this claim.

5.4.1. An Economic Derivation of the Stochastic Approach to Price Indexes

Consider the basic cost minimisation problem of economic theory shown in (5.1) above. The well known first order condition for obtaining a minimum to this problem is shown below where λ is the Lagrange Multiplier for period t.

$$p_i^t = \lambda \frac{\partial U(x^t)}{\partial x_i^t}, \quad i = 1,...,N \tag{5.17}$$

It is straightforward to show that, if the utility function is homogeneous of degree one, $U(kx) = kU(x) \;\; k > 0$, and hence, the cost function is linearly homogeneous in utility, $C(p^t, U) = c(p^t)U$, then λ is equal to the unit cost function, $\lambda = c(p^t)$.[55] This is shown in Section 5.8.2 of the Appendix. In this case we can rewrite equation (5.17).

$$p_i^t = c(p^t) \frac{\partial U(x^t)}{\partial x_i^t}, \quad i = 1,...,N \tag{5.18}$$

A natural interpretation of (5.18) is that the *quality-adjusted price* of each good is equal to the unit cost function where the 'quality adjustment' that we apply is dividing the price of the good by its marginal utility.

We can take the logarithms of (5.18) to derive an equation in levels, which has the appearance of a TPD or CPD equation discussed previously.

$$\log(p_i^t) = \log(c(p^t)) + \log\left(\frac{\partial U(x^t)}{\partial x_i^t}\right), \quad i = 1,...,N, \; t = 0,1,...,T \tag{5.19}$$

Equally we can put (5.18) into log-relative form, in which case we have a set of equations similar in structure to the stochastic approach in price relatives outlined above.

[55] Note that the assumption that preferences are homothetic over the time period $t = 0,1,...,T$ is strong especially if T is large.

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \log\left(\frac{c(p^t)}{c(p^{t-1})}\right) + \log\left(\frac{\partial U(x^t)}{\partial x_i^t} \Bigg/ \frac{\partial U(x^{t-1})}{\partial x_i^{t-1}}\right),$$

$$i = 1,...,N\ ,\ t = 1,...,T \qquad (5.20)$$

The use of economic theory provides a new way of looking at the basis of the stochastic approach. We discuss the similarities between the economic equations represented by (5.19) and (5.20) and the conventional stochastic approach in the next section.

5.4.2. The Economics of the Stochastic Approach

Equations (5.19) and (5.20) are particularly useful in illuminating the economics of the stochastic approach to index numbers. The first point to note is that the basic intuition of the stochastic approach, that price levels or price relatives give an indication of the inflation rate is certainly validated in terms of economic theory. Equations (5.19) and (5.20) show that prices do indeed relate directly to the (unit) cost function, as well as to marginal utility. In the following sections we discuss the economics of the basic stochastic approach.

a. When is the Basic Stochastic Approach Economically Correct?

An important point to note is that by comparing equations (5.20) with (5.8) and (5.19) with (5.11) we see that the implicit assumption of the conventional stochastic approach in terms of economic theory is that the marginal utility of each good is fixed across time. To see the specific case in which the stochastic approach in relatives and levels is correct, suppose that the utility function takes the following Linear (Random) Utility form, where x_i^t is the quantity of the good consumed, a_i

is a good specific quality or taste parameter which is fixed across time and e_i^t is a random component which is time and good dependent.[56]

$$U(x^t) = \sum_{i=1}^{N} (a_i e_i^t) x_i^t \tag{5.21}$$

This utility function has the convenient property that the marginal utility for each good is fixed except for a stochastic component.

$$\frac{\partial U(x^t)}{\partial x_i^t} = a_i e_i^t \tag{5.22}$$

If we substitute this into (5.19) then it can readily be seen that we obtain a model which has the form of the TPD method shown in (5.11).

$$\log(p_i^t) = \log(c(p^t)) + \log(a_i) + \log(e_i^t), \quad i = 1,...,N, \; t = 1,...,T \tag{5.23}$$

Comparing equations (5.11) and (5.23) we see that the price level parameter in (5.11) is equal to the unit cost function, $c(p^t) = \delta^t$, and for the good specific term we have, $\log(a_i) = \gamma_i$ and for the random component, $\log(e_i^t) = u_i^t$. Similarly, if we substitute (5.22) into (5.20) then we obtain the set of equations shown below, which is very much analogous to the stochastic approach in relatives of (5.8), where the ratio of cost functions is represented by the parameter $\delta^{t,t-1}$.

[56] We can interpret the a_i parameter as representing 'average' tastes with some random shocks, represented by e_i^t, to these tastes each period. We assume that the consumer knows the realisation of e_i^t prior to solving the cost minimisation problem so optimising behaviour is preserved.

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \log\left(\frac{c(p^t)}{c(p^{t-1})}\right) + \log\left(\frac{e_i^t}{e_i^{t-1}}\right), \quad i = 1,\ldots,N\ ,\ t = 1,\ldots,T \tag{5.24}$$

This shows that the conventional stochastic approach, in price relatives and levels, is exact when the marginal utility of each good is held fixed across time. Importantly, it is also required that the error term in each model have the same distribution.

b. Interpreting the Error Term in the Stochastic Approach

Let us further investigate the theoretical basis of the standard stochastic approach by using the Cobb-Douglas (Random) utility function. We again have good specific taste or quality parameters, represented by α_i , and random components, represented by e_i^t. In order to ensure linear homogeneity we require that $\sum_{i=1}^{N} \alpha_i e_i^t = 1$.

$$U(x^t) = \prod_{i=1}^{N} (x_i^t)^{\alpha_i e_i^t} \tag{5.25}$$

Note that unlike the Linear Utility functional form used above, the Cobb-Douglas functional form allows for diminishing marginal utility. We can easily calculate the marginal utility for this utility function and for illustrative purposes we will substitute it into (5.20) and look at the stochastic approach in price relatives.

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \log\left(\frac{c(p^t)}{c(p^{t-1})}\right) + \log\left(\frac{U(x^t)}{U(x^{t-1})}\right) - \log\left(\frac{x_i^t}{x_i^{t-1}}\right) + \log\left(\frac{e_i^t}{e_i^{t-1}}\right),$$
$$i = 1,\ldots,N\ ,\ t = 1,\ldots,T \tag{5.26}$$

What is interesting is that, for the Cobb-Douglas functional form, we can see what the error term represents in the stochastic approach in price relatives. Comparing the model (5.26) with (5.8) above we see that,

$$\varepsilon_i^{t,t-1} = \log\left(\frac{U(x^t)}{U(x^{t-1})}\right) - \log\left(\frac{x_i^t}{x_i^{t-1}}\right) + \log\left(\frac{e_i^t}{e_i^{t-1}}\right), \qquad i = 1,...,N\ ,\ t = 1,...,T \quad (5.27)$$

If the Cobb-Douglas utility function is a reasonable representation of the utility function then the error term will tend be large for those goods for which consumption has grown relatively slowly compared with the change in utility. Those goods whose consumption has grown slowly compared with the growth in utility or 'average' consumption growth are those goods which have the highest price relatives.

5.4.3. An Economic Justification for Stochastic Quantity Indexes

In economic measurement we are often at least as concerned about quantity change as price change. We could of course easily derive an *implicit* stochastic quantity index by deflating total revenues by a stochastically estimated price index. However, given the discussion in the preceding section, we can also show that it is possible to justify the direct estimation of stochastic quantity indexes. The literature on the stochastic approach has (to my knowledge) not discussed methods for calculating stochastic quantity indexes.[57]

The primary reason for the lack of interest in stochastic quantity indexes seems to be that there is no justification for stochastic quantity indexes which is analogous to the 'quantity theory of money' justification for stochastic price indexes. Here, we provide a different economic justification for stochastic quantity

[57] However, Clements and Izan (1987) do note that they believe there results are equally applicable to quantity or productivity measurement as to the measurement of prices.

indexes arising from the desire to estimate the change in utility level, the economic quantity measure, across time.

In economically justifying stochastic quantity indexes we follow the dual approach to that outlined above. Again assuming the cost function is homothetic and using Shephard's Lemma we have the following well known result.

$$x_i^t = \frac{\partial c(p^t)}{\partial p_i^t} U(x^t), \qquad i = 1,...,N,\ t = 0,1,...,T \tag{5.28}$$

Equation (5.28) is clearly analogous to equation (5.18) above. As we did for this equation, we can write (5.28) in the form of either levels or relatives.

$$\log(x_i^t) = \log(U(x^t)) + \log\left(\frac{\partial c(p^t)}{\partial p_i^t}\right), \qquad i = 1,...,N,\ t = 0,1,...,T \tag{5.29}$$

$$\log\left(\frac{x_i^t}{x_i^{t-1}}\right) = \log\left(\frac{U(x^t)}{U(x^{t-1})}\right) + \log\left(\frac{\partial c(p^t)}{\partial p_i^t} \Big/ \frac{\partial c(p^{t-1})}{\partial p_i^{t-1}}\right),\ i = 1,...,N,\ t = 1,...,T \tag{5.30}$$

Analogously to the discussion above we have an equation relating quantities, utility, and the partial derivative of the unit cost function. These equations show that under various assumptions about the derivative of the unit cost function a conventional stochastic approach in quantities could be justified as an estimator of the growth in utility. It can be readily inferred, that for various specifications of the unit cost function, different stochastic approaches will be exact as we discussed for prices above. However, in the interests of brevity we will not repeat this discussion for the estimation of stochastic quantity indexes.

5.5. The Conventional Economic Approach to Preference Estimation

In the discussion above we have shown how the stochastic approach to index numbers can be motivated from an economic perspective. We have endeavoured to show economically how a set of stochastic equations can be derived. However, from an economic perspective the assumptions that were required on the form of the utility or cost function were rather restrictive. There are other ways in which we can use economic theory, and parametric representations of technology, to estimate economic price and quantity indexes. As these are estimated stochastically a standard error is also naturally produced.

In this section we briefly discuss some of these methods. We focus on methods for estimating price indexes, which mean that we estimate the parameters of the cost function. While we could also investigate the estimation of utility functions and hence quantity indexes, we focus on the cost function representation because this is what most of the literature has done and also because the two discussions are very much parallel.

We look at two particular functional forms for the cost function; the Cobb-Douglas and the Translog functional forms. We start with the case where we have a complete panel of prices, no missing observations, and in the final section we discuss how this general approach could be expanded to take account of new and disappearing goods which generate 'holes' in the data.

5.5.1. The Cobb-Douglas Cost Function

The Cobb-Douglas function is one of the simplest and most useful functional forms known to economists. Let us suppose that the cost function is linearly homogeneous and that the unit cost function takes the Cobb-Douglas form as in (5.31) where $\sum_{i=1}^{N} \alpha_i = 1$.

$$c_{CD}(p^t) = \prod_{i=1}^{N} (p_i^t)^{\alpha_i}, \quad t = 0,1,...,T \tag{5.31}$$

Using Shephard's Lemma it can easily be seen that the parameters of the Cobb-Douglas cost function are in fact equal to the expenditure shares.

$$s_i^t = \alpha_i, \quad i = 1,...,N, \; t = 0,1,...,T \tag{5.32}$$

Thus we can easily obtain an estimate of the cost function by first estimating these parameters and then recovering the cost function itself. Note that the question of weighting the equations does not arise here. This is because we are estimating the parameters of the cost function rather than the cost function itself. If we estimate the parameters over some set of goods and time periods, and obtain $\hat{\alpha}_i \;\; i = 1,...,N$, then we can estimate the log of the cost-of-living index for Cobb-Douglas preferences between periods 0 and 1, denoted by $\log(\hat{P}_{1,0}^{CD})$, using the following expression.

$$\log(\hat{P}_{1,0}^{CD}) \equiv \log(\hat{c}_{CD}(p^1)) - \log(\hat{c}_{CD}(p^0)) = \sum_{i=1}^{N} \hat{\alpha}_i \log(p_i^1 / p_i^0) \tag{5.33}$$

In order to obtain the actual cost-of-living index we need to take the exponent of this expression. Additionally, as the log of the price index is a linear function of the parameters we can straightforwardly estimate the variance of the log of the cost-of-living index.

$$Var(\log(\hat{P}_{1,0}^{CD})) = \sum_{i=1}^{N} \sum_{j=1}^{N} Cov(\hat{\alpha}_i, \hat{\alpha}_j)(\log(p_i^1 / p_i^0)\log(p_j^1 / p_j^0)) \tag{5.34}$$

$$= \sum_{i=1}^{N} Var(\hat{\alpha}_i)(\log(p_i^1 / p_i^0))^2 \tag{5.35}$$

Here (5.35) follows from (5.34) as the covariance between the estimates is zero because the good specific dummy variable are uncorrelated. Notice how the variance of the log price index is derived in (5.35) and what it represents. It is a linear combination of the variance estimates from estimating the equations shown in (5.32). This means that if the Cobb-Douglas functional form fits the data very well and expenditure shares are close to constant through time then the variance will generally be low. The critical point in this case is that the variance of the index estimate will reflect how well the hypothesised functional form fits the data. This is quite different from the standard error that arose in the basic stochastic approach and equation (5.10) suggested by Selvanathan and Rao (1994). However, also note that from (5.35) that if the change in price relatives is large, in either direction, then the variance of the estimate will be increased. It is in this sense that the variance of the cost-of-living index is greater when there is greater dispersion in prices.

To obtain an approximation to the variance of the actual cost-of-living index, not the log cost-of-living index, we can use the Delta Method, which is discussed in Section 5.8.3 of the Appendix. In this situation, an application of the Delta Method means we linearize the exponential transformation, using a first order Taylor Series approximation, and calculate the variance of this linear function.

a. The Cobb-Douglas Cost Function and the Tornqvist Price Index

It is interesting to note that if we estimate the parameters of the Cobb-Douglas cost function, α_i , over only two periods, say 0 and 1, then the ratio of the cost functions will give the well known Tornqvist Price Index. This is because if we estimate α_i over just two periods, using OLS, then $\hat{\alpha}_i$ will simply be the arithmetic average of the two periods expenditure shares for the good, $\hat{\alpha}_i = 0.5\times(s_i^0 + s_i^1)$, as shown in Section 5.8.4 of the Appendix. Thus the estimate of the cost-of-living index ($\hat{P}_{1,0}^{CD}$) will have the Tornqvist ($P_{1,0}^{T}$) form.

$$\log(\hat{P}_{1,0}^{CD}) \qquad = \log(\hat{c}_{CD}(p^1)) - \log(\hat{c}_{CD}(p^0)) \tag{5.36}$$

$$= \sum_{i=1}^{N} \hat{\alpha}_i \log(p_i^1 / p_i^0) \quad (5.37)$$

$$= \sum_{i=1}^{N} 0.5 \times (s_i^0 + s_i^1) \log(p_i^1 / p_i^0) \quad (5.38)$$

$$\equiv \log(P_{1,0}^T) \quad (5.39)$$

Again we can provide the standard error for this cost-of-living index using the covariance matrix of the parameter estimates and the Delta Method.

5.5.2. The Translog Cost Function

The problem with the Cobb-Douglas functional form is that it places a priori restrictions on the patterns of substitution. For the Cobb-Douglas cost function the elasticity of substitution is equal to one for all goods.[58] Because of this restriction, which is inherent in the Cobb-Douglas cost function, we are less confident that it can represent complex patterns of substitution and complementarity between goods and across time and hence give an accurate representation of the cost-of-living index. It is for this reason that much of the literature on price and quantity measurement has focused on flexible representations of preferences (Diewert, 1976).

The Translog function is a flexible functional form, as defined by Diewert (1976), and can be thought of as a second-order Taylor Series approximation in logarithms about the point where the price vector is equal to one. The Translog unit cost function has the following form.

$$\log(c_{TLOG}(p^t)) = \sum_{i=1}^{N} \alpha_i \log(p_i^t) + \sum_{j=1}^{N} \sum_{i=1}^{N} \beta_{j,i} \log(p_j^t) \log(p_i^t), \quad t = 0,1,...,T \quad (5.40)$$

[58] More formally this means that for goods i and j we have, $\partial \log(x_i^t / x_j^t) / \partial \log(p_i^t / p_j^t) = -1$.

Using Shephard's Lemma we have the following relationship between expenditure shares and the parameters of the unit cost function.

$$s_i^t = \alpha_i + \sum_{j=1}^{N} \beta_{j,i} \log(p_j^t), \quad i = 1,...,N, \; t = 0,1,...,T \tag{5.41}$$

The share equations can be estimated using standard linear regression techniques. However, an important point is that there are a range of restrictions that must be imposed on the parameters of the cost function to make it consistent with economic theory. Firstly, the second derivates of the cost function must be symmetric.

$$\beta_{j,i} = \beta_{i,j}, \quad i, j = 1,...,N \tag{5.42}$$

In order to satisfy linear homogeneity of the cost function in prices we also require,

$$\sum_{j=1}^{N} \beta_{j,i} = 0, \quad i = 1,...,N \tag{5.43}$$

We also need to reflect the fact that expenditure shares sum to one. This implies the following constraints.

$$\sum_{i=1}^{N} \alpha_i = 1 \tag{5.44}$$

$$\sum_{i=1}^{N} \beta_{j,i} = 0, \quad j = 1,...,N \tag{5.45}$$

There are a range of different ways in which these linear restrictions can be imposed but these are relatively straightforward and while easing the estimation they will not affect the results.

Using the same approach as for the Cobb-Douglas case above we can obtain an estimate of the cost-of-living index, say between periods 0 and 1, using the estimated parameters for the unit cost function.

$$\log(\hat{P}^{TLOG}_{1,0}) = \log(\hat{c}_{TLOG}(p^1)) - \log(\hat{c}_{TLOG}(p^0)) \tag{5.46}$$

$$= \sum_{i=1}^{N} \hat{\alpha}_i \log(p_i^1 / p_i^0)$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{\beta}_{i,j} (\log(p_i^1)\log(p_j^1) - \log(p_i^0)\log(p_j^0)) \tag{5.47}$$

Again, as the log of the cost-of-living index is linear in the estimated parameters it is relatively easy to obtain the standard error of this index as we did for the Cobb-Douglas case. Using the covariance matrix of the parameter estimates we can obtain an approximate standard error for the cost-of-living index using the Delta Method.

In comparing the Cobb-Douglas and Translog approaches, the advantage of the latter is clearly a greater flexibility in the functional form, so that more complex interrelationships can be accommodated. However, this flexibility comes at a cost which is the far greater number of parameters. With the restrictions imposed the Translog cost function has $N(N-1)/2$ parameters to estimate while the Cobb-Douglas cost function has just N parameters. The Translog approach will be impractical when the number of goods is very large and the number of time periods is relatively small. Indeed for the parameters to be identified in the Translog case we must have the number of observations greater than the number of parameters, $NT > N(N-1)/2$ or $T > (N-1)/2$. Clearly when N is large this may require an impractically long time series.

5.5.3. The Conventional Economic Approach with New and Disappearing Goods

As discussed previously, one of the great advantages of the stochastic approach to index numbers is that in various formulations it can accommodate new and disappearing goods. Indeed, this is why the TPD (and CPD) method, of equation (5.11), is so useful. An important question then is, can the conventional approach to preference estimation accommodate changes in the set of available goods and hence give an estimate of the cost-of-living index in this circumstance?

Here it is argued that it is indeed possible to use the conventional approach to account for the effects of missing prices due to new and disappearing goods but there are added complications. To discuss this further let us introduce some notation. Denote the set of goods available in period t by I^t and suppose we have a time series $t = 0,1,...,T$. Then in order for this approach to be meaningful we must define a (unit) cost function over all the goods available in all the time periods, $I^* = I^0 \cup ... \cup I^T$.

$$c(p^t) = \min{}_x \left\{ \sum_{i \in I^t, i \in I^*} p_i^t x_i^t + \sum_{i \notin I^t, i \in I^*} \hat{p}_i^t x_i^t : U(x) \geq 1 \right\}, \quad t = 0,1,...,T \qquad (5.48)$$

We can proceed in conventional fashion and specify a functional form for the unit cost function which includes all the goods available in all periods. However, along with the parameters of the cost function we are also required to estimate the (reservation) prices for those goods which were not present in a given period. It is only with estimates of both the reservation prices and the parameters of the cost function that we can determine the cost-of-living index.

Another important aspect of this approach is that we must choose a functional form for the cost function which depends on prices in such a way that it allows the zero consumption of commodities. To see where this does not work, consider the Cobb-Douglas cost function. Here the expenditure share of a good is strictly positive regardless of the price of the good – that is there is no price which will generate zero consumption. In the more flexible Translog case however, a reservation price which is finite may exist.

The approach advocated here in allowing for the influence of new and disappearing goods on the cost-of-living index has to my knowledge not been applied in practice.[59] The estimation proceeds on the basis of a parametrically specified cost function defined over the entire panel of goods. The zero consumption of goods which were not available in a given period is regarded as an equilibrium outcome, generated by the high notional price for the good in that period. These reservation prices are required to be estimated along with the parameters of the cost function. This should be practicable as long as the cost function is flexible enough to allow zero consumption.

There are some advantages of this approach over that advocated by Hausman (1981, 1997, 1999, 2003), which for a new good estimates the market demand curve over the post-introduction period and then derives the cost function by Roy's Identity.[60] In order to estimate the market demand curve, only data from the post-introduction period is used. The estimated parameters of the market demand function, and the prices of the various alternative goods that existed in previous periods, are then used to extrapolate the demand curve backwards in order to estimate the reservation price for the new good. In our framework the reservation price is actually estimated as part of the system, from both post and pre-introduction data. This approach to new and disappearing goods appears interesting though has yet to be tested. Again, it should be noted that because a flexible functional form has to be used, and we also have to estimate each of the reservation prices, the number of parameters in the model may be very large.

5.5.4. Summary

The conventional economic approach to preference estimation has a natural appeal. A theoretically strong and flexible representation of preferences can be hypothesised and estimated from which we can derive the cost-of-living index. We have also described how this approach can be applied when there are new and

[59] Fisher and Shell (1972, p. 24) advocate this general approach where the reservation prices are regarded as unknowns which we solve for along with the optimal quantities and Diewert (1980) has outlined a practical procedure, including functional forms, which could be used.

[60] This approach was discussed in greater detail in Chapter 2, Section 2.4.

disappearing goods. However, one disadvantage of this approach over the basic stochastic approach is that we are estimating the cost function indirectly. That is, we first estimate the parameters of the cost function and then derive the cost function from these parameters and the relevant prices. As a result a large number of parameters need to be estimated and burdensome calculations are required to obtain the price index and its variance. In the next section we consider an attractive alternative method which avoids this problem and sits somewhere between the conventional economic approach outlined in this section and the stochastic approach of Section 5.3.

5.6. An Alternative Approach Using the CES Cost Function

Rather than estimating the parameters of the cost function it would be preferable to estimate the price index or price level directly as does the basic stochastic approach. Certain functional forms are likely to be more amendable to this direct estimation than others. Here we consider an approach based upon the Constant Elasticity of Substitution (CES) unit cost function shown in (5.49) for period t.[61]

$$c_{CES}(p^t) = \left(\sum_{i=1}^{N} a_i \, (p_i^t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{5.49}$$

Using Shephard’s Lemma we have the following result relating expenditure shares, prices and the unit cost function.

$$s_i^t = \frac{a_i \, (p_i^t)^{1-\sigma}}{\sum_{i=1}^{N} a_i \, (p_i^t)^{1-\sigma}} = a_i \left(\frac{p_i^t}{c_{CES}(p^t)} \right)^{1-\sigma}, \qquad i = 1,\ldots,N \,,\; t = 0,1,\ldots,T \tag{5.50}$$

[61] Parts of this section were inspired by the discussion of Feenstra and Reinsdorf (2004).

The economic intuition of the relationship is that expenditure shares are determined by ‘real’ prices, that is prices relative to the unit cost function, and a good specific component. Over time any change in expenditure shares is due to changes in the price of a good relative to the ‘composite economic good’.

$$\frac{s_i^t}{s_i^{t-1}} = \left(\frac{p_i^t}{p_i^{t-1}} \bigg/ \frac{c_{CES}(p^t)}{c_{CES}(p^{t-1})} \right)^{1-\sigma}, \quad i = 1,\dots,N\,,\ t = 1,\dots,T \tag{5.51}$$

The basic relationship expressed in (5.50) is particularly useful as it allows us to construct a set of equations where the unit cost function, or ratio of unit cost functions, can be estimated directly.

5.6.1. A Stochastic CES Price Index in Levels

We first consider the case where we estimate the CES cost-of-living index using a stochastic approach in levels. Let us take the logarithms of (5.50) to obtain the equation below.

$$\log(s_i^t) = \log(a_i) + (1-\sigma)\log(p_i^t) - (1-\sigma)\log(c_{CES}(p^t))\,,$$
$$i = 1,\dots,N\,,\ t = 0,1,\dots,T \tag{5.52}$$

This is readily estimable, with the addition of an error term. Note also that as we are estimating this equation in levels, it can accommodate changes in the domain of goods over time. One problem with (5.52), however, is that the equation is nonlinear, the log of the unit cost function and the elasticity parameter enter multiplicatively. The nonlinearity of this equation is not likely to pose insurmountable difficulties to estimation. However, one alternative is to rearrange the model as we do below, putting prices on the LHS. Here, if the parameters are chosen appropriately, the model is linear.

$$\log(p_i^t) = -\left(\frac{1}{1-\sigma}\right)\log(a_i) + \left(\frac{1}{1-\sigma}\right)\log(s_i^t) + \log(c_{CES}(p^t)),$$

$$i = 1,\ldots,N\,,\, t = 0,1,\ldots,T \qquad (5.53)$$

However, this model is a little theoretically unappealing as the expenditure share, or more precisely, the quantity of the good consumed, are the endogenous variables and hence should appear on the LHS of the regression equation. This is because the relationship was derived from the optimisation problem where prices are taken as given and the consumer chooses quantities. We could justify the use of the model in (5.53) on the grounds of simplicity if the results from this model are not too different from those of (5.52).

Clearly this latter model is very much analogous to the TPD method discussed above. The models (5.52) and (5.53) offer a very appealing refinement of the idea underlying the stochastic approach in that the estimated price index is influenced by the degree of substitution. If $\sigma > 1$ then this raises the RHS of equation (5.53), as $\log(s_i^t) < 0$ because $s_i^t < 1$. This means that part of any price increases over time is absorbed by this term and does not get reflected in the unit cost parameter. On the other hand if $\sigma < 1$ then this lowers the LHS and magnifies the effect of any price changes. Finally, if $\sigma = 1$ then we have Cobb-Douglas preferences and from (5.50) $s_i^t = a_i$, which means that the first two terms on the RHS cancel. This implies that the cost-of-living index is estimated by a mean of prices.

One important consideration that we have not discussed yet is whether to impose the constraint linking the good-specific parameters (a_i), the elasticity of substitution (σ), and the unit cost function ($c(p^t)$). That is whether we should estimate the model with (5.49) imposed as a constraint. We need to impose this constraint if we have complete data for all the goods that enter the unit cost function. The difficulty with imposing this constraint, however, is that the function linking these parameters is highly nonlinear which greatly complicates estimation. One approach would be to not impose this constraint on the basis that the data used for any estimation is partial and contains only a sample of the set of goods which enters the unit cost function.

5.6.2. A Stochastic CES Price Index in Relatives

Let us put equation (5.50) in relative form and take the logarithms. This is helpful as it eliminates the parameter a_i, which is troublesome as it relates in a complex nonlinear way to the unit cost function.

$$\log\left(\frac{s_i^t}{s_i^{t-1}}\right) = (1-\sigma)\log\left(\frac{p_i^t}{p_i^{t-1}}\right) - (1-\sigma)\log\left(\frac{c_{CES}(p^t)}{c_{CES}(p^{t-1})}\right),$$
$$i = 1,\dots,N\,,\ t = 1,\dots,T \qquad (5.54)$$

This gives a method for estimating the cost-of-living index, represented by the CES cost function. However, this equation is again nonlinear in the parameters with the elasticity and relative price level parameters entering multiplicatively. In the interests of simplicity we could rearrange this equation into a linear form keeping in mind the reservations we had about this approach above.

$$\log\left(\frac{p_i^t}{p_i^{t-1}}\right) = \log\left(\frac{c_{CES}(p^t)}{c_{CES}(p^{t-1})}\right) + \left(\frac{1}{1-\sigma}\right)\log\left(\frac{s_i^t}{s_i^{t-1}}\right), \quad i = 1,\dots,N\,,\ t = 0,1,\dots,T \ (5.55)$$

If we contrast this equation with the stochastic approach in relatives, equation (5.8) above, we see that the error term in (5.8) is equal to the last term on the RHS of (5.55). Interestingly, this term need not sum to one. In practice an error term could be appended to this equation and it could form the basis of a stochastically estimated price index. Because the a_i parameters of the unit cost function are not included in model (5.55) we do not need to impose any additional constraints on the estimation of this equation.

a. Good Specific Price Trends Again

Interestingly, as was noted by Feenstra and Reinsdorf (2004), the equation (5.55) is similar to the *good specific price trend model* discussed above, where the relative log expenditure shares term takes the role of good specific price trend. Furthermore, as emphasised by Feenstra and Reinsdorf (2004), suppose we choose to estimate equation (5.55) using WLS, for periods 0 and 1, and that the weights used were the normalised logarithmic mean of expenditure shares shown below.[62]

$$\tilde{w}_i = \frac{(s_i^1 - s_i^0)/(\log(s_i^1) - \log(s_i^0))}{\sum_{i=1}^{N} (s_i^1 - s_i^0)/(\log(s_i^1) - \log(s_i^0))}, \qquad i = 1,\ldots,N \tag{5.56}$$

Then it can readily be seen that the sum of the good specific price trend term is equal to zero just as in the econometric model discussed above.

$$\sum_{i=1}^{N} \tilde{w}_i \left(\frac{1}{1-\sigma}\right) \log\left(\frac{s_i^1}{s_i^0}\right) = 0, \qquad t = 0,1,\ldots,T \tag{5.57}$$

This provides some economic backing for the form of the good specific price trend model. We now turn to a summary and some conclusions regarding our discussion.

5.7. Conclusion

This chapter has attempted a reconciliation between the stochastic and economic approaches to index numbers. It has been shown how closely related these two methods are in that the basic stochastic approach can be interpreted as a method for estimating the cost-of-living index under certain conditions.

[62] These weights have a particular appeal in this context because they are the weights for the price relatives in the Sato-Vartia Price Index, which is an exact price index for the CES cost function.

Adopting this unified economic-stochastic approach to index numbers leads us to think of a number of natural enhancements that could be made to the stochastic approach. Firstly, it seems reasonable to estimate stochastic quantity indexes in much the same spirit as price indexes. Secondly, a number of alternative methods, closely related to economic theory are available. We have discussed various methods for estimating the parameters of the cost function. However, perhaps the most attractive option is the approach suggested in Section 5.6 using the CES functional form. The main advantage of this approach is that the log of the price index, and its standard error, is automatically produced when the regression is run rather than having to be derived in a rather complex way. It offers a generalisation of the basic stochastic approach. Interestingly, the standard error from such an approach will reflect both the variability of prices and the fit of the functional form to the data.

The stochastic approach offers a new way of dealing with difficult problems such as new and disappearing goods. This chapter has emphasised that just because we calculate the index from a regression does not necessarily mean that we cannot interpret the index within a cost-of-living framework. In fact stochastically estimated indexes can be closely related to economic theory. This means that the basic approaches can be studied in the context of our economic measurement goals and more general and flexible methods developed.

5.8. Appendix

5.8.1. Deriving the Cramer-Rao Lower Bound in Equation (5.16)

The likelihood function, under the assumption of normally distributed errors, $\varepsilon_i^{t,t-1} \sim N(0,\sigma_i^2)$, is shown below, along with the second derivatives of this function with respect to $\delta_*^{t,t-1}$.

$$\log L = c + -\frac{T}{2}\sum_{i=1}^{N} \log\sigma_{*i}^2 - \sum_{t=1}^{T}\sum_{i=1}^{N}\frac{1}{2\sigma_{*i}^2}\left(\log\left(\frac{p_i^t}{p_i^{t-1}}\right) - \delta_*^{t,t-1}\right)^2 \tag{5.58}$$

$$\frac{\partial^2 \log L}{\partial(\delta_*^{t,t-1})^2} = -\sum_{i=1}^{N}\frac{1}{\sigma_{*i}^2} \tag{5.59}$$

From the Cramer-Rao Theorem we know that the lower bound on the variance of $\delta_*^{t,t-1}$, the asymptotic variance, is equal to the following value.

$$Asy.Var(\delta_*^{t,t-1}) = \frac{1}{-E\left(\dfrac{\partial^2 \log L}{\partial(\delta_*^{t,t-1})^2}\right)} \tag{5.60}$$

We obtain the result in the text by using equations (5.59) and (5.60).

5.8.2. Interpreting the Lagrange Multiplier

Using (5.17), if we multiply both sides by x_i^t and sum over all goods, this gives the following relationship for period t.

$$\sum_{i=1}^{N} p_i^t x_i^t = \lambda\left(\sum_{i=1}^{N} \frac{\partial U(x^t)}{\partial x_i^t} x_i^t\right) \tag{5.61}$$

As we assume that the utility function is homogeneous of degree one, $U(kx) = kU(x) \quad k > 0$, then the non-$\lambda$ term on the RHS of (5.61) will equal the utility level.[63] The LHS of (5.61) is equal to total expenditure. In the case where the utility function is homogeneous of degree one then the cost function is linearly homogeneous in utility, $C(p^t, U) = c(p^t)U$ (Diewert, 2001, pp. 48-49). By rearranging (5.61) and using these results we can see that λ is in fact equal to the unit cost function.

$$\lambda = \frac{\left(\sum_{i=1}^{N} p_i^t x_i^t\right)}{\left(\sum_{i=1}^{N} \frac{\partial U(x^t)}{\partial x_i^t} x_i^t\right)} \tag{5.62}$$

$$= \frac{C(p^t, U(x^t))}{U(x^t)} \tag{5.63}$$

$$= \frac{c(p^t)U(x^t)}{U(x^t)} \tag{5.64}$$

$$= c(p^t) \tag{5.65}$$

5.8.3. The Delta Method

In a number of places we are interested in the variance of the exponential transformation of a variable. In general suppose we have an estimated random variable $\hat{a}$, for which we know the variance, and we wish to find the variance for the transformation $f(\hat{a})$. Then applying the Delta Method we can take a first order

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[63] This is due to Euler's Theorem, an outline of which can be found in many mathematical economics textbooks, see for example Simon and Blume (1994, p. 491, Theorem 20.4).

Taylor Series approximation of $f(\hat{a})$ about the expected value of $\hat{a}$, let us define $a^* = E(\hat{a})$.

$$f(\hat{a}) \cong f(a^*) + \frac{\partial f(a^*)}{\partial a^*}(\hat{a} - a^*) \qquad (5.66)$$

Then we can calculate the variance using the following formula.

$$Var(f(\hat{a})) \cong Var(\hat{a}) \left(\frac{\partial f(a^*)}{\partial a^*} \right)^2 \qquad (5.67)$$

In the case of the exponential function, $f(\hat{a}) = \exp(\hat{a})$, we have the following formula, where in practice we can replace a^* by $\hat{a}$.

$$Var(\exp(\hat{a})) \cong Var(\hat{a}) \exp(2a^*) \qquad (5.68)$$

5.8.4. The Least Squares Estimation of the Cobb-Douglas Cost Function

Let us suppose that we estimate the parameters of the Cobb-Douglas cost function over periods $t = 0,1,...,T$ for goods $i = 1,...,N$. Then let us minimise the following sum of squared residuals, from equation (5.32) given in the text, to obtain the OLS estimates.

$$SSR = \sum_{t=0}^{T} \sum_{i=1}^{N} (s_i^t - \alpha_i)^2 \qquad (5.69)$$

Minimising SSR gives the first order condition shown below.

$$\frac{dSSR}{d\alpha_i} = 2\sum_{t=0}^{T} (s_i^t - \alpha_i) \tag{5.70}$$

At the minimum $dSSR / d\hat{\alpha}_i = 0$ so the OLS estimates take the following form.

$$\hat{\alpha}_i = \frac{1}{T+1}\sum_{t=0}^{T} s_i^t \tag{5.71}$$

For two time periods, 0 and 1, we have the result discussed in the text.

CHAPTER 6

6. Conclusion

This thesis has covered a lot of ground. It began by surveying, defining and conceptualising the quality change and new and disappearing goods problem and moved onto outlining various solutions, and then undertaking both empirical and theoretical investigations of these methods. The issues involved are certainly not simple either conceptually or practically. However, the thesis has emphasised that we should look to the future with optimism and confidence as our knowledge and ability to deal with these issues is now well advanced and continues to develop. In this regard let us summarize the contribution of this thesis to this area by reiterating its major results and conclusions.

Chapter 2 surveyed the issues of quality change and new and disappearing goods in the context of price measurement. While the primary role of the paper was to outline various approaches to the measurement problem some novel results were also outlined. In appraising conventional statistical agency methods of quality adjustment we defined what we meant by quality differences. More specifically we took differences in marginal utility to represent quality differences at the margin. While the 'quality' of a good has been closely identified with the effect on utility there has been little formalization of this concept. As noted in the text there are other ways in which the quality of a good can be conceptualized in relation to utility such as looking at the goods share of total utility. A more detailed exploration of these issues in the literature would help to clarify the assumptions that are made in different approaches to quality adjustment.

Perhaps the most compelling point made in Chapter 2 is that the standard approach to quantity-quality change, where the package size of a good changes, is likely to be wrong. Due to the concavity of the price-package-size function the adjustment of prices by the change in package size will lead to an overestimate of quality change when package size increases and an underestimate when package size decreases. It would be useful if this result was replicated by other researchers in different product areas. If similar results were obtained then the standard approach of quality-adjusting by package size should be abandoned and more sophisticated approaches, based upon hedonic package size models, could be used instead.

The observation in Chapter 3 that the hedonic regression time-dummy method fails monotonicity is worrying. This means that a price index calculated using this method can be lower when we increase a price for a later period, leaving all other prices unchanged. This violates what seems like an important intuitive principle and undermines the use of this method. Interestingly, however, it may be the way in which we have applied that monotonicity axiom that is in some sense faulty – in terms of economic theory. This case provides an interesting conflict between the economic and axiomatic approach to index numbers analogous to that discussed in Reinsdorf and Dorfman (1999). The paper is useful in outlining this conflict and alerting practitioners in this area to the undesirable axiomatic properties of this procedure. Indeed it will be interesting to see how practitioners react to this finding.

Empirical research on scanner data sets has burgeoned in recent years (For example: Dalén, 1997; Haan, 2001; Reinsdorf, 1999; Silver and Webb, 2002). However, little of this attention has focused on the effect of new and disappearing goods on the cost-of-living index despite this being an important and enduring measurement problem. Chapter 4 looked at the effect of new and disappearing goods for a large Australian scanner data set. The approach taken is to apply a method derived from Feenstra (1994) and Balk (1999) to estimate these effects. A large difference is found between the matched goods price index and the index which properly accounts for new and disappearing goods. The difference is between 1.2 and 2.4 percentage points per year. This bias is confirmed by a simplified reservation price method – though this method gives a smaller estimate of the bias. These results indicate that there may be significant gains in consumer welfare from expansions in the available set of commodities. This gain is not currently being recorded by the statistical authorities. This area certainly warrants further research and it seems advisable to reserve our judgment until further replication of these types of studies has been undertaken over different time periods and product ranges. It is anticipated that there will be significant advances made in this area both because we now have a wider range of tools than was previously available but perhaps more important because we now have access to high frequency scanner data.

The stochastic approach to index numbers has been undergoing somewhat of renaissance recently, primarily because of its usefulness in cases where there are changes in the domain of goods. This has proved particularly advantageous in the spatial context (Aizcorbe and Aten, 2004; Rao, 2004) and in time may also prove equally useful in this regard in the time series context. Chapter 5 attempts to further foster this rehabilitation of the stochastic approach by showing that it can be interpreted as a method for estimating the economic cost-of-living index under certain circumstances. This should broaden the appeal of this approach, however, it does emphasize some of the rather restrictive assumptions that are required to motivate this link. Perhaps the most important contribution made by this chapter is to suggest what amounts to a generalization of the stochastic approach, based upon the Constant Elasticity of Substitution functional form. This allows for the cost function to be directly estimated from a regression and importantly allows for different degrees of substitution to be reflected in the estimation of the price index. This approach would benefit from empirical testing but shows some promise in more firmly grounding the stochastic approach in economic theory.

The aim of the thesis has been to look at some interesting and pressing concerns in economic measurement. It is hoped that the chapters of this thesis have shed light on these issues and contributed to improving our shared knowledge in these areas.

CHAPTER 7

7. Bibliography

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