

Learning geometry problem solving by studying worked-examples: effects of learner guidance and expertise

Author:

Bokosmaty, Sahar

Publication Date:

2012

DOI:

<https://doi.org/10.26190/unsworks/15806>

License:

<https://creativecommons.org/licenses/by-nc-nd/3.0/au/>

Link to license to see what you are allowed to do with this resource.

Downloaded from <http://hdl.handle.net/1959.4/52246> in <https://unsworks.unsw.edu.au> on 2024-04-25



**Learning Geometry Problem Solving by Studying
Worked-Examples: Effects of Learner Guidance
and Expertise**

Sahar Bokosmaty

Bachelor of Mathematics
Graduate Diploma of Education
Master of Mathematics Education
Master of Business Administration

Thesis submitted in fulfilment of the requirements for
the degree of Doctor of Philosophy

March 2012

**School of Education
Faculty of Arts and Social Sciences
University of New South Wales
Sydney, Australia**

PLEASE TYPE**THE UNIVERSITY OF NEW SOUTH WALES
Thesis/Dissertation Sheet**

Surname or Family name: Bokosmaty

First name: Sahar

Other name/s:

Abbreviation for degree as given in the University calendar: PhD

School: School of Education

Faculty: Arts and Social Sciences

Title: Learning Geometry Problem Solving by Studying Worked-

Examples: Effect of Guidance and Expertise

Abstract 350 words maximum: (PLEASE TYPE)

Research has demonstrated that instruction that relies heavily on worked examples is more effective for novices as opposed to instruction consisting of problem-solving. However, excessive guidance for expert learners may reduce their performance.

This study investigated optimal degrees of guidance using geometry worked examples. Three conditions were used. In the Theorem & Step Guidance condition, students were told the steps to find each angle, the measure of the angle, and the theorem used to justify the answer. In the Step Guidance condition, learners were told the sequence of steps needed to reach the answer, but not told the theorem required to make a step. The Problem Solving condition required learners to solve problems with no guidance.

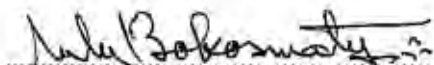
It was hypothesized that by using Step Guidance, a new concept would be more readily incorporated into existing knowledge held in long-term memory compared to a Problem Solving approach or a Theorem & Step Guidance approach. Problem Solving would impose the heavy cognitive load associated with problem solving search while providing information concerning well-known theorems would be redundant. In other words, as long recognised by cognitive load theory, most students need to learn to recognise problem states and the moves associated with those states and this information is provided by Step Guidance without additional, redundant information.

A series of geometry instruction experiments supported these hypotheses. The results of these experiments revealed that for students who already understand the relevant theorems, learning to solve problems primarily consists of learning to recognise problem states and their associated moves. Information concerning theorems only should be provided if students have yet to learn and automatise theorem schemas.

Declaration relating to disposition of project thesis/dissertation

I hereby grant to the University of New South Wales or its agents the right to archive and to make available my thesis or dissertation in whole or in part in the University libraries in all forms of media, now or here after known, subject to the provisions of the Copyright Act 1968. I retain all property rights, such as patent rights. I also retain the right to use in future works (such as articles or books) all or part of this thesis or dissertation.

I also authorise University Microfilms to use the 350 word abstract of my thesis in Dissertation Abstracts International (this is applicable to doctoral theses only).


Signature
Witness03/03/2012
Date

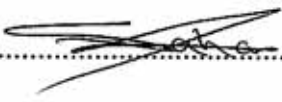
The University recognises that there may be exceptional circumstances requiring restrictions on copying or conditions on use. Requests for restriction for a period of up to 2 years must be made in writing. Requests for a longer period of restriction may be considered in exceptional circumstances and require the approval of the Dean of Graduate Research.

FOR OFFICE USE ONLY

Date of completion of requirements for Award:

Originality Statement

'I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where due acknowledgment is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis. I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation and linguistic expression is acknowledged.'

Signed..........

Date.....03/03/2012.....

Acknowledgements

I would like to acknowledge everyone who contributed to the completion of this thesis.

My supervisor, Professor John Sweller for his immeasurable support, inspiration, and continued encouragement; my gratitude for him is beyond words. My co-advisor Professor Slava Kalyuga for his valuable insight and guidance. John and Slava were very cooperative and responded promptly to emails and queries. I am grateful for their patience and persistence especially during the writing of this thesis.

My mum, her soul did not leave me a minute during this journey. I followed her advice and the path that she taught me in life to complete this task. I only wish she was here now to see my achievements!

My dad, who always wanted me to be “Dr. Sahar” and continuously reminded me of his trust in my abilities. He worked very hard all his life to provide me with the highest educational standard in order to reach this goal. I am very happy that I was able to fulfil his dream.

My sisters Boriada & Sawsan, my sisters in law, Nabiha, Samira, Samia, Sanaa, & Missou, My special nephews Ramzi & Rami, and my brothers in law Dr Fadi & Walid. The support that they all provided my family and me during this period was beyond expectations.

My overseas friends: Fatima, Nadia, Samar, Suzy, and Majed. The genuine and enormous help that they provided my family during my absence is unforgettable. I am very thankful for your sincere friendship. A special acknowledgment to Rana, Chee, & Linda for following my progress step by step.

My previous manager Chris, who always believed in my capabilities and encouraged me to reach my ultimate potential, the regular professional conversation enhanced my commitment to the task.

My friend Kusai and his family for providing a loving family support that pulled me up in hard days.

My friend Joe for creating the essential network that facilitated the substantial amount of participants to conduct the experiments of this thesis.

Last but not least, my dearest husband Ghazi for his unconditional love and continuous support; thank you Ghazi for being a great (father & a mother) to our beautiful children during my absence. I would not dream of raising Layal & Khaled any differently than you did by yourself.

My gorgeous daughter Layal and my lovely son Khaled, whom I left as little children, and now they have become teenagers. They passed through tough days with no mum around but reacted positively by working academically very hard and exceeded every one's expectations. Layal & Khaled, I am very proud of you especially when you were able to attain the "Honour list" on most of the school results. The completion of my PhD would have not been possible without the continuous support that you gave me by being solely responsible for yourselves. I hope that I will make up these days for you!

Table of Contents

Originality Statement.....	i
Acknowledgements	ii
Table of Contents	iv
Abstract	ix
List of Figures	xi
List of Tables.....	xiii
Appendices.....	xv
Chapter One	1
Human Cognition and the Categories of Knowledge.....	1
1.1 Introduction	1
1.2 Working Memory	2
1.3 Long-Term Memory	4
1.4 Schemas.....	6
1.5 Automation	8
1.6 Biologically Primary and Biologically Secondary Knowledge.....	10
1.7 Human Cognitive Architecture.....	13
1.7.1 Information store principle	14
1.7.2 Borrowing and reorganising principle.....	14
1.7.3 Randomness as genesis principle	15
1.7.4 The narrow limits of change principle	16

1.7.5 The environmental organising and linking principle	16
1.8 Summary	17
 Chapter Two	 19
Cognitive Load Theory	19
2.1 Introduction	19
2.2 Element Interactivity.....	20
2.3 Understanding and Rote Learning.....	23
2.4 Altering Intrinsic Cognitive Load and Element Interactivity	26
2.5 Extraneous Cognitive Load and Element Interactivity.....	27
2.6 Germane Cognitive Load	32
2.8 Instructional Design Effects	34
2.8.1 The Worked Example Effect.....	35
2.8.2 The Split-Attention Effect.....	39
2.8.3 The Redundancy Effect	43
2.8.4 The Expertise Reversal Effect.....	46
2.9 Summary.....	49
 Chapter Three	 52
Learning and Instruction in Geometry Problem Solving.....	52
3.1 Issues in Teaching Geometry	52
3.2 The Role of Geometry Schemas in Problem Solving	56
3.3 Features of Geometric Schemas.....	58
3.4 Research in Cognitive Load Aspects of Learning Geometry	61

3.4.1 Introduction	61
3.4.2 Worked Examples in Learning Geometry.....	62
3.5. Summary.....	68
 Chapter Four.....	 69
 Learner Levels of Expertise and Different Forms of Instructional	
Guidance in Worked Examples	69
4.1 Introduction	69
4.2 Expertise Reversal Effect.....	70
4.3 Some Worked Example Experimental Results.....	74
4.4 Summary.....	84
 Chapter Five.....	 87
Evaluation of Cognitive Load and Instructional Efficiency	87
5.1 Introduction	87
5.2 Objective Evaluation of Cognitive Load.....	88
5.3 Subjective Evaluation of Cognitive Load	90
5.4 Measurement of Instructional Efficiency.....	92
5.5 Objective of the Study.....	94
 Chapter Six	 95
Experiment 1	95
6.1 Introduction	95
6.2 Method.....	96

6.2.1 Participants	96
6.2.2 Material.....	96
6.2.3 Procedure	97
6.3 Results	102
Variables	102
Similar test results	102
Transfer test results	103
6.4 Discussion	104
 Chapter Seven	 106
 Experiment 2	 106
 7.1 Introduction	 106
 7.2 Method.....	 107
7.2.1 Participants	107
7.2.2 Materials and procedure.....	107
7.3 Results	109
Variables	109
Similar test results	110
Transfer results	112
7.4 Discussion	113
 Chapter Eight.....	 116
 Experiment 3	 116
 8.1 Introduction	 116
 8.2 Method.....	 117
8.2.1 Participants	117
8.2.2 Material and Procedure.....	118
8.2.3 Rating of cognitive load associated with task performance	119
8.3 Results	119
Variables	119
Similar test results	120

Transfer test results	123
Ratings of cognitive load.....	126
Relative efficiency of instructional conditions.....	129
8.4 Discussion	135
 Chapter Nine	 138
 Experiment 4	 138
 9.1 Introduction	 138
 9.2 Method.....	 140
9.2.1 Participants	140
9.2.2 Material.....	141
9.2.3 Procedure	142
9.2.4 Rating of cognitive load associated with task performance	142
9.3 Results	143
Variables	143
Similar test results	143
Transfer test results	146
Ratings of cognitive load.....	147
Relative efficiency of instructional conditions.....	150
9.4 Discussion	155
 Chapter Ten	 158
 General Discussion and Conclusion.....	 158
 10.1 Theoretical Overview	 158
 10.2 Review of the Experiments	 162
 10.3 Limitations of the Study and Future Research	 166
 10.4 Conclusion	 167
 References.....	 170
 Appendices.....	 187

Abstract

Research has demonstrated that instruction that relies heavily on worked examples is more effective for novices as opposed to instruction consisting of problem-solving. However, excessive guidance for expert learners may reduce their performance.

This study investigated optimal degrees of guidance using geometry worked examples. Three conditions were used. In the Theorem & Step Guidance condition, students were told the steps to find each angle, the measure of the angle, and the theorem used to justify the answer. In the Step Guidance condition, learners were told the sequence of steps needed to reach the answer, but not told the theorem required to make a step. The problem solving condition required learners to solve problems with no guidance.

It was hypothesized that by using Step Guidance, a new concept could be more readily incorporated into existing knowledge held in long-term memory compared to a Problem Solving approach or a Theorem & Step Guidance approach. Problem Solving would impose the heavy cognitive load associated with problem solving search while providing information concerning well-known theorems would be redundant. In other words, as long recognised by cognitive load theory, most students need to learn to recognise problem states and the

moves associated with those states and this information is provided by Step Guidance without additional, redundant information.

A series of geometry instruction experiments supported these hypotheses. The results of these experiments revealed that for students who already understand the relevant theorems, learning to solve problems primarily consists of learning to recognise problem states and their associated moves. Information concerning theorems only should be provided if students have yet to learn and automatise theorem schemas.

List of Figures

Figure 1. A worked example presented to the Theorem & Step Guidance group.	99
Figure 2. A worked example presented to the Step Guidance group.	100
Figure 3. A worked example presented to the Step Guidance group for Experiment 2.	109
Figure 4. Interaction between instructional format and level of learner expertise for overall similar test scores of Experiment 2.....	111
Figure 5. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 2.	113
Figure 6. Interaction between instructional format and level of learner expertise for overall similar test scores of Experiment 3.....	121
Figure 7. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 3.	124
Figure 8. Interaction between instructional format and level of learner expertise for the ratings of cognitive load of Experiment 3.	128
Figure 9. Interaction between instructional format and level of learner expertise for the relative efficiency of the instructional conditions of Experiment 3.	131
Figure 10. Representation of relative instructional efficiency (E) associated with learning through Theorem & Step Guidance (TSG), Step Guidance (SG) and Problem Solving (PS) for students with a lower prior knowledge level (Year 8) and higher prior knowledge level (Year 9, red)(Experiment 3).....	134
Figure 11. Interaction between instructional format and level of learner expertise	

for overall similar test scores of Experiment 4.....	144
Figure 12. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 4.	146
Figure 13. Interaction between instructional format and level of learner expertise for the ratings of cognitive load of Experiment 4.	150
Figure 14. Interaction between instructional format and level of learner expertise for the relative efficiency of the instructional conditions of Experiment 4.	152
Figure 15. Representation of relative instructional efficiency (E) associated with learning through Theorem & Step Guidance (TSG), Step Guidance (SG) and Problem Solving (PS) for students with lower prior knowledge level (Year 7) and higher prior knowledge level (Year 10, red) (Experiment 4)	154

List of Tables

Table 1 Description of Van Hiele Levels 1-4.....	54
Table 2 Means and Standard Deviations for the Similar and Transfer Test Scores for Different Instructional Conditions (Experiment1)	104
Table 3 Means and Standard Deviations for the Similar and Transfer Test Scores for Different Instructional Conditions and Levels Learner Prior Knowledge (Experiment 2).....	110
Table 4 Means and Standard Deviations for the Similar and Transfer Test Scores Different Instructional conditions and Levels of Learner Prior Knowledge (Experiment 3).....	120
Table 5 Means and Standard Deviations for the Ratings of Cognitive Load for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 3).....	129
Table 6 Means and Standard Deviations for the Relative Instructional Efficiency for Different Instructional conditions and Levels of Learner Prior Knowledge (Experiment 3).....	130
Table 7 Means and Standard Deviations for the Similar and Transfer test Scores for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4)	145
Table 8 Means and Standard Deviations for the Ratings of Cognitive Load for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4).....	148

Table 9 Means and Standard Deviations for the Relative Instructional Efficiency for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4)	151
---	-----

Appendices

Appendix A	188
Similar and Transfer Test Questions of Experiments 1 & 2	
Appendix B.....	189
Sets of problems given to the Theorem & Step Guidance group of Experiments 1, 2, & 3	
Appendix C.....	190
Sets of problems given to the Step Guidance group of Experiment 1	
Appendix D	191
Sets of problems given to the Problem Solving group of Experiments 1, 2, & 3	
Appendix E.....	192
New sets of problems given to the Step Guidance group of Experiments 2 & 3	
Appendix F	193
New sets of Tests Questions used in Experiment 3	
Appendix G	194
Subjective Rating of Cognitive Load for Learning	
Appendix H	195
Sets of problems used for each instructional Condition (Theorem & Step Guidance, Step Guidance, and Problem Solving) of Experiment 4	
Appendix I.....	196
Similar and Transfer Test Questions of Experiment 4	

Chapter One

Human Cognition and the Categories of Knowledge

1.1 Introduction

“Human cognitive architecture is peculiar” (Sweller, 2003, p.215). It consists of a working memory that has limitations when dealing with novel material, but can process intricate and extensive material that has been previously learned and stored in long-term memory. Massive amounts of information are stored in long-term memory in schematic form. Long-term memory is associated with working memory in the sense that it directs or misdirects the way working memory processes material. These two systems along with a sensory memory system are at the core of our cognitive activities that can be very simple or extremely complex (Sweller, 2003).

Cognitive load theory (CLT) is basically concerned with how humans process information and the instructional consequences that follow. During complex tasks incorporating extensive information, learners may be overwhelmed with many elements. Learning cannot take place if learners cannot process all the information and the interactions of the elements presented. The theory suggests that only when the conditions of learning are aligned with human cognitive

architecture does learning take place efficiently (Paas, Tuovinen, Tabbers, & Van Gerven, 2003; Paas, Renkl, & Sweller, 2004; Paas, Renkl, & Sweller, 2003; Sweller, 1988, 1999).

Our ultimate goal as educators is to help learners to deal with complex tasks, facilitate the acquisition of skills, and become proficient experts. This is achievable through appropriate instructional designs. Based on recent studies in the field of cognitive science we are more aware now of how we process information. This research has been conducted since the 1970s and focused on cognitive structures and their relations. Moreover, it has led to knowledge of human cognitive architecture (Sweller, 2004).

1.2 Working Memory

Cognitive load theory focuses on the critical relations between working memory load and instructional design. It suggests that our cognitive architecture includes as one of its critical components, working memory which is the structure that is used to hold and process information we are provided with (Baddeley, 1992; Ericsson & Kintsch, 1995). Working memory is related to consciousness. We are only conscious of what is held in working memory and nowhere else. On one hand, working memory has limited capacity when dealing with novel material, but on the other hand it can process intricate formerly learned information (Sweller, 1999).

The most important two features of working memory are its limited capacity, as indicated by Miller (1956) and its limited duration, discussed by Peterson and Peterson (1959). Material that is learned well and understood does not suffer from either of these limitations (Ericsson & Kintsch, 1995).

Information enters working memory via two routes: from long term memory if it has been previously learned or from sensory memory if it is new (Sweller, 2004). How to process the information in working memory depends on the source and this leads to instructional design issues. Based on what Peterson and Peterson (1959) found, when learners are presented with new information, instruction has to be designed to compensate for the limited duration of working memory otherwise it will be lost within seconds. In addition, according to Miller (1956), working memory can hold between 5 and 9 elements of novel unfamiliar information, or even less depending on the nature of processing (e.g., if some information must be contrasted or combined) (Sweller, 2004). Instruction also should take this limitation into account.

Information that enters working memory from long-term memory has different characteristics to information entering working memory from the environment. There are no known limits for the amount of information that working memory can process if it comes from long-term memory. Sweller (2004) gives an example of the word restaurant that is stored in long-term memory. The information related to this word includes food, the building, service, tables, chairs

etc. All can be moved from long-term memory to working memory without overloading it and processed as a single element.

1.3 Long-Term Memory

It is common knowledge that we all possess a long term-memory since we are able to recall things learned a long time ago. Its importance to cognitive functioning has been clarified in the last few decades. De Groot (1965) studied long-term memory in higher cognitive functioning. He showed that expert chess players rely on previously learned moves when encountering similar conditions and configurations in new games. They store those moves in long-term memory and this is how they defeat beginner or weekend players. He found that the skills of master chess players have nothing to do with thinking ahead and considering more moves than beginner players. Upon giving less able players a few seconds to reproduce a board configuration taken from a real game, they did not perform as well as master players who could usually place most pieces correctly.

Knowledge of a large number of moves that are stored in long-term memory as schemas changes the characteristics of working memory. Chase and Simon (1973) confirmed that the difference between experts and novices was not in their working memory capacity, since the board recall results were the same for novices and experts when random configurations were used. Simon and Gilmarin (1973) estimated that chess grand masters could learn up to 100,000

configurations. Consequently, they can reproduce configurations that they are familiar with but they do not perform any better than beginner players when dealing with unfamiliar, random configurations. This knowledge of moves is stored in long-term memory after years of practice leading to high levels of expertise. It might be the only learnable factor contributing to differences in levels of skill among players (Ericsson & Charness, 1994).

The same results have been obtained in other complex tasks. Egan and Schwartz (1979) displayed electronic wiring diagrams for a short period of time to expert and beginner electronic technicians who were asked to reproduce the same diagrams. The performance of expert technicians was better than that of the novice. However, upon repeating the experiment using random diagrams, the difference faded. Chiesi, Spilich, and Voss (1979) provided students with some prose about baseball. Learners with some knowledge of baseball performed better at recalling details of baseball games than those with less knowledge.

These experiments and studies have shown that the difference between expert problem solvers and beginners is not knowledge of refined strategies, but exposure to and knowledge of a huge number of different problem states and their associated moves (Sweller, 1999). Long-term memory allows us to solve problems, perceive, and think efficiently. Deliberate practice and rehearsal lead to high levels of intellectual performance (Ericsson & Charness, 1994).

The above characteristics of human cognitive architecture have significant implications for instructional design issues. Unlimited amounts of complex information can be stored in the human cognitive system. Long-term memory can store very complex, intricate procedures and facts. Human cognitive skills do not come only from the ability to perform complex reasoning activities in working memory but from stored knowledge in long-term memory. The finding that working memory has limited capacity and duration suggests that humans are able to deal with intricate reasoning only when the information they are presented with includes elements that are stored in long-term memory. Only then can they perform well. As a result, instructional designs that require learners to engage in complex reasoning processes that deal with unfamiliar elements are ineffective (Sweller, van Merriënboer, & Paas, 1998).

1.4 Schemas

When information is stored in long term memory, it is categorized according to how it is going to be used in schematic form (Chi, Glaser, & Rees, 1982). By definition a schema is a cognitive structure that allows us to consider several elements as a single element that is categorized according to how it will be used (Sweller, 1999). When one sees a tree, one immediately perceives it as a tree even though each tree is different from every other tree in colour, number, shape, branches. A tree schema, stored in long-term memory, allows us to categorize this information according to how it is used and treated as a single

element (Sweller, van Merriënboer, & Paas, 1998). The learning process and outcomes are defined by schemas. Our ability to read is possible because of the enormous number of schemas stored in long-term memory. Regardless of the text, we can recognize an infinite variety of shapes as the letter 'a'. Combinations of letters that form different words and combination of words that form phrases and letters are stored in higher-order schemas. Consequently when we read, we can ignore all the other details and focus on the meaning (Sweller, 1999).

Only since the 1980s have schemas become important to modern cognitive theory and in particular to problem solving theories. Due to the studies conducted by Larkin, McDermott, Simon, and Simon (1980), and Chi, Glaser, and Rees (1982) who suggested that schemas provide learners with the ability to sort out many elements of information as one element that needs less working memory capacity when being dealt with, thus diminishing the load on working memory. The role of schemas in expert problem solving has become evident. Tens of thousands of schemas permit expert problem solvers to recognize certain problem situations in relation to suitable moves. Hence schema theory suggests that in order to be skilled in any domain, one has to acquire specific schemas and store them in long-term memory. The tens of thousands of configurations stored in the form of schemas in long-term memory allow chess masters to defeat novice players upon recalling problem states and the corresponding moves (De Groot, 1965). The same mechanism applies to all areas of expertise (Sweller, 1999).

Decreasing the load on working memory is another essential function for schemas, in addition to storing and organizing information. In spite of the fact that working memory has a limited capacity in the sense that the number of elements that can be process is limited, the size, complexity, and sophistication is not (Sweller 2004). Stored schemas may include a huge amount of information. The restaurant schema mentioned previously is a good example. It is held as a single entity, but it includes wide knowledge, everything from food to the structure of a building. Though the number of elements (or schemas) that working memory can process is limited, there are no limits on the size of an element. As a result, the two functions of schemas can be summarized as the storage and organization of information and a decrease of working memory load.

1.5 Automation

Information can be processed consciously or automatically. Conscious processing of information has the characteristics described previously. However, automatic processing circumvents working memory (Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977). With practice, knowledge may become automated and less conscious effort is required for information to be processed. A clear example is related to reading text. When reading, a competent reader does not consider the individual letters that make up the text. Processing letters becomes automated in childhood. However, processing each letter consciously is essential

when young children are learning to read (Sweller, van Merriënboer, & Paas, 1998).

Thus, automation has the same consequence for working memory as schema acquisition; they both reduce the load on working memory. Kotovsky, Hayes and Simon (1985) indicated that when rules are memorized to the extent that they can be repeated easily, then problem solution becomes easier since the rules are not processed consciously and planning a solution takes place in what is now a working memory with a reduced load. When rules are not automated, effort is exerted in working memory to recall them and reaching solutions becomes difficult (Sweller, van Merriënboer, & Paas, 1998). The experiments conducted by Kotovsky, Hayes, and Simon (1985) reflected the essential role of automation in problem solving.

Working memory capacity increases when a learner has a more automated schema. For example, when the schemas related to letters, words, and phrases are automated, the capacity of working memory is used to comprehend the text. In contrast, less proficient readers whose schemas are not fully automated, may be able to read the text fluently but they may not comprehend the text fully because they do not have enough working memory capacity to derive meaning from it (Sweller, van Merriënboer, & Paas, 1998). This implies that instructional designs should not just focus on the construction of schemas and storing of information, but also on the automation of these schemas that leads to problem solving (van Merriënboer, 1997).

To sum up, a powerful long-term memory, a limited working memory, and learning mechanisms that involve schema construction and automation are the constituents of our cognitive system (Sweller, van Merriënboer, & Paas, 1998).

However, in order to understand the way by which human cognition handles different types of information, we need to classify knowledge into broad categories that have different evolutionary characteristics. Each category of knowledge may be attained, systematized and saved in specific ways and involve different learning processes. We need to know how we handle different categories of knowledge in order to understand which features of human cognition are important when devising instructional procedures (Sweller, Ayres, & Kalyuga, 2011).

1.6 Biologically Primary and Biologically Secondary Knowledge

Geary's (2002, 2007a, 2007b, 2008: see also Sweller, 2008) evolutionary description of educational psychology proposes that working memory limitations may be crucial when dealing with new information that we have not evolved to acquire easily and rapidly (biologically secondary knowledge) though it may be important culturally. However, these limitations have less significance when acquiring new information which our brain has evolved to acquire in an automated and implicit way (biologically primary knowledge).

Sweller (2009) explains that humans acquire primary knowledge without awareness and with no direct instruction since they have evolved to attain that knowledge. Taking part in a functioning society is all that is required to acquire those skills. Primary knowledge includes basic skills such as generic problem solving strategies. It is easy for us to gain extensive biologically primary knowledge independently without the help of some kind of educational milieu and with no apparent working memory load.

Some examples that reflect our capacity to acquire knowledge on our own without the need for direct and clear teaching is our ability to recognise people (e.g., Bentin, Deouell, & Soroker, 1999) and the skill of learning to articulate (e.g., Kuhl, 2000). Just by being integrated within a listening/ speaking society, we learn to speak quickly without being aware of this and with no effort. Since humans have evolved to process biologically primary knowledge in a mostly unconscious manner, the limitations of working memory do not apply to processing this type of knowledge.

Biologically secondary knowledge has emerged relatively recently, with insufficient time for humans to have developed primary modular structures for dealing with this kind of information. Humans have not evolved to attain biologically secondary type of knowledge in an implicit and unconscious way. Biologically secondary knowledge is acquired with conscious effort as opposed to biologically primary knowledge. Learning biologically secondary knowledge usually needs instruction, since this information has not been biologically

programmed. Direct and clear instruction is considered as a replacement for the biological programming which is present when dealing with information that we have evolved to acquire (Sweller, 2009).

According to Sweller (2008), Geary's differentiation between biologically primary knowledge and biologically secondary knowledge represents an exceptional development in our discipline. Geary's theory provides answers to some issues having insightful instructional implications. For example, it has been proposed in some learning and instruction theories that learners must construct their own knowledge without getting extensive help from others. Indeed, according to Geary, we humans acquire a huge bulk of information without direct instruction. We learn to form our lips, tongue, breath, and voice and simply utter sounds by just being part of a community that listens and speaks. One can compare the ease with which we attain information outside the context of a learning environment to the difficulty we often face within a learning environment. The logical reason could be attributed to the unsuitable instructions used to teach learners. It could be suggested that if educational institutions used the same procedures that we are exposed to in the external world, then education would be easy and fast. However, evidence does not support this view.

Geary's distinction between biologically primary and biologically secondary knowledge provides a rationale. This differentiation between different types of knowledge explains that learners are motivated to learn biologically primary information without explicit instruction. It also clarified why learning is

achieved effortlessly and effectively. On the other hand, some information needs external motivation and its acquisition is assisted by clear and direct instruction. Our failure to distinguish between these two types of information has led to the misconception that biologically secondary knowledge can be attained by immersion in the same way we attain biologically primary knowledge. When learners deal with biologically secondary knowledge, they lack the motivation and the genetically inspired ability to understand information on their own. They need the motivation and the external instructions which are not required when dealing with biologically primary knowledge. Thus, Sweller (2008) explains that cognitive load theory applies only to biologically secondary knowledge that is usually taught in educational institutions. The evolutionary human cognitive architecture (Sweller, 2003, 2004; Sweller & Sweller, 2006) around which cognitive load theory revolves, pertains to the knowledge that is dealt with in educational institutions rather than the knowledge that is addressed by our biologically primary systems.

1.7 Human Cognitive Architecture

The human cognitive architecture discussed in Sections 1.2 to 1.5 above applies only to biologically secondary knowledge. That architecture has been unified by relating it further to biological evolution. Sweller (2009) explains that our cognitive system has developed to deal with information in the same way evolution by natural selection processes information. The two are considered

examples of systems that process information naturally. For that reason, the familiar procedures of evolutionary biology can be employed in order to know more about human cognition which is often considered as an information processing system while evolutionary biology is usually not analysed in terms of information processing.

Below are the five principles that govern both evolution by natural selection and human cognition.

1.7.1 Information store principle

This principle deals with the huge information stock that is essential to the performance of both evolutionary biology and human cognition. With respect to human cognition, long-term memory supplies us with that stock of information, while a genome serves that purpose in the case of evolution by natural selection (Sweller, 2009).

1.7.2 Borrowing and reorganising principle

Most information in the information store is acquired by borrowing and reorganizing information from different stores. Evolutionary biology makes use of sexual reproduction by a method that requires reorganization. As for the information in long-term memory, it is mostly acquired from other people by replicating what we hear or read. Information is moved and reorganized based on

information that is previously held in long-term memory before the new information is stored (Sweller, 2009).

1.7.3 Randomness as genesis principle

New information is not generated by the borrowing principle except insofar as reorganisation occurs, but rather by the randomness as genesis principle (Sweller, 2006). This principle is the driving force of creativity according to evolutionary biology. Discrepancies between genomes can be attributed to changes in genetic material that happen haphazardly. However, the method is not just a haphazard mutation method. It is a random generate-and-test method (Sweller, 2009). The efficiency of a random novel genetic code is examined. Information that is useful is kept, while useless information is discarded. The same applies to long-term memory. Information that is not acquired according to the borrowing and reorganizing principle is haphazardly generated while solving problems. When we are attempting to solve a problem we resort to our schemas that are stored. When these schemas are not available, we either do not succeed in solving the problem or new alternatives are produced by using a haphazard generate-and-test method which involves keeping successful moves and discarding failed ones. This principle explains the origin of novel information (Sweller, 2006).

1.7.4 The narrow limits of change principle

Due to the randomness element of the randomness as genesis principle, the production of novel information is slow because the addition of large amounts of random information to an information store is likely to destroy its effectiveness. In evolutionary biology, effective changes in genetic material occur slowly and over a long period of time. As for human cognition, the limitations in capacity and duration of working memory apply only to new information (Sweller, 2006).

1.7.5 The environmental organising and linking principle

When information is organized in the information store, the limits that are related to the narrow limits of change principle vanish. Large amounts of information, whether genetic or cognitive, may be used to organize and connect to the environment. For that reason, there are no limits when working memory has to deal with organized information from long-term memory thus allowing us to relate to our environment. There is no reason why information, whether in long-term memory or a genome, to be limited since it has been tested previously for efficiency. Thus, this principle allows us to interact and function in our society and environment (Sweller, 2006).

1.8 Summary

According to Sweller (2006), the above principles are the foundation for cognitive load theory. The role of instruction according to the above principles is to acquire information that is functional in the long-term memory store. He also suggests that learning takes place by the borrowing principle, and as a result, the main role of instruction is to focus on how written, spoken and diagrammatic information should be offered to learners. He adds that giving attention to methods that focus on discovery and inquiry are not effective due to the randomness as genesis principle. When structuring information, we should keep in mind the narrow limits of change principle which relates to the limits of working memory. Finally, when information is organized via the environmental organizing and linking principle in long-term memory, it becomes the foundation of our interaction with the environment. Instructional effects that have been generated by cognitive load theory take into account these principles.

From a scientific angle, Geary's conceptualization is interesting, yet serious from an instructional design angle. Knowledge can be separated into biologically primary knowledge which humans acquire through evolutionary mechanisms and biologically secondary knowledge that is crucial for reasons related to culture.

The most important implication of the above theoretical outline of cognitive load theory is the need to develop instructional strategies that take the

limited capacity of our working memory into consideration. Even though working memory limitations might be serious when dealing with new information that is related to culture that humans cannot attain on their own (i.e., biologically secondary knowledge), these limitations have less effect when attaining new information which the human brain has evolved to process (i.e., biologically primary knowledge).

Because human cognition can be described as a natural information processing system, the five principles discussed in this chapter provide an outline of human cognitive architecture. Cognitive load theory is an instructional theory that makes use of this cognitive architecture (Sweller et al., 2011).

Chapter Two

Cognitive Load Theory

2.1 Introduction

The cognitive architecture discussed in chapter one is essential when designing instruction. Sweller et al. (2011) explained that according to what we know about human cognitive architecture, we can decide on categories of instruction that are expected to be efficient. They suggested that since general learning and problem solving are part of biologically primary knowledge that is acquired at a young age and cannot be taught at a later stage in life, then school-based learning may be limited to knowledge that is specific to certain domains that is not acquired automatically as biologically primary knowledge. They also suggested that based on the architecture explained in the previous chapter, the knowledge that is related to specific domains is most efficiently acquired from other people, therefore direct, clear, and explicit instruction should be used via the borrowing and reorganizing principle.

The limitations of working memory have to be kept in mind when designing instruction for learners to obtain new information. Therefore, information has to be presented in a way that reduces processing that is not

needed for learning. Hence, cognitive load theory revolves around methods that ensure that new information is presented to learners in a way that diminishes this unneeded load and at the same time, increases the features of cognitive load that result in learning. This chapter discusses the categories of cognitive load that might be imposed by instructional procedures along with some cognitive load effects.

2.2 Element Interactivity

Changes in performance occur after learning and practicing due to changes in the schemas that are stored in long-term memory. After schema acquisition and automation, a learner is able to move the schemas that incorporate many elements from long-term memory to working memory allowing him or her to perform better and with minimal effort (Sweller, 1999). However, we need to take the material to be learned into account. Some material is difficult to learn while other material can be learned easily. Sweller (2003) proposed that all information can be placed on a continuum based on the degree of element interactivity. Elements that do not interact are found at one extreme, and each element can be learned individually and independently of other elements. As a result, cognitive load is low. On the other hand, elements that need to interact with other elements in order for understanding to occur are found on the other extreme of the continuum. Consequently, cognitive load is high.

Sweller et al., (2011) defined interacting elements as those that are processed at the same time in working memory since they are related. Anything that is processed or must be processed is an element. Elements are typically schemas. Almost all schemas consist of sub-schemas or sub-elements. Before a schema is acquired, its sub-elements must be dealt with in working memory as separate elements. Later when these sub-elements are acquired and integrated into a schema, this schema is then considered as a single element in working memory. As a result, working memory load is diminished when different lower-level schemas are processed into a smaller number of higher-level schemas.

Element interactivity of some material is low since elements can be learned one at a time and so the load imposed by the nature of the material (intrinsic cognitive load) is low. This type of material does not need a lot of working memory resources. On the other hand, some materials contain elements that cannot be learned separately. This type of material has high element interactivity and may impose a high intrinsic cognitive load. In this case, more working memory resources are needed until the elements are incorporated into schemas (Sweller et al., 2011).

Intrinsic cognitive load (Sweller 1994; Sweller & Chandler 1994) is associated with the level of difficulty of the material to be learned and the information to be comprehended. Regardless of the manner by which it is given to the learners or the type of activities they might participate in order to maximize learning, intrinsic cognitive load is stable and cannot be changed except by

altering the level of knowledge of the learner or the whole assignment itself (Sweller, 2010). The interaction between the type of material to be learned and the expertise of the learners determines intrinsic cognitive load through element interactivity.

Consider this example: $\frac{a+b}{c} = d$ solve for a. For a beginner algebra

learner, each symbol is a separate element that needs to be dealt with and learned at the same time in order to solve the equation. If compared to learning the chemical symbols, this equation has a higher element interactivity and it imposes a heavier cognitive load than learning the symbols though it might be more difficult to learn the symbols since there are many symbols. For example, one can learn that the symbol of iron is Fe, and that Cu is the symbol for copper. Learning these two symbols or any other symbol does not impose a heavy cognitive load since the elements here do not interact with each other. They can be learned in isolation. Consequently, the algebraic equation imposes a higher intrinsic cognitive load.

The manner in which the material is presented (extraneous cognitive load) is also related to element interactivity. Various instructional procedures need learners to deal with a narrow number of elements at the same time. In this case, element interactivity is not high along with extraneous cognitive load. However, when instructional procedures need learners to deal with several elements at the same time, then element interactivity is high along with extraneous cognitive

load. For example, if a learner has to solve a problem such as $a + b = c$ and the solution is not found in long-term memory, he/she is likely to use a means-end strategy. He/she has to simultaneously consider the problem state $a + b = c$, the goal state (a being the subject of the equation), extract the difference between the current state and the goal state ($+ b$ is found on the left side of the equation and should be removed) and finally to find rules that can be employed to eliminate the differences between the current state and the goal state (subtracting b from both sides of the equation). A learner is unlikely to find this solution if he/she does not consider all these elements. Using such a strategy entails high element interactivity which involves several interacting elements (Sweller et al., 2011).

Germane cognitive load can also be defined in relation to intrinsic cognitive load. It is defined as the load that is not imposed by the material to be learned but rather refers to the working memory resources which are devoted to relevant or germane information that is needed for learning. Consequently, germane cognitive load is also linked to element interactivity. Hence, the different effects of cognitive load theory can be explained based on the concept of element interactivity (Sweller, 2010).

2.3 Understanding and Rote Learning

Sweller (2003) defined understanding as the processing of all elements that interact at the same time in working memory. When elements of a certain

task are processed individually, in isolation, and without learning relations between them, the result is rote memory. When learning by rote, only one element or just a very few are processed at the same time. Sweller (1999) explained that if a student is presented with a problem such as $\frac{a}{b} = c$ solve for a, he/she might learn that in order to solve it he/she has to move the denominator to the right-hand side resulting in $a = b \times c$. However, if this is all he/she learned, then the result is learning by rote. The student has to learn that all problems of such a type are solved by multiplying both sides of the equation by the denominator, cancelling the denominator on the left side and thus moving it to the right side. The student has to realize that the problem is not solved just by moving the denominator to the right side of the equation. Learning by rote is changed to learning by understanding when the student relates the process of multiplying and cancelling the denominator on the left side to the strategy of moving it to the right side. However, the consequence for learning by understanding is an increase in working memory load due to an increase in the number of elements that must be processed. There are few elements involved in moving a denominator on the left hand side to the numerator on the right hand side of an equation. More elements are involved if the student learns how to multiply and cancel on the left hand side.

In conclusion, the level of difficulty in learning material with low element interactivity depends on the number of elements, though understanding them is easy due to the absence of interaction between the elements. As for the understanding of material with high element interactivity, the elements need to be

processed and held at the same time in working memory which has limitations in capacity and duration. Therefore, if the element interactivity of information is high, learning with understanding (rather than just rote learning) is not an easy task.

However, working memory load depends not just on the characteristics of the material learned, but also on the characteristics of the student. Many elements for one learner might be a single element for another learner. If a learner acquires an intricate complicated schema in a certain area, it acts as one unit of information that encapsulates many other elements. If another learner has to process the same material in the absence of relevant schemas, then each element of the material along with its interaction is dealt with individually in working memory resulting in a heavy load on working memory. Therefore, actual element interactivity is determined by the interaction between the material and the learner.

Sweller (2003) suggested that in order for working memory to deal with many interacting elements simultaneously, it would have had to evolve into a larger structure. However, this did not happen, of course. As a result, when humans are presented with material of high element interactivity, they are not able to understand it. Rote learning becomes the only option. What our cognitive architecture tends to do in this case is integrate the new information that is high in element interactivity within a schema and deal with it as a single element in working memory, thus circumventing the limitation of working memory. This

occurs after extensive learning over a long period of time. This is what is meant by learning through schema acquisition and automation.

2.4 Altering Intrinsic Cognitive Load and Element Interactivity

According to Sweller (2006c), there are two ways to decrease intrinsic cognitive load. The first one is to manipulate the material to be learned, and the second by schema acquisition and automation. Pollock, Chandler, and Sweller (2002) suggested manipulating the material through what is called the isolated-interactive elements procedure which involves presenting the material to the learners as separate elements to learn at the initial phase of instruction followed by the fully interactive elements at the subsequent phase. They conducted a two phase experiment in which two groups were used. In the first phase the isolated-interacting elements group studied isolated elements instructions and the second phase involved interacting elements. The second group, the interacting elements only group, was given instructions that involved interacting elements during both phases. The results showed that the performance of the isolated-interacting elements instruction group was better than that of the interacting elements only group since the cognitive load imposed by this method of instruction was lower. The assumption was that in the first phase, the isolated-interacting elements group did not completely comprehend the difficult concept presented to them; however, the basic schemas were acquired. The students may have acquired some of the necessary elements without processing in working memory all interacting

elements that are needed in order to understand the material. Afterwards, learning the interactions between these elements was achieved in the second phase. On the other hand, the interacting elements may have surpassed learner working memory capacity in the case of the interacting elements only group resulting in decreased learning at either the first or the second phase.

Sweller and his associates (Clarke, Ayres, & Sweller, 2005; Sweller, 2006c) also proposed teaching material of high element interactivity in separate elements and later working on the interaction between them. The same method was used by Ayres (2006) in algebra. The result was that the material to be learned imposed a decreased cognitive load when it was presented to learners in isolation while the load increased if the elements of the same material were given at the same time. However, Ayres also found that learners with more knowledge of the subject did not perform as well if the elements were separated. He explained this as a result of a reduction in germane cognitive load which affected schema acquisition (see the section below on germane cognitive load).

2.5 Extraneous Cognitive Load and Element Interactivity

As indicated above, another source for cognitive load other than the intrinsic quality of the material is the way in which the material is presented to the learner; in other words, the instructional design (Paas, Renkl, & Sweller, 2003).

Suboptimal instructional procedures may impose an extraneous cognitive load (Sweller, 2010).

Beckmann (2010) states that while in respect to intrinsic cognitive load, element interactivity is a critical factor, until recently, little attempt has been made to relate element interactivity to extraneous cognitive load (Sweller, 2010).

Sweller (2010) proposed that element interactivity is also behind extraneous cognitive load. The suggestion is that when element interactivity is decreased with no change to what has to be learned then the load is extraneous. However, if element interactivity is changed by changing what has to be learned, then the load is intrinsic (Beckmann, 2010). Schnotz and Kurschner (2007) provided an example that if the objective of a certain task is to understand the ideas in text which contains jargon, then the load is extraneous. However, if the objective is to learn the jargon, then the load is intrinsic. As a result, the same material can impose both intrinsic and extraneous cognitive load depending on the objective.

Sweller (2010) concluded that element interactivity underlies not just intrinsic cognitive load, but also extraneous and germane cognitive load (which will be discussed later). He elaborated that instructional procedures that aim at making learning easier all focus on decreasing the elements that must be learned at the same time. Many cognitive load effects (also discussed below) indicate in a

very clear-cut way the elements that interact and that are required in order for the learners to process information.

Many conventional instructional procedures impose an extraneous load since they do not take into consideration the structure of information nor cognitive architecture. Paas, Renkl, and Sweller, (2003) provided an example of extraneous cognitive load. When learners are asked to search for references or locate something without suggesting where to search, this inflicts a heavy extraneous cognitive load. This is due to the fact that working memory is used for procedures that are not related to schema acquisition and automation.

Efficient learning consists of the construction of automated schemas held in long-term memory. Sweller (1999) suggested that when a student learns that the sum of the interior angles of a triangle is 180° and vertically opposite angles are equal, a worked example can be provided to show how to find the value of unknown angles using these theorems. If then the student is provided with a lot of practice problems, a schema is acquired that tells the student that regardless of the triangle and the size of the angles, they always add up to 180° . These schemas can be moved back to working memory after being stored in long-term memory whenever they are needed. However, at this point, automation may not yet have fully taken place. The learner must recall the relations between the interior angles of a triangle and that their sum is 180° . If a learner needs to put effort into remembering the characteristics of a theorem, then this activity needs working

memory resources. Thus, the resources available to use the theorem to solve problems will be reduced.

Sweller (1999) continued that if a problem is presented to a learner which involves two theorems, the sum of interior angles of a triangle equals 180° and vertically opposite angles are equal, then solving it depends on the knowledge-base of the learner. The schemas may have been acquired which permit the learner to identify and isolate the interior angles of a triangle keeping in mind that regardless of the size of the angles, they always add up to 180° . These schemas may be present in long-term memory and can be moved to working memory when needed. If the learner is highly knowledgeable in this area, then the previously learned schemas produce the moves smoothly and effortlessly.

In contrast, a student who is presented with this problem for the first time (meaning he had not acquired the schemas related to this) must go through problem solving search different from the schema-based strategy used with the previous learner. The first step is to recognize the goal of the problem: Find a value for an exterior angle. The second goal is to find a theorem that will provide a value for the goal. The student in this case does not know of any theorem that can provide him with this value and has to find another angle that can be connected to the goal angle by a theorem and can act as a subgoal. Consequently, a value for the goal can be obtained if a value for the subgoal is known. Once the student discovers that the goal angle (the exterior angle) can be connected to the interior angle (the subgoal angle) by the vertically opposite angles theorem, the

answer is found. In order for this realization to take place, the student has to conduct a new search. The search for the value of the subgoal (the interior angle) must replace searching for the value of the goal (exterior angle). Knowledgeable students will use the theorem of the interior angles of a triangle adding up to 180° . Thus the value of the subgoal is found followed by a value of the goal using the theorem that the vertically opposite angles are equal. The problem is hence solved. This strategy is called means-end analysis in which a learner needs to find a subgoal (a vertically opposite angle in the case of the problem at hand) related to a goal (interior angle, the value of which needs to be found). This strategy must be distinguished from a schema-based strategy. The latter tells the learner the solution while the former requires some search. Though the means-end strategy is helpful in problem solving, it is not related to learning and schema construction. Moreover, the heavy load that it imposes by this method hinders learning. Consequently, instructional alternatives to problem solving are required. The best designs are those that allow working memory resources to be allotted to schema acquisition and automation instead of other extraneous activities (Sweller, 1998).

When high intrinsic and high extraneous cognitive load are combined, working memory may be significantly overloaded, and hence learning may not take place. Since intrinsic cognitive load cannot be modified for a given learning task, it is important to design instruction in a way that reduces extraneous cognitive load. However, if the intrinsic cognitive load is low because of low element interactivity, suboptimal instructional procedures that result in a high

extraneous cognitive load may not be as harmful since the total cognitive load may be still within the limits of working memory. For example, if students are presented with material that requires them to mentally integrate texts and diagrams in a low intrinsic cognitive environment, the result may not be adverse since the load is within the limits of working memory capacity.

2.6 Germane Cognitive Load

Germane cognitive load is also explained in terms of element interactivity but under different conditions than intrinsic and extraneous cognitive load. Both the presentation of the material to be learned and the material itself, affect extraneous and intrinsic cognitive load, respectively. Germane cognitive load, on the other hand, is affected by the features of the learner. It has to do with the resources of working memory that are assigned to handling the intrinsic load. Germane load does not impose a source of load in itself. Germane cognitive load becomes high when extraneous load is low and intrinsic load is high since the learner has to dedicate significant working memory resources to deal with the material at hand. When extraneous load is intensified, germane load in turn is diminished since the learner uses the resources of her/his working memory to handle the extraneous load that is imposed by the instructional procedures instead of handling the intrinsic material. In conclusion, germane load is directly related to the resources of the working memory that are dedicated to the elements that interact together and which influence the intrinsic load. When the learners

dedicate more resources to dealing with extraneous load, fewer resources will be free to deal with the intrinsic load resulting in reduced learning (Sweller, 2010).

As a result, germane cognitive load and learning is boosted when instruction is structured in a way to permit the resources of the working memory to handle the element interactivity related to intrinsic cognitive load. However, when instruction necessitates learners to dedicate working memory resources to handling elements associated with extraneous load, fewer resources will be free to handle intrinsic load and hence, germane cognitive load will be lower (Sweller, 2011).

Sweller (2010) suggests that the benefit of this formulation is that it gets rid of potential contradictions. In experimental situations, we can only measure alterations in overall cognitive load rather than separate types of it. It has also been suggested that both extraneous and germane cognitive load are complementary. When extraneous cognitive load diminishes, the assumption is that germane load increases. Nevertheless, the question that needs to be posed is why does overall cognitive load change if germane load makes up for extraneous load? The current formulation resolves this contradiction. What determines total cognitive load is both intrinsic and extraneous load. If one alters and the other stays stable, overall cognitive load alters. For example, overall cognitive load increases when extraneous cognitive load goes up. At the same time, germane cognitive load decreases since working memory resources must be used to deal with extraneous rather than intrinsic cognitive load. As a result, less resources are

available for dealing with intrinsic cognitive load. The complementarity between germane and extraneous cognitive load is explained in this manner as overall cognitive load changes.

2.8 Instructional Design Effects

Cognitive load theory clarifies the learning results by looking into the effectiveness and restrictions of human cognitive architecture and developing principles and instructions based on our understanding of the human brain (Paas, Renkl, & Sweller, 2003).

CLT proposes that attention needs to be paid to the effects of learning materials and instruction on cognitive processes. Designers must take into account intrinsic load (imposed by the material itself), germane load (the working memory resources needed), and extraneous load (the quality of the material and how it is presented) when designing activities and presenting material (Sweller, 2010). Based on recent advances of CLT, there are several application procedures suggested. Each procedure is a result of a cognitive load effect which shows that the instructional procedure that is based on cognitive load theory principles makes learning or problem solving easier if weighed against traditional procedures (Sweller et al., 2011). These effects are stable relationships between instructional procedures, learner characteristics and learning outcomes (Kalyuga, 2009b, pp.43). Several of these effects represent instructional methods for reducing

extraneous cognitive load. Those directly related to the studies of this thesis will be briefly discussed in the following sections.

2.8.1 The Worked Example Effect

This effect takes place when students do not gain knowledge from solving problems as much as they do from studying a worked example related to the same problem (Cooper & Sweller, 1987). A worked example is a problem and its solution presented to students. Its solution is presented step-by-step in order to teach students the solution techniques (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985). It is suggested that schema acquisition is superior when worked examples are used instead of problem-solving methods. It was observed that the more worked examples are used the better and more efficient is the performance of learners (Cooper & Sweller, 1987; Sweller & Cooper, 1985). Zhu and Simon (1987) stated that not only do worked example decrease the time needed for students to learn but they also aid in schema acquisition and automation if they are well structured.

It has already been stated that extraneous load does not impede learning when the material has a low intrinsic load, but when the material is high in intrinsic load, it is essential in this case to diminish extraneous load so that learning is not affected (Van Merriënboer & Sweller, 2005). Using worked examples is an efficient way of diminishing the extraneous load since learners can

allocate all working memory capacity to studying the worked-out solution, and hence construct the necessary schema related to such problems in long-term memory (Sweller, 2004). The remaining cognitive capacity may be used in other activities that add to the learning and transfer performance.

Sweller (2010) explained that learners solving problems must deal with a large number of interacting elements in order to use a means-end strategy. However, learners using worked examples only consider a problem state and the move that takes them to the next state. In this case, the search for other problem states and moves which is typical of problem solving and which is characterized by a large number of interacting elements is eliminated thus diminishing extraneous cognitive load. Only the interacting elements that are related to each problem state and its linked move remain when dealing with a worked example. However, this comprises intrinsic rather than extraneous cognitive load on the condition that no extraneous elements are included in the design of the worked example. If the resources of working memory are allocated to dealing with each problem state and the moves related to it instead of a large number of potential moves, germane cognitive load is increased compared to problem solvers who must allocate resources to a large number of interacting elements related to potential but inappropriate moves.

Research has indicated that learning from worked examples is more efficient than problem solving especially in the early phases of learning. The theory presented focuses on an analysis of the various types of cognitive load and

the alterations that take place during the different phases of skill acquisition. As a result, a fading procedure is proposed in which problem-solving elements are consecutively integrated with worked examples till learners are capable of solving problems on their own (Renkl & Atkinson, 2003).

Cooper (1998) also states that cognitive load is low when studying worked examples since students may focus on only two problem states at a time and the transformation (rule operator) that relates them. He also suggested that an effective technique when presenting worked examples could be to provide them to students with similar conventional problems in an alternating pattern. In this way, students study the example carefully since they are not permitted to go back to it once they begin solving the related problem. As a result, students' attention is directed to the type of the problem and the related solution steps. In addition, in this way, they can also know if they learned the method or not based on their success in solving the problem.

A study was conducted by Atkinson and Renkl (2004) to test whether the fading procedure is more efficient than learning by problem pairs. It was conducted in two classrooms of a physics lesson (electricity). In one classroom, an example-problem was given. There were four tasks in the fading group. The first one was a worked out-example. In the second, the last solution step was removed. For the third task, the last two steps were removed (backward fading of solution steps). In the final task, all three steps were omitted and a to-be-solved

problem was given to the learners. The results showed that the fading group performed better than the example-problem pairs group.

This study was followed by two further experiments. In the first one, students were given either the fading or the example-problem condition. A forward fading procedure was used in this study (i.e. the first solution step was removed, then the second, etc.). The result was that near but not far transfer performance was enhanced since during the learning stage, a lower number of mistakes was made.

In the second experiment, similar conditions were used with respect to example-problem pairs and forward fading. Moreover, the condition of backward fading was used in order to compare the two types of fading procedures. The students were given the same number of problems in the forward fading, backward fading, or the example-problem condition. The same results were obtained on near transfer but what was also noticed was the positive result on far transfer favouring the backward fading condition.

Two essential suggestions can be made. First, increasing problem solving during learning and acquisition is possible since intrinsic load decreases gradually due to schema construction and automation. The second suggestion is that in contrast to the early stages of acquisition, when learners acquire understanding, activities that need learners to generate self-explanations become extraneous since cognitive resources are devoted to knowledge that has already been acquired thus

leading to a redundancy effect. Self-explanations and worked examples become extraneous and problem-solving germane after schemas have been acquired.

Therefore, attention has to be paid to what the particular goal is. The conclusion is that problem solving should be introduced after examples are studied. Then, an example is given and one worked example step is eliminated. After that, the number of worked example steps that are eliminated are increased gradually until just the problem to be solved is left. A study was conducted in order to test the effectiveness of this fading procedure. The implications were that the positive effects that fading had on learning verify the expertise reversal effect (discussed below), (Renkl & Atkinson, 2003).

According to Sweller (2003) the worked example procedure proved to be efficient for almost all classes of problems. Experiments showed that learners who were presented with a large number of worked examples learned more than those who were presented with the same number of problems to solve (Carroll, 1994; Cooper & Sweller, 1987).

2.8.2 The Split-Attention Effect

Some worked examples do not take into consideration the limitations of working memory by, for example, requiring learners to split their attention between many sources of information (Sweller, 2003). Some of the materials that are given to

learners may include a picture and written information above, below or to one side. Such instruction may lead to a split in attention if the learner has to mentally integrate both sources in order to process and comprehend the material. Relating to one source only will impede understanding (Cooper, 1998). A good example might be a geometry worked example with diagrams and texts. A learner cannot get meaning from either if he/she separately analyses them. He/she has to integrate both. This kind of integration needs working memory resources since learners have to locate referents that may not be obvious when learning geometry.

Instructional materials are often designed with a split-attention layout, and this adds to the extraneous cognitive load if weighed against layouts that are physically incorporated. Sweller (2010) provided the example of a geometry worked example having a diagram with a list of steps under the diagram leading to the solution of the problem. If one of the steps suggest that, Angle ABC = Angle XYZ, students have to locate the two angles. The interacting elements related to Angle ABC are the statement “Angle ABC” and almost all the angles on the diagram. The student must go over all the angles till he/she locates the correct one. In order not to check angles that have been checked before, he or she needs to keep in mind the ones he or she checked before. The same applies to Angle XYZ. When the student locates both, he or she has to work on proving why they are equal. This process entails extraneous cognitive load.

Sweller (2010) suggested that extraneous cognitive load related to searching for the angle should be removed. This could be done by incorporating

the statement “Angle ABC = Angle XYZ” within the diagram. It should be put in a suitable place on the diagram instead of below the diagram. It can also be done by making use of arrows to reduce the search.

If this technique is used, the students only have to check the angles that are needed instead of a list of angles. Consequently, element interactivity related to extraneous load is diminished.

Sweller, Chandler, Tierney, and Cooper (1990) demonstrated that incorporating text into diagrams aids learning. Two groups of students were involved in these experiments. The first group was given separate text and diagrams while the other group was given the same problem in which the text was incorporated into diagrams. The experiment was divided into two stages. During the first stage, students were given either the conventional or the modified instruction and they were allowed to use up as much time as needed to understand it. The second stage was the assessment stage. The results showed that the group that was presented with the incorporated information performed better in both stages. They spent less time to understand the material which suggests a lower cognitive load due to the incorporated material which in turn made learning easier since they performed better than the other group on the later tests. Presenting the solution on the diagram facilitates learning and gets rid of split attention since learners are presented with only one source of information.

However, this effect is not just related to texts and diagrams. It is present whenever there are multiple sources of information that learners have to make use of at the same time. This includes two or more sources of textual information. Chandler and Sweller (1992) demonstrated this effect with the traditional format of educational psychology research papers in which the outcomes of the experiments and the discussions are conventionally presented in separate sections even though readers have to refer to both in order to comprehend the results thoroughly. The split attention was reduced when the results and the discussion were integrated into a single entity.

Chandler and Sweller (1996) described a split-attention situation created by referring to software and a hardcopy of a manual in order to understand how the software functions. The best alternative is not to use the computer when learning but rather to refer to diagrams in the manual. Chandler and Sweller (1996) found that learners who first studied the manual without the presence of a computer did better than those who simultaneously used both the computer and the hardcopy manual.

Taking into consideration the split attention effect in the experimental design used in this thesis, multiple sources of information were replaced with a single integrated source (e.g., marking equal angles on the figure rather than saying “angle ABC= angle XYZ”) in order to reduce the extraneous cognitive load. The details of the experimental design will be discussed in Chapter 5.

2.8.3 The Redundancy Effect

The redundancy effect may occur when learners can understand in isolation multiple sources of information without mentally integrating them (Sweller et al., 2011). When a diagram can be understood without the need of written or spoken text that simply redescibes it, including such texts is an example of redundancy.

Leahy, Chandler, and Sweller (2003) conducted an experiment in which a temperature outline (a diagram) was made self-explanatory by integrating it with part of the text and given to one group. The other group was presented with the same material along with audio instructions. Since the diagram was self-explanatory, the audio text was redundant. Attending to redundant information increases cognitive load and affects the performance of the students. Thus, the performance of the first group was superior to that of the second group.

The split attention effect occurs only when integration of dissimilar sources is needed in order for learners to understand the information. However, when a diagram can be understood and provides the information needed for the learner, including text that only repeats the information presented in the diagram will be ineffective. The text in this case is redundant. Removing the text may boost learning (Sweller, 2003).

Integrating sources that are redundant increases cognitive load as opposed to integrating sources that cannot be understood separately. Chandler and Sweller (1991) used a diagram that represents the blood circulation in the human body to show that integrating the text with the diagram is not important for understanding. Integration in this case is redundant since the diagram can be understood separately, without the text. As a result, if a learner realizes that the integrated text is redundant and decides to ignore it, cognitive load is low. However, if he or she decides to study both sources and mentally integrate them, cognitive load is high.

Chandler and Sweller (1991) conducted several experiments investigating the redundancy effect. The results of the first experiment aimed at comparing conventional instructions with integrated instruction showed that integrated instructional formats were significantly better than conventional split-source formats. The latter needed mental integrations that resulted in learners' attention being misdirected and created a heavy cognitive load. As a result, less cognitive resources were available for learners to make use of and acquire the knowledge needed. On the other hand, integrated instructions decreased cognitive load and permitted learners to focus on acquiring knowledge of the material at hand which continued to positively affect their performance over a period of time. The second experiment compared conventional instructions with integrated instructions in areas that did not really require integration of sources in order for learning to take place. The results of the experiment were different from those of Experiment 1. Integration did not prove to be useful. The third experiment demonstrated that

learning was hampered when novice learners were instructed to mentally integrate redundant textual information with diagrams that were self-explanatory since their attention and cognitive resources were unnecessarily directed to this task. Removing the redundant information enhanced learning and performance.

This hypothesis was tested in another experiment. The group that was presented with a diagram only performed the best. This group spent the least time to process the instructions which favoured the reduced cognitive load hypothesis. The findings showed that when sources did not need to be integrated in order to be comprehensible, a redundant source might require removal. Instruction time was diminished by simply eliminating insignificant sources of information. Moreover, learning was improved. Similar results were obtained when conducting experiments in different domains.

One might think that adding redundant information should not have any negative consequence. However, such information inflicts an extraneous load since processing the redundant information and connecting it to the other sources requires the use of working memory resources. The learner might discover the redundancy only when it is too late (Sweller, 2003).

2.8.4 The Expertise Reversal Effect

Kalyuga, Ayres, Chandler, and Sweller (2003) suggested that when the material given to learners is new to them, the available working memory capacity is very limited. When learners become more knowledgeable in the domain, working memory restrictions are decreased because schemas are constructed and stored in long-term memory. Instructional techniques are developed in order to make schema acquisition and automation easier. However, the effectiveness of many instructional design effects may reverse once learners develop expertise. This is referred to as the expertise reversal effect.

It has already been mentioned that when novel material is given to learners, it is processed in working memory that is very limited in capacity. However, when schemas stored in long-term memory are activated, the limitations of working memory could be lifted. All the above instructional principles aimed at decreasing working memory load and hence making schema construction and automation easier. Nevertheless, learner's expertise contributes to the efficiency of these practices. Instructional principles that are good for inexperienced learners may not be as effective when used with experienced learners. On the contrary, they might have negative implications (Kalyuga, Ayres, Chandler, & Sweller, 2003). Kalyuga, Chandler, and Sweller (1998) provided evidence for this in an experiment that demonstrated the split-attention effect using novices. With an increase in expertise, the difference in students' performance was first reduced and then reversed when they were presented with

separate or integrated formats. It is more efficient for experts to be given the sources of information separately with no integration. The split attention effect was transformed into a redundancy effect. Learning was better achieved in the case of expert learners when only a diagram was given instead of a diagram and text. A group of beginning learners were only given diagrams with no text. They did not do well since the text was important to these learners in order to understand the diagram. After some time of practicing in the required domain, their level of expertise increased and the importance of the text diminished since schemas were constructed and took over from the text which became redundant and imposed an extraneous cognitive load. Other experiments in different areas obtained the same results. Cognitive load effects vanished and then reversed with increased expertise.

The degree to which schemas are taken into working memory in order to deal with the information presented is affected by the level of expertise of the learner in a specific domain. Learners who do not have experience need the guidance that related schemas in long-term memory otherwise offer. Therefore, including guidance in the instructions replaces the missing schemas and might lead to schema construction and at the same time, diminishes the load on working memory (Sweller, 1999; Sweller et al., 1998). However, if instructions do not offer the needed guidance, problem-solving search occurs, but it imposes a heavy load on working memory (Kalyuga et al., 2003).

On the other hand, learners who have some level of expertise in a certain domain, bring in their schemas while dealing with a specific problem. They are in no need of any kind of guidance in the instruction. However, if guidance is offered and cannot be ignored, it may cause a redundancy effect. Extraneous cognitive load may be imposed on working memory even if the learner is aware of its redundancy and manages to ignore it as much as possible. Therefore, it is better to reduce instructional guidance offered to learners who are experienced. Consequently, instructional guidance that is important for learners who are inexperienced and which leads to better performance when compared to the performance of learners who are not offered instructional guidance might reflect negatively on those who are experienced and are offered the same guidance (Kalyuga, et al., 2003).

In another study, the relations between levels of learner knowledge in a certain field and levels of guidance in instructions were explored. The group consisted of mechanical trade apprentices who did not have any experience and were given worked examples to study or problems to solve. It was shown that the worked examples were more beneficial for the inexperienced apprentices and their performance was superior to those who solved problems. The higher the level of experience was, the more redundant worked examples became and better problem solving proved to be (Kalyuga, Chandler, Tuovinen, & Sweller, 2001).

Sweller (2010) stated that the expertise reversal effect relied on the condition of interacting elements. When learners are novices, intrinsic load is

imposed by the interacting elements since these elements are important for understanding. When students' expertise increases, these same elements are redundant and impose an extraneous cognitive load since they are not important anymore for understanding. For example, for understanding, it might be important for a novice to study problem solutions or read statements related to diagrams. The elements that interact here are related to learning and thus make up intrinsic load. When working memory resources are allocated to the same elements, germane cognitive load is increased. The same elements that were once intrinsic to learning become extraneous once the expertise of learners is increased. Thus, these elements should be removed. In conclusion, the level of learner expertise affects the type of load imposed.

Taking into account the expertise reversal effect, the experimental design of this study was intended to test the amount of guidance needed in geometry problem solving for novice as well as expert learners. Later chapters will discuss the best instructional design for each.

2.9 Summary

Learners have an advantage when studying worked examples rather than solving problems resulting in the worked example effect. Currently, the evolutionary approach in cognitive load theory highlights the borrowing and

reorganizing principle which provides basis for the worked example effect (Sweller et al., 2011).

There are many reasons why the split-attention effect is significant. Information that must be processed concurrently should be given to the learners without spatial or temporal separation. This effect explains why the worked example effect sometimes does not occur. Worked examples should be presented in a way that does not impose extraneous cognitive load. Finally, the split-attention effect flows directly from cognitive load theory. When learners have to keep information in working memory in order to search for referents, an extraneous cognitive load is imposed (Sweller et al., 2011).

The results of many experiments show that presenting information in more than one form may hinder rather than improve learning. From a cognitive load perspective, when learners are given the same information but in different forms such as auditory and written forms, they have to find the relations between them and in this case, a heavy extraneous load is imposed on working memory which affects learning. The concept of redundant information relies on the level of learner's expertise. Information that is important for novices might be redundant for experts imposing an extraneous cognitive load and leading to the expertise reversal effect (Sweller et al., 2011).

Based on evidence from studies conducted within a cognitive load perspective, procedures that work well for novice learners may inhibit learning

for experts. Thus the expertise reversal effect suggests that novice learners have to be provided with information that enhances learning while more expert learners may find this information redundant and learning is inhibited since they have to find connections between elements of information presented to them and what they already know. However, in the case of novice learners, instructions have to make up for the lack of learners' knowledge (Sweller et al., 2011).

Cognitive load theory is applied best in domains having challenging content. The implication is that extraneous cognitive load has to be minimal when designing instruction in order to allow learning to take place effectively. The next chapter discusses why students find it difficult to use their prior knowledge while solving geometry problems and how this knowledge is built and acquired along with its application during problem solving.

Chapter Three

Learning and Instruction in Geometry Problem Solving

3.1 Issues in Teaching Geometry

Mathematics education has been engaged with Euclidean geometry and geometric proof from the time of the Greeks up to twentieth century Western culture. A major part of learning mathematics in secondary schools is related to Euclidean geometry and the proofs that support it. However, towards the last part of the twentieth century, many school mathematics curricula stopped focusing on both Euclidean geometry and proof (e.g., van Dormolen, 1977). Healy and Hoyles (1999) stated that “Proof is at the heart of mathematical thinking, and deductive reasoning, which underpins the process of proving, exemplifies the distinction between mathematics and the empirical sciences (p. 1)”.

Research shows that students often do not comprehend the reason behind mathematical proof, and they establish their own understanding based on practical evidence. Geometry is one of the mathematical areas that students find difficult to learn (Senk, 1985). Healy and Hoyles (1999) also demonstrated that many students have a poor understanding of mathematical proof. Other studies have

examined the specific difficulties which students experience with proof. Some studies have focused on students' understanding of the concept of proof (for example, Galbraith, 1981; de Villiers, 1991) whereas others (for example, Duval, 1991) have looked at students' difficulty with construction of proofs, in particular deductive proofs.

Students' concept of mathematical proof and the difficulties they face in constructing proofs are both related to students' cognitive readiness. The learners' cognitive readiness for mathematical proof is indirectly suggested in the sequence of mathematical reasoning skills. The existence of hierarchical levels of geometry understanding was proposed by Pierre van Hiele (1959, 1984) and Dina van Hiele-Geldof (1957, 1984), who suggested that students progress sequentially from a simple identification level to a more deductive level. Table 1 lists the van Hiele levels (levels 1-4).

Evidence for this progressive chain of geometry learning is strengthened by studies based on Hiele's theory, however, it has also been proposed that these levels are not disconnected but rather students can move from one level to another depending on the concepts taught (Usiskin, 1982; Fuys, Geddes, & Tischler, 1988). The emphasis on class inclusion as a signifier of Level 3 thinking has also been questioned (for example, Pegg, 1992), since it appears that understanding of this concept depends on the learning environment and is not a natural part of students' progress in geometry.

Table 1 Description of Van Hiele Levels 1-4

Level	Description
1	Shapes are recognized by their visual appearance alone.
2	One or more properties of a geometric shape are recognised.
3	Relationships between properties are recognized, with simple steps of deductive reasoning. The concept of class inclusion is understood.
4	The significance of deductive reasoning and the concept of necessary and sufficient conditions are understood.

It has also been suggested that a progressive chain model of mathematical development where diagrams are the means by which visualisation and symbolism intermingle results in the need for definitions and proofs. The argument is that the cognitive reconstruction, which is needed to shift from diagrams to formal proof requires a certain degree of cognitive development (Tall, 1995).

In addition to the diagrams, verbal information is involved when a geometry proof problem is presented to students. Also, such presentations usually include physical movements indicating the parts of the diagram related to the

problem at hand. Students have to view, listen, and make notes of the lecture. As a result, different elements of the instruction have to be integrated within the learner's working memory (Sweller, 1998). This integration process imposes cognitive load and may affect students' learning. Many experiments have indicated that significant problems in geometry learning are caused by the lack of thorough understanding of the problem, symbols, and proofs that are based only on visual elements (e.g., Chazan, 1993; Healy & Hoyles, 2000), and the lack of the necessary knowledge needed to complete the proofs. Duval (1998) and Healy and Hoyles (1998) noted that instruction in numerical operations and algebra are easier than those in geometry that involve tasks that are demanding for teachers and students alike.

Consequently, providing a good and solid basis in the early years of education, which includes not only practical evidence but also basic steps of deductive reasoning is crucial for students' readiness for formal proof. Hoyles (1998) suggested that it is essential to take into consideration how proof is presented to students. Therefore, presented instructions should be designed to assist students in comprehending geometry proofs and theorems considering the processing limitations of human cognitive architecture.

3.2 The Role of Geometry Schemas in Problem Solving

The poor use of prior knowledge when solving problems in geometry could be one of the reasons for students' difficulties in learning this domain. The understanding of the role of prior knowledge that students use when trying to solve problems has become an important focus of research studies (e.g., Byers & Erlwanger, 1985; Lester, 1994). The outcomes of these studies show that succeeding in problem solving is greatly dependent on the relations that students make between elements of mathematical knowledge (Chinnappan, 1998). Consequently, if we need to understand why students find it difficult to use their prior knowledge while solving geometry problems, we have to focus on how this knowledge is built and acquired along with its application during problem solving (Board of Senior Secondary Studies, 1995). Resnick and Ford (1981) pointed out that the failure of students to trigger and apply what they had previously learned was due to their poor mathematical knowledge.

Researchers in this domain are concerned with the relations students make between dissimilar mathematical concepts while integrating them into significant wholes and structures. This is what constitutes mathematical knowledge, and it not only makes students relate previously learned material to new problems, but also determines how the information should be used to search for a problem solution (Alexander & Judy, 1988; Prawat, 1989; Lawson & Chinnappan, 1994). (see also Chinnappan 1998).

The manner in which mathematical knowledge is stored in long-term memory determines how well students are able to activate it. If we use this notion in the field of geometry, it means that we have to focus on the geometric knowledge structures that students retrieve while solving a problem, to what degree they use them, and how efficient the use is (Chinnappan, 1998).

The quality and function of mathematical knowledge that students make use of while attempting to solve problems was discussed by Mayer (1975), Kintsch and Greeno (1985), and Halford (1993). They referred to schemas and mental models when examining the geometry knowledge used by students when solving problems. Chinnappan (1998) defined a schema as a group of basic notions, the connections between these notions, and the manner and timing of using them. Thus, the mathematical notions that students understand are classified into schemas that form the knowledge base for dealing with later mathematical problems. The term schema has also been used in a similar context by Nesher and Herschkovitz (1994) when studying students' work while solving word problems. As students think about and try practicing what they have learnt, their mathematical schemas are changed through a process of construction and reconstruction leading to altering their mathematical schemas. According to Sweller (1989), the manner in which they sort and manipulate problems is affected by how complex these schemas are.

Chinnappan (1998) stated that to know more about how geometric information is structured one should take into consideration a schema theorem,

which suggests a geometric schema that could develop gradually around a geometric form. He gives an example of the right-angled triangle schema (RTS). Ideas, notions and information are constructed in relation to the central part of the RTS which is the right-angled triangle. One section of the RTS is Pythagoras' theorem, which deals with the lengths of the sides of a triangle. Another example is the sameness of the base angles if the triangle is a right isosceles triangle (45 degrees each). This knowledge, along with their own experiences aid students in working with problems involving right-angled triangles. Thus, the RTS system would include not just the geometric shape but the knowledge and aspects that relate to right-angled triangles.

3.3 Features of Geometric Schemas

Organisation and spread are two major features of geometric schemas (Chinnappan, 1998). Organisation deals with the network of ideas while spread deals with the size of the network. The greater the extent of organisation and spread, the more complex the geometric schema is. However, knowledge of the organisation and spread of schemas may not be sufficient to yield a solution to a given problem. Chinnappan (1998) stated that studying mental models related to a problem helps researchers know more about how students incorporate geometric schemas while trying to find a solution to a problem. Halford (1993) suggested that mental models are depictions which are active when a student is in the process of solving a problem. During this phase, mental models elicit cognitive

procedures that involve reasoning and making decisions related to what schemas should be triggered and the manner in which knowledge should be used while trying to find a solution to a particular problem. According to English and Halford (1995), the mental models that students should be helped to build must deal with the crucial connections between mathematical schemas that have been learned before and parts of a problem to be solved.

Chinnappan (1998) conducted a study in which he used both structures, schemas and mental models, to study how students use their knowledge when solving a plane Euclidean geometry problem. The purpose of the study was to identify the schemas used when solving a geometric problem, find out the frequency of activation, and come up with a clear picture of the nature of the mental models students deploy and/or build when solving problems. The study took into consideration levels of learner prior knowledge by distinguishing low and high achievers. Chinnappan assumed that more complicated geometric schemas would be stimulated and used more often by high achievers along with mental models that show better organised knowledge of the problem at hand if compared to low achievers.

The experiment showed that the high achievers used more geometric schemas than the low achievers while solving problems. Another result of the analysis was that the schemas of high achievers were more often stimulated than those of the low achievers. A third feature of the analysis of the students' records showed that the geometric schemas used by the high achieving students were

more complicated and diverse than those used by low achieving peers. The analysis provides evidence for the premise that the schemas used by high achievers are not only more frequent than those used by their low achieving counterparts, but they are also more sophisticated in quality.

As for the mental models built by activating geometric schemas during problem solving, it was suggested that both high and low achievers would engage in this activity. A careful study of students' solutions revealed that a systematic examination of the problem was conducted by high achievers. However, Chinnappan reported that 60 percent of the low achievers' attempts to reach solutions required some kind of direction when solving a problem. The study also reported two cases of high achievers who were able to make use of a novel way of solving a problem which entailed the activation of triangle schemas. In contrast, low achievers did not invent novel procedures. Chinnappan explained the disparity between the two groups' employment and launch of the geometric schemas by their different approaches to problem solving. The high achievers dealt with the structural aspects of the problem and succeeded in structuring mental models that expressed the relationship between the givens and the objectives of the problems. After that, the students looked for the related geometric schemas in their memories to reduce the gap between the givens and objectives. The low achievers did not delve deeply into the features of the problem and could not identify the relations between the essential aspects of the problem and their own geometric schemas.

Chinnappan's study emphasized the importance of organising information in chunks or schemas to acquire better insight into when and how to use the information efficiently while solving problems. The mental models created provide evidence for this insight.

3.4 Research in Cognitive Load Aspects of Learning Geometry

3.4.1 Introduction

Cognitive load theory has been employed to examine instructional techniques that facilitate schema acquisition and automation. It has been suggested that techniques that require students to participate in activities that do not aim at schema acquisition and automation are not effective since they require cognitive resources that cannot be allocated to learning. When novel problems are given to students, they may not be able to make use of already acquired schemas to come up with solutions. However, they may solve the problems using a means-end strategy that is effective in reaching problem goals. Nevertheless, this technique is not directly related to schemas and schema acquisition. In order to attain a problem solving schema, students should distinguish each problem state and learn appropriate moves associated with each state. When using means-ends analysis, on the other hand, they must find relations between a problem state and the goal state, extract differences between them, and find problem operators that affect those differences positively. These cognitive operations must be done at the

same time while also bearing in mind any sub-goals. For novice learners, the problem states or operators are not likely to be automated and thus must be processed consciously in working memory. Such difficult tasks may enforce a heavy cognitive load that is not related to schema acquisition and thus may hinder learning. Learners who are given geometry problems and who use this strategy will not reach their ultimate learning goal which is schema acquisition and automation (Sweller, 1994). Instructional techniques that can be used in order to substitute conventional problem solving will be discussed below.

3.4.2 Worked Examples in Learning Geometry

Tarmizi and Sweller (1988) demonstrated that studying worked examples was a more beneficial instructional method than solving problems because the provided guidance reduced cognitive load otherwise generated by a means-ends strategy involved in problem solving. However, the guidance offered should not itself impose extraneous cognitive load due its presentation format. If this happens, the benefits of the worked examples and guidance could disappear. Five experiments in geometry were conducted to provide evidence for this premise.

Experiment 1 included thirty-three year nine top mathematical students, who were not previously exposed to the geometry theorems related to circles but would be able to use the explanations and solve the problems given in this field. The experiment demonstrated that the means-ends method used by students who

were working on conventional problems interfered with learning due to the cognitive load that accompanied this method. The use of goal-free problems reduced this load and facilitated the acquisition of schemas. Conversely the guided-solution group was forced to pay attention to the information on the diagram and the text that presented the guidance separately from the diagram, and at the same time mentally integrate them. However, the goal-free problems did neither require participants to pay attention to integration, nor the employment of means-ends analysis. Learning outcomes (as measured by the speed and precision of student responses) hence improved for the goal-free problem group. It was proposed that the positive implications of guidance were eliminated because of the heavy cognitive load that was imposed by the extra material.

Experiment 2 included twenty-four year eight top mathematical students who had no experience in circle geometry. This experiment compared the outcomes of goal-free problems against those of a guided-solution group. Even though less guidance was provided in Experiment 2 than in Experiment 1, the results were similar: the goal-free group performed better than the guided-solution group proving that incorporating many sources of information imposed a heavy cognitive load and negatively affected learning.

Experiment 3 included twenty year nine top mathematical students who were not previously exposed to the geometry theorems related to circles. The experiment was conducted to compare the efficiency of worked examples and problem solving in geometry instruction. Contrary to what Sweller and Cooper

(1985) and Cooper and Sweller (1987) found using algebra worked examples, the results of Experiment 3 were the same as those of Experiments 1 and 2. It demonstrated that the guidance presented to the worked example group did not make learning easier; it actually impeded it.

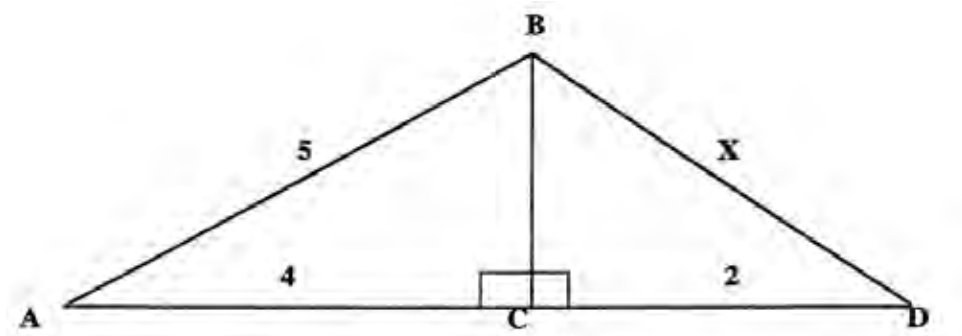
Based on the results of the above experiments, Experiment 4 was conducted and it included thirty year nine top mathematical students who were not previously exposed to the geometry theorems related to circles but had enough background to use the explanations given in order to solve problems in this field. The experiment was conducted to demonstrate that providing students with geometry worked examples that diminish the requirement to integrate many split sources of information should boost the facilitatory effect of worked examples. In order to accomplish this, the information that was related to guidance was presented at appropriate locations on the geometric figure contrary to the split-source format. The results demonstrated that the group that studied the modified geometry worked examples in which the textual explanatory material was put on the diagram performed better than the other two groups. Extraneous cognitive load was reduced because students did not have to allocate additional cognitive resources to integrate split sources of information. These results matched those of Sweller and Cooper (1985) and Cooper and Sweller (1987).

Experiment 5 replicated the results of Experiment 4 using different tasks, thus proving again that using modified formats of geometry worked examples reduced cognitive load and improved learner problem solving skills.

A conventional succession of instructions may be followed by learners in the early phases of learning geometry and trigonometry. For example, when learners are introduced to geometry, they learn about parallel line theorems (alternate, corresponding, and co-interior) and they start solving simple problems that require them to use one theorem at a time. At a later stage, they have to start using more than one theorem. Most of these problems are categorised as transformation problems (see Greeno, 1978). In these problems the initial state is changed into a goal state, and to solve them and find the answer to the goal state, learners have to calculate a subgoal or several subgoals. In order to solve these transformation problems, backward-working strategies such as means-ends analysis might be used by learners (see Mayer, 1983). However, such a strategy imposes a high cognitive load since learners have to go back to goals and subgoals. An instructional design that has been examined by researchers in different fields like trigonometry (Owen & Sweller, 1995) and geometry (Ayres, 1993) is the employment of no-goal or goal-free problems. By eliminating certain goals in a problem and requiring students to figure out all the unknowns instead of certain goals, a goal-free context is set and the application of means-end analysis is reduced and in turn so is cognitive load (Ayres, 1993).

The two stage transformation problem below needs Pythagoras' Theorem to be employed twice in order to be solved. Before figuring out (X), the goal, the subgoal (side BC) has to be calculated. A learner might calculate the subgoal (BC) first without thinking about (BD) which is working forward. The solution is uncomplicated since there are no other sides that need calculations. By applying

the rule that says to calculate the side of two adjacent triangles the subgoal will be calculated. The outcome of having a simple path to reach a solution is that a forward working strategy might be generated by instructional techniques. Moreover, backwards-working strategies might also be used since in simple solution paths, cognitive load is reduced (Ayres, 1993).



Example of a 2-stage Pythagorean Problem

Ayres (1993) designed a study in order to examine the usefulness of no-goal problems in the field of Pythagoras' Theorem and forward and backward strategies. The study included fifty-six girls from grade eight in a Sydney High School having average abilities in mathematics. The Theorem of Pythagoras had been taught to them and the participants had previously solved problems that need a single step. The experiments consisted of two groups of twelve problems each containing two unknown sides. The problems were similar to the two-stage transformation problem above. The first group of problems was used for the acquisition stage while the second was used for testing. In order to figure out these two sides, the participants had to employ two applications of Pythagoras Theorem and the subgoal (the side which was common to both triangles) had to

be found before the goal. During acquisition, the focus of participants in the forward working strategy group was directed towards the subgoal first when they were required to calculate the first unknown side. Then they were asked to find the value of (X) which is the goal. Thus they would work forward once their focus was directed towards the subgoal. The backward working group, on the other hand, was told that to find (X) they had to calculate an unknown side, and they were asked to calculate that side followed by instructions for calculating (X). As for the conventional group, participants were asked to calculate (X) for all problems. Participants in the final group, the no-goal group, were required to calculate all unknown sides and (X) was not referred to. The results showed that the no-goal method is quite efficient when working on transformation problems. Cognitive load is diminished by eliminating the goal. The results of forward and backward working strategies were similar to those of the conventional group.

In another study, different conditions of variability of learning tasks in a computer-based environment for geometrical problem solving were examined focusing on the effects they had on performance, transfer, and cognitive load (Paas & Van Merriënboer, 1994). Four conditions were compared. They included low- and high-variability conventional practice problems to be solved followed by worked examples. They were compared with a low- and a high-variability worked condition, which included worked examples to be studied. The second condition included low and high-variability worked examples to be studied. The results showed that students who studied worked examples benefited the most from the high-variability condition. They required less time and mental effort while

practicing and demonstrated better transfer performance compared to students who first solved conventional problems followed by studying worked examples.

3.5. Summary

Students' poor use of mathematical knowledge, the manner in which geometric knowledge is stored in long-term memory, and how it is activated should be considered when discussing why students find it difficult to work on generating proofs in geometry. Mental models and schemas are used by students when attempting to solve mathematical problems. However, high achievers use more geometric schemas that are more complex and more frequently accessed than those of low achievers. As the students gather more experience and become more expert at solving geometric problems, their performance improves. The following chapter will discuss why exposure to worked examples while solving a geometric problem is an effective instructional technique since it diminishes extraneous cognitive load, thus ensuring that more working memory resources are available for activities that facilitate learning and transfer. The chapter will also discuss how to optimize the design and delivery of worked examples in order to foster learning of geometry problem solving.

Chapter Four

Learner Levels of Expertise and Different Forms of Instructional Guidance in Worked Examples

4.1 Introduction

An essential mathematical activity is demonstrating mathematical proof. Therefore, all students should understand not only the skills needed to read and construct proofs but also the concepts that are associated with these proofs which they find difficult. Though it has been around 100 years ago since Poincare (1913) stated that students did not perform well at finding proofs, classroom instruction on complex problem-solving skills and on the heuristic skills nowadays is still facing this problem (Heal & Hoyles, 1998; Reiss, Klieme, & Heinze, 2001).

Proving is a cognitive activity that involves exploring, inducing, and deducing the logic needed. It is crucial for a learning environment that focuses on proofs to foster heuristic problem-solving processes. Presenting students with worked examples that demonstrate finding a proof using heuristic processes can provide them with such a learning environment (Hilbert, Renkl, Kessler, & Reiss, 2008). However, the effectiveness of worked examples may depend on levels of

learner prior knowledge. The following section will examine this issue in more detail.

4.2 Expertise Reversal Effect

Learners' prior knowledge is an important factor influencing learning processes, and it has to be taken into consideration when designing instruction. What is effective for beginning learners might in fact be harmful to expert learners. As levels of learner knowledge in a specific domain change, the efficiency of instructional methods may reverse. Various instructional materials and participants have been used to demonstrate this effect. The implication of this effect is that as learners become more experienced in a certain domain, there is a need to modify instructional methods and procedures (Kalyuga & Renkl, 2010).

Cognitive load theory explains this interaction between the effectiveness of different instructional methods and levels of learner expertise by disparities between the organized knowledge base of the learners and provided instructional guidance. This disparity might be due to either the fact that instructional guidance, in particular at the initial stages of learning, does not compensate for an inadequate learner knowledge base or to the overlap between the already existing knowledge of advanced learners and the instructional guidance given. When such gaps in knowledge exist, in the first case, novice learners have to take on certain search practices that might lead to heavy levels of extraneous cognitive load. As

for advanced learners, redundant instructional guidance needs to be assimilated and cross-referenced with already existing knowledge structures. This results in the consumption of additional cognitive resources. Consequently, instructional guidance has to be minimal to permit these learners to make good use of their knowledge in an effective way. As a result, instructional guidance that is important for novice learners might hamper learning for more experienced learners when it interferes with the recovery and use of available knowledge structures, especially if redundant explanations cannot be neglected. So, for an optimal cognitive load, instructional support for advanced learners has to be eliminated when acquisition is in progress while novice learners must be provided with essential support (Kalyuga & Renkl, 2010).

Oksa, Kalyuga, and Chandler (2010) conducted a study comparing the instructional efficiency of Modern English explanations of parts of plays written by Shakespeare integrated within original Elizabethan English Text. The first experiment showed that an explanatory notes group reported lower cognitive load and performed better on a comprehension post-test than the control group when students did not have any prior knowledge of the text. In Experiment 2 the same material was given to a group of Shakespearean experts and a reverse effect was demonstrated. The control group's performance was better than the performance of the experimental group since the explanation turned out to be redundant for the expert participants. Experiment 3 replicated the results of Experiment 1 but with a different Shakespearean text. The results showed the advantages of explanatory notes. The efficiency of instructions depended on the level of expertise of the

learners, thus showing an expertise reversal effect in the text comprehension area. Integrated explanatory text was beneficial for low-level knowledge learners. On the other hand, for learners who had more advanced and automated knowledge structures, the need to process information that was redundant for them may have generated an unnecessary working memory load. Thus, guidance could become harmful for experienced learners.

Nückles, Hübner, Dümer, and Renkl (2010) conducted another study in a non-technical area of journal writing in a developmental psychology courses. Journal writing is learner reflection on what has been studied previously. It was established previously that instructional support in the form of prompts relevant to suitable cognitive and metacognitive strategies could maximize productive, germane cognitive load and improve learning. In Experiment 1, students were asked to respond to each weekly session by writing a journal entry for a whole term. The experimental group was given cognitive and metacognitive prompts, whereas the control group did not get any prompts. During the first half of the term, the experimental group performed better than the control group and used more strategies in their writings. However, towards the end of the term, the number of cognitive and metacognitive strategies used by the experimental group lessened while the number of cognitive strategies used by the control group increased. The control group performed better than the experimental group whose members were not as stimulated to learn as well as at the beginning of the term.

In Experiment 2, the prompts were gradually eliminated to avoid their negative consequences. Once the students in the experimental group started using the prompted strategies, they were eliminated gradually. As for the control group, the prompts were given all the time. The results indicated that the group with the fading prompts performed better and used more strategies than the group with permanent prompts that used increasingly less cognitive strategies. The results thus showed an expertise reversal effect in writing-to-learn. At the start of the term the students made good use of the prompts in applying strategies. However, as the students became more skilled in writing journals and in applying these strategies, the guidance became more redundant and caused an extraneous cognitive load hampering the ability of the students to apply the desired strategies. The gradual elimination of the prompts as a form of instructional guidance with increased student's skill was more efficient than continuously providing the prompts.

Blayney, Kalyuga, and Sweller (2010) investigated the interactions between the isolated-interactive element instructional formats and levels of learner expertise with undergraduate accounting students in their first year. The results obtained showed that the expertise of the learners interacted with instructions that provided isolated or interactive elements of information at the beginning stages of instruction. The isolated-interactive effect suggests that learning could be improved by giving learners elements of information sequentially, in isolation at the beginning of tasks instead of presenting this information in a fully interactive form. Novice learners gained from this

technique. However, more advanced learners did not benefit from it since they were not given the chance to make good use of their knowledge base. Since becoming an expert is a gradual process, changing instructional formats from isolated to interacting elements requires the elimination of intermediate formulas slowly.

Based on the results of the previous experiments, it is expected that novice learners would benefit from guided instruction that decreases their mostly random search for solutions steps. However, for experienced learners, studying detailed instructions with guidance and integrating this information with their available knowledge that already provides the same information may add to the extraneous load. On the other hand, less knowledgeable students would benefit more from worked examples that provide them with required guidance rather than from problem solving with less guidance.

4.3 Some Worked Example Experimental Results

Conventional problems include a given and a goal statement. Worked examples, in addition to the given and the goal statement, present the steps leading to the required solution. Based on available research, instruction that depends on worked examples is more efficient than solving problems, especially for novices, because less time and mental effort is usually required (Van Gog, Kester, & Paas, 2011).

Van Gerven, Paas, van Merriënboer, and Schmidt (2002) compared the effect of studying examples on problem solving in elderly and young learners. Thirty psychology students were part of the first group of participants. The second group included 24 elderly participants. The results showed that worked examples were more effective for training the elderly than conventional problems. In addition, elderly participants devoted more working memory resources to learning when studying worked examples than solving conventional problems.

Bobis, Sweller, and Cooper (1985) conducted a series of five experiments to establish the knowledge needed to solve algebra problems and methods to accelerate the acquisition of this knowledge. The results of the first experiment with Year 9 and 11 secondary school students, and university mathematics students, demonstrated that more experienced students had better representations of algebraic equations compared to less experienced learners. The following experiments focused on the effect of worked examples on the acquisition of the knowledge required for effective problem solving. The results showed that worked examples needed less time to be processed than solving conventional problems. Moreover, the problems that followed the study of worked examples were solved faster with less mathematical mistakes. It was also established that using example-problem pairs (i.e., solving a similar problem right after studying an example) was more effective than studying examples only.

Another study was conducted to test the efficiency of three instructional procedures that were based on examples (example-problem pairs, problem-

example pairs, and fading worked examples) in order to teach serial and parallel electrical circuit analysis to learners with low or high levels of prior knowledge (Reisslein, Atkinson, Seeling, & Reisslein, 2006). It was shown that learners with less prior knowledge gained the most from example-problem pairs. Learners with high levels of prior knowledge gained most from problem-example pairs. This study supported the expertise reversal effect, which suggested that the effectiveness of instructional procedures in example-based learning depended on levels of learner prior knowledge.

While studying example-based problems, learners usually face difficulties if they are not given enough support and guidance. The result of this would be superficial acquisition of problem categories and solution processes. The cause of this problem might be that conventional worked examples present problem categories and category-specific solution processes in a molar format. In this format, problem categories and solution processes are considered as central elements of analysis that cannot be separated into parts. Learners might be overcome by these difficulties because of the many structural problem characteristics and steps in a solution that they must deal with at the same time leading to a high intrinsic cognitive load. In order to work out this problem, a modular example presentation was created that concentrated on the function of single structural problem characteristics and single solution steps (Catrambone, 1994; Gerjets, Scheiter, & Catrambone, 2004). Gerjets, Scheiter, and Catrambone (2006) conducted two experiments to compare traditional molar worked examples

that focused on problems and their solution procedures with modular worked examples thus reducing intrinsic load.

In Experiment 1, 96 university students of different majors were instructed in complex probability concepts. They had studied elementary concepts of probability in high school and had little exposure to more complex concepts. The results showed that modular worked examples were superior to molar worked examples. Learners who studied modular examples required less learning time, retrieved less examples, performed better on post-tests, and indicated less cognitive load than the learners in the molar examples group. However, there was no evidence to support the idea that more instructional explanations benefit learners. On the contrary, the more explanations associated with the examples, the less efficient they were with no indications of better performance. As for cognitive load, detailed examples reduced the effort needed to understand the material and enhanced the learners' feeling of success. The conclusion was that though students benefited from modular work-examples, the two groups did not benefit from the additional support that was given. The instructional explanations might have been redundant for learners in both approaches but the reasons for this were different. Learners who studied modular work examples had more cognitive resources to use while engaging in self-explaining activities. Even though more time was used to study these detailed examples, the effort they exerted was not necessary to guarantee their understanding of the underlying principle for the solution approach. However, learners who studied molar examples would have gained from instructional explanations given, but they abstained from doing this.

These students were not able to come up with the explanation of the solution steps on their own. Thus instructional explanations might sometimes hamper self-explanatory activities.

In Experiment 2, 91 students from different majors participated. They were all familiar with basic concepts of probability. The purpose of the experiment was to investigate the efficiency of asking students to give elaborated self-explanatory steps towards the solutions. Every second example was substituted with a medium-detailed example and a prompt requiring learners to add self-explanations. The suggestion was that self-explanation prompts would enhance learning for students who were given modular examples. These participants were supposed to have enough cognitive resources to come up with the self-explanatory steps. Previously, it was assumed that learners presented with molar examples would not have the needed cognitive resources, and so would not gain from self-explanation prompts. However, based on the results of the first experiment confirming that self-explanations might lessen the effort needed by learners to understand the material, self-explanation prompts in molar examples may reduce the possibility of illusions of understanding leading to inefficient instructional explanations. The assumption was that if students were asked to provide self-explanations, they would realize that they did not understand the material and as a result, they might put in more effort and time to process the examples. Consequently, it was not evident whether the self-explanation prompts for molar examples might be inefficient due to the lack of cognitive resources or

because they resulted in a greater need to process instructional material (Gerjets et al., 2006).

At first it was suggested that instructional explanations might enhance learning of molar examples. However, the results of Experiment 1 indicated that performance was not enhanced when instructional explanations were used whether in molar or modular examples even though learners had the mistaken notion that they would understand better if more instructional explanations were provided. Instructional explanations may not have been required for both methods but for different causes. Learners working with modular examples have sufficient resources to engage in self-explanatory tasks on their own which means that they do not need instructional explanations. Learners who worked on molar examples abstained from studying instructional explanations because they had illusions of understanding. It was suggested that if they were encouraged to come up with these explanations on their own, then these illusions would be surmounted. Experiment two proved this to be wrong. Performance was even worse than expected. Learners dealing with modular examples were obliged to come up with self-explanations for materials that they comprehended. These results were in line with the redundancy effect discussed earlier which states that learning is hampered when learners study materials they already know (Gerjets et al., 2006).

Research that has been conducted has indicated that for novices solving problems as an instructional tool is not as efficient as worked examples. The latter requires less time and mental effort in order to get to superior transfer which

depends on schema acquisition because both the information related to the types of problem and the related operators are stored in problem schemata. Thus students perform better on transfer problems when they study worked examples. They identify the structural characteristics of a problem they have been exposed to and they use the solution steps that are related to that problem. However, in order for students to join problem operators that were learned earlier and go through far transfer and solve problems of different categories, they need to understand a procedure. They need to recognize why certain steps are used in a certain order along with the principles of the domain. To make it simpler they need to recognize why certain structural problems are related to a certain operator (Van Gog, Paas, & van Merriënboer, 2004). Thus using Young's (1983) wording, a lot of the worked examples used in earlier research do not permit far transfer since they do not foster the increase in mental models that enhance performance, learning and logical thinking. Paas and Van Merriënboer (1994) call the examples that present the given state, the solution steps, and the goal state as the product method. As a result, these examples might be referred to as product-oriented worked examples since the reason for choosing and applying the operators is not incorporated in a schema for it is not part of the examples. When students are able to come up with sufficient interpretations themselves, the information grows into their knowledge structures. A process method of worked examples which presents to the learners the 'why' and 'how' information and application of operators might have positive effects on students' understanding and far transfer that are supported by the mental models that are a result of such information (Van Gog, Paas, & van Merriënboer, 2004). The manner how the solution is reached, i.e.

(how) along with (why) it was reached are considered essential since learners often asked them when being presented with worked examples (Hoogveld et al., 2002; Hoogveld et al. 2003).

Van Gog and colleagues (van Gog, Paas, & van Merriënboer, 2006a, 2006b) developed the concept of process-oriented worked-out examples. The difference between them and the conventional worked examples is that they include the (how) and the (why). Van Gog et al. (2006a) distinguish between recurrent (i.e., algorithmic) and non-recurrent (i.e., heuristic strategies) essential skills that make up a complex cognitive skill. Information on (why) should be included when learners are presented with recurrent constituent skills. However, when heuristic strategies are presented, information on how to apply these strategies should be added along with why to apply them in order to enlighten learners about the sense of the task. Heuristic examples are a special type of worked examples that are associated with problems that do not require students to provide the steps towards the solution. Students are supposed to present the reason behind the solution given and they should also identify how to investigate the problem so as to comprehend the conjecture. After the investigation, a heuristic worked out example provides students with an accurate and specific proof for the conjecture. The common feature of heuristic examples and process-oriented examples is the addition of heuristic strategies. As for the difference between them, both recurrent and non-recurrent skills are taught in process-oriented examples while heuristic examples focus on non-recurrent skills (van Gog, Paas, & van Merriënboer, 2006a, 2006b).

While product-oriented worked examples give only a problem solution, process-oriented worked examples provide the students with the underlying principle behind the solution which is very important in order to reach efficient transfer. A study was conducted in the field of electrical circuit trouble shooting. It included 82 students in their fifth year of secondary education. They did not have any experience in applying their knowledge of electrical circuits to troubleshooting. Before the experiments, they were randomly allocated to four conditions corresponding to product-product, process-process, product-process, and process-product training sequences. Two sessions of training worked-examples were given to the students, and each one was followed by a session of transfer test problems. However, the study showed that germane cognitive load would at first be imposed when studying process-oriented worked examples and the result would be better efficiency on the first test than studying product-oriented worked examples. However, carrying on with studying process-oriented worked-examples imposed an extraneous cognitive load that impeded learning and resulted in less efficiency on the second test in comparison with studying product-oriented worked examples. The hypothesis was that on the first test, the process-process and process-product conditions would score better than the product-product and product-process conditions. However, when learners become familiar with the solution procedure, processing the additional process-oriented information would become redundant and this could affect the effectiveness of transfer. Eliminating this information and presenting learners with product-oriented worked examples would lead to improved effectiveness. This suggests

that on the second test, the process-process condition would be less effective. The second hypothesis suggested that the process-product condition could be more effective than the product-product or the product-process conditions. The results confirmed the hypothesis discussed above (van Gog, Paas, & van Merriënboer, 2008).

Hilbert, Renkl, Kessler, and Reiss (2008) conducted an experiment to test the efficiency of heuristic examples in helping learners to enhance their knowledge about mathematical skills and analyse the advantages of self-explanation prompts. It was found that self-explanation prompts were beneficial to learners in helping them to attain conceptual knowledge. In addition, it also benefited learners' skills. Another finding was that prompting learners to recognize the phases of problem solving was helpful in the acquisition of not only proving skills but also conceptual knowledge about the proving process. Finally, another result was that the efficiency of heuristic examples was weakened by the gaps to be filled in the examples. Studying heuristic examples proved to be more beneficial than working with related geometry tasks. It was also found that learners who had better conceptual knowledge were more successful in solving geometry tasks that required solving. The importance of this study was that it demonstrated the efficiency of example-based learning in the acquisition of both algorithmic and heuristic knowledge in mathematics.

A quasi-experimental design was used in a study that investigated the effects of using worked examples in a primary school mathematics curriculum

(Van Loon-Hillen, Van Gog, & Brand-Gruwel, 2010). In this study, a realistic instructional setting was used and the study lasted for three weeks. The results did not show any significant differences in performance between the worked example group and the problem solving group. However, the worked examples group required less time to reach the same level of performance.

4.4 Summary

Worked examples represent one of the most efficient methods of instruction that effectively decrease extraneous cognitive load which permits more working memory resources to be allocated to activities that foster learning and enhance transfer. However, as students advance through training, the effectiveness of worked examples may decline leading to the expertise reversal effect. The main implication of this effect for the design of example-based learning is that the levels of instructional guidance in examples should be appropriately managed. It may involve presenting learners initially with fully guided worked examples along with self-explanation guidelines, and then gradually removing this guidance as learners become more proficient in the task domain. For example, in the successive presentations of examples, the explained sub-steps could be gradually replaced with problem-solving questions until eventually, students are required to solve novel problems (a fading technique), (Renkl, Atkinson, & Große, 2004).

In conclusion, research has demonstrated that instruction which relies heavily on worked examples is more effective for novice learners as opposed to instruction consisting of problem-solving tasks. However, unnecessary instructional guidance provided in worked examples might become redundant and impose an extraneous cognitive load on more expert learners and thus affect their performance.

It has been observed that students find it difficult to transfer knowledge even between problems in the same domain. Process approaches and product approaches are two different methods that bring about transfer. In traditional product-oriented worked examples, solution steps are provided to demonstrate how to reach the goal state with no explanation why these problem-solving steps are used. In contrast, process-oriented worked examples provide learners with the solution steps and strategies (how) and the principles (why) explaining reasons behind choosing those steps.

Eliminating search that is not necessary and enhancing schema construction are reasons why worked examples are efficient. In order for students to succeed in solving problems, they need to identify the structural problem features and the category to which a problem belongs, and remember the steps related to solving it and reaching a solution. Transfer performance depends on the acquisition of specific schemas since the information concerning types of problems and their related steps are stored in problem schemas.

The purpose of this study was to investigate optimal degrees of guidance when using geometry worked examples. Research has demonstrated that instruction that relies heavily on worked examples is more effective for novices as opposed to instruction consisting of problem-solving. However, excessive guidance for expert learners may reduce their performance. Three conditions were used: Theorem & Step Guidance; Step Guidance; and Problem Solving. In the Theorem & Step Guidance condition, students were provided with both the steps necessary to find each angle and the theorem used to justify the step. In the Step Guidance condition, learners were presented only with the sequence of steps needed to reach the answer without indicating the theorem required to make a step. The problem solving condition required learners to solve the problems with no guidance.

It was hypothesized that the Problem Solving condition would impose a heavy cognitive load associated with search while providing information concerning well-known theorems in the Theorem & Step Guidance approach would be redundant for experts but might be beneficial for novices. In other words, as long recognized by cognitive load theory, most students need to learn to recognize problem states and the moves associated with those states and this information is provided by Step Guidance without additional redundant information. Moreover, information concerning theorems should only be provided to students who have yet to learn and automate theorem schemas.

Chapter Five

Evaluation of Cognitive Load and Instructional Efficiency

5.1 Introduction

The core goal of cognitive load theory is the creation of learning environments that make optimal use of cognitive resources and reduce extraneous load in order for learning to take place (e.g., Paas, Renkl, & Sweller, 2004; Paas, Tuovinen, Tabbers, & van Gerven, 2003). The method of presenting information to learners affects the cognitive load they experience when acquiring this information. Consequently, it is important to measure cognitive load in order to provide empirical evidence that would support a cognitive load-based interpretation of results.

It has been proposed that cognitive load is a concept associated with two aspects of learner performance on a certain task: mental load and mental effort (Paas, Tuovinen, et al., 2003). The cognitive resources that are needed for a certain task make up mental load which is a result of how the task is presented in terms of its information and structure (Paas, 1992). On the other hand, the cognitive resources that are devoted to a certain task make up mental effort (Paas, 1992; Paas, Tuovinen, et al., 2003). Various suggested cognitive load

measurement tools can be classified into objective and subjective measures (Brünken, Plass, & Leutner, 2003; Brünken, Seufert, & Paas, 2010; Kalyuga, 2009b)

5.2 Objective Evaluation of Cognitive Load

Objective measures are based on observing the behaviour or performance of the learner, or associated physiological signs. These measures are not very commonly used when evaluating cognitive load theory within a cognitive load framework. Some examples of objective measures are heart-rate variability (Paas & van Merriënboer, 1994a) and cognitive papillary response (van Gerven, Paas, van Merriënboer, & Schmidt, 2004). The effects of the variation in levels of cognitive load on physiological measurements have been usually studied in laboratory conditions. Brünken et al. (2003) observed an indirect relation between levels of cognitive load and increased heart rate as a result of high stress. Pass and van Merriënboer (1994a) did not find a relation between heart-rate measures and the changes in levels of cognitive load. Moreover, this method interfered with the learning process and was not very convenient in real learning situations.

Another objective but direct option for evaluating cognitive load is the dual-task method. It assumes that less cognitive resources that are involved in a primary task would result in more available resources thus leading to better performance on a secondary task. As a result, quality of performance on a

secondary task could be used as an indicator of cognitive load involved in the primary learning task. Brünken, Steinbacher, Plass, and Leutner (2002) tested the efficiency of a dual-modality learning condition as opposed to a single modality condition when studying the human cardiovascular system (primary task). A visual-monitoring secondary task was presented alongside the primary task. This secondary task involved students acting in response to changes in the colours of letters on a screen. Results showed that their response was faster when the primary task was presented in a dual-modality audiovisual format than when the primary task was presented in a visual-only format. This proved that the modality effect was related to the levels of cognitive load that were brought about by the method of presenting the materials.

Nevertheless, the dual task method has rarely been used in cognitive load research (Paas, Tuovinen, et al., 2003), though the viability of this method has been established (e.g., Brünken et al., 2003; Chandler & Sweller, 1996; Marcus et al., 1996; Sweller, 1988; van Gerven, Paas, van Merriënboer, & Schmidt, 2006). The main reason is that this method interferes with the learning processes involved in the primary task.

Another way of objective evaluation of cognitive load is based on the analysis of learner verbal think-aloud reports collected simultaneously with performing the learning task. Learners should be taught and trained on how to think aloud prior to the learning task. Moreover, certain questioning skills along with careful rubrics for the analyses of the reports are required in order to gather

relevant data. These concurrent verbal reports could help in revealing learners' cognitive processes, but they could hinder the learning process itself. As a result, they might not be efficient when dealing with high cognitive load conditions (Kalyuga, 2009b).

5.3 Subjective Evaluation of Cognitive Load

The subjective measures of cognitive load have been commonly used in many research studies. The reason behind this is the fact that this method is simple to apply, valid, uncomplicated, straightforward, and does not interfere with learning. Moreover, its results are highly correlated with more advanced objective methods of cognitive load evaluation (Ayres, 2006b; Paas, 1992). It suggests that learners are capable of introspecting their cognitive processes and indicating the magnitude of their mental effort on a numerical scale. Paas (1992) used a one-dimensional 9-point symmetrical category rating scale (Likert-type scale) for assessing the learners' mental effort in different phases of learning and performance. Later on and in other studies, different scales were used, such as a 7-point scale (e.g., Kalyuga et al., 2004; Marcus et al., 1996; Tindall-Ford et al., 1997) and a 5-point scale (e.g., Halabi, Tuovinen, & Farley, 2005; Salden et al., 2004). It should be mentioned that in the majority of studies conducted within a cognitive load framework, mental effort was related to instructional tasks (e.g., Marcus et al., 1996) and hardly ever was it related to performance tasks (e.g.,

Paas & van Merriënboer, 1993) or to both (e.g., Kester, Kirschner, & van Merriënboer, 2006; Paas & van Merriënboer, 1994b).

There are a variety of questions that are used to direct learners to scale their mental effort. In some studies learners were required to assess their mental effort while in others they were asked to assess the difficulty or complexity of the tasks presented to them. It was suggested that the more difficult the task was to learners the higher mental effort was put in. Nevertheless, it was observed that when tasks were very difficult to learners, they would not be stimulated to put in the required mental effort, and as a result, the assessment for task complexity might not be representative of the mental effort exerted. In addition to this, the average numerical value on the scale reflected overall cognitive load and did not show the value of any of the separate types (extraneous, intrinsic, or germane) of cognitive load. However, such separate measures could be obtained in controlled studies by changing one factor of cognitive load while keeping the others fixed (e.g., Ayres, 2006; Sweller, 2010).

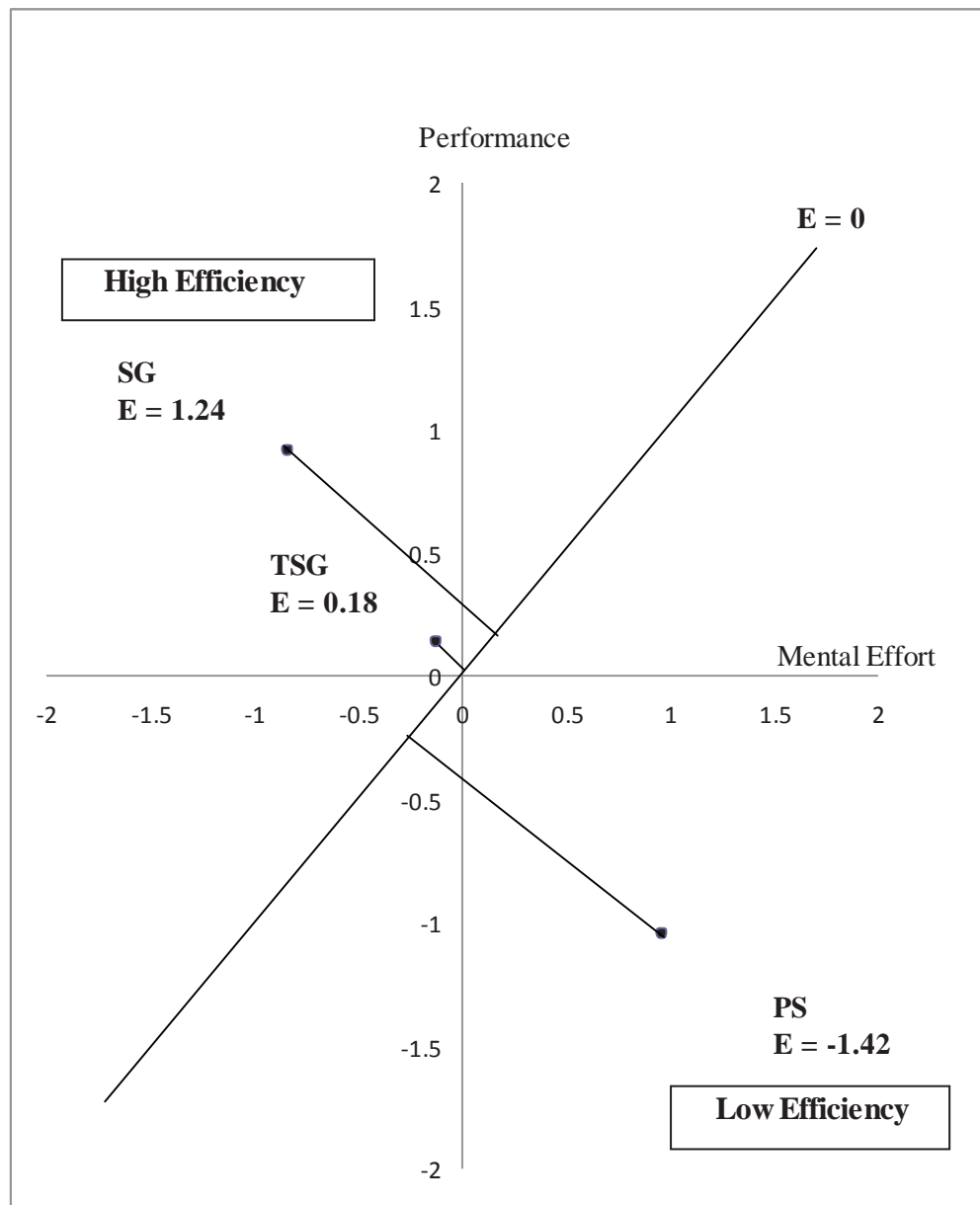
To assess cognitive load in this thesis, subjective ratings of mental effort were used. It is an uncomplicated and useful method which can be employed in classroom settings. A 9-point scale was used to detect differences of ratings of mental effort. The mental effort ratings of the test performance stages were gathered (except for Experiment 1 & 2). The evaluation of task difficulty was used since it was difficult for young learners to grasp the notion of mental effort.

5.4 Measurement of Instructional Efficiency

For efficiency, both the learning result and learners' cognitive load are essential. A learning condition is considered more useful if it has a higher average score on learners' performance than an alternative condition. On the other hand, when two instructional conditions record the same average performance by learners, then they would be both considered at the same levels of usefulness. However, when mental effort is considered, the learning condition that requires a lower level of mental effort is the one with a higher level of instructional efficiency. Accordingly, the learning condition that needs more mental effort is considered to be less efficient than the one that requires learners to exert less mental effort.

Within a cognitive load framework, Paas and van Merriënboer (1993) suggested a method for evaluating instructional efficiency. The formula $E = \frac{(P - R)}{\sqrt{2}}$ in which (E) represents the relative efficiency of the instructional condition, (P) the standardized z-scores for test performance scores, and (R) the ratings of cognitive load related to test tasks. Based on this formula, a learning condition would be more efficient when lower subjective ratings of cognitive load related to performing test tasks are obtained together with higher performance scores. Representing the cognitive load and performance z-scores in a cross of axes makes the graphical analysis of the formula simpler by using $\sqrt{2}$ in the denominator (like the following figure). For any point on the graph, relative

efficiency can be calculated by determining the distance from this point to the line of zero efficiency ($P = R$; or $E = 0$). Over this line is the high efficiency area (higher performance with comparatively lower cognitive load) with $E > 0$. Under this line is the lower efficiency area (lower performance with higher cognitive load) with $E < 0$ (Paas & van Merriënboer, 1993; van Gog & Paas, 2008).



5.5 Objective of the Study

Though research shows that for novices, instructions that depend on worked examples are more effective than those that consist of problem solving, disproportionate guidance for expert learners may reflect negatively on their performance. The purpose of this study is to examine the degree of guidance needed in geometry worked examples as levels of expertise change. The next chapter discusses the four experiments conducted and the results obtained.

Chapter Six

Experiment 1

6.1 Introduction

This experiment was designed to investigate the effects of varying the amount of guidance provided in geometry worked examples. Three conditions were used: Theorem & Step Guidance, Step Guidance and a Problem Solving condition. The subject area was finding angles that are held between parallel lines. In the Step Guidance worked examples, learners were told which angle they had to find for each step but not told the theorem they had to use to find the angle. Learners were required to ‘complete’ that aspect of the move themselves. In the Theorem & Step Guidance worked examples, learners were guided in each step and given the theorem behind each move. It was hypothesized that for students who knew the relevant theorems, Step Guidance worked examples that described the procedure required to solve a problem could be more readily incorporated into existing knowledge in long-term memory compared to Problem Solving or a Theorem & Step Guidance approach. In the case of a Problem Solving approach, the use of a means-ends strategy is likely to impose an extraneous cognitive load. In the case of a Theorem & Step Guidance approach, an explanation of the

relevant theorem is likely to impose an extraneous cognitive load due to the redundancy effect.

6.2 Method

6.2.1 Participants

The participants were 45 Year 8 students attending a private school in Beirut, Lebanon. The students were aged between 13 and 14 years, and were all at the same Mathematical level, as determined by their class teachers. The grading of students by class teachers according to mathematical skills is standard practice and is part of the school curriculum in Lebanese schools. These students had not been exposed previously to finding angles using properties of parallel lines. Students were randomly assigned into three equivalent groups of 15.

6.2.2 Material

For the acquisition phase, three theorems were selected from the parallel lines topic that forms part of the mathematics curriculum material suitable for students at this stage. The three theorems were related to finding the angles associated with parallel lines. The selected angles were alternate angles, corresponding angles and co-interior angles; these angles were formed when parallel lines are presented. The figures forming these parallel lines were not previously sighted by any students involved in the experiment. The same figures were then reproduced

with steps and full solutions including the theorems used or with steps to follow with no theorem indicated, or with no solution at all. The reproduced figures were identical in size, including angle size, and retained the same angle name, for each figure category (see Figures 1 & 2).

The test material included finding angles based on two parallel lines, with 4 problems similar to the acquisition problems, with exactly the same figure as in the acquisition problem, but with a different measure of the given angle, and 3 transfer problems in which the given angles, the angles to be found, and the transversals were in different positions than in the corresponding acquisition problems. The test problems can be found in Appendix A.

6.2.3 Procedure

The experiment consisted of a learning phase (25 minutes) and a test phase (35 minutes). It was conducted over one school session, with each child tested individually. Two days prior to the experiment, a revision session occurred (45 minutes) to review the prerequisite material needed to conduct the experiment. The session was similar for all groups and was given to all the students in order to remind them of the geometric terminology used in the experiment (bisector of an angle, supplementary angles add up to 180° , complimentary angles add up to 90° , parallel lines, transversal, alternate angles, corresponding angles, co-interior angles). Participants were informed that they would be given sets of geometric figures related to angles formed between parallel lines and a transversal.

Participants were also advised that they would be allowed the same fixed time for learning. The participants were then randomly assigned to one of the three instructional groups.

During the *learning phase*, students in each group were presented with three pairs of problems. In the Theorem & Step Guidance condition, each pair consisted of a worked example followed by an identical problem to solve with only a change in the measure of the given angle. For example, students were shown a Theorem & Step Guidance example indicating steps to follow in order to find the measure of an angle formed by one pair of parallel lines that are cut by a transversal, with a related angle that measured 130° . Additionally, students were given the reason/theorem used behind each step. Then they were asked to solve an identical problem where the related angle had a measure of 140° . None of the worked steps were indicated on the paired problem. All problem pairs were similar in their content. The set of problems used in the Theorem & Step Guidance condition can be found in Appendix B.

An example of a Theorem & Step Guidance worked example is presented in Figure 1.

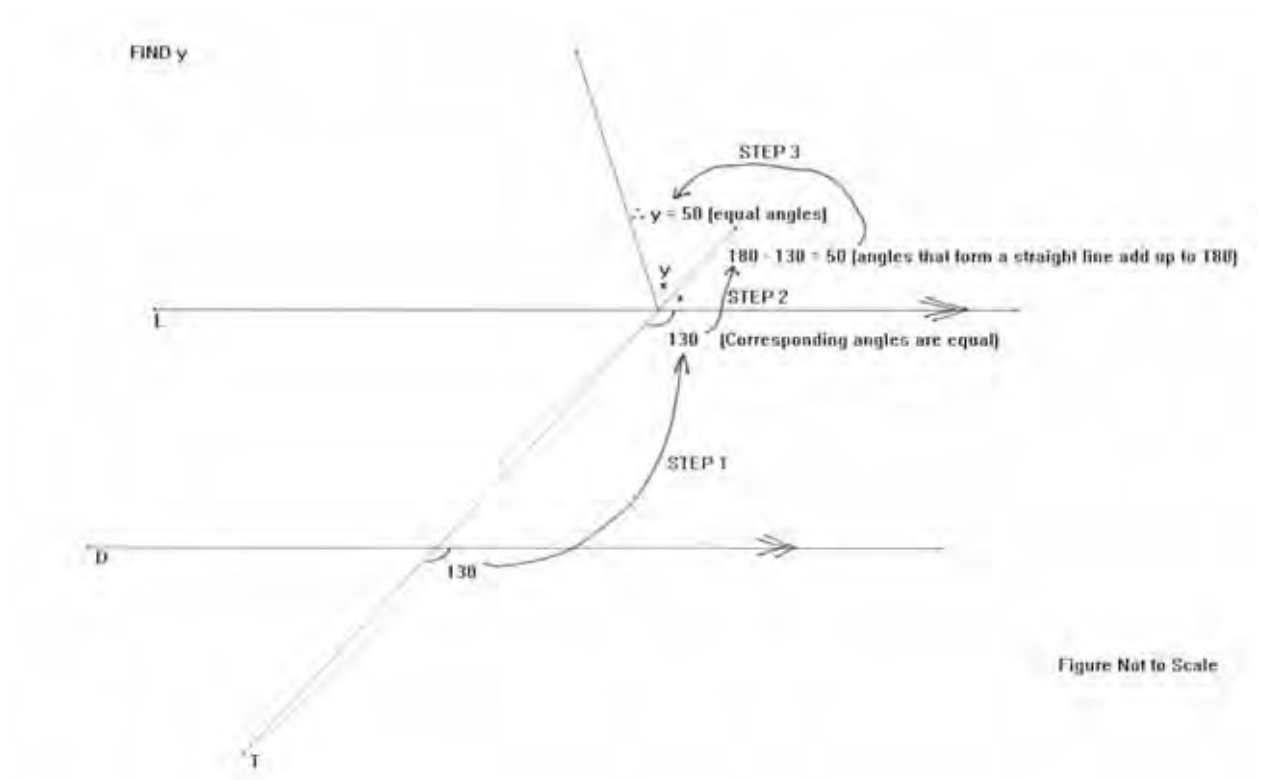


Figure 1. A worked example presented to the Theorem & Step Guidance group.

In the Step Guidance condition, learners were given the same figure and angle measure (130°) and were provided with the guidance for each step but not told the theorem they had to use. Learners were required to ‘complete’ that aspect of the move themselves. Then, as was the case for the Theorem & Step Guidance group, they were asked to solve the paired problem where the related angle has a measure of 140° . The set of problems used in the Step Guidance condition can be found in Appendix C.

An example of the Step Guidance condition is presented in Figure 2.

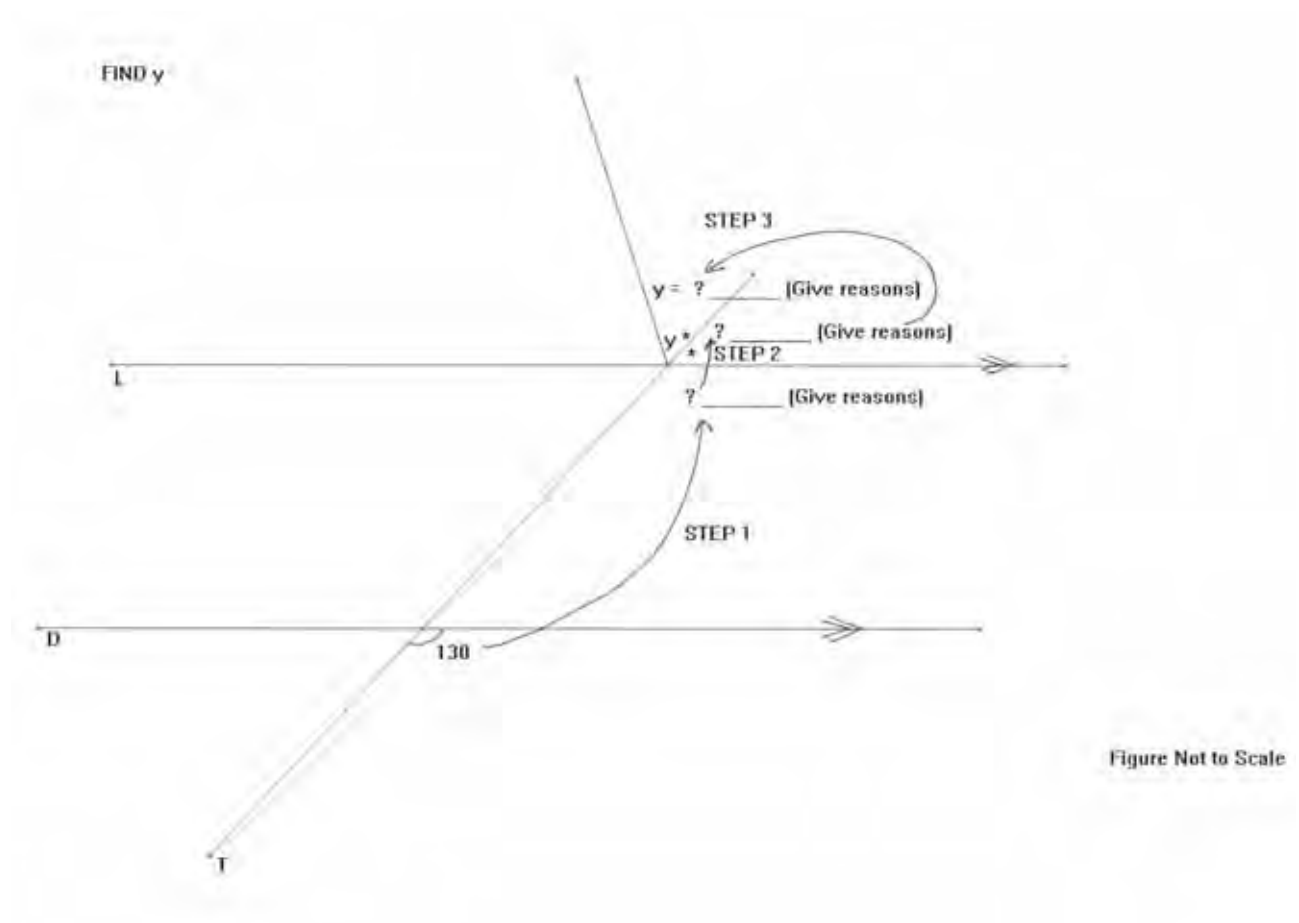


Figure 2. A worked example presented to the Step Guidance group.

Participants in the problem solving condition were just given the figure showing the parallel lines and the angle that they needed to find without any solution information. Then they were asked to solve the same pair problem as the other two groups. The set of problems used in the problem solving condition can be found in Appendix D.

Each group studied a set of three problem pairs with each problem immediately followed by the next problem presented on a single sheet of paper with sufficient space to write a solution. Students were asked to work on the first

problem that included finding the size of a certain angle under one of the three conditions (Theorem & Step Guidance or Step Guidance or Problem Solving) for four minutes and then solve the paired problem. The time needed to solve the paired problem was measured up to a maximum of four minutes. Students were stopped if they had not solved the paired problem within four minutes. Students who finished the paired problem in less than four minutes were asked to review their work and wait till the time expired to make sure that all students took the same time for each problem. If students gave an incorrect solution they were asked to try again within the four-minute time limit. This procedure was followed for all problem pairs for all groups. The initial acquisition problem was available to students while they were solving the subsequent paired problem.

A *test phase* immediately followed the learning phase. It consisted of four problems similar to the acquisition phase problems and three transfer problems. All testing tasks were administered on an individual basis.

Since each problem had three solution steps, the test score was determined by allocating up to 3 marks for each test problem since there were 4 problems, the highest score that participant could achieve in the similar test was 12 points. Half a mark was allocated for a correct solution step and half a mark was allocated for a correct explanation. Three marks were allocated for a correct task solution step with correct explanations. Two marks were allocated if one error occurred in a solution step and explanation, and one mark was allocated if two errors occurred in solution steps and explanations. The transfer test score was determined using the same marking system as the similar test problems, providing a score out of 9

for each participant since there were 3 transfer problems, the highest score that participant could achieve was 9 points.

Each problem was provided on a separate sheet of paper. Participants were asked to provide written reasoning about their solutions. They were asked to work as rapidly and as accurately as possible. Students who finished the test in less than the allocated time (35 min), were asked to review their work and wait till the time expired to make sure that all students took the same time for each task. No feedback was given to participants until the whole experiment was completed. The sheets used during the learning phase were not available to participants during the test phase.

6.3 Results

Variables

The dependent variables under analysis were the similar and transfer test scores. The independent variables were instructional designs (Problem Solving condition, Step Guidance condition and Theorem & Step Guidance condition). Means and standard deviation are provided in Table 2.

Similar test results

A one-way analysis of variance (ANOVA) indicated a significant difference between the experimental conditions on the similar test scores, $F(2, 42) = 8.20$,

$MSE = 2.66$, $p = .001$, $\eta_p^2 = 0.28$. According to a Tukey HSD post-hoc test, the Step Guidance condition significantly outperformed the Problem Solving condition, $p < .001$. The Step Guidance condition marginally outperformed the Theorem & Step Guidance condition, $p = .06$. There was no significant difference between the Problem Solving condition and Theorem & Step Guidance condition, $p = .23$. (A 0.20 effect size was considered small; 0.40 medium, and 0.60 large. These classifications, based on Hattie, 2009, were used through this thesis.).

Transfer test results

An ANOVA indicated a significant difference between the experimental conditions on the transfer test scores, $F(2, 42) = 21.07$, $MSE = 1.81$, $p < .001$, $\eta_p^2 = 0.50$. According to a Tukey HSD post-hoc test, the Step Guidance condition significantly outperformed the Problem Solving condition, $p < .001$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition, $p < .001$. The Step Guidance condition marginally outperformed the Theorem & Step Guidance condition, $p = .09$.

Table 2 Means and Standard Deviations for the Similar and Transfer Test Scores for Different Instructional Conditions (Experiment1)

Instructional Condition	<i>n</i> = 45		
	Theorem & Step Guidance <i>n</i> = 15	Step Guidance <i>n</i> = 15	Problem Solving <i>n</i> = 15
Total Scores for Similar Test			
<i>M</i>	8.67	10.07	7.67
<i>SD</i>	2.13	0.88	1.63
Total Scores for Transfer Test			
<i>M</i>	4.00	5.07	1.93
<i>SD</i>	1.65	0.88	1.39

Note: The maximum score was 12 for the similar test and 9 for the transfer tests.

6.4 Discussion

The results of Experiment 1 showed a significant advantage of the Theorem & Step Guidance condition over the Problem Solving condition on transfer problems but not on similar problems while the Step Guidance condition was more effective than the Problem Solving condition on both similar and transfer problems. There was no significant difference between the Theorem & Step Guidance condition and the Step Guidance condition on both the similar and transfer test, but the means favoured the Step Guidance condition on both tests.

The results of the first experiment revealed that learning to solve problems primarily consists of learning to recognise problem states and their associated moves. It was hypothesized that the performance of the Step Guidance condition would be higher than the Theorem & Step Guidance condition, providing an example of the redundancy effect due to the presence of unnecessary information in the Theorem & Step Guidance condition that imposes an extraneous cognitive load. It was also hypothesized that a normal worked example effect (the Theorem & Step Guidance and the Step Guidance conditions superior to the Problem Solving condition) will be revealed. The results supported this hypothesis. The next experiment was conducted to test whether the expertise reversal effect might influence these findings.

Chapter Seven

Experiment 2

7.1 Introduction

Experiment 1 compared the effectiveness of the Step Guidance, Theorem & Step Guidance, and the Problem Solving conditions. The results of the experiment supported the effectiveness of both the Step Guidance and the Theorem & Step Guidance conditions over the Problem Solving condition. The results suggested that under at least some conditions, learning to recognise a problem state and its associated moves and learning the relevant theorems are important.

The purpose of Experiment 2 was to examine the expertise reversal effect. It was hypothesized that more expert learners might perform better under a Step Guidance condition as they need to learn to identify problem states and do not need to learn the relevant theorem at that stage because it has already been learned. Emphasising the theorem might impose an unnecessary extraneous cognitive load. It also can be hypothesised that novices might perform best in the Theorem & Step Guidance condition, as they may require an emphasis on the theorem as well as the solution steps.

7.2 Method

7.2.1 Participants

Ninety students from Year 8 of a Lebanese private school participated in this study. The students were aged between 13 and 14 years. They were divided into two groups: Low achievers and high achievers. The students were chosen according to their school Mathematics level. The high achievers were students in the highest level of Mathematics Year 8 class at the school and the low achievers were students who attended the lowest level of Year 8 Mathematics. Students at each ability level were divided randomly into three groups of 15. These students had not been exposed previously to finding angles using the properties of parallel lines. The researcher depended on the school evaluation of each student's level of achievement in Mathematics, and no prior knowledge test was conducted to test the participants' familiarity with the parallel lines.

7.2.2 Materials and procedure

The same materials and procedure were used as in Experiment 1 with only one change in the structure of the Step Guidance condition in order to decrease the possibility of a redundancy effect that might be generated by the diagram. The Step Guidance problems of Experiment 1 were presented in Figure 2 and the Step Guidance problems of Experiment 2 were presented as in Figure 3. The new sets of Step Guidance condition problems as used in Experiment 2 are provided in Appendix E. When comparing Figures 2 and 3, it may be seen that the

requirement for students to provide the relevant reason or theorem when making a step was eliminated in Experiment 2. This was done because Step Guidance should emphasise the relevant steps and de-emphasise the theorems used to make each step in order to reduce the redundancy effect. In addition, the arrows were deleted for each step and the word Step was written closer to its associated angle in order to reduce a possible split attention effect. As for the test phase, it was also the same as in Experiment 1 except for a modification in the marking: 1 mark was allocated for a solution step (rather than half) with no marks for an explanation as students were not required to give reasons or explanations. The total scores of the similar test were out of 12 (the highest score participant could achieve was 12, since there were 4 similar test problems) and the total scores of the transfer test were out of 9 (the highest score participant could achieve was 9, since there were 3 transfer test problems). In all other respects, the materials and procedure was identical for the two experiments.

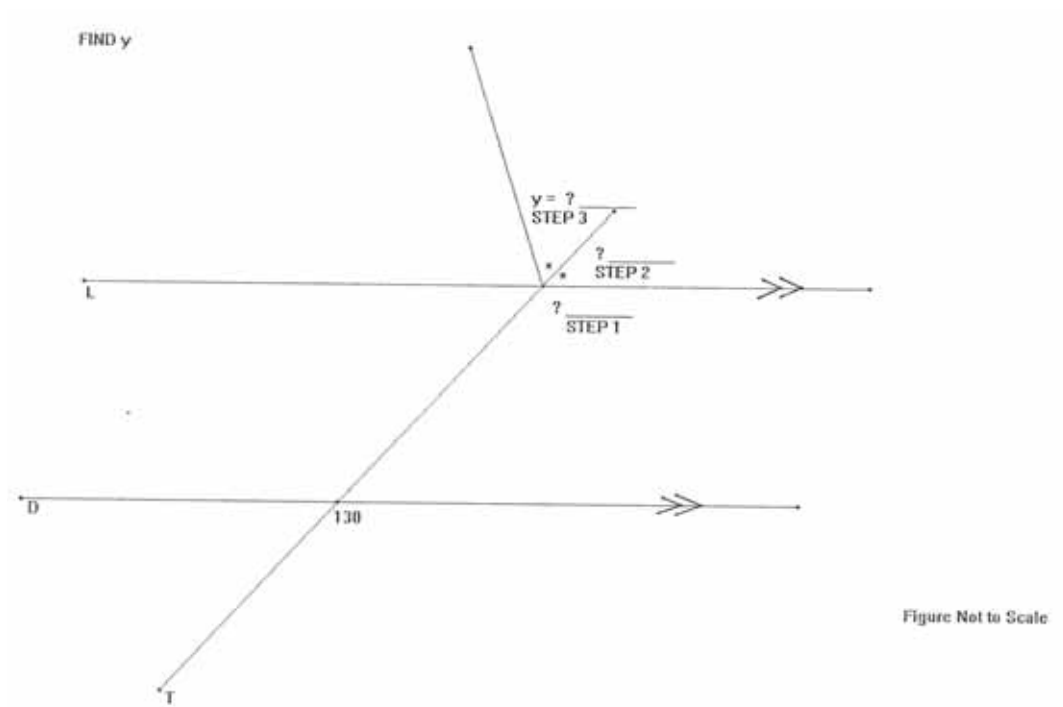


Figure 3. A worked example presented to the Step Guidance group for Experiment 2.

7.3 Results

Variables

The dependent variables under analysis were the similar and transfer test scores. Means and standard deviation are provided in Table 3. The independent variables were the instructional design (Theorem & Step Guidance, Step Guidance, or Problem Solving) and the level of learner expertise (lower and higher levels).

Similar test results

A two-way analysis of variance (learner level of expertise X instructional conditions) for the scores of the similar test indicated a significant main effect for the three instructional conditions, $F(2, 84) = 11.07$, $MSE = 3.63$, $p < .001$, $\eta_p^2 = 0.21$. There was no significant main effect for the level of expertise, $F(1, 84) = 1.77$, $MSE = 3.63$, $p = .19$, $\eta_p^2 = 0.02$. The interaction between the three instructional conditions and the learners' expertise levels was not significant, $F(2, 84) = 0.73$, $MSE = 3.63$, $p = .49$, $\eta_p^2 = 0.02$, (see Figure 4).

Table 3 Means and Standard Deviations for the Similar and Transfer Test Scores for Different Instructional Conditions and Levels Learner Prior Knowledge (Experiment 2)

Expertise Level	Lower learners <i>n</i> = 45			Higher learner <i>n</i> = 45		
	Theorem & Step Guidance <i>n</i> = 15	Step Guidance <i>n</i> = 15	Problem Solving <i>n</i> = 15	Theorem & Step Guidance <i>n</i> = 15	Step Guidance <i>n</i> = 15	Problem Solving <i>n</i> = 15
Total Scores for Similar Test						
<i>M</i>	8.47	9.53	6.67	8.80	9.60	7.87
<i>SD</i>	1.64	1.46	1.92	1.37	1.99	2.72
Total Scores for Transfer Test						
<i>M</i>	3.00	4.00	1.73	3.40	4.33	1.80
<i>SD</i>	1.25	1.51	1.03	1.35	1.45	1.52

Note: The maximum score was 12 for the similar test and 9 for the transfer tests.

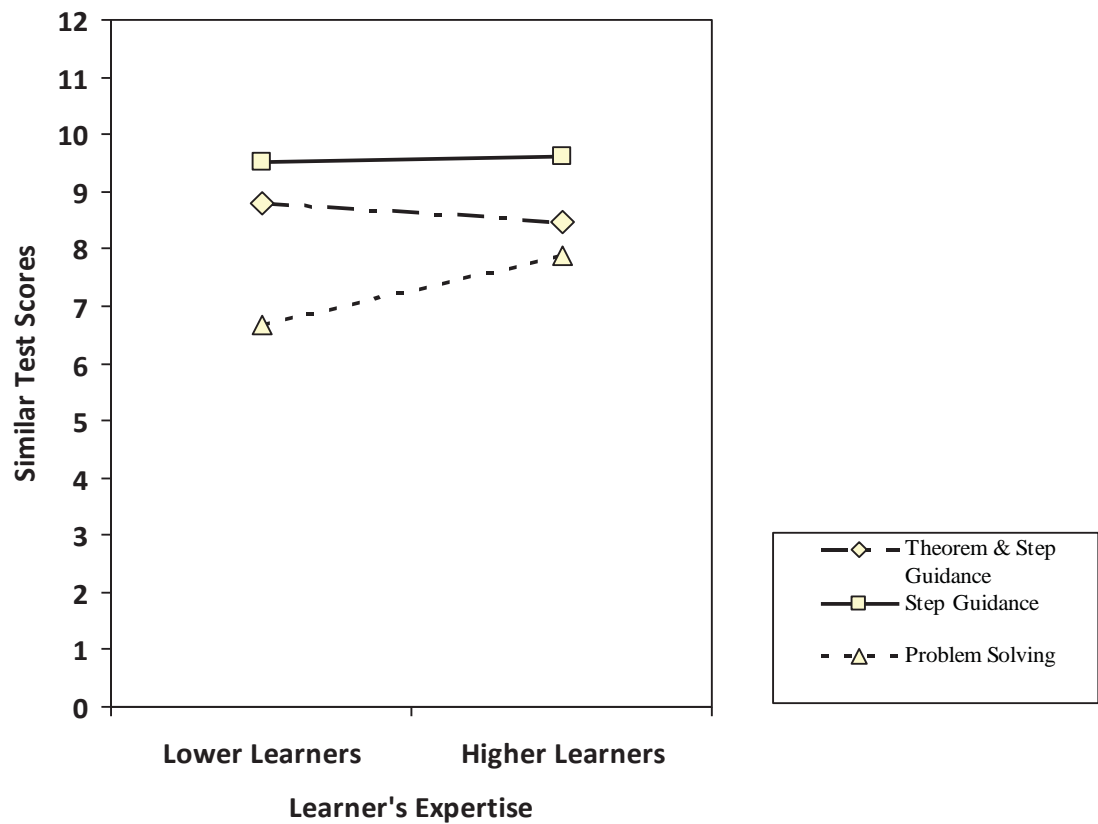


Figure 4. Interaction between instructional format and level of learner expertise for overall similar test scores of Experiment 2.

According to a Tukey HSD post-hoc test, the Step guidance condition significantly outperformed the Problem solving condition, $p < .001$. The Theorem & Step Guidance condition significantly outperformed the Problem solving condition, $p < .05$. There was no significant difference between the Step guidance and the Theorem & Step Guidance conditions, $p = .15$.

Transfer results

A two-way analysis of variance (learner level of expertise X instructional conditions) for the transfer test scores indicated a significant main effect for the three instructional conditions, $F(2, 84) = 23.52$, $MSE = 1.86$, $p < .001$, $\eta_p^2 = 0.36$. There was a non-significant main effect for the level of expertise, $F(1, 84) = 0.86$, $MSE = 1.86$, $p = .36$, $\eta_p^2 = 0.01$. There also was a non-significant interaction between the three instructional conditions and the learners' expertise levels, $F(2, 84) = 0.13$, $MSE = 1.86$, $p = .88$, $\eta_p^2 = 0$, (see Figure 5).

According to a Tukey HSD post-hoc test, the Step Guidance condition significantly outperformed the Problem Solving condition, $p < .001$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition, $p < .001$. The Step Guidance condition significantly outperformed the Theorem & Step Guidance condition, $p < .05$.

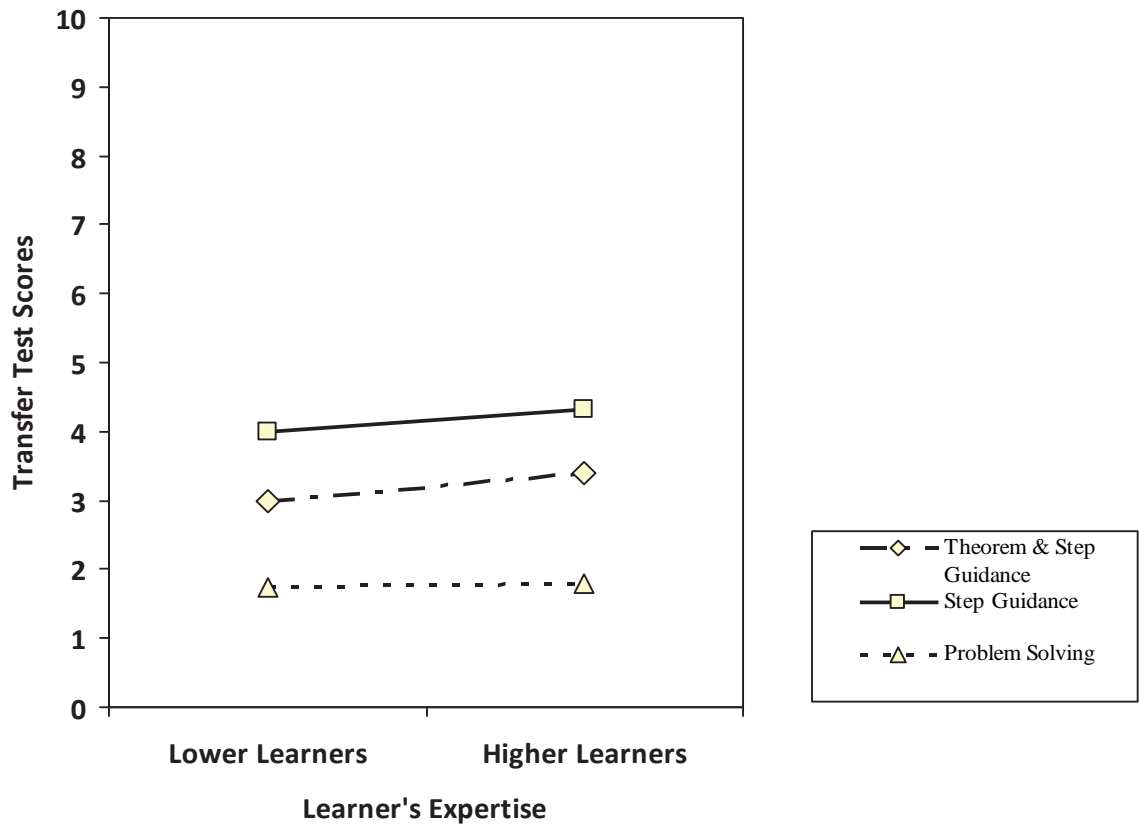


Figure 5. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 2.

7.4 Discussion

The results revealed significant differences between the experimental groups on the similar and transfer scores, while no significant differences were obtained between levels of learner expertise. The Step Guidance condition significantly outperformed the Problem Solving condition on both similar and

transfer problems. Moreover, the Step Guidance condition significantly outperformed the Theorem & Step Guidance condition on transfer problems with no difference on this measure using similar problems as there was a ceiling effect on one of the similar problems. 72 of the 90 students attained a full mark on this question, thus it was eliminated in the next experiment.

No significant difference was obtained on the expertise factor. A possible reason is that students were chosen from the same maths year but divided into low achievers and high achievers based only on the school classification of students who were placed in either a lower or higher maths class. No prior knowledge test was carried out to obtain two groups of learners with distinctively different levels of prior knowledge. This reason might explain the absence of differences between the two levels of maths ability on the dependent variables.

A second experiment with a larger N again indicated consistency in the results. There was a significant advantage of both Step Guidance and Theorem & Step Guidance conditions over the Problem Solving condition, this time on both the similar test and transfer problems. Furthermore, in this experiment, the Step Guidance condition was superior to the Theorem & Step Guidance condition on the transfer although not on the similar problems.

It was hypothesized that the Step Guidance condition might outperform the Theorem & Step Guidance condition providing an example of the redundancy effect due to the presence of the theorems that impose an unnecessary extraneous cognitive load. In the case of the transfer problems, the results supported this hypothesis.

It was also hypothesized that an interaction might occur revealing an expertise reversal effect, but due to the lack of a significant difference between different knowledge levels of the students used, an expertise reversal effect failed to be obtained. Based on these results, the next experiment was designed to test the expertise reversal effect using two different years or grades rather than different levels of math classes in the same year. Students in Year 8 and 9 were used in the experiment. While Year 8 students had never been exposed to the properties of parallel lines held by a transversal, Year 9 students had learnt this topic in school. It was expected that the increased difference in levels of expertise in Experiment 3 compared to Experiment 2 would yield an increased difference in knowledge levels leading to an expertise reversal effect.

Chapter Eight

Experiment 3

8.1 Introduction

In Experiment 2, a geometry problem was presented using Theorem & Step Guidance, Step Guidance and Problem Solving conditions. The results revealed that presenting a geometry worked example using Step Guidance proved to be a more efficient instructional method than presenting it in Theorem & Step Guidance or Problem Solving conditions. Furthermore, it was demonstrated that students performed significantly better on transfer problems, and marginally better on similar problems, if the learning phase was presented with a Step Guidance condition rather than being presented with Theorem & Step Guidance or Problem Solving conditions. Nevertheless, as indicated in the previous chapter, Experiment 2 did not yield an expertise reversal effect, possibly because there was an insufficient gap between levels of expertise. Thus, Experiment 3 was designed to test this possibility.

The purpose of Experiment 3 was to investigate if the redundancy effect would apply to more knowledgeable learners with greater mathematical skills and exposure to properties of parallel lines. Participants were two groups of students

from Year 8 and Year 9. In this experiment, familiarity with parallel lines was assumed to be an important factor determining the knowledge structures available in a learner's long-term memory and, consequently, the effectiveness of a specific instructional technique. Therefore, possible interactions between alternative instructional techniques and levels of learner expertise using parallel lines were investigated. It was hypothesized that Year 8 students will perform better using the Theorem & Step Guidance condition and Year 9 students will perform better using the Step Guidance condition. As Year 8 students had not been exposed to the properties of parallel lines, having the theorem associated with each step might improve their understanding of the problem, whereas the presence of the theorem for Year 9 students might be redundant, thus imposing an extraneous cognitive load that reduces performance.

8.2 Method

8.2.1 Participants

The participants were 180 Year 8 students and 180 Year 9 students attending a private school in North Sydney, Australia. Year 8 students were aged between 13 and 14 years, Year 9 students were aged between 14 and 15 years. The students from each year were at the same Mathematical level, as determined by their class teachers. Students were chosen from the intermediate ability level of each class. The grading of students by class teachers according to mathematical skills is

standard practice and is part of the school curriculum in Sydney schools. Year 8 students had not been exposed previously to finding angles using properties of parallel lines, but Year 9 students had learnt the properties of parallel lines in school.

Students were randomly assigned into three equivalent groups of 60 with one group guided in each step including the angle and the theorem behind each move (Theorem & Step Guidance), another group presented with the angle they had to find for each step but not the theorem they had to use to find the angle (Step Guidance), and the third group learned under Problem Solving conditions.

8.2.2 Material and Procedure

The same material and procedure that was used in Experiment 2 was used again in this experiment, except that there was a ceiling effect in one of the similar test question in Experiments 1 and 2, thus that problem was eliminated in Experiment 3. The new sets of questions for Experiment 3 are provided in Appendix F. The marking system was also the same as in Experiment 2, except the total similar score was out of 9 rather than out of 12 (the highest score that participant could achieve was 9, since there were 3 similar test problems) as one problem was eliminated because it was too easy. The total transfer score was out of 9 (the highest score that participant could achieve was 9, since there were 3 transfer test problems) as in the previous experiment. The time for the learning phase remained the same as in Experiment 2 (25 minutes).

8.2.3 Rating of cognitive load associated with task performance

Immediately after the acquisition phase, each participant was asked to estimate how easy or difficult it was to learn the material and answer the questions on a nine-point scale by placing an “X” in the space provided related to one of the nine numbers (1 being extremely easy, 9 being extremely difficult, (see Appendix G). The scores obtained from this rating scale were used as indicators of cognitive load associated with the learning tasks. These rating were also used for calculating relative instructional efficiency of instructional conditions.

8.3 Results

Variables

The dependent variables under analysis were similar and transfer test scores (Means and standard deviations are provided in Table 4), subjective ratings of cognitive load (Means and standard deviations are provided in Table 5), and relative instructional efficiency measures, (Means and standard deviations are provided in Table 6). The independent variables were instructional design and the level of learner expertise.

Similar test results

A 2 (Learner Expertise: Year 8 vs. Year 9 students) x 3 (Instructional Design: (Theorem & Step Guidance vs. Step Guidance, vs. Problem Solving) analysis of variance for the scores of the similar test indicated a significant main effect for instructional condition, $F(2, 354) = 233.00$, $MSE = 0.95$, $p < .001$, $\eta_p^2 = 0.57$. There was a significant main effect for the level of expertise, $F(1, 354) = 33.40$, $MSE = 0.95$, $p < .001$, $\eta_p^2 = 0.09$. There also was a significant interaction between instructional condition and learner expertise, $F(2, 354) = 3.55$, $MSE = 0.95$, $p = .03$, $\eta_p^2 = 0.02$, (see Figure 6).

Table 4 Means and Standard Deviations for the Similar and Transfer Test Scores Different Instructional conditions and Levels of Learner Prior Knowledge (Experiment 3)

Expertise Level		Year 8 <i>n</i> =180		Year 9 <i>n</i> =180		
Instructional Condition	Theorem & Step Guidance	Step Guidance	Problem Solving	Theorem & Step Guidance	Step Guidance	Problem Solving
	<i>n</i> = 60	<i>n</i> = 60	<i>n</i> = 60	<i>n</i> = 60	<i>n</i> = 60	<i>n</i> = 60
Total Scores for Similar Test						
<i>M</i>	5.73	6.47	3.97	6.02	7.42	4.52
<i>SD</i>	1.10	0.93	1.09	0.93	0.81	0.97
Total Scores for Transfer Test						
<i>M</i>	4.28	5.23	2.30	4.55	6.08	2.72
<i>SD</i>	1.12	0.87	0.79	1.00	0.81	0.76

Note: The maximum score was 9 for both the similar and transfer tests.

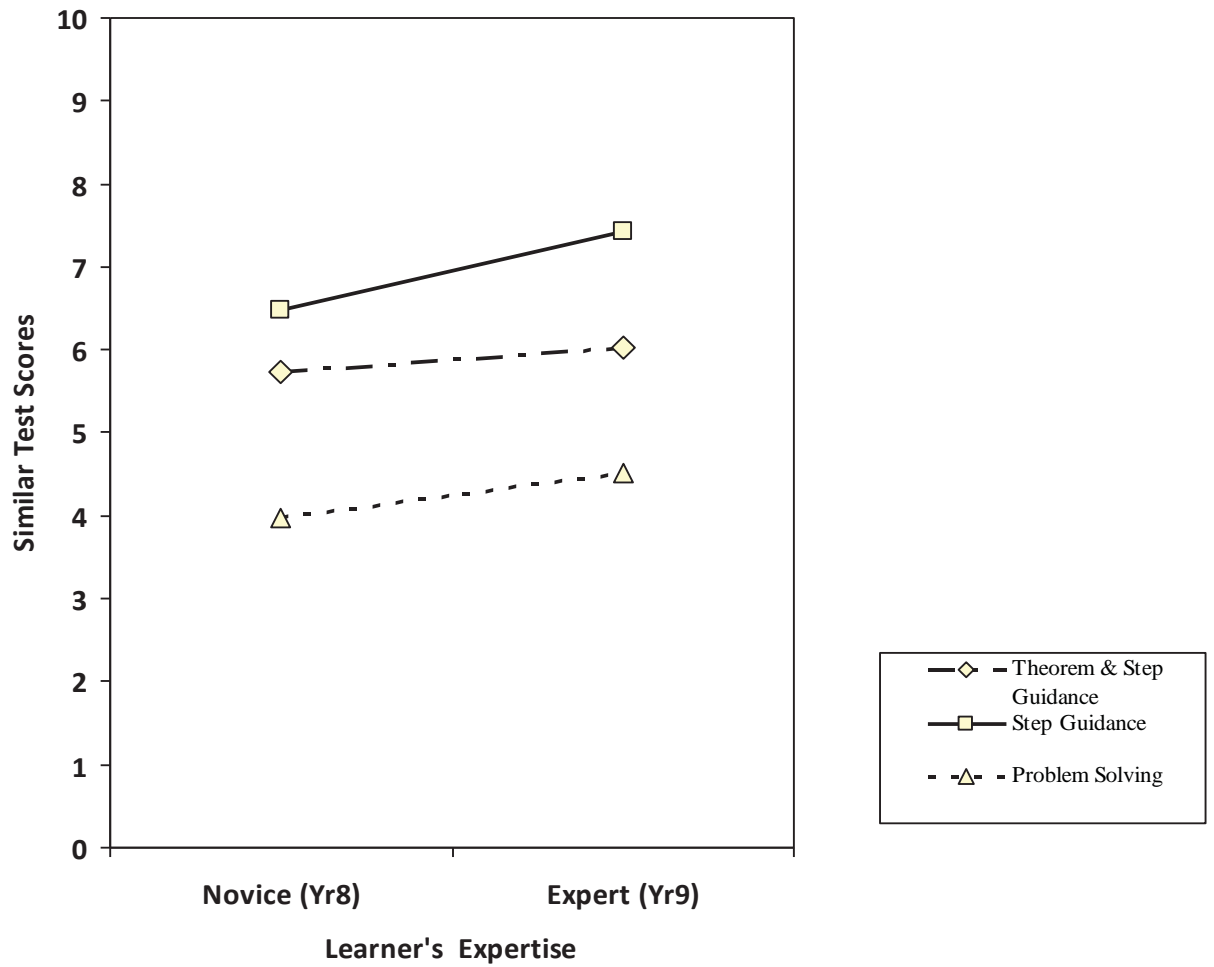


Figure 6. Interaction between instructional format and level of learner expertise for overall similar test scores of Experiment 3.

Following the significant interaction, a simple effects test for the more expert learners (Year 9), demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions on the similar test, $F(1, 118) = 77.47$, $MSE = 0.76$, $p < .001$, $\eta_p^2 = 0.40$. The Step Guidance condition

significantly outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 75.17$, $MSE = 0.90$, $p < .001$, $\eta_p^2 = 0.39$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition.

For the novice learners (Year 8), there was a significant difference between the Step Guidance and the Theorem & Step Guidance conditions on the similar test, $F(1, 118) = 15.52$, $MSE = 1.04$, $p < .001$, $\eta_p^2 = 0.12$. The Step Guidance condition significantly outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 77.99$, $MSE = 1.20$, $p < .001$, $\eta_p^2 = 0.40$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition.

Since the pattern of significance for the simple effects tests was identical for novices and experts, that pattern cannot be used to indicate why a significant interaction was obtained. Instead, the relative effect sizes will be used.

The effect of the Step Guidance over the Theorem & Step Guidance condition for novices was $\eta_p^2 = .12$, and for experts was $\eta_p^2 = .40$. Since the effect size for experts was larger than the effect size for novices, this difference is likely to have contributed to the significant interaction. In contrast, the effect size of the Theorem & Step Guidance condition over the Problem Solving condition for novices was $\eta_p^2 = .40$, and for experts was $\eta_p^2 = .39$. They both have a similar

effect size and so these effect size results did not contribute much to the explanation of the significant interaction. Accordingly, it can be seen that the significant interaction was largely caused by the larger advantage of the Step Guidance condition over the Theorem & Step guidance condition for experts compared to novices.

Transfer test results

A two-way analysis of variance (learner level of expertise X instructional conditions) for the scores of the transfer test indicated a significant main effect for the three instructional conditions, $F(2, 354) = 375.09$, $MSE = 0.81$, $p < .001$, $\eta_p^2 = 0.68$. There was a significant main effect for the level of expertise, $F(1, 354) = 29.19$, $MSE = 0.81$, $p < .001$, $\eta_p^2 = 0.08$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2, 354) = 3.42$, $MSE = 0.81$, $p = .03$, $\eta_p^2 = 0.02$, (see Figure 7).

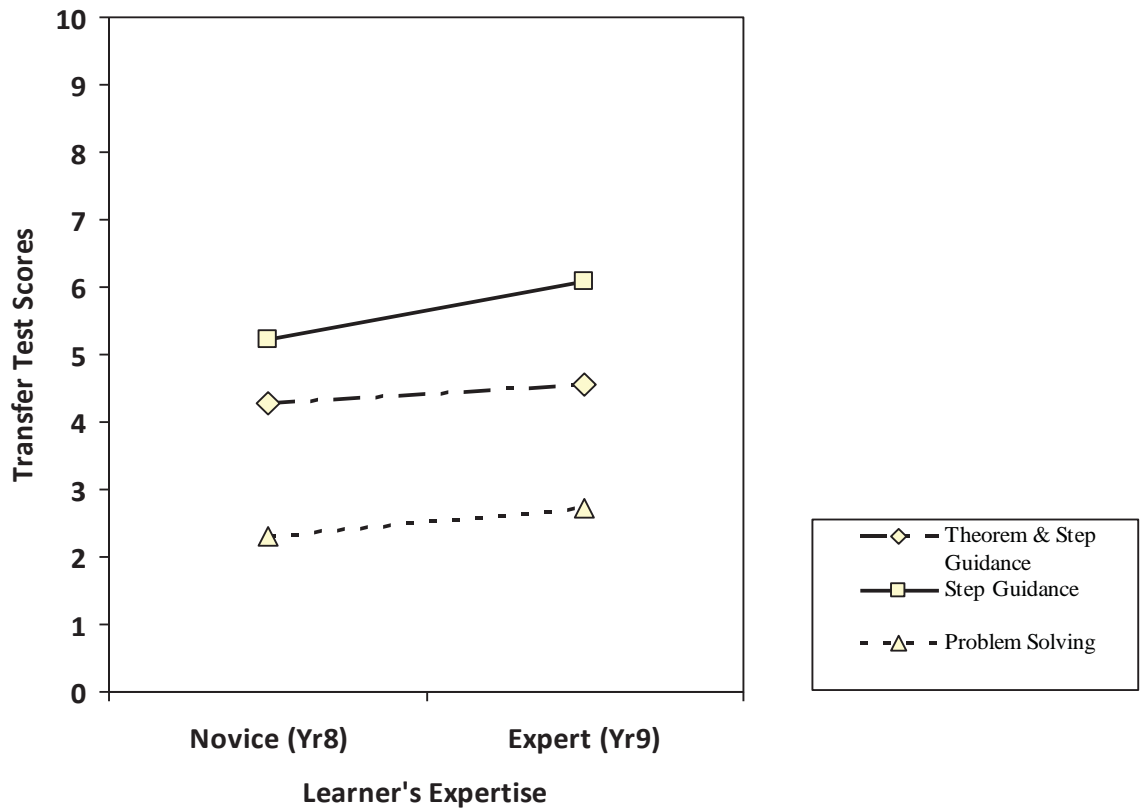


Figure 7. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 3.

Following the significant interaction, simple effects test for the expert learners (Year 9) demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions for the transfer test, $F(1, 118) = 85.42$, $MSE = 0.83$, $p < .001$, $\eta_p^2 = 0.42$. The Step Guidance condition significantly outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the

Problem Solving condition, $F(1, 118) = 127.89$, $MSE = 0.79$, $p < .001$, $\eta_p^2 = 0.52$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition.

For the novice learners (Year 8), there was a significant difference between the Step Guidance, and the Theorem & Step Guidance groups for the transfer test, $F(1, 118) = 27.33$, $MSE = 0.99$, $p < .001$, $\eta_p^2 = 0.19$. The Step Guidance condition significantly outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 128.01$, $MSE = 0.92$, $p < .001$, $\eta_p^2 = 0.52$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition.

Similarly to the similar test, the pattern of significance for the simple effects tests was identical for novices and experts and so that pattern cannot be used to indicate why a significant interaction was obtained. Instead, the relative effect sizes will be used.

The effect of the Step Guidance over the Theorem & Step Guidance condition for novices was $\eta_p^2 = .19$, and for experts was $\eta_p^2 = .42$. Since the effect size for experts was larger than the effect size for novices, this might explain the reason behind the significant interaction. As for the effect of the Theorem & Step Guidance condition over the Problem Solving condition for novices was $\eta_p^2 = .52$, and for experts was $\eta_p^2 = .52$. They both have identical

effect sizes and so these effect size results do not contribute to an explanation of the significant interaction. Rather, the significant interaction was caused by the larger advantage of the Step Guidance condition over the Theorem & Step guidance condition for experts compared to novices.

Ratings of cognitive load

A two-way analysis of variance (learner level of expertise X instructional conditions) for the ratings of cognitive load, indicated a significant main effect for the three instructional conditions, $F(2, 354) = 271.11$, $MSE = 1.42$, $p < .001$, $\eta_p^2 = 0.61$. There was a significant main effect for the level of expertise, $F(1, 354) = 39.41$, $MSE = 1.42$, $p < .001$, $\eta_p^2 = 0.10$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels, $F(2, 354) = 23.81$, $MSE = 1.42$, $p < .001$, $\eta_p^2 = 0.12$ (See Figure 8). Means and standard deviations for different instructional conditions and levels of prior knowledge are provided in Table 5.

Following the significant interaction a simple effects test for the expert learners (Year 9), demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions for the ratings of cognitive load, $F(1, 118) = 93.09$, $MSE = 1.61$, $p < .001$, $\eta_p^2 = 0.44$. The Step Guidance condition significantly demonstrated a lower cognitive load rating than the Theorem & Step Guidance condition. There also was a significant difference

between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 94.94$, $MSE = 1.70$, $p < .001$, $\eta_p^2 = 0.45$. The Theorem & Step Guidance condition significantly demonstrated a lower cognitive load rating than the Problem Solving condition.

For the novice learners (Year 8), an F-test demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions for the rating of cognitive load, $F(1, 118) = 5.61$, $MSE = 1.82$, $p = .02$, $\eta_p^2 = 0.05$. The Step Guidance condition significantly demonstrated a lower cognitive load rating than the Theorem & Step Guidance condition. There also was a significant difference between the Step Guidance and the Problem Solving condition, $F(1, 118) = 68.40$, $MSE = 1.73$, $p < .001$, $\eta_p^2 = 0.37$. The Theorem & Step Guidance condition significantly demonstrated a lower cognitive load rating than the Problem Solving condition.

Since the pattern of significance for the simple effects tests was identical for novices and experts, that pattern cannot be used to indicate why a significant interaction was obtained. Instead, the relative effect sizes will be used.

The effect of the Step Guidance over the Theorem & Step Guidance condition for novices was $\eta_p^2 = .05$, and for experts was $\eta_p^2 = .44$. Since the effect size for experts was larger than the effect size for novices, this difference is likely to contribute heavily to the significant interaction. As for the effect of the Theorem & Step Guidance condition over the Problem Solving condition, for novices $\eta_p^2 = .37$, and for experts $\eta_p^2 = .45$. Since the effect size for experts was

larger than the effect size for novices, this difference also may contribute to some extent to the significant interaction. From this pattern of effect sizes, it can be concluded that the significant interaction was largely caused by the larger advantage of the Step Guidance condition over the Theorem & Step guidance condition for experts compared to novices.

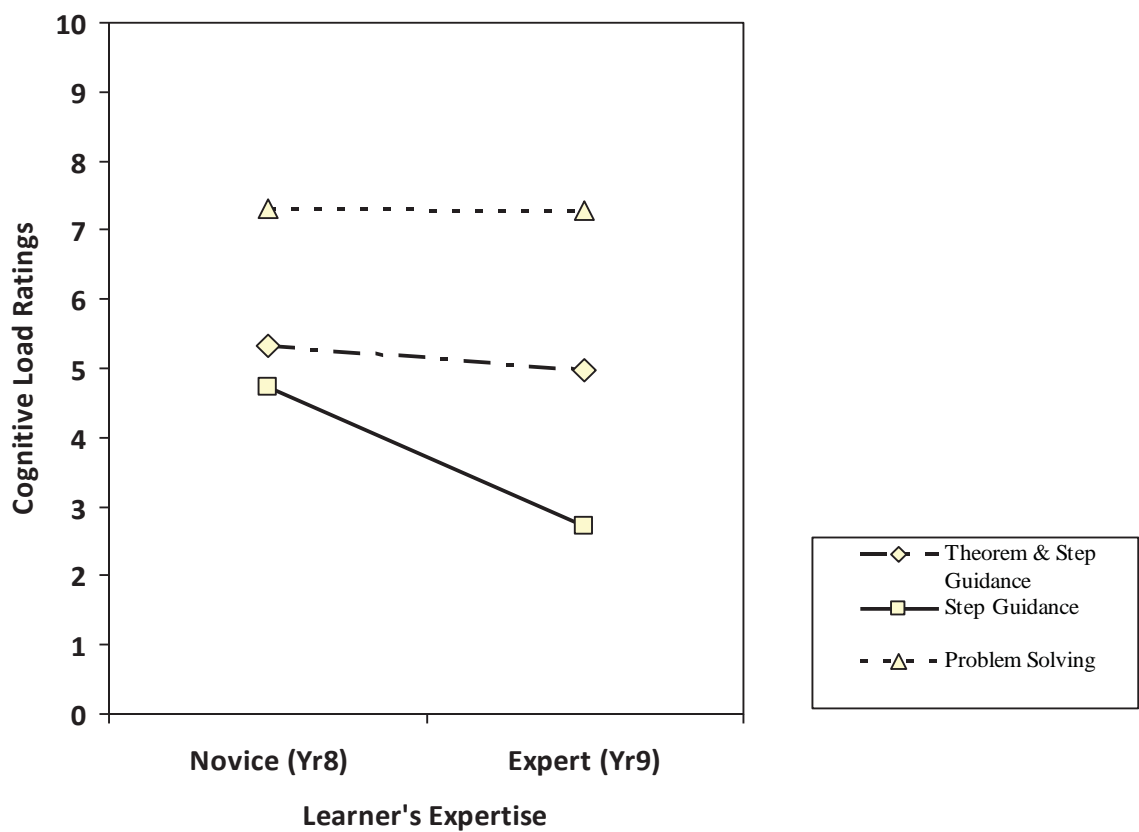


Figure 8. Interaction between instructional format and level of learner expertise for the ratings of cognitive load of Experiment 3.

Note. Ratings were made on 9-point scales (1=extremely easy, 9=extremely difficult)

Table 5 Means and Standard Deviations for the Ratings of Cognitive Load for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 3)

Expertise Level		Year 8 <i>n</i> =180		Year 9 <i>n</i> =180		
Instructional Condition	Theorem & Step Guidance <i>n</i> = 60	Step Guidance <i>n</i> = 60	Problem Solving <i>n</i> = 60	Theorem & Step Guidance <i>n</i> = 60	Step Guidance <i>n</i> = 60	Problem Solving <i>n</i> = 60
Ratings of Cognitive Load						
<i>M</i>	5.32	4.73	7.30	4.97	2.73	7.28
<i>SD</i>	1.58	1.07	0.98	1.64	0.73	0.85

Note. Ratings were made on 9-point scales (1=extremely easy, 9=extremely difficult)

Relative efficiency of instructional conditions

Cognitive load theory considers the quality of learning and instruction in terms of efficiency which takes both performance and learners' cognitive load into consideration. According to this methodology, instructional conditions that show a more favourable relationship between mental effort and performance (students invest less mental effort to achieve higher performance) are considered more efficient than instructional conditions that show a less favourable relationship between mental effort and performance (students invest more mental effort to achieve lower performance). The formula $E = \frac{(P - R)}{\sqrt{2}}$ that was suggested by Paas and van Merriënboer (1993) for calculating the instructional efficiency was used in this study, where (E) represents the relative efficiency of the instructional

condition, (P) the standardized z-scores for test performance scores, and (R) the ratings of cognitive load related to test tasks.

A two-way analysis of variance (learner level of expertise X instructional conditions) for the scores of the relative efficiency indicated a significant main effect for the three instructional conditions, $F(2,354) = 370.61$, $MSE = 0.58$, $p < .001$, $\eta_p^2 = 0.68$. There was a significant main effect for the level of expertise, $F(1,354) = 46.00$, $MSE = 0.58$, $p < .001$, $\eta_p^2 = 0.12$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2,354) = 13.93$, $MSE = 0.58$, $p < .001$, $\eta_p^2 = 0.07$ (See Figure 9). Means and standard deviations are provided in Table 6.

Table 6 Means and Standard Deviations for the Relative Instructional Efficiency for Different Instructional conditions and Levels of Learner Prior Knowledge (Experiment 3)

Expertise Level		Year 8 <i>n</i> =180		Year 9 <i>n</i> =180		
Instructional Condition	Theorem & Step Guidance <i>n</i> = 60	Step Guidance <i>n</i> = 60	Problem Solving <i>n</i> = 60	Theorem & Step Guidance <i>n</i> = 60	Step Guidance <i>n</i> = 60	Problem Solving <i>n</i> = 60
Relative Instructional Efficiency						
<i>M</i>	0.06	0.67	-1.54	0.31	1.81	-1.31
<i>SD</i>	0.99	0.66	0.68	0.97	0.56	0.62

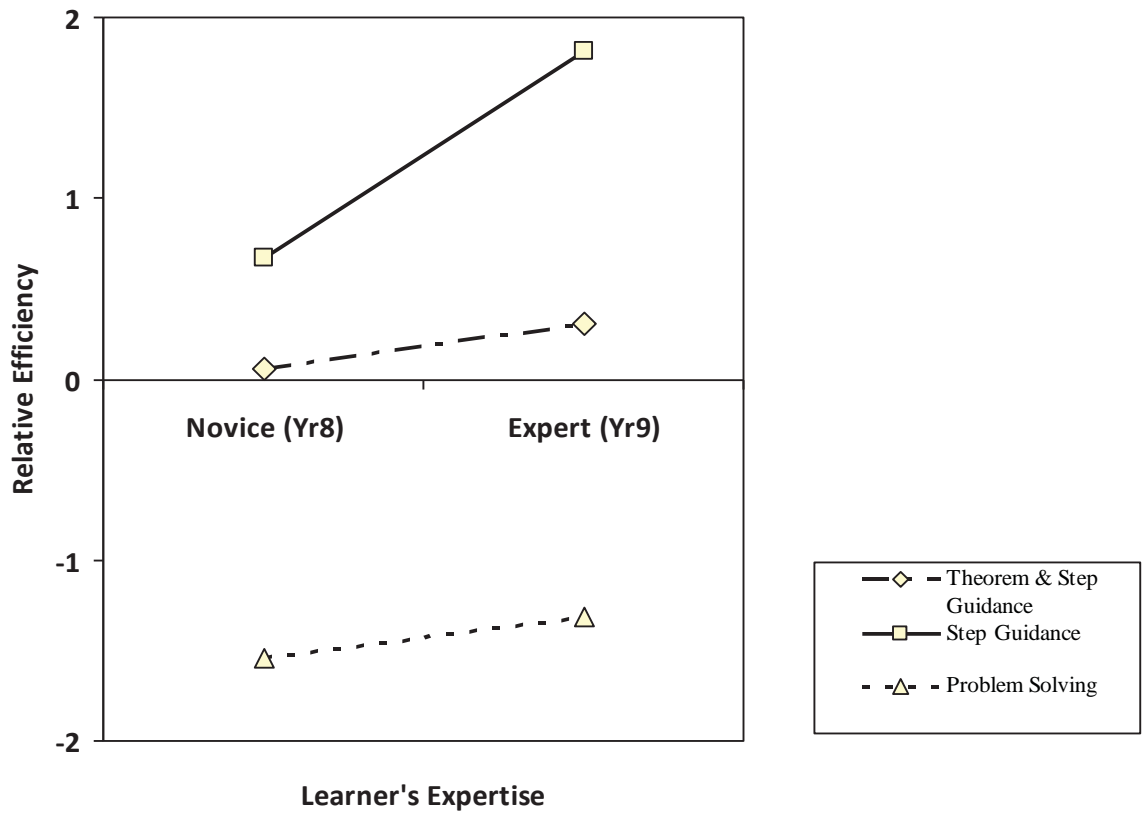


Figure 9. Interaction between instructional format and level of learner expertise for the relative efficiency of the instructional conditions of Experiment 3.

Following the significant interaction, simple effects tests for the expert learners (Year 9), demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions for the relative efficiency of experimental conditions, $F(1, 118) = 107.97$, $MSE = 0.62$, $p < .001$, $\eta_p^2 = 0.48$. The Step Guidance condition was a relatively more effective instructional design than the Theorem & Step Guidance condition. There also was

a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 119.87$, $MSE = 0.66$, $p < .001$, $\eta_p^2 = 0.50$. The Theorem & Step Guidance condition was a relatively more effective instructional design than the Problem Solving condition.

For the novice learners (Year 8) a significant difference was obtained between the Step Guidance, and the Theorem & Step Guidance conditions for the relative efficiency of experimental conditions, $F(1, 118) = 15.74$, $MSE = 0.70$, $p < .001$, $\eta_p^2 = 0.12$. The Step Guidance condition was relatively a more effective instructional design than the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 118) = 107.02$, $MSE = 0.72$, $p < .001$, $\eta_p^2 = 0.48$. The Theorem & Step Guidance condition was relatively a more effective instructional method than the Problem Solving condition.

Since the pattern of significance for the simple effects tests was identical for novices and experts, that pattern cannot be used to indicate why a significant interaction was obtained. Instead, the relative effect sizes will be used.

The effect of the Step Guidance over the Theorem & Step Guidance condition for novices was $\eta_p^2 = .12$, and for experts was $\eta_p^2 = .48$. Since the effect size for experts was larger than the effect size for novices, this might explain the reason behind the significant interaction. As for the effect of the Theorem & Step Guidance condition over the Problem Solving condition, for novices $\eta_p^2 = .48$, and for experts $\eta_p^2 = .50$. Since the effect size for experts was

similar to the effect size for novices, this difference is unlikely to contribute a great deal to the significant interaction. This pattern of effect sizes did not contribute much to the significant interaction. In view of that, it can be concluded that the significant interaction originated from the larger advantage of the Step Guidance condition over the Theorem & Step guidance condition for experts compared to novices.

Relative instructional efficiency values were positive with lower level group learners for the Theorem & Step Guidance and Step Guidance conditions and negative for the Problem solving condition (see Figure 10). With higher level learners, relative instructional efficiency values were positive with lower level group learners for the Theorem & Step Guidance and Step Guidance conditions and negative for the Problem solving condition (see Figure 10).

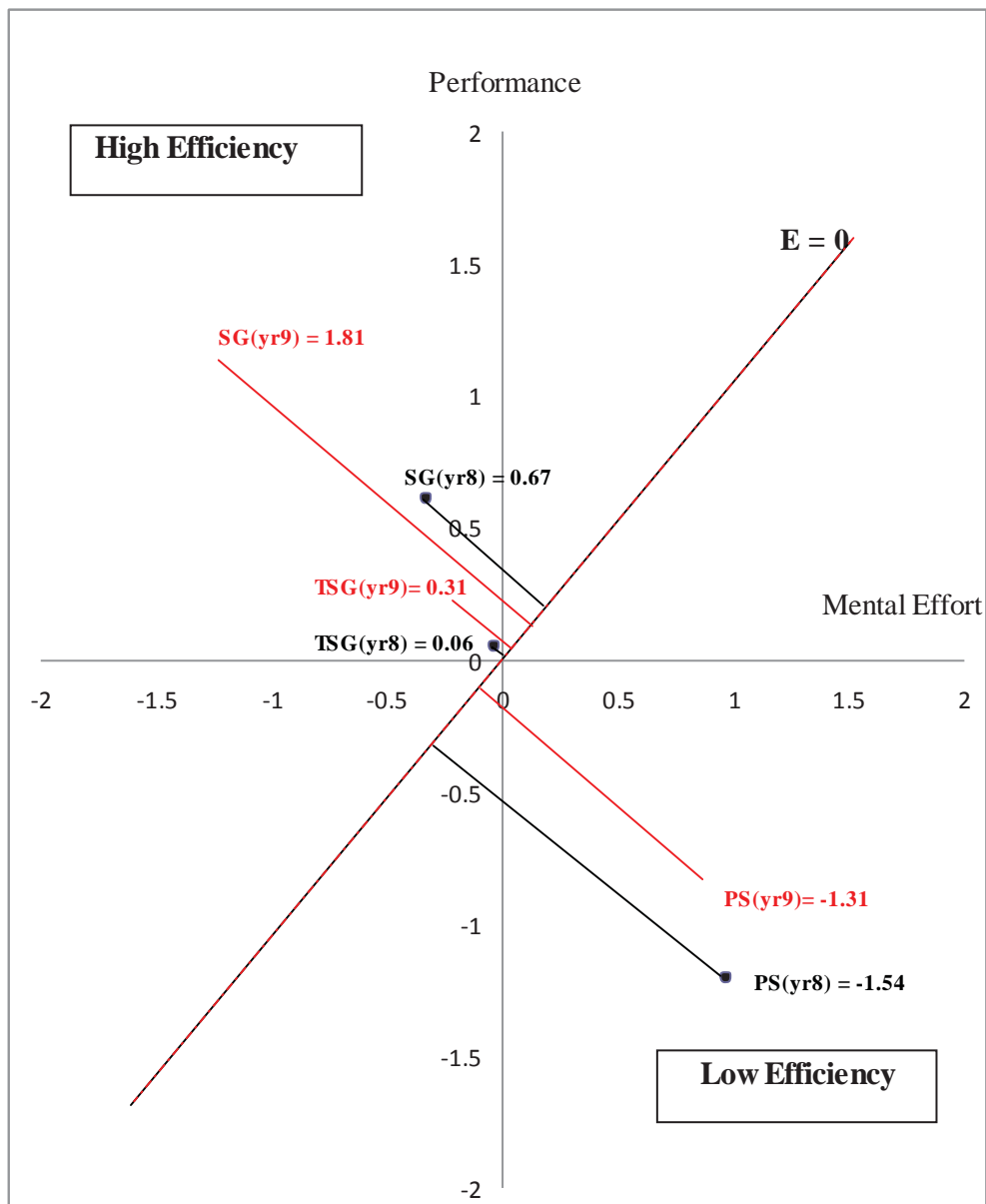


Figure 10. Representation of relative instructional efficiency (E) associated with learning through Theorem & Step Guidance (TSG), Step Guidance (SG) and Problem Solving (PS) for students with a lower prior knowledge level (Year 8) and higher prior knowledge level (Year 9, red)(Experiment 3).

8.4 Discussion

Experiment 3 was designed to test for an expertise reversal effect taking into consideration the instructional methods and the learners' prior knowledge. It was hypothesized that it would be more important for Year 8 than Year 9 students to be presented worked examples that include theorems under the Theorem & Step Guidance condition relative to the other conditions, as they had not been exposed to the properties of parallel lines previously, and having the theorem associated with each step might improve their understanding of the problem. On the other hand, Year 9 students may perform better using the Step Guidance condition compared to the other conditions as the presence of the theorem might be redundant since they have learnt these theorems at school. The presence of these theorems might impose an extraneous cognitive load that reduces performance.

The results indicated an advantage of the Step Guidance condition over the Theorem & Step Guidance condition which in turn was superior to the problem solving condition on all measures for both Year 8 and Year 9 students. However a significant, ordinal interaction was demonstrated. The effect size of the advantage of step guidance over a combination of step guidance and the theorem was larger for Year 9 than Year 8. It can be concluded that the significant interaction originated from this larger advantage of the Step Guidance condition over the Theorem & Step guidance condition for Year 9 compared to Year 8.

While the advantage of the Step Guidance condition over the Theorem and Step Guidance condition was reduced for Year 8 compared to Year 9, it was

not reversed. The reason might be that the parallel line theorems are comparatively easy to understand, even for novice learners and the available knowledge structures about the theorems in their long-term memory might have allowed them to solve the problem without any theorem guidance. Therefore, providing a theorem in a worked example might be redundant even for Year 8 students and that might have imposed an extraneous load that caused the instructional condition to be less effective.

The reported significantly lower ratings of cognitive load for the Step Guidance condition than the other two conditions (Theorem & Step Guidance and Problem Solving conditions) for both Year 8 and 9 students support the cognitive load explanation of the findings. This result might imply that reducing the amount of redundant information associated with presenting the theorem released sufficient cognitive resources for effective learning of geometric problem solving.

The significant difference in instructional efficiency associated with the performance task also provided support for the cognitive load explanation of the findings based on the redundancy effect (e.g. Chandler & Sweller, 1991; Sweller, 2003). Redundant information accompanying the Theorem & Step Guidance condition could become a source of extraneous cognitive load. The removal of this essentially redundant source of information may enhance solving geometric problems.

The results did not support the hypothesis that the less knowledgeable learners would excel under the Theorem & Step Guidance condition, as the Step Guidance condition was the best instructional method for both Year 8 and Year 9. Since the results revealed an expertise reversal effect based on an ordinal but not a dis-ordinal interaction, an attempt to widen the difference between the expertise levels may be needed to obtain a dis-ordinal interaction. As can be seen in the ordinal interaction plots of Figures 6, 7, 8, and 9, decreasing the levels of expertise, could result in the lines eventually crossing. Therefore, Experiment 4 was designed to widen the expertise difference by using students from Years 7 & 10 rather than Years 8 & 9. In addition, a more difficult Geometry Topic, Circle Geometry, with more complex, more difficult theorems was chosen in order to increase the importance of including the theorem in worked examples for novice learners.

Chapter Nine

Experiment 4

9.1 Introduction

Experiment 3 was designed to test for an expertise reversal effect taking into consideration the instructional methods and the learners' prior knowledge. It was hypothesized that the advantage of the Step Guidance condition over the Theorem & Step Guidance condition would be reduced for Year 8 compared to Year 9 students. The results indicated an advantage of the Step Guidance condition over the Theorem & Step Guidance condition which in turn was superior to the problem solving condition on all measures for both Year 8 and Year 9 students. However a significant, ordinal interaction was demonstrated.

While the advantage of the Step Guidance condition over the Theorem and Step Guidance condition was reduced for Year 8 compared to Year 9, it was not reversed. Since the results revealed an expertise reversal effect based on an ordinal but not a dis-ordinal interaction, an attempt to widen the difference between the expertise levels may be needed to obtain a dis-ordinal interaction. As the graphs presented in Experiment 3 illustrate an ordinal interaction, decreasing the levels of expertise could result in the lines ultimately crossing. Therefore, Experiment 4 was designed to widen the expertise difference by using students from Years 7 & 10 rather than Years 8 & 9 as the gap between levels of expertise

in Experiment 3 may not have been large enough. Thus, by choosing Years 7 & 10, we might be able to obtain a dis-ordinal interaction.

In addition, a more difficult Geometry Topic, Circle Geometry, with more complex theorems was chosen in order to increase the importance of including the theorem in worked examples for novice learners. In Experiment 3 the theorems chosen were finding angles that were held between two parallel lines. These theorems are part of the Mathematical curriculum of Year 9 Students. Accordingly, it was understandable that Year 8 students found these theorems comparatively easy to understand as the available knowledge structures about the theorems in their long-term memory might have allowed them to solve the problem without any theorem guidance. Taking this into consideration along with the findings of Experiment 3, the theorems chosen in Experiment 4 were based on circle geometry, topic that is part of the Mathematical curriculum for Year 10.

This topic was chosen to test if presenting guidance with the theorem might have a positive effect on novices' learning. Circle geometry is considered a difficult topic and thus, Experiment 4 was designed to test for the expertise reversal effect. It was hypothesized that Year 7 would perform better than Year 10 when using the Theorem & Step Guidance condition as the available knowledge structure about these theorems in Year 7 students' long-term memory might not allow them to solve the problem without any theorem guidance. It was also hypothesized that Year 10 would perform better using the Step Guidance procedure as the information concerning the theorem in the Theorem and Step Guidance condition would be redundant and inhibit students' learning. In

summary, Experiment 4 was designed to investigate whether a dis-ordinal interaction between the level of expertise (Year 7 & 10) and the instructional methods (Theorem & Step Guidance and Step Guidance) could be obtained.

9.2 Method

9.2.1 Participants

The participants were 60 Year 7 students and 60 Year 10 students attending a private school in North Sydney, Australia. Year 7 students were aged between 12 and 13 years, Year 10 students were aged between 15 and 16 years. The students who belonged to each level were at the same level of mathematical skills, as determined by their class teachers. The grading of students by class teachers according to mathematical skills is standard practice and is part of the school curriculum in Sydney schools. Year 7 students had not been exposed previously to circular geometry, but Year 10 students had learnt the properties of circular geometry prior to the experiment.

Students from each year were randomly assigned into three equivalent groups of 20. One group was guided at each solution step with the answer for the corresponding angle and the theorem behind each move (Theorem & Step Guidance). The students in another group were presented with the angle they had to find at each step but not provided the theorem they had to use to find the angle

(Step Guidance). The third group represented the Problem Solving (control) condition.

9.2.2 Material

For the acquisition phase, theorems were selected from the circle geometry topic that forms a part of the mathematics curriculum material suitable for students at this stage. The theorems were related to finding the angles associated with arcs in a circle. The selected angles were angles that subtended the same arc, a central angle, an angle at the circumference, an angle formed by a tangent and a secant and angles in a cyclic quadrilateral (these angles were formed when several lines cut a circle). The geometry associated with these figures depicting these angles had not been previously taught to Year 7 students involved in the experiment, while Year 10 students had learnt the properties of circle geometry in school and were previously exposed to such figures. Depending on the experimental condition, the same figures were supplemented either with steps and full solutions including the theorems used or with steps to follow with no theorem indicated, or with no solution at all. The figures presented to each experimental group were identical in size, including angle sizes, and retained the same angle names (the problems presented to each group are found in Appendix H).

The test material included finding angles based on angles formed by lines, chords and tangents to a circle. Three problems were similar to the acquisition problems, with almost the same figures as in the acquisition problems, but with different measures of the given angles. Following the similar questions, three

transfer problems were given that did not include any measurements of angles; participants were asked to prove two angles to be equal using the properties of circle geometry. The marking system was identical to Experiment 3. The test problems can be found in Appendix I.

9.2.3 Procedure

The same procedure was used as in the previous experiment. It consisted of a learning phase (25 minutes) and a test phase (35 minutes). It was conducted over one school session, with each child tested individually. Two days prior to the experiment, a revision session occurred (45 minutes) to review the prerequisite material needed to conduct the experiment. The session was similar for all groups and was given to all the students in order to remind them of the geometric terminology used in the experiment (radius, diameter, centre, circumference, tangent, secant, chord, arc, right angle, straight angle, revolution angle, vertical opposite angles, sum of measures of angle in a triangle and equal angles).

9.2.4 Rating of cognitive load associated with task performance

Immediately after the acquisition phase, each participant was asked to estimate how easy or difficult it was to learn the material and answer the questions on a nine-point scale by placing an “X” in the space provided related to one of the nine numbers (1 being extremely easy, 9 being extremely difficult, (see Appendix G). The scores obtained from this rating scale were used as indicators of cognitive

load associated with the learning tasks. These rating were also used for calculating relative instructional efficiency of instructional conditions.

9.3 Results

Variables

The dependent variables under analysis were similar test and transfer test scores (Means and standard deviations are provided in Table 7), subjective ratings of cognitive load (Means and standard deviations are provided in Table 8), and relative instructional efficiency measures (Means and standard deviations are provided in Table 9). The independent variables were instructional designs (Theorem & Step guidance, Step guidance, or Problem solving) and the level of learner expertise (Year 7 and Year 10 students).

Similar test results

A two-way analysis of variance (learner level of expertise X instructional conditions) for the scores on the similar test indicated a significant main effect for the three instructional conditions, $F(2, 114) = 38.39$, $MSE = 1.31$, $p < .001$, $\eta_p^2 = 0.40$. There was a significant main effect for the level of expertise, $F(1, 114) = 8.67$, $MSE = 1.31$, $p = .004$, $\eta_p^2 = 0.07$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2, 114) = 10.75$, $MSE = 1.31$, $p < .001$, $\eta_p^2 = 0.16$ (See Figure 11).

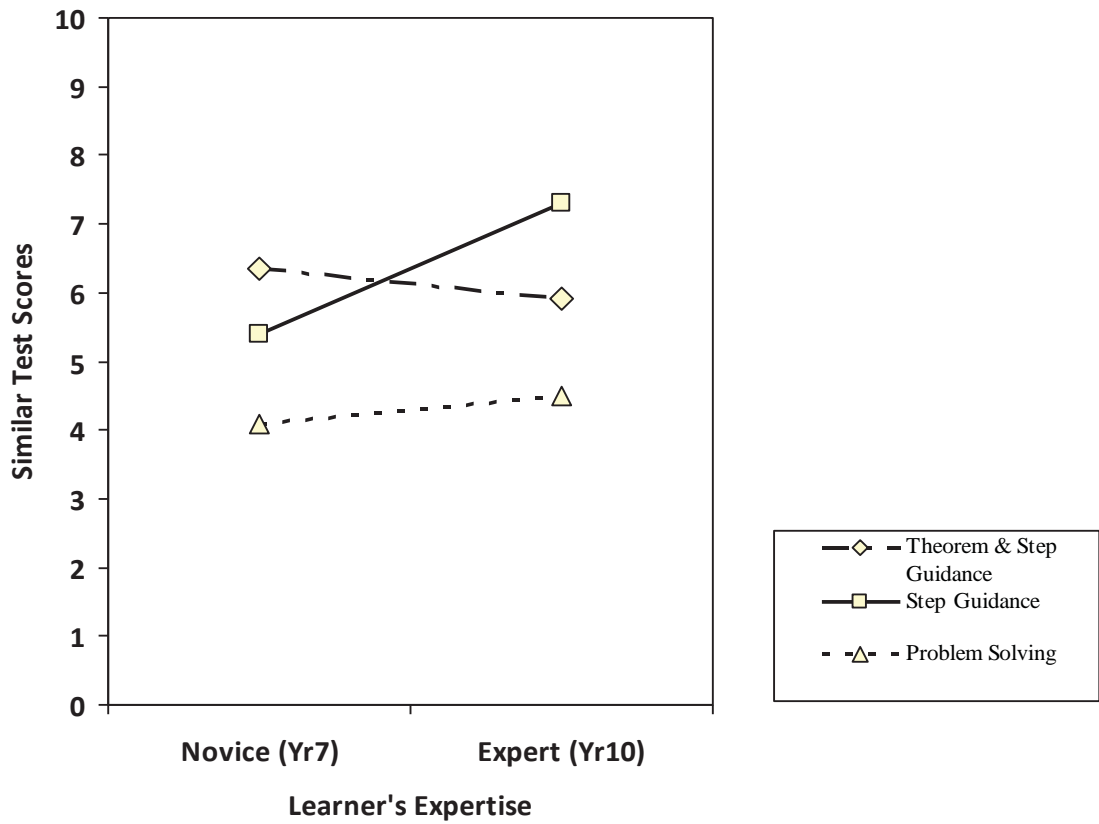


Figure 11. Interaction between instructional format and level of learner expertise for overall similar test scores of Experiment 4.

Following the significant interaction, a simple effects test for the expert learners (Year 10), demonstrated a significant difference between the Step Guidance, and the Theorem & Step Guidance conditions, $F(1, 38) = 19.60$, $MSE = 1.00$, $p < .001$, $\eta_p^2 = 0.34$. The Step Guidance condition outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1,$

38) = 15.92, $MSE = 1.23$, $p < .001$, $\eta_p^2 = 0.30$. The Theorem & Step Guidance condition significantly outperformed the Problem Solving condition.

For the novice learners (Year 7), there was a significant difference between the Theorem & Step Guidance condition, and the Step Guidance condition, $F(1, 38) = 5.59$, $MSE = 1.61$, $p = .023$, $\eta_p^2 = 0.13$. The Theorem & Step Guidance condition outperformed the Step Guidance condition. There also was a significant difference between the Step Guidance and the Problem Solving condition, $F(1, 38) = 9.36$, $MSE = 1.81$, $p = .004$, $\eta_p^2 = 0.20$. The Step Guidance condition significantly outperformed the Problem Solving condition.

Table 7 Means and Standard Deviations for the Similar and Transfer test Scores for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4)

Expertise Level		Year 7 $n = 60$			Year 10 $n = 60$	
Instructional Condition	Theorem & Step Guidance $n = 20$	Step Guidance $n = 20$	Problem Solving $n = 20$	Theorem & Step Guidance $n = 20$	Step Guidance $n = 20$	Problem Solving $n = 20$
Total Scores for Similar Test						
<i>M</i>	6.35	5.40	4.10	5.90	7.30	4.50
<i>SD</i>	1.04	1.47	1.21	1.12	0.86	0.10
Total Scores for Transfer Test						
<i>M</i>	5.45	4.50	2.30	4.75	6.10	2.60
<i>SD</i>	0.89	1.32	0.86	1.16	0.97	1.05

Note: The maximum score was 9 for both the similar and transfer tests.

Transfer test results

A two-way analysis of variance (learner level of expertise X instructional conditions) for the scores of the transfer test indicated a significant main effect for the three instructional conditions, $F(2, 114) = 91.23$, $MSE = 1.11$, $p < .001$, $\eta_p^2 = 0.62$. There was a significant main effect for the level of expertise, $F(1, 114) = 4.33$, $MSE = 1.11$, $p = .04$, $\eta_p^2 = 0.04$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2, 114) = 11.99$, $MSE = 1.11$, $p < .001$, $\eta_p^2 = 0.17$ (see Figure 12).

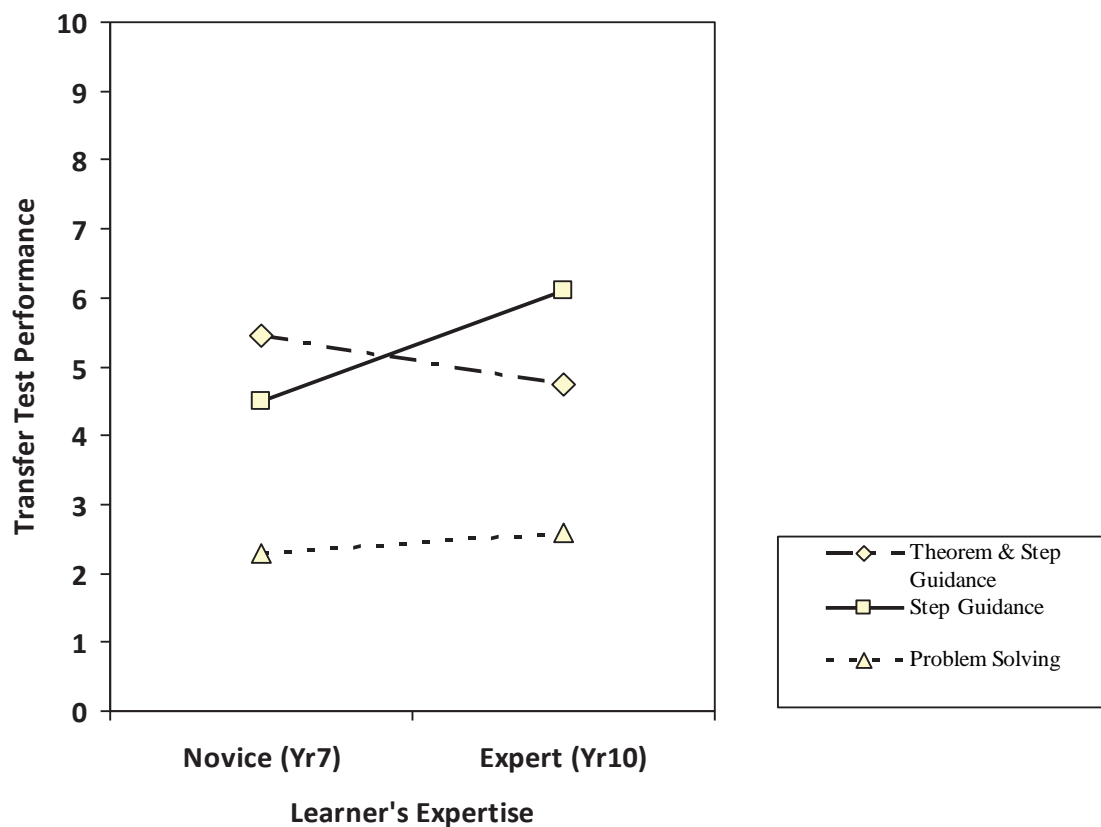


Figure 12. Interaction between instructional format and level of learner expertise for overall transfer scores of Experiment 4.

Following the significant interaction, a simple effect test for the expert learners (Year 10) demonstrated a significant difference between the Step Guidance and the Theorem & Step Guidance conditions, $F(1, 38) = 15.90$, $MSE = 1.15$, $p < .001$, $\eta_p^2 = 0.30$. The Step Guidance condition outperformed the Theorem & Step Guidance condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 38) = 37.74$, $MSE = 1.23$, $p < .001$, $\eta_p^2 = 0.50$. The Theorem & Step Guidance condition outperformed the Problem Solving condition.

For the novice learners (Year 7), there was a significant difference between the Theorem & Step Guidance, and the Step Guidance conditions, $F(1, 38) = 7.15$, $MSE = 1.26$, $p = .011$, $\eta_p^2 = 0.16$. The Theorem & Step Guidance condition outperformed the Step Guidance condition. There also was a significant difference between the Step Guidance and the Problem Solving conditions $F(1, 38) = 38.97$, $MSE = 1.24$, $p < .001$, $\eta_p^2 = 0.51$. The Step Guidance condition significantly outperformed the Problem Solving condition.

Ratings of cognitive load

A two-way analysis of variance (learner level of expertise X instructional conditions) for the ratings of the cognitive load indicated a significant main effect for the three instructional conditions, $F(2, 114) = 57.97$, $MSE = 1.86$, $p < .001$,

$\eta_p^2 = 0.50$. There was a significant main effect for the level of expertise, $F(1, 114) = 8.69$, $MSE = 1.86$, $p = .004$, $\eta_p^2 = 0.07$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2, 114) = 8.14$, $MSE = 1.86$, $p < .001$, $\eta_p^2 = 0.13$ (see Figure 13). Means and standard deviations for different instructional conditions and levels of prior knowledge are provided in Table 8.

Table 8 Means and Standard Deviations for the Ratings of Cognitive Load for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4)

Expertise Level		Year 7 <i>n</i> = 60		Year 10 <i>n</i> = 60		
Instructional Condition	Theorem & Step Guidance <i>n</i> = 20	Step Guidance <i>n</i> = 20	Problem Solving <i>n</i> = 20	Theorem & Step Guidance <i>n</i> = 20	Step Guidance <i>n</i> = 20	Problem Solving <i>n</i> = 20
Ratings of Cognitive Load						
<i>M</i>	5.05	5.10	7.20	4.95	2.95	7.25
<i>SD</i>	1.10	1.94	0.95	1.85	0.83	1.07

Note. Ratings were made on 9-point scales (1=extremely easy, 9=extremely difficult)

Following the significant interaction, a simple effect test for the expert learners (Year 10), demonstrated a significant difference between the Theorem & Step Guidance, and the Step Guidance conditions, $F(1, 38) = 19.51$, $MSE = 2.05$,

$p < .001$, $\eta_p^2 = 0.34$. The Step Guidance condition demonstrated a significantly lower cognitive load rating than the Theorem & Step Guidance condition. There also was a significant difference between the Step Guidance and the Problem Solving conditions $F(1, 38) = 202.48$, $MSE = 0.91$, $p < .001$, $\eta_p^2 = 0.84$. The Step Guidance condition demonstrated a lower cognitive load rating than the Problem Solving condition. There was a significant difference between the Theorem & Step Guidance and the Problem Solving conditions $F(1, 38) = 23.19$, $MSE = 2.28$, $p < .001$, $\eta_p^2 = 0.38$. The Theorem & Step Guidance condition demonstrated a lower cognitive load rating than the Problem Solving condition.

For the novice learners (Year 7), a simple effect test demonstrated a non-significant difference between the Theorem & Step Guidance, and the Step Guidance conditions, $F(1, 38) = 0.10$, $MSE = 2.49$, $p = .921$, $\eta_p^2 = 0$. The Theorem & Step Guidance and the Step Guidance conditions demonstrated almost identical cognitive load ratings. There was a significant difference between the Step Guidance and the Problem Solving condition, $F(1, 38) = 18.83$, $MSE = 2.34$, $p < .001$, $\eta_p^2 = 0.33$ and a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 38) = 45.75$, $MSE = 1.06$, $p < .001$, $\eta_p^2 = 0.54$. Both the Step Guidance and the Theorem & Step Guidance conditions demonstrated lower cognitive load rating than the Problem Solving condition.

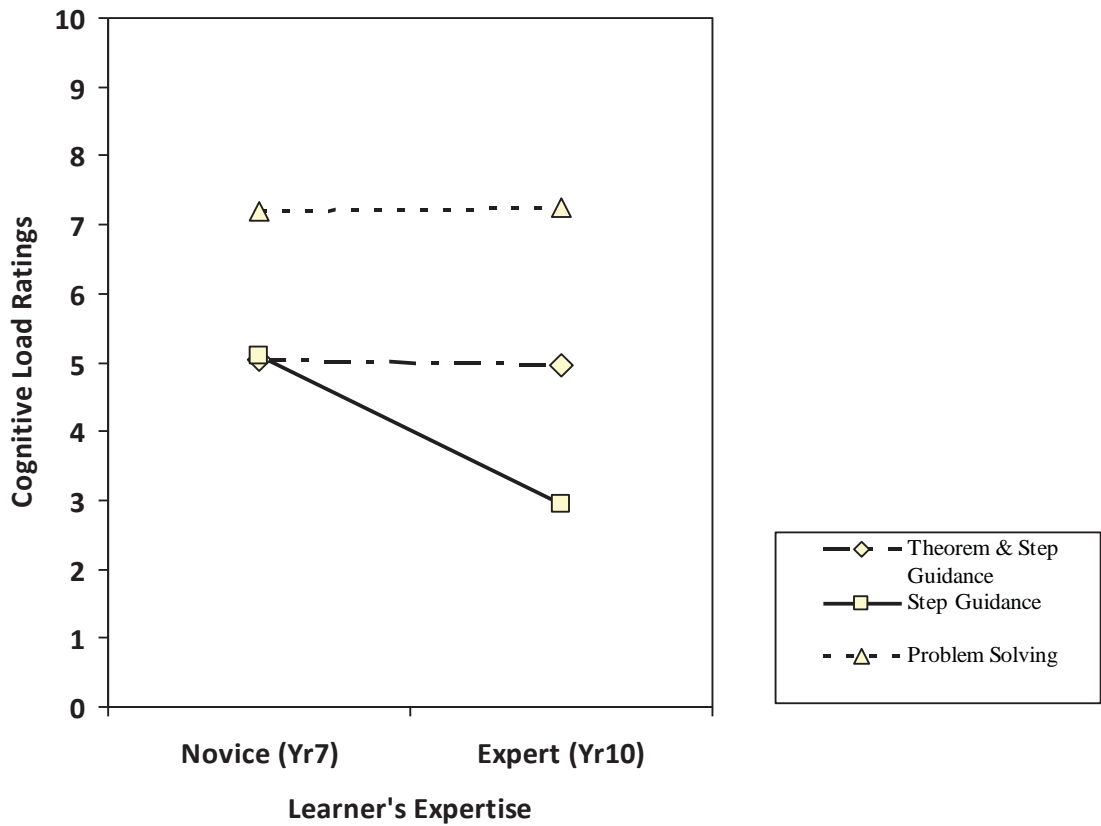


Figure 13. Interaction between instructional format and level of learner expertise

for the ratings of cognitive load of Experiment 4.

Note. Ratings were made on 9-point scales (1=extremely easy, 9=extremely difficult)

Relative efficiency of instructional conditions

A two-way analysis of variance (learner level of expertise X instructional conditions) for the relative efficiency scores indicated a significant main effect for the three instructional conditions, $F(2, 114) = 68.87$, $MSE = 0.87$, $p < .001$, $\eta_p^2 = 0.55$. There was a significant main effect for the level of expertise, $F(1, 114) =$

8.76, $MSE = 0.87$, $p = .004$, $\eta_p^2 = 0.71$. There also was a significant interaction between the three instructional conditions and the learners' expertise levels $F(2, 114) = 10.83$, $MSE = 0.58$, $p < .001$, $\eta_p^2 = 0.16$ (see Figure 14). Means and standard deviations are provided in Table 9.

Table 9 Means and Standard Deviations for the Relative Instructional Efficiency for Different Instructional Conditions and Levels of Learner Prior Knowledge (Experiment 4)

Expertise Level		Year 7 <i>n</i> =60		Year 10 <i>n</i> =60		
Instructional Condition	Theorem & Step Guidance <i>n</i> = 20	Step Guidance <i>n</i> = 20	Problem Solving <i>n</i> = 20	Theorem & Step Guidance <i>n</i> = 20	Step Guidance <i>n</i> = 20	Problem Solving <i>n</i> = 20
Relative Instructional Efficiency						
<i>M</i>	0.58	0.11	-1.47	0.34	1.71	-1.33
<i>SD</i>	0.75	1.28	0.75	1.16	0.67	0.80

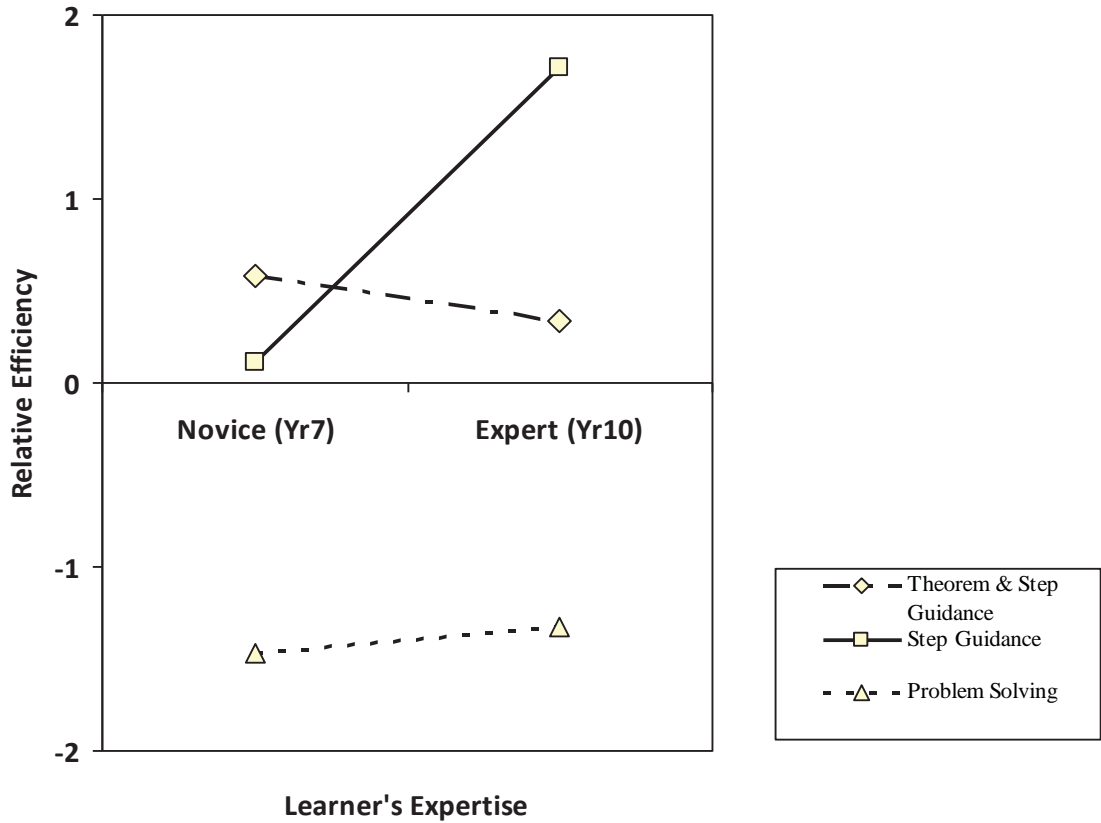


Figure 14. Interaction between instructional format and level of learner expertise for the relative efficiency of the instructional conditions of Experiment 4.

Following the significant interaction, simple effect tests for the expert learners (Year 10), demonstrated a significant difference between the Theorem & Step Guidance, and the Step Guidance conditions, $F(1, 38) = 20.85$, $MSE = 0.90$, $p < .001$, $\eta_p^2 = 0.35$. The Step Guidance condition was relatively more efficient than the Theorem & Step Guidance condition. There was a significant difference between the Step Guidance and the Problem Solving conditions, $F(1, 38) = 169.13$, $MSE = 0.54$, $p < .001$, $\eta_p^2 = 0.82$. The Step Guidance condition was a

relatively better instructional method than the Problem Solving condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 38) = 27.70$, $MSE = 1.00$, $p < .001$, $\eta_p^2 = 0.42$. The Theorem & Step Guidance condition was a relatively more efficient instructional method than the Problem Solving condition.

For the novice learners (Year 7), a non-significant difference was obtained between the Step Guidance and the Theorem & Step Guidance conditions, $F(1, 38) = 1.99$, $MSE = 1.10$, $p = .166$, $\eta_p^2 = 0.05$. There was a significant difference between the Step Guidance and the Problem Solving condition, $F(1, 38) = 22.70$, $MSE = 1.10$, $p < .001$, $\eta_p^2 = 0.37$. The Step Guidance condition was relatively more efficient than the Problem Solving condition. There also was a significant difference between the Theorem & Step Guidance and the Problem Solving condition, $F(1, 38) = 74.20$, $MSE = 0.57$, $p < .001$, $\eta_p^2 = 0.66$. The Theorem & Step Guidance condition was relatively more efficient than the Problem Solving condition.

For both lower and higher knowledge level groups, relative instructional efficiency values were positive for the Theorem & Step Guidance and Step Guidance conditions and negative for the Problem solving condition (see Figure 15).

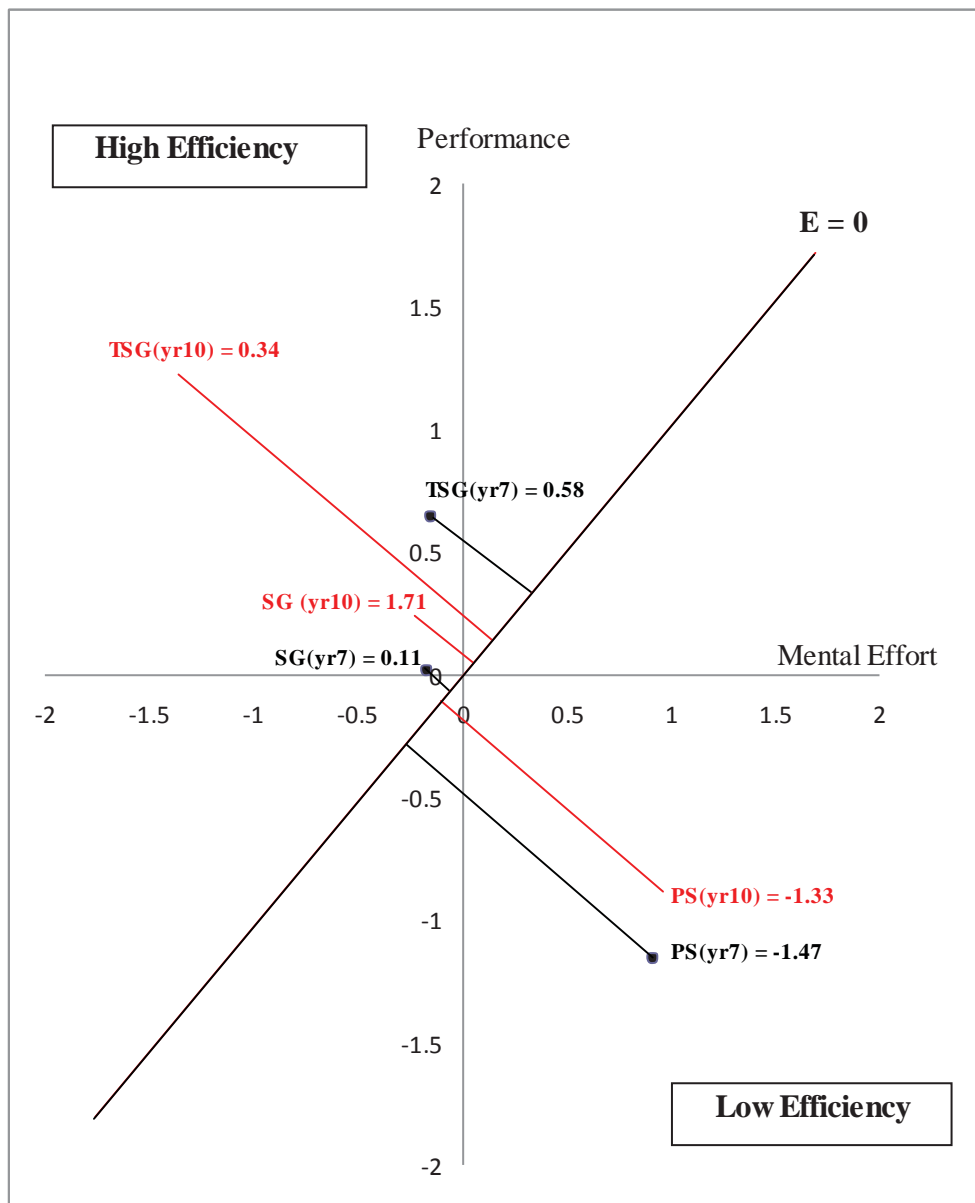


Figure 15. Representation of relative instructional efficiency (E) associated with learning through Theorem & Step Guidance (TSG), Step Guidance (SG) and Problem Solving (PS) for students with lower prior knowledge level (Year 7) and higher prior knowledge level (Year 10, red) (Experiment 4)

9.4 Discussion

A significant interaction between the effectiveness of alternative instructional technique in learning geometric problem solving and levels of learner prior knowledge was demonstrated in this experiment. The results supported our hypothesis that when the difference between the levels of learner expertise increased, a cross over interaction would be revealed. Also it was hypothesised that experts would perform better using the Step Guidance format as the theorems would impose extraneous cognitive load, and novices would perform better using the Theorem & Step Guidance format as the presence of the Theorem might help them to better understand the material. The overall similar test and transfer test results for experts demonstrated that Step Guidance group outperformed the Theorem & Step Guidance group and both these groups outperformed the Problem Solving group. The overall similar test and transfer test results for novices demonstrated that the Theorem & Step Guidance condition outperformed the Step Guidance condition and both these groups outperformed the Problem Solving condition.

The superiority of the Step Guidance condition was demonstrated for learners with higher levels of prior knowledge as their available knowledge structures may have reduced the level of intrinsic load sufficiently to allow processing this information without theorem guidance. However, this condition was relatively less effective for novice learners as their insufficient knowledge in long-term memory may have resulted in an increased extraneous cognitive load

caused by the need to search for relevant explanations. The relevant information (the geometric theorems) in the Theorem & Step Guidance condition provided such explanations and thus enhanced novice learning. However, this information was redundant for more knowledgeable students and inhibited their learning in comparison to the format without the theorem guidance.

Novices and experts that used the Theorem & Step Guidance conditions reported similar levels of load (novices 5.05, experts 4.95) but novices achieved a higher level of efficiency (0.58) as most of the effort they invested was germane. In contrast, the experts who invested the same amount of effort achieved a relatively lower level of efficiency (0.34), as most of the effort they invested was extraneous. Since they already knew the theorems, it was a redundant effort and it did not have a positive effect on learning.

The calculated relative instructional efficiency measures associated with the overall performance task supported our hypothesis. The positive efficiency values for the Step Guidance condition (1.71 for higher prior knowledge level and 0.11 for lower prior knowledge level) and the Theorem & Step Guidance condition (0.34 for higher prior knowledge level and 0.58 for lower prior knowledge level) and the negative relative instructional efficiency values for the Problem Solving condition (-1.33 for the higher prior knowledge level and -1.47 for the lower prior knowledge level) demonstrated the overall advantage of worked examples over problem solving (in accordance with the worked example effect). The relative standing of the efficiency measures for different experimental

conditions and levels of learner expertise demonstrated an expertise reversal effect.

These results correspond with previous studies (van Gog, Paas, & van Merriënboer, 2006a, 2006b) that have suggested that when learners become familiar with the solution procedure, processing any additional information would become redundant and inhibit the effectiveness of transfer. We hypothesised that eliminating this information and presenting learners with product-oriented worked examples would lead to improved effectiveness. The data supported that hypothesis.

Chapter Ten

General Discussion and Conclusion

10.1 Theoretical Overview

Cognitive load theory provides the basis of this study since the theory explains the relation between designing instruction and human cognitive architecture (Sweller et al, 1998; van Merriënboer & Sweller, 2005). Working memory is the structure that is used to hold and process information we are provided with. The limitations of working memory which is one of the most important characteristics of human cognitive architecture are exceeded when more than a few pieces of information are processed at the same time (Baddeley, 1992; Cowan, 2001; Miller, 1956)

Long-term memory is another essential element of our cognitive architecture. It can store unlimited amounts of complex information and has no duration and capacity limitations. It holds information in the form of schematic knowledge structures by which our cognitive abilities are determined. A schema is a cognitive structure that allows us to consider several elements as a single element that is categorized according to how it will be used. Schemas provide learners with the ability to encapsulate many elements of information into a single unit that needs less working memory capacity than multiple units, thus

diminishing the load on working memory. Another method to reduce cognitive load is by automation. With practice, skills can become automated and do not require much controlled processing in working memory (Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977). Consequently, less conscious effort is required for this information to be processed and more complex cognitive activities can take place since working memory resources become available.

Instructional procedures should be aligned with these characteristics of human cognitive architecture. The focal point of cognitive load theory is to enhance learning by generating activities that control and manage cognitive load. Reducing extraneous cognitive load that results from instructional designs that require learners to take part in cognitive activities that are not related to learning is crucial (Sweller et al., 1998). As a result of the reduction, more working memory resources will be allocated to productive and meaningful learning. Nevertheless, cognitive load depends on the learner's existing domain-specific knowledge base. For example, when advanced learners who already have sufficient knowledge to process information are provided with detailed instructional guidance designed for novices, the excessive guidance may become redundant. As a result, cognitive resources will be used to integrate the redundant instructions with the learner's available knowledge structures, thus cognitive resources will be diverted from productive higher-order activities. For example, while providing both theorems and solution steps is beneficial for novices, it might become harmful for advanced learners. The present findings demonstrate

this expertise reversal effect (e.g., Kalyuga, 2007; Kalyuga et al., 2003) by indicating the differential effectiveness of guided forms of instruction depending on the degree of learner familiarity with the learning contents.

The degrees of guidance used in worked examples were examined in four experiments that used geometry materials. Geometry is one of the most difficult mathematical areas for students. It has been demonstrated that many students have a poor understanding of mathematical proof and find it difficult to construct mathematical proofs. In order to enhance students' cognitive readiness for proof, it is essential to provide them during their early years with a good basis that includes basic steps of deductive reasoning. Also, it is important to demonstrate the succession of theorems needed in order to solve problems instead of focusing on the theorems themselves. Hoyles (1998) proposed that it is important to consider how proof is presented to students. Therefore, instructional presentations should be designed to help students in comprehending geometry proofs and theorems by taking into consideration the limitations of human cognitive architecture.

The reason why worked examples are efficient when dealing with geometry problems is that they eliminate unnecessary search processes and enhance schema construction. However, in order for students to succeed in problem solving, they need to identify the structural problem features and the category to which a problem belongs, and remember the steps related to reaching

a solution for this category of problems. Acquiring specific schemas also leads to transfer performance since the information concerning types of problems and their related steps is stored in problem schemas. Furthermore, cognitive load theory suggests that considering learner prior knowledge is crucial in designing instructional presentations for novel material. Learning is made easier when the context of the new complex material is related to previously constructed and stored schemas.

However, while using theorems as a means of providing explanation and needed reasoning behind each step might be beneficial for novice learners, it could impose an additional extraneous cognitive load on more experienced students since the redundant information might cause total cognitive load to exceed learner working memory capacity.

The reported experiments compared two formats of geometry worked examples: Theorem & Step Guidance and Step Guidance formats. In both formats the corresponding material was placed next to the angle related to the problem to avoid split-attention as a typical source of extraneous cognitive load in geometric worked-examples presentations. However, integrating explanations that are redundant for more experienced learners may increase their experienced cognitive load. Unfortunately, these learners might discover this redundancy only after they have already processed it (Sweller, 2003).

The studies presented in this thesis were based on the assumption that, according to the expertise reversal effect, the effectiveness of presenting the steps and the associated theorem related to each step would depend on the levels of learner's prior knowledge. While beneficial to novices, this method might impede learning of more experienced learners due to the redundancy effect. Presenting the steps only could be a better option for these learners.

10.2 Review of the Experiments

The results of the experiments described in Chapters 6-9 supported the assertion of cognitive load theory that instructional designs that lead to unnecessary cognitive load might deprive learners of cognitive resources that are required for effective learning. The experiments confirmed the superiority of the Step Guidance presentation over the Theorem & Step Guidance for more knowledgeable learners and the superiority of the Theorem & Step Guidance over the Step Guidance for less knowledgeable learners for whom assimilation of the theorem potentially represented a substantial cognitive load.

Experiment 1 demonstrated that the Theorem & Step Guidance example was superior to the conventional Problem Solving format on transfer but not on similar problems while the Step Guidance format was superior to the Problem Solving condition on both similar problems and transfer. Thus the results of this

experiment divulge the normal effect of worked examples in learning geometric problem solving: geometry worked examples were more effective than conventional problem solving. Even though there were no significant differences between the two worked example formats on both the similar and transfer tests, this experiment indicated the possibility of a potential redundancy effect associated with theorem guidance since the Step Guidance condition demonstrated a more comprehensive superiority over the Problem Solving condition than the Theorem & Step Guidance condition and the means favoured the Step Guidance condition on both tests.

These results were replicated in Experiment 2 with students who were chosen from the same maths year but divided into low achievers and high achievers based only on the school classification of students who were placed in either a lower or higher maths class. In this experiment, the worked-examples effect was also demonstrated favouring the two worked-examples conditions (Theorem & Step Guidance and Step Guidance) over the problem solving condition on both similar and transfer problems for both low and high achievers. Results also indicated that students who were presented with the Step Guidance format outperformed those who were given the Theorem & Step Guidance format on transfer problems only. The redundant information related to the theorems may have affected learners in the Theorem & Step Guidance condition. However, no expertise reversal effect was revealed in this experiment, as participants could not be clearly distinguished into two expertise levels in the selected area of geometry based on more objective characteristics.

Experiment 3 was designed to include a larger gap between expertise levels. Year 8 and 9 students were chosen for this experiment. The experiment replicated the worked example effect by demonstrating the superiority of both the Theorem & Step Guidance and Step Guidance conditions over the Problem Solving condition. Moreover, it also revealed the advantage of the Step Guidance format over the Theorem & Step Guidance format using both less and more knowledgeable learners, this time on both similar and transfer problems. In addition, a significant ordinal interaction between these formats and levels of learner expertise was demonstrated due to a stronger effect of Step Guidance over Theorem & Step Guidance for more knowledgeable than less knowledgeable learners. Since the result of Experiment 3 revealed an expertise reversal effect based on an ordinal but not a dis-ordinal interaction, an attempt to further widen the difference between the expertise levels was undertaken in Experiment 4 in order to attempt to obtain a dis-ordinal interaction.

Experiment 4 revealed a significant cross-over interaction between the levels of learner expertise and the instructional methods. In this experiment, the superiority of the Step Guidance condition over the Theorem & Step Guidance and Problem Solving conditions was demonstrated with more knowledgeable learners; additionally the superiority of the Theorem & Step Guidance condition over the Step Guidance and Problem Solving conditions was demonstrated with novice learners. The availability of written theorem information enhanced the learning of novices by guiding their attention to the theorems that they needed to learn. On the other hand, the same theorem information was redundant for expert

learners and inhibited their performance. The results of this experiment reveal that for students who already understand the relevant theorems, learning to solve problems primarily consists of learning to recognise problem states and their associated moves. Information concerning theorems should only be provided if students have yet to learn and automate them.

In summary, novices often have difficulty in acquiring new information. Knowledge of geometric theorems is crucial to the learning of unfamiliar and difficult geometry problems. It is important that a beginner problem solver receive sufficient guidance in the applicable geometric theorems for effective learning. The Theorem & Step Guidance worked examples were hypothesised to be an efficient teaching method in this situation. However, students who already have sufficient knowledge of the relevant geometric theorems (theorem schemas) may be able to apply these theorems when solving corresponding geometric problems without consciously processing this information. Such more knowledgeable students may have an advantage in using their available resources to learn and automate the relevant problem solving steps by studying Step Guidance examples rather than wasting these resources on processing unnecessary theorem information in the Theorem & Step Guidance worked examples. To conclude, learning is a process of acquiring schemas through integrating new information with existing knowledge structures. If a task is too complex to process, the number of elements of information that need to be processed might exceed working memory capacity limits and learning may be inhibited. Learning how to solve a geometric problem usually involves high levels

of element interactivity and is therefore a difficult task to learn and master. For expert problem solvers, simultaneously processing many elements or schemas may not be problematic; however novice problem solvers are usually faced with a considerably higher cognitive load. Reducing any unnecessary extraneous load caused by random search processes will free up working memory resources available to novice problem solvers. Providing Theorem & Step Guidance examples may help to reduce this load for novice learners. For more expert learners, eliminating theorem guidance that is redundant for them may reduce extraneous cognitive load. It is the responsibility of instructional designers to eliminate an unnecessary cognitive load.

10.3 Limitations of the Study and Future Research

The redundancy effect that was demonstrated in Experiment 4 of this study indicated that the effectiveness of Theorem & Step Guidance and Step Guidance as methods of instructional support with different degrees of guidance might rely on prior geometry proficiency and levels of learner expertise. Consequently, it is important to be able to assess the level of learner expertise and accordingly provide a level of guidance and support tailored to specific learner needs. Further studies are required to examine the effectiveness of this kind of adaptive learning environment that focuses on individually tailored levels of support and guidance.

The most important instructional implication of this study is the necessity to develop learning environments that assess levels of learner' prior knowledge and accordingly alter mathematical support levels. Specific knowledge structures that may influence the instructional effectiveness of geometry examples need to be examined by conducting more research. Instructional materials that are learner-controlled but still include possible mathematical support could also be considered as an alternative approach that would overcome the complexity of assessing learner levels of expertise correctly.

One of the limitations of this study is the lack of a prior knowledge test. Prior knowledge of the relevant theorems and problem solving steps need to be evaluated in future studies to have clear and objective measures of actual levels of learner expertise in specific geometry task areas. Using a test like NAPLAN can provide this information, or WRAT-4 where a more global measure of math expertise is required.

10.4 Conclusions

The results of this study have not only theoretical significance but also practical implications for the development of instructional materials and mathematical instruction when teaching geometric problem solving.

Based on extensive research in cognitive load theory, worked examples represent a very effective instructional technique. In this study, Step Guidance and Theorem & Step Guidance methods also had a significant effect on learning geometry since these methods allowed learners to avoid random search activities and thus reduced extraneous cognitive load by providing relevant knowledge and activating already constructed schemas.

A new factor examined in this study was the relation between knowledge of geometric theorems and learning to solve geometry problems with worked examples. Since for the students who already understand the relevant theorems, learning to solve problems primarily consists of learning to recognise problem states and their associated moves, information concerning theorems should only be provided to students who have yet to learn and automate theorem schemas.

The results of this research also provide an additional, evidence-based justification for the effectiveness of using worked examples in teaching problem-solving steps in specific task domains in order to reduce extraneous cognitive load and achieve the best learning outcomes. In addition, the idea that studying the theorems always benefits solving a geometric problem was tested. Providing theorems could have either positive or negative effects depending on levels of learner expertise. Learning geometric problem solving may be affected by different presentation formats and degrees of guidance that relate to the redundancy and expertise reversal effects as proposed by cognitive load theory.

Of course, these results should not be interpreted as indicating that instruction in the use of geometric theorems can never be beneficial. They may benefit younger learners during early stages while building their mathematical knowledge or may provide useful reference information for more advanced learners. Nevertheless, presenting theorem guidance within worked examples appears to have negative consequences for more knowledgeable students. Knowledge levels are critical when designing instruction.

References

- Alexander, P., & Judy, J. (1988). The interaction of domain-specific and strategic knowledge in academic performance. *Review of Educational Research* 58, 375- 404.
- Ayres, P. (1993). Why goal-free problems facilitate learning. *Contemporary Educational Psychology*, 18, 376-381.
- Ayres, P. (2006a). Impact of reducing intrinsic cognitive load on learning in a mathematical domain. *Applied Cognitive Psychology*, 20, 287-298.
- Ayres, P. (2006b). Using subjective measures to detect variations of intrinsic cognitive load within problems. *Learning and Instruction*, 16, 389-400.
- Baddeley, A. D. (1992). Working memory. *Science*, 255, 556-559.
- Beckmann, J. (2010). Taming a beast of burden-On some issues with the conceptualisation and operationalisation of cognitive load. *Learning and Instruction*, 20, 250-264.
- Bentin, S., Deouell, L. Y., & Soroker, N. (1999). Selective visual streaming in Face recognition: Evidence from developmental prosopagnosia. *NeuroReport*, 10, 823-827.
- Blayney, P., Kalyuga, S., & Sweller, J. (2009). Interactions between the isolated-interactive elements effect and levels of learner expertise: experimental evidence from an accountancy class. *Instructional Science*, 38, 277-287.
- Board of Senior Secondary Studies, (1994). *Examiner's Report* (Mathematics 1). Brisbane.

- Bobis, J., Sweller, J., & Cooper, M. (1993). Cognitive load effects in a primary school geometry task. *Learning and Instruction, 3*, 1-21.
- Brunken, R., Plass, J., & Leutner, D. (2003). Direct measurement of cognitive load in multimedia learning. *Educational Psychologist, 38*, 53-61.
- Brünken, R., Seufert, T., & Paas, F. (2010). Measuring Cognitive Load. In J. Plass, R. Moreno, R. Brunken (Eds), *Cognitive Load Theory*. New York: Cambridge University Press.
- Brünken, R., Steinbacher, S., Plass, J. L., & Leutner, D. (2002). Assessment of cognitive load in multimedia learning using dual-task methodology. *Experimental Psychology, 49*, 109-119.
- Byers, V., & Erlwanger, S. (1985). Memory in mathematical understanding. *Educational Studies in Mathematics 16*, 259-281.
- Carroll, W. (1994). Using worked examples as an instructional support in the algebra classroom. *Journal of Educational Psychology, 86*, 360-367.
- Catrambone, R. (1994). Improving examples to improve transfer to novel problems. *Memory and Cognition, 22*, 606-615.
- Chandler, P., & Sweller, J. (1991). Cognitive load theory and the format of instruction. *Cognition and Instruction, 8*, 293-332.
- Chandler, P., & Sweller, J. (1992). The split-attention effect as a factor in the design of instruction. *British Journal of Educational Psychology, 62*, 233-246.
- Chandler, P., & Sweller, J. (1996). Cognitive load while learning to use a computer program. *Applied Cognitive Psychology, 10*, 151-170.

- Chase, W. G., & Simon, H. A. (1973). Perception in chess. *Cognitive Psychology*, 4, 55-81.
- Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, 24, 359-387.
- Chi, M., Glaser, R., & Rees, E. (1982). Expertise in problem solving. In R. Stenberg (Ed.), *Advances in psychology of human intelligence*. (pp. 7- 75). Hillsdale, NJ: Erlbaum.
- Chiesi, H., Spilich, G., & Voss, J. F. (1979). Acquisition of domain-related information in relation to high and low domain knowledge. *Journal of Verbal learning and Verbal Behaviour*, 1, 257-273.
- Chinnappan, M. (1998). Schemas and mental models in geometry problem solving. *Educational Studies in Mathematics*, 36, 201-217.
- Clarke, T., Ayres, P., & Sweller, J. (2005). The impact of sequencing and prior knowledge on learning mathematics through spreadsheet applications. *Educational Technology, Research and Development*, 53, 15-24.
- Cooper, G. (1998). *Research into Cognitive Load Theory and Instructional Design at UNSW*. Retrieved from http://www.arts.unsw.edu.au/education/CLT_NET_Aug_97.HTML.
- Cooper, G., & Sweller, J. (1987). Effects of schema acquisition and rule automation on mathematical problem-solving transfer. *Journal of Educational Psychology*, 79, 347-362.

- Cowan, N. (2001). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences*, 24, 87-185.
- De Groot, A. (1965). *Thought and choice in chess*: The Hague, Netherlands: Mouton. (Original work published in 1964).
- De Villiers, M. (1991). Pupils' needs for conviction and explanation within the context of geometry. *Pythagoras*, 26, 18-27.
- Duval, R. (1991). Structure du raisonnement déductif et apprentissage de la démonstration. *Educational Studies in Mathematics*, 22, 233-261.
- Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Eds.), *Perspectives on the Teaching of Geometry for the 21st Century: An ICMI study* (pp. 37-52). Dordrecht: Kluwer.
- Egan, D. E., & Schwartz B. J. (1979). Chunking in recall of symbolic drawings. *Memory and Cognition*, 7, 149-158.
- English, L. D., & Halford, G. S. (1995). *Mathematics Education: Models and processes*. Hillsdale, NJ: Lawrence Erlbaum.
- Ericsson, K. A., & Charness, N. (1994). Expert performance: Its structure and acquisition. *American Psychologist*, 49, 725-747.
- Ericsson, K. A., & Kintsch, W. (1995). Long-term working memory. *Psychological Review*, 102, 211-245.
- Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education* [monograph number 3]. Reston, VA: NCTM.

- Galbraith, P. L. (198). Aspects of proving: A clinical investigation of process. *Educational Studies in Mathematics* 12, 1-28.
- Geary, D. C. (2002). Principles of evolutionary educational psychology. *Learning and Individual Differences*, 12, 317-345.
- Geary, D. C. (2007a). Educating the evolved mind: Conceptual foundations for an evolutionary educational psychology. In J. S. Carlson & J. R. Levin (Eds.), *Psychological perspectives on contemporary educational issues* (pp. 1-99). Greenwich, CT: Information Age.
- Geary, D. C. (2007b). Educating the evolved mind: Reflections and refinements. In J. S. Carlson & J. R. Levin (Eds.), *Psychological perspectives on contemporary educational issues* (pp. 1-99). Greenwich, CT: Information Age.
- Geary, D. C. (2008). An evolutionarily informed education science. *Educational Psychologist*, 43, 179-195.
- Gerjets, P., Scheiter, K., & Catrambone, R. (2004). Designing instructional examples to reduce intrinsic cognitive load: Molar versus modular presentation of solution procedures. *Instructional Science*, 32, 33-58.
- Gerjets, P., Scheiter, K., & Catrambone, R. (2006). Can learning from molar and modular worked examples be enhanced by providing instructional explanations and prompting self-explanations? *Learning and Instruction*, 16, 104-121.
- Greeno, J. G. (1978). A study of problem solving. In R Glaser (Ed.), *Advances in Instructional Psychology (Vol II)*. Hillsdale, NJ: Lawrence Erlbaum.

- Halabi, A. K., Tuovinen, J. E., & Farley, A. A. (2005). Empirical evidence on the relative efficiency of worked examples versus problem-solving exercises in accounting principles instruction. *Issues in Accounting Education*, 20, 21-32.
- Halford, G. S. (1993). *Children's understanding: The development of mental models*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Hattie, J. (2009). *Visible Learning; A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.
- Healy, L., & Hoyles, C. (1998). *Justifying and proving in school mathematics*. Technical Report on the Nationwide Survey, London: Institute of Education, University of London.
- Healy, L. & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education*, 31, 396-428.
- Hoyles, C., & Healy, L. (1999). Linking informal argumentation with formal proof through computer-integrated teaching experiment. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, 3, (pp.105-112). Israel: Haifa University.
- Hilbert, T. S., Renkl, A., Kessler, S., & Reiss, K. (2008). Learning to prove in geometry: Learning from heuristic examples and how it can be supported. *Learning and Instruction*, 18, 54-65.
- Hoogveld, A. W. M., Paas, F., Jochems, W. M. G., & Van Merriënboer, J. J. G. (2002). Exploring teachers' instructional design practices from a systems design perspective. *Instructional Science*, 30, 291-305.

- Hoogveld, A. W. M., Paas, F., & Jochems, W. M. G. (2003). Application of an instructional systems design approach by teachers in higher education: individual versus team design. *Teaching and Teacher Education, 19*, 581-590.
- Kalyuga, S. (2007). Expertise reversal effect and its implications for learner-tailored instruction. *Educational Psychology Review, 19*, 509-539.
- Kalyuga, S. (2009). Instructional designs for the development of transferable knowledge and skills: A cognitive load perspective. *Computers in Human Behavior, 25*, 332-338.
- Kalyuga, S. (2009b). *Managing cognitive load in adaptive multimedia learning*. Hershey, PA: Information Science Reference.
- Kalyuga, S., Ayres, P., Chandler, P., & Sweller, J. (2003). The expertise reversal effect. *Educational Psychologist, 38*, 23-32.
- Kalyuga, S., Chandler, P., & Sweller, J. (1998). Levels of expertise and instructional design. *Human Factors, 40*, 1-17.
- Kalyuga, S., Chandler, P., Tuovinen, J., & Sweller, J. (2001). When problem solving is superior to studying worked examples. *Journal of Educational Psychology, 93*, 579-588.
- Kalyuga, S., & Renkl, A. (2010). Expertise reversal effect and its instructional implications: introduction to the special issue. *Instructional Science, 38*, 209-215.
- Kalyuga, S., & Sweller, J. (2004). Measuring knowledge to optimize cognitive load factors during instruction. *Journal of Educational Psychology, 96*, 558-568.

- Kester, L., Kirschner, P. A., & van Merriënboer, J. J. G. (2006). Just-in-time information presentation: Improving learning a troubleshooting skill. *Contemporary Educational Psychology, 31*, 167-185.
- Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review 92*, 109-129.
- Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive Psychology, 17*, 248-294.
- Kuhl, P. (2000). A new view of language acquisition. *The Proceedings of the National Academy of Sciences, USA, 97, 22*, 11850-11857.
- Larkin, J., McDermott, J. R., & Simon, D., & Simon, H. (1980). Models of competence in solving physics problems. *Cognitive Science, 4*, 317-348.
- Lawson, M. J., & Chinnappan, M. (1994). Generative activity during geometry problem solving: Comparison of the performance of high-achieving and low-achieving students. *Cognition and Instruction, 12*, 61-93.
- Leahy, W., Chandler, P., & Sweller, J. (2003). When auditory presentations should and should not be a component of multimedia instruction. *Applied Cognitive Psychology, 17*, 401- 418.
- Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education, 25*, 660-675.
- Marcus, N., Cooper, M., & Sweller, J. (1996). Understanding instructions. *Journal of Educational Psychology, 88*, 49-63.
- Mayer, R. E. (1975). Information processing variables in learning to solve problems. *Review of Educational Research, 45*, 525-541.

- Mayer, R. E. (1983). *Thinking, Problem Solving, Cognition*. W.H. Freeman & Co: NY.
- Miller, G. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63, 81-97.
- Nesher, P., & Hershkovitz, S. (1994). The role of schemes in two-step problems: Analysis and research findings. *Educational Studies in Mathematics*, 26, 1-23.
- Nückles, M., Hübner, S., Dümer, S., & Renkl, A. (2009). Expertise reversal effects in writing-to-learn. *Instructional Science*, 38, 237-258.
- Oksa, A., Kalyuga, S., & Chandler, P. (2009). Expertise reversal effect in using explanatory notes for readers of Shakespearean text. *Instructional Science*, 38, 217-236.
- Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems? *Journal of Educational Psychology*, 77, 272-284.
- Paas, F. (1992). Training strategies for attaining transfer of problem solving skill in statistics: A cognitive load approach. *Journal of Educational Psychology*, 84, 429-434.
- Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist*, 38, 1-4.
- Paas, F., Renkl, A., & Sweller, J. (2004). Cognitive load theory: Instructional implications of the interaction between information structures and cognitive architecture. *Instructional Science*, 32, 1-8.

- Paas, F., Tuovinen, J., Tabbers, H., & Van Gerven, P. (2003). Cognitive load measurement as a means to advance cognitive load theory. *Educational Psychologist*, 38, 63-71.
- Paas, F., & van Merriënboer, J. J. G. (1993). The efficiency of instructional conditions: An approach to combine mental effort and performance measures. *Human Factors*, 35, 737-743.
- Paas, F., & van Merriënboer, J. J. G. (1994a). Variability of worked examples and transfer of geometrical problem solving skills: A cognitive load approach. *Journal of Educational Psychology*, 86, 122-133.
- Paas, F., Van Merriënboer, J. J. G., & Adam (1994b). Measurement of cognitive load in instructional research. *Perceptual and Motor Skills*, 79, 419-430.
- Pegg, J. (1992). Assessing students' understanding of the primary and secondary levels in the mathematical sciences. In M. Stephens and J. Izard (Eds.) *Reshaping Assessment Practices: Assessment in the Mathematical Sciences Under Challenge* (pp.368-385). Melbourne: A.C.E.R.
- Peterson, L.R., & Peterson M.J. (1959). Short-term retention of individual verbal items. *Journal of Experimental Psychology*, 58, 193-8.
- Poincare, H. (1913). Mathematical creation. In J. R. Newman (Ed.). *The world of Mathematics*, Vol. 3 (pp. 2041-2050). New York, NY: Simon and Schuster.
- Pollock, E., Chandler, P., & Sweller, J. (2002). Assimilating complex information. *Learning and Instruction*, 12, 61-86.
- Prawat, R. (1989). Promoting access to knowledge, strategy and disposition in students. *Review of Educational Research*, 59, 1-42.

- Reiss, K., Klieme, E., & Heinze, A. (2001). Prerequisites for the understanding of proofs in the geometry classroom. In M. v. d. Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the international group for the psychology of mathematics education*, 4, 97-104. Utrecht, The Netherlands: Utrecht University.
- Reisslein, J., Atkinson, R. K., Seeling, P., & Reisslein, M. (2006). Encountering the expertise reversal effect with a computer-based environment on electrical circuit analysis. *Learning and Instruction*, 16, 92-103.
- Renkl, A., & Atkinson, R. K. (2003). Structuring the transition from example study to problem solving in cognitive skill acquisition: A cognitive load perspective. *Educational Psychologist*, 38, 15-22.
- Renkl, A., Atkinson, R. K., & Große, C. S. (2004). How fading worked solution steps works: A cognitive load perspective. *Instructional Science*, 32, 59-82.
- Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Hillsdale, NJ: Lawrence Erlbaum.
- Salden, R. J. C. M., Paas, F., Broers, N. J., & van Merriënboer, J. J. G. (2004). Mental effort and performance as determinants for the dynamic selection of learning tasks in air traffic control training. *Instructional Science*, 32, 153-172.
- Schneider, W., & Shiffrin, R. (1977). Controlled and automatic human information processing: I. Detection, search and attention. *Psychological Review*, 84, 1-66.

- Schnotz, W., & Kurschner, C. (2007). A reconsideration of cognitive load theory. *Educational Psychology Review*, 19, 469-508.
- Senk, S. (1985). How well do students write geometry proofs? *Mathematics Teacher*, 78, 448- 456.
- Shiffrin, R., & Schneider, W. (1977). Controlled and automatic human information processing: II. Perceptual learning, automatic attending, and a general theory. *Psychological Review*, 84, 127-190.
- Simon, H., & Gilmarin, K. (1973). A simulation of memory for chess positions. *Cognitive Psychology*, 1, 29-46.
- Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, 12, 257-285.
- Sweller, J. (1989). Cognitive technology: Some procedures for facilitating learning, problem solving in mathematics and science. *Journal of Educational Psychology*, 81, 457- 466.
- Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, 4, 295-312.
- Sweller, J. (1999). *Instructional design in technical areas*. Melbourne: ACER Press.
- Sweller, J. (2003). Evolution of human cognitive architecture. *The Psychology of Learning and Motivation*, 43, 215-266.
- Sweller, J. (2004). Instructional design consequences of an analogy between evolution by natural selection and human cognitive architecture. *Instructional Science*, 321, 9-31.

- Sweller, J. (2006a). The worked example effect and human cognition. *Learning and Instruction, 16*, 165-169.
- Sweller, J. (2006b). Discussion of emerging topics in cognitive load research: Using learner and information characteristics in the design of powerful learning environments. *Applied Cognitive Psychology, 20*, 353-357.
- Sweller, J. (2006c). How the human system deals with complexity. In J. Elen & R. E. Clark (Eds.), *Handling complexity in learning environments* (pp. 25). Amsterdam: Elsevier.
- Sweller, J. (2008). Instructional implications of David C. Geary's evolutionary educational psychology. *Educational Psychologist, 43*, 214-216.
- Sweller, J. (2009). Cognitive bases of human creativity. *Educational Psychology, 21*, 11-19.
- Sweller, J. (2010). Element interactivity and intrinsic, extraneous, and germane cognitive load. *Educational Psychology Review, 22*, 123-138.
- Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive Load Theory*. New York: Springer.
- Sweller, J., Chandler, P., Tierney, P., & Cooper, M. (1990). Cognitive load and selective attention as factors in the structuring of technical material. *Journal of Experimental Psychology, 119*, 176-192.
- Sweller, J., & Chandler, P. (1994). Why some material is difficult to learn. *Cognition and Instruction, 12*, 185-233
- Sweller, J., & Cooper, G. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction, 2*, 59-89.

- Sweller, J., & Sweller, S. (2006). Natural information processing systems. *Evolutionary Psychology*, 4, 434-458.
- Sweller, J., Van Merriënboer, J. J. G., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational Psychology Review*, 10, 251-296.
- Tall, D. O., (1995). Mathematical growth in elementary and advanced mathematical thinking. *Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education*, Recife, Brazil, I, 61-75.
- Tarmizi, R., & Sweller, J. (1988). Guidance during mathematical problem solving. *Journal of Educational Psychology*, 80, 424-436.
- Tindall-Ford, S., Chandler, P., & Sweller, J. (1997). When two sensory modes are better than one. *Journal of Experimental Psychology: Applied*, 3, 257-287.
- Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (CDASS Project Report), 231 pages.
- Van Dormolen, J. (1977). Learning to understand what a proof really means. *Educational Studies in Mathematics*, 8, 27-34.
- Van Gerven, P. W. M., Paas, F., van Merriënboer, J. J. G., & Schmidt, H. G. (2006). Modality and variability as factors in training the elderly. *Applied Cognitive Psychology*, 20, 311-320.
- Van Gerven, P. W. M., Paas, F., Van Merriënboer, J. J. G., & Schmidt, H. G. (2002). Cognitive load theory and aging: Effects of worked examples on training efficiency. *Learning and Instruction*, 12, 87-105.

- Van Gog, T., Kester, L., & Paas, F. (2011). Effects of worked examples, example-problem, and problem-example pairs on novices' learning. *Contemporary Educational Psychology*, 36, 212-218.
- Van Gog, T., Paas, F., & van Merriënboer, J. J. G. (2004). Process-oriented worked examples: Improving transfer performance through enhanced understanding. *Instructional Science*, 32, 83-98.
- Van Gog, T., Paas, F., & van Merriënboer, J. J. G. (2006a). Effects of process-oriented worked examples on troubleshooting transfer performance. *Learning and Instruction*, 16, 154-164.
- Van Gog, T., Paas, F., & Van Merriënboer, J. J. G. (2006). Enhancing troubleshooting transfer performance: Effects of product-oriented versus process-oriented worked examples. In G. Clarebout & J. Elen (Eds.), *Avoiding simplicity, confronting complexity: Advances in studying and designing (computer-based) powerful learning environments* (pp. 339-347). Rotterdam, The Netherlands: Sense Publishers.
- Van Gog, T., Paas, F., & Van Merriënboer, J. J. G. (2008). Effects of studying sequences of process-oriented and product-oriented worked examples on troubleshooting transfer efficiency. *Learning and Instruction*, 18, 211-222.
- Van Gog, T., & Paas, F. (2008). Instructional efficiency: Revisiting the original construct in educational research. *Educational Psychologist*, 43, 16-26.
- Van Hiele, P. M. (1959/1984). The child's thought and geometry. In D. Fuys, D. Geddes and R. Tischler (Eds.). *English translation of selected writings of Dina Van Hiele- Geldof and Pierre M. van Hiele*. Brooklyn, NY: School of Education, Brooklyn College.

- Van Hiele-Geldof, D. (1957/1984). The didactics of geometry in the lowest class of secondary school. In D. Fuys, D. Geddes & R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M van Hiele* (pp. 1- 214). Brooklyn: Brooklyn College.
- Van Loon-Hillen, N., Van Gog, T., & Brand-Gruwel, S. (2010). Effects of worked examples in a primary school mathematics curriculum. *Interactive Learning Environments*, 18, 1-11.
- Van Merriënboer, J.J.G. (1997). *Training Complex Cognitive Skills: A Four-Component Instructional Design Model for Technical Training*. Englewood Cliffs, NJ: Educational Technology Publications.
- Van Merriënboer, J. J. G., Kester, L., & Paas, F. (2006). Teaching complex rather than simple tasks: Balancing intrinsic and germane load to enhance transfer of learning. *Applied Cognitive Psychology*, 20, 343-352.
- Van Merriënboer, J. J. G., & Paas, F. (1990). Automation and schema acquisition in learning elementary compute programming: Implications for the design of practice. *Computer in Human Behavior*, 6, 273-289.
- Van Merriënboer, J. J. G., & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational Psychology Review*, 17, 147- 177.
- Yerushalmy, M., & Chazan, D. (1993). Overcoming visual obstacles with the aid of the Supposer. In: Schwartz, Yerushlami & Wilson (Eds.), *The Geometric Supposer. What is it a case of?* Hillsdale: Lawrence Erlbaum Associates, pp. 26-56.

- Young, R.M. (1983). Surrogates and mappings: Two kinds of conceptual models for interactive devices. In: D. Gentner & A.L. Stevens (Eds), *Mental Models* (pp. 35–52). Hillsdale, NJ: LEA.
- Zhu, X., & Simon, H. A. (1987). Learning mathematics from examples and by doing. *Cognition and Instruction*, 4, 137-166.

Appendices

Appendix A

Similar and Transfer Test Questions of Experiments 1 & 2

Sets of Similar Test Problems

NAME _____

FIND y

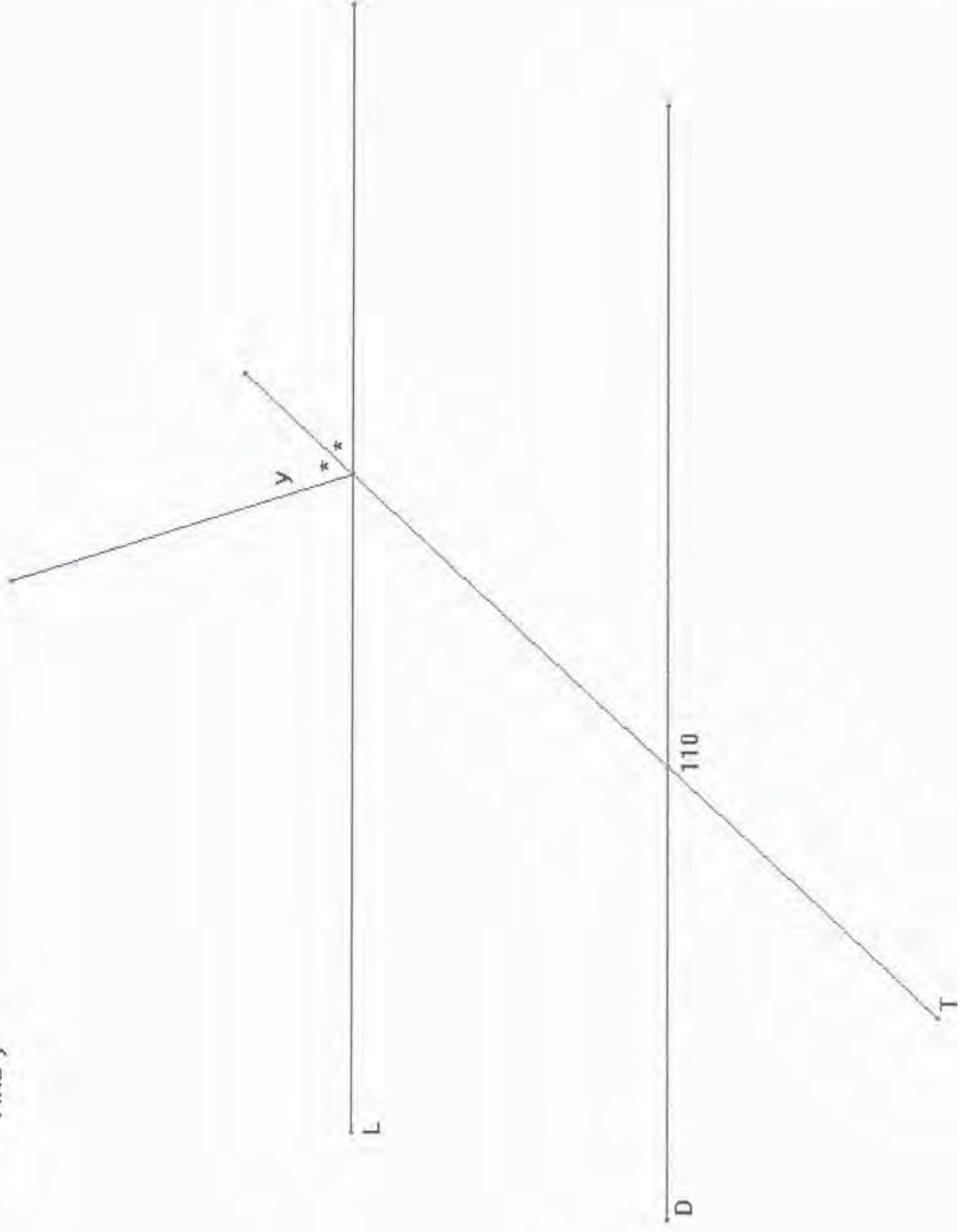


Figure Not to Scale

FIND y

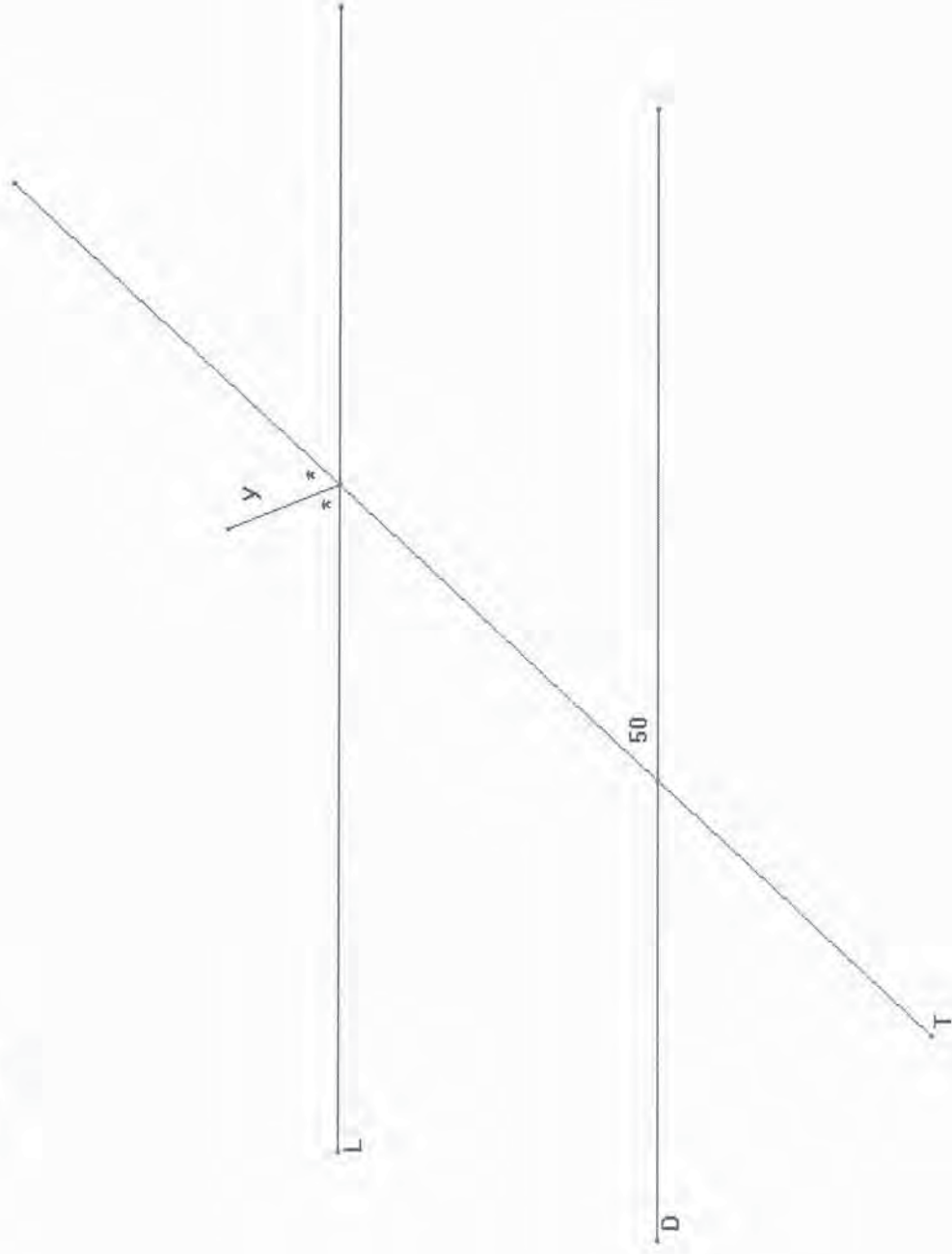


Figure Not to Scale

FIND y

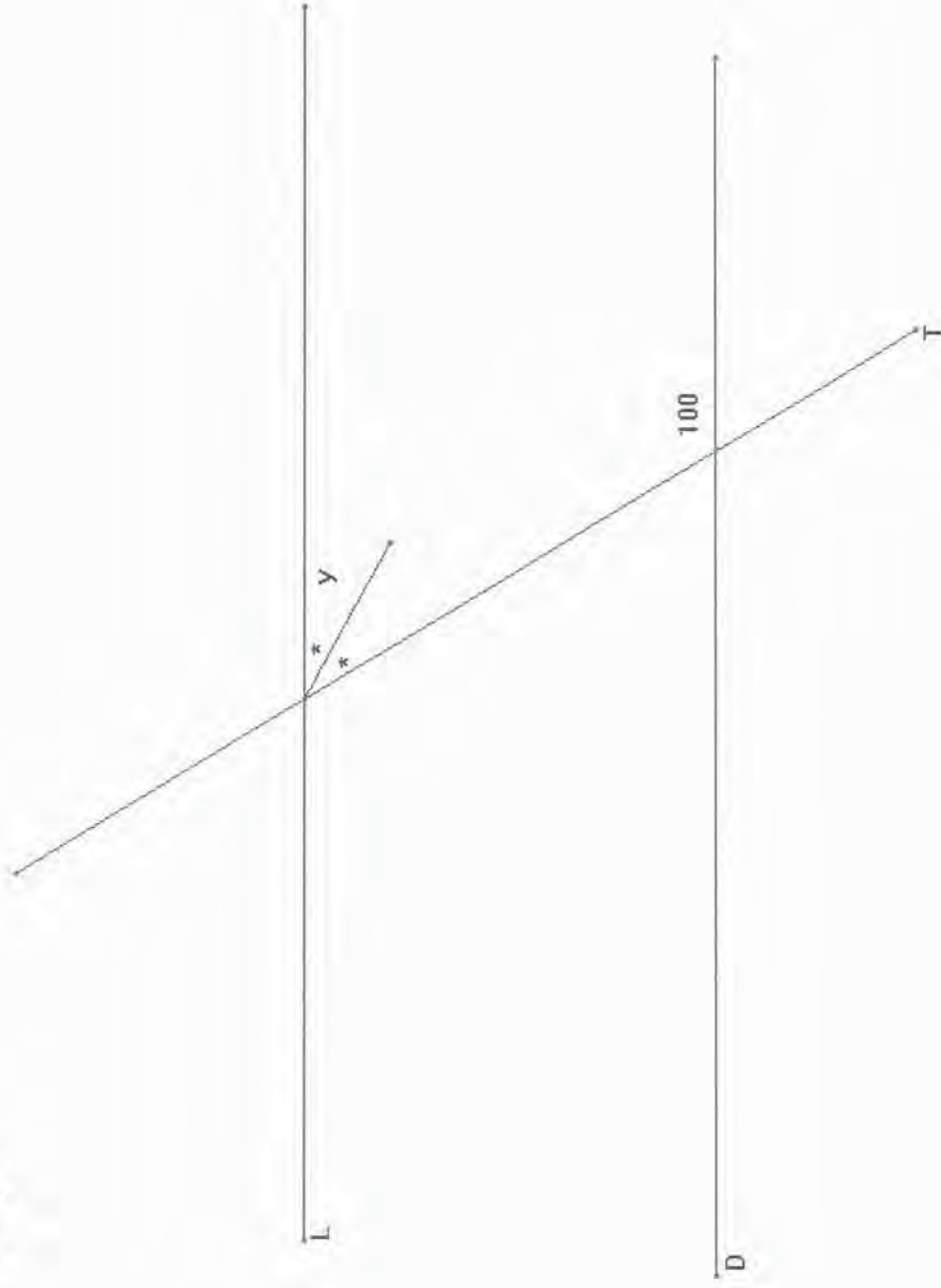


Figure Not to Scale

FIND y

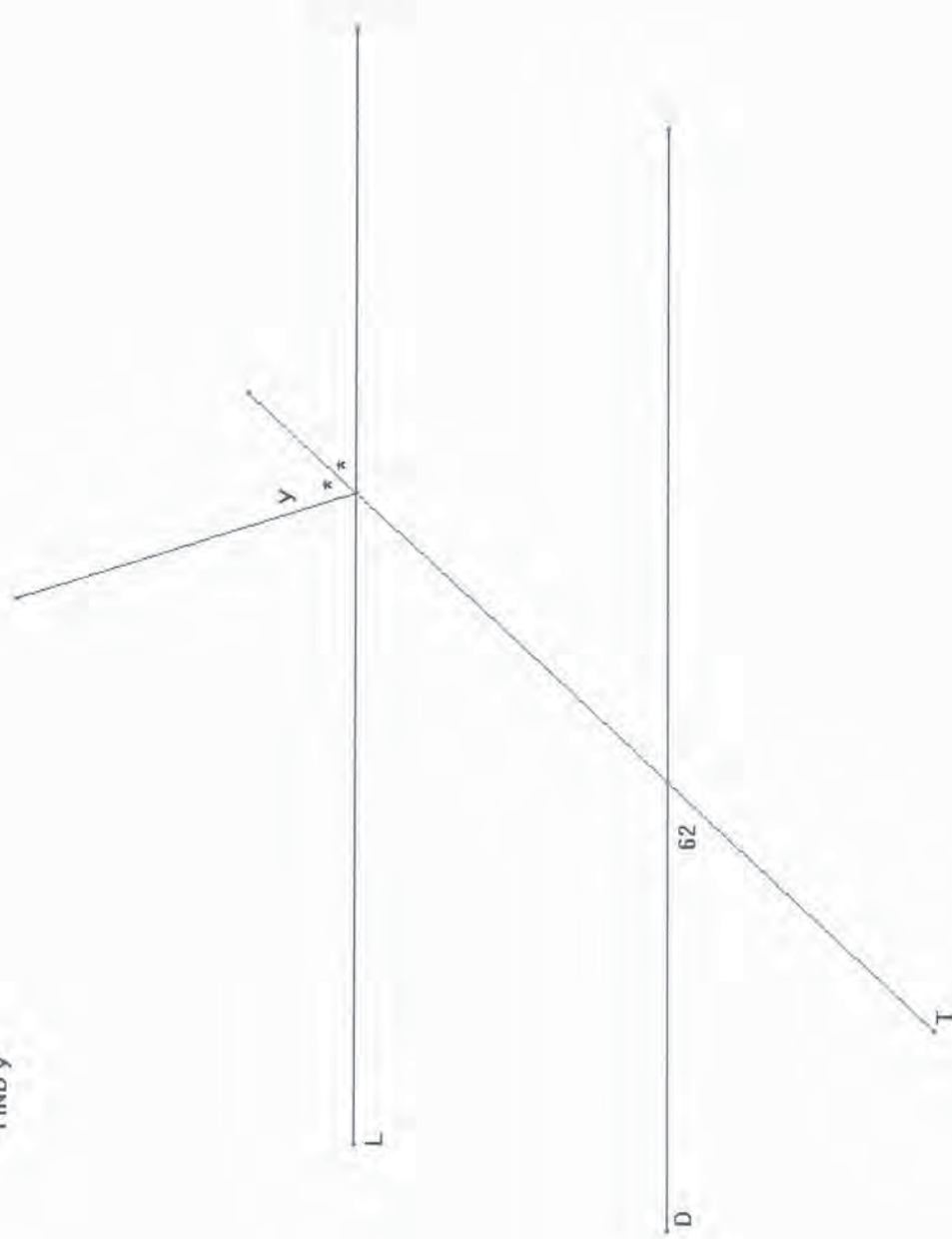


Figure Not to Scale

Sets of Transfer Test Problems

NAME _____

FIND y

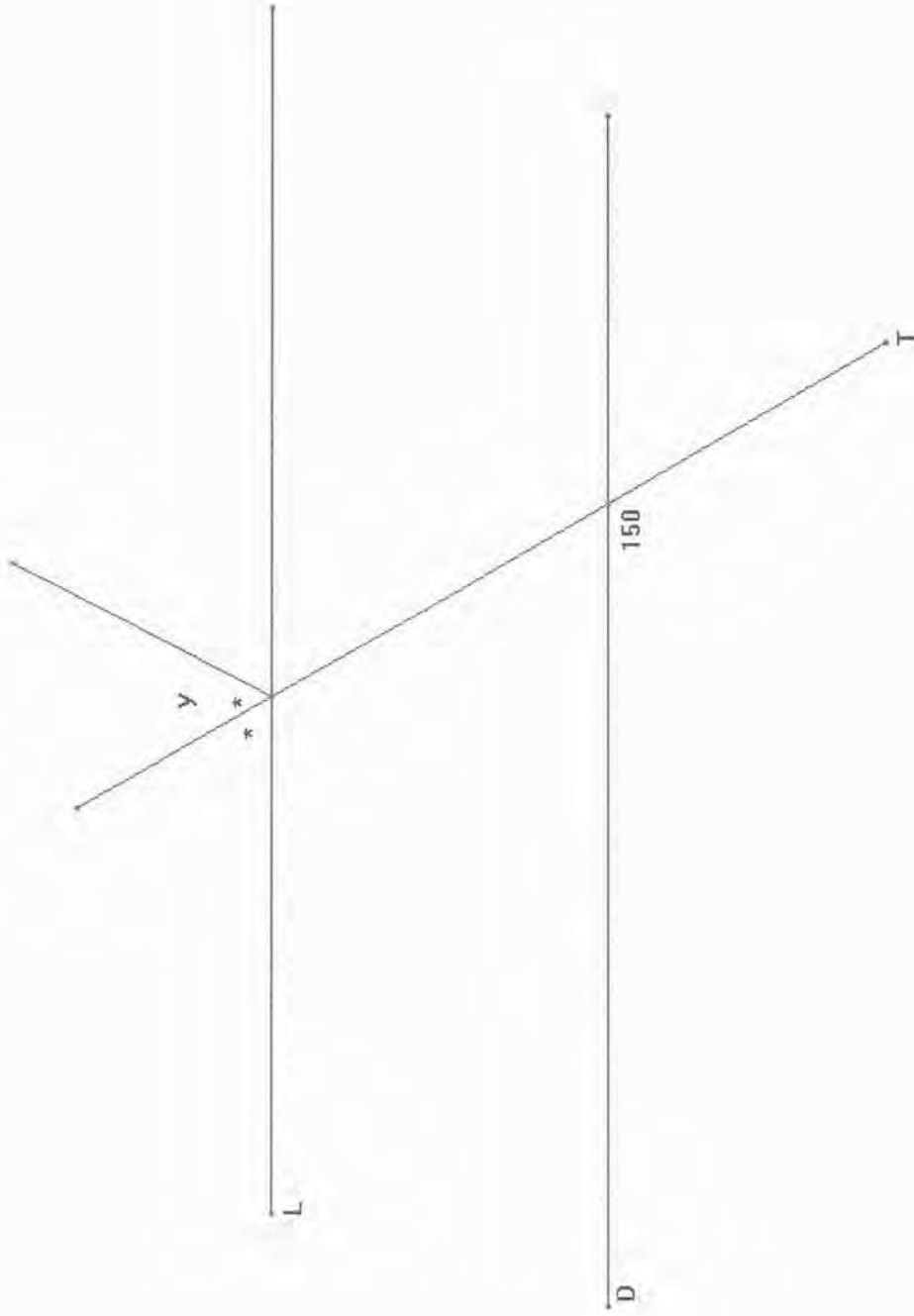


Figure Not to Scale

FIND y

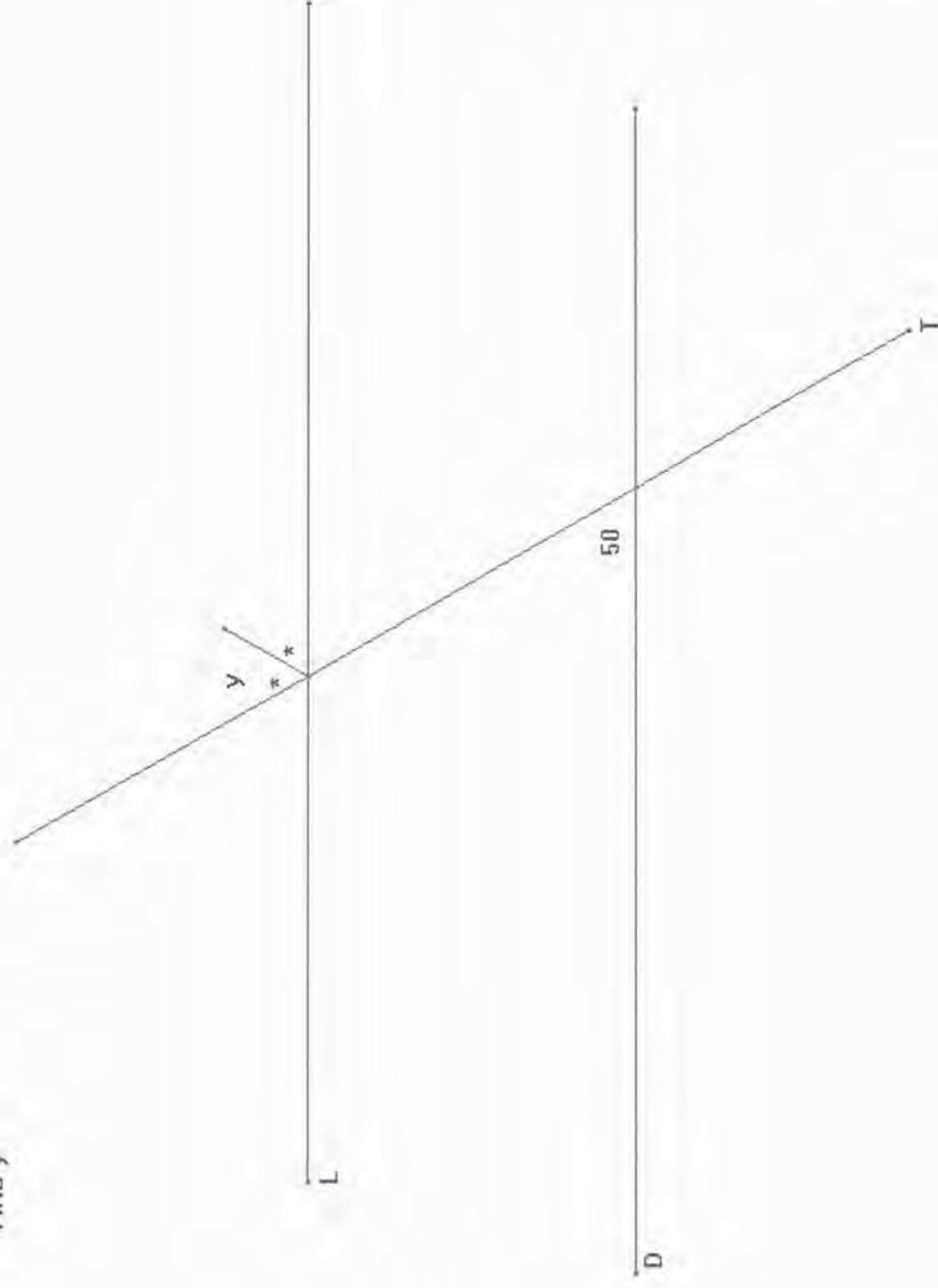


Figure Not to Scale

FIND y

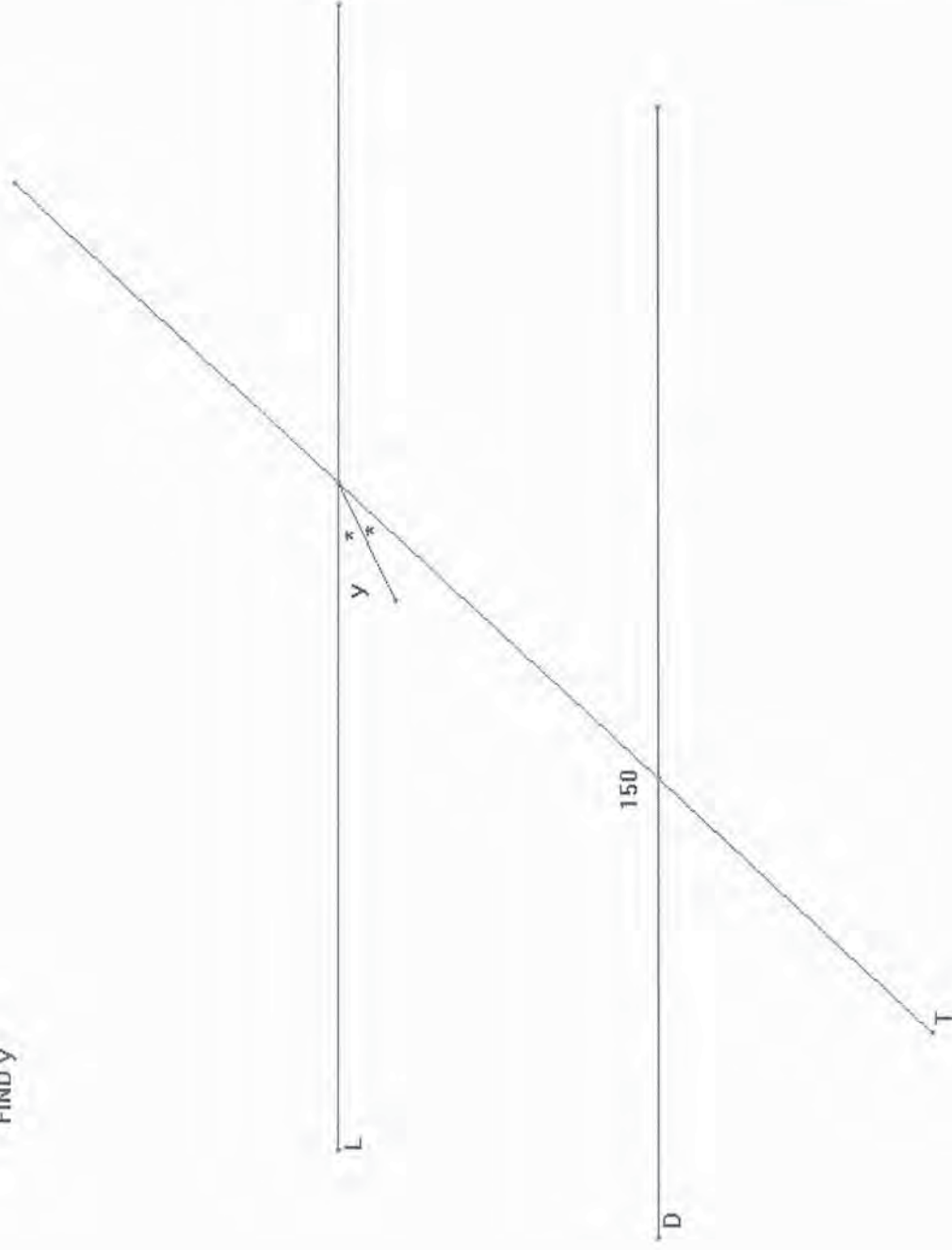


Figure Not to Scale

Appendix B

Sets of problems given to the Theorem & Step Guidance group of Experiments 1, 2, & 3

NAME _____

FIND y

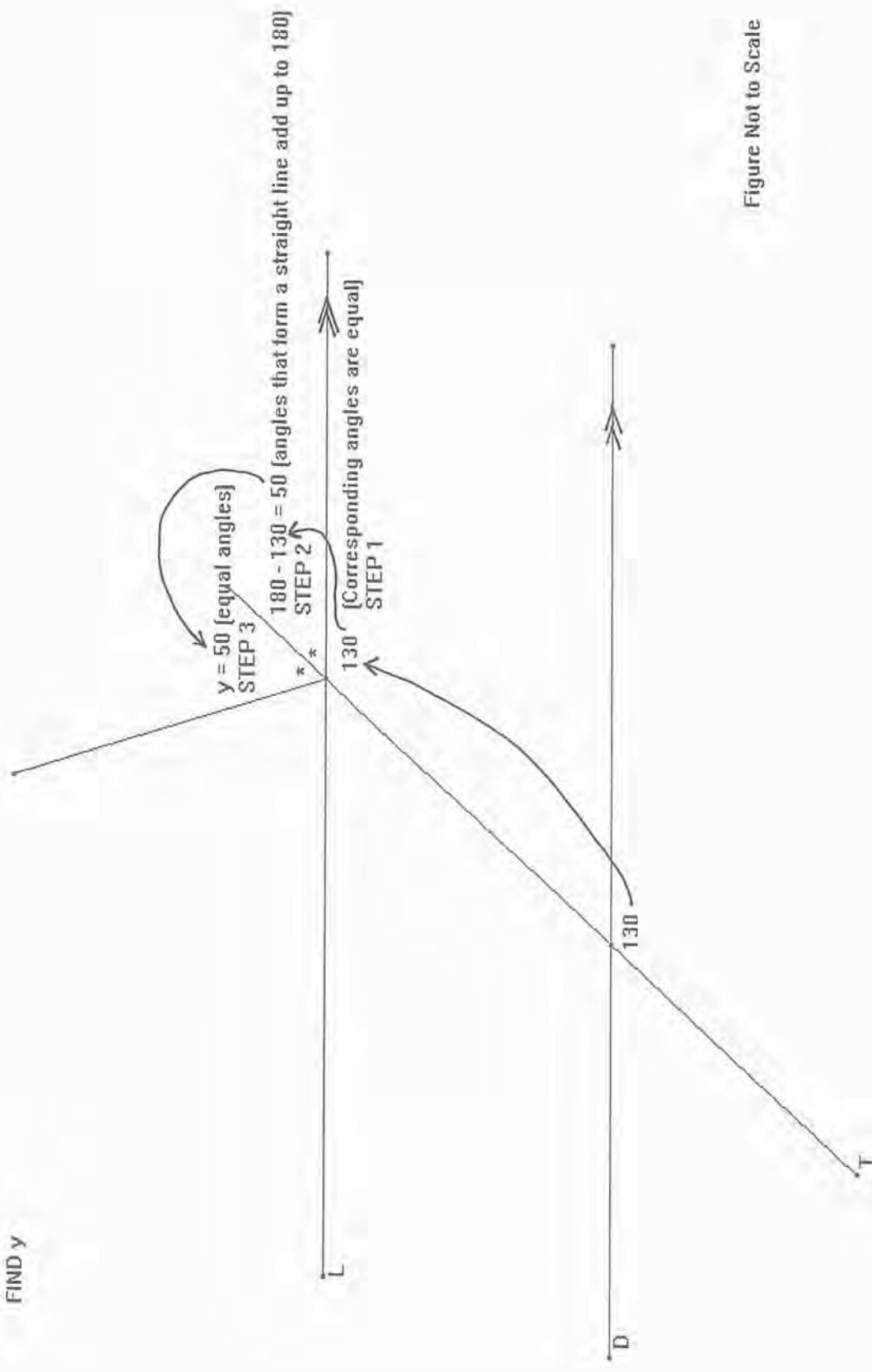


Figure Not to Scale

FIND y

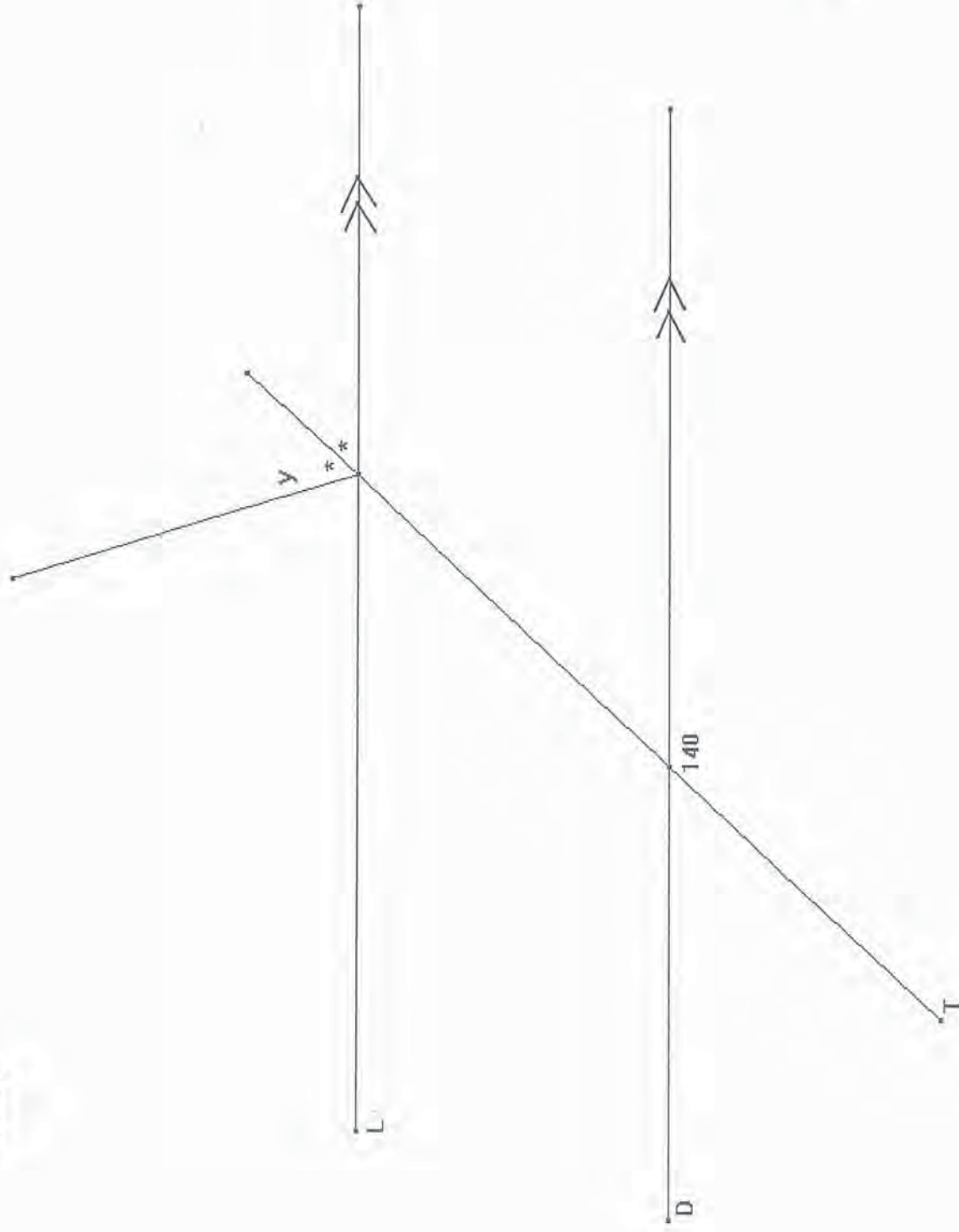


Figure Not to Scale

FIND y

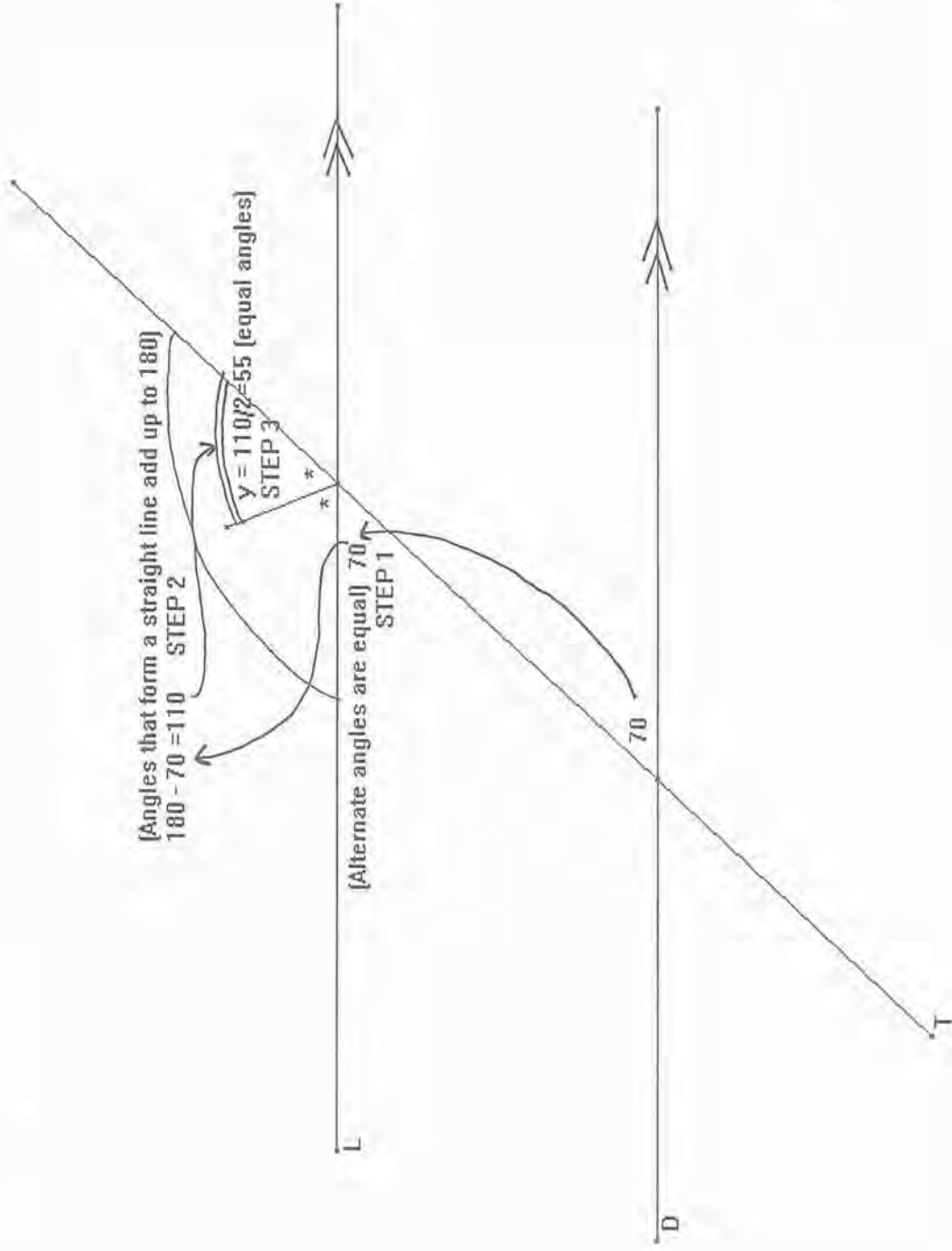


Figure Not to Scale

FIND y

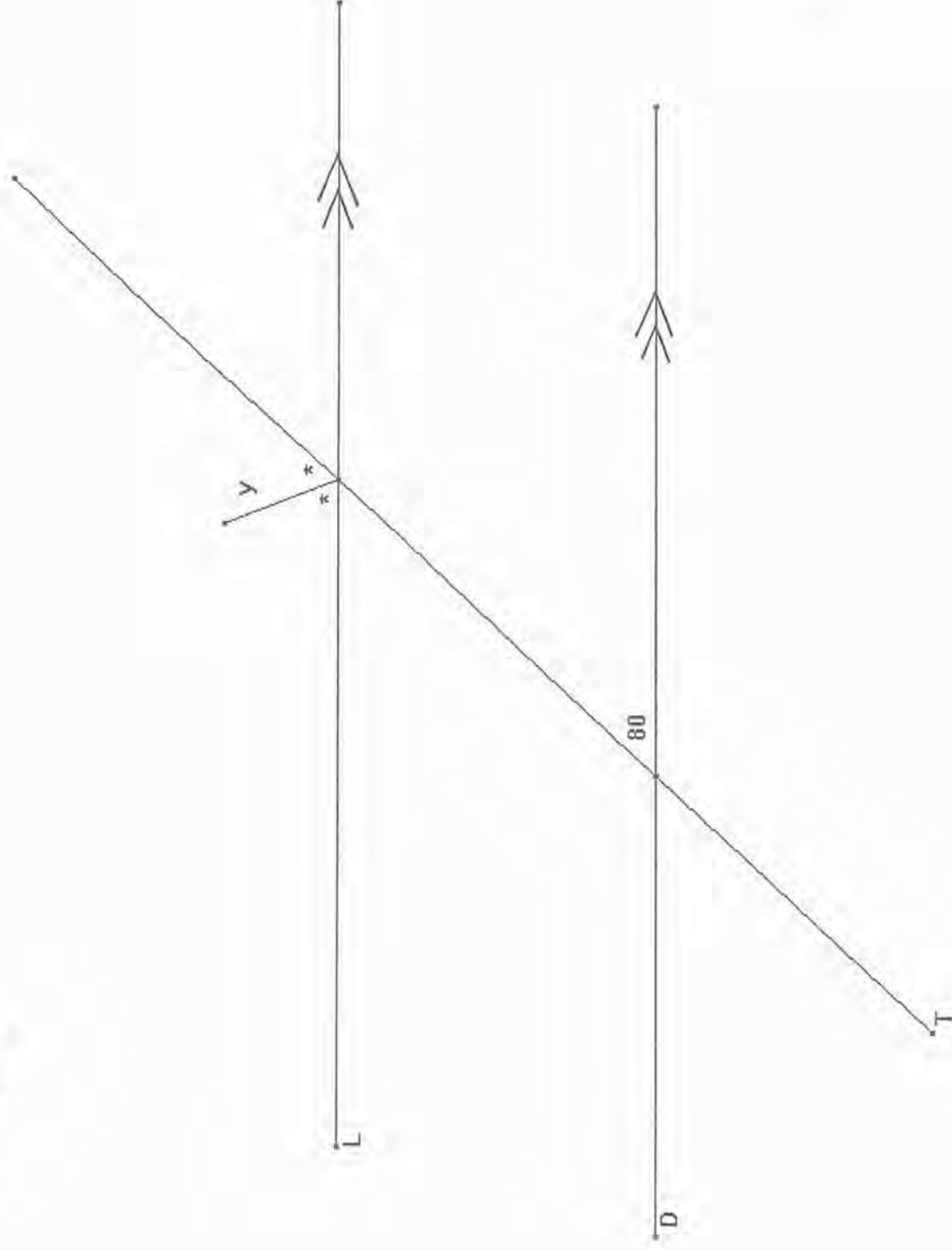


Figure Not to Scale

FIND y

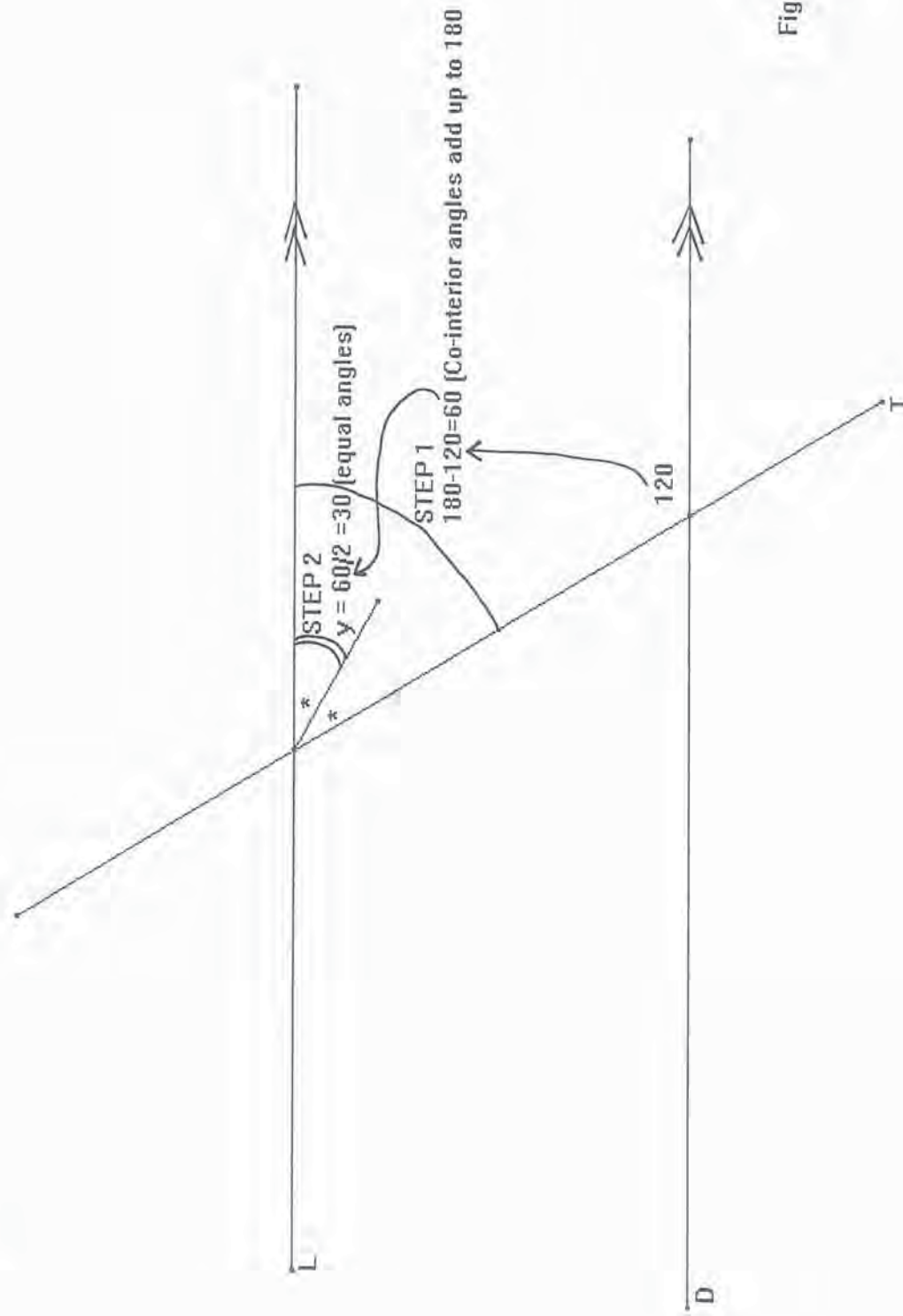


Figure Not to Scale

FIND y

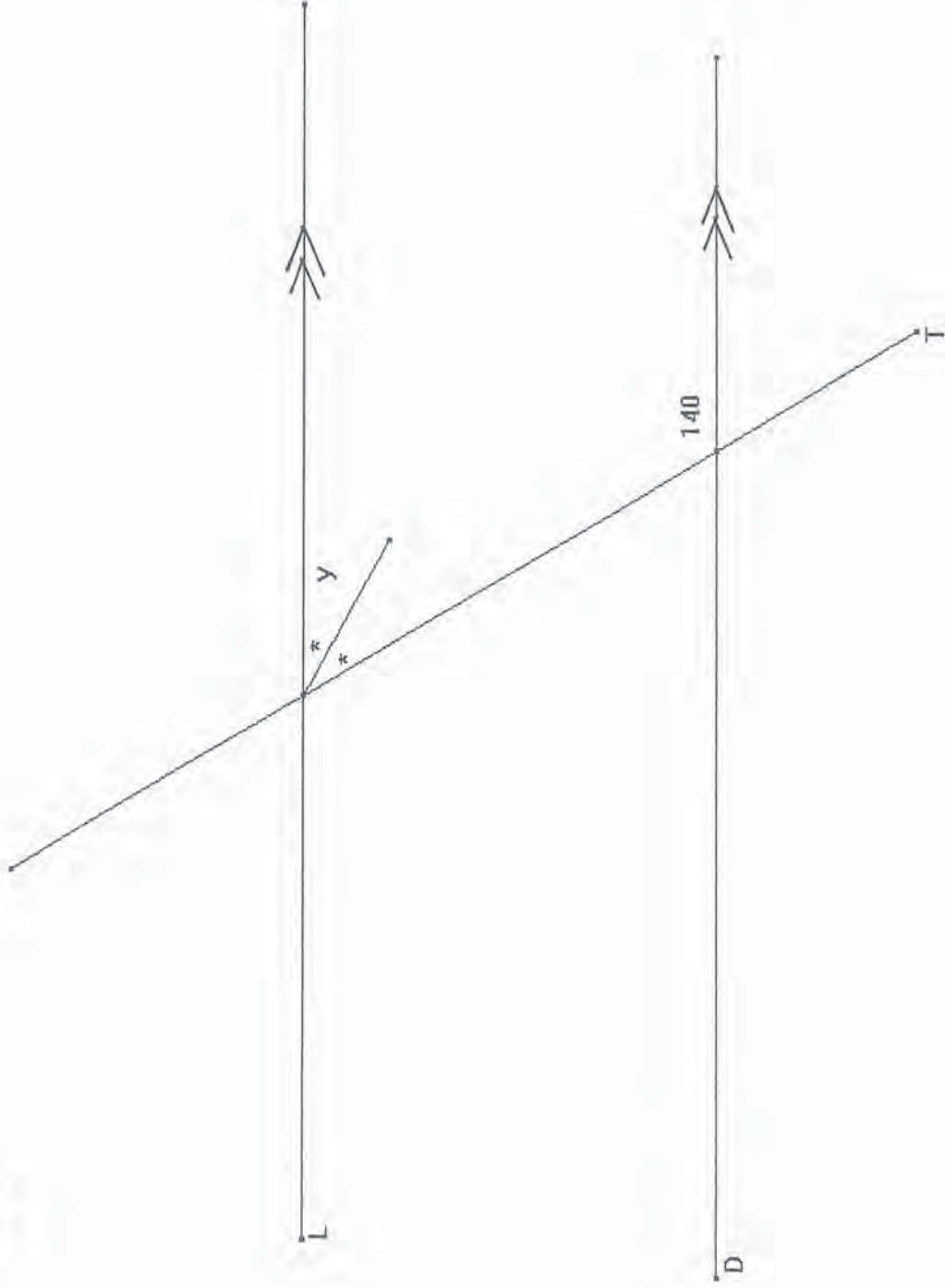


Figure Not to Scale

Appendix C

Sets of problems given to the Step Guidance group of Experiment 1

FIND y

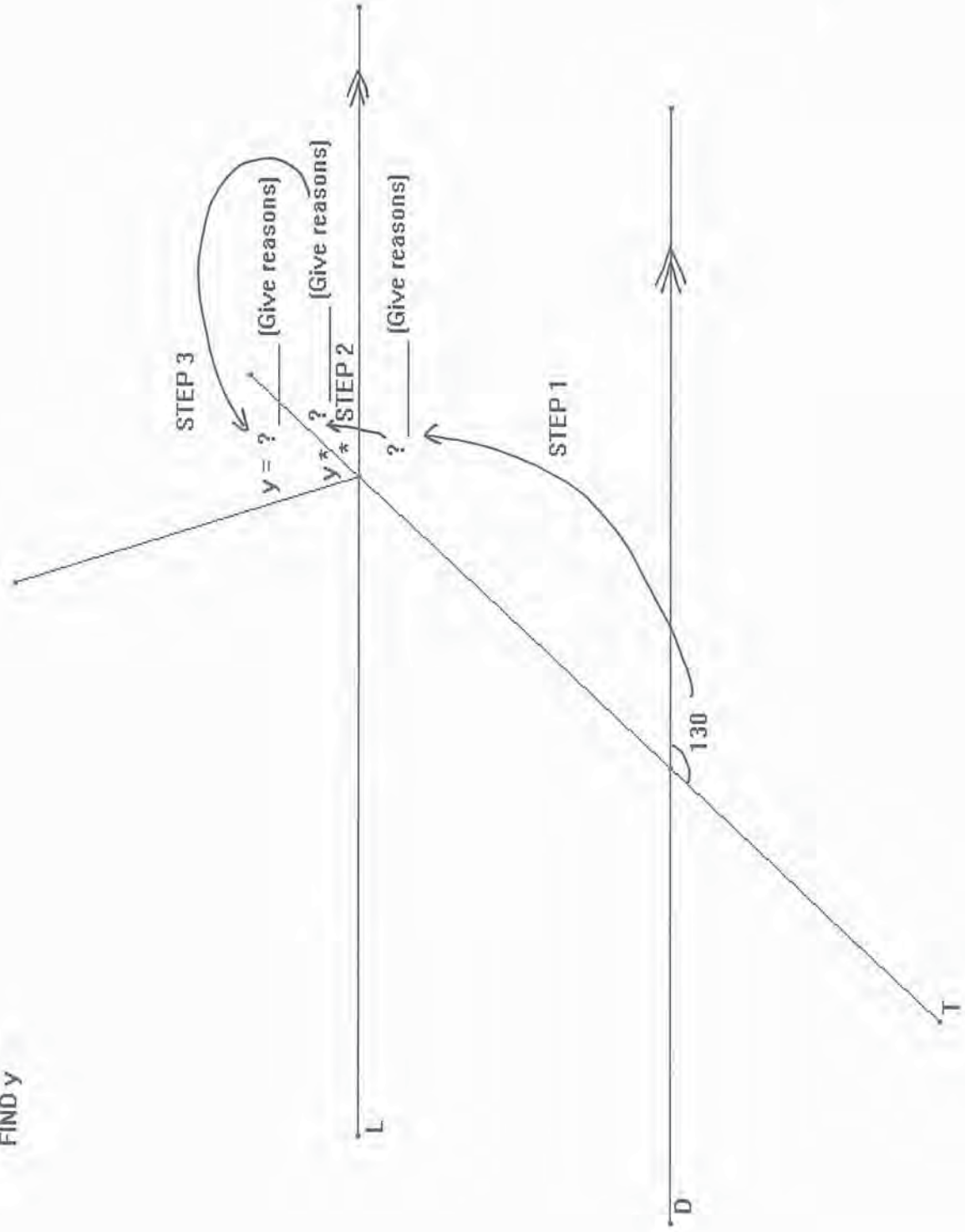


Figure Not to Scale

FIND y

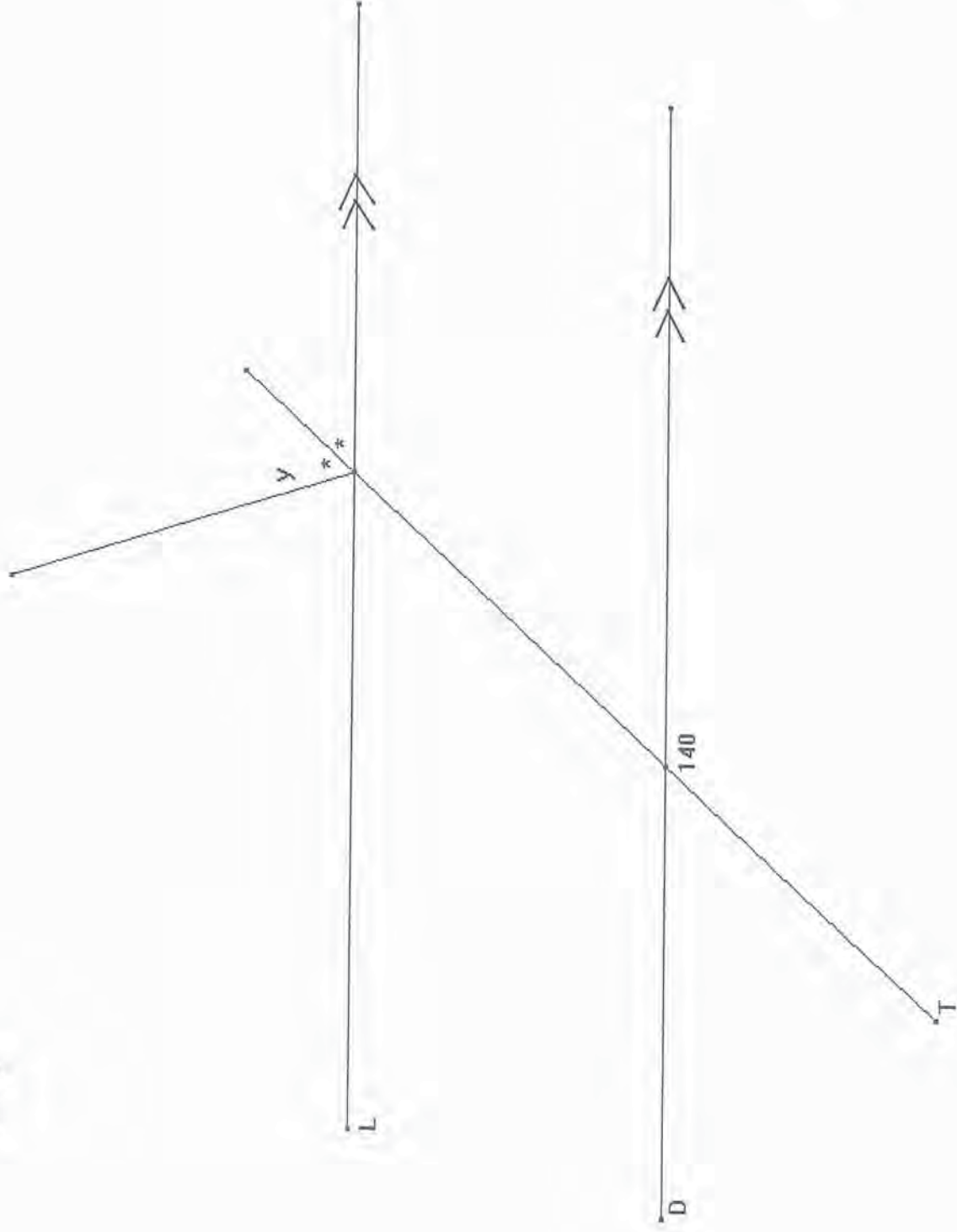


Figure Not to Scale

FIND y

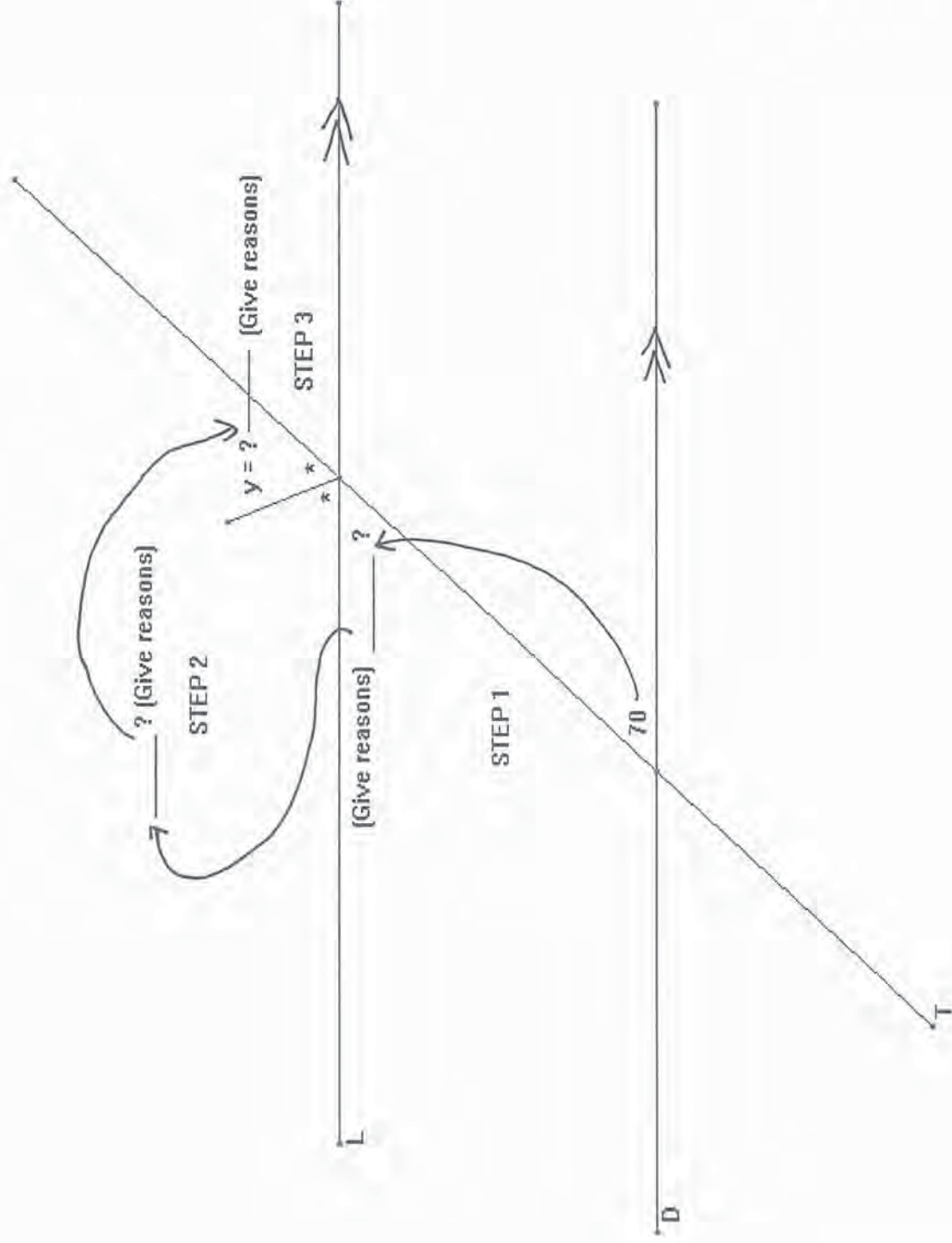


Figure Not to Scale

FIND y

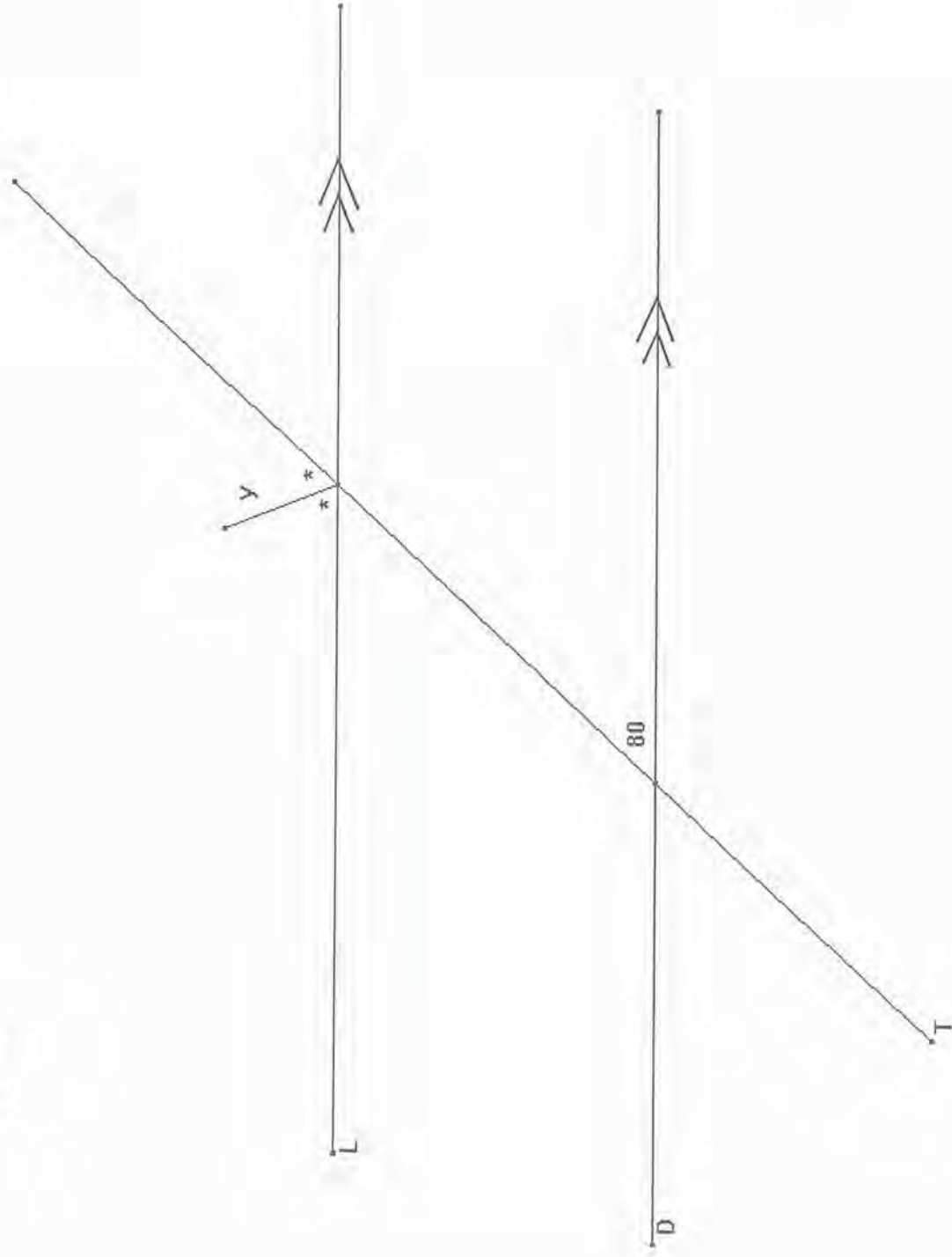


Figure Not to Scale

FIND y

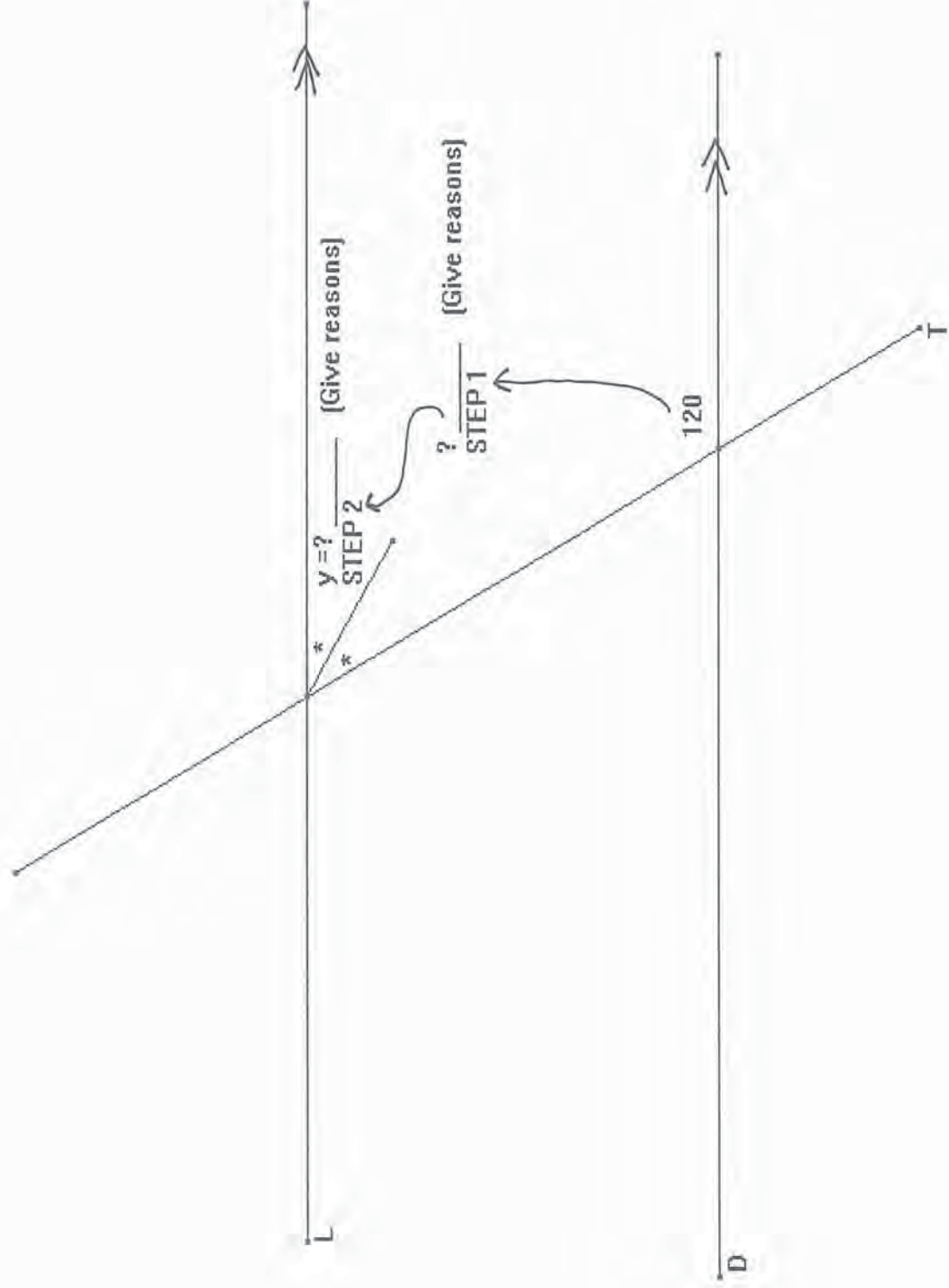


Figure Not to Scale

FIND y

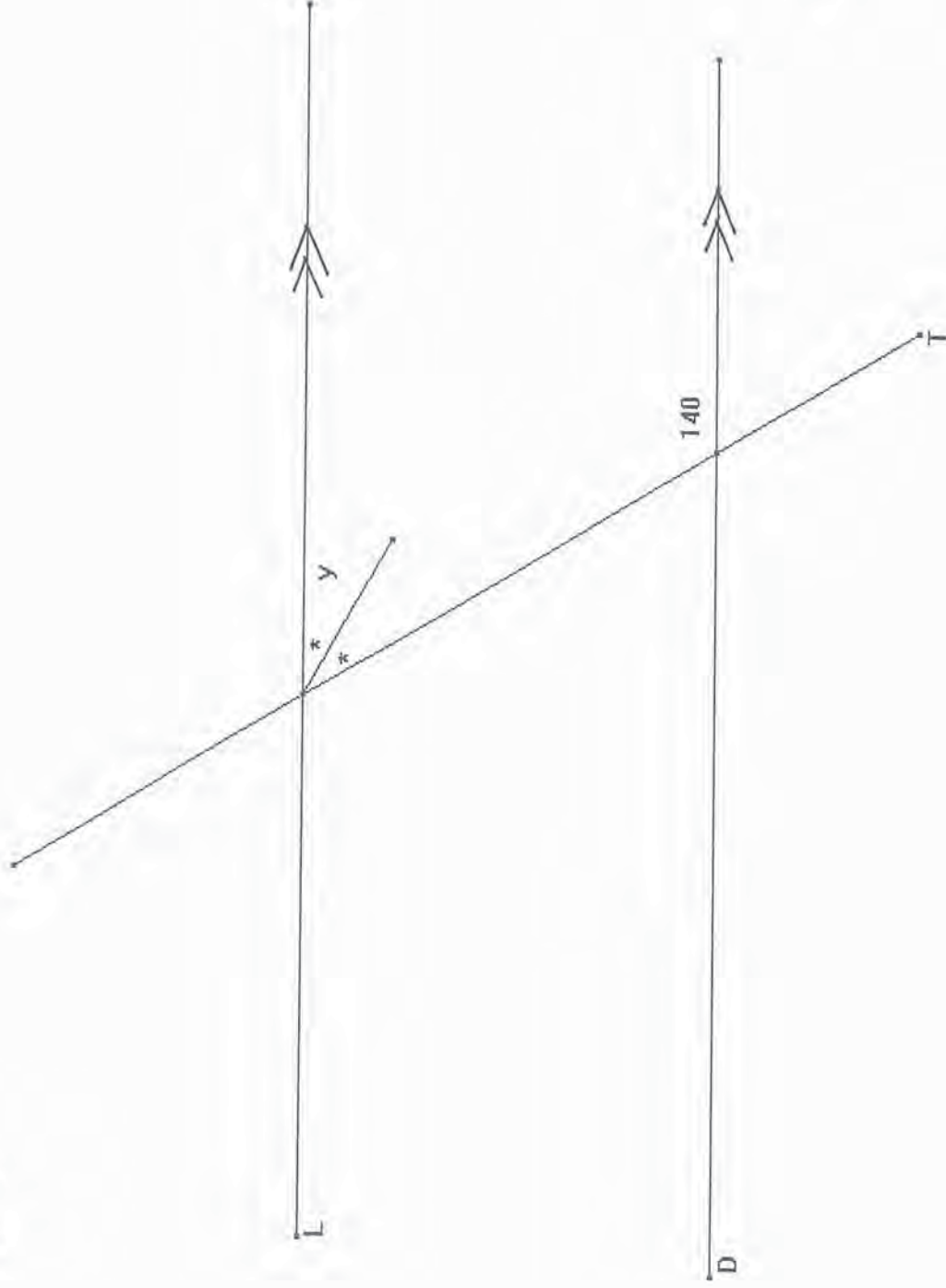


Figure Not to Scale

Appendix D

Sets of problems given to the Problem Solving group of Experiments 1, 2, &3

NAME _____

FIND y

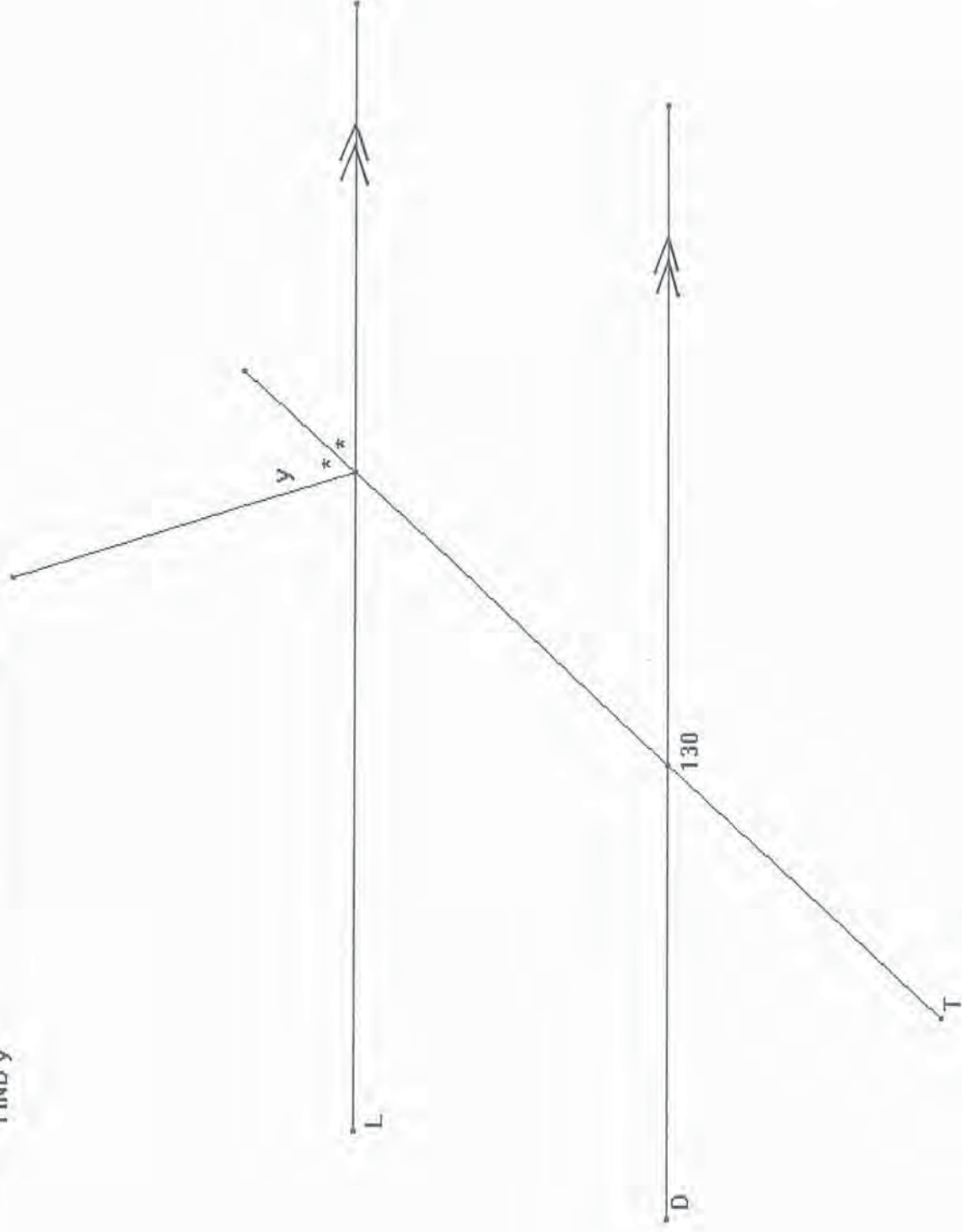


Figure Not to Scale

FIND y

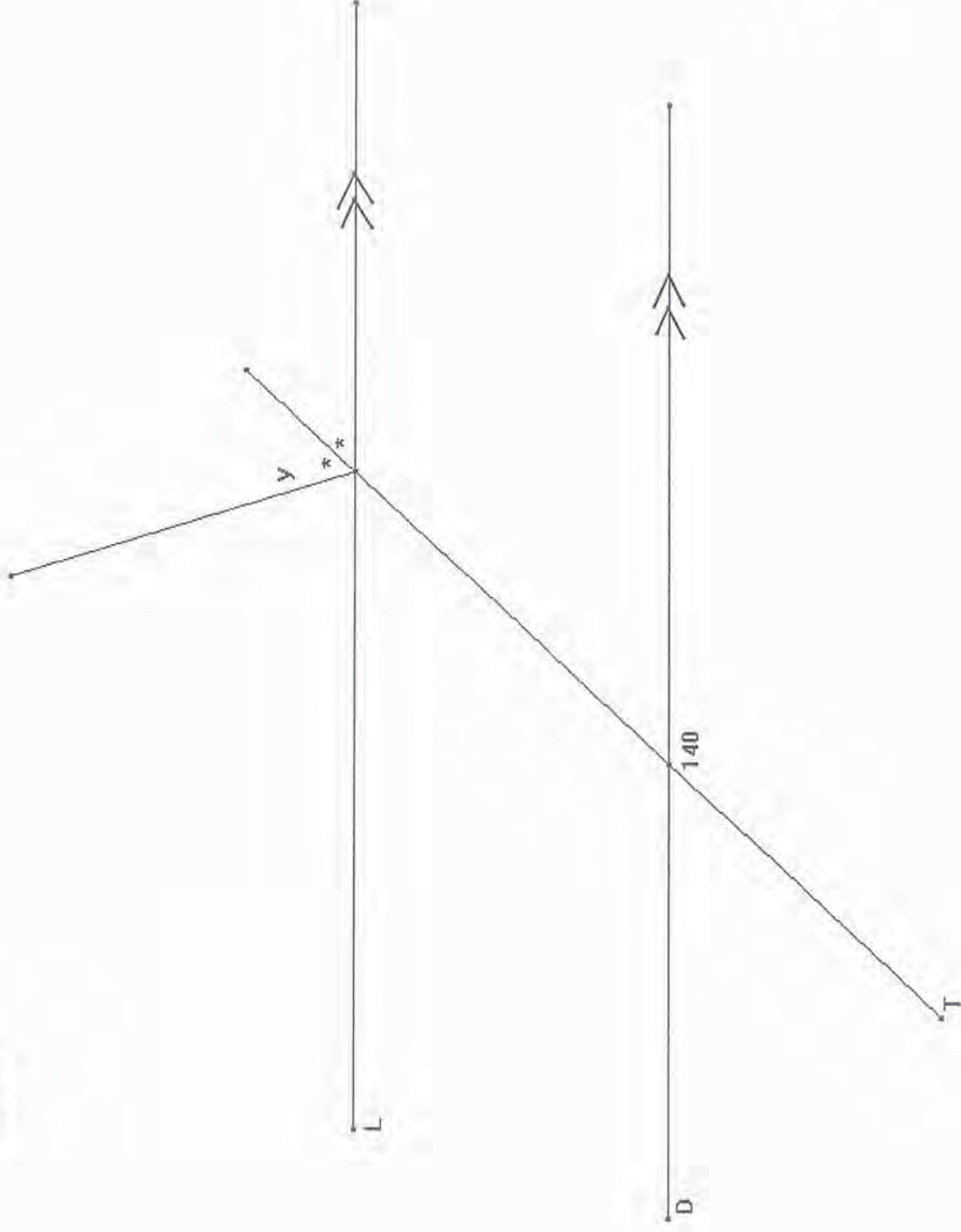


Figure Not to Scale

FIND y

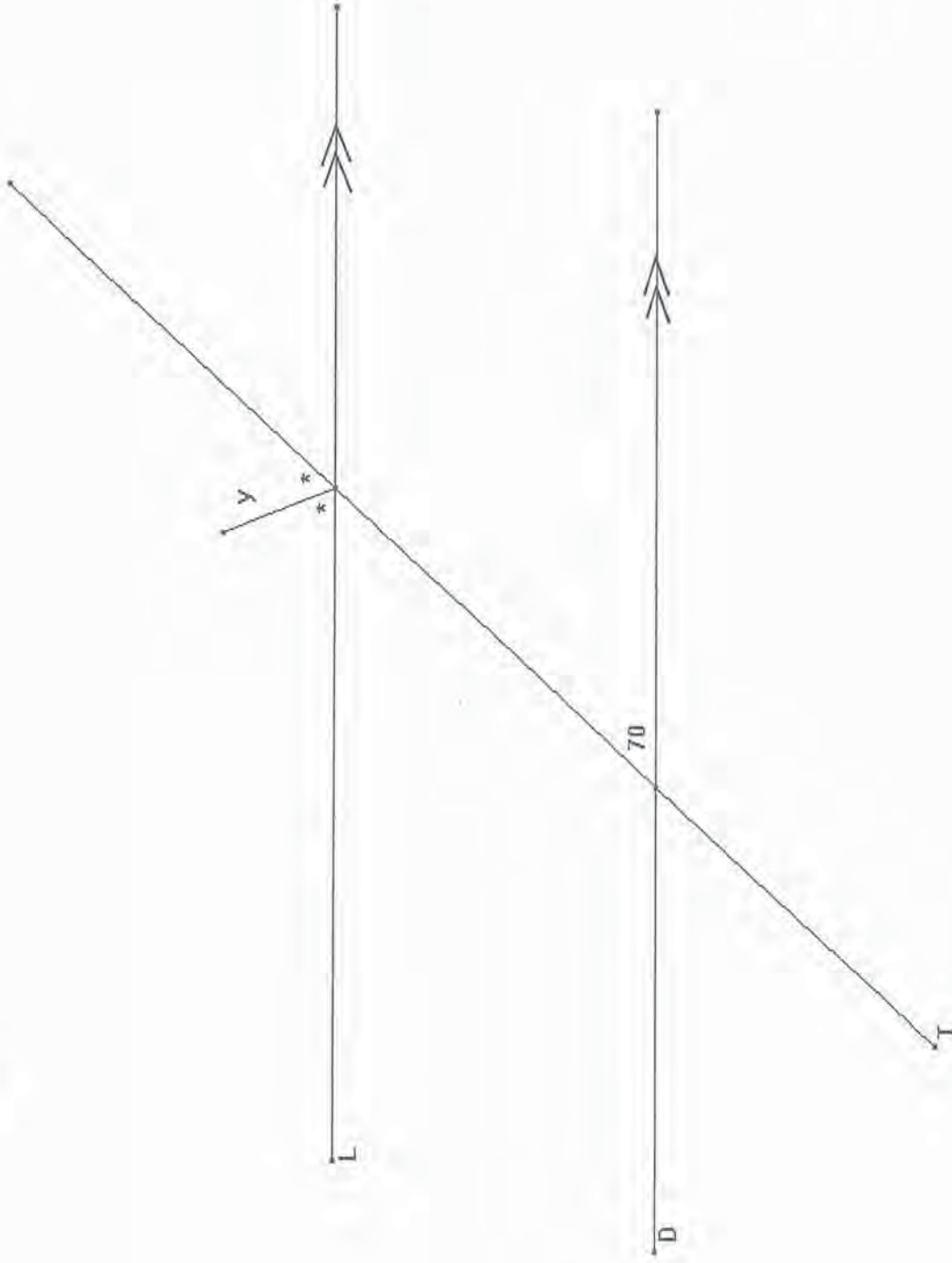


Figure Not to Scale

FIND y

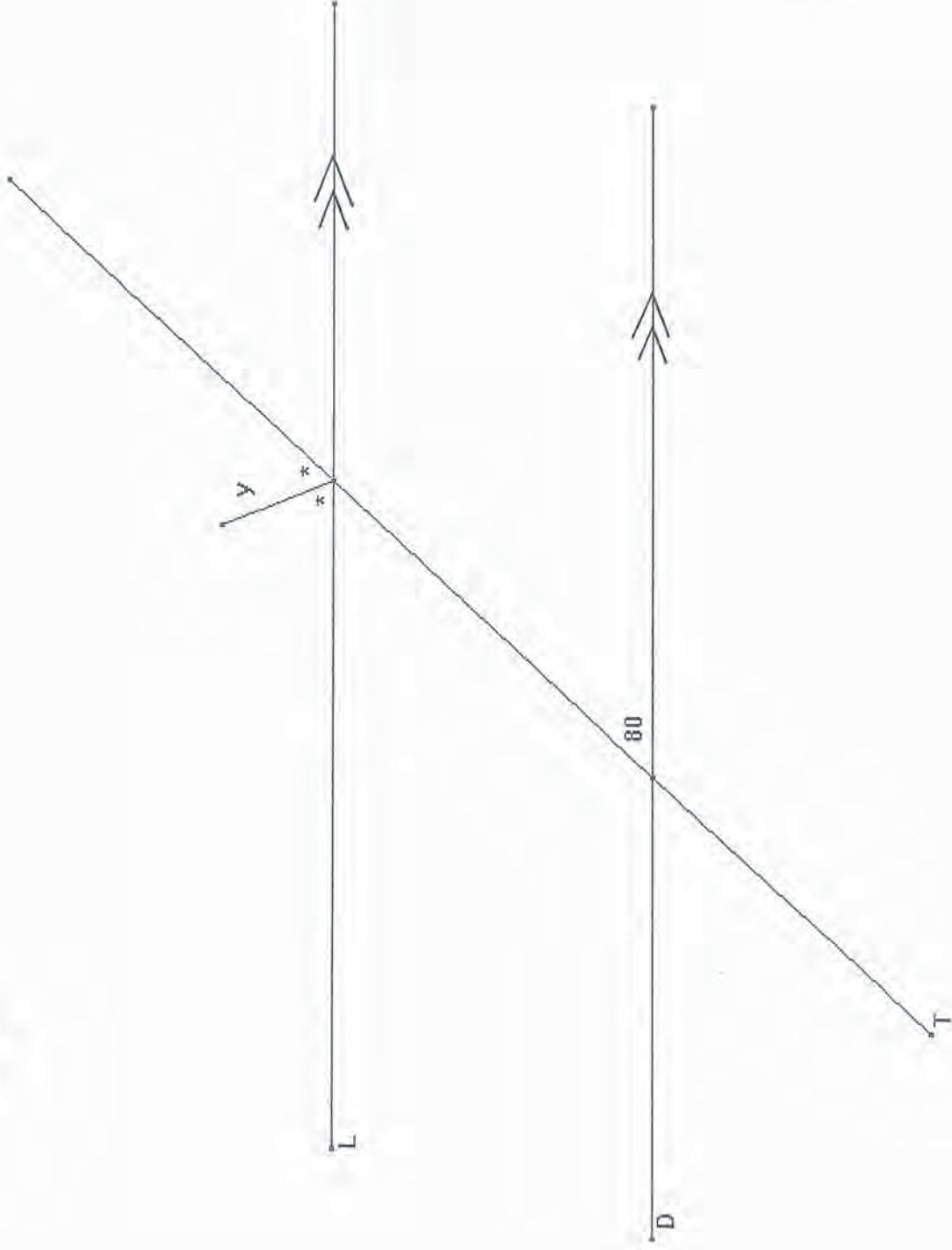


Figure Not to Scale

FIND y

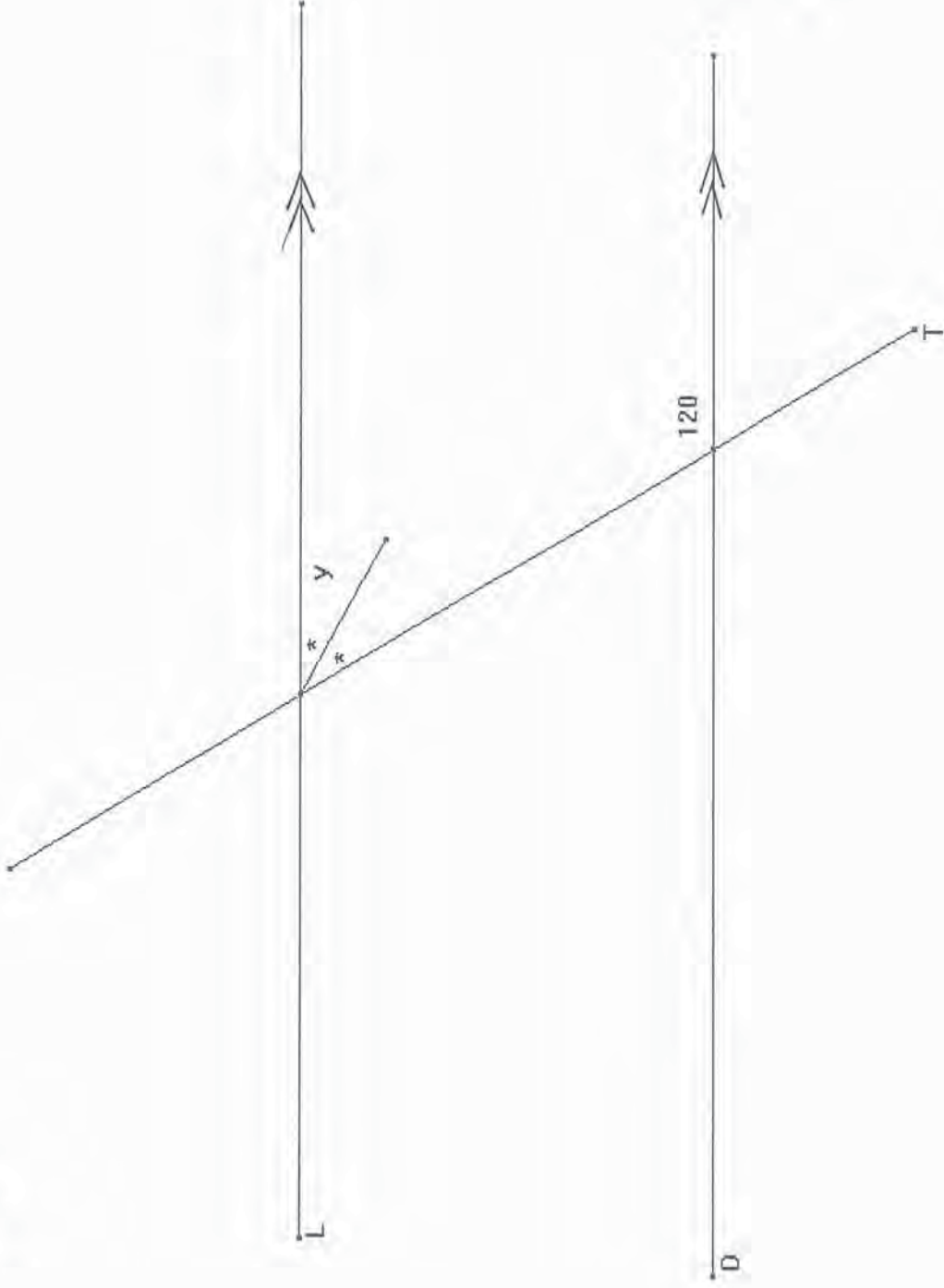


Figure Not to Scale

FIND y

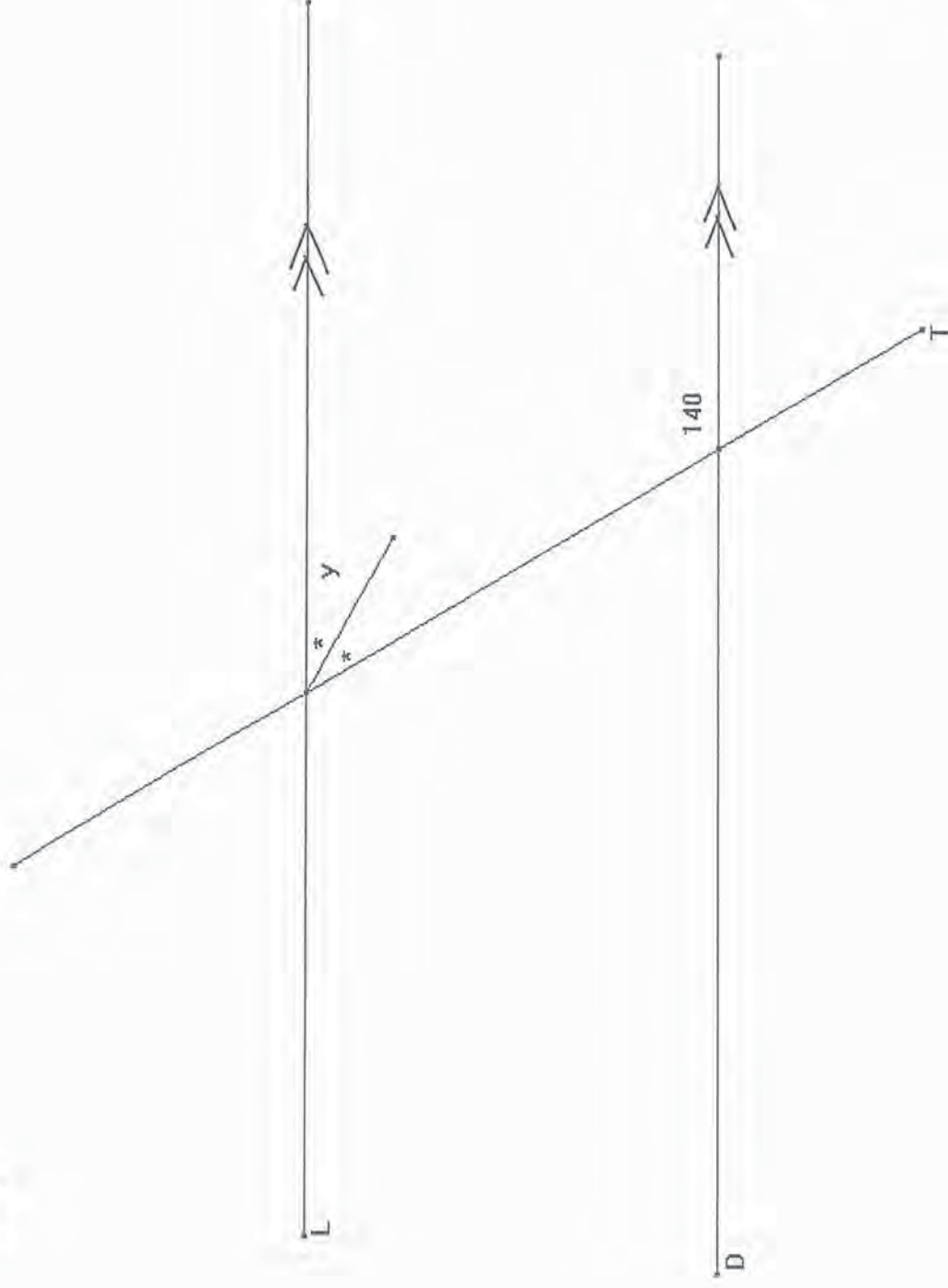


Figure Not to Scale

Appendix E

New sets of problems given to the Step Guidance group of Experiments 2 & 3

NAME _____

FIND y

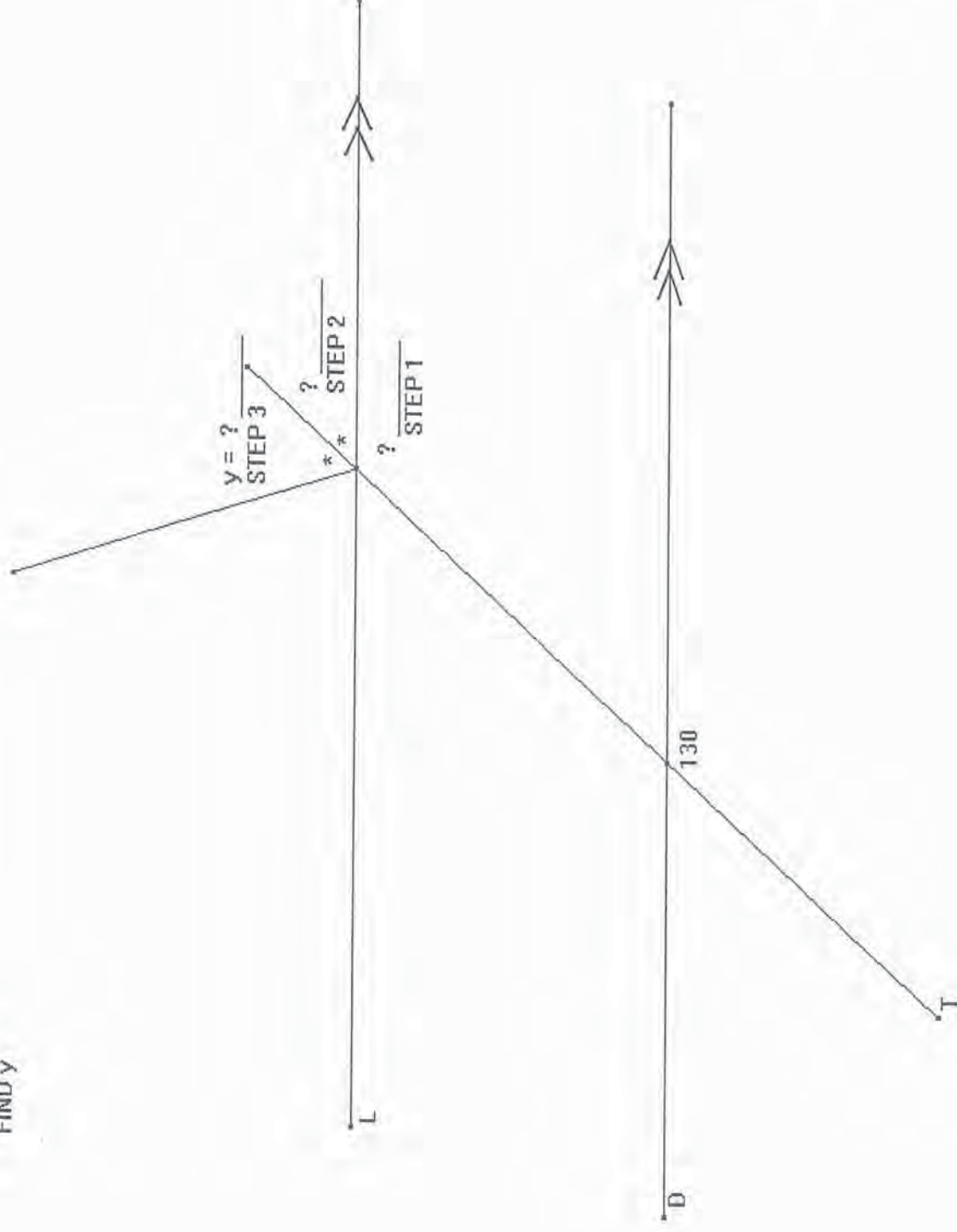


Figure Not to Scale

FIND y

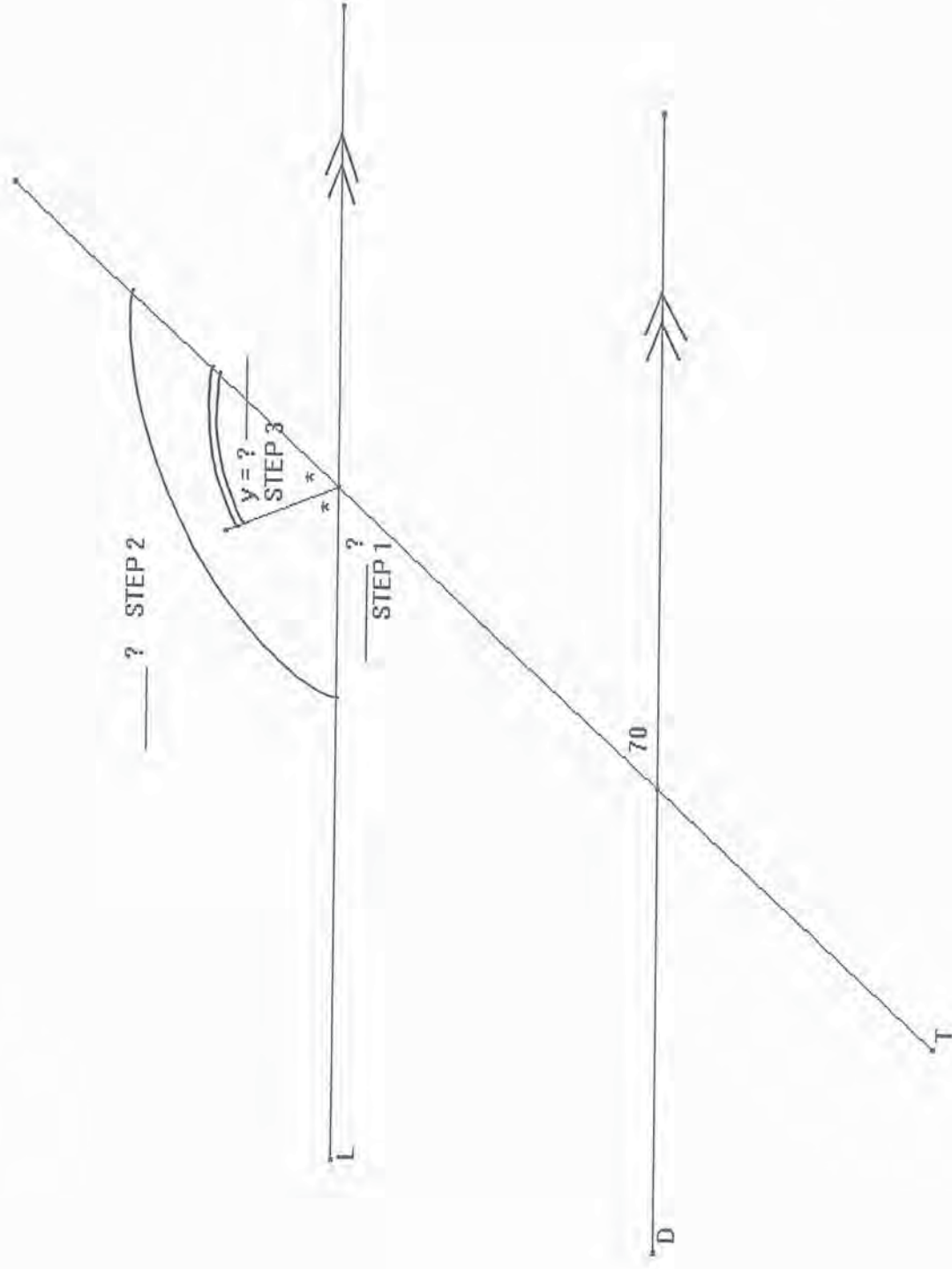


Figure Not to Scale

FIND y

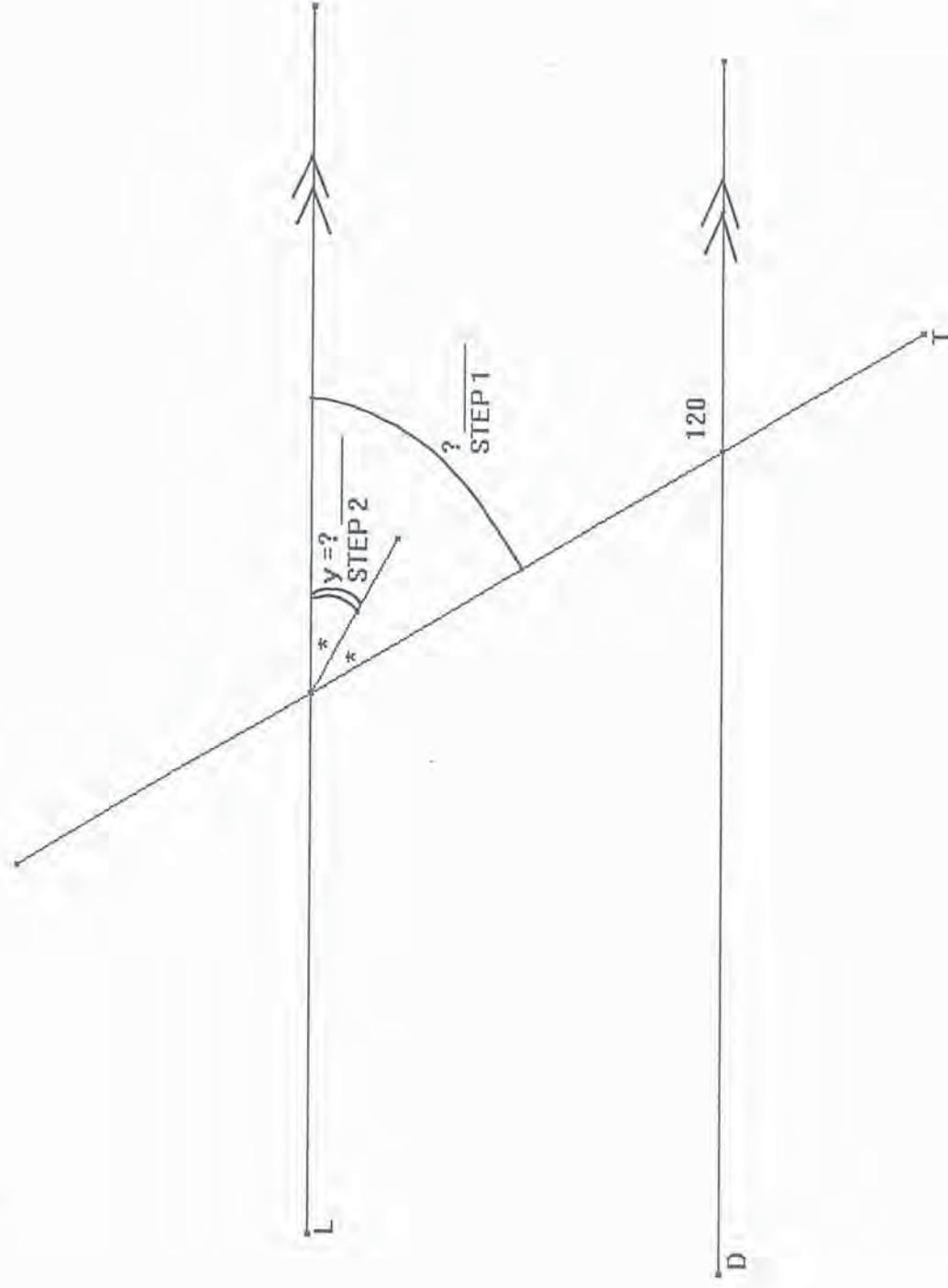


Figure Not to Scale

Appendix F

New sets of Tests Questions used in Experiment 3

Sets of Similar Test Problems

NAME _____

FIND y

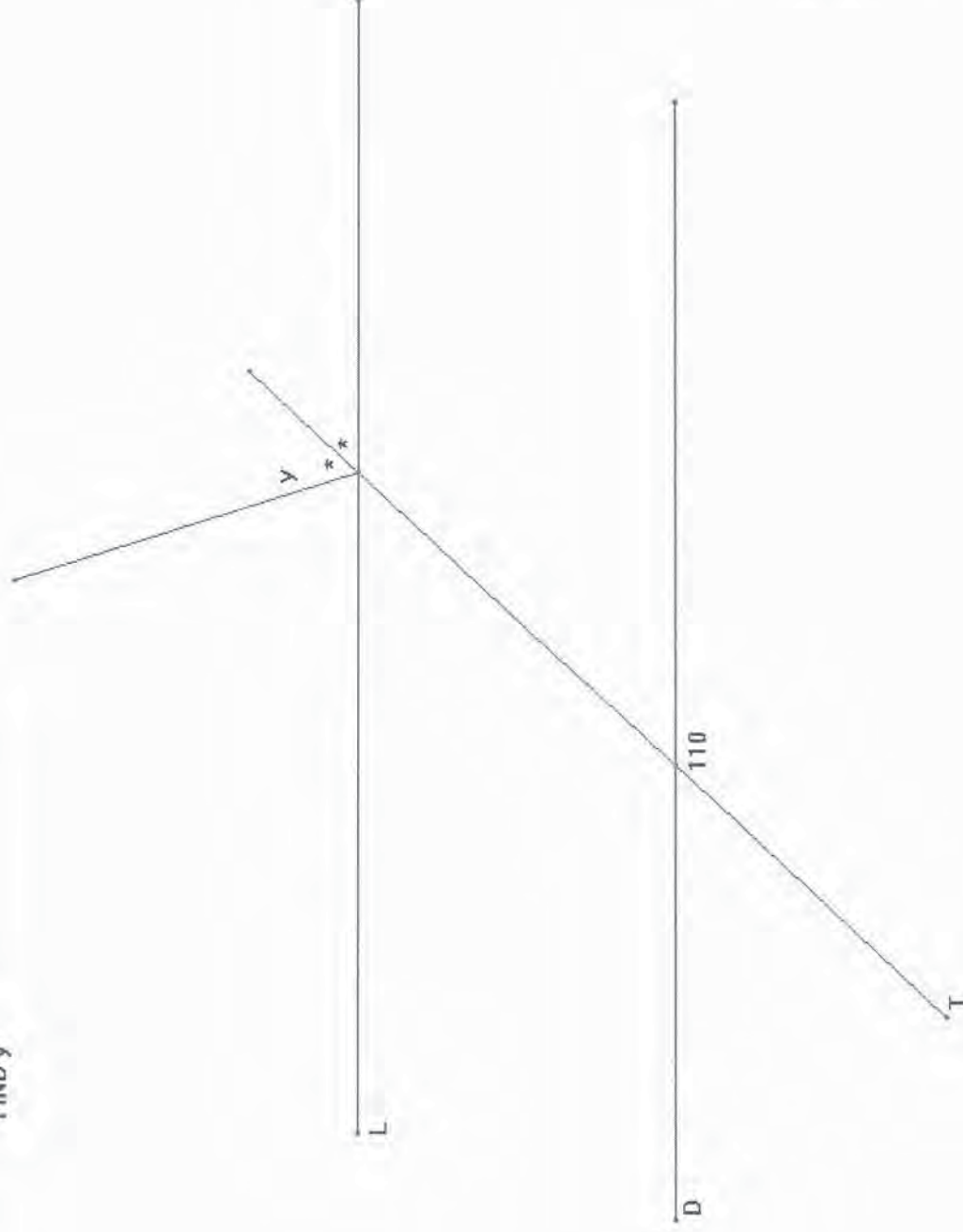


Figure Not to Scale

FIND y

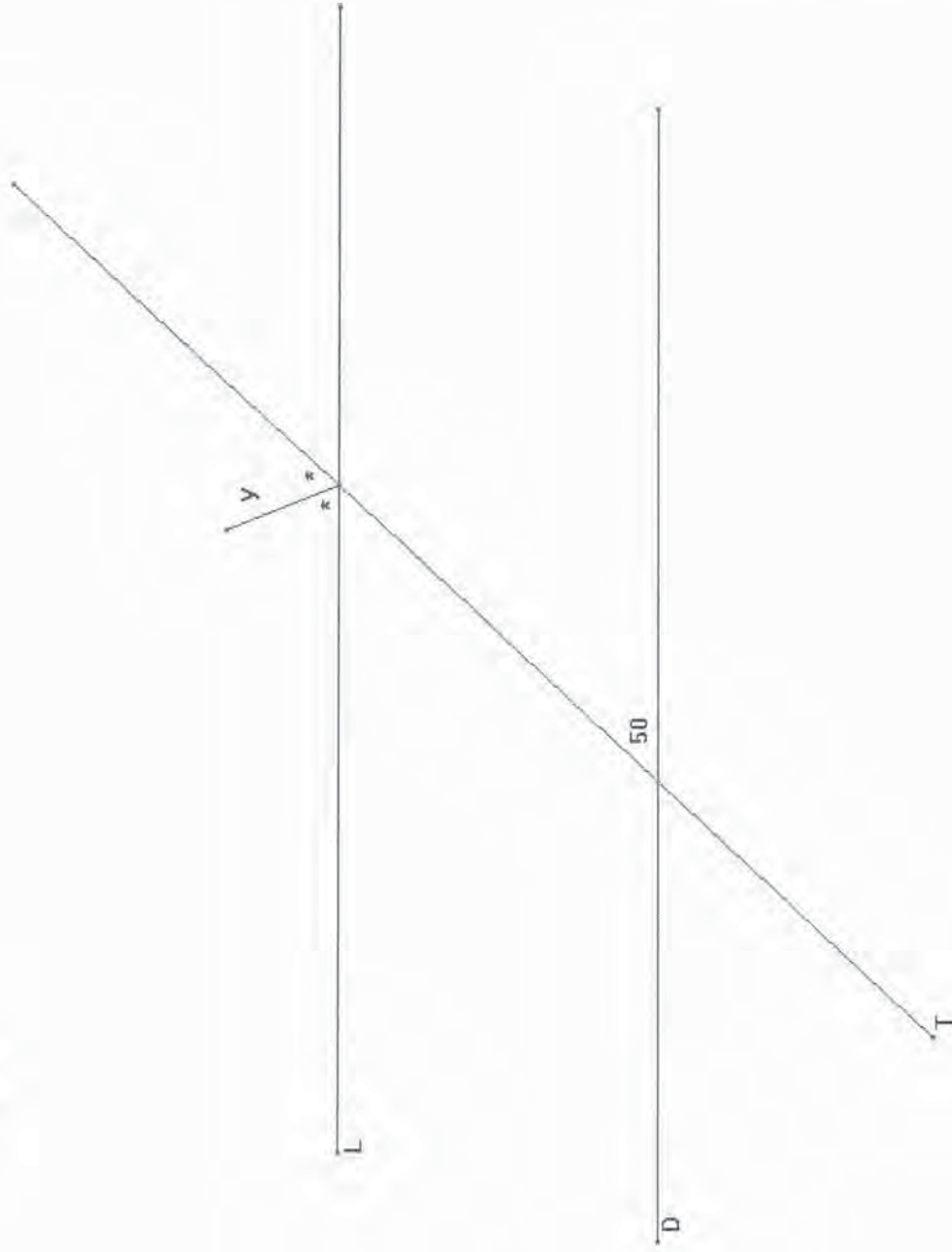


Figure Not to Scale

FIND y

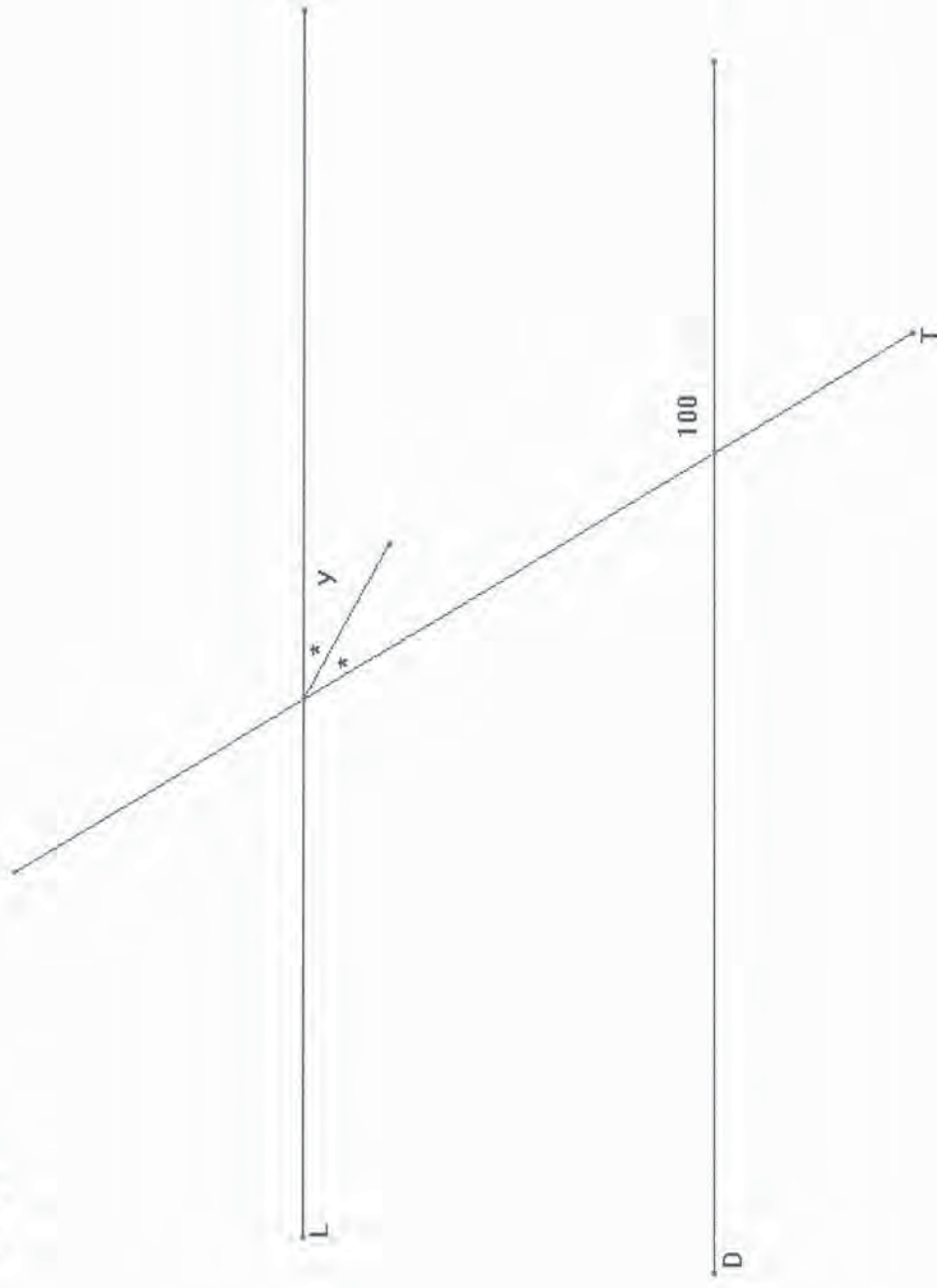


Figure Not to Scale

Sets of Transfer Test Problems

NAME _____

FIND y

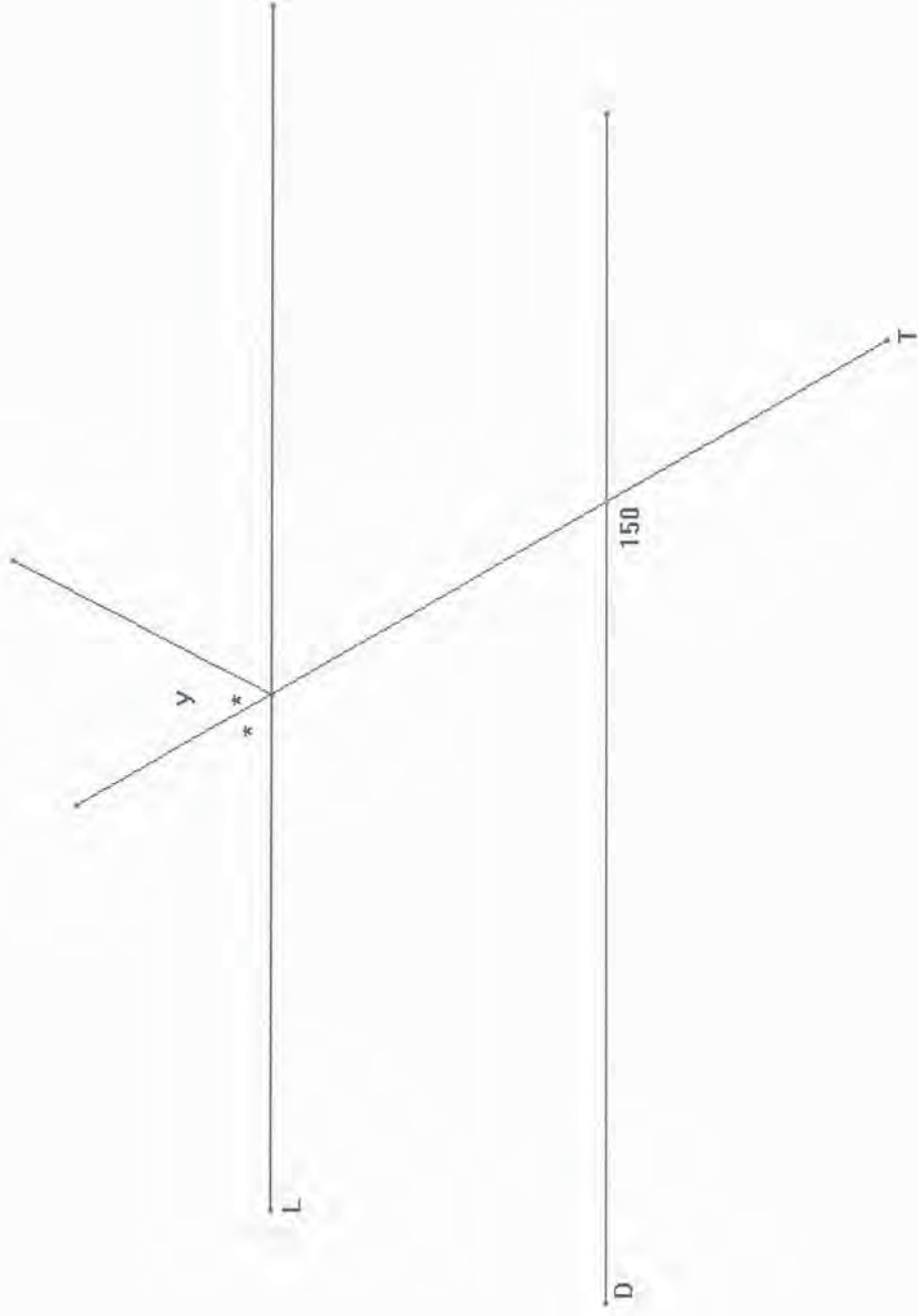


Figure Not to Scale

FIND y

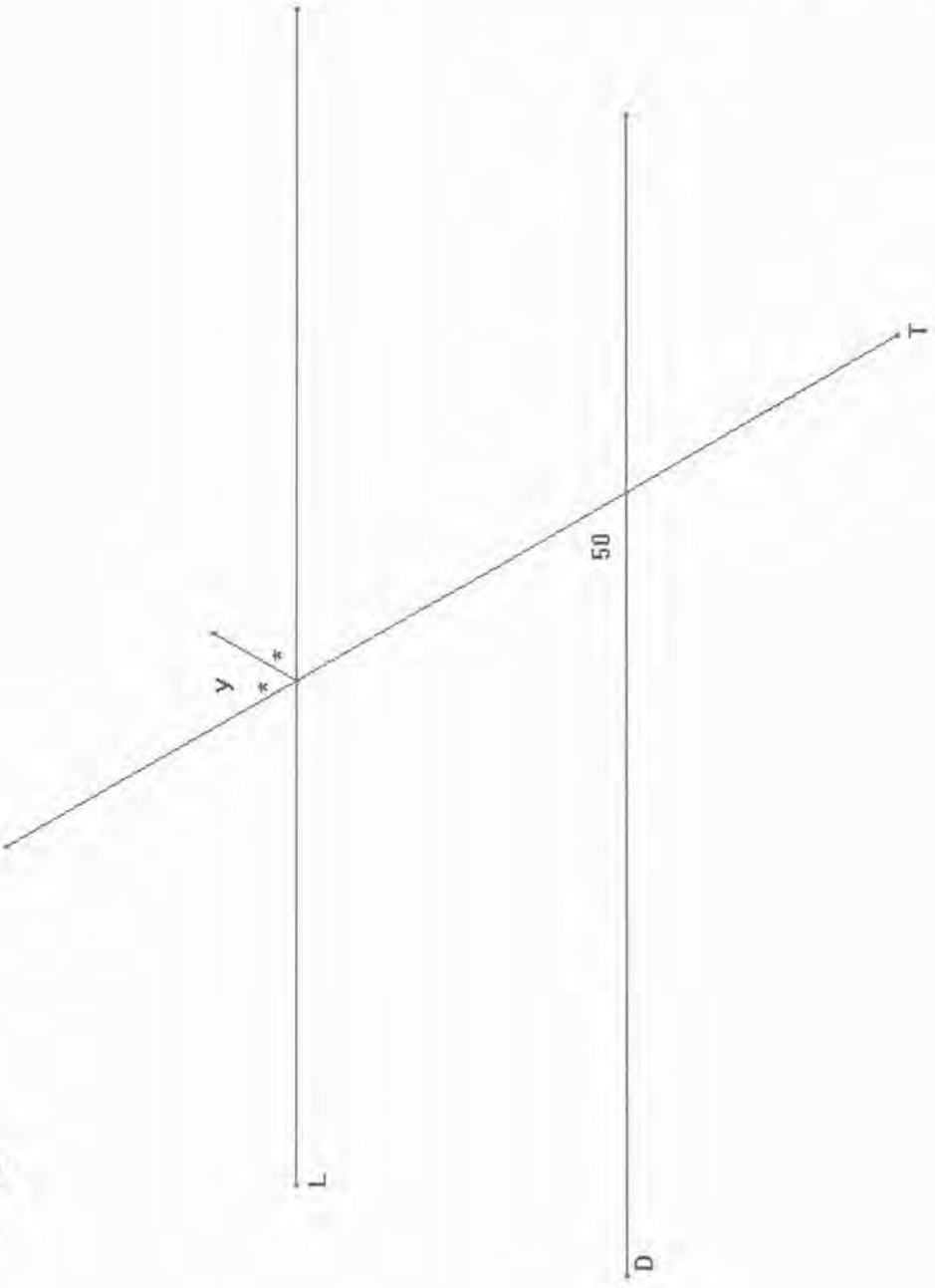


Figure Not to Scale

FIND y

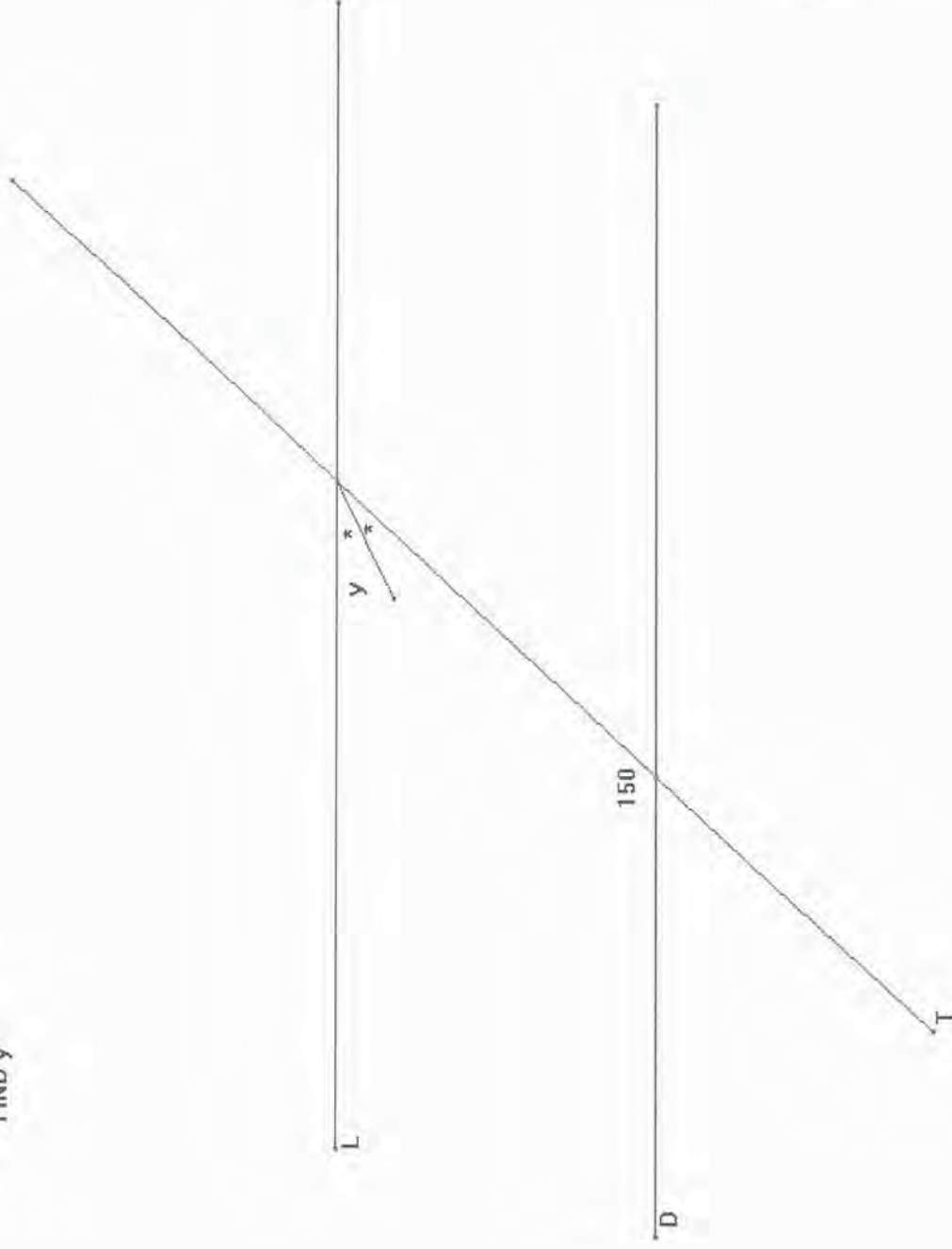


Figure Not to Scale

Appendix G

Subjective Rating of Cognitive Load for Learning

Appendix H

Sets of problems used for each instructional Condition (Theorem & Step Guidance, Step Guidance, and Problem Solving) of Experiment 4

Sets of Problems Given to the Theorem & Step Guidance group

Name _____
Find y



Figure not to Scale

Find y

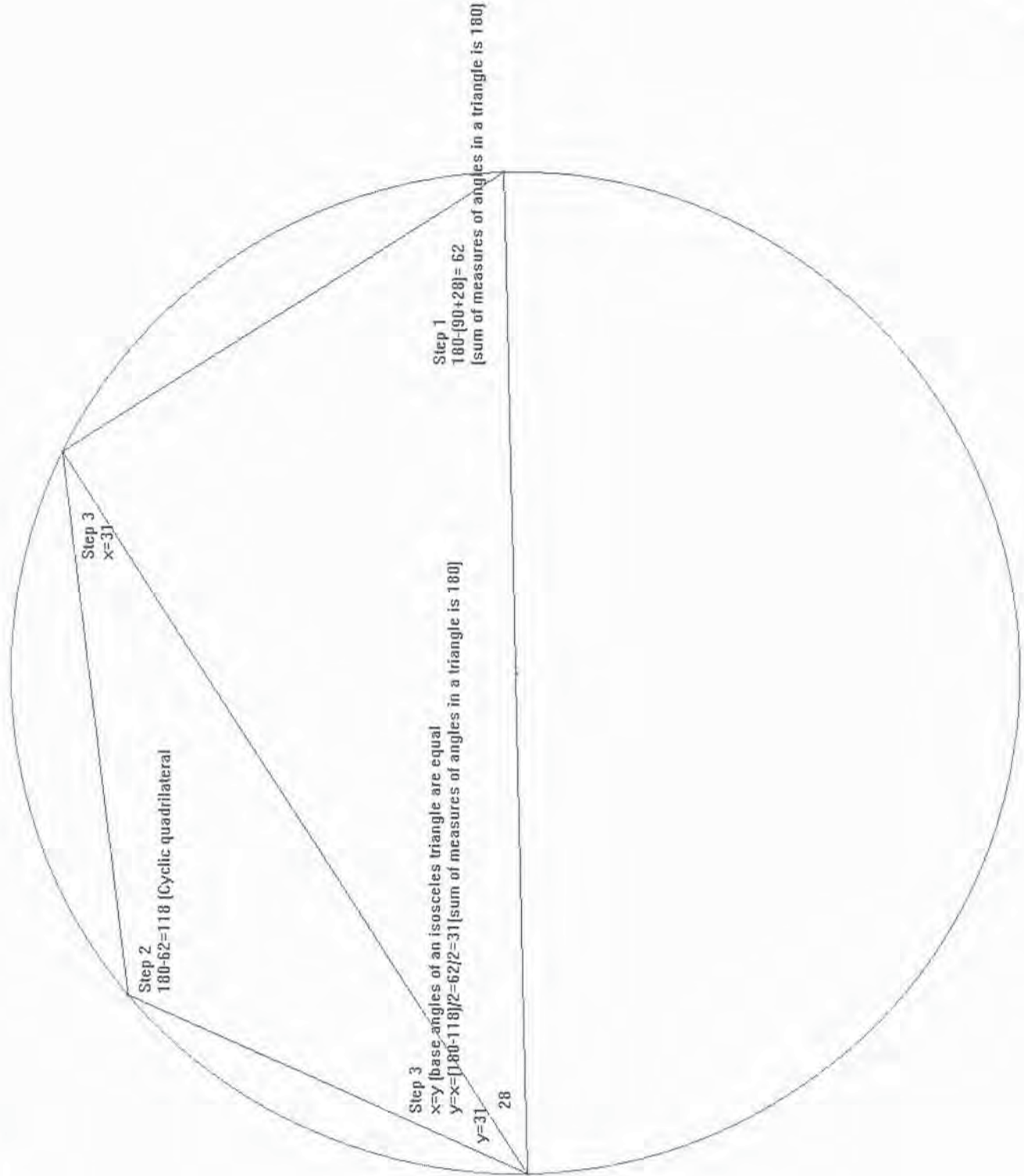


Figure Not to Scale

Find x, y, z

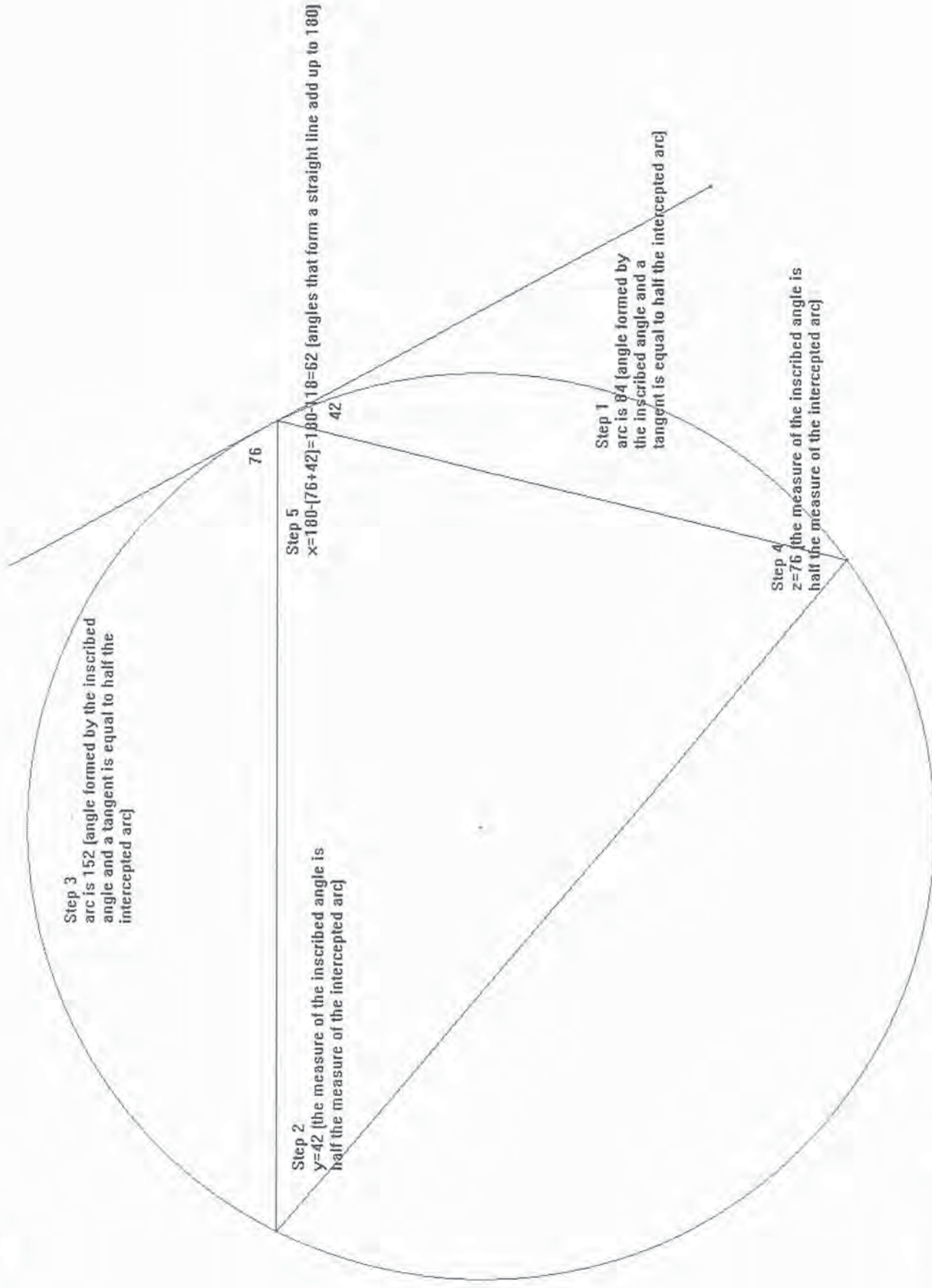


Figure not to scale

Sets of Problems Given to the Step Guidance group

Name _____
Find y _____

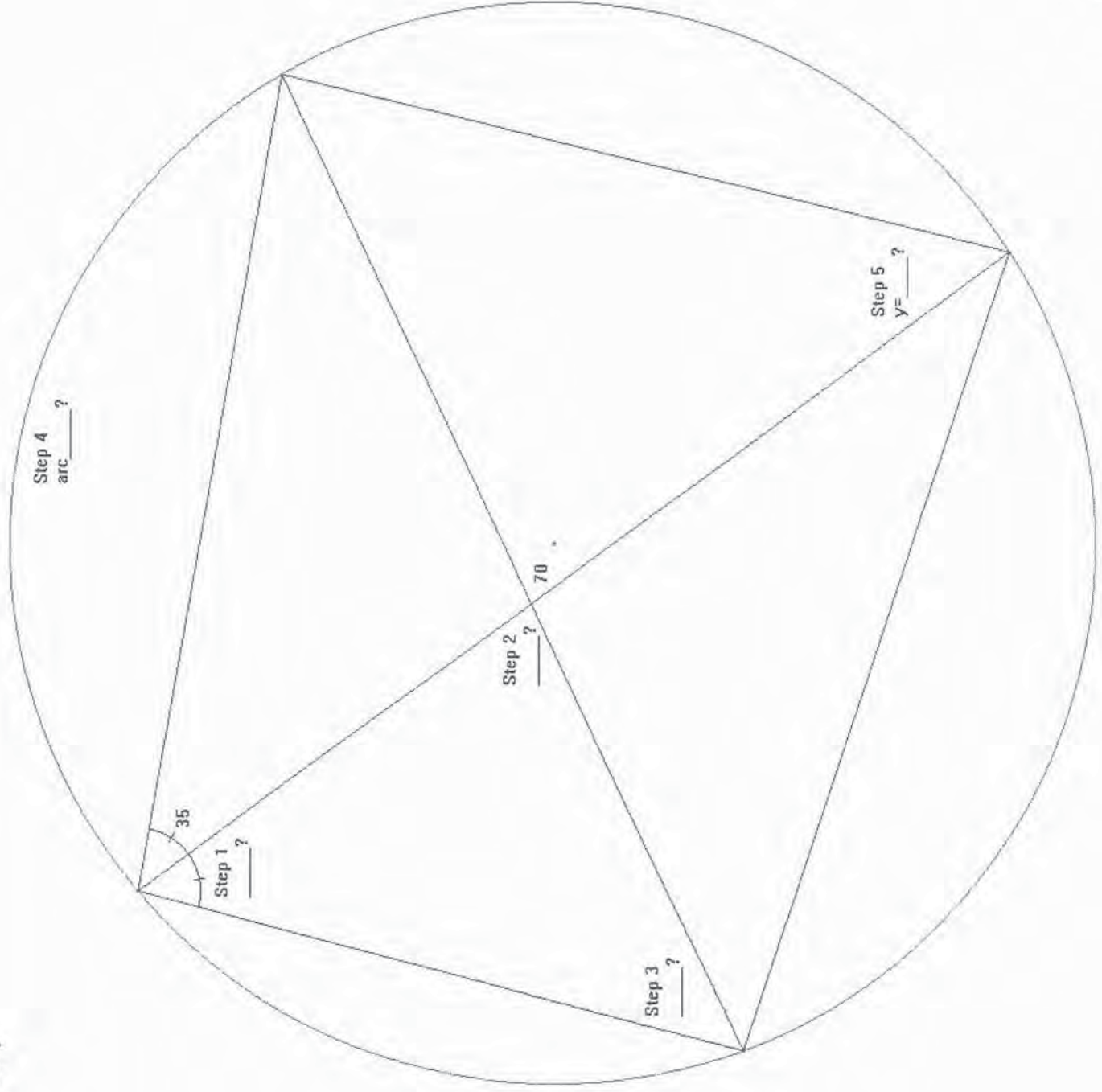


Figure not to scale

Find y

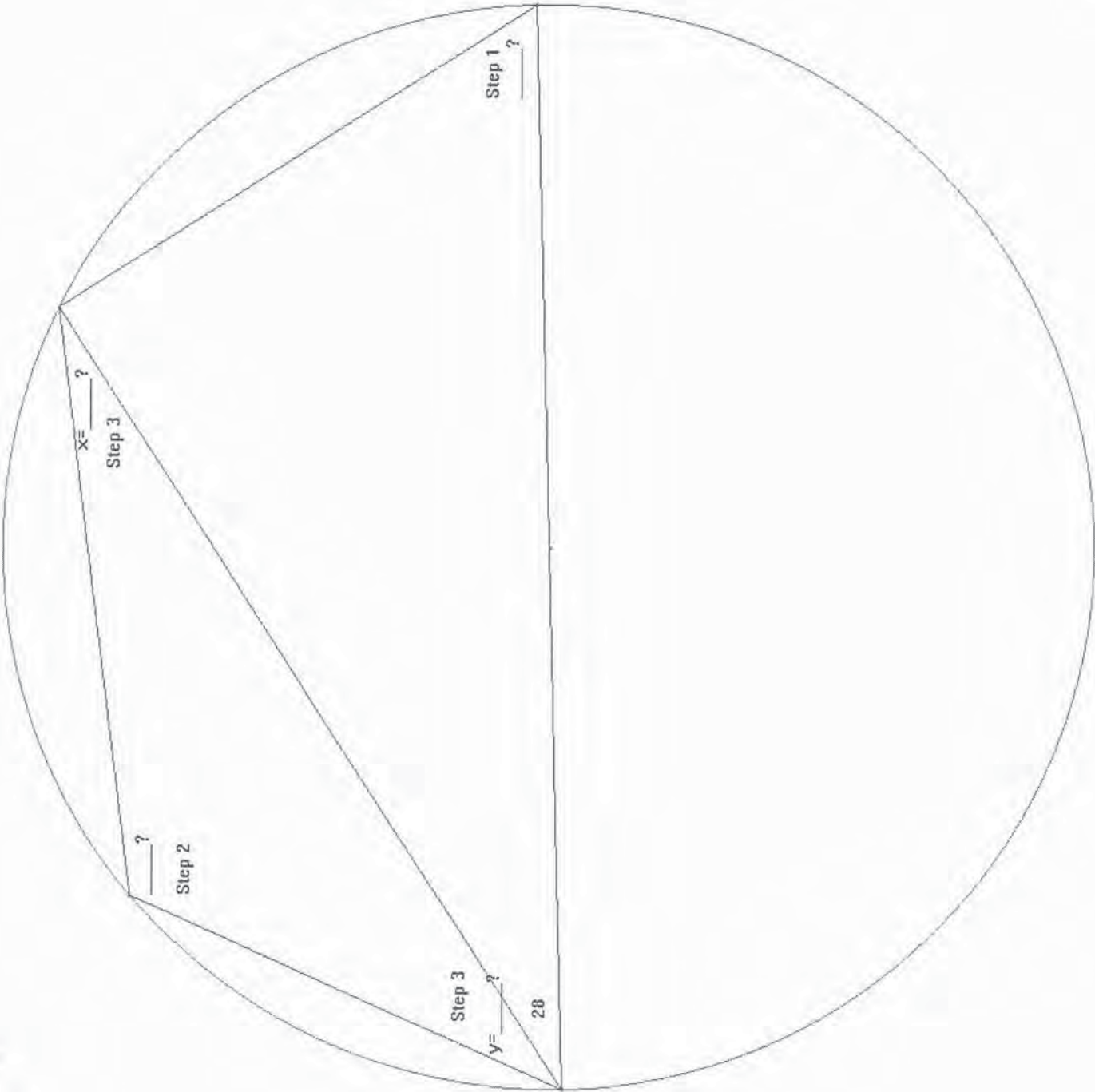


Figure not to scale

Find x, y, z

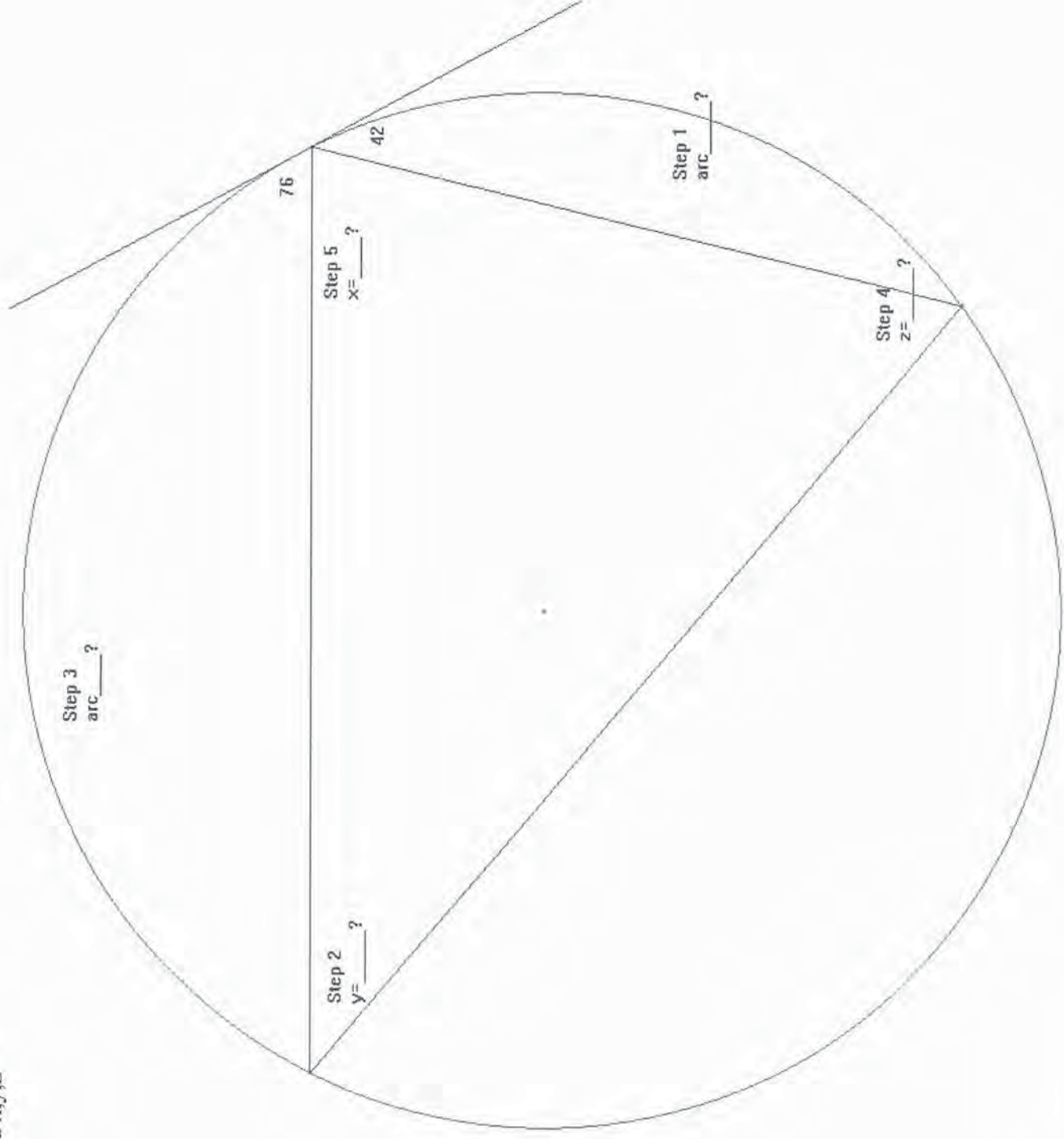
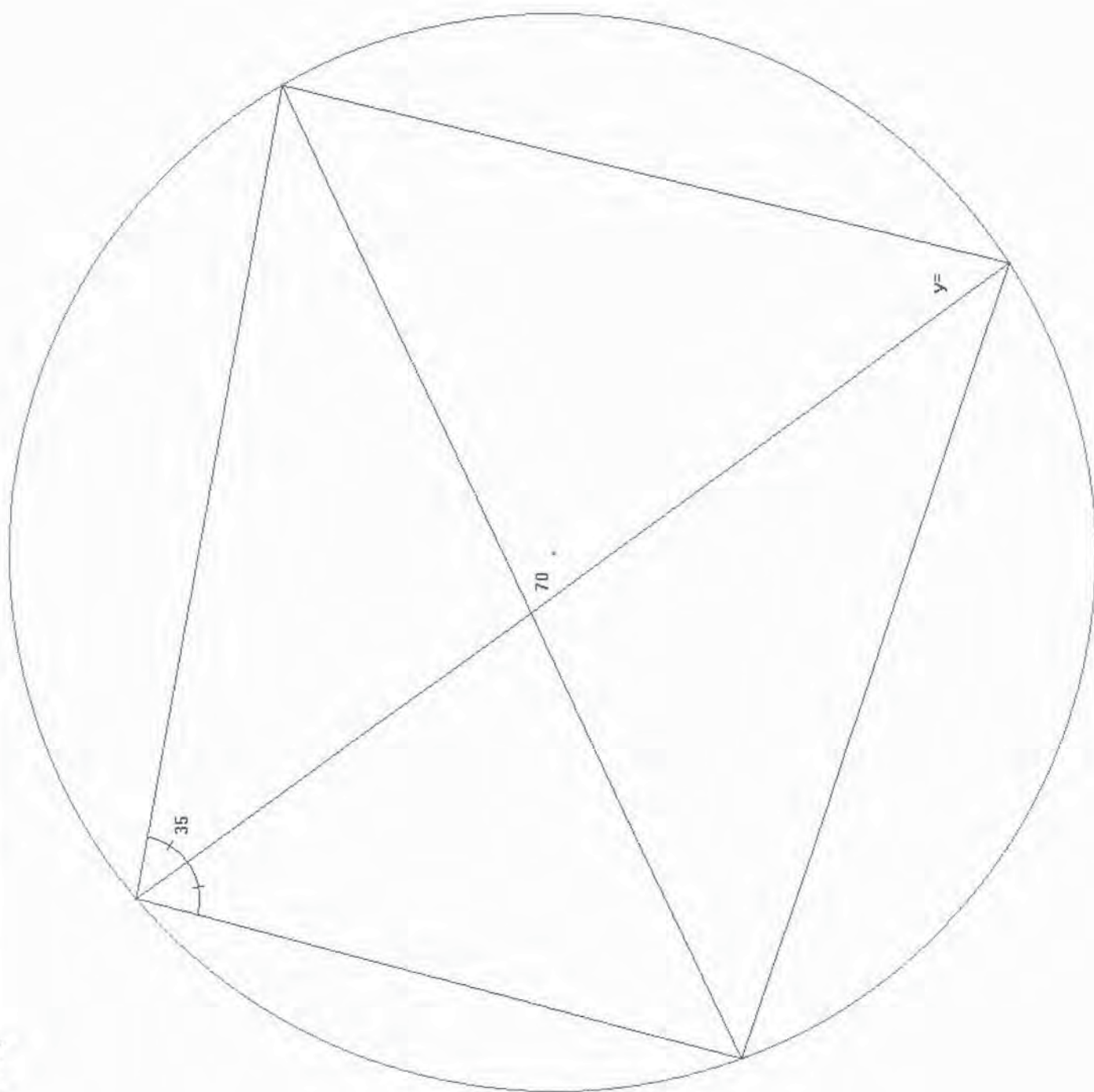


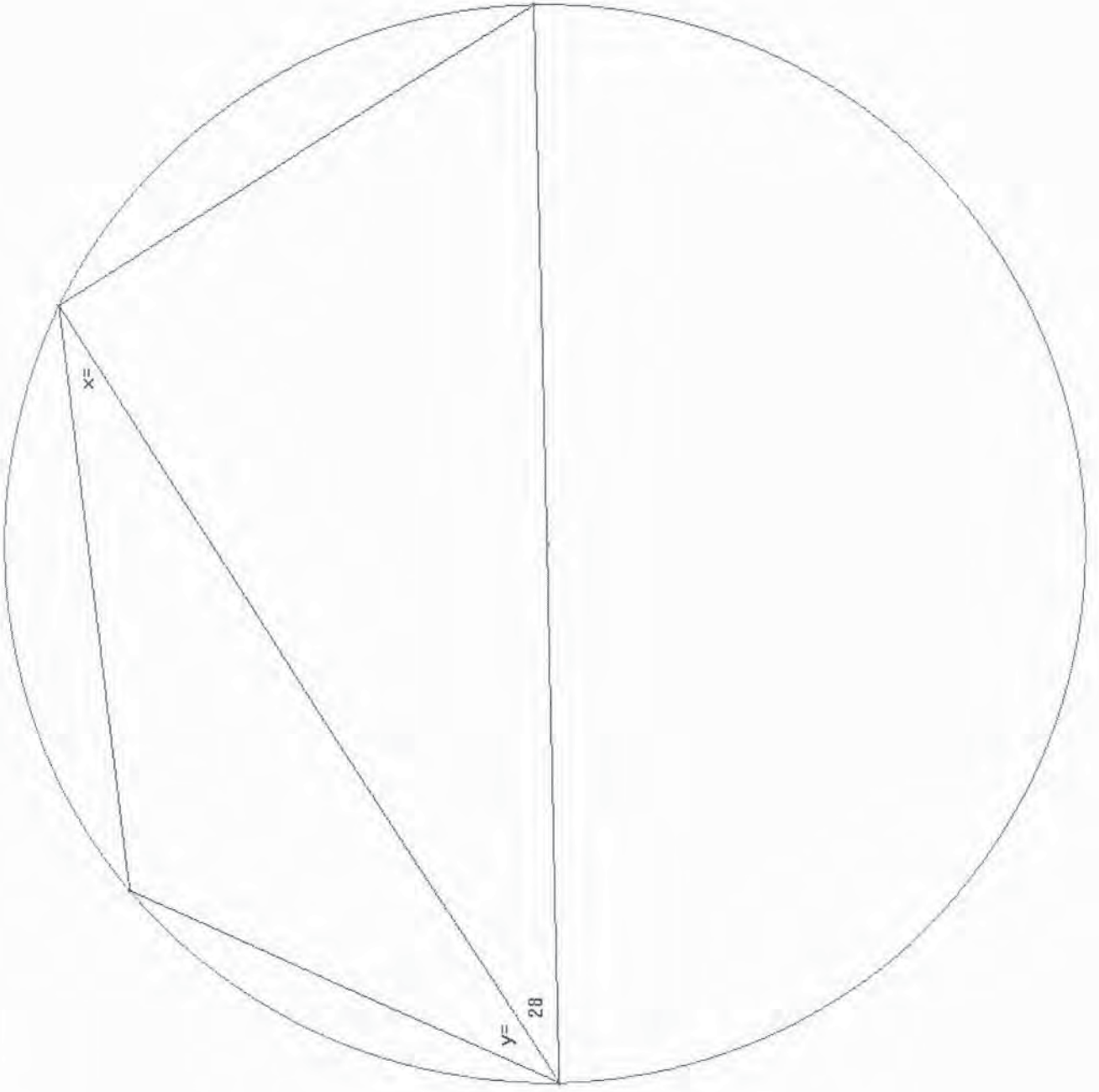
Figure not to scale

Sets of Problems Given to the Problem Solving group

Name _____
Find y



Find y



Find x, y, z

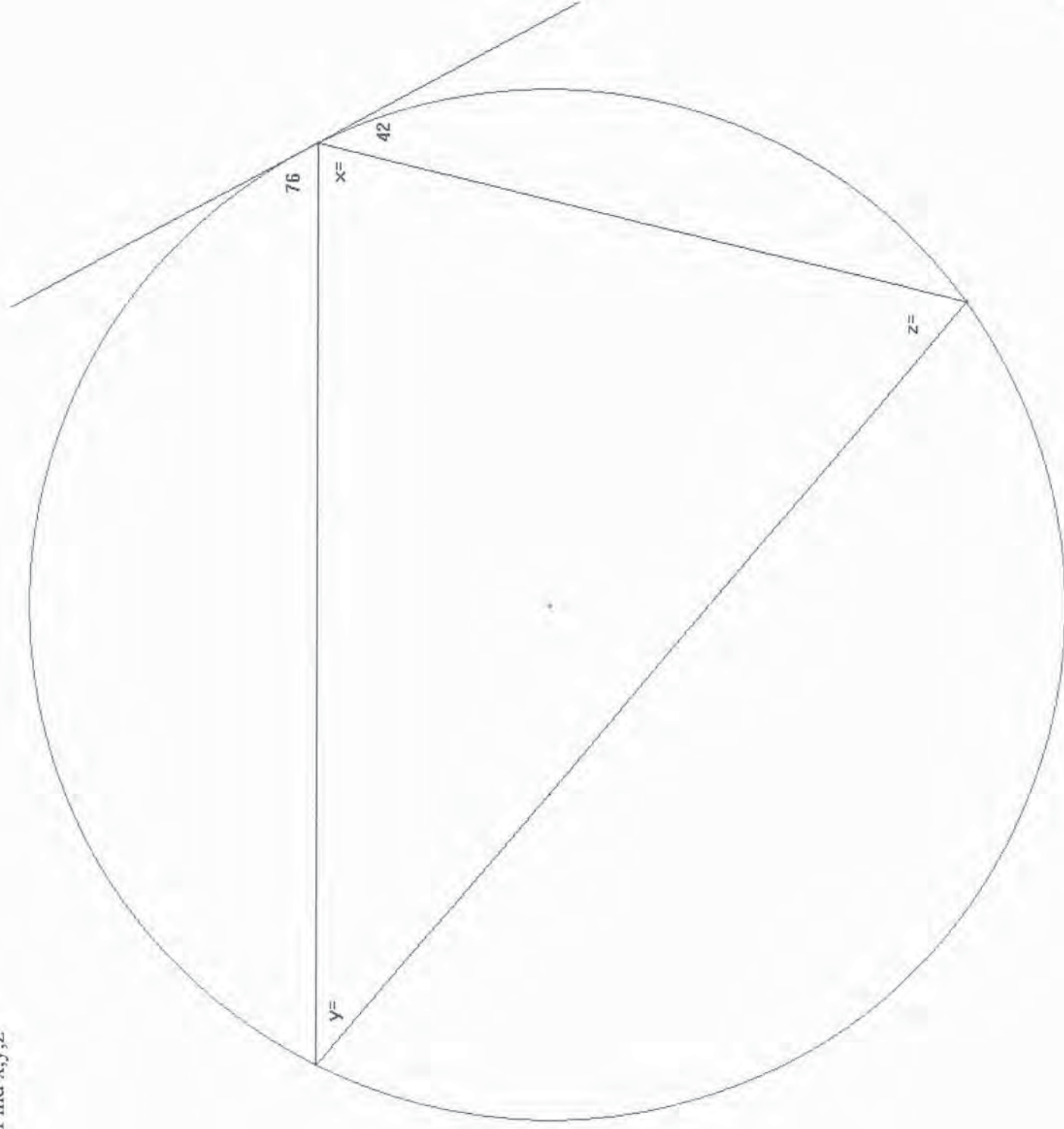


Figure not to scale

Appendix I

Similar and Transfer Test Questions of Experiment 4

Similar Test Problems

Name _____
Find y

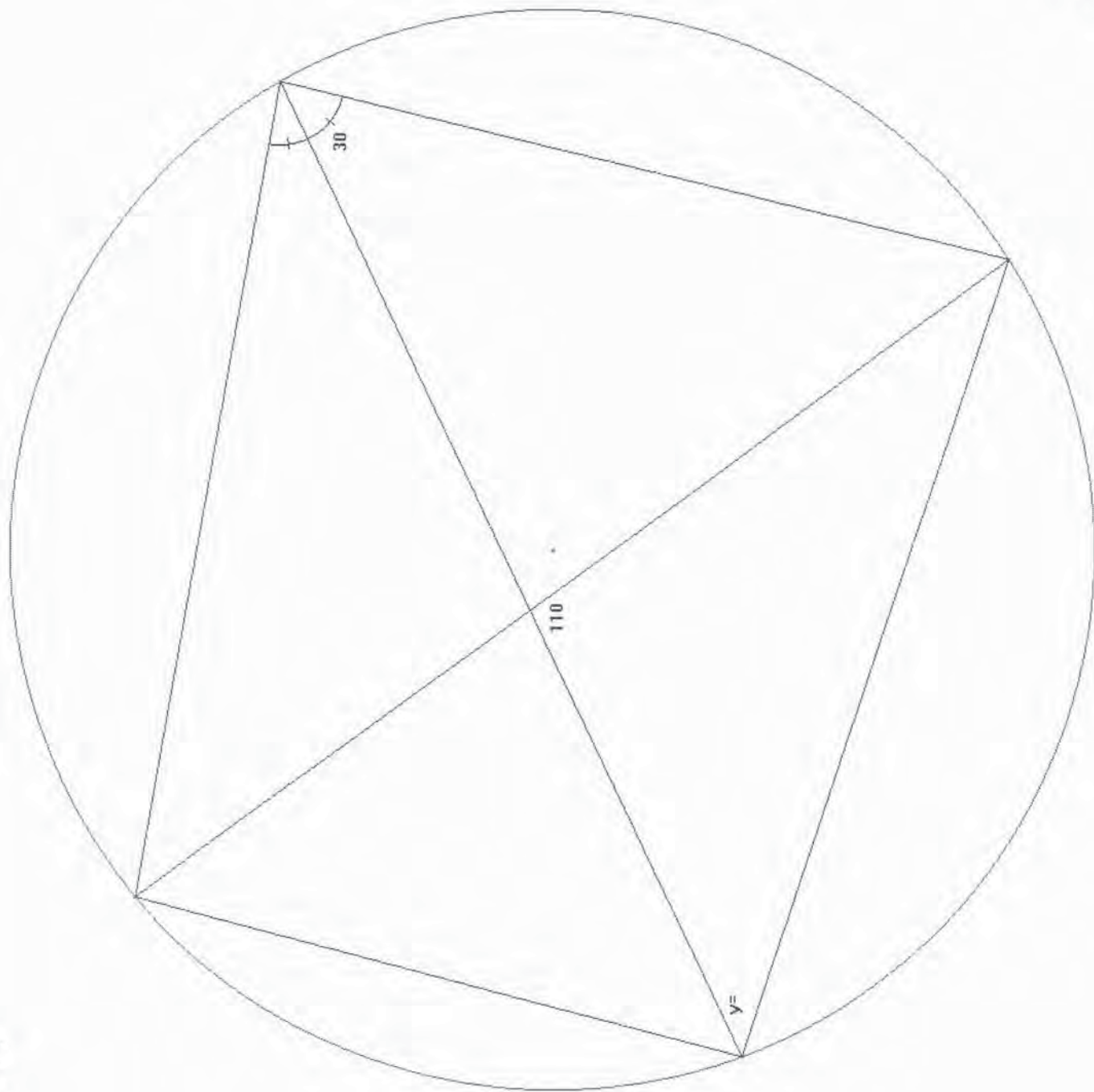


Figure not to scale

Find y

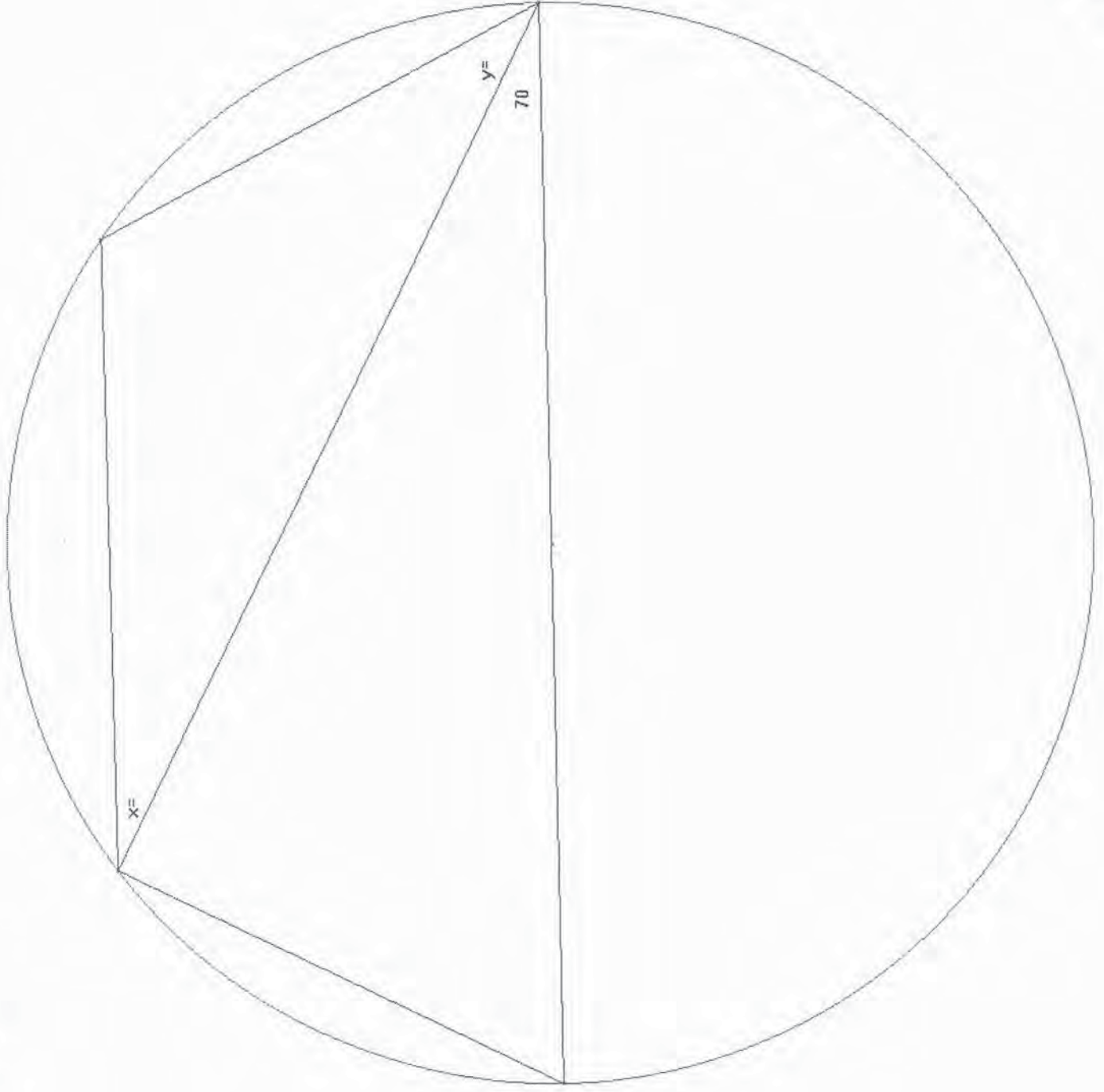


Figure not to scale

Find x, y, z

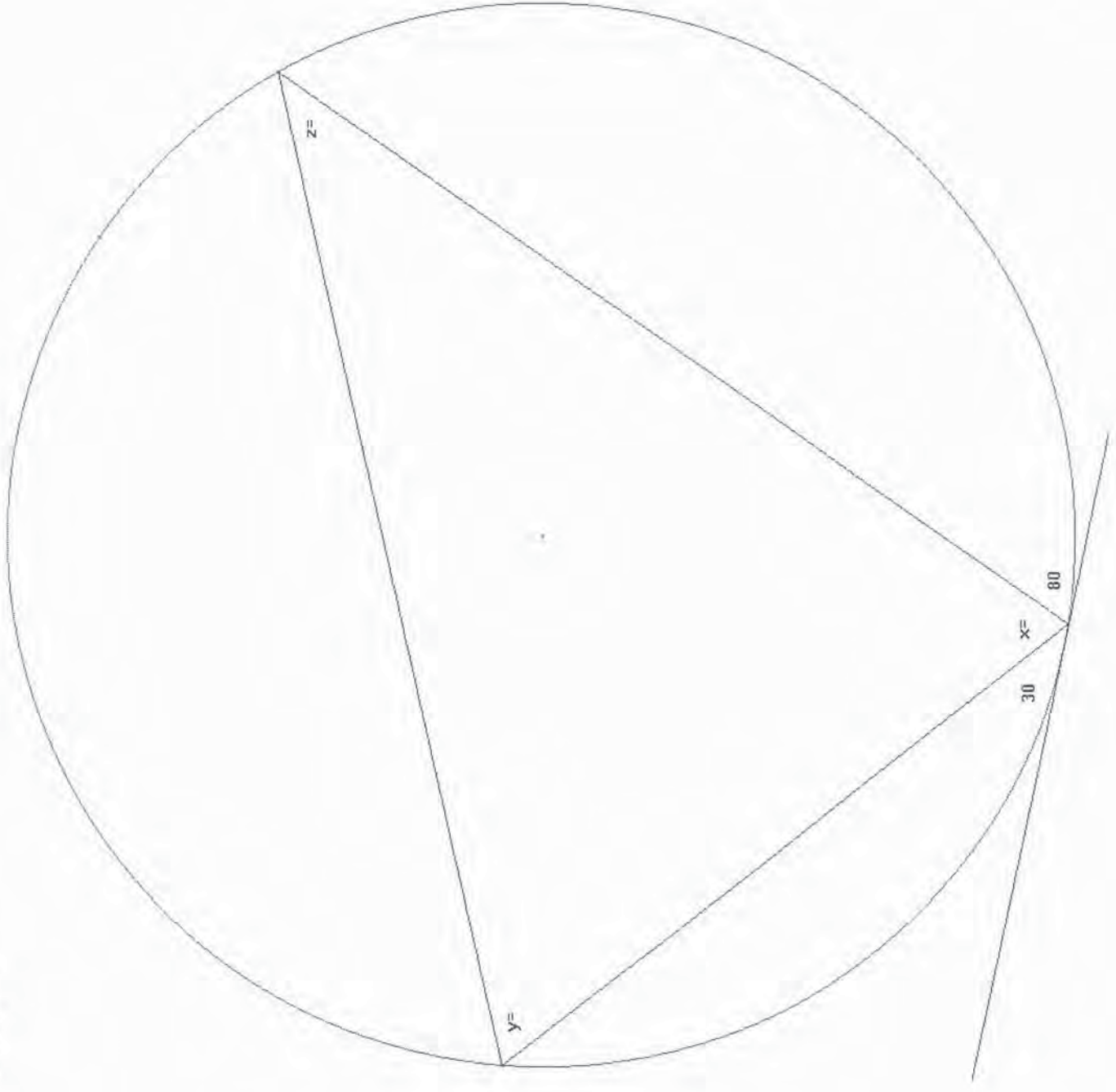


Figure not to scale

Transfer Test Problems

Name _____
Prove that $x = y$

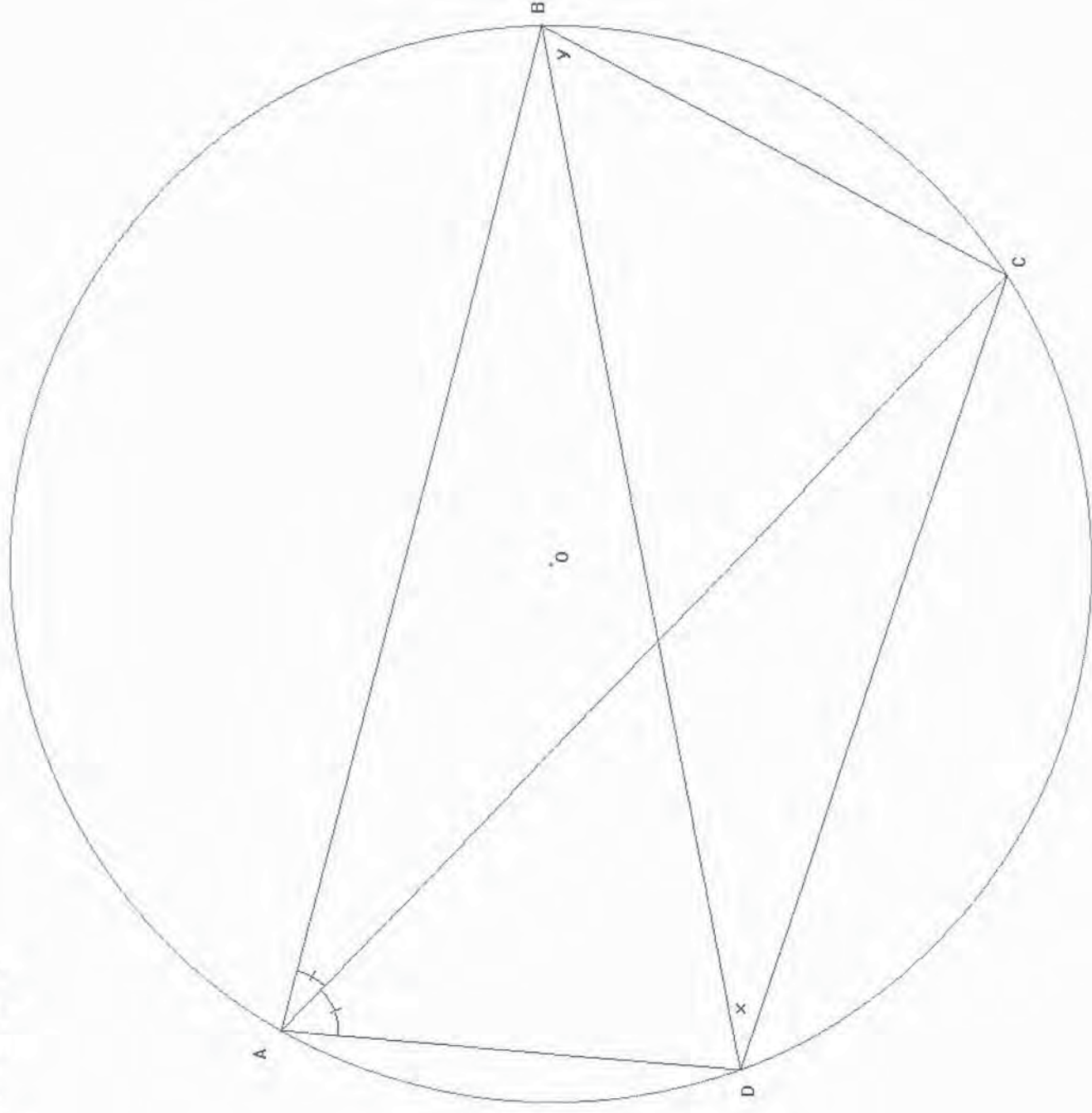


Figure not to scale

Prove that $x = y$

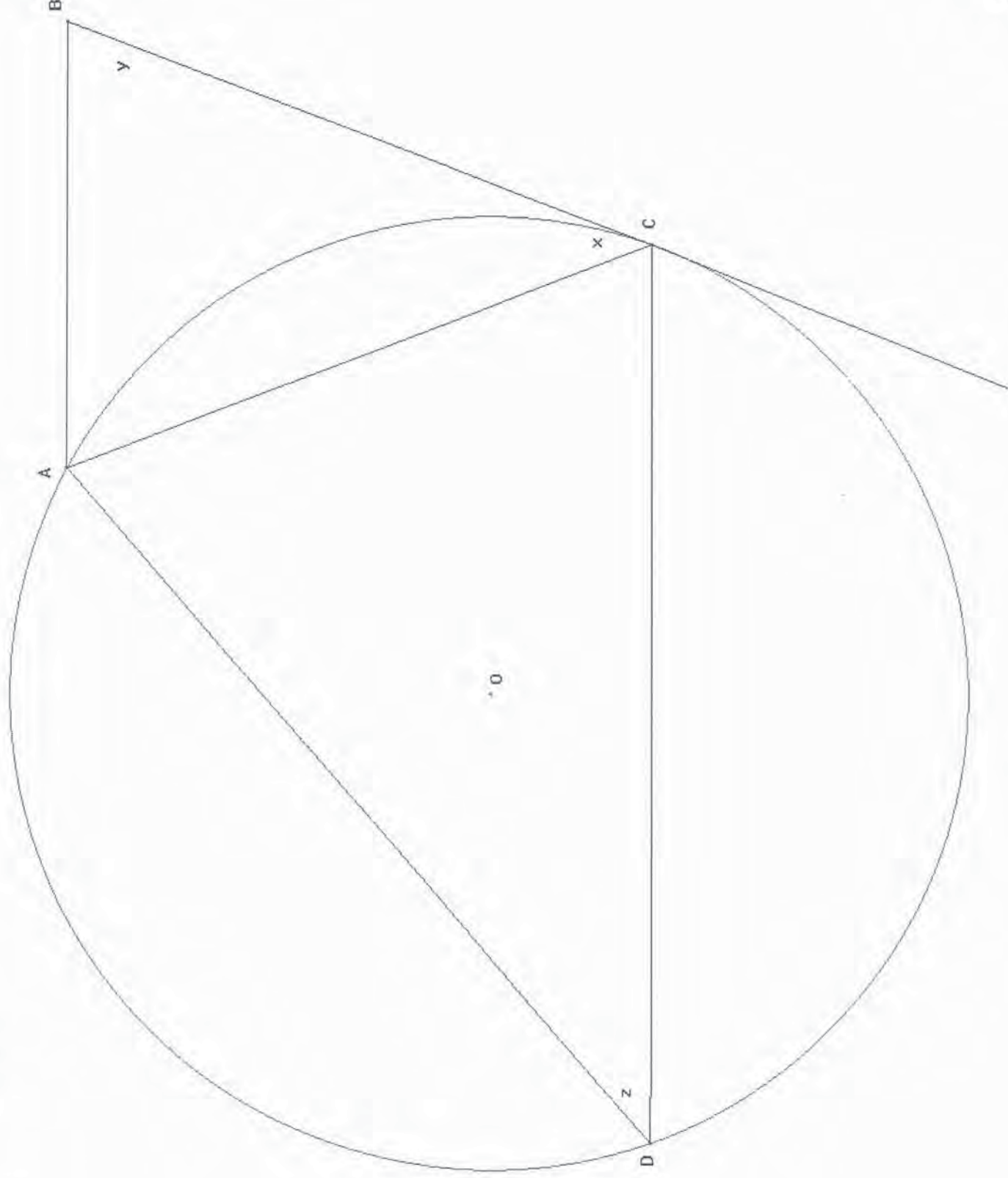


Figure not to scale

Find that $x = y$

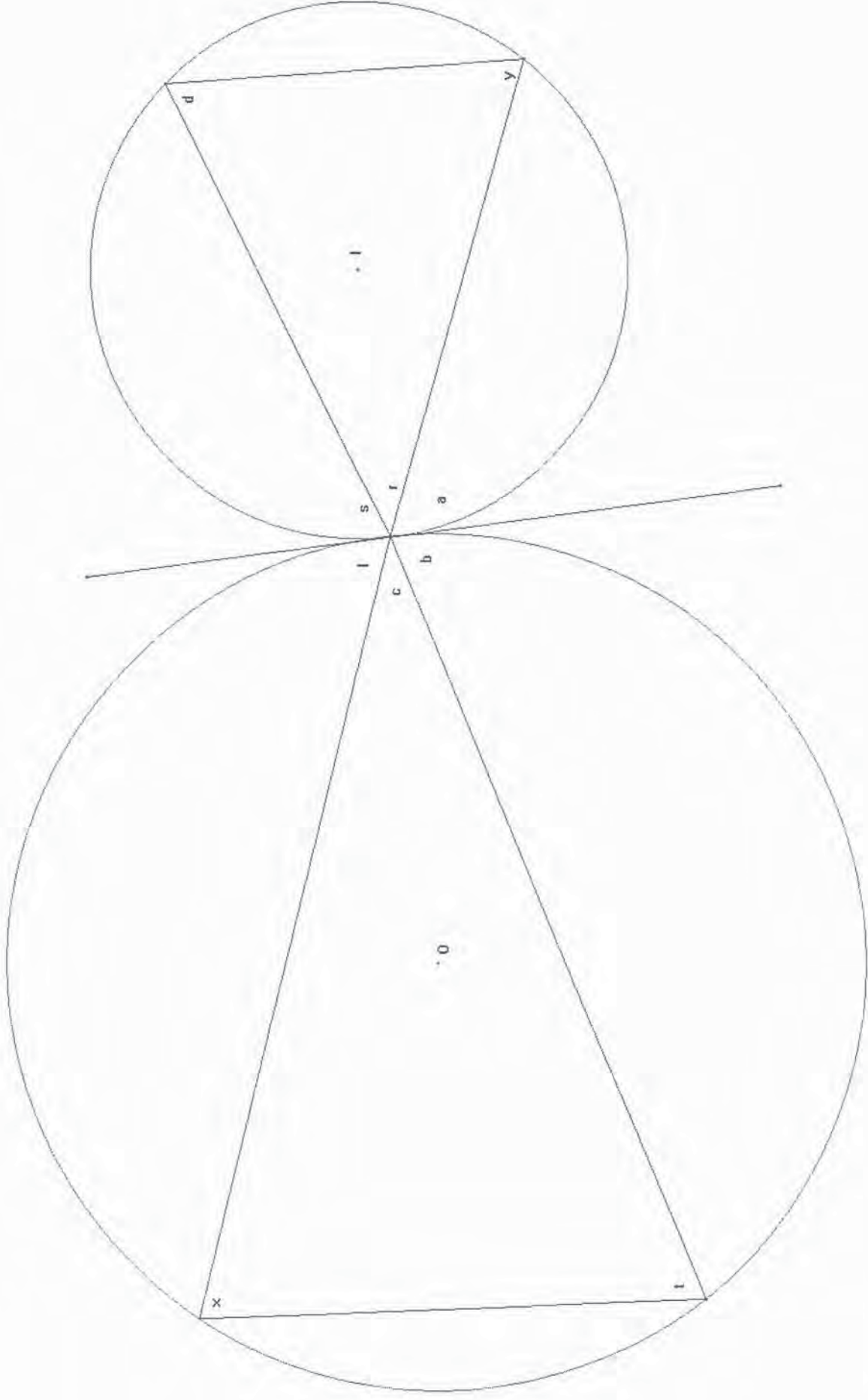


Figure not to scale