Radar rainfall estimation: consideration of input and structural uncertainty

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Abstract

Accurate spatial and temporal distribution of rainfall is important for hydrological applications. Rain gauges and weather radars are most widely used sensors for rainfall measurement. This research is a step towards improved rainfall estimates by reducing the weaknesses that restrict their ability to represent the rainfall field properly. This thesis focuses on the methods to minimize the uncertainty of radar rainfall estimates in parametric approach. Considering the limitations of the parametric approach, a nonparametric method has been developed. This research also demonstrates a new technique to merge radar and gauge rainfall. Finally, studying several existing merging methods, the optimal method has been suggested.

In parametric approach, the uncertainty in reflectivity to rainfall conversion parameters originates from the inaccurate representation of spatial distribution of rainfall from rain gauge at radar grid resolution, and from incompatible temporal resolutions of radar-gauge pairs. An error model has developed to compute the spatial uncertainty of gauge. Then SIMEX method uses this error model to determine the uncertainty present in the conversion parameters. The uncertainty related to temporal resolutions is addressed by using the optimal temporal resolutions of radar reflectivities for a specified gauge temporal resolution that results best radar estimate across ten Australian weather radars. A nonparametric (NPR) method for converting radar reflectivity into rainfall is suggested that uses the bandwidth of reflectivity and rainfall pairs. Compared to parametric method the NPR method performs better in reducing error at around 90% of the locations. The NPR has smaller errors than spatially interpolated gauge rainfall in regions with low gauge density.

Methods to merge radar and gauge rainfall are investigated to avoid the concerns about the biases in radar rainfall and difficulties of gauges to capture spatial representation of rainfall. The proposed dynamic combination method uses weights that vary in space and time and is calculated from the error covariance matrix. The performance of the new method is better than the individual radar or gauge estimates and is about 20% more accurate than the traditional parametric method. Comparing this new and other existing merging methods, an optimal method is suggested depending on the attributes of interest.

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Summary

Accurate spatial and temporal distribution of rainfall is important for hydrological applications. Rain gauges and weather radars are most widely used sensors for rainfall measurement. This research is a step towards improved rainfall estimates by reducing the weaknesses that restrict their ability to represent the rainfall field properly. This thesis focuses on the methods to minimize the uncertainty of radar rainfall estimates in parametric approach. Considering the limitations of the parametric approach, a nonparametric method has been developed. This research also demonstrates a new technique to merge radar and gauge rainfall. Finally, studying several existing merging methods, the optimal method has been suggested.

In parametric approach, the uncertainty in reflectivity to rainfall conversion parameters originates from the inaccurate representation of spatial distribution of rainfall from rain gauge at radar grid resolution, and from incompatible temporal resolutions of radar-gauge pairs. An error model has developed to compute the spatial uncertainty of gauge. Then SIMEX method uses this error model to determine the uncertainty present in the conversion parameters. The uncertainty related to temporal resolutions is addressed by using the optimal temporal resolutions of radar reflectivities for a specified gauge temporal resolution that results best radar estimate across ten Australian weather radars.

A nonparametric (NPR) method for converting radar reflectivity into rainfall is suggested that uses the bandwidth of reflectivity and rainfall pairs. Compared to parametric method the NPR method performs better in reducing error at around 90% of the locations. The NPR has smaller errors than spatially interpolated gauge rainfall in regions with low gauge density.

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1. Introduction and literature review

1.1 Motivation

Accurate rainfall measurement plays a significant role in hydrology and water management (Barros et al., 2015; Cramer et al., 2014; Reisinger et al., 2014). Rainfall is the most important input that controls the reliability and accuracy of the hydrological model in predicting stream flow and runoff. Therefore, a key challenge of hydrology is the accurate measurement and prediction of the spatial and temporal distribution of rainfall (Liechti et al., 2013; Looper and Vieux, 2012; Wright et al., 2014). Meteorological events affect human life. Extreme rainfall within short time causes flood. In recent years, extreme events are becoming more frequent, causing damage and loss of life (Stocker, 2014) that creates significant challenges for adaptation in many parts of Australia and the rest of the world as well.

Rain gauges are a widely used instrument for getting rainfall information at a fixed location, though gauges have limitations in representing the spatial distribution of rainfall field. There is increasing interest in getting good estimates of rainfall at ground level by using remote sensing tools (Looper and Vieux, 2012; Nikolopoulos et al., 2015). With the improvement of hydro-meteorological models and short-term prediction systems, it is necessary to have real-time rainfall field with full spatial coverage. Therefore, the quantitative rainfall prediction system has gained more popularity compared to the qualitative rainfall prediction systems, such as weather radars. Recent progress is helping to improve the quality of weather forecast systems where rainfall forecasting based on the radar data is one of the main inputs (Liechti et al., 2013; Rendon et al., 2013; Seo et al., 2013; Smith et al., 2011). Rainfall is one of the most important inputs to hydrological analysis and modelling. Any error in this input will propagate through the model and will introduce uncertainty in subsequent predictions (Hossain et al., 2004; Morin et al., 2005; Pessoa et al., 1993; Sharif et al., 2002; Vieux and Bedient, 1998; 2007; Zhu et al., 2013). Therefore, the accurate characterization of radar rainfall errors is important to avoid propagation of uncertainties in input radar rainfall into predictions from hydrologic models. The motivation of this research is to develop an approach to improve rainfall estimates. The goal of this research is to develop a set of techniques to estimate rainfall from radar and enhanced techniques for merging radar and gauge estimates. It is expected
that this research will be a step towards improved spatial and temporal estimates of rainfall distribution.

This chapter is organized into five sections, starting with background information on the research, including research context, followed by research aims, thesis outline, and presentation of the research contribution.

1.2 Research background

This section reviews the literature on radar research and identifies the knowledge gaps. In addition, the introductory sections of Chapter 2 to Chapter 5 provide a brief summary of the relevant literature for the problems addressed in individual chapters.

1.2.1 Reflectivity – rainfall relationship

Weather radar emits and receives electromagnetic waves during its operation. The emitted electromagnetic energy transmitted through the atmosphere is backscattered by raindrops or other obstacles on its way. A fraction of this electromagnetic energy returns to the radar. The strength of the electromagnetic wave backscattered in the atmosphere is termed as ‘Reflectivity’. The radar rainfall estimation includes the uncertainty associated with the reflectivity measurement process that includes ground clutter, beam blocking, anomalous propagation, hail, bright band, attenuation, range-dependent bias (Chumchean et al., 2006b; Villarini and Krajewski, 2010). The process of radar rainfall estimation includes measurement of reflectivity, removal of errors caused during reflectivity measurement, conversion of the estimated reflectivity into rainfall and adjustment depending on gauge rainfall measurement (Chumchean et al., 2006a).

The transformation of radar reflectivities into ground rainfall contains considerable uncertainty (Anagnostou et al., 1999; Hossain et al., 2004; Knox and Anagnostou, 2009; Krajewski and Smith, 1995). The calibration of the weather radar is an essential step in obtaining accurate rainfall estimates at the ground from the radar reflectivities measured aloft. The exact manner in which the reflectivity-rainfall conversion is carried out affects the accuracy of the radar rainfall estimates. There are a number of methods used to estimate rainfall from reflectivity; the parametric method (Hasan et al., 2014; Verrier et al., 2013), nonparametric approaches (Calheiros and Zawadzki, 1987). Nonparametric method includes probability matching method
(Rosenfeld and Amitai, 1998; Rosenfeld et al., 1993; Seed et al., 1996) and Kernel-based nonparametric method (Hasan et al., 2016). Among all these methods, the parametric method, known as a power-law relationship \( Z = AR^b \) between reflectivity \( Z (\text{mm}^6/\text{m}^3) \) and ground rainfall rate \( R (\text{mm/h}) \) is widely used to estimate radar rainfall (Krajewski and Smith, 2002; Mapiam et al., 2009). This relationship has two coefficients, the pre-factor \( A \) and the exponent \( b \). Some parts of the world use drop size distributions (DSDs) obtained by disdrometers to estimate Z-R relationship parameters (Prat and Barros, 2009; Sánchez-Diezma et al., 2000; Verrier et al., 2013). However, DSD measurements are not commonly available in developing countries (Mapiam et al., 2009). Therefore, the Z-R relationship parameters are routinely estimated by using a least square regression calibration technique aimed to minimize the deviations between radar reflectivity and ground based gauge measurement (Chumchean et al., 2004; Hasan et al., 2014; Mapiam et al., 2009). Previous studies on Z-R relationships indicated that several factors, such as different climates, seasons, regions, rainfall types, rainfall intensity and drop size distributions (DSDs) significantly affect Z-R relationship parameters (Chumchean et al., 2006b; Lee et al., 2005; Morin et al., 2005; Steiner et al., 2004; Uijlenhoet, 2001). However, in many parts of the world, including Australia, gauge based calibration of the Z-R relationships is routinely used. Furthermore, in developing countries where disdrometer data is not available and even sub-daily rainfall measurements are infrequent, daily rainfall gauges are the sole source of information on ground rainfall estimates (Mapiam et al., 2009). In these cases, gauge densities are also likely to be very low compared to developed countries such as the United States of America and Europe.

The uncertainty in the Z-R relationship introduces discrepancy between radar and gauge rainfall. Because of the inappropriate Z-R relationship and spatial subgrid scale gauge rainfall variability, this parametric relationship often introduces biases between radar and point gauge rainfall (Ciach et al., 2007; Jordan et al., 2003).

The conventional Doppler radars gives one-dimensional picture of precipitation by sending and receiving only horizontal pulse of energy. In contrast, the dual-polarization radars measure both the horizontal and vertical dimensions of precipitation by sending and receiving both horizontal and vertical pulse of energy. Therefore, the Dual-polarization radar gives the improved information about the type and the intensity of precipitation compared to Doppler radar (Bringi et al., 2011; Uijlenhoet et al., 2003).
The radar network in Australia is mostly comprised of reflectivity only radars and as yet there are no operational dual-polarisation radars in the operational network. Therefore, this research deals with the data available from Doppler radar. The Australian Bureau of Meteorology does not operate a disdrometer under each radar in the network and therefore the primary technique for estimating the parameters for the Z-R relationship depends on using 30-minute gauge accumulations and radar reflectivity at the gauge locations. Furthermore, the radar network is mostly comprised of reflectivity only radars and as yet there are no operational dual-polarisation radars in the operational network. The data that has been used is the 1.5 Constant Altitude Plan Position Indicator (CAPPI) data. It is a horizontal cross section through the three-dimensional radar field. Therefore, the radar data being used in this study is at a height of 1.5 km. The reflectivity data obtained from the Australian Bureau of Meteorology (BoM) processed through quality control that includes vertical profile reflectivity correction, bright band and range dependent error correction which ensure the physics of reflectivity data. The following section discusses the radar rainfall estimation error caused by the uncertainty in the gauge measurement.

1.2.2 Spatial variability of the Z-R relationship

Since the beginning of research on radar rainfall estimation (Marshall and Palmer, 1948), the accuracy of radar rainfall has been quantified with reference to point gauge measurement at the ground (Anagnostou et al., 1999; Habib and Krajewski, 2002). These point gauge measurements are therefore considered as the ‘ground reference’ (Lebel and Amani, 1999; Wolff et al., 2005). Gauge data and radar pixels are representative of a different support size and there is a spatial discrepancy between the two data sources, since radar rainfall estimates are provided as spatial averages with a resolution of 1-4 km² (Krajewski and Smith, 2002). In contrast, rain gauges measure precipitation at fixed point locations, and are thus often too sparse to properly represent the spatial variability and areal structure of rainfall (Morrissey et al., 1995; Rodríguez-Iturbe and Mejía, 1974). Rain gauges measurements also contain a variety of errors including wind effects, evaporation and mechanical error (Groisman and Legates, 1994), spatial sampling error (Morrissey et al., 1995) that may introduce considerable uncertainty depending on the spatial variability. Moreover, point gauge fails to present areal rainfall in many cases and certain number of gauges are required to represent areal rainfall conditioned on accuracy level (Villarini and Krajewski, 2008). The term uncertainty is used to illustrate the errors
resulting from the approximation of an aerial estimate using a point estimate. Therefore, the dense rain gauge network is more appropriate to properly model the radar rainfall uncertainty (Krajewski et al., 2010a). An important question is whether the Z-R relationship parameters remain the same if the errors associated with spatial averaging of the gauge rainfall over the radar grid scale is considered? The next section describes the impact of temporal resolution of radar reflectivity on radar rainfall estimates.

### 1.2.3 Temporal variability of the Z-R relationship

Gauge generally provides rainfall estimates with relatively smaller and bias-free measurement errors (Delrieu et al., 2009; Hasan et al., 2014; Krajewski and Smith, 2002; Lebel and Amani, 1999). Therefore, Z-R relationship derived from the reflectivity-gauge relationship found to be superior to DSD derived one. However, the choice of reflectivity – rainfall pairs is mainly reliant on the temporal resolution of gauge measurement and radar reflectivity. Radar provides rainfall estimates at high temporal resolution which is one of the reasons that radars are widely used for hydrological simulation (Habib et al., 2008; Schuurmans et al., 2007; Smith et al., 2007; Smith et al., 2012; Wright et al., 2014). However, studies on radar temporal resolution are mainly focused on finding the optimum temporal resolution that can effectively simulate the catchment hydrological behaviour (Atencia et al., 2011; Mapiam et al., 2014; Morin et al., 2001). The Z-R relationship parameters differ significantly with the temporal resolution of gauge rainfall (Mapiam et al., 2009). For the real-time application, the Z-R relationship needs to be calibrated with a specific time interval to get the best radar rainfall estimates (Alfieri et al., 2010). Commonly, the Z-R relationship is calibrated using either hourly or daily accumulated data, assuming the radar reflectivity remains constant over the gauge accumulation period. However, radar has a temporal resolution of either six minutes or ten minutes depending on the type of radar used to measure reflectivity. Also, radar reflectivities vary significantly from one scan to another, though hourly averaged reflectivity is traditionally used in Z-R relationship calibration process. Therefore, it is important to investigate how do radar rainfall estimates vary with the temporal discretization of reflectivity? Next section reviews the alternate way getting rainfall estimates from radar reflectivities.
1.2.4 Nonparametric radar rainfall estimates

The uncertainties in converting the radar reflectivity to rainfall rates have hampered the widespread use of radars in hydrology (Villarini and Krajewski, 2010). Z-R relationships are typically complex. Theoretically, reflectivity and rainfall intensity should be proportional to the 6th and 3.7th moments of the raindrop diameter respectively (Marshall and Palmer, 1948). Hence, radar reflectivity is more sensitive to rain drop diameter than to rainfall rate. Moreover, it has been observed that the Z-R relationship can be non-injective, such that a reflectivity value can correspond to samples having different drop size distributions and rainfall intensities (Ochou et al., 2011; Uijlenhoet, 2001). The commonly used power-law, with only two free parameters (A and b), is unable to capture this complexity. To overcome this limitation by making the best use of available records of radar reflectivity and ground rainfall, a nonparametric method for converting radar reflectivity into rainfall was proposed by Calheiros and Zawadzki (1987). It assumes the same probability for gauge measured rainfall and radar-derived estimates (Rosenfeld et al., 1993). This probability matching method (PMM) removes limitations related to the sampling volume. In addition, PMM also eliminates collocation and timing errors because it does not consider the actual timing when the Z-R pair occurred (Piman et al., 2007). Later, the Window Probability Matching Method (WPMM) (Rosenfeld et al., 1994) was developed, which alleviates limitations of the PMM method by considering homogeneous rainfall regions. In the WPMM, the probability distribution of reflectivity is matched with gauge rainfall over small spatial extents and time windows. Finally, the Window Correlation Matching Method (WCMM) method (Piman et al., 2007) was developed by introducing a space window to correct timing and collocation errors. In the estimation of rainfall accumulation using this method, the WPMM and bias-corrected regression based Z-R relationship have similar skill (Rosenfeld and Amitai, 1998) although there remains probability of overestimation of low rainfall intensities and underestimation of high rainfall intensities. The PMM has a drawback regarding the estimation of high rainfall rates. The maximum rain rate predicted by the PMM cannot exceed the observed maximum rain rate. To eradicate this issue, a least square regression based Z-R relationship is suggested (Seed et al., 1996). Piman et al., (2007) found that WCMM method provides better rainfall estimates of mean aerial and point rainfall than PMM and WPMM. However, all these methods do not consider the joint distribution of
reflectivity and rainfall. Also, these methods ignore the actual inter-association of radar-gauge pairs. Another limitation of these methods is the extrapolation of reflectivity when larger than observed reflectivity is recorded. The use of all the reflectivity – rainfall pairs is the major limitation of least square regression based traditional Z-R relationship method. Therefore, it is important to develop a method that addresses these limitations and uses the bandwidth of reflectivity and rainfall pair instead of utilizing all the reflectivity. The next section reviews the methods of getting improved rainfall estimates by combining radar and gauge estimates.

1.2.5 Radar and rain gauge combination

Though radar and gauge rainfall estimates are fairly correlated (Creutin et al., 1988), gauges provide accurate rainfall estimates at point locations whereas radar provides rainfall estimates over a widespread domain (Berndt et al., 2014; Hasan et al., 2014). However, concerns about the biases in radar rainfall estimates hampers their direct use in hydrological studies (Mandapaka et al., 2009; Mandapaka et al., 2010; Seo and Breidenbach, 2002; Smith et al., 2007; Villarini et al., 2008; Zhang and Smith, 2003). On the other hand, the spatial representativeness of gauge rainfall can be poor as gauges are often sparsely distributed over a catchment, particularly in areas of high relief where there may be large rainfall gradients. Given these potential weaknesses in both data sources, what can be done to provide better rainfall estimates?

The idea of combining multiple outputs is widely used in the field of econometrics (Bates and Granger, 1969; Genre et al., 2013; Hall and Mitchell, 2007; Timmermann, 2006). Model combination approaches have been applied in hydrology and climate science, including streamflow simulation and forecasting (Ajami et al., 2006; Georgakakos et al., 2004), rainfall-runoff modelling (Shamseldin and O'Connor, 1999) as well as weather and climate models (Chowdhury and Sharma, 2009; Khan et al., 2014; Krishnamurti et al., 1999; Krishnamurti et al., 2000).

Radar and gauge merging can improve rainfall estimates by exploiting their strengths and correcting for their shortcomings (Burlando et al., 1996; Goudenhoofdt and Delobbe, 2009; Haberlandt, 2007; Schiemann et al., 2011). Radar has high spatial and temporal resolution regarding areal precipitation estimation, whereas rain gauge offers a better precision at the point scale. The idea of merging radar and gauge measurements is
not new and a number of different methods have been developed in this regard. Previous studies were mostly focused on the application of gauge data for correcting systematic errors in radar rainfall estimates. The most common application is Mean Field Bias (MFB) correction of radar rainfall (Chumchean et al., 2006b; Seo, 1998a; Seo, 1998b; Seo et al., 1999; Smith and Krajewski, 1991; Steiner et al., 1999). It considers a multiplicative adjustment factor calculated as a ratio between accumulated radar rainfall and accumulated gauge rainfall (Kitzmiller et al., 2013; Seo et al., 2013). Although MFB correction improves radar rainfall quality (Rabiei and Haberlandt, 2015), it underestimates rainfall in some situations (Chumchean et al., 2006b). A similar method is suggested by Seo and Breidenbach (2002) to correct spatially varying non-uniform bias in radar estimates by considering a small bin within the radar domain. In this local bias correction method, radar-gauge pairs are collected from fixed radius of influence, then those pairs are used to correct bias (Seo et al., 2013) which is the main difference from MFB method. Based on the number of rain gauges and their distance from the radar, Chumchean et al., (2006b) present a method to update the current MFB estimate by applying a Kalman filter. However, the application of bias correction depends mostly on the availability and quality of gauge data (Seo et al., 2013). In real time, the number of rain gauges available is often very small. Therefore, spatial averaging of gauged locations is required to apply MFB method to the ungauged region (Chumchean et al., 2006b; Kitzmiller et al., 2013; Rabiei and Haberlandt, 2015; Seo, 1998a; 2002; Seo et al., 2013; Seo, 1998b; Seo et al., 1999; Smith and Krajewski, 1991; Steiner et al., 1999).

Several geostatistical methods have been proposed for radar and gauge merging, such as ordinary kriging, co-kriging, kriging with external drift (KED) (Berndt et al., 2014; Creutin et al., 1988; Delrieu et al., 2014; Goudenhoofdt and Delobbe, 2009; Haberlandt, 2007; Hwang et al., 2012; Krajewski, 1987; Schuurmans et al., 2007; Sideris et al., 2014; Velasco-Forero et al., 2009) and wavelet analysis (Kalinga and Gan, 2012). The KED method outperformed other kriging-based methods (Delrieu et al., 2014; Haberlandt, 2007). This geostatistical merging is done by applying two different approaches (Jewell and Gaussiat, 2015). In the first approach, initially the gauge rainfall is spatially interpolated independently of the radar, and then the difference between radar and spatially interpolated gauge rainfall is added to the radar rainfall to get an estimate at an unknown location. This idea is used in kriging with radar-based error correction (Ehret
et al., 2008) and in conditional merging (Sinclair and Pegram, 2005) also. In the second approach, before the generation of the interpolated field, the radar estimate at ungauged locations is formulated using weights. The weights are determined from the relationship between the measured radar and gauge rainfall.

It has to be mentioned that the studies of radar and gauge merging consider that the gauge rainfall is the primary true source and the radar data is the auxiliary information. Radar data can be used to improve the spatial interpolation of gauge measurements (Goudenhoofdt and Delobbe, 2009; Rabiei and Haberlandt, 2015). This assumption is generally required because of the uncertainties and errors that result from converting the radar reflectivity to rainfall intensity. However, gauge and radar both have their own limitations regarding the rainfall pattern and amount. It is commonly admitted that the most useful information from the radar is the spatial pattern of the storm events rather than the magnitude of the rainfall (Méndez-Antonio et al., 2009). On the contrary, gauge measurements are more accurate but only representative of a very small area (Goudenhoofdt and Delobbe, 2009; Martens et al., 2013). Although there are uncertainties in radar rainfall estimates, it does contain useful information about the temporal distribution of rainfall (Martens et al., 2013; Shucksmith et al., 2011). There are also differences in combination approaches in whether the weights vary in time (dynamic weighting) or are optimised to provide a single weight at each location (static weighting). It is therefore important to develop an effective method to combine the radar and gauge estimates that can account for their respective strength and weakness. There is a need to investigate the optimal radar - gauge combination method depending on the merits and demerits of the method on a wide range of practical setting.

1.3 Research aims

Given the research background and questions, there remains immense needs and vital opportunities to improve rainfall estimates. Hence, the central question to be addressed in this study is:

*How to improve the spatial and temporal distribution of rainfall estimates?*

This thesis aims to address the knowledge gaps identified in the previous sections by utilizing the state-of-the-art tools, data sources, algorithms and statistical methods. The specific objectives of this research are, therefore, to:
• Correct biases in the Z–R relationship because of uncertainty in rain gauge network.
• Investigate the impact of discretization of radar reflectivities on radar rainfall
• Improve techniques to estimate radar rainfall using a nonparametric relationship.
• Improve techniques to estimate rainfall at the ungauged location by merging radar and gauge rainfall.
• Investigate the various approaches for radar and gauge combination and recommend the best method to improve spatial distribution of rainfall in a given context.

1.4 Thesis outline

The thesis is structured into two main sections to answer the questions raised in the research background. The first section presents methods to improve radar rainfall estimates. The second part examines the approaches to combine radar and gauge estimates. Figure 1-1 shows the overall thesis organization.

Chapter 1 explores the research background, research question, objective of the research, brief methodology, research context and expected contribution.

Chapter 2 provides details of a methodology that examines whether the Z-R relationship parameters remain same if consideration was given to the nature of the errors associated with spatial averaging of the gauge rainfall over the radar grid scale.

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<th>Part 1: Radar - Gauge Relationship</th>
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<td>Develop an approach to estimate rainfall at the ungauged location by merging radar and gauge rainfall.</td>
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<td>Investigate the various approaches for radar and gauge combination</td>
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<td>Water Resources Research, (2016), (accepted, in press)</td>
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**Figure 1-1:** Thesis organization.
Chapter 3 presents, the impact of radar temporal resolution on the accuracy of rainfall estimate and how radar rainfall estimates vary with the discretization of reflectivities.

Chapter 4 comprises two major sections which are kernel regression based nonparametric method and radar and gauge combination method. The NPR method addresses limitations of the existing nonparametric method. This chapter also presents an effective method to combine the radar and gauge estimates that can account for their respective strength and weakness.

Chapter 5 focuses on different approaches of combining radar and gauge estimates. It also suggests the best radar and gauge combination method that improves spatial distribution of rainfall.

Chapter 6, the last chapter, provides the summary and conclusion of this study. To accomplish the answer to the research questions and objectives, this chapter synthesises the overall findings followed by the contributions to the body of knowledge. At the end, several future research directions are suggested followed by the limitations of this research.

1.5 Research contribution

The study contributes in the following ways to radar rainfall estimation:

- Bias correction method addressing the uncertainty of point rain gauge networks
- Methods to reduce the uncertainty in radar rainfall estimation
- Techniques to estimate radar rainfall using nonparametric relationship
- Techniques to estimate rainfall at the ungauged location by merging the radar and gauge estimates
- Dynamic and static radar-gauge combination approach
2. Correcting bias in radar Z-R relationships due to uncertainty in point rain gauge networks

2.1 Introduction

One of the key challenges in hydrology is to accurately measure and predict the spatial and temporal distribution of rainfall. Rain gauges measuring at point locations are often considered as the ‘ground reference’ for grid based radar rainfall calibration. Usually, no consideration is given to the uncertainty in the measurement that varies depending on the number of rain gauges that fall within each grid cell. If this uncertainty in the rain gauge network measurements is ignored, the Z-R relationship used to convert reflectivity (Z) to rainfall (R) will be biased.

The radar estimates rainfall over 1 square kilometre at a height of 1000 m above the radar and the rain gauge observes rainfall at a point on the ground. It is recognized that radar-rainfall estimates are associated with significant uncertainties that arise from various factors such as: hardware calibration, significant variations and non-uniqueness in the relationship between radar-measured reflectivity (Z) and rainfall rate (R), range-related effects, beam overshooting and partial beam filling, bright band, anomalous propagation of the radar beam, beam blockage, ground clutter and spurious returns, random errors, and the non-uniformity in vertical profiles of reflectivity (VPR) (Austin, 1987; Krajewski et al., 2010b; Villarini and Krajewski, 2010). Therefore, there are numerous sources of doubts associated with radar-based rainfall estimates that produce the differences between the radar estimate and the gauge observation. The term “differences” would imply an additive error definition. Therefore, to avoid any specific error definition, the discrepancies between radar-rainfall and the corresponding gauge rainfall termed here as uncertainty, to indicate the discrepancies as well as sources of discrepancies. The term uncertainty widely used by several researchers in the field of radar hydrology (AghaKouchak et al., 2010; Anagnostou et al., 1999; Benoit et al., 2009; Chaubey et al., 1999; Ciach et al., 2007; Habib et al., 2008; Knox and Anagnostou, 2009; Krajewski and Smith, 1995; Krajewski et al., 2010b; Morrissey et al., 1995; Rico-Ramirez et al., 2015; Steiner, 1996; Villarini et al., 2008b; Zappa et al., 2010).

This chapter aims to address whether the A parameter in the radar Z-R relationship needs to change to account for uncertainty in the point gauge rainfall network. It has been
reported that the parameter \( A \) carries most of the variability in the Z-R relationship, whereas the uncertainty in \( b \) can be seen as second-order (Chumchean et al., 2003b; Chumchean et al., 2006b; Steiner et al., 1999). Marshall and Palmer (1948) proposed the power law relation, \( Z = 200 \, R^{1.6} \) with specified values for \( A \) equal to 200 and \( b \) equal to 1.6. Since then, several studies have been conducted to find appropriate \( A \) and \( b \) parameter values in different settings. Given the need for consistent estimates of rainfall, for operational purposes the Australian Bureau of Meteorology (BOM) specifies a fixed value for \( b \) equal to 1.53 for most of the weather radars that form its rainfall measuring network. Fixing \( b \) for operational networks has also previously been used by other researchers (Steiner and Smith, 2004; Verrier et al., 2013). The Simulation Extrapolation method (SIMEX) (Cook and Stefanski, 1994) has been used to investigate the significance of point gauge uncertainty on the Z-R relationship parameter \( A \). The SIMEX method involves developing a relationship between the gauge uncertainty distribution and input gauge rainfall, which can then be used to assess the bias in the parameters of the Z-R relationship. Moreover, for the case of radar rainfall estimation mean field bias is often known as bias which is the ratio, reported by several authors (Borga et al., 2002; Chumchean et al., 2006b; Ciach et al., 2000; Holleman, 2007). The term “error” used is the standard error. The standard error of the mean is the standard deviation of those sample means over all possible samples (of a given size) drawn from the population. The standard error of the mean can refer to an estimate of that standard deviation, computed from the sample of data being analysed at the time.

The chapter is organized in seven sections. Section 2.2 outlines the logic behind the SIMEX approach. Section 2.3 describes the radar and gauge data used in this study, whilst Section 2.4 presents the development of an error model for the lowest radar pixel resolution (1 km\(^2\)). The application of SIMEX method on radar Z-R relationship is presented in Section 2.5 followed by results and discussion in Section 2.6. The main findings are summarized in Section 2.7.

### 2.2 Simulation Extrapolation (SIMEX)

SIMEX is a method for parameter estimation that attempts to ascertain model parameters taking into account the error distribution associated with each predictor variable. It estimates parameter values that should have resulted if the covariates were error-free. The general idea behind the method is that if the error in the predictors causes
bias in the parameter estimates, then adding more error should cause the parameter estimates to become even more biased (Benoit et al., 2009). A relationship between the bias and the added error can be developed and this can be extrapolated back to the case where there is no error. This process is shown in Figure 2-1 where the naive estimate is the estimation of some parameter $\beta$ using the recorded data, including its inherent errors. Estimates to the right of the naive estimate represent the cases where more error has been added to give even more biased estimates of $\beta^*$.  

![Image](image.png)

**Figure 2-1:** Synthetic example of the SIMEX approach.

The trend of the bias versus the error is then extrapolated back to the case of no error, which is known as the SIMEX estimate. The error variance is multiplied by a factor $\lambda$, termed the Multiple of Error Variance. The difference between two consecutive $\lambda$’s is a variance inflation factor.

Consider a simple linear regression $Y = \beta X$. Suppose that, instead of observing the covariate $X$, an erroneous covariate $W$ is observed with an error distribution $U$ having a zero mean and variance $\sigma^2_u$ (or, $W = X + U$). The estimate for the coefficient $\beta$ (referred to as $\beta_{naive}$ and termed the naive estimate) is likely to be different from the estimate that would result if $W$ were error free (or, equal to $X$). In SIMEX, more erroneous realisations of $W$ with variance of $\lambda \sigma^2_u$ are drawn (these being termed $W^*$), and the resulting parameters (with an expected value of $\beta^*$, (shown with a circle in Figure 2-1 are
estimated. Finally, a relationship is developed with multiple of error variances (\( \lambda \)) and the expected value \( \beta^* \) corresponding to each \( \lambda \). The relationship is then extrapolated to the condition where no additive error is present, i.e. \( \lambda = -1 \), giving an unbiased SIMEX estimate of the target parameter which is denoted as \( \beta_{\text{SIMEX}} \).

SIMEX has been popular in statistical research due to its ability to produce an efficient parameter estimate for linear, nonlinear, logistic and nonparametric models (Carroll et al., 1996; Carroll et al., 1999; Cook and Stefanski, 1994; Holcomb, 1999; Marcus and Elias, 1998; Staudenmayer and Ruppert, 2004). Chowdhury and Sharma (2008) applied the SIMEX method in hydrology to determine the bias in the Sacramento Rainfall Runoff Model’s key storage parameters using synthetic data. They found that compared to naive estimates, SIMEX estimates were closer to the true values. SIMEX has also been used to reduce biases in parameter estimate of future droughts, which are due to uncertainties and errors in General Circulation Model (GCM) projections of rainfall (Woldemeskel et al., 2012). To date, SIMEX has not been applied in radar-rainfall calibration and this chapter aims to test the utility of the method in this context.

It is important to note that the SIMEX method requires the prior knowledge of the error distribution associated with the intended model parameter. Chowdhury and Sharma (2007) illustrates the SIMEX method using a synthetic example with additive error in a linear regression model. However, Tian et al., (2013) report that the multiplicative error models are more suitable than additive error models when dealing with precipitation measurements. Therefore, in this chapter, a multiplicative point gauge rainfall measurement error has been considered. The following section discusses the data used in this study and this is followed by the development of the multiplicative error model.

### 2.3 Data

In this research, the reflectivity data were obtained from the Australia Bureau of Meteorology for the Terrey Hills radar (Sydney, Australia) during the period from November 2009 to December 2011. The Terrey Hills radar is an S-band Doppler radar with six minutes temporal resolution and one km spatial resolution. The radar covers a region of 256 km by 256 km extent, with bandwidth of one degree and wavelength of 10.7 cm. The climatological freezing levels in Sydney are about 2.5 km (Chumchean et al., 2003b). Therefore, the 1.5 km Constant Altitude Plan Position Indicator (CAPPI)
reflectivity data were used to avoid bright band effects. The noise and hail effect was nullified by only considering reflectivity ranges from 15 dBz to 53 dBz (Chumchean et al., 2004; Chumchean et al., 2006a).

The rain gauge network used for the calibration of the radar includes 150 tipping bucket gauges located within 128 km of the radar as shown in Figure 2-2a. The Australian Bureau of Meteorology operates and maintains these stations, with the tipping bucket size being either 0.2 or 0.5 mm. In this study, a threshold on the minimum rainfall intensity of one mm/h was used to avoid quantification error at low rainfall intensity (Chumchean et al., 2004; Chumchean et al., 2006b).

For this research, storms are defined as discrete meteorological events through visual inspection of the time series of radar images. The start of each storm was chosen such that there was significant rainfall over a spatially coherent area (i.e. scattered small systems were not used in the analysis). The end of each storm was found when there was no longer any significant rainfall occurring in the region. Thus the duration of each storm is allowed to vary and the final dataset is compiled from all hours belonging to any storm. The selected 107 storm events lead to 1259 hours of rainfall and contain a wide range of storm durations varying between 1 to 79 hours, although 59% of the storm event duration ranges are less than 10 hours. It is therefore concluded that the data set contains a good mix of convective and stratiform events. The radar reflectivity and gauge rainfall data are extracted with the same event start and end time, assuming no time lag between radar and gauge rainfall. The radar rainfall software MAPVIEW (Chumchean et al., 2006b; Seed and Jordan, 2002) was used to import the reflectivity files and to accumulate the rainfall.

2.4 Error Model

The rain gauge data cannot be realistically assumed as error-free when calibrating the Z-R relationship. The SIMEX technique allows to account for the various sources of errors in the rainfall totals and to remove any corresponding bias from our estimate of the parameter $A$. As outlined above, there are two main sources of error in the rain gauge observations with respect to the radar-rainfall relationship. The first one is due to recording errors in the tipping bucket gauge. The second one is the error introduced by not properly sampling the spatial variability of rainfall at the scale of the radar pixels.
These two error components can be estimated separately and the total error calculated by combining them.

2.4.1 Measurement error model

As discussed previously, rain gauge measurements contain a variety of errors including wind effects, evaporation and mechanical error (Groisman and Legates, 1994) as well as the spatial sampling error (Morrissey et al., 1995). Using a network of rain gauges, Habib et al., (2001) investigated tipping bucket sampling error with the emphasis on gauge ability to present temporal variability for the case of small scale rainfall. Their research showed sampling interval and bucket size are the dominant factors that control gauge performance and its associated error. In contrast, Ciach (2003) found that gauge measurement error are highly reliant on rainfall intensity and timescale. However, both studies found that as the rainfall rate and/or accumulation time increases, the gauge measurement error generally decreases.

For the error in tipping bucket gauge measurements, the error model from Ciach (2003) is adopted. In this model, the gauge measurement uncertainty is a function of the rainfall intensity and the time period over which the rainfall is being aggregated. Shorter time scales were found by Ciach (2003) to have larger relative errors than longer time aggregations where the errors in the tipping buckets tended to cancel out. The tipping bucket error model is presented in Equation 2-1 and based on Ciach (2003) where the error is defined as the standard error (\(\sigma_g\)) of the gauge measurement.

\[
\sigma_g(T, P_T) = e_0(T) + \frac{R_0(T)}{R_i}
\]

where, \(e_0\) and \(R_0\) are the model coefficients at time scale \(T\), which for the one hour rainfall totals take the value of 0 and 0.16 respectively (Ciach (2003)). The rainfall rate is denoted by \(R_i\) in units of mm/hr.

2.4.2 Spatial disaggregation error model

Zawadzki (1973) provided an analytical description of the errors and the fluctuations of areal rainfall considering uniformly distributed rain gauges. Rodríguez-Iturbe and Mejía (1974) considered the spatial distribution of rain gauge and developed a method to estimate the standard error of areal mean for stratified and random rain gauge
network configurations. Morrissey et al., (1995) further developed the model for any spatial configuration of measuring sites by considering the correlation between a number of gauges as well as their spatial distribution. Krajewski et al., (1998) attempted to estimate the radar rainfall uncertainty at sub-pixel scales. Later, the radar rainfall estimation error was differentiated from rain-gauge sampling error by introducing an error separation technique. Other studies (Krajewski et al., 2000; Villarini et al., 2008a) used a variance reduction factor (VRF) to compute the accuracy of the pixel-average rainfall. Jensen and Pedersen (2005) studied rainfall variation for single radar pixel (500 by 500 m) of nine gauges and their results showed that within a pixel, the gauge rainfall varies up to 100% among neighbouring stations. The studies of Villarini et al. (2008a) used the VRF as an indicator of uncertainties associated with the average of point measurements with the approximation of the true areal value. They suggest that VRF depends on the network density, its configuration and the spatial correlation of the sampled process. Villarini and Krajewski (2008) pointed out that the spatial sampling uncertainty should be considered if a single gauge is used to approximate the areal estimates. The authors proposed that a minimum number of gauges are required to represent areal rainfall, for example they found that 25 gauges are required to estimate the true areal rainfall with 20% accuracy for a basin size of 12 km by 16 km. The radar rainfall estimation method as currently used is strictly valid only when the rainfall measured by a single gauge represents that measured in a radar pixel, as one could then argue that the spatial sampling uncertainty is relatively uniform. However, there are clearly significant scale differences between these two measurements.

Despite the extensive previous research on rainfall spatial variability, a simple error model was not found that could account for the error from incorrectly sampling the sub-grid variability at the radar pixel scale. Therefore, a model has been developed using data for the study region. Our hypothesis is that the uncertainty in the rainfall spatial variability should be a function of rainfall rate, gauge density and the size of the area that the point rainfall is being assumed to represent. The desirable characteristics of the error model are:

- The uncertainty in the gauge rainfall should approach zero as the area over which spatial variability becomes smaller;

- As the number of gauges in the area increases, the standard deviation should represent the true variability across the studied area;
As the rainfall intensity increases, the standard deviation should also increase.

For our study area, a relationship between the number of gauges and the standard deviation of the rainfall across a finite analysis area was formulated. Because of the relatively low density of gauges across a large area, most radar pixels only contain at most a single gauge and therefore a relationship between the sub-grid spatial variability and the gauge rainfall could not be determined. To overcome this, increasing levels of spatial aggregation were considered so that multiple gauges were contained in each aggregated grid cell. For the consideration of spatial aggregation, the same area was divided into subgrid sizes of 2 km by 2 km, 4 km by 4 km, 8 km by 8 km, 16 km by 16 km and 32 km by 32 km. Figure 2-2 shows the location of point gauges around the Sydney Terrey Hills radar. The highlighted area is enlarged to illustrate the number of rain gauges per cell for different subgrid sizes.

![Figure 2-2](image)

**Figure 2-2:** Location of point gauges around Sydney Terrey Hills Radar. The highlighted area shows enlarged view of different number of rain gauges for subgrid sizes of b) 8 km x 8 km c) 16 km x 16 km and d) 32 km x 32 km respectively.

The process is start by standardizing the data results across all rainfall intensities. To this end, the coefficient of variation (CV) (Pedersen et al., 2010) was computed as
\[ CV = \frac{\sigma}{R} \]  

where, \( \sigma \) is the standard deviation of rainfall in each subgrid and \( R \) is the mean rainfall for that subgrid. For each rainfall event and for all combinations of aggregation area, the numbers of gauges recording rainfall and the CV of the rainfall across those gauges were calculated for each subgrid. The final data set of area, number of gauges and CV values was then used to formulate an empirical relationship using the Eureqa model exploration framework (Schmidt and Lipson, 2013). Eureqa gives a number of alternate relationships depending on their associated complexity and a R-square goodness of fit. The adopted functional relationship is presented in Figure 2-3 for different area aggregations from 4 km\(^2\) to 1024 km\(^2\). This relationship was then extrapolated back to the case of 1 km\(^2\) which is the radar pixel resolution the rainfall is derived at, and this final relationship is also shown in Figure 2-3. Then, for any radar pixel, the number of gauges within the pixel is counted and the corresponding CV ascertained. When combined with the average rainfall for the gauges in the radar pixel for that time period, the CV is an estimate of the spatial uncertainty in the gauge rainfall (\( \sigma_{cv} \)).

**Figure 2-3:** Illustration of Eureqa modelled spatial variability uncertainty (\( \sigma_{cv} \)) for different subgrid sizes having different number of gauges in each grid. This model is used to extrapolate spatial variability uncertainty for a subgrid size of 1 km by 1 km.
2.4.3 Total error model

Finally, the gauge measurement error is estimated using Equation 2-1, and the spatial variability uncertainty is estimated from Figure 2-3 for each radar pixel of 1 km$^2$. The total uncertainty ($\sigma_u$) from the gauge rainfall for a 1 km$^2$ radar pixel is dependent on the rainfall rate as well as the gauge density and given by

$$\sigma_u = \sqrt{\left(\sigma_g^2 + \sigma_{cv}^2 \right)}$$

(0-3)

By combining the error model with the recorded rainfall, new rainfall simulations can be created that are consistent with the original recorded data but account for the uncertainty due to spatial variability and gauge measurement error. A multiplicative model for the rainfall error is used:

$$R'_{t,k} = R_{t,k} \times \varepsilon_{t,k}$$

(0-4)

where $R'$ is the simulated rainfall at time $t$ and location $k$ based on the observed rainfall $R$. The error $\varepsilon$ is defined using a lognormal distribution with zero mean in the log space and a total variance expressed as:

$$\varepsilon_{t,k} \sim LN\left(0, \sigma^2_{u,t,k}\right)$$

(0-5)

Here, $\sigma^2_u$ is the error variance of the point gauge rainfall uncertainty obtained from the error model and LN is the lognormal distribution, which is appropriate to use for the assumed multiplicative error structure (Tian et al., 2013).
Figure 2-4: Sample estimates of modelled rainfall uncertainty for 300 realizations based on error model for a) gauge measurement uncertainty, b) spatial variability uncertainty and c) total uncertainty. In panel d) the probability density function averaged across all rainfall rates is shown for the gauge measurement uncertainty and spatial variability uncertainty.

The results from applying this error model to the measured gauge rainfalls are presented in Figure 2-4, where 300 realisations are compared to the observed hourly rainfall intensities. Figure 2-4a shows the random errors introduced by the gauge measurement error ($\sigma_g$), whilst Figure 2-4b shows the spatial variability ($\sigma_{cv}$) and Figure 2-4c presents the total error (i.e. based on Equation 2-3). The distribution of modelled percent uncertainties for gauge measurement uncertainty and spatial variability uncertainty are presented in Figure 2-4d. It is clear that the spatial variability uncertainty dominates the total uncertainty estimate. The gauge measurement uncertainty is quite small, although proportionally is more important for the lower rainfall intensities. The average spatial variability uncertainty is around 15% as shown in Figure 2-4d.

2.5 Use of SIMEX to improve the radar Z-R relationship

This section presents the algorithm that was used to estimate the unbiased A parameter using the SIMEX method. Suppose, instead of observing the gauge rainfall R,
the erroneous rainfall $W$ is observed, where, $W = R \times \delta$ and $\delta$ is the multiplicative error distribution given by Equation 2-5 (assuming $\delta \sim LN(0, \lambda \sigma^2_u)$). Here, $\sigma^2_u$ is the error variance of the point gauge rainfall uncertainty obtained from the error model and $\lambda$ is the multiple of error variance. In the simulation step, additional independent measurement errors with variance $\lambda \sigma^2_u$ are generated and multiplied with the original $W$, thereby creating data sets with successively larger measurement error variance realizations $W^*$. Next, sample estimates for the Z-R parameter $A$ are obtained, and their expected value denoted $A^*$. It is assumed that the radar reflectivity $Z$, radar power law parameter $b$ and realization $W^*$ are related through the relationship $Z^b = A^b W^*$. The value for $A$ is estimated through a linear regression of $Z^b$ onto $W^*$.

In the implementation of SIMEX adopted here, seven values for the error variance factors, $\lambda = [0, 0.5, 1.0, 1.5, 2.0, 2.5, \text{and } 3.0]$ were used. For each $\lambda$, simulations were done for 300 runs, resulting in 300 biased parameter estimates $A$. The expected estimate for each $\lambda$, denoted as $A^*$, is the mean of the 300 bias parameter estimates $A$. Finally, the SIMEX estimate is obtained from the relationship between the expected estimate of $A^*$ with multiple of error variance $\lambda$ and extrapolating back to a value of $\lambda = -1$. A quadratic function was chosen for the extrapolation.

The algorithm for unbiased $A$ parameter estimation using SIMEX method is given below:

1. Specify the multiple of error variance factor $\lambda$, ranging from 0 to 3, with steps every 0.5.
2. Specify the number of realizations (i). In this case, 300 realizations have been selected.
3. Generate random normal deviates $\delta^*_i$ for $\lambda = \lambda_1$, where, $\delta^*_i \sim LN(0, \lambda_1 \sigma^2_u)$
4. Estimate the erroneous realization, $W^*_i = R \times \delta^*_i$
5. Estimate the slope $A_{i=1}$ from $Z^b_p = A^b_{i=1} W^*_i$, where, $Z^b_p = Z^b, b = 1.53$
6. The expected estimate $A^*_i$ is the average of 300 realizations, $A^*_i = \frac{1}{n} \sum_{i=1}^{n=300} A_{i=1}$
7. Repeat the steps 3 to 6 for $W^* = [W^*_2, W^*_3, W^*_4, ..., W^*_m]$ and $\lambda = [\lambda_2, \lambda_3, \lambda_4, ..., \lambda_m]$
8. Develop an empirical relationship \( A^* = f(\lambda) \). Here \( f(.) \) is the relationship between the expected value of \( A^* \) and the multiple of error variance (\( \lambda \)) which could be a linear or a nonlinear relationship.

9. Extrapolate to get \( A_{SIMEX} = f(\lambda = -1) \)

To illustrate the SIMEX process, take the example of a recorded hourly rainfall of 30 mm. The assumption in this chapter is that this recorded rainfall amount is uncertain. To sample this uncertainty, random noise is added to the measurement of 30 mm using a log normal distribution with a mean of zero and total error variance \( \lambda \sigma^2 u \) which is the combined gauge measurement error and spatial uncertainty multiplied by the SIMEX error variance factor. From this log-normal distribution a perturbation factor is sampled (e.g. 0.828) and used to create a new rainfall amount (30 * 0.828 = 24.84 mm). This process is repeated for all recorded rainfall amounts to create a new set of data (W) that is consistent with the observed data and its error. From the new data set, the value for the parameter \( A \) is estimated using recorded reflectivity (Z) and rearranging the Z-R relationship to give \( Z^{1/b} = A W \). This process is repeated 300 times and the values of \( A \) averaged to give \( A^* \) for some value of \( \lambda \).

The whole process described above (Step 3 to 6 in the algorithm) is repeated for the other values of \( \lambda \) (from 0 to 3) with the assumption that as more error is added to the recorded rainfall amounts (i.e. \( \lambda \) increases) the value found for \( A^* \) will become more biased. The next step is to fit a relationship between the \( A^* \) values and \( \lambda \) values. Finally, this can be extrapolated back to the case of \( \lambda = -1 \) where there is no error in the measurements.
2.6 Results and Discussion

2.6.1 SIMEX results

Gauge measurement uncertainty \((\sigma_g)\) and spatial variability uncertainty \((\sigma_{cv})\) are combined to estimate point gauge uncertainty \((\sigma_u)\) (Figure 2-4c). It is found that spatial variability uncertainty dominates (Figure 2-4b) compared to the gauge measurement uncertainty (Figure 2-4a). In addition, the same number of gauges in a larger area gives higher spatial variability uncertainty in the area of interest (Figure 2-3).

The results of the SIMEX analysis are shown in Figure 2-5. In this case the naive estimate is the traditional estimate of the parameter \(A\) using the point gauge data and not allowing for any errors in the gauge data. To the right of the naive \(A\) parameter estimate are the cases where more error has been included by applying the error model developed in Section 2.4 in the SIMEX framework. For each value of the multiple of error variance \((\lambda)\), the \(A\) values for each of the 300 simulations are shown, along with the mean estimate \(A^*\). The fitted quadratic function is shown that provides the relationship between the mean estimates \((A^*)\) and the multiple of error variance \((\lambda)\). Finally the SIMEX estimate, which

**Figure 2-5:** Relationship between SIMEX error variance and estimated value of \(A\). The naive estimate is found at \(\lambda = 0\) and SIMEX estimate at \(\lambda = -1\).
represents the case where there is no error in the rainfall, is shown with the dotted line which is the quadratic function extrapolated to $\lambda = -1$.

As can be seen in Figure 2-5, the naive estimate of the $A$ parameter is 167.6 and the SIMEX estimate is 178.8 – a difference of approximately 7% in the parameter estimates. Thus for a typical (average) reflectivity, the SIMEX rainfall intensity will be approximately four percent lower than if errors in gauge rainfall were not included (i.e. naive estimate). Although this is a small change compared to the other sources of error in hydrologic modelling, the important point is that this bias can be easily addressed using the method in this chapter and therefore increases the accuracy of radar reflectivity measurements for characterising rainfall fields. The parameter $A$ estimated by applying the SIMEX method is significantly different from the currently used biased naive estimate and shows that the consideration of point gauge uncertainty has an impact on the radar rainfall estimation. This gauge uncertainty is classically ignored in radar Z-R relationship development by considering point gauge as a ‘ground reference’. Our result indicates that it is necessary to consider point gauge network uncertainty when developing radar Z-R relationships.

It is important to note that the correct Z-R relationship is also affected by random fluctuations introduced by variation in the rainfall drop size distribution both spatially and temporarily (Lee et al., 2007). Jaffrain and Berne (2012) reported that the spatial variability of the DSD within a radar pixel leads to variability of power law parameters, with deviations of between -2% to +15% in the pixel rainfall amount. The authors also reported relative variability in the Z-R relationship parameters $A$ (-18% to 1%) and $b$ (-4% to 1%).

### 2.6.2 Sensitivity of SIMEX method to assumptions

One of the main assumptions in the application of the SIMEX method is the form of the error distribution associated with the covariate. To test this, alternate error models for the gauge rainfall from Eureqa were considered. All models led to a similar value for the extrapolated value of the spatial variability ($\sigma^2_{cv}$) for the case of a single gauge in the 1 km x 1 km grid cell. When these models are used in the SIMEX framework, there are some differences in the final value for $A^*$, which are dependent on the magnitude of the errors in the adopted error model. In general, when errors are smaller, the difference between the two results is also smaller, something that would be expected from the
rationale behind SIMEX. While this indicates that the SIMEX estimates are sensitive to the choice of the error model, one can also conclude that the presence of error is leading to a bias in a consistent direction for all models, pointing to the need for considering the uncertainty associated with the rain gauge network used in the formulation of any Z-R relationship.

The second major assumption in the SIMEX method is the choice of an extrapolation method for the bias. In this chapter, a quadratic extrapolation function was used, but other extrapolation functions could also be used. Figure 2-6 illustrates that the choice of the extrapolation function can have a small impact on the resulting parameter estimate. In this study, linear, quadratic and polynomial extrapolation functions have been tested. The quadratic extrapolation was adopted since all the methods lead to comparable results which provide confidence in the robustness of the extrapolation. It is clear that the uncertainty introduced by the extrapolation is much smaller than the sensitivity of the method to the adopted error model.

**Figure 2-6:** SIMEX estimate of parameter A using a variety of functions to extrapolate to the no error case at $\lambda = -1$.

Other assumptions made in the SIMEX analysis include the number of realizations as well as the appropriate range and the increment for the values of the error variance factor. The choice of 300 realizations for the SIMEX method is arbitrary and could be
varied. Previous hydrological applications of the SIMEX approach have used 300 realizations (Chowdhury and Sharma, 2008) and 500 realizations (Chowdhury and Sharma, 2007) and therefore 300 realizations were adopted for this current chapter. The sensitivity tests have been carried out on the number of realizations and found that it only has a limited impact with variations of generally less than 1% in the value of the parameter A. However, if SIMEX is used in other applications, then such parameters may become more important depending on the characteristics of the covariate-response sample. Similarly, a multiplicative error has been assumed for simplicity in the analysis reported here, and partly to maintain consistency with other studies on input uncertainty in the literature (McMillan et al., 2011; Renard et al., 2010; Tian et al., 2013). While choices such as these could be fine-tuned depending on the situation being analysed, the intention of this chapter was to introduce the reader to the idea that point gauge uncertainty may affect radar rainfall relationships and demonstrate that SIMEX is an appropriate choice for resolving this issue.

2.7 Conclusion

Accurate spatial rainfall data is essential to get the best output from hydrological models. Generally, radar rainfall estimation accuracy is evaluated by comparing point gauge rainfall without considering gauge uncertainty. This chapter makes a unique contribution as this chapter examines the impacts of uncertainty in the Z-R relationship and clarifies that point gauge uncertainty is a factor that may affect the radar rainfall estimation process. It is identified that there are two main sources of error: the first one being due to recording errors in the tipping bucket gauge and the second one being the error introduced by not properly sampling the spatial variability of rainfall at the scale of the radar pixels. To address both sources of uncertainty, an error model is developed that quantifies the total error in the rain gauge measurements. The innovation in this error model is to consider large spatial aggregations of the data to overcome the low gauge density in the operational network, where at best there is only a single gauge for each radar pixel.

The SIMEX method is shown to be able to account for this input uncertainty and provide an unbiased estimate of the parameter A of the Z-R relationship. An example is included to demonstrate the readers to explain the SIMEX capability of obtaining true value from the biased estimates. It is found that the SIMEX method leads to changes in
the A parameter which will translate to biases in the resulting rainfall rates estimated from the Z-R relationship. Although the changes are small, they reflect a consistent bias in radar rainfall estimates that can be corrected using the ability of SIMEX to incorporate gauge uncertainty during the calibration of radar rainfall measurements. One main advantage of the SIMEX framework is that if different error models are found to be appropriate for other radar networks, then these can easily be used in place of the error model adopted in this chapter. In addition, if a number of Z-R relationships are found to be appropriate for a particular location to reflect the physics of rainfall and meteorological of storm events, then a different error model could be used for each Z-R relationship. Finally, to know whether it is reasonable to ignore a source of error in modelling, first the impact of that error needs to be quantified. The SIMEX method provides a framework for this decision to be made.

This chapter investigates the effects of point gauge rainfall uncertainty on parameter bias in the Z-R relationship. An error model is developed to compute point gauge rainfall uncertainty at the radar grid resolution. This error model has two components: 1) the error in the gauge measurement itself, and 2) the error introduced by the gauge not capturing the spatial variability within a radar pixel. The Simulation Extrapolation method (SIMEX) is used to determine the extent of parameter bias present in the rainfall-reflectivity relationship as a result of this uncertainty. When considering the point gauge rainfall uncertainty, the rainfall estimates obtained from SIMEX corrected A parameter, it is around four percent less than the average radar rainfall estimates. This indicates that SIMEX method significantly improves radar A parameter value, though, in reality, there is no true value exist for parameter A

This study demonstrates the uncertainty in rain gauge measurements can be easily considered to produce unbiased Z-R relationships. Future work could couple this with the uncertainty in the radar reflectivity (Jordan et al., 2003), particularly in the extrapolation between the levels of the CAPPI measurements and the ground level.
3 Radar rainfall estimation: influence of input temporal discretization

3.1 Introduction

Accurate radar rainfall estimates are needed for characterizing the spatial and temporal distribution of rainfall. Uncertainties in converting the radar reflectivity to rainfall rates have hampered the widespread use of radars in hydrology. This chapter aims to address whether the temporal resolution of radar reflectivity needs to change to better calibrate Z-R relationship parameters. In most studies, the Z-R relationship is calibrated using either hourly or daily accumulated radar and rain gauge data, assuming radar reflectivity remains constant over the gauge accumulation period (Bellon et al., 2007; Chumchean et al., 2003b; Haberlandt, 2007). However, the radar reflectivity data is usually retrieved at a temporal resolution of either six minutes or ten minutes depending on the type of radar used to measure reflectivity. Moreover, radar reflectivities vary significantly from one scan to another. The mean hourly reflectivity is estimated by taking the average of individual scans that lie within the same hour. The impact of gauge temporal resolution on the Z-R relationship parameters was studied in the past, and found that the Z-R relationship parameters differ significantly with the temporal resolution of gauge rainfall (Mapiam et al., 2009). Alfieri et al. (2010) showed that best radar rainfall estimates can be found by calibrating the Z-R relationship with a certain time interval. Therefore, it is important to study the impact of radar temporal resolution on the Z-R relationship parameter. The key focus of this study is to investigate how radar rainfall estimates vary with the temporal discretization of reflectivity.

The rest of this chapter is organized as follows. Section 3.2 presents the data used in this study. The Z-R relationship calibration method is presented in Section 3.3. The results are discussed in Section 3.4. Finally, summary and conclusions are presented in Section 3.5.

3.2 Radar and gauge data

In this research, collocated data of radar reflectivity and gauge rainfalls were obtained from the Australian Bureau of Meteorology. The data collection period was February 2015 to June 2015. The Bureau of Meteorology operates a network of fifty weather radars. The half-hourly co-located gauge and radar data from ten radars located
in different regions of Australia for the purpose of this study (Table 3-1) have been used in this study. The reflectivity samples contain the full set of quality control corrections, including partial beam blocking, bright band effects, and Vertical Profile Reflectivity (VPR) correction. The reflectivity ranges from 15 dBZ to 53 dBZ were used in this study to cancel out the impacts of noise and hail on the retrievals (Chumchean et al., 2006a; Hasan et al., 2014).

<table>
<thead>
<tr>
<th>Radar number</th>
<th>Radar name (location)</th>
<th>Longitude</th>
<th>latitude</th>
<th>Number of Gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Grafton (Grafton, NSW)</td>
<td>152.95</td>
<td>-29.62</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>Appin (Wollongong, NSW)</td>
<td>150.88</td>
<td>-34.26</td>
<td>121</td>
</tr>
<tr>
<td>3</td>
<td>Lemon Tree (Newcastle, NSW)</td>
<td>152.03</td>
<td>-32.73</td>
<td>61</td>
</tr>
<tr>
<td>4</td>
<td>Yarrawonga (Yarrawonga, Victoria)</td>
<td>146.02</td>
<td>-36.03</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>Marburg (Brisbane, QLD)</td>
<td>152.54</td>
<td>-27.61</td>
<td>277</td>
</tr>
<tr>
<td>6</td>
<td>Kurnell (Sydney, NSW)</td>
<td>151.23</td>
<td>-34.01</td>
<td>114</td>
</tr>
<tr>
<td>7</td>
<td>Mt. Stapylton (Brisbane, QLD)</td>
<td>153.24</td>
<td>-27.72</td>
<td>274</td>
</tr>
<tr>
<td>8</td>
<td>Serpentine (Perth, WA)</td>
<td>115.87</td>
<td>-32.39</td>
<td>29</td>
</tr>
<tr>
<td>9</td>
<td>Terrey Hills (Sydney, NSW)</td>
<td>151.21</td>
<td>-33.70</td>
<td>117</td>
</tr>
<tr>
<td>10</td>
<td>Mt. Koonya (Hobert, TAS)</td>
<td>147.81</td>
<td>-43.11</td>
<td>32</td>
</tr>
</tbody>
</table>

The significant sources of uncertainty in radar rainfall estimation are partial beam filling and overshooting low clouds at far range. Moreover, there is uncertainty in reflectivity measurements at far range which lead to uncertainty in rainfall estimation (Chumchean et al., 2003a; Chumchean et al., 2004; Krajewski et al., 2011). For these reasons only, the reflectivity-gauge accumulation pairs that lie within a 128 km range from radar are used in this study. The number of rain gauges within this radius of each radar is different. The location of radar and rain gauge networks around Marburg radar is shown in Figure 3-1. The ground rainfall measurements are taken from rain gauges with tipping bucket sizes of 0.2 or 0.5 mm, operated by the Australian Bureau of Meteorology. The minimum rainfall intensity of 0.5 mm per 30 minutes was applied as threshold limit for rainfall detection (Alfieri et al., 2010; Chumchean et al., 2004). To determine the
performance of calibration parameters, the data (radar reflectivity-gauge accumulation pairs) of each radar are divided into calibration and validation data of the same length. The required parameters (A, b) are calculated using the calibration data and their performance was evaluated using the validation data. The uncertainties as discussed in previous chapter could be incorporated but not done as they detract from the stated objective of this work.

![Marburg radar and rain gauge network](image)

**Figure 3-1:** Topographic map of the study region showing the Marburg radar and rain gauge locations.

### 3.3 Methodology

This section will describe the method for calibrating Z-R relationship depending on the temporal resolution of radar reflectivity. The three Z-R relationships on an event basis using the co-located gauge and radar data have been calibrated. The number of
reflectivity samples provided per gauge sample depends on the radar temporal resolution. For each gauge observation, the set of reflectivity values includes one radar scan taken before the gauge accumulation period, all of the scans during the gauge period, and the radar scans immediately after the gauge period. This allows for maximum flexibility when deciding how to interpolate or integrate the reflectivities over time. For example, a radar with a six minute resolution provides seven reflectivity samples for each gauge sample of thirty minutes: one before, five during and one after the gauge period. Generally, gauge rainfall recorded at 30 minutes interval from 0 to 30 minutes and/or 31 to 60 minutes. In contrast, radar temporal resolution does not necessarily start at 0 minutes. A gauge sample with timestamp 03:30 indicates an accumulation from 03:00 to 03:30. The corresponding reflectivity samples from six minutes radar might be for 02:56 (one before), 03:02, 03:08, 03:14, 03:20, 03:26 (during), and 03:32 (one after). Depending on radar resolution (either 6 minutes or 10 minutes), the 30 minutes gauge accumulation contains either five or seven reflectivity scans. Therefore, the radar scans before and after the 30 minutes window to match gauge rainfall with radar rainfall duration have been considered.


Figure 3-2: Radar reflectivity over the gauge accumulation period a) at different times b) accumulated reflectivity (Case I) c) constant over radar temporal resolution (Case II) d) blue circle shows interpolated reflectivity (Case III).

The Z-R relationship parameters have been estimated for three different reflectivity resolutions:

Case I: It is the traditional method for calibrating the radar Z-R relationship parameters where reflectivities are averaged across the gauge accumulation period (Figure 3-2b). Reflectivity samples are averaged over thirty minutes to match the gauge accumulation period.

Case II: Radar reflectivities are constant over the radar temporal resolution (Figure 3-2c). In this study, reflectivity samples remain constant over its temporal resolution of six minutes. Therefore, radar has five reflectivity samples for thirty minutes gauge accumulation period.

Case III: Radar reflectivities are interpolated for one minute time step using cubic spline interpolation (Hastie and Tibshirani, 1990) from reflectivity samples (Figure 3-2d). The reflectivity samples outside the thirty minutes gauge accumulation period are excluded from further processing.

The Z-R relationship parameters $A$ and $b$ are determined using the derivative-free “Nelder-Mead” optimization algorithm (Kelley, 1999; Nash, 2014). A quantile matching objective function has been used to optimize all the radars. The optimized $A$ and $b$ parameters provide the minimum difference between quantile of gauge and radar estimated rainfall. The quantiles are calculated for the percentile ranges between 0 to 1 with step of 0.01. The Bounds are placed on the upper and lower limits of parameters $A$ and $b$ to minimize the computational expense. In this study, parameter $A$ is restricted in the range 50 to 400 and the parameter $b$ is allowed to vary between 1.00 and 1.70.

3.4 Results

The calibrated Z-R parameters were applied to the validation data. The impact of these parameters on the accuracy of radar rainfall estimates was evaluated by comparing them to the gauge rainfall of the validation set. The calibrated Z-R relationship parameters
for ten radars are summarized in Figure 3-3 and Figure 3-4. Figure 3-3 shows calibrated parameter $A$ and Figure 3-4 shows calibrated parameter $b$ for all radars.

For most of the radars, parameter $A$ is lowest for Case-III and highest for Case I. For Case II, parameter $A$ lies between Case I and Case III. Therefore, parameter $A$ increases with the increase in temporal resolution of reflectivity. For the parameter $b$, the scenario is opposite (Figure 3-4). Parameter $b$ is minimum when reflectivity is averaged over half-hour temporal resolution and maximum for one minute resolution.

**Figure 3-3:** Calibrated parameter $A$ for different radars. Blue, black and red circles are for Case I, Case II and Case III respectively.
Figure 3-4: Calibrated parameter $b$ for different radars. Blue, black and red circles are for Case I, Case II and Case III respectively.

The impact of these calibrated parameters on the accuracy of radar rainfall estimates is then validated by comparing radar rainfall with the gauge rainfall. The performance is evaluated using the root mean square error (RMSE), mean absolute error (MAE), mean field bias (Bias) and log error by comparing with the gauge observations. These measures are calculated as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (R_{\text{est}} - R_{g})^2}$$  

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |R_{\text{est}} - R_{g}|$$  

$$\text{Bias} = \frac{\sum_{i=1}^{n} R_{\text{est}}}{\sum_{i=1}^{n} R_{g}}$$  

$$\text{logerror} = 10 \log_{10} \left( \frac{R_{\text{est}}}{R_{g}} \right)$$

where $R_{g}$ represents gauge rainfall and $R_{\text{est}}$ represents radar rainfall. MAE gives equal weights to all the data points whereas RMSE gives higher weights to higher rainfall rates.
A Bias larger than one indicates an overestimation of ground rainfall and Bias value equals to one indicates a perfect match of radar and gauge accumulation.

Table 3-2: Summary of vital statistics for all radar

<table>
<thead>
<tr>
<th>Radar number</th>
<th>MAE (mm/h)</th>
<th>RMSE (mm/h)</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>1.50</td>
<td>2.59</td>
<td>0.94</td>
</tr>
<tr>
<td>Case II</td>
<td>1.47</td>
<td>2.53</td>
<td>0.98</td>
</tr>
<tr>
<td>Case III</td>
<td>1.47</td>
<td>2.53</td>
<td>0.96</td>
</tr>
<tr>
<td>Case I</td>
<td>1.01</td>
<td>1.70</td>
<td>0.99</td>
</tr>
<tr>
<td>Case II</td>
<td><strong>0.99</strong></td>
<td><strong>1.68</strong></td>
<td><strong>1.00</strong></td>
</tr>
<tr>
<td>Case III</td>
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<td><strong>1.68</strong></td>
<td>0.99</td>
</tr>
<tr>
<td>Case I</td>
<td>1.08</td>
<td>1.86</td>
<td>0.96</td>
</tr>
<tr>
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<td><strong>1.05</strong></td>
<td><strong>1.82</strong></td>
<td><strong>0.97</strong></td>
</tr>
<tr>
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<td><strong>1.80</strong></td>
<td>0.96</td>
</tr>
<tr>
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<td>0.70</td>
<td>1.15</td>
<td>0.92</td>
</tr>
<tr>
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<td><strong>0.68</strong></td>
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<td><strong>0.96</strong></td>
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<td><strong>1.00</strong></td>
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<td>0.97</td>
</tr>
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<td>0.96</td>
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<tr>
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<td>0.98</td>
</tr>
<tr>
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<td><strong>2.23</strong></td>
<td>0.99</td>
</tr>
<tr>
<td>Case III</td>
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<td>2.28</td>
<td><strong>1.00</strong></td>
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<tr>
<td>Case I</td>
<td>0.77</td>
<td>1.25</td>
<td>0.92</td>
</tr>
<tr>
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<td><strong>0.75</strong></td>
<td><strong>1.24</strong></td>
<td><strong>0.94</strong></td>
</tr>
<tr>
<td>Case III</td>
<td>0.77</td>
<td>1.25</td>
<td>0.92</td>
</tr>
<tr>
<td>Case I</td>
<td>0.97</td>
<td>1.69</td>
<td>0.96</td>
</tr>
<tr>
<td>Case II</td>
<td><strong>0.94</strong></td>
<td><strong>1.67</strong></td>
<td><strong>0.97</strong></td>
</tr>
<tr>
<td>Case III</td>
<td>0.96</td>
<td>1.68</td>
<td><strong>0.97</strong></td>
</tr>
<tr>
<td>Case I</td>
<td>0.53</td>
<td>0.73</td>
<td>0.83</td>
</tr>
<tr>
<td>Case II</td>
<td><strong>0.46</strong></td>
<td><strong>0.69</strong></td>
<td><strong>0.96</strong></td>
</tr>
<tr>
<td>Case III</td>
<td>0.48</td>
<td>0.71</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 3-2 shows the summary of the validation metrics. The best result in all three cases is shown in bold font. Regarding MAE, Case II shows the best performance for all radars. Similar results have been obtained for RMSE and Bias. Case I underestimate the rainfall rates and led to biased estimates compared to other two cases. It is due to reason that, Case I gives lowest A parameter value and highest b parameter value. For a few of the radars, Case III was also found best. From Table 3-2, it is evident that the best performance of the radar calibration can be obtained when the radar is calibrated using the reflectivity samples of its temporal resolution instead of accumulating the reflectivity over the gauge accumulation period which is traditionally used for Z-R relationship calibration.

The parameter A varies significantly in time and space compared to parameter b. The values of parameter A vary between 100 to 300 (Figure 3-3) and parameter b between 1.40 to 1.55 (Figure 3-4). In the Z-R relationship, higher reflectivity gives higher rainfall estimates. If there is significant variation in consecutive radar scan, the effect of higher reflectivity maximized because of the temporal discretization which results in smaller value of the parameter A (Case III). Consequently, this gives higher radar rainfall
estimates. In contrast, temporal accumulation minimizes the effect of higher reflectivity and thereby overestimates parameter A (Case I).

Parameter A has distinct characteristics depending on the rainfall intensity at different geographic locations. Therefore, it was estimated for New South Wales (NSW) and Tasmania. The value of parameter A in Tasmania was found around 100 for mean annual rainfall of 550 mm. For NSW it was between 200 to 250 for five radars, where the mean annual rainfall is 1150 mm. This indicates that rainfall intensity has a significant effect on the radar Z-R relationship parameter.

### 3.5 Summary and Conclusions

Radar and rain gauge provide rainfall estimates at different temporal resolution. The accurate conversion of radar reflectivities into ground rainfall estimates depends on the accuracy of the conversion parameters. This study has evaluated the impact of radar temporal resolution on the Z-R relationship parameters. The calibration was done for the Z-R relationship for three different temporal resolution of radar reflectivities such as averaged over the gauge accumulation period, constant over radar temporal resolution and interpolated over the gauge accumulation period, while keeping the temporal resolution of gauge accumulation fixed.

These approaches have been tested on ten radars across Australia. The outcomes of this calibration process show that the estimated parameters A and b significantly differ depending on the radar temporal resolution. For the same temporal resolution, parameter A and b show distinct characteristics. For example, radar reflectivity of thirty minutes temporal resolution, the parameter A is lowest while the parameter b is highest. Most importantly, the parameter A and b exhibit consistent characteristics across all radars. In the Z-R relationship, parameter A controls the bias and parameter b controls the highest rainfall.

It is further examined to identify which temporal resolution gives the most accurate radar rainfall estimates. The application of these calibrated parameters over several radars confirms that neither the accumulation of reflectivity over a gauge accumulation period, nor the discretization of reflectivity into finer resolution, improves the radar rainfall estimates. The consideration of constant radar reflectivity over its temporal resolution gives most accurate radar rainfall estimates.
4 Improving radar rainfall estimation by merging point rainfall measurements within a model combination framework

4.1 Introduction

It has been shown that improved rainfall estimates can be achieved through a combination of radar and gauge measurements by exploiting their strengths and correcting for their shortcomings (Burlando et al., 1996; Krajewski, 1987; Schiemann et al., 2011). This chapter proposes two innovative methods for improving radar rainfall estimation by combining gauge and radar measurements. The idea of merging radar and gauge measurement is not new and a number of different methods have been developed. Previous studies were mostly concerned about the application of gauge data for correcting systematic errors in radar rainfall estimates.

While the value of correcting raw radar rainfall estimates using simultaneous ground rainfall observations is well known, approaches that use the complete record of both gauge and radar measurements to provide improved rainfall estimates are much less common. In most situations, the process of radar rainfall estimation involves 1) the measurement of reflectivity, 2) the removal of errors caused during its measurement, 3) the conversion of estimated reflectivity into rainfall, and 4) an adjustment depending on gauge rainfall measurements (Chumchean et al., 2006b). Uncertainties are associated with each of these steps. In practice, when considering long duration rainfall periods and/or multiple storm types, steps 1) and 2) can be affected by phenomena such as ground clutter, beam blockage, anomalous propagation, hail, bright band, attenuation, range-dependent bias, range degradation, vertical profile of reflectivity, temporal and spatial sampling errors (Chumchean et al., 2006b; Villarini and Krajewski, 2010). There are also errors introduced by rainfall variability and precipitation drift as well as the uncertainties of relating point rainfall measurements to radar measurements across a gridded domain. Aim of this chapter is to address the uncertainties in converting the radar reflectivity to rainfall rates that have hampered the widespread use of radars in hydrology (Villarini and Krajewski, 2010).

This chapter presents a method for combining spatially interpolated gauge rainfall with a radar rainfall estimate. Such combination is accomplished without making
assumptions on the relationship between radar reflectivity and gauge rainfall. Furthermore, it considers the dependence between the radar and gauge estimates and thereby shows improvement over either the radar or gauge estimates taken individually.

In merging the radar and gauge products, there are parallels with recent work in combining multiple climate models or seasonal forecasts (Chowdhury and Sharma, 2009). One of the important findings from these studies is that dynamic weighting of the models (i.e. where the combination weights change with time) provides superior performance compared to static weighting schemes (Chowdhury and Sharma, 2010; Devineni and Sankarasubramanian, 2010). Therefore, dynamic weighting has been proposed to merge the radar and gauge data, which to our knowledge is a new contribution to the field. Dynamic weighting requires estimates of the error in each of the models at every location and every time step. The proposed method weights two different sources of information (radar and gauge estimates) based on their temporal distribution of errors without giving priority to one over another. An improved radar-rainfall relationship has been proposed that calculates the uncertainties for different rainfall intensities.

Radar-rainfall relationships are very complex. The Z-R relationship depends on the drop size distribution of the rainfall as well as the rainfall regime and geographical location (Hazenberg et al., 2011; Lee and Zawadzki, 2005; Steiner et al., 2004). For example, Marshall and Palmer (1948) found that, theoretically, reflectivity and rainfall intensity should be proportional to the 6th and 3.7th moments of the raindrop diameter respectively. Hence, radar reflectivity is more sensitive to rain drop diameter than to rainfall rate. Moreover, it has been observed that the Z-R relationship can be non-injective, such that a reflectivity value can correspond to samples having different drop size distributions and rainfall intensities (Ochou et al., 2011; Uijlenhoet, 2001). While DSDs obtained by disdrometers can be used for obtaining the Z-R relationship (Prat and Barros, 2009; Verrier et al., 2013), these measurements are not available in many parts of the world (Hasan et al., 2014; Mapiam et al., 2009) and statistical calibration of the Z-R relationship is required. The commonly used power-law, with only two free parameters (A and b), is unable to capture this complexity. To circumvent this limitation and make the best use of available records of radar reflectivity and ground rainfall, a
nonparametric method has been proposed to model the full complexity of the Z-R relationship.

The proposed kernel-based nonparametric method (NPR) is in some ways an extension of PMM that address these limitations, except that a conditional (and not a quantile matching) approach is used. The proposed NPR method uses the bandwidth of reflectivity and rainfall pair instead of utilizing all the reflectivity – rainfall pair which is the major limitation of least square regression.

In this chapter, a nonparametric relationship between radar reflectivity and the expected ground rainfall estimate has been proposed. The resulting radar rainfall estimate is then combined with an interpolated rain gauge field using a dynamic weighting scheme. The combination addresses the known sensitivity of nonparametric approaches to limited record lengths, as the combination will automatically assign a greater weight to the approach that exhibits a lower error. It is important to note that the combination approach does not rely on this choice and could be implemented with any other interpolation method as long as estimates of variance in the fitted surface are available.

In summary, this chapter aims to answer two main questions with respect to the merging of radar and rain gauge data. Firstly, can a nonparametric approach provide better estimates than a traditional gauge adjusted Z-R relationship? Secondly, what is an effective method of combining the radar and gauge based estimates that can account for their respective strength and weakness? The rest of this chapter is organized as follows. The proposed methods are presented in Section 4.2. Section 4.3 describes the study area and the data used. The rainfall estimation results are discussed in Section 4.4. Finally, Summary and conclusions are presented in Sections 4.5.

4.2 Methodology

This section describes the proposed methods to merge radar and gauge estimates. First, the nonparametric rainfall estimation method (NPR) is discussed. The copula-based rain gauge interpolation (CSI) method is then presented followed by details of the dynamic combination of both products.
4.3 Radar rainfall relationships

4.3.1 Nonparametric rainfall estimation (NPR)

The NPR method aims to estimate ground rainfall for a given reflectivity by using a parametric kernel function. The application of the NPR method requires the kernel bandwidth and the conditional covariance matrix to be estimated from past observed rainfall and reflectivity data. Then, the conditional mean and conditional weight for each kernel are estimated. For a given radar reflectivity, the local weighted average of the conditioned kernels gives the expected rainfall.

To illustrate the NPR method, the problem of estimating ground rainfall ($R_{NPR}$) from the given radar reflectivity ($Z$) has been considered. The rainfall and reflectivity are in units of mm/h and dBZ respectively. The past observed radar rainfall record contains rainfall ($r_i$) and corresponding reflectivity ($z_i$) for each time step $i$, where $i = 1, \ldots, n$. The conditional probability density of $R_{NPR}$ given $Z$ can be written as (see Sharma et al. (1997) for details):

$$
\hat{f}(R_{NPR}|Z) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{n} \exp \left( -\frac{(R_{NPR} - b_i)^2}{2\lambda^2 S_c} \right)
$$

(4-1)

where $S_c$ is the conditional covariance formed on the basis of past observed rainfall-reflectivity pairs that measure the spread of the conditional probability density. $S_c$ is defined as:

$$
S_c = S_{RZ} - \frac{S_{RR}^2}{S_{ZZ}}
$$

(4-2)

where $S_{RZ}$, $S_{RR}$ and $S_{ZZ}$ are sample covariance and respective variances of the two variables; $\lambda$ is the bandwidth, adopted as the Gaussian reference bandwidth for standardised data (see Silverman, (1986) for details) and is estimated as:

$$
\lambda = 1.06n^{-1/5}
$$

(4-3)

Next, $b_i$ is the conditional mean associated with the $i$th kernel. It is equivalent to the projection of the observed response related to that kernel ($r_i$) along the slope of a linear regression fit as dictated by the deviation ($Z-z_i$):
\[ b_i = r_i + (Z-z_i) \frac{S_{RZ}}{S_{ZZ}} \]  

(4-4)

\[ w_i \] is the conditional weight for each kernel or the fractional probability that a kernel contributes to the overall makeup of the conditional probability density of \( R|Z \):

\[ w_i = \frac{p_i}{\sum_{i=1}^{n} p_i} \]  

(4-5)

where \( p_i \propto \exp \left( -\frac{(Z-z_i)^2}{2\lambda^2S_{ZZ}} \right) \)

The expected rainfall for a given radar reflectivity can then be estimated as the mean of the conditional PDF in (4-1):

\[ R_{NPR} = \sum_{i=1}^{n} (w_i b_i) \]  

(4-6)

In the above description, the Gaussian reference bandwidth is assumed for simplicity. Alternate bandwidths that vary depending on the local probability density associated with each data point can be formulated based on the approach proposed by Sharma et al. (1998). While the formulation above uses a single conditioning variable \((Z)\), one could include additional covariates such as gradients in the reflectivity field in \( x \) and \( y \) directions. Extensions of the conditional kernel density estimate presented above to include additional covariates would then require use of partial weights that scale the Euclidean distances in Equation 4-5 and prescribe the importance each covariate has to the response (Sharma and Mehrotra, 2014). In the NPR method, the local weighted average is calculated with more weight being given to the values closer to target reflectivity and less weight for the values further away from target reflectivity. Therefore, the NPR method is expected to provide better rainfall estimate compared to a parametric radar-gauge relationship. The method is sensitive to the past observed data set and therefore it is necessary to have enough reflectivity-rainfall pairs to obtain statistically robust rainfall estimates. However, the NPR method can extrapolate based on \((Z-z_i)\) and the slope \( S_{RZ}/S_{ZZ} \) as per Equation 4-4. Readers are referred to Sharma et al. (1997) for further details and illustrations of how conditional probability density estimates are
derived from the joint PDF. The NPR method is developed using non zero reflectivity – gauge rainfall pairs. The threshold for reflectivity is 15 dBZ and for gauge rainfall is 0.2 mm/h. The data-pairs with either value less than the threshold were excluded to minimize the error due to noise at low intensities. While the conditional expectation can assume negative values (in which case they would be set equal to zero), no such cases were encountered in the data that was used in our study.

**4.3.2 Parametric rainfall estimation (Z-R relationship)**

In the results presented later, a traditional Z-R relationship has also been fitted to the available radar and gauge data for comparison to the new NPR method (details are provided in Section 4.3). This was achieved by pooling all data available and using a linear regression to determine the value for the A parameter, holding the b parameter constant at the operational value of 1.53 (Hasan et al., 2014). Although the operational Z-R relationship could have been used, this would not have provided a valid comparison for the NPR method since the operational relationship is obtained using a different time period. There are some differences in the resulting radar gauge relationship \( Z=183.87 R^{1.53} \) compared to the operational one \( Z=194 R^{1.53} \) for this radar, which are likely due to differences in the data length and time window. It is worth noting that the operational relationship is also based on a gauge-radar adjustment. The use of a fixed b parameter follows from recommendations in Henschke et al. (2009). In the remainder of this chapter, this fitted radar - gauge relationship (designated as "parametric Z-R") is used to compare the performance of the proposed NPR and combination methods.

**4.3.3 Copula-based rain gauge interpolation (CSI)**

The gauge data needs to be interpolated on to the same pixels as the radar to combine the gauge and radar rainfall estimates. Interpolation of rain gauge measurements onto a regular grid can be achieved in a number of ways. Previous studies have investigated inverse distance weighting (Hwang et al., 2012; Kurtzman et al., 2009), Thiessen polygons (Okabe et al., 2009), nearest neighbour (Isaaks and Srivastava, 1990), thin plate smoothing splines (Hutchinson, 1998), genetic algorithms (Chang et al., 2005; Huang et al., 1998), kriging (Cressie, 1988; Goovaerts, 2000), and conditional bias penalized kriging (Seo, 2013; Seo et al., 2014). Among these methods, kriging is most
widely used. It uses the information from nearby locations to estimate the target parameter at a particular location. However, there are several drawbacks in kriging related to underlying assumptions of Gaussianity, and the difficulty in accommodating high-order types of non-stationarity such as changing variance across the spatial domain (Bárdossy and Li, 2008b). Another problem in the context of rainfall interpolation is that variograms are affected by skewed distributions and outliers (Li et al., 2011). Copulas have been shown to overcome these problems. In particular, they allow the spatial dependence between random variables to be captured from their univariate distributions, excluding the information from marginal distributions. Moreover, copulas are not sensitive to measurement outliers and do not require transformations into a Gaussian space, which can result in intractable biases for high-order properties (Bárdossy, 2006b; Bárdossy and Li, 2008b). However, as is the case with most approaches, subjective parameter choices within a Copula framework can lead to model structural uncertainty, leading to overly “rough” or “smooth” spatially interpolated rainfall. A way around this was recently proposed by Wasko et al. (2013), who presented a combinatorial algorithm to form a spatial rainfall estimate that consistently outperforms a stand-alone Copula. This copula based spatial interpolation (CSI) method has been used here as a means of interpolating gauge rainfall data throughout this chapter.

The primary focus of this thesis is not to develop a new spatial interpolation method but to apply the combined copula interpolation algorithm developed by Wasko et al. (2013) using the spatial copula toolbox (Kazianka, 2013b) which is referred hereafter as the copula based spatial interpolation (CSI). The CSI is used to create a grid of rainfall values at all the pixel locations for each 30 minute periods considered (more details on the data are given in the next section). In CSI, a rainfall estimate at each radar pixel is interpolated using copula parameters estimated using all available data (global) as well as a local neighbourhood. The global and local estimates are then combined using the errors at each unknown location (Wasko et al., 2013) to produce a rainfall estimate at that location. This is repeated for all pixel locations and for all individual 30 minute periods. In this approach, the rainfall at any particular radar pixel is interpolated using the gauge observations at that time step. For the purposes of testing the CSI method, leave-one-out cross-validation is used (Goovaerts, 2000). In this approach, the rainfall at
any pixel with a coincident rain gauge is interpolated by ignoring the gauge observation at that particular pixel and interpolating the rainfall using the remainder of the gauge observations at that time step. For computational ease, a Gaussian copula using a Gaussian univariate distribution is assumed. An exponential correlation structure is used throughout. The parameters are estimated using a by maximum likelihood approach. As the copula is a Gaussian there are no “copula specific” parameters that need to be calibrated. The parameters are re-estimated for each interpolation scheme, there is no specific parameters value. Readers are referred to Kazianka (2013a) and Wasko et al. (2013) for further details of the copula-based combinatorial approach used.

It should be noted that the weight estimation process in this study, while having some similarities to kriging based methods such as KED and conditional merging (Sinclair and Pegram, 2005), is significantly different in its makeup and form. The proposed method weights two different estimation approaches (NPR and copula-based interpolation). The weight is computed based on the temporal distribution of radar and gauge estimation errors, instead of the spatial distribution of radar and gauge estimation errors of individual rainfall periods. The conditional merging method performs a spatial interpolation on the radar rainfall which is not the case here. Moreover, the number of gauge measurements available has a significant effect on the variogram and the merged estimates (Jewell and Gaussiat, 2015). In contrast, our combination weight estimation process is independent of the gauge density which only affects the CSI estimates in a way that is similar to kriging. While it is possible to replace the CSI estimates with any of the kriging based methods discussed above, this is not attempted here and is recommended for future research.

4.3.4 Combination method

At each radar pixel, there are two rainfall estimates, the first from the NPR method and the second from the CSI method. The final rainfall is estimated by combining the CSI and NPR values according to the confidence in each estimates. This confidence comes from the past observed estimation accuracy of each method and is expressed as weights. The two sources of information are merged using a dynamic combinatorial algorithm with weights that vary in both space and time. The weight for any specific period is calculated based on the error covariance matrix that is formulated from the radar
and spatially interpolated rainfall errors of similar reflectivity periods in a cross-validation setting. The application of the combination method to any particular pixel requires the past observed records of NPR and CSI estimation error at that pixel or near that pixel. The overall process is illustrated in Figure 4-1.

![Figure 4-1: Flowchart for weighted combination method.](image)

The weights are estimated using leave one out cross-validation. The weight for any 30 minute periods is calculated based on the error covariance matrix constructed from similar previous reflectivity measurements at the nearest radar pixel with a coincident rain gauge. The similar reflectivities are identified through the application of a nonparametric $k$-nearest neighbour (KNN) approach to the past observed data (Lall and Sharma, 1996). The covariance matrix of errors is formulated from the $k$-similar periods in the past observed dataset, not including the current NPR and CSI estimates. In the combination algorithm, the sum of weights is equal to one and weights cannot be negative (Timmermann, 2006). The final rainfall estimate at a particular location and time is the weighted sum of the NPR and CSI estimates.

The combination method is illustrated through the following example. Suppose that we want to find the weights for each of the CSI and NPR methods for an observed reflectivity $Z_{t,x}$ at location $x$ and time $t$. The corresponding rainfall estimates from the CSI and NPR methods are $R_{CSI,t,x}$ and $R_{NPR,t,x}$ at the same location and time. There are also the sets of all observed, CSI and NPR estimates from the nearby gauged location for $n$ times, which is referred as $R_k$, $R_{CSI}$ and $R_{NPR}$ respectively. The idea is to find $W$, which
is a vector of two weights, \( W_{CSI} \) and \( W_{NPR} \), where \( W_{CSI} + W_{NPR} = 1 \). The procedure adopted is as follows:

1) Form the matrix \( \mathbf{Y} = [\mathbf{Z}_h \hspace{1em} \mathbf{R}_h \hspace{1em} \mathbf{R}_{CSI} \hspace{1em} \mathbf{R}_{NPR}] \) of past observed reflectivity (\( \mathbf{Z}_h \)), observed rainfall and the corresponding rainfall estimates from the CSI and NPR methods from the closest gauged pixel.

2) The reflectivity periods that are similar to the observed reflectivity (\( Z_{t,x} \)) in the past observed records are estimated by calculating the smallest absolute distance (\( d_i \)) between \( \mathbf{Z}_h \) and \( Z_{t,x} \) in the matrix \( \mathbf{Y} \).

\[
d_i = \min |Z_h - Z_{t,x}|
\]

where, \( i \) is the \( i \)th element of vector \( \mathbf{Z} \). The distances \( d_i \) are sorted in ascending order and the first \( k \) periods are selected from matrix \( \mathbf{Y} \).

3) For each of the \( k \) periods, calculate the error for CSI and NPR methods (\( e_{CSI} \) and \( e_{NPR} \)) as the difference between \( \mathbf{R}_h \) and \( \mathbf{R}_{CSI} \) or \( \mathbf{R}_{NPR} \) respectively.

\[
e_{CSI} = \mathbf{R}_h - \mathbf{R}_{CSI}
\]

\[
e_{NPR} = \mathbf{R}_h - \mathbf{R}_{NPR}
\]

4) Find the covariance matrix of the estimation errors, \( \Sigma_e \). The size of the covariance matrix is 2 by 2 with the diagonals representing the variance of the errors over the \( k \) neighbours and the off-diagonals the covariance of the errors from the two methods.

\[
\Sigma_e = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
\sigma^2_{CSI} & \rho \sigma_{CSI} \sigma_{NPR} \\
\rho \sigma_{CSI} \sigma_{NPR} & \sigma^2_{NPR}
\end{bmatrix}
\]

where, \( \sigma^2_{CSI} \) and \( \sigma^2_{NPR} \) are the variance of the estimation errors, \( e_{CSI} \) and \( e_{NPR} \) respectively, \( \rho \) is the correlation between estimation errors.

5) The dynamic weights (\( W' \)) for combining CSI and NPR estimates can be obtained minimizing quantity in (11) subject to the constraint that the weights add up to unity.

\[
\min (W' \Sigma_e W) \text{ such that } W'I = 1
\]

where, \( W' = [W_{CSI} \hspace{1em} W_{NPR}] \), \( I \) is a \( m \times 1 \) column vector of ones where \( m \) denotes the number of methods with \( m=2 \) in this case, the weights being constrained to lie between 0 and 1.
Readers are referred to Timmermann (2006) and Khan et al. (2014) for further details concerning the derivation of Equation 4-11.

6) The final combined rainfall estimate is then the weighted average of the different methods:

$$R_{\text{COMBINATION}} = W_{\text{CSI}} R_{\text{CSI},x} + W_{\text{NPR}} R_{\text{NPR},x}$$  \hspace{1cm} (4-12)

The covariance matrix at any particular location and time is unique for each rainfall observation. The main free parameter in this method is the number of neighbours to use in calculating the error covariance matrix. In this case, 200 neighbours are selected. The main sources of uncertainty in the combination method arise from how the weights for the two methods are determined. In this study, a k-nearest neighbours approach was adopted. This means that the primary uncertainty is in the number of neighbours to be used in estimating the covariance matrices of the gauge and radar error estimates. The adopted value of 200 neighbours was chosen following a sensitivity analysis of the relations of the RMSE to the number of neighbours. As the number of neighbours increases, the errors rapidly decrease until about 100 neighbours are used. After this the rate of decrease slows until the relationship flattens with about 200 neighbours. The adopted value of 200 was therefore chosen, in agreement with Khan et al. (2014). This value constitutes a trade-off between large biases and the need for too much data. In our study, most of the gauge locations contain around 300 observations, and very few of them contain less than 200 observations. At the few locations with less than 200 data points, data from the nearest gauge are used.

4.4 Quality metrics

The errors are assessed using the root mean square error (RMSE), mean absolute error (MAE), median absolute error (MedianAE), bias and mean relative error (MRE) (Bennett et al., 2013; Wang et al., 2012). MedianAE denotes the median of absolute error (AE). Rainfall often has a highly skewed distribution. Hence, the RMSE and MAE error estimates will be strongly influenced by the highest rainfall rates. This is why MRE and MedianAE also chosen to use, which are expected to be less influenced by the magnitude of rainfall (Wasko et al., 2013). These measures are formulated as follows:
The RMSE is a commonly used performance measure that provides the overall skill measure of a method. The overall systematic error of a method is assessed by bias whereas, Conditional Bias (CB) measures how an estimated rainfall differs from the gauge estimate (Ciach et al., 2000; Habib et al., 2008). The conditional bias conditioned to rainfall rate and radar reflectivity are formulated as follows:

\[
CB_g(a < R_g \leq b) = \frac{\sum_{i=1}^{n} R_{est} | (a < R_g \leq b)}{\sum_{i=1}^{n} R_g | (a < R_g \leq b)} \tag{4-18}
\]

\[
CB_R(a < dBZ \leq b) = \frac{\sum_{i=1}^{n} R_{est} | (a < dBZ \leq b)}{\sum_{i=1}^{n} R_g | (a < dBZ \leq b)} \tag{4-19}
\]

where \(R_g\) represents gauge rainfall and \(R_{est}\) represents estimated rainfall estimated using one of the four methods, \(CB_g\) is the conditional bias conditioned to gauge rainfall rate, \(CB_R\) is the conditional bias conditioned to radar reflectivity and \(a, b\) represent the lower and upper bounds of the moving window.
4.5 Study region and data

The NPR, CSI and combination methods have been tested over the Sydney region where the Australian Bureau of Meteorology operates the Terrey Hills radar and a relatively dense network of tipping bucket rain gauges. The S-band Terrey Hills radar covers a 256 km by 256 km region. It is a Doppler radar with a wavelength of 10.7 cm and a bandwidth of 1 degree. The radar precipitation estimates have a temporal resolution of 6 minutes and spatial resolution of 1 km. Of the approximately 65000 km$^2$ area covered by the radar, 55% is on land. The remainder of the domain, which is over the Pacific Ocean, has been disregarded from the analyses due to the absence of gauge observations. Data from November 2009 to December 2011 (two years) were used to analyse the performance of the proposed methods. The gauge data was collected from the Australian Bureau of Meteorology and Sydney Water and the radar data was obtained from the Australian Bureau of Meteorology.

The rainfall periods were selected by visual inspection of the radar images using the radar rainfall software Mapview (Seed and Jordan, 2002) and resulted in 1442 half-hour rainfall periods available for the analyses. Noise and hail effects (Chumchean et al., 2004; Chumchean et al., 2006a) were avoided by limiting the reflectivity values to between 15 dBZ to 53 dBZ. In the Sydney area, the climatological freezing level is 2.5 km (Chumchean et al., 2003b). Therefore, bright band effects were avoided by selecting 1.5 km Constant Altitude Plan Position Indicator (CAPPI) reflectivity data. To minimise errors at very low rainfall intensities, a minimum threshold of 1 mm/h was adopted (Chumchean et al., 2004; Chumchean et al., 2006b).

Within the study region, there are 282 tipping bucket gauges with bucket sizes of either 0.2 or 0.5 mm. The locations of these gauges are shown in Figure 2a, overlaid on the topography of the study region. The gauge density is high within 50 km of the radar and most of the gauges are located near the coast and to the south of the radar. Further inland, gauge densities are lower, in particular in the Blue Mountains where elevations reach up to 1600 m above sea level.
Figure 4-2: a) Topography of the study region showing the location of the Terrey Hills radar and rain gauge locations and b) mean annual rainfall across the study domain.

Mean annual rainfall, based on a standard 30-year climatology (1961-1990), is presented in Figure 2b. There is a strong east-west rainfall gradient with the driest areas located at the foothills of the Blue Mountains and higher rainfalls observed on the tops of the ranges. The area of highest rainfall is found to the south of Sydney in the Wollongong area and is due to the Illawarra Escarpment where the land rises steeply close to the coast.

4.6 Results

The performance of the different rainfall estimation methods is presented in this section, looking at aggregated results and temporal and spatial variations. The variation of the results with respect to rainfall rate and reflectivity is then presented.

4.6.1 Performance of the proposed methods

For each half-hour rainfall period, calculations were done for the radar rainfall using the parametric Z-R and NPR methods and the gauge rainfall using the CSI method. For each period, the NPR and CSI rainfall estimates are then merged using the dynamic
combination approach. The performance of all four methods (parametric Z-R, NPR, CSI and combination) are evaluated by comparing them with the gauge observations and are presented in Table 4-1.

The results displayed in Table 4-1 (aggregated over all 1442 rainfall periods) show that the NPR method is superior to the parametric Z-R method, with improvements in four error statistics (RMSE, MAE, MedianAE and MRE). Additional tests (not shown here) have been carried out for coarser timescales; these tests indicate that the performance of parametric Z-R and NPR methods is unchanged. The overall bias from the NPR method is similar to the parametric Z-R method. It is suspected that this could be due to the fact that the entire data is used in formulating the parametric Z-R, whereas this happens in a spatially distributed fashion in the case of the NPR. It should be noted that any bias larger than one indicates an overestimation of ground rainfall.

**Table 4-1:** Performance measure of different estimation methods. Values in bold represent the best result in each category.

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE (mm/h)</th>
<th>MAE (mm/h)</th>
<th>BIAS</th>
<th>MedianAE (mm/h)</th>
<th>MRE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parametric Z-R</td>
<td>4.37</td>
<td>2.64</td>
<td>0.95</td>
<td>1.53</td>
<td>76.08</td>
</tr>
<tr>
<td>NPR</td>
<td>3.97</td>
<td>2.41</td>
<td>0.95</td>
<td>1.43</td>
<td>73.44</td>
</tr>
<tr>
<td>CSI</td>
<td>4.17</td>
<td>2.22</td>
<td>0.79</td>
<td>1.10</td>
<td>59.85</td>
</tr>
<tr>
<td>Combination</td>
<td><strong>3.56</strong></td>
<td><strong>1.99</strong></td>
<td>0.86</td>
<td><strong>1.06</strong></td>
<td><strong>55.42</strong></td>
</tr>
</tbody>
</table>

The uncertainties in the radar-rainfall relationships (both parametric and nonparametric) are clear when the CSI results are compared to the radar-derived ones. The CSI method has smaller MRE than both parametric Z-R and NPR methods. The best performance is obtained from the combination method, with improvements in four error statistics (RMSE, MAE, MedianAE and MRE). Compared to the parametric Z-R
method, the NPR has improved RMSE by 10% while an even larger improvement (20%) is obtained using the combination method.

To illustrate the differences in the four methods, Figure 4-3 compares the rainfall estimates for a single period on 01 November 2010 at 06:30 UTC. For small reflectivities the rainfall estimates are higher using the NPR method than with the parametric Z-R method, which is evident from the larger areas of light blue in Figure 4-3b than in Figure 4-3a. In contrast, for the highest reflectivities the NPR method leads to lower rainfall estimates compared to the parametric Z-R which is evident from the smaller number of red points in Figure 4-3b than in Figure 4-3a. For the gauge based CSI method, the non-zero measurements at gauges in the northeast and centre of the study area have led to quite different patterns in the rainfall than the radar estimates (Figure 4-3c). To illustrate the dynamic weighting in combination method, the estimation at the locations marked by A, B, C, D, and E are evaluated shown in Figure 4-3. The performance of the combination method depends on the weights of the participating methods. Around location A, the CSI method estimates high rainfall (Figure 4-3c). At this location, the CSI and NPR methods have an average weight of 0.16 and 0.84 respectively. The combination method therefore reflects NPR and results in low rainfall amounts. The same happens at location B. The opposite scenario can be found at nearby radar locations around the Sydney region where gauge density is high and the average CSI weight is around 0.68. The Combination rainfall patterns are thus similar to the CSI method. As there is a fair number of gauges situated around the location D (in the Blue Mountain region), the average weight of CSI method will be higher at that location. The combination method thereby displays rainfall patterns similar to CSI and estimates intense rainfall for the particular period. At location E (near Newcastle region) both methods have similar weights. As both the methods estimate high rainfall at this location, the combination method just complies with them.

There are few gauges in the western part of the domain and therefore there is limited information available to the CSI method in this area. This demonstrates the influence of the gauge network arrangement on interpolation methods, in particular relative to the spatial pattern of any particular storm period. The radar can provide more information on the spatial pattern and rainfall amounts in these poorly gauged areas. For
this particular rainfall period, the error from the CSI method (Figure 4-3c) is approximately equivalent to the NPR method. As seen for the overall results in Table 1, the combination method leads to the smallest error (Figure 4-3d).

**Figure 4-3**: Rainfall estimation for an individual storm event on 01 November 2010 at 06:30 UTC from a) Parametric Z-R b) NPR c) CSI and d) Combination method.

Another example of improvement for the combination method is shown in Figure 4-4 for a single period on 21 July 2011 at 09:00 UTC. In Figure 4-4, at location A, the NPR method shows intense rainfall whereas the CSI method shows moderate rainfall. As the weight is high for NPR, the combined figure shows the rainfall pattern similar to NPR, but with lower intensities. Locations near the radar have high gauge density. At location
B, the combination method shows a rainfall rate similar to the CSI method as it has higher weight for this location. Locations in the Blue Mountains have fewer gauges and hence the CSI method has a larger weight compared to NPR. As a result, in D the combination method shows rainfall patterns driven by CSI. The NPR average weight is 0.60 around region C, resulting in the combination method rainfall pattern being similar to that from the NPR method.

Figure 4-4: Rainfall estimation for an individual storm period on 21 July 2011 at 09:00 UTC from a) Parametric Z-R b) NPR c) CSI and d) Combination method.
As shown in Figure 4-3, the spatial structure of the gauge network and the spatial patterns of rainfall periods lead to differences between the four methods. This spatial information is as important as the magnitude of the rainfall error in any hydrological analysis using the rainfall fields. In Figure 4-5 the performance of each method across the study domain is presented. At each gauge location, the method that leads to the best results has been identified. In Figure 4-5a the parametric Z-R and NPR methods are compared. It is also found that out of 282 gauge locations, the NPR method performs better than the parametric Z-R method at 246 gauge locations. For the same high reflectivity, the parametric method gives higher rainfall rates compared to the NPR method. This may be due to the relatively limited number of the Z-R pair dataset available for high reflectivities that is used to develop the NPR method. Therefore, in the high rainfall region the parametric method has a relatively smaller error compared to NPR.

The study region has a mixture of areas with high and low gauge densities. One high gauge density area is marked with “D” and one of the sparse gauge density area is marked with “S” in Figure 4-5b. Also the performance of NPR with the gauge-based CSI estimation is shown in Figure 4-5b. The NPR method provides more reliable estimates at 138 locations, particularly in the areas of low gauge density. The CSI method is reliable in the high gauge density area. This is to be expected given that the interpolation has much more information available in these areas.

Finally, all four methods are compared in Figure 4-5c. The CSI method continues to lead to the best estimates in the densely gauged areas. The combined method produces better estimates in regions of lower gauge densities, showing the added value that the radar information can provide. The better performance at these locations leads to the overall smaller errors from this method as seen in Table 4-1.
Figure 4-5: Spatial representation of the best RMSE between a) Parametric Z-R and NPR b) NPR and CSI c) Parametric Z-R, NPR, CSI and combination method. Areas with high gauge density (D) and sparse gauge density (S) are shown in panel b).
Clearly gauge density is an important factor in determining the effectiveness of the different methods. To further assess this relationship, Table 4-2 separates the results according to the gauge density, with the regions shown in Figure 4-6. The sparsely gauged area (S1) to the north west of Sydney has a density of 1 gauge per 612 km$^2$. The densely gauged region (D) has a gauge density of 1 gauge per 34 km$^2$. The gauge density over the full domain is 1 gauge per 127 km$^2$. The RMSE has been estimated for the parametric Z-R, NPR, CSI and combination methods across each of these three regions by comparing them with the observed gauge rainfall. Between the parametric Z-R, NPR and CSI methods, the NPR method has the smallest errors in the sparse region. For the dense region the CSI method minimises the errors as demonstrated previously in Figure
4-5b. The combination method is found to be superior to all other methods for the sparse regions. The magnitude of the values presented in Table 4-2 confirms the efficiency of the NPR and combination method.

**Table 4-2**: RMSE (mm/h) across different regions. Values in bold represent the best result in each category.

<table>
<thead>
<tr>
<th>Method</th>
<th>Sparse Region S1</th>
<th>Sparse Region S2</th>
<th>Sparse Region S3</th>
<th>Dense Region D</th>
<th>Whole Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parametric Z-R</td>
<td>4.01</td>
<td>4.22</td>
<td>4.74</td>
<td>4.18</td>
<td>4.37</td>
</tr>
<tr>
<td>NPR</td>
<td>3.46</td>
<td>3.58</td>
<td>4.44</td>
<td>3.82</td>
<td>3.97</td>
</tr>
<tr>
<td>CSI</td>
<td>4.54</td>
<td>3.98</td>
<td>5.01</td>
<td>3.76</td>
<td>4.17</td>
</tr>
<tr>
<td>Combination</td>
<td><strong>3.28</strong></td>
<td><strong>3.18</strong></td>
<td><strong>4.26</strong></td>
<td><strong>3.37</strong></td>
<td><strong>3.56</strong></td>
</tr>
</tbody>
</table>

In addition to the original sparse region (S1), two other sparse regions with different numbers of rain gauges denoted by S2 and S3 have been investigated, as shown in Figure 4-6. The total number of gauges in S3 region is around 3 times higher compared to S1 region. From Table 4-2, it is clear that the conclusions remain the same even though the sparse region is changed. The NPR method performs better than the parametric Z-R method in terms of RMSE. Most importantly, the RMSE of combination method is the lowest among all other methods, whether we apply the combination method for sparse or dense region.

### 4.6.2 Spatial and temporal variation of weights

The results presented in the preceding sections have mainly considered the aggregate performance of each of the methods over all 1442 half-hourly rainfall periods. Also of interest is how the combination of the NPR and CSI method varies over all the periods. The dynamic weighting ensures that for any location the best method for a particular rainfall period is chosen. Information about how the weighting changes in time
and space is valuable in providing further information about how the methods perform. The mean weight assigned to the NPR method at each gauge location is represented in Figure 4-7a. In general, NPR weights are higher in the low gauge density regions. The kriging based merging method considers radar rainfall as a secondary information that improves the spatial interpolation of gauge rainfall estimates (Goudenhoofdt and Delobbe, 2009; Rabiei and Haberlandt, 2015). In contrast, the spatial and temporal variation of the NPR weights (Figure 4-7) considers radar rainfall as potentially equally or more important than the gauge rainfall depending on the information content in the radar data. The exception is the area of gauges to the northeast of the radar where, consistent with the results shown in Figure 4-5b, the CSI method leads to lower errors and therefore is more strongly weighted when combining both approaches.

We also have information on the temporal variation of the weights for any particular location. To demonstrate the utility of this information, three representative rain gauges X, Y and Z are highlighted in Figure 4-7a. The gauge “X” was chosen to represent a high rainfall area, whereas gauges “Y” and “Z” are located in dense and sparsely gauged areas respectively. The mean weights and temporal variation of the NPR weights at these gauge locations are shown in Figure 4-7b to 4-7d. At gauge X, the weight ranges from 0.38 to 0.80 (Figure 4-7b) with a mean of 0.61. This gauge has relatively high rainfall but is located in a sparsely gauged region. At this location, radar provides better information on the magnitude of the rainfall totals compared to the gauge based estimates. Therefore, the weights are generally greater than 0.50.
Figure 4-7: The NPR mean weights a) at different locations. The variation of the NPR weights for different storm periods are shown for b) location X, c) location Y and d) location Z. The weights for the periods presented in Figure 4-3 are shown in blue colour symbol (Figure 4-5b-d).

At gauge Y the NPR weights are much lower, ranging from 0.11 to 0.40 with a mean of 0.32 (Figure 4-7c). As there are many other gauges surrounding this gauge, the spatial interpolation is generally successful. Finally, at gauge Z, in the sparsely gauged region, the NPR weights range from 0.74 to 1.0 with a mean of 0.84. The mean NPR weights at other gauges in this area (Figure 4-7a) are quite similar. When the weight of
NPR method is equal to one, spatial interpolation does not provide any useful information. This occurs for periods that are particularly spatially non-uniform over the domain. The CSI method does not capture highly spatially varied rainfall patterns well (such as when a high rainfall is observed adjacent to low rainfall in neighbouring gauges). The NPR estimates are calculated independently at each location and can therefore better capture rainfall periods with high spatial heterogeneity.

### 4.6.3 Performance for different storm properties

The results presented above explore the performance of the different methods with respect to gauge density. In this section, the relationship between the proposed methods and several rainfall characteristics such as rainfall rate and reflectivity have been considered. For each characteristic, the data set is binned into several classes.

The impact of rainfall rate on the performance of each of the estimation methods has been analysed by dividing the mean rainfall across the region into 4 mm/h bin size. For the different intensity rainfall periods, the CSI method leads to the lowest bias (Figure 4-8). This is likely due to two reasons. The first is that there is less variation between adjacent gauges and hence the spatial interpolation is more successful. Secondly, the errors in the NPR method are quite high for small reflectivity periods (Figure 4-8d). Additional data may assist in better defining the NPR relationship at these rainfall rates. Improvements from the NPR are evident by the larger reductions in bias than the RMSE. It is worth noting that the rainfall estimates from smaller and higher reflectivity bins using the parametric Z-R have the largest uncertainty (Figure 4-8d).

For different rainfall rates, the combination method leads to the best performance, substantially reducing the MRE, RMSE and Bias compared to using the radar alone and providing a modest improvement over the CSI method.
Figure 4-4: The conditional performance measure of different estimation methods, conditioned to rainfall rate and reflectivity a) RMSE conditioned to rainfall rate b) RMSE conditioned to reflectivity c) Bias conditioned to rainfall rate d) Bias conditioned to reflectivity.

Similar to rainfall rate, reflectivity has been divided into 2dBZ bin size. The results presented in Figure 8 show that the NPR method performs better compared to the parametric Z-R method for all reflectivity bins. The results presented in Figure 8 shows that high reflectivity bins lead to higher error for parametric Z-R compared to NPR. Similar to rainfall rates, the combination method leads to lowest RMSE for all reflectivity bins.

For higher rainfall rates, NPR leads to the lower estimation of ground rainfall compared to parametric Z-R. This is likely due to the lack of observed data points in the higher reflectivity band which is used to develop NPR from the past observed data set. Therefore, it is important to have a good number of past observed data to get an accurate rainfall estimate from all reflectivity bins using NPR. Improvements from NPR for the
different rainfall rates are evident by the large reductions in RMSE and bias (Figure 4-8). It is worth noting that the rainfall estimates from the higher reflectivity bins using parametric Z-R have the largest uncertainty (Figure 4-8). However, the combination method has been tested by incorporating the past observed records at the gauge of interest, rather than in the cross-validation setting used in the results section. If data from the gauge of interest are used, then the uncertainty of the covariance matrix is smaller. The combination method provides better results with further improvement in RMSE of around 10%. If implemented in this way, the combination method can be used in real time to quality control gauge data by leaving out the measurement at a particular time step and comparing the gauge observation with the estimate from the combination approach.

4.7 Summary and Conclusion

Two new methods have been proposed in this chapter, namely the nonparametric Z-R relationship and the dynamic combination approach for the radar and spatially interpolated gauge rainfalls. In this section, uncertainties in each of these methods are discussed in more detail.

The advantage of the traditional parametric relationship (Z=AR\(^b\)) is that only two parameters need to be estimated. In parametric estimation, every data point influences the inference of the parameters A and b. The disadvantage of a parametric approach is however that a single relationship may not account for all the variations in the data if an incorrect function form is used. In the nonparametric approach, the aim is to use the sample of Z and R to specify the underlying relationship. The idea is that the data points closest to the target point will lead to better regression estimates. As with any statistical approach, the sample Z and R values need to properly represent the true population, which often requires large amounts of data. In radar rainfall, the Z-R pairs give thousands of data points, which is ideal for applying nonparametric approaches. When there is insufficient data, such as in the case of the highest rainfall rates, parametric approaches can extrapolate and may perform better.

The combination method provides better rainfall estimates compared to the radar or gauge estimates. The advantage of the proposed combination is that it can be used...
with existing rain gauge and radar networks to estimate rainfall at ungauged locations. The improvement due to the combination method depends on uncertainties in the estimation of the covariance matrix.

This study presents a new method for rainfall estimation from radar reflectivity with greater efficiency compared to a traditional parametric Z-R relationship method. The uncertainty of the parametric Z-R relationship parameters introduces biases into the rainfall estimates. Given the large amount of data available from the radar, a nonparametric approach is attractive. The proposed NPR method is shown to produce more reliable rainfall estimates and to reduce errors by approximately 10\% compared to a traditional Z-R relationship, with improvements in the rainfall estimates at around 90\% of the locations. In addition, the rainfall estimated using NPR has smaller errors than when spatially interpolated from gauges in the areas of low gauge density.

Further improvements are demonstrated when radar and gauge rainfall estimates are combined. The combination method uses dynamic weights based on the errors from each method. The rainfall estimate from the combination method is about 20\% more accurate than the traditional parametric Z-R relationship method and has the lowest error over the entire study domain.

It is shown that over densely gauged areas spatial interpolation can provide good rainfall estimates. The benefit of a radar and gauge combination approach is evident in sparsely gauged areas or when storm periods have large spatial heterogeneity. Under these conditions the dynamic combination of radar and gauge data is recommended. Thus the combination method is not only useful in reducing errors in spatial rainfall estimates, but also in diagnosing the strengths and weaknesses of a particular gauge and radar network.
5 Merging radar and in-situ rainfall measurements: An assessment of different combination algorithms

5.1 Introduction

The concept of combining different estimates of the same event has gained wide acceptance. The rationale behind the combination of multiple sources is that the individual model/method contains errors from various sources. Each source of error affects each estimation method in a particular way. The combination of several model estimates, therefore, allows minimizing these errors, resulting in better predictions compared to the single best model (Ajami et al., 2006; Stock and Watson, 2004).

A number of different methods have been developed to merge radar and gauge rainfall estimates such as Mean Field Bias (Berndt et al., 2014; Borga et al., 2002; Seo and Breidenbach, 2002; Smith and Krajewski, 1991), Kalman Filter (Chumchean et al., 2006a) and geostatistical approaches. The geostatistical approaches include Kriging, Co-kriging, Kriging with external drift (Berndt et al., 2014; Goudenhoofdt and Delobbe, 2009; Haberlandt, 2007; Sideris et al., 2014; Velasco-Forero et al., 2009). This geostatistical merging is done by applying two different approaches (Jewell and Gaussiat, 2015). In the first approach, initially the gauge rainfall is spatially interpolated independently of the radar, and then the difference between radar and spatially interpolated gauge rainfall is added to the radar rainfall to get an estimate at an unknown location. This idea is used in kriging with radar-based error correction (Ehret et al., 2008) and conditional merging (Sinclair and Pegram, 2005). In the second approach, before the generation of the interpolated field, the radar estimate at ungauged locations is formulated using weights. The weights are determined from the relationship between the measured radar and gauge rainfall at gauged locations. All those studies consider the radar data as secondary information which is used to improve the spatially interpolated gauge rainfall (Goudenhoofdt and Delobbe, 2009). There are also differences in combination approaches whether the weights vary in time (dynamic weighting) (Chowdhury and Sharma, 2010; Krishnamurti et al., 1999) or not (static weighting) (Allard et al., 2012; Robertson et al., 2004; Timmermann, 2006). Hasan et al. (2016) used a weighted combination approach to merge radar and spatially interpolated gauge rainfall estimates, assuming that the errors from the multiple estimates are correlated. This study investigates the effect of different merging techniques such as static and dynamic
weighing as well as point (Chowdhury and Sharma, 2009; Genre et al., 2013; Smith and Wallis, 2009; Timmermann, 2006) and density based merging (Allard et al., 2012; Clemen and Winkler, 2007; Liu et al., 2012; Ranjan, 2009), given that mean and variance of radar and gauge estimates are known. This chapter examines several merging methods that differ in the consideration of correlation among the estimation errors, their distribution and the application of dynamic and static weighting. This study does not give priority to gauge measurements over radar rainfall estimates, rather considers that both radar and gauge provide useful information. The importance of each is given based on the estimation error. While kriging based approaches are not explicitly included in our comparison, in principle they have similarities to the Copula based approach (Bárdossy, 2006a; Bárdossy and Li, 2008a; Durante and Sempi, 2010) which is included.

The previous studies on radar and gauge combination approaches did not consider the implication of error independence, error distributions and static vs dynamic weighting on the merged estimates. The focus of this chapter is to investigate the merits and demerits of alternative combination approach in the context of radar rainfall estimation. It is expected the outcomes reported here can be used as a guide when choosing the optimal merging algorithm in a wide range of practical settings.

The rest of this chapter is organized as follows. The different merging approaches are presented in section 5.2. Section 5.3 presents the data used in this study. The performance of the different merging approaches is discussed in Section 5.4. Finally, discussion and conclusion are presented in Section 5.5.

### 5.2 Merging methods

Merging is based on the idea that the weighted average of multiple methods can provide a better quantity of interest than any single method (Kim et al., 2007; Looper and Vieux, 2012). This is particularly the case when the errors from the different methods are complementary, in the sense that one method tends to perform well where other methods are weak. A literature review of combination methods suggests that there are a number of approaches available to determine these weights. In this chapter, six different methods that have been found to be useful in previous studies is considered for evaluation. As discussed above, three main distinctions can be made between the methods:
• Correlated or independent errors from the multiple methods
• Static or dynamic weighting
• Gaussian or non-Gaussian error distributions around the estimates

According to these three criteria, the five methods are summarised in Table 5-1 along with details of their previous applications.

**Table 5-1: Different merging approaches**

<table>
<thead>
<tr>
<th>Name</th>
<th>Error correlation structure</th>
<th>Weighting type</th>
<th>Error distribution</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error variance method</td>
<td>Independent errors</td>
<td>Dynamic</td>
<td>Gaussian</td>
<td>(Bates and Granger, 1969; Smith and Wallis, 2009; Wasko et al., 2013)</td>
</tr>
<tr>
<td>Error covariance method</td>
<td>Correlated errors</td>
<td>Dynamic</td>
<td>Gaussian</td>
<td>(Khan et al., 2014; Timmermann, 2006)</td>
</tr>
<tr>
<td>Error rotation method</td>
<td>De-correlated errors</td>
<td>Dynamic</td>
<td>Gaussian</td>
<td>(Fortuna and Capson, 2004; Khan et al., 2014; Shih-Hau et al., 2008)</td>
</tr>
<tr>
<td>PDF overlap method</td>
<td>Independent errors</td>
<td>Dynamic</td>
<td>Gaussian and Non-Gaussian</td>
<td>(Liu et al., 2012)</td>
</tr>
<tr>
<td>Probability aggregation method</td>
<td>Conditional Independence errors</td>
<td>Static</td>
<td>Gaussian</td>
<td>(Allard et al., 2012; Clemen, 1989; Clemen and Winkler, 2007; Gneiting and Ranjan, 2013; Ranjan, 2009)</td>
</tr>
</tbody>
</table>

The following example illustrates the workings of these merging methods. Let \( R_A \) and \( R_B \) be the estimates from two different methods A and B, \( e_A \) and \( e_B \) are the estimation
errors, $\rho$ is the correlation between estimation errors. The estimation errors are the differences between observations ($R_O$) and corresponding estimates (i.e. $e_A = R_O - R_A$ and $e_B = R_O - R_B$). $\sigma^2_A$ and $\sigma^2_B$ are the variance of the estimation errors. $W_A$ and $W_B$ are the weights in the combined estimate. The basic equation for combination, assuming two methods, A and B is

$$R_{COM,t} = W_{A,t}R_{A,t} + W_{B,t}R_{B,t}$$  \hfill (5-1)

where for time step $t$, $R_{A,t}$ and $R_{B,t}$ are the estimates obtained from the two different methods A and B. $W_{A,t}, W_{B,t}$ are the weights applied to each method to provide a combined estimate $R_{COM,t}$. The question is to determine the time-varying weights $W_{A,t}$ and $W_{B,t}$.

The following section will describe each of these methods, their advantages and drawbacks and the procedure for estimating the weights. To make a fair comparison, a common dataset have been used to compare all methods and have divided the dataset into calibration and validation periods. The required weights are calculated using the calibration dataset. Finally, the performance of different methods is compared using the results obtained from the validation dataset.

### 5.2.1 Error variance method

In this approach, the estimates are combined according to weights that are calculated from variances of estimation errors (Smith and Wallis, 2009). In the combined estimate, the method with better estimates is given greater weight. One of the major assumptions of this approach is that the errors from the multiple methods are uncorrelated. It is also assumed that the errors of individual estimates are normally distributed and unbiased. The combined estimates are unbiased because the sum of the weights equals to one (Timmermann, 2006).

The weights of method A and B can be estimated by Equation 5-2 and Equation 5-3, given below:

$$W_A = \frac{\sigma^2_B}{\sigma^2_A + \sigma^2_B}$$  \hfill (5-2)
\[ W_B = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_B^2} \]  

(5-3)

5.2.2 Error covariance method

The previous method assumed uncorrelated errors, while it is clearly not the case in many circumstances. Therefore, it is possible to extend the method by including the correlation between estimation errors. The estimates from different methods are combined using the weighted average of estimates. The weight is calculated using a variance and covariance of estimates (Timmermann, 2006). Minimising the quantity in Equation 5-4, given below, can estimate weights of methods A and B as:

\[
\min (W' \sum_e W) \text{ such that } W'I = 1
\]  

(5-4)

where, \( W' = [w_A \ w_B] \), \( \sum_e \) is the covariance matrix of the estimation errors, off-diagonals are the covariance of the errors and diagonals are the variance of the errors from the two methods. \( b \) is a \( m \) by 1 column vector of ones with \( m \) denoting the number of methods. For two methods (A and B), value of \( m \) is two. The weights constrained to add up to unity and lie between 0 and 1. Readers are referred to Timmermann (2006) for further details concerning the derivation of Equation 5-4.

5.2.3 Error rotation method

One way to process correlated errors is to transform them to de-correlated values (Fortuna and Capson, 2004; Qian and Fowler, 2007; Shih-Hau et al., 2008). This method is similar to the error covariance method, except that the correlation between errors is forced to zero. The de-correlation is done by applying principal component analysis (PCA) (Genre et al., 2013; Stock and Watson, 2002; Stock and Watson, 2004). In the rotated space, the de-correlated estimation errors are used to estimate the combination weights. Finally, those weights are rotated back onto the original space.

The procedure adopted is as follows:

1. Compute uncorrelated errors (\( \hat{e}_A, \hat{e}_B \)) by applying PCA to the estimation errors (\( e_A, e_B \)).
2. Calculate the weights ($\hat{W}_A$, $\hat{W}_B$) in the transformed space using Equation 5-4.
3. Back-transform the calculated weights ($\hat{W}_A$, $\hat{W}_B$) into the original space by applying the inverse rotation and rescale them so that the weights add to one.

5.2.4 PDF overlap method

The idea behind this approach is to determine an empirical probability density function (PDF) that has the maximum overlap with the PDFs of the estimates of each individual method used (Liu et al., 2012). It is an iterative process where the area of the combined PDF is determined through a numerical optimization that maximizes the area of the combined PDF. The proportion of the area contributing the combined PDF determines the weights. This idea is illustrated in Figure 5-1 where the two separate estimates are assumed to have Gaussian distributions.

Although Figure 5-1 illustrates the idea using two assumed Gaussian distributions, the method is general such that any distribution can be used. In this study, both the Gaussian and Non-Gaussian cases method have been investigated. Suppose, PDF$_A$ and PDF$_B$ represent the PDF of the two estimates by methods A and B. PDF$_{COM}$ is the PDF of the combined estimate. The weight for method A ($W_A$) is estimated by the overlapping area between PDF$_{COM}$ and PDF$_A$. The overlapping area between the PDF$_{COM}$ and PDF$_B$ gives the weight of method B ($W_B$). The summation of $W_A$ and $W_B$ equals one.

The procedure adopted is as follows:
1) Assume initial weights for methods A and B such that they add up to one. The weight iteration is initialized by setting $W_A=W_B=0.50$.
2) Calculate the cumulative distribution functions (CDF) as $F_A$, $F_B$ from the respective PDF$_A$ and PDF$_B$.
3) Find the combined CDF from the two individual estimates

$$F_{COM} = W_A F_A + W_B F_B$$  \hspace{1cm} (5-5)

4) Estimate the overlapping area contributed by methods A and B as:

$$A_{OA} = \sum_{j=1}^{n} \min[p_{COM}^j, p_A^j]$$  \hspace{1cm} (5-6)
where, \( A_{OA} \) and \( A_{OB} \) are the overlapping area contributed by the PDF\(_A\) and PDF\(_B\) respectively. \( p_{COM}^j \), \( p_A^j \), and \( p_B^j \) are the probability related to the Combination, A and B respectively at any section \( j \) (\( j=1,2,\ldots,n \)). \( p_{COM}^j \), \( p_A^j \), and \( p_B^j \) can be estimated by subtracting the probability of two consecutive sections from their respective CDF.

5) Estimate the total overlapping area \( A_{COM} \) as:

\[
A_{COM} = A_{OA} + A_{OB}
\]

(5-8)

6) Estimate the weight of PDF\(_A\) and PDF\(_B\) from the overlapping area as:

\[
W_A = \frac{A_{OA}}{A_{OA} + A_{OB}}
\]

(5-9)

\[
W_B = \frac{A_{OB}}{A_{OA} + A_{OB}}
\]

(5-10)

7) Then follow the steps provided in 3 to 5 using the weight calculated in step 5. Each iteration offers new sets of weights along with the total overlapping area. Finally, the stopping criterion of the weight maximization approach is given by a pre-specified tolerance (\( \delta \)) (Liu et al., 2012) defined as the summation of the difference in weights produced in two consecutive iterations of the methods A and B.

\[
\delta = |W_A^{i+1} - W_A^i| + |W_B^{i+1} - W_B^i|
\]

(5-11)

The PDF overlap method is sufficiently general to be implemented with any distribution. Section 5.3 explains the practical example of Gaussian and non-Gaussian PDF overlap methods.
Figure 5-1: Illustration of the PDF overlap (Gaussian) approach. PDF$_A$ and PDF$_B$ represent the PDF of the two estimates by methods A and B. PDF$_{COM}$ is the PDF of the combined estimate. A$_{OA}$ and A$_{OB}$ denote the overlapping area contributed by methods A and B.

5.2.5 Probability aggregation method

The probability aggregation method merges two different sources of information in a probabilistic framework. Previous studies of probability relationship between two variables suggest that it can be applied to combine any distribution that represents the same physical quantity (Allard et al., 2012; Mariethoz et al., 2009; Tarantola, 2005). There are many ways of probability aggregation reported in (Allard et al., 2012) such as additive method (linear pooling, beta-transformed linear pooling), multiplication of probabilities (log-linear pooling, generalized logarithmic pooling, maximum entropy approach), multiplication of odds (Tau-model, Nu-model) etc. The linear pool is a method of combining individual probability forecasts linearly according to their weights. Ranjan and Gneiting (2010) reported that linear pool is sub-optimal. Methods based on the product of probabilities are superior to all other methods (Allard et al., 2012). Therefore,
in this chapter, the product of probabilities (log-linear pooling) method have been applied. The following example illustrates this probability aggregation method.

Let PDF\(_A\) and PDF\(_B\) represent the PDFs of estimates by methods A and B. PDF\(_A(x;R_A)\) is the PDF ordinate for method A at location \(x\) derived as a Normal distribution centered at the estimated mean \(R_A\). Similarly, PDF\(_B(x;R_B)\) is the PDF for method B at the same location \(x\) using a Normal distribution with mean \(R_B\) that can be derived using an exogenous variable, in this case the radar reflectivity \(Z\) as explained later. Using the concept of probability aggregation, these two distributions can be combined into a single one using the following equation:

\[
\text{PDF}_{\text{COM}}(x) = \frac{[\text{PDF}_A(x;R_A)]^{w_A}[\text{PDF}_B(x;R_B)]^{w_B}}{\int [\text{PDF}_A(x;R_A)]^{w_A}[\text{PDF}_B(x;R_B)]^{w_B} dx}
\]  

(5-12)

where PDF\(_{\text{COM}}(x)\) is the PDF of the combined estimate, and the denominator is a normalizing factor that ensures the resulting PDF integrates to unity.

It is important to note that the weight estimation in this method differs from that in the other methods. The weight remains constant over all the events and the approach is termed a static method. The required weights are calculated using the calibration dataset. More details on obtaining this static weight are presented in the next section.

5.3 Data and application

This section presents the results from an evaluation of the six radar and rain gauge merging approaches over the Sydney region. Radar data is obtained from the Sydney Terrey Hills radar. This radar is an S-band Doppler radar with a bandwidth of 1 degree and wavelength of 10.7 cm. It covers a 256 km by 256 km region with one kilometer spatial resolution and six minutes temporal resolution. Total 1992 no of half-hour rainfall events have been selected by visual inspection from the period from November 2009 to December 2011. In order to avoid noise and hail effects, radar reflectivities were limited between 15 dBZ to 53 dBZ (Chumchean et al., 2004; Chumchean et al., 2006a).

There are 282 gauges located within the study region with data during the analysis period (Figure 5-2). The gauges have a bucket size of either 0.2 mm or 0.5 mm size. A minimum threshold value of 1 mm/h is adopted for gauge rainfall to avoid errors from
low intensity events (Chumchean et al., 2004; Chumchean et al., 2006b). Three different validation approaches, namely temporal, spatial and spatio-temporal are used to assess performance of merged estimates. In the temporal validation, the available data divided into two periods one for calibration (1035 half-hour rainfall events) and another one for validation (957 rainfall events). In the spatial validation (Goudenhoofdt and Delobbe, 2009), the available gauges are spatially divided into a calibration set (n=158) and a validation set (n=124) as shown in Figure 5-3. In the spatio-temporal validation, the two sets of gauges are used with different time period for calibration and validation. The rainfall event selection is subjective and randomly selected for calibration and validation.

![Study region around the Sydney Terrey Hills radar.](image)

**Figure 5-2**: Study region around the Sydney Terrey Hills radar.

Radar rainfall at the radar pixel location are estimated using a nonparametric radar rainfall estimation method (NPR method)(Hasan et al., 2016). The NPR method provides better radar rainfall estimates compared to the parametric Z-R relationship (Hasan et al., 2016).
The kernel based NPR method models the joint pdf between reflectivity and rainfall using kernel smoothing (Silverman, 1986). Each pair of radar reflectivity and rainfall values corresponds to the mean of a Gaussian kernel. The standard deviation (or bandwidth) of kernels is given by the conditional covariance. For each kernel, conditional weight and conditional mean are estimated. The expected radar rainfall can be obtained from the locally weighted average of all kernels. The gauge observations have been interpolated using the copula-based spatial interpolation (CSI) technique described in Wasko et al. (2013). For computational ease, a Gaussian copula using a Gaussian univariate distribution is assumed. An exponential correlation structure is used throughout. The parameters are estimated using a by maximum likelihood approach. As the copula is a Gaussian there are no “copula specific” parameters that need to be calibrated. The parameters are re-estimated for each interpolation scheme, there is no specific parameters value. The gauge observations at any radar pixel location is interpolated excluding coincident gauge observations at the pixel and using only the remaining observations. The interpolated rainfall is the weighted contribution of the nearest neighbour gauges with weights depending on the covariance of the nearest measurements and the distance between radar pixel and gauge locations (Delrieu et al., 2014; Jewell and Gaussiat, 2015; Velasco-Forero et al., 2009). The leave-one-out cross-validation technique (Goovaerts, 2000) have been used to implement the CSI method developed by Wasko et al. (2013) and Kazianka (2013a). Future research can be carried out by replacing the CSI estimates with any other interpolation technique such as kriging, as long as they provide mean and variance.

In the following section, methods A and B will be substituted by the CSI and NPR methods respectively. Then, rainfall estimates (R_A and R_B), variance of estimation errors (σ^2_A and σ^2_B), weights in the combined estimate (W_A and W_B) represent the CSI and NPR methods respectively. The weights in error variance method are obtained using Equations 5-2 and 5-3. Similarly, weights in error covariance method are estimated using Equation 5-4. The error covariance matrix in Equation 5-4 is obtained from CSI and NPR estimation errors (Hasan et al., 2016). The error rotation method combines de-correlated CSI and NPR estimates. The PDF overlap and probability aggregation methods require a PDF estimate of the rainfall. The CSI and NPR PDFs are formulated from the mean and
variance of the respective estimates (CSI and NPR) with the assumption of a Normal distribution. The Gaussian PDF merging is performed using the two normal distribution PDFs (PDF\textsubscript{CSI} and PDF\textsubscript{NPR}). The non-Gaussian PDF Overlap also have been tested here. The non-Gaussian PDF Overlap is similar to the Gaussian PDF Overlap method except that the non-Gaussian NPR PDF is used in the combination instead of a Gaussian NPR PDF. The non-Gaussian NPR PDF is derived from the conditional distribution of the NPR estimate (Equation 5-13). Given radar reflectivity (Z), the conditional probability density of rainfall (\( R_{\text{NPR}} \)) can be estimated as (see Hasan et al., 2016; Sharma et al., 1997) for details:

\[
\hat{f}(R_{\text{NPR}}|Z) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{(2\pi\lambda^2 S_c)^{1/2}} w_i \exp \left\{ -\frac{(R_{\text{NPR}} - b_i)^2}{2\lambda^2 S_c} \right\}
\]

(5-13)

where \( w_i \) is the conditional weight and \( b_i \) is the conditional mean associated with the \( i \)th kernel, \( \lambda \) is the bandwidth, \( S_c \) is the conditional covariance of rainfall-reflectivity pairs that measure the spread of the conditional probability density.

In most of the cases, the non-Gaussian NPR PDF provides longer tail distributions which have advantages in the estimation of extremes. As the use of a Gaussian PDF may result in probability density for negative values of rainfall, the widely used reflection technique have been incorporated (Jones, 1993; Karunamuni and Alberts, 2005a; Karunamuni and Alberts, 2005b; Silverman, 1986) at the boundary to avoid any negative expected values. However, this issue was not faced with the sample data used in our study, as no negative estimates were noted.
Figure 5-3: Spatial validation. Red colour represents validation gauges and blue colour represents calibration gauges.

The probability aggregation method requires pre-specification of CSI and NPR weights \((W_A, W_B)\) in the combination process. The weight has been calibrated by using 64 possible pairs of \(W_A, W_B\) on a calibration dataset. The weights are allowed to vary from 0 to 3.5 with a step of 0.50. The RMSE of each pair are computed by comparing to the observation. The pair that gives minimum RMSE is used for validation. The results of the sensitivity analysis are shown in Figure 5-4. The CSI and NPR weights are shown in two axes. Depending on the \(W_A\) and \(W_B\) pairs RMSE varies from 2.7 mm/h to 4.5 mm/h. In general, RMSE increases when the weight \((W_A, W_B)\) increase.
5.4 Results

The radar and gauge rainfall are calculated at each radar pixel using the NPR and CSI methods for each half-hour rainfall event. Then these two estimates are combined using the six different combination approaches described in section 5.2. The performance of these combination methods is evaluated using the root mean square error (RMSE), mean absolute error (MAE) and percent bias (PBIAS), median absolute error (MedianAE) by comparing with the gauge observations. These measures are calculated as follows:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (R_{\text{est}} - R_{g})^2}
\]

\( (5-14) \)
\[
\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |R_{\text{est}} - R_g|
\]

(5-15)

\[
\text{PBIAS} = 100\times \frac{1}{n} \sum_{i=1}^{n} \left( \frac{R_{\text{est}} - R_g}{R_g} \right)
\]

(5-16)

\[
\text{logerror} = 10 \log_{10} \left( \frac{R_{\text{est}}}{R_g} \right)
\]

(5-17)

\[
\text{AE} = |R_{\text{est}} - R_g|
\]

(5-18)

where \(R_g\) represents gauge rainfall and \(R_{\text{est}}\) represents the rainfall estimated using one of the six methods. A summary of the results for the different combination approaches is given in Table 2 based on the validation period.

The results displayed in Table 5-2 show that the PDF overlap (non-Gaussian) method is superior to the rest of the approaches in terms of RMSE. The probability aggregation method gives the best result in terms of MAE, PBIAS, and MedianAE among the methods. It is important to note that RMSE gives a relatively high weight to large errors, as the errors are squared before they are averaged. In contrast, the MAE is a linear score where all the individual differences are weighted equally in the average like Median AE (Median absolute error) (Chai and Draxler, 2014). The non-Gaussian PDF overlap method gives higher PBIAS compared to all other methods which might have influenced the RMSE. Therefore, in density based combination approach, the probability aggregation method provides better performance among all other methods even though the PDF overlap (non-Gaussian) method results in lower RMSE than the probability aggregation method.

**Table 5-2:** Temporal (T), Spatial (S) and Spatio-Temporal (ST) validation performances of different estimation methods with the best estimates in bold and the second best shown in italics.

<table>
<thead>
<tr>
<th></th>
<th>RMSE (mm/h)</th>
<th>MAE (mm/h)</th>
<th>PBIAS</th>
<th>MedianAE (mm/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>3.30</td>
<td>3.40</td>
<td>2.02</td>
<td>-1.89</td>
</tr>
<tr>
<td>S</td>
<td>3.40</td>
<td>3.20</td>
<td>1.95</td>
<td>10.11</td>
</tr>
<tr>
<td>ST</td>
<td>3.20</td>
<td>2.02</td>
<td>1.95</td>
<td>-1.51</td>
</tr>
</tbody>
</table>

82
The probability aggregation and error variance methods are better in terms of PBIAS where the former yields positive PBIAS and the latter yields negative PBIAS. Apart from the probability aggregation method, the error rotation method gives good result in all performance criteria. On the other hand, the Gaussian and non-Gaussian PDF overlap methods show dissimilarity in performance measures. The non-Gaussian PDF overlap method gives better combination estimates (in two performance criteria, i.e. RMSE and MAE) compared to the Gaussian based overlap method. The results clearly indicate that among all merging methods presented, the error variance method is best for point based merging and the probability aggregation method is best for density based merging. It can be observed that the magnitudes of the MAE and RMSE in Table 5-2 are not markedly different, suggesting that the influence of large outliers (which would have resulted in greater RMSEs) is low.

The combination method has been assessed using three different validation approaches, namely temporal, spatial and spatio-temporal, to confirm whether same results hold at the ungauged location. The performance of the different combination methods remains consistent in the four assessment criteria, namely RMSE, MAE, PBIAS, MedianAE.

The performance of each of the estimation methods was next analysed with respect to rainfall rate by dividing into three categories: low (less than 2 mm/h), medium (2~5 mm/h), and high (5~20 mm/h). The result presented in Table 5-3 shows that performance of merging approaches is dependent on rainfall rates. For low rainfall rate, the error variance method shows better performance in the MAE, while probability

<table>
<thead>
<tr>
<th></th>
<th>2.60</th>
<th>3.50</th>
<th>3.53</th>
<th>2.40</th>
<th>2.26</th>
<th>2.35</th>
<th>36.98</th>
<th>39.44</th>
<th>38.01</th>
<th>1.55</th>
<th>1.40</th>
<th>1.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.15</td>
<td>3.09</td>
<td>3.07</td>
<td>1.91</td>
<td>1.82</td>
<td>1.85</td>
<td>-3.38</td>
<td>-3.76</td>
<td>-2.20</td>
<td>1.09</td>
<td>0.96</td>
<td>1.03</td>
</tr>
<tr>
<td>4</td>
<td>3.08</td>
<td>3.05</td>
<td>3.03</td>
<td>1.95</td>
<td>1.83</td>
<td>1.92</td>
<td>15.23</td>
<td>20.61</td>
<td>14.60</td>
<td>1.19</td>
<td>1.03</td>
<td>1.16</td>
</tr>
<tr>
<td>5</td>
<td>2.97</td>
<td>3.01</td>
<td>2.92</td>
<td>1.86</td>
<td>1.79</td>
<td>1.82</td>
<td>11.32</td>
<td>19.88</td>
<td>11.74</td>
<td>1.13</td>
<td>0.99</td>
<td>1.10</td>
</tr>
<tr>
<td>6</td>
<td>3.15</td>
<td>3.19</td>
<td>3.09</td>
<td>1.98</td>
<td>1.90</td>
<td>1.94</td>
<td>19.60</td>
<td>27.01</td>
<td>20.39</td>
<td>1.20</td>
<td>1.07</td>
<td>1.15</td>
</tr>
<tr>
<td>7</td>
<td><strong>2.93</strong></td>
<td><strong>2.93</strong></td>
<td><strong>2.87</strong></td>
<td>1.90</td>
<td>1.85</td>
<td>1.86</td>
<td>26.48</td>
<td>34.73</td>
<td>27.20</td>
<td>1.24</td>
<td>1.14</td>
<td>1.19</td>
</tr>
<tr>
<td>8</td>
<td>3.00</td>
<td>3.05</td>
<td>2.99</td>
<td><strong>1.84</strong></td>
<td><strong>1.78</strong></td>
<td><strong>1.81</strong></td>
<td><strong>5.30</strong></td>
<td><strong>4.18</strong></td>
<td><strong>-2.00</strong></td>
<td><strong>1.08</strong></td>
<td><strong>0.94</strong></td>
<td><strong>1.02</strong></td>
</tr>
</tbody>
</table>

* method: 1- CSI, 2- NPR, 3- Error variance, 4- Error covariance, 5- Error rotation, 6- PDF overlap (Gaussian), 7- PDF overlap (non-Gaussian), 8 - Probability aggregation
aggregation method and the PDF overlap (non-Gaussian) method is better in medium and high rainfall rates respectively.

**Table 5-3:** Temporal (T), spatial (S) and spatio-temporal (ST) validation MAE (mm/h) of different estimation methods with respect to rainfall rate thresholds, i.e. (low < 2 mm/h, Medium: 2–5 mm/h, High: 5–20 mm/h). Values in bold represent the best result in each category with the second best shown in italics.

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>S</td>
<td>ST</td>
</tr>
<tr>
<td>1</td>
<td>0.96</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>1.42</td>
<td>1.29</td>
<td>1.41</td>
</tr>
<tr>
<td>3</td>
<td>0.76</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>4</td>
<td>0.99</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>6</td>
<td>1.01</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>1.14</td>
<td>1.12</td>
<td>1.11</td>
</tr>
<tr>
<td>8</td>
<td>0.84</td>
<td>0.74</td>
<td>0.73</td>
</tr>
</tbody>
</table>

method: 1- CSI, 2- NPR, 3- Error variance, 4- Error covariance, 5- Error rotation, 6- PDF overlap (Gaussian), 7- PDF overlap (non-Gaussian), 8 - Probability aggregation

The spatial structure of PBIAS is also studied, which is presented in Figure 5-5. The non-Gaussian PDF overlap (Figure 5-5d) and error rotation method (Figure 5-5b) produce higher positive PBIAS while the error variance (Figure 5-5a) and probability aggregation (Figure 5-5c) methods produce smaller PBIAS. The error variance based combination method provides smaller negative PBIAS, whereas probability aggregation provides positive PBIAS at most locations. The process of forcing a zero-correlation influences the covariance matrix in the error rotation method and results in higher positive PBIAS (Figure 5-5b) compared to the error variance case (Figure 5-5a). The negative PBIAS indicates lower rainfall rates that may underestimate river discharge and thereby flash flood peaks.
Figure 5-5: Percent Bias (PBIAS) of the following estimation methods: a) Error variance b) Error rotation c) Probability aggregation and d) PDF overlap (Non-Gaussian). Blue circles indicate positive bias and red circles indicate negative bias.

The NPR method gives longer PDF tails compared to that of the CSI method. This has a significant effect on the PBIAS in the non-Gaussian PDF method, which is the highest among all other methods. Synthetic tests confirmed that if one of the PDF has a thick tailed distribution, the overlap will be very long and moments get biased towards the longer one. The PDF overlap method is designed to generate the maximum overlapping area. Therefore, the area optimization process moves towards the wider PDF and produces higher PBIAS.
Figure 5-6: a) Histogram of gauge rainfall (in decibel). The log error distribution (in decibel) of different combination methods with respect to rainfall rates b) Error variance c) Error rotation d) Probability aggregation e) PDF overlap (non-Gaussian). Histogram of the logerror distribution (in decibel) for different combination methods f) Error variance g) Error rotation h) Probability aggregation i) PDF overlap (non-Gaussian). Three different rainfall values are denoted by the symbol A, B and C.
To get an accurate rainfall estimates from radar-gauge merging, it is important to know the impact of rainfall magnitude on the selection of the merging method. In Figure 5-6, the error distribution of the different combination methods with respect to rainfall rates are presented. The gauge rainfall and the corresponding errors are presented in units of decibels (Figure 5-6). To illustrate the difference among the methods, three different rainfall values have been picked denoted by the symbol A, B and C. In the error variance method, all three errors lie on the positive axis whereas, for the error rotation method they lie both in the positive and negative axis. The histogram of logerror shows that the errors are less sparse/scattered and skewness is low in probability aggregation and PDF overlap (non-Gaussian) methods. But, PDF overlap (non-Gaussian) method provides smaller mean logerror compared to the probability aggregation method. RMSE in Table 5-2 also comply with the mean logerror.

**Figure 5-7:** Variation of NPR weight for different methods for all the events a) across all gauges and b) across different gauges.
The logerror is negative in the error variance method which is associated with underestimation of rainfall. In contrast, positive logerror in the PDF overlap (non-Gaussian) method gives overestimation of rainfall. The spatial distribution of PBIAS (Figure 5-6) also indicates the same outcome. A logerror equal to zero implies that the estimates are unbiased.

Results presented above show that the performance of merged estimates varies among the methods. The merged estimates depend on the weights given by CSI and NPR estimates. Therefore, the variation of NPR weight (Figure 5-7) have been investigated to understand the variation in merged estimates in different methods. The NPR weight in the Gaussian PDF overlap method has a narrower band whereas the non-Gaussian PDF overlap method shows a wider range of variability. Table 5-2 shows that prior to merging, PBIAS for NPR method is around 37% and for CSI method it is 2%. Therefore, the allocation of high weight to the NPR method resulted higher PBIAS in non-Gaussian PDF overlap. The NPR weight in the Gaussian PDF overlap method is around 0.50 (Figure 5-7), and therefore, this method does not differentiate between radar and gauge rainfall. The error covariance method gives higher combination weights with a wider range of variability to the NPR method compared to the error variance and error rotation methods. Although the NPR weight in the error rotation method is similar to that of the error variance method, the later shows more variability. It is important to note that estimation errors are uncorrelated for error variance, correlated for error covariance, and de-correlated, for the error rotation method. Therefore, the correlated errors give a wider range of variability compared to uncorrelated and de-correlated case. These observations have been summarised in the Table 5-4 below.
### Table 5-4: Advantages and disadvantages of different merging approach.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages/Disadvantages</th>
<th>Key Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error variance</td>
<td>• simple and easy</td>
<td>• smaller RMSE for low rainfall rate</td>
</tr>
<tr>
<td></td>
<td>• dynamic weight</td>
<td>• smaller negative PBIAS on spatial domain</td>
</tr>
<tr>
<td></td>
<td>• do not consider correlations</td>
<td></td>
</tr>
<tr>
<td>Error covariance</td>
<td>• consider correlations</td>
<td>• gives a wider range of variability in combination weights</td>
</tr>
<tr>
<td></td>
<td>• dynamic weight</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• stable estimation of the covariance matrix depends on data length</td>
<td></td>
</tr>
<tr>
<td>Error rotation</td>
<td>• consider correlations</td>
<td>• larger variability in logerror distribution</td>
</tr>
<tr>
<td></td>
<td>• dynamic weight</td>
<td>• higher PBIAS on spatial domain</td>
</tr>
<tr>
<td></td>
<td>• stable estimation of the covariance matrix depends on data length</td>
<td></td>
</tr>
<tr>
<td>PDF overlap (Gaussian)</td>
<td>• dynamic weight</td>
<td>• higher RMSE</td>
</tr>
<tr>
<td></td>
<td>• iterative weight estimation process</td>
<td>• higher PBIAS</td>
</tr>
<tr>
<td>PDF overlap (non-Gaussian)</td>
<td>• dynamic weight</td>
<td>• Smaller RMSE</td>
</tr>
<tr>
<td></td>
<td>• iterative weight estimation process</td>
<td>• Smaller MAE for high rainfall rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• higher PBIAS on spatial domain</td>
</tr>
<tr>
<td>Probability aggregation</td>
<td>• static weight</td>
<td>• Smaller MAE, PBIAS, MedianAE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Smaller RMSE for medium rainfall rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• smaller positive PBIAS on spatial domain</td>
</tr>
</tbody>
</table>

### 5.5 Discussion and Conclusion

In this chapter, several radar and gauge rainfall-merging approaches have been compared. The performance of these methods has been evaluated through several attributes of interest in assessing spatial rainfall fields. Depending on the attribute of
interest, different advantages and disadvantages can be noted for each of the merging methods evaluated. It is emphasised that the conclusions drawn from the above results are based on the use of a fairly long dataset in a validation setting. Hence, it is expected that the features of the above methods to be replicated in similar applications elsewhere.

The main conclusions drawn from the results presented above are:

1. PDF overlap based methods versus non-PDF overlap methods: While the PDF overlap methods are elegant and follow our intuitive expectation of the method to trust, they are difficult to implement and can result in fitting or sampling uncertainty especially when data is limited. The overlap methods result in the highest bias across all the approaches considered, while attaining the best “fit” in terms of RMSE. Significant advantages are observed especially when focusing on the high rainfall. The improved RMSE obtained with these methods may be due to the better fit in the upper tails of the distribution.

2. Non-dynamic (static) versus dynamic merging methods: The only non-dynamic method considered in our study is the probability aggregation method, which has one of the better performances across all the methods considered. The downside of this method is the complexity in its implementation, with the need to estimate weights as exponents using an optimisation approach. Furthermore, there is an intuitive disadvantage here as this method is sub-optimal using calibration data simply because the weights do not vary with time. However, given the superior performance of the method, we feel there is merit in investigating alterations that use the product weighting approach while allowing the weights to vary with time or the estimated rainfall.

3. Covariance versus non-covariance based methods: The covariance based methods assessed in our study have a clear advantage over the non-covariance based approaches. This is not a new finding and has been reported by others in different contexts (Khan et al., 2014; Timmermann, 2006). However, the percentage bias estimated is lower for the simpler non-covariance based approach. It should be mentioned that the order of the performance across the
various methods was found to remain the same when the calibration and the validation datasets were inter-changed. Hence, while the above conclusion may be a result of sampling uncertainty, it is more likely to be an outcome of the simplicity of the method being adopted.

4. Covariance versus rotation based approach: An interesting outcome of our study was the clear advantage of the rotation based approach over the one that used the covariance matrix alone. It should be noted that both methods require the use of the error covariances, while a further correction is involved in the rotation method. Clear improvements in all statistical attributes were visible, as were improvements when the assessment was done over the various rainfall thresholds.

In conclusion, each of the method evaluated for merging gauge based observations with radar derived rainfall have their own advantages and disadvantages. The error variance method is computationally inexpensive, straightforward and simple to implement. Calculation errors are much less likely to be made in the implementation of the error variance method compared to all other methods as is based on a simple mathematical calculation. In contrast, the error covariance and error rotation method use more complex algorithms, while resulting in better results. The choice of which merging method to use is a function of the stability with which the parameters of the approach can be estimated. It is recommended to use more complex covariance based approaches (including the rotation method) when sufficient number of events and gauge observations are available. When this is not the case, the simpler alternatives (error variance and probability aggregation) should be adopted.
6 Synthesis

Accurate quantitative rainfall estimation is vital for obtaining reliable output from hydrological simulation and to better understand critical hydrologic processes. This thesis investigates techniques to minimize the uncertainty in radar rainfall estimates and then examines various techniques of combining radar and gauge rainfall. Though summaries were provided at the end of each chapter, the main findings of the thesis are presented here. Future research directions and limitations of this research are also provided at the end of this Chapter. The following sections summarise the outcomes against the objectives defined in Chapter 1 of the thesis.

6.1 Thesis overview and conclusions

6.1.1 Radar - gauge relationship

Uncertainty in radar rainfall estimates is a major concern that has hampered its suitability for hydrological applications. The errors in the radar rainfall estimates mainly originate from the uncertainty in the Z-R relationship parameters that convert measured reflectivity aloft into ground rainfall. In the Z-R relationship calibration process, gauge measurement is assumed to be ‘ground reference’. The uncertainty in the gauge measurement varies depending on the number of rain gauges that fall within each radar grid cell. The errors in gauge measurement can bias the Z-R relationship parameters that eventually affect the accuracy of the radar rainfall estimates. In Chapter 2, an error model is developed to compute point gauge rainfall uncertainty at radar grid resolution. The problem of sparse gauge network is circumvented in the error model by using the large spatial aggregation of the data over different subgrid sizes. This error model has two components, the error in the gauge measurement itself, and the error introduced by the gauge not capturing the spatial variability within a radar pixel. The extent of bias present in the Z-R relationship parameter is determined by using the Simulation Extrapolation (SIMEX) method. The result obtained for the point gauge rainfall uncertainty, presented in Chapter 2, demonstrates a decrease in the average radar rainfall estimates. This finding suggests that gauge uncertainty affects the radar Z-R relationship parameter, consequently, radar rainfall estimates. Although the changes are small, this consistent bias in radar rainfall estimates can be corrected using the SIMEX method, which can also be used to define different error models for other radar networks.
In addition to the spatial variability of gauge rainfall, radar temporal resolution is another factor that affects the accuracy of radar rainfall estimates. Chapter 3 presents the uncertainty in radar rainfall estimates related to the inappropriate temporal resolution of reflectivity in the radar Z-R relationship. Radars and rain gauges provide rainfall estimates at unequal temporal resolutions. However, the radar reflectivities are assumed to remain constant over the gauge accumulation period while calibrating the reflectivity-rain rate relationship, though, the selection of gauge accumulation period is arbitrary. Chapter 3 investigates the impact of temporal resolution of radar reflectivity on the Z-R relationship and how radar rainfall estimates differ with the temporal discretization of radar reflectivity. The quantile matching technique has been used to calibrate the relationship between reflectivities to rainfall rate on an event basis using the co-located gauge and radar data. The reflectivity and corresponding gauge data from ten radars across Australia were used in the analysis. For the fixed gauge temporal resolution, three Z-R relationships were calibrated depending on radar temporal resolutions, for example, the averaged reflectivity over the gauge accumulation period, constant reflectivity over the radar temporal resolution and interpolated reflectivity over the gauge accumulation period. The Z-R relationship resulting from constant radar reflectivity over the radar temporal resolution gives most accurate radar rainfall estimates that are consistent across all the radars.

Chapter 4 presents a nonparametric method (NPR) for rainfall estimation from radar reflectivity with greater efficiency compared to using a traditional parametric Z-R relationship. The advantage of the traditional parametric relationship \(Z=AR^b\) is that only two parameters \((A \text{ and } b)\) need to be estimated. In parametric estimation, every data point influences the inference of the parameters \(A\) and \(b\). The disadvantage of a parametric approach is that a single relationship may not account for all the variations in the data if an incorrect function form is used. The uncertainty of the parametric Z-R relationship parameters introduces biases into the rainfall estimates. A nonparametric method is developed to avoid the limitations and to model the full complexity of the Z-R relationship. This method makes the best use of the available records of radar reflectivity and ground rainfall. The idea in the nonparametric approach is that the data points closest to the target point will lead to better regression estimates. As with any statistical approach,
the sample reflectivity and rainfall values need to properly represent the true population, which often requires large amounts of data. When there is insufficient data, such as in the case of the highest rainfall rates, parametric approaches can extrapolate and may perform better. In Chapter 4, the NPR method is shown to produce more reliable rainfall estimates and to reduce errors by approximately 10% compared to a traditional Z-R relationship, with improvements in the rainfall estimates at around 90% of the locations. Also, the NPR results in smaller errors compared to the spatially interpolated gauge rainfall, particularly in the sparse region.

6.1.2 Radar - gauge combination

Uncertainty in the spatial interpolation of gauge rainfall is a major concern, especially when using sparsely distributed gauge network. In Chapter 4, approaches are developed to merge radar and gauge data by exploiting the strength and minimizing the weaknesses of each of the methods and thus, improving the depiction of the rainfall field by taking into consideration the dependence between both estimates. It is commonly thought that the most useful information from the radar is the spatial pattern of the storm events rather than the magnitude of the rainfall. On the contrary, gauge measurements represent the small scale. This thesis presents a method for combining the spatially interpolated gauge rainfall with the radar rainfall without making assumptions about the relationship between radar reflectivity and gauge rainfall. Rather, the emphasis is given on quantifying the errors in the rainfall field resulting from gauge and radar. Then these errors are combined efficiently using a dynamic weighting method without discarding the intensity information from the radar.

The dynamic weighting based merging method is a new contribution to the field. Dynamic weighting requires estimates of the error in each of the models at every location and every time step. In a traditional radar-rainfall relationship, this error is not explicitly quantified. This thesis proposes an improved radar-rainfall relationship that calculates the uncertainties for different rainfall intensities. The two sources of information are combined using a dynamic combinational algorithm with weights that vary in both space and time. The weight for any specific event is calculated based on an error covariance matrix. The error covariance matrix is formulated from errors in radar and spatially interpolated rainfall errors of similar reflectivity events in a cross-validation setting. The
advantage of the proposed combination is that it can be used with existing rain gauge and radar networks to estimate rainfall at ungauged locations.

Compared to the radar or gauge estimates, the proposed combination method provides better rainfall estimates that depend on the covariance matrix. The rainfall estimate from the combination method is about 20% more accurate than the traditional parametric Z-R relationship method and has the lowest error over the entire study domain. Although spatial interpolation can provide good rainfall estimates over densely gauged areas, combination approaches provide more accurate estimates in sparsely gauged areas or when storm events have significant spatial heterogeneity. Under these conditions, the dynamic combination of radar and gauge data is recommended. Thus, the combination method is not only useful in reducing errors in the spatial distribution of rainfall estimates, but also in diagnosing the strengths and weaknesses of a particular gauge and radar network.

This thesis further examines the implication of different combination approaches, such as error independence (correlated or independent errors), error distributions (Gaussian or non-Gaussian error) and weighting (static vs dynamic) on the spatial distribution rainfall field. The outcomes reported in Chapter 5 can be used as guidelines to select the optimal merging algorithm in a broad range of practical settings.

Each of the methods evaluated has advantages and disadvantages. The error variance method is straightforward, simple to implement and computationally inexpensive. Compared to all other methods, the error variance method is based on a simple mathematical calculation. Therefore, errors are much less likely to be made in the implementation. In contrast, the error covariance and the error rotation method give better results by using more complex algorithms. The choice of which merging method to use is a function of the stability with which the parameters of the approach can be estimated. The use of the more complex covariance based approaches (including the rotation method) is recommended when a sufficient number of events and gauge observations are available. Otherwise, the simpler alternatives (error variance and probability aggregation) should be adopted.
6.2 Research limitations and future research

Limitations of the research have been discussed in each chapter, and this section summarises the main limitations identified and suggests ways that future research may overcome them.

The rain gauge data used in this study has the temporal resolution of thirty minutes, whereas radar has the temporal resolution of six minutes. Therefore, the radar-gauge relationship parameter suggested can be further improved by testing the methodologies with higher temporal resolution of radar and gauge data pairs.

This study ignores the error in the spatiotemporal sampling and variability in vertical reflectivity profile (VPR). Although rainfall can move substantial lateral distance or evaporate before reaching the ground, this research assumes reflectivity measured aloft corresponds to gauge measurement at the ground. These problems can be avoided by using the three-dimensional reflectivity data instead of CAPPI data.

In chapter 2, the Simulation Extrapolation (SIMEX) method is used to estimate the extent of bias in the Z-R relationship parameter. In fact, some of the parameters of the SIMEX method, such as the incremental steps of the variance inflation factor, the number of simulations, extrapolation methods and the confidence intervals, require further investigation.

This thesis has only focused on the accuracy of overall rainfall estimates. Special consideration was not given to the extreme rainfall events.

The implementation of methodologies presented in this thesis in another region will require pre-determination of location-specific factors. This thesis has not studied the regional and seasonal variation rainfall estimates.

There remains significant scope for further improvement of spatial and temporal distribution of rainfall estimates using radar and gauge measurements which was outside the scope of the thesis:
• Examine the impact of geographical locations on the variation of multiplicative bias in a radar Z-R relationship.
• Investigate the bias in a radar Z-R relationship for estimating extreme rainfall events.
• Investigate the method of combining the multiple weather radar rainfall estimates.
• Extend the radar and gauge combination method to other sources of rainfall information, in particular, satellite and the numerical weather prediction.
• Examine the optimal resolution of rainfall estimates required for hydrological simulation.

6.3 Summary

As the outcome of the research presented in this thesis, four journal papers, four conference posters/oral presentations have been prepared. The main conclusions of the research presented in this thesis can be summarised as follows:

In summary, this research has produced:

• An approach for correcting the bias in the Z-R relationship evolved from the uncertainty in point rain gauge networks.
• An approach to reduce the uncertainty in radar rainfall estimates originated from the temporal resolution of reflectivity-rainfall pairs.
• An approach for estimating rainfall from radar reflectivity using the nonparametric relationship between reflectivity and gauge measurement.
• An approach to estimate rainfall at the ungauged location by merging the radar and gauge estimates.
• An approach to merge radar and gauge estimates using the weights that vary in space and time.
• A comparative study on different radar and gauge merging methods.
References


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