Cognitive load theory and mathematics education

Author:
Awawdeh, Majeda

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Cognitive Load Theory and Mathematics Education

Majeda Awawdeh

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Master of Mathematics Education

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Faculty of Arts and Social Sciences
University of New South Wales
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Declaration

ORIGINALITY STATEMENT

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where due acknowledgement is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation, and linguistic expression is acknowledged.

Signed ……………………………………………..............

Date ……………………………………………..............

Acknowledgement

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This thesis was completed with devotion, courage, and passion. It is dedicated to all women who long to fly.

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Abstract

Cognitive load theory uses the immense size of human long-term memory and the significantly limited capacity of working memory to design instructional methods. Five basic principles: information store principle, borrowing and reorganizing principle, randomness as genesis principle, narrow limits of change principle, and environmental linking and organizing principle explain the cognitive basics of this theory.

The theory differentiates between three major types of cognitive load: extraneous load that is caused by instructional strategies, intrinsic cognitive load that results from a high element interactivity material and germane load that is concerned with activities leading to learning. Instructional methods designed in accordance with cognitive load theory rely heavily on the borrowing and reorganizing principle, rather than on the randomness as genesis principle to reduce the imposed cognitive load. As learning fractions incorporates high element interactivity, a high intrinsic cognitive load is imposed. Therefore, learning fractions was studied in the experiments of this thesis.

Knowledge held in long-term memory can be used to reduce working memory load via the environmental linking and organizing principle. It can be suggested that if fractions are presented using familiar objects, many of the interacting elements that constitute a fraction might be embedded in stored knowledge and so can be treated as a single element by working memory. Thus, familiar context can be used to reduce cognitive load and so facilitate learning.

In a series of randomized, controlled experiments, evidence was found to argue for a contextual effect. The first three experiments of this thesis were designed to test the main hypothesis that presenting students with worked examples concerning fractions would enhance learning if a real-life context was used rather than a geometric context. This hypothesis was tested using both a visual and a word-based format and was supported by the results.

The last two experiments were intended to test the context effect using either worked examples or problem solving. The results supported the validity of the previous hypothesis using both instructional methods.

Overall, the thesis sheds some light on the advantages of using familiar objects when mastering complex concepts in mathematics.

Introduction

Teaching the concept of fractions has been of interest to many researchers in mathematics education owing to its complexity (Hunting, Davis, & Bigelow, 1991; Kieren, 1980; 1993; Mack, 1990; 1993; 2001; Nesher, 1989; Steffe & Olive, 1993; Streefland, 1991; 1993). This thesis tested the general hypothesis that learning fractions should be facilitated using realistic, familiar materials. This hypothesis was tested using worked examples as an instructional method. Since learning concepts in an abstract form should be the ultimate desired outcome, it was further hypothesized that to achieve such an abstraction, a real-life context should be followed by a more abstract, geometric context. Lastly, it was hypothesized that the effect of real-life contexts should also apply to problem-solving as opposed to using worked examples.

Five experiments tested these hypotheses within a cognitive load theory perspective. The first chapter of this thesis outlines human cognitive architecture and its main structural elements; long-term and working memory with a review of several suggested memory models. The second chapter delineates the main principles of cognitive load theory, the cognitive load categories, and the interaction between them. Methods suggested by cognitive load theory to reduce cognitive load are also described in this chapter. In the third chapter the complexity of the fraction concept is discussed in general and from a cognitive load perspective. Chapter 4 reviews research into the effect of context on enhancing learning, identifies a need for further research, and provides a rationale for the study. Chapters 5 to 9 describe the five experiments in this study.

The results provide evidence to support the hypothesis of a contextual effect when learning fractions. The final chapter discusses these results from a cognitive load theory approach and indicates the educational implications of the findings along with suggestions for further research.

Chapter 1: Cognitive architecture

1.1 Introduction

The term ‘cognitive architecture’ has been defined (Sweller, 2003) as ‘the manner in which cognitive structures are organized’ (p. 219). Anderson (1996) described human cognitive architecture as ‘an abstract, hierarchical structure which characterizes humans apart from other species’ (p. 357). Stillings and his colleagues (Stillings et al., 1995) referred to this term as the ‘information processing capacities and mechanisms of a system that is an integral part of the structure’ (p.16). A few years earlier, Simon (1989) claimed that human cognitive architecture consists of basic design descriptions of the human information processing system.

Researchers consider that human cognitive characteristics play a significant part in understanding and predicting the range and nature of tasks that humans are capable of doing (Stillings et al., 1995; Sweller, 2003; 2004). Examining cognitive theories shows that human cognition is considered to be a unique system (Sweller, 1994; 2003; 2004; Sweller & Sweller, 2006). From Stillings’ quantitative perspective (1995), humans have the largest brain of all mammals, taking body weight into account. This trait is not the sole unique characteristic of the human brain: more important is its high degree of flexibility that enables a large range of capabilities.

Many cognitive theories consider that evolution provides the design engine of human cognitive architecture (Laird, Newell, & Rosenbloom, 1987; Sweller, 2003; 2004; Sweller & Sweller, 2006). It is understood that evolution by natural selection selects and chooses systems that help in the survival of a species, including human beings. Sweller (2003; 2004) postulated that similar information structures underlie both evolution by natural selection and human cognitive architecture.

1.2 Information structures

From an evolutionary point of view, knowledge can be classified into two categories; biologically primary and biologically secondary knowledge (Geary, 2002; 2005; 2007). Generally, all tasks that human beings have evolved to learn by natural processes are considered primary knowledge. For example, acquiring the ability to speak and understand one's mother tongue or the ability of a baby to crawl and then walk. Speaking or understanding a second language is considered as secondary knowledge. Moreover, writing and reading one's first language is also considered as secondary knowledge. In short, Sweller and Sweller (2006) claimed that skills that we need to learn in schools or other institutions can be classified as secondary knowledge, whilst skills that we have evolved to do naturally are categorized as primary. He speculated therefore, that the process of obtaining secondary knowledge is totally different from the process of acquiring primary knowledge and thus, needs different mechanisms. Sweller and Sweller (2006) suggested that since human beings have not evolved to acquire secondary knowledge, they must have a general mechanism to handle this type of knowledge. Acquiring primary knowledge must have a unique system for each particular type of this knowledge. For example, a different mechanism is used for speaking than for listening. In other words, they understood that learners use the same mechanism to acquire all categories of secondary knowledge. However, learners' information processing mechanisms consist of several processes and cognitive capacities that might vary because of individual differences (for a broader review see chapter 2).

From a different perspective, Sweller (1994) proposed that all information can be classified according to the degree to which its elements interact; at one extreme, that information can have no interaction or low interaction among its elements. This category includes learning simple facts such as the names of people or objects or learning to count up to ten. At the other extreme, information could have high element interactivity such as learning complex mathematics equations or learning the grammar of a second language, or any kind of learning that can be called 'learning with understanding'. This kind of acquisition requires all elements that interact to be processed concurrently, a situation that might result in a high cognitive load (Sweller, 2003).

Additionally, knowledge has been classified by cognitive science researchers into two types; declarative knowledge and procedural knowledge. Declarative knowledge is known also as 'knowing–that' (Brien & Eastmond, 1994). It is used to represent objects or events. It was defined by Anderson (1983; 1993) as all facts that one can know or the factual knowledge that people can describe. Procedural knowledge was identified as 'knowing- how' knowledge. It is used to represent operations and activations of declarative knowledge (Brien & Eastmond, 1994). In the same paper, Brien and Eastmond claimed that the distinction between these two types of knowledge is not always clear since they can sometimes overlap. In addition, it was suggested that one can acquire knowledge that is procedural and declarative at the same time. Anderson (1983) argued that knowledge could be obtained first in a declarative form, and transformed later into a procedural one.

The features of those mechanisms required for dealing with information structures is discussed next.

1.3 Memory

1.3.1 Introduction

Hermann Ebbinghaus was the first person to study human memory experimentally (Baddeley, 1982). Assuming that the meaning of a word helps to remember it, he created ‘nonsense’ syllables to eliminate the effect of meaning. Basically, by using ‘nonsense’ syllables, he intended to neutralize the meaning factor. One of Ebbinghaus’ important discoveries was the limit of memory which he called ‘memory span’ (Klatzky, 1975). Other investigators suggested various models for the human memory system.

1.3.2 Memory models

1.3.2.1 Memory model of Waugh-Norman and Norman

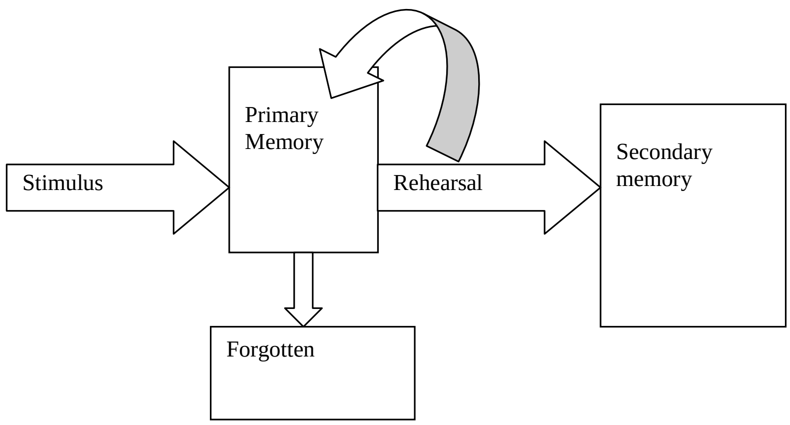

Figure 1: Waugh and Norman’s Model (1965, p. 93)

Waugh and Norman, D.A.(1965) suggested a memory model (see Figure 1) in which they distinguished between two kinds of memories; primary memory and secondary memory. This model suggested that a stimulus that enters primary memory might be lost quickly unless it is rehearsed. Rehearsal can be either observable or hidden. When an item enters primary memory and is rehearsed it remains in primary memory and may enter secondary memory. Once it is in secondary memory, there is no need further for rehearsal. Secondary memory is considered as a more permanent store than primary memory. In addition, primary memory capacity is assumed to be limited (Houston, 1981).

1.3.2.2 Atkinson and Shiffrin's model

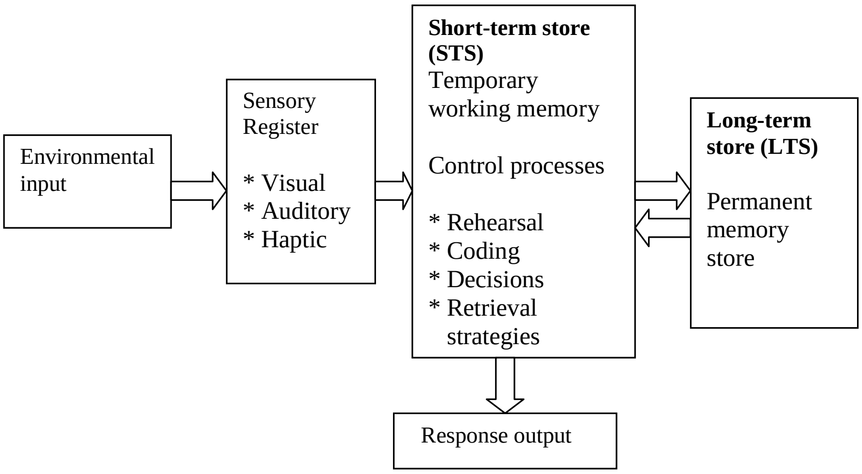

Figure 2: Atkinson and Shiffrin's Model (1971, p. 2)

Atkinson and Shiffrin (1968) suggested a similar model of memory (see Figure 2), but it is more complex than both the Norman and the Waugh-Norman models. Specifically, it adds a third component; the sensory register. The concept of

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short-term storage and long-term storage was introduced by Atkinson and Shiffrin. Their model consisted of three components: sensory register, short-term storage and long-term storage.

Sensory-register: in this model Atkinson and Shiffrin (1968) proposed that information passes from the environment through a number of transitory sensory memories before reaching the short-term store. It can be thought of as a pathway for the short-term store.

Short-term storage: is considered a temporary storage area in which information remains for a period of time that is usually under the control of the subject. The information can be kept in the short-term storage by rehearsal. However, the short-term storage capacity was thought to be limited (Atkinson & Shiffrin, 1971). The short-term store is defined in this model as 'a system in which decisions are made, problems are solved and information flow is directed' (p. 3).

Long-term storage: is regarded as a long-term or permanent memory store. Information is transferred from the short-term storage to the long-term storage by rehearsal. Atkinson and Shiffrin (1971) stated that the longer an item has been maintained in the short-term store by rehearsal, the more likely it is to be transformed to the long-term store. The long-term store is assumed to be unlimited both in its capacity and its duration. In other words, it can hold a massive amount of information that can last for a long period.

The flow of information through the memory system, as conceptualized by Atkinson and Shiffrin is shown in Figure 3.

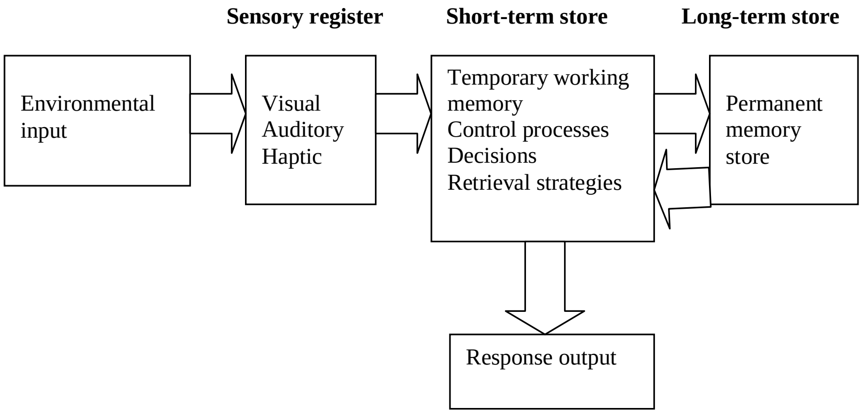

Figure 3: Flow of information through the memory system, as conceptualized by Atkinson and Shiffrin.

1.3.3 Long-Term Memory

Although long-term memory is considered a fundamental component in all memory models, its vitality in controlling human beings' cognitive activities was disregarded until the noticeable phenomenon that characterizes experts in chess was first discovered by De Groot (1965) and Chase and Simon (1973). This study of a chess master's ability to remember chess board arrangements provided sound evidence for long-term memory's crucial role. De Groot (1965) claimed that memory capacity is the factor which makes the difference between experts and beginners in chess.

With respect to most memory theories, two basic presumptions regarding long-term memory can be found: firstly, it is assumed that this storage is unlimited in its capacity (Baddeley, 1986; Newell & Simon, 1972), secondly, it is believed that items stored in long-term memory can last permanently (Cowan, 1988) and are ready to be retrieved once a stimulus is provided. Baddeley (1982) claimed that information in long-term memory never disappears, but becomes less accessible.

Sweller (2003) proposed that human beings are unaware of the content of their long-term memory until information is brought into working memory for processing a task.

1.3.4 Working memory

1.3.4.1 Development of the term Working Memory (historical review)

Working memory in cognitive psychology and neouropsychology refers to a system that has been evolved for the short-term maintenance and management of information needed for complex tasks performance such as learning (Baddeley, 1998). The standard definition of working memory refers to the momentary storage of information that is processed to perform cognitive tasks (Baddeley, 1986).

Working memory is also called short-term memory, primary-memory and immediate-memory (Klatzky, 1975). The term working memory was used by Galanter and Pribram

(1960) in their book *Plans and the Structure of Behavior.* They considered working memory as a central component of cognitive psychology. A comprehensive survey of the historical development of working memory can be found in Logie (1996). He summarized the history of working memory in seven stages, named by him as seven ages.

Age 1- working memory as contemplation: Working memory was referred to as early as the seventeenth century. The philosopher John Locke distinguished between short-term storage named, 'idea in view' and a permanent storage named 'storehouse of ideas'. This distinction is quite analogous to the distinction between short-term and long-term memory used later.

Age 2 – working memory as a primary memory: This expression was used by Waugh and Norman (1965). They assumed a limited capacity for the primary memory and claimed that rehearsal is crucial for maintaining information in primary memory and transferring it from primary to secondary memory, long-term memory.

Age 3- working memory as a short-term memory: Until this stage, working memory was regarded as a passive storage of information rather than as an active processor of information. An innovation was attributed to Atkinson and Shiffrin (1968) who referred to short-term memory as a combination of storage and control processes. They supposed a limited capacity working memory and argued for a flexible system that could function for storage as well as process information.

Age 4- working memory as a processor: Craik and Lockhart (1972) contributed a new meaning for working memory as a product of cognitive processing rather than as a separate entity. In other words, working memory was recognized by them as a process, rather than a fixed or passive part of the human cognitive architecture.

Age 5- working memory as a constraint on language comprehension: This age is considered by Logie (1996) as one of the major, current ages of working memory in research into language learning. This age was determined by Daneman and Carpenter (1980) who developed a task to measure working memory capacity.

Age 6- working memory as activation, attention, and expertise: In this age, working memory was regarded as an independent entity with limited attention, activation and capacity (Cowan, Winkler, Teder, & Näätänen, 1993). However, this limitation could be expanded as a function of expertise. This assumption was the core of a model developed by Ericsson and Kintsch (1995). The central theme of this model is that working memory capacity is larger when acting within one's area of expertise.

Age 7- working memory as multiple components: This view sees working memory as a workplace, rather than an entryway. In this age the most important research was conducted by Baddeley and Hitch (1974) and Baddeley (1990). A model of working memory was created by Baddeley and Hitch (1974). It contains 3 components which will be discussed later.

Working memory attracted many researchers in the last few decades. An important distinction between the two functions of working memory, firstly as a passive storage of information and secondly as an active processor of information, was made by many researchers. Logie (1996) made a clear distinction between working memory as a workspace - here working memory is considered as a set of cognitive functions that can store and process information and, a working memory as a gateway for information passing from sensory input to long term storage.

Atkinson and Shiffrin (1968) assumed that short-term storage could be considered as a 'working-memory' that receives inputs from the sensory register and the long-term store. They speculated that working memory is an essential and intrinsic part of human cognition.

Working memory tasks were defined by Daneman and Carpenter (1980) as tasks that required storage and manipulation of information at the same time. From Baddeley's point of view (1996a; 1998), working memory provides a crucial bridge between perception, attention, memory and action and was considered as a complex system that involves a range of interacting sub-components. Moreover, working memory was seen as a system in which information can be affected and stored in many active forms rather than as a passive store house (Klatzky, 1975).

Researchers suggested that working memory is not a unitary system; they proposed that there are different working memories to deal with different types of information. There are separate systems for verbal and spatial information (Smith & Jonides, 1998; Smith & Milner, 1989). Each system, verbal and spatial has three different functional components:

- A storage component of which the contents decays very quickly.
- A rehearsal component that can reactivate the decaying contents of the storage component.
- An executive component, that controls and directs the processing of the contents of working memory. There are situations that require the storage rehearsal component to be active such as remembering a phone number until dialing it. Other situations need the executive part to be involved such as rehearsal of a mental map of an area while trying to reach it (Smith & Jonides, 1998).

1.3.4.2 Working memory architecture

1.3.4.2.1 Baddeley's model

A working memory architecture was suggested by Baddeley and Hitch (1974), Baddeley (1986; 1992a; 1992b; 1996a; 1998) and was updated by Baddeley and Hitch (2000). It was proposed that working memory consisted of a 'central processor' and two kinds of storage; a visuo-spatial sketchpad and a phonological loop. These two types of storage are quite similar to the 'verbal working memory' and 'spatial working memory' nomination made by Smith and Jonides (1998).

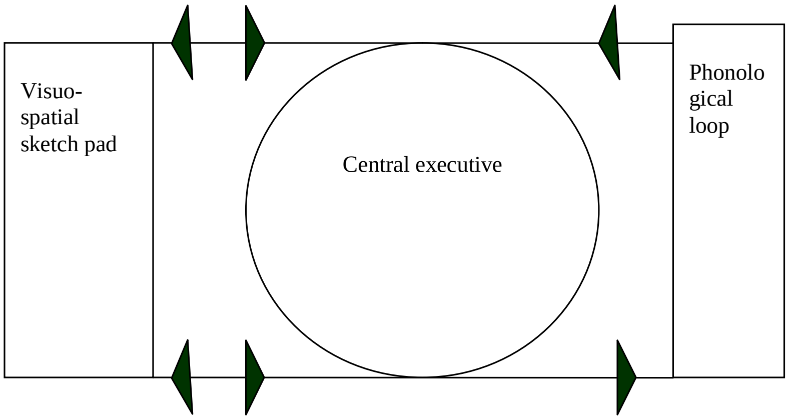

Figure 4: Working memory model by Baddeley (1990, p. 71)

This model consisted of three main components: a visuo-spatial storage named visuo-spatial 'sketch-pad,' a temporary verbal storage labeled 'phonological loop' and a coordinator function termed 'central executive.'

Phonological loop: It is assumed to contain a temporary, passive storage system. Audio-based information enters this passive storage and can be held there. This information decays within a couple of seconds unless rehearsed (Baddeley, 1996a). Baddeley (1998) assumed that the phonological loop plays a significant role both in the development of language in children and in learning a second language.

Kemps and his colleagues (Kemps, De Rammelaere, & Desmet, 2000) asserted that this component contains two subsystems; a passive phonological store and an active mechanism that refreshes memory traces.

Visuo-spatial sketch pad: This component is assumed to be more complicated than the previous one. Visual information enters the visuo-spatial sketch pad but, Baddeley (1996a) claimed, the use of visual imagery is less automatic than phonological information, a situation that results in heavier demands on the central executive. The visuo-spatial sketch pad, claimed Logie and his colleagues (Logie, Zucco, & Baddeley, 1990) is responsible for the temporary maintenance and manipulation of visuo-spatial information. In addition, its resources are different from the phonological loops' resources.

Central executive: It is responsible for the intentional control of working memory and works as a coordinator for all functions of working memory components (Baddeley, 1996a). It is also assumed to be an active component. The central executive is capable of holding both spatial information of an object, for example, its location, and information about its visual appearance such as color, shape or other features (Baddeley, 1998). Baddeley (1998) proposed that the phonological loop and the sketch pad involve the interaction of a number of brain regions. Subsequently, Baddeley (1998) provided evidences for the fractionation of the central executive into a number of subsystems.

Kemps and his colleagues (Kemps et al., 2000) claimed that the central executive is an intentional control system that is limited in its resources. They asserted it was responsible for three functions: firstly, transmission of information from short- term memory to long-term memory; secondly, selection of information to be stored; and finally, coordination of the other components of the working memory system.

Other researchers such as Pascual- Leone and his colleagues (Pascual-Leone & Baillargeon, 1994; Pascual-Leone & Morra, 1991) considered working to be an active subset of long-term memory.

1.3.4.2.2 Working Memory multi-component model of Baddeley

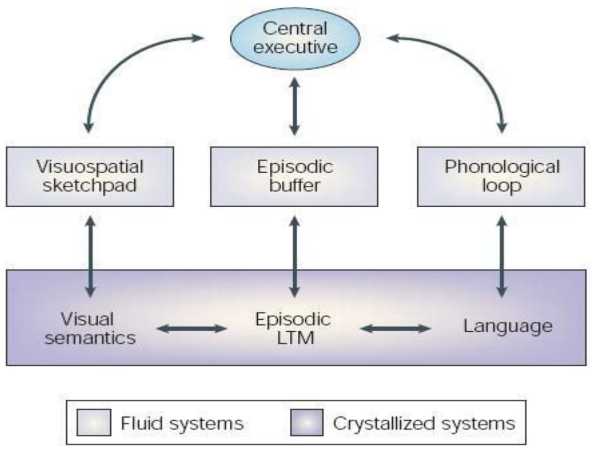

Figure 5: Multi-component Model of Working Memory (Baddeley & Hitch, 2000, p. 418)

Baddeley and Hitch (2000) added a new element to this working memory model, the episodic buffer. This element is assumed to be a temporary memory and limited capacity storage subsystem. Its limited capacity is determined by the number of chunks it can hold.

1.3.4.2.3 Logie's model of working memory

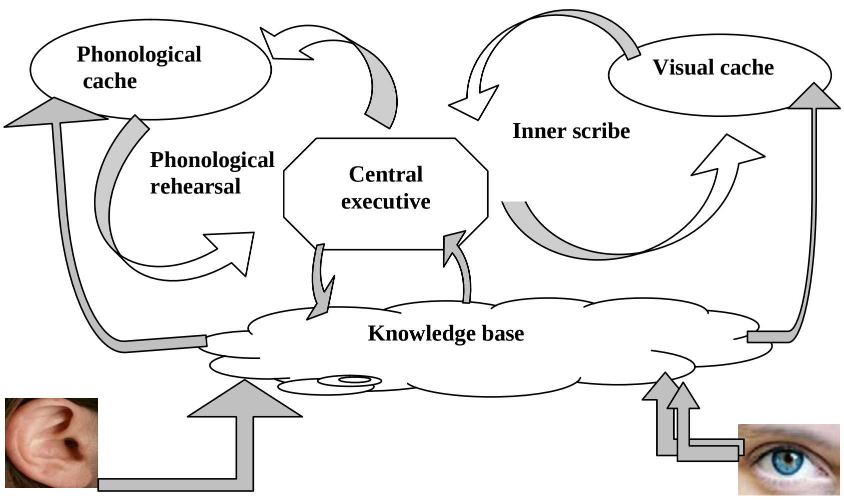

Figure 6: Logie's model of working memory (Logie, 1995, p. 127)

Logie (1995) suggested an additional model for working memory. It contains three components: the phonological loop, the visuo-spatial working memory, and the central-executive. The most important contribution of this model is that it emphasizes the importance of prior knowledge stored in long-term memory. Logie's modification for the working memory model clarifies the utility of previous information held in long term memory to initial processing of new information received by the two secondary subsystems of the working memory system.

Cowan (1988) mentioned the existence of an intentional filter after the sensory storage, which allowed selected information to pass to a higher level of processing. It was defined as 'a mechanism that, once set, can block the processing of some stimuli and allow the subject to further process other stimuli easily' (p. 172).

1.3.4.3 Working memory capacity

Notwithstanding that memory capacity has improved through adaptive processes in human evolution, as proposed by Cowan (2001), its limitation still has a serious effect on human cognition (Stillings et al., 1995).

Hermann Ebbinghaus (1885) was the first to study human memory. One of his important discoveries was that if a list was short enough, seven or less items, it could be learnt in one reading. If the list expands to eight or more items, the learning time increases dramatically. He called the seven item limit 'memory span.' Miller (1956) in his well-known paper 'The Magical Number Seven Plus or Minus Two' suggested that the memory span is between five and nine items. Cowan (2001) claimed that the capacity limit of short term memory is four plus or minus two items. He identified memory capacity as the maximum number of chunks that can be recalled in a particular situation.

According to Baddeley (1986; 1998) a memory span is assumed to be limited to the number of items that can be rehearsed in a repeating loop, before the decay of their stored representations. Formerly, Simon (1974) found that the number of items one can recall immediately after reading or listening was about seven one-syllable words, about seven two-syllable words and about six three-syllable words. In the same context, Baddeley and Hitch (2000) argued that a memory span varies depending on the information to be stored. They found that working memory span for sentences is almost 16 words, whereas working memory span for unrelated words is only approximately six. However, other researchers such as Shiffrin (1973) and Peterson and Peterson (1959) claimed that the duration items last in short term storage is limited and it does not vary according to the type of items.

Not only is the amount of information that working memory holds limited, but also the duration of this information. Working memory allows humans to maintain information in an active state for a brief period of time ranging between 0-60 seconds (Smith & Jonides, 1998; Smith, Jonides, Marshuetz, & Koeppe, 1998). Furthermore, Baddeley (1998) emphasized that the limited duration of an item in working memory was dependent on how long it could remain active without rehearsal in working memory. He hypothesized that working memory span is limited by the rate at which recently stored information loses activation (Baddeley & Hitch, 2000).

Broadbent (1975) concluded that short term memory capacity is limited not only when dealing with new information, but also when recalling stored information from long-term memory, a view contrary to Ericsson and Kintsch (1995).

The limitation of human working memory capacity cannot be ignored; neither can its limited duration. Therefore there should be several ways to bypass these limitations.

The first method to circumvent working memory limits is chunking.

1.4 Chunking

Chunking is the ability to combine several items into larger units such as joining letters to create words. It is a mechanism used to conquer the limitations of working memory allowing the storage of as much possible information at one time. The term ‘chunk’ was used first by Miller (1956). Chase and Simon (1973) in their work on chess. They hypothesized that chess experts perceive a chess board arrangement as one chunk; this ability of chunking and then un-chunking when retrieving information from their memory, enables them to outperform weaker players who may have a fewer number or smaller sized chunks. Miller(1956) claimed that the number of chunks of information is constant for immediate memory. He proposed that the immediate memory span is independent of the size of a chunk. Nonetheless, Cowan (1988) suggested that the claim made by Miller (1956), that subjects can remember at one time seven items or chunks, is meaningless unless a clear definition of a chunk is provided.

Chunks were defined by many researchers and various suppositions concerning the number of elements a chunk can hold were proposed:
Following Miller (1956), Simon (1974) suggested one way to define a chunk is by empirically determining the span for a type of item and dividing it by seven.

Newell (1972) claimed that the smallest units of information held in long-term memory are symbols. These stored symbols act as internal representation for stimulus patterns. He suggested that chunks are patterns of stimuli that become recognizable as particular symbols through learning. Klatzky (1975) claimed that the understanding of chunks is complicated owing to an incongruous definition. On the one hand a chunk is defined as whatever short term memory holds seven of; on the other hand, it is claimed that the span of short term memory is seven chunks. This resulted, he claimed, in an untenable definition 'seven of whatever short term memory holds seven of.' A simpler definition for chunking can be found in Stillings and his colleagues' study (Stillings et al., 1995). They defined chunking as using multiple elements as a single unit in long term memory. In this light, for Stillings, a chunk is a complex element that can be stored as a single unit in long-term memory. For Anderson (1983; 1993) chunks are cognitive units that encode a set of elements in a specific connection. He hypothesized that chunks contain no more than five elements. However, because complex structures can be created by hierarchical structures, one chunk can appear as an element of another chunk, as Anderson proposed.

Recently, Cowan (2001, p. 89) defined a chunk as 'a collection of concepts that have strong association to one another and much weaker associations to other chunks concurrently in use.' In his two papers, Cowan (1995; 2001) pointed to the fact that we tend to arrange telephone number in groups of three or four digits as an indicator of how many elements can be reasonably held and maintained in short term memory, at one time, to allow the formatting of new chunks in long-term memory.

1.5 Schemas

The word schema has its root in Greek language. The Greek word 'σχημα' means 'form' or 'shape' (Marshall, 1995). Piaget (1928) and Barttlet (1932) provided the foundation of modern schema theory. According to Piaget (1952), a schema is a coordinated combination between cognitive functions and physical actions that work together to respond to any perceived experience that could be related to the schema. He speculated that a schema needs three phases to be formed and applied; repetition, recognition and generalization. He assumed that repetition of situations helps to alter schemas. Recognition- individuals with increasing experience are able to classify and distinguish among stimuli; these discriminations become part of the schema itself.

Generalization- with experience, the small details fade away and the schema become more generalized and can be applied to a larger variation of experience (Piaget, 1952).

In light of his remembering theory, Barttlet (1932) hypothesized that a schema holds and organizes previous experiences. Both Piaget and Barttlet emphasised the assumption that a schema is a memory structure that develops from previous experiences and is influenced by personal prior knowledge and interpretations.

Skemp (1971, p. 39) defined a schema as ‘ the general psychological term for a mental structure’. He contemplated two functions for a schema; integration of previously stored knowledge; and operating as a mental mechanism to acquire new knowledge. The first function is essential to organizing existing knowledge in a categorized mode, or more specifically in a schematic way. This function, for example, enables people to bring existing knowledge related to a specific concept, once this concept is encountered again. The second function of a schema acts as a tool for further learning. According to Skemp (1971) almost all new material people learn is based on previously learned knowledge. Therefore, learners’ schemas are structured in a hierarchal system. However, to enable such an effective system to operate efficiently, existing schemas need to be general and adaptable enough to assimilate new schemas appropriately (Skemp, 1971).

Recently, Marshall (1995, p. 39) proposed that ‘a schema is a vehicle of memory, allowing organization of an individual’s similar experiences’. This organization occurs in such a way that an individual can recognize and classify additional experiences (‘identification knowledge’ - using Marshall’s term.) Learners can enter or create a mental model for a problem (‘elaboration knowledge’); here Marshall included verbal and visual information. The third type of knowledge is planning knowledge. Individuals can draw conclusions, estimate, create goals, and develop a plan. This knowledge (labeled as ‘execution knowledge’) claimed Marshall (1995), leads them to utilize the skills they need to implement a proper solution. From this problem solving outline that involves several types of knowledge, suggested by Marshall (1995), the emphasis placed on creating a new schema or having accessibility to an existent schema becomes apparent. Additionally in her definition, there is a significance placed on the role of one’s prior knowledge when establishing or ‘organizing’ schemas. Marshall (1995) in her book *Schemas in Problem Solving* denoted a few features of schemas such as a storage mechanism, a network, levels of connectivity, flexibility, variations in size, and overlap and embedding.

Cognitive load theory relies heavily on schemas to reduce cognitive load. From this perspective, it is assumed that schemas allow many elements to be treated as a single element in working memory; as a result, more working memory capacity becomes free (Sweller, 2003). Schemas can be defined as mental structures that we use to organize knowledge. Besides, schema theory assumes that expertise in any area is a function of the acquisition of particular schemas stored in long-term memory (Sweller, 2003). In fact, it is believed that experts can perform better in their area of expertise due to the fact that they have at their disposal schemas regarding this particular area (van

Merrienboer, Kirschner, & Kester, 2003). Moreover, it is supposed that a learner is considered an expert depending on the number and sophistication of schemas held in the learner's long-term memory and his or her ability to automate stored schemas. It is also assumed that the ability to automate a schema is a critical condition for utilizing schemas and subsequently reducing cognitive load. Newell (1990) and Anderson (1983; 1993; 1996) referred to automated schemas that connect specific conditions to particular actions as 'production rules.'

In short, schema theory specifies that knowledge is stored in long-term memory in a schematic form and a schema enables us to deal with several elements as a single one according to the way it will be used. As a result, less working memory capacity is occupied and more working memory capacity is free (Sweller, 2003). This issue is crucial considering the limited capacity of working memory. Schema theory assumes the more specific schemas are stored in long-term memory, the more skillful one can be in a particular area (Sweller, 2003). In other words, experts can perform better in their area of expertise because they have at their disposal schemas regarding those particular areas (van Merrienboer et al., 2003).

1.6 Automation

Automation was defined by Sweller (2003) as the ability to process information without conscious working memory control. In many of his studies, Anderson (1980; 1983; 1993; 1996; 2005) underlined the significance of automation as an influential factor in human cognitive architecture. Furthermore, he claimed that this process is gradual. It is also supposed that even highly controlled tasks are able to be automated with sufficient practice (Leahey & Harris, 1989). Shiffrin and Schneider (1977) and Schneider and Shiffrin (1977) suggested two basic conditions for an automatic process. Firstly, an automatic process should not occupy working memory capacity and secondly, an automated process once started cannot be stopped.

In their research on isomorphic problems of the Tower of Hanoi puzzle, Kotovsky and his colleagues (Kotovsky, Hayes, & Simon, 1985) stated that 'once automation has occurred, the solution is obtained very rapidly' (p. 284). In fact, they found that subjects were 16 times quicker when solving problems using automated schemas than when solving problems with no available automated schemas.

From a Cognitive load theory point of view, because automation is an unconscious action, it does not require working memory manipulation. Therefore, it reduces the load on working memory which is assumed to be limited (Cooper & Sweller, 1987; Sweller, 2003). As a result, a learner is able to process more information, thus enhancing learning as an ultimate outcome.

In other words, automatic cognitive processes require less working memory resources than non-automated processes and therefore, having an automated schema is very likely to facilitate the learning process. Bruning and his colleagues (Bruning, Schraw, Norby, & Rouning, 2004) claimed that activities such as reading fluently, appropriate placing of fingers by expert typists, and driving while talking with a friend sitting beside you, need fewer resources and less attention as a result of having automated schemas for such activities.

In the domain of algebra, Cooper and Sweller (1987) suggested that automation is essential for effective learning, in particular when solving transfer problems. They suggested that rule automation plays a major role in problem solving expertise. However, they stated that learners need time and effort for both schema acquisition as well as schema automation.

In research on expertise, possessing automated schemas was suggested as differentiating experts from novices by many researchers (Cooper & Sweller, 1987; Larkin, McDermott, Simon, & Simon, 1980b; Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977). Furthermore, Larkin and her colleagues (Larkin et al., 1980b) found that the degree of automation in the process of applying physics principles, was the major factor behind the difference in performance between experts and novices.

Interestingly, Feldon (2007a) in his study into the role that automaticity plays in classroom teaching considered automaticity as a double-edged sword. On the one hand, it is beneficial as it reduces the overall level of cognitive load needed to process multiple, complex classroom interactions. On the other hand, Feldon argued that automaticity may lead teachers to resort to an automated but unintended behavior when experiencing a high level of cognitive load. He also noted that although developing automaticity reduces levels of cognitive load and therefore provides large benefits, there are concerns about the ability to adapt to new or rapidly changing situations if a person acquires automated schemas that result in automated behavior. Another point stated by Feldon (2007a) is the possibility that if a skill becomes automated before an optimal level of effectiveness has been attained, learners will be unable to perform adaptively. In short, Feldon (2007a; 2007b) argued that automated skills are not subject to controlled processes and therefore can be handled by minimal cognitive effort. However, for this reason they are not easily modified.

This point was also made by Skemp (1971). Skemp mentioned multiple advantages of automated schemas and the positive role schemas play in facilitating learning and understanding. However, he also considered a few possible negative effects. The first disadvantage that has to be considered is time. He argued that schematic learning takes a long time particularly in Mathematics. The second drawback, according to Skemp, is that schemas have a highly selective effect on learners' experience. New to-be-learned material can be acquired better if it fits in an existing schema, however by contrast, if it is not adequate for an existing schema, most probably it will not be learned. Therefore, stable schemas can either hamper learning or facilitate learning, since a stable schema is robust to the extent to which it rejects new situations or experiences that do not correspond to the schema. This situation has been referred to as *mental set or Einstellung*. Briefly, this term is explained by Sweller and his colleagues (Cooper & Sweller, 1987; Sweller, 2003; Sweller & Gee, 1987) as a situation when a learner approaches a new problem using previously practiced methods even when an easier and more effective technique can be used. This demonstrates how an automated schema can hinder learning instead of facilitating it.

Concisely, schemas are defined as mental frameworks that humans use to organize knowledge. Chase and Simon (1973) in their research on the superior memory of chess experts, proposed that chess experts store a large number of specific patterns of chess pieces as a chunk in long-term memory. This enables them to match each new position of pieces to a chunk stored in their long-term memory and act accordingly. It could be assumed that a chess expert has schemas that allow him or her to classify chess board positions according to the moves required (Sweller, 2003; 2004). Sweller assumed that schema construction and automation provide humans with a learning mechanism, so learning includes the building of schemas. It is clear that schemas can be strengthened by using them. Van Merrienboer and Ayres (2005) claimed that when schemas are applied they are, practically, strengthened. Automation happens when knowledge is processed unconsciously rather than consciously in working memory (Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977). Sweller, van Merrienboer and Paas (1998) suggested that the automation of basic schemas is essential for the construction of higher level schemas, especially when the higher level schemas are built upon more basic ones.

On the other hand, however, researchers have cautioned that automated schemas can have negative consequences, in particular when they are automated before reaching generalization (Feldon, 2007a). A major drawback that was considered by Skemp (1971) is that despite schemas being used as a key mechanism for acquiring knowledge and assimilating new experiences, they can act to hinder learning if the new knowledge does not fit in an existing schema. Furthermore, schemas can hinder effective learning when the ‘ Einstellung’ effect occurs (Sweller, 2003)

1.7 Summary of Chapter 1

This chapter contained an outline of human cognition architecture. An analogy between human cognition architecture and its evolution, and the way in which this system operates to store new information is used currently by Cognitive load theory (Sweller, 2003; Sweller, 2004). Sweller (2006b) based on Geary (2002; 2005) drew a distinction between two types of information; primary which can be learned spontaneously, and secondary which needs effort and educational institutions to be acquired.

From a cognitive load theory perspective, knowledge can also be classified into two categories according to the degree to which its elements interact; high-element interactivity and low-element interactivity (Sweller, 2003; Sweller & Chandler, 1994).

Human memory consists of two main structures, long-term memory and working memory. Studies in this field resulted in a few suggested models for memory; Waugh and Norman's model (1965) included a primary and secondary memory. Rehearsal was claimed to be the main mechanism that allows the transfer of items to secondary memory. Atkinson and Shiffrin (1968) added a third component to the previous model, the sensory register. Furthermore, the primary and secondary memories were replaced by short-term and long-term stores respectively. Baddley and Hitch (1974) were the first to introduce a model for working memory and subsequently updated it (Baddeley & Hitch, 2000). The initial model consisted of three components; central processor (central executive), visuo-spatial sketch pad and phonological loop. The last two components were considered as storage areas, whereas the central executive was assumed to be an active coordinator of all working memory functions. In the updated model (Baddeley & Hitch, 2000), the main change was adding a fourth element, the episodic buffer, which was assumed to act as a temporary memory which is limited in its capacity. Logie (1995) suggested a model which contained three elements; phonological loop, the visuo-spatial working memory and the central executive. The main contribution of this model is its emphasis on the significance of prior-knowledge.

The key feature of working memory is its limited capacity which is assumed to vary between five and nine chunks as Miller (1956) proposed or between two to six as suggested by Cowan (2001). Its duration was supposed to be limited and it may extend to 60 seconds according to Smith and his colleagues (Smith & Jonides, 1998; Smith et al., 1998). In addition, Baddeley (1998) emphasized the limited duration of working memory and focused on rehearsal as a necessary mechanism to keep an item active. In short, most researchers agreed on the relatively narrow margins of working memory.

There were a few suggested methods to circumvent these limitations such as chunking, and constructing and automating schemas. Chunking combines many elements into a single one. Since a chunk can be handled in working memory as a sole element irrespective of the numbers of individual elements it contains, more elements can be processed simultaneously in working memory. Schemas are central to overcoming the limited capacity of working memory. A schema is defined as a coordinated combination of cognitive functions and physical actions (Piaget, 1952). Schemas also hold and organize previous experiences and can be affected by personal prior-knowledge (Bartlett, 1932). Similarly, Marshall (1995) stated that schemas allow the organization of an individual's experiences. She also emphasized the significant role of personal prior-knowledge when constructing and arranging new schemas. Cognitive load theory considers schemas as a major factor that determines how expert a learner is in a particular domain; the greater the number of schemas that individuals hold in their long-term memory and the more complicated these schemas are, then the more expert they are (Sweller, 2003). However, utilizing schemas depends on having these schemas automated so they can be processed unconsciously in working memory, i.e. with no need to invest cognitive load to activate them. Anderson (1996; 2005) and Sweller (2003; 2004; 2006b) agreed that schemas can be automated as a function of time and effort in practicing. Automation can significantly reduce cognitive load and as a result facilitate learning in terms of time and effort (Kotovsky et al., 1985).

Chapter 2: Cognitive Load Theory

2.1 Introduction

Human cognitive architecture provides a base for cognitive load theory. Relatively recently, that theory has become one of the most influential theories in instructional psychology with applications in various areas of education. The fundamental assumption of this theory is that for instructional methods to be effective, instruction designers need to take human cognitive architecture into account. It also emphasizes the necessity for instructional techniques to be designed in alignment with the basic operational principles of the human cognitive system (Chandler & Sweller, 1991; 1996; Sweller, 1988; 1989; 1993; 1994; 2003; 2004; 2006b; Sweller & Chandler, 1991; 1994; Sweller & Sweller, 2006). In effect, cognitive load theory integrates knowledge regarding human cognitive unnecessary cognitive load to facilitate and enhance learning processes.

2.2 Cognitive Architecture Principles and Cognitive Load Theory

There are five basic principles of human cognitive architecture that underlie cognitive load theory. In this thesis an analysis from a biological evolutionary perspective by Sweller and Sweller (2006) is outlined (see also Sweller, 2003; 2004; 2006b).

2.2.1 Long term memory and the information store principle

As mentioned in chapter one, long-term memory as a long-term storage space of information is one of the main systems of human cognitive architecture. In this light, the information store principle places a heavy role on long-term memory assuming that it provides the function for governing most human cognitive activities. From an evolutionary point of view, Sweller (2006b) and Sweller and Sweller (2006) stated that similar to a genome which supplies the large amount of information required when functioning biologically in a complex environment, long-term memory provides an ample storage of information that is essential for controlling the majority of human cognitive activities.

2.2.2 Borrowing and reorganizing principle

Sweller and Sweller (2006) claimed that the importance of imitation in human cognition was indicated by the discovery of a mirror neuron system in humans. Furthermore, a study by Iacoboni and his colleagues (Iacoboni et al., 1999) on the mirror neuron system found that this system was as active when subjects were required to observe an action, as it was when they were asked to perform motor actions. A further study by Tettamanti and his colleagues (Tettamanti et al., 2005) added that a condition of listening to a description of an action resulted in mirror neuron system activation. According to Sweller and Sweller (2006), these two studies reflect the fact that imitation is a significant technique to obtain information. Although imitation is biologically primary, the information sought by imitation is not necessarily primary. As long as humans have an ability to imitate, they can use it to acquire secondary knowledge such as learning mathematical equations. This principle provides an explanation for the source of the massive amount of information stored in long-term memory. Sweller and Sweller (2006) claimed that most of the information in long-term memory is acquired by imitating other's actions, listening to what they say or reading what they write. In other words, we borrow stored information held in other people's long-term memory storage, but usually this information needs to be reconstructed and modified by the borrowers to fit their own knowledge stored in their long-term memory. Sweller (2003; 2004) argued that schema theory reflects this reconstruction and modification process (see chapter 1 for details). Briefly, schemas have two functions; organizing information in long-term memory and reducing working memory load. Automation has a function of reducing working memory load too. Going back to the borrowing principle, Sweller and Sweller (2006) claimed that the borrowing or transmitting of information is never exact, rather, it is affected by previous knowledge. In this way, schemas are constructed and modified by previously stored personal knowledge and result in unique rather than copied schemas. However, this principle does not explain the procedure of acquiring original knowledge.

2.2.3 Randomness as genesis principle

The randomness principle may provide an explanation for how information that is transmitted by borrowing from one person to another was established in the first instance. Sweller (2003; Sweller & Sweller, 2006) suggested that random generation followed by tests of effectiveness is the most likely way. When dealing with familiar information, long-term memory can act as a central executive. However, the situation is different when handling new material owing to the fact that knowledge structures are unavailable to help organize this new information (Sweller, 2004).

During problem solving, random moves are generated when knowledge is unavailable. These random moves are subject to effectiveness tests with useful ones being retained while non-beneficial ones are ignored. Retained moves can be incorporated into long-term memory and this knowledge then may be transmitted to others via the borrowing principle (Sweller, 2004).

2.2.4 The narrow limits of change principle

As suggested by Cognitive load theory, the processes of human cognitive architecture are analogous to the processes of evolution by natural selection (Sweller, 2003; 2004; Sweller & Sweller, 2006). One of the basic assumptions of evolution by natural selection is that alterations to a genome occur as a result of random mutations followed by testing of effectiveness. Changes that contribute to survival are kept, and failed ones are not. Basically, these changes are assumed to be incremental and slow. In the same manner, our cognitive architecture adopts changes to long-term memory. As a result of random generation followed by testing, these changes are slow and gradual. According to the narrow limits of change principle, working memory's limited capacity ensures that changes to long-term memory are limited and prolonged (Sweller, 2003; 2004; Sweller & Sweller, 2006).

Instructional designers need to take into account this factor of not overloading working memory, or exceeding its limited capacity. As discussed previously (see chapter 1), human cognition is limited by seven items to be held in working memory (Miller, 1956) and by four items to be processed (Cowan, 2001). In order to avoid these limits, schemas held in long-term memory need to be utilized. A schema can permit multiple elements to be treated as a single one in working memory and less effort will be faced by working memory. Therefore, experts in an area do not suffer these limitations because their schemas have already been acquired and automated. The problem of working memory limits applies only to novices when dealing with novel information (Sweller, 2003; 2004).

2.2.5 The environment organizing and linking principle

The limited capacity of working memory appears when processing new material that is not organized into schemas. Capacity limitations disappear when dealing with previously stored and organized material (Sweller, 2004).

Sweller and Sweller (2006) claimed that the environment organizing and linking principle explains how humans are able to transfer large amounts of information stored in long-term memory to be processed in working memory when required to be activated. This ability was recognized and explained by Ericsson and Kintch (1995). They proposed the concept 'Long Term Working Memory'. This term was defined by them as a mechanism based on skilled use of storage in long-term memory. The main characteristic of long-term working memory is that learners can acquire memory skill to answer expected demands for working memory in a specific domain. It is assumed that in order to recall information stored in long-term working memory, only the link concerning the particular structure needs to be available in working memory. Sweller and Sweller (2006) claimed that experts, according to this principle, can transfer enormous amounts of organized information from their long-term memory to working memory as long as they are performing in their field of expertise.

In short, according to Sweller (2003; 2004) and Sweller and Sweller (2006), the borrowing and randomness as a geneses principles, are the learning mechanisms we use to create our information storage. The narrow limit of change principle ensures that these mechanisms function appropriately without destroying the information store. Then, when the stored information needs to be activated, the environment organizing and linking principle helps to convey the required information from long-term memory to the working memory to be processed adequately.

2.3 Categories of Cognitive Load

Cognitive load can be classified into three categories: intrinsic, extraneous, and germane cognitive load. The total cognitive load is determined by the sum total of these three sources of load.

Throughout this thesis the term ‘mental effort’ will be used as an index of cognitive load. It was defined by Paas and his colleagues as the amount of capacity or resources that is invested to answer the demands of a given task. Mental effort can be used as an index of cognitive load. It can be measured by either using rating scales or by using physiological parameters (Paas, van Merriënboer, & Adam, 1994).

2.3.1 Intrinsic Cognitive Load

This type of load depends on the number of elements of to-be-learned material that must be processed simultaneously in working memory (van Merrienboer & Ayres, 2005). It is believed that high element interactivity causes high intrinsic cognitive load. It was assumed that this type of cognitive load was not controlled by instructional techniques. However recently, the possibility of reducing intrinsic cognitive load by manipulating the material element interactivity and the subject-task interaction, has been suggested (Paas, Renkl, & Sweller, 2003).

Sweller (2006c) speculated that intrinsic cognitive load can be reduced in two main ways; firstly, by manipulating the to-be-learned material, and secondly by schema acquisition and automation. Manipulating material is discussed first.

Pollock, Chandler, and Sweller (2002) were the first researchers who tested techniques for reducing intrinsic cognitive load using a strategy called the 'isolated-elements procedure', by which they dissected the material into isolated elements and presented it to subjects one by one, enabling them to learn the elements partially and gradually rather than simultaneously. Similarly, to avoid high intrinsic cognitive load, Sweller and his colleagues (Clarke, Ayres, & Sweller, 2005; Sweller, 2006c) suggested teaching complicated material as isolated elements, and teaching the interactivity between elements later. Using the same strategy, Ayres (2006) examined the impact of reducing intrinsic cognitive load in Algebra. He found that when using an instructional strategy that isolates the elements of the to-be-learned material, a reduced cognitive demand was indicated, rather than when using an integrated-elements strategy (all elements together). However, he found that subjects with more prior knowledge benefited less from isolated tasks than subjects with less prior knowledge. Ayres (2006) attributed these results to a decrease that occurred in germane cognitive load, a matter that hindered schemas acquisition (see the section on germane cognitive load for a further explanation).

Acquiring schemas provides the second method to manipulate intrinsic cognitive load; when several elements are treated as a single element in working memory, the number of interactions between elements is decreased. Therefore, by enhancing schema acquisition, a reduction in intrinsic cognitive load can be achieved. Many studies indicated the former method to be effective (e.g. Kalyuga, Ayres, Chandler, & Sweller, 2003; Renkl & Atkinson, 2003) Another technique to reduce intrinsic cognitive load was providing pre-training as used by Mayer and his colleagues (Mayer, Mathias, & Wetzell, 2002; Mayer & Moreno, 2003) and also Clarke and his colleagues (Clarke et al., 2005). They found that providing subjects with training on relevant elements before approaching the full learning procedure enhanced the subsequent learning process. This tactic aimed to build subjects' prior knowledge before attaining the actual learning. In effect, this maneuver consolidates the argument for constructing schemas to minimize intrinsic cognitive load.

In summary, almost all techniques intended to reduce intrinsic cognitive load share a common feature; they strengthen the supposition that prior knowledge embedded in schema acquisition and automation is crucial when dealing with complicated materials that have high element interactivity. Moreover, it is assumed that the degree of intrinsic cognitive load depends on the learner familiarity with the field of the new material to be learned. Therefore, utilizing prior knowledge was demonstrated to be efficient (Ayres, 2006).

Intrinsic cognitive load is related to materials that have high element interactivity; therefore, the following section discusses this type of material.

2.3.1.1 The Element Interaction of the to-be-learned material

It is assumed that the difficulty of acquiring information depends on the degree to which its elements interact during the learning process. The more elements that have to be processed synchronously in working memory during the learning process, the more intrinsic cognitive load is imposed, and the less chance this material has to be understood. In effect, understanding, claimed Sweller and his colleagues (Pollock et al., 2002; Sweller, 2003; 2004) is a function of processing all required elements simultaneously in working memory. As indicated by Chandler and Sweller (1996), material imposes high intrinsic demands to be comprehended if it requires a large number of individual elements to be processed concurrently in working memory, or if its individual elements depend on each other to a high extent.

Therefore, processing all elements separately does not result in understanding without a comprehensive processing of the interaction (Paas et al., 2003; Pollock et al., 2002; Sweller, 2003; 2004; van Merrienboer & Sweller, 2005).

Intrinsic cognitive load cannot be managed by instructional interventions without affecting understanding (Pollock et al., 2002).Pollock, Chandler and Sweller found that complicated information that is rich in element interactivity can be handled by the 'isolated-interacting elements' method which enables subjects to study the material as isolated elements at an initial phase and then, after they acquire a schema for each individual element, comprehensive instructions can be provided to explain the interactivity of all elements at a latter stage.

2.3.2 Extraneous Cognitive Load

Extraneous cognitive load is assumed to be controlled by instructional methods. Because this type of cognitive load interferes with learning, reducing this kind of cognitive load by designing suitable instructional methods has been a primary purpose of cognitive load theory (van Merrienboer & Ayres, 2005). When dealing with material which is low in element interactivity, reducing extraneous cognitive load may not be necessary because the total cognitive load may not exceed working memory capacity. In contrast, when teaching in an area with high elements interactivity, reducing extraneous cognitive load is assumed to be critical (Sweller & Chandler, 1994).

Sweller and Sweller (2006) asserted that an instructional method imposes a high extraneous load by ignoring the narrow limits of change principle i.e. ignoring the limitation of working memory or by relying on the randomness as genesis principle rather than the borrowing principle. Any aimed change for the information store, claimed Sweller, is likely to be ineffective if an instructional method was not designed according to cognitive architecture principles since it may cause a high extraneous cognitive load.

Cognitive load theory suggests various techniques to manipulate extraneous cognitive load such as goal free problems (Ayres, 1993; Sweller, 1988; Sweller & Cooper, 1985; Sweller & Levine, 1982; Sweller, Mawer, & Ward, 1983; Tarmizi & Sweller, 1988) , presenting materials using dual mode instruction (Jeung, Chandler, & Sweller, 1997; Mayer & Moreno, 1998; Moreno & Mayer, 1999; Mousavi, Low, & Sweller, 1995; Tindall-Ford, Chandler, & Sweller, 1997) and an emphasis on using worked examples instead of conventional problems (Cooper & Sweller, 1987; Paas, 1992; Paas & van Merrienboer, 1994; Sweller & Cooper, 1985). In a hypermedia environment, Schwartz and his colleagues (Schwartz, Andersen, Hong, Howard, & McGee, 2004) stated that when setting up an educational website, instructional designers need to take into account the cognitive demands a website structure places upon learners' working memory.

There are several effects due to these instructional methods used to reduce cognitive load under the umbrella of cognitive load theory: the goal-free effect, the split-attention effect, the modality effect, the redundancy effect, the problem completion effect, the expertise reversal effect, the element interactivity effect, and the worked example effect (Sweller, 2003). The two last mentioned effects are central to this thesis, thus they will be discussed later.

The table shown in Figure 7 was presented in van Merreinboer & Sweller (2005, p. 151). It summarizes the effects and outlines the reasons behind reducing extraneous cognitive load.

Effect	Description	Extraneous load
Goal-free effect	Replace conventional problems with goal-free problems that provide learners with an a-specific goal	Reduces extraneous cognitive load caused by relating a current problem state to a goal state and attempting to reduce differences between them; focus learners' attention on problem states and available operators
Worked example effect	Replace conventional problems with worked examples that must be carefully studied	Reduces extraneous cognitive load cause by weak-method problem solving; focus learners' attention on problem states and useful solution steps
Completion problem effect	Replace conventional problems with completion problems, providing a partial solution that must be completed by the learners	Reduces extraneous cognitive load because giving part of the solution reduces the size of the problem space; focus attention on problem states and useful solution steps
Split attention effect	Replace multiple sources of information with a single, integrated source of information	Reduces extraneous cognitive load because there is no need to mentally integrate the information sources
Modality effect	Replace a written explanatory text and another source of visual information such as a diagram(unimodal) with a spoken explanatory text and a visual source of information (multimodal)	Reduces cognitive load because the multimodal presentation uses both the visual and auditory processor of working memory
Redundancy effect	Replace multiple sources of information that are self-contained with one source of information	Reduces extraneous cognitive load caused by unnecessarily processing redundant information

Figure 7: Some effects studied by cognitive load theory and why they reduce extraneous cognitive load.

2.3.3 Germane Cognitive Load

This category is associated with processes that are relevant to learning such as schemas acquisition and automation. This type of cognitive load was discussed by Sweller, van

Merrienboer and Paas (1998). It is defined as follows: 'germane load is load that directly contributes to learning, that is, to the learner's construction of cognitive structures and processes that improve performance.' (van Merrienboer, Kester, & Paas, 2006, p. 344). Paas and van Merrienboer (1994) in their experiments found that although high variability during the training period caused high cognitive load, the subjects' performance on transfer problems was enhanced and resulted in better schema acquisition. Since then, germane cognitive load has been regarded as beneficial and essential for schema construction as long as the total cognitive load does not exceed working memory limitations. Furthermore, van Merreienboer and Sweller (2005) suggested that to improve learning, instructional methods should be manipulated in such a way that reduces extraneous cognitive load and as a consequence frees germane cognitive load encouraging learners to invest the free working memory capacity to construct schemas. Paas and his colleagues (Paas et al., 2003) argued that despite the effectiveness of germane cognitive load, its value depends on learner motivation.

There were two key directions of research into germane cognitive load firstly, using contextual interference conducted by van Marrienboer and his colleagues (DE Croock, van Merrienboer, & Paas, 1998; van Merrienboer, DE Croock, & Jelsma, 1997; van Merrienboer, Schuurman, DE Croock, & Paas, 2002) . In these studies, it was found that high contextual interference increased cognitive load during the training phase, however, performance was shown to be better in the test phase. The other approach was using self-explanation. Renkl and his colleagues used this method to investigate germane cognitive load (Renkl & Atkinson, 2001; Renkl, Stark, Gruber, & Mandl, 1998). They concluded that students who explained the procedures of worked examples reported higher mental effort, but outperformed students who were not asked for self-explanations.

2.4 Interrelation between categories of cognitive load

Researchers assume interactions between these categories of cognitive load. Paas, Renkl and Sweller (2003) suggested an asymmetric and recurring relation between cognitive load categories; they supposed that human cognition during learning deals first with the intrinsic cognitive load, then any remaining working memory capacity will be consumed to handle extraneous and germane cognitive loads. Once working memory capacity is available to free germane cognitive load by reducing extraneous cognitive load, this available germane cognitive load can be used for schema construction and automation. Once a learner acquires the needed schemas, he or she gains expertise in the specific field; subsequently less intrinsic cognitive load will be encountered. Eventually, total cognitive load is minimized and learning is enhanced and the newly learned material will be used to construct more advanced schemas, then a new cycle commences (Paas et al., 2003).

Therefore, using minimal working memory resources to handle extraneous cognitive load is quite essential to free germane cognitive load and stimulate this cycle to occur.

In a different field, Scott and Schwartz (2007) found that when the imposed cognitive load was high and extraneous, performance suffered. However, when the cognitive load was high and germane, performance was enhanced. Nonetheless, Germane cognitive load should not exceed the limits of working memory capacity otherwise it will decrease rather than increase performance (see Große & Renkl, 2006). In their study into learning mathematics, Große and Renkl (2006) found that learning was decreased when students were asked to provide self-explanations. Sweller (2006a) in his commentary on this study attributed these results to an increase in germane cognitive load beyond the limits of working memory with insufficient time given to accommodate this expansion in cognitive load.

Researchers assume that these three categories are additive and if the total cognitive load exceeds the available working memory capacity, learning is likely to be compromised.

Paas and van Merrienboer (1994, p. 123) suggested a schematic representation of the cognitive load construct (

Figure 8).

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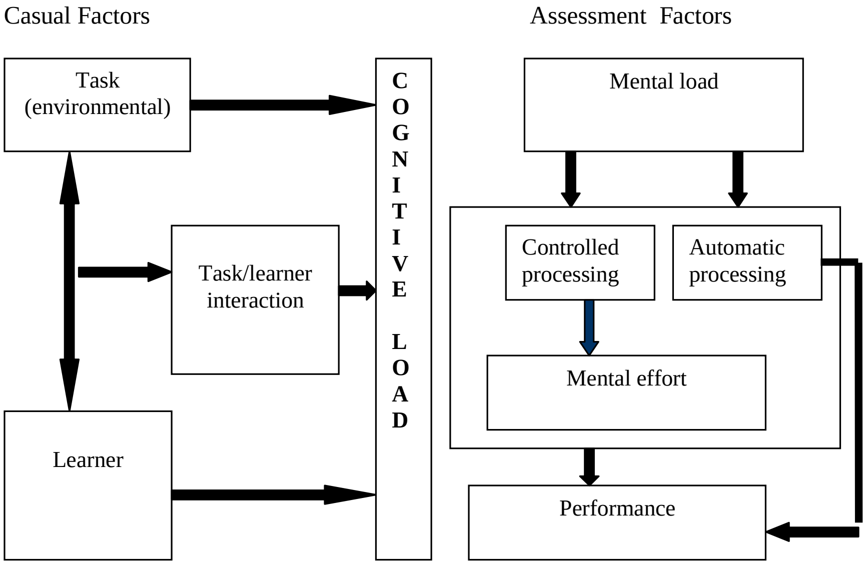


Figure 8: Cognitive load construct (Paas & van Merrienboer, 1994, p. 123)

They assumed that there are several factors that affect the invested mental effort such as the task environment characteristics (e.g. task structure, type of reward system and time pressure), the learner characteristics (e.g. the learner cognitive capability and the previous knowledge of the learner), and the interaction between them (e.g. motivation or personal expectations of performance).

2.5 Worked examples

A worked example basically contains a problem with a procedure for solving the problem. It is a solved problem with a step-by-step solution that the learner needs to study.

Cognitive load theory assumes that schemas can be acquired more easily and rapidly by using worked examples as an instructional method compared with problem-solving techniques (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985). Sweller and Cooper (1985) argued that studying worked examples supports problem solving schema construction and automation more than solving the equivalent problems.

During the last few years, the educational advantages of worked examples have been provided (Sweller & Cooper, 1985; Sweller et al., 1998). The success of worked examples has been demonstrated in several domains; algebra (Chung & Tam, 2005; Cooper & Sweller, 1987; Große & Renkl, 2006; Nathan, Mertz, & Ryan, 1994; Sweller & Cooper, 1985), geometry (Paas & van Merrienboer, 1994; Zhu & Simon, 1987), computer programming (Sweller & Tuovinen, 1999; Trafton & Reiser, 1993), Physics(Sweller & Ward, 1990), engineering (Van Gog, Paas, & van Merrienboer, 2006) and statistics (Paas, 1992).

There are several factors that influence the effectiveness of worked example as an instructional method. To benefit from worked examples, the quantity of presented worked examples is consequential. Sweller and his colleagues found that providing students with many worked examples is more effective than providing them with a few worked examples followed by conventional practice (Cooper & Sweller, 1987; Sweller & Cooper, 1985). In a more recent study, Reed and Bolstad (1991) demonstrated that two examples facilitate learning more than a single worked example. Sweller (2006a) indicated in his commentary paper that one worked example is unlikely to facilitate learning.

Not only is the number of worked examples important, but also other features are crucial. Researchers have emphasized the critical role of the worked example's structure (Mwangi & Sweller, 1998; Ward & Sweller, 1990). Furthermore, Zhu and Simon (1987) stated that using worked example as an instructional method can significantly reduce the teaching time. However, they emphasized carefully planned worked examples. A well designed worked example helps to build and automate an appropriate schema. According to cognitive load theory worked examples, to be effective, must direct the learners' attention aptly. In geometry, for example, Tarmizi and Sweller (1988) demonstrated that worked examples that required learners to split their attention between a diagram and the information given separately were less effective than worked examples that integrated the information in the diagram; this effect was named the 'split attention effect'. Therefore, they concluded that integrating textual explanation into a diagram is vital to circumvent the split attention effect. In a further study by Ward and Sweller (1990) it was indicated that using well designed worked examples that take into account other aspects of cognitive load facilitate schema acquisition and automation.

Worked examples are supposed to encourage schema acquisition and enhance the learning process. Paas and his colleague (Paas & van Merrienboer, 1994) indicated that adding variability to worked examples during the acquisition phase enhanced the schema acquisition process; however, adding variability during conventional problem practice had a reverse effect.

Chinnappan and Lawson (1996) using trigonometry problem solving, compared the effectiveness of worked examples that were designed to highlight reading, planning and checking versus worked examples that did not make any reference to these management processes. They found that students performed better in test problems, both near and far transfer, when they were encouraged to pay attention to strategic processes during the acquisition phase.

Sweller and Tuovinen (1999) found that familiarity with the domain results in less benefit from worked examples compared to exploration problems. They suggested that familiarity with an area enables learners to use their previously built schemas rather than using the provided worked examples during learning. In other words, existing schemas could eliminate the worked example effect. Therefore, worked examples ought to be reduced as learner expertise increases in a specific domain. This was one of the recommendations of Paas, Renkl and Sweller (2003) following the results of Renkl and

Atkinson's study (2001; 2003) on the relation between learner levels of expertise and the degree learners benefit from worked examples.

2.5.1 Why are worked examples advantageous?

An answer to this question can be found in Sweller (2006a) in his commentary on several recent studies on worked examples. From a cognitive load theory perspective, worked examples require less working memory capacity than solving problems because they eliminate the need to search for problem solutions. Worked examples, as a result, minimize extraneous cognitive load. It is supposed that the effectiveness of a worked example depends on the extent to which cognitive load is reduced. As mentioned formerly in this chapter, the borrowing principle is a fundamental plank by which long-term memory content is modified and learning occurs. Worked examples, as an instructional method, provide a lucid case of how instructional techniques can benefit from this principle. Relying on the borrowing principle rather than the randomness as a genesis principle is exemplified by using worked examples rather than conventional problem solving as a teaching procedure (Sweller, 2006a).

2.6 Contextual learning

Contextual learning emerges from several theories and movements. Contextual learning has been studied in various educational contexts and its implementation can be observed in several instructional designs in different areas. Contextual learning revolves around the idea that learning takes place in a meaningful environment rather than laboratory settings. Three of these movements: Gibson's information pickup theory, situated learning theory and the realistic mathematics education approach are related to this thesis. Thus, a brief outline of them is presented next. Then, contextual learning from a cognitive load theory perspective will be discussed.

The 'information pickup theory' (Gibson, 1987) suggests that human cognition integrates environmental factors when perceiving an object. Gibson centered his theory applications in aviation training and claimed that context is a critical factor in learning processes. He emphasized including realistic environmental settings in instructional material to enhance and facilitate perception since, he claimed, learning is unlikely to occur in isolation from an environment (Gibson, 1986)

'Situated learning' (Lave, 1988; Lave & Wenger, 1991) is a leaning theory which was developed by Jean Lave and Etienne Wenger in the late 1980s and early 1990s. The main argument of this theory is that learning is situated. In other words, Lave (1988) claimed that information can be acquired only in real-situations and ongoing experiences in daily life. Therefore, knowledge needs to be presented in settings and applications that would normally involve that knowledge. According to this theory, knowledge cannot be learned in an abstract, decontextualised presentation.

'Realistic mathematics education' started around 1970, in the Netherlands. Its foundations were laid by Freudenthal (1905-1990). Freudenthal believed that mathematics must be connected to reality, and the experiences of the children. He underlined the idea of mathematics as a human activity. From Freudenthal's point of view, mathematics education for children should depend on mathematizing everyday reality (Gravemeijer & Terwel, 2000). This process can be done by: (1) Horizontal mathematization that leads from the world of life to the world of symbols. (2) Vertical mathematization which is the process of reorganization within the mathematical system itself (Gravemeijer, 1999). The starting point should be found in situations that need to be organized. Instructional designers should look for problem situations that could function as informal starting points, from which cognitive growth is possible (Gravemeijer & Doorman, 1999).

Context problems are defined as problem situations that are experientially real to students. The student's final understanding of formal mathematics should be rooted in their understanding of these experientially real, everyday-life phenomena (Freudenthal, 1991). At the same time, a shift is made from problems put in terms of everyday life contexts to a focus on the mathematical concepts and relations. Gravemeijer (1999) argued that to make such a shift possible for students, they need to develop a mathematical framework of reference that enables them to look at these types of problems, mathematically.

In the realistic mathematics education approach, context problems play a main role. It is claimed that well chosen context problems offer opportunities for students to develop informal knowledge to go through mathematical symbols. In the context related questions, the context gives meaning to concepts. Moreover, in school settings, students' informal knowledge can serve as a basis for the development of understanding of mathematics (Mack, 1990).

Several research studies have been carried out in order to examine the effect of a real-life context to generate understanding of mathematics concepts, in general, and fractions in particular (Mack, 1990). Results showed that students came to school with a store of informal knowledge about fractions that enabled them to solve problems presented in a real-life context, and they could build on informal knowledge when they could match problems represented symbolically to problems presented in the context of real-life situations.

In this light, from the realistic mathematics education point of view, making mathematical concepts more realistic enables students to link new concepts to an existing schema (Empson, 1999; Mack, 1990; 1993; 2001; Streefland, 1991; 1993).

2.7 Cognitive Load Theory and familiarity with the context

Recent movements in cognitive load theory have been interested in familiarity with context as a factor that reduces cognitive load in order to facilitate learning (Carlson, Chandler, & Sweller, 2003; Marcus, Cooper, & Sweller, 1996). Van Merriënboer, Kirschner, and Kester (2003) suggested that learners should be presented realistic tasks. Perry and his colleagues using aviation contexts (Perry, Stevens, Wiggins, & Howell, 2007) credited the superiority of learners' performance who studied icons connected with familiar meanings, to the meaningful associations that participants have in their long-term memory.

This approach is rooted in schema theory and emerges from the premise of automated schemas freeing working memory capacity (Sweller, 2003). These automated schemas are assumed to provide the contents of human long-term memory which is believed to have an unlimited capacity (see chapter 1 for a detailed review). Moreover, it is proposed that previously constructed and automated schemas are ready to be retrieved and used in working memory once a stimulus is presented (Anderson, 1996; Cowan, 1988).

Cognitive load theory relies heavily on schemas to reduce cognitive load. In effect, it is assumed that schemas allow many elements to be treated as a single element in working memory and as a result, more working memory capacity is free (Sweller, 2003). Furthermore, it is assumed that experts can perform better in their area of expertise due to the fact that they have at their disposal schemas regarding this particular area (van Merrienboer et al., 2003).

Cognitive load theory assumes that when knowledge has to be acquired in a complex environment, more considerations have to be taken into account when designing instructional methods than in a simple environment. Hence, considering the limitations of the human processing system becomes critical. Sweller and Sweller (2006) argued that the environmental organizing and linking principle allows humans to use unlimited amounts of pre-stored information to process and link to the environment. Therefore, Sweller strongly recommends that relying on the environment organizing and linking principle is beneficial when designing effective instructional material and in particular, when dealing with high element–interactivity material. According to this principle, knowledge held in long-term memory is critical to intellectual functioning.

In several recent studies, with the intention to reduce intrinsic cognitive load when dealing with complex material, it was found that learners who had prior-knowledge such as pre-training regarding a specific field outperformed learners who lacked this kind of knowledge (Clarke et al., 2005; Mayer et al., 2002; Mayer & Moreno, 2003). In these experiments subjects were provided with initial information during a pre-training period in such a way that learners commenced their actual learning phase with preliminary knowledge. The 'isolated-element procedure' used by Pollock and her colleagues (Pollock et al., 2002) to reduce intrinsic cognitive load also emphasizes the significance of schemas as a critical factor in enhancing subsequent learning (see also chapter 1). In Pollock et al.'s study, by isolating the to-be-learned material's elements into individual parts and enabling subjects to construct 'partial' schemas regarding these isolated elements beforehand, intrinsic cognitive load was minimized and eventually subjects learned more efficiently (Ayres, 2006). Again, in these experiments, knowledge held in long-term memory was critical to subsequent learning.

2.8 The expertise reversal effect

2.8.1 Introduction and definition

The expertise reversal effect as defined in Kalyuga et al. (2003, p. 23) ‘is the reversal of cognitive load effects with expertise’. In other words, effects that can be obtained when considering human cognitive architecture to generate more effective instructional techniques are not applicable when learners are highly knowledgeable. Cognitive load theorists provided evidence that learners’ knowledge or expertise plays a critical role in the extent to which they benefit from an instructional method. Thus, instructional designers need to be aware of learners’ level of expertise to produce effective instructions (Chandler & Sweller, 1991; Kalyuga, Chandler, & Sweller, 1998; Kalyuga, Chandler, & Sweller, 2000; Kalyuga, Chandler, & Sweller, 2001; Tuovinen & Sweller, 1999; Yeung, Jin, & Sweller, 1998) and also (Chung, 2007; 2008; Feldon, 2007b; Nilsson & Mayer, 2002; Schwartz, Verdi, Morris, Lee, & Larson, 2007; Scott & Schwartz, 2007; Taft & Chung, 1999). In effect, instructional designs that are appropriate for novices need to be modified to be effective for experts (Kalyuga et al., 2003; Kalyuga & Sweller, 2004). From another perspective, Feldon (2007b) argued that expert-appropriate support during instruction requires adaptation to be beneficial for novices.

Before expanding further on this effect, the concept of expertise will be discussed.

2.8.2 Experts versus novices

De Groot (1965) followed by Chase and Simon (1973) were the first to attribute expertise to large amounts of domain-specific knowledge held in long-term memory. Chase and Simon in their study into chess masters realized that the difference between chess masters and less capable players was their ability to memorize chess board arrangements. In other words, chess masters hold more chess board configurations in their long-term memory and are able to recall them when required. However, this superiority disappears when these masters are faced with random board configuration. Based on schema theory (see Chapter 1 for a detailed review), experts have schemas that allow them to store more information and retrieve it if required as a stimulus is provided.

Feldon (2007b) suggested other factors that differentiate experts from non-experts; he stated that the knowledge structure of experts is different from non-experts'. Feldon in the same paper assumed that experts' schemas are more abstract and general than novices'. Another factor mentioned by Feldon (2007b) that differentiates experts form non-experts is the strategies they use to solve problems. Experts work forwards, for example they consider the requirements and the limitations of a problem to achieve the optimal solution and use conceptual schemas, whereas learners with less expertise classify problems based on surface structures and work backwards from the required solution to determine their strategy. According to Larkin and her colleagues (Larkin, McDermott, Simon, & Simon, 1980a; Larkin et al., 1980b) novices might use trial and errors strategies to obtain the required solution while experts work more systematically.

Lawson and Chinnappan (1994) compared high-achieving to low-achieving student protocols while solving geometry problems. They found that high-achievers had access to greater amounts of geometric knowledge and were more able to use this knowledge effectively. Moreover, high-achievers managed their problem-solving process efficiently. In another study, Lawson and Chinnappan (2000) indicated that knowledge connectedness in high-achieving students was superior to low-achieving students. In other words, high-achievers retrieved related knowledge more readily and were able to link given knowledge schemas to related information.

Feldon (2007b, p. 103) summarized the characteristics of expertise in the following manner:

- Experts possess extensive conceptual and strategic knowledge.
- Experts use effective automated procedures that allow them to outperform non-experts consuming less time and experiencing less effort.
- Experts' working memory has more capacity than novices'. It allows them to successfully attempt more complex problems.

2.8.3 Why and how does the expertise reversal effect occur?

To comprehend the reason behind the expertise reversal effect, human cognitive architecture, in particular the limited capacity of working memory and the unlimited capacity of long-term memory need to be considered (Kalyuga et al., 2003; Sweller, 2003; 2004). Kalyuga and his colleagues (Kalyuga et al., 2003) asserted that if learners are less experienced in a particular domain, they either lack schemas, or the schemas that they possess are insufficiently developed to incorporate the problem situation. Consequently, for learning outcomes to be achieved, instructional guidance is essential. In contrast, as learners' experience in a particular domain increases, not only do they hold more schemas but also these stored schemas become more complex. In addition they are likely to have more accessibility to the stored schemas when necessary. Thus, once a stimulus is provided they are able to approach the required schema readily. Therefore, the need for instructional support is unnecessary and may create a negative impact resulting in a reversal effect.

Sweller (2004) suggested that the extent to which instructional aid is required depends on the extent to which related knowledge is available in the information store. Thus, if the needed information is already available in long-term memory, the provided information by an instructional method becomes redundant. It might interfere with learning and the desired positive effect from the aid offered by the instructional technique reversed. In fact, Sweller (2004) assumed that the necessity of support provided by instructional methods is a function of the learners' level of expertise.

Specifically, as the level of expertise rises, the amount of instructional aid should be reduced, or else a reversed effect is likely to arise.

2.8.4 Evidence for the expertise reversal effect

Kalyuga and his colleagues (Kalyuga et al., 2003; Sweller, 2003; 2004) were the first to demonstrate this effect. Extensive evidence can be found across several research fields and several cognitive load effects are shown to be reversed when learners are knowledgeable. For example, worked examples were found to be unhelpful and distracting rather than effective. Kalyuga and his colleagues (Kalyuga, Chandler, & Sweller, 2001) in their study into mechanical trade training found that subjects with less expertise in the domain performed better under a worked example condition compared to those under a problem solving condition, whereas the situation was reversed concerning more expert participants. In the domain of data programming, Tuovinen and Sweller's (1999) study demonstrated that learners with no previous knowledge benefited more from worked examples while those with more prior-knowledge had fewer advantages.

In a hypermedia environment, Schwartz and his colleagues (Schwartz et al., 2007) found that providing navigators with a navigation map of a site (which is intended to reduce extraneous cognitive load during navigation) was advantageous when geographic maps was presented and the learners were unfamiliar with the geography of the area. By contrast, when they were familiar with the geographic features of the area, learners performed better when navigation maps of a site were not provided. Similarly Scott and Schwartz (2007) verified that navigators benefit more from a site map only if their metacognitive skills were relatively low. Mayer and Moreno (2003) and Mayer (2001) confirmed the expertise reversal effect and its impact on designing a multimedia environment.

The expertise reversal effect was also confirmed in languages studies. Chung's (2007) study in Chinese as a second language demonstrated an interaction between modality and learners' expertise. He found that the mixed format (visual-audio mode) worked better than the visual only format when learners lacked experience. Chung (2007; 2008) explained that as learners' level of expertise increases, their cognitive structures change accordingly. Therefore, some of the information that was essential when their level of expertise was low becomes redundant when expertise rises. Hence, when forced to process this unnecessary information they experience more cognitive load. Consequently, their performance suffers. Chung argued that the optimal method of learning is a function of learners' characteristics in addition to the instructional format (Chung, 2007; Taft & Chung, 1999). Yeung, Jin and Sweller (1998) reported similar results. In their study they found adding a textual note to a to-be-learned text was beneficial for less knowledgeable learners, but detrimental for more knowledgeable learners.

Other studies found evidence for an expertise reversal effect in various areas. Kalyuga, Chandler and Sweller (1998) found that including explanatory material in an integrated format was beneficial for less knowledgeable learners, but inferior for more knowledgeable participants. Kalyuga, Chandler and Sweller (2000) obtained similar findings in their study on visual and auditory format.

2.9 Summary of Chapter 2

This chapter outlined five principles of human cognitive architecture that are relevant to instruction: the information store principle, the borrowing principle, the randomness principle, the narrow limits of change principle and the environment organizing and linking principle (Sweller, 2006b; Sweller & Sweller, 2006). The information store principle guarantees a large amount of information held in long-term memory which is assumed to be unlimited in its capacity and duration. The borrowing and reorganizing principle acts as a mechanism to acquire this knowledge by borrowing it from other's long-term memory while the randomness as a geneses principle explains how novel information can be generated initially. The narrow limits of change principle ensures that changes to long-term memory are minor and incremental. Once this new information is stored it becomes a part of the information stored in long-term memory and it can be used for appropriate actions via the environmental organizing and linking principle and can, in turn, be transmitted to another person (Sweller, 2003; 2004;

2006c). Cognitive load theory relies on these principles to design effective instructional methods.

Cognitive load theory distinguishes between three types of cognitive load: intrinsic which is related to the material-to-be-learned itself, extraneous which is linked to the instruction methods used to present this material, and germane cognitive load which is associated with the cognitive load learners invest in the learning process in order to construct and automate schemas. These three sources of cognitive load are additive and the total cognitive load faced by a learner is determined by their sum total.

The main goal of cognitive load theory is to control and maintain the faced cognitive load within the margins of working memory. Working memory is assumed to be limited in its capacity and duration. Manipulating intrinsic cognitive load which is associated with the material's complexity can be achieved by two tactics; isolating the material elements (Ayres, 2006; Pollock et al., 2002; Sweller, 2006c) or using a pre-training phase to enable learners to build prior-knowledge or sub-schemas to help them to handle the complexity of the material (Clarke et al., 2005; Mayer et al., 2002; Mayer & Moreno, 2003). These techniques are based on the assumption that learners' prior-knowledge is a significant factor, particularly when dealing with a high element-interactivity material.

Extraneous cognitive load can be reduced by several instructional methods based on effects such as the split-attention and worked example effects (Sweller, 2003). This thesis is particularly concerned with the worked example effect.

Germane cognitive load needs to be optimized to construct and automate related schemas to enhance the learning process. Learners need to have free working memory capacity at their disposal to generate germane cognitive load and so, minimizing both extraneous and intrinsic cognitive load is a necessary condition to allow learners to optimize germane cognitive load within the limits of working memory. Triggering germane cognitive load can be achieved by using contextual interference (DE Croock et al., 1998; van Merrienboer et al., 1997; van Merrienboer et al., 2002) and by using self-explanation (Renkl & Atkinson, 2001; Renkl et al., 1998).

These three types of cognitive load are assumed to be additive and the total imposed cognitive load is determine by the sum of all three types of cognitive load (Paas et al., 2003).

There are various factors that affect the invested mental effort in learning. Factors that are related to the task to be performed (such as a task structure, time pressure and reward system), characteristics of the learner (i.e. the learner's previous knowledge and intellectual capability) and the interaction between them (Paas et al., 1994).

Cognitive load theory has provided evidence that learning can be facilitated in terms of time and effort when worked examples are used as an instructional method principally because it relies on the borrowing and reorganizing principle rather than the randomness as genesis principle (Sweller, 2006a; 2006b; Sweller & Sweller, 2006).

Several educational theories and approaches originating from contextual and situated learning movements such as information pick-up theory (Gibson, 1986; 1987) situated learning theory (Lave, 1988; Lave & Wenger, 1991) and the realistic mathematics education approach (Freudenthal, 1991; Gravemeijer & Doorman, 1999) emphasize that learning occurs in realistic environmental settings and is unlikely to be achieved in isolation from a relevant environment. These movements encourage using familiar contexts to present a learning material and suggest the use of careful contextual problems when introducing a new concept. Such a starting point has to function as an informal starting point that enables cognitive development.

Cognitive load theory assumes that familiarity with the context reduces cognitive load (Carlson et al., 2003; Marcus et al., 1996; van Merrienboer et al., 2003). This assumption emerges from the heavy weight given to existing and automated schemas as a factor that reduces cognitive load. Having prior-knowledge, whether this knowledge was acquired as a result of pre-training or as a result of familiarity with the context has proved to be an advantage in many studies (e.g. Ayres, 2006; Clarke et al., 2005; Mayer et al., 2002; Mayer & Moreno, 2003; Pollock et al., 2002).

The interaction between learners' level of expertise and the extent to which support should be provided by an instructional design should be taken into account if an effective instruction is to be effective. Instructional methods that are effective for less knowledgeable learners might be unhelpful or even, as a result of the expertise reversal effect, have negative effect for more knowledgeable learners. Concisely, effects that can be obtained when learners are considered as novices can disappear or be reversed when learners have more knowledge in a particular domain (Feldon, 2007b; Kalyuga et al., 2003; Kalyuga et al., 1998; Kalyuga, Chandler, & Sweller, 2001; Kalyuga & Sweller, 2004).

Chapter 3: Fractions

3.1 Introduction

The fraction concept is one of the more complex concepts that primary school students encounter. In essence, the complexity of this concept emerges from the several sub-constructs that a fraction contains. Kieren (1980) outlined five sub-constructs for the fraction concept: part-whole, quotient, ratio, operator and measure. Each sub-construct is associated with a different meaning and has its own challenging issues. Furthermore, the fraction concept is high in element-interactivity a matter that elevates intrinsic cognitive load (Sweller & Chandler, 1994). Learners need to understand the meaning of the numerator, the denominator, and also the specific relation between these two elements to understand the meaning behind the concept. Grasping these interactions is a highly demanding task for primary school students, and we have merely considered a single sub-construct, the part-whole sub-construct. However, if a holistic perception is to be reached, all sub-constructs and all different meanings in addition to the interaction between them needs to be thoroughly assimilated.

3.2 Schemas involved in the acquisition of rational numbers

According to Piaget (1952), thinking is exemplified by mental representations and manipulations. Humans constantly acquire knowledge and store it in schematic symbols. A new learned concept emerges from a particular combination of the novel to-be-learned concept with previously stored knowledge. Fieshbein (1989) suggested that learning occurs as a result of manipulating or modifying an existing schema. Tzur (1999) claimed that learning is a procedure that aims to rearrange previously constructed schemas. According to Mack (1993), learning is a result of the relations that learners make between new material and their prior-knowledge.

Research that deals with rational numbers indicates several schemas central to learning fractions (Hunting et al., 1991; Kieren, 1993; Nesher, 1989; Steffe & Olive, 1993).

Schemas of whole number: Children use their previously constructed schemas about whole numbers. They use these schemas to build on so they can understand the meaning behind the elements of the fraction concept. Then a child integrates this procedure with the division operation.

Schemas of division: These schemas include; halving, dealing (equal-sharing) and folding schemas. Halving is the first to be developed in children and can be used to divide continuous or discrete quantities. Dealing is another type of primitive division that occurs when children use the ‘one for me, one for you’ procedure that results in equal shares. Folding is a method used to produce multiple numbers of parts every time one more fold occurs.

Schemas of measurement: According to this schema, children can perceive fractions as quantities that can be measured.

Schemas of comparison: This schema is the basis for multiplication using whole numbers. For fractions, it enables learners to manipulate the 'unit' concept.

Schemas of interrelation: This schema allows children to perceive relations between parts and wholes.

3.3 Mental processes involved in the acquisition of rational numbers

Acquiring rational number knowledge needs several stages (Kieren, 1993). Kieren suggested a recursive model in which each external circle is built on and can organize an internal one. The development from an internal to a more external circle occurs as a result of social interaction throughout the learning process. In this model, the most internal and initial level is called 'Ethno mathematical knowledge'. At this level, knowledge is obtained as a result of natural environmental experiences. Kieren claimed that at this stage rational number concepts include only 'half' and 'quarter'. While at this point, children can deal with sharing continuous quantities, they do not use a formal mathematical language. The second circle is the ‘Intuitive level’ which occurs in educational systems. At this level, children learn a fraction as a part of a whole by dividing a whole into equal parts. The next outer circle in this suggested model is ‘Technical symbols.’ During this stage, children deal with mathematical language and use symbols and algorithms. The last and the most external level is called ‘Axiomatic deductive knowledge’. At this phase, students learn the relations between the various sub-constructs of fractions. This circle is supposed to embrace all previous ones. Moreover, a learner is very unlikely to thoroughly understand fractions unless reaching this final level (Kieren, 1993).

3.4 Sub-constructs of the rational number

Not only can the fraction concept be presented and perceived in many different meanings and sub-constructs, but also the interactions between these sub-constructs must be understood in order to gain a holistic understanding (Chinnappan, 2005; Hunting et al., 1991; Pitkethly & Hunting, 1996). Researchers identified five sub-constructs (Behr, Wachsmuth, Post, & Lesh, 1984; Kieren, 1981; 1988; 1993; Nesher, 1989). A brief outline of each is presented next.

Rational number as a part-whole: In this sub-construct, a whole is divided to n-parts. Each part is called 1/n and k-parts are called k/n. The proposition of a whole that equals n/n is central. In this sub-construct, the procedure of dividing the whole into equal parts is considered a main principle. Many researchers claim that this sub-construct is the most dominant, since it is presented as an initial model in educational systems. This model was criticized by Mack (1993) and Streefland (1991; 1993) because a fraction, in this sub-construct, is perceived as having a static and nominal meaning. In addition, students concentrate more on the fraction as a procedure rather than as a meaningful object (see also Khateeb, 2002).

Rational number as a quotient: In this model, a/b is the result of dividing two integers. This quotient exemplifies a quantity and can answer the question ‘how much’. If we consider the fraction 2/3, as an example, according to this sub-construct, 2/3 is the result when dividing two wholes by three. Or, using the fair-sharing schema, it is the portion each person receives when two items are shared equally and wholly among three sharers.

Rational number as an operator: This sub-construct emphasizes the meaning of a rational number as an expansion-contraction factor. This meaning is supposed to be beneficial when learning about the fixed value of different names or presentations of the same fraction.

Rational number as a ratio: In this model, a/b represents a relation between ‘a’ and ‘b’ as two quantities. It is usually represented as a:b.

Rational number as a measure: This model represents a type of division. In this presentation, a/b is perceived as the number of times a quantity ‘b’ is included into a quantity ‘a’.

3.5 Complexity of fractions: A cognitive load theory perspective

Knowledge, as mentioned in previous chapters can be classified into two main categories. The first is knowledge that can be incorporated entirely in individual, isolated elements. This type can be processed element by element in working memory, therefore the process does not place a heavy load on working memory, and so it is undemanding. Intrinsic cognitive load that is associated with learning this type of material is minor. On the other hand, there is a different type of knowledge which is rich in element interactivity and demands all elements be processed simultaneously in working memory to be assimilated appropriately. To mentally process this kind of information, a high intrinsic cognitive load is imposed.

The degree of a material complexity depends on the number of elements it contains. Sweller and Chandler (1994) discussed the difficulty of to-be-learned materials. They stated that difficulty is a function of the number of elements and the degree of interactions among these elements. In particular, they described fractions as a troublesome concept. They claimed that fractions cause a heavy intrinsic cognitive load. Since fractions have several elements that have to be manipulated concurrently in working memory, the cognitive load that has to be invested in such a process is likely to surpass the limited capacity of working memory. Hence, learning is congested.

3.6 Suggested methods to teach fractions

Teaching complex concepts to children such as fractions has long been a concern amongst educators. Researchers have tried to find an efficient method to teach mathematical concepts. There have been many suggestions. Steiner and Stoecklin (1997) noted that fraction comprehension is context dependent. Streefland (1991) proposed using realistic situations to teach mathematical concepts in general and fractions (Streefland, 1993) in particular. Other researchers (Kieren, 1988; Mack, 1993; Streefland, 1991; 1993) argued in favor of using informal real-life knowledge that children have acquired to facilitate leaning processes. In essence, Streefland (1991; 1993) suggested that fractions should be taught through fair–sharing problems which can be recognized as a real-life situation (Mack, 1993). According to Empson (1999), fair-sharing problems provide a context about which young children have knowledge. He claimed that equal-sharing tasks facilitate children’s use of knowledge about partitioning to think about fractions in mathematical ways. This schema was used also by Khateeb (2002) who found that using a fair-sharing schema, which is deeply rooted in children’s real-life practices, allows children to understand a fraction as an entity rather than a procedure of dividing and shading shapes. Nowlin (1996) found that using problems in contexts was more helpful than using problems with no context in learning division with fractions. Sharp and Adam (2002) in their study into learning division with fractions found that utilizing students' prior-knowledge about division facilitates learning. In this case, by using the common-denominator rather than the invert-multiply procedure to divide fractions, they took advantage of the strong whole-number schema that students have acquired to build a further expansion for fractions.

In conclusion, most suggested strategies to facilitate learning rational numbers count on building on existing schemas, in other words, on children's prior-knowledge. This prior knowledge can be acquired either by realistic daily practices (Empson, 1999; Khateeb, 2002; Kieren, 1988; Mack, 1993; 2001; Steiner & Stoecklin, 1997; Streefland, 1991; 1993) or as a result of previous, school-based learning (Cooper & Sweller, 1987; Sharp & Adams, 2002; Sweller, 2003; Sweller & Chandler, 1994).

3.7 Summary of Chapter 3

The rational number is known as a difficult concept. It contains several sub-constructs: part-whole, quotient, ratio, operator and measure (Kieren, 1980) with each one providing a different meaning. Moreover, it is high in element-interactivity, which increases intrinsic cognitive load (Sweller & Chandler, 1994).

There are a number of schemas that may be associated with learning fractions: schemas of whole numbers, schemas of division, schemas of measurement, schemas of comparison, and schemas of interrelation (Hunting et al., 1991; Kieren, 1993; Nesher, 1989; Steffe & Olive, 1993).

Several mental processes are involved in comprehending fractions. Kieren’s suggested recursive model (Kieren, 1993) includes four levels: ethno mathematical knowledge, the intuitive level, the technical symbols stage, and axiomatic deductive knowledge. Throughout these four levels children develop their mathematical knowledge about fractions.

From a cognitive load theory perspective, fractions impose a high intrinsic cognitive load because of their high element interactivity (Sweller & Chandler, 1994). Based on cognitive load theory, there are methods appropriate to teaching fractions efficiently. One technique is to use realistic contexts or realistic settings to enhance or motivate appropriate schemas held in long-term memory (Empson, 1999; Khateeb, 2002; Kieren, 1988; Mack, 1993; 2001; Nowlin, 1996; Streefland, 1991; 1993), or to use previously learned material (Sharp & Adams, 2002; Sweller, 2003; Sweller & Chandler, 1994).

Chapter 4: Research in the field

4.1 Research in the field

Several studies have been carried out in order to examine the effect of a real-life context to generate understanding of mathematics concepts, in general (Carraher, Carraher, & Schliemann, 1985; 1987; Koedinger & Nathan, 2004; Saxe, 1988) and fractions in particular (Empson, 1999; Mack, 1990; 1993; Rittle-Johnson & Koedinger, 2005). Results showed that students came to school with a store of informal knowledge about mathematical concepts that enables them to solve problems presented in a real-life context, and build on informal knowledge when they could match problems represented symbolically to problems presented in the context of real-life situations.

The importance of context also can be seen from the work of Freudenthal (1991) and Gravemeijer (1999; Gravemeijer & Doorman, 1999). Context problems are defined as problems that are experientially real to students. According to Freudenthal, a student's final understanding of formal mathematics should be rooted in their understanding of experientially real, everyday life phenomena. From this point of view, making mathematical concepts more realistic enables students to link new concepts to an existing schema (Empson, 1999; Mack, 1990; 1993; Streefland, 1991; 1993) .

Koedinger and his colleagues (1997) introduced the cognitive tutor algebra curriculum and software within the Pittsburgh urban mathematics project, based on the ‘Adaptive control of thought theory’, known as ACT theory (Anderson, 1996; Anderson, Corbett, Koedinger, & Pelletier, 1995; Koedinger & Anderson, 1990). The key feature of this project is to help students learn abstract algebraic representations by bridging existing knowledge using situations or familiar word problems. The Pittsburgh urban mathematics project curriculum employs real-world situations designed to make mathematics more meaningful and accessible for children. The continuous success of the cognitive tutor and the Pittsburgh urban mathematics project provide evidence of the effectiveness of using familiar contexts to facilitate learning materials (Koedinger et al., 1997; Ritter, Anderson, Koedinger, & Corbett, 2007).

Goldstone and Sakamoto’s participants (2003) showed better abstract understanding of a simulation when concrete rather than idealised graphical elements were used as illustrations. However, for learners who did not originally show strong evidence of comprehending the abstraction, the reverse was shown to be the case. Goldstone and Sakamoto concluded that increasing the concreteness and surface-level similarity between two domains can distract learners from taking a more abstract perspective. In Koedinger and Nathan’s (2004) study, money as a familiar context was used as cover stories for algebraic problems. This resulted in a better performance when solving algebra story problems than when solving the equivalent algebraic equations. Nonetheless, Koedinger and Nathan attributed these differences to students’ difficulties in understanding the formal symbolic representation of quantitative relations, a difficulty that might be minimised when solving a real-world story problem.

Rittle-Johnson and Koedinger's (2005) study on the design of learning scaffolds showed that providing students with contextual scaffolds enabled them to perform better in adding and subtracting fractions. Also, students made fewer conceptual errors when a problem was presented in a real-world context. Rittle-Johnson and Koedinger stated that contextual scaffolds seemed to elicit prior-knowledge that helped learners to implement different approaches for solving problems.

Goldstone and Son (2005) showed that transferring scientific principles was better when learners started with concrete representations (using a representation of ants that search for apples to eat) and switched to idealised representations (ants were presented as lines and apples as dots) in the learning phase. Goldstone and Son demonstrated that the concrete fading strategy allowed grounded principles to become less specific context related. This study emphasises the fact that real-world physical experiences are highly effective if transfer is to be achieved. On the contrary, a study conducted by Sloutsky, Kaminski and Heckler (2005) provided evidence that concrete symbols may hinder learning while abstractness may have benefits. Taking into account that materials used in their experiments did not capture all aspects of the to-be-learned material, they acknowledged that in order to facilitate learning, concrete representations have to communicate relevant aspects of the to-be-learned material.

Using cognitive load theory, familiarity with context has been examined as a factor that reduces cognitive load in order to facilitate learning (Carlson, Chandler, & Sweller, 2003; Marcus, Cooper, & Sweller, 1996; van Merriënboer, Kirschner, and Kester (2003). In effect, real-life experience can result in expertise and familiarity. This approach is rooted in schema theory and emerges from the premise of automated schemas freeing working memory capacity (Sweller, 2003).

4.2 Rationale for this study

Mathematical knowledge in abstract form should be an ultimate goal of teaching mathematics, nevertheless, there are grounds for suggesting that when initially exposed to new concepts, and procedures, learners should be presented with the new material within a familiar, concrete context. Cognitive load theory provides a theoretical rationale for presenting novel material in more rather than less familiar form (Carlson et al., 2003; Marcus et al., 1996). Moreover, using worked example as an instructional method can add to the efficiency of this strategy since the imposed extraneous cognitive load is likely to be minimized.

4.3 A brief description of experiments conducted in this thesis

In this thesis five experiments have been conducted to test the general hypothesis that novel material should be presented in more rather than less familiar form. The first three experiments examined the context effect in learning fractions using worked examples as an instructional method in both diagrammatic and word-based format, while the last two investigated the effect of context comparing a problem solving instructional technique vs. a worked example instructional technique.

Experiment 1 compared the effect of a real-life context using pizzas and cakes to demonstrate fractions as a cover story vs. a geometric context using circles and rectangles. It was hypothesized that the realistic context would be superior.

Experiment 2 investigated the context sequence of worked examples. Four different sequences were examined; a Real-Real group was given two phases with realistic materials in both phases, a Real-Geometric was given in the first phase materials in a realistic context followed by second phase materials in a geometric context. The third group, Geometric-Real group, had its materials in the first phase in a geometric context followed by materials in a realistic context, and a Geometric-Geometric group was given two phases with geometric materials in both phases. It was hypothesized that the Real-Geometric condition will be the most beneficial sequence on the assumption that once the concept was learned using realistic materials, it would be easier to transfer it to abstract materials than if learning commenced with geometric objects.

Experiment 3 was intended to generalize the results of the first two experiments by examining the same sequences using worked examples in a wording rather than a visual format. All diagrammatic illustrations were removed from the presented worked examples. Also, the test problems were categorized into two categories based on a transfer continuum; near transfer and more distant transfer problems and a superiority of a real-geometric condition was anticipated in transfer problems based on the previous experimental results.

Experiment 4 examined the effect of using a real-life context compared with an abstract context as a cover story using a problem solving instructional technique vs. a worked example instructional technique. Since contextual problems are used to stimulate learners to benefit from prior knowledge held in their long-term memory and activate schemas already constructed and stored in long-term memory, it was hypothesized that the context would have an effect on the students' performance in the test problems regardless of the instructional method. Moreover, a worked example effect was anticipated.

Experiment 5 replicated the previous experiment with participants who had less previous knowledge about fractions. In this experiment, a superiority of a realistic context was hypothesized in both near and more distant transfer problems. Also, a worked examples effect was predicted.

A detailed description of each experiment is provided in the following chapters.

Chapter 5: Experiment 1

5.1 Introduction

This experiment tested the consequences of presenting worked examples in real-life, familiar contexts rather than geometric contexts. The subject area was comparing fractions. It was hypothesized that by using real-life cover stories from learners' real-life to explain fractions, the new concepts could be more readily assimilated into existing knowledge held in long-term memory compared to the more traditional geometric contexts.

5.2 Method

5.2.1 Participants

Thirty-two students from Year five (approximately 10 years old) of a Sydney public school participated in this study. They were allocated randomly into two equivalent groups of 16 students each with the same number of each gender in each group. No participant had been taught about fractions comparison previously.

5.2.2 Material and procedure

All testing was carried out on an individual basis. The experiment consisted of two phases; a learning phase and a test phase. For the learning phase, students of each group were presented three pairs of worked examples, each pair consisting of a solved example followed by an identical problem to solve. For example, students were shown a worked example indicating that 1/3 is greater than 1/5. Then they were asked to solve a problem asking whether 1/3 is greater than, less than or equal to 1/5. All problem pairs were similar in their content. At the beginning of the each experiment, it was explained orally that for all comparisons of fractions, the same size object can be assumed.

The fractions that were compared in the worked examples were:

1/3 and 1/5

2/3 and 4/6

3/8 and 2/4

A part-whole sub-construct was used to introduce fractions for both of the two groups. The part-whole model requires the interpretation of fractions as part of a whole and is commonly used in teaching fractions. For example, the fraction, 3/4 is frequently described in class as three parts out of four equal parts.

Each group studied a set of three worked examples immediately followed by an identical problem with three minutes allocated to each of the worked examples and another three minuets to each associated problem. Students were asked to study the first worked example within three minutes then solve the paired problem. Time needed to solve the paired problem was measured up to a maximum of three minutes. Then they were asked to rate the difficulty of each pair using the provided nine-point scale.

Students were stopped after three minutes. If they had not succeeded in solving the problem, a correct solution was provided by the examiner. The same procedure was followed for all pairs of worked examples. The worked example was available while solving the paired problem. The first group studied the examples in a real-life context using the context of pizzas, that were always round, or cakes, that were always rectangular, as a cover story. The whole set of worked examples presented for the real-life context group can be found in Appendix A1. A representative pair of worked examples which was provided to the real-context group is shown in Figure 9

Problem 3	Solution 3	
Who will eat more pizza; Sam who eats 3 slices of large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (3/4 of one large pizza)?	Draw one large pizza.  Divide this pizza into 8 equal slices.  Mum said that Sam can eat 3 slices, which is 3/8 of the pizza. He will eat this portion:  Sam will eat 3/8.	Draw another large pizza.  Divide this pizza into 4 equal slices.  Mum said that Tim can eat 2 slices of the four slices. This is his portion: Tim will eat 2/4.  Before eating, a few friends came and mum re-divided the pizza into 8 parts, but she said that Tim still can eat the same amount of pizza. Now Tim's portion will look like this: . Tim can eat 4 slices of 8 (4/8) which is the same amount as 2 slices of 4 (2/4)
Conclusion	Now compare: Sam will eat 3/8 but, Tim will eat 2/4 which is the same as 4/8. Conclusion: Sam will eat less pizza than Tim. So, 3/8 is less than 2/4.	

Figure 9: Worked examples: Real-Context group (Worked example number 3)

This worked example was followed by an identical problem to solve shown in Figure 10

Who will eat more pizza; Sam who eats 3 slices of a large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (3/4 of one large pizza)?

Figure 10: The associated problem to be solved: the Real-Context group (Worked example number 3)

The second group (labeled 'geometric group' studied the examples in a geometric context using geometric shapes (Two variations, circles and squares, were used as cover stories) unconnected to familiar objects. Identical geometric shapes were used for the two sets of worked examples. For instance, if a round pizza was used to represent a whole for the first worked example of the real-life group, a geometric circle was used to represent a whole for the first worked example of the geometric group.

A representative pair of worked examples which was provided to the geometric group is shown in Figure 11

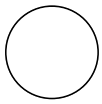
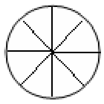
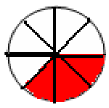
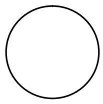
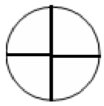
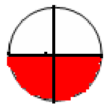
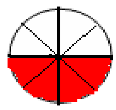
Problem 3	Solution 3	
What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?	Draw a circle:  Divide this circle into 8 equal parts  Paint three parts of these eight parts red.  You have 3/8 painted red.	Draw another circle:  Divide this circle into 4 equal parts.  Paint two parts of these four parts red.  You have 2/4 painted red. Repeat halving this circle (add 2 lines)  Now you have a circle divided into 8 parts, 4 of them painted red. You have 4/8 painted red which is the same area as 2/4.
Conclusion	Now compare: In the first circle you have 3/8 painted red whereas, in the second circle you have 2/4 painted red which is identical to 4/8 painted red. Conclusion: we have less red area in the first circle than the second circle. So, 3/8 is less than 2/4.	

Figure 11: Worked example: Geometric-Context group (Worked example 3)

This worked example was followed by an identical problem to be solved shown in Figure 12.

What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?

Figure 12: The associated problem to be solved: the Geometric-Context group (Worked-example number 1)

Each worked example of the real-life set was similar in length and structure to the equivalent worked example of the geometric set (see Appendix A2). Each student received one problem after each worked example, on a single sheet of paper with sufficient space after it to write a solution. In each worked example or problem, students were asked to compare two fractions. If a student gave an incorrect solution he or she was told to try again within the three minute time limit. If the student did not provide a correct solution within the time limit, he or she was given the correct solution with an explanation and then moved to the next pair. The explanation that the student was provided with was similar to the explanations used in the solved worked examples. The time each student needed to reach the correct solution was measured up to a maximum of three minutes. The acquisition score was determined using the following marking system (which was used throughout this thesis for all phases) providing a score out of six:

Two marks were allocated for a correct solution with a correct explanation.

One mark was allocated if a correct solution was provided, with an incorrect or no explanation.

Zero marks if a student failed to provide a correct solution.

Examples of correct and incorrect explanations provided by students can be found in Appendix A3.

Mental effort refers to the amount of capacity or resources that are invested to answer the demands of a given task. It can be used as an index of cognitive load, and can be measured by using rating scales. Self–ratings of task difficulty as a method of measuring mental effort have been used for this purpose previously, for example (Kalyuga et al., 1998; Kalyuga et al., 2000; Kalyuga, Chandler, Tuovinen, & Sweller, 2001; Paas & van Merrienboer, 1993; Paas et al., 1994). The real-life measure can be hypothesized to reduce cognitive load because people, having appropriate schemas, can combine multiple elements into a single element. In the case of the geometric context, such schemas may not be available. Evidence compatible with this suggestion would be provided if we find that the cognitive load reduced for real-life materials compared to geometric materials.

All students recorded a subjective rating for task difficulty after each one of the three problems to be solved that followed a paired worked example. A nine-point scale shown in Figure 13 was used. The participants were asked 'How easy or difficult was this problem to solve? Tick the appropriate answer.'

Very-very easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Very-very difficult

Figure 13: Mental effort rating scale

A test phase immediately followed the learning phase. It consisted of 10 arithmetic problems to solve (the whole set of test problems can be found in Appendix A4. The test problems had no context associated with them, for example:

Is 1/5 greater than, less than or equal to 1/6?

How many eighths are equal to 1/4?

Is 1/5 greater than, less than or equal to 1/6?

How many eighths are equal to 1/4?

The two groups were given a maximum of two minutes to solve the given problems. Each problem was provided on a separate sheet of paper with the participant asked to solve it using paper and pencil. Participants were asked to provide written reasoning about their solutions. They were asked to work as rapidly and as accurately as possible.

No feedback was given to participants until the whole experiment was completed. The worked examples sheet was not available to participants during the test-problem phase. Each question was allocated two, one or zero points respectively (using the same marking system as the acquisition phase) providing a score out of 20 for each participant.

5.3 Results and Discussion

The variables under analysis were learning scores, test scores and mental effort. During the acquisition phase cognitive load (mental effort) was examined. A mean of mental effort ratings ranging from one (very-very easy) to nine (very-very difficult) was computed for each participant (see Table 1).

The instructional efficiency for each student was calculated using Paas and van Merrienboer's (Paas & van Merrienboer, 1993; Paas et al., 1994) formula:

$$E = \frac{|P - R|}{\sqrt{2}}$$

Where E = efficiency, P = the test performance z-score (test phase) and R = the mental effort z-score (acquisition phase).

Table 1: Means and standard deviations (in parentheses)

Variable/Group	Real-Context	Geometric-Context
Mental effort (out of 9)	2.166 (1.03)	3.58 (1.03)
Acquisition Time (in minutes)	8.277 (0.77)	8.958 (0.13)
Acquisition score (out of 6)	3.87 (1.5)	2.75 (1.52)
Test score (out of 20)	16.75 (2.69)	13.43 (3.20)
Efficiency	0.93 (1.18)	-0.93 (1.24)

An independent t-test was used to compare means of this variable (mental effort) between the two groups. It revealed a significant difference between the two groups' mean mental effort, t (30) = 3.87, partial η 2 = 0.33, indicating a reduced mental effort by the real-life context group. (The 0.05 level of significance is used through this thesis).

A 2 (groups) x 2 (learning and test scores) analysis of variance with repeated measures on the second factor was conducted for the test-score. It indicated a significant main effect between groups, $F(1, 30) = 10.89$, $MSe = 7.24$, partial $\eta 2 = 0.27$. The interaction between groups and the test scores was significant, $F(1, 30) = 5.01$, $MSe = 3.82$, partial $\eta 2 = 0.14$. Following the significant interaction, simple effects tests revealed a significant difference between the two groups in favor of the real life context group in the learning scores during the acquisition phase, $t(30) = 2.1$, partial $\eta 2 = 0.13$, and also a significant effect on the test problems, $t(30) = 3.67$, partial $\eta 2 = 0.29$ favoring the real-life context group. The difference in effect sizes explains the significant interaction.

The means and standard deviations for efficiency are given in Table 1. There was a significant difference between the means of the two groups, $t(30) = 4.26$, partial $\eta 2 = 0.38$ in favor of the real-life context group.

We had hypothesized that by being able to use contextual information held in long-term memory (the environmental linking and organizing principle) students would be able to learn new mathematical concepts more readily than students required to use less familiar, geometrical, contextual information. These results support that hypothesis. They indicate that instruction using a familiar, real-life context was a much more efficient form of instruction than using a less familiar geometrical context.

Chapter 6: Experiment 2

6.1 Introduction

While the use of a concrete, realistic context proved superior in Experiment 1, it might be expected that there may be some conditions under which using a geometric context is beneficial. Ultimately, in mathematics, we require students to be able to use fractions using any appropriate materials including novel materials. Once a concept has been acquired using familiar material, subsequently presenting the information in geometric form may be of benefit. Experiment 2 tested this hypothesis by presenting information concerning fractions using familiar real world materials followed by the same information using geometric materials (the real-geometric group). In this experiment, three control groups were used: real-real, geometric-geometric and geometric-real.

6.2 Method

6.2.1 Participants:

Sixty Year five students (10-11 year old) from a Sydney, private school were allocated randomly into four equivalent groups of 15 students each: real-real, geometric-geometric, real-geometric, and geometric-real groups. Each group contained, approximately, the same number of boys and girls. Teacher information indicated that all students had approximately the same level of numeric ability.

6.2.2 Material and procedure

Students were tested on an individual basis. The experiment consisted of two phases; an acquisition phase and a test phase. For the acquisition phase, students in the real-real group were presented four pairs of problems, each pair consisting of a solved example followed by the similar problem to solve. For the real-real group all of these problems were presented in real-life contexts: dividing and sharing pizzas and cakes. For the geometric-geometric group, the four pairs of problems were presented in geometric contexts: dividing a circle or a rectangle into equal parts and filling in one or more parts (the same variations were used as in Experiment 1). The real-geometric group had their first two pairs of problems presented in a real-life context with the next two pairs presented in a geometric context. The geometric-real group was presented with two pairs of problems in a geometric context, followed by two pairs of problems presented in a real-life context. Each student had three minutes to study each worked example, and another three minutes to solve the associated problem. In total, each student was allocated 24 minutes to complete the four problems. Timing for the acquisition phase was the same for all groups. Table 2 explains the context sequence of each group.

Table 2: Context sequence for the four groups (acquisition phase)

Group/Worked-example	Worked-example 1	Worked-example 2	Worked-example 3	Worked-example 4
Real-Real	Real-Life context	Real-Life context	Real-Life context	Real-Life context
Geometric-Geometric	Geometric context	Geometric context	Geometric context	Geometric context
Real-Geometric	Real-Life context	Real-Life context	Geometric context	Geometric context
Geometric-Real	Geometric context	Geometric context	Real-Life context	Real-Life context

The four worked examples had approximately the same number of words and the same number of steps to be solved. The problem following each worked example was presented on a single sheet of paper with sufficient space to permit working. Learners were asked to follow the worked example when solving the problem. The worked example was available while solving the paired problem. If a student gave an incorrect solution within three minutes he or she was told to try again. If the correct solution was not obtained within three minutes, the student was given the solution with an explanation similar to the explanations used in the worked examples and then moved to the next pair. To measure mental effort, a nine-point scale was used with participants being asked ‘How easy or difficult to understand were the worked examples?’ see Figure 13. The time each student needed to reach the correct solution was measured up to the maximum of three minutes. Students were stopped at three minutes. An identical marking scheme to Experiment 1 was used, providing a score out of eight for each participant over the four acquisition phase problems. Worked examples pairs in a real-life context can be found in Appendix B1. Worked examples pairs in a geometric context are in Appendix B2.

The fractions compared in the learning phase were:

1/3 and 1/5

2/3 and 4/6

3/8 and 2/4

6/10 and 4/5

In the test phase, the four groups were given a common test to examine the effect of the acquisition phase. This test contained 10 context-free arithmetic problems of the type used in Experiment 1 to solve immediately after the acquisition period with up to two minutes for each problem (see Appendix A4). The time that each participant took to solve each problem was measured up to two minutes. If a student failed to solve a problem within two minutes, the next problem was presented. Solutions to unsolved problems were not provided until after the test was completed. Each problem was provided on a separate sheet of paper to be solved using pen and paper. Students were asked to write explanations concerning their solutions to avoid random solutions.

A marking scheme identical to Experiment 1 provided a score out of 20 for the 10 problems each participant was presented. No feedback during the test was given to participants until the whole test was completed. The worked examples sheet was not available to participants during the test-problem phase.

6.3 Results and Discussion

In the acquisition phase, the variables under analysis were cognitive load (mental effort), time spent to solve the problems associated with the worked examples, and learning score. In the test phase, the variables under analysis were time needed to solve the test problems and test score. Table 3 indicates the means and the standard deviations of each variable for each group.

Table 3: Means and standard deviations (in parentheses) for Experiment 2

Variable/Group	Real-Real	Real-Geometric	Geometric-Real	Geometric – Geometric
Cognitive load/ mental effort (out of 9)	2.42 (1.40)	3.33 (1.03)	4.13 (1.24)	3.83 (1.50)
Learning time (minutes)	11.73 (0.53)	11.66 (0.45)	11.73 (0.56)	11.90 (0.28)
Learning score (out of 8)	5.33 (1.72)	6.13 (1.68)	3.67 (2.06)	3.4 (2.61)
Test time (seconds)	508.2 (146.1)	511.53 (95.47)	645.20 (188.73)	576.2 (123.79)
Test score (out of 20)	12.67 (3.22)	14.67 (4.23)	9.8 (4.33)	8.73 (2.96)
Efficiency	0.863 (1.06)	0.705 (1.24)	-0.76 (1.09)	-0.80 (1.24)

A 2 (real vs. geometric1st) x 2 (real vs. geometric 2nd) ANOVA was run on the cognitive load (mental effort estimates). It indicated a significant difference between real and geometric presentations on the 1st two pairs, $F(1, 56) = 10.8$, $MSe = 27.3$, partial $\eta 2 = 0.16$. A significant difference between real and geometric presentations was not found for the 2nd two pairs $F(1, 56) = 0.83$, $MSe = 27.3$, nor was there an interaction between the two sub-phases, $F(1, 56) = 3.25$, $MSe = 27.3$.

With respect to the learning time variable a 2x2 ANOVA indicated no significant effect due to the 1st two pairs, $F(1, 56) = 0.93$, $MSe = 0.22$, 2nd two pairs, $F(1, 56) = 0.17$, $MSe = 0.22$, nor was there a significant interaction, $F(1, 56) = 0.93$, $MSe = 0.22$.

An identical 2x2 ANOVA was conducted on the learning score variable for the learning problems. It revealed a significant difference between real and geometric presentations of the 1st two pairs $F(1, 56) = 17.23$, $MSe = 4.12$, partial $\eta 2 = 0.24$, but no significant difference was found between the 2nd two pairs $F(1, 56) = 0.25$, $MSe = 4.12$. The interaction between the 1st and the 2nd two pairs was not significant, $F(1, 56) = 1.01$, $MSe = 4.12$.

An analysis on the test time variable indicated a significant main effect of the 1st two pairs in test times, $F(1, 56) = 7.5$, $MSe = 20351.33$, partial $\eta 2 = 0.12$, but neither a significant difference between the 2nd two pairs $F(1, 56) = 0.75$, $MSe = 2035.33$, nor a significant interaction between the two phases was found, $F(1, 56) = 0.96$, $MSe = 2035.33$.

A 2 (real vs. geometric on the 1st two pairs of the worked examples in the acquisition phase) x 2 (real vs. geometric on the 2nd two pairs of worked examples in the acquisition phase) ANOVA on the test score revealed a significant difference between the real and geometric presentation of the 1st two pairs (sub-phase 1), $F(1, 56) = 20.8$, $MSe = 13.96$, partial $\eta 2 = 0.27$. There was neither a significant difference between real and geometric presentation due to the 2nd two pairs (sub-phase 2), $F(1, 56) = 0.23$, $MSe = 13.96$, nor a significant interaction between the two sub-phases, $F(1, 56) = 2.52$, $MSe = 13.96$.

The instructional efficiency was calculated using Paas and van Merrienboer's (Paas & van Merrienboer, 1993; Paas et al., 1994) formula. The relative efficiency means and standard deviations are shown in Table 2. A 2(1st two pairs) x 2(2nd two pairs) ANOVA on the efficiency measures indicated a significant effect of the 1st two pairs, $F(1, 56) = 27.34$, $MSe = 1.35$, partial $\eta 2 = 0.33$. Neither a significant effect due to the 2nd two pairs, $F(1, 56) = 0.1$, $MSe = 1.35$, nor a significant interaction effect was found, $F(1, 56) = 0.44$, $MSe = 1.35$.

In summary, the results of the second experiment support the results of the first experiment. Those results indicate that worked examples are best presented in a real-life context. There was no evidence from Experiment 2 of a superiority of geometric presentation following a realistic presentation. All differences between groups depended on whether geometric or realistic materials were used in the first two pairs of examples and problems. There were no differences between geometric and realistic presentations when these were located in the second pair of examples and problems.

Chapter 7: Experiment 3

7.1 Introduction

The results of the second experiment showed that there was a significant difference in performance between the four groups with the Real-Real and Real- Geometric groups superior to the Geometric-Geometric and the Geometric-Real groups. The Geometric-Geometric and Geometric-Real groups had significantly lower test scores in the test phase and experienced more mental effort during the acquisition phase. Nevertheless, a significant difference between the Real-Real and the Real-Geometric test score was not obtained. However, the Real-Geometric group did not have significantly higher test scores than the Real-Real group. Therefore, Experiment 3 had two goals; firstly, to generalize the results further by testing whether the results of the previous experiments would still be obtained using word-based rather than diagrammatic worked examples, and secondly to test the hypothesis that a Real- Geometric condition will be superior to a Real-Real condition for more distant transfer problems.

7.2 Method

7.2.1 Participants

Forty five Year four students were assigned randomly to three groups of 15 students each: Real-Real, Real-Geometric, and Geometric-Geometric. Because this experiment was conducted at the end of the school year, Year four students had the same age range of 10-11 years as the previous experiments' Year five participants who were tested at the beginning of the school year. Since there were neither theoretical grounds nor empirical evidence for a Geometric-Real advantage, this combination was omitted.

7.2.2 Material and procedure

The procedure of this experiment was similar to the procedure of Experiment 2 except that for the learning phase, all worked examples were presented in a word-based format. No figures were used to present the examples. Students in each group were presented six pairs of worked examples, each pair consisting of a solved example followed by a problem to solve. Figure 14 shows a representative sample of the worked examples that were used in Real-Context condition. Figure 15 demonstrates a representative sample of worked examples that were used in Geometric-Context condition (the whole two sets are given in Appendix C1 and Appendix C2 respectively).

<table>
<tr><td>Problem 3</td><td colspan="2">Solution 3</td></tr>
<tr><td>Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?</td><td>Mum divided Mark’s cake into 3 equal parts. She said that Mark can eat 2 of these 3 parts.
Mark will eat 2/3 of his cake.

To sort out this situation mum halved Mark’s portion. He now can eat 4 pieces out of 6, instead of 2 out of 3 but that is still the same amount of cake.
He is allowed to eat 4/6 of his cake.</td><td>Mum divided Jack’s cake into 6 equal parts. She allowed Jack to eat 4 of these 6 parts: Jack will eat 4/6 of his cake.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: Mark is allowed to eat 2/3 of his cake, which is the same amount as 4/6 of the cake. Jack is allowed to eat 4/6 of the cake. Both cakes are the same size.
Conclusion: If we have 2/3 and we halve it we will get 4/6. Mark and Jack will eat the same amount of cake because both can eat 4/6 of the whole cake.</td></tr>
</table>

Figure 14: A representative example of real-context condition worked examples

<table>
<tr><td>Problem 3</td><td colspan="2">Solution 3</td></tr>
<tr><td>One group of children, who painted 2/3 of a rectangle blue, got into an argument with another group of children who painted 4/6 of a same size rectangle blue.
They said that the children in the second group got more blue than the first. Are they right?</td><td>The first group of children drew a rectangle and divided it into 3 equal parts.
They painted 2 of these parts blue.
They have 2/3 of this rectangle painted blue.
____
To sort out this argument, the teacher suggested that the first group halve the 3 parts of the rectangle.
They have now 4 blue parts out of 6.</td><td>The second group of children drew a rectangle.
They divided it into 6 equal parts.
They painted 4 out of these 6 parts blue.
They have 4/6 of the rectangle painted blue.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: In the first rectangle you have 2/3 of it painted blue, which is the same amount as 4/6. Both of them had 4/6 painted blue.
Conclusion: if we have 2/3 and we halve it we will have 4/6. It will still have the same painted area. So 2/3 is equal to 4/6.</td></tr>
</table>

Figure 15: A representative example of geometric-context condition worked examples

The fractions used in the worked examples during the acquisition phase were:

1/3 and 1/5; 1/5 and 1/9; 2/3 and 4/6; ½ and 2/4; 3/8 and 2/4; 6/10 and 4/5

For the test phase, students were presented 12 arithmetic problems classified into two categories: Eight near transfer problems, and four more distant transfer problems. An example of a near transfer problem is (the whole set of the test problems can be found in Appendix C3):

Is ¾ greater, less than or equal to 5/8?

To solve the near transfer problems, students needed to use similar procedures as in the learning phase such as numerically doubling the numerator and denominator of a given fraction to enable it to be compared with another given fraction with the same, larger numerical value in the denominator.

An example of a more distant transfer problem is:

How many ninths are equal to 2/3?

To solve the more distant transfer problems, procedures beyond the ones used in the acquisition phase were required such as combining fractions rather than re-dividing or enlarging the numbers of a fraction by a factor of three instead of two to enable comparisons.

7.3 Results and Discussion

In the acquisition phase, the variables under analysis were cognitive load (mental effort) that was imposed by the learning materials, learning time and learning score. In the test phase, the variables under analysis were test time and test score (the total test score), the near transfer questions score, the more distant transfer questions score and the instructional method efficiency. Table 4 indicates the means and the standard deviations of each variable for each group.

Table 4: Means and standard deviations (in parentheses) for Experiment 3

Variable/Group	Real- Real	Real-Geometric	Geometric-Geometric
Cognitive load (mental effort out of 9)	2.77 (1.24)	2.87 (1.40)	3.62 (1.79)
Learning time (in minutes)	16.67 (0.94)	14.67 (1.33)	16.57 (1.5)
Learning score (out of 12)	9.20 (2.21)	9.80 (2.04)	5.60 (2.58)
Test time	653.67 (249.23)	658.47 (232.78)	556.80 (232.06)
Global test score (out of 24)	15.93 (3.70)	18.27 (2.34)	9.87 (5.055)
Near transfer questions test score (out of 16)	11.40 (3.54)	13.73 (2.71)	7.80 (4.63)
More distant transfer questions test score (out of 8)	3.07 (2.22)	4.67 (1.34)	0.93 (0.96)
Efficiency	0.3948 (1.03)	0.7304 (0.75)	-1.1252 (1.62)

A one-way analysis of variance followed by Tukey post-hoc tests were used to analyse these variables. Tests on the cognitive load measure revealed no significant difference among the three groups, $F(2, 44) = 1.46$ $MSe = 80.54$. A one-way ANOVA on the learning time revealed a significant difference among the three groups, $F(2, 44) = 11.66$, $MSe = 1.63$, partial $\eta 2 = 0.36$. A Tukey post-hoc test indicated that the real-geometric group needed significantly less time than both the real-real group and the geometric-geometric group. Tests on the learning score revealed a significant difference among groups $F(2, 44) = 14.75$, $MSe = 5.25$, partial $\eta 2 = 0.41$. A Tukey post-hoc test indicated that both the real-real and the real-geometric groups did not differ significantly, but achieved significantly higher scores than the geometric-geometric group.

Variables analyzed in the test phase were the test time, the global test score (out of 24 based on 12 questions), the near transfer questions score (out of 16 based on eight questions) and the more distant transfer questions score (out of eight based on four questions). The scoring system, both in the acquisition and test phase, was identical to the one used in the previous experiments.

A one-way analysis of variance followed by Tukey post-hoc tests were run on the test time, the global test score, the near transfer test score and the more distant transfer test score. Tests on the test time variable revealed that the difference among the three groups was non significant. $F(2, 44) = 0.87$, $MSe = 56718.46$. Tests revealed a significant difference among the 3 groups on the global test score, F (2, 44) = 18.93 MSe = 14.89, partial $\eta 2$ = 0.47. A Tukey post-hoc showed that the real-real and the real-geometric group did not differ significantly, but performed significantly better in the test than the geometric-geometric group. Moreover, tests revealed a significant difference among the three groups on the near transfer test scores, F (2, 44) = 9.72 MSe = 13.78, partial $\eta 2$ = 0.32. A Tukey post-hoc test showed that the real-real and real-geometric groups did not differ significantly but obtained significantly higher scores than the geometric-geometric group in the near transfer questions.

The same analysis was conducted on the more distant transfer questions scores. It revealed a significant difference among the three groups, F (2, 44) = 20.62 MSe = 2.55, partial $\eta 2$ = 0.49. A Tukey post-hoc test indicated that all three groups differed significantly from each other with the real-geometric group having the highest score and the geometric-geometric having the lowest score.

Since the analysis conducted on the learning times revealed a significant difference among the three groups, favoring the real-geometric group, a substitute analysis of covariance (ANCOVA) was run on each variable to control for the effect of learning time. The analyses revealed that the covariate, learning time, was non-significantly related to other measures used in the ANOVA. An identical pattern of results was obtained when controlling for the effect of learning time as when running the previously indicated ANOVA’s.

These results are consistent with the results of the previous two experiments. The real-real and the real-geometric conditions performed significantly better when solving test problems. In the test problems, in this experiment, there is a transfer continuum. The first eight problems tested for near transfer while, the last four problems tested for more distant transfer. The superiority of a real-geometric condition over a real-real condition is significant when solving the more distant transfer problems. This experiment indicates that the superiority of a real-life over a geometric context can be obtained not only in a visual format but also in a wording-based format. It also indicates that when solving more distant transfer problems it is advantageous to commence with realistic problems before moving to geometric problems.

Chapter 8: Experiment 4

8.1 Introduction

Presenting worked examples in a real-life context proved to be a more efficient instructional method than presenting them in a geometric context, both in a diagrammatic and word-based format in the previous experiments. Furthermore, it was demonstrated that students perform better on more distant transfer problems if these realistic worked examples were followed by worked examples presented in a geometric context rather than having all of the worked examples in a realistic context only. The first three experiments were conducted using the worked-example method as an instructional technique. Testing the context effect under a different instructional method would strengthen the findings and generalize the results. Thus, Experiment 4 and Experiment 5 were designed to confirm and generalize the hypothesis that a real-life context facilitates learning fractions more than a geometric context, using problem solving as an instructional method. Experiment 4 was designed to examine the effect of using a real-life context compared with a geometric context as a cover story using problem solving rather than studying worked examples during instruction. It was hypothesized that a realistic context would have an equally beneficial effect on students' performance on the test problems following problem solving practice compared to practice studying worked examples.

8.2 Method

8.2.1 Participants

Sixty Year five (10-11 years old) students from a Sydney public school were assigned randomly to four equal groups, realistic problem solving, geometric problem solving, realistic worked examples, and geometric worked examples, of 15 students each (each group contained approximately an equal number of boys and girls).

8.2.2 Procedure

Each participant was given two booklets, an acquisition booklet at the beginning of the acquisition phase, and a test booklet when commencing the test phase. These booklets contained A4 sheets of paper with sufficient space for students to write their solutions.

For the acquisition phase, the problem solving groups' booklets (realistic problem solving and geometric problem solving) contained four pairs of problems to solve. Each pair contained two similar, but not identical, problems Similarity between problems sets was determined in terms of the schemas and procedures that need to be used to solve them. An example of one set of problems pairs is: 1/3, 1/5 and ¼, 1/6. Every aspect of the problems given to the two groups was identical except the context. The two booklets can be found in Appendix D3 and Appendix D4 respectively.

The two worked examples groups' booklets, realistic worked examples and geometric worked examples (The booklets can be seen in Appendix D1 and Appendix D2 respectively), contained four pairs of examples and problems consisting of an example followed by a similar, but not identical, problem to solve (in total, eight problems half of which consisted of worked examples). All problems, including the problems embedded in the worked examples, were identical to the problems presented to the problem solving groups.

The realistic groups had a realistic context, sharing a pizza or a cake as a cover story. The geometric groups, problems were in a geometric context, dividing and shading a geometric shape. All students in the two problem solving groups were given a maximum of three minutes to solve each problem. Following each problem, they were asked to rate how easy or difficult they found the problem on a nine-point scale provided on the next page. Next, they were given a minute to read a detailed solution provided on the next page.

Students in the worked examples groups were told to study the given worked example for three minutes and then were asked to rate the difficulty of understanding the worked example using the nine-point scale provided on the next page. After that, they had one minute to re-read the given solution. The procedure for each corresponding problem was identical to that for the problem solving groups. Thus, all participants in all of the four groups had an equivalent time for the acquisition phase. The students were told to turn the page over when their time was up. The learning booklets for all groups were designed to look similar in terms of length and space occupied.

The score on the problems in the acquisition phase for the two worked examples groups (realistic and geometric) was determined by giving two marks if a correct answer and a correct explanation were given, one marks if the correct answer was combined with an incorrect explanation or a failure to provide an explanation, and zero marks if an incorrect answer or no answer was given. Regarding the four problems, a maximum score of eight was therefore available for each student. The learning score for the two problem solving groups (realistic and geometric) was also out of eight, since only the scores of each second problem corresponding to the problem presented to the worked example groups was analyzed.

An example from the realistic worked example group booklet is shown in Figure 16 (the complete set can be found in Appendix D1):

Who will eat more pizza; Beverly who eats 3 slices of a large pizza divided into 6 equal parts (3/6 of one large pizza), or Clair who eats 2 slices of a large pizza divided into 3 equal parts (2/3 of one large pizza)?

Figure 16: A representative example of the real-life context problems

The respective problem in the geometric worked example booklet is shown in Figure 17 (the complete set can be found in Appendix D2)

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Figure 17

> What will have more red; one circle divided into 6 equal parts with 3 parts painted red (3/6 of a circle is painted red), or a circle divided into 3 equal parts with 2 parts painted red (2/3 of a circle is painted red)?

Figure 17: A representative example of the geometric context problems

The cognitive load average during the acquisition phase was calculated by adding up all ratings for all problems or worked examples pairs and then dividing the total by eight. The booklets for the test phase were identical for all groups. They contained 12 arithmetic problems, eight problems that test for near transfer followed by four more distant transfer problems (same problems as in the previous experiment). All participants were given two minutes to solve each problem. The experimenter told the students after a two minute period that they had to turn the page and start the next one. The complete set of test problems can be found in Appendix C3.

The test score was determined in the same way as the acquisition score, providing a score out of 16 for the near problems and a score out of eight for the more distant transfer problems.

8.3 Results and Discussion

In the acquisition phase, the variables under analysis were cognitive load and problem solving score. In the test phase, the variables under analysis were near transfer problems score and more distant transfer problems score. Efficiency was calculated according to Paas and van Merrienboer's (Paas & van Merrienboer, 1993; Paas et al., 1994) formula. The means and standard deviations are displayed in Table 5.

Table 5: Means and standard deviations (in parentheses) of the variables and instructional efficiency across groups for Experiment 4

Variable/Group	Realistic worked examples	Geometric worked examples	Realistic problem solving	Geometric problem solving
Acquisition cognitive load (/ 9)	3.08 (2.09)	3.00 (1.28)	2.61 (1.25)	2.41 (0.84)
Acquisition problems score(/8)	4.27 (2.50)	4.80 (1.67)	3.90 (1.44)	3.50 (1.19)
Near transfer problems score (/16)	11.20 (3.19)	10.26 (3.24)	10.20 (4.62)	11.40 (3.06)
More distant transfer problems score(/8)	5.40 (1.95)	2.93 (1.87)	4.33 (2.44)	2.40 (1.72)
Efficiency	0.35 (1.30)	-0.37 (1.02)	0.26 (1.11)	-0.240 (0.73)

A 2 (context-realistic vs. geometric) x 2 (method- worked examples vs. problem solving) ANOVA was conducted on the acquisition cognitive load. There was neither a significant main effect of the context, $F(1, 56) = 0.14$, $MSe = 2.07$, partial $\eta^2 = 0.002$ nor of the method on the average of cognitive load the students experienced in the acquisition phase, $F(1, 56) = 2.02$, $MSe = 2.07$, partial $\eta^2 = 0.035$. No interaction between the context and the method was found to be significant, $F(1, 56) = 0.03$, $MSe = 2.07$, partial $\eta^2 = 0.001$.

The same analysis was conducted on the acquisition score. It revealed no significant main effect of the context, $F(1, 56) = 0.06$, $MSe = 497.77$, partial $\eta^2 = 0.001$ nor was the method shown to have a significant main effect on the acquisition score, $F(1, 56) = 3.40$, $MSe = 497.77$, partial $\eta^2 = 0.06$. The interaction between the context and the method was found to be non-significant, $F(1, 56) = 0.82$, $MSe = 497.77$, partial $\eta^2 = 0.01$.

An identical 2 x 2 ANOVA was used to analyze the near transfer test score. Neither a significant context main effect, $F(1, 56) = 0.02$, $MSe = 12.84$, partial $\eta^2 = 0.00$ nor a significant method main effect was found, $F(1, 56) = 0.005$, $MSe = 12.84$, partial $\eta^2 = 0.00$. The interaction between the context and the method was also non-significant, $F(1. 56) = 1.33$, $MSe = 12.84$, $\eta^2 = 0.02$.

A 2 x 2 ANOVA was conducted on the more distant transfer test problem scores. There was a significant main effect of the context, $F(1, 56) = 17.87$, $MSe = 4.06$, partial $\eta^2 = 0.24$. However, neither a significant method main effect on the performance of these problems, $F(1, 56) = 2.36$, $MSe = 4.06$, partial $\eta^2 = 0.04$, nor a significant interaction between context and method was found, $F(1. 56) =0.26$, $MSe = 4.06$, partial $\eta^2 = 0.005$.

For the instructional efficiency, the same analysis was used. It revealed a significant main effect of the context, $F(1, 56) = 4.94$, $MSe = 1.13$, partial $\eta^2 = 0.08$. Neither a significant method main effect, $F(1, 56) = 0.003$, $MSe = 1.13$, partial $\eta^2 = 0.00$, nor an interaction was found $F(1, 56) = 0.17$, $MSe = 1.13$, partial $\eta^2 = 0.003$.

It was hypothesized that a context and a worked example effect such that students would benefit from real-life context and worked example conditions. The results supported the hypothesis partially. They indicated that the real-life context was beneficial only when students attempted more distant transfer problems but not when solving for nearer transfer problems. The reason could possibly be attributed to the fact that these students had previous knowledge concerning fractions. They had finished a textbook chapter on fractions a short time before conducting the experiment. The near transfer problems may have been too easy for students resulting in a ceiling effect. Furthermore, a worked example effect was not obtained in this experiment. This result can possibly be explained by the expertise reversal effect. The previous knowledge students had about fractions made them more experts in this field and thus the expertise reversal effect may have eliminated the worked example effect.

Chapter 9: Experiment 5

9.1 Introduction

In the previous experiment, a context effect was obtained favoring a realistic context over a geometric context in more distant transfer problem scores, but not in near transfer problem scores. As mentioned previously, the result could be due to the fact that the students had specific prior knowledge regarding fractions. Furthermore, a worked examples effect was not obtained in this experiment. Thus, Experiment 5 was designed to investigate if a context-method interaction could be obtained on more distant transfer as well as near transfer problem scores, if the participants had less prior knowledge. It was hypothesized that the students' performance would be affected by the context regardless of the technique used during the acquisition phase.

9.2 Method

9.2.1 Participants

Sixty Year four (9-10 years old) students from a Sydney public school were assigned randomly to four equal groups, realistic worked examples, geometric worked examples, realistic problem solving, and geometric problem solving, of 15 students each. Each group contained approximately the same number of males and females.

9.2.2 Procedure

The procedure was identical to that of the previous experiment.

9.3 Results and Discussion

The variables under analysis in the acquisition phase were the cognitive load mean, obtained by adding up all self rating outcomes, using a nine-point scale after each problem, and then dividing this total by the number of problems. The second variable to be analyzed in this phase was the acquisition score. In the test phase the variables under analysis were the near transfer problem test scores, the more distant transfer problem test scores, and the instructional efficiency. The means of all variables and standard deviations are displayed in Table 6

Table 6: Means and standard deviations (in parentheses) for all variables under analysis and instructional efficiency across groups, for Experiment 5

Variable/Group	Realistic worked examples	Geometric worked examples	Realistic problem solving	Geometric problem solving
Acquisition cognitive load (/ 9)	2.78 (1.64)	3.85 (1.19)	3.72 (1.52)	4.05 (2.21)
Acquisition problems score (/8)	4.67 (2.84)	2.27 (1.53)	4.07 (2.12)	3.47 (2.10)
Near transfer problems score (/16)	11.73 (4.73)	8.067 (4.16)	10.87 (3.04)	7.80 (3.74)
More distant transfer problems score (/8)	4.60 (2.26)	1.27 (1.44)	2.13 (1.46)	0.80 (.77)
Efficiency	1.14 (0.99)	-0.41 (0.72)	-0.07 (0.90)	-0.65 (1.00)

A 2 (context-realistic vs. geometric) x 2 (method- worked examples vs. problem solving) ANOVA was conducted on the acquisition cognitive load. There was neither a significant main effect of the context, $F(1, 56) = 2.61$, $MSe = 2.83$, partial $\eta^2 = 0.05$ nor of the method, $F(1, 56) = 1.69$, $MSe = 2.83$, partial $\eta^2 = 0.03$ on the average cognitive load the students experienced in the acquisition phase. The interaction between the context and the method was found to be non- significant, $F(1, 56) = 0.72$, $MSe = 2.83$, partial $\eta^2 = 0.01$.

The same analysis was conducted on the outcomes of the students' performance in the acquisition phase problems. It revealed a significant main effect of the context, $F(1, 56) = 10.17$, $MSe = 682.30$, partial $\eta 2 = 0.15$. Neither a main effect of the method, $F(1, 56) = 0.16$ $MSe = 682.30$, partial $\eta 2 = 0.00$ nor the interaction were found to be significant, $F(1, 56) = 3.07$ $MSe = 682.30$, partial $\eta 2 = 0.05$.

An identical analysis on the near transfer problem scores showed a significant main effect of the context, $F(1, 56) = 10.80$, $MSe = 15.75$, partial $\eta 2 = 0.16$. Neither a significant main effect of the method, $F(1, 56) = 0.30$, $MSe = 15.75$, partial $\eta 2 = 0.00$, nor of the interaction between context and method were found, $F(1, 56) = 0.08$, $MSe = 15.75$, partial $\eta 2 = 0.00$.

The context F (1, 56) = 32.98, MSe = 2.48, partial $\eta 2$ = 0.37 and the method, F (1, 56) = 13.03, MSe = 2.48, partial $\eta 2$ = 0.19, appeared to have a significant main effect on the students' performance when solving more distant transfer problems. The interaction between context and method was shown also to be significant F (1, 56) = 6.06, MSe = 2.48, partial $\eta 2$ = 0.10. Following the significant interaction, simple effect tests showed that there was a significant difference between the realistic and geometric contexts under worked example conditions, F (1, 59) = 7.31 MSe = 4.38, partial $\eta 2$ = 0.11 in favor of the realistic context, and a significant difference between realistic and geometric context under problem solving conditions, F (1, 59) = 4.03 MSe = 4.38, partial $\eta 2$ = 0.07 in favor of the realistic context. The significant interaction can be attributed to differential effect sizes.

To calculate instructional efficiency, the more distant transfer test problem scores were used since differences in the near transfer test problem scores were found to be non-significant. An analysis of instructional efficiency revealed a significant main effect of the context, F (1, 56) = 20.24, MSe = 24.08, partial $\eta 2$ = 0.27. Neither a significant main effect of method, F (1, 56) = 2.58, MSe = 24.08, partial $\eta 2$ = 0.04, nor a significant interaction was shown, F (1, 56) = 1.05, MSe = 24.08, partial $\eta 2$ = 0.02.

In this experiment it was hypothesized that the context will affect students' performance when learning the fraction concept regardless of the instructional method. The results support the hypothesis by indicating that in both near and more distant transfer test problems, students in the real-life context condition performed better than students in the geometric condition regardless of whether a worked example or a problem solving technique was used in the acquisition phase. Moreover, a worked example effect was obtained in this experiment.

Chapter 10: General Discussion

10.1 Summary of results

In a series of experiments, learning fractions was investigated from a cognitive load perspective. Cognitive load theory has provided considerable evidence that extraneous cognitive load can be reduced by using worked examples as an instructional method. Furthermore, cognitive load theory argues strongly in favor of relying on prior-knowledge to introduce novel material. Familiarity with the context of new material is assumed to connect to previously constructed and stored schemas, thus it is likely to facilitate learning particularly when dealing with complex, novel material.

This study examined whether presenting worked examples in a real-life context facilitates learning a complex concept. Since cognitive load theory suggests that the level of intrinsic cognitive load depends on the interaction between new material and a learner's previously stored knowledge, linking to-be-learned material with learners' prior-knowledge may play a crucial role in reducing the total cognitive load. Consequently, it was hypothesized that worked examples presented in a real-life context can connect to existing schemas from daily experiential practice and therefore enhanced learning can be predicted.

The series of experiments conducted in this study provided evidence of contextual effects on learning fractions. A real-life context was significantly more beneficial compared to a geometric context. Worked examples presented in real-life contexts, were a significantly more efficient instructional method in facilitating learning the fraction concept amongst grade five students, than the same examples presented in a geometric context.

In the first experiment, students reported less cognitive load and performed better in both the acquisition and the test phase when they learned to compare fractions using worked examples presented in a realistic context in contrast to students who practiced worked examples presented in a geometric context. The second experiment supported the results of the first experiment. In this experiment, the context sequence was tested. Four sequences were examined: a realistic context followed by a realistic context again, a realistic context followed by a geometric context, a geometric context followed by a realistic context, and a geometric context followed by a geometric context again. It was found that the differences among conditions were determined by the first presentation. There was no evidence to suggest that the context had an effect in the second stage of worked examples. However, the real-geometric group had the highest test score. This trend may have resulted from improved performance on more difficult test questions.

This hypothesis was examined in the third experiment in which a real-life context followed by a geometric context was examined, against a realistic followed by a realistic and a geometric followed by a realistic context, including test questions that test for more distant transfer. It was found that the real-real and the real-geometric conditions performed significantly better in the test problems than the geometric-real group. Moreover, the real followed by geometric context was shown to be more advantageous than the real-real context when testing for more distant transfer. Furthermore, the third experiment generalized the results to a word-based format. In the first two experiments, the worked examples were presented using a diagrammatic form whereas, in the third experiment a word-only format was used to present all worked examples in both contexts.

In the fourth and the fifth experiments another factor was tested. In both experiments, besides the context effect, an instructional method effect was investigated; worked examples versus problem solving. In Experiment 4, it was found that the real-life context groups performed better in more distant transfer problems but not in nearer transfer problems although a worked example effect could not be obtained. These results could be explained by a possible expertise reversal effect since participants in this experiment had learned about fractions a short time prior to running the experiment.

Therefore, in the fifth experiment the same hypotheses were examined using participants who did not have classroom-based prior-knowledge about fractions. In this experiment, both a context and a method effect were obtained. Students in real-life context groups outperformed students under geometric context conditions both when testing for nearer and further transfer problems regardless of the method used. Furthermore, when testing for further transfer problems, students who learned using worked examples were superior to students who had to solve problems in the acquisition phase. The five experiments' results were consistent and provided some evidence to indicate that a real-life context was advantageous when learning fractions.

10.2 The context effect

The major finding of the experiments of this thesis is that a fraction concept expressed in terms of familiar, real-life objects is more easily dealt with by children than when expressed in the more commonly used, geometrical representations. While children in upper primary school are likely to be familiar with circles and rectangles, they appear to be better able to think geometrically when dealing with round pizzas and rectangular cakes than circles and rectangles. When taught fractions, a fraction of a round pizza or rectangular cake is significantly easier to grasp by a child than a fraction of a circle or rectangle. For most educated adults, a 'circle' (which incorporates a higher order concept, Skemp, 1971, p.25) is just as real, and entrenched in long-term memory as a pizza (a round pizza is considered to incorporate a lower order concept than a circle, Skemp, 1971, p.25). Most adults are unlikely to have difficulty retrieving our schema for a circle from long-term memory and a quarter of a circle is just as meaningful as a quarter of a pizza (Skimp, 1971). For children learning fractions, their ability to conceptualize a quarter of a circle seems more difficult than conceptualizing a quarter of a pizza. Therefore, presenting new material in a familiar context can make a significant difference.

Furthermore, in Experiment 4 students benefited from a real-life context more than a geometric context only when solving more distant transfer problems. These results could possibly be attributed to the fact that participants had some previous knowledge about fractions since they learned fractions shortly before conducting the experiment. Students learned fractions using the traditional method of dividing and shading parts of geometric shapes. Therefore, these results could be explained by two factors. Firstly, the nearer transfer problems were easier than the further transfer problems and may have been too easy to generate cognitive load effects. Conversely, this issue did not affect the further transfer problems, where the superiority of students under a real-life context condition was found to be significant. Secondly, classroom prior-knowledge (which was presented in a more geometric context) might have interfered, thus eliminating a context effect.

These results are, to some extent, consistent with other studies that emphasize contextual significance from other perspectives such as Mack (1990; 1993), Empson (1999), Streefland (1991; 1993) and Gravemeijer (1999; Gravemeijer & Doorman, 1999). However, the results are not in complete agreement with Mack's, Empson's and Streefland's assertion that a sharing schema rather than context is the reason behind the effect. The results suggest that the difference is due to the context and this can be explained by two points: firstly, in these experiments sharing was used solely in one worked example (in Experiments 1 and 2) and in two worked examples out of six in Experiment 3, while in the rest of the worked examples an emphasis on dividing rather than on sharing was made in both contexts. Therefore, it can be suggested that it is not only a sharing schema, but also the context (whether realistic or geometric) that is likely to be the cause of difference.

It can be argued that a real-life context plays a role in activating a sharing schema, however, this can be true only in the sharing worked examples (only one worked example implied a sharing context in Experiments 1, 2, 4 and 5 and two in Experiment 3) not in the rest of the worked examples, and hence it can be argued that the obtained effect is more likely to be accredited to the context rather than a sharing schema. The second point that supports this assertion is that participants were tested on arithmetic problems not on sharing. The test problems were solely arithmetic and not associated with any context.

The main effect of this study was that a real-life familiar context facilitates the acquisition process and therefore enhances learning. These results are in concurrence with other studies' findings that emphasize the contextual role in learning mathematics such as Koedinger and Nathan (2004). In this study, money as a familiar context was used as a cover story for teaching algebraic word problems. Participants of that study who solved word problems associated with money as a context outperformed their counterparts who had to solve equivalent non-contextual equations. Furthermore, using familiar word problems to connect to learners' prior-knowledge formed the core of the 'Pittsburgh urban mathematics project' (Koedinger et al., 1997). This project was developed to teach algebra by designing word problems using cover stories from students' environmental contexts. The success of this project shows the importance of linking to familiar prior knowledge. Similarly, Goldstone and Sakamoto (2003) demonstrated that better abstract understanding was shown when concrete (real) rather than idealized (abstract) illustrations were used in a simulation. In addition, Rittle-Johnson and Koedinger (2005) in their study into fractions found that fewer errors were made when a real-world contextual scaffold was provided.

Nevertheless, children do need to learn to deal with fractions of circles and rectangles as abstract entities. Some evidence was obtained to suggest that presenting learners with more familiar, concrete examples first, followed by the more important, geometric versions, facilitated learning, especially when dealing with more difficult, transfer problems. Findings confirm that moving from familiar realistic to less familiar geometric objects is important, even when dealing with seemingly well-known geometric objects such as circles or rectangles. To some extent, this evidence is in line with other studies such as Goldstone and Son (2005) who found that a concrete fading strategy helped the abstraction of scientific principles compared to a concrete only strategy.

Familiarity with the context was explored from a cognitive load theory perspective as a factor that reduces cognitive load, and consequently elevates performance. This approach is entrenched in schema theory suggesting that once to-be-learned material is incorporated in a pre-existing schema, several elements of the new material can be processed as a single element in working memory, resulting in a low level of cognitive load (Carlson et al., 2003; Marcus et al., 1996; van Merrienboer et al., 2006; van Merrienboer et al., 2003). Findings are in complete agreement with this hypothesis and have provided further evidence to support it.

10.3 The worked example effect

The last two experiments of this thesis introduced further findings; in Experiment 4, the performance on the test problems of students who were presented worked examples in the acquisition phase was similar to students who were asked to solve the equivalent problems. In other words, a worked example effect was not obtained in this experiment. This result could possibly be explained by the expertise reversal effect (Kalyuga et al., 2003). Since students had learned about fractions a short time before participating in this experiment, they might have gained a sufficient level of expertise for worked examples to be partially redundant. Consequently, the worked example effect was eliminated.

Experiment 5 found a worked example effect. Students who learned to compare fractions by studying worked examples and then solving near transfer problems performed better in the test phase than students who learned using the problem solving conventional method. These findings are in accord with many studies in this field (e.g. Cooper & Sweller, 1987; Paas, 1992; Paas & van Merrienboer, 1994; Sweller & Cooper, 1985).

10.4 Theoretical issues

The results conform with the evolution-based cognitive architecture used recently by cognitive load theory. That architecture places a central emphasis on a very large store of information held in long-term memory – the information store principle. Information held in that store can be dealt with readily via the environmental organizing and linking principle. Information not well represented in that store must be dealt with by a limited working memory via the narrow limits of change principle. For children, round pizzas and rectangular cakes are well entrenched in the long-term memory allowing the environmental organizing and linking principle to closely align that knowledge in long-term memory with the external environment. In turn, that knowledge can be readily processed in working memory when learning to deal with fractions. In contrast, information concerning circles and rectangles may not be as well entrenched in long-term memory. Less emphasis can be placed on the environmental linking and organizing principle with commensurately more emphasis on the narrow limits of change principle exemplified by a limited working memory. That increased working memory load due to the use of abstract circles and rectangles interferes with learning.

By relying on the borrowing principle by using worked example as an instructional method, the imposed extraneous cognitive load was minimized increasing free working memory to handle the complex concepts. In addition, using a familiar, real-life context enabled a smooth alignment with previously stored knowledge and less weight was placed on the narrow limits of change principle.

10.5 Educational implications

In general, learning about fractions is best done in a familiar environment. From a practical perspective, until children's knowledge of circles and rectangles is as well entrenched as their knowledge of pizza and cake shapes, they should be taught fractions using more rather than less familiar concepts.

The results of this research might have more general educational implications. The use of worked examples has been well researched and found to be a beneficial instructional technique. Several factors have been found to contribute positively to well-designed worked examples. However, worked examples that incorporate a contextual factor have not been previously examined. This has been investigated here by using randomized, controlled experiments. In this research, the context appeared to have a considerable effect on learning the fraction concept. Distinctively, worked examples presented in a real-life context were found to be advantageous. Such worked examples reduce extraneous cognitive load because they connect to previously experienced and stored knowledge and activate previously constructed schemas, and hence they should decrease the level of intrinsic cognitive load as well as extraneous cognitive load.

This research emphasized the importance of connecting to prior-knowledge. This prior-knowledge could be used to enhance the learning process. Familiar contexts are highly beneficial in connecting to already stored schemas and activate these schemas to lessen the cognitive load. There was some evidence to indicate that a familiar daily life context, if used as a cover story for teaching fractions at an initial stage followed by a geometrical context afterwards, was beneficial. This evidence can be generalized to other material, particularly complex concepts that are characterized by their high-element interactivity across the domain of mathematics. This approach, based on cognitive load theory, has shown to be promising in a specific area. There may be grounds to suggest that the approach may be successfully implemented in other complex domains.

10.6 Limitations of the study

This study has some limitations. Firstly, only two contextual variations (pizzas and cakes as opposed to circles and rectangles) were used in the experiments. The results would have been more generalized if more contextual variations had been used.

Secondly, the experimenter had to repeatedly emphasize that the pizzas being dealt with were homemade pizzas and therefore they were not yet divided, unlike a normal purchased pizza which is usually divided into eighths. Thus, another context should have been chosen instead, such as a pie instead of a pizza. Moreover, it was assumed that children are familiar with the two chosen variations (pizzas and cakes). However, it would have been more informative if the degree of familiarity was examined and thus, further tests could have been conducted on the results.

Thirdly, more worked examples are needed to generate more general schemas, but the allocated time for each experiment was limited as a result of being conducted in schools.

Finally, the obtained results have to be interpreted cautiously. Since a school curriculum was run in parallel to the experiments, it was not possible to determine how students' school learning could have interfered with the content of the experiments.

Despite these limitations, there are grounds to argue that the results appear to have a solid theoretical base and significant educational implications. However, these results would be reinforced by further research.

10.7 Further research

The main aim of this study was to investigate the context effect. Cognitive load theory suggests several methods to handle extraneous cognitive load. The context effect in this research was examined using a worked examples method and in the last two experiments, worked example as opposed to problem solving. A further study to explore the context effect using other instructional methods that reduce extraneous cognitive load would be beneficial and would add to the generalization of these results across other instructional methods such as completion problems. In fact, using completion problems instead of worked examples or conventional problems could test for the completion problem effect and demonstrate that the current results hold across instructional techniques. Using goal-free problems might also attain the same objective.

In line with recent research into variability and germane cognitive load (van Merrienboer et al., 2006) it would be interesting to examine the context effect with methods that induce germane cognitive load. Van Merrienboer and his colleagues (van Merrienboer et al., 2006) suggested a training design approach intended to achieve transfer learning in complex tasks using methods that stimulate germane cognitive load. Variability and limited guidance were used in their study to induce germane cognitive load. A possible study might be to explore a variability effect using random versions of real-life contexts and random practice of geometric contexts versus a contextual fading strategy that starts with complete real-life contexts and ends with abstract contexts to achieve transfer learning using complex material. Results may show which variation is the most effective to balance intrinsic and germane cognitive load and enhance transfer learning.

The results were obtained by testing students immediately after the acquisition phase. It would be appropriate, therefore, to conduct a longitudinal study to confirm whether a long-term contextual effect can be obtained. In such a study, an intervention program could be created in two versions; one that incorporates a real-life context along with the program content, and a second one that uses a geometric context to teach fractions. In this case, more worked examples can be used, thus more general schemas might be generated, and hence an enduring contextual effect can be hypothesized. To examine a short-term effect, an immediate test can be conducted. Then, a follow up test after a period of time could be carried out to investigate a long-term contextual effect. However, it should be taken into account that while running this kind of program, students are expected to gain expertise; therefore, an adaptive program that decreases the guidance level as a function of the degree of expertise should be applied. In other words, a completion strategy would be an appropriate method for this program (Renkl & Atkinson, 2003; van Merrienboer, 1990).

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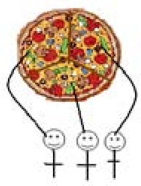
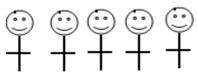
Appendix A1: Worked examples set for the realistic context condition used in Experiment 1

Problem 1 (real-life context group)	Solution 1 (real-life context group)	
Where will one get more pizza; at a table with three children sharing one large pizza, with each child getting 1/3, or at a table with five children sharing one large pizza with each child getting 1/5?	Draw a circle for one large pizza:  Draw 3 stick children  Divide this pizza into 3 equal parts.  Give each child one part. Each child has 1/3 of the pizza 	Draw another circle for another large pizza.  Draw 5 stick children:  Divide this pizza into 5 equal parts.  Give each child one part. Each child has 1/5 of the pizza 
Conclusion	Now compare: each child in the first situation gets more pizza than each child in the second situation. Conclusion: the less sharers, the more pizza each sharer gets. So one gets more pizza at a table with 3 children sharing one pizza, than at a table with 5 children sharing one pizza. Therefore, 1/3 is greater than 1/5.	

Similar problem

Now use the worked-example to solve this problem:

Where will one get more pizza; at a table with three children sharing one large pizza, with each child getting 1/3, or at a table with five children sharing one large pizza with each child getting 1/5?

Worked-example number 2: Real life context group

Problem 2 (real-life context group)	Solution 2 (real-life context group)	
Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?	Here is Mark's birthday cake:  Mum divided this cake into 3 equal parts. She said that Mark can eat 2 of these 3 parts:  Mark will eat 2/3 of his cake. Mark couldn't eat such big parts so mum halved his portion but, he insisted on eating all of it. Now it looks like this:  It is 4/6 of the cake.	Here is Jack's birthday cake:  Mum divided this cake into 6 equal parts. She allowed Jack to eat 4 of these 6 parts:  Jack will eat 4/6 of his cake
Conclusion	Now compare: Mark is allowed to eat 2/3 of his cake, which is the same amount as 4/6 of the cake. Jack is allowed to eat 4/6 of the cake. Both cakes are the same size. Conclusion: If we have 2/3 and we halve it we will got 4/6. Mark and Jack will eat the same amount of cake because both can eat 4/6 of the whole cake.	

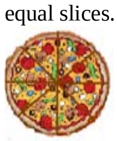
Similar problem

Now use the worked example to solve this problem

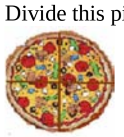
Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake. They both have the same size birthday cakes. Who do you think will eat more cake?

Worked-example number 3: Real life context group

<table>
<tr><td>Problem 3</td><td colspan="2">Solution 4</td></tr>
<tr><td>Who will eat more pizza; Sam who eats 3 slices of large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (3/4 of one large pizza)?</td><td>Draw one large pizza.

Divide this pizza into 8 equal slices.

Mum said that Sam can eat 3 slices, which is 3/8 of the pizza.
He will eat this portion:

Sam will eat 3/8.</td><td>Draw another large pizza.

Divide this pizza into 4 equal slices.

Mum said that Tim can eat 2 slices of the four slices. This is his portion: Tim will eat 2/4.

Before eating, a few friends came and mum re-divided the pizza into 8 parts, but she said that Tim still can eat the same amount of pizza.
Now Tim's portion will look like this:

Tim can eat 4 slices of 8 (4/8) which is the same amount as 2 slices of 4 (2/4).</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: Sam will eat 3/8 but, Tim will eat 2/4 which is the same as 4/8.
Conclusion: Sam will eat less pizza than Tim.
So, 3/8 is less than 2/4.</td></tr>
</table>

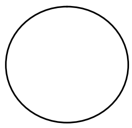
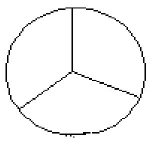
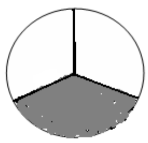
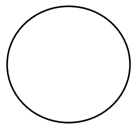
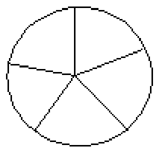
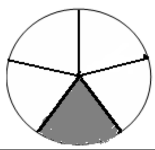
Similar problem

Now use the worked-example to solve a similar problem

Who will eat more pizza; Sam who eats 3 slices of large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (3/4 of one large pizza)?

Appendix A2: Worked examples set for the geometric context condition used in Experiment 1

Worked-example number 1- Geometric context group

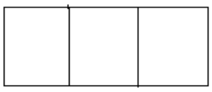
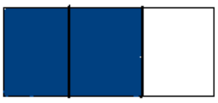
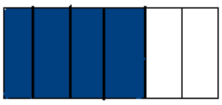
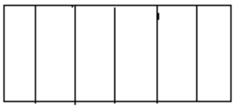
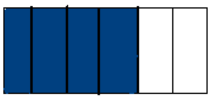
Problem 1	Solution 1	
Which results is a larger shaded area; if you divide a circle into three parts and shade one part resulting in 1/3 of the circle being shaded or if you divide a circle into five parts and shade one part resulting in 1/5 of the circle being shaded?	Draw a circle:  Divide this circle into 3 equal parts  Shade one part of these three parts. You have 1/3 of the circle shaded. 	Draw another circle:  Divide this circle into 5 equal parts.  Shade one part of these five parts. You have 1/5 of the circle shaded. 
Conclusion	Now compare: you have more shading in the first circle than in the second circle. Conclusion: The less parts, the more the shaded area. So a circle divided into 3 parts with 1 part shaded has more shading than a circle divided into 5 parts with 1 part shaded. Therefore, 1/3 is greater than 1/5.	

Similar problem

Now use the worked-example to solve a similar problem

Which results is a larger shaded area; if you divide a circle into three parts and shade one part resulting in 1/3 of the circle being shaded or if you divide a circle into five parts and shade one part resulting in 1/5 of the circle being shaded?

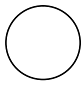
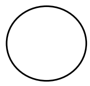
Worked-example number 2: Geometric context group

Problem 2	Solution 2	
One group of kids, who painted blue 2/3 of a rectangle, got into an argument with another group of kids who painted blue 4/6 of a same size rectangle. They said that the kids in the second group got more blue than the first. Are they right?	Draw a rectangles :  Divide this rectangle into 3 equal parts  Paint 2 of these 3 parts blue.  You have 2/3 painted blue. Re-divide this rectangle(add two more lines):  You have the same area, 4/6 painted blue.	Draw another rectangle :  Divide this rectangle into 6 equal parts.  Paint 4 of these 6 parts blue.  You have 4/6 painted blue.
Conclusion	Now compare: In the first rectangle you have 2/3 of it painted blue, which is the same amount as 4/6. Both of them had 4/6 painted blue. Conclusion: if we have 2/3 and we halve it we will got 4/6. It will still be the same painted area. So 2/3 is equal to 4/6.	

Similar problem
Now use the worked-example to solve a similar problem

One group of kids who painted blue 2/3 of a rectangle, got into an argument with another group of kids who painted blue 4/6 of a same size rectangle. They said that the kids in the second group got more blue than the first. Are they right?

Worked-example number 3: Geometric context group

Problem 3	Solution 3	
What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red) ?	Draw a circle:  Divide this circle into 8 equal parts  Paint three parts of these eight parts red.  You have 3/8 painted red.	Draw another circle:  Divide this circle into 4 equal parts.  Paint two parts of these four parts red.  You have 2/4 painted red. Now, repeat halving this circle (add 2 lines)  Now you have a circle divided into 8 parts, 4 of them painted red. You have 4/8 painted red which is the same area as 2/4.
Conclusion	Now compare: In the first circle you have 3/8 painted red whereas, in the second circle you have 2/4 painted red which is identical to 4/8 painted red. Conclusion: we have less red area in the first circle than the second circle. So, 3/8 is less than 2/4.	

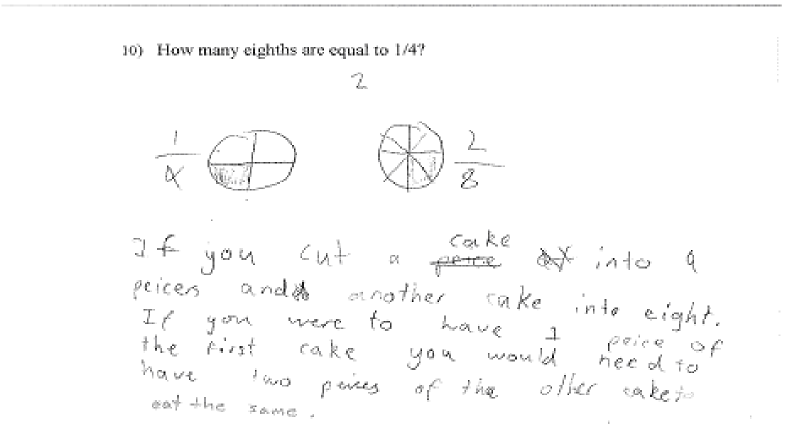
Similar problem

Now use the worked-example to solve a similar problem

What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?

Appendix A3: Samples of students' explanations

A correct answer with a correct explanation:

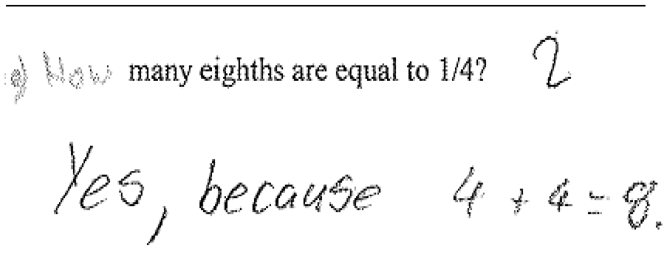

10) How many eighths are equal to 1/4?

2

$\frac{1}{4}$ $\frac{2}{8}$

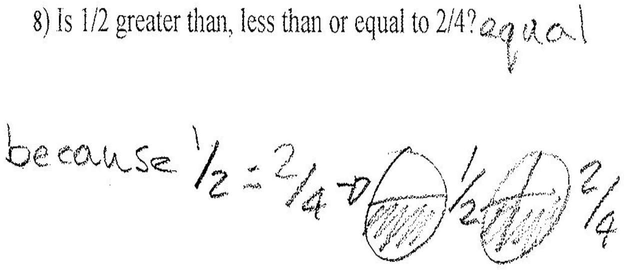
If you cut a ~~peice~~ cake into 4 peices and another cake into eight. If you were to have 1 peice of the first cake you would need to have two peices of the other cake to eat the same.

A correct answer with an incorrect explanation:


How many eighths are equal to 1/4? 2

Yes, because 4 + 4 = 8.

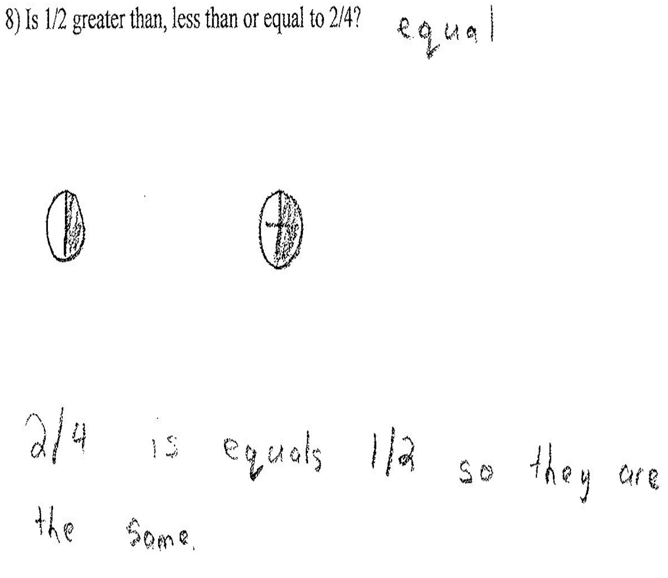
A correct answer with a correct explanation:



8) Is 1/2 greater than, less than or equal to 2/4? equal

because 1/2 = 2/4 → 1/2 2/4

A correct answer that lacks an explanation:



8) Is 1/2 greater than, less than or equal to 2/4? equal

2/4 is equals 1/2 so they are the same.

A correct answer with an incorrect explanation

8) Is 1/2 greater than, less than or equal to 2/4?

$$\frac{1+1}{2+2} = \frac{2}{4}$$

An incorrect answer with an incorrect explanation:

8) 1/2 greater than, less than or equal to 2/4?

No 1/2 is bigger than 2/4 because you get more of the pizza.

A correct answer with a correct explanation:

5) Is 1/3 greater than, less than or equal to 1/8? 1/3 is greater.

1/3 1/8

The less shares the greater the amount.

Appendix A4: Test problems for Experiments 1 and 2

1. Draw a diagram to show 1/3 then draw another diagram to show a fraction larger than 1/3

2. Draw a diagram to show 1/2, and then draw another diagram to show a fraction equal to 1/2.

3. Draw a diagram to show 3/4 then draw another diagram to show a fraction less than 3/4.

4. Is 1/5 greater than, less than or equal to 1/6?

5. Is 1/3 greater than, less than or equal to 1/8?

6. Is 2/3 greater than, less than or equal to 3/4?

7. Is 5/6 greater than, less than or equal to 2/3?

8. Is 1/2 greater than, less than or equal to 2/4?

9. Is 3/4 greater than, less than or equal to 6/8?

10. How many eighths are equal to 1/4?

Appendix B1: Worked example for real-life context conditions used in Experiment 2

The first three worked example were identical to the worked-examples used in Experiment 1 (see Appendix A). The fourth worked example is as follows:

Worked example number 4- Real-life context group

<table>
<tr><td>Problem 4</td><td colspan="2">Solution 4</td></tr>
<tr><td>Does one have more pizza, if one eats 6 slices from a large pizza divided into 10 equal slices, or if one eats 4 slices from a large pizza divided into 5 equal slices?</td><td>Draw one large pizza:

Divide this pizza into 10 equal slices:

Here one will eat 6 slices of 10, meaning 6/10 of a large pizza:
</td><td>Draw another large pizza:

Divide this pizza into 5 equal parts.

Here one will eat 4 slices of 5, meaning 4/5 of a large pizza:

Try to halve these parts, and you will have:

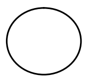
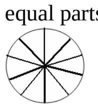
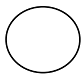
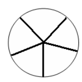
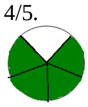
You have 8 slices of large pizza divided into 10 equal parts.
You have 8/10 of a large pizza.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: in the first situation one eats 6 slices of 10 (6/10 of a pizza).In the second situation one eats 4 slices of 5 (4/5 of a pizza) which is equal to 8 slices of 10 (8/10 of pizza).
Conclusion: One can eat less from the first pizza than the second one.
Solution: 6/10 is less than 4/5.</td></tr>
</table>

Similar problem
Now use the worked-example to solve a similar problem

Does one have more pizza, if one eats 6 slices from a large pizza divided into 10 equal slices, or if one eats 4 slices from a large pizza divided into 5 equal slices?

Appendix B2: Worked example for geometric context conditions used in Experiment 2

The first three worked example were identical to the worked-examples used in Experiment 1 (see Appendix A2).The fourth worked example is as follows:

Problem 4	Solution 4	
Who will have more green; a child who drew a circle, divided it into ten equal parts, and painted six parts green, or a child who drew a circle, divided it into five equal parts and painted four parts green?	Draw a circle:  Divide this circle into 10 equal parts:  Paint 6 of these ten parts green. 	Draw another circle:  Divide this circle into 5 equal parts.  Paint 4 of these five parts green. You have 4/5.  Halve each part of this circle and you will get:  Now you have 8 of 10 parts painted green. You have 8/10 painted green.
Conclusion	Now compare: in the first circle you have 6/10 painted green, in the second circle you have 4/5 painted green which is equal to 8/10 painted green. Conclusion: In the first circle you have less green than the second circle. Solution: 6/10 is less than 4/5.	

Similar problem

Now use the worked-example to solve a similar problem

Who will have more green; a child who drew a circle, divided it into ten equal parts and painted six parts green, or a child who drew a circle, divided it into five equal parts and painted four parts green?

Appendix C1: Worked examples used for real-life context conditions used in Experiment 3

Problem 1	Solution 1	
Where will one get more pizza; at a table with three children sharing one large pizza, with each child getting 1/3, or at a table with five children sharing one large pizza with each child getting 1/5?	When sharing one large pizza among 3 children, each one will get 1/3.	When sharing one large pizza among 5 kids, each one will get 1/5.
Conclusion	Now compare: each child in the first situation gets more pizza than each child in the second situation. Conclusion: the less sharers, the more pizza each sharer gets. So one gets more pizza at a table with 3 children sharing one pizza, than at a table with 5 children sharing one pizza. Therefore, 1/3 is greater than 1/5.	

Similar problem

Now use the worked-example to solve a similar problem

Where will one get more pizza; at a table with three children sharing one large pizza, with each child getting 1/3, or at a table with five children sharing one large pizza with each child getting 1/5?

Worked example number 2- Real life context group

Problem 2	Solution 2	
Where will one get more pizza; at a table with five children sharing one large pizza, with each child getting 1/5, or at a table with nine children sharing one large pizza with each child getting 1/9?	When sharing one large pizza among 5 children, Each one gets 1 slice out of 5 equal slices. Each child gets 1/5.	When sharing one large pizza among 9 children, Each one gets one slice out of 9 equal slices. Each child gets 1/9.
Conclusion	Now compare: each child in the first situation gets more pizza than each child in the second situation. Conclusion: the less sharers, the more pizza each sharer gets. So one gets more pizza at a table with 5 children sharing one pizza, than at a table with 9 children sharing one pizza. Therefore, 1/5 is greater than 1/9.	

Similar problem
Now use the worked-example to solve a similar problem

Where will one get more pizza; at a table with five children sharing one large pizza, with each child getting 1/5, or at a table with nine children sharing one large pizza with each child getting 1/9?

Worked example number 3- Real life context group

Problem 3	Solution 2	
Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?	Mum divided Mark’s cake into 3 equal parts. She said that Mark can eat 2 of these 3 parts. Mark will eat 2/3 of his cake. To sort out this situation mum halved Mark’s portion. He now can eat 4 pieces out of 6, instead of 2 out of 3 but that is still the same amount of cake. He is allowed to eat 4/6 of his cake.	Mum divided Jack’s cake into 6 equal parts. She allowed Jack to eat 4 of these 6 parts: Jack will eat 4/6 of his cake.
Conclusion	Now compare: Mark is allowed to eat 2/3 of his cake, which is the same amount as 4/6 of the cake. Jack is allowed to eat 4/6 of the cake. Both cakes are the same size. Conclusion: If we have 2/3 and we halve it we will get 4/6. Mark and Jack will eat the same amount of cake because both can eat 4/6 of the whole cake.	

Similar problem

Now use the worked-example to solve a similar problem

Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?

Worked example number 4- Real life context

Problem 4	Solution 4	
Who will eat more pizza; Sam who eats 1 slice of a large pizza divided into 2 equal parts (1/2 of one large pizza) , or Tim who eats 2 slices of large pizza divided into 4 equal parts (2/4 of one large pizza)?	Sam’s pizza is divided into 2 equal parts. He can eat 1 of these 2. He can eat ½ of the pizza. To find out which portion is a bigger, Sam halves his pizza again, now he has 4 equal parts and his portion is 2 out of these 4. Sam’s portion is 2/4 of the pizza.	Tim’s pizza is divided into 4 equal parts. He can eat 2 out of these 4 parts. He can eat 2/4 of the pizza.
Conclusion	Now compare: Tim will eat 2/4 of a pizza but Sam will eat 1/2 which is the same as 2/4. Conclusion: Sam will eat the same amount of pizza as Tim. So, 1/2 is the same as 2/4.	

Similar problem

Now use the worked-example to solve a similar problem

Who will eat more pizza; Sam who eats 1 slice of a large pizza divided into 2 equal parts (1/2 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (2/4 of one large pizza)?

Worked example number 5- Real life context group

Problem 5	Solution 5	
Who will eat more pizza; Sam who eats 3 slices of a large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (2/4 of one large pizza)?	Sam's pizza is divided into 8 slices. He can eat 3 slices out of these 8. He can eat 3/8 of the pizza.	Tim's pizza is divided into 4 equal slices. He can eat 2 slices out of these 4. He can eat 2/4 of the pizza. To figure out this problem mum halved Tim's pizza again. Now it is divided into 8 equal slices. But still Tim can eat the same portion. Tim's portion now is 4 slices out of 8, 4/8 of the pizza.
Conclusion	Now compare: Sam will eat 3/8 but Tim will eat 2/4 which is the same as 4/8. Conclusion: Sam will eat less pizza than Tim. So, 3/8 is less than 2/4.	

Similar problem
Now use the worked-example to solve a similar problem

Who will eat more pizza; Sam who eats 3 slices of a large pizza divided into 8 equal parts (3/8 of one large pizza), or Tim who eats 2 slices of large pizza divided into 4 equal parts (2/4 of one large pizza)?

Worked example number 6- Real life context group

Problem 6	Solution 6	
Does one have more pizza, if one eats 6 slices from a large pizza divided into 10 equal slices, or if one eats 4 slices from a large pizza divided into 5 equal slices?	In the first situation: One can eat 6 slices out of 10 from a large pizza divided into 10 equal slices. One eats 6/10 of the pizza.	In the second situation: One can eat 4 out of 5 slices from a large pizza divided into 5 equal slices. One eats 4/5 of the pizza. To figure out which portion is more, We can halve each of the slices of the second pizza, Instead of 4 out of 5 We will have now 8 out of 10 Instead of 4/5 now we have 8/10 but still it is the same amount of pizza.
Conclusion	Now compare: in the first situation one eats 6 slices of 10 (6/10 of a pizza). In the second situation one eats 4 slices of 5 (4/5 of a pizza) which is equal to 8 slices of 10 (8/10 of pizza). Conclusion: One can eat less from the first pizza than the second one. Solution: 6/10 is less than 4/5.	

Similar problem
Now use the worked-example to solve a similar problem

Does one have more pizza, if one eats 6 slices from a large pizza divided into 10 equal slices, or if one eats 4 slices from a large pizza divided into 5 equal slices?

Appendix C2: Worked examples used for the geometric context conditions used in Experiment 3

Problem 1	Solution 1	
Which results in a larger colored area; if you divide a circle into three parts and color one part resulting in 1/3 of the circle being colored or if you divide a circle into five parts and color one part resulting in 1/5 of the circle being colored?	When dividing a circle into 3 equal parts and coloring in one of these parts, you will have 1/3 of the circle colored in.	When dividing a circle into 5 equal parts and coloring in one of these parts, you will have 1/5 of the circle colored in.
Conclusion	Now compare: you have more color in the first circle than in the second circle. Conclusion: The less parts, the more the colored area. So a circle divided into 3 parts with 1 part colored has more color than a circle divided into 5 parts with 1 part colored. Therefore, 1/3 is greater than 1/5.	

Similar problem
Now use the worked-example to solve a similar problem

Which results in a larger colored area; if you divide a circle into three parts and color one part resulting in 1/3 of the circle being colored or if you divide a circle into five parts and color one part resulting in 1/5 of the circle being colored?

Worked example number 2- Geometric context

Problem 2	Solution 2	
Which results in a larger colored area; if you divide a circle into five equal parts and color one part resulting in 1/5 of the circle being colored or if you divide a circle into nine equal parts and color one part resulting in 1/9 of the circle being colored?	When dividing a circle into 5 equal parts, you are coloring in 1 of these 5 parts. Resulting in 1/5 of the circle being colored in.	When dividing a circle into 9 equal parts and coloring in 1 of these 9 parts. Resulting in 1/9 of the circle being colored in.
Conclusion	Now compare: you have more color in the first circle than in the second circle. Conclusion: The less parts, the more the colored area. So a circle divided into 5 equal parts with 1 part colored has more color than a circle divided into 9 equal parts with 1 part colored. Therefore, 1/5 is greater than 1/9.	

Similar problem
Now use the worked-example to solve a similar problem

Which results in a larger colored area; if you divide a circle into five equal parts and color one part resulting in 1/5 of the circle being colored or if you divide a circle into nine equal parts and color one part resulting in 1/9 of the circle being colored?

Worked example number 3- Geometric context

Problem 3	Solution 3	
One group of children, who painted 2/3 of a rectangle blue, got into an argument with another group of children who painted 4/6 of a same size rectangle blue. They said that the children in the second group got more blue than the first. Are they right?	The first group of children drew a rectangle and divided it into 3 equal parts. They painted 2 of these parts blue. They have 2/3 of this rectangle painted blue. To sort out this argument, the teacher suggested that the first group halve the 3 parts of the rectangle. They have now 4 blue parts out of 6.	The second group of children drew a rectangle. They divided it into 6 equal parts. They painted 4 out of these 6 parts blue. They have 4/6 of the rectangle painted blue.
Conclusion	Now compare: In the first rectangle you have 2/3 of it painted blue, which is the same amount as 4/6. Both of them had 4/6 painted blue. Conclusion: if we have 2/3 and we halve it we will have 4/6. It will still have the same painted area. So 2/3 is equal to 4/6.	

Similar problem
Now use the worked-example to solve a similar problem

One group of children, who painted 2/3 of a rectangle blue, got into an argument with another group of children who painted 4/6 of a same size rectangle blue. They said that the children in the second group got more blue than the first. Are they right?

Worked example number 4- Geometric context

Problem 4	Solution 3	
What will have more red; one circle divided into 2 equal parts with 1 part painted red (1/2 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?	The first circle is divided into 2 equal parts. 1 part is painted red. ½ of the circle is painted red. To find out which circle has more red, we halve each of the 2 parts to divide the circle into 4 equal parts. Now you have a circle divided into 4 equal parts with 2 of them painted red. You have 2/4 painted red, which is the same area as ½.	The second circle is divided into 4 equal parts. 2 parts are painted red. 2/4 of the circle is painted red.
Conclusion	Now compare: In the second circle you have 2/4 painted red whereas, in the first circle you have 1/2 painted red which is identical to 2/4 painted red. Conclusion: we have the same red area in the two circles. So, 1/2 is equal to 2/4.	

Similar problem
Now use the worked-example to solve a similar problem:

What will have more red; one circle divided into 2 equal parts with 1 part painted red (1/2 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?

Worked example number 5- Geometric context

Problem 5	Solution 5	
What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?	The first circle is divided into 8 equal parts. We paint 3 parts out of these 8 red. We have 3/8 of a circle painted red.	The second circle is divided into 4 equal parts. We paint 2 parts out of these 4 red. We have 2/4 of this circle painted red To compare between the red areas in the two circles, we halve each of the 4 parts to give 8 equal parts. Now the red area will be 4 out of 8 parts. It is 4/8 of the circle.
Conclusion	Now compare: In the first circle you have 3/8 painted red whereas, in the second circle you have 2/4 painted red which is identical to 4/8 painted red. Conclusion: we have less red area in the first circle than the second circle. So, 3/8 is less than 2/4.	

Similar problem
Now use the worked-example to solve a similar problem

What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?

Worked example number 6- Geometric context

Problem 6	Solution 6	
Who will have more green; a child who drew a circle, divided it into ten equal parts and painted six parts green, or a child who drew a circle, divided it into five equal parts and painted four parts green?	In the first situation: Dividing a circle into 10 equal parts and painting 6 of these parts green will result in 6/10 of the circle area being green.	In the second situation: Dividing a circle into 5 equal parts and painting 4 out of these 5 parts green will result in 4/5 of the circle area being green. To figure out which area is larger, We halve each part of the second circle by adding 5 more lines, resulting in 10 parts. 8 of the 10 parts are green so 8/10 of the second circle is green.
Conclusion	Now compare: in the first circle you have 6/10 painted green, in the second circle you have 4/5 painted green which is equal to 8/10 painted green. Conclusion: In the first circle you have less green than the second circle. Solution: 6/10 is less than 4/5.	

Similar problem
Now use the worked-example to solve a similar problem

Who will have more green; a child who drew a circle, divided it into ten equal parts and painted six parts green, or a child who drew a circle, divided it into five equal parts and painted four parts green?

Appendix C3: Test problems used in Experiments 3, 4 and 5

(Near Transfer problems are 1 to 8, more distant transfer problems are 9 to 12)

1. Draw a diagram to show 1/3 then draw another diagram to show a fraction larger than 1/3.
2. Draw a diagram to show 3/4 then draw another diagram to show a fraction less than 3/4.
3. Draw a diagram to show two equal fractions.
4. Is 1/4 greater, less than or equal to 1/5?
5. Is 2/5 greater, less than or equal to 4/10?
6. Is ¾ greater, less than or equal to 5/8?
7. Is 2/3 greater, less than or equal to 5/6?
8. Is ¾ greater than, less than or equal to 6/8?
9. How many ninths are equal to 2/3?
10. How many tenths are equal to 2/5?
11. How many thirds are equal to 2/6?
12. Is 3/5 greater, less than or equal to 4/7?

Appendix D1: The complete learning booklet of the realistic worked examples condition used in Experiments 4 and 5

Cover Page

Learning booklet
Worked examples group- Realistic context (RWE)

In this booklet you have 4 pairs of worked examples.
You need to study each one carefully because you need to solve a similar problem in the next page.

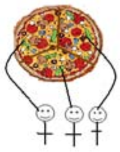
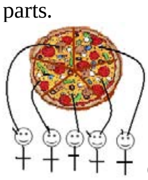
When you solve, you need to explain your answer.

You have 3 minutes to study the worked example and 3 minutes to solve the similar problem following it.
You need to write on the same sheet of paper.
Please read the question carefully, solve as accurately and as neatly as you can.
You do NOT need to write your name
This is your number please keep it to the next stage.

N.B- In this study we are talking about a home made pizza, which we can cut into as many slices as we like ☺

Boy or Girl Group- RWE Number _______

Page 1

Problem 1 (real-life context group)	Solution 1 (real-life context group)	
Where will one get more pizza; at a table with three children sharing one large pizza, with each child getting 1/3, or at a table with five children sharing one large pizza with each child getting 1/5?	Draw a circle for one large pizza:  Draw 3 stick children: Divide this pizza into3 equal parts:  Give each child one part. Each child has 1/3 of the pizza 	Draw another circle for another large pizza.  Draw 5 stick children: Divide th~~is pi~~zza into 5 equal parts.  Give each child one part. Each child has 1/5 of the pizza 
Conclusion	Now compare: each child in the first situation gets more pizza than each child in the second situation. Conclusion: the less sharers, the more pizza each sharer gets. So one gets more pizza at a table with 3 children sharing one pizza, than at a table with 5 children sharing one pizza. Therefore, 1/3 is greater than 1/5.	

Page 2

Now you have 1 minute to read the answer again

Solution:

You will get more pizza at a table with 3 children sharing one large pizza (1/3), than at a table with 5 children sharing one large pizza (1/5) because the fewer the number of sharers, the more pizza a person can get.

<u>Conclusion</u>: 1/3 is greater than 1/5

Page 3

Solve a similar problem, use the example! (You have 3 minutes)

Where will you get more pizza; at a table with 4 children sharing one large pizza, with each child getting 1/4, or at a table with 6 children sharing one large pizza with each child getting 1/6?

Solution:

Write your final answer in the box
Score_________

Page 4

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extremely Easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Worked example number 1									

Page 5

Now you have 1 minute to read the correct answer

Solution:
You will get more pizza at a table with 4 children sharing one large pizza (1/4), than at a table with 6 children sharing one large pizza (1/6) because the fewer the number of sharers, the more pizza a person can get.

Conclusion: ¼ is greater than 1/6

Page 6

Problem 2	Solution 2	
Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?	Here is Mark's birthday cake:  Mum divided this cake into 3 equal parts. She said that Mark can eat 2 of these 3 parts:  Mark will eat 2/3 of his cake. Mark couldn't eat such big parts so mum halved his portion but, he insisted on eating all of it. Now it looks like this:  It is 4/6 of the cake.	Here is Jack's birthday cake:  Mum divided this cake into 6 equal parts. She allowed Jack to eat 4 of these 6 parts:  Jack will eat 4/6 of his cake
Conclusion	Now compare: Mark is allowed to eat 2/3 of his cake, which is the same amount as 4/6 of the cake. Jack is allowed to eat 4/6 of the cake. Both cakes are the same size. Conclusion: If we have 2/3 and we halve it we will got 4/6. Mark and Jack will eat the same amount of cake because both can eat 4/6 of the whole cake.	

Page 7

Now you have 1 minute to read the answer again

Solution

Mark can eat two pieces of his cake which was divided into 3 equal pieces (2/3). If we divide again the whole cake, cut each piece into two pieces, Mark's portion can be seen as 4 pieces out of 6 (4/6) which is the same as Jack's portion.

Conclusion: 2/3= 4/6

Page 8

Solve a similar problem, use the example! (You have 3 minutes)

Tim who can eat 1/4 of his birthday cake, argued with Ali who is allowed to eat 2/8 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?

Solution:

Write your final answer in the box

Score_________

Page 9

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Worked example number 2									

Page 10

Now you have 1 minute to read the correct answer

Solution

Tim can eat one piece of his cake which was divided into 4 equal pieces (1/4). If we divide again the whole cake, cut each piece into two pieces, this quarter can be seen as 2 pieces out of 8 (2/8) which is the same as Ali’s portion.

Conclusion: ¼ = 2/8

Page 11

Problem 3	Solution 3	
Who will eat more pizza; Sam who eats 3 slices of a large pizza divided into 8 equal parts 3/8 of one large pizza), or Rakin who eats 2 slices of large pizza divided into 4 equal parts (2/4 of one large pizza)?	Draw one large pizza.  Divide this pizza into 8 equal slices.  Mum said that Sam can eat 3 slices, which is 3/8 of the pizza. He will eat this portion:  Sam will eat 3/8.	Draw another large pizza.  Divide this pizza into 4 equal slices.  Mum said that Rakin can eat 2 slices of the four slices. This is his portion: Rakin will eat 2/4.  Before eating, a few friends came and mum re-divided the pizza into 8 parts, but she said that Rakin still can eat the same amount of pizza. Now Rakin's portion will look like this:  Rakin can eat 4 slices of 8 (4/8) which is the same amount as 2 slices of 4 (2/4).
Conclusion	Now compare: Sam will eat 3/8 but, Rakin will eat 2/4 which is the same as 4/8. Conclusion: Sam will eat less pizza than Rakin. So, 3/8 is less than 2/4.	

Page 12

Now you have 1 minute to read the answer again

Solution:

Rakin eats 2 slices of a pizza which is divided into 4 equal slices (2/4). If we cut the whole pizza again, each slice into 2, he would have 4 slices of a pizza divided into 8 equal parts (4/8). That is more than Sam's amount of 3 slices out of 8 (3/8).

<u>Conclusion</u>: 2/3 is greater than 3/6

Page 13

Solve a similar problem, use the example. (You have 3 minutes)

Who will eat more pizza; Beverly who eats 3 slices of a large pizza divided into 6 equal parts (3/6 of one large pizza), or Clair who eats 2 slices of a large pizza divided into 3 equal parts (2/3 of one large pizza)?

Solution

Write your final answer in the box

Score_________

Page 14

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extre mely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficul t	Very difficu lt	Extre mely diffic ult
Worked example number 3									

Page 15

Now you have 1 minute to read the correct answer

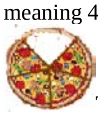
Solution:
Clair can eat 2 slices of a pizza divided into 3 equal slices (2/3). If we cut the whole pizza again, she will have 4 slices of a pizza divided into 6 equal parts (4/6). That is more than Beverly's amount of 3 slices out of 6 (3/6).

Conclusion: 2/3 is greater than 3/6

Page 16

<table>
<tr><td>Problem 4</td><td colspan="2">Solution 4</td></tr>
<tr><td>Do you have more pizza, if you eat 6 slices from a large pizza divided into 10 equal slices, or if you eat 4 slices from a large pizza divided into 5 equal slices?</td><td>Draw one large pizza:

Divide this pizza into 10 equal slices:

Here one will eat 6 slices of 10, meaning 6/10 of a large pizza:
</td><td>Draw another large pizza:
 Divide this pizza into 5 equal parts.
 Here one will eat 4 slices of 5, meaning 4/5 of a large pizza:
 Try to halve these parts, and you will have:
 You have 8 slices of large pizza divided into 10 equal parts.
You have 8/10 of a large pizza.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: in the first situation one eats 6 slices of 10 (6/10 of a pizza).In the second situation one eats 4 slices of 5 (4/5 of a pizza) which is equal to 8 slices of 10 (8/10 of pizza).
Conclusion: One can eat more from the second pizza than the first one.
Solution: 4/5 is greater than 6/10.</td></tr>
</table>

Page 17
Now you have 1 minute to read the answer again

Solution:

Each of the 4 slices of a large pizza which is cut into 5 equal parts (4/5) can be cut in half again. It will give us 8 slices out of 10 (8/10), which is more than 6/10.

Conclusion: 4/5 is greater than 6/10

Page 18
Solve a similar problem, use the example. (You have 3 minutes)

Do you have more pizza, if you eat 3 slices from a large pizza divided into 10 equal slices (3/10), or if you eat 2 slices from a large pizza divided into 5 equal slices (2/5)?

Solution:

Write you final answer in the box

Score_________

Page 19

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extremely Easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Dificult	Very difficult	Extremely Difficult
Worked example number 4									

Page 20

Now you have 1 minute to read the correct answer

Solution:

Each of the 2 slices of a large pizza which is cut into 5 equal parts (2/5) can be cut in half again. It will give us 4 slices out of 10 (4/10), which is more than 3/10.

Conclusion: 2/5 is greater than 3/10.

Appendix D2- The complete learning booklet of the geometric worked examples condition used in Experiments 4 and 5

Cover Page

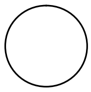
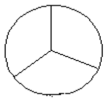
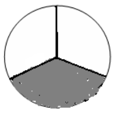
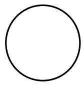
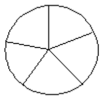
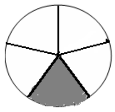
Learning booklet
Worked examples group- Geometric context (GWE)

In this booklet you have 4 pairs of worked examples.
You need to study each one carefully because you need to solve a similar problem in the next page.
When you solve, you need to explain your answer.
You have 3 minutes to study the worked example and 3 minutes to solve the similar problem following it.
You need to write on the same sheet of paper.

Please read the question carefully, solve as accurately and as neatly as you can.
You do NOT need to write your name
This will be your number, please keep it to the next stage.

Boy or Girl Group- GWE Number _______

Page 1

<table>
<tr><td>Problem 1</td><td colspan="2">Solution 1</td></tr>
<tr><td>Which results in a larger colored area; if you divide a circle into three parts and color one part resulting in 1/3 of the circle being colored or if you divide a circle into five parts and color one part resulting in 1/5 of the circle being colored?</td><td>Draw a circle:

Divide this circle into 3 equal parts

Color one part of these three parts. You have 1/3 of the circle colored.
</td><td>Draw another circle:

Divide this circle into 5 equal parts.

Color one part of these five parts. You have 1/5 of the circle colored.
</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: you have more color in the first circle than in the second circle.
Conclusion: The less parts, the more the colored area. So a circle divided into 3 parts with 1 part colored has more color than a circle divided into 5 parts with 1 part colored. Therefore, 1/3 is greater than 1/5.</td></tr>
</table>

Page 2

Now you have 1 minute to read the answer again

Solution:

One shaded part of a circle which is divided into three equal parts (1/3), is more than one shaded part of a circle divided into five equal parts (1/5) because the fewer the number of parts the bigger the shaded area.

<u>Conclusion</u>: 1/3 is greater than 1/5

Page 3
Solve a similar problem, use the example! (You have 3 minutes)

Which results in a larger colored area; if you divide a circle into 4 equal parts and color one part to get 1/4 of the circle being shaded or if you divide a circle into 6 parts and color one part to get 1/6 of the circle being shaded?

Solution:

Write your answer in the box
Score___________

Page 4
How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extremely Easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Worked example number 1									

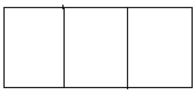
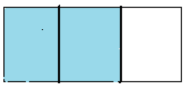
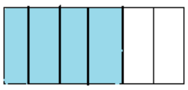
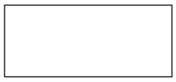
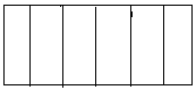
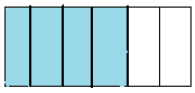
Page 5
Now you have 1 minute to read the correct answer

Solution

One shaded part of a circle which is divided into four equal parts (1/4), is more than one shaded part of a circle divided into 6 equal parts (1/6) because the fewer the number of parts the bigger the shaded area.

<u>Conclusion</u>: ¼ is greater than 1/6.

Page 6

Problem 2	Solution 2	
One group of children, who painted 2/3 of a rectangle blue, got into an argument with another group of children who painted 4/6 of a same sized rectangle blue. They said that the children in the second group got more blue than the first. Are they right?	Here is a rectangle  This rectangle is divided into 3 equal parts  Two of these 3 parts are painted blue.  2/3 is painted blue. Each of the three parts can be divided into 2 (add three more lines):  We have the same area, 4/6 painted blue.	Here is another rectangle :  This rectangle is divided into 6 equal parts.  Four of these 6 parts are painted blue.  4/6 is painted blue.
Conclusion	Now compare: In the first rectangle you have 2/3 painted blue, which is the same amount as 4/6. Both of them had 4/6 painted blue. Conclusion: if we have 2/3 and we halve it we will have 4/6. It will still have the same painted area. So 2/3 is equal to 4/6.	

Page 7

Now you have 1 minute to read the answer again

Solution:

The first group painted two parts of a rectangle blue which is divided into 3 equal parts (2/3). If they divided each part into two, they would get 4 blue parts out of 6 parts (4/6) which is the same as the second group's blue area.

Conclusion: 2/3=4/6.

Page 8

Solve a similar problem, use the example! (You have 3 minutes)

One group of children, who painted 1/4 of a rectangle blue, got into an argument with another group of children who painted 2/8 of a same size rectangle blue. They said that the children in the second group got more blue than the first. Who do you think had more blue area?

Solution:

Write your final answer here

Score__________

Page 9

How easy or difficult was the worked-examples to be understood and solved (tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Worked-example number 2									

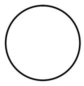
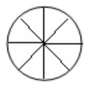
Page 10
Now you have 1 minute to read the correct answer

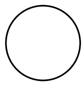
Solution

The first group painted one part of a rectangle blue which is divided into 4 equal parts (1/4). If they divided each part into two, they would get 2 blue parts out of 8 parts (2/8) which is the same as the second group's blue area.

Conclusion: ¼ =2/8

Page 11

<table>
<tr><td>Problem 3</td><td colspan="2">Solution 3</td></tr>
<tr><td>What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?</td>
<td>Draw a circle:

Divide this circle into 8 equal parts

Paint three parts of these eight parts red.

You have 3/8 painted red.</td>
<td>Draw another circle:

Divide this circle into 4 equal parts.

Paint tow parts of these four parts red

You have 2/4 painted red.
Repeat halving this circle (add 2 lines)

Now you have a circle divided into 8 parts, 4 of them painted red. You have 4/8 painted red which is the same area as 2/4.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: In the first circle you have 3/8 painted red whereas, in the second circle you have 2/4 painted red which is identical to 4/8 painted red.
Conclusion: We have less red area in the first circle than the second circle.
So, 3/8 is less than 2/4.</td></tr>
</table>

Page 12

Now you have 1 minute to read the detailed answer again

Solution:

Let us assume that we have two red parts out of a circle which is divided into 4 equal parts (2/4). If we cut each part into 2 parts, we would get 4 red parts out of 6 parts (4/8). That is more than 3 red parts out of 8 (3/8).

<u>Conclusion</u>: 2/4 is greater than 3/8.

Page 13

Solve a similar problem (You have 3 minutes)

What will have more red; one circle divided into 6 equal parts with 3 parts painted red (3/6 of a circle is painted red), or a circle divided into 3 equal parts with 2 parts painted red (2/3 of a circle is painted red)?

Solution:

Write your final answer here

Score_____________

Page 14

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Worked-example number 3									

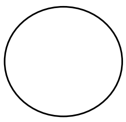
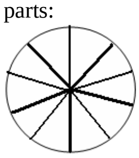
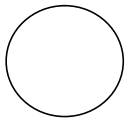
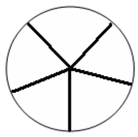
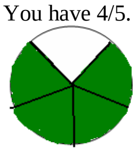
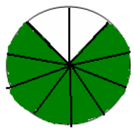
Page 15
Now you have 1 minute to read the correct answer

Solution:

Let us assume that we have two red parts out of a circle which is divided into 3 equal parts (2/3). If we cut each part into 2 parts, we would get 4 red parts out of 6 parts (4/6). That is more than 3 red parts out of 6 (3/6).

Conclusion: 2/3 is greater than 3/6.

Page 16

<table>
<tr><td>Problem 4 (geometric group)</td><td colspan="2">Solution 4 (geometric group)</td></tr>
<tr><td>Who will have more green; a child who drew a circle, divided it into 10 equal parts and painted 6 parts green (6/10), or a child who drew a circle, divided it into 5 equal parts and painted 4 parts green (4/5)?</td><td>Draw a circle:

Divide this circle into 10 equal parts:

Paint 6 of these ten parts green.
</td><td>Draw another circle:

Divide this circle into 5 equal parts.

Paint 4 of these five parts green. You have 4/5.

Halve each part of this circle and you will get:

Now you have 8 of 10 parts painted green. You have 8/10 painted green.</td></tr>
<tr><td>Conclusion</td><td colspan="2">Now compare: in the first circle you have 6/10 painted green, in the second circle you have 4/5 painted green which is equal to 8/10 painted green.
Conclusion: In the first circle you have less green than the second circle.
Solution: 6/10 is less than 4/5.</td></tr>
</table>

Page 17

Now you have 1 minute to read the detailed answer again

Solution

If we paint green 4 parts out of a circle which is divided into 5 equal parts we would have 4/5 of the circle painted green. If we divide each part into two parts we would have 8 green parts out of 10 (8/10) that is a greater green area than 6/10.

<u>Conclusion:</u> 4/5 is greater than 6/10.

Page 18

Solve a similar problem, use the example! (You have 3 minutes)

Who will have more green; a child who drew a circle, divided it into 10 equal parts and painted 3 parts green (3/10), or a child who drew a circle, divided it into 5 equal parts and painted 2 parts green (2/5)?

Solution:

Write your final answer here

Score__________

Page 19

How easy or difficult did you find the worked-examples to be understood and solved (tick one box)?

	Extre mely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficu lt	Extre mely diffic ult
Worked-example number 4									

Page 20

Now you have 1 minute to read the correct answer

Solution:

If we paint green 2 parts out of a circle which is divided into 5 equal parts we would have 2/5 of the circle painted green. If we divide each part into two parts we would have 4 green parts out of 10 (4/10) that is a greater green area than 3/10.

Conclusion: 2/5 is greater than 3/10.

Appendix D3- The complete learning booklet of the realistic problem solving condition used in Experiments 4 and 5

Cover Page

Learning booklet
Problem solving group-Realistic context (RPS)

In this booklet you are to solve 4 pairs of problems.
You need to give an answer and an explanation.
You will be given 3 minutes for each problem
You need to write on the same sheet of paper.
Please read the question carefully, solve as accurately and as neatly as you can.
You do NOT need to write your name

This will be your number, please keep it to the end of the study.

N.B- In this study we are talking about a home made pizza, which we can cut into as many slices as we like ☺

Boy or Girl Group- RPS Number _______

Page 1

Problem 1:

Where will you get more pizza; at a table with 3 children sharing one large pizza, with each child getting 1/3, or at a table with 5 children sharing one large pizza with each child getting 1/5?

Solution:

Write your final answer in the box
Score__________

Page 2

Now you have 1 minute to read the correct answer

Solution:

You will get more pizza at a table with 3 children sharing one large pizza (1/3), than at a table with 5 children sharing one large pizza (1/5) because the fewer the number of sharers, the more pizza a person can get.

<u>Conclusion</u>: 1/3 is greater than 1/5

Page 3
Solve a similar problem (You have 3 minutes)

Where will you get more pizza; at a table with 4 children sharing one large pizza, with each child getting ¼ of a pizza, or at a table with 6 children sharing one large pizza with each child getting 1/6 of a pizza?

Solution:

Write your final answer in the box

Score_________

Page 4

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Problem 1									

Page 5

Now you have 1 minute to read the correct answer

Solution:

You will get more pizza at a table with 4 children sharing one large pizza (1/4), than at a table with 6 children sharing one large pizza (1/6) because the fewer the number of sharers, the more pizza a person can get.

<u>Conclusion</u>: ¼ is greater than 1/6

Page 6

Problem 2

Mark who can eat 2/3 of his birthday cake, argued with Jack who is allowed to eat 4/6 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?

Solution:

Write your final answer here

Score__________

Page 7

Now you have 1 minute to read the correct answer

Solution:

Mark can eat two pieces of his cake which was divided into 3 equal pieces (2/3). If we divide again the whole cake, cut each piece into two pieces, Mark's portion can be seen as 4 pieces out of 6 (4/6) which is the same as Jack's portion.

Conclusion: 2/3= 4/6

Page 8

Solve a similar problem (You have 3 minutes)

Tim who can eat 1/4 of his birthday cake, argued with Ali who is allowed to eat 2/8 of his birthday cake about who is allowed to eat more cake! They both have the same size birthday cakes. Who do you think will eat more cake?

Solution:

Write your final answer in the box

Score_________

Page 9

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Extreme ly easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Diffi cult	Very difficu lt	Extre mely difficu lt
Problem 2									

Page 10

Now you have 1 minute to read the correct answer

Solution:

Tim can eat one piece of his cake which was divided into 4 equal pieces (1/4). If we divide again the whole cake, cut each piece into two pieces, this quarter can be seen as 2 pieces out of 8 (2/8) which is the same as Ali's portion.

Conclusion: ¼ = 2/8

Page 11
Problem 3

Who will eat more pizza; Sam who eats 3 slices of a large pizza divided into 8 equal parts (3/8 of one large pizza), or Rakin who eats 2 slices of a large pizza divided into 4 equal parts (2/4 of one large pizza)?

Solution:

Write your final answer here
Score_________

Page 12

Now you have 1 minute to read the correct answer

Solution:

Rakin eats 2 slices of a pizza which is divided into 4 equal slices (2/4). If we cut the whole pizza again, each slice into 2, he would have 4 slices of a pizza divided into 8 equal parts (4/8). That is more than Sam's amount of 3 slices out of 8 (3/8).

<u>Conclusion</u>: 2/3 is greater than 3/6

Page 13

Solve a similar problem (You have 3 minutes)

Who will eat more pizza; Beverly who eats 3 slices of a large pizza divided into 6 equal parts (3/6 of one large pizza), or Clair who eats 2 slices of a large pizza divided into 3 equal parts (2/3 of one large pizza)?

Solution:

Write your final answer in the box

Score_________

Page 14

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Problem 3									

Page 15

Now you have 1 minute to read the correct answer

Solution:

Clair can eat 2 slices of a pizza divided into 3 equal slices (2/3). If we cut the whole pizza again, she will have 4 slices of a pizza divided into 6 equal parts (4/6). That is more than Beverly's amount of 3 slices out of 6 (3/6).

<u>Conclusion</u>: 2/3 is greater than 3/6

Page 16
Problem 4

Do you have more pizza if you eat 6 slices from a large pizza divided into 10 equal slices (6/10), or if you eat 4 slices from a large pizza divided into 5 equal slices (4/5)?

Solution:

Write your final answer here
Score__________

Page 17

Now you have 1 minute to read the correct answer

Solution:

Each of the 4 slices of a large pizza which is cut into 5 equal parts (4/5) can be cut in half again. It will give us 8 slices out of 10 (8/10), which is more than 6/10.

<u>Conclusion</u>: 4/5 is greater than 6/10

Page 18
Solve a similar problem (You have 3 minutes)

Do you have more pizza if you eat 3 slices from a large pizza divided into 10 equal slices (3/10), or if you eat 2 slices from a large pizza divided into 5 equal slices (2/5)?

Solution

Write you final answer in the box

Score_________

Page 19

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Problem 4									

Page 20

Now you have 1 minute to read the correct answer

Solution:

Each of the 2 slices of a large pizza which is cut into 5 equal parts (2/5) can be cut in half again. It will give us 4 slices out of 10 (4/10), which is more than 3/10.

<u>Conclusion</u>: 2/5 is greater than 3/10.

Appendix D4: The complete learning booklet of the geometric problem solving condition used in Experiments 4 and 5

Cover page

Learning booklet (GPS) group

In this booklet you are to solve 4 pairs of problems.
You need to give an answer and an explanation.
You have 3 minutes for each problem
You need to write on the same sheet of paper.
Please read the question carefully, solve as accurately and as neatly as you can.

You do NOT need to write your name

This will be your number please keep it to the next stage

Boy or Girl Group GPS Number _______

Page 1

Problem 1:

Which results in a larger colored area; if you divide a circle into three parts and color one part resulting in 1/3 of the circle being shaded or if you divide a circle into five parts and color one part resulting in 1/5 of the circle being colored?

Solution:

Write your final answer in the box

Score _________

Page 2

Now you have 1 minute to read the correct answer

Solution

One shaded part of a circle which is divided into three equal parts (1/3), is more than one shaded part of a circle divided into five equal parts (1/5) because the fewer the number of parts the bigger the shaded area.

<u>Conclusion</u>: 1/3 is greater than 1/5

Page 3

Solve a similar problem (you have 3 minutes)

Which results in a larger colored area; if you divide a circle into four parts and color one part to get 1/4 of the circle being shaded or if you divide a circle into six parts and color one part to get 1/6 of the circle being colored?

Solution:

Write your answer in the box

Score___________

Page 4

How easy or difficult did you find learning to solve these problems (please tick one box)?

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	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Extremely difficult
Problem 1									

Page 5

Now you have 1 minute to read the correct answer

Solution:

One shaded part of a circle which is divided into four equal parts (1/4), is more than one shaded part of a circle divided into 6 equal parts (1/6) because the fewer the number of parts the bigger the shaded area.

Conclusion: ¼ is greater than 1/6

Page 6

Problem 2:

One group of children, who painted 2/3 of a rectangle blue, got into an argument with another group of children who painted 4/6 of a same size rectangle blue. They said that the children in the second group got more blue than the first. Are they right?

Solution:

Write your final answer here
Score_______

Page 7

Now you have 1 minute to read the correct answer

Solution

The first group painted two parts of a rectangle blue which is divided into 3 equal parts (2/3). If they divided each part into two, they would get 4 blue parts out of 6 parts (4/6) which is the same as the second group's blue area.

Conclusion: 2/3=4/6.

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Solve a similar problem (You have 3 minutes)

One group of children, who painted 1/4 of a rectangle blue, got into an argument with another group of children who painted 2/8 of a same size rectangle blue. They said that the children in the second group got more blue than the first. Are they right?

Solution

Write your final answer here
Score__________

Page 9

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Extremely easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Very-very difficult
Problem 2									

Page 10

Now you have 1 minute to read the correct answer

Solution:

The first group painted one part of a rectangle blue which is divided into 4 equal parts (1/4). If they divided each part into two, they would get 2 blue parts out of 8 parts (2/8) which is the same as the second group's blue area.

Conclusion: ¼ =2/8

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Problem 3

What will have more red; one circle divided into 8 equal parts with 3 parts painted red (3/8 of a circle is painted red), or a circle divided into 4 equal parts with 2 parts painted red (2/4 of a circle is painted red)?

Solution:

Write your final answer here

Score__________

Page 12

Now you have 1 minute to read the correct answer

Solution:

Let us assume that we have two red parts out of a circle which is divided into 4 equal parts (2/4). If we cut each part into 2 parts, we would get 4 red parts out of 6 parts (4/8). That is more than 3 red parts out of 8 (3/8).

<u>Conclusion</u>: 2/4 is greater than 3/8.

Page 13

Solve a similar problem (You have 3 minutes)

What will have more red; one circle divided into 6 equal parts with 3 parts painted red (3/6 of a circle is painted red), or a circle divided into 3 equal parts with 2 parts painted red (2/3 of a circle is painted red)?

Solution:

Write your final answer here

Score_____________

Page 14

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Very very easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Very very difficult
Problem 3									

Page 15

Now you have 1 minute to read the correct answer

Solution:

Let us assume that we have two red parts out of a circle which is divided into 3 equal parts (2/3). If we cut each part into 2 parts, we would get 4 red parts out of 6 parts (4/6). That is more than 3 red parts out of 6 (3/6).

Conclusion: 2/3 is greater than 3/6.

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Problem 4:

Who will have more green; a child who drew a circle, divided it into 10 equal parts and painted 6 parts green (6/10), or a child who drew a circle, divided it into 5 equal parts and painted 4 parts green (4/5)?

Solution:

Write your final answer here

Score__________

Page 17

Now you have 1 minute to read the correct answer

Solution:

If we paint green 4 parts out of a circle which is divided into 5 equal parts we would have 4/5 of the circle painted green. If we divide each part into two parts we would have 8 green parts out of 10 (8/10) that is a greater green area than 6/10.

<u>Conclusion:</u> 4/5 is greater than 6/10.

Page 18

Solve a similar problem (You have 3 minutes)

Who will have more green; a child who drew a circle, divided it into 10 equal parts and painted 3 parts green (3/10), or a child who drew a circle, divided it into 5 equal parts and painted 2 parts green (2/5)?

Solution:

Write your final answer here

Score__________

Page 19

How easy or difficult did you find learning to solve these problems (please tick one box)?

	Very-very easy	Very easy	Easy	Rather easy	Neither easy nor difficult	Rather difficult	Difficult	Very difficult	Very very difficult
Problem 4									

Page 20

Now you have 1 minute to read the correct answer

Solution:

If we paint green 2 parts out of a circle which is divided into 5 equal parts we would have 2/5 of the circle painted green. If we divide each part into two parts we would have 4 green parts out of 10 (4/10) that is a greater green area than 3/10.

<u>Conclusion:</u> 2/5 is greater than 3/10.